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Gravitational Waves with High Frequency and Higher Curvature Gravity

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JCAP08 (2023) 023

A.Biswas, A. Kar, **BHL**, H.Lee, W.Lee, S.**Scopel,** L.Velasco-Sevilla, L.Yin
JCAP 09 (2024) 007

S. Kang, **BHL**, W. Lee, **S. Scopel**, G. Tumurtushaa **work in progress**

W.Lee, M. Mishra, G. Tumurtushaa, L.Yin, BHL, **work in progress**

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A. Biswas, A. Kar, **BHL**, H. Lee, W. Lee, **S. Scopel**, Velasco-Sevilla, L. Yin **JCAP08 (2023) 023**

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S. Kang, **BHL**, W. Lee, **S. Scopel**, G. Tumurtushaa **work in progress**

W.Lee, M. Mishra, G. Tumurtushaa, L.Yin, **BHL, work in progress**

I. Motivation : Is beyond Einstein Gravity needed?

1. Theoretical Aspect: - GR is an effective theory valid below $M_{UV} \sim M_{Pl} \sim 10^{19} GeV$

Ex) String theory $\xrightarrow[E \lesssim M_{Pl}]{} \sum_{n \geq 0} (\sim R)^n$ Gauss-Bonnet term $R^2_{GB} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\varrho\sigma}$

Ex) Einstein-dilaton-Gauss-Bonnet (EdGB) **theory** $S = \int d^d x \sqrt{-g} \frac{1}{16\pi G} [-2\Lambda + R + \alpha(\phi) R^2_{GB}]$

Einstein Gravity : linear in R, is the simplest, but **incomplete** theory of gravity.

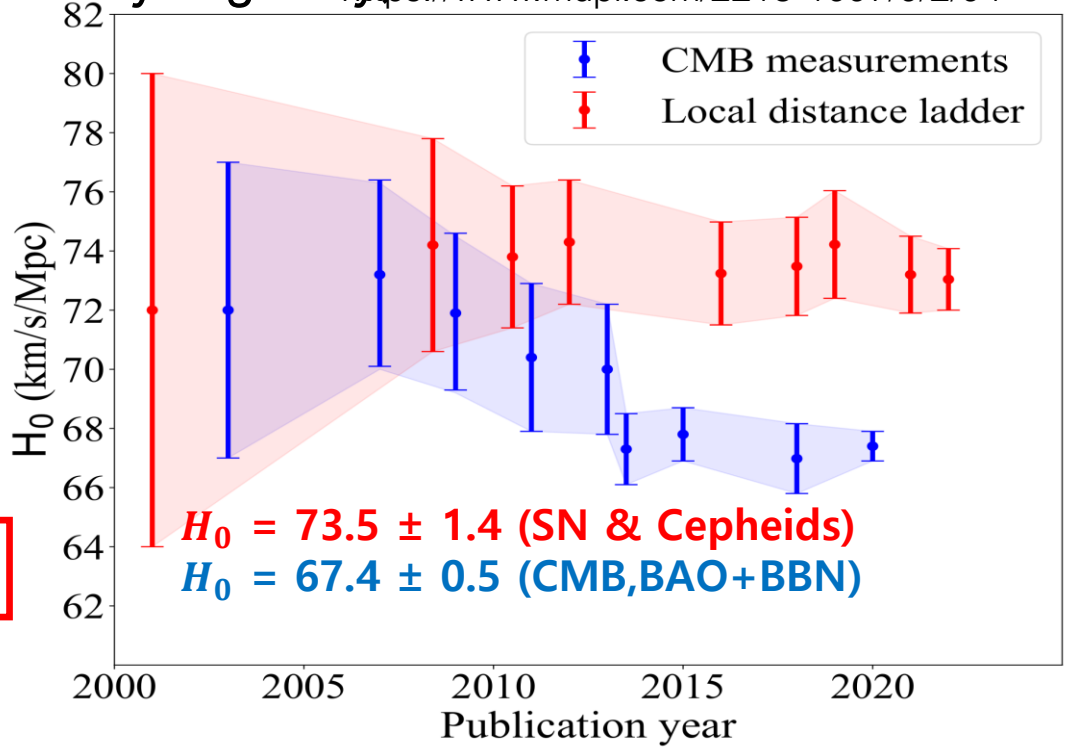
There MUST be higher orders of curvature terms.

2. Observational Aspect:

1) H_0 tension (~5.8σ) Colgaine, BHL W.Lee, Sheikh-Jabbari, Thakur, JCAP (2022)
2) Cosmic Birefringence(~3σ) Kochappan, Yin, B-HL, Ghosh PRD (2025)
3) S_8/σ_8tension (2-3σ)

Some observations challenging the ΛCDM

https://www.mdpi.com/2218-1997/9/2/94

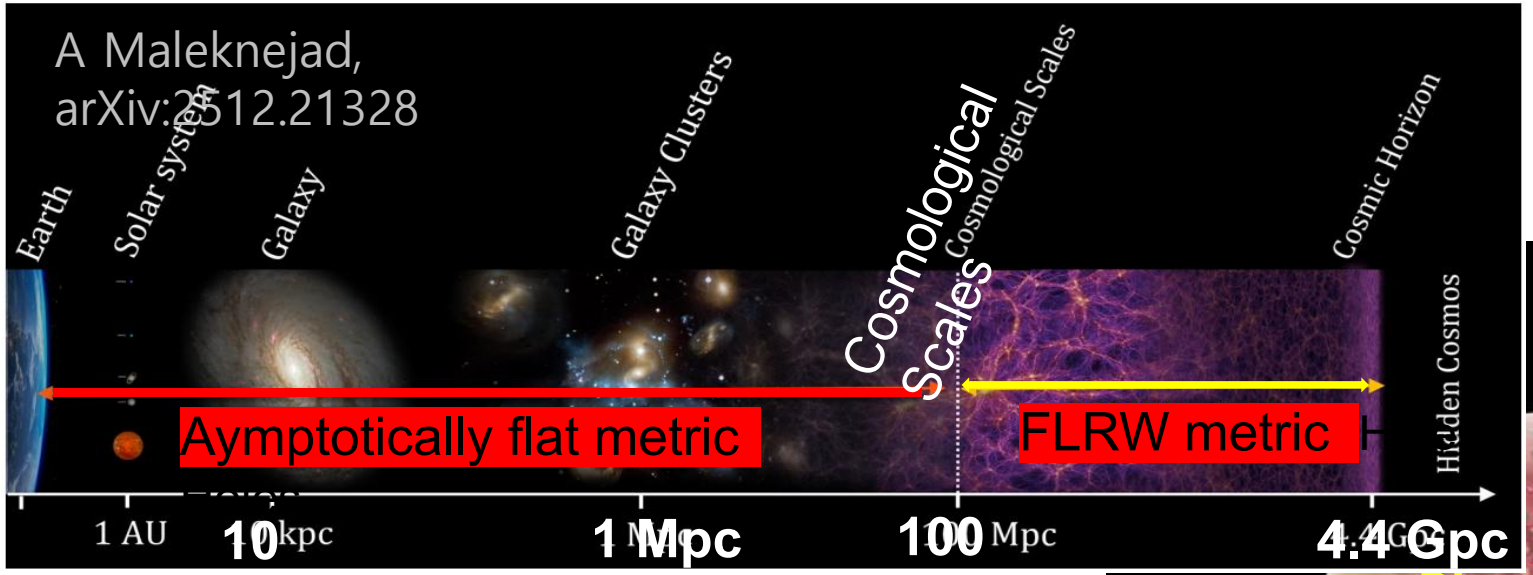


3. A simplest theory beyond Einstein

Einstein-Dilaton-Gauss-Bonnet (EdGB) **Gravity theory**

III. EdGB Cosmology

ΛCDM (Λ -Cold Dark Matter)/Standard Cosmology

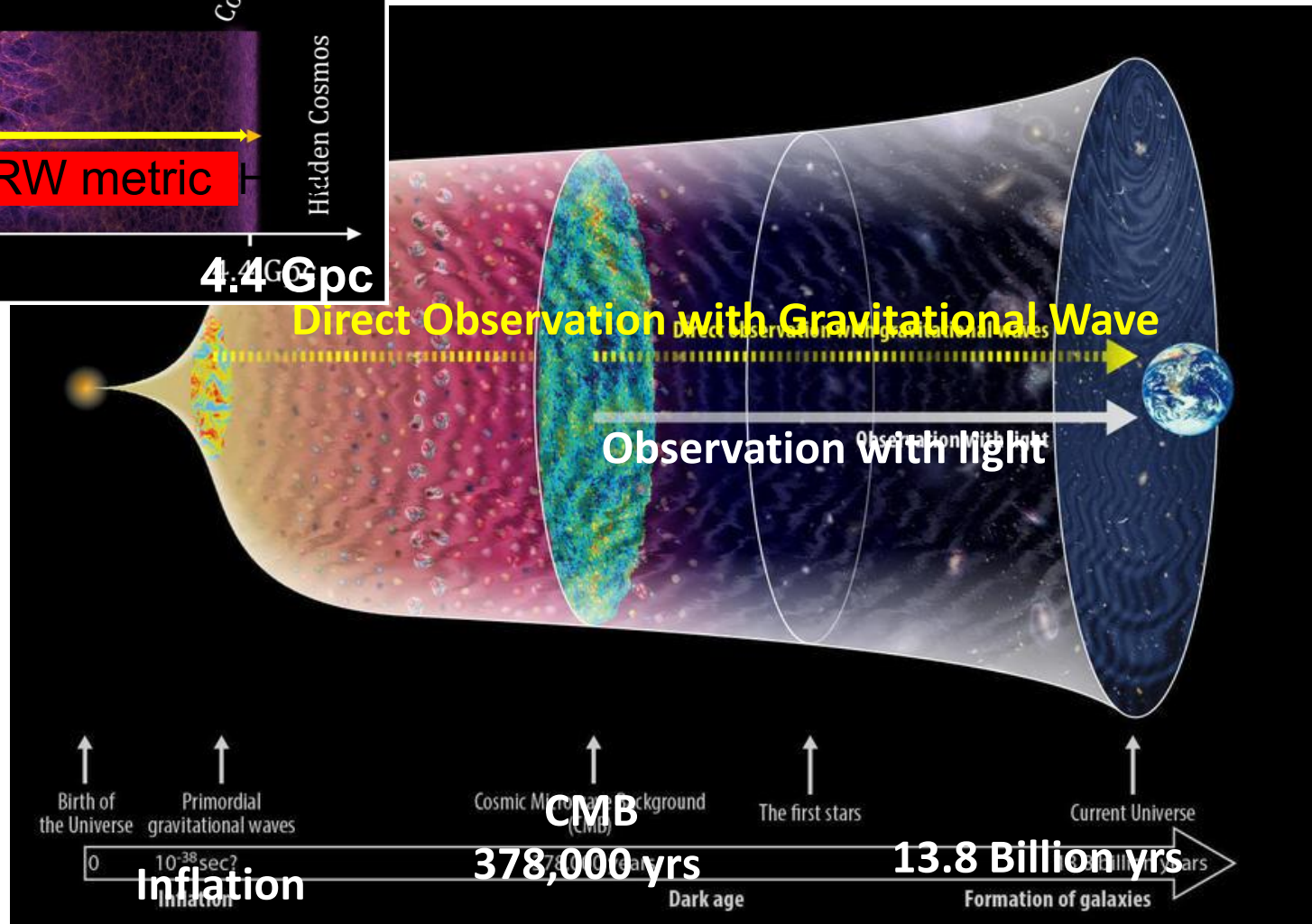


Observation : Universe is

1) Homogeneous & Isotropic
2) Expanding

FLRW metric (spatially flat) ,

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$



Observational Tool

1) Light (electromagnetic waves)
ex) Cosmic Microwave Background (CMB)
Can observe upto decoupling time

2) Gravitational Waves (2015~)
Ex) Cosmic Gravitational Microwave Background (CGMB)
Can observe upto deep into the early universe.

ΛCDM : based on Einstein Gravity

II Einstein-dilaton-Gauss-Bonnet (EdGB) Gravity

The general theory with quadratic curvature terms with the 2nd order e.o.m

(in $d > 4$) the Einstein-Gauss-Bonnet (EGB)- Λ Gravity (GB-AdS)

$$S_{EGB-\Lambda} = \int d^d x \sqrt{-g}\left[\frac{1}{2\kappa}(R - 2\Lambda + \alpha R^2_{GB}) + \mathcal{L}_m^{matt}\right]$$

Note : $\Lambda = -\frac{(d-1)(d-2)}{2\ell^2}$

$\kappa = 8\pi G$, $g = \det g_{\mu\nu}$

$R^2_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ Gauss-Bonnet (G-B) term

$[\alpha] = [\text{length}]^2$

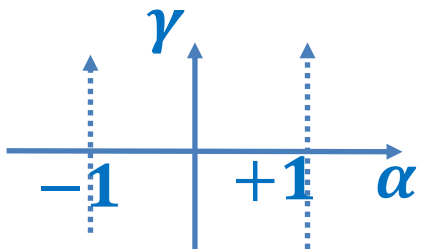
(**In $d = 4$**) the Einstein-Dilaton-Gauss-Bonnet (EDGB) Gravity

$$S_{EdGB} = \int d^4 x \sqrt{-g}\left[\frac{1}{2\kappa}(R + f(\phi)R^2_{GB}) + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m^{matt}\right]$$

$\boldsymbol{f(\phi)}$: the coupling function; $f(\phi) = \alpha e^{\gamma\phi}$ (*) or $f(\phi) = \sim\phi^n$ etc.

(*) α can be absorbed by r $\rightarrow$ r/$\sqrt{\alpha}$. Only sign of α is important

$\rightarrow$ reduce (it to 2 or) 1 parameters $(\boldsymbol{\alpha}, \boldsymbol{\gamma})$ or 1 $(\text{sgn}(\boldsymbol{\alpha}), \boldsymbol{\gamma})$



Note

For $\alpha = 0$ (or $\gamma = 0$), DEGB becomes the Einstein (+scalar) theory.

III-1) EdGB Black Holes

Guo,N.Ohta & T.Torii, (2008); (2009); Ohta &Torii, (2009);(2010);
Maeda,Ohta Y.Sasagawa, (2009);(2011) Ohta & Torii, (2013). Kanti et al

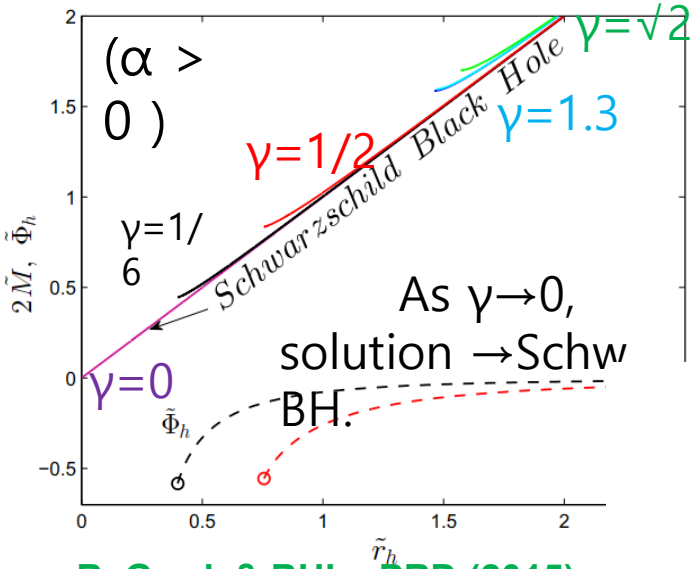
BHL, W. Lee, D. Rho, PRD (2019)

minimum mass

Soliton Star? Black Holes

M=0 M_{min} $\rightarrow \infty$

→New Phase?



S. Koh, BHL, Tumurtushaa **PRD98 (2018)**

S. Koh, BHL, Tumurtushaa **PRD 95 (2017)**

S. Koh, BHL, W. Lee, Tumurtushaa **PRD**

B. Gwak & BHL, PRD (2015).

B.Gwak, BHL, D. Rho, PL.B (2016)

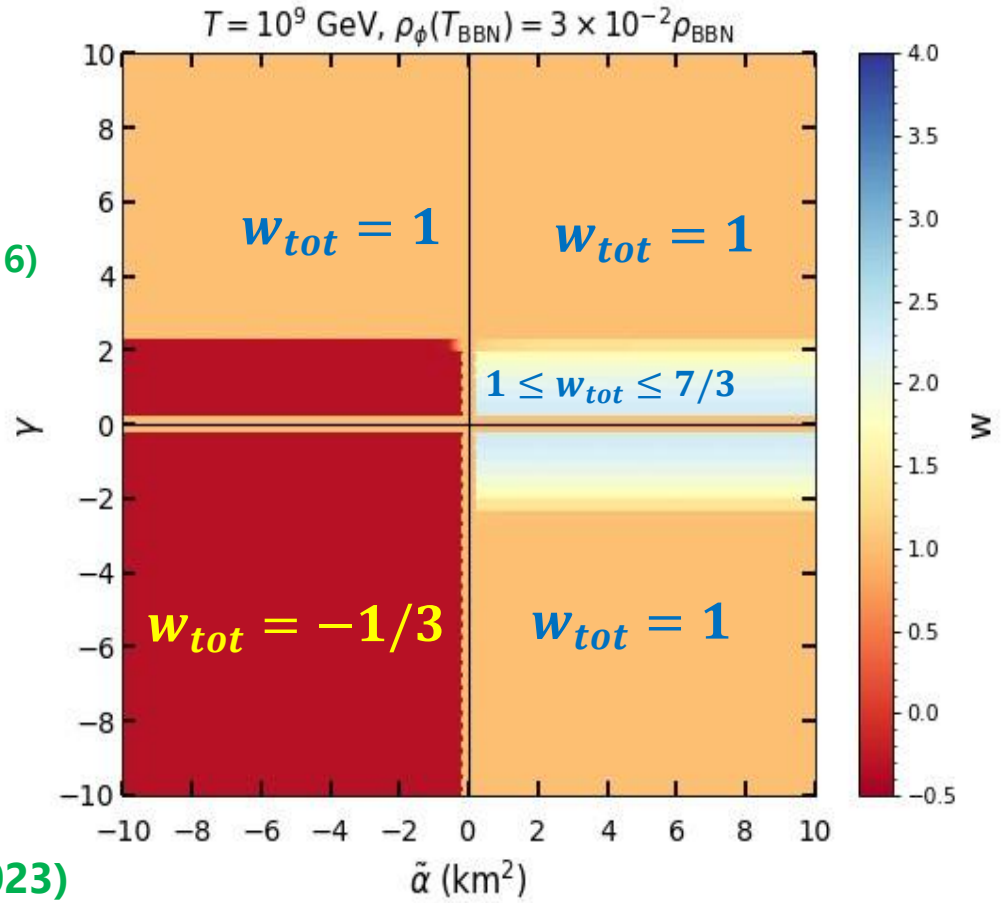
III-2) EdGB Cosmology

NEW PHASE

NEW PHASE→ | ←Rad Dom→ | ← Matt → | ← Λ(DE)

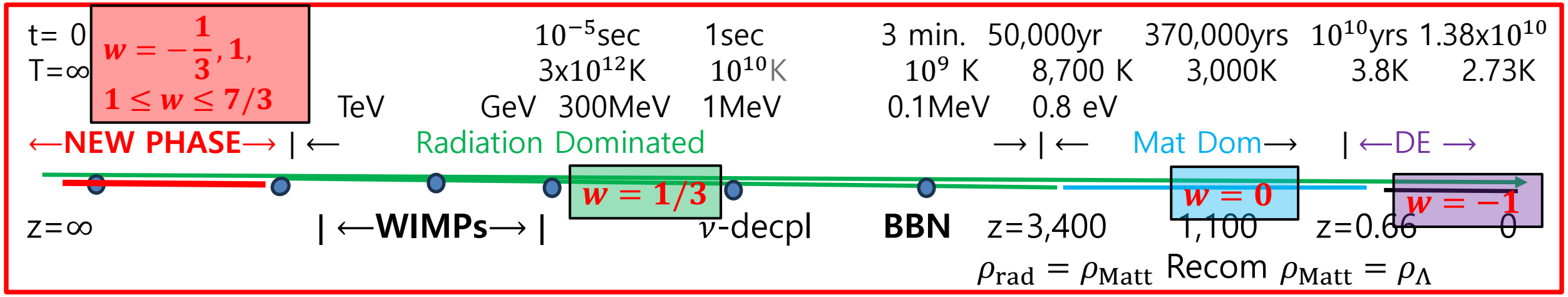
$w_{tot} = -\frac{1}{3}, 1,$ $w = \frac{1}{3}$ $w = 0$ $w = -1$

$1 \leq w_{tot} \leq 7/3$



Biswas, Kar, **BHL**, H.Lee, W.Lee, **Scopel**, Velasco-Sevilla, L. Yin **JCAP (2024)**

Biswas, Kar, **BHL**, H. Lee, W. Lee, **Scopel**, Velasco-Sevilla, L. Yin **JCAP08 (2023)**



Friedmann-Lemaitre-Robertson-Walker (FLRW) metric,

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \quad \text{spatially flat}$$

The Einstein and scalar Eqs.

$$H^2 = \frac{\kappa}{3}\left(\rho_{\{\phi+GB\}} + \rho_m\right) = \frac{\kappa}{3}\rho_{tot}$$

$$\dot{H} = -\frac{\kappa}{2}\left[\left(\rho_{\{\phi+GB\}} + p_{\{\phi+GB\}}\right) + (\rho_m + p_m)\right]$$

$$\equiv -\frac{\kappa}{2}(\rho_{tot} + p_{tot}) = -\frac{\kappa}{2}\rho_{tot}(1 + w_{tot})$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) + V'_{GB} = 0$$

$H = \frac{\dot{a}}{a}$ Hubble Parameter

$$\rho_m(p_m) \equiv \rho_{\rm rad}(p_{\rm rad})$$

$$w_{tot} = \frac{p_{tot}}{\rho_{tot}} = \frac{p_{rad} + p_\phi + p_{GB}}{\rho_{rad} + \rho_\phi + \rho_{GB}}$$

Deceleration parameter

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} \equiv -H^2 q$$

$$= -\frac{\kappa}{6}\left(\rho_{\{\phi+GB\}} + 3p_{\{\phi+GB\}}\right) + (\rho_m + 3p_m)$$

$$= -\frac{1}{2}H^2(1 + 3w_{tot})$$

Note Treat G-B & ϕ as "matters".

Then $\rho_{GB}\ p_{GB};\ \rho_{\{\phi+GB\}}$;& $p_{\{\phi+GB\}}$:
NOT necessarily +tive.

Bdry Conditions at BBN ($T_{BBN} \simeq 1\ MeV$) for $\phi, \dot{\phi}, a, \dot{a}$

$\phi_{BBN} = 0$ (shift of $\phi_{BBN} \Leftrightarrow \alpha$-scaling) Choose

$\dot{\phi}_{BBN} \ge 0$ ($\epsilon \equiv \frac{\rho_\phi(T_{BBN})}{\rho_{tot}(T_{BBN})} \le 3\times10^{-2} \leftarrow N_{eff} \le 2.99 \pm 0.17$)

IV. Cosmic Gravitational Microwave Background (CGMB)

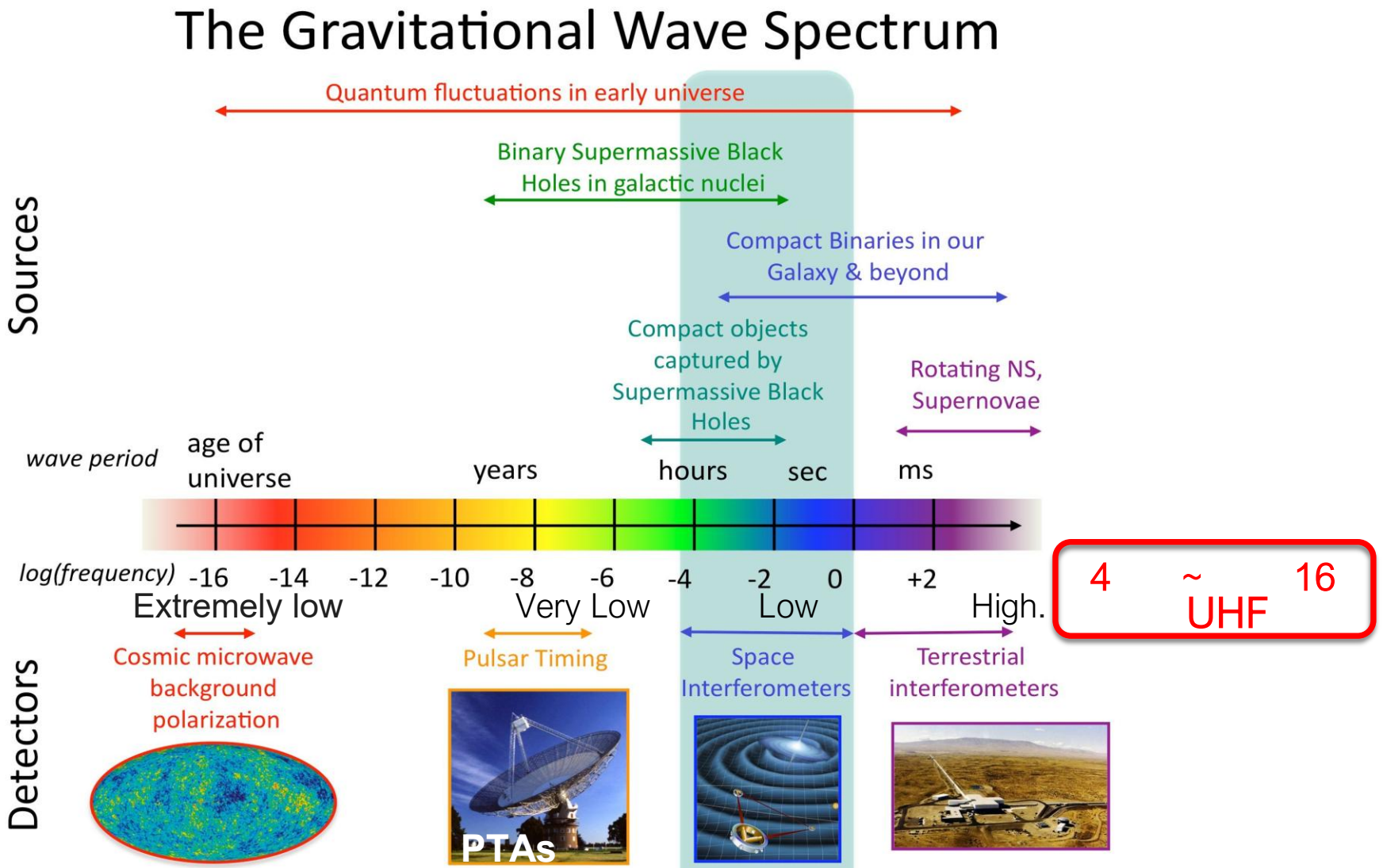
http://science.gsfc/nasa.gov

Sources

- PBH ($m \lesssim 10^8 g$)
- Cos Ph Tr
- Preheating
- Cosmic Strings.
- from the Sun:
- **Plasma in Early Universe (CGMB)**

Detection

ADMX, HAYSTAC, CAPP



4 ~ 16 UHF

"tensor-to-scalar ratio" $(r < 0.032)$ an upper bound

$h_c \approx 2\times10^{-15}$
$f \approx 3\times10^{-8}$ Hz.

LISA
TAIJI
TIANJIN

LIGO/Virgo/KAGRA
$\Omega_{GW} \lesssim 5.8\times10^{-9}$
in 20-90 Hz band.

Summary of Sources

Agarwal et.al, LRR (2021)

1) Summary of late universe sources.

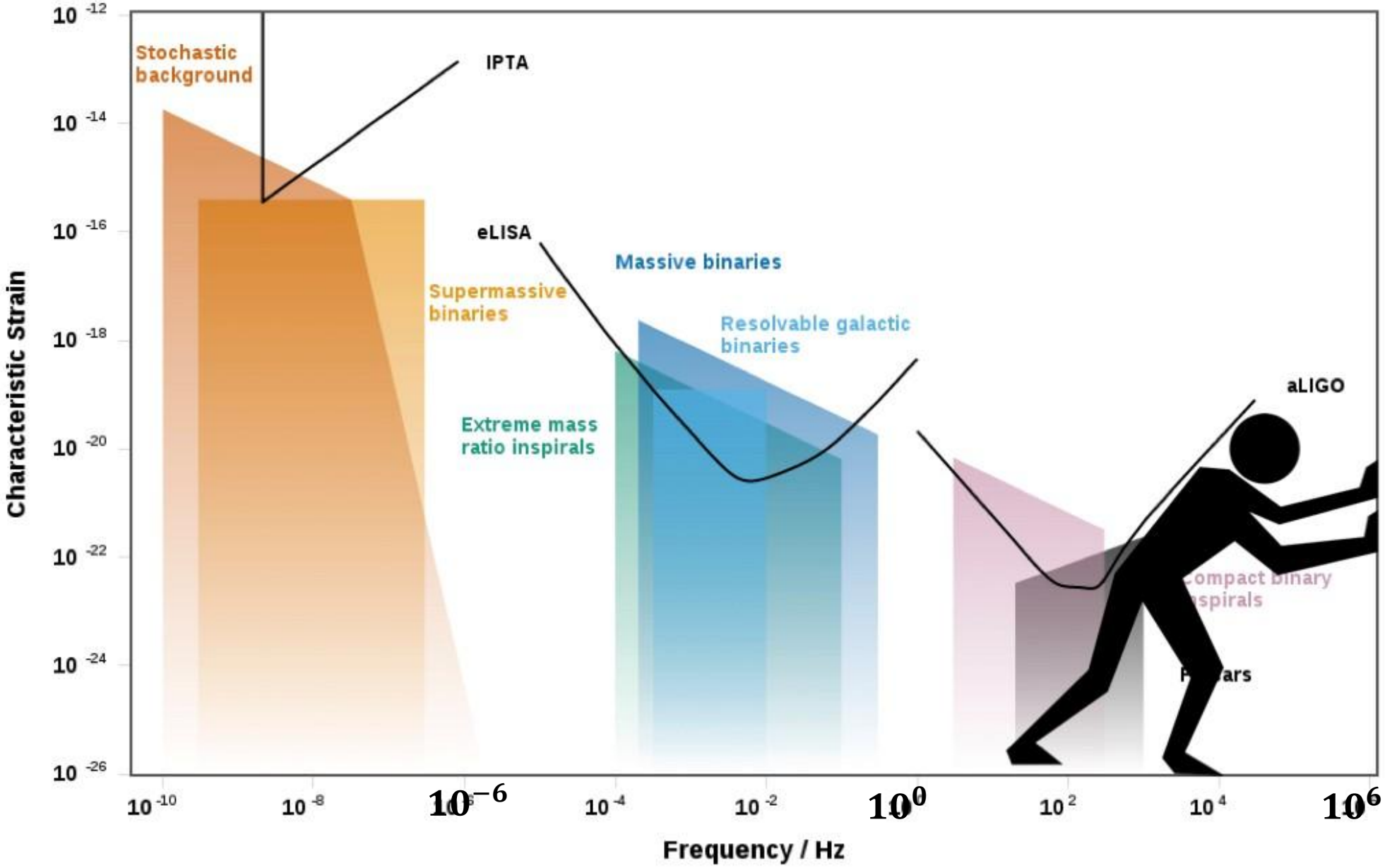
	Source	Typical frequency	Amplitude
NS	Neutron star mergers	$\lesssim (1-5)\,\mathrm{kHz}$	$h(f) \lesssim 10^{-26}\,\mathrm{sec}$
	Phase transitions in neutron star mergers	$\simeq 0.6\,\mathrm{MHz} \times \left(\frac{0.1}{v_w}\right)\left(\frac{1\,\mathrm{ms}}{\tau}\right)$	$h_{c,\mathrm{sto}} \simeq 1.5 \times 10^{-24} v_f^2 \left(\frac{100\,\mathrm{Mpc}}{D}\right)$
	Disk around supermassive BHs	$\simeq 3.3 \times 10^{19}\,\mathrm{Hz}$	$h_{c,\mathrm{sto}} \lesssim 3 \times 10^{-44}$
	Sun	$\simeq 10^{14}\,\mathrm{Hz}$	$h_{c,\mathrm{sto}} \lesssim 3 \times 10^{-42}$
BH	Primordial BH mergers	$\lesssim \frac{4400}{(m_1+m_2)}\,\mathrm{Hz}$	$h(f) \approx 10^{-31}\,\mathrm{sec}\left(\frac{\mathrm{kpc}}{D}\right)\left(\frac{m_{\mathrm{PBH}}}{10^{-5}M_\odot}\right)^{5/6}\left(\frac{f}{\mathrm{GHz}}\right)^{-7/6}$
	Primordial BH mergers: SGWB	$\lesssim \frac{4400}{(m_1+m_2)}\,\mathrm{Hz}$	$h_{c,\mathrm{sto}} \approx 5 \times 10^{-31}\left(\frac{f_{\mathrm{ISCO}}}{\mathrm{GHz}}\right)^{-1.07}$
	Primordial BH mergers: memory	$\lesssim \frac{4400}{(m_1+m_2)}\,\mathrm{Hz}$	$h(f) \approx 5 \times 10^{-25}\,\mathrm{sec}\left(\frac{f_{\mathrm{ISCO}}}{f}\right)\left(\frac{m_{\mathrm{PBH}}}{10^{-5}M_\odot}\right)\left(\frac{\mathrm{kpc}}{D}\right)$
	Primordial BH hyperbolic encounters	$\simeq 0.5\,\mathrm{GHz}\left(\frac{10^{-5}M_\odot}{m_{\mathrm{PBH}}}\right)\left(\frac{R_S}{r_p}\right)^{3/2}$	$h(f) \approx 10^{-24}\,\mathrm{sec}\left(\frac{f}{\mathrm{GHz}}\right)^{2/3}\left(\frac{m_{\mathrm{PBH}}}{10^{-5}M_\odot}\right)^{5/3}\left(\frac{\mathrm{Mpc}}{D}\right)$
Cpt Obj	Exotic compact objects	$\lesssim C^{3/2}\left(\frac{6\times 10^{-3}\,\mathrm{M}_\odot}{M}\right)\mathrm{MHz}$	$h(f) \approx 10^{-31}\,\mathrm{sec}\left(\frac{f}{\mathrm{GHz}}\right)^{-7/6}\left(\frac{m_{\mathrm{PBH}}}{10^{-5}M_\odot}\right)^{5/6}\left(\frac{\mathrm{kpc}}{D}\right)$
SupRad	Superradiance: annihilation	$\simeq 5\,\mathrm{MHz}\left(\frac{\mu}{10^{-8}\,\mathrm{eV}}\right)$	$h_S(f) \approx 5 \times 10^{-30}\,\mathrm{sec}\left(\frac{m_{\mathrm{PBH}}}{10^{-5}M_\odot}\right)\left(\frac{\mathrm{kpc}}{D}\right)$ $h_{V,T}(f) \approx 10^{-26}\,\mathrm{sec}\left(\frac{m_{\mathrm{PBH}}}{10^{-5}M_\odot}\right)\left(\frac{\mathrm{kpc}}{D}\right)$
	Superradiance: nonlinear effects	$\simeq 5\,\mathrm{MHz}\left(\frac{\mu}{10^{-8}\,\mathrm{eV}}\right)$	$h(f) \approx 10^{-27}\,\mathrm{sec}\left(\frac{m_{\mathrm{PBH}}}{10^{-5}M_\odot}\right)\left(\frac{\mathrm{kpc}}{D}\right)$

2) Summary of stochastic sources

Agarwal et.al, LRR (2021)

	Source	Frequency Range	Amplitude $\Omega_{\rm GW}(f)$	Characteristic Strain $h_{c,\rm sto}$
Inf	Inflation: vacuum amplitude	flat in the range $(10^{-16} - 10^{8})$ Hz	$\lesssim 10^{-16}$	$\lesssim 10^{-32}\left(\frac{\rm MHz}{f}\right)$
	Inflation: extra-species	$(10^{5} - 10^{8})$ Hz	$\lesssim 10^{-10}$	$\lesssim 10^{-29}\left(\frac{\rm MHz}{f}\right)$
	Inflation: broken spatial reparametrization	Blue in the range $(10^{-16} - 10^{8})$ Hz	$\lesssim 10^{-10}$	$\lesssim 10^{-29}\left(\frac{\rm MHz}{f}\right)$
	Inflation: secondary GW production	Flat or bump	$\lesssim 10^{-8}$	$\lesssim 10^{-28}\left(\frac{\rm MHz}{f}\right)$
(P)RH	Preheating	$(10^{6} - 10^{9})$ Hz	$\lesssim 10^{-10}$	$\lesssim 10^{-29}\left(\frac{\rm MHz}{f}\right)$
	Oscillons	$(10^{6} - 10^{9})$ Hz	$\lesssim 10^{-10}$	$\lesssim 10^{-29}\left(\frac{\rm MHz}{f}\right)$
CGMB	Cosmic gravitational microwave background	$f_{\rm peak} \sim (10 - 100)$ GHz	$\Omega_{\rm GW}(f_{\rm peak}) \lesssim 10^{-6}$	$h_c(f_{\rm peak}) \lesssim 10^{-31}\left(\frac{\rm MHz}{f}\right)$
PhT	Phase transitions	$\lesssim 10^{9}$ Hz	$\lesssim 10^{-8}$	$\lesssim 10^{-28}\left(\frac{\rm MHz}{f}\right)$
	Defects	Scale invariant	$\Omega_{\rm rad,0}\,\frac{v^4}{M_{\rm Pl}^4}\,F_U$	$10^{-26}\,\frac{v^4}{M_{\rm Pl}^4}\,F_U\left(\frac{\rm MHz}{f}\right)$
	Gauge textures	$\sim 10^{11}\,\frac{v}{M_{\rm Pl}}$ Hz	$\lesssim 10^{-4}\,\frac{v^4}{M_{\rm Pl}{}^4}$	$\lesssim 10^{-26}\,\frac{v^4}{M_{\rm Pl}^4}\left(\frac{\rm MHz}{f}\right)$
PBH	Grand unification primordial BH evaporation	$(10^{18} - 10^{15})$ Hz	$\lesssim 10^{-8}$	$\lesssim 10^{-28}\left(\frac{\rm MHz}{f}\right)$

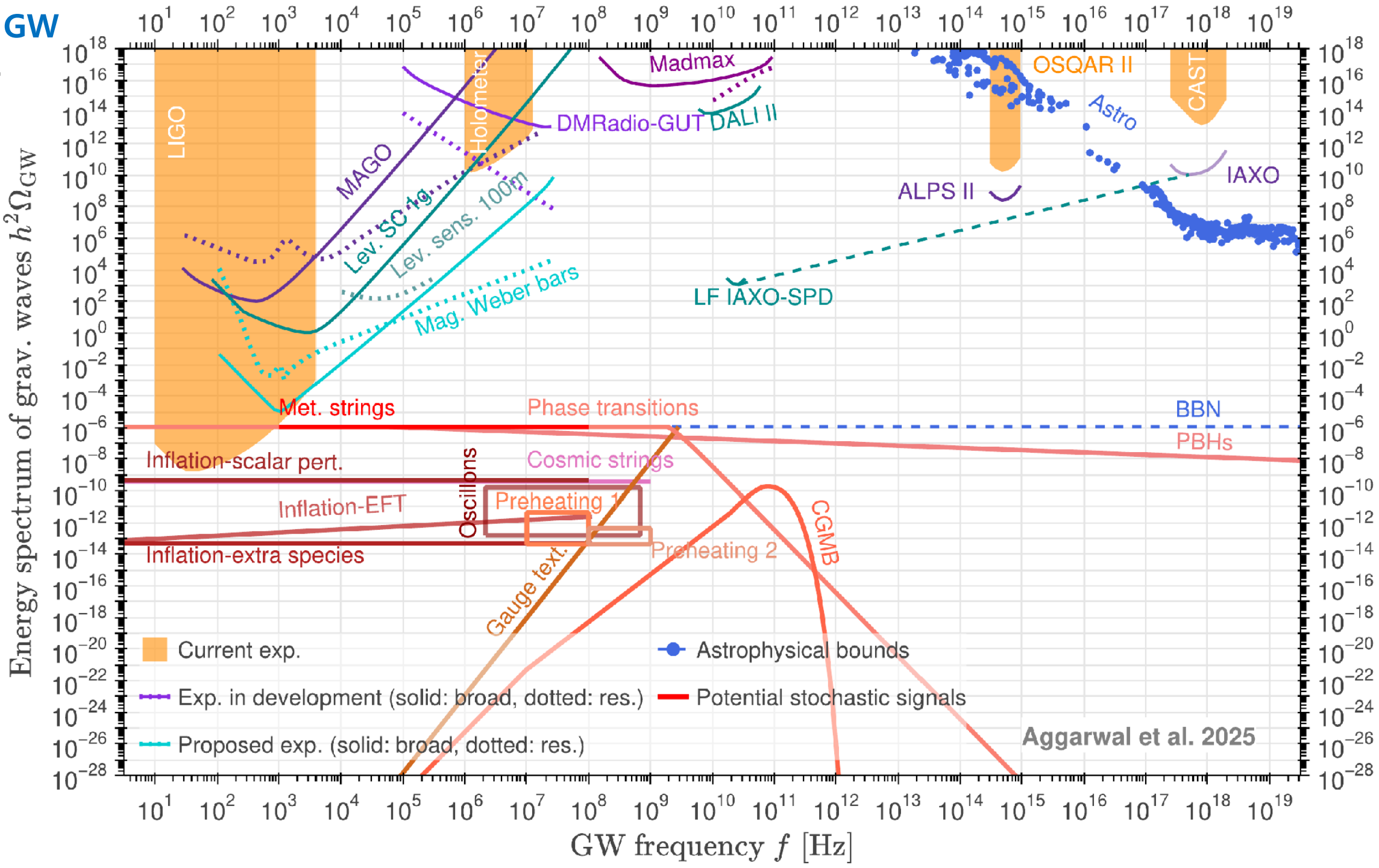
The Gravitational Wave Spectrum



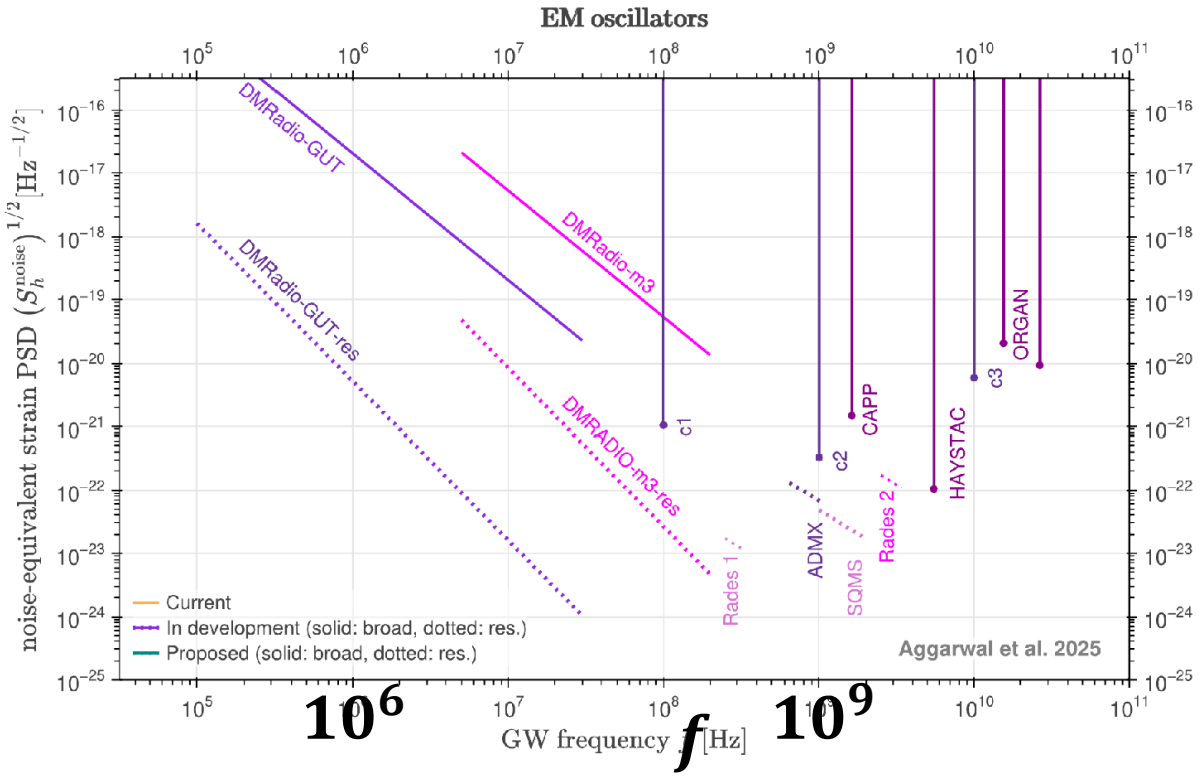
https://www.ctc.cam.ac.uk/activities/UHF-GW.php

Gravitational Waves Initiative

High-freq GW
Sources & detectors



EM oscillators



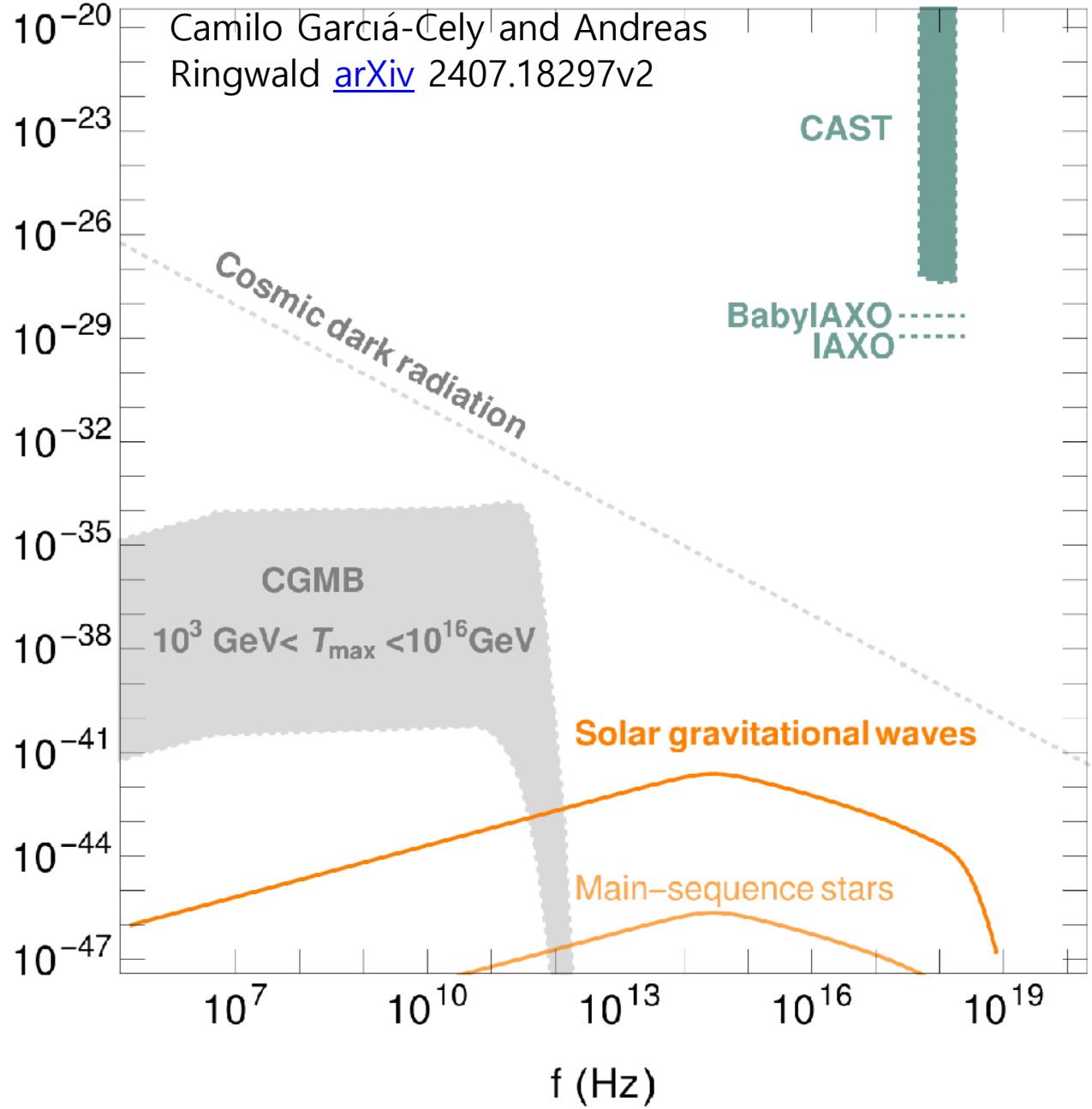
10^6 f 10^9



GW **axions**

Berlin et.al, PRD 105, 116011(2022)

Characteristic GW strain h_c vs freq f



Cosmic Gravitational Microwave Background (CGMB)

Any thermal plasma emits GWs : stochastic GW background (SGWB)

Energy liberated into GW radiation,

$$\Omega_{GW}(f,T_0)h^2 \equiv \frac{1}{\rho_{crit}(T_0)}\frac{d\rho_{GW}(T_0)}{d\ln f}h^2$$
$$\propto \int_{T_{EWCO}}^{T_{max}} dT \left(\frac{g_{*0}}{g_*(T)}\right)^{\frac{4}{3}} T^2 \hat{k}(f,T)^3 \frac{\eta(\hat{k},T)}{\sqrt{\rho(T)}}\beta(T) \propto \frac{T^3}{\sqrt{\rho}}$$

Note

- UV-dominated, by GWs emitted at high T, i.e., $\boldsymbol{kT_{max}}$.
- **For $T_{max} < M_P$**, GWs are **out-of-equilibrium**
- $\eta(\hat{k},T)$ **peak at $\hat{k}_{peak} \simeq 3.92$ (at production)**
  or $f_{peak} \simeq 74$GHz (today)

The BBN bound:
$\Omega(f,T_0)h^2 < 1.3\times10^{-6}$

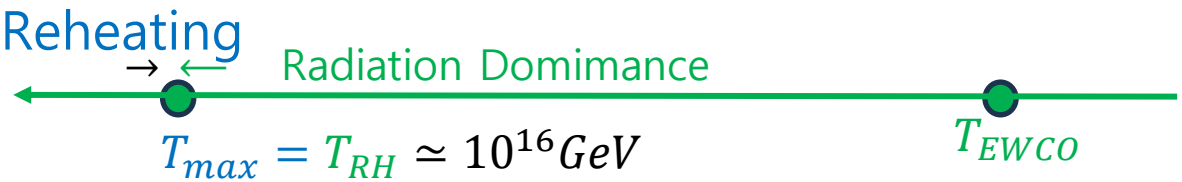
$$\hat{k}(f,T) = \left[\frac{g_{*S}(T)}{g_{*S}(T_0)}\right]^{\frac{1}{3}}\frac{2\pi f}{T_0}$$

$$f = \frac{1}{2\pi}\left[\frac{g_{*S}(T_0)}{g_{*S}(T_{EWCO})}\right]^{\frac{1}{3}}\left(\frac{T_0}{T_{EWCO}}\right)k_{EWCO}$$

$\eta(k,T)$:the shear viscosity of the plasma
T_{EWCO} = 160 GeV $\quad \hat{k} = k/T$
$h = H_0/(100km/s/Mpc) \quad T_0 = 2.7K$
f: freq. measured today; $\quad \lambda = 30\sqrt{3}/\pi^4$,
$\Omega_{\gamma,0} = 2.47\times10^{-5}$ photon density (now)
$g_{*S}(T_0) = 3.91,\ g_{*S}(T_{EWCO}) = 106.75$
$g_{*0} = 2, \quad \beta = \left(1 + \frac{1}{3}\frac{d\ln g_{*s}}{d\ln T}\right)$
$\rho_{crit} = 3H^2/(8\pi G)$

J. Ghiglieri and M. Laine, JCAP (2015)
Ghiglieri, Jackson, Laine, Zhu, JHEP (2020)

For ΛCDM



$$\frac{d\Omega_{GW}}{d\ln f} \propto \frac{T^3}{\sqrt{\rho}} \propto T,$$

rad. Dom. for all $T_{max} > T_{EWCO}$,
- Ω_{GW} is a monotonically growing fn of T_{max},
- Ωh^2 at $T_{max} = T_{RH}(10^{16} GeV)$
- the BBN bound

the EdGB scenario for CGMB bounds



For **slow roll (w = −1/3)**, "slow-roll regime"

$$\frac{d\Omega_{GW}}{d\ln f} \propto \frac{T^3}{\sqrt{\rho}} \propto T^2$$

$\rho_{\rm rad}$, $|\rho_{\rm GB}| \propto T^4$, while $\rho_{\rm rad} + \rho_{\rm GB} \propto T^2$

strong enhancement

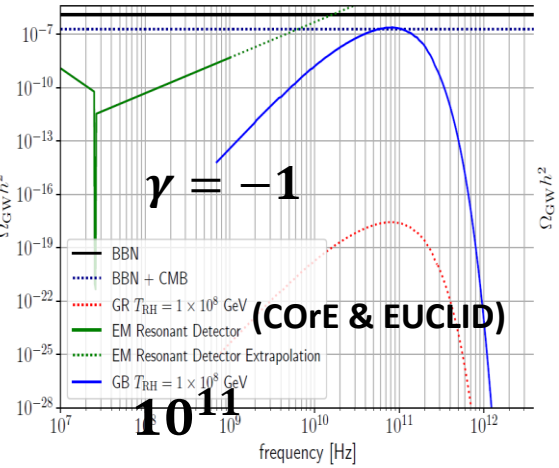
$$\boldsymbol{\Omega_{GW}(f_{peak})h^2} \gg (\Lambda CDM).$$

The BBN bound puts bounds on

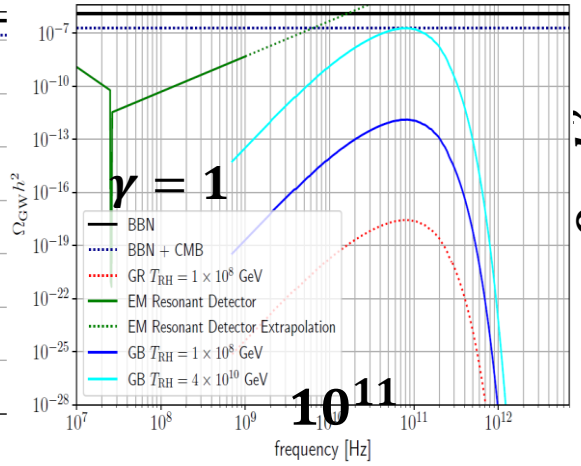
$$T_{RH} \simeq 10^8 - 10^9 \text{ GeV} \ll 10^{16} \text{ GeV}.$$

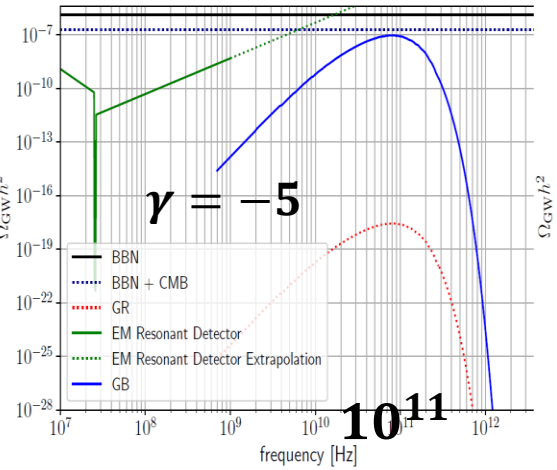
(GW peak not exceeding the BBN upper limit)

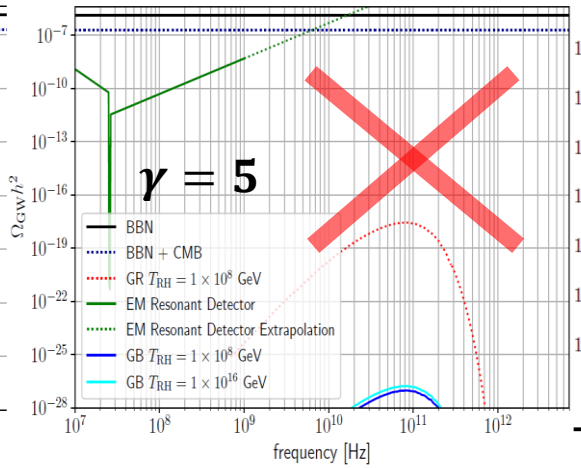
$\tilde{\alpha} = -1\text{km}^2$ $T_{RH} = T_{max} = 1\times10^8\text{GeV}$



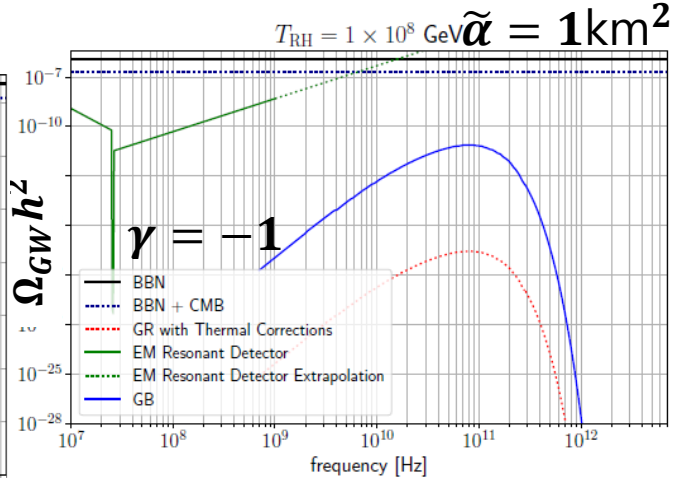
(COrE & EUCLID)

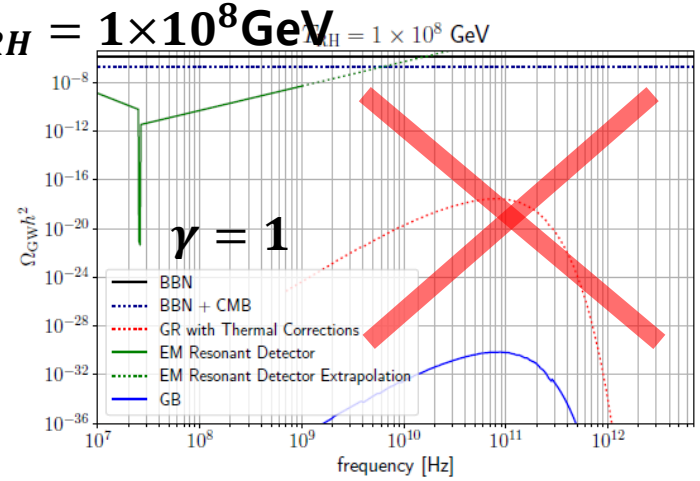


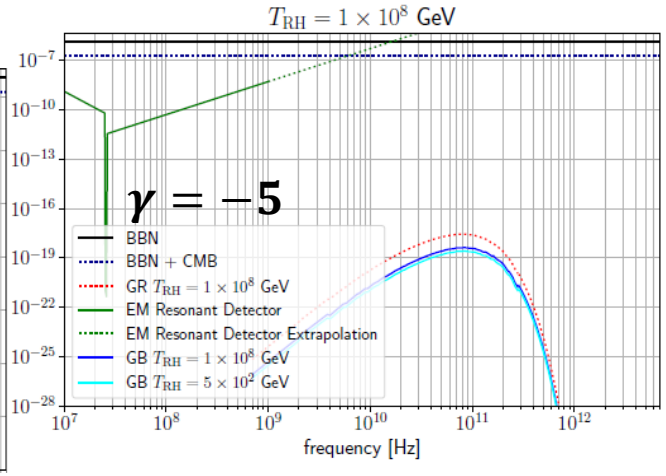


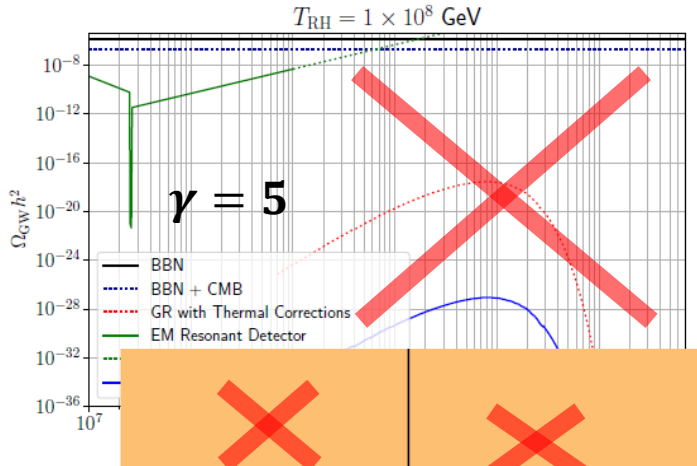


$\tilde{\alpha} = 1\text{km}^2$ $T_{RH} = 1\times10^8\text{GeV}$









EM Resonant Detector

GR $T_{RH} = 1\times10^8$ GeV

BBN limit

$\rho_\phi(T_{BBN}) = 3\times10^{-2}\,\rho_{BBN}$

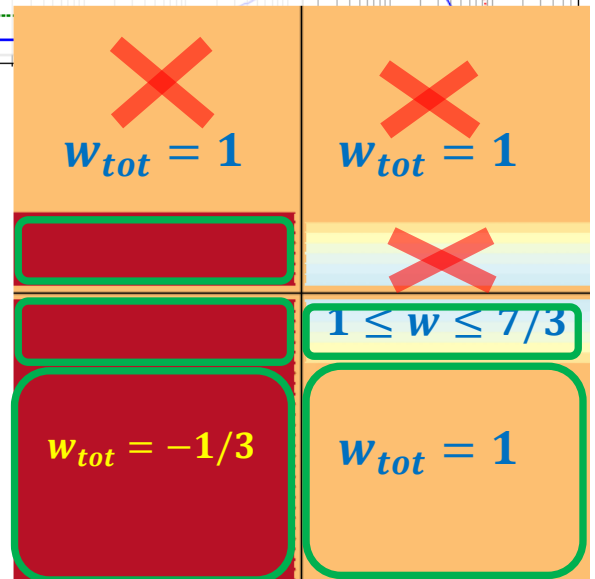
$\rho_\phi(T_{BBN}) = 10^{-4}\,\rho_{BBN}$

$\rho_\phi(T_{BBN}) = 0$

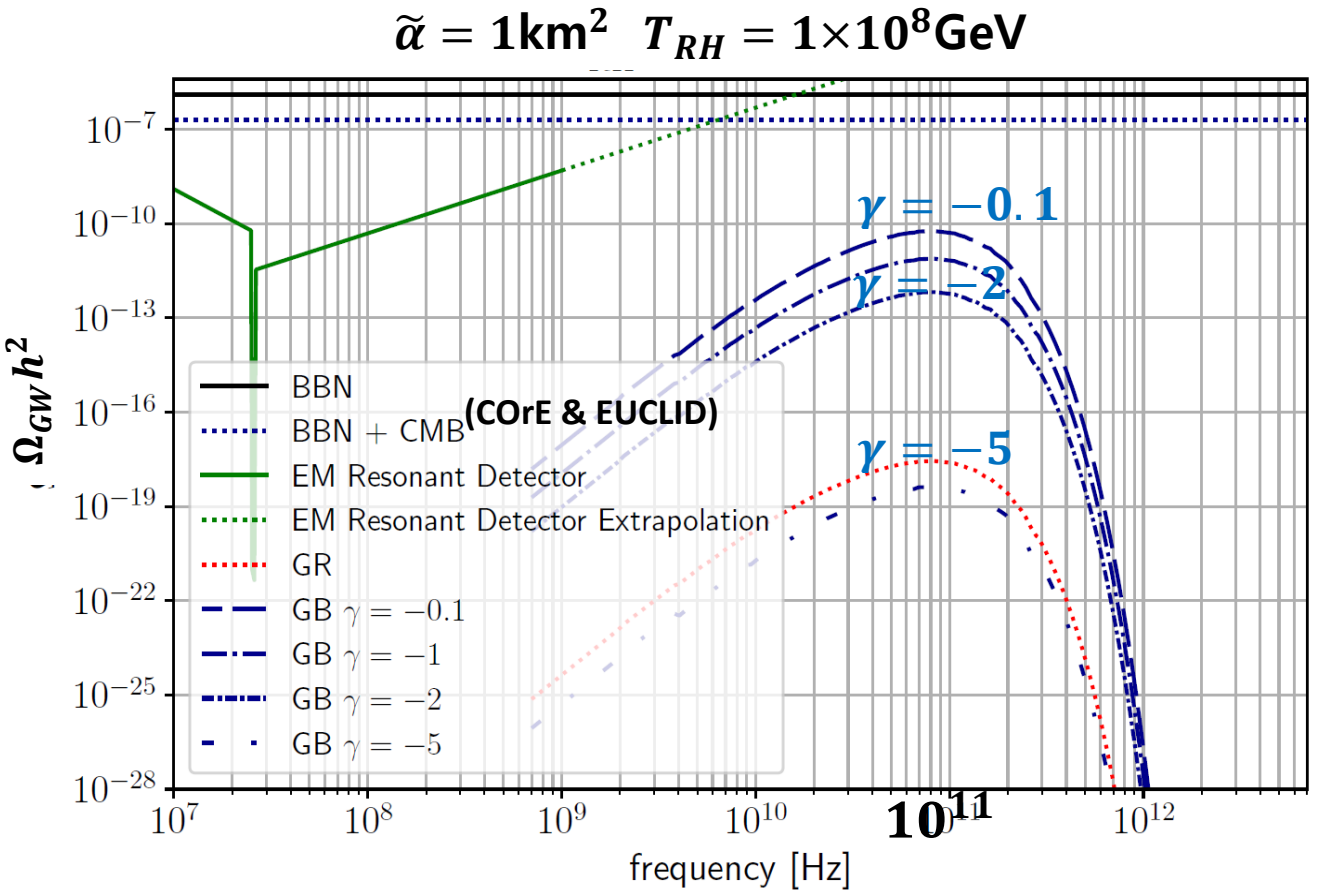
The system follows the z axis with w = −1/3 before jumping to the attractor,
-an enhancement of the GW signal,

As $|\gamma| \to 0$ detaches "later" from the z<0 axis,and the metastable slow-roll lasts longer.

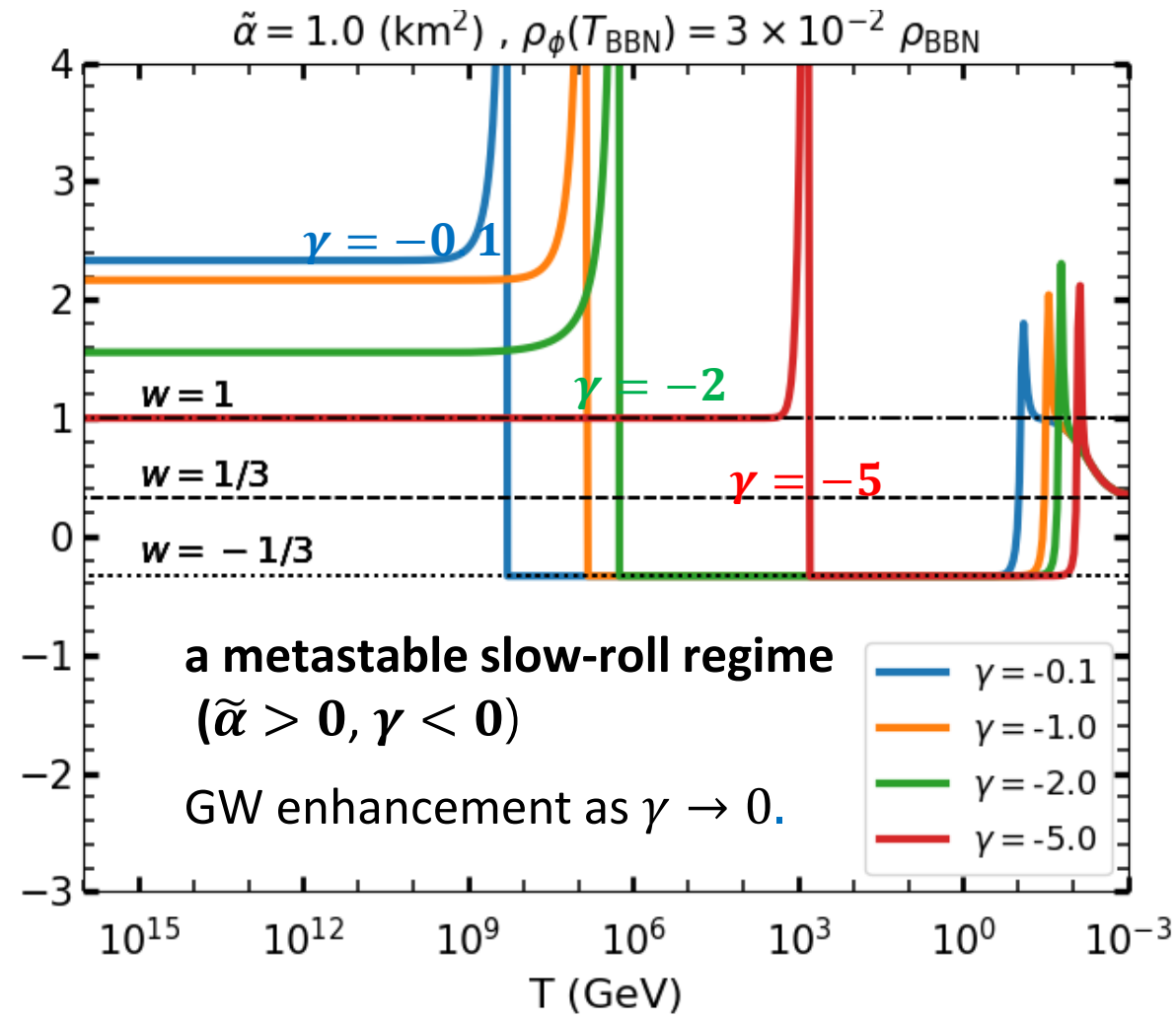
GW signal is way below the BBN bounds.



Biswas,Kar,**BHL**,Lee,Lee,**Scopel**,Velasco-Sevilla,Yin **JCAP (2024)**



The signal is enhanced as $|\gamma| \to 0$ because the metastable slow–roll solution with w=-1/3 lasts longer.



As $|\gamma| \to 0$ the system follows the metastable solution w = −1/3 for a larger interval of T before jumping to a different regime, implying an increasing GW stochastic background.

V. Summary

High Freq GWs from the Modified Gravity beyond Einstein

- **Theoretical Aspect effective theory**→ There MUST be higher orders of curvature terms.
- **Observational Aspect** - challenging the ΛCDM

In dim>4, GB term is dynamical, allowing 2nd order e.o.m.

$$S_{EGB-\Lambda} = \int d^d x \sqrt{-g}\left[\frac{1}{2\kappa}(R - 2\Lambda + \alpha R^2_{GB}) + \mathcal{L}_m^{matt}\right]$$

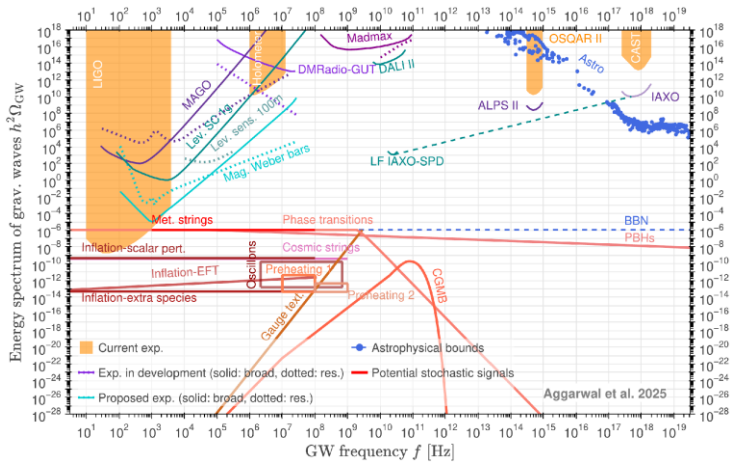
In dim=4 the Dilaton-Einstein-Gauss-Bonnet (dEGB) Gravity

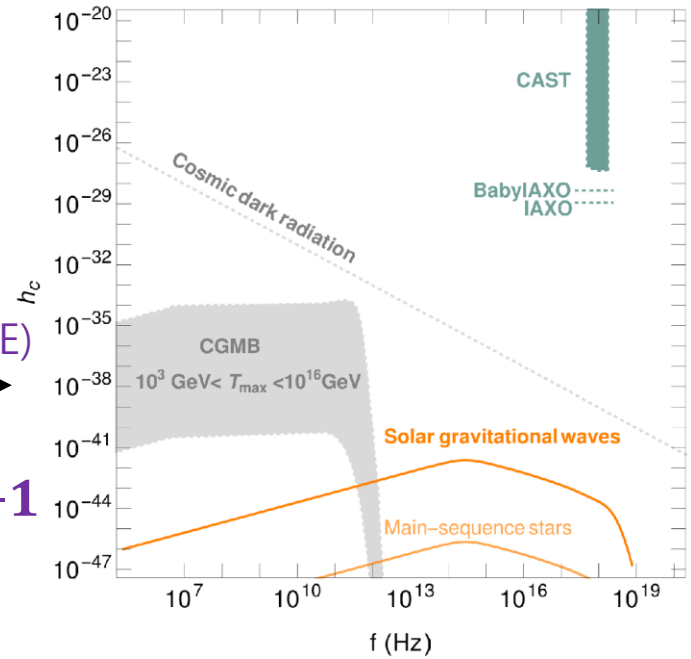
$$S_{dEGB} = \int d^4 x \sqrt{-g}\left[\frac{1}{2\kappa}R + f(\phi)R^2_{GB} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m\right]$$

$$f(\phi) = \alpha e^{\gamma\phi}$$

How to confirm? – BHs, Cosmology, **GWs**, etc.

GWs,





EdGB Black Holes

minimum mass

Soliton Star? Black Holes

M=0 M_{min} $\longrightarrow \infty$

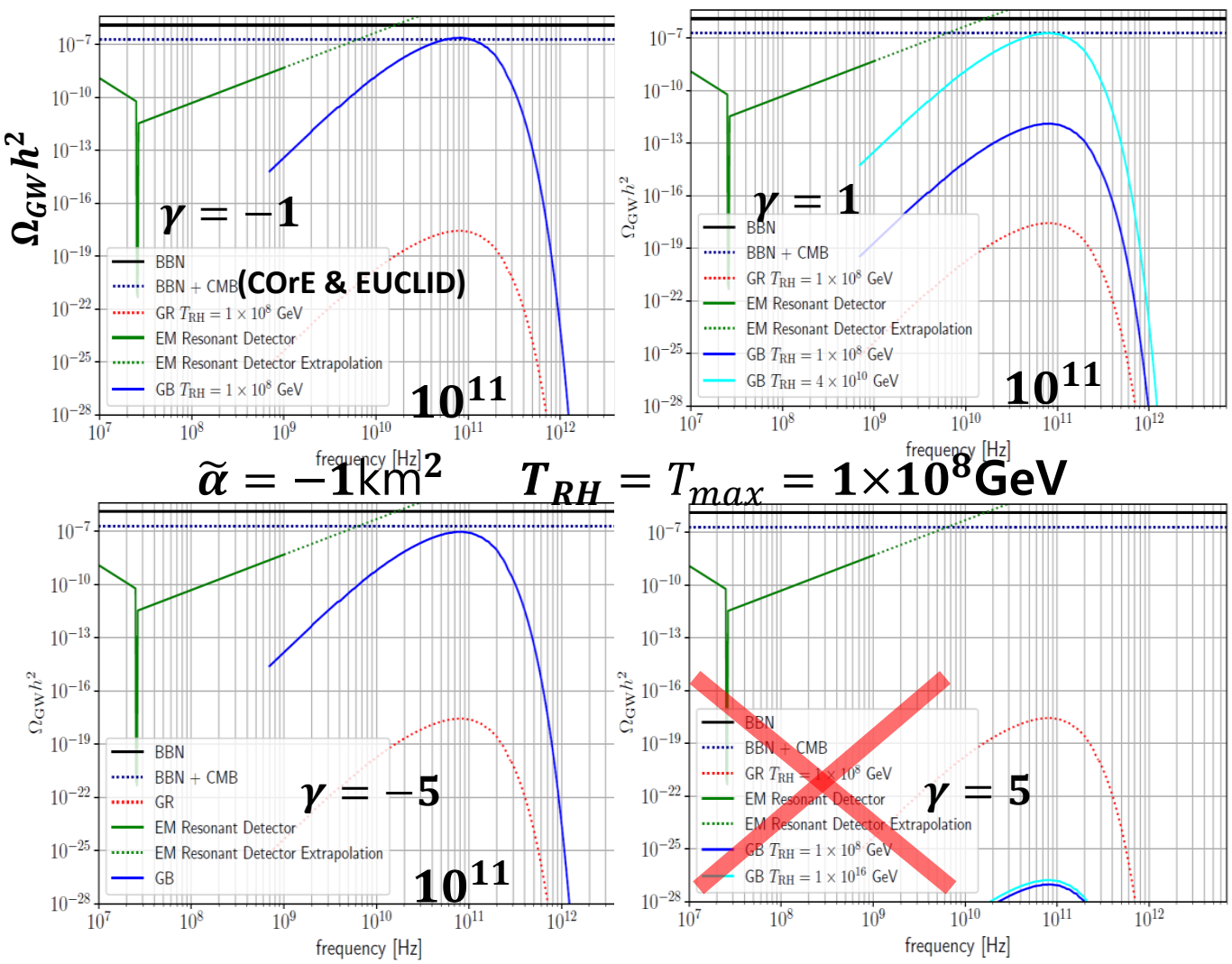
EdGB Cosmology

NEW PHASE→ | ←Rad Dom→ | ← Matt → | ← Λ(DE)

$w_{tot} = -\frac{1}{3}, 1,$ $w = \frac{1}{3}$ $w = 0$ $w = -1$

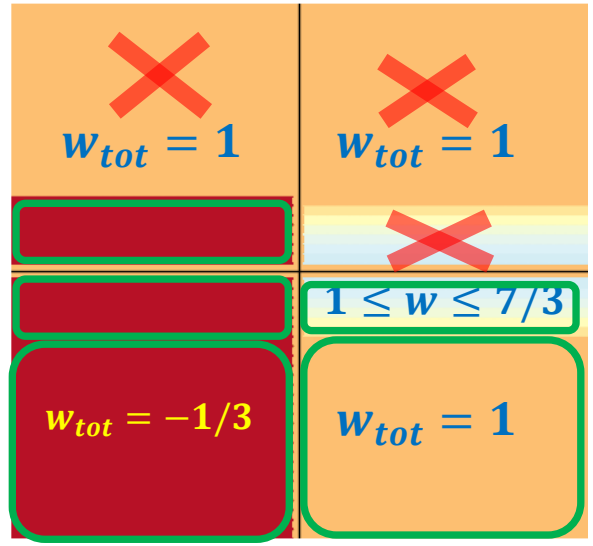
$1 \le w_{tot} \le 7/3$

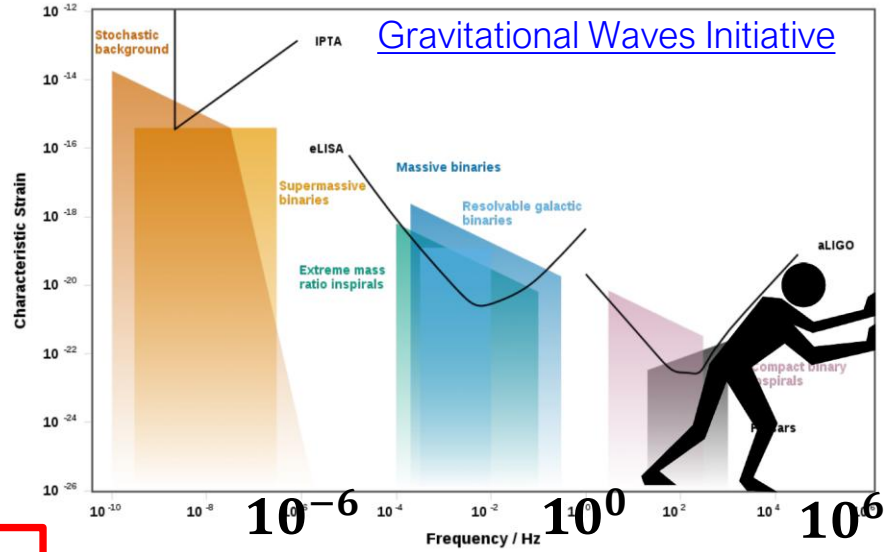
CGMB w = −1/3 give a strong enhancement of the SGWB



(COrE & EUCLID)

$\tilde{\alpha} = -1\text{km}^2$ $T_{RH} = T_{max} = 1\times10^8\text{GeV}$





EM Resonant Detector

GR $T_{RH} = 1 \times 10^8$ GeV

BBN limit

HF GW needs further investigation

Thank you!