



Pion Transition Form Factor in Lattice QCD with Blending method

arxiv: 2607.27868

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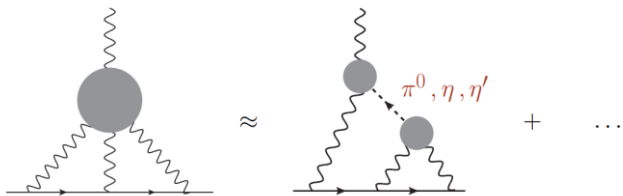
Outline



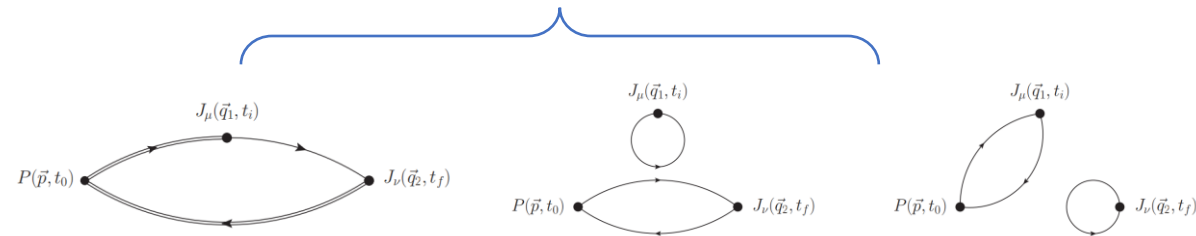
01. Pion transition form factor
02. General distillation and blending method
03. Lattice setting and computational cost
04. Connected and Disconnected correlation functions
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Pion transition form factor

Pion transition form factor:



$$\mathcal{M}_{\mu\nu}(P, q_1) = i \int d^4 x e^{iq_1 x} \langle \Omega | T \{ J_\mu(x) J_\nu(0) \} | \pi^0(P) \rangle = \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \mathcal{F}_{\pi^0 \gamma^* \gamma^*}(q_1^2, q_2^2)$$



Connected and Disconnected sign problem:

A. Gérardin, H. B. Meyer, and A. Nyffeler, “Lattice calculation of the pion transition form factor $\pi^0 \rightarrow \gamma^* \gamma^*$ ”, Phys. Rev. D 94, 074507 (2016),

➡ The calculation results of these two Feynman diagrams **have opposite signs**, meaning they cancel each other out.

T. Lin, M. Bruno, X. Feng, L.-C. Jin, C. Lehner, C. Liu, Q.-Y. Luo, Lattice QCD calculation of the π^0 -pole contribution to the hadronic light-by-light scattering in the anomalous magnetic moment of the muon, arXiv:2411.06349 [hep-lat] (2024).

➡ The calculation results of these two Feynman diagrams **have the same sign**, meaning that they add up to each other.

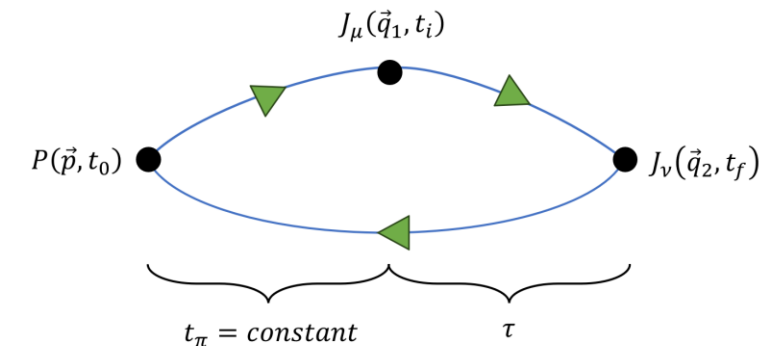
Pion transition form factor



Start with the following equation:

$$C_{\mu\nu}^3(t_x, t_y, t_0) = -\frac{a^6 Z_\pi}{2E_\pi} \sum_{\vec{x}, \vec{y}} \langle \Omega | T \{ J_\mu(\vec{x}, \tau) J_\nu(\vec{y}, 0) \} | \pi^0 \rangle e^{-i\vec{q}_1(\vec{x}-\vec{y})} e^{-m_\pi t_\pi}$$

$$\tau = t_x - t_y, \quad t_\pi = \min(t_x - t_0, t_y - t_0), \quad Z_\pi = \langle \Omega | \mathcal{O}_{\pi^0}(\vec{Z}, 0) | \pi^0 \rangle$$



$$P = (m_\pi, \vec{0}) = q_1 + q_2$$

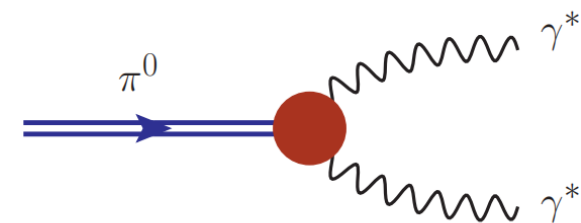
$$q_1 = (\delta, \vec{q}_1)$$

$$q_2 = (m_\pi - \delta, -\vec{q}_1)$$

Get pion transition form factor:

$$A_{\mu\nu}(\tau) = \lim_{t_\pi \rightarrow \infty} \frac{1}{V} C_{\mu\nu}^3(\tau, t_\pi) e^{m_\pi t_\pi} = \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta A(\tau)$$

$$\tilde{A}(\tau) = \frac{A(\tau)}{1 - e^{-E_\pi(T-2t_\pi-|\tau|)}}, \quad \mathcal{F}_{\pi^0 \gamma^* \gamma^*} = \frac{2E_\pi}{Z_\pi} \left\{ \int_{-\infty}^0 d\tau e^{\delta\tau} \tilde{A}(\tau) e^{-E_\pi\tau} + \int_0^\infty d\tau e^{\delta\tau} \tilde{A}(\tau) \right\}$$



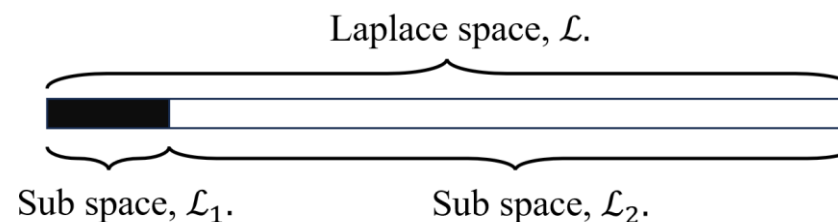
General Distillation and Blending Method



Distillation:

M. Peardon, J. Bulava, J. Foley, C. Morningstar, J. Dudek, R. G. Edwards, B. Jo'o, H.-W. Lin, D. G. Richards, and K. J. Juge (Hadron Spectrum Collaboration), Phys. Rev. D 80, 054506 (2009).

$$-\tilde{V}_{xy}^2(t) = 6\delta_{xy} - \sum_{j=1}^3 \left(\tilde{U}_j(x, t) \delta_{x+j, y} + \tilde{U}_j^\dagger(x - j, t) \delta_{x-j, y} \right). \quad \mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 \longrightarrow \phi_i = \begin{cases} \phi_i \in \mathcal{L}_1 & \text{only } Ne \text{ vectors.} \\ \phi_i \in \mathcal{L}_2 & \text{have } [\mathcal{L}] - Ne \text{ vectors. (ignore)} \end{cases}$$



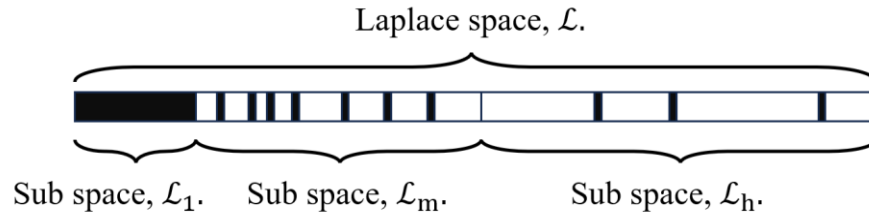
General Distillation:

$$P_i(t_x, \vec{x}; t_y) = \sum_{\vec{w}} G(t_x, \vec{x}; t_y, \vec{w}) |\phi_i(t_y, \vec{w})\rangle, \quad \phi_i(t_y, \vec{w}) \in \mathcal{L}_1$$

General Distillation and Blending Method

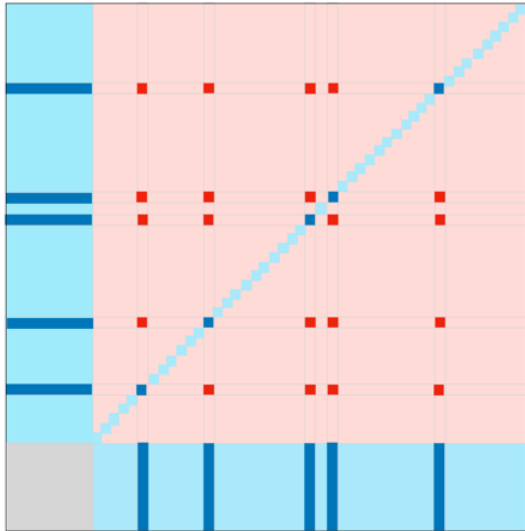


Blending:



Blending Vector: $\{\phi_i(t, \vec{x})\}$

$$P_{ij}(t_x, t_y) = \sum_{\vec{z}, \vec{w}} \langle \phi_i(t_x, \vec{z}) | G(t_x, \vec{z}; t_y, \vec{w}) | \phi_j(t_y, \vec{w}) \rangle,$$



Operator:

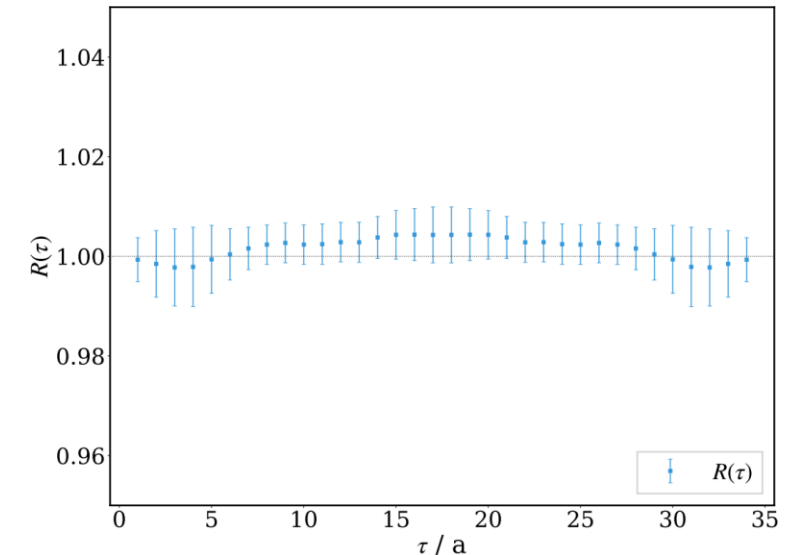
$$J_4 = \frac{1}{2} [\delta_{x,y+\hat{n}_4 a} (1 + \gamma_4) U_4^\dagger(y) - \delta_{x+\hat{n}_4 a, y} (1 - \gamma_4) U_4(x)],$$

$$\mathcal{O}_{ij}(t_x; t_y) = \sum_{\vec{z}, \vec{w}} \Omega_{ij}^{(2)} \langle \phi_i(t_x, \vec{z}) | J_4(t_x, \vec{z}; t_y, \vec{w}) | \phi_j(t_y, \vec{w}) \rangle.$$

$$\Omega_i^{(1)} = \Omega_i^{(1,m)} = \omega_{m0}, \quad \text{for } \phi_i \in \mathcal{L}_m;$$

$$\Omega_{ij}^{(2)} = \begin{cases} \omega_{m0} & \text{for } i = j, \phi_i \in \mathcal{L}_m \\ \omega_{m0}\omega_{m1} & \text{for } i \neq j, \phi_i, \phi_j \in \mathcal{L}_m \\ \omega_{m0}\omega_{n0} & \text{for } i \neq j, \phi_i \in \mathcal{L}_m; \phi_j \in \mathcal{L}_n; n \neq m. \end{cases}$$

$$\omega_{mm'} = \frac{[\mathcal{L}_m] - m'}{N_m - m'}$$



Lattice setting



Ensemble	$a[\text{fm}]$	$L^3 \times T$	$m_\pi[\text{MeV}]$	$m_K[\text{MeV}]$	N_{conf}
C24P29	0.10530(18)	$24^3 \times 72$	292.7(1.2)	509.4(1.1)	200

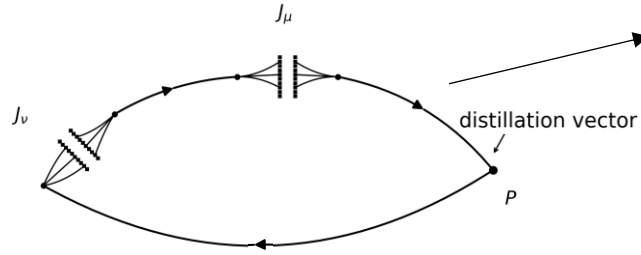
TABLE I. This table summarizes the parameters of the ensemble used in this study. We use a single ensemble in CLQCD, a is the lattice spacing, $L^3 \times T$ is the lattice volume, m_π is the pion mass, m_K is the kaon mass, N_{conf} is the number of configurations we used.

Cost:

- 1.The number of inversions performed is $200(\text{conf}) * 72(T) * 650(\text{BLD vectors})$ of BLD perambulators.
- 2.The contraction time can be significantly reduced.
- 3.For Blending the most important thing is storage of data, because the main advantage of Blending is its ability to store a large volume. Every conf use 29GB for vector; 522GB for perambulator; 463GB for disconnected perambulator

Connected Correlation Functions

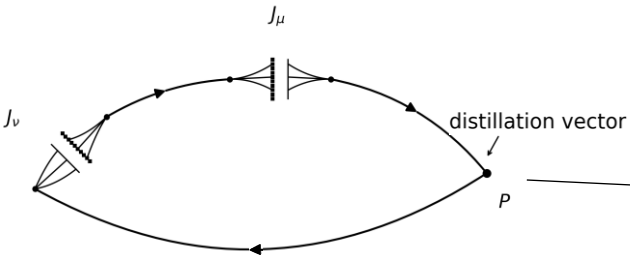
$$C_{\mu\nu}^3 = \left\langle \sum_{\vec{x}, \vec{y}} e^{-i\vec{q}_1(\vec{x}-\vec{y})} \sum_{ijmnl} \Omega_{ijmn} \phi_i(\tau, \vec{x}) P_{ij}(\tau, 0) \phi_j^\dagger(0, \vec{y}) \gamma_\nu \phi_m(0, \vec{y}) P_{mk}(0, \vec{y}; t_0) \gamma_5 \phi_n^\dagger(\tau, \vec{x}) P_{nl}^\dagger(\tau, \vec{x}; t_0) \gamma_\mu \delta_{kl} \right\rangle.$$



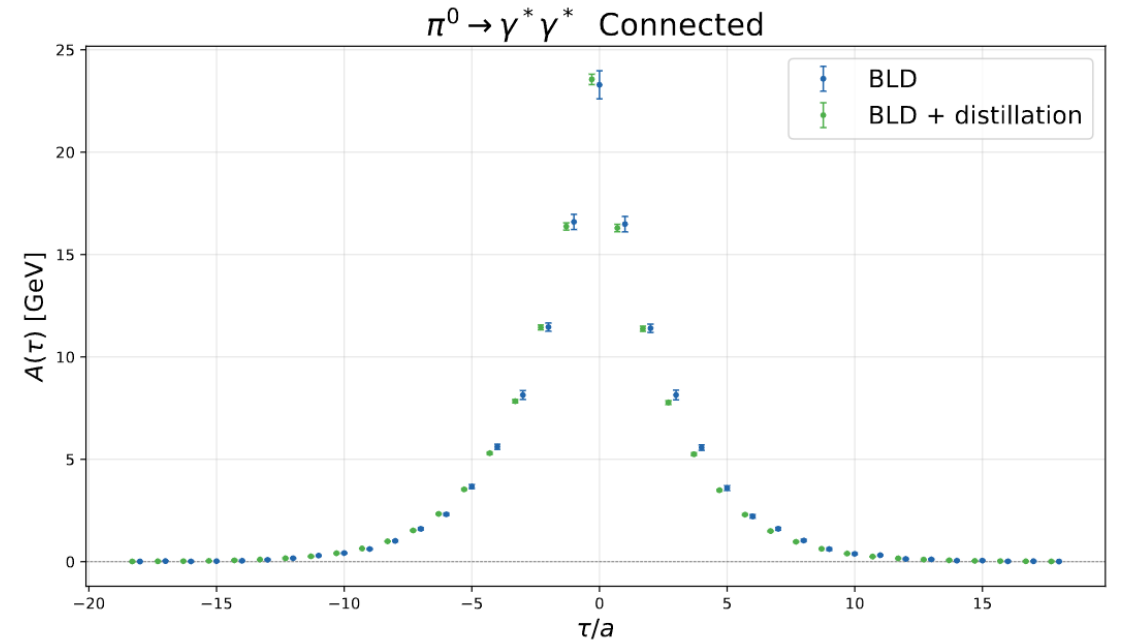
$$\Omega_{ijmn} = \begin{cases} \Omega_{in}^{(2)} \Omega_{jm}^{(2)}, & \text{for } \tau \neq 0 \\ \Omega_{ijmn}^{(4)}, & \text{for } \tau = 0. \end{cases}$$

$$\Omega_{ijmn}^{(4)} = \prod_{s=1}^{N_{\text{sub}}} \prod_{x=0}^{r_s(i,j,m,n)-1} \omega_{sx}, \quad \omega_{sx} = \frac{[\mathcal{L}_s] - x}{N_s - x}.$$

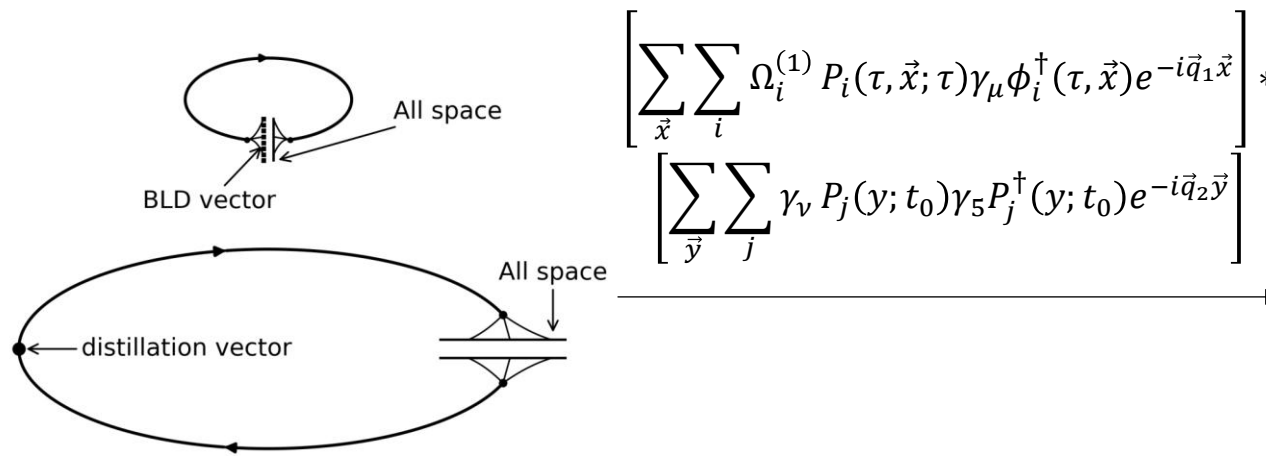
$$r_s(i, j, m, n) = |\{a \in \{i, j, m, n\}: \phi_a \in \mathcal{L}_s\}|,$$



$$C_{\mu\nu}^3 = \left\langle \sum_{\vec{x}, \vec{y}} e^{-i\vec{q}_1(\vec{x}-\vec{y})} \sum_{ijkl} \Omega_{ij} \phi_i(\tau, \vec{x}) P_{ij}(\tau, 0) \phi_j^\dagger(0, \vec{y}) \gamma_\nu P_k(0, \vec{y}; t_0) \gamma_5 P_l^\dagger(\tau, \vec{x}; t_0) \gamma_\mu \delta_{kl} \right\rangle.$$

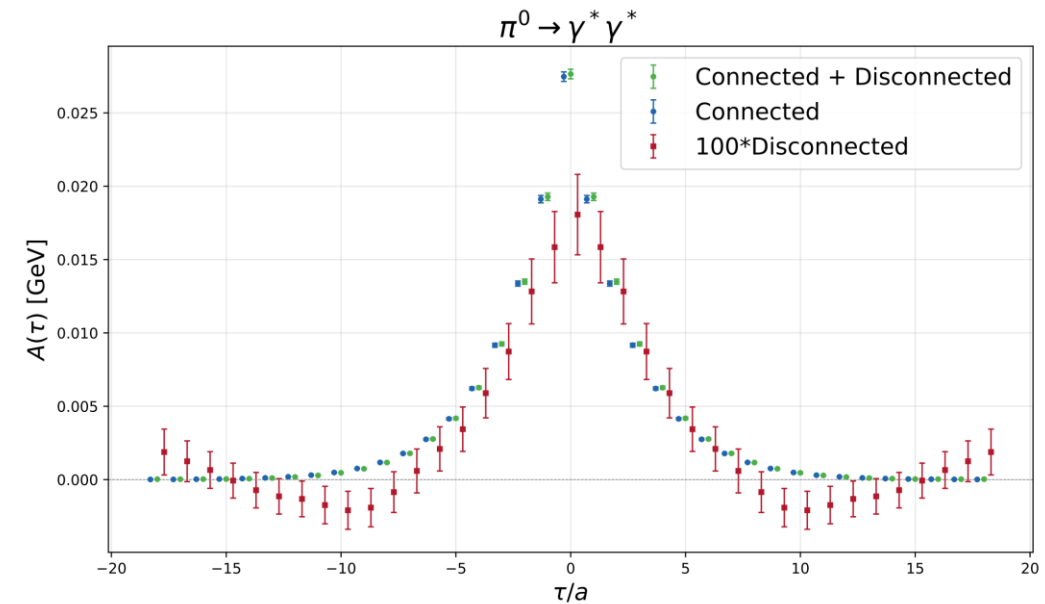


Disconnected Correlation Functions



Quark loop:

$$P_i(t_x, \vec{x}; t_y) = \sum_{\vec{w}} G(t_x, \vec{x}; t_y, \vec{w}) |\phi_i(t_y, \vec{w})\rangle, \quad t_x = t_y$$



Result

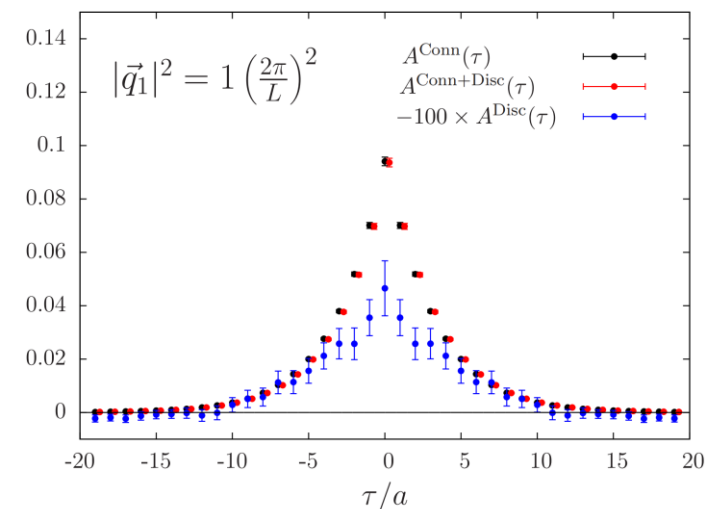


$$\mathcal{F}_{\pi^0\gamma^*\gamma^*} = \frac{2E_\pi}{Z_\pi} \left\{ \int_{-\tau_c}^0 d\tau e^{\delta\tau} \tilde{A}(\tau) e^{-E_\pi\tau} + \int_0^{\tau_c} d\tau e^{\delta\tau} \tilde{A}(\tau) \right\}$$

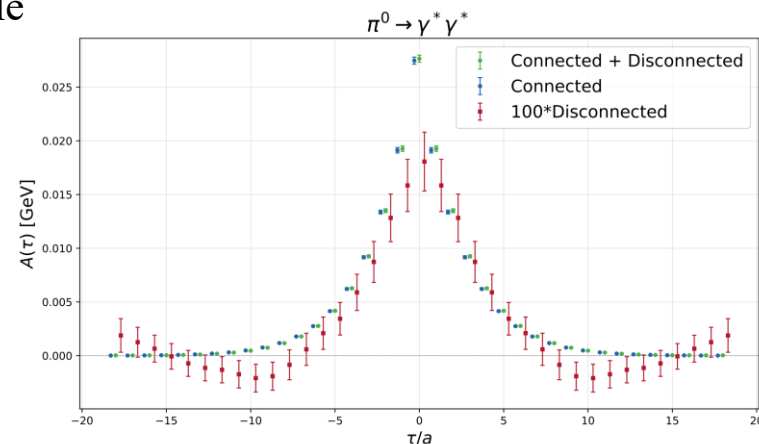
$\mathcal{F}_{\pi^0\gamma^*\gamma^*} \times 10^3 [\text{GeV}^{-1}]$			
	BLD-con	con	dis
$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q^2, -Q^2)$	250.1(5.6)	243.2(3.2)	1.6(1.0)
$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q^2, -Q^2/2)$	277.0(6.8)	268.3(3.5)	2.2(2.3)
$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q^2, 0)$	342(12)	326.7(5.6)	4.5(6.8)

Bare pion transition form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q^2, -Q^2)$ for different Q^2 configurations with $\vec{q}_1 = (0,0,1)$ and $t_\pi \approx 1.9$ fm of the C24P29 ensemble used in this study. The fixed integration interval $\tau_c = t_\pi$.

$\mathcal{F}_{\pi^0\gamma^*\gamma^*} \times 10^3 [\text{GeV}^{-1}]$				
	$t_\pi = 12$	$t_\pi = 14$	$t_\pi = 16$	$t_\pi = 18$
$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q^2, -Q^2)$	2.15(53)	1.86(58)	1.59(61)	1.40(63)
$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q^2, -Q^2/2)$	2.23(73)	1.81(80)	1.43(85)	1.17(86)
$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q^2, 0)$	2.3(1.3)	1.6(1.4)	1.0(1.5)	0.6(1.5)

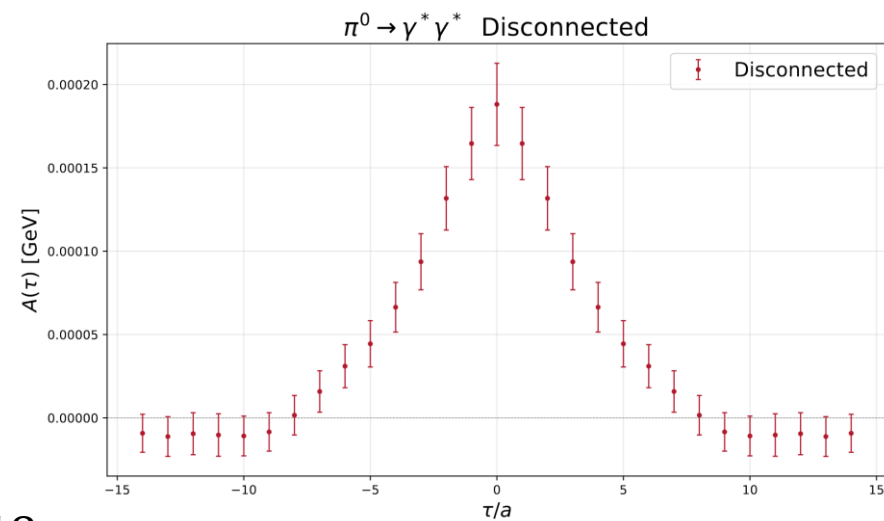
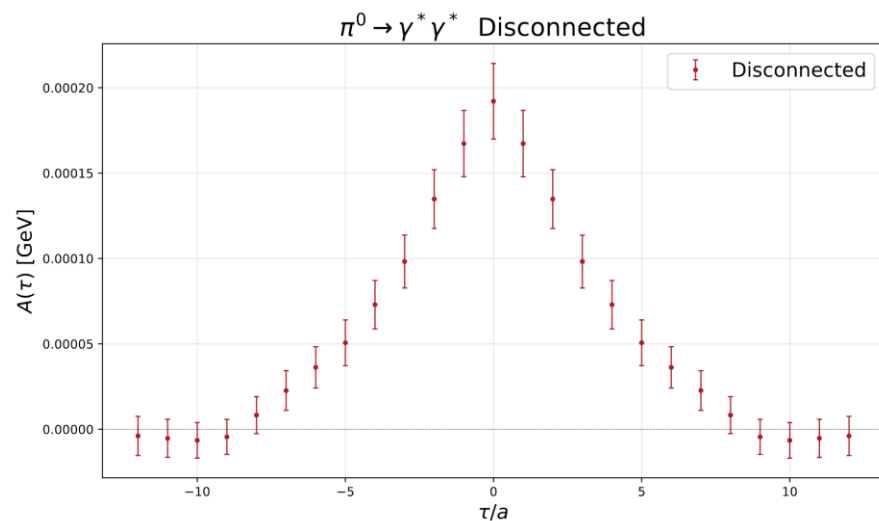


A. Gérardin, H. B. Meyer, and A. Nyffeler, “Lattice calculation of the pion transition form factor $\pi^0 \rightarrow \gamma^* \gamma^*$ ”, Phys. Rev. D 94, 074507 (2016),

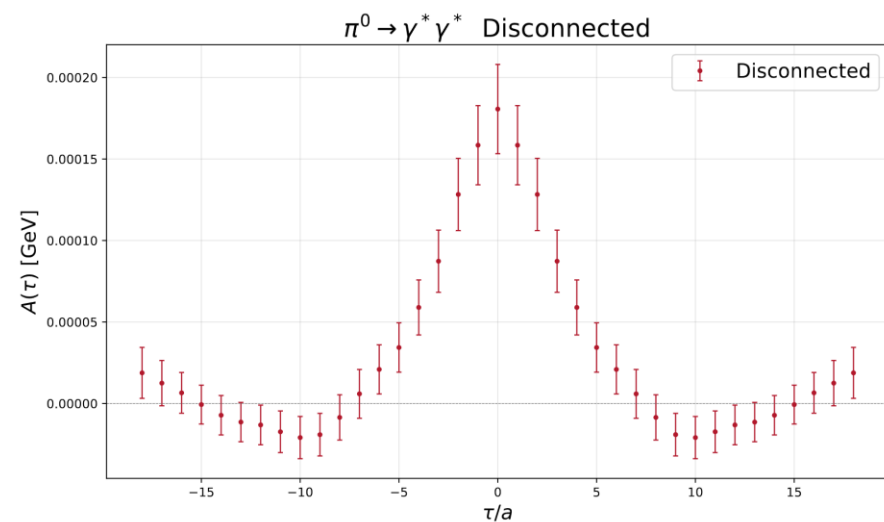
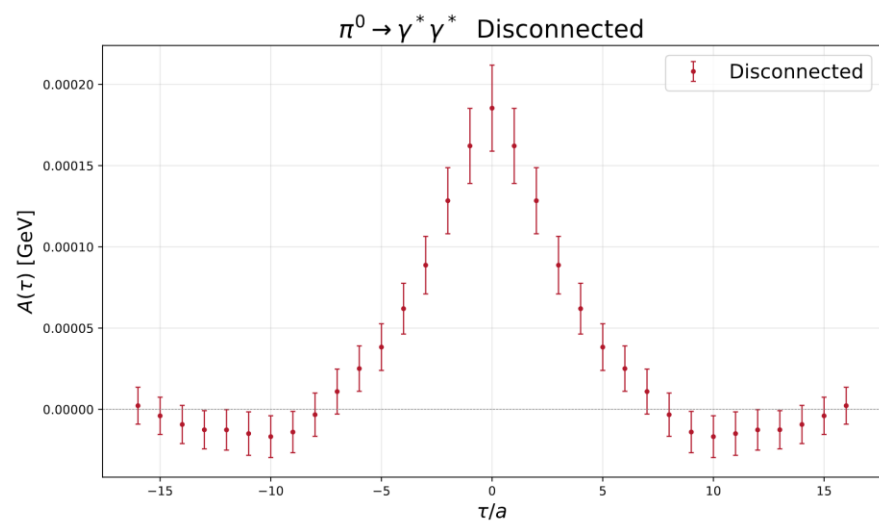


Results of the disconnected contribution for different t_π and fixed integration interval $\tau_c/a = 12$.

Result



$t_\pi = 12, 14, 16, 18$



Summary



1. General distillation combined with blending provides a new method for treating connected and disconnected contributions to the pion transition form factor.
2. Under the framework of truncated integration, we can obtain that the signs of the two contracted graphs are the same in the short-distance regime.
3. Due to the poor quality of the long-distance signals, it is impossible to directly determine what the relative symbols of the connected and disconnected graphs.



Thank you for your attention!

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