Black Hole Superradiance

Shou-Shan Bao（鲍守山）

ssbao@sdu.edu.cn

山东大学(青岛)

第四届新物理数值计算与模拟前沿讲习班@新乡

Outline

Superradiance-History

- **Superradiance** 最早由 ***Robert H. Dicke*** 在1954年的经典论文中提出：

N个激发原子共同辐射时，

- 普通情况下

$$I \propto N$$

- 而若原子保持相干，则

$$I \propto N^2$$

PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

Coherence in Spontaneous Radiation Processes

R. H. Dicke
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received August 25, 1953)

By considering a radiating gas as a single quantum-mechanical system, energy levels corresponding to certain correlations between individual molecules are described. Spontaneous emission of radiation in a transition between two such levels leads to the emission of coherent radiation. The discussion is limited first to a gas of dimension small compared with a wavelength. Spontaneous radiation rates and natural line breadths are calculated. For a gas of large extent the effect of photon recoil momentum on coherence is calculated. The effect of a radiation pulse in exciting "super-radiant" states is discussed. The angular correlation between successive photons spontaneously emitted by a gas initially in thermal equilibrium is calculated.

量子光学中的奠基论文

集体相干增强辐射（coherent emission）

Superradiance-History

➢ **旋转超辐射**：Yakov Zel’dovich（1971）

让一束电磁波照射一个旋转的金属圆柱体。若圆柱具有有限电阻（耗散），且入射波的方位角量子数 m 满足

$$0<\omega<m\Omega$$

ω 为波频率，Ω 为旋转角速度

此时，圆柱体的吸收系数变为负值，

旋转体的转动动能将转移给散射波，

从而实现对电磁波的**放大**。

GENERATION OF WAVES BY A ROTATING BODY

Ya.B. Zel'dovich
Institute of Applied Mathematics, USSR Academy of Sciences
Submitted 9 July 1971
ZhETF Pis. Red. 14, No. 4, 270 - 272 (20 August 1971)

An axially-symmetrical body rotating inside a resonator cavity is capable of amplifying definite oscillation modes inside the resonator, transferring the rotation energy to these oscillations.

- **波从旋转体提取转动能，**这是黑洞超辐射最直接的前身。
- 并没有使用*superradiance*这个词。他的论文用的是*amplification*

Superradiance-History

➢2020年首次在实验室内验证 Zel‘dovich 效应

- 系统：旋转的声波吸收体（泡沫盘）
- 结果：精确测量到当 $\omega<m\Omega$ 时声波被放大，最高能量增益约 30%。

➢ 2023年首次在经典电磁体系中证实

- 困难：机械旋转速度通常远低于电磁波频率
- 结果：观测了电磁场被机械旋转导体增强的现象
- 验证了Zel‘dovich 条件

nature physics

ARTICLES

https://doi.org/10.1038/s41567-020-0944-3

Check for updates

Amplification of waves from a rotating body

Marion Cromb[1], Graham M. Gibson[1], Ermes Toninelli[1], Miles J. Padgett[1]✉, Ewan M. Wright[2] and Daniele Faccio[1,2]✉

In 1971, Zel'dovich predicted that quantum fluctuations and classical waves reflected from a rotating absorbing cylinder will gain energy and be amplified. This concept, which is a key step towards the understanding that black holes may amplify quantum fluctuations, has not been verified experimentally owing to the challenging experimental requirement that the cylinder rotation rate must be larger than the incoming wave frequency. Here, we demonstrate experimentally that these conditions can be satisfied with acoustic waves. We show that low-frequency acoustic modes with orbital angular momentum are

nature communications

Article

https://doi.org/10.1038/s41467-024-49689-w

Amplification of electromagnetic fields by a rotating body

Received: 12 December 2023

Accepted: 11 June 2024

Published online: 27 June 2024

Check for updates

M. C. Braidotti[1], A. Vinante[2], M. Cromb[3], A. Sandakumar[3], D. Faccio[1,4] & H. Ulbricht[3]✉

In 1971, Zel'dovich predicted the amplification of electromagnetic (EM) waves scattered by a rotating metallic cylinder, gaining mechanical rotational energy from the body. This phenomenon was believed to be unobservable with electromagnetic fields due to technological difficulties in meeting the condition of amplification that is, the cylinder must rotate faster than the frequency of the incoming radiation. Here, we measure the amplification of an electromagnetic field, generated by a toroid LC-circuit, scattered by an aluminium cylinder spinning in the toroid gap. We show that when the Zel'dovich condition is met, the resistance induced by the cylinder becomes negative implying amplification of the incoming EM fields. These results reveal the connection between the concept of induction generators and the physics of this fundamental physics effect and open new prospects towards testing the Zel'dovich mechanism in the quantum regime, as well as related quantum friction effects.

Superradiance-History

- **Misner最早将 superradiance 引入 Kerr 黑洞**

  首次提出：Kerr 黑洞可以放大标量波；

  $$\omega < m\Omega_H$$

  时发生波放大；并把这一现象称为**superradiance**

  超辐射条件也被称为 **Zel'dovich–Misner condition**

- **超辐射不稳定性（Superradiant Instability）**

  - BH Bomb: (Press&Teukolsky1972)The wave grows exponentially with time, until the mirror breaks.
  - 这也是第一篇正式发表、明确使用黑洞 "superradiant scattering" 的论文

Stability of Kerr black holes against scalar perturbations

Show affiliations

Misner, Ch.

The author is introducing the notion of "superradiance'" for a phenomenon of amplification of waves by a rotating (Kerr) black hole.

nature

Explore content ˅ About the journal ˅ Publish with us ˅ Subscribe

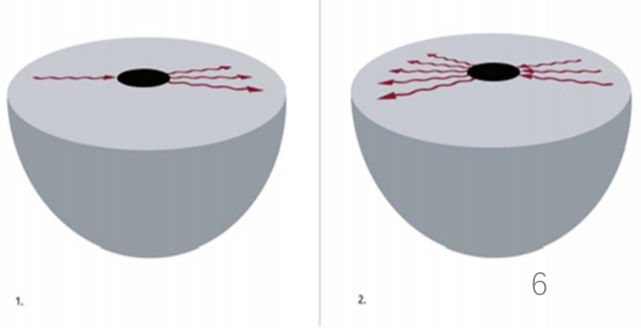
nature > letters > article

Letter | Published: 28 July 1972

Floating Orbits, Superradiant Scattering and the Black-hole Bomb

WILLIAM H. PRESS & SAUL A. TEUKOLSKY

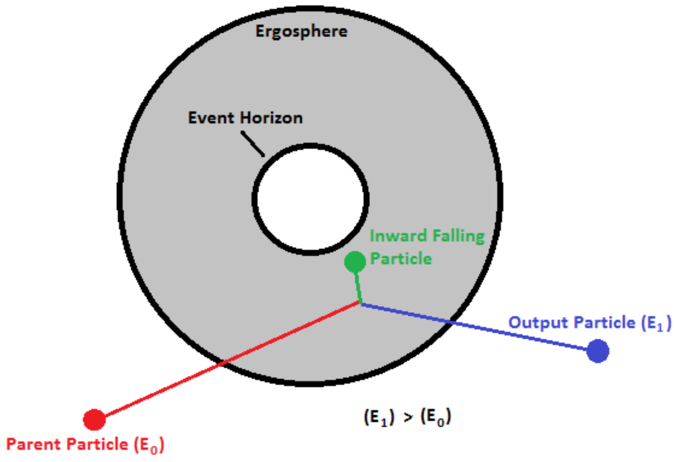
Nature **238**, 211–212 (1972) | Cite this article



Superradiance-History

➢Penrose (1971)

- Penrose Process: Particle energy extraction
- Superradiance: *wave analogue* of the Penrose process



Letters to Editor | Published: 08 February 1971

Extraction of Rotational Energy from a Black Hole

R. PENROSE & R. M. FLOYD

Nature Physical Science **229**, 177–179 (1971) | Cite this article

Superradiance-History

- ➢**Teukolsky** (1972)推导出了著名**Teukolsky Equation**，统一描述：scalar、Dirac、Maxwell、gravitational wave 在 Kerr 背景上的传播，证明了Kerr 黑洞上的各种场方程都可以分离变量。

Rotating Black Holes: Separable Wave Equations for Gravitational and Electromagnetic Perturbations
Physical Review Letters **29**, 1114 (1972)
Perturbations of a Rotating Black Hole. I. Fundamental Equations for Gravitational, Electromagnetic, and Neutrino-Field Perturbations
Astrophysical Journal **185**, 635 (1973)

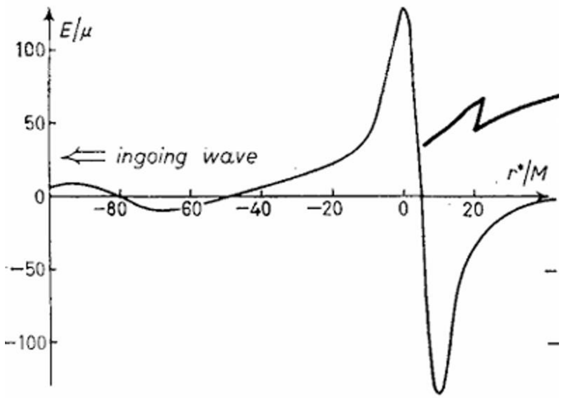
- ➢**Starobinsky**（1973）计算了标量场、电磁波和引力波的反射系数，第一个给出了完整解析结果，其中引力的放大率，发现放大因子随着自旋迅速增加。

A. Starobinski, "Amplification of waves during reflection from a rotating black hole," *Zh. Eksp. Teor. Fiz.* **64** (1973) 48. (Sov. Phys. - JETP, 37, 28, 1973).

A. A. Starobinski and S. M. Churilov, "Amplification of electromagnetic and gravitational waves scattered by a rotating black hole," *Zh. Eksp. Teor. Fiz.* **65** (1973) 3. (Sov. Phys. - JETP, 38, 1, 1973).

- ➢**Massive Scalar：** T. Damour, N. Deruelle & R. Ruffini (1976) 得到在超辐射条件下，存在超辐射束缚态:

$$\omega = \omega_0 - i\Gamma/2 \qquad \omega_0 \gg \Gamma, \quad \omega_0 < \mu, \qquad \Gamma < 0 \quad \propto e^{-\Gamma t}$$



Outline

Kerr Black Hole

➢ Kerr BH

$$ds^2 = \left(1 - \frac{2r_g r}{\Sigma}\right)dt^2 + \frac{4ar_g r}{\Sigma}\sin^2\theta dt d\varphi - \frac{\Sigma}{\Delta}dr^2$$
$$- \Sigma d\theta^2 - \left[(r^2 + a^2)\sin^2\theta + 2\frac{r_g r}{\Sigma}a^2\sin^4\theta\right]d\varphi^2,$$

$$a = J/M, \quad r_g = GM$$

$$\Omega_H = \frac{a}{r_+^2 + a^2}$$

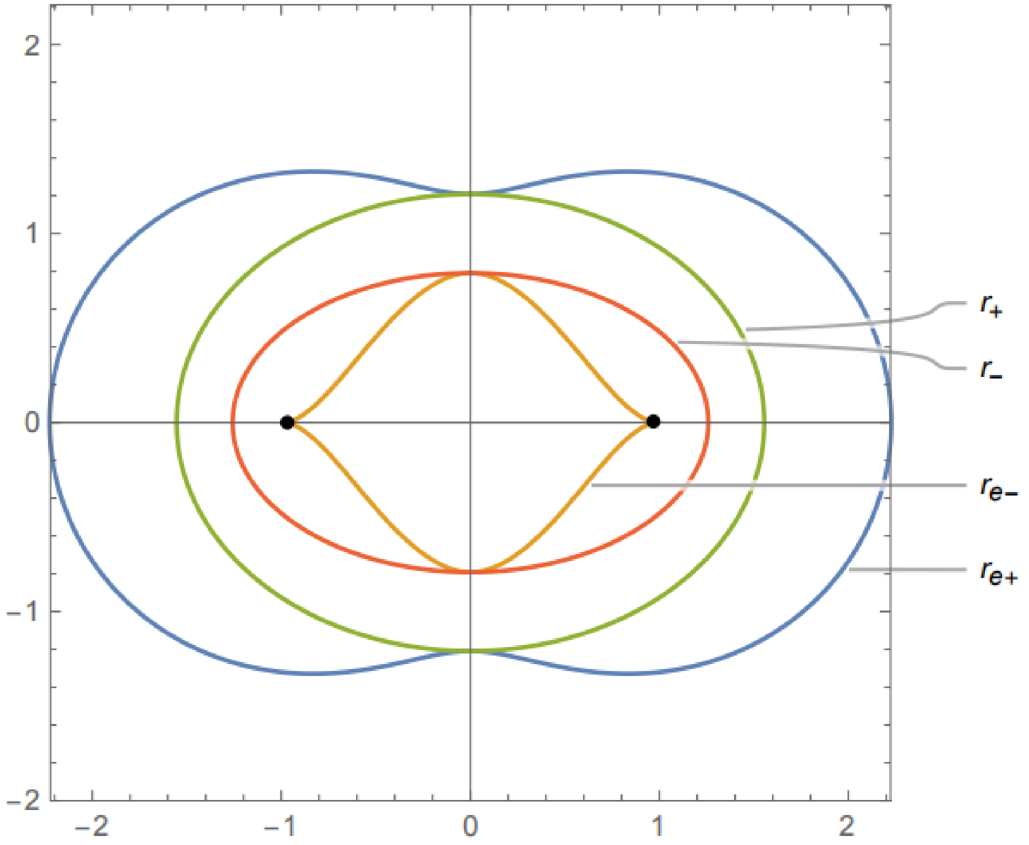
$$\Delta = r^2 - 2r_g r + a^2$$

$$G = \hbar = c = 1$$

$$\Sigma = r^2 + a^2\cos^2\theta$$

Horizons: $\Delta = 0 \Rightarrow r_\pm = r_g \pm \sqrt{r_g^2 - a^2}$

Ergosphere: $1 - \frac{2r_g r}{\Sigma} = 0 \Rightarrow r_{E,\pm} = r_g \pm \sqrt{r_g^2 - a^2\cos\theta^2}$



Kerr Black Hole

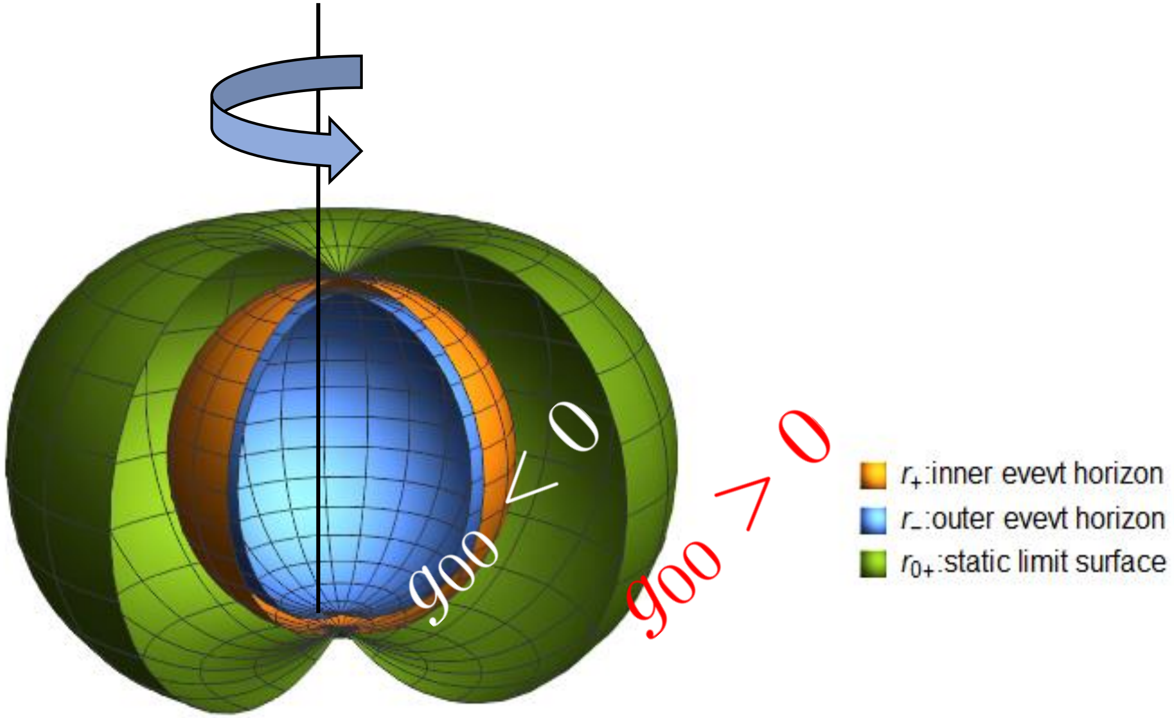
- **If a particle is at rest in Minkowski space**

$$ds^2 = c^2 dt^2 > 0 \text{ Time-like}$$

- **If a particle is at rest in Kerr space**

$$r > r_{0+} : ds^2 = g_{00} dt^2 > 0 \text{ Time-like}$$

$$r < r_{0+} : ds^2 = g_{00} dt^2 < 0 \text{ Space-like}$$



The physical worldline can not be space-like.

- Superradiance is a process to extract energy from the BH in the ergoregion.

Scalar Superradiance

➢ Klein-Gordon Equation

$$(\nabla^2 + \mu^2)\phi = 0$$

$$\nabla^2\phi = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\, g^{\mu\nu}\partial_\nu\phi)$$

➢ Separation of Variables

$$\Phi = \sum_{nlm} e^{-i\omega_{nlm}t} e^{im\phi} S_{lm}(\theta) R_{nlm}(r) + \text{ c.c.}$$

Press &Teukolsky,(1973)

where

- Angular Part S: Spheroidal Harmonics
- Radial Part R(r): Schrodinger-like Equation

Angular Wave Function

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dS_{lm}}{d\theta}\right)+\left[a^2\left(\omega^2-\mu^2\right)\cos^2\theta-\frac{m^2}{\sin^2\theta}+\Lambda_{lm}\right]S_{lm}(\theta)=0$$

$$\gamma^2 = a^2(\omega^2-\mu^2)$$

- When $a \to 0$ ($\gamma \to 0$), $S_{lm} \to Y_{lm}$ 退化为球谐函数
- If $\gamma \ll 1$, we have $\Lambda_{lm} = l(l+1) + O(\gamma^2)$ (see Seidel1989)
- Numerically we can solve the eq with *continued fraction*
- *SpheroidalEigenvalue* in *Mathematica*

Radial Wave Function

$$\frac{d}{dr}\left(\Delta\frac{dR_{lm}}{dr}\right)+\left[\frac{\omega^2\left(r^2+a^2\right)^2-4Mam\omega r+m^2a^2}{\Delta}-\left(\omega^2a^2+\mu^2r^2+\Lambda_{lm}\right)\right]R_{lm}(r)=0$$

The Radial wave function is expressed as a *Schrodinger-like* equation

$$dr_* = \frac{r^2+a^2}{\Delta}\,dr \qquad R^* = \sqrt{r^2+a^2}\,R \qquad \Rightarrow \qquad \boxed{\frac{\mathrm{d}^2R^*}{\mathrm{d}\,r_*^2} - V(r)\;R^* = 0}$$

$$V_{lm} = \frac{4r_g mar\omega - a^2m^2}{(r^2+a^2)^2} + \frac{\Delta}{r^2+a^2}\left(\mu^2 + \frac{\Lambda + a^2(\omega^2-\mu^2)}{r^2+a^2} - \frac{\omega^2(r^2+a^2)}{\Delta} + \frac{3r^2+a^2-4r_gr}{(r^2+a^2)^2} - \frac{3\Delta r^2}{(r^2+a^2)^3}\right)$$

Superradiant Scattering

- **Tortoise Coordinate**

$$r_* = r - \frac{r_+ + r_-}{r_+ - r_-}\left[r_- \log\left(\frac{r - r_-}{2M}\right) - r_+ \log\left(\frac{r - r_+}{2M}\right)\right]$$

$$dr_* = \frac{r^2 + a^2}{\Delta}\, dr \quad \Rightarrow \quad \begin{array}{ll} r \to r_+, & r_* \to -\infty; \\ r \to \infty, & r_* \to r; \end{array}$$

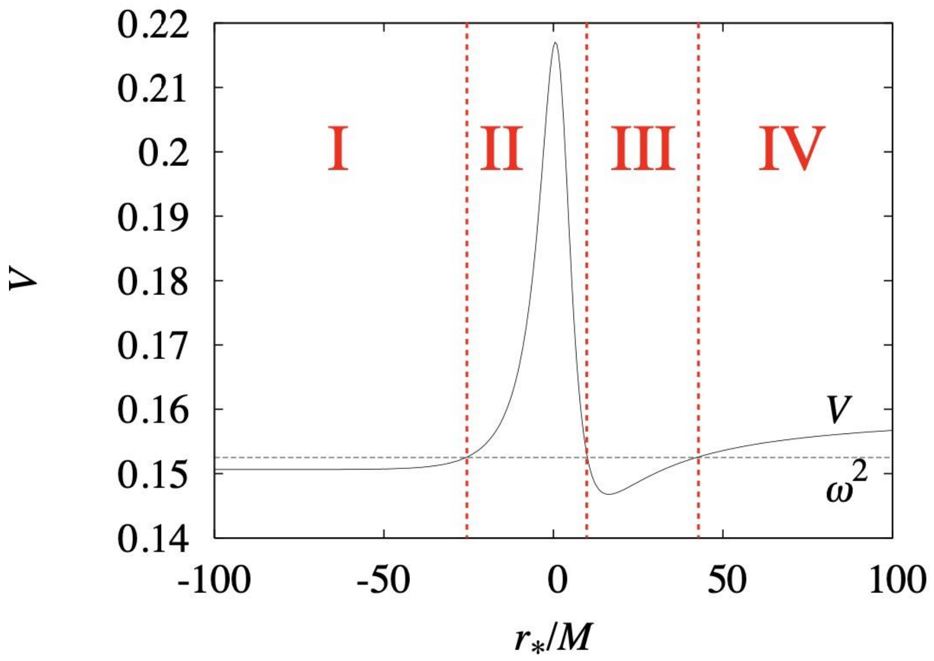
$$\Delta = r^2 - 2M\,r + a^2 = (r - r_+)(r - r_-)$$

- **Superradiance Condition**

$$\frac{\mathrm{d}^2 R^*}{\mathrm{d}\, r_*^2} - V(r)\ R^* = 0$$

$$r \to r_+, \qquad V \to -(\omega - \omega_c)^2; \qquad k_H = \omega - \omega_c$$

$$r \to \infty, \qquad V \to \mu^2 - \omega^2; \qquad k_\infty = \sqrt{\omega^2 - \mu^2}$$

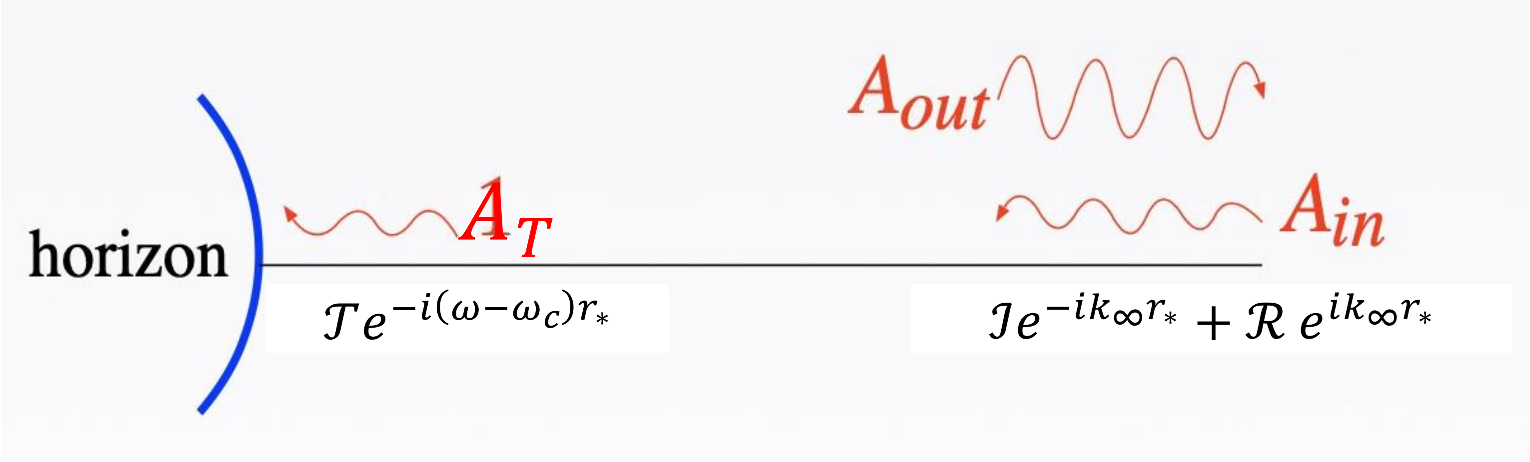


Superradiant Scattering

➢ **Superradiance Condition**

$$\frac{d^2 R^*}{d r_*^2} - V(r)\ R^* = 0$$

$$R^* \sim \begin{cases} \mathcal{I}e^{-ik_\infty r^*} + \mathcal{R}e^{ik_\infty r^*}, & \text{as } r^* \to \infty, \quad \text{or } r \to \infty \\ \mathcal{T}e^{-ik_H r^*} + \mathcal{O}e^{ik_H r^*}, & \text{as } r^* \to -\infty, \quad \text{or } r \to r_+ \end{cases}$$



The energy flux direction is opposite to the phase velocity

Wronskian

$$2(\omega - \omega_c)|\mathrm{T}|^2 = 2k_\infty(|I|^2 - |R|^2)$$

⬇

Superradiance $|R|^2 > |I|^2$

Amp factor $Z_{lm} = \left|\frac{R}{I}\right|^2 - 1 > 1$

⬇

Superradiance condition

$$\boldsymbol{\omega < \omega_c = m\Omega_H = \frac{am}{r_+^2 + a^2}}$$

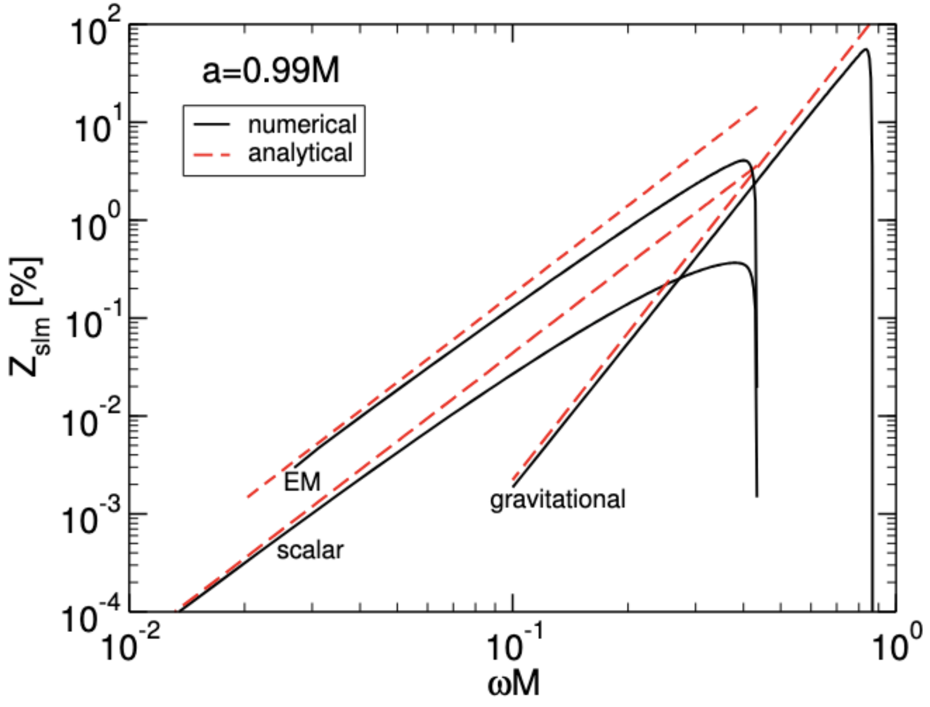
Similar analysis of the fermion shows that the reflection coefficient is always less than unity, indicating that superradiance cannot occur.

Superradiant Scattering

$$Z_{0lm} = -8Mr_+(\omega - m\Omega_{\rm H})\omega^{2l+1}(r_+ - r_-)^{2l}\left[\frac{(l!)^2}{(2l)!(2l+1)!!}\right]^2 \prod_{k=1}^{l}\left[1 + \frac{M^2}{k^2}\left(\frac{\omega - m\Omega_{\rm H}}{\pi r_+ T_H}\right)^2\right]$$

$$Z_{slm} = Z_{0lm}\left[\frac{(l-s)!(l+s)!}{(l!)^2}\right]^2$$

- The maximum amplification factors are about 0.4%, 4.4% and 138% for scattering of **massless** scalar, EM and GWs, respectively



A. Starobinski, "Amplification of waves during reflection from a rotating black hole," *Zh. Eksp. Teor. Fiz.* **64** (1973) 48. (Sov. Phys. - JETP, 37, 28, 1973).

A. A. Starobinski and S. M. Churilov, "Amplification of electromagnetic and gravitational waves scattered by a rotating black hole," *Zh. Eksp. Teor. Fiz.* **65** (1973) 3. (Sov. Phys. - JETP, 38, 1, 1973).

Outline

I. Introduction to Superradiance——History

II. Kerr Black Hole Superradiance

- Superradiant Scattering
- Superradiant Bound-state
- Other BH QBS/superradiance

III. Ultralight ALP and Superradiance Cloud

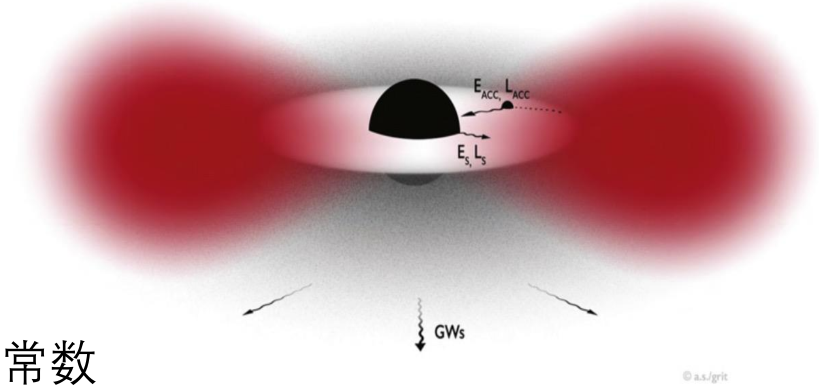
IV. Detection and Signal of Superradiance

V. Summary

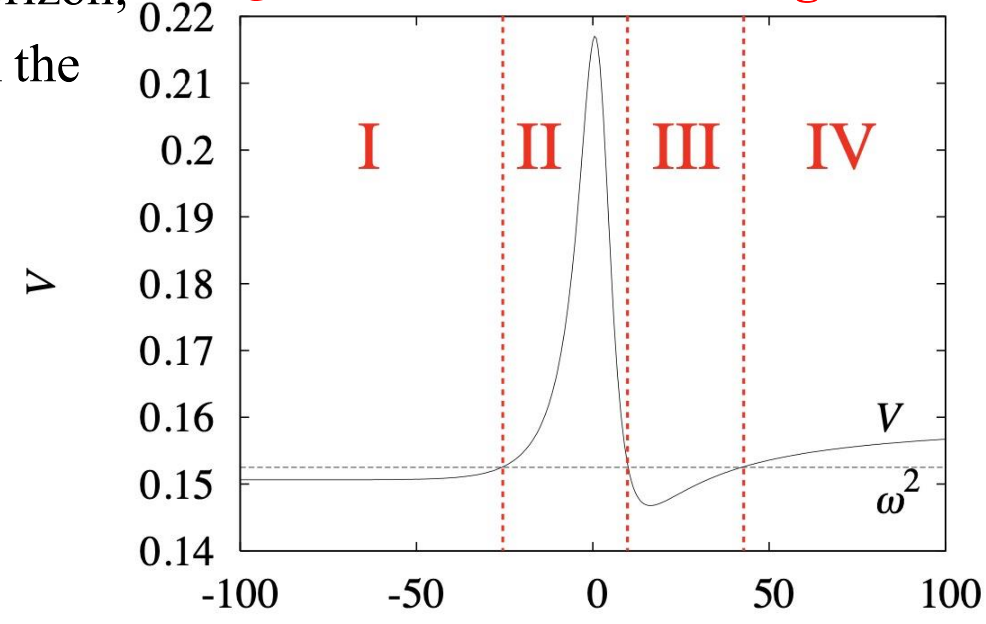
Superradiance with bound state

- For massive scalar field, a potential (m>0) well locates near the horizon;
- The field forms a bound state rotating BH and extract energy from the BH continuously

Hydrogen-like state： Gravitational Atom



Quasi-Bound States in region III



定义引力结构常数

$$\alpha = \frac{GM\mu}{\hbar c} \simeq 0.75 \times \left(\frac{M}{M_\odot}\right) \times \left(\frac{\mu}{10^{-10}\ \mathrm{eV}}\right)$$

$$\omega_{nlm} = \mu - \mu\frac{\alpha^2}{2n^2} - \mu\frac{\alpha^4}{8n^4} + \mu\frac{(2l-3n+1)\alpha^4}{n^4(l+1/2)} + \mu\frac{2\tilde{a}m\alpha^5}{n^3 l(l+1/2)(l+1)}.$$

Baumann et al JCAP12(2019)006

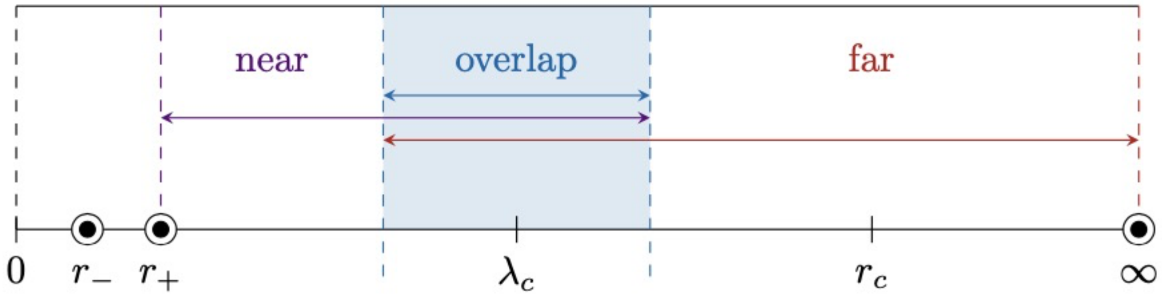
定义波尔半径

$$r_B = \frac{r_g}{\alpha^2}$$

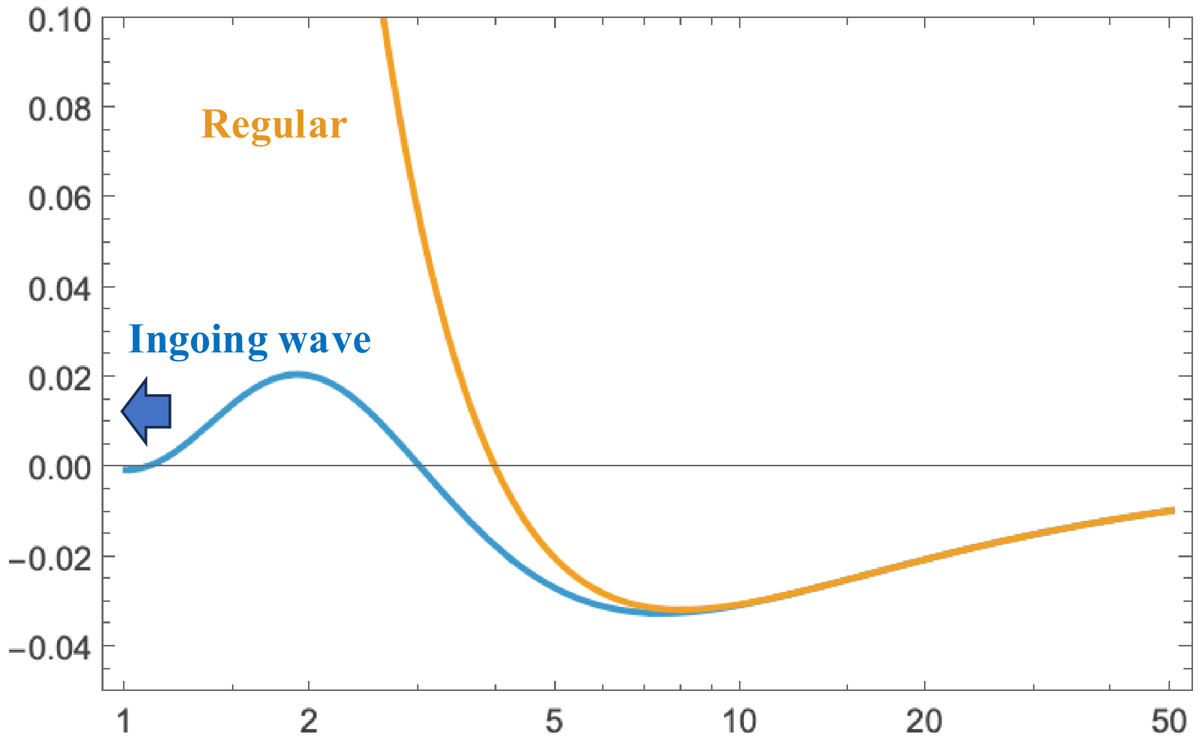
$\alpha \ll 1$ 意味着标量场在黑洞周围形成非常弥散的“云”，经典“引力原子”的描述非常适用。

Superradiance with bound state

➢ **Detweiler**(1980)年的经典结果:



$1/\mu$ $1/(\alpha\mu)$



$$\omega = \omega_0 + \omega_1 \delta\lambda$$

$$\omega_0 = \mu\left(1 - \frac{r_g^2\mu^2}{2\bar{n}^2}\right), \quad \omega_1 = \mu\frac{r_g^2\mu^2}{\bar{n}^3}$$

$$\delta\lambda^{(0)} = -ip\,(4\kappa b)^{2l+1}\frac{(n+2l+1)!(l!)^2}{n!\left[(2l)!(2l+1)!\right]^2}\prod_{j=1}^{l}(j^2+4p^2),$$

Superradiance spectrum

$$\frac{d^2}{dr^2}(rR)+\left[\omega^2-\mu^2+\frac{2M\mu^2}{r}-\frac{l(l+1)}{r^2}\right]rR=0.$$

Detweiler, 1980

$$l' = l + \epsilon$$

BSS, Xu and Zhang, PRD 106, 064016(2022);

- **Regulator:**

$$\frac{\Gamma(-2l-1)}{\Gamma(-l)}=\lim_{\epsilon\to 0}\frac{\Gamma(-2l'-1)}{\Gamma(-l')}=\frac{1}{2}\times\frac{(-1)^{l+1}l!}{(2l)!}$$

VS

$$\frac{\Gamma(-2l-1)}{\Gamma(-l)}=\frac{(-1)^{l+1}l!}{(2l)!}$$

- **NLO:**

$$\epsilon\simeq-\frac{8\left(r_g\mu\right)^2}{2l+1}$$

$$\omega=\omega_0+\omega_1(\epsilon+\delta\lambda^{(1)})$$

Fine structure (ϵ)

More accuracy ($\delta\lambda^{(1)}$)

BH-Cloud Evolution

➤ **Numerical: Continuous Fraction vs Shooting**

$$R(r) = (r - r_+)^{-i\sigma}(r - r_-)^{i\sigma+\chi-1}e^{qr}\sum^{\infty} a_n\left(\frac{r - r_+}{r - r_-}\right)^n,$$

$$\sigma = \frac{2r_+(\omega - \omega_c)}{r_+ - r_-},$$

$$q = \pm\sqrt{\mu^2 - \omega^2}, \quad \text{and} \quad \chi = \frac{\mu^2 - 2\omega^2}{q}.$$

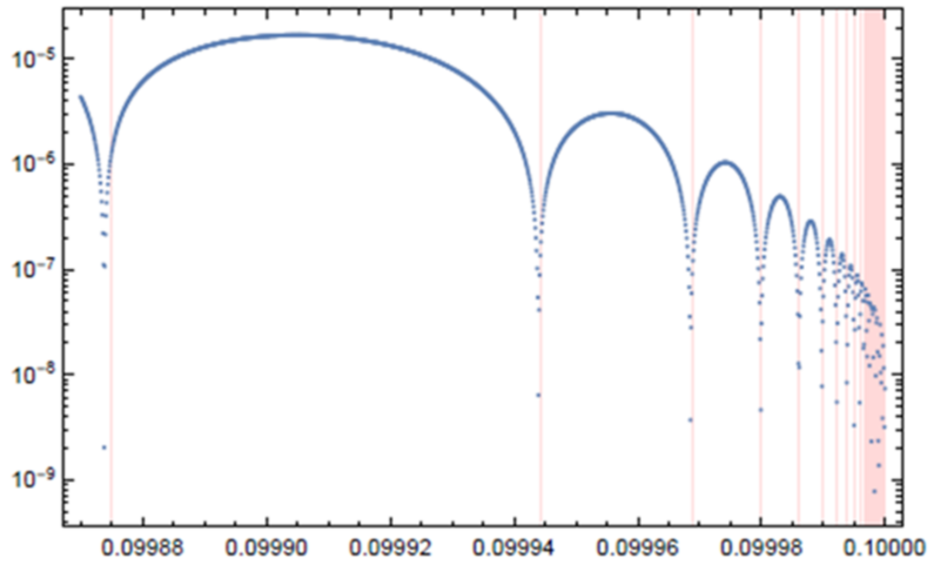
$$\alpha_0 a_1 + \beta_0 a_0 = 0,$$

(Leaver1985, Dolan2007)

$$\alpha_n a_{n+1} + \beta_n a_n + \gamma_n a_{n-1} = 0, \qquad n \in \mathbb{N},$$

$$\frac{a_{n+1}}{a_n} = -\frac{\gamma_{n+1}}{\beta_{n+1}-}\frac{\alpha_{n+1}\gamma_{n+2}}{\beta_{n+2}-}\frac{\alpha_{n+2}\gamma_{n+3}}{\beta_{n+3}-}\ldots.$$

$$\beta_0 - \frac{\alpha_0\gamma_1}{\beta_1-}\frac{\alpha_1\gamma_2}{\beta_2-}\frac{\alpha_2\gamma_3}{\beta_3-}\ldots = 0.$$



- The details can be found in Ref.[Dolan2007];
- The analytical result can be used as the initial value for numerical calculation.

Superradiance spectrum

- Leading order

At small α with $a = 0.99$,

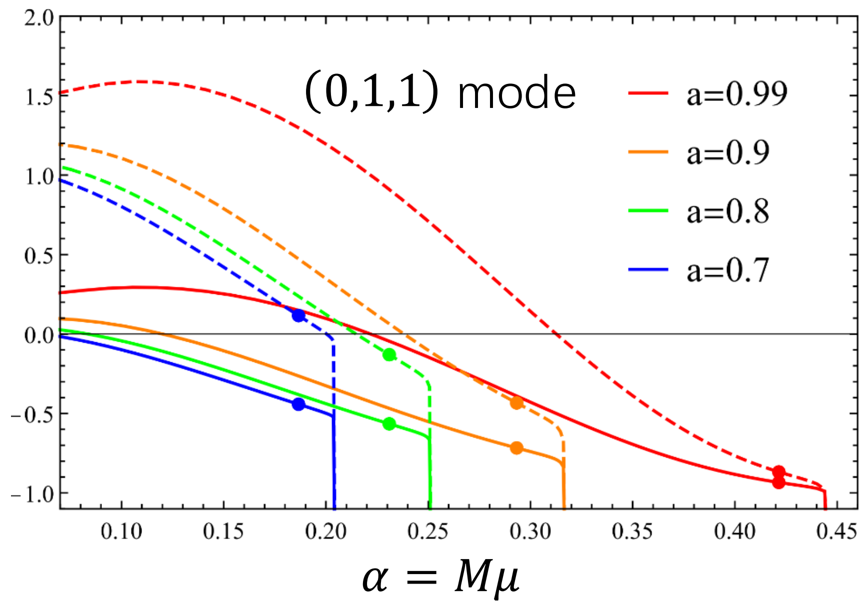
Err. of original result $\sim$ 150% $\quad \frac{ana}{num} - 1$

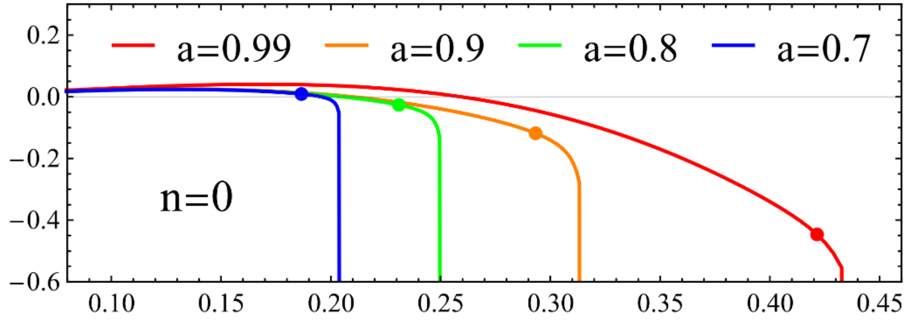
Err. of corrected result $\sim$ 30%

We confirm a mistake in the LO calculation of $\omega_{nlm}^{(I)}$

- Next-to-leading order

At small α, **err.** $< 5\%$ $\quad \frac{ana}{num} - 1$





BSS, Xu and Zhang**,** PRD (2022)

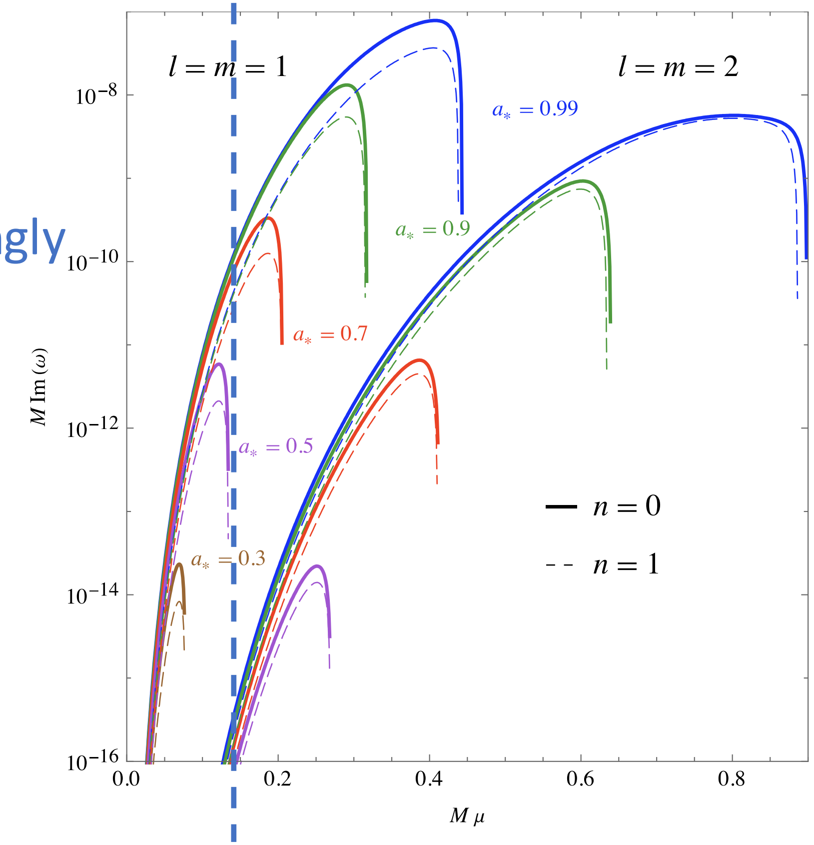
Superradiance Rate

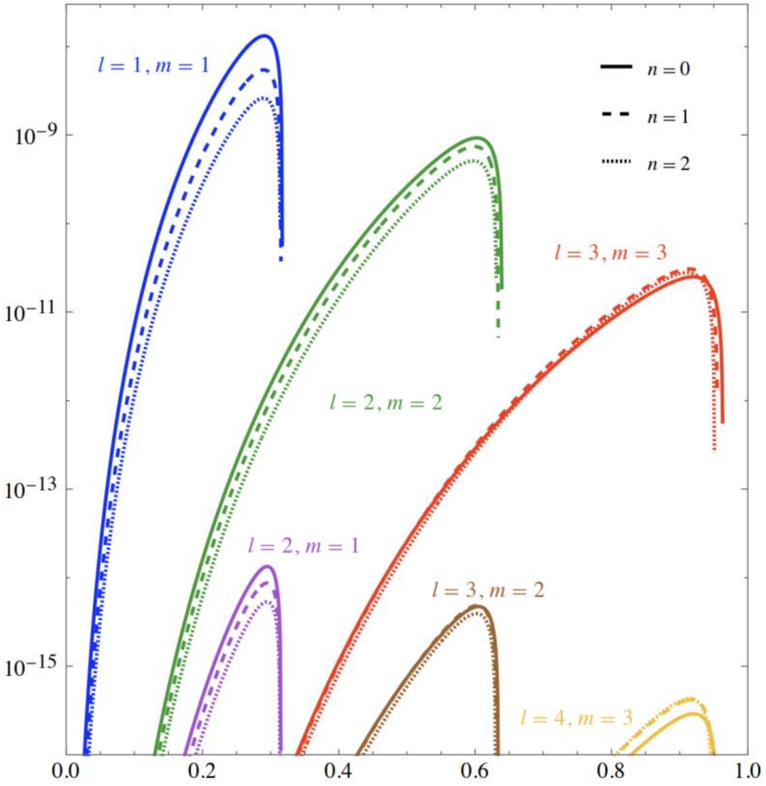
- Superradiance condition

$$Re(\omega) < \omega_c = \frac{am}{r_+^2 + a^2}$$

$$2GM\mu \sim 1$$

- $Im(\omega)$ dependent on a (not) strongly
- (0,1,1) are the most important, modes with $m \neq l$ can be ignored
- In each l-stage, $(0, l, l)$ is the fastest, followed by $(1, l, l)$
- (n, l, l) modes with different l have very different rates





Outline

KN Superradiance spectrum

超辐射不稳定性不仅存在于Kerr黑洞外的标量场，其他情况也存在 **Superradiant QBS**

KN BH mass is normalized to 1, BH charge $Q = 0.02$

- **Kerr-Newman BH:** Charged and Rotating

Following Detweiler's Matching scheme, the superradiance spectra for KNBH is obtained by Furuhashi & Nambu,2004;

- We corrected the expression with the ½ factor and introduce the improvement from ϵ_l,
- and obtained expression is more consistent with the numerical results.

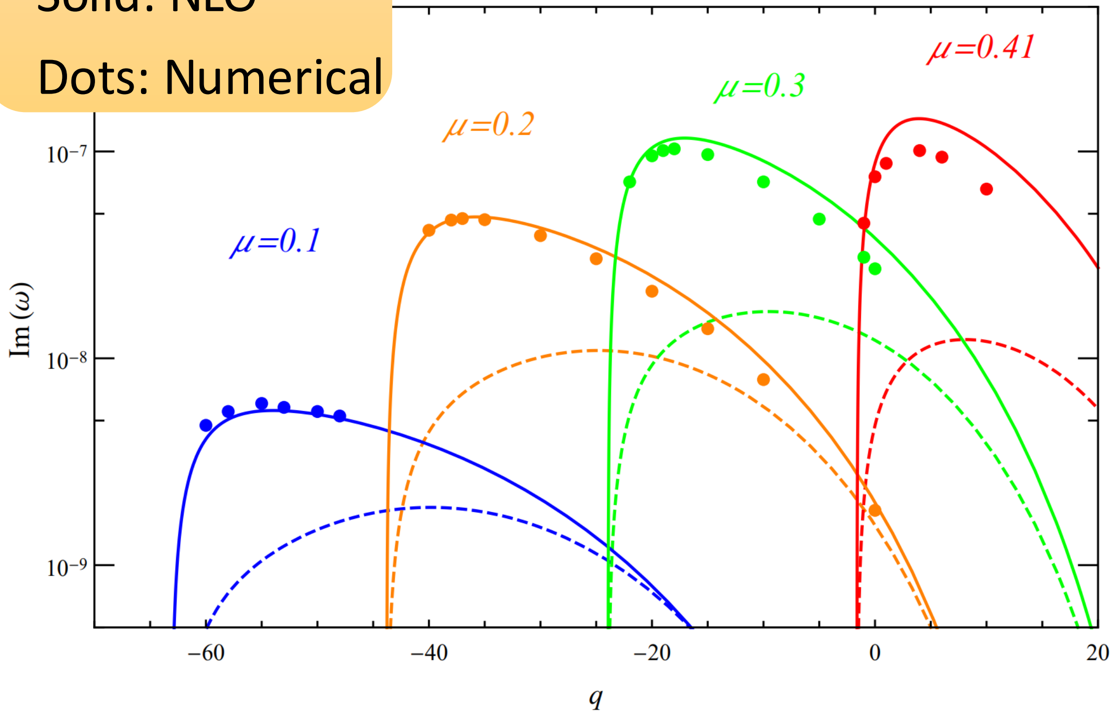
BSS, Xu and Zhang: Phys.Rev.D 107 (2023) 6, 064037

Dashed: LO

Solid: NLO

Dots: Numerical

(0,1,1) mode

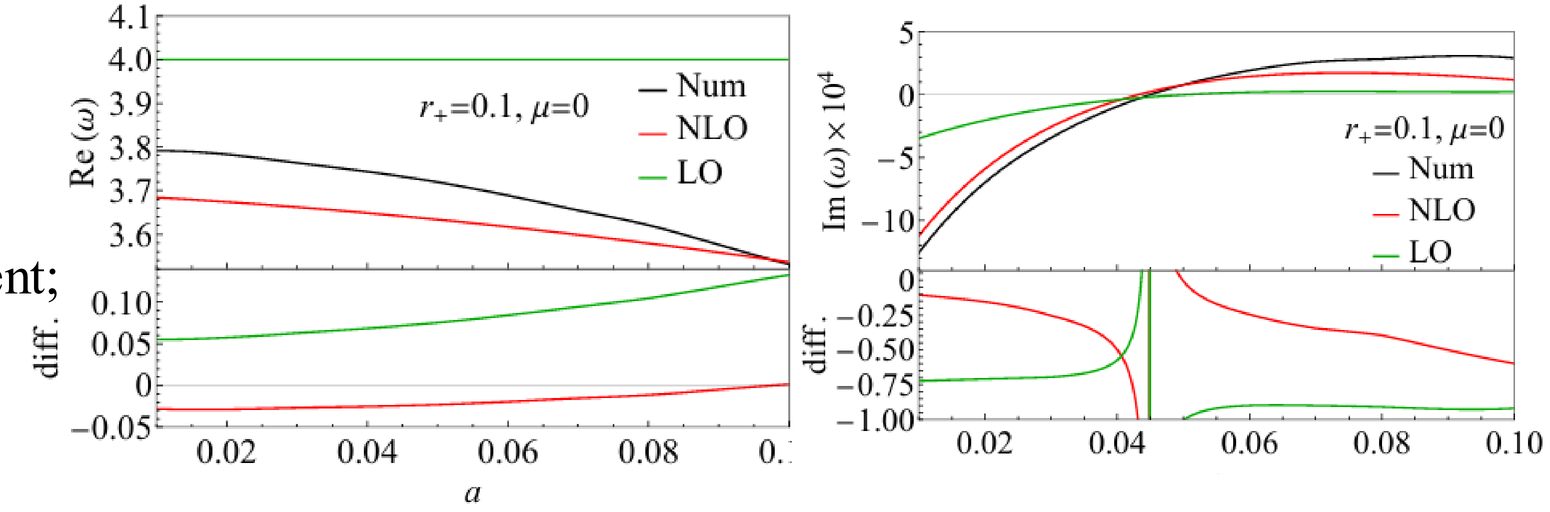


K-AdS Superradiance spectrum

➢ **Kerr-anti-de Sitter BH: $\Lambda < 0$**

- In Ref.[Cardoso & Dias,2004], the space near Horizon is treated as pure Kerr background, while in far region the space is pure AdS space.

- We **improve** the expression:
  - ✓ The real part is BH spin-dependent;
  - ✓ The spectra is more accurate.



XChu,YChu,**BSS** and Zhang, PRD 111, 043039 (2025)

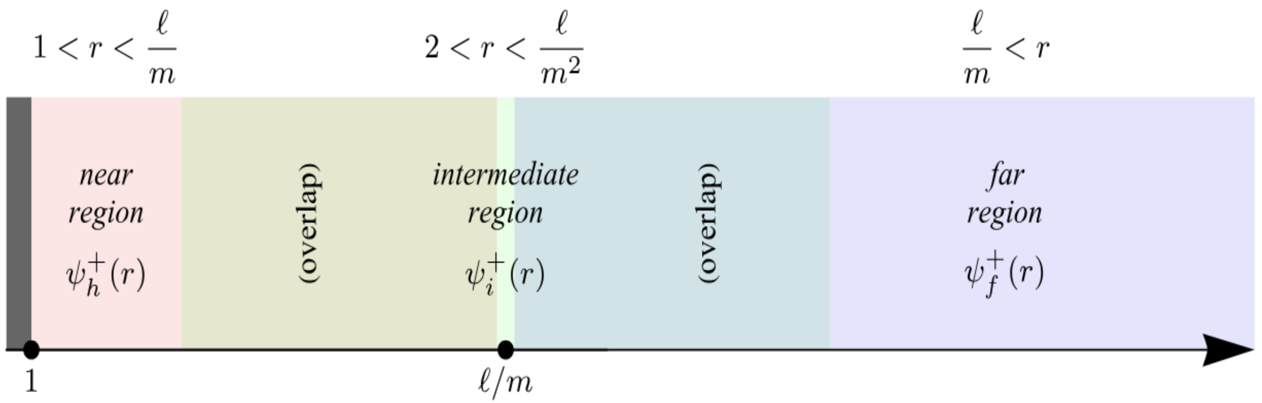
Dirac QBS

- **Fermion** $$\left[\gamma^{\mu}(\nabla_{\mu} - iqA_{\mu}) + i\mu\right]\Psi = 0. \quad |\mathcal{R}|^2 = |\mathcal{I}|^2 - \frac{\omega}{\sqrt{\omega^2 - \mu_e^2/2}}|\mathcal{T}|^2 .$$

- The reflection coefficient is always less than unity, showing that superradiance cannot occur.
- Due to the well in the potential, there are long-lived quasi-bound-states

$$\Delta^{1/2}\frac{dR_-}{dr} - i\frac{K}{\Delta^{1/2}}R_- = (\lambda + i\mu r)R_+,$$
$$\Delta^{1/2}\frac{dR_+}{dr} + i\frac{K}{\Delta^{1/2}}R_+ = (\lambda - i\mu r)R_-.$$

$$(\partial_r^2 + P_\pm(r)\partial_r + Q_\pm(r))\psi^\pm = 0,$$
$$P_+(r) = \frac{1}{2r} + \frac{1}{2(r-1)} - \frac{1}{r - i\ell/m},$$
$$Q_+(r) = -p^2 + \frac{\ell^2}{r} - \frac{\ell^2 + m^2 - 2\omega^2 + \frac{\ell\omega/m}{1-i\ell/m}}{r-1} + \frac{\omega^2 + i\omega/2}{(r-1)^2} + \frac{\ell\omega/m}{1-i\ell/m}\frac{1}{r-i\ell/m},$$



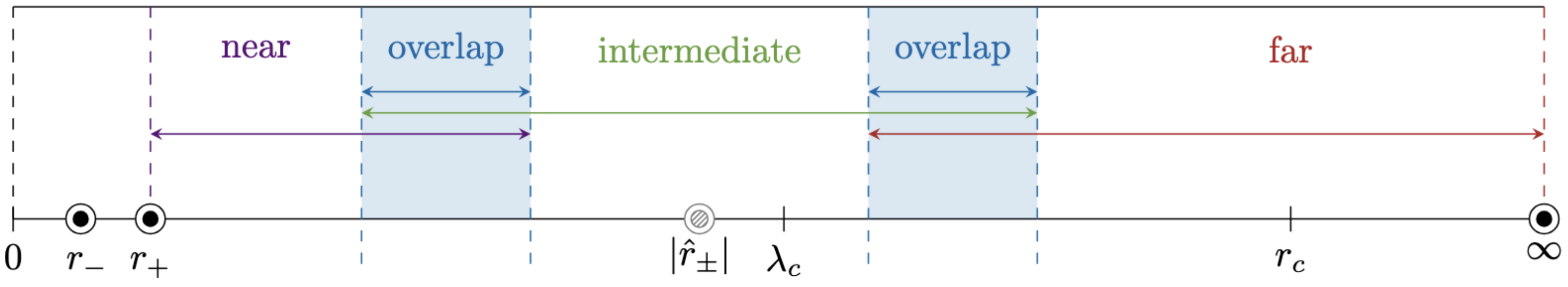
✓ The mixing between + and – prevent the matching

✓ Matrix-matching scheme is proposed in

Chen, Yang, **BSS**, Tang and Wu, PRD111,125006(2026)

(Superradiant) QBS

- **Kerr-Proca:** Baumann et al 2019; Guo, Jia, **BSS**, Hzhang and Xzhang, PRD 110 (2024) 8, 083029



- **静态带电黑洞的超辐射** $|R|^2 = |I|^2 - \frac{\omega - qQ/r_+}{\sqrt{\omega^2 - \mu^2}}|T|^2.$ $\omega - q\Phi_H < 0$

- **Kerr-dS** $|R|^2 = |I|^2 - \frac{\omega - m\Omega_H}{\omega - m\Omega_c}|T|^2,$ $m\Omega_c < \omega < m\Omega_H.$

- Higher dimension, Beyond Einstein… more discussion can be found in **arXiv:1501.06570v8**

Outline

I. Introduction to Superradiance——History

II. Kerr Black Hole Superradiance

III. Ultralight ALP and Superradiance Cloud

- Ultralight Axionlike Particle
- Energy Density distribution
- Velocity distribution

IV. Detection and Signal of Superradiance

V. Summary

Strong CP and Axionlike Particle

- **The SM is not CP invariant.**

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

- **The vacuum of QCD is non-trivial.**

$$|\theta\rangle = \sum_i e^{-in\theta}|n\rangle, \qquad \langle\theta|\theta\rangle = \int [DA, D\psi, Dc] e^{iS_{\rm QCD}} = \int [\cdots] \exp^{i\int d^4x \mathcal{L}_{\rm QCD}+\mathcal{L}_{\rm gf}+\mathcal{L}_{\rm gh}+\mathcal{L}_\theta}$$

- **The theta-term can be induced by chiral transformation**

$$\mathcal{L}_{\rm mass} = -\bar{q}_L M q_R - \bar{q}_R M^\dagger q_L \qquad \bar{\theta} = \theta_{\rm QCD} + \arg\det(M_u M_d)$$

It's natural to have the term in SM

Strong CP and Axionlike Particle

- **Hadron electric dipole momentum** $\bar{\theta} G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \Longrightarrow \mathcal{L}_{\mathrm{EDM}} = -\frac{\mathrm{i}}{2} d_n \bar{n}\sigma^{\mu\nu}\gamma_5 n F_{\mu\nu} \simeq -d_n \frac{\boldsymbol{S}\cdot\boldsymbol{E}}{|\boldsymbol{S}|}.$

LQCD $d_n = -(1.5 \pm 0.7) \times 10^{-3}\bar{\theta}\mathrm{e\ fm} - (0.2 \pm 0.1)d_u + (0.78 \pm 0.03)d_d + (0.0027 \pm 0.016)d_s$

Exp $|d_n| \lesssim 10^{-26}\, e\cdot\mathrm{cm}.$ → $|\bar{\theta}| \lesssim 10^{-10}.$

- **Strong CP problem：** θ is dimensionless but why is so small?

BESIII arXiv: 2506.19180v2 $|d_\Lambda| < 6.5 \times 10^{-19}\ e\ \mathrm{cm}.$

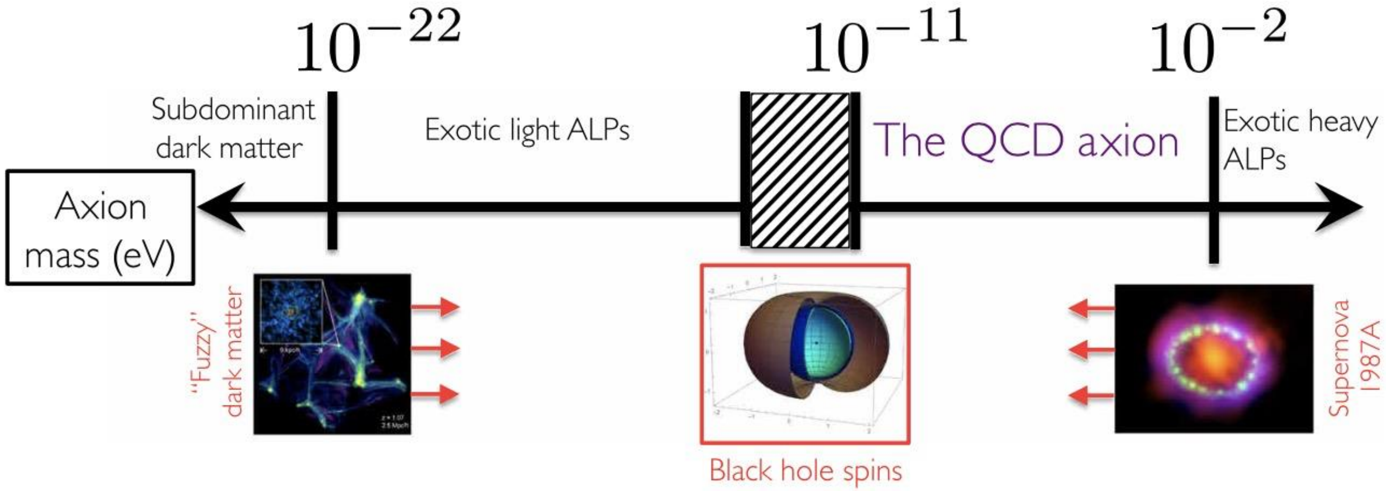
- **PQ symmetry and Axion**

$$q \to e^{i\alpha\gamma^5} q, \quad \frac{a}{f_a} \to \frac{a}{f_a} + \alpha$$

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{2}(\partial a)^2 + \frac{g_s^2}{32\pi^2}\left(\bar{\theta} + \frac{a}{f_a}\right) G\tilde{G}$$

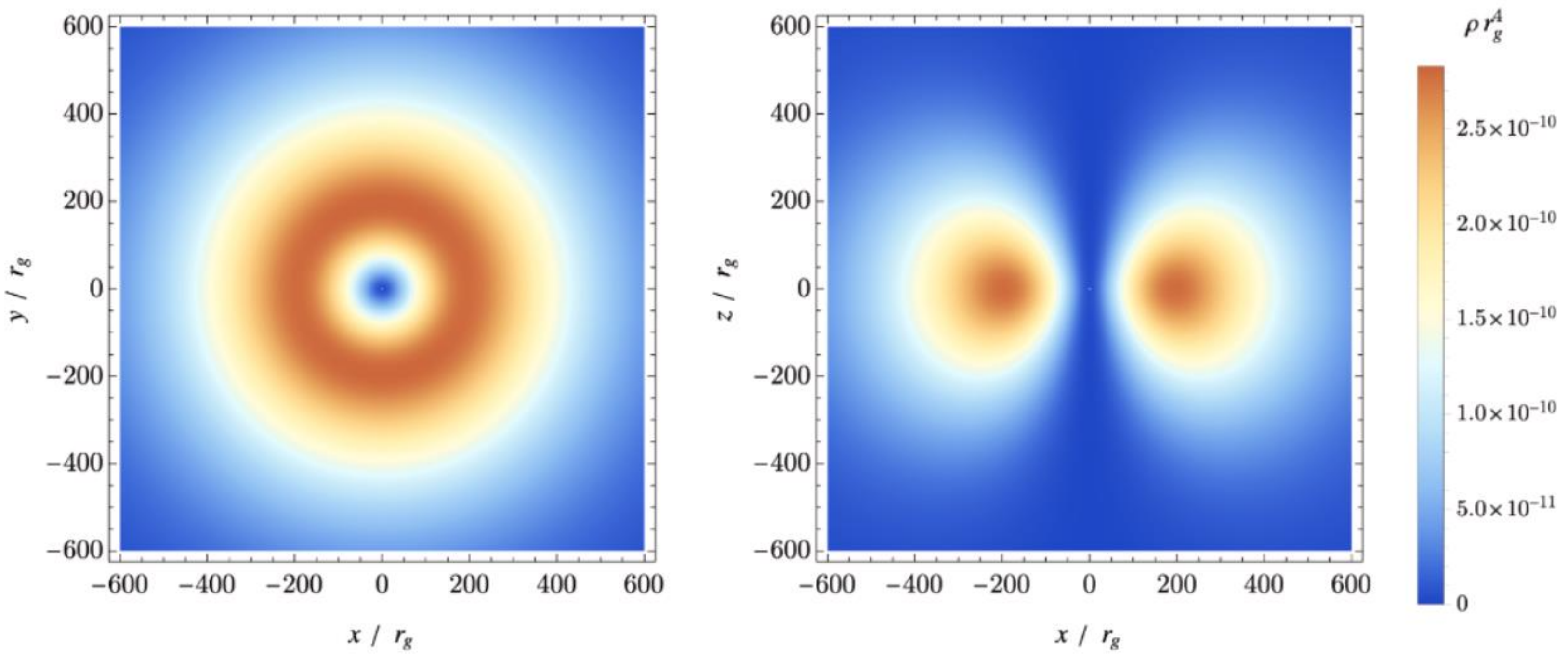
Spontaneous Symmetry Breaking $\left\langle \bar{\theta} + \frac{a}{f_a} \right\rangle = 0.$

- **Ultralight ALP as DM candidate**



Energy Density Distribution

Guo, Zhou, **BSS** and Zhang, Phys.Lett.A 572 (2026) 131316



(a) $t = 0, z = 0$

(b) $t = 0, y = 0$

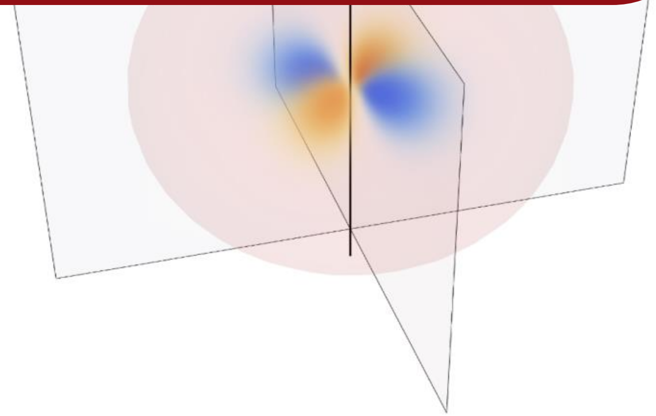
The energy density reaches its maximum at $r_p = 2r_b$ on the equatorial plane.

Assuming a BH with mass $M = 10M_\odot$ and a condensate with the particle number N_011 = 10^77, the maximum energy density is approximately 1.17×10^{29} GeV⁄cm^3, which is many orders of magnitude larger than the local dark matter halo density of 0.39 GeV⁄cm^3 in the Milky Way.

Energy Density Distribution

Time evolution of the cloud

$$\delta\rho(t, r, \theta, \varphi) = \rho - <\rho>$$



All components rotate together

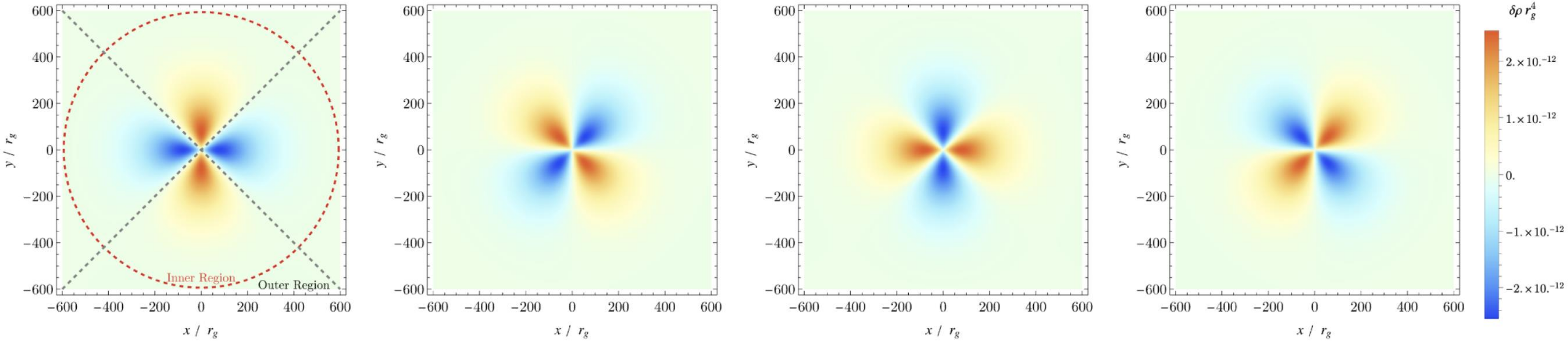
- T = π/ω
- 8 region divided by $\delta\rho$=0
- Resembles rigid body rotation

$$v_p \simeq \mu r_p \sim (r_g \mu)^{-1} = 10 > 1$$

Key insight:

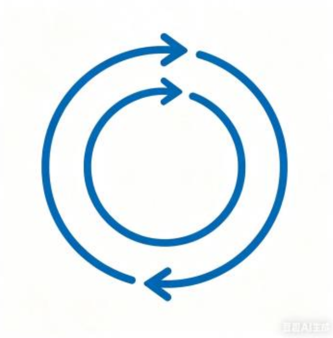
- NOT rigid rotation (would be superluminal)
- Apparent rotation = collective local motion
- Local velocity: radial + polar + azimuthal

Guo, Zhou, **BSS** and Zhang, Phys.Lett.A 572 (2026) 131316



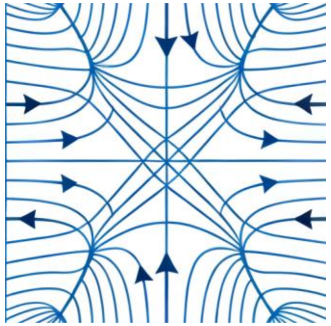
(e) $\delta\rho : t = 0,\ z = 0$ (f) $\delta\rho : t = T/4,\ z = 0$ (g) $\delta\rho : t = T/2,\ z = 0$ (h) $\delta\rho : t = 3T/4,\ z = 0$

Local Velocity Distribution



Apparent rotation

沿方位角方向，表现为围绕黑洞的简单环形旋转模式。



Local Velocity

分布结构更为复杂，包含径向、**极向和方位角三个分量，且各分量的**动力学行为存在显著差异。

Apparent Velocity

$$v^{\phi} = \omega r \simeq \mu r$$

Local Velocity

$$v^{i} = \frac{P^{i}}{\rho}, \quad P^{i} = T^{\nu}_{\sigma}\,(e_{t})^{\sigma}\left(e^{i}\right)_{\nu}$$

Local Velocity Distribution

Asymptotic Expressions ($r \gg r_g$)

$$v^r \approx \frac{1 - r/r_\mathrm{p}}{\mu r} \sin\left(2\omega_{011} t - 2\varphi\right),$$

$$v^\theta \approx \frac{\cot\theta}{\mu r} \sin\left(2\omega_{011} t - 2\varphi\right),$$

$$v^\varphi \approx \frac{2}{\mu r \sin\theta} \sin^2\left(\omega_{011} t - \varphi\right),$$

This demonstrates that the condensate's apparent rotation arises from a collective effect of local motions.

Values: The velocities decrease proportionally to $1/r$.

$$v_r \rightarrow r_g \mu/2 \sin 2(\omega t - \varphi)$$

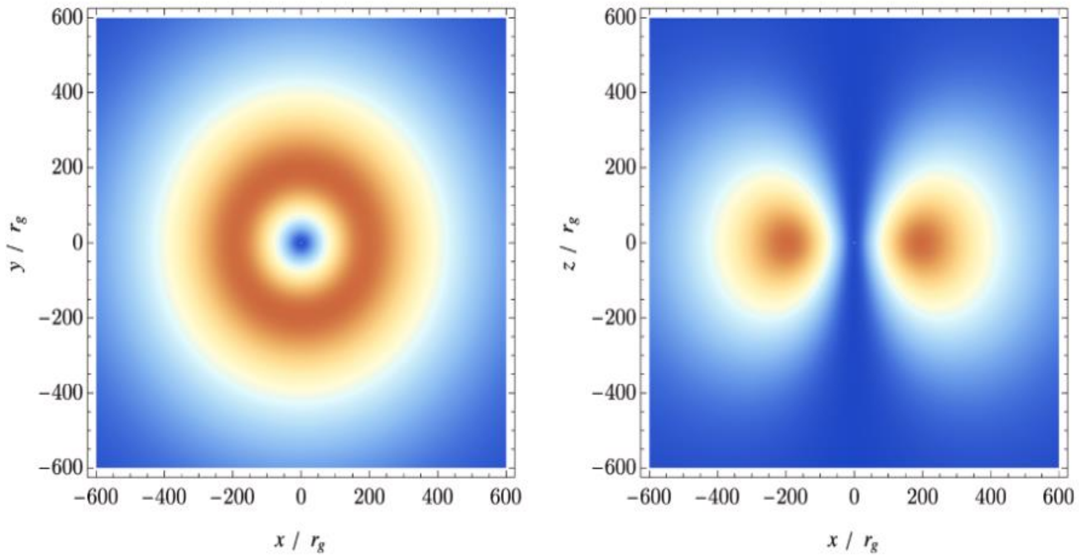
$$v_{\theta,\phi} \rightarrow 0$$

$r \gg r_p, \ v_r > v_{\theta,\phi}$ Energy-flux oscillate along radial direction.

Direction: The radial velocity v_r changes sign at $r = r_\mathrm{p}$, enegy fly into/outfrom r_p, gives the peak of ρ. The v_θ reverses across the equatorial plane; The v_φ keeps its direction in the entire space, aligned with the BH spin.

Local Velocity Distribution

Asymptotic Expressions ($r \gg r_g$)



(a) $t = 0, z = 0$　　(b) $t = 0, y = 0$

This demonstrates that the condensate's apparent rotation arises from a collective effect of local motions.

Values: The velocities decrease proportionally to $1/r$.

$$v_r \to r_g \mu/2 \sin 2(\omega t - \varphi)$$

$$v_{\theta,\phi} \to 0$$

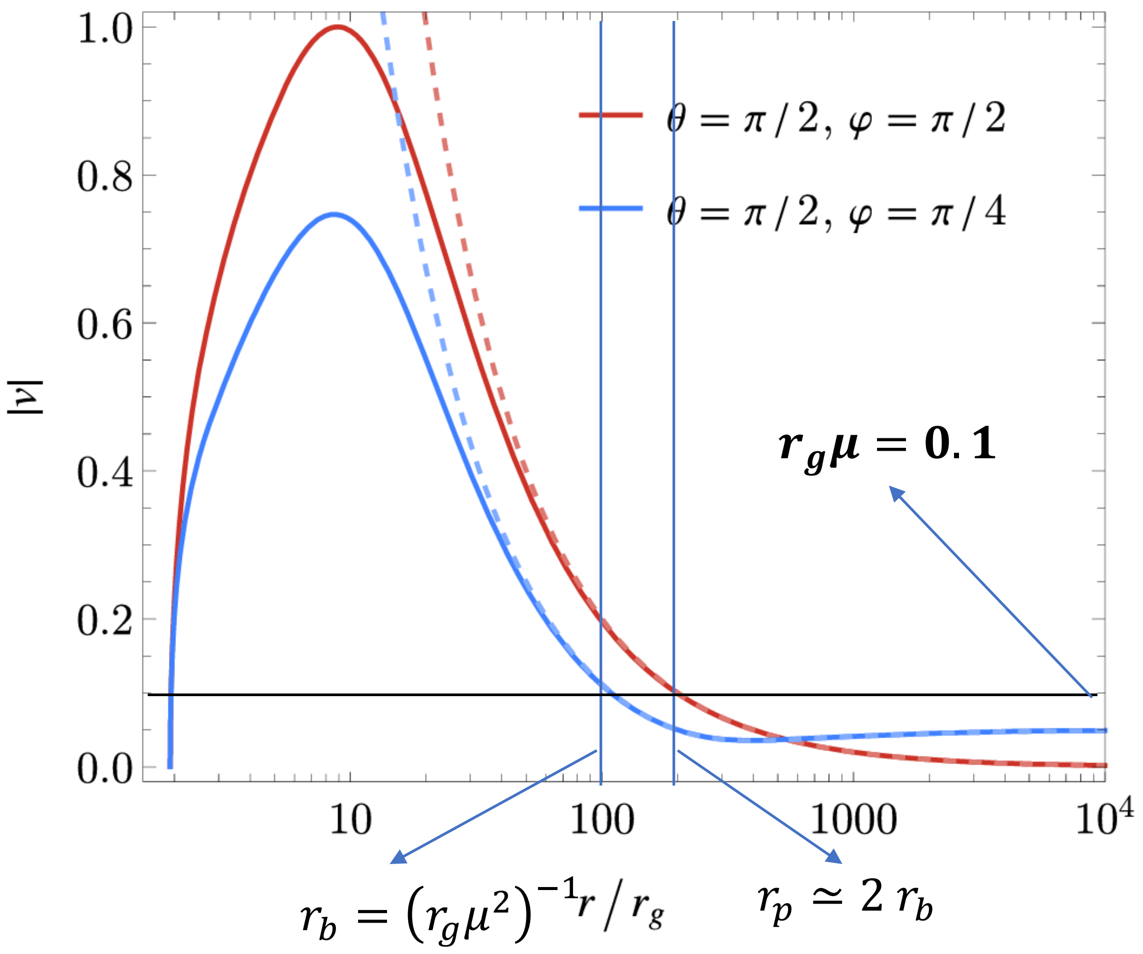
$r \gg r_p,\ v_r > v_{\theta,\phi}$ Energy-flux oscillate along radial direction.

Direction: The radial velocity v_r changes sign at $r = r_\mathrm{p}$, enegy fly into/outfrom r_p, gives the peak of ρ.

The v_θ reverses across the equatorial plane;

The v_φ keeps its direction in the entire space, aligned with the BH spin.

Local Velocity Distribution



Agreement

Behavior in far region $r > r_b$ is consistent with non-relative approximation

- $|\boldsymbol{v}| \to (\boldsymbol{r_g\mu/2})\, \boldsymbol{sin\, 2(\omega t - \varphi)}$ for $\boldsymbol{r} \to \infty$

Disagreement

In the near region, the cloud is relative;

- $|\boldsymbol{v}| < \mathbf{1}$ in the entire region;
- $|\boldsymbol{v}| \to \mathbf{0}$ close to the horizon.

Outline

BH-Cloud Evolution

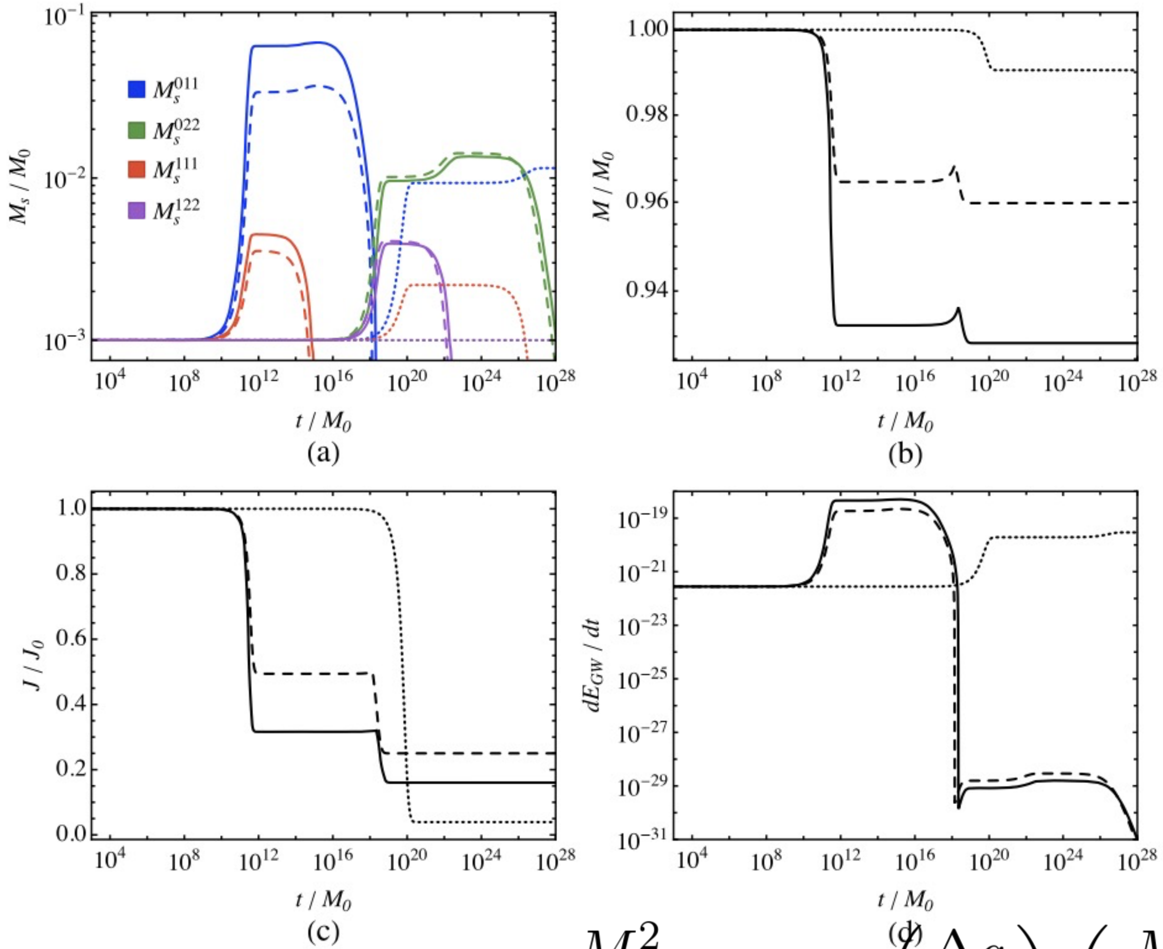
Guo, **BSS**&Zhang, PRD107,075009(2023)

$$\dot{M} = -\sum_{nlm} 2M_s^{(nlm)}\omega_I^{(nlm)},$$

$$\dot{J} = -\sum_{nlm} 2mM_s^{(nlm)}\omega_I^{(nlm)}/\omega_R^{(nlm)}$$

$$\dot{M}_s^{(nlm)} = 2M_s^{(nlm)}\omega_I^{(nlm)} - \dot{E}_{\rm GW}^{(nlm)}$$

$$\dot{J}_s^{(nlm)} = 2mM_s^{(nlm)}\omega_I^{(nlm)}/\omega_R^{(nlm)} - m\dot{E}_{\rm GW}^{(nlm)}/\omega_R^{(nlm}$$

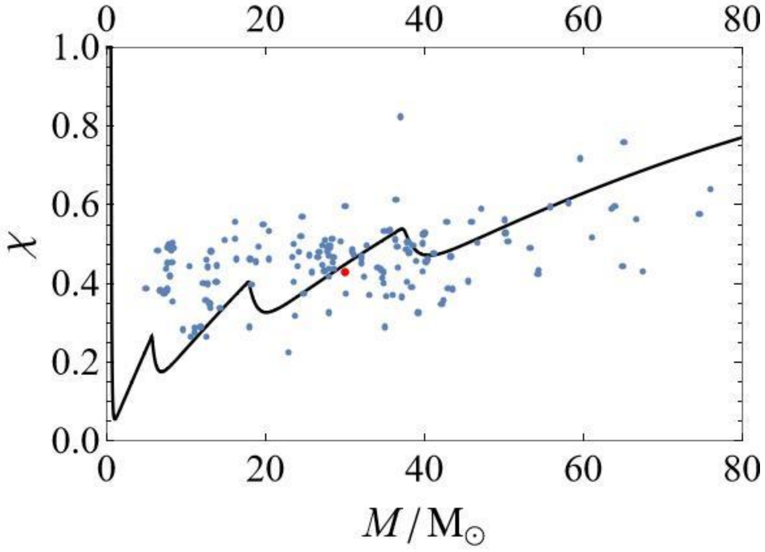


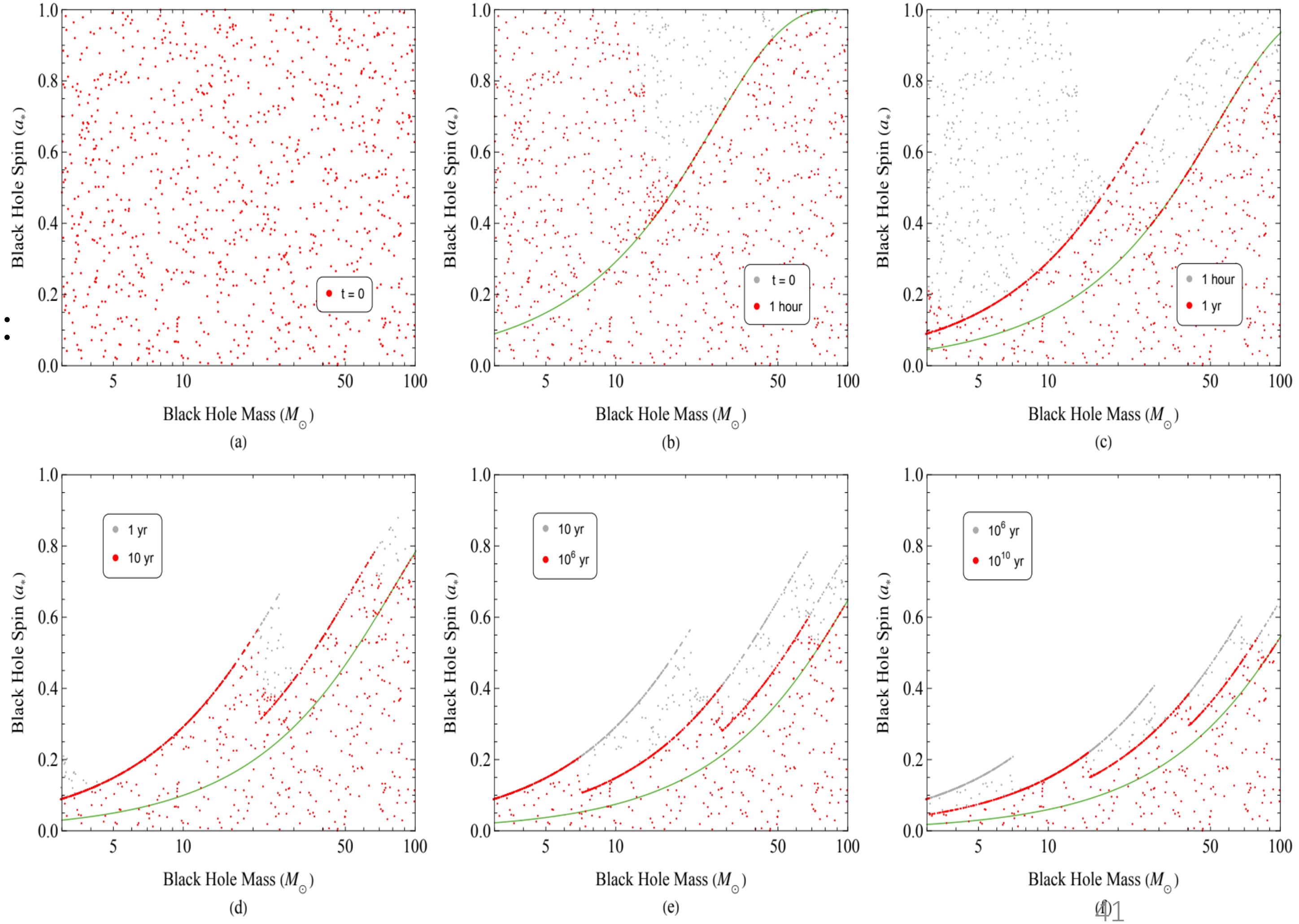
- The evolution of BH :
  - BH mass decreases slightly(a few percent)
  - BH spin decreases significantly(~100%)
  - In each l stage, the BH spin and mass are almost **constants**

$$\Delta J \sim \Delta a \frac{M_{bh}^2}{M_{\rm pl}^2} = 10^{76}\left(\frac{\Delta a}{1}\right)\left(\frac{M_{bh}^2}{M_{sun}^2}\right)$$

Regge Trajectory

1. The region above each curve is the corresponding exclusion region.
2. The BHs favor the trajectory.
3. Regge Plot can be used to constrain the ALP:

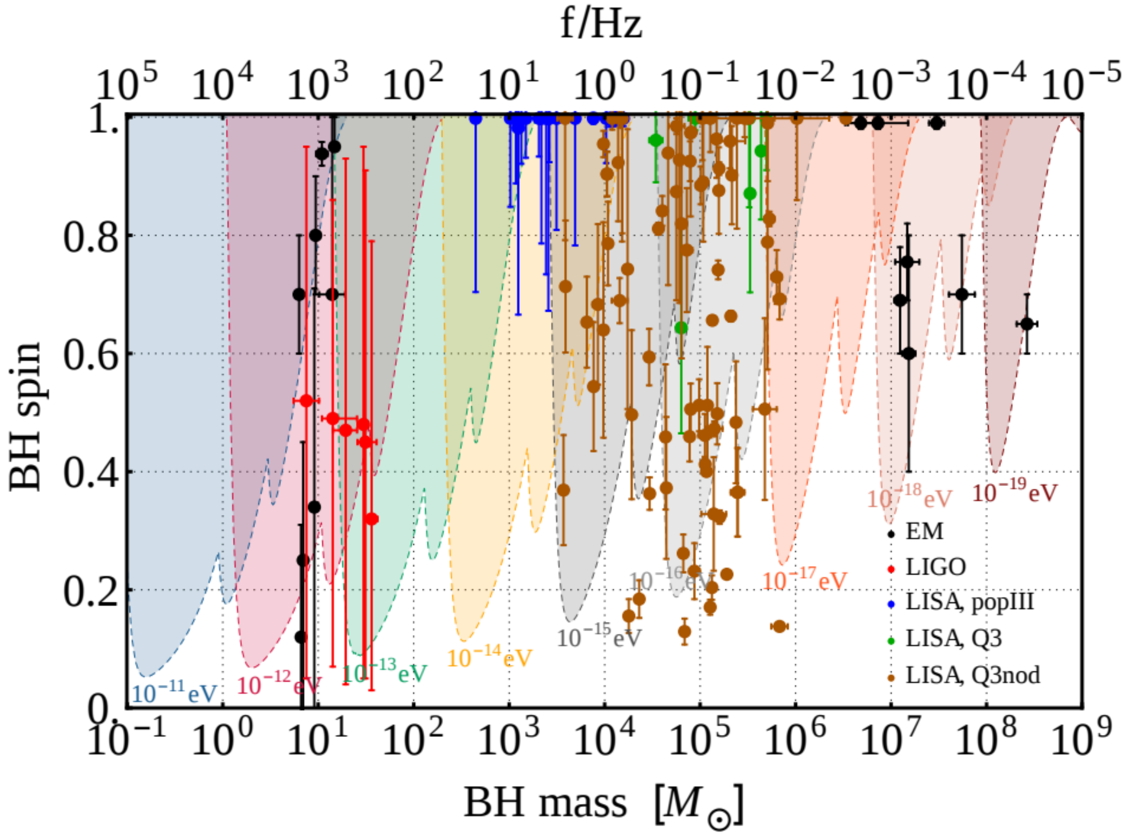




Regge Trajectory

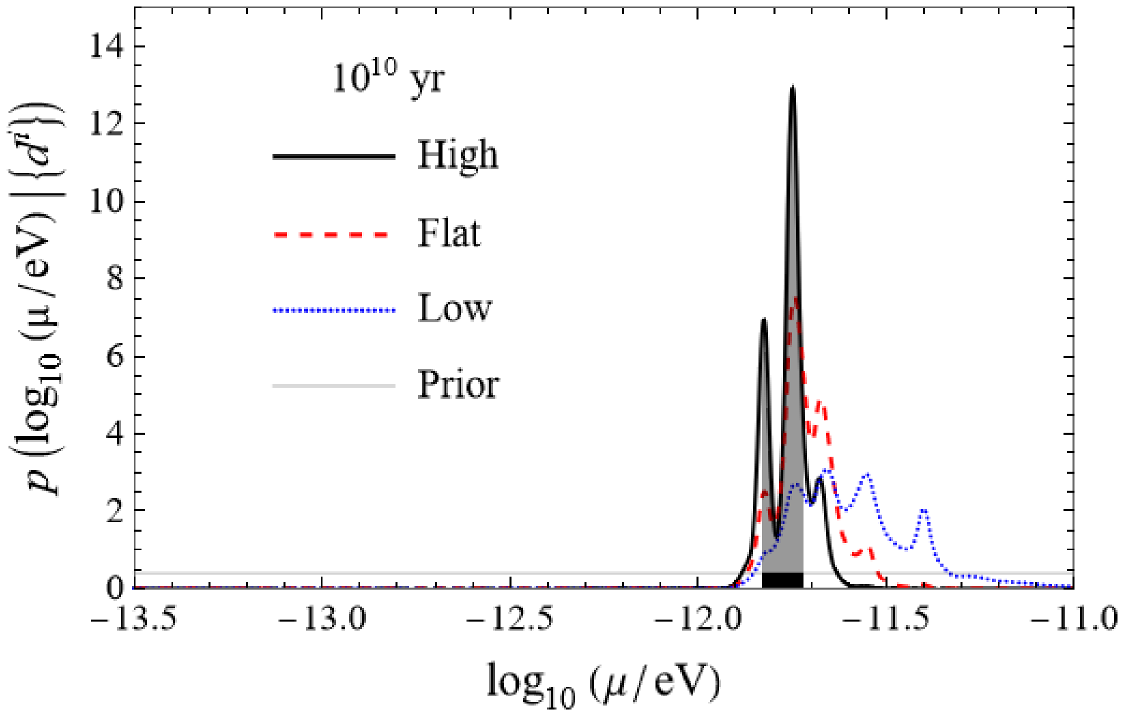
➢ 质量-自旋分布的 Regge Gap

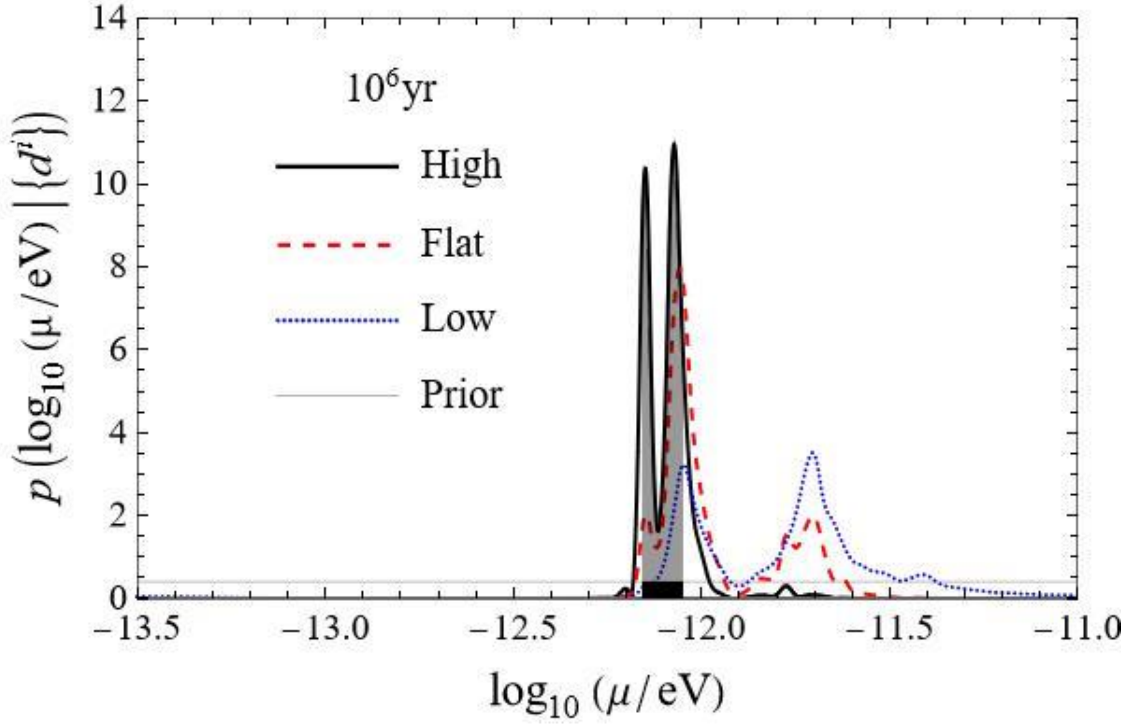
- 超辐射发生的条件是波的频率小于黑洞视界的角速度（$\omega<m\Omega_H$）。对于特定质量 μ 的玻色子，只有特定质量和自旋组合的黑洞才会触发超辐射。
- 超辐射会迅速提取黑洞的角动量，使其自旋大幅下降，直到不满足超辐射条件为止。
- 观测效应：如果在“黑洞质量-自旋”二维分布图（Regge Plane）上，发现某个特定质量区间内缺乏高自旋黑洞（即出现空洞），这就强烈暗示了特定质量的超轻玻色子的存在。



Regge Trajectory

➢ **Favor Region for ALP** Cheng, Zhang and **BSS**,PRD 107, 063021(2023)





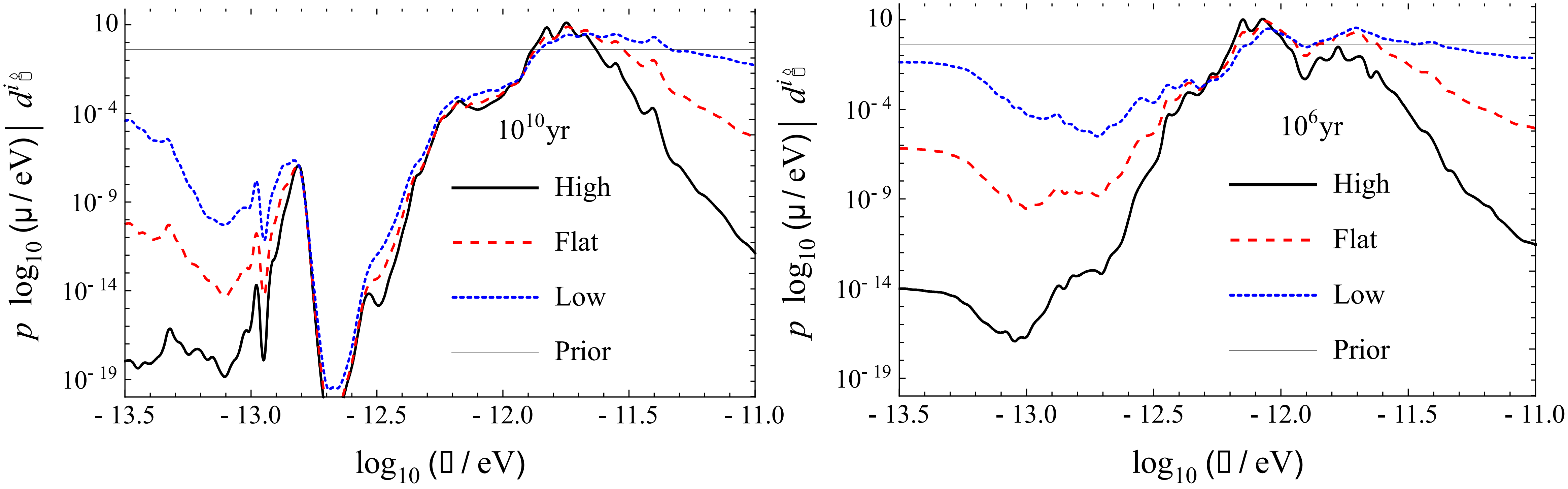
$$-11.83 \leq \log_{10}(\mu/\text{eV}) \leq -11.72$$

BF ~ 16.54

$$-12.16 \leq \log_{10}(\mu/\text{eV}) \leq -12.05$$

BF ~16.45

Regge Trajectory



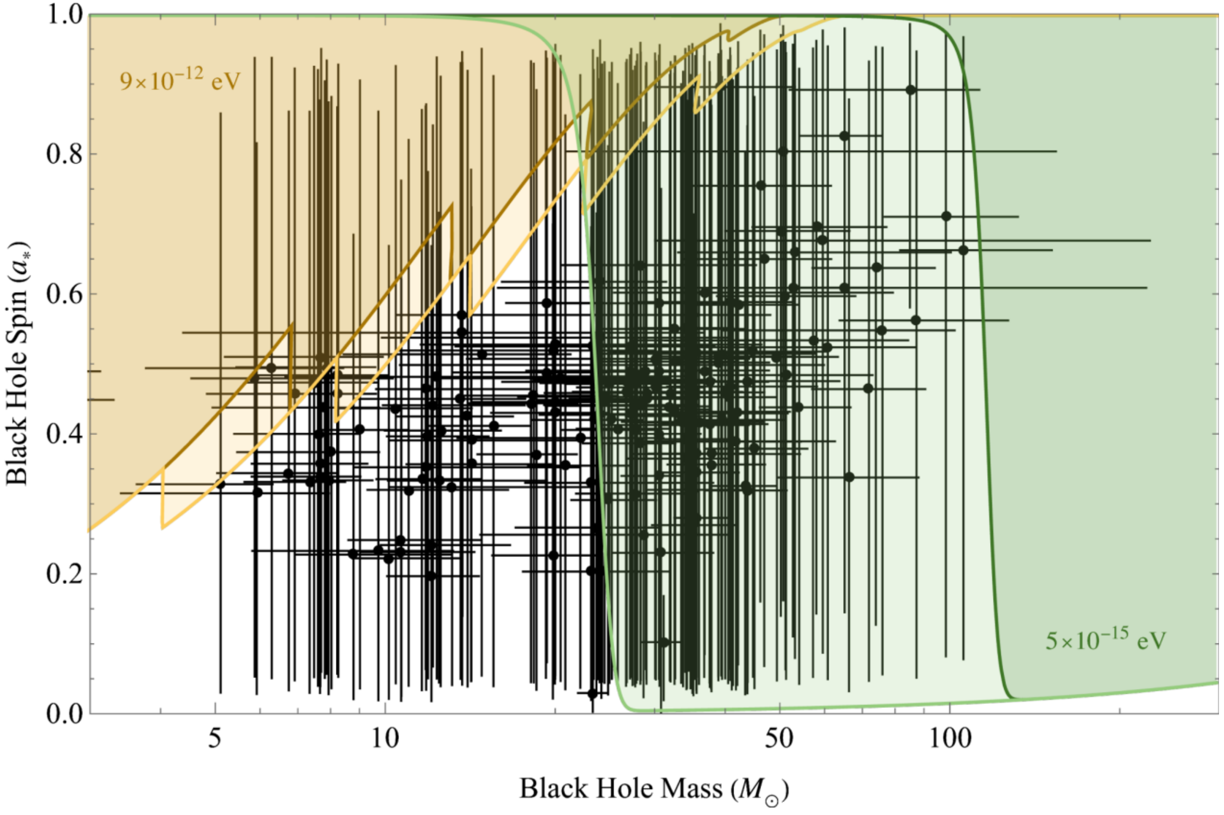
➢ **Disfavored region** but can not Excluded: −12.89 ≤ log10(μ/eV) − 12.57,

Ng *et al*, Phys. Rev. Lett. 126, 151102 (2021)

Regge Trajectory

➢ **Massive vector superradiance**

Guo, Jia, **BSS**, Hzhang and Xzhang:PRD **110**, 083029(2024)



For each mass, the darker and lighter curves correspond to 10^6 and 10^{10} years evolution time, respectively

$$5\times10^{-15}\ \text{eV}<\mu<9\times10^{-12}\ \text{eV}$$

- Large uncertainty make the conclusion crude;
- More data could help to improve the constraint.

Outline

Gravitational Waves from Superradiance

➢ 连续引力波 (Continuous Gravitational Waves, CWs)

- 这是超辐射最标志性的信号。玻色子云通过两种主要机制发射近乎单色的连续引力波：
- **玻色子湮灭：** 云中的两个玻色子湮灭成一个引力子。发出的引力波频率严格等于玻色子的康普顿频率。
- **能级跃迁：** 类似于氢原子发光，玻色子从高能级跃迁到低能级时释放引力子。
- **观测特征：** 信号极其持久（可持续数年到数百万年），频率非常稳定，但由于黑洞自旋被逐渐提取，频率会有极微小的长期漂移（Spin-down）。目前LIGO正在利用定向搜索和全天空盲搜寻找这种信号。

➢ 随机引力波背景 (Stochastic Gravitational Wave Background, SGWB)

- 宇宙中存在海量的旋转黑洞。如果超轻玻色子存在，许多黑洞都会经历超辐射并 emit 连续引力波。这些无法被单独分辨的信号叠加在一起，会形成随机引力波背景。
- **观测特征：** 具有特定的能谱形状（与双星并合产生的背景不同）。未来的脉冲星计时阵列（PTA）和空间引力波探测器有望在特定频段探测到这一背景。

GW radiation

➢ **The oscillatory monochromatic cloud will emit GWs**

Due to the separation of scales between the size of the cloud and the BH size for $\mu M \ll 1$, the GW emission can be analyzed taking the source to lie in a **nonrotating/flat** background

$$\dot{E}_{\rm GW}^{(011)} = \frac{484+9\pi^2}{23040}\left(\frac{M_s^{(011)}}{M}\right)^2 (M\mu)^{14},$$

$$\dot{E}_{\rm GW}^{(111)} = \frac{128(484+9\pi^2)}{23914845}\left(\frac{M_s^{(111)}}{M}\right)^2 (M\mu)^{14},$$

$$\dot{E}_{\rm GW}^{(022)} = \frac{1024+49\pi^2}{5423886846}\left(\frac{M_s^{(022)}}{M}\right)^2 (M\mu)^{18},$$

$$\dot{E}_{\rm GW}^{(122)} = \frac{1024+49\pi^2}{15032385536}\left(\frac{M_s^{(122)}}{M}\right)^2 (M\mu)^{18}.$$

R. Brito, V. Cardoso and P. Pani, Class. Quantum Grav. 32 (2015) 134001

H. Yoshino and H. Kodama, PTEP 2014 043E02

Guo, **BSS** and Zhang, PRD107,075009(2023)

Multi-Modes:

(011) dominated

(111) sub-dominated

GW radiation

- Numerically solve the Teukolsky formalism for the gravitational perturbations.

Guo, **BSS**&Zhang, PRD107,075009(2023)

$$\mathcal{T}_{\mu\nu}(\phi_i,\phi_j) = \frac{1}{2}\left[\partial_\mu\phi_i\partial_\nu\phi_j - g_{\mu\nu}\left(\frac{1}{2}\partial_\rho\phi_i\partial^\rho\phi_j - \frac{1}{2}\mu^2\phi_i\phi_j\right) + (i\leftrightarrow j)\right], \quad (47)$$

$$\begin{aligned}\frac{dE_{\rm GW}}{dt} = \frac{1}{8\pi}\sum_{\tilde{l}}\Bigg\{ &\frac{N_{011}^2}{\omega^{(011)2}}\frac{|U_{\tilde{l}2}^{(\tilde{\omega}_1)}|^2}{\tilde{\omega}_1^2} + \frac{N_{111}^2}{\omega^{(111)2}}\frac{|U_{\tilde{l}2}^{(\tilde{\omega}_2)}|^2}{\tilde{\omega}_2^2} + 4\frac{N_{011}N_{111}}{\omega^{(011)}\omega^{(111)}}\frac{|U_{\tilde{l}2}^{(\tilde{\omega}_3)}|^2}{\tilde{\omega}_3^2} \\ &+ 4\sqrt{\frac{N_{011}^3N_{111}}{\omega^{(011)3}\omega^{(111)}}}\frac{|U_{\tilde{l}2}^{(\tilde{\omega}_1)}||U_{\tilde{l}2}^{(\tilde{\omega}_3)}|}{\tilde{\omega}_1\tilde{\omega}_3}\cdot\cos\left[\tilde{\omega}_4(t-r_*) - \phi_{\tilde{l}2}^{(\tilde{\omega}_3)} + \phi_{\tilde{l}2}^{(\tilde{\omega}_1)}\right] \\ &+ 2\frac{N_{011}N_{111}}{\omega^{(011)}\omega^{(111)}}\frac{|U_{\tilde{l}2}^{(\tilde{\omega}_1)}||U_{\tilde{l}2}^{(\tilde{\omega}_2)}|}{\tilde{\omega}_1\tilde{\omega}_2}\cdot\cos\left[2\tilde{\omega}_4(t-r_*) - \phi_{\tilde{l}2}^{(\tilde{\omega}_2)} + \phi_{\tilde{l}2}^{(\tilde{\omega}_1)}\right] \\ &+ 4\sqrt{\frac{N_{011}N_{111}^3}{\omega^{(011)}\omega^{(111)3}}}\frac{|U_{\tilde{l}2}^{(\tilde{\omega}_2)}||U_{\tilde{l}2}^{(\tilde{\omega}_3)}|}{\tilde{\omega}_2\tilde{\omega}_3}\cdot\cos\left[\tilde{\omega}_4(t-r_*) - \phi_{\tilde{l}2}^{(\tilde{\omega}_2)} + \phi_{\tilde{l}2}^{(\tilde{\omega}_3)}\right]\Bigg\}.\end{aligned}$$

$$\tilde{\omega}_1 = 2\,\omega^{011}$$

$$\tilde{\omega}_2 = 2\,\omega^{111}$$

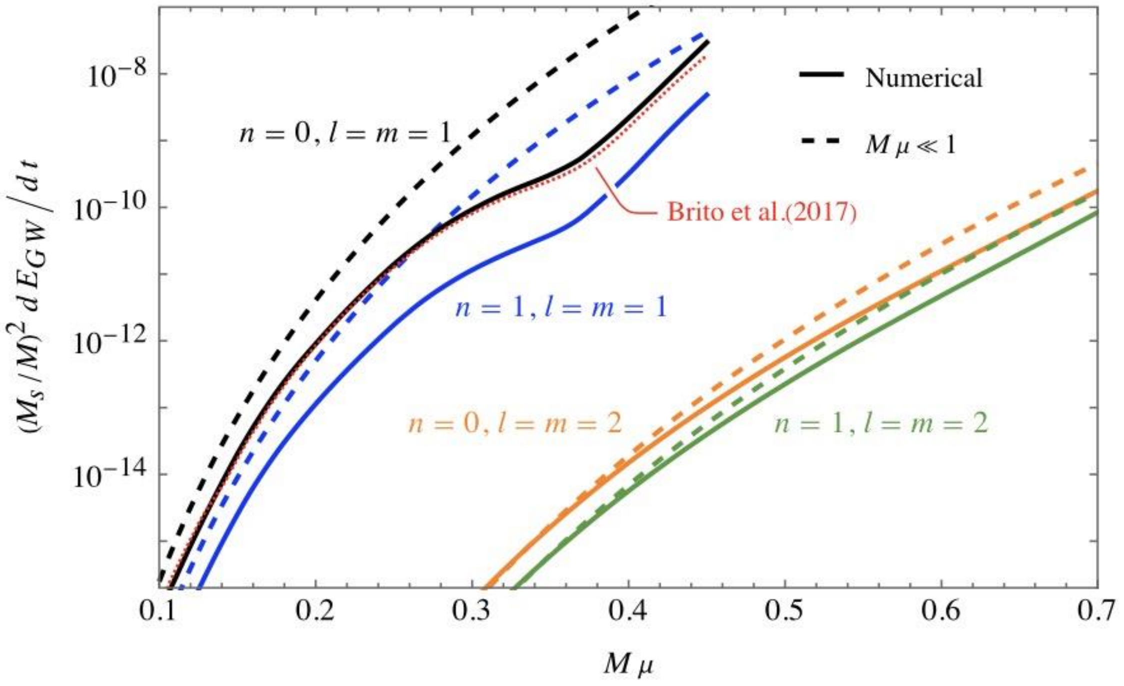
$$\tilde{\omega}_3 = \omega^{011} + \omega^{111}$$

$$\tilde{\omega}_4 = \omega^{111} - \omega^{011}$$

$$\boldsymbol{\tilde{\omega}_4 \ll \tilde{\omega}_{123}}$$

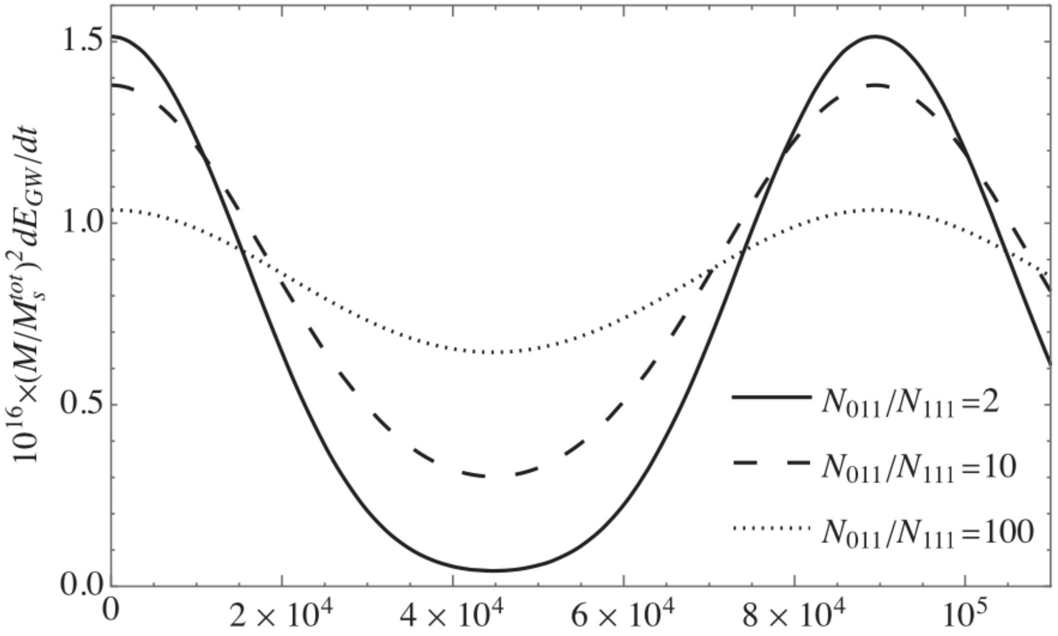
GW radiation

Monochromatic



Guo, **BSS** and Zhang, PRD107,075009(2023)

Polychromatic

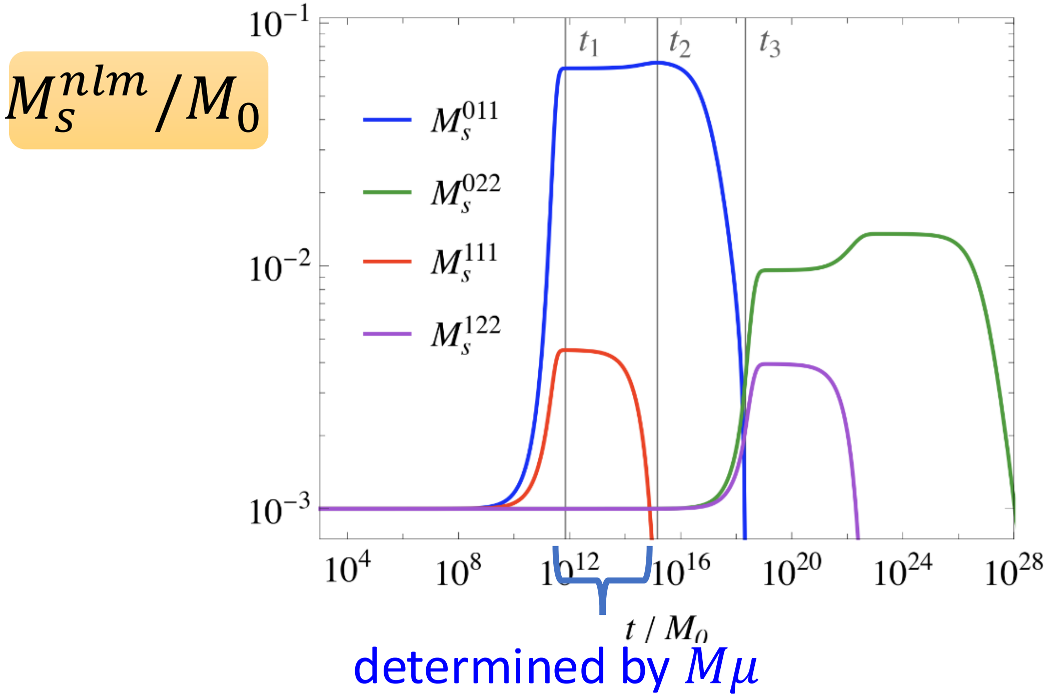


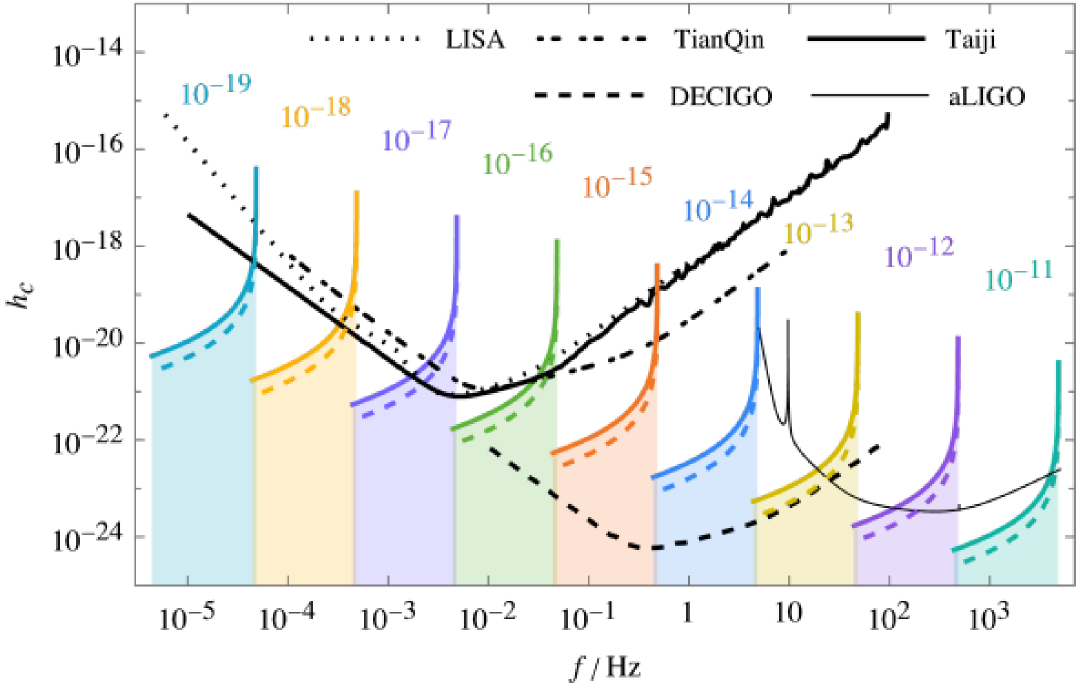
GW Beatlike Pattern: Observable

$$\approx 6.0\times 10^4 \ \sec\left(\frac{10^{-16}\ \mathrm{eV}}{\mu}\right)\left(\frac{0.1}{M\mu}\right)^2,$$

GW observation

M_s^{nlm}/M_0

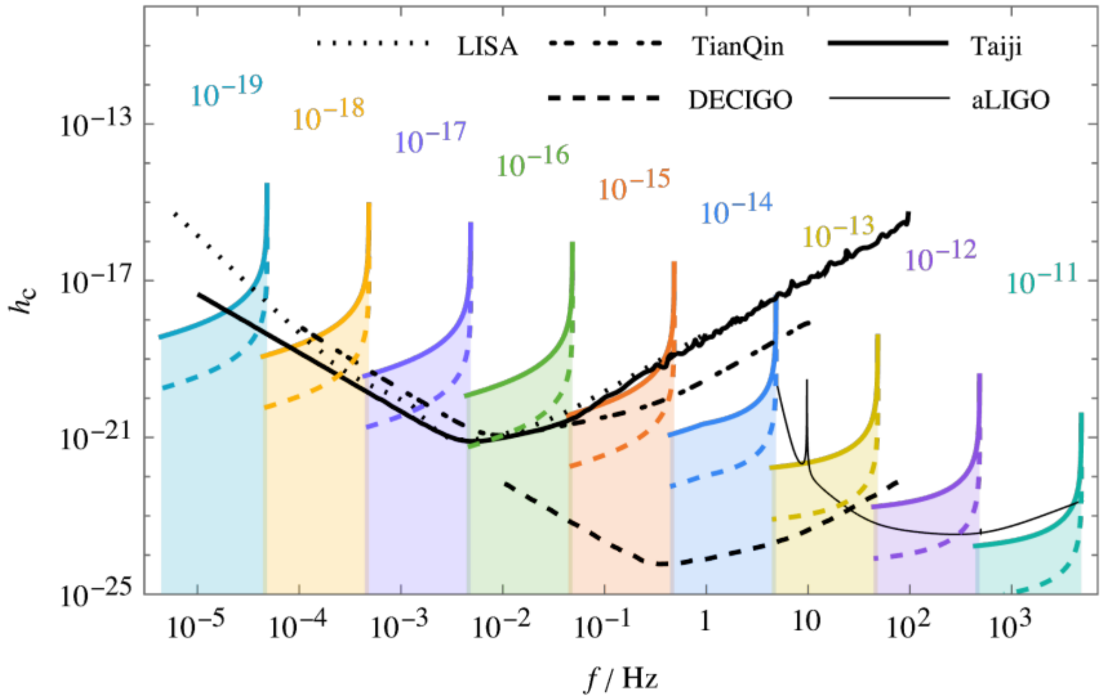




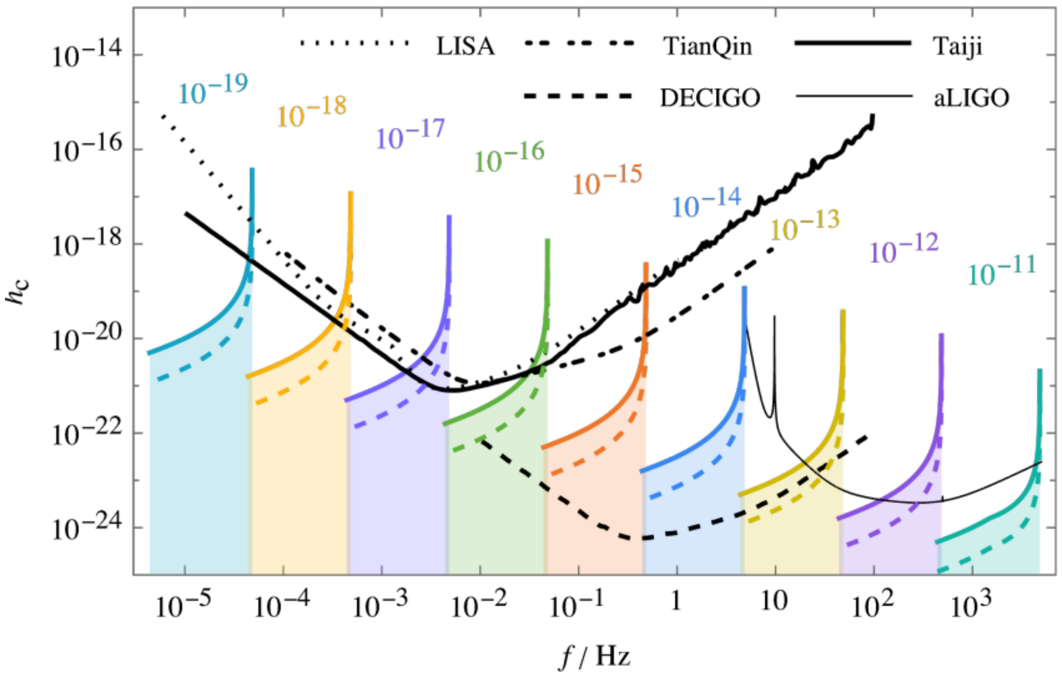
- Parameters: $M\mu = 0.17$ (so $a_C = 0.6$), $M_s/M = 0.1$, $N_{111}/N_{011} = 0.1$
- The red shift ranges from 0.001 to 10
- The current and future GW telescope can cover a large range of scalar mass.

GW radiation from vector superradiance

J=m=1 stage $GM\mu = 0.1$



J=m=2 stage $GM\mu = 0.25$



Guo, Jia, **BSS**, HZhang and XZhang, Phys.Rev.D 110 (2024) 8, 083029

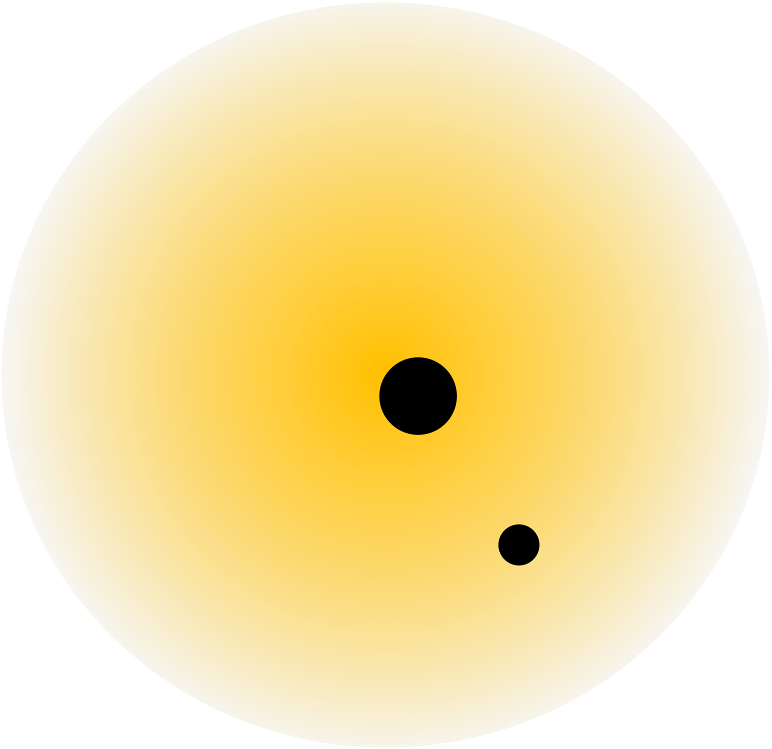
Scalar (n,l,m) → Vecor (n,j,l,m)

Outline

Detection and Signal

➢ **动力学与轨道摄动效应：**超辐射云对周围天体的引力拖拽和动力学摩擦，会改变天体的轨道演化。

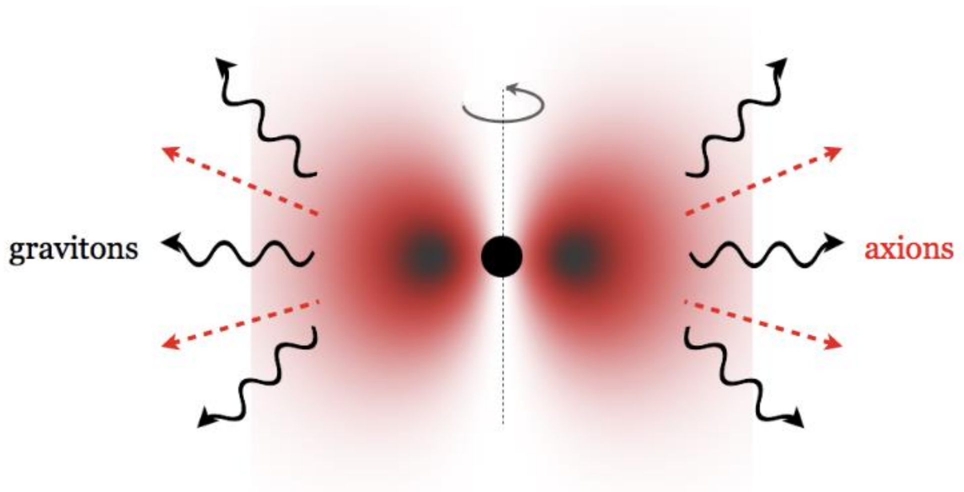
- 动力学摩擦：脉冲星在穿过或靠近玻色子云时，会受到引力动力学摩擦，导致其轨道衰减速度比纯广义相对论预测的更快。
- 多极矩摄动：玻色子云通常不是完美的球对称，也不是静态的，这会导致脉冲星轨道的近星点进动率发生异常变化。
- 云的动力学摩擦和引力摄动会改变EMRI的轨道相位演化。未来的空间引力波探测器对EMRI的轨道相位极其敏感，有望借此探测超大质量黑洞周围的超轻暗物质云。



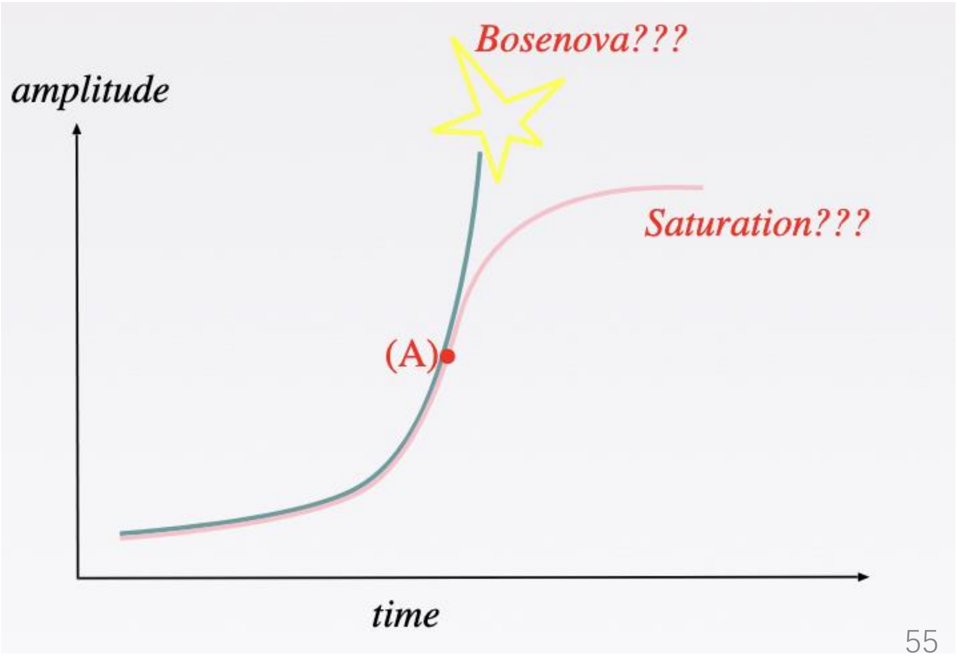
Detection and Signal

➢ “玻色子新星” (Bosenova) 爆发

- 如果玻色子场存在较强的非线性自相互作用（如轴子的周期性势能），当云的质量增长到一定程度时，会发生类似“超新星”的坍缩爆发。
- 观测特征：在极短时间内释放强烈的宽频引力波脉冲，随后云的质量大幅减少并重新生长，形成周期性的引力波爆发。



Yoshino & Kodama, PTP 128 (2012), 153-190



Detection and Signal

➢电磁波与多信使信号

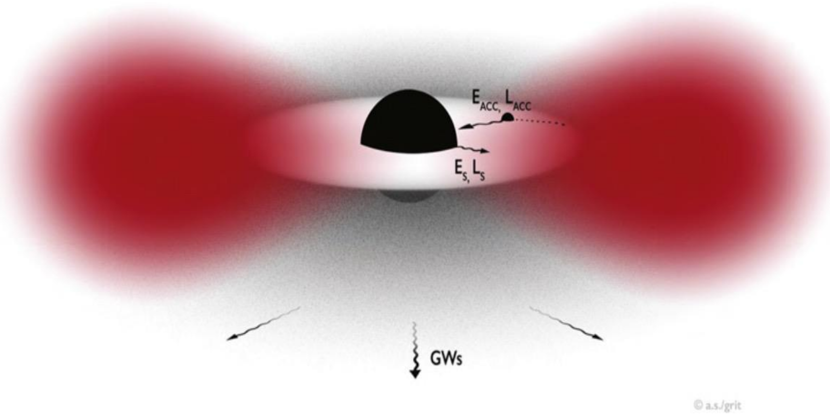
玻色子云可以通过与黑洞磁层或周围等离子体的相互作用产生电磁信号。

1. Axion-Photon Interaction
   - 在黑洞周围的强磁场中，轴子可以通过Primakoff效应转化为光子,这可能在黑洞喷流或磁层中产生高度偏振的单色谱线。
   - 光子通过Axion 云时相位发生变化，可以通过黑洞照片（EHT）观测[Yifan Chen et al, PRL.130,111401]
   - 超辐射云的不稳定性或“玻色子新星”爆发导致的瞬时轴子-光子转换，可能是某些快速射电暴(FRBs) 或未知射电瞬变源的起源。

2. 吸积盘与喷流特征的调制
   - 吸积盘的存在会改变黑洞的超辐射演化； [Guo, **BSS**, Li and Zhang, JCAP09(2025)066]
   - 超辐射云的存在也会改变喷流特征

Summary

- A brief review BH Superradiance: $\omega < \omega_c$
- The QBS for mass field around BHs: $\omega = \omega_r + i\,\omega_i$
  - Gravitational Atom: $\omega_r = \mu(1 - \alpha^2/2\bar{n}^2)$
  - Superradiant instability: $\omega_i > 0$
  - Longlived QBS for Dirac Fields: $-\omega_i \ll \omega_r$



- Ultralight ALP can be detected through BH superradiance:
  - Regge Plot: BH spin-mass plane.
  - GWs: SGWB, beat, DF.

Thank You!

- There are many unclear questions: Accretion, Non-linear effect, effect on BBHs.

Superradiant Scattering

- Superradiance is a process to extract energy from the BH.

黑洞面积随质量和角动量的变化由下式给出：

$$dA \sim dM - \Omega_H dJ$$

其中 dM 是黑洞质量的变化， dJ 是黑洞角动量的变化； ΩH 是黑洞的转动频率。
如果一个频率满足 $0<\omega<m\Omega$的波被黑洞吸收，

$$dM \sim \omega, \qquad dJ \sim m$$

视界面积将会减少，违反第二定律。

Outline

Outlook and Open Issues

一、 非线性动力学与饱和机制（理论计算瓶颈）

目前的超辐射研究大多基于线性微扰理论，这只能描述超辐射云初始的指数增长阶段。当场云的能量变得巨大时，非线性效应将占据主导，但这部分的物理图像仍不清晰；

- 非线性导致变量不可分离；
- 需要在极长时间尺度上精确求解非线性方程演化，仍是数值相对论的巨大挑战。
- 需要在极长空间尺度上精确求解非线性方程演化：超轻玻色子的康普顿波长可能比黑洞视界大几个数量级，同时分辨视界尺度和巨大的云尺度，需要极高的计算资源。
- 多自由度（多模式、多场）：黑洞周围可能同时激发多个量子数 (l,m) 的模式或者场，这些自由度之间如何演化和演变？目前缺乏完整的非线性多模式演化图景；

二、 观测与数据解释的简并性（天体物理挑战）

通过观测黑洞的“质量-自旋”分布（Regge Plot）来寻找超轻玻色子，但这面临简并性问题：

- 自旋测量的系统误差较大。如果自旋测量本身不准，基于此排除的“轴子质量窗口”就不可靠。
- 缺乏对黑洞初始条件的知识，无法给出确定答案。

Outlook and Open Issues

三、 复杂天体物理环境的干扰（现象学不确定性）

真实宇宙中的黑洞环境极其复杂，这些环境效应如何影响超辐射云尚未被完全量化：

- 吸积盘与等离子体效应：吸积盘的物质流和强磁场会如何干扰超辐射云？对于矢量场（如暗光子），周围等离子体会赋予其一个等效的“等离子体质量”，这会动态改变共振条件，甚至可能完全破坏超辐射不稳定性。
- 双星系统中的“电离”与潮汐摄动：如果旋转黑洞处于双星系统中，伴星的引力潮汐力会像“电离”氢原子一样，将超辐射云中的玻色子剥离（Cloud Ionization）。伴星的存在是会抑制超辐射，还是会通过共振激发产生新的可观测效应？

四、 基础物理与量子层面的延伸

- 修改引力理论中的超辐射
- 量子超辐射与量子引力的关系