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The Chinese University of Hong Kong, Shenzhen

Lattice Parton Collaboration

Gluon Helicity PDF in Continuum Limit from Lattice QCD

Speaker: Shi-Yi Zhong

2026 October 6th

In collaboration with Jian-Hui Zhang, Peng Sun,
Liu-Ming Liu, Yi-Bo Yang et al.

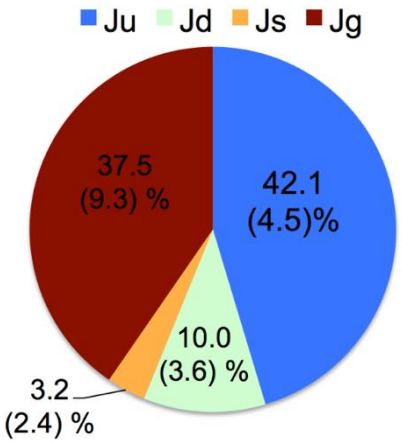
Background

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Proton spin decomposition

Ji Decomposition

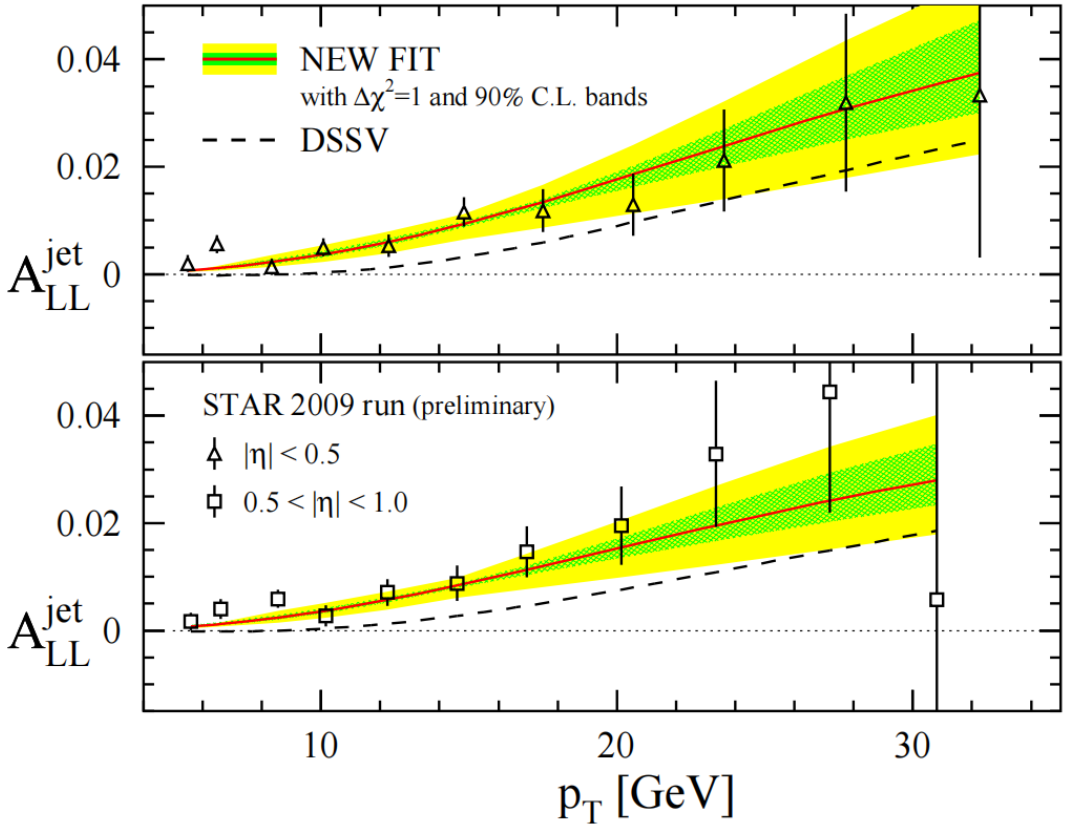
$$\frac{1}{2}\Delta\Sigma + L_q^z + J_g = \frac{\hbar}{2}$$



Jaffe-Manohar decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

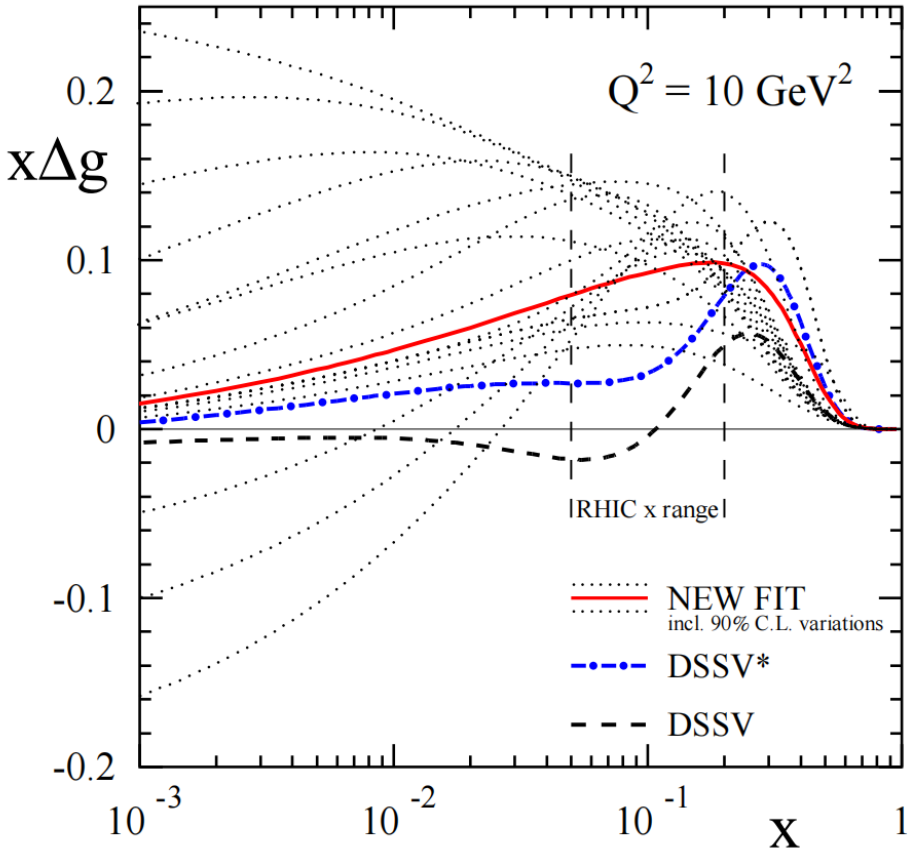
Double-spin asymmetry

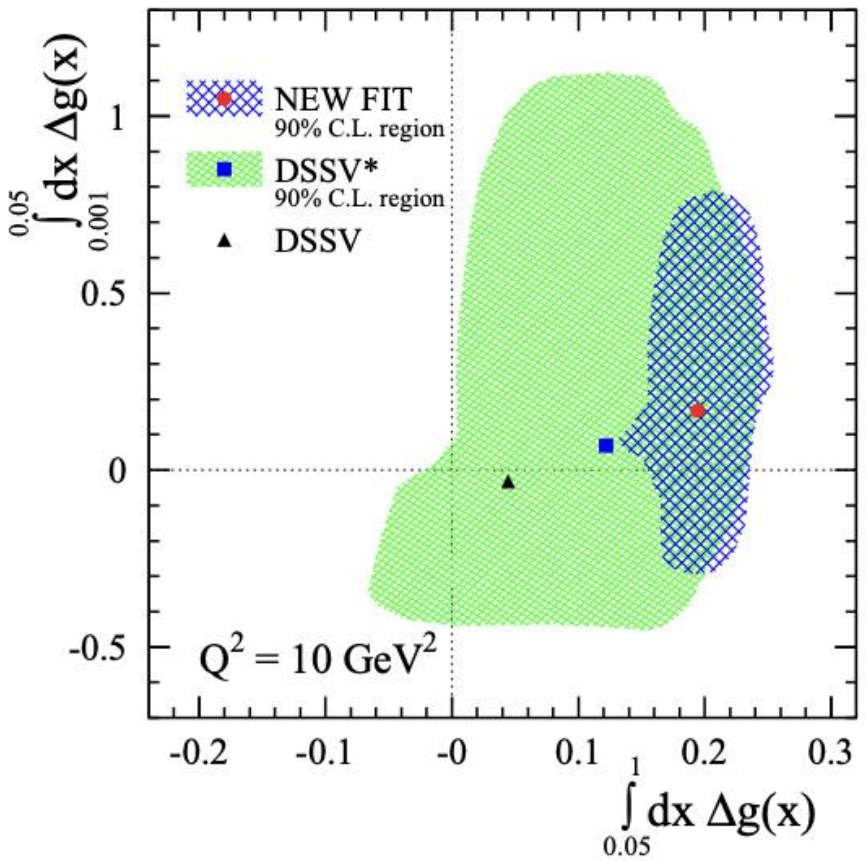


C. Alexandrou, et al. *Phys. Rev. D* 101, 094513, **2020**.
D. de Florian, et al., *Phys. Rev. Lett.* 113, 012001 (**2014**).

Background

Gluon helicity distribution from global fit



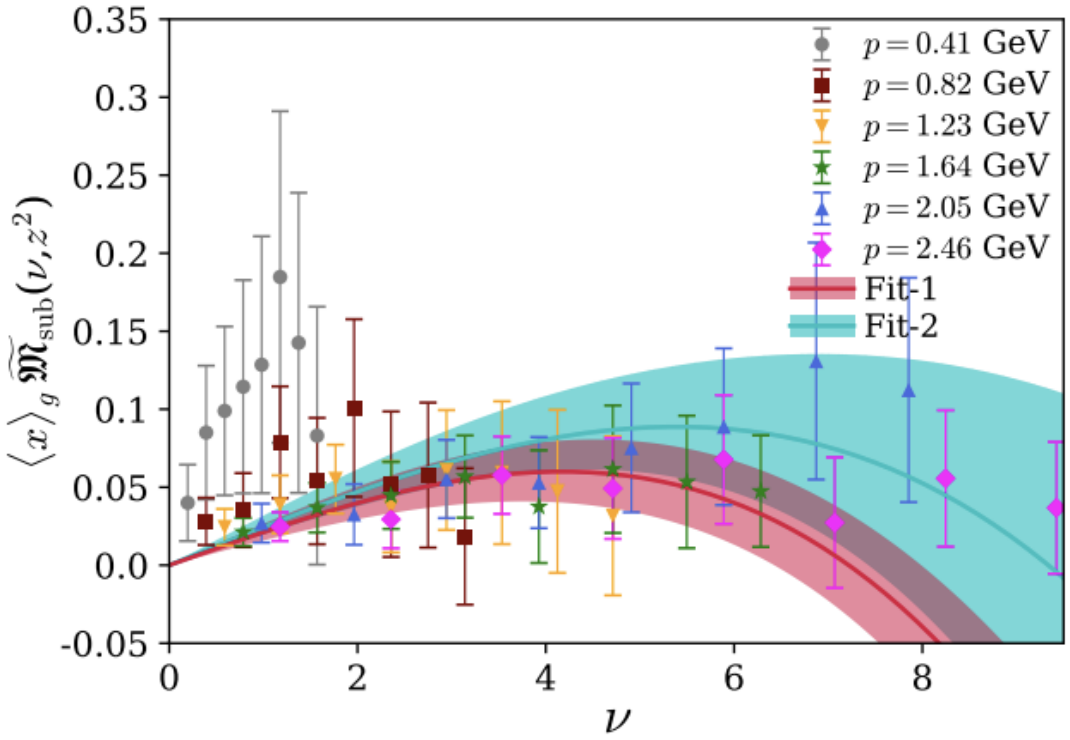


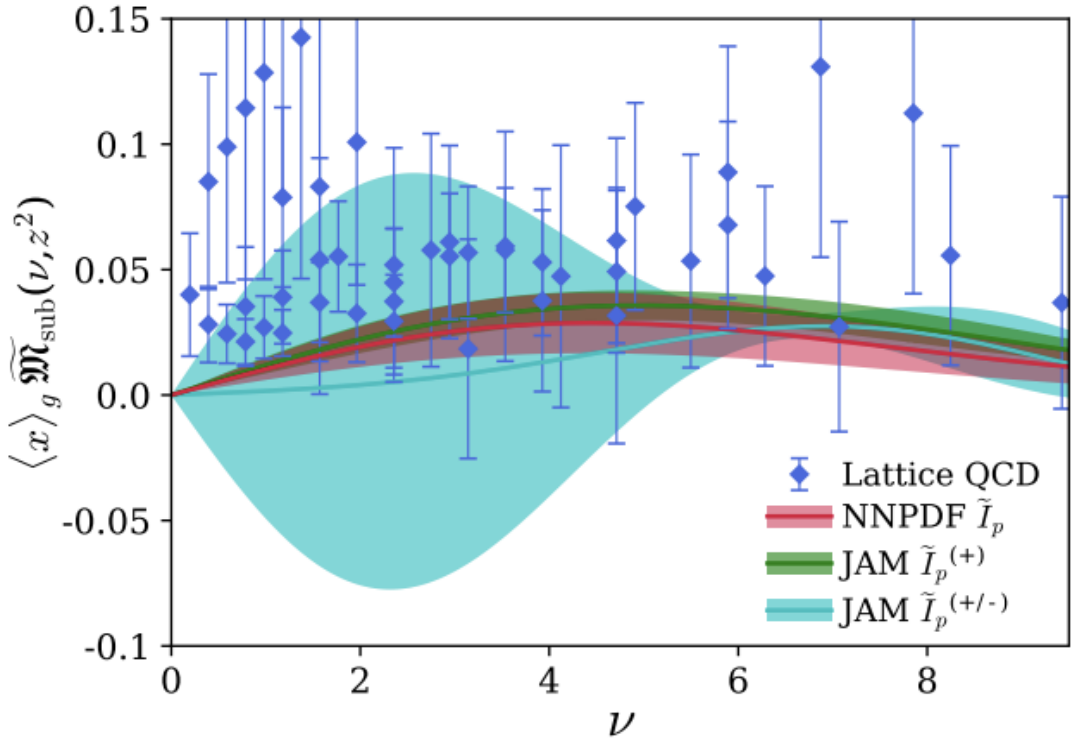
D. de Florian, et al., *Phys. Rev. Lett.* 113, 012001 (**2014**).

Previous Work

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Ioffe-time pseudo-distribution





ID	a (fm)	m_π (MeV)	$L^3 \times N_t$	$N_{\rm cfg}$	$N_{\rm srcs}$
$a094m358$	0.094(1)	358(3)	$32^3 \times 64$	1901	64

C. Egerer, et al. , Phys. Rev. D 106, 094511 (**2022**).

Gluon Helicity PDF

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Quasi PDF

$$\tilde{h}_B(z, P_z, 1/a) = \langle PS|F^{0i}W(z,0)\tilde{F}_{0i} + F^{ij}W(z,0)\tilde{F}_{ij} - \{(z \leftrightarrow -z)\}|PS\rangle$$

Gluon helicity PDF

$$\Delta\tilde{g}(x, P_z, 1/a) = \frac{i}{2xP_z}\int \frac{dz}{2\pi} e^{ixzP_z}\tilde{h}(z, P_z, 1/a)$$

Total gluon helicity

$$\Delta\tilde{G}(P_z, 1/a) \equiv \int dx\, \Delta\tilde{g}(x, P_z, 1/a) = \frac{1}{2P_z}\int_0^\infty dz\, \tilde{h}(z, P_z, 1/a).$$

Balitsky, et al., *J. High Energ. Phys.* 193 (**2022**).
Pang, Z., Yao, F. & Zhang, JH. *J. High Energ. Phys.* 222 (**2024**).

Lattice Setup

CLQCD	Sea Type	$N_L^3 \times N_T$	Lattice Spacing (fm)	$m_\pi^{(s)}$(MeV)	Valence Type	Ncfg	Nsource
C32P29	Clover (2+1) + TITLS	$32^3 \times 64$	0.1052	292.4(1.1)	Clover	980	64
E32P29		$32^3 \times 64$	0.0897	287.3(2.5)		1700	64
F32P30		$32^3 \times 96$	0.0775	300.4(1.2)		800	96

Zhi-Cheng Hu et al. Phys. Rev. D 109, 054507 (**2024**).

Bare Matrix

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Feynman-Hellmann method

2pt

$$C_2(t_f;\mathcal{H}) = \left\langle \sum_{\vec{y}} \mathcal{H}(t_f,\vec{y}) \sum_{\vec{x}} \mathcal{H}^\dagger(0,\vec{x}) \right\rangle$$

3pt

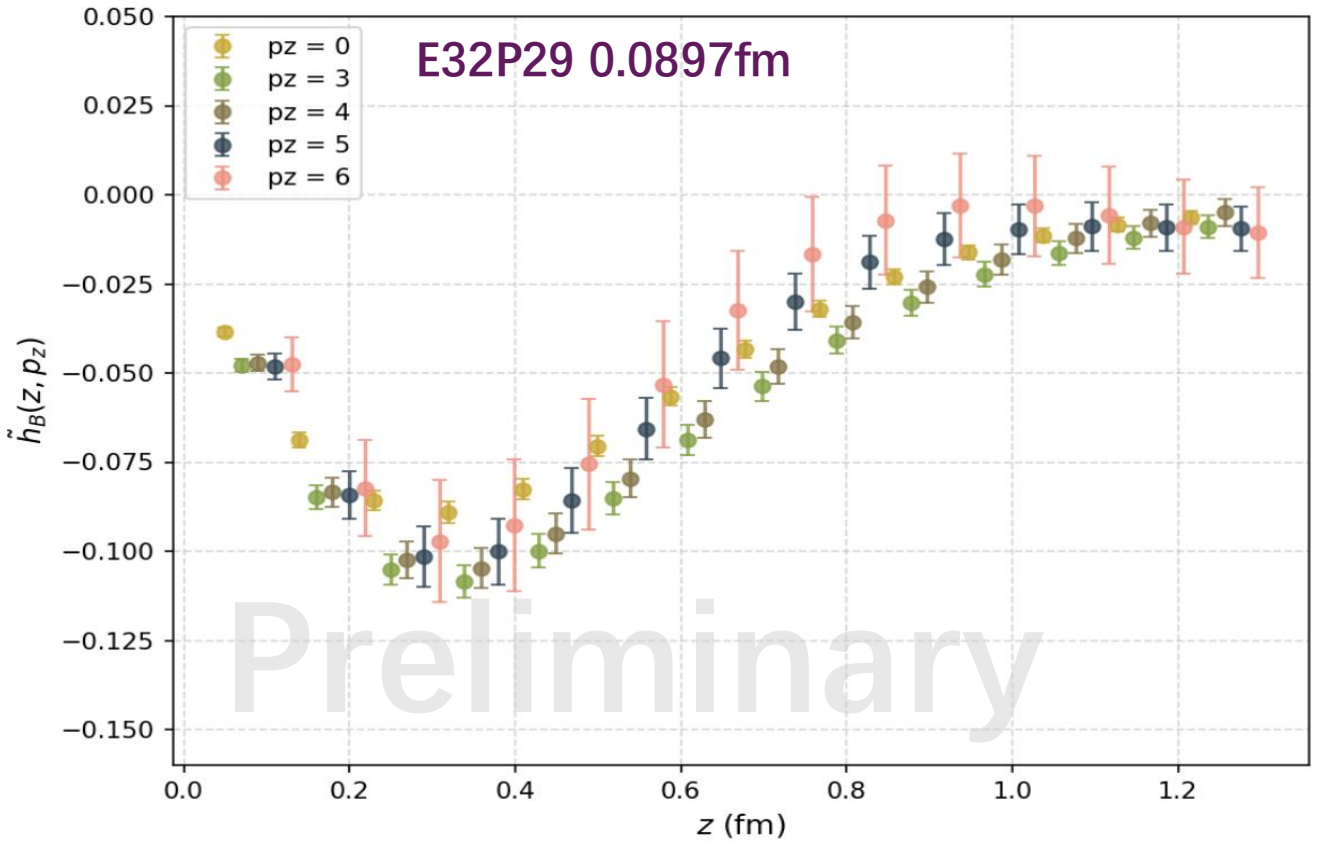
$$SC_3(t_f;\mathcal{H}) = \left\langle \sum_{\vec{y}} \mathcal{H}(t_f,\vec{y}) \sum_{t\in(t_f,0),\vec{z}} O(t,\vec{z}) \sum_{\vec{x}} \mathcal{H}^\dagger(0,\vec{x}) \right\rangle$$

Bare matrix

$$\tilde{h}_B(z,p_z) = \langle O \rangle_H = \Delta R(t_f) + \mathcal{O}(e^{-\delta m t_f})$$

$$\Delta R(t_f) \equiv \frac{SC_3(t_f;H)}{C_2(t_f;H)} - \frac{SC_3(t_f-1;H)}{C_2(t_f-1;H)}$$

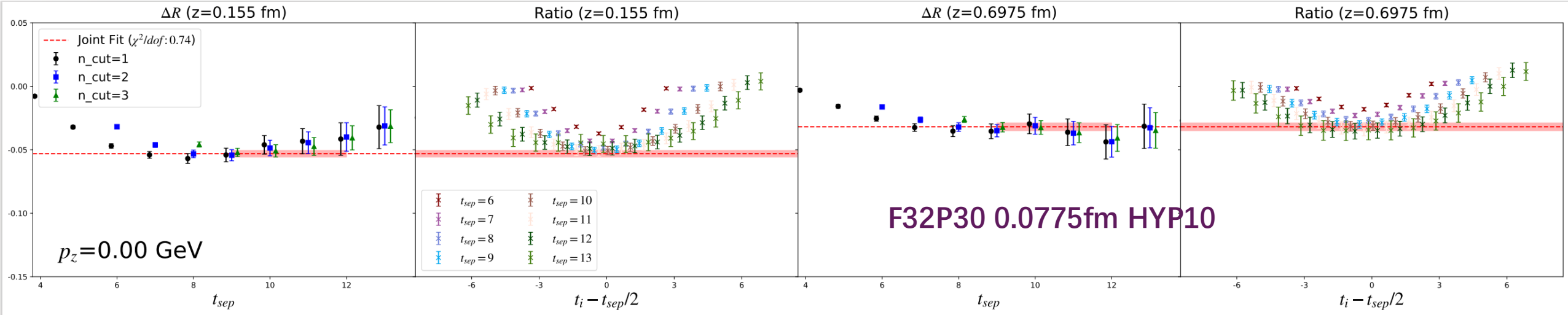
Bare matrix

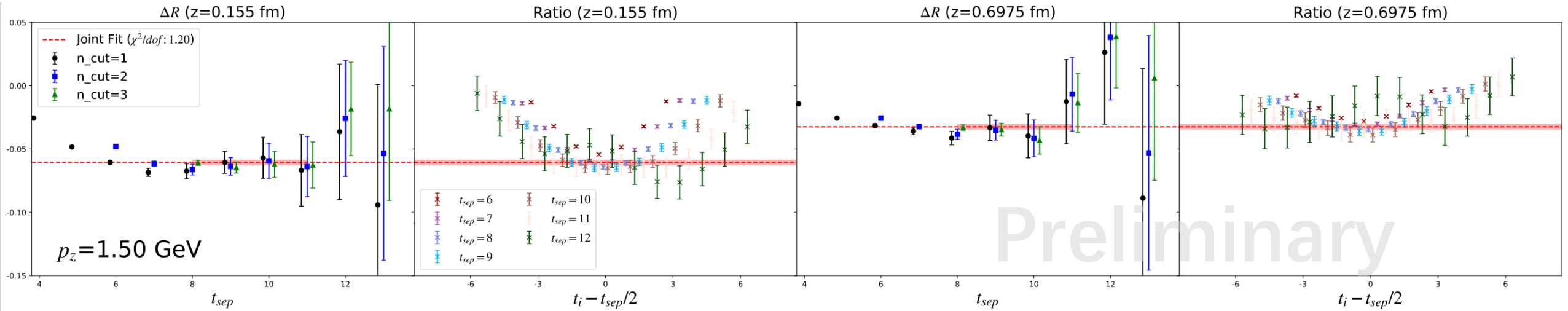


C. C. Chang, et al., Nature 558, 91 (2018).

Bare Matrix

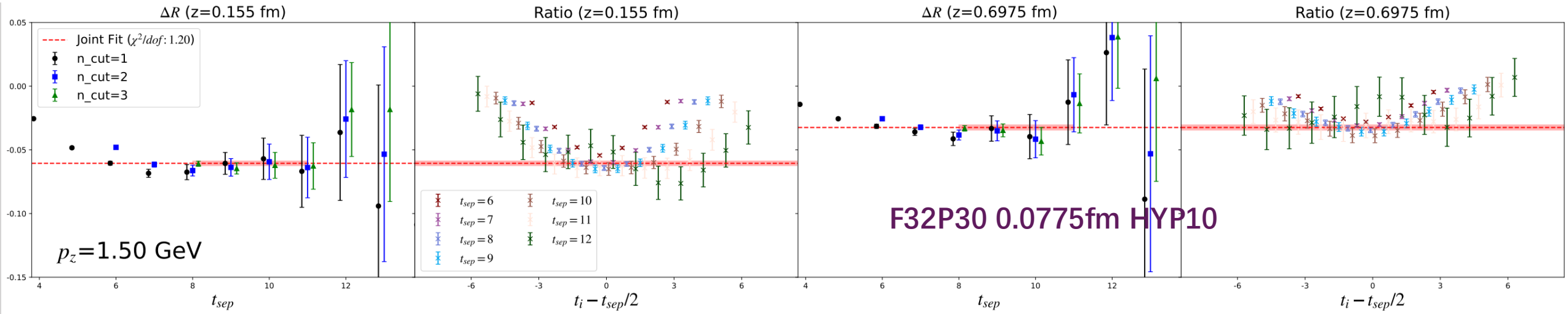
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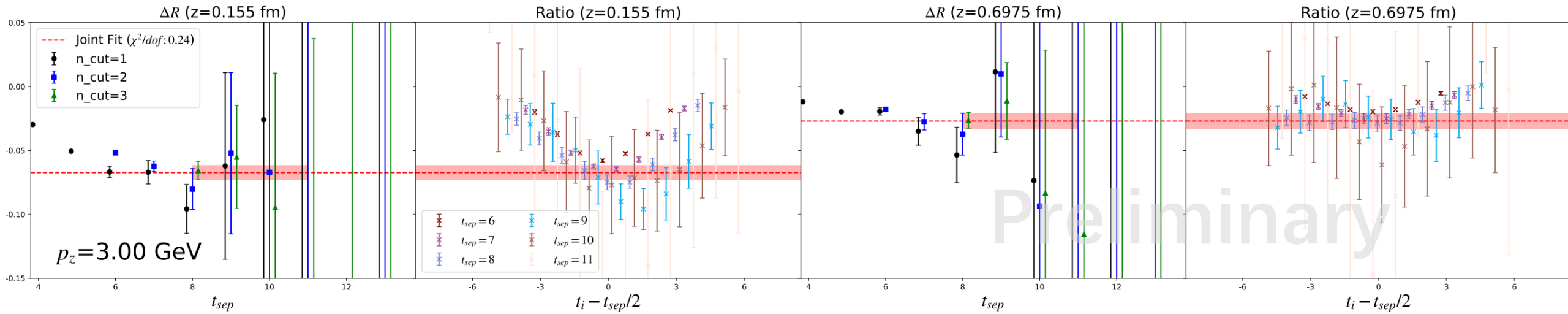




Bare Matrix

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Self Renormalization

Symmetrization

$$\tilde{h}_{\rm sym}(z, P_z, 1/a) = \frac{\tilde{h}_B(z, P_z, 1/a)/z}{\left[\tilde{h}_B(z, P_z, 1/a)/z\right]_{z\to 0}}$$

Renormalization

$$\ln \tilde{h}_{\rm sym}(z, P_z = 0, 1/a) = \ln\left[Z_R(z, 1/a, \mu; k, \Lambda_{\rm QCD}, m_0, d)\right]$$

$$+ \begin{cases} \ln\left[\tilde{h}^{\rm pert}_{sym,\overline{\rm MS}}(z,\mu)\right] & \text{if } z_0 \le z \le z_1 \\ g(z) - m_0 z & \text{if } z_1 < z \end{cases}$$

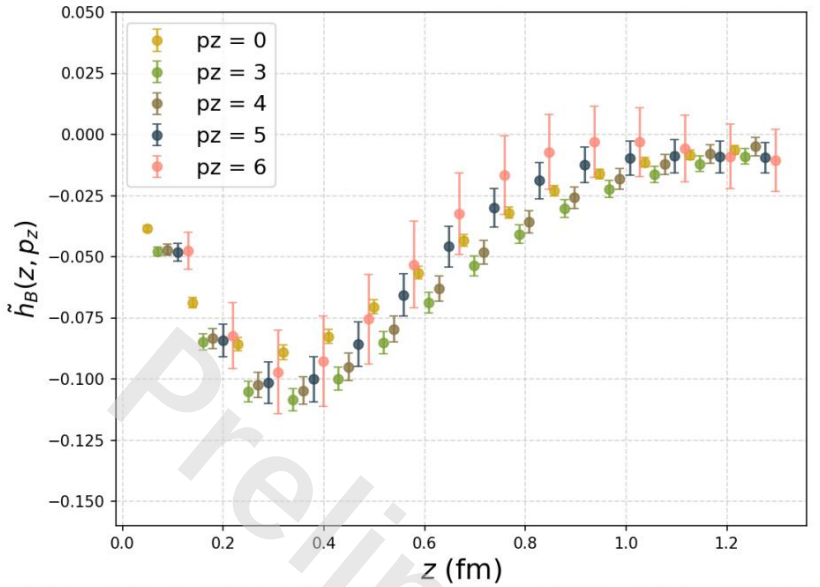
$$\tilde{h}^{\rm pert}_{sym,\overline{\rm MS}}(z,\mu) = 1 + \frac{\alpha_s(\mu) C_A}{4\pi}\left(\frac{19}{6}\ln\frac{z^2\mu^2}{4e^{-2\gamma_E}} - \frac{5}{6}\right)$$

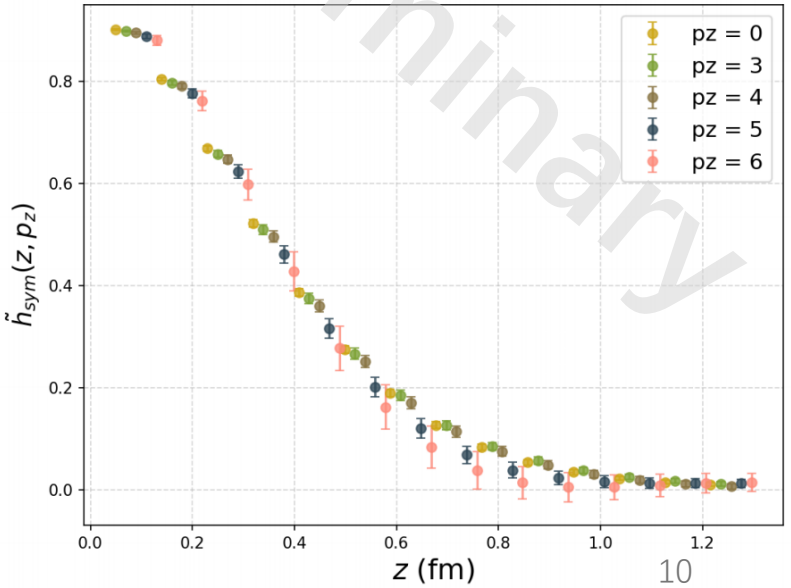
$$Z_R(z, 1/a, \mu; k, \Lambda_{\rm QCD}, m_0, d) = \exp\left\{\frac{kz}{a\ln[a\Lambda_{\rm QCD}]} + m_0 z\right.$$

$$\left. + \frac{14 C_A}{3 b_0}\ln\left[\frac{\ln(1/a\Lambda_{\rm QCD})}{\ln(\mu/\Lambda_{\rm QCD})}\right] + \frac{1}{2}\ln\left[\left(1 + \frac{d}{\ln[a\Lambda_{\rm QCD}]}\right)^2\right]\right\}$$

Yao, F., Ji, Y. & Zhang, JH. *J. High Energ. Phys.* 21 (**2023**).
Ji et al. *Nucl. Phys. B*, 964, 115311 (**2021**),
Deng et al. In progress.

Bare matrix

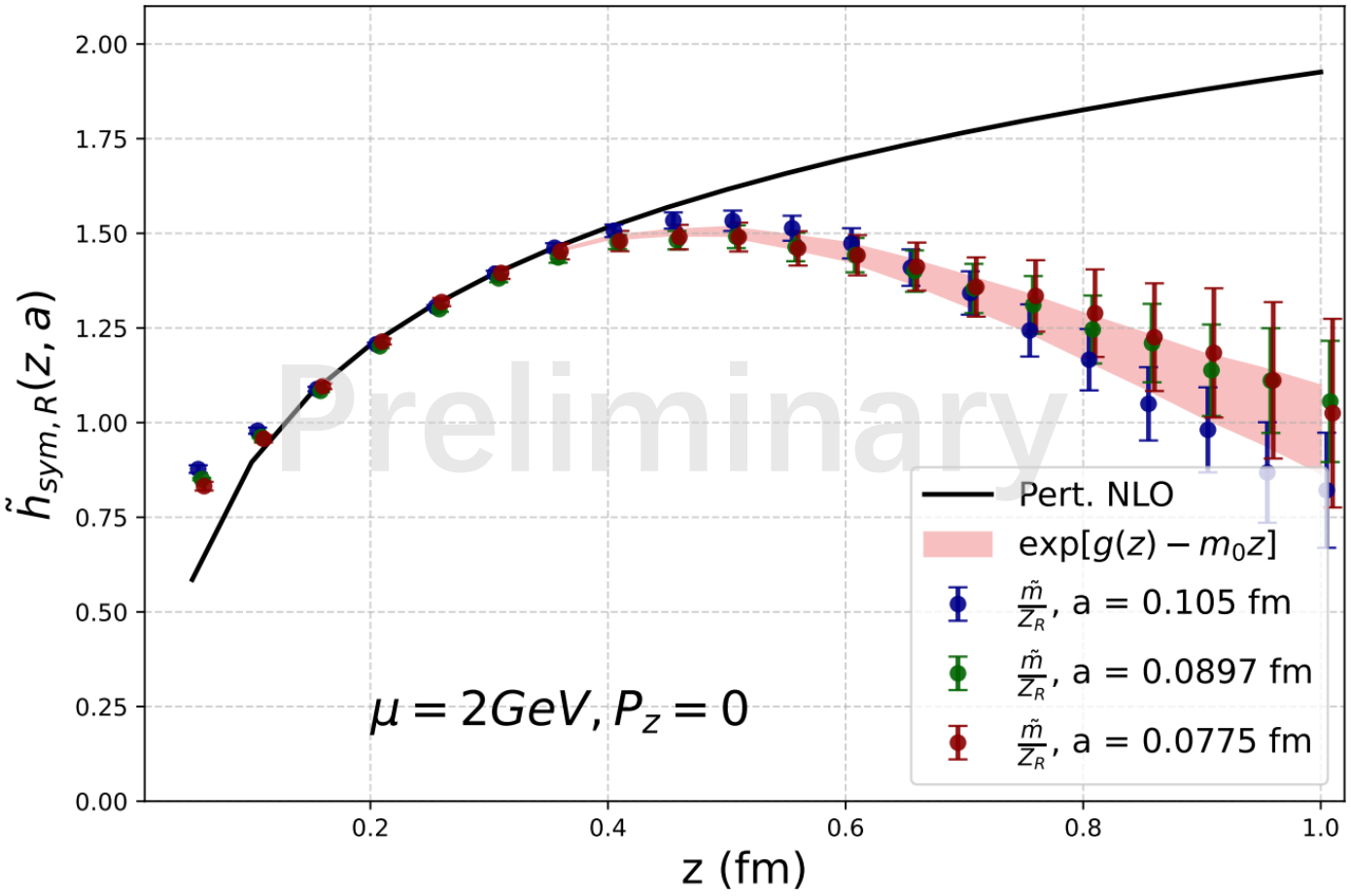




Preliminary

Renormalization

Renormalized Matrix



Preliminary

$$\mu = 2GeV$$
$$\alpha_s(\mu = 2GeV) = 0.296$$

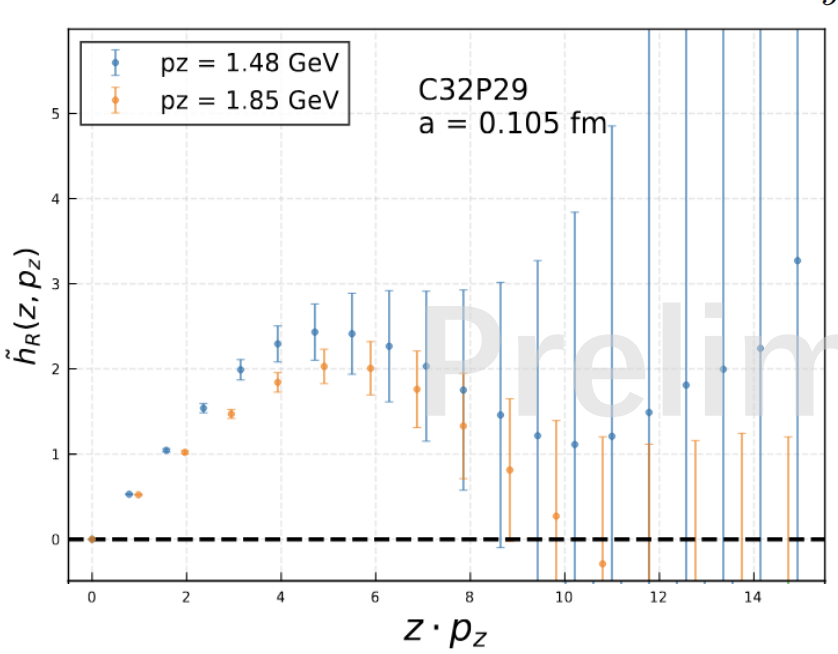
k	1.61(0.24)
$\Lambda_{\rm QCD}$(GeV)	0.032(0.043)
m_0(GeV)	−0.0086(0.23)
d	−1.66(0.79)
χ^2/d.o.f.	0.57

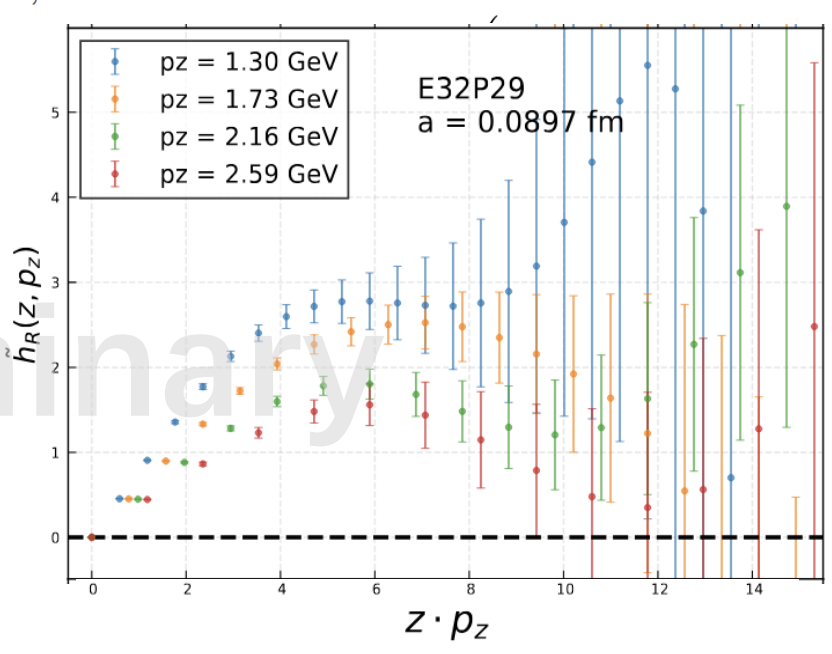
$$\tilde{h}_{\rm sym}(z, P_z, 1/a) = \frac{\tilde{h}_B(z, P_z, 1/a)/z}{\left[\tilde{h}_B(z, P_z, 1/a)/z\right]_{z\to 0}}$$

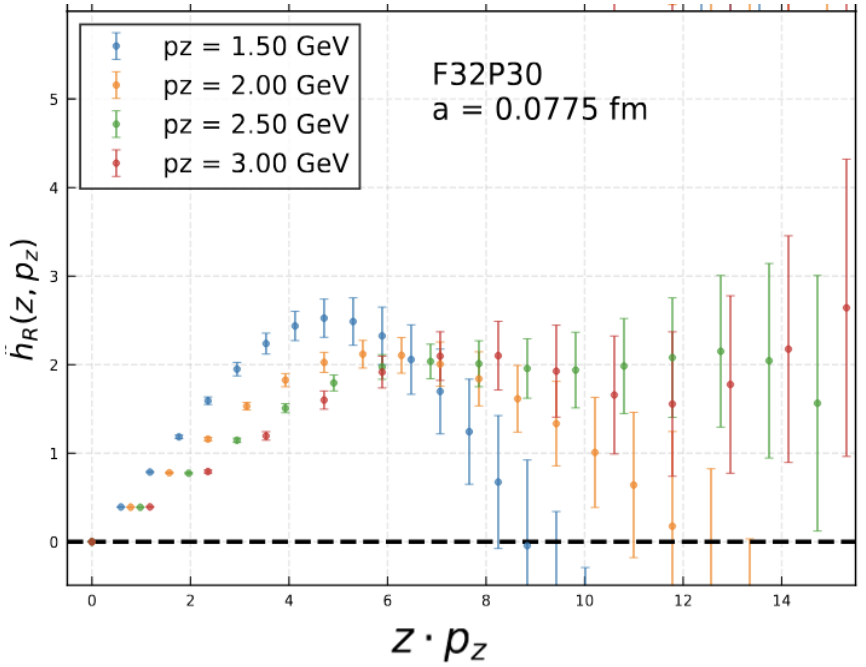
$$\tilde{h}^R_{sym} = \frac{\tilde{h}_B(z, p_z, 1/a)}{z_R}$$

Renormalized Matrix

$$\tilde{h}_R^{\text{ratio}}(z, P_z) = z \left[\frac{\tilde{h}_{sym}(z, P_z, 1/a)}{\tilde{h}_{sym}(z, P_z = 0, 1/a)} \theta(z_1 - |z|) + \frac{\tilde{h}_{sym}(z, P_z, 1/a)}{Z_R(z, 1/a)\, \tilde{h}^{\text{pert}}_{sym,\overline{\text{MS}}}(z, \mu)} \theta(|z| - z_1) \right]$$

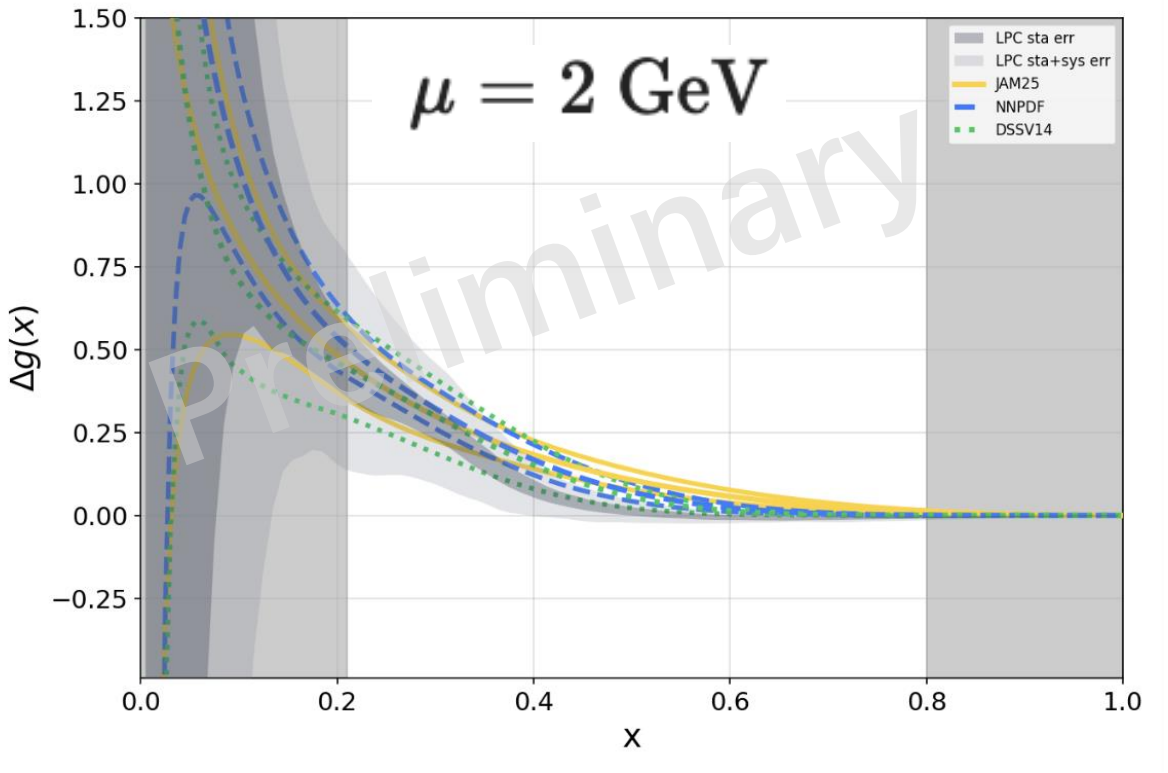


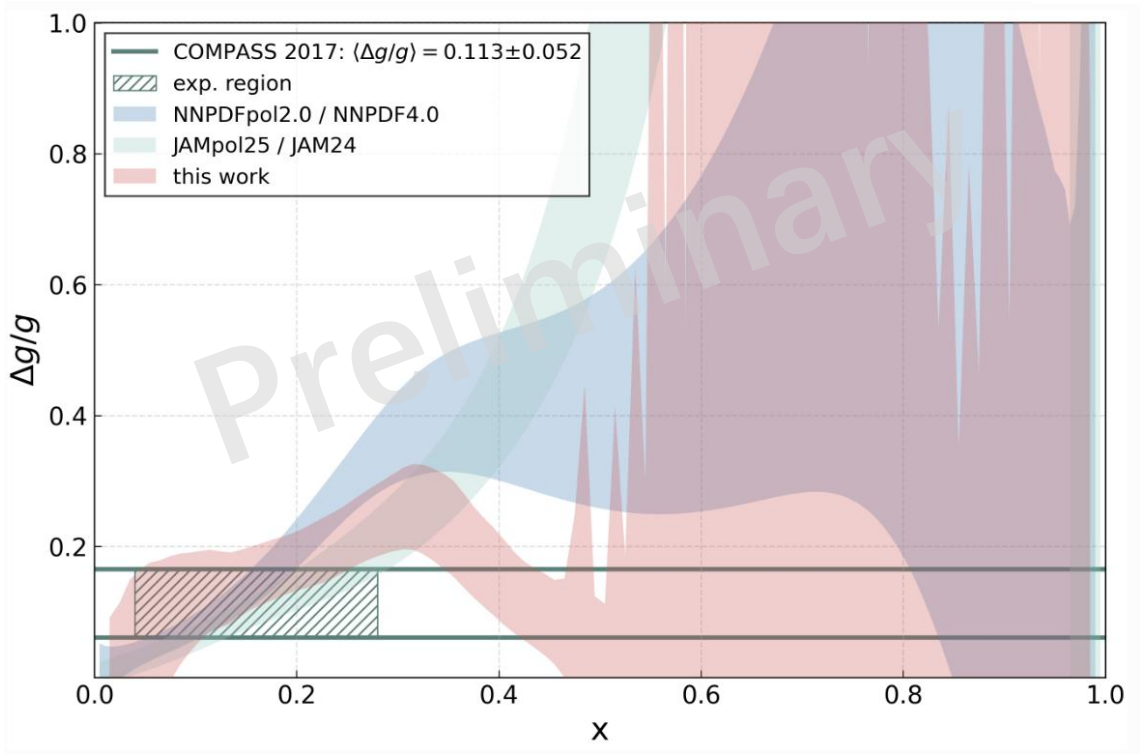




Preliminary

Infinite Momentum & Continuum Limit





JAM: 2025
NNPDF: 2025
DSSV14: 2019

Systematic error

- Renormalization
- Lambda extrapolation
- Infinite momentum

J. Cruz-Martinez, *et al. J. High Energ. Phys.* **2025**, 168 (2025).
D. de Florian, et al., Phys. Rev. D 100, 114027 (**2019**).
C. Chen, et al., arXiv:2510.26425 (**2025**).
C. Adolph, et al., Eur. Phys. J. C 77, 4716 (**2017**).

Summary and Outlook

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Summary

- Extracted the first *ab initio* gluon helicity PDF entirely from Lattice QCD and compared it with global fits.
- Achieved continuum and large-momentum limits using gauge configurations at three distinct lattice spacings.
- Implemented a self-renormalization scheme for purely imaginary, antisymmetric observables.

Outlook

- Investigate the pion mass dependence of the gluon helicity PDF.
- Further enhance the signal-to-noise ratio and systematically reduce uncertainties.

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LaMET 2026

Lattice Parton Collaboration

Thank you !

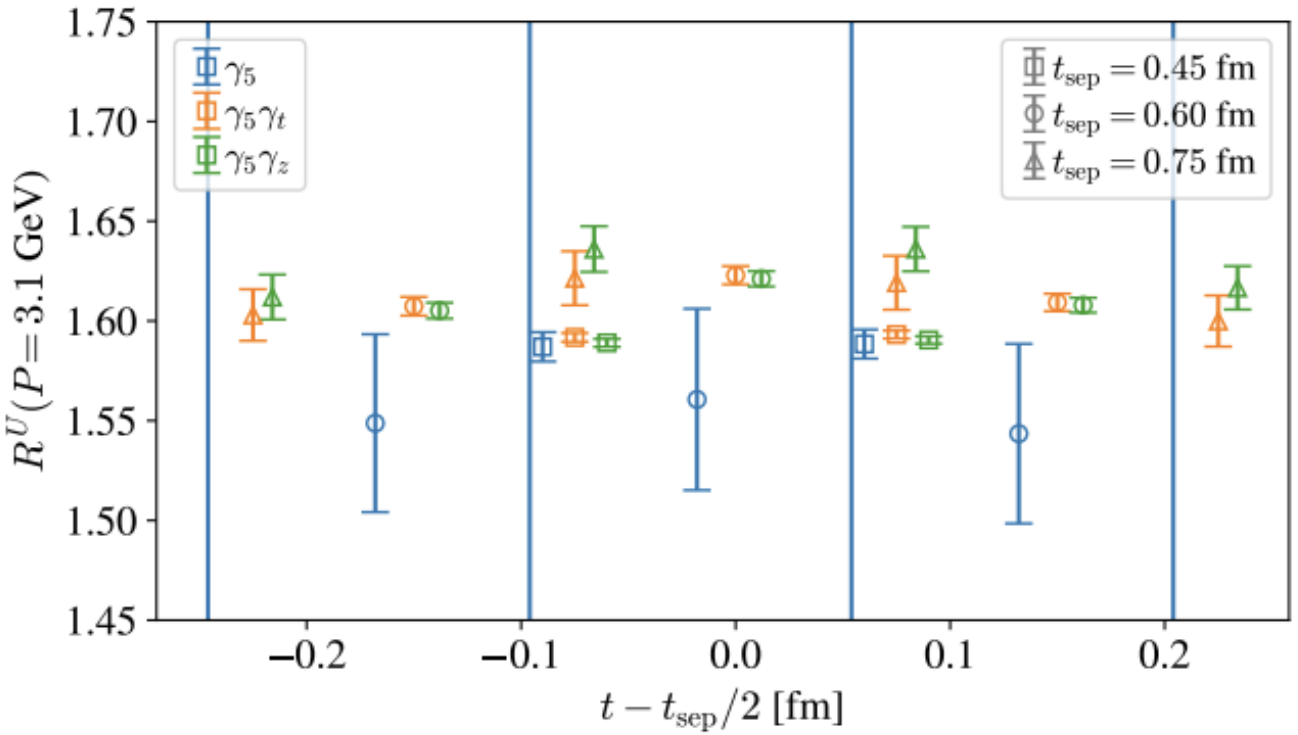
Speaker: Shi-Yi Zhong

2026 July 6th

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Backup

Vector current in the nucleon state

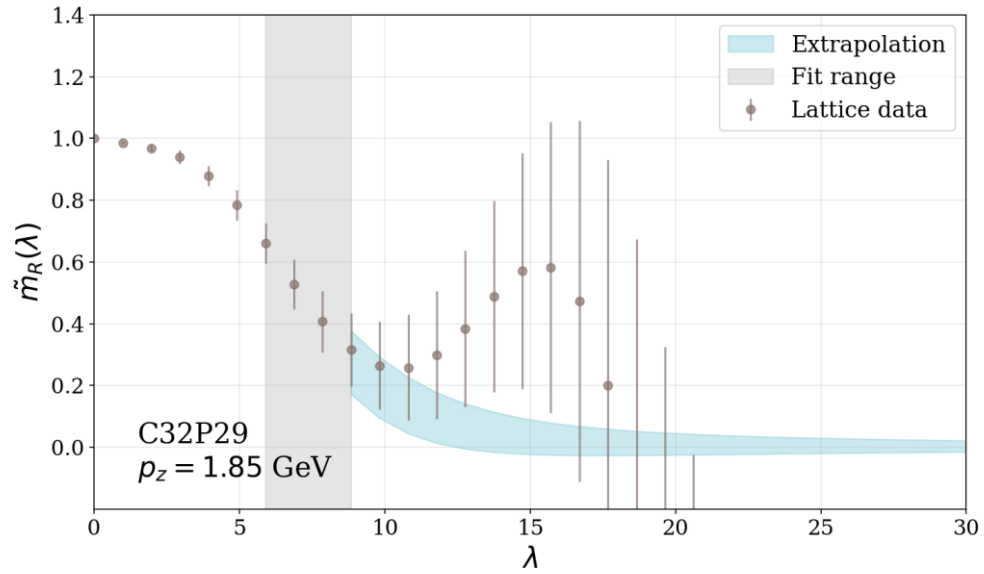


$$C_{3\mathrm{pt}}(\vec{P}, t, t_{\mathrm{sep}}) = \sum_{\vec{x}} e^{i\vec{x}\cdot\vec{P}} \mathrm{Tr}\left[\mathcal{P}_+ \langle N_\Gamma(t_{\mathrm{sep}}, \vec{x}) \times \overline{u}(t)\gamma_t u(t)\ \overline{N}_\Gamma(0)\rangle\right],$$

Zhang, R., Grebe, A.V., Hackett, D.C., Wagman, M.L., & Zhao, Y. *Phys. Rev. D* 112, L051502 (**2025**).

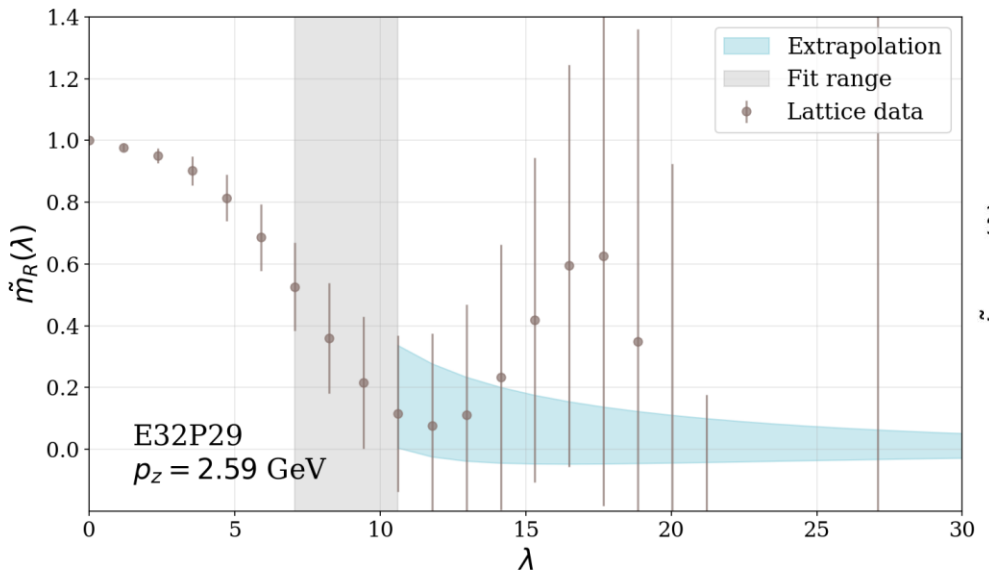
Backup

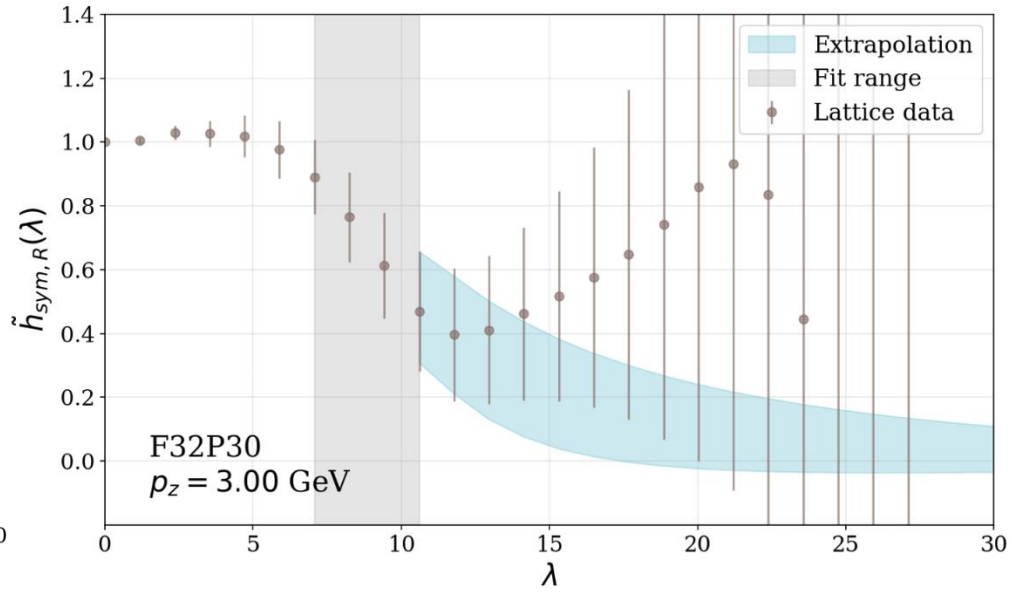
$$\tilde{m}_R(\lambda) = l_1 \lambda^{-a_1} e^{-\lambda/\lambda_0},$$



Ensemble	P_z (GeV)	default range (aP_z)	new range (aP_z)
C32P29	1.48	[9, 13]	[7, 13]
	1.85	[8, 12]	[6, 12]
E32P29	1.3	[9, 13]	[7, 13]
	1.73	[9, 13]	[7, 13]
	2.16	[6, 10]	[4, 10]
	2.59	[6, 10]	[4, 10]
F32P30	1.50	[9, 13]	[7, 13]
	2.00	[9, 13]	[7, 13]
	2.50	[8, 12]	[6, 12]
	3.00	[7, 11]	[5, 11]

TABLE III. λ extrapolation with the baseline and an adjusted fitting window; the resulting systematic error is given as the difference.

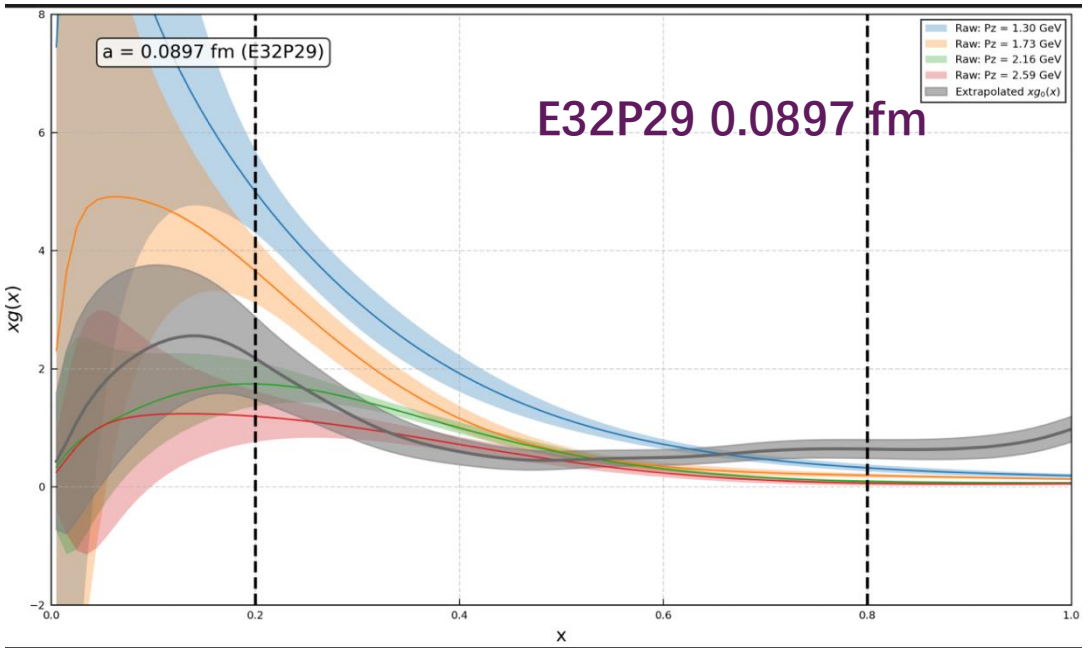


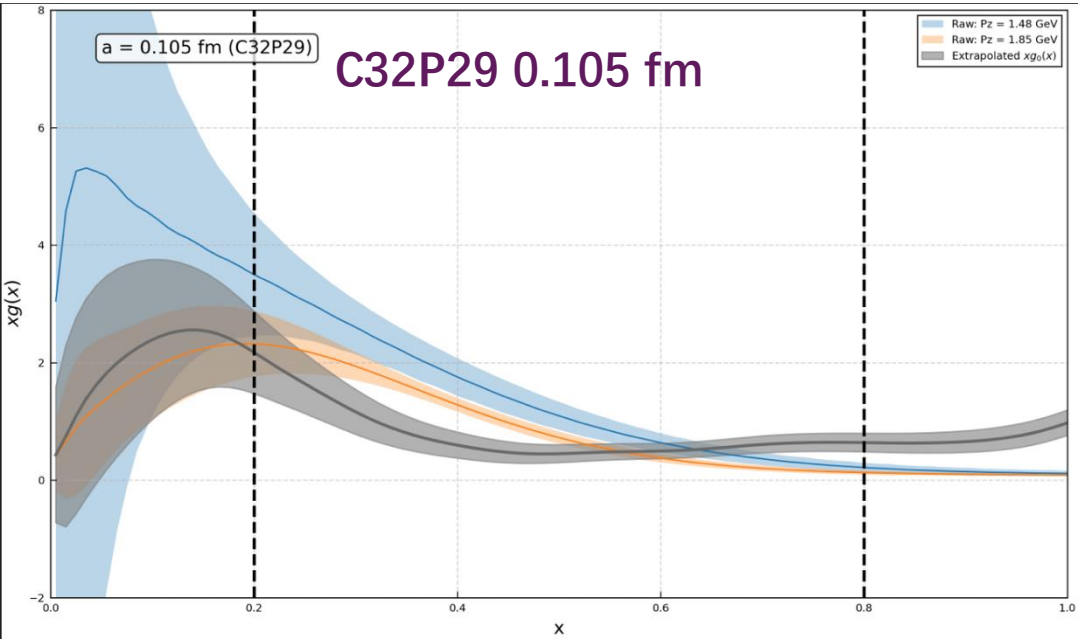


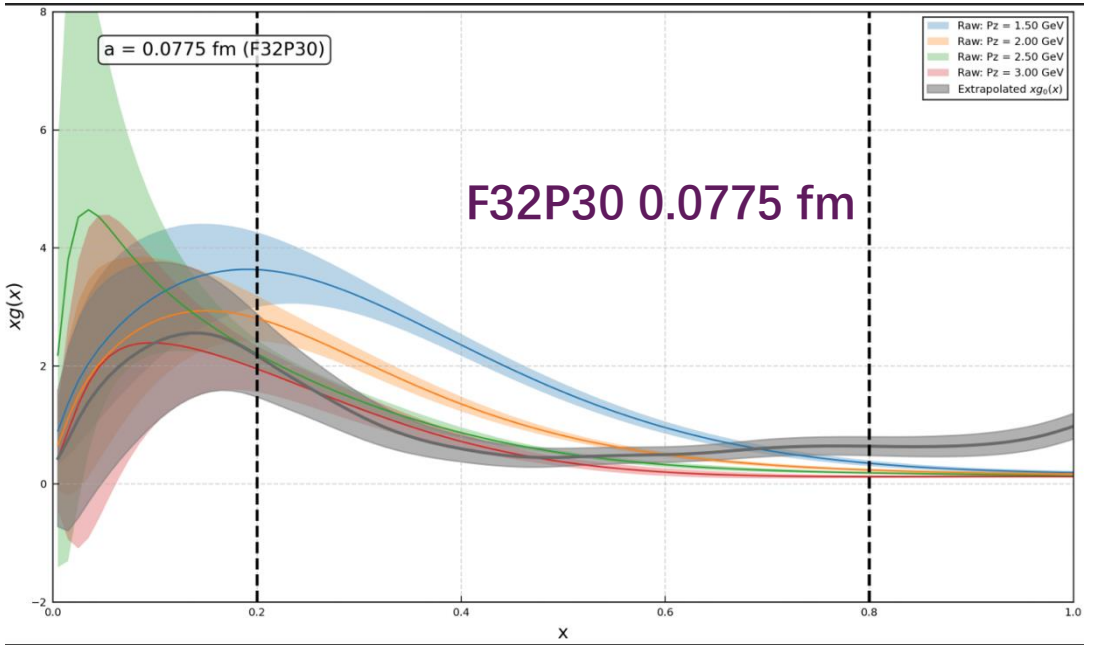
Backup

Extrapolation

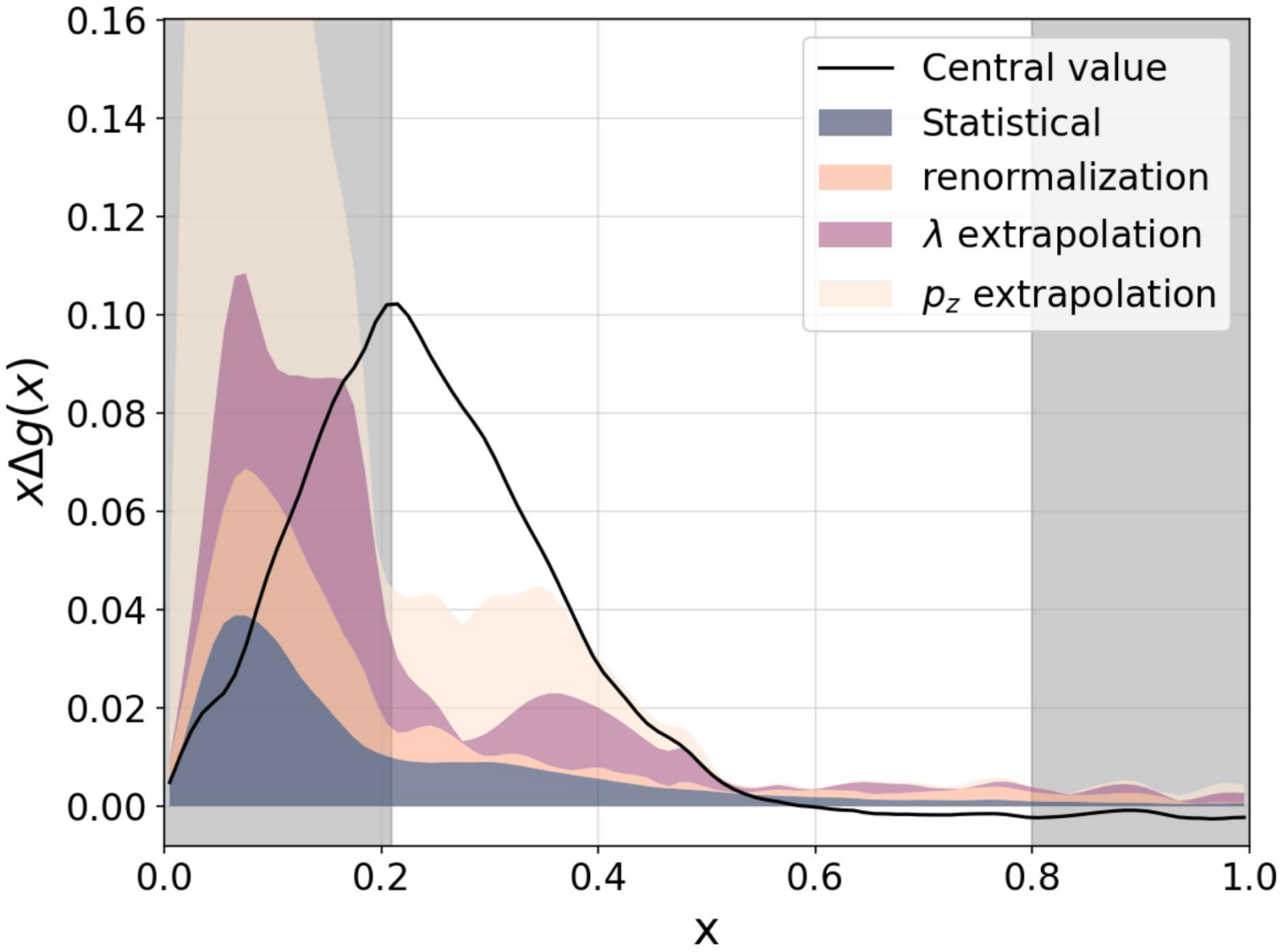
$$xg(x, P_Z, a) = xg_0(x) + a^2 f(x) + ah(x) + \frac{d(x)}{P_z^2}$$







Backup

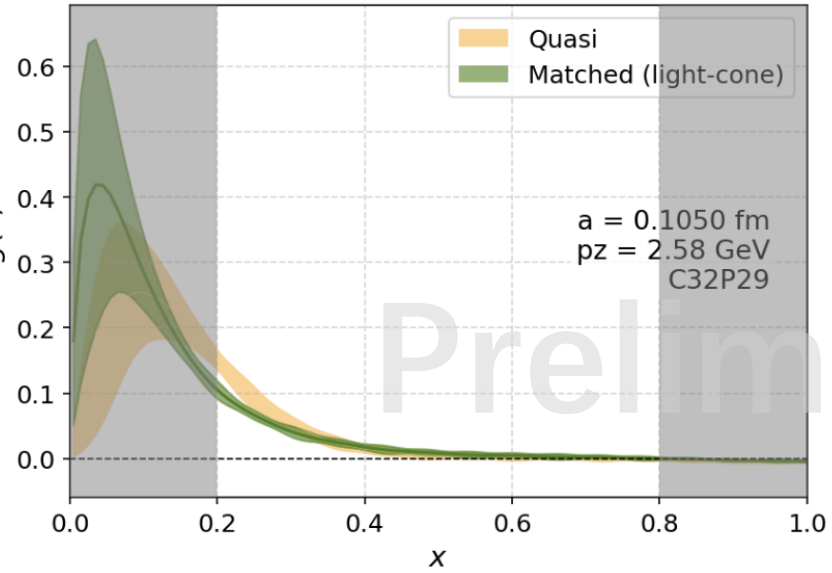

Central value
Statistical
renormalization
λ extrapolation
p_z extrapolation
$x\Delta g(x)$
x
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0.0
0.2
0.4
0.6
0.8
1.0

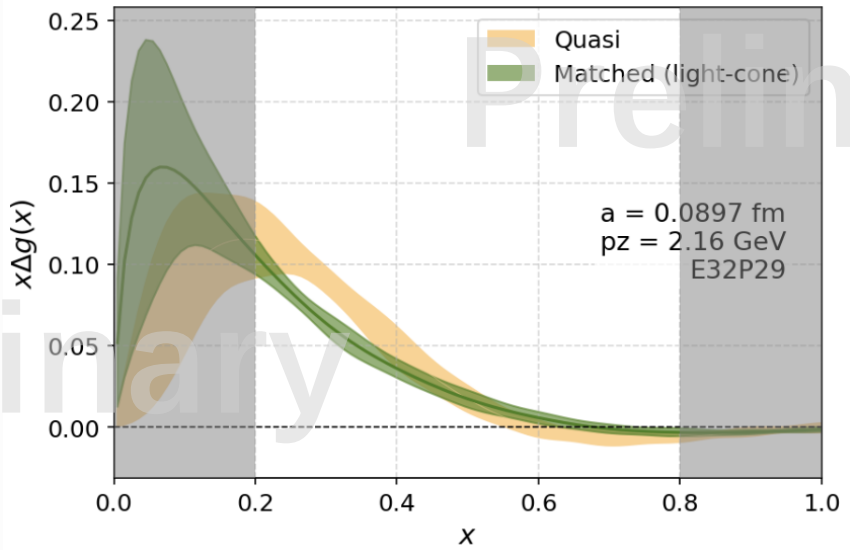
Matching

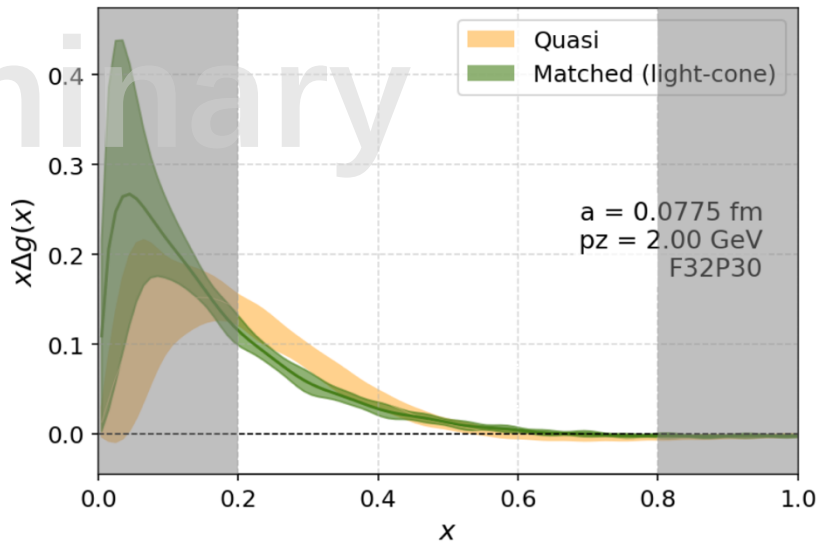
$$C_{g/g}^{\text{ratio}}(x,\rho) = \delta(1-x) + \frac{\alpha_s C_A}{2\pi}\frac{1}{x}[xc(x,\rho)]_{(1-x)+}$$

$$c(x,\rho) = \begin{cases} \dfrac{2x(2x^2-3x+2)}{x-1}\ln\dfrac{x-1}{x} + \dfrac{12x^2-12x+10}{3(x-1)}, & x>1, \\ \dfrac{2x(2x^2-3x+2)}{1-x}\ln\dfrac{4x(1-x)}{\rho^2} + \dfrac{-30x^3+45x^2-24x+10}{3(1-x)}, & 0<x<1, \\ \dfrac{2x(2x^2-3x+2)}{1-x}\ln\dfrac{1-x}{-x} + \dfrac{12x^2-12x+10}{3(1-x)}, & x<0. \end{cases}$$

$$C_{\text{hybrid}}(\xi,\lambda_s,\frac{\mu}{yP_z}) = C_{\text{ratio}}\left(\xi,\frac{\mu}{yP_z}\right) + \frac{\alpha_s(\mu)C_A}{2\pi}\frac{13}{6}\left(-\frac{1}{|1-\xi|} + \frac{2\text{Si}((1-\xi)|y|\lambda_s)}{\pi(1-\xi)}\right)_+,$$







Preliminary
Preliminary

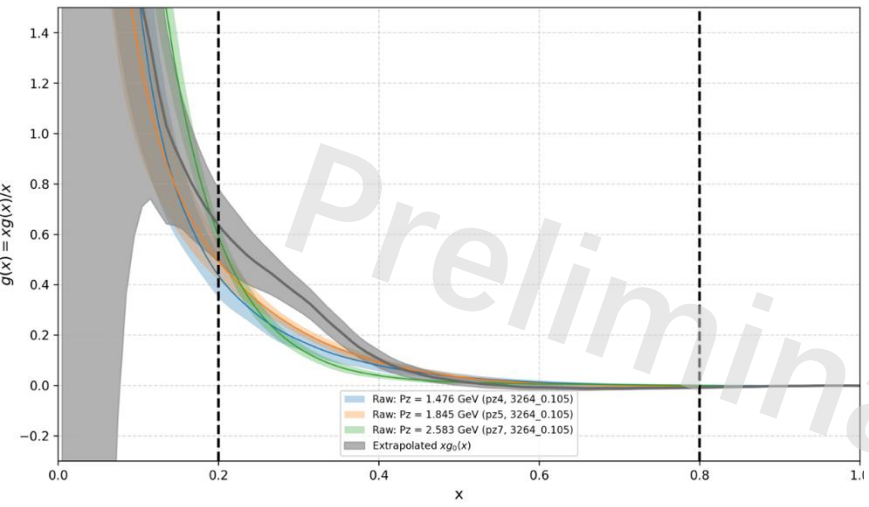
Infinite Momentum & Continuum Limit

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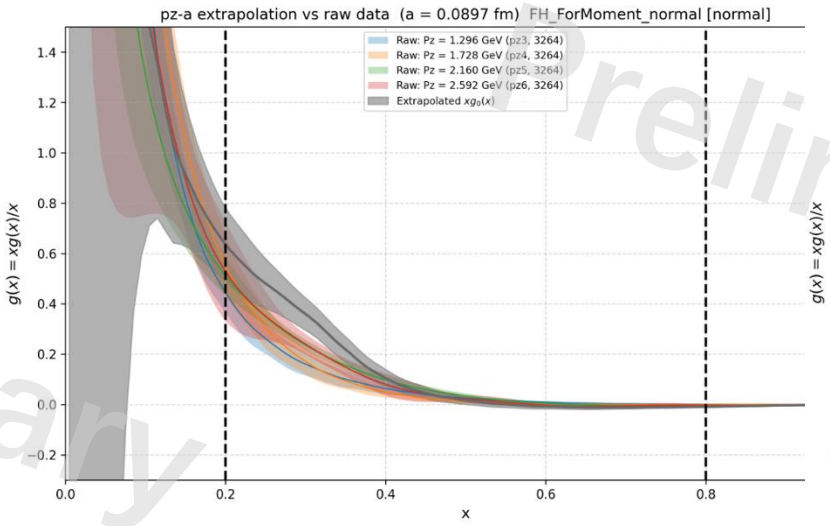
Extrapolation $$xg(x, P_Z, a) = xg_0(x) + a^2 f(x) + a^2 P_z^2 h(x) + \frac{d(x)}{P_z^2}$$

Distribution after Fourier transformation and matching

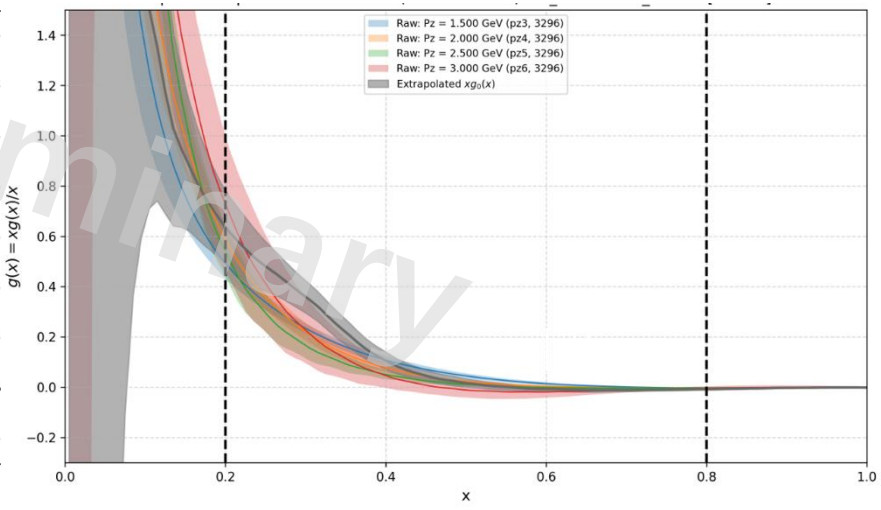
C32P29 0.105 fm



E32P29 0.0897 fm



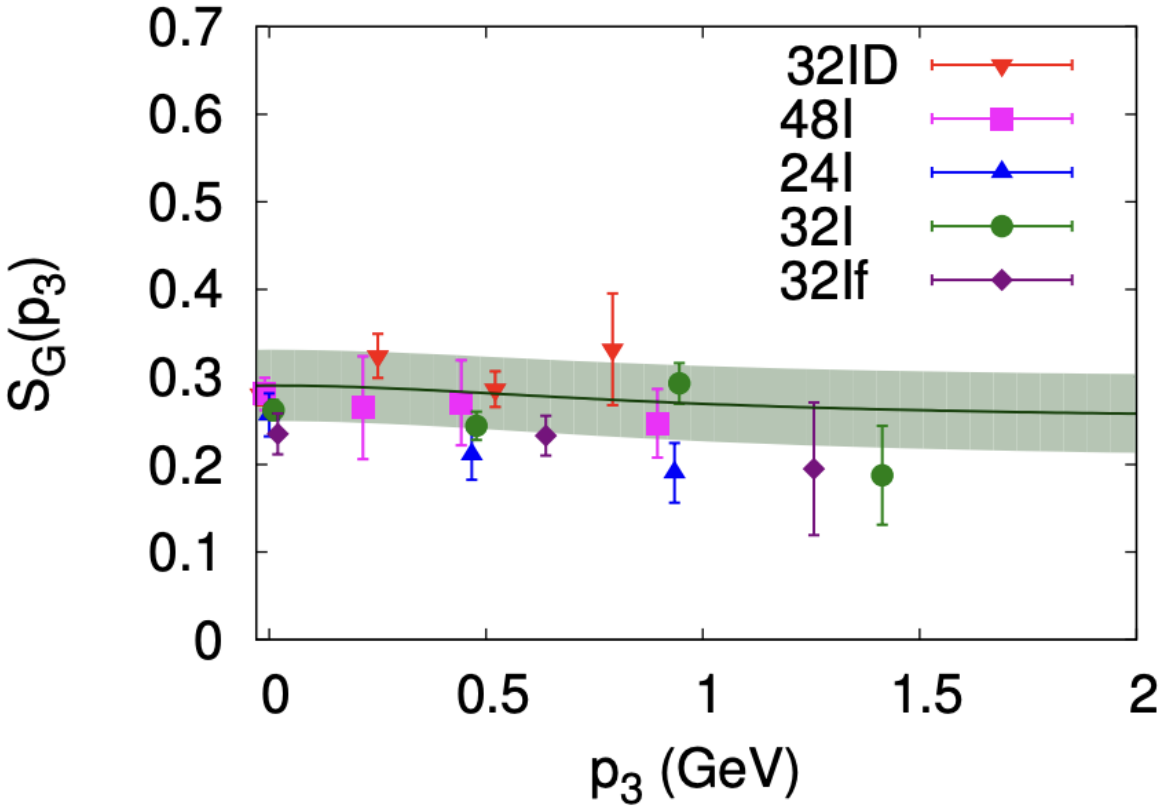
F32P30 0.0775 fm



Preliminary

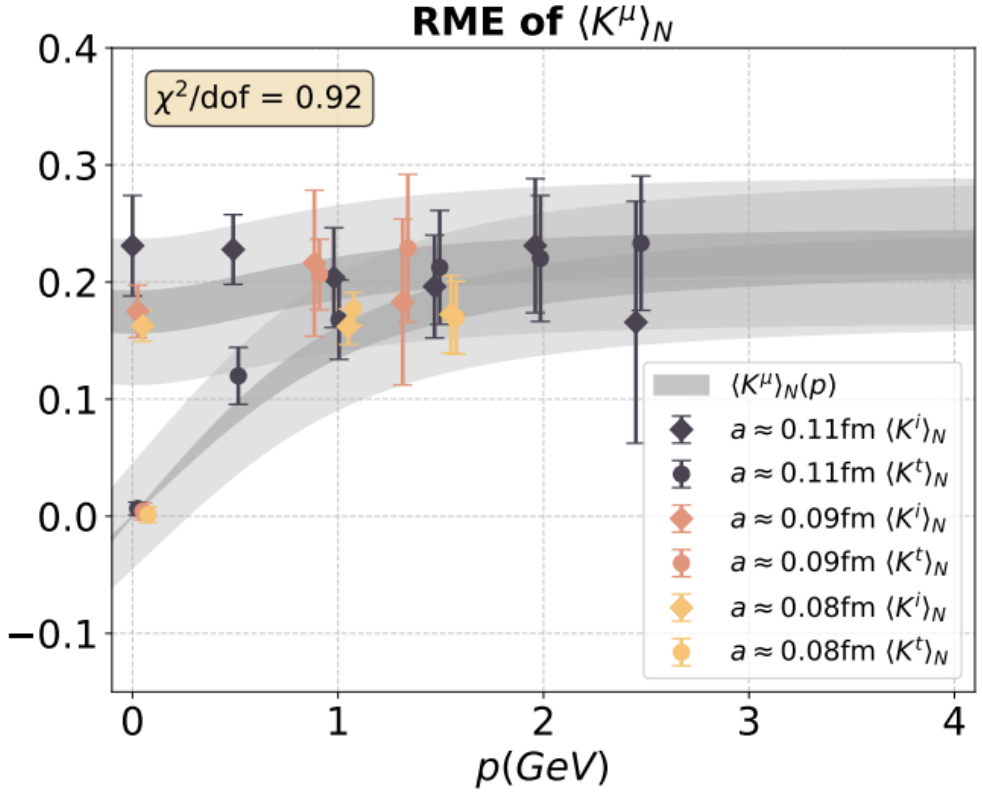
Previous Work

Total gluon spin



$$\Delta G \propto \langle \vec{E} \times \vec{A} \rangle_{\text{C.G.}} = S_G|_{\text{IMF}} = 0.251(47)^{\text{sta.}}(16)^{\text{sym.}}$$

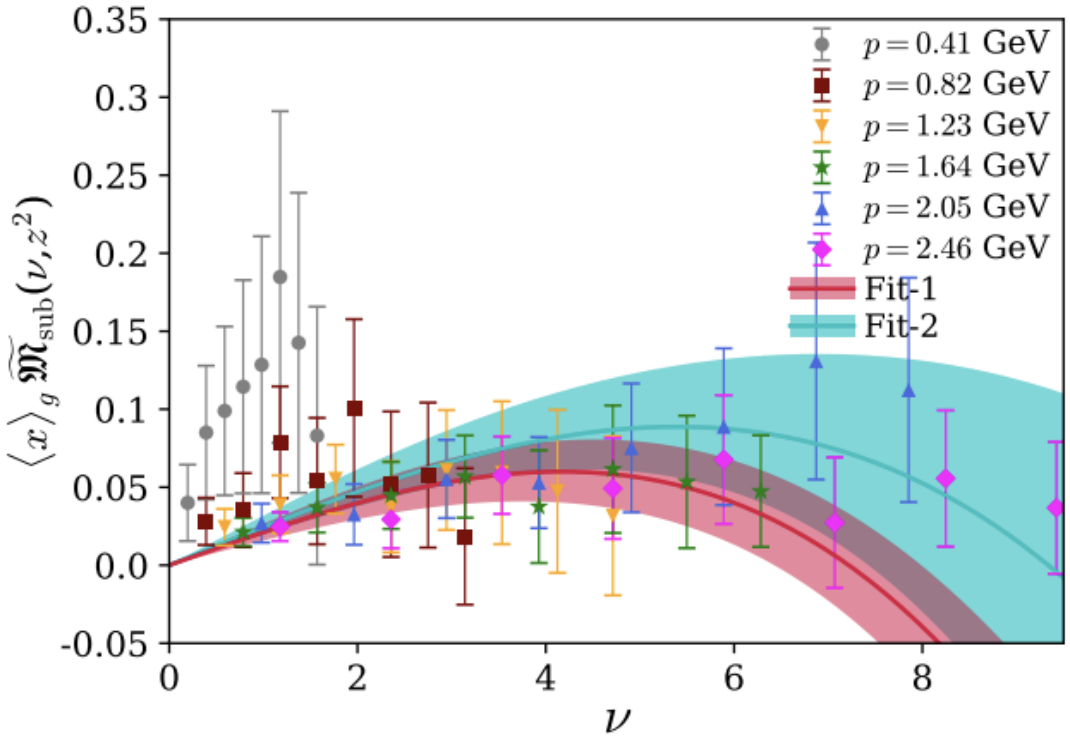
Y.-B. Yang, et al., Phys. Rev. Lett. 118, 102001 (2017).



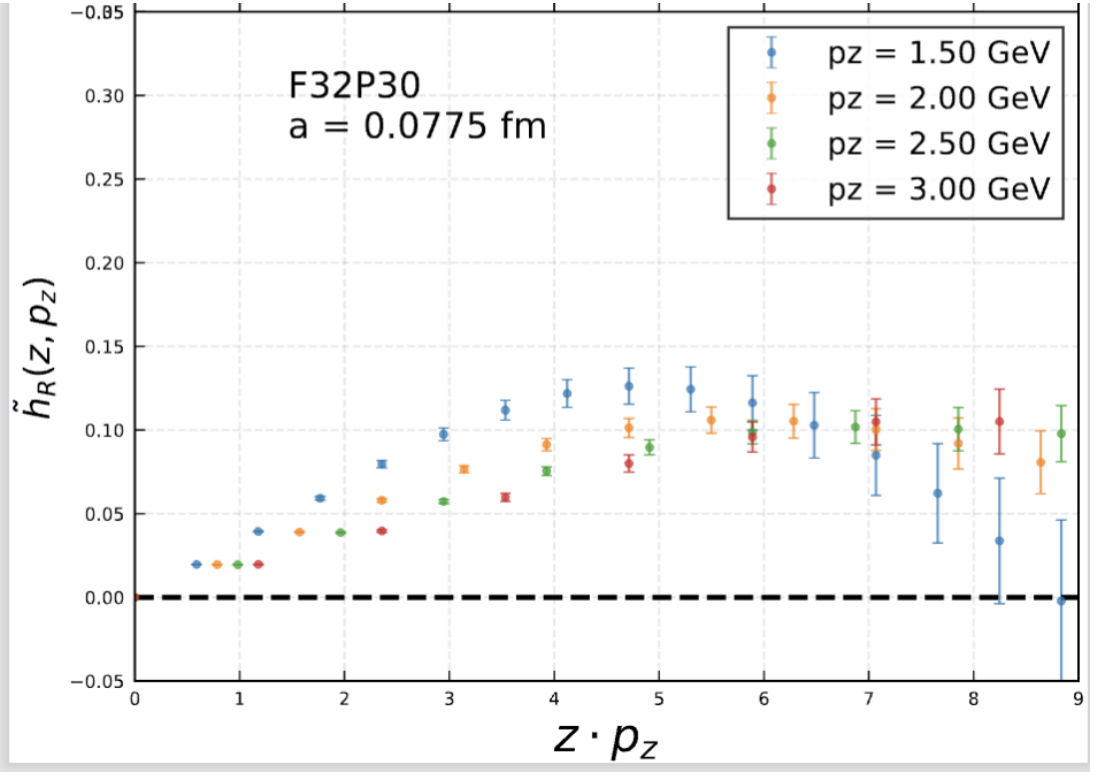
$$\Delta G^{\overline{\text{MS}},\ 10\ \text{GeV}^2} = 0.231(17)^{\text{sta.}}(44)^{\text{sym.}}$$

D.-J. Zhao, et al. arXiv preprint arXiv:2512.24315 (2025).

Backup



ID	a (fm)	m_π (MeV)	$L^3 \times N_t$	$N_{\rm cfg}$	$N_{\rm srcs}$
$a094m358$	0.094(1)	358(3)	$32^3 \times 64$	1901	64



C. Egerer, et al. , Phys. Rev. D 106, 094511 (**2022**).