随机引力波背景的探测

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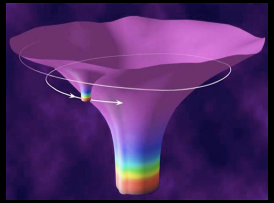
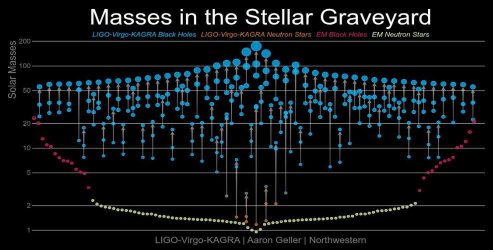
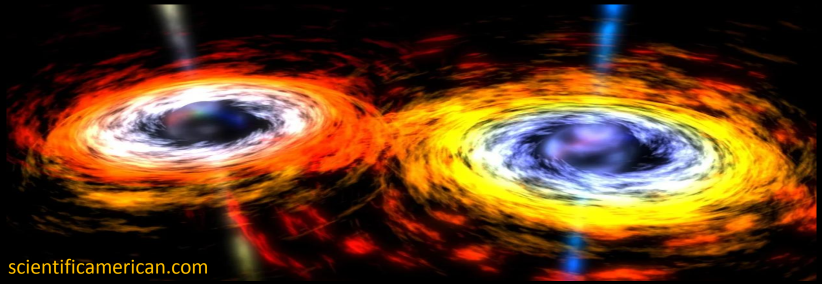
国际理论物理中心-亚太地区

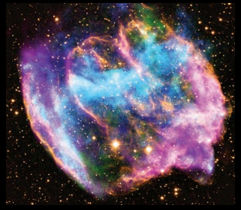
第四届新物理数值计算与模拟前沿讲习班

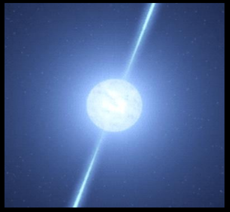
引力波谱

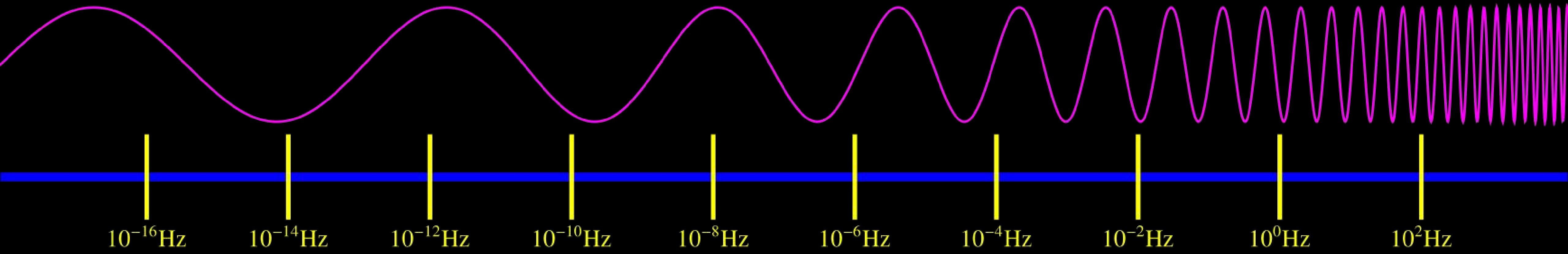




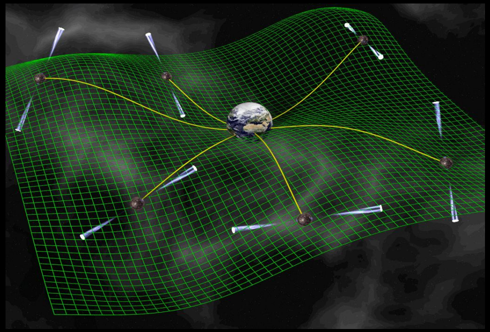




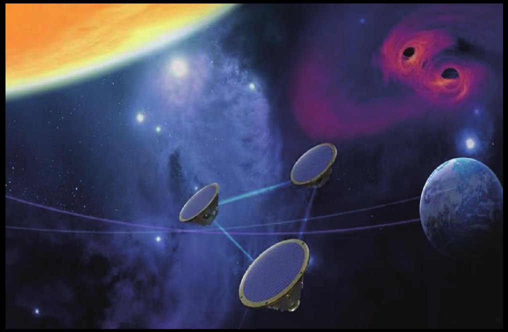




原初引力波探测

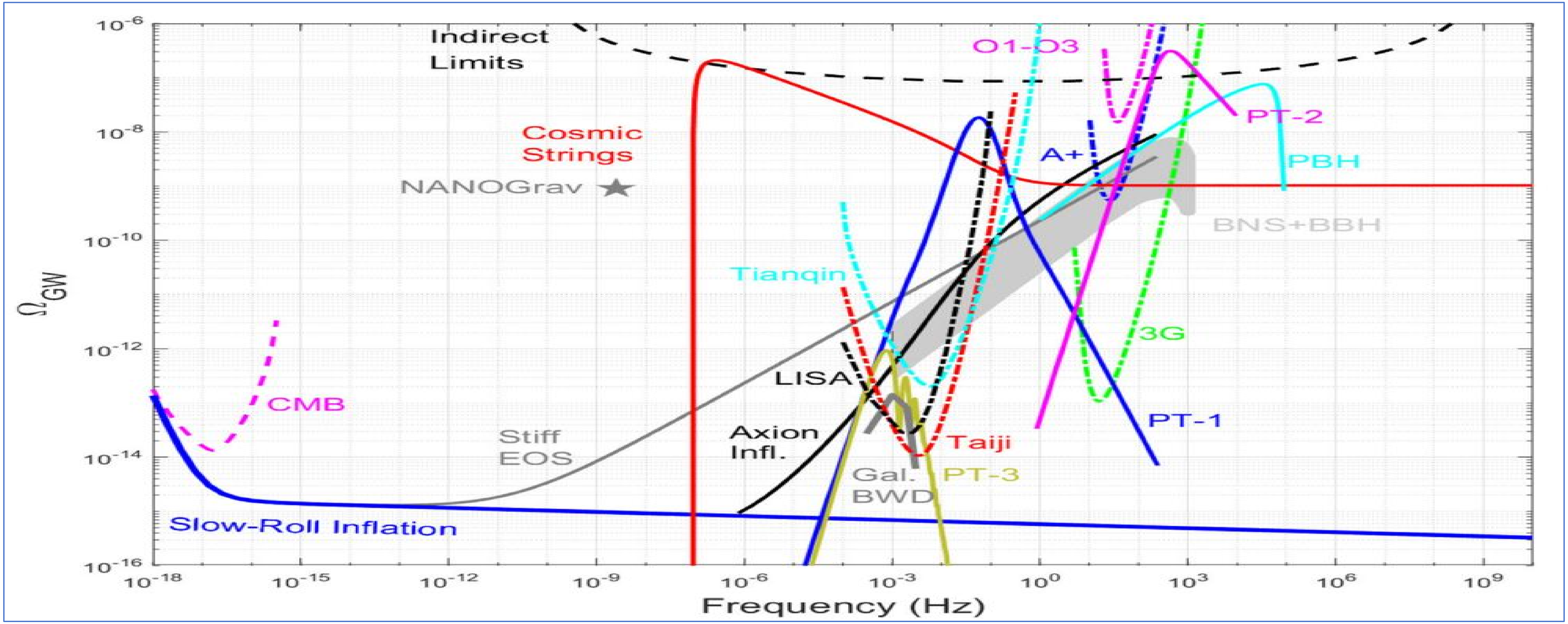


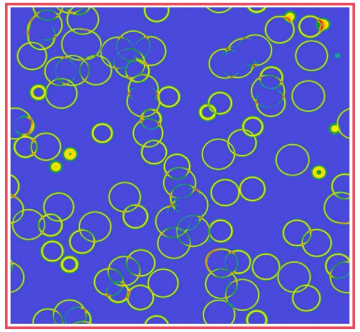
脉冲星测时阵列



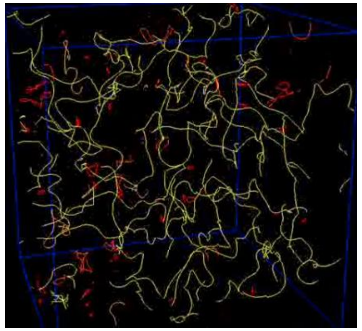
中国空间引力波探测（太极）

LIGO/Virgo/KAGRA

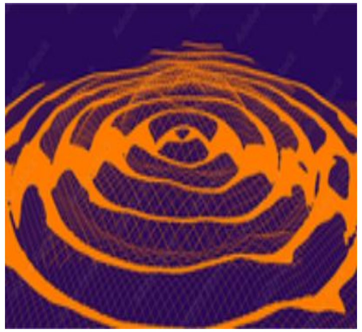




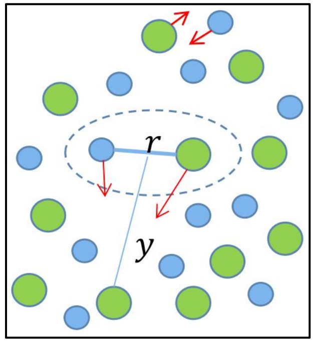
宇宙相变



宇宙弦等

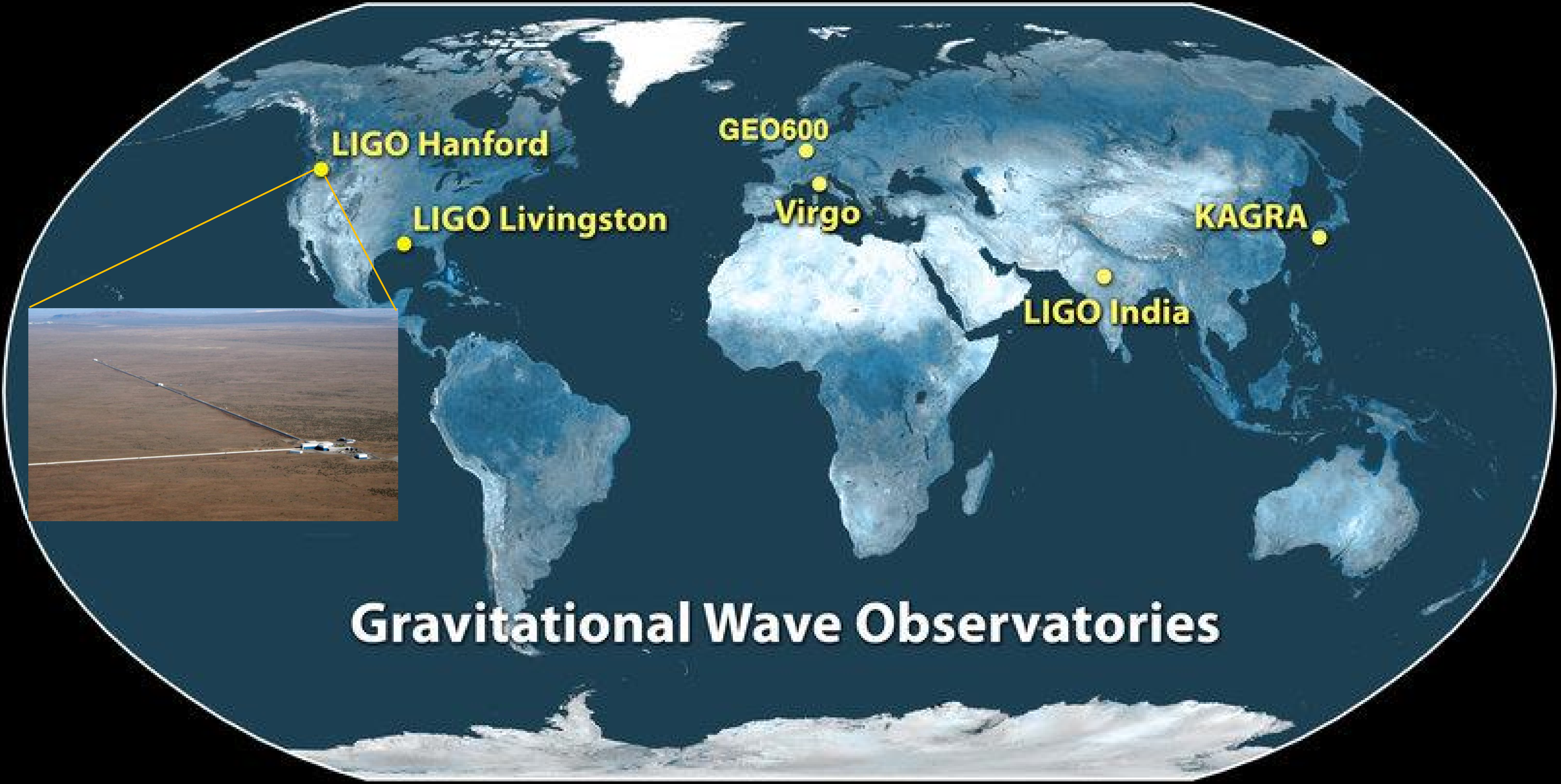


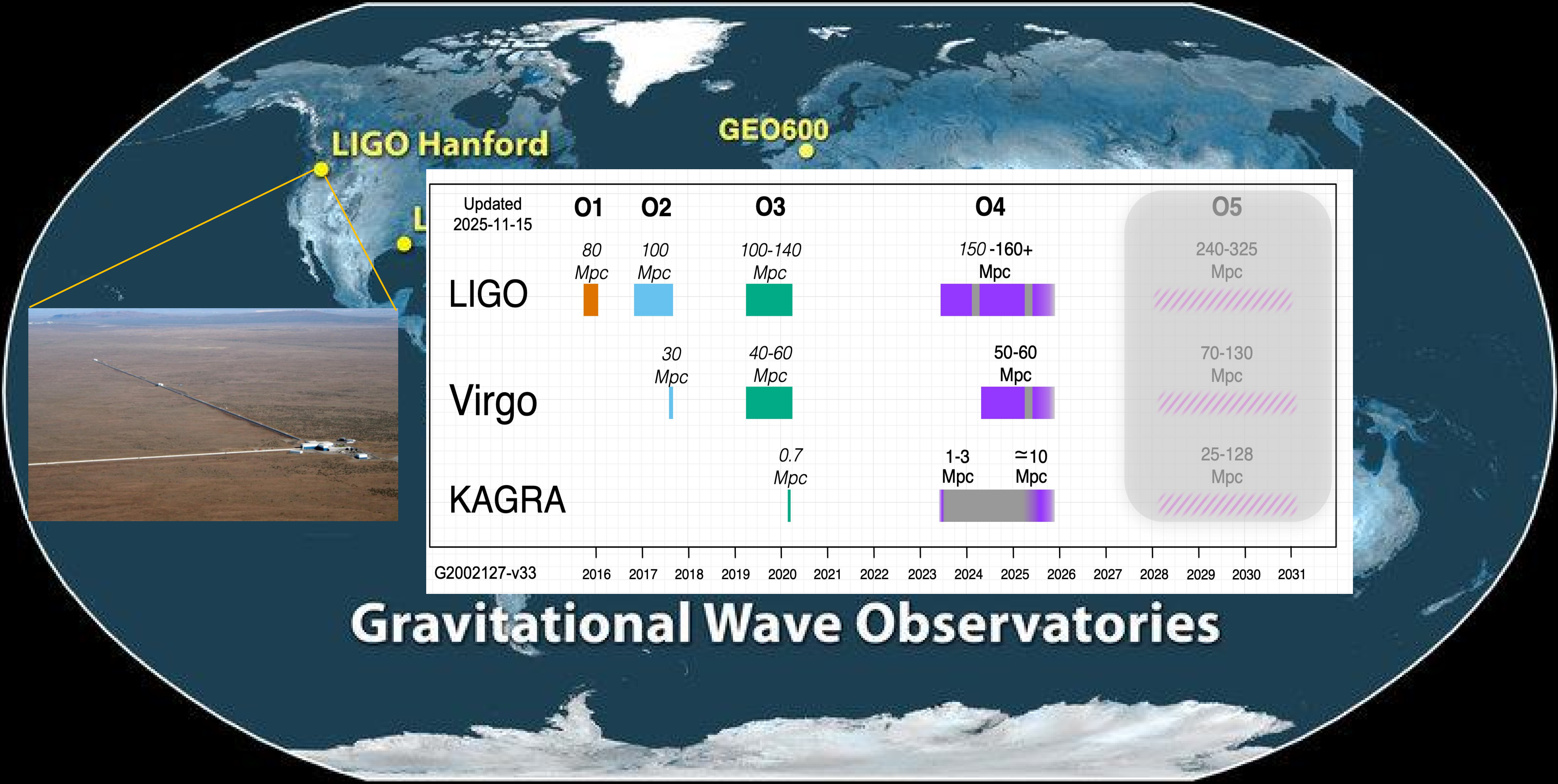
标量诱导引力波



原初黑洞并合


End-station @ 4 km
Mid-station @ 2 km
4 km (2.5 mi.)


LIGO Hanford
LIGO Livingston
GEO600
Virgo
KAGRA
LIGO India
Gravitational Wave Observatories


LIGO Hanford
GEO600
Updated
2025-11-15
O1
O2
O3
O4
O5
LIGO
80 Mpc
100 Mpc
100-140 Mpc
150 -160+ Mpc
240-325 Mpc
Virgo
30 Mpc
40-60 Mpc
50-60 Mpc
70-130 Mpc
KAGRA
0.7 Mpc
1-3 Mpc
≃10 Mpc
25-128 Mpc
G2002127-v33
2016
2017
2018
2019
2020
2021
2022
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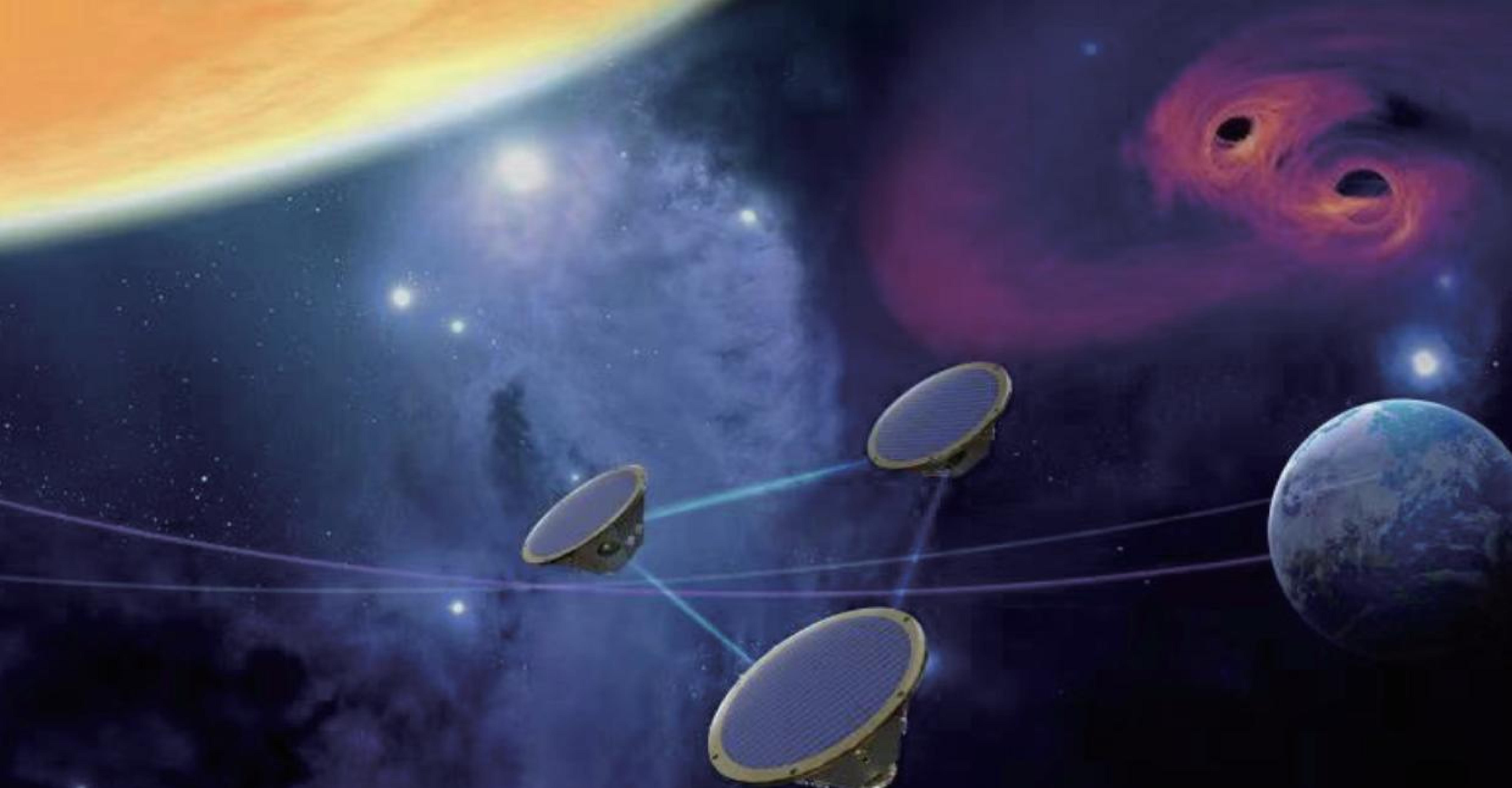
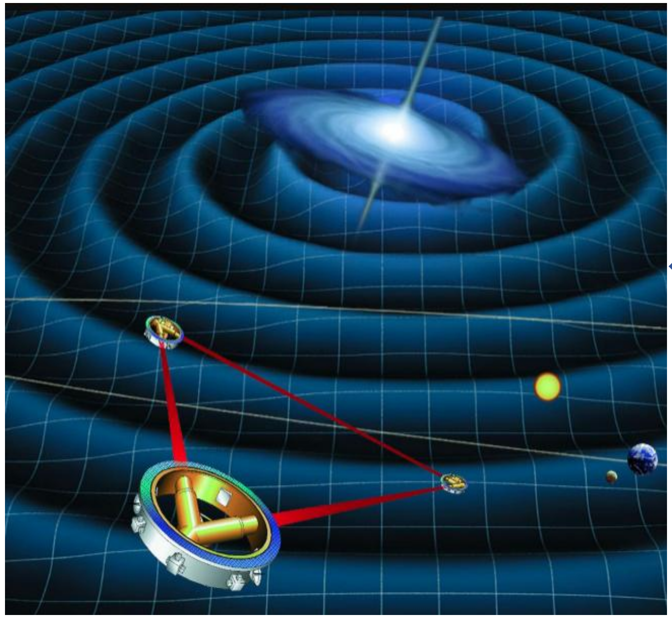
2029
2030
2031
Gravitational Wave Observatories



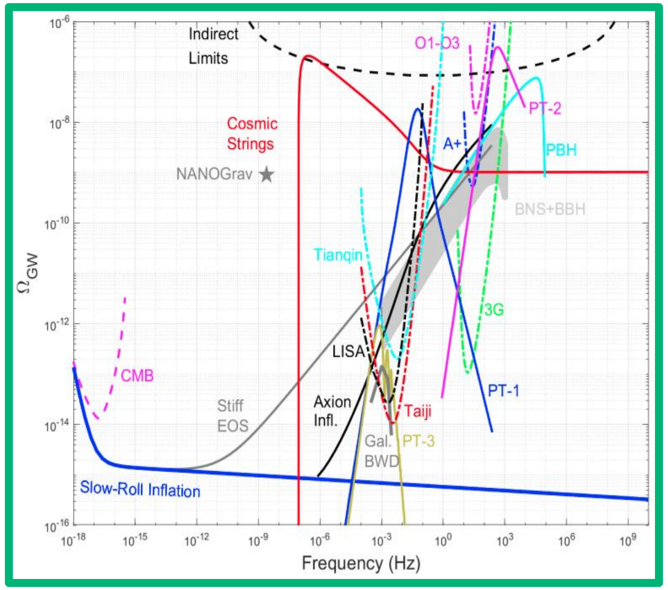
空间引力波探测：太极、天琴、LISA

Example: Phase Transition

Theorist



LIGO, LISA/Taiji/Tianqin, PTA, ...



Gravitational Wave Spectrum

α

β

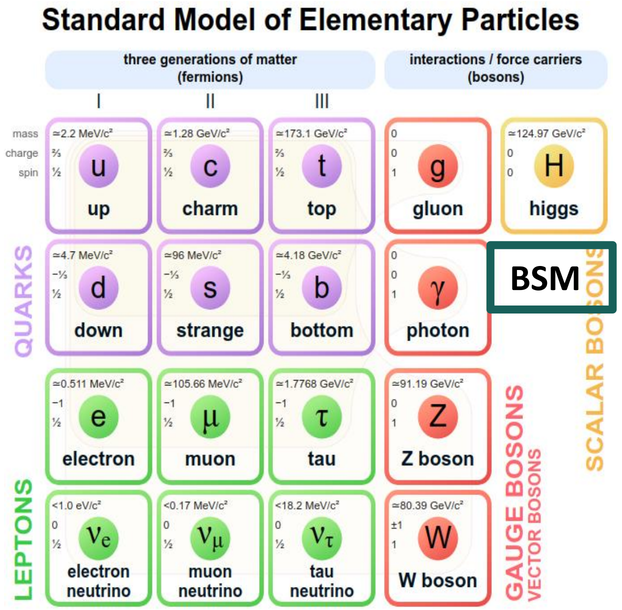
v_w

T_*

g_s

...

Phase Transition Parameters



Particle Physics Model

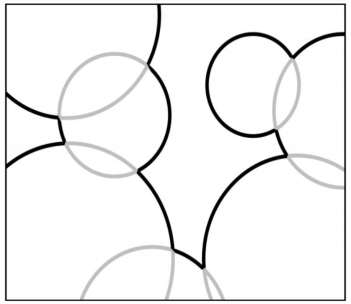
Experimentalist

Questions to Answer

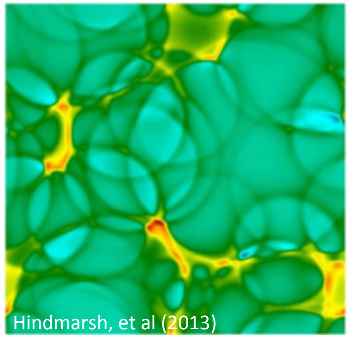
- Set limits when signal is absent
- Parameter estimation when signal is discovered
  - What is precise shape of the signal spectrum?
  - What are the values of alpha, beta, vw, T*, etc (PT as an example)?
  - What is the underlying particle physics model?
    - What are the values of the model parameters?
    - What this implies for collider experiments?
  - ...

Key Observables

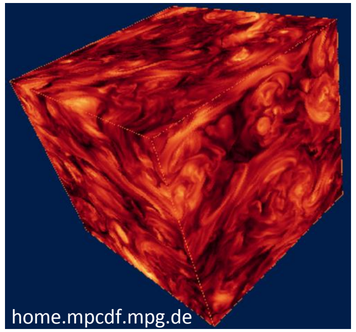
bubble collision



sound waves



MHD



$$\Omega_{\rm coll}(f)h^2 = 1.67\times 10^{-5}\Delta\left(\frac{H_{\rm pt}}{\beta}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2 \times\left(\frac{100}{g_*}\right)^{1/3} S_{\rm env}(f),$$

$$\Omega_{\rm sw}(f)h^2 = 2.65\times 10^{-6}\left(\frac{H_{\rm pt}}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3} \times v_w\left(\frac{f}{f_{\rm sw}}\right)^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}\Upsilon(\tau_{\rm sw}),$$

$$h^2\Omega_{\rm turb}(f) = 3.35\times 10^{-4}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm turb}\,\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{100}{g_*}\right)^{1/3} v_w\, S_{\rm turb}(f)$$

Chiara Caprini et al JCAP [1512.06239]

Energy density Spectrum

$$\Omega_{\rm GW}(f) = \frac{d\rho_{\rm GW}}{\rho_c d\log f}$$

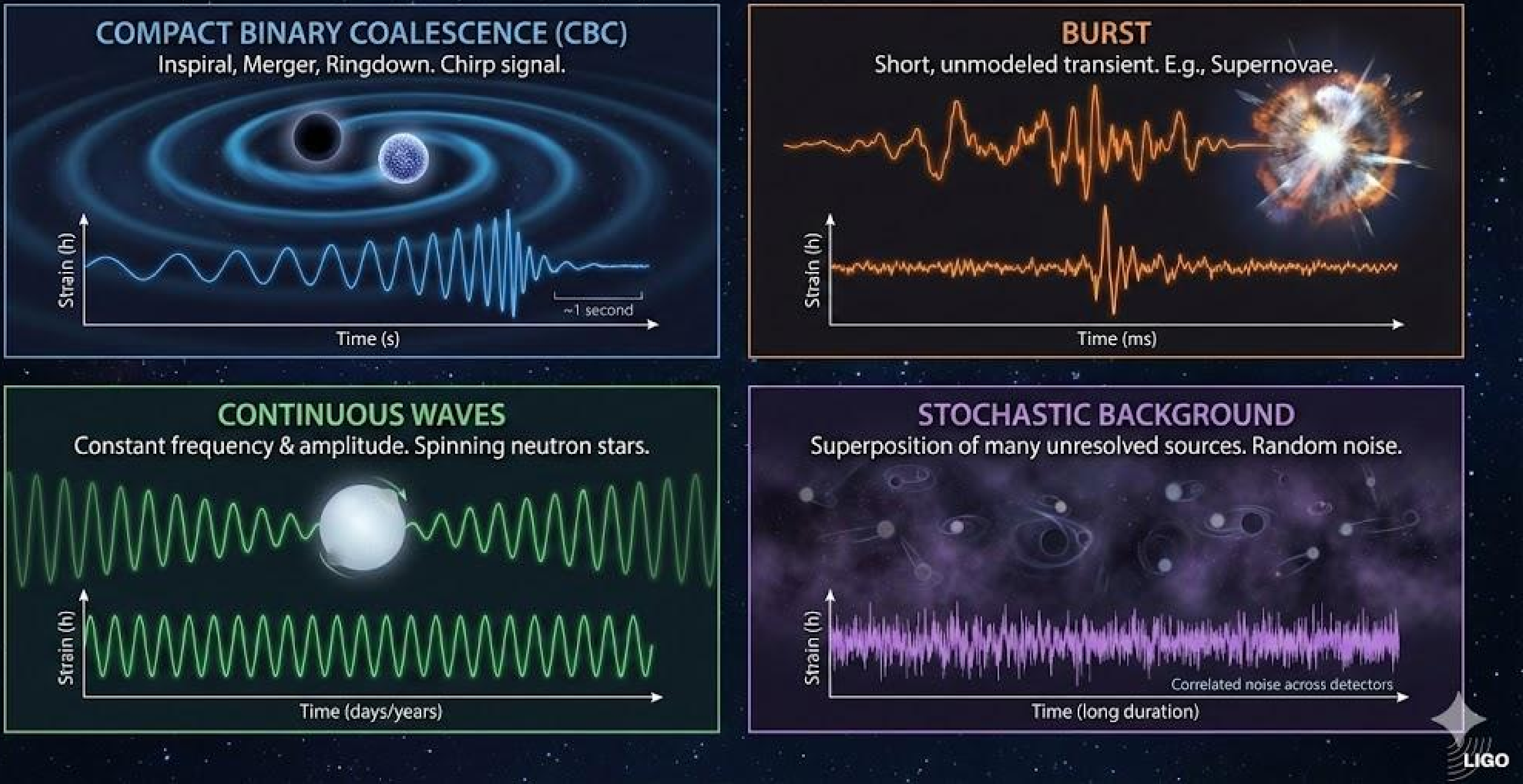
$$\Upsilon = 1-(1+2\tau_{\rm sw}H_{\rm pt})^{-1/2}$$ (RD)

HG, Sinha, Vagie, White, JCAP [2007.08537]

$$\Upsilon = \frac{2[1-y^{3(w-1)/2}]}{3(1-w)}$$

Yang Xiao, HG, Jia-Hang Hu, Jin Min Yang, Yang Zhang [2410.23666]

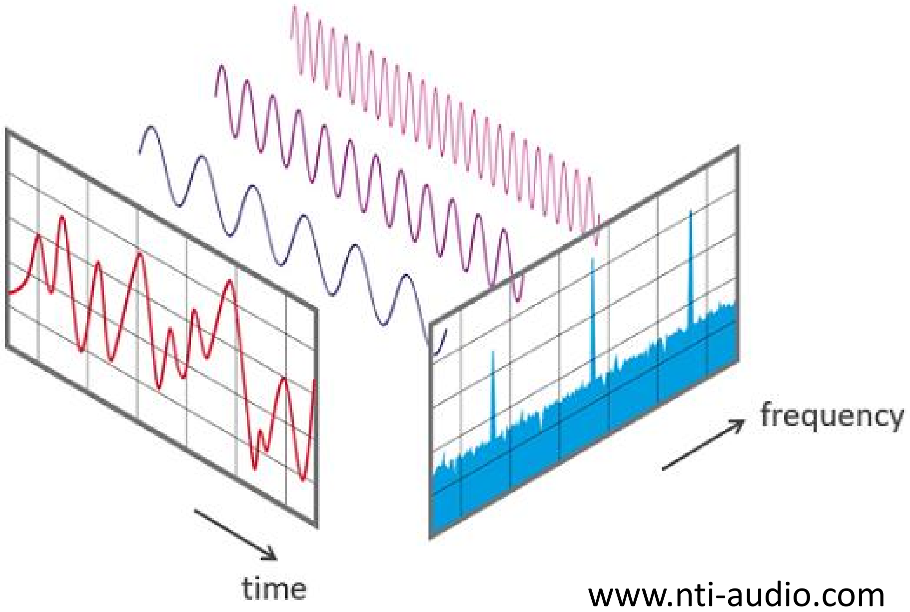
FOUR TYPES OF GRAVITATIONAL WAVES DETECTED BY LIGO



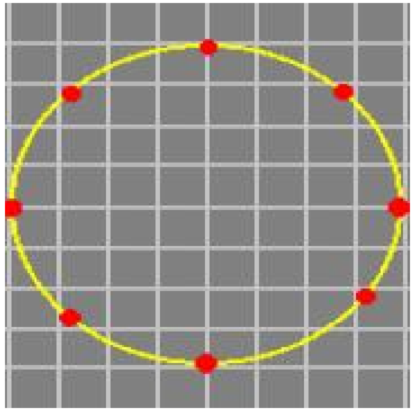
Plane Wave

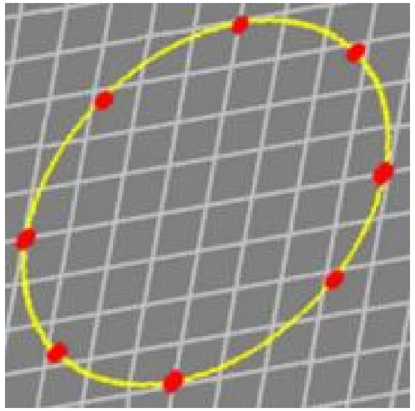
- A special gauge, the TT gauge, can be chosen in vacuum (caution: extended sources).
- Only 2 degrees of freedom (+, x).

$$h_{ij}^{\mathrm{TT}}(t,z) = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij} \cos[\omega(t - z/c)]$$



www.nti-audio.com





https://www.johnstonsarchive.net/relativity/pictures.html

Detector Response

- From GW to detector outputs

$$h_{ij}(t,\mathbf{x}) = \sum_{A=+,\times} \int_{-\infty}^{\infty} df \int_{S^2} d^2\hat{\mathbf{n}}\, \tilde{h}_A(f,\hat{\mathbf{n}})\, e^A_{ij}(\hat{\mathbf{n}}) e^{-2\pi i f(t-\hat{\mathbf{n}}\cdot\mathbf{x}/c)}$$

$$h_I(t) = d_I^{ij} h_{ij}(t, \mathbf{x}_I)$$

$$h_I(t) = \sum_A \int df \int d^2\hat{\mathbf{n}}\, \tilde{h}_A(f,\hat{\mathbf{n}}) F_I^A(\hat{\mathbf{n}}) e^{-2\pi i f(t-\hat{\mathbf{n}}\cdot\mathbf{x}_I/c)}$$

$$F_I^A(\hat{\mathbf{n}}) = d_I^{ij} e^A_{ij}(\hat{\mathbf{n}})$$

$$s_I(t) = h_I(t) + n_I(t)$$

different detectors/channels

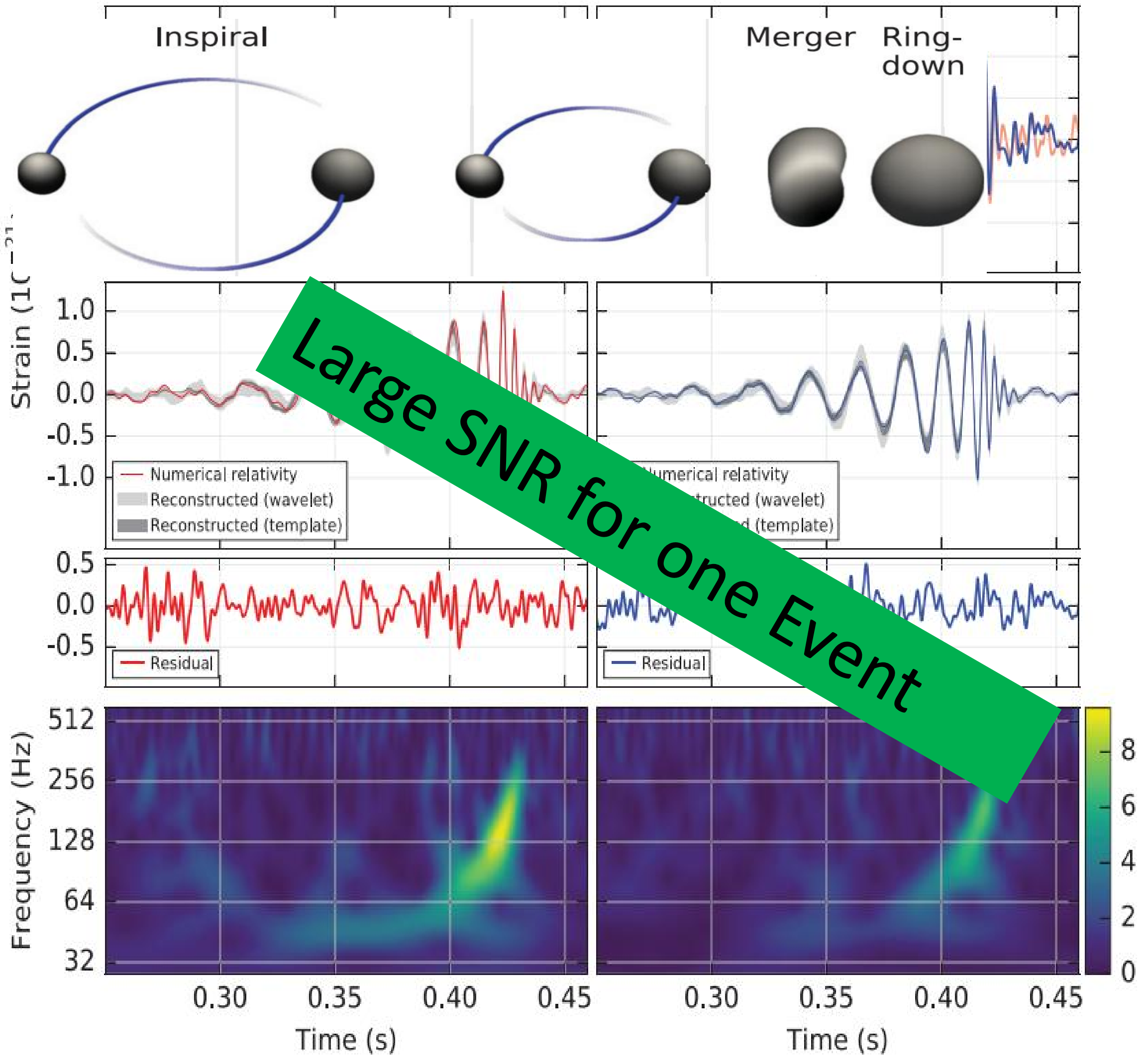
$$\langle n_I^2(t)\rangle = \int_0^{\infty} df\, P_I(f)$$

$$\langle \tilde{n}_I^*(f)\tilde{n}_J(f')\rangle = \frac{1}{2}\delta(f-f')\delta_{IJ} P_I(f)$$

- Detectors: LIGO (Hanford, Livingston), Virgo, KAGRA, LIGO-India...
- Space-based dectors: Taiji, Tianqin, LISA; TDI channels (A, E, T)

Stochastic GW Background

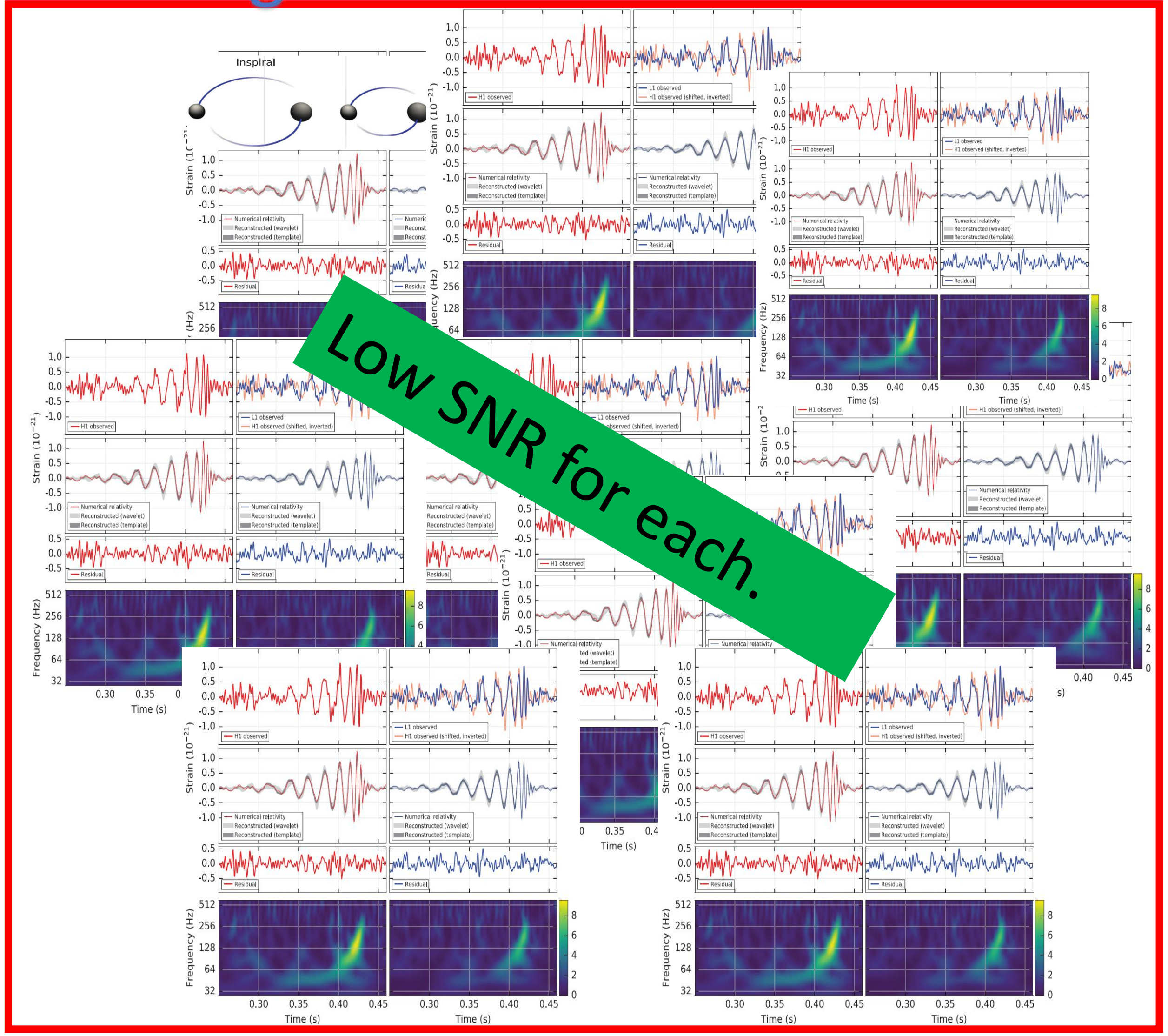
SGB ≠



LIGO, Phys. Rev. Lett. 116, 061102

Stochastic GW Background

SGB ~



Similar to a noise.

But how to detect a noise?

LIGO, Phys. Rev. Lett. 116, 061102

Stochastic GW Background

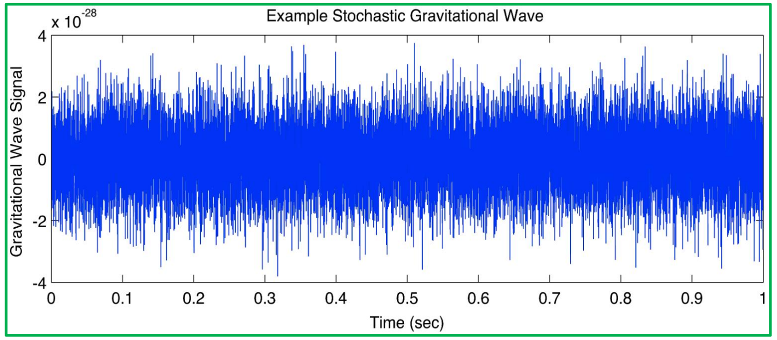
- Assumptions (applies for cosmic sources)

$$\langle \tilde{h}^*_A(f, \hat{\mathbf{n}}) \tilde{h}_{A'}(f', \hat{\mathbf{n}}') \rangle = \delta(f - f') \frac{\delta^2(\hat{\mathbf{n}}, \hat{\mathbf{n}}')}{4\pi} \delta_{AA'} \frac{1}{2} S_h(f)$$

- Stationary
- Gaussian
- Isotropic
- Unpolarized

$$S_h(f) = \frac{3H_0^2}{4\pi^2} \frac{\Omega_{\rm GW}(f)}{f^3}$$

$$\Omega_{\rm GW}(f) \equiv \frac{f}{\rho_c} \frac{d\rho_{\rm GW}}{df}$$

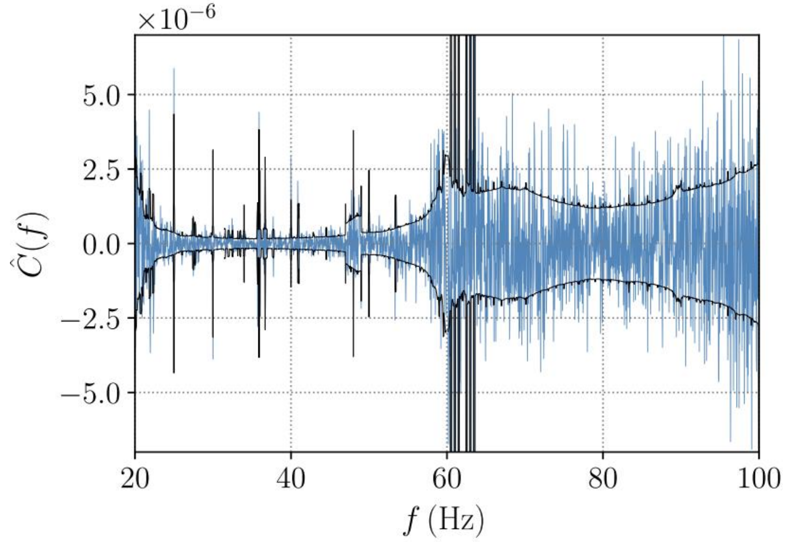


Question: how to simulate GW?

LIGO-Virgo-KAGRA

$$\hat{C}^{IJ}(f) = \frac{2}{T}\frac{\mathrm{Re}[\tilde{s}_I^\star(f)\tilde{s}_J(f)]}{\gamma_{IJ}(f)S_0(f)}$$

$$\langle \hat{C}^{IJ}(f)\rangle = \Omega_{\rm GW}(f)$$



- Glitches and gating
- Narrow spectral artifacts
- Non-Gaussian noise
- Magnetic noise
- Calibration uncertainty

LVK collaborations, PRD [2101.12130]

PT searches:

Cosmological and HEP implications: [2510.26848]
Romero, ..., HG, ..., PRL [2102.01714]

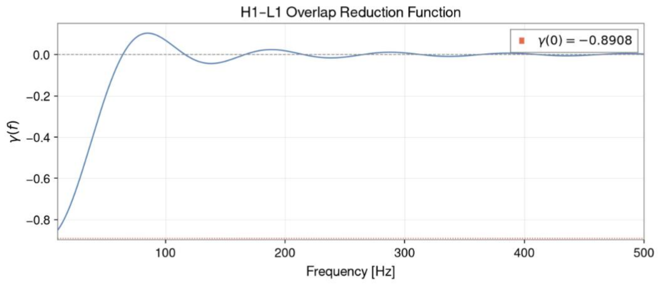
Cross-Correlation

- Single detector/channel output

$$\tilde{h}_I(f) = \sum_A \int_{S^2} d^2\hat{\mathbf{n}}\, \tilde{h}_A(f,\hat{\mathbf{n}}) F_I^A(\hat{\mathbf{n}}) e^{2\pi i f \hat{\mathbf{n}}\cdot\mathbf{x}_I/c}$$

- cross correlation of outputs

$$\left\langle \tilde{h}_I^*(f)\tilde{h}_J(f')\right\rangle = \frac{1}{2}\delta(f-f')\Gamma_{IJ}(f)S_h(f)$$



overlap reduction function

$$\gamma_{IJ}(f) = \frac{\Gamma_{IJ}(f)}{\Gamma_{II}(0)} = \frac{\Gamma_{IJ}(f)}{2/5}$$

➢ orf describes the reduction of cross-correlation from mis-aligned, separated detectors

Cross-Correlation

For a detector pair (IJ), segment t:

$$\hat{C}_t^{IJ}(f_k) = \frac{2}{T_{\rm seg}} \frac{\mathrm{Re}\left[\tilde{s}_{I,t}^*(f_k)\, \tilde{s}_{J,t}(f_k)\right]}{\gamma_{IJ}(f_k)\, S_0(f_k)}$$

↓

$$\hat{C}_k^{IJ} \equiv \hat{C}^{IJ}(f_k) = \frac{\sum_t \hat{C}_t^{IJ}(f_k)\,[\sigma_t^{IJ}(f_k)]^{-2}}{\sum_t [\sigma_t^{IJ}(f_k)]^{-2}}$$

$$\left[\sigma^{IJ}(f_k)\right]^2 = \left[\sum_t \frac{2T_{\rm seg}\Delta f\, \gamma_{IJ}^2(f_k) S_0^2(f_k)}{P_{I,t}(f_k) P_{J,t}(f_k)}\right]^{-1}$$

↓

$$\left\langle \hat{C}_k^{IJ} \right\rangle = \Omega_M(f_k|\Theta)$$

$$\mathrm{Var}[\hat{C}_k^{IJ}] = [\sigma_k^{IJ}]^2$$

SNR

- cross-correlation estimator

$$Y = \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt'\, s_1(t) s_2(t') Q(t-t')$$

filter

- SNR

$$\frac{S}{N} \equiv \frac{\langle Y \rangle}{\sqrt{\langle Y^2 \rangle - \langle Y \rangle^2}} = T^{1/2} \frac{\int_{-\infty}^{\infty} df\, S_h(f) \Gamma(f) \tilde{Q}(f)}{\left[\int_{-\infty}^{\infty} df\, |\tilde{Q}(f)|^2 P_1(f) P_2(f)\right]^{1/2}}$$

- Optimized SNR

$$\frac{S}{N} = \left[2T \int_0^{\infty} df\, \Gamma^2(f) \frac{S_h^2(f)}{P_1(f) P_2(f)}\right]^{1/2}$$

optimal filter

$$\tilde{Q}_{\rm opt}(f) \propto \frac{\Gamma(f) S_h(f)}{P_1(f) P_2(f)}$$

Bayesian Analysis

We can now construct the likelihood

$$p(\hat{C}_k^{IJ}|\mathbf{\Theta}) \propto \exp\left[-\frac{1}{2}\sum_{IJ}\sum_{k}\left(\frac{\hat{C}_k^{IJ} - \Omega_{\mathrm{M}}(f_k|\mathbf{\Theta})}{\sigma_{IJ}(f_k)}\right)^2\right]$$

- Specify a prior
- Specify a model (astro, cosmo)
- Sample the posterior

- Model comparison (Bayes factor)
- Parameter estimation

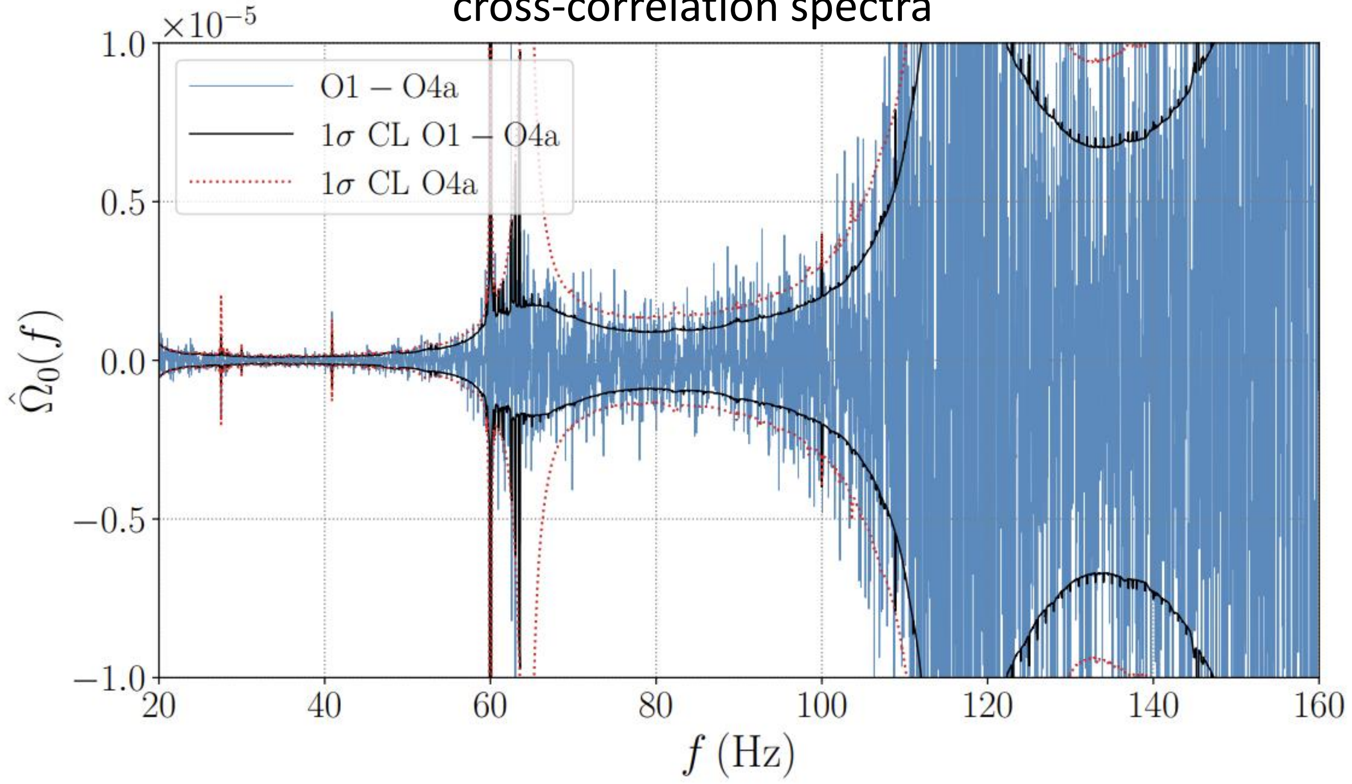
Bayes's theorem

$$p(H_i \mid D, I) = \frac{p(H_i \mid I)\, p(D \mid H_i, I)}{p(D \mid I)}$$

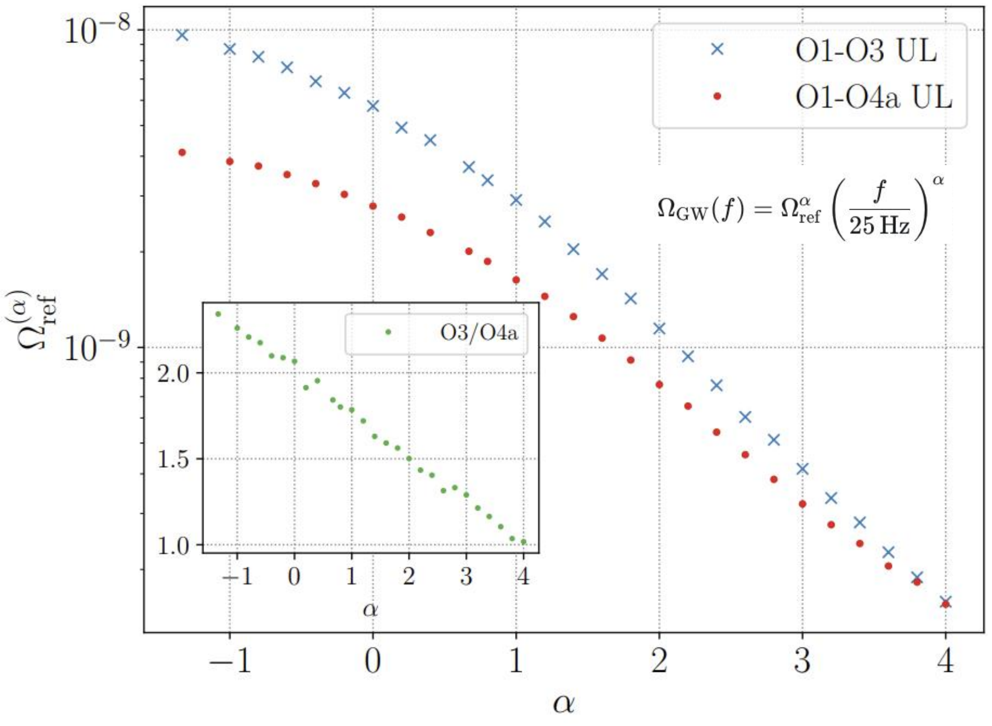
Cross-Correlation: O4a

LVK [2508.20721]

cross-correlation spectra



upper limits for power law models



Consistent with noise only scenario. Exclusion for power law models obtained

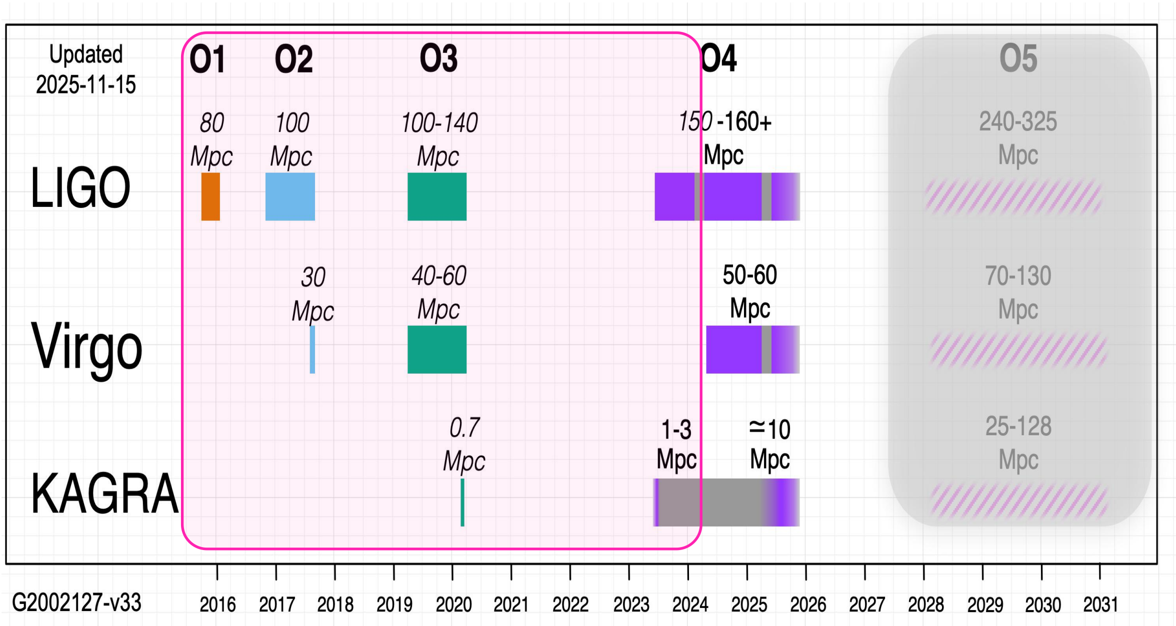
Cosmological Sources in O4a Analysis

Cosmological and High Energy Physics implications from gravitational-wave background searches in LIGO-Virgo-KAGRA's O1-O4a runs

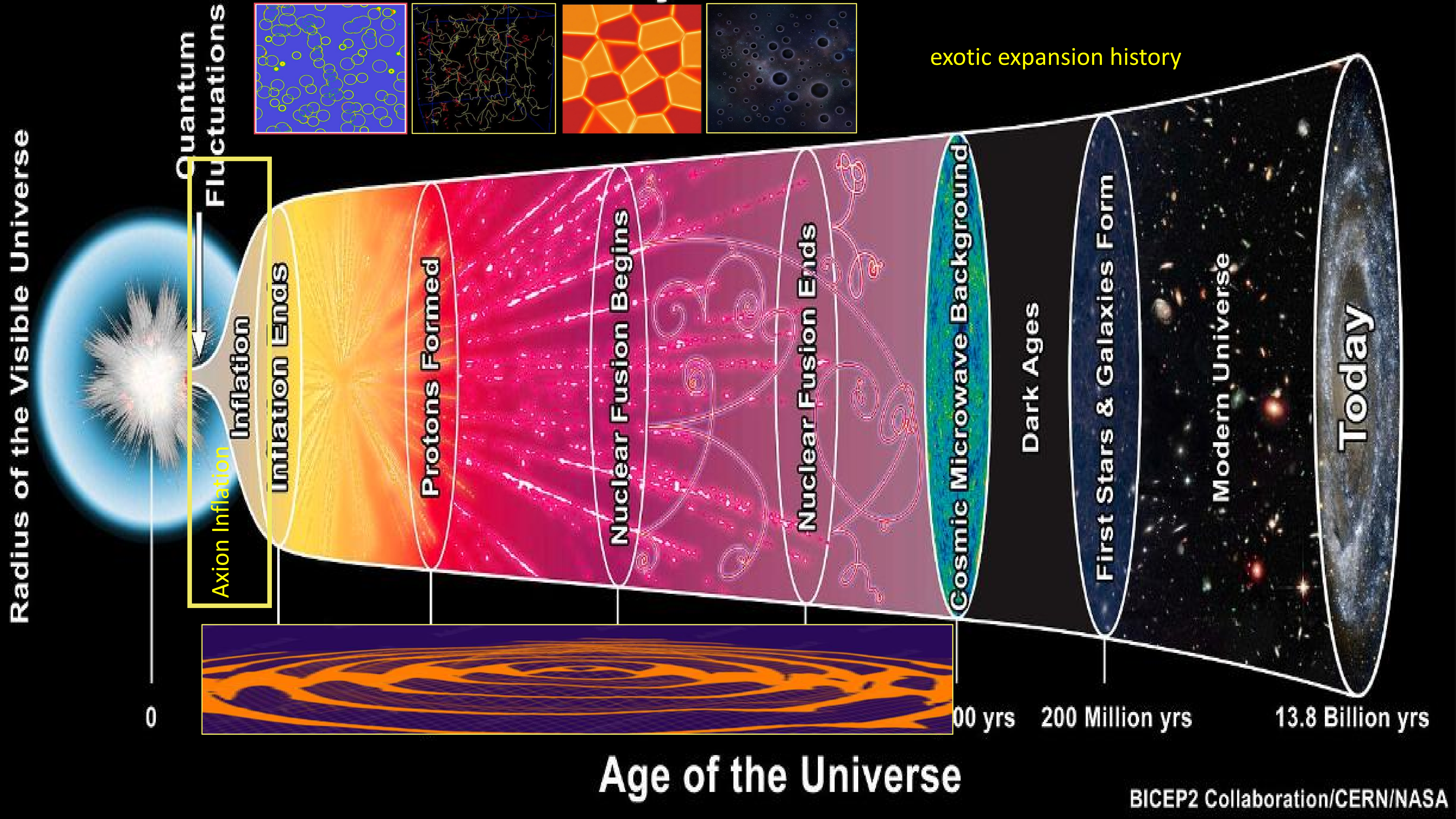
The LIGO Scientific Collaboration, The Virgo Collaboration, and The KAGRA Collaboration*

arxiv:2510.26848

- Cosmological phase transitions
- Cosmic strings
- Domain walls
- Stiff equation of state
- Axion Inflation
- Second order scalar perturbations
- Primordial black holes
- Parity violation (not covered here)

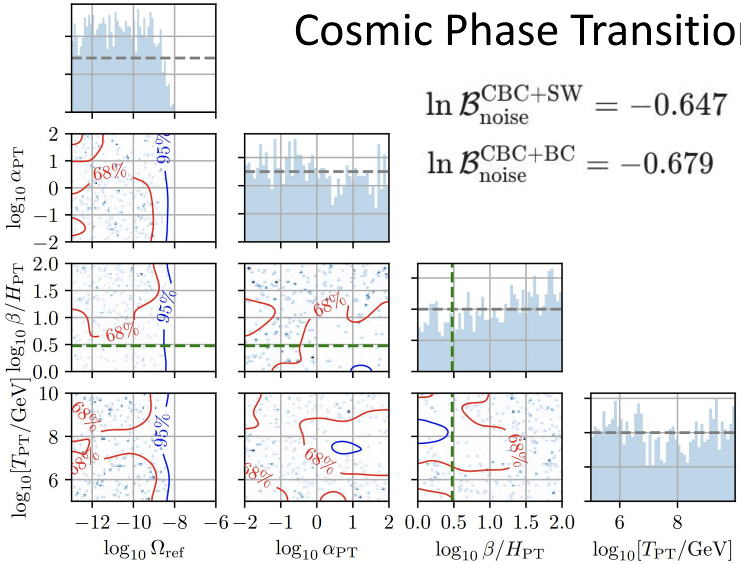


O4a: 2023-05-24 to 2024-01-16 16:00:00 UTC


exotic expansion history
Quantum Fluctuations
Radius of the Visible Universe
Axion Inflation
Inflation
Inflation Ends
Protons Formed
Nuclear Fusion Begins
Nuclear Fusion Ends
Cosmic Microwave Background
Dark Ages
First Stars & Galaxies Form
Modern Universe
Today
0
00 yrs
200 Million yrs
13.8 Billion yrs
Age of the Universe
BICEP2 Collaboration/CERN/NASA

Representative Results

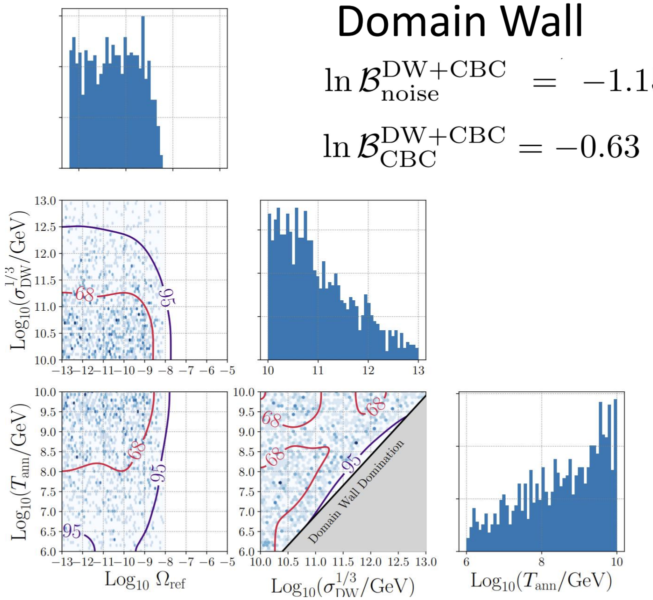
Cosmic Phase Transition



$$\ln \mathcal{B}^{\rm CBC+SW}_{\rm noise} = -0.647$$

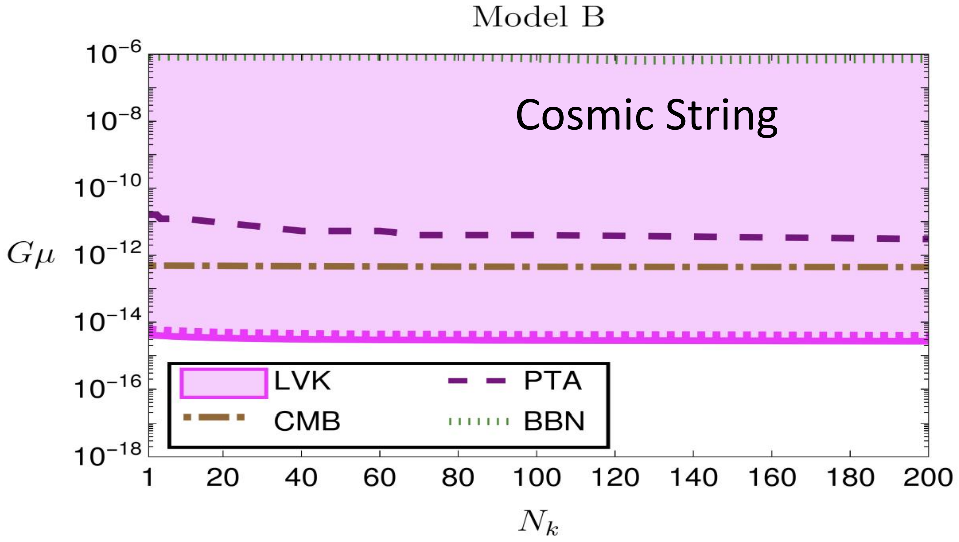
$$\ln \mathcal{B}^{\rm CBC+BC}_{\rm noise} = -0.679$$

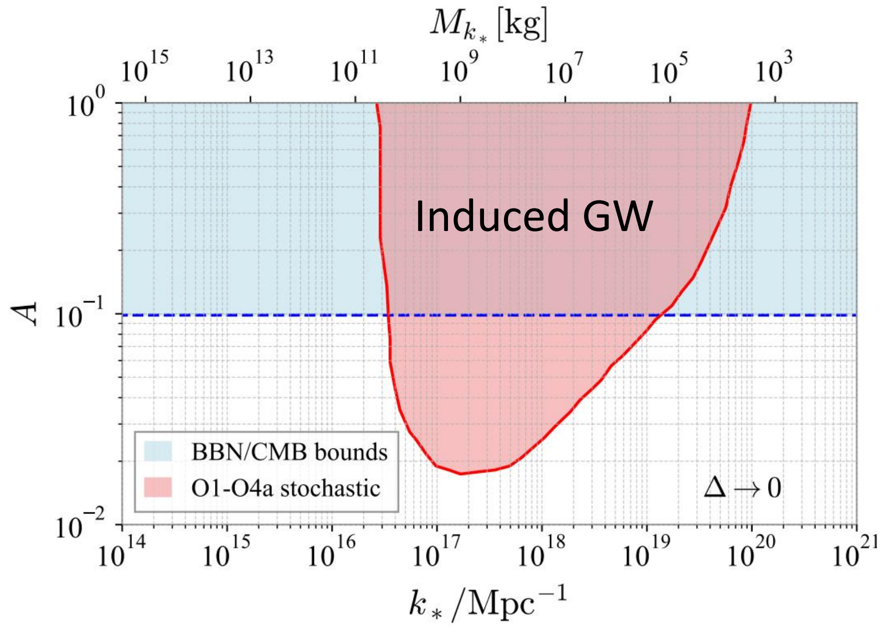
Domain Wall



$$\ln \mathcal{B}^{\rm DW+CBC}_{\rm noise} = -1.15$$

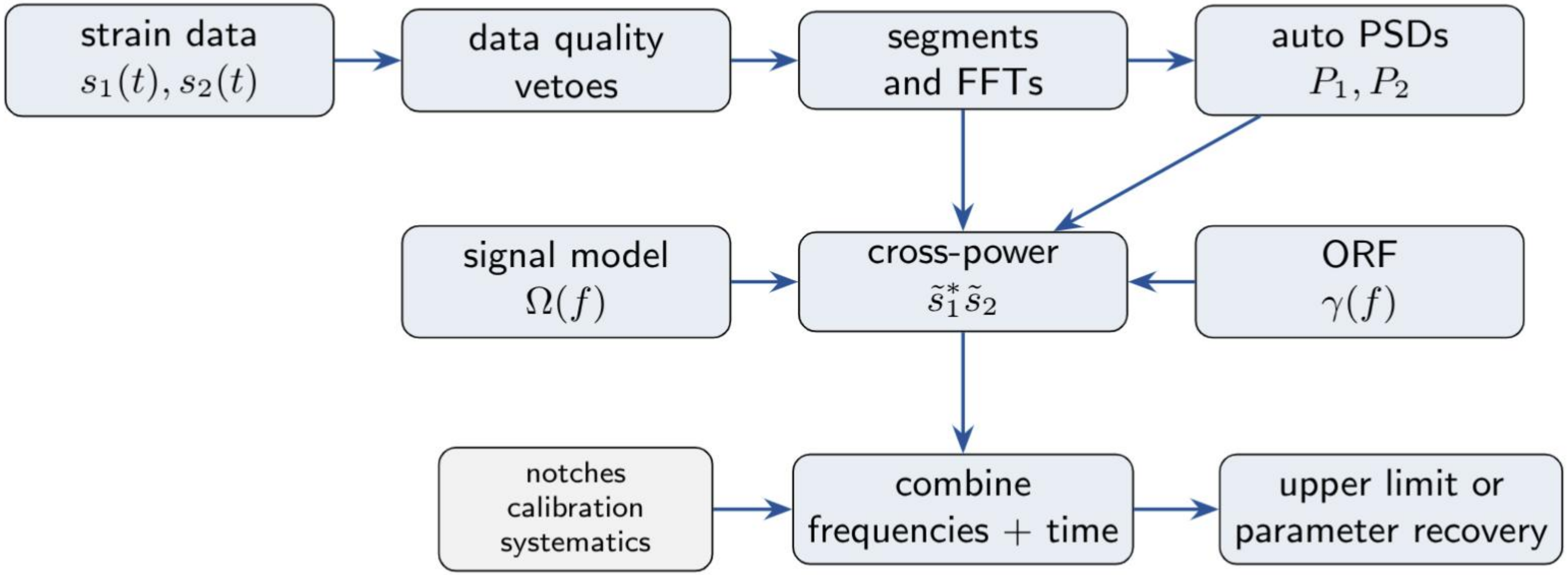
$$\ln \mathcal{B}^{\rm DW+CBC}_{\rm CBC} = -0.63$$





pyGWB

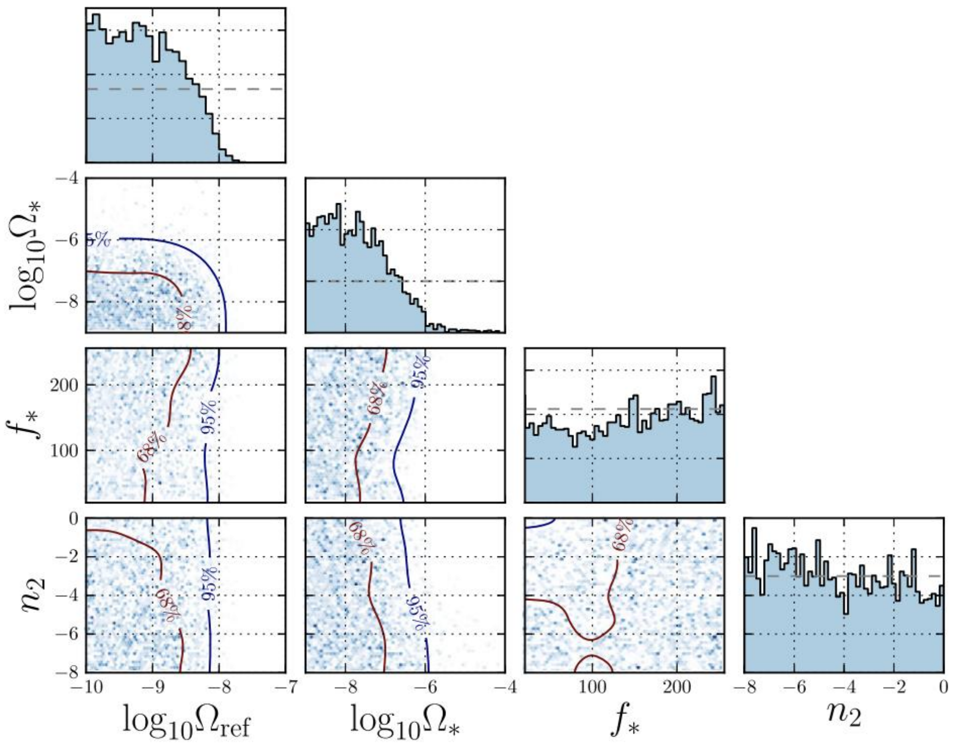
https://github.com/a-renzini/pygwb



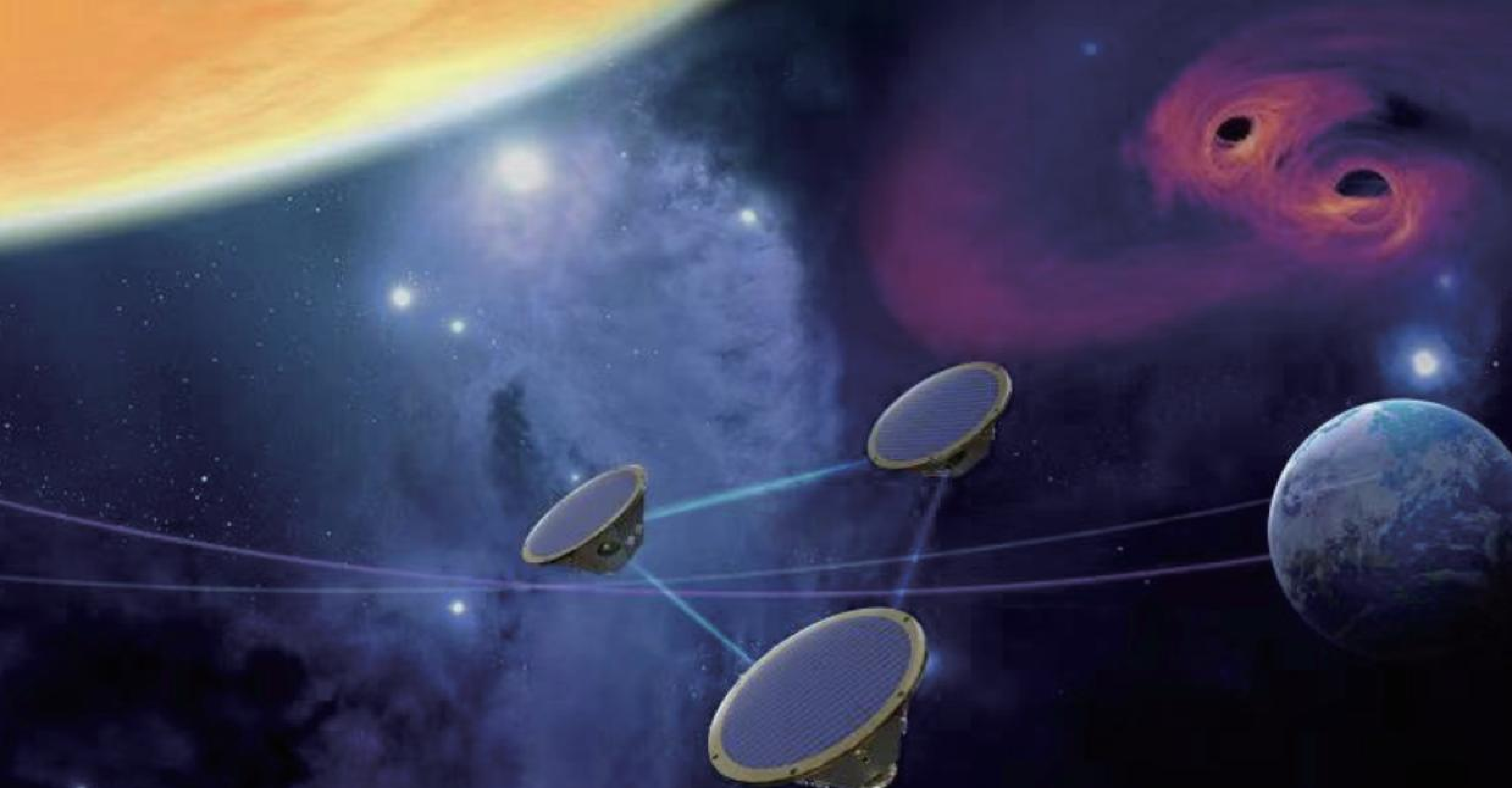
Renzini, et al, ApJ [2303.15696]

Reproducing O3 PT Studies

- Phase transitions + CBC



Romero et al, PRL [2303.15696]



空间引力波探测：太极、天琴、LISA

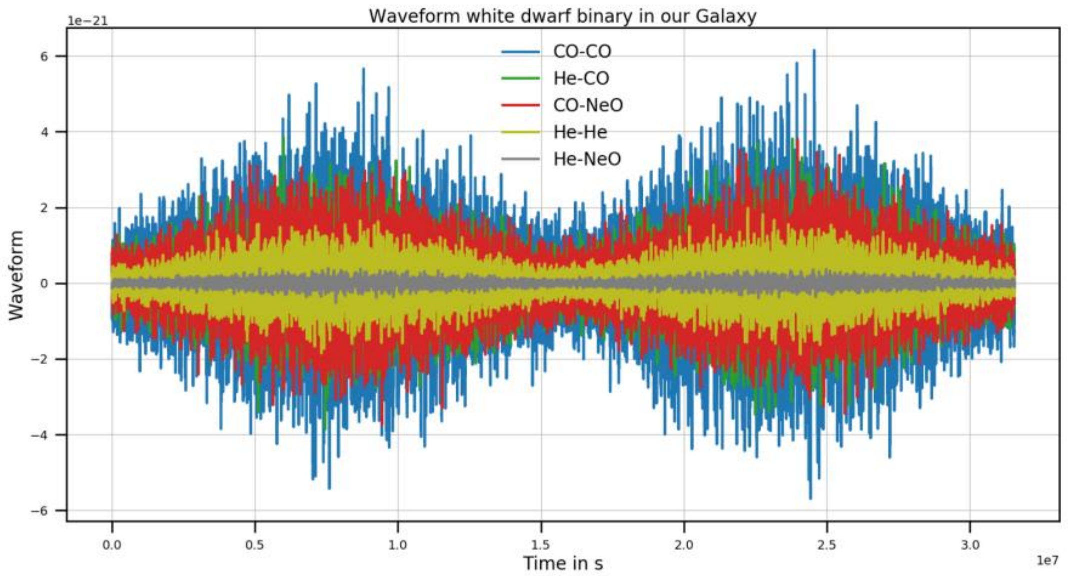
Sources

Cosmological

- Inflation
- Phase transitions (EW, GUT, etc)
- Cosmic strings, Domain walls
- SIGW
- PBH

Astrophysical

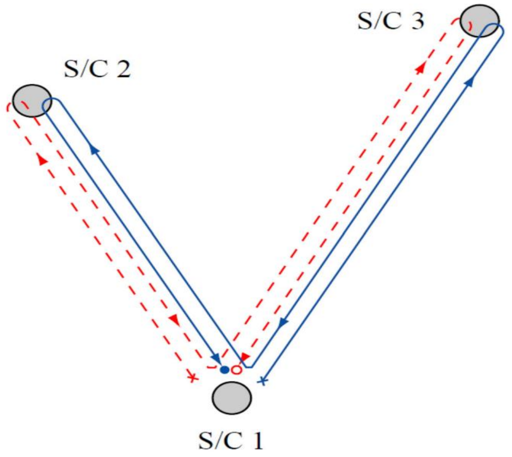
- Galactic DWD
- BBH, BNS, etc
- EMRI
- SMBH binary

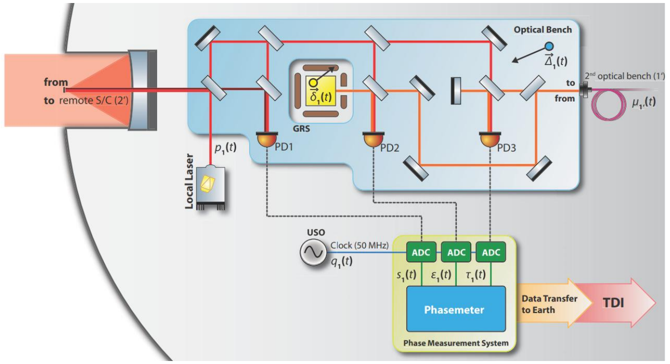


Detection in Space

Stochastic GW detection in space:

- Complicated, and correlated noise
- Complications from time-delay interferometry
- Solution: null channel method, or with a detector network





Cosmo SGB detectable down to $\Omega_{GW} \sim O(10^{-13})$

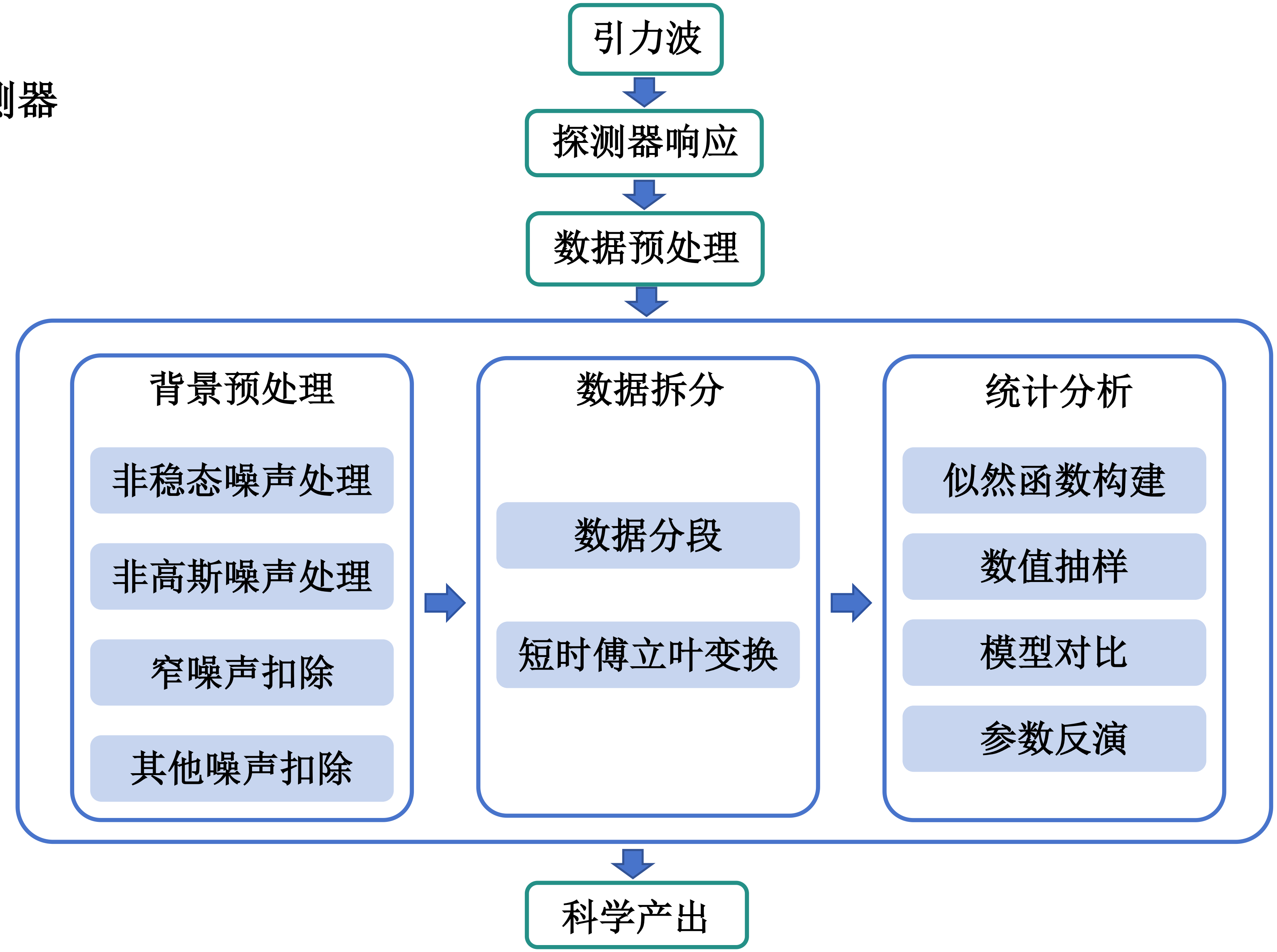
Boileau et al, MNRAS [2105.04283]

Studies on PT detection in space:

Gowling, Hindmarsh, Hooper, Torrado, JCAP [2209.13551]
Gowling, Hindmarsh, JCAP [2106.05984]
Boileau, et al, JCAP [2209.13277]
Huang, Bi, Cao, Huang, CPC [2309.14045]
Lewicki, et al, PRD [2403.03769]
Caprini, et al, JCAP [2403.03723]

数据处理流程

基于单探测器

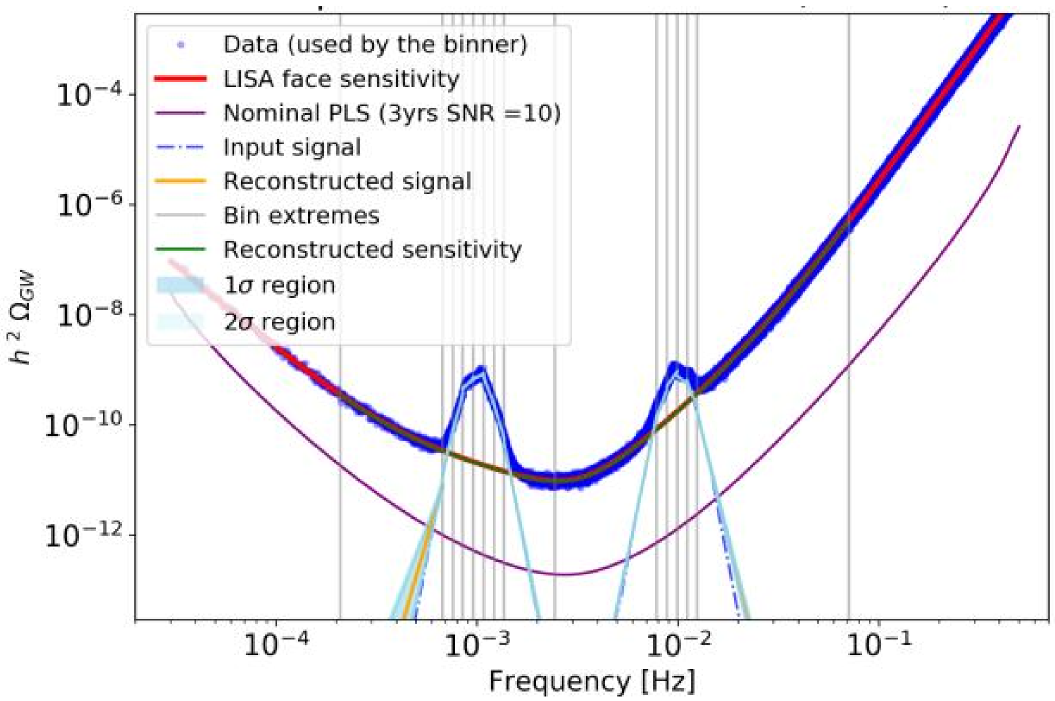


Agnostic Detection

Fit to data bin by bin: SGWBinner

$$h^2\,\Omega_{\rm GW}\,(f,\,\vec{s}_i) = \; 10^{\alpha_i}\,\left(\frac{f}{\sqrt{f_{{\rm min},i}\,f_{{\rm max},i}}}\right)^{p_i}\;\theta\,(f - f_{{\rm min},i})\;\theta\,(f_{{\rm max},i} - f)$$

$$h^2\,\Omega_{\rm GW}\,(f,\,\vec{s}_i) = 10^{\alpha_i}\;\theta\,(f - f_{{\rm min},i})\;\theta\,(f_{{\rm max},i} - f)$$



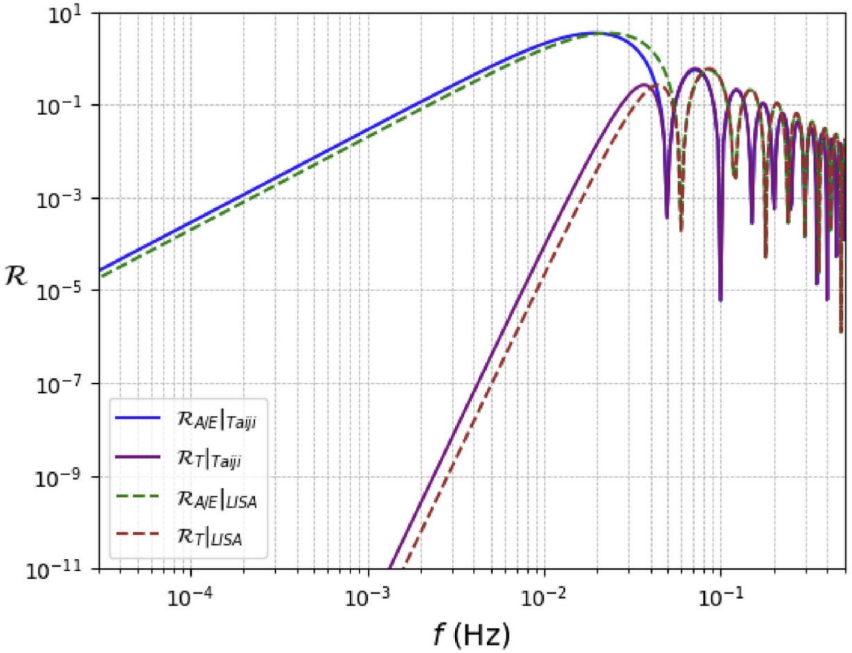
Other Methods

- Excess power method: Karnesis, Petiteau, CQG [1906.09027]
- Principal component analysis: Pieroni, Barausse, JCAP [2004.01135]
- ...

Null Channel Method

Adams, Cornish, PRD [1002.1291]

TDI: cancel dominant noise from laser frequency fluctuation

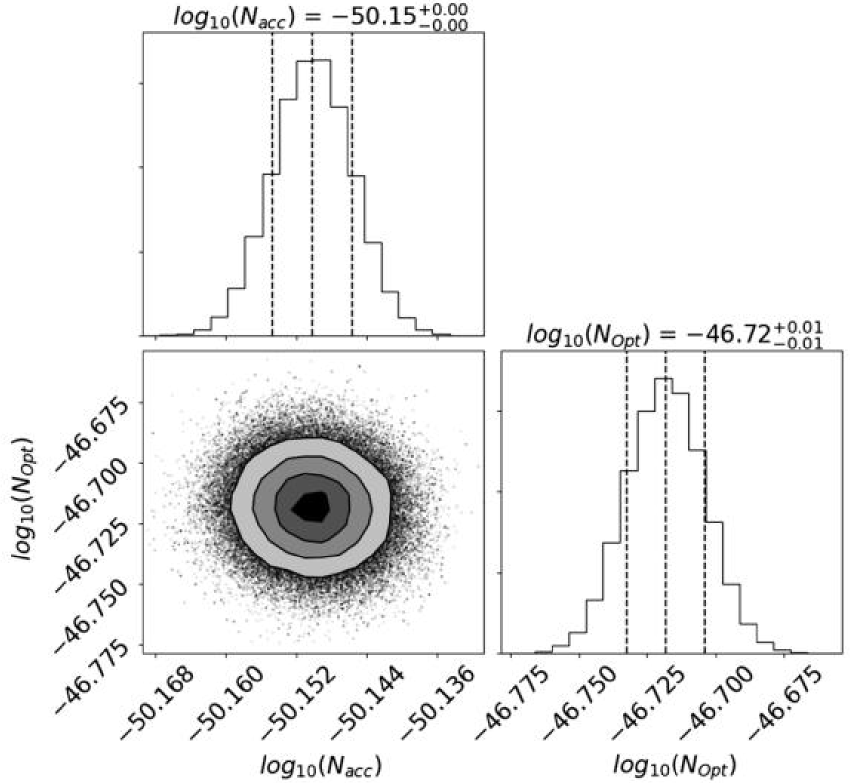


Shuo Guan, HG, Dian Jiao, Qingyuan Liang, Lei Wu, Yang Zhang, PRD [2511.00996]

$$\begin{cases} A = \frac{1}{\sqrt{2}}(Z - X) \\ E = \frac{1}{\sqrt{6}}(X - 2Y + Z) \\ T = \frac{1}{\sqrt{3}}(X + Y + Z). \end{cases}$$

$$P_a(f) = S_a(f) + N_a(f)$$

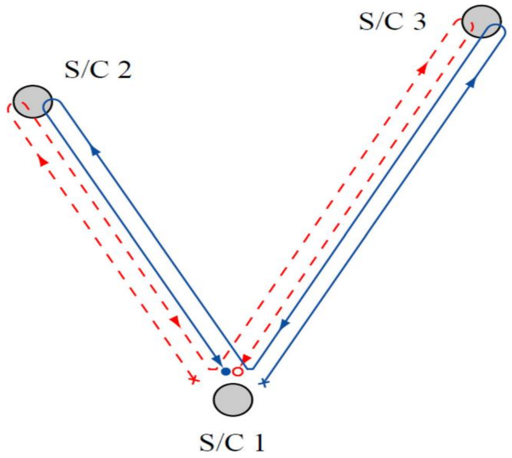
$$S_a(f) = \frac{3H_0^2}{4\pi^2}\frac{\Omega_{\rm GW}(f)}{f^3}\mathcal{R}_a(f)$$



Boileau, Christensen, Meyer, Cornish, PRD [2011.05055]

TDI optimization: Wang, Li, Xu, Fan, PRD [2201.10902], ...

Detector Response



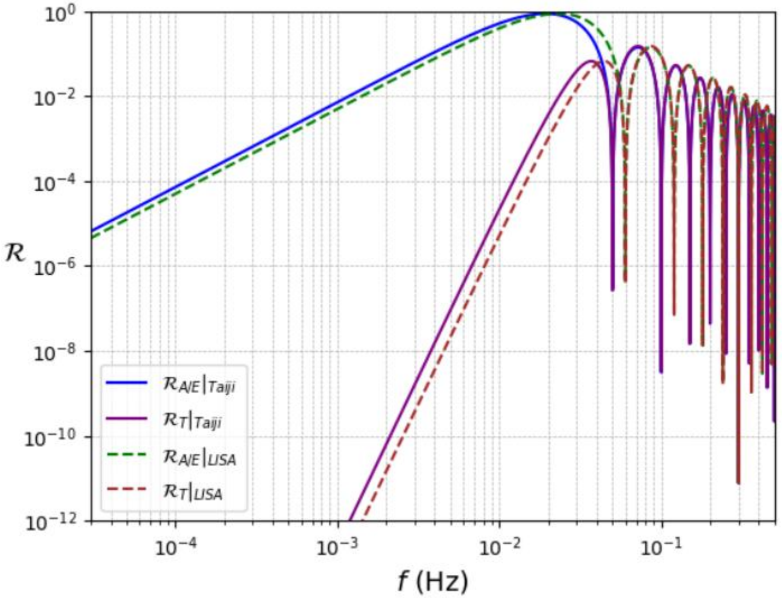
3 orthogonal TDI channels

TDI: time-delay interferometry
Tinto, Dhurandhar, LRR, 2021

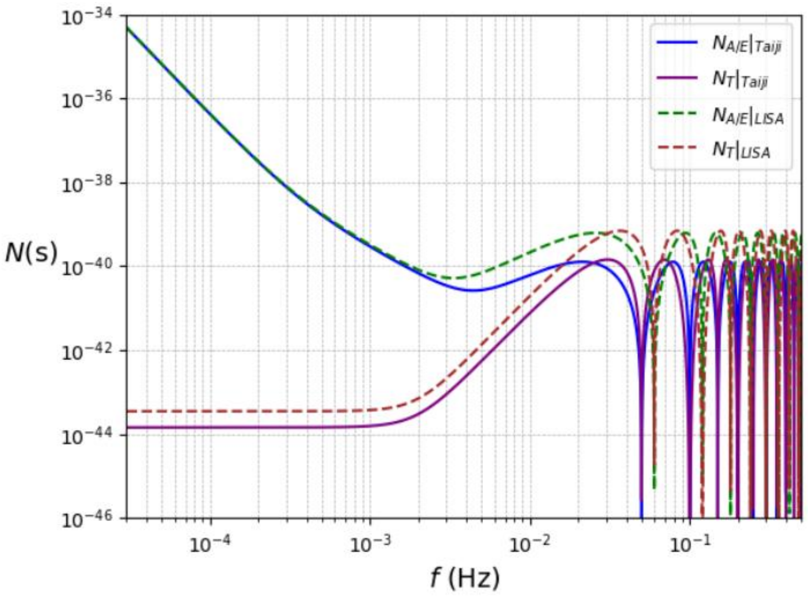
$$\langle \tilde{d}_a(f)\tilde{d}_b^\star(f')\rangle = \frac{1}{2}P_a(f)\delta_{ab}\delta(f-f')$$

$$P_a(f) = \frac{3H_0^2}{4\pi^2}\frac{\Omega_{\rm GW}}{f^3}R_a(f) + N_a(f)$$

Response functions



Noise spectra



Noise Sources

There are many noise sources. The two usually considered：

$$N_1(f) = \frac{1}{L^2}\left\{4S_{\rm OMS}(f) + 8\left[1+\cos^2\left(\frac{f}{f_*}\right)\right] \times \frac{1}{(2\pi f)^4} S_{\rm acc}(f)\right\}|W(f)|^2,$$

$$N_2(f) = -\frac{1}{L^2}\left[2S_{\rm OMS}(f) + \frac{8}{(2\pi f)^4} S_{\rm acc}(f)\right] \times \cos\left(\frac{f}{f_*}\right)|W(f)|^2.$$

$$N_A = N_E = N_1 - N_2,$$

$$N_T = N_1 + 2N_2,$$

- Optical path-length fluctuation

$$\sqrt{S_{\rm OMS}(f)} = \delta x \sqrt{1+\left(\frac{2\ {\rm mHz}}{f}\right)^4}\left(\frac{\rm m}{\sqrt{\rm Hz}}\right)$$

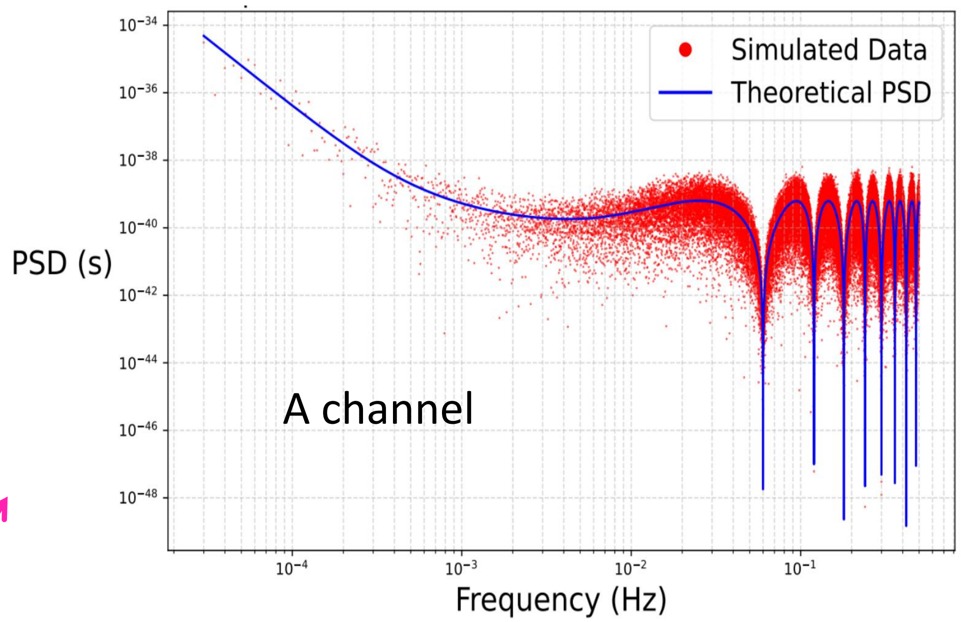
- Test mass acceleration noise

$$\sqrt{S_{\rm acc}(f)} = N_{\rm acc}\sqrt{1+\left(\frac{0.4\ {\rm mHz}}{f}\right)^2}\sqrt{1+\left(\frac{f}{8\ {\rm mHz}}\right)^4} \times \left(\frac{\rm m}{{\rm s}^2\sqrt{\rm Hz}}\right),$$

$$\delta x = 8 \times 10^{-12}$$

$$N_{\rm acc} = 3 \times 10^{-15}$$

Likelihood



The core of the statistical analysis: likelihood

both signal and noise behave as random variables

$$\mathcal{L} = \prod_{\kappa=1}^{N_0} \prod_{k=1}^{N/2} \frac{1}{8\pi^3 \sigma_A^2 \sigma_E^2 \sigma_T^2} \exp\left[- \sum_{a=A,E,T} \frac{\left| \tilde{d}_a^\kappa(f_k) \right|^2}{2\sigma_a^2} \right]$$

$$\sigma_a^2 = \frac{T f_s^2}{4} \left[\frac{3H_0^2}{4\pi^2} \frac{\Omega_{\rm GW}}{f^3} R_a(f) + N_a(f) \right]$$

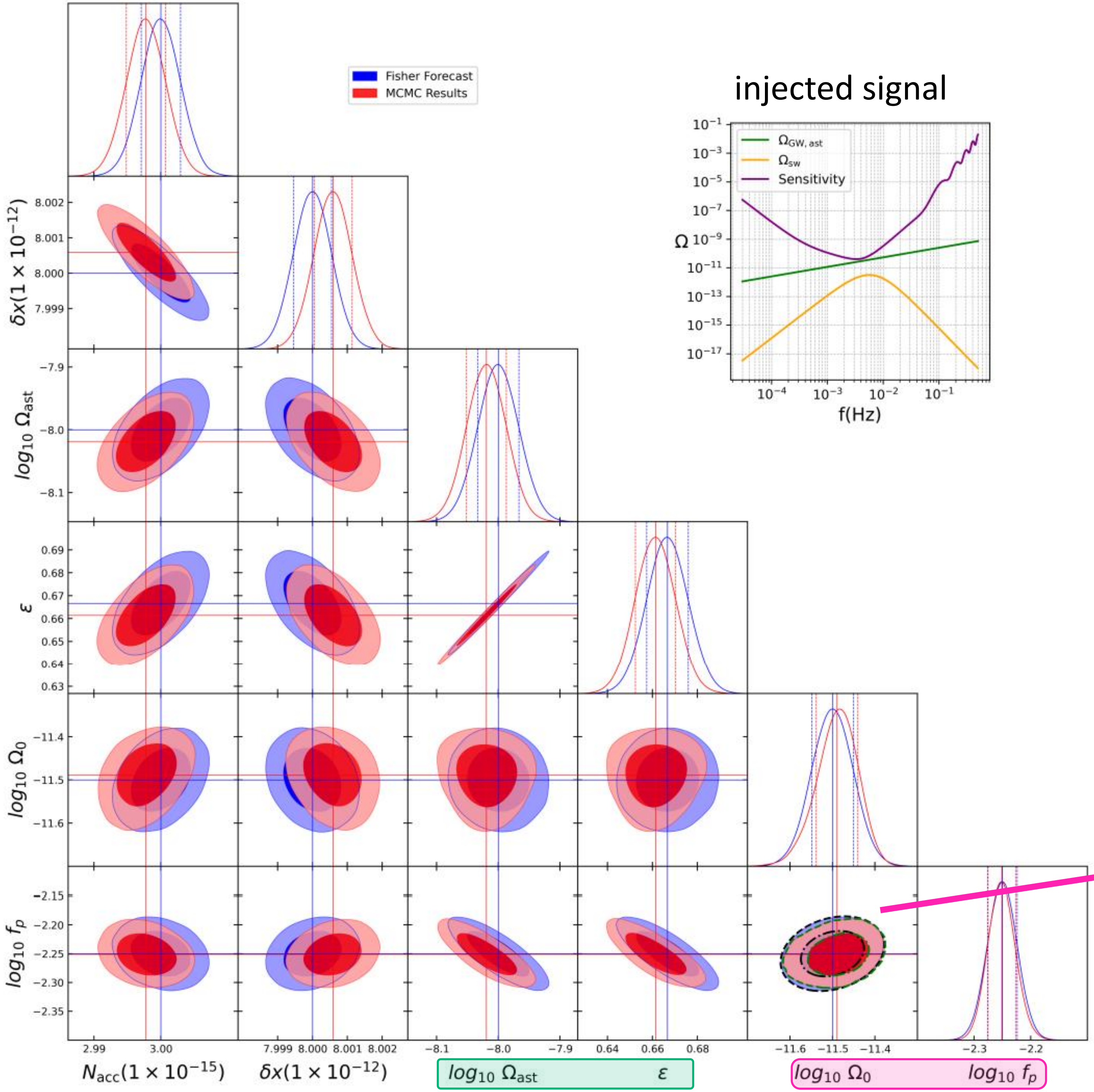
$N_0 = 126$: number of segments

$N = 10^6$: frequency bins in one segment

$T = 1.26 \times 10^8\,\mathrm{s}$: observation time

$f_s = 1\,\mathrm{Hz}$: sampling frequency

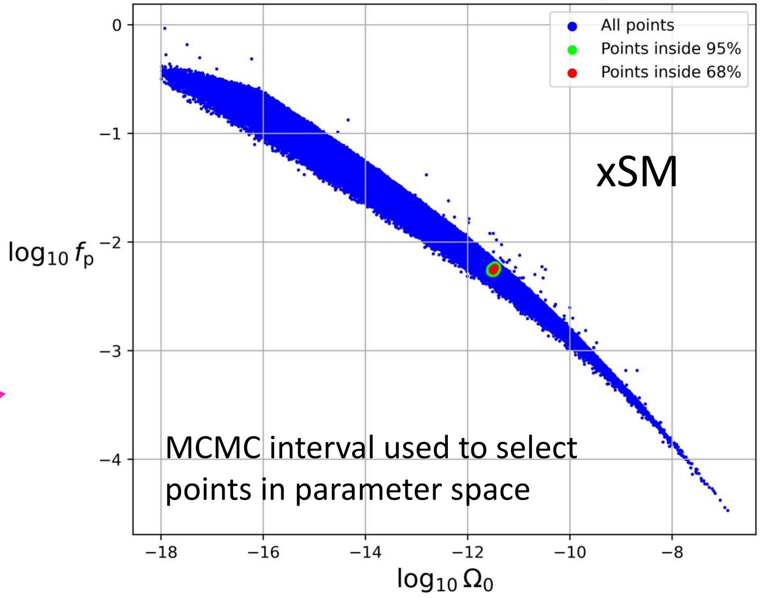
astrophysical background (extra-galactic BBH, BNS)
astrophysical foreground (double white dwarf)
cosmological background (PT)

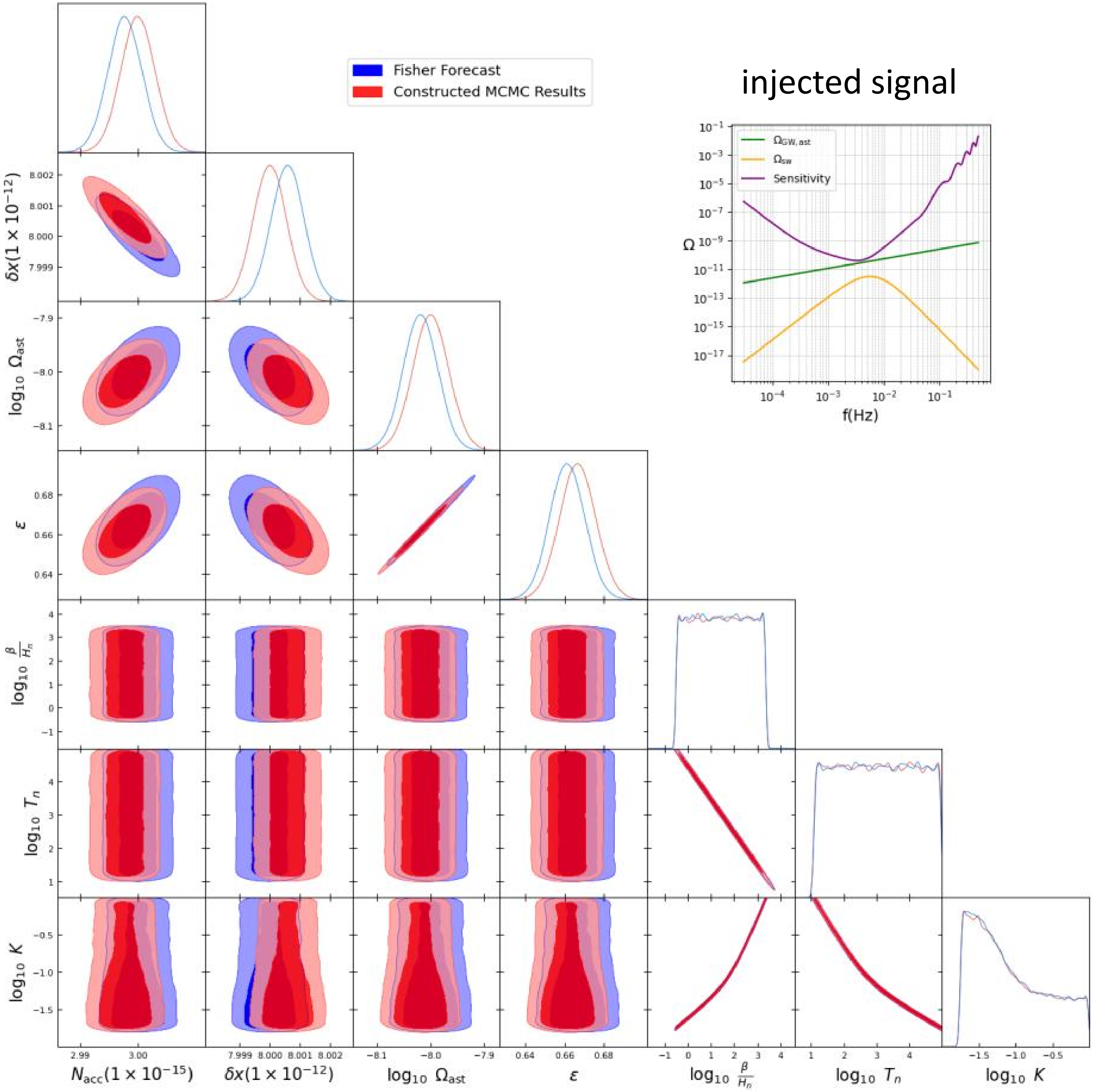


Simplified SW spectrum with no parameter degeneracy

$$\Omega_{\mathrm{sw}}(f) = \Omega_0 \left(\frac{f}{f_{\mathrm{p}}}\right)^3 \left(\frac{7}{4 + 3\,(f/f_{\mathrm{p}})^2}\right)^{7/2}$$

$$\Omega_{\mathrm{astro}}(f) = \Omega_{\mathrm{ast}} \left(\frac{f}{f_{\mathrm{ref}}}\right)^{\varepsilon}$$





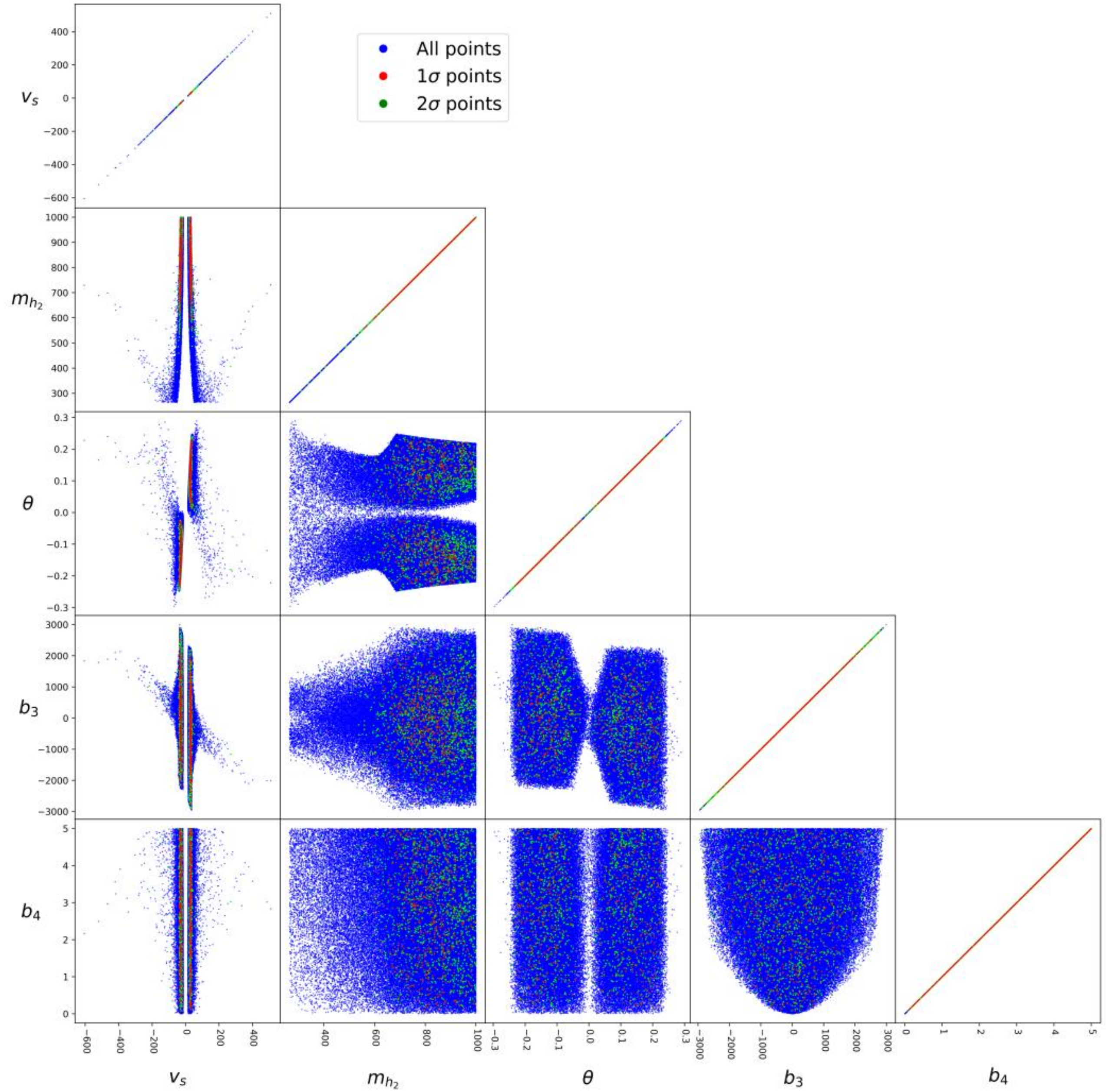
$$\Omega_{\rm sw}(f)h^2 = 2.65\times10^{-6}\left(\frac{H_{\rm n}}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3}v_{\rm w}\left(\frac{f}{f_{\rm p}}\right)^3\left(\frac{7}{4+3(f/f_{\rm p})^2}\right)^{7/2}\Upsilon(\tau_{\rm sw}),$$

$$f_{\rm p} = \frac{19}{v_{\rm w}}\left(\frac{\beta}{H_{\rm n}}\right)\left(\frac{T_{\rm n}}{100\ {\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}10^{-6}\ {\rm Hz}$$

Parameter degeneracy forbids a determination of phase transition parameters.

Future solutions: more observables

xSM Model



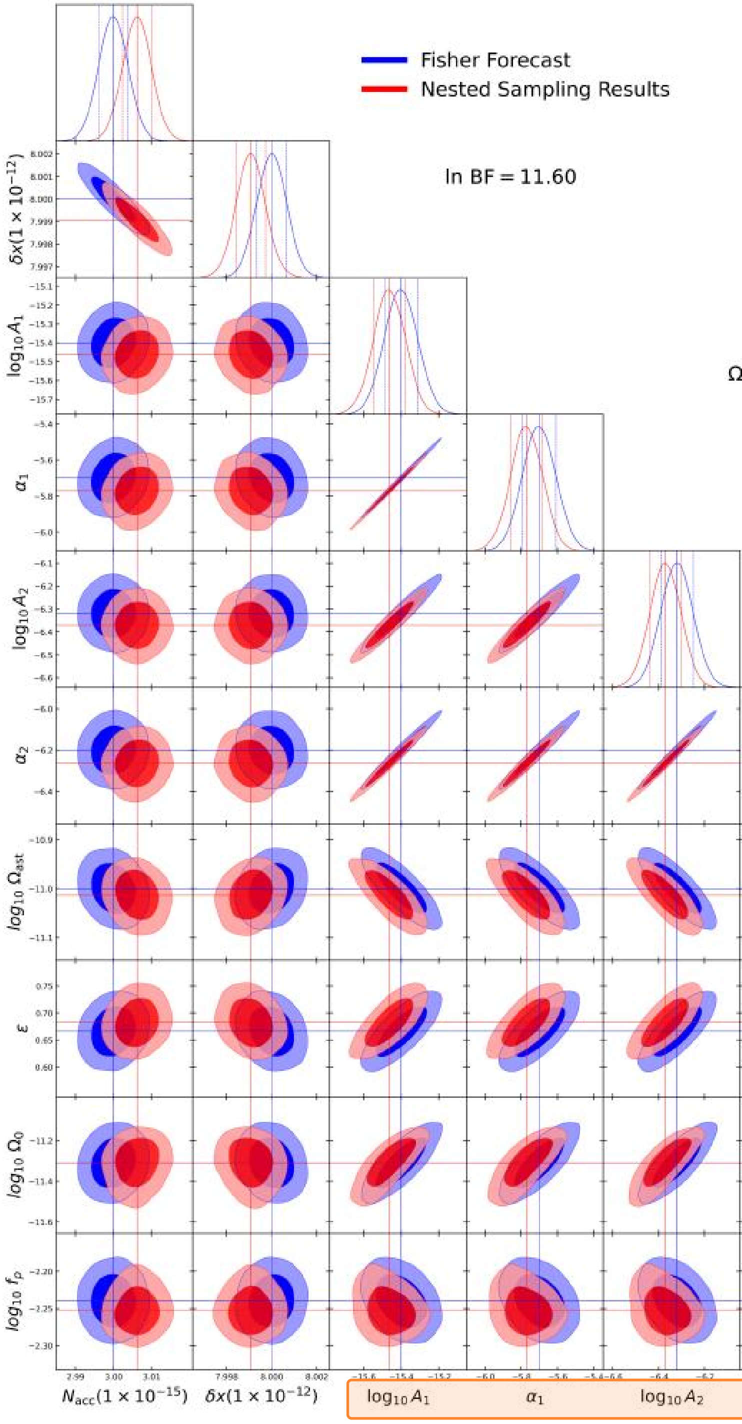
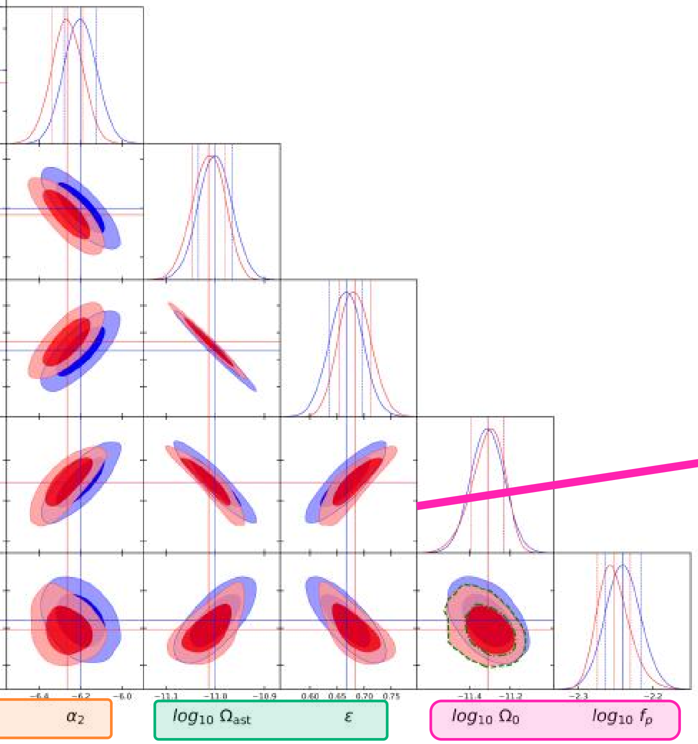
Barger, Langacker, McCaskey, Ramsey-Musolf, Shaughnessy, PRD [0706.4311]

$$V(\phi, S) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 + \frac{a_1}{2} H^\dagger H S + \frac{a_2}{2} H^\dagger H S^2 + \frac{b_2}{2} S^2 + \frac{b_3}{3} S^3 + \frac{b_4}{4} S^4$$

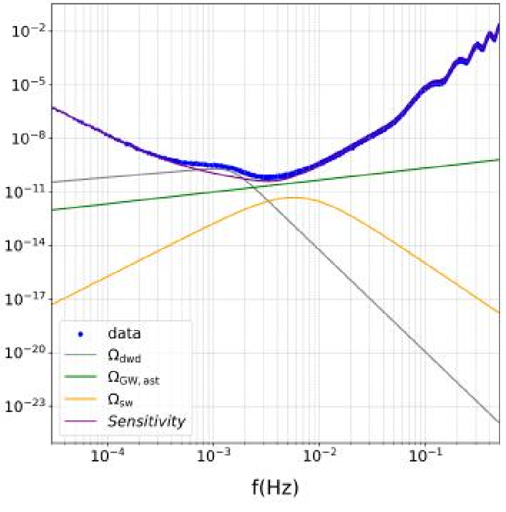
$$v_s, \quad m_{h_2}, \quad \theta, \quad b_3, \quad b_4$$

Points survive pheno constraints.

Shuo Guan, HG, Dian Jiao, Qingyuan Liang, Lei Wu, Yang Zhang, PRD [2511.00996]

injected signal

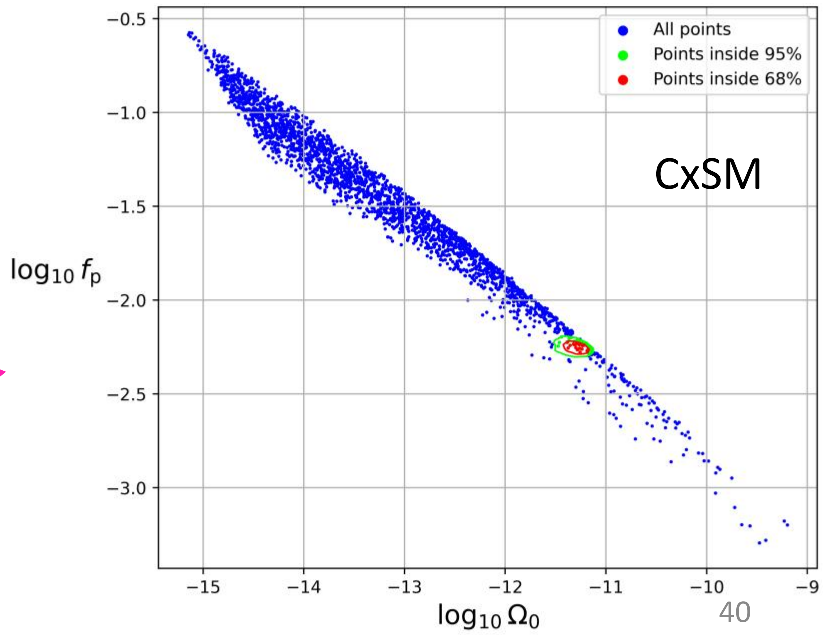


Simplified SW spectrum with no parameter degeneracy

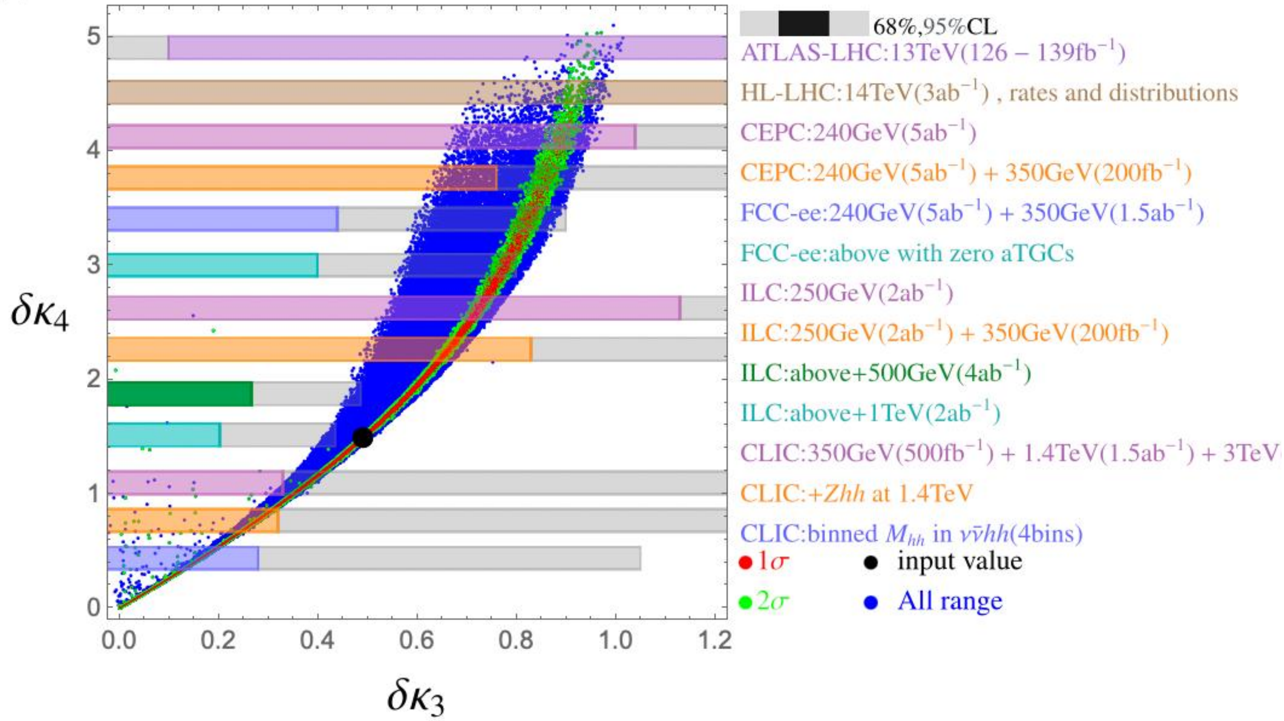
$$\Omega_{\rm sw}(f) = \Omega_0 \left(\frac{f}{f_{\rm p}}\right)^3 \left(\frac{7}{4+3\,(f/f_{\rm p})^2}\right)^{7/2}$$

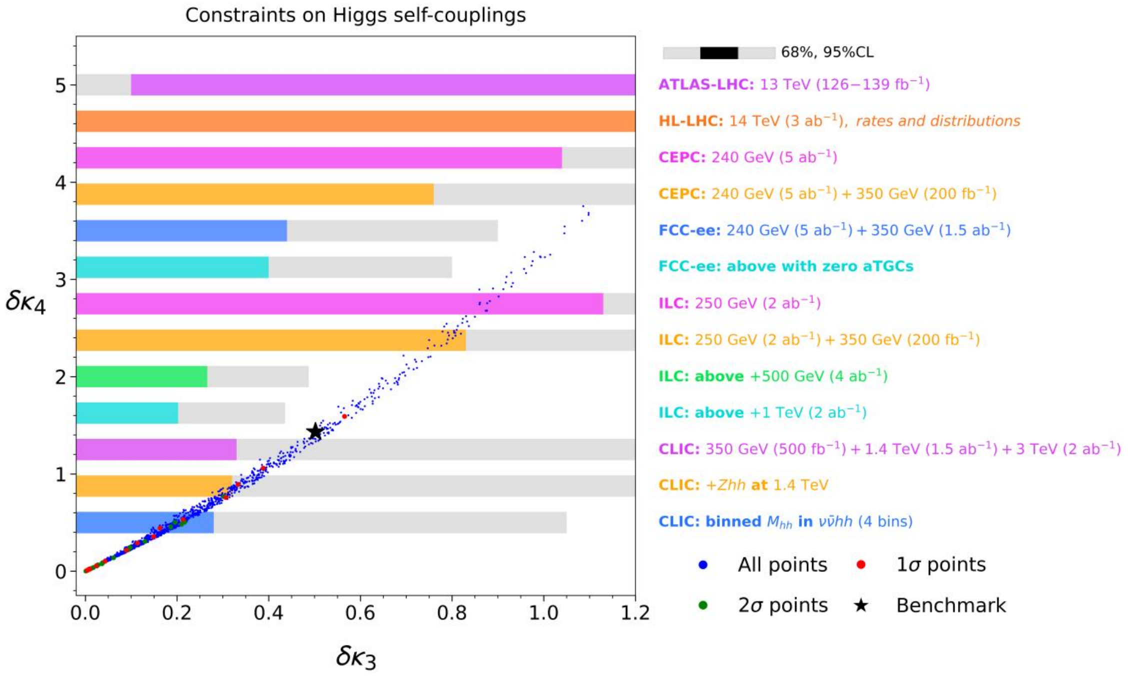
$$\Omega_{\rm astro}(f) = \Omega_{\rm ast}\left(\frac{f}{f_{\rm ref}}\right)^{\varepsilon}$$

$$\Omega_{\rm dwd}(f) = \frac{A_1\,(f/f_*)^{\alpha_1}}{1 + A_2\,(f/f_*)^{\alpha_2}}$$



Higgs Self-Couplings

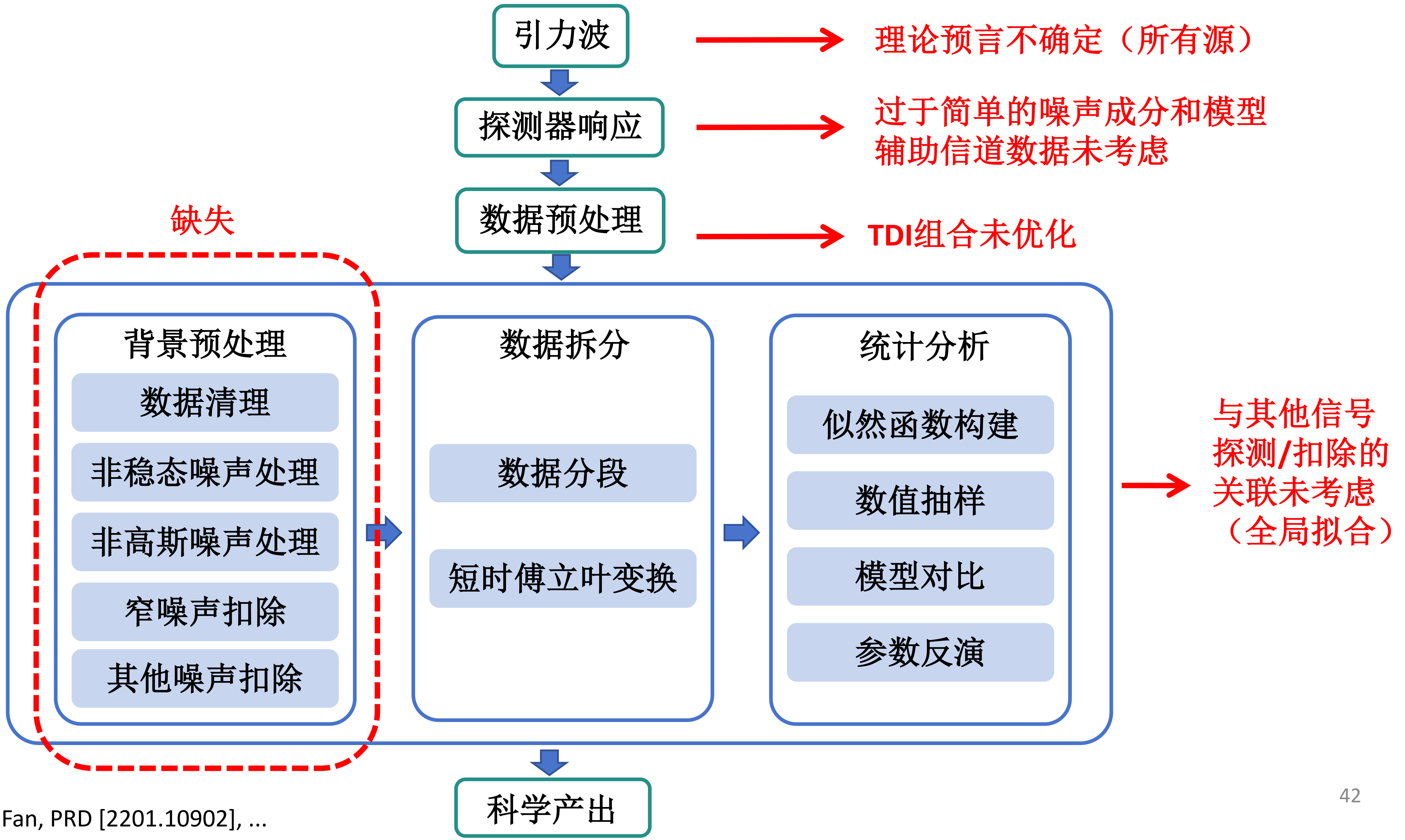




Shuo Guan, HG, Dian Jiao, Qingyuan Liang, Lei Wu, Yang Zhang, PRD [2511.00996]

Qingyuan Liang, Ligong Bian, HG, Yongcheng Wu, PRD [2511.21488]

未来挑战



TDI优化：Wang, Li, Xu, Fan, PRD [2201.10902], ...