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Leading-order dynamical QED corrections to thermodynamics of isospin-asymmetric QCD

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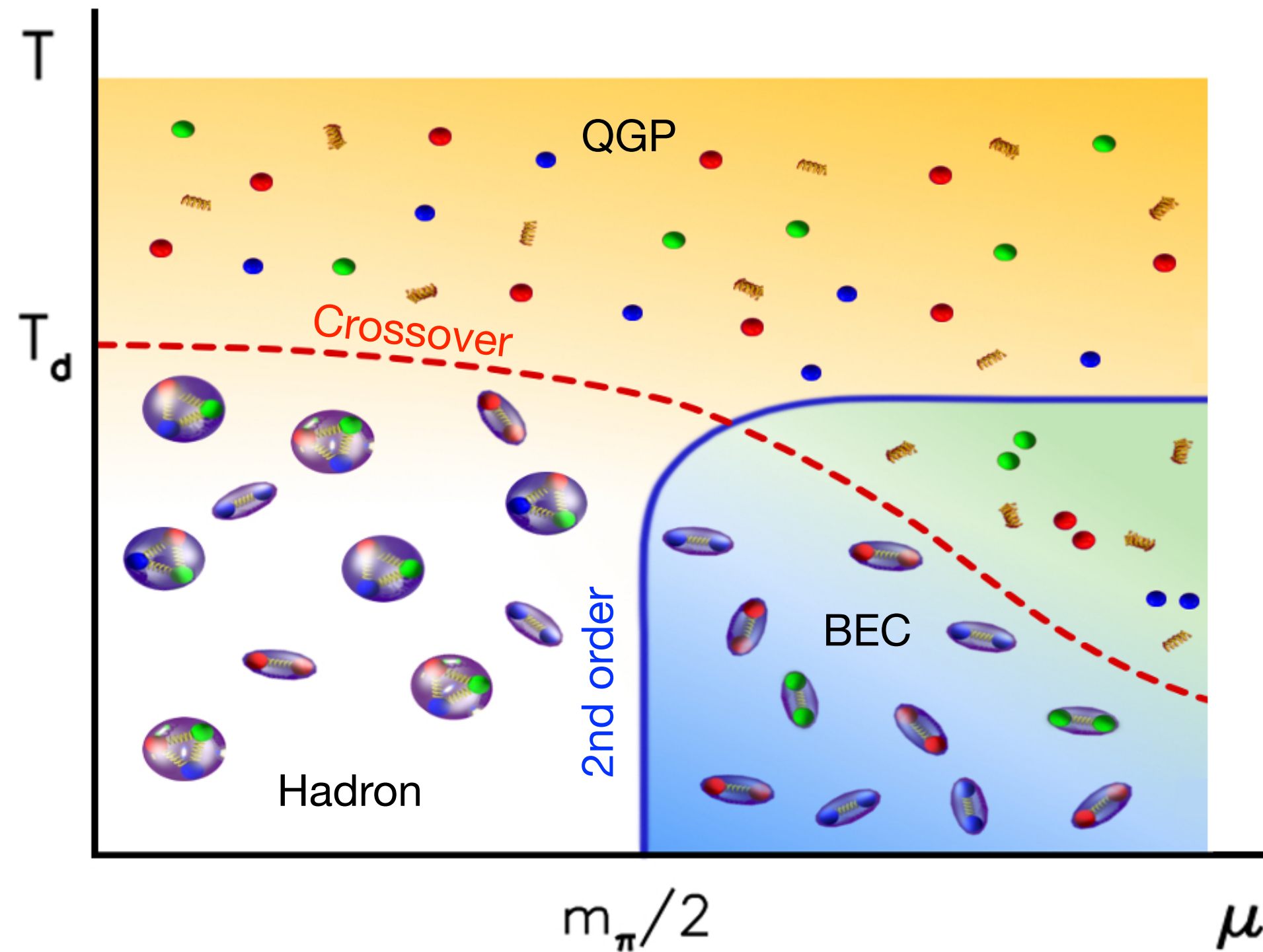
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in collaboration with Gergely Endrődi, Ádám Fischer and Gergely Markó

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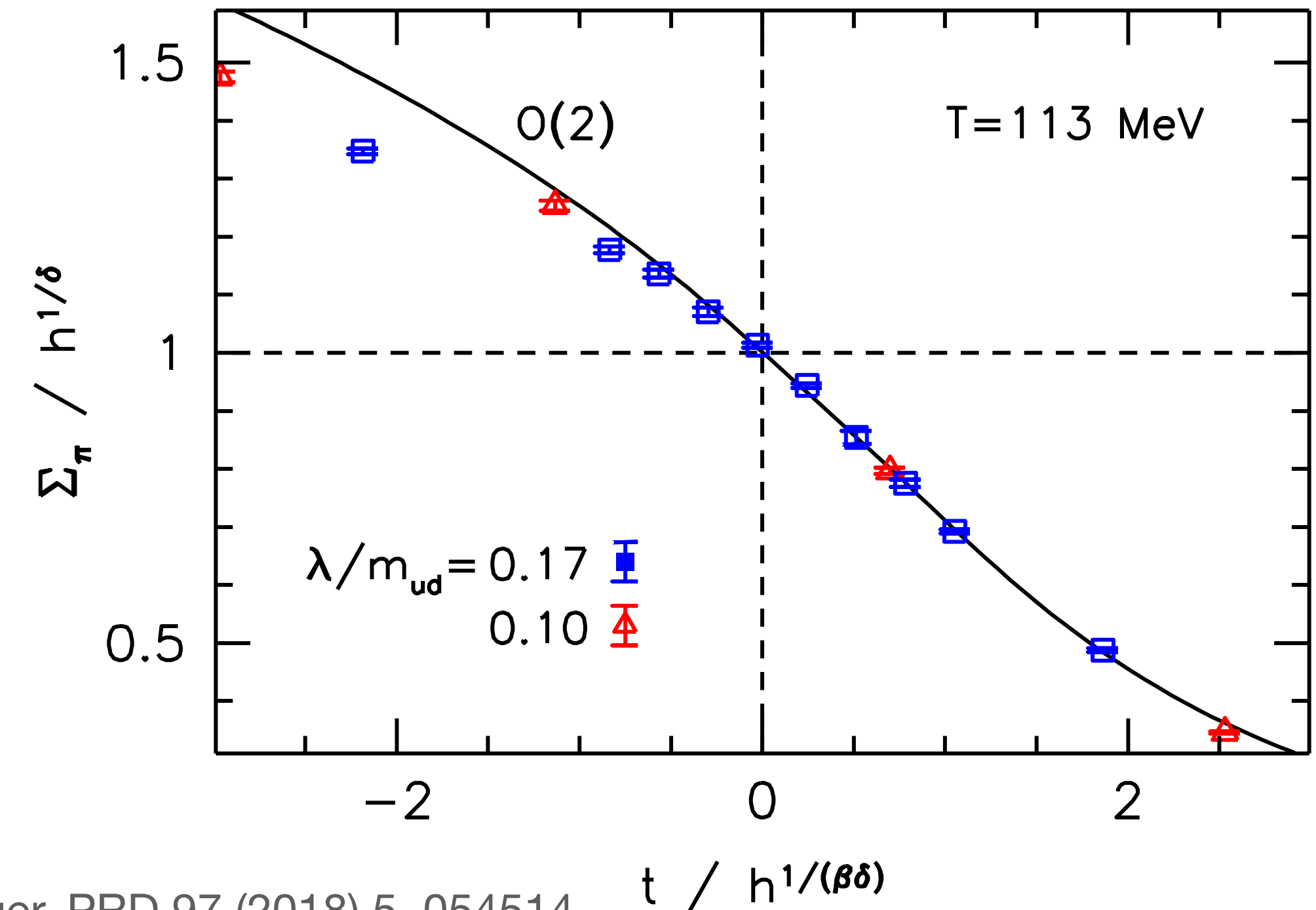
QCD phase transition at nonzero isospin

QCD phase diagram in $T - \mu_I$ plane



B. Brandt, G. Endrodi, S. Schmalzbauer, PRD 97 (2018) 5, 054514

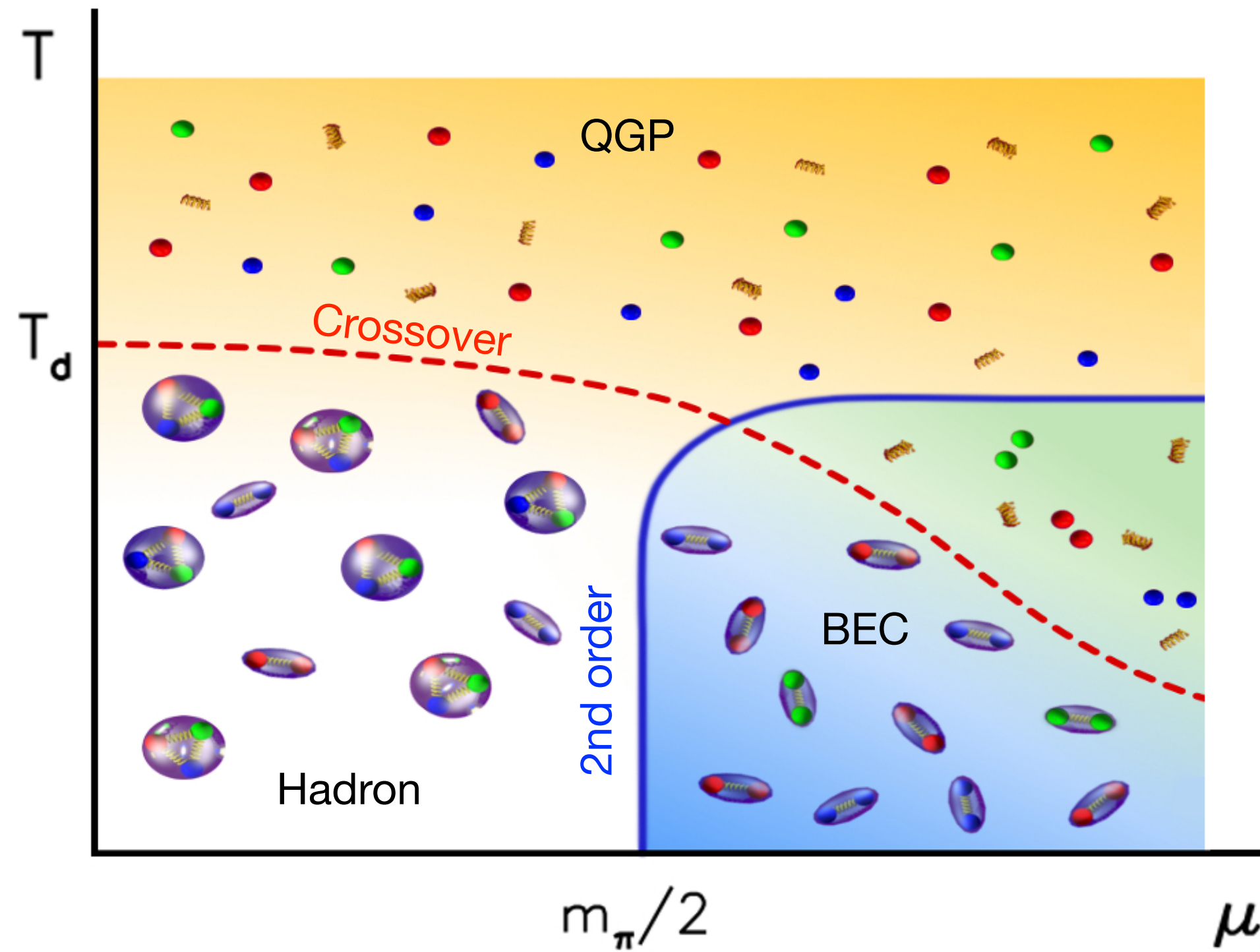
$O(2)$ scaling of order parameter



- Sign-problem-free simulations at $\mu_I \neq 0$ + Pionic λ driving spontaneous breaking at finite V
- Second order $O(2)$ transition at onset of charged-pion BEC

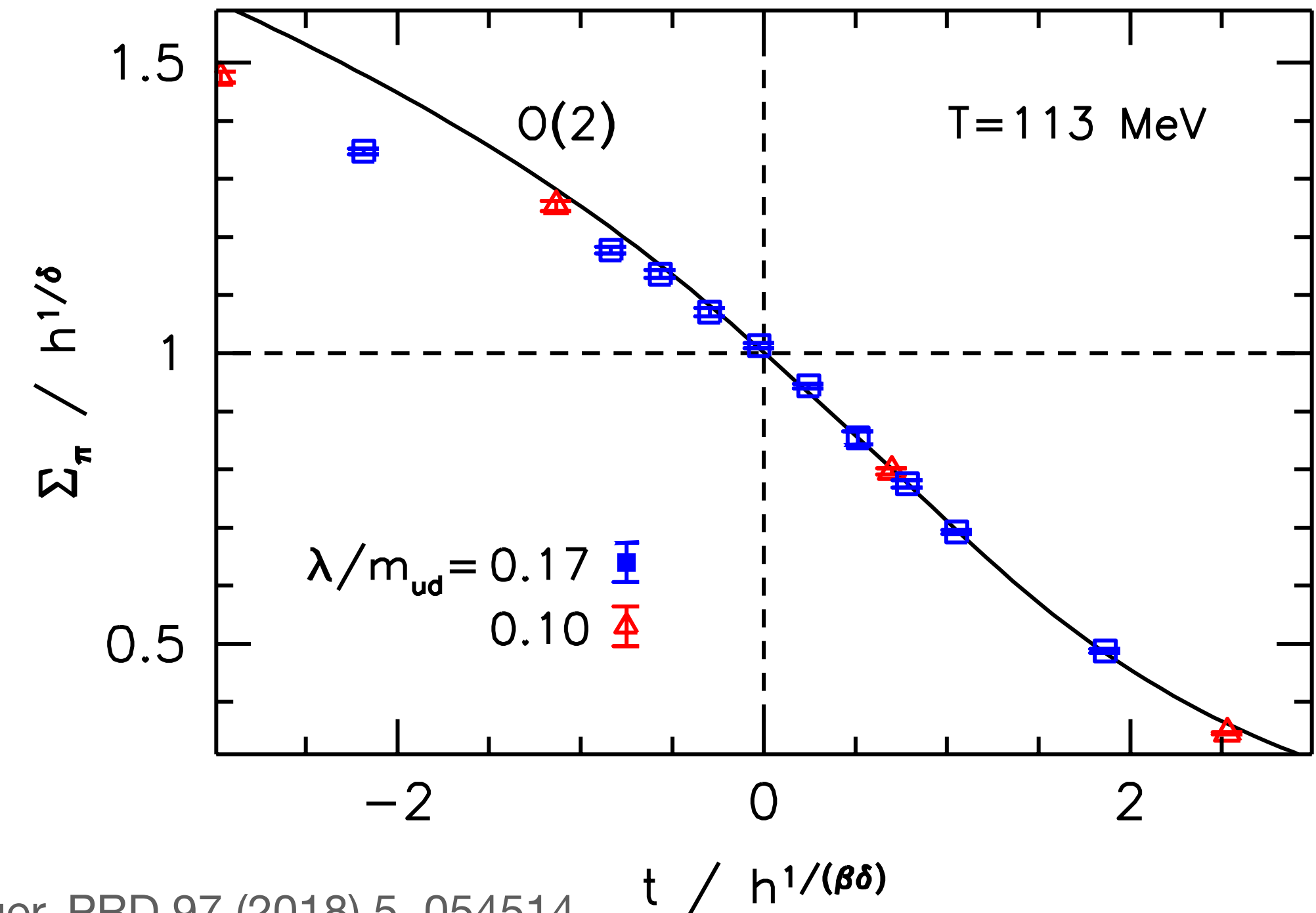
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$O(2)$ scaling of order parameter



What if dynamical photons are included in isospin-asymmetric QCD?

1st order transition driven by dynamic photons

- Charged pion $\langle \pi^\pm \rangle$ coupled to dynamic photons :

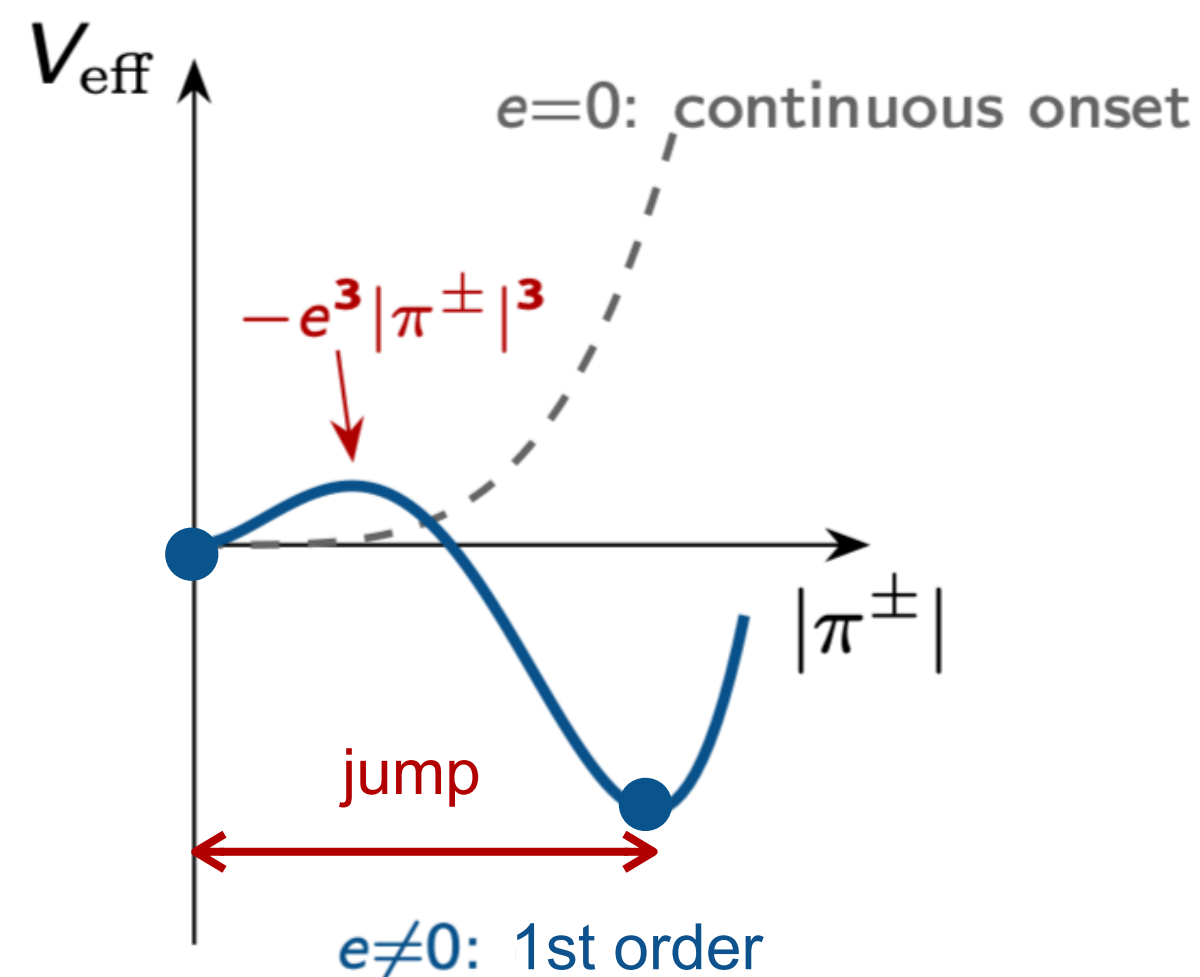
$$F[\pi^\pm, A^{EM}] \sim |D_\mu \pi^\pm|^2, \text{ with } D_\mu = \partial_\mu - ieA_\mu^{EM}$$

- Cubic term appears after gauge fluctuations integrated :

Halperin, Lubensky, Ma, PRL 32 (1974) 292

$$V_{\text{eff}}(\pi^\pm) \supset -e^3 |\pi^\pm|^3$$

I. Giannakis, et.al., PRL 93 (2004) 232301

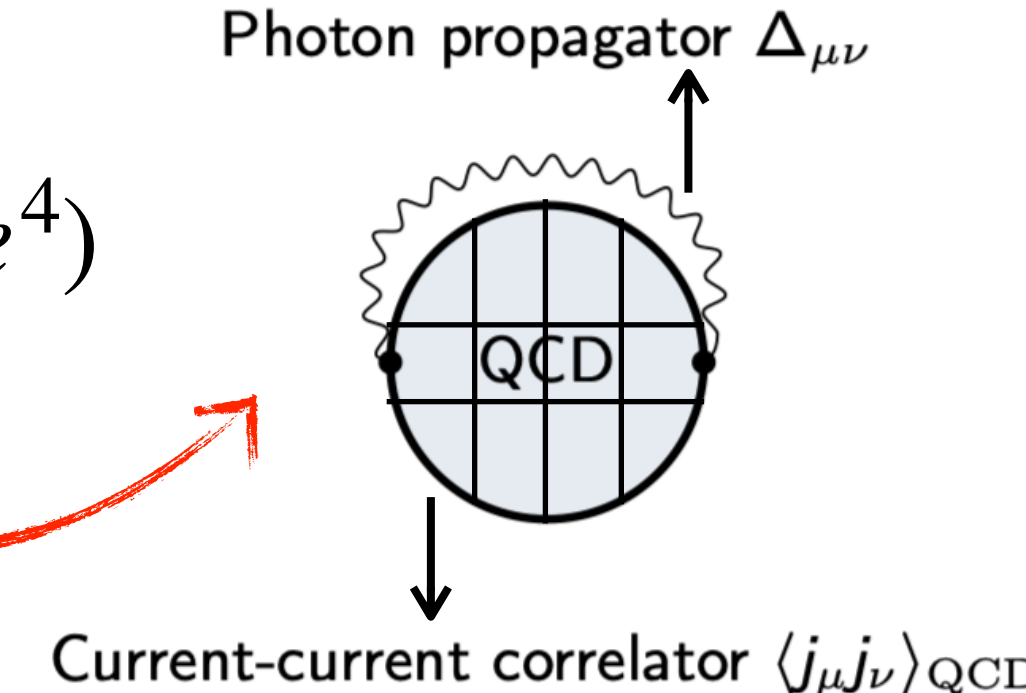


- $e=0$: single minimum slides off zero continuously, no barrier $\Rightarrow$ 2nd order
- $e \neq 0$: barrier and rival minimum appear $\Rightarrow$ 1st order

Strategy: expansion in e around QCD at $\mu_I \neq 0$

☹️ Complex action for QCD+QED at $\mu_I \neq 0$: unequal charges $q_u \neq q_d$ break $u \leftrightarrow d$ pairing

😊 Taylor expand the free-energy density: $f(\mu_I, e) = f(\mu_I, 0) + \frac{e^2}{2} \underbrace{\left. \frac{\partial^2 f}{\partial e^2} \right|_{e=0}}_{\text{(the pineapple, } \mathcal{P})} + \mathcal{O}(e^4)$



The 'pineapple': leading order dynamical QED effect, as photon propagator $\otimes$ vector current-current correlator

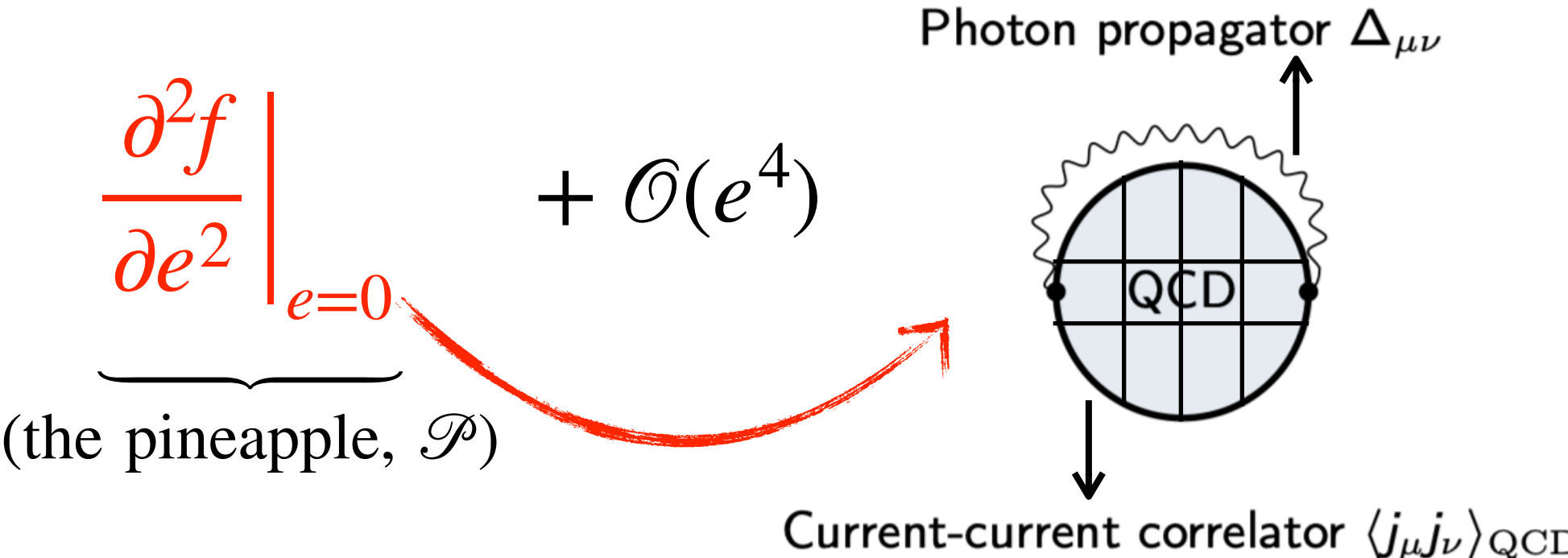
$$\text{Pineapple Diagram} = \left. \frac{\partial^2 f}{\partial e^2} \right|_{e=0} = \sum_x \Delta_{\mu\mu}(x) \langle j_\mu(x) j_\mu(0) \rangle = \frac{1}{V_4} \sum_{k \in \text{BZ} \cap \text{QED}_{L/TL}} \frac{\Pi_{\mu\mu}(k)}{\hat{k}^2}$$

(Feynman-gauge photon, lattice momentum $\hat{k}$)

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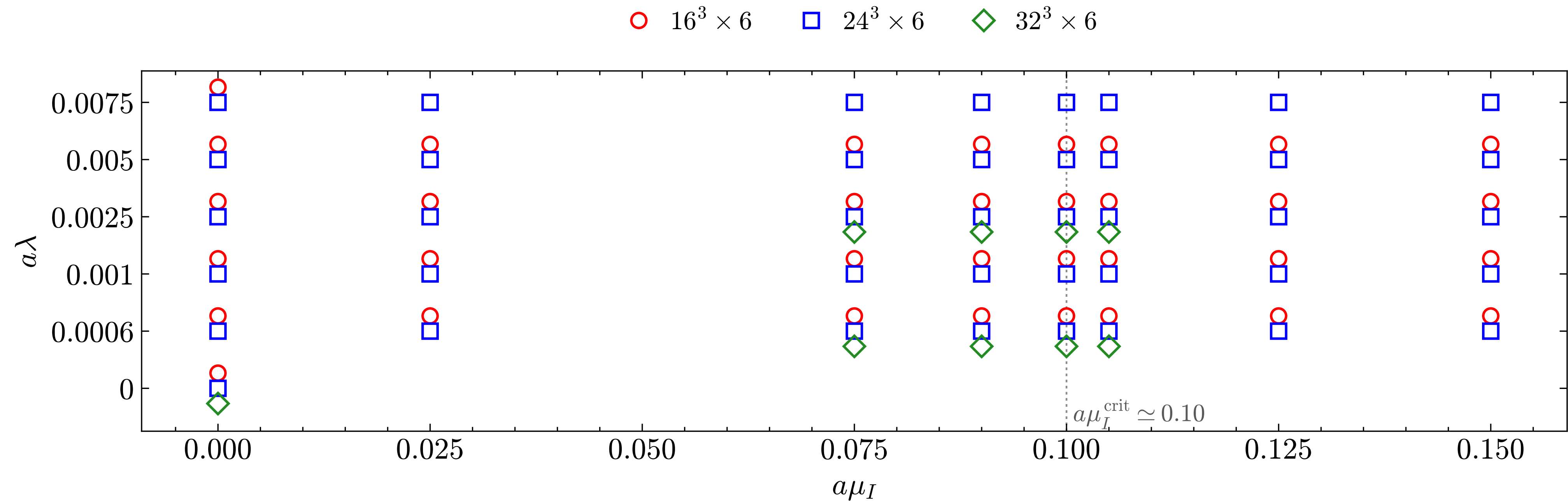
LO dynamical QED effect on thermodynamics:

Pressure
$$P(\mu_I, e) = -f(\mu_I, e) = P(\mu_I, 0) - \frac{e^2}{2} \text{ (QCD loop) } + \mathcal{O}(e^4)$$

Isospin density
$$n_I(\mu_I, e) \equiv -\frac{\partial f(\mu_I, e)}{\partial \mu_I} = n_I(\mu_I, 0) - \frac{e^2}{2} \frac{\partial \text{ (QCD loop) }}{\partial \mu_I} + \mathcal{O}(e^4)$$

Simulation details

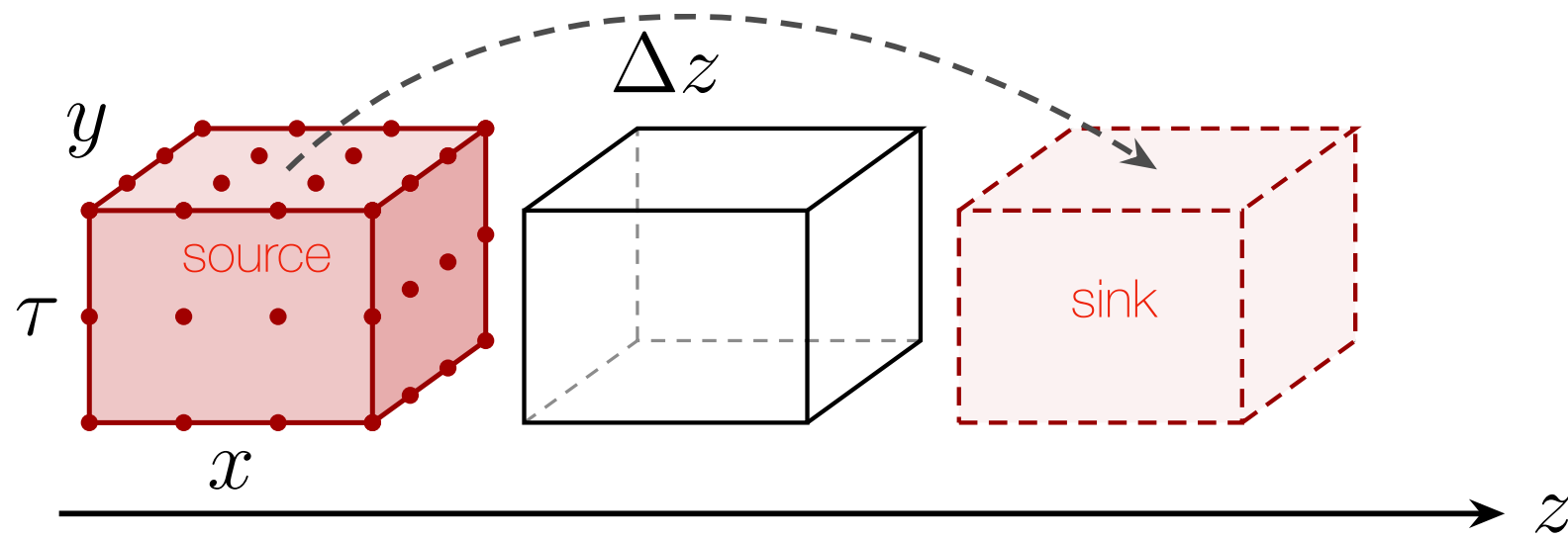
- (2+1)-f pure-QCD staggered ensembles at finite μ_I and λ :



- Fixed $\beta_g = 3.45$, $N_\tau = 6$, at physical point $m_s/m_l \approx 28.15$
- Three spatial volumes $N_s = 16, 24, 32$
- Scan along μ_I direction for each $a\lambda \in [0.0006, 0.0075]$ with $a\mu_I \in [0, 0.15]$

Simulation details

- Full (2+1)-f current-current correlator : connected + disconnected + tadpole

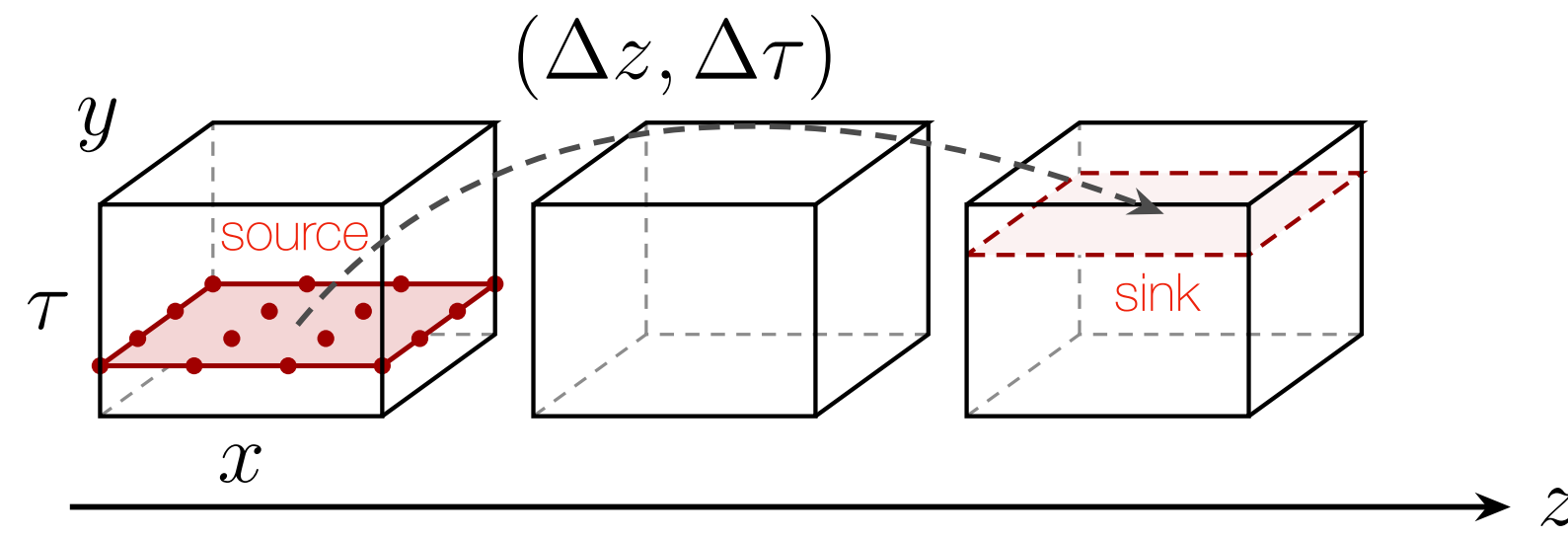


Wall source

$$C_{\mu\mu}^{1D}(\Delta z) = \sum_{x,y,\tau} C_{\mu\mu}^{4D}(x,y,\Delta z,\tau)$$

$$\Pi_{\mu\mu}(0,0,k_z,0)$$

✗ 1D projection cannot give $\mathcal{P}$

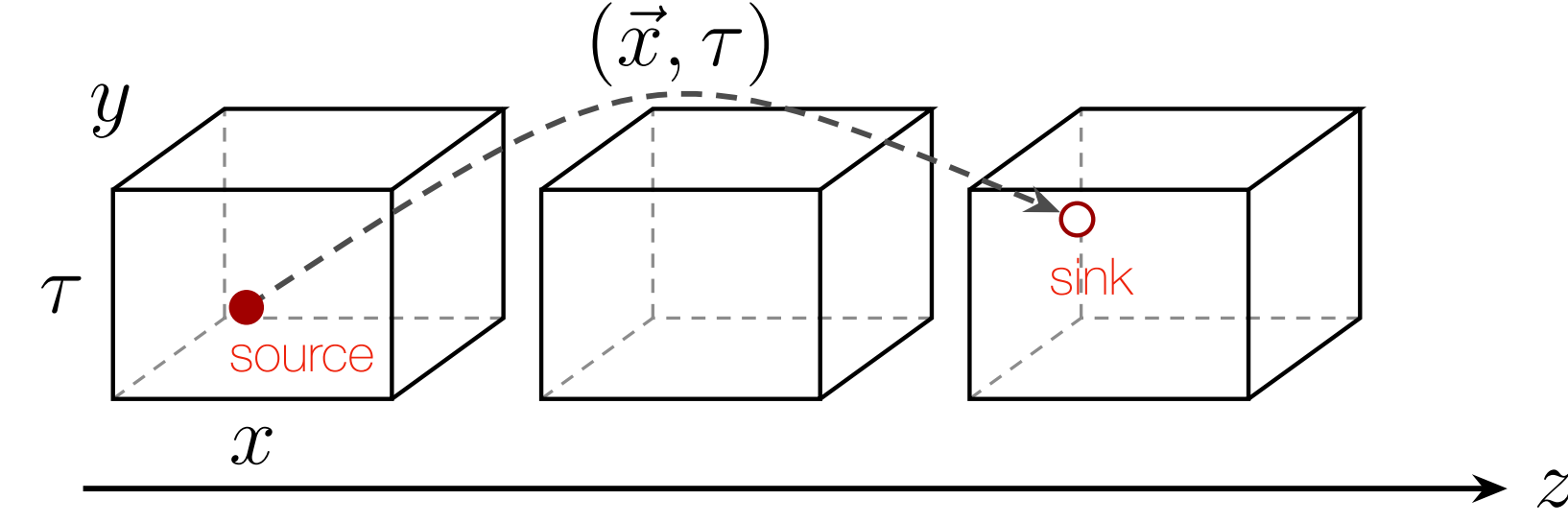


Planar source (new)

$$C_{\mu\mu}^{2D}(\Delta z, \Delta\tau) = \sum_{x,y} C_{\mu\mu}^{4D}(x,y,\Delta z, \Delta\tau)$$

$$\Pi_{\mu\mu}(0,0,k_z,k_\tau)$$

⚠ rest of the BZ from spatial symmetry
 $\Rightarrow$ model-dependent (interp. + extrap.)



Point source

$$C_{\mu\mu}^{4D}(\vec{r}, \tau) = \langle j_\mu(x,y,z,\tau) j_\mu(0) \rangle$$

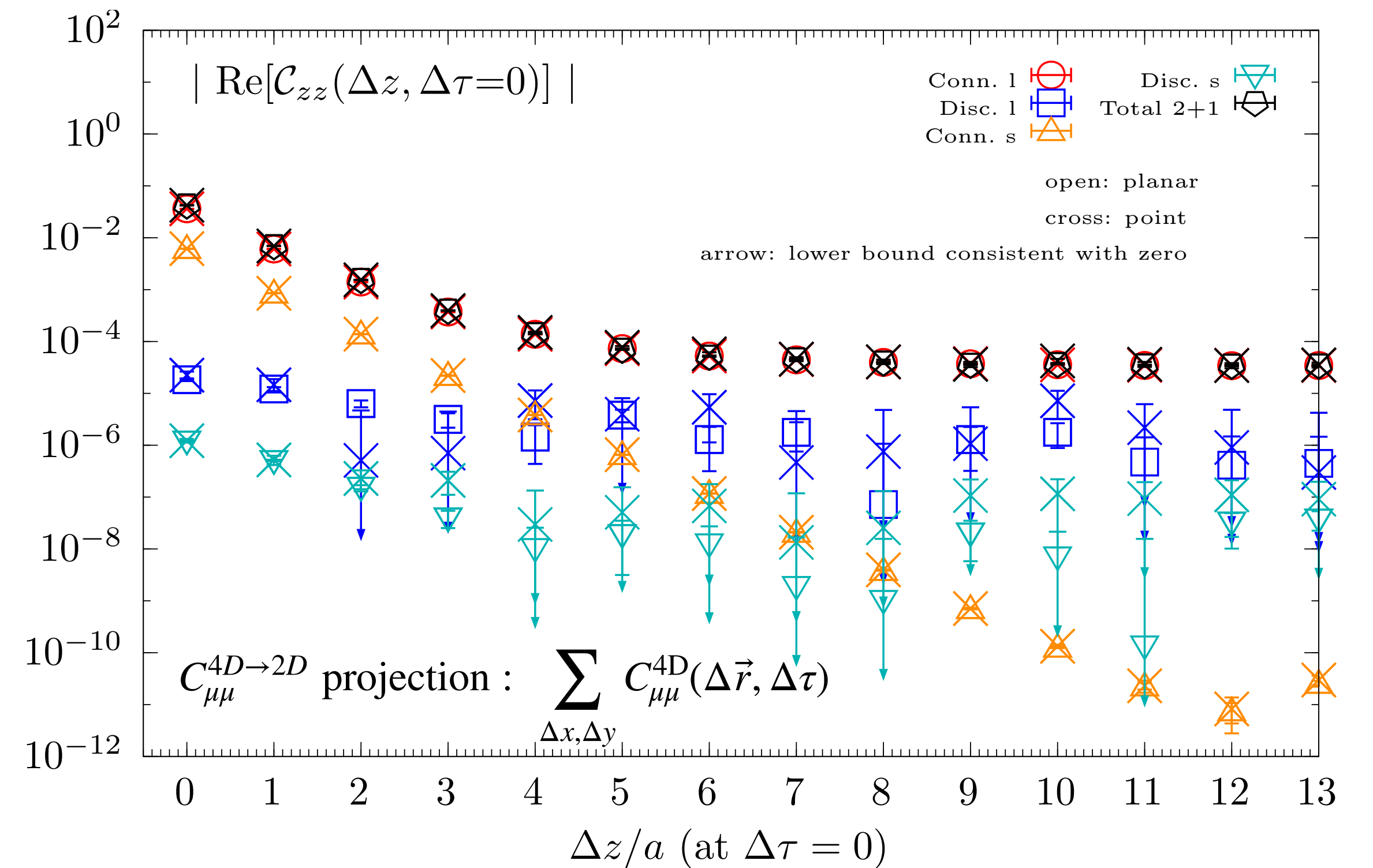
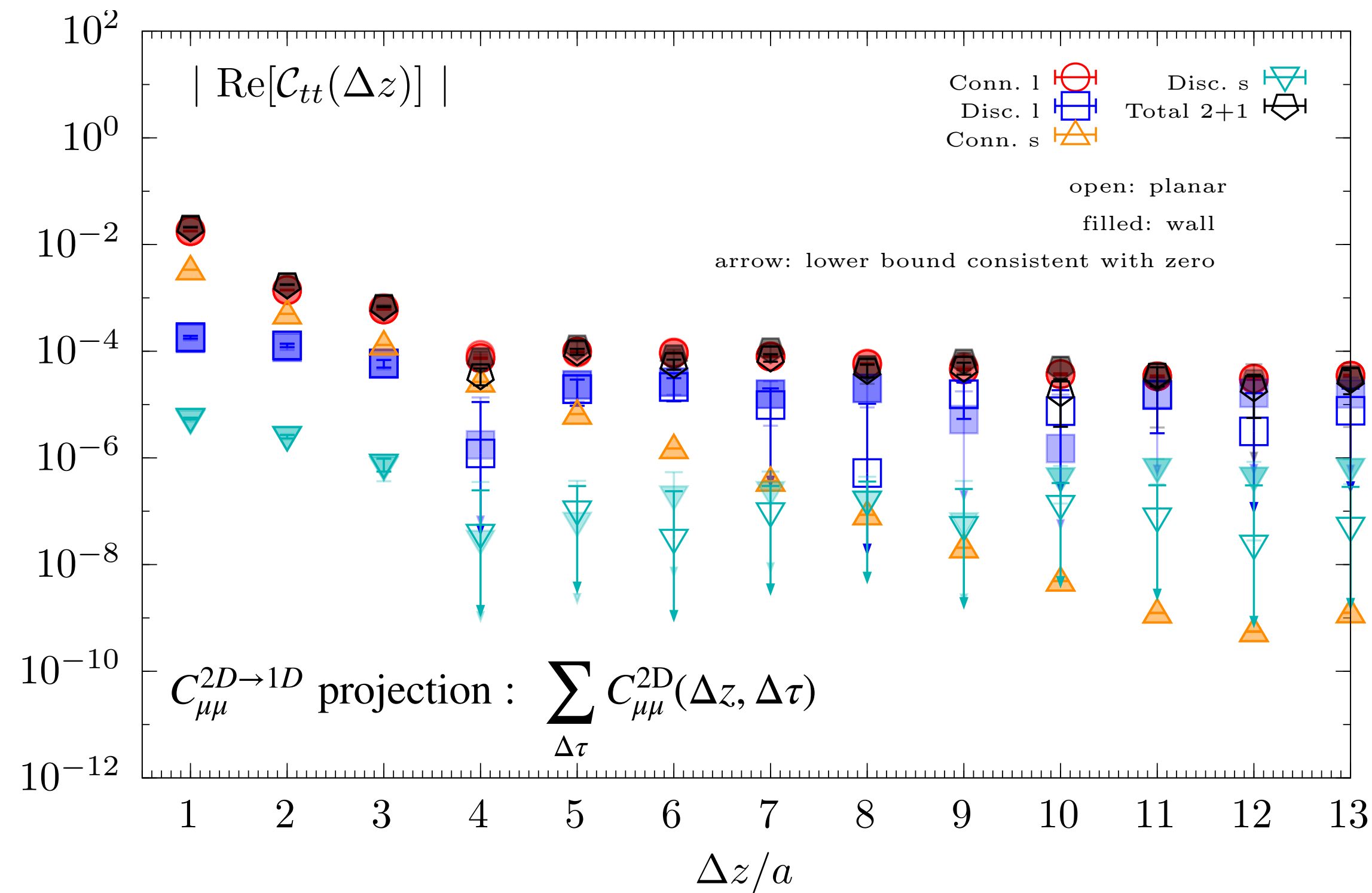
$$\Pi_{\mu\mu}(k_x, k_y, k_z, k_\tau)$$

✓ no reconstruction, with fine precision

Validation: point v.s. planar v.s. wall source

Current-current correlator $C_{\mu\mu}$ using different sources

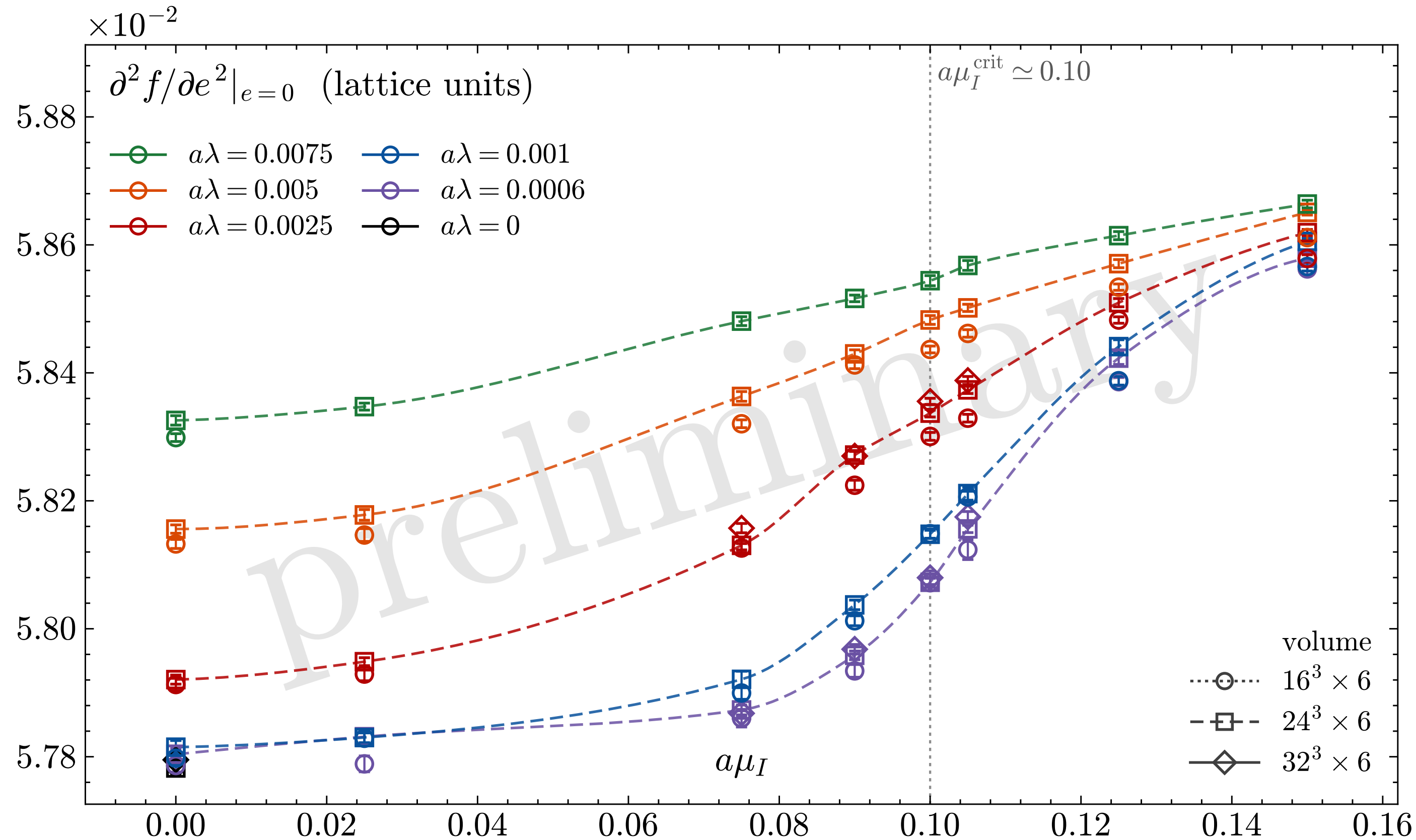
● wall-src. $C_{\mu\mu}^{1D}(\Delta z)$ ○ planar-src. $C_{\mu\mu}^{2D}(\Delta z, \Delta\tau)$ × point-src. $C_{\mu\mu}^{4D}(\Delta\vec{r}, \Delta\tau)$



$C_{\mu\mu}^{nD}$ from different sources agree channel-by-channel in 1D and 2D projections

Bare pineapple $\mathcal{P}$ across (λ, μ_I) grid

$$\text{QCD} = \frac{\partial^2 f}{\partial e^2} \Big|_{e=0} = \frac{1}{V_4} \sum_{k \in \text{BZ} \cap \text{QED}_L} \frac{\Pi_{\mu\mu}(k_1, k_2, k_3, k_4)}{\hat{k}^2} \quad \text{with} \quad \Pi_{\mu\mu}(k) = \sum_x e^{-ik \cdot x} C_{\mu\mu}^{4D}(x)$$



— ‘Bare’: $\mathcal{P} = \mathcal{P}_{\text{med}}(\mu_I, \lambda) + \mathcal{P}_{UV}(\lambda)$

— Flatter below $\mu_I^{\text{crit.}}$ as $\lambda \rightarrow 0$

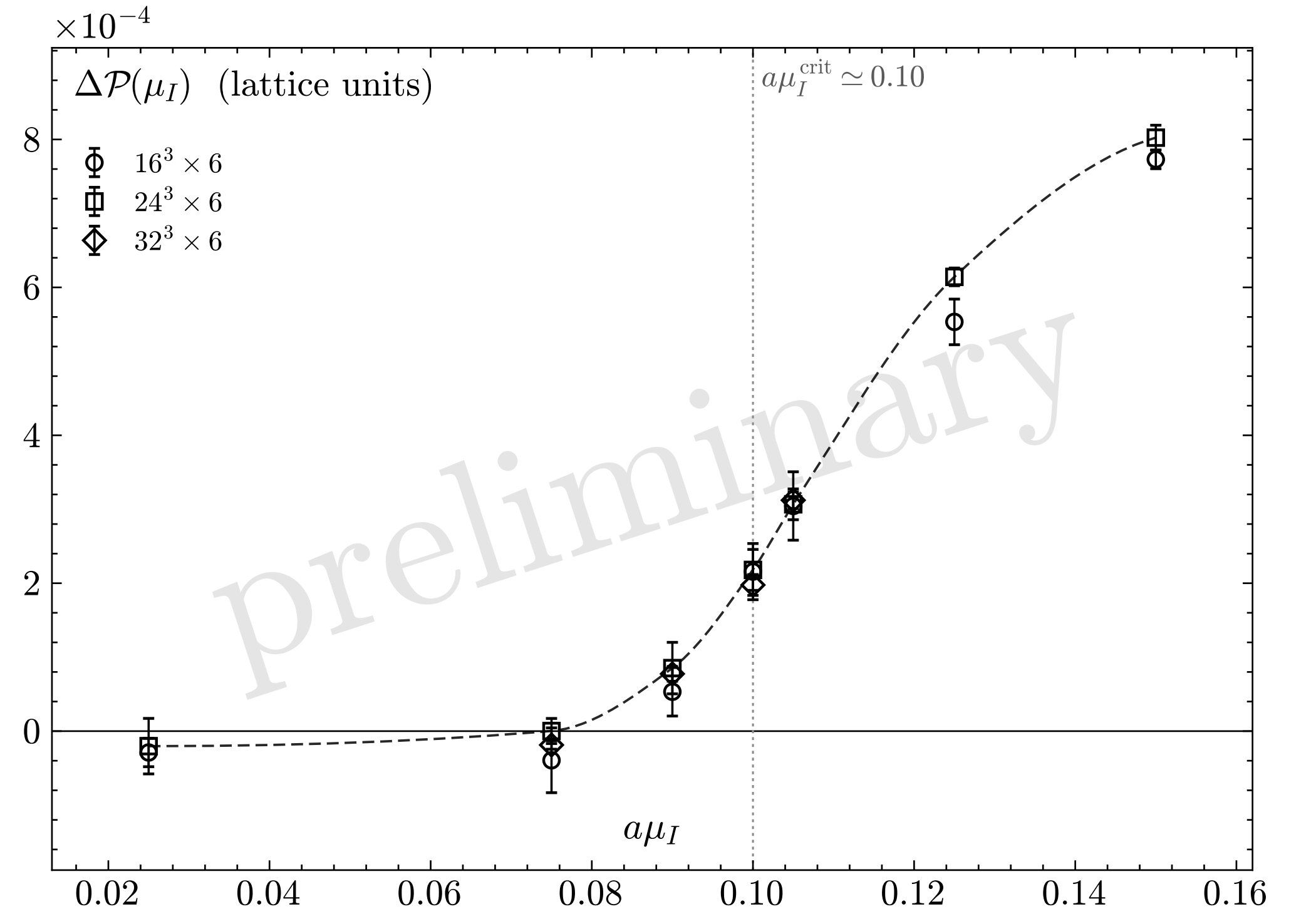
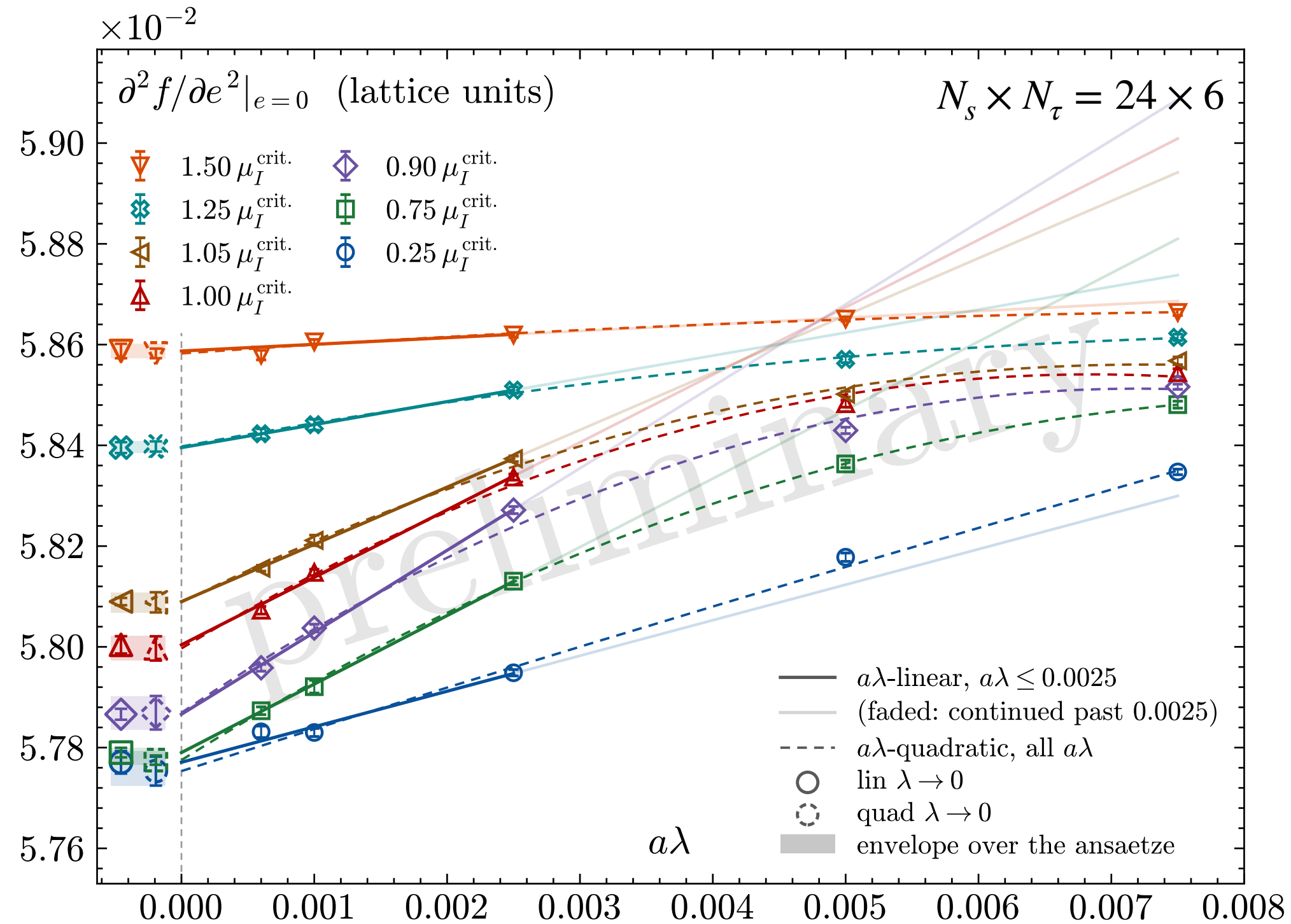
— Steeper rise across $\mu_I^{\text{crit.}}$ as $\lambda \rightarrow 0$

— Slight volume effect

Pineapple $\mathcal{P}$ after $\lambda \rightarrow 0$ extrapolation

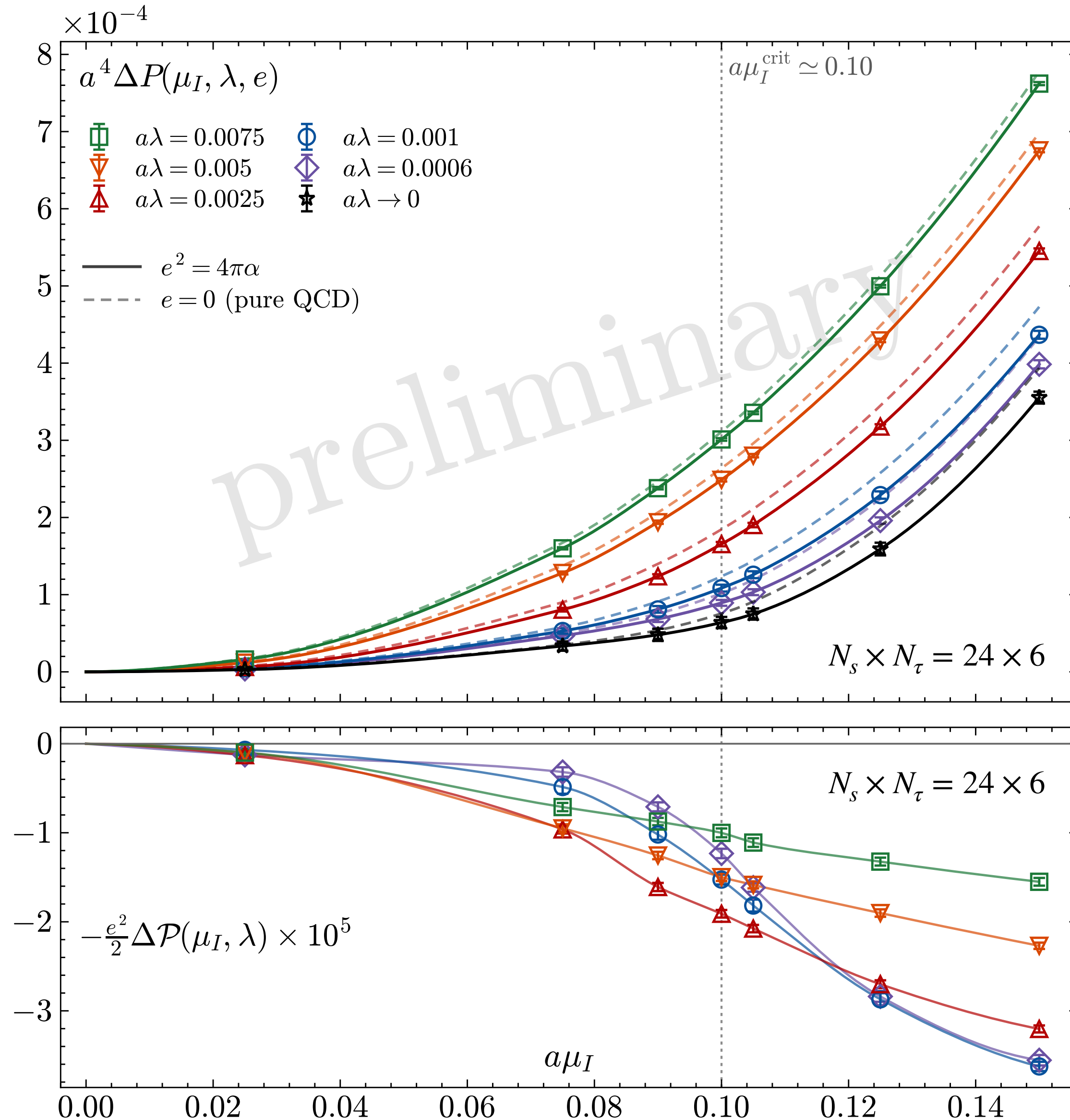
$\lambda \rightarrow 0$ extrap. via linear and quadratic ansatz

UV-subtracted $\Delta\mathcal{P}(\mu_I) \equiv \mathcal{P}(\mu_I, \lambda \rightarrow 0) - \mathcal{P}(\mu_I = 0, \lambda = 0)$



— Silver-Blaze-like plateau appears after $\lambda \rightarrow 0$ extrap., with a sharp rise across the onset

Pressure with dynamical QED correction



$$\Delta P(\mu_I, \lambda, e) = \int_0^{\mu_I} n_I(\mu'_I, \lambda) d\mu'_I - \frac{e^2}{2} \left[\mathcal{P}(\mu_I, \lambda) - \mathcal{P}(0, \lambda) \right]$$

Pure QCD pressure by integrating

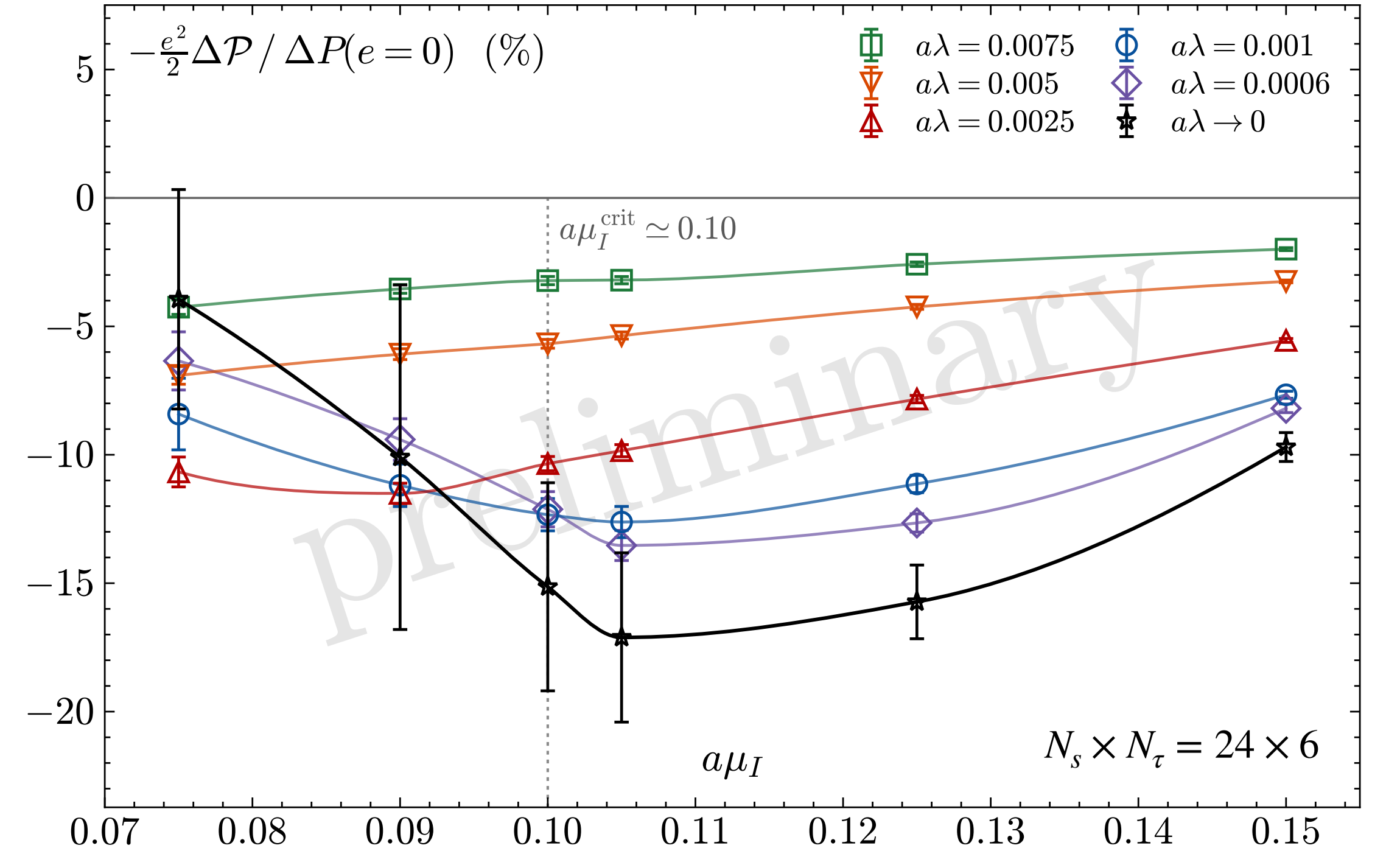
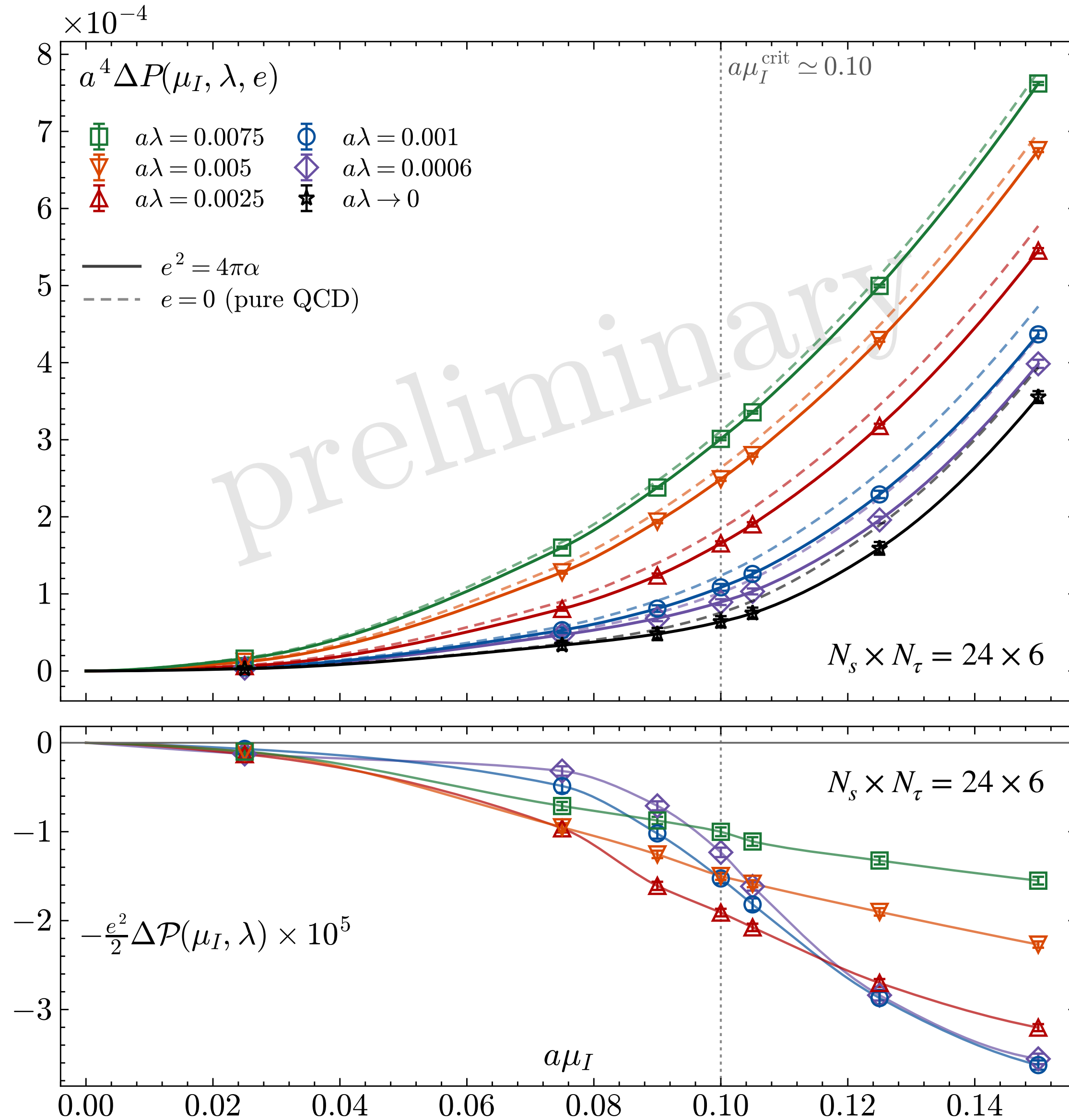
$$n_I(\mu_I, \lambda) \equiv - \frac{\partial f(\mu_I, \lambda)}{\partial \mu_I}$$

LO e expansion coefficient

$$P(\mu_I, \lambda, e) = P(\mu_I, \lambda, e = 0) - \frac{e^2}{2} \mathcal{P}(\mu_I, \lambda) + \mathcal{O}(e^4)$$

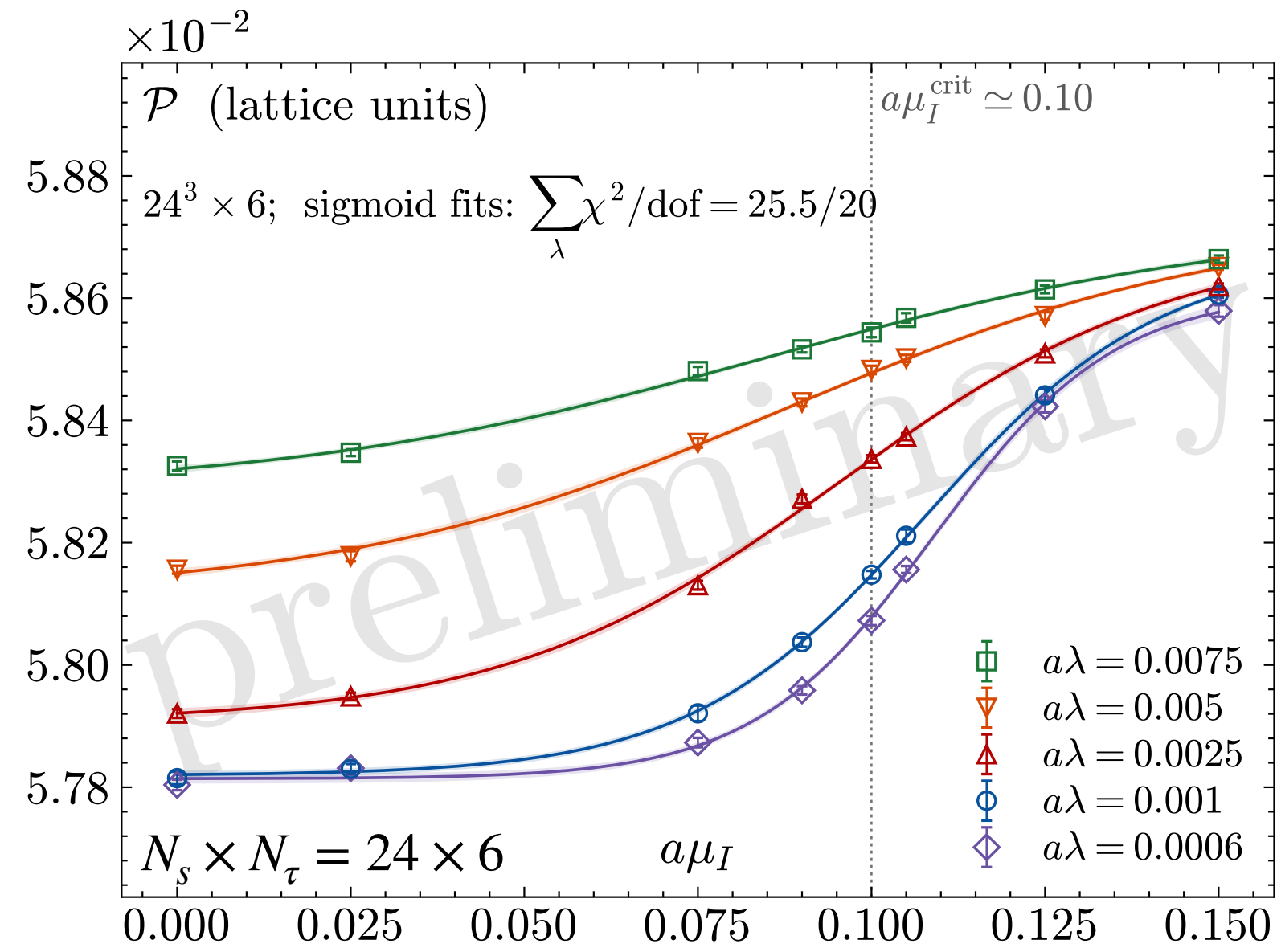
QED effect lowers the pressure at every (μ_I, λ)

Pressure with dynamical QED correction



Relative QED contribution peaks around $\mu_I^{\text{crit.}}$ as $\lambda \rightarrow 0$

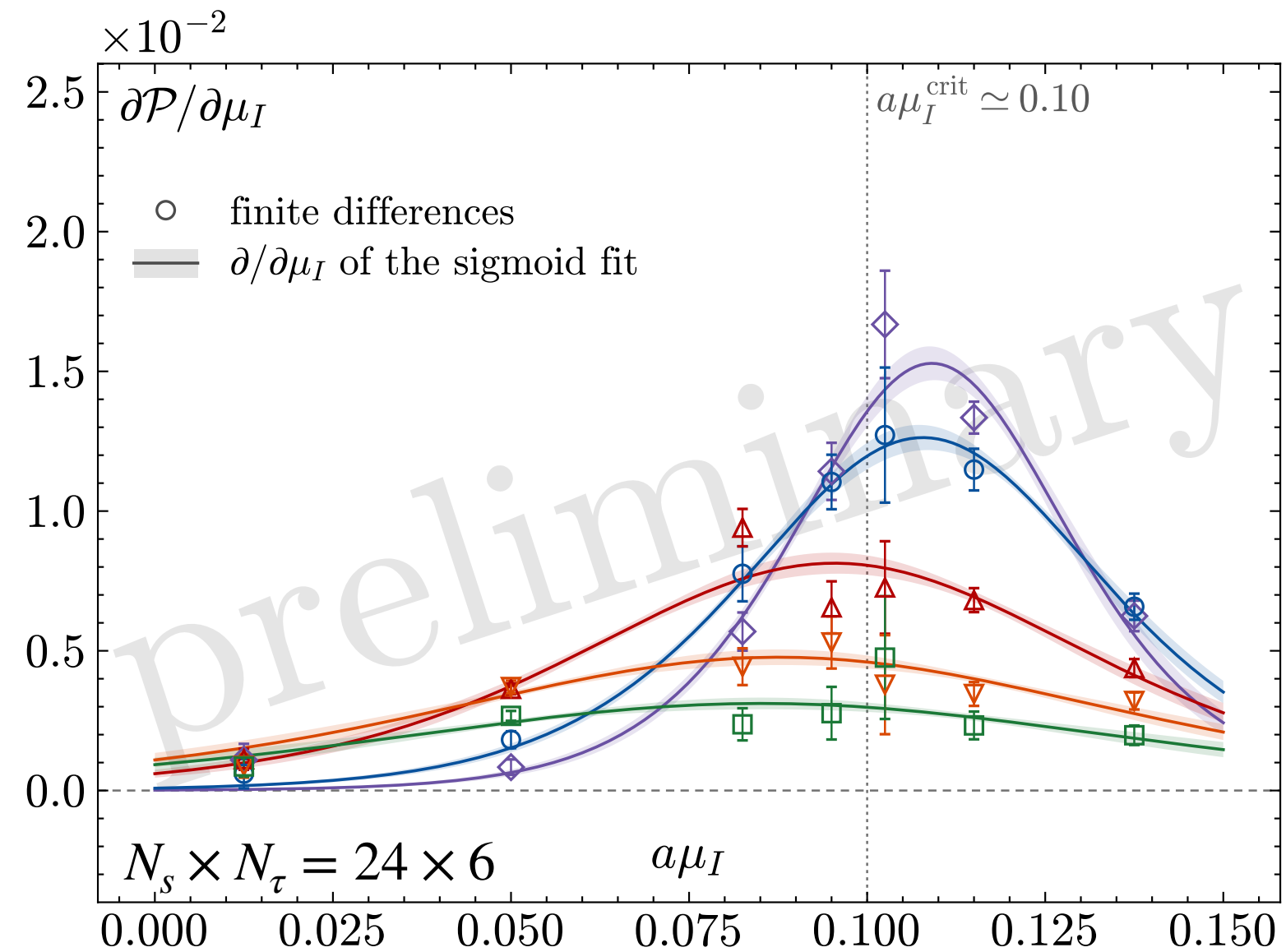
Isospin density at $\mathcal{O}(e^2)$: the pressure slope



$$n_I(\mu_I, e) \equiv -\frac{\partial f(\mu_I, e)}{\partial \mu_I} = \frac{\partial P(\mu_I, e)}{\partial \mu_I} = n_I(\mu_I, e=0) - \frac{e^2}{2} \frac{\partial \mathcal{P}(\mu_I)}{\partial \mu_I} + \mathcal{O}(e^4)$$

$\Downarrow \ni \left\langle \frac{\partial C_{\mu\mu}^{4D}(x; \mu_I)}{\partial \mu_I} \right\rangle_U$

Higher-point operator needed, but not yet measured



- $\frac{\partial \mathcal{P}}{\partial \mu_I}$ is zero below $\mu_I^{\text{crit.}}$ as $\lambda \rightarrow 0$, with sharp peak around $\mu_I^{\text{crit.}}$
- Peak grows and narrows as $\lambda \rightarrow 0$
- Need to be confirmed by direct measurements

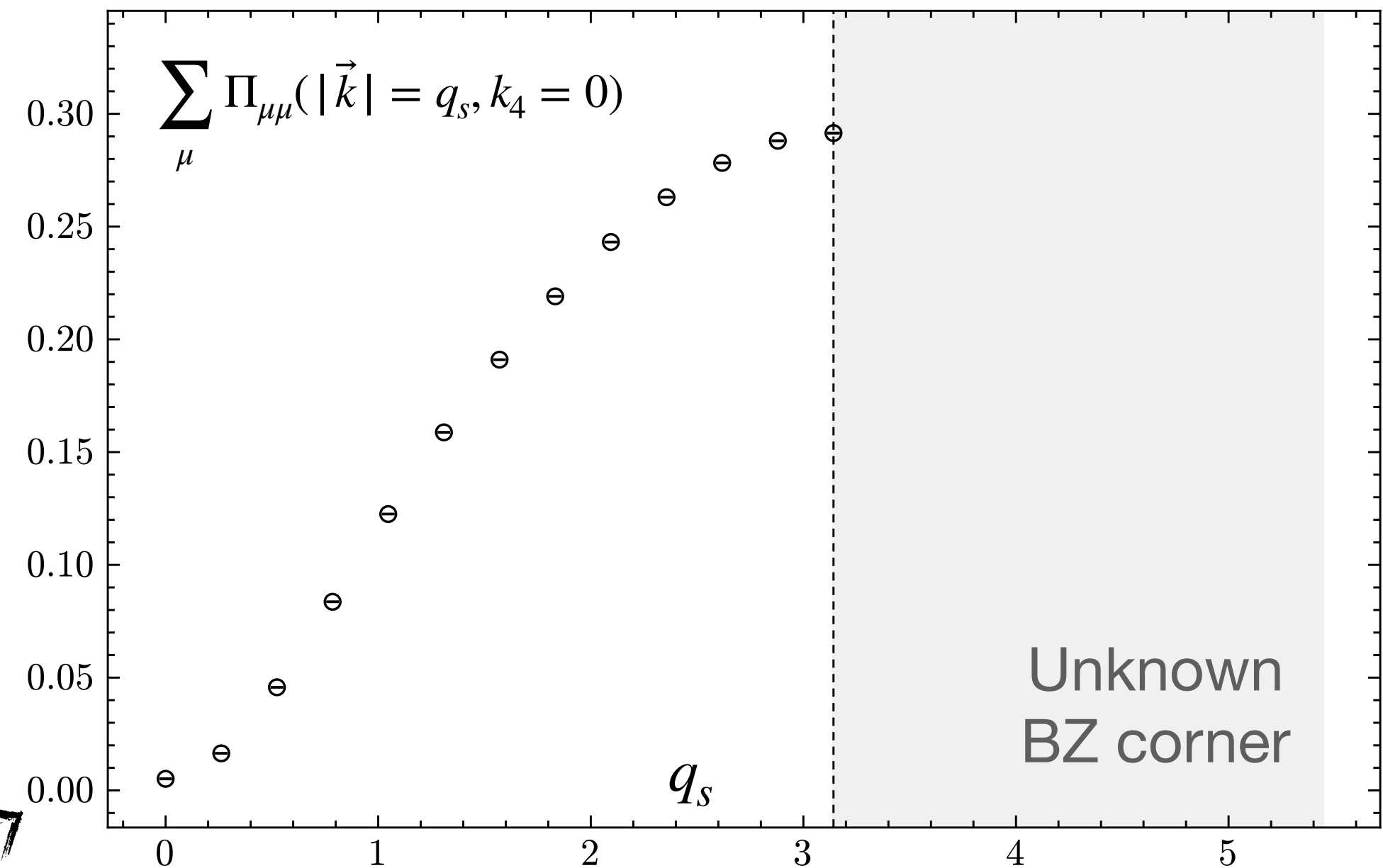
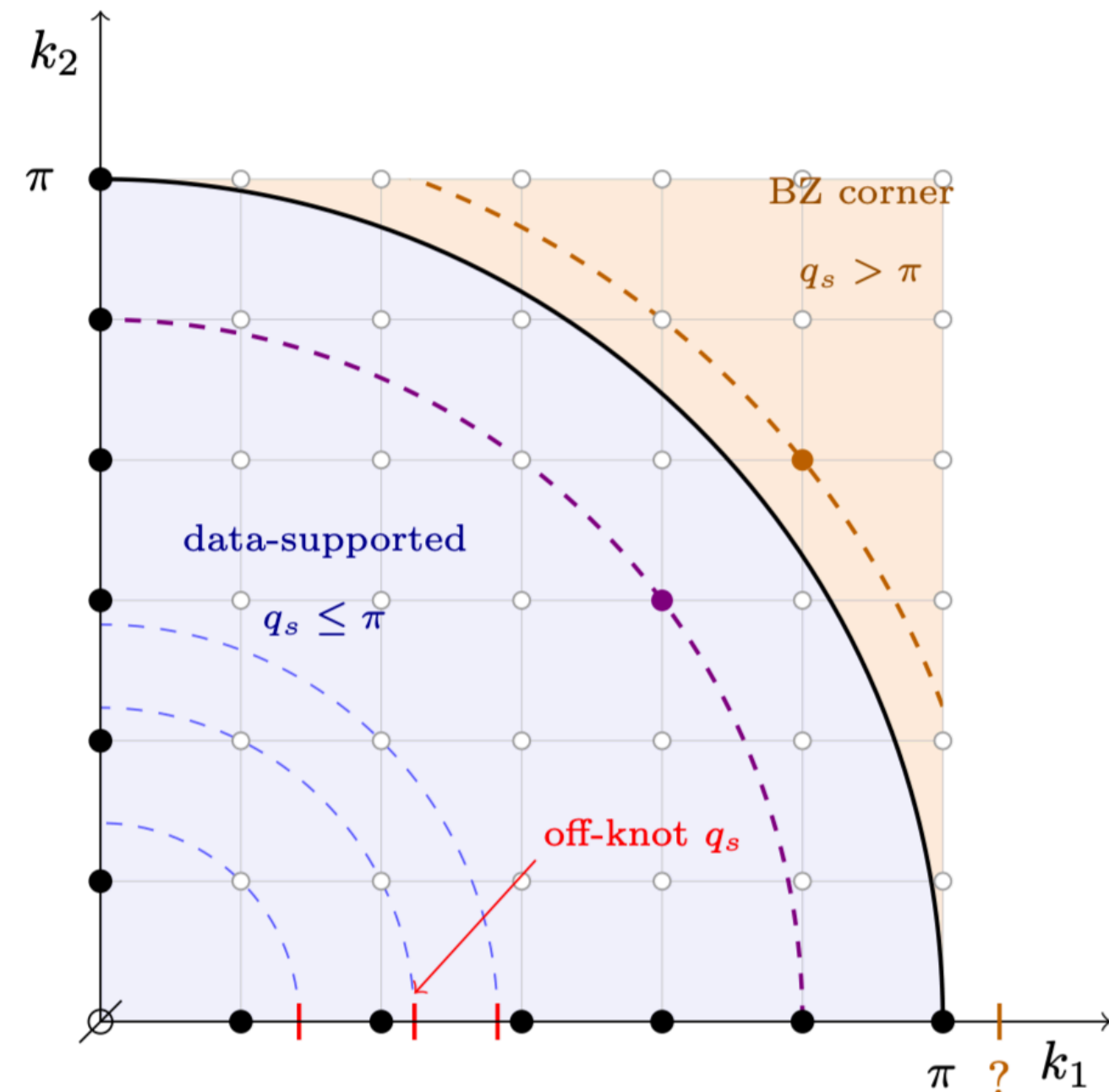
Summary & outlook

- ☒  is calculated from full 4D current-current correlator with point source
- ☒ μ_I independence of pineapple before onset of BEC, while sharp rise across transition region
- ☐ Fine lattice spacings to control the continuum limit; improved $\lambda \rightarrow 0$ extrapolation
- ☐ Direct calculation of the pineapple derivative w.r.t. various parameters ($\mu_I, \lambda, m, \dots$)

Backup

From $C^{2D}(z, \tau)$ to Brillouin-zone sum

$$\text{QCD} = \frac{1}{V_4} \sum_{k \in \text{BZ} \cap \text{QED}_{L/TL}} \frac{\sum_{\mu} \Pi_{\mu\mu}(\vec{k}, k_4)}{\hat{k}^2}$$

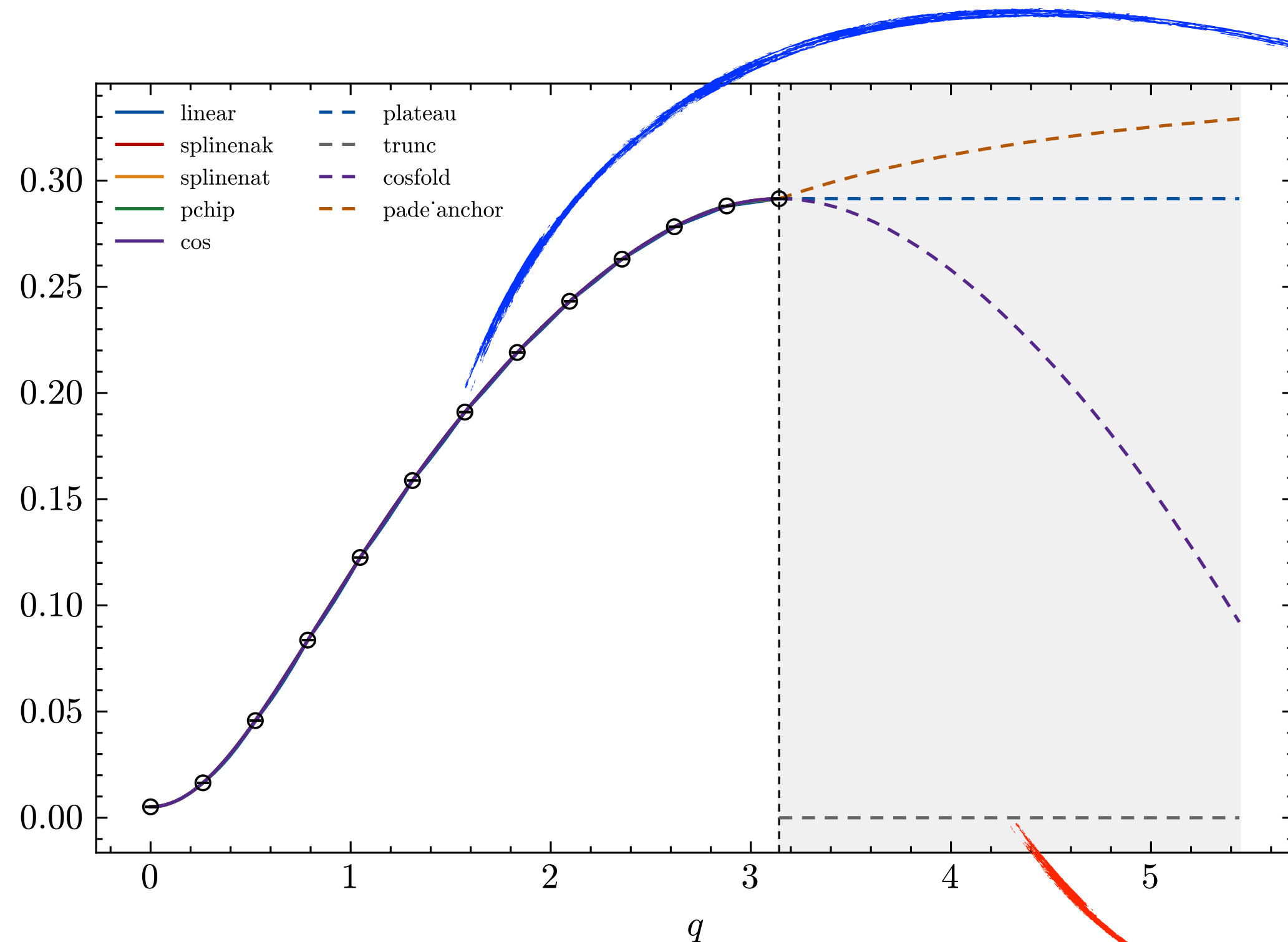


$$q_s = \frac{2\pi}{N_s} \sqrt{\sum_{i=1}^3 m_i^2}, \text{ with only one of } m_i \in \left\{ \frac{-N_s}{2} + 1, \dots, \frac{N_s}{2} \right\} \text{ and other zero}$$

😞 2D planar correlator fixes $\Pi_{\mu\mu}(k)$ exactly only on the axis slice $k = \{(0,0,k_3,k_4), (0,k_2,0,k_4), (k_1,0,0,k_4)\}$

From $C^{2D}(z, \tau)$ to Brillouin-zone sum

😊 Reconstruction of $\Pi(\vec{k}, k_4)$ via $O(3)$ spatial symm.



● Interior $|\vec{k}| \leq \pi$

interpolate in q^2 via ansatz { linear, spline, PCHIP, cosine poly. }

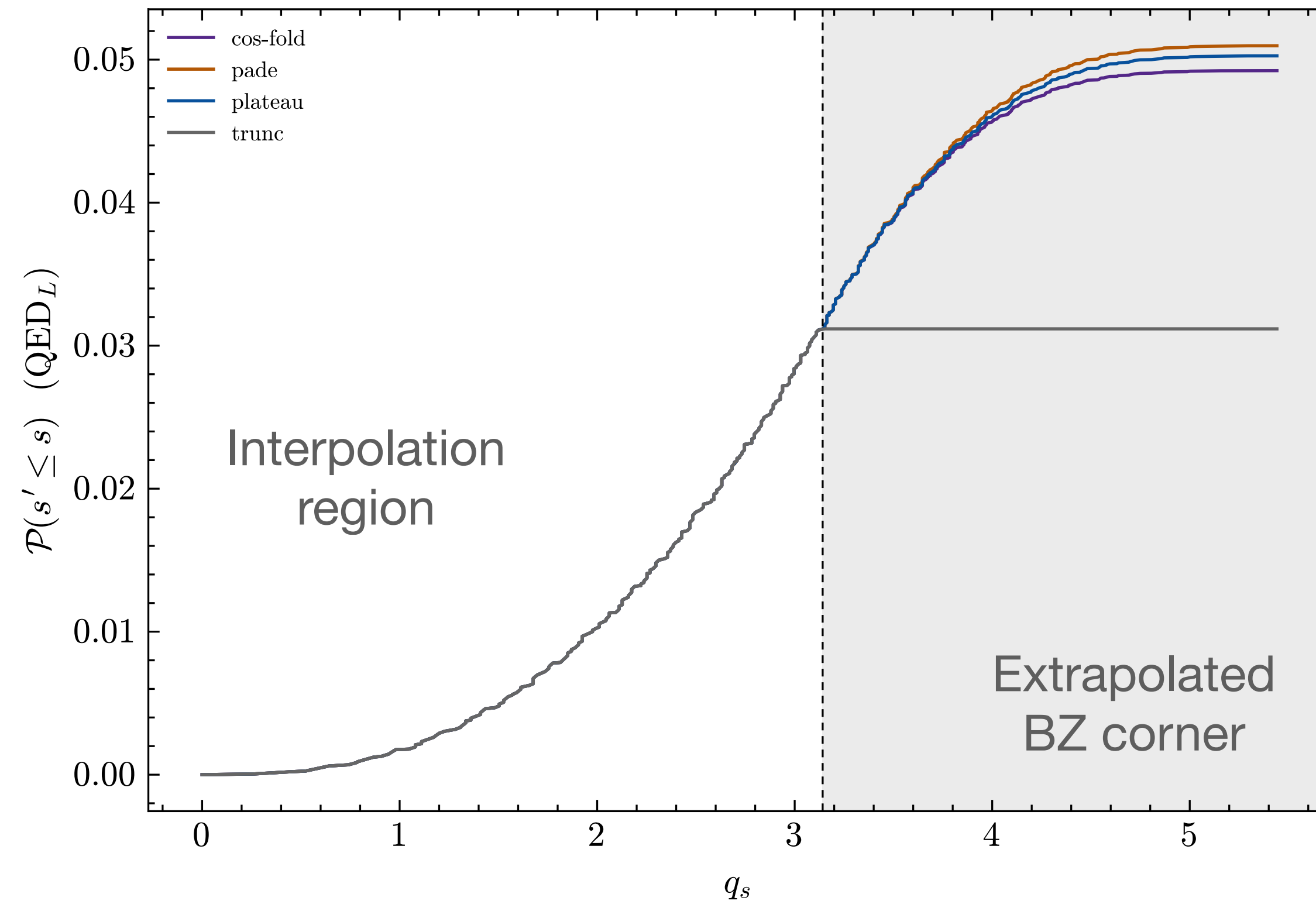
● BZ corner $\pi < |\vec{k}| \leq \sqrt{3}\pi$

extrapolate in q^2 via ansatz { plateau, cos-fold, Pade, drop-corner }

$$\text{QCD} = \frac{1}{V_4} \sum_{k \in \text{BZ} \cap \text{QED}_{L/TL}} \frac{\sum_{\mu} \Pi_{\mu\mu}(\vec{k}, k_4)}{\hat{k}^2} \text{ can be constructed completely!}$$

Result: cumulative contribution to pineapple

Accumulate momentum sum ordered by q_s :
$$P(q'_s \leq q_s) = \frac{1}{V_4} \sum_{k_4, |\vec{k}'| \leq q_s} \frac{\sum_{\mu} \Pi_{\mu\mu}(\vec{k}, k_4)}{\hat{k}^2}$$



- Interior $|\vec{k}| \leq \pi$: different interp. ansatz almost coincide — correlator data well shape the curve profile
- BZ corner $\pi < |\vec{k}| \leq \sqrt{3}\pi$: contributes about 35%; different schemes split; UV-sensitive piece