

An Infrared Topological Approach to Baryon Structure

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林 凡, 王昕杨 arXiv:2601.20274v3 [hep-th]

安徽理工大学, 基础物理研究中心

Introduction

QCD Anomalies, Solitons, and the Infrared Topological Paradigm

Quantum Chromodynamics

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_f (i\not{D} - m_f) \psi_f - \frac{1}{4} G_{\mu\nu}^a G^{\mu\nu,a}$$

Perturbative UV: Asymptotic Freedom

$$\beta(g) < 0, \quad \alpha_s(Q^2) \propto \ln^{-1}(Q^2/\Lambda_{\text{QCD}}^2)$$

Weakly coupled quarks and gluons at high momentum transfer.

Non-Perturbative IR: Core Triad

Yang-Mills Mass Gap



Color Confinement

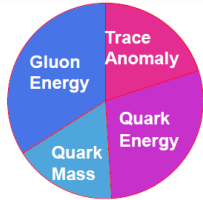


Chiral Symmetry Breaking

Nucleon Structure: Mass and Spin

Trace Anomaly & Mass Generation

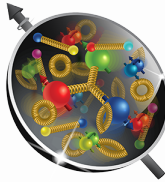
$$T^\mu{}_\mu = \frac{\beta(g)}{2g} G_{\mu\nu}^a G^{\mu\nu,a} + \sum_f m_f (1 + \gamma_m) \bar{\psi}_f \psi_f$$



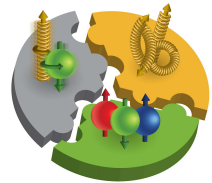
Proton Mass

Axial Anomaly & Spin Structure

$$\partial_\mu j^{5,\mu} = 2im\bar{\psi}\gamma^5\psi + \frac{g^2 N_f}{8\pi^2} \text{Tr}(F_{\mu\nu} \tilde{F}^{\mu\nu})$$



Proton Structure

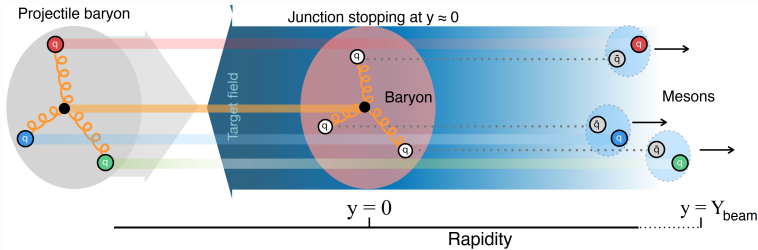


Proton Spin

Nucleon Structure: The Mysterious Gluon Structure

Y-Shaped Flux Junction

Lattice and recent experiment both show that the 3-quark system inside the baryons forms a static Y-shaped flux junction.



The Central Objective

What type of gluon field structure produces the junction, and how does it serve as a universal skeleton unifying color confinement, trace-anomaly mass, and spin?

STAR Collaboration, Science 393 (2026) 6812, ads5962; T. T. Takahashi et al., PRL 86, 18 (2001); F. Okiharu et al., PRL 94, 192001 (2005) et al.

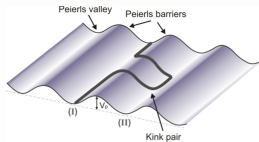
Condensates & Topological Defects

Infrared Condensate Phase

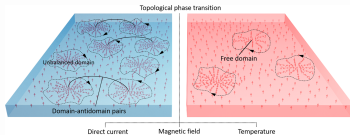
As the energy scale flows from the UV to the deep IR, matter tends to evolve from a microscopic quantum state toward a more collective and condensed state.

Emergent Topological Defects Across Dimensions

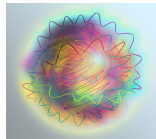
In the IR, topological defects emerge as universal degrees of freedom governing long-range dynamics.



Kinks (1+1D)



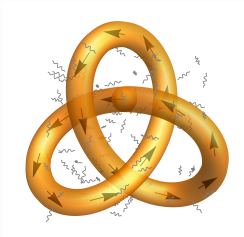
Vortices (2+1D)



Glueballs (3+1D)

Central Paradigm: The Gluon Knot

- **Fundamental IR Degree of Freedom:** Gluon knots act as stable topological degrees of freedom of Yang–Mills theory in IR — Glueball.
- **Baryon Frame Architecture:** Physical baryons embed gluon knots as their intrinsic gluon field frame.
- **Unified Mechanism:** Directly unifies chromoelectric flux tube formation (confinement) with dynamical chiral symmetry breaking.



Glueball



Baryon

Chiral Effective Field Theory & Skyrmion

Chiral Symmetry Breaking & χ EFT

$$\mathcal{L}_\chi = \frac{f_\pi^2}{4} \text{Tr}(\partial^\mu U^\dagger \partial_\mu U) + \dots, \quad U(x) \in \text{SU}(N_f)$$

Spontaneous symmetry breaking ($\text{SU}(N_f)_L \times \text{SU}(N_f)_R \rightarrow \text{SU}(N_f)_V$) gives rise to pseudo-Nambu-Goldstone bosons (π, K).

Topological Solitons (Skyrmions)

$$\pi_3(\text{SU}(N_f)) \cong \mathbb{Z} \quad \longleftrightarrow \quad B^\mu = \frac{i}{24\pi^2} \epsilon^{\mu\nu\sigma\rho} \text{Tr}(U^\dagger \partial_\nu U U^\dagger \partial_\sigma U U^\dagger \partial_\rho U)$$

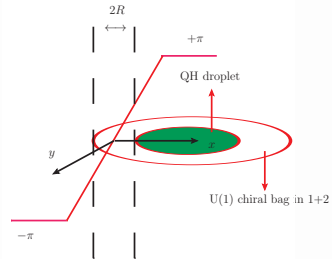
Baryons emerge as Skyrmions, with their topological charge mapped directly to baryon number through anomaly matching.

T. H. R. Skyrme, Proc. R. Soc. A 260, 127 (1961); E. Witten, Nucl. Phys. B 223, 422 (1983) et al.

One-Flavor Baryons on η' Domain Walls

Quantum Hall Droplets, Chiral Bags, and Vortex Solitons

- η' domain walls support $SU(N_c)_{-1} \leftrightarrow U(1)_{N_c}$ Chern–Simons level-rank duality.
- Baryons emerge naturally as Quantum Hall droplets / chiral bags via the Cheshire Cat Principle.
- Chern–Simons–Higgs effective description realizes baryons as quantum vortex excitations.



$$S_A[a, \phi] = \int d^3x \left[|\partial_\mu \phi - i a_\mu \phi|^2 - V(\phi^* \phi) + \frac{N_c}{4\pi} \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho \right]$$

Z. Komargodski, arXiv:1812.09253; Y.-L. Ma et al., PRL 123, 172301 (2019); F. Lin & Y.-L. Ma, JHEP 05, 270 (2024); PRD 112, 014009 (2025)
et al.

Yang–Mills Theory

Abelian Projection, CFN Decomposition, and Knots

't Hooft Projection & Magnetic Monopoles

$$\mathcal{L}_{\text{YM}} = -\frac{1}{2g^2} \text{Tr}(G_{\mu\nu}G^{\mu\nu}), \quad G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$$

Abelian Projection / Dominance & Topological Singularities

$$\text{SU}(N) \xrightarrow{\text{Maximal Abelian Gauge (MAG)}} \text{U}(1)^{N-1}$$

Partial gauge fixing isolates diagonal abelian components. Singularities of the gauge transformation generate chromomagnetic monopoles in the infrared:

$$\pi_2 \left(\text{SU}(N)/\text{U}(1)^{N-1} \right) \cong \mathbb{Z}^{N-1} \longleftrightarrow \text{Chromomagnetic Monopoles}$$

't Hooft Projection

$$\text{Non-Abelian SU}(N) \longleftrightarrow \text{Abelian U}(1)^{N-1} + \text{Chromomagnetic Monopoles}$$

G. 't Hooft, Nucl. Phys. B 190, 455 (1981); T. T. Wu & C. N. Yang, Phys. Rev. D 14, 437 (1976) et al.

Cho–Faddeev–Niemi (CFN) Decomposition

Local Color Frame

$$m_i(x) = g(x)H_i g^\dagger(x) = m_i^a T^a \in \text{SU}(N)/\text{U}(1)^{N-1}, \quad i = 1, \dots, N-1$$

The gauge field decomposes into chromomagnetic monopole, diagonal, and off-diagonal fields:

$$A_\mu^a = C_{i,\mu} m_i^a + H_\mu^a + X_\mu^a$$
$$H_\mu^a = f^{abc} \partial_\mu m_i^b m_i^c, \quad X_\mu^a = (\rho_{ij} f^{abc} + \sigma_{ij} d^{abc}) \partial_\mu m_i^b m_j^c$$

- **Monopole field (H_μ^a):** carries chromomagnetic charges

$$g_m = \oint_\Sigma F^H \cdot d\sigma, \quad F_{i,\mu\nu}^H = f^{bcd} m_i^b \partial_\nu m_j^c \partial_\mu m_j^d$$

- **Abelian-Higgs Multiplets:** diagonal $\text{U}(1)^{N-1}$ field $C_{i,\mu}$ and off-diagonal complex scalars $\phi_{ij} = \rho_{ij} + i\sigma_{ij}$, $|\phi_{ij}|^2 = |\rho_{ij}|^2 + |\sigma_{ij}|^2 > 0$.

Y. M. Cho, PRD 21, 1080 (1980); L. Faddeev & A. J. Niemi, PRL 82, 1624 (1999); PLB 449, 214 (1999) et al.

Chromomagnetic Charges & CFN Vortex Tubes

Topological Charge Matching

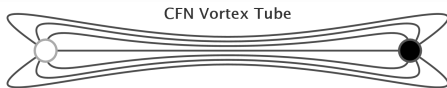
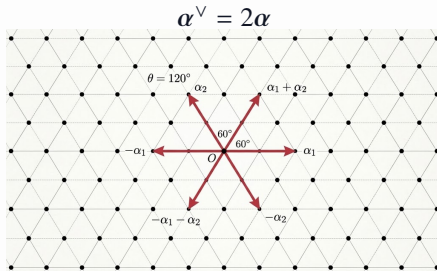
- Monopole field (H_μ^a):

$$g_m = \oint_\Sigma \mathbf{F}^H \cdot d\boldsymbol{\sigma} = 2\pi\xi_\alpha \alpha^\vee$$

- Abelian-Higgs Multiplets:

$$\alpha^{(ij)} \cdot \Phi_{\text{CFN}} = 2\pi n, \quad \implies \quad \Phi_{\text{CFN}} = \pi \zeta_\alpha \alpha^{\vee(ij)}$$

chromomagnetic charges and CFN flux tubes obey matching co-root lattice conditions $\zeta_\alpha = 2\xi_\alpha$. Monopoles act as physical end-points confining $M\bar{M}$ pairs.



CFN Vortex Tube Terminated by $M\bar{M}$ Monopoles

Fractional Center Vortex

Fractional Flux Quantization

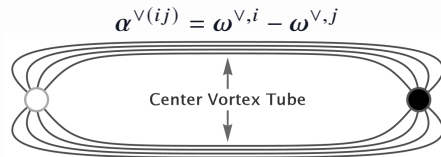
$$\mathbb{Z}_N \text{ Vacuum Symmetry} \longrightarrow \omega^i \cdot \Phi_{\sigma_i} = 2\pi \left(n_i - \frac{n}{N} \right) \longrightarrow \Phi_{\sigma_i} = 2\pi \omega^{\vee, i}$$

An intact CFN root vortex fractionates into a pair of center vortices with fundamental co-weight fluxes.

Energy Minimization in SU(3)

$$\frac{E_{\sigma}}{E_{\text{CFN}}} \approx \frac{|\Phi_{\sigma_i}|^2 + |\Phi_{\sigma_j}|^2}{|\Phi_{\text{CFN}}|^2} = \frac{N-1}{N} = \frac{2}{3} < 1$$

Fractionated center-vortex pairs are energetically favored over intact CFN vortices.



Fractionation into Center Vortices

Center Vortices Network & Confinement

Center Vortex

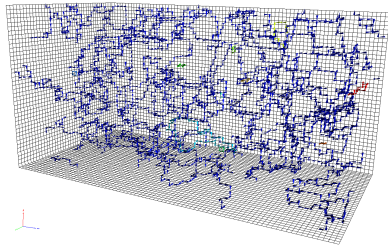
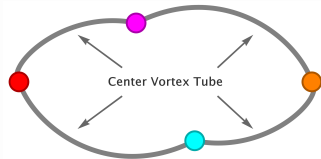
$$z_k = e^{i\frac{2\pi k}{N}} \in \mathbb{Z}_N \subset \text{SU}(N)$$

Wilson Loop

$$W(C) \longrightarrow z_k W(C)$$

Random center vortex piercings induce a statistical phase factor, naturally yielding the **Area Law**:

$$\langle W(C) \rangle \sim \exp [-\sigma \cdot \text{Area}(C)]$$

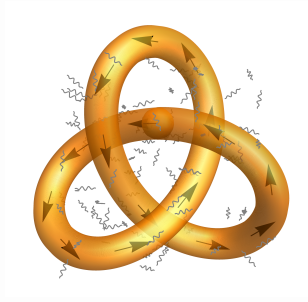
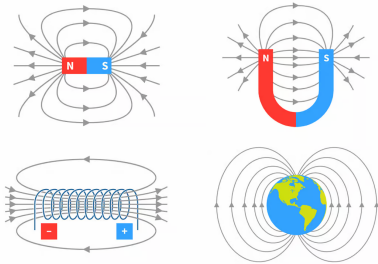


G. 't Hooft, Nucl. Phys. B 190, 455 (1981); J. Greensite, Prog. Part. Nucl. Phys. 51, 1 (2003) et al.

Topological Gluon Knots

Topological Stabilization of the Chromomagnetic Network

Non-trivial knot topology provides the essential helicity and topological charge required to dynamically stabilize chromomagnetic monopoles and the vortex network.



M. Ünsal & L. G. Yaffe, PRD 78, 065035 (2008); Y. Hayashi, Y. Tanizaki, M. Ünsal, PRL 133, 171902 (2024); JHEP 02, 126 (2026) et al.

Faddeev–Niemi Theory

Effective Infrared Action

$$\mathcal{L}_{\text{YM}} = -\frac{1}{2g^2} \text{Tr}(G_{\mu\nu}G^{\mu\nu}) \xrightarrow{\text{Infrared}} \mathcal{L}_{\text{FN}} = (\partial_\mu m_i^a)^2 + \frac{1}{e_i^2} (f^{bcd} m_i^b \partial_\nu m_j^c \partial_\mu m_j^d)^2$$

Integrating out massive off-diagonal gluons yields an IR effective Lagrangian governed by local Cartan unit vectors $m_i^a \in \text{SU}(N)/\text{U}(1)^{N-1}$.

Gluon Knots as Stable Hopf Solitons

$$\mathcal{H} = \sum_{i=1}^{N-1} \mathcal{H}_{(ii)} = \frac{1}{4\pi^2} \sum_i \int_{\mathbb{S}^3} A_i^H \wedge F_i^H \quad \longleftrightarrow \quad \pi_3 \left(\text{SU}(N)/\text{U}(1)^{N-1} \right) \cong \mathbb{Z}$$

Topological Stability & Mass Gap & Glueball

The Hopf invariant $\mathcal{H}_{(ii)}$ guarantees stability via the Vakulenko-Kapitanski bound ($E \geq c |\mathcal{H}_{(ii)}|^{3/4}$), establishing **gluon knots as stable IR degrees of freedom (Glueball), also explaining the Yang–Mills mass gap.**

L. Faddeev & A. J. Niemi, PRL 82, 1624 (1999); Nature 387, 58 (1997). A.F. Vakulenko & L.V. Kapitanski, Dokl. Akad. Nauk USSR 246, 840 (1979) et al.

Baryon Structure

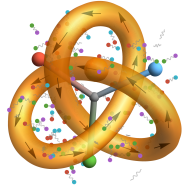
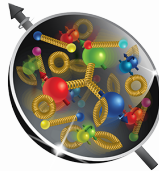
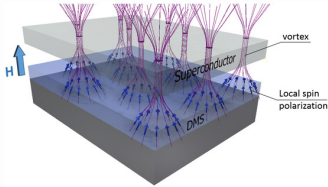
Mass Generation, Spin Currents, and Hadronic Spectra

Baryon Structure & Dual Ginzburg–Landau Theory

Dual Superconductivity Formulation

$$\mathcal{L}_{\text{GL}} = -\frac{1}{4}M_{\mu\nu}^2 + \sum_{\alpha} \left[|D_{\mu}\chi_{\alpha}|^2 - V(|\chi_{\alpha}|^2) \right]$$

Lattice QCD confirms a weak Type-II dual superconductor. Chromomagnetic monopole condensation squeezes chromoelectric fields into a **Y-shaped flux tube junction** centered at the knot (**Abelian / Monopole dominance**).



S. Maedan & T. Suzuki, Prog. Theor. Phys. 81, 229 (1989); A. D' Alessandro & M. D' Elia, Nucl. Phys. B 164, 236 (2007); N. Cardoso et al., arXiv:1004.0166; STAR Collaboration, Science 393 (2026) 6812, ads5962. et al.

Baryon Mass

Gluon Condensate & Trace Anomaly

$$T^\mu{}_\mu = \frac{\beta(g)}{2g} G_{\mu\nu}^a G^{\mu\nu,a} + \sum_f m_f (1 + \gamma_m) \bar{\psi}_f \psi_f$$

The gluon knot condensate directly provides the dominant $\sim 40\%$ trace anomaly contribution to nucleon mass (**Yang–Mills Mass Gap**).

Magnetic Catalysis & Dynamical Mass

$$\langle 0 | \bar{\psi} \psi | 0 \rangle \sim |g \mathbf{B}|$$

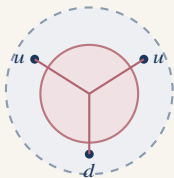
Concentrated chromomagnetic fields inside the gluon core catalyze chiral symmetry breaking (χ SB), generating constituent quark masses.

R. J. Crewther, PRL 28, 1421 (1972); Y.-B. Yang et al., PRL 121, 212001 (2018); V. P. Gusynin et al., PRL 73, 3499 (1994); PRD 60, 025003 (1999); et al.

Mass and Charge Radii

Experimental Estimation of Proton Radii

$$r_{\text{charge}} \approx 0.84 \text{ fm}$$



$$r_{\text{mass}} \approx 0.55 \text{ fm}$$

Baryon Structure



Temperature of Phase Transition

Peripheral chiral cloud melts first ($T_\chi \sim 130 \text{ MeV}$); dense knot core dissociates at higher $T_d \sim 170, \text{ MeV}$, naturally explaining the hierarchy $T_\chi \leq T_d$.

D. Kharzeev, PRD 104, 054015 (2021); V. D. Burkert et al., Nature 557, 396 (2018); Ding H T, Hegde P, Kaczmarek O, et al. PRL, 2019, 123(6): 062002. Hanada, Masanori, et al. PTEP 2024.4 (2024): 041B02.et al.

Vacuum Topology & Axial Anomaly

Homotopy Isomorphism & Hopf-Winding Relation in SU(3)

$$\pi_3 \left(\text{SU}(N)/\text{U}(1)^{N-1} \right) \cong \pi_3(\text{SU}(N)) \cong \mathbb{Z}$$

$$\frac{1}{16\pi^2} \sum_{i=1}^{N-1} \int A_i^H \wedge F_i^H = \frac{1}{4} \sum_{i=1}^{N-1} \mathcal{H}_{(ii)} = \frac{\mathcal{H}}{4} \equiv N(g) = \frac{1}{24\pi^2} \int_{\mathbb{S}^3} \epsilon^{ijk} \text{Tr} \left(g \partial_i g^\dagger g \partial_j g^\dagger g \partial_k g^\dagger \right)$$

Opposite component charges yield $\mathcal{H} = \mathcal{H}_{(11)} + \mathcal{H}_{(22)} = 0$, selecting it as the topologically trivial ground state of nucleons.

Axial Anomaly & Instanton Overlap & Quark Zero Modes (Banks–Casher Relation)

$$\partial_\mu j^{5,\mu} = 2im\bar{\psi}\gamma^5\psi + \frac{g^2 N_f}{8\pi^2} \text{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) \quad \implies \quad \langle \bar{\psi}\psi \rangle = -\frac{\pi}{V} \text{sign}(m) \overline{\nu(0)}$$

Fluctuations in $\mathcal{H}$ drive vacuum tunneling, inducing Dirac zero modes and dynamical χ SB.

S. L. Adler, PR 177, 2426 (1969); J. S. Bell & R. Jackiw, Nuovo Cim. A 60, 47 (1969); T. Banks & A. Casher, NPB 169, 103 (1980) et al.

Topological Current of Gluon Spin

Jaffe–Manohar Spin Decomposition

$$\mathbf{J}_{\text{QCD}} = \int d^3x \left[\psi_f^\dagger \frac{\boldsymbol{\Sigma}}{2} \psi_f + \psi_f^\dagger (\mathbf{x} \times -i\boldsymbol{\nabla}) \psi_f + \mathbf{E}_a \times \mathbf{A}_a + E_a^i (\mathbf{x} \times \boldsymbol{\nabla}) A_a^i \right]$$

Topological charges of the gluon knot drive a spatial current $\mathbf{H} \propto \mathbf{E}_i^H \times \mathbf{A}_i^H$, mapping directly into the gluon spin operator.

Ji Decomposition & Poynting Vector

$$\mathbf{J}_{\text{QCD}} = \int d^3x \left[\psi_f^\dagger \frac{\boldsymbol{\Sigma}}{2} \psi_f + \psi_f^\dagger (\mathbf{x} \times -i\mathbf{D}) \psi_f + \mathbf{x} \times (\mathbf{E} \times \mathbf{B}) \right]$$

Time-varying chromomagnetic fields inside the knot induce orthogonal chromoelectric fields, generating Poynting vector angular momentum.

R. L. Jaffe & A. Manohar, Nucl. Phys. B 337, 509 (1990); X. Ji, Phys. Rev. Lett. 78, 610 (1997) et al.

Gluon Knots in Glueball and Mesonic Spectra

Light Scalar Meson $f_0(500)$ (σ)

Identified as the fundamental gluon knot, setting the baseline non-perturbative scale for chiral symmetry breaking and trace anomaly.

Heavy Quarkonia (J/ψ , Υ)

Heavy $Q\bar{Q}$ states form chromoelectric flux tubes tightly bound and wrapped by gluon knots.

Glueball Classification

Glueball spectrum is systematically classified by distinct knot topological invariants and linking numbers.

Vector vs Pseudoscalar Mesons

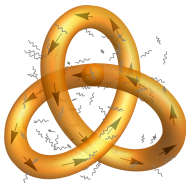
$m_\sigma + m_\pi \approx m_\rho$ supports transient knot components in vector mesons (ρ, ω), **whereas pseudoscalars (π, K) is very light.**

G. E. Brown & M. Rho, PRL 66, 2720 (1991); Y.-L. Ma & M. Rho, PPNP 113, 103791 (2020); Y.-L. Li et al., PRD 95, 114011 (2017) et al.

Conclusions

Toward a Unified Picture of Confinement and Baryon Structure

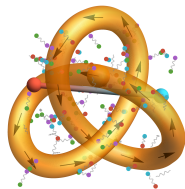
- Gluon knots (glueballs) emerge as fundamental non-perturbative degrees of freedom in the IR Yang–Mills theory, providing a unified picture of confinement, Abelian / Monopole dominance, and the mass gap.
- The framework naturally unifies quark confinement through the dual Meissner effect and Y-junction flux tubes, chiral symmetry breaking through magnetic catalysis and Dirac zero modes, and baryon mass and spin through the trace anomaly and gluon Poynting angular momentum.



Glueball



Baryon



Heavy Meson

Thank You for Your Attention!