Explicit Lorentz-invariant QED pair production rates from superstrings

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The talk is based on the following papers:

- J. X. Lu,"Magnetically-enhanced open string pair production," JHEP **12**, 076 (2017);
- J. X. Lu, "Some aspects of interaction amplitudes of D branes carrying worldvolume fluxes," Nucl. Phys. B **934**, 39 (2018);
- J. X. Lu, "A possible signature of extra-dimensions: The enhanced open string pair production," Phys. Lett. B **788**, 480 (2019);
- Q. Jia and J. X. Lu,"Remark on the open string pair production enhancement," Phys. Lett. B **789**, 568 (2019);
- J. X. Lu, "A note on the open string pair production of the D3/D1 system," JHEP **10**, 238 (2019);
- Q. Jia, J. X. Lu, Z. Wu and X. Zhu, "On D-brane interaction & its related properties,"Nucl. Phys. B 953 (2020)114947;
- J. X. Lu and Nan Zhang, "More on the open string pair production" , Nucl. Phys. B **977** (2022) 115721;
- J. X. Lu, "Understanding the open string pair production of Dp/D0 system", JHEP **11**, 019(2023);
- J. X. Lu, "The open string pair production, its enhancement and the physics behind," PLB 848 (2024)138397;
- J. X. Lu, "The open string pair production revisited" PLB 872 (2026) 140125;
- J. X. Lu, "Explicit Lorentz invariant QED pair production rates from superstrings", arXiv:2608.08712.

Outline

- Motivation/introduction
- The general setup
- Explicit LI QED PPR from superstrings in diverse dimensions
- Conclusion and discussion

String Theory (ST)/QFT

In this talk, I address a question, from top-down approach, on how to obtain the usual QFT results from the computations of ST.

There have been many previous examples in perturbative computations such as KKLT relation and the double-copy approach.

In particular, we focus here on how to obtain:

the usual Schwinger pair production rate for a pair of charged/ anti-charged massive particles (scalars, spinors or vectors)

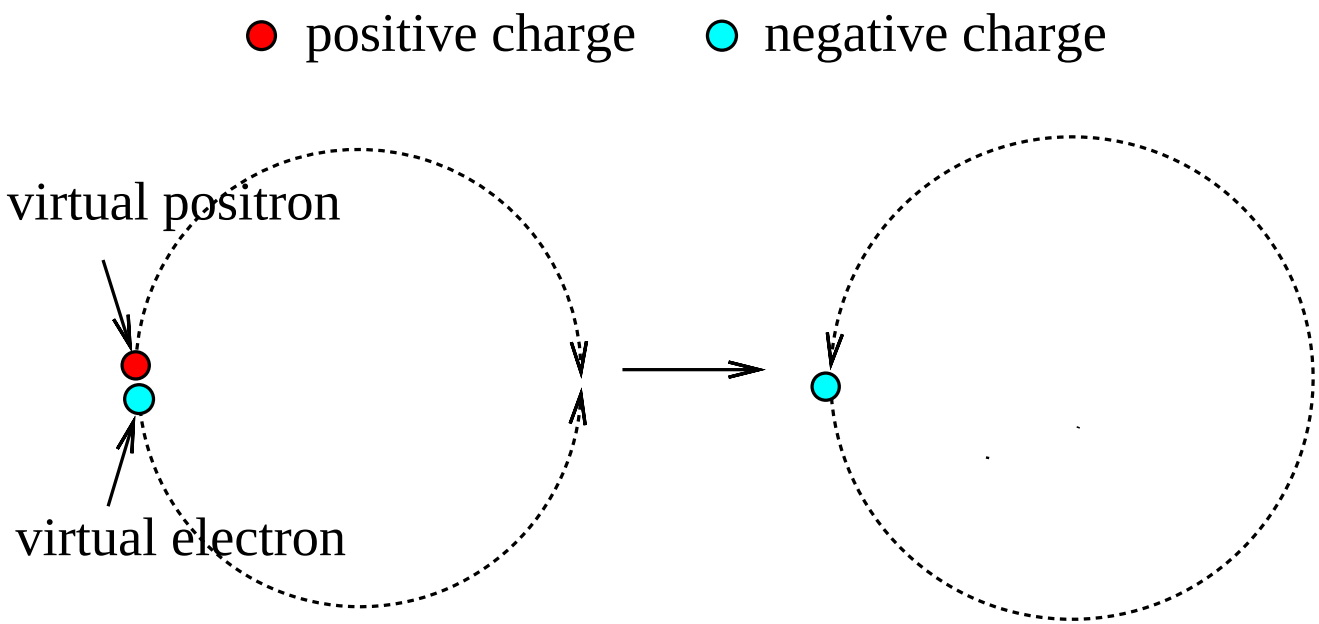
- in the most general allowed constant EM background,
- in an explicit Lorentz invariant form,
- & in diverse dimensions $2 \leq d \leq 7$.

Some of the results are new and have not been given before.

The QED Vacuum Picture

VACUUM QUANTUM FLUCTUATIONS

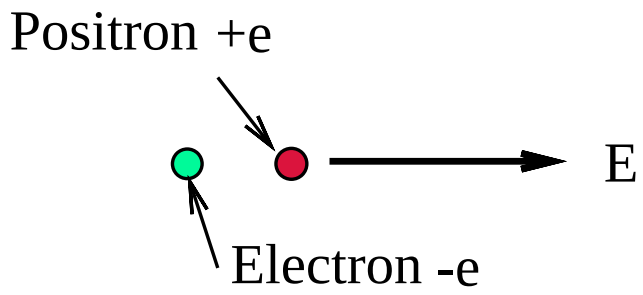
An anti-charge moving forward in time equivalent to a charge moving backward in time



$$(\delta t \sim 10^{-21} s)$$

QED Vacuum Fluctuations

Schwinger effect [Schwinger '51].



The decay rate of the QED vacuum is

$$\mathcal{W}_{\text{spinor}}^{\text{Schwinger}} = 2\,\frac{(eE)^2}{(2\pi)^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\,e^{-n\pi\frac{m_e^2}{eE}}\,, \tag{1.1}$$

while the actual experimental pair production rate [Nikishov'70] is

$$\mathcal{W}_{\text{spinor}}^{\text{QED}} = 2\,\frac{(eE)^2}{(2\pi)^3}\,e^{-\frac{\pi m_e^2}{eE}}\,. \tag{1.2}$$

J. S. Schwinger, "On gauge invariance and vacuum polarization," Phys. Rev. **82**, 664 (1951)
A. I. Nikishov, Sov. Phys. JETP 30, 660 (1970); Nucl. Phys. B21, 346 (1970).

D branes

For the purpose of this talk, we consider non-perturbative D-branes in oriented 10D Type II superstring theories.

D branes are a particular type of string solitons and play a vital role in the development of non-perturbative string theory.

- They were first found as string solitons and classified by Duff and myself along with others in a series of publications:

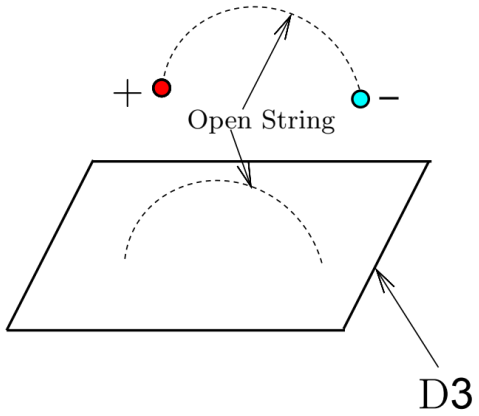
1. M. J. Duff and J. X. Lu, "The Selfdual type IIB superthreebrane," PLB273 (1991) 409-414;
2. M. J. Duff and J. X. Lu, "Type II p-branes: The Brane scan revisited," NPB390 (1993) 276-290;
3. M. J. Duff and J. X. Lu,"Black and super p-branes in diverse dimensions," NPB416 (1994) 301-334;
4. M. J. Duff, R. R. Khuri and J. X. Lu, "String solitons," Phys. Rept. 259 (1995), 213-326

- They were also found in a completely different way by Polchinski et al by applying T-duality, which was found from closed strings, to open strings:

1. J. Dai, R. G. Leigh and J. Polchinski, "New Connections Between String Theories," MPLA4(1989)2073;
2. J. Polchinski, "Dirichlet Branes and Ramond-Ramond charges," PRL75(1995)4724

A single isolated D-brane

Consider a single D brane, say, a D3 brane, mimicking our own $(1+3)$-dimensional world. Its quantum fluctuations:



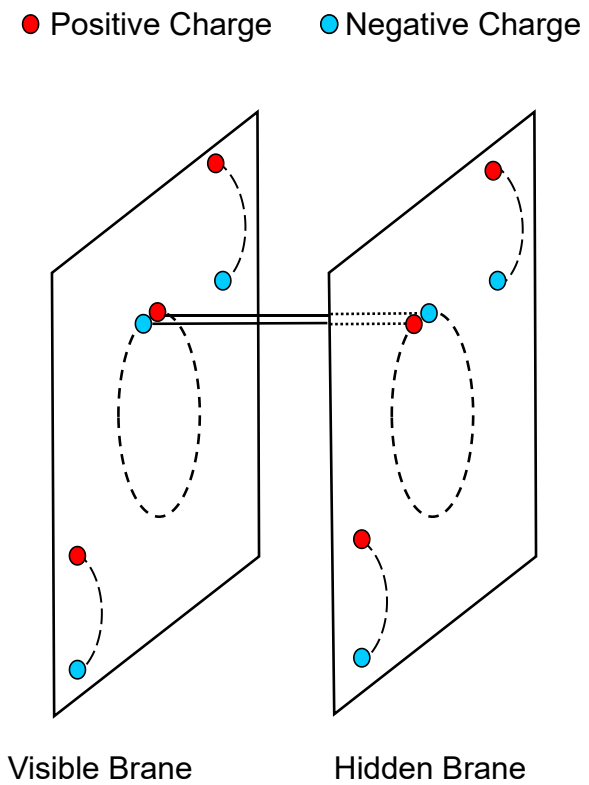
The brane observer would have $\mathcal{W}^{\mathrm{QED}} \neq 0$ if adding E on the brane.

However, unlike QED, string theory says

$$\mathcal{W}^{\mathrm{String}} = 0 \Longrightarrow \mathcal{W}^{\mathrm{QED}} = 0. \tag{1.3}$$

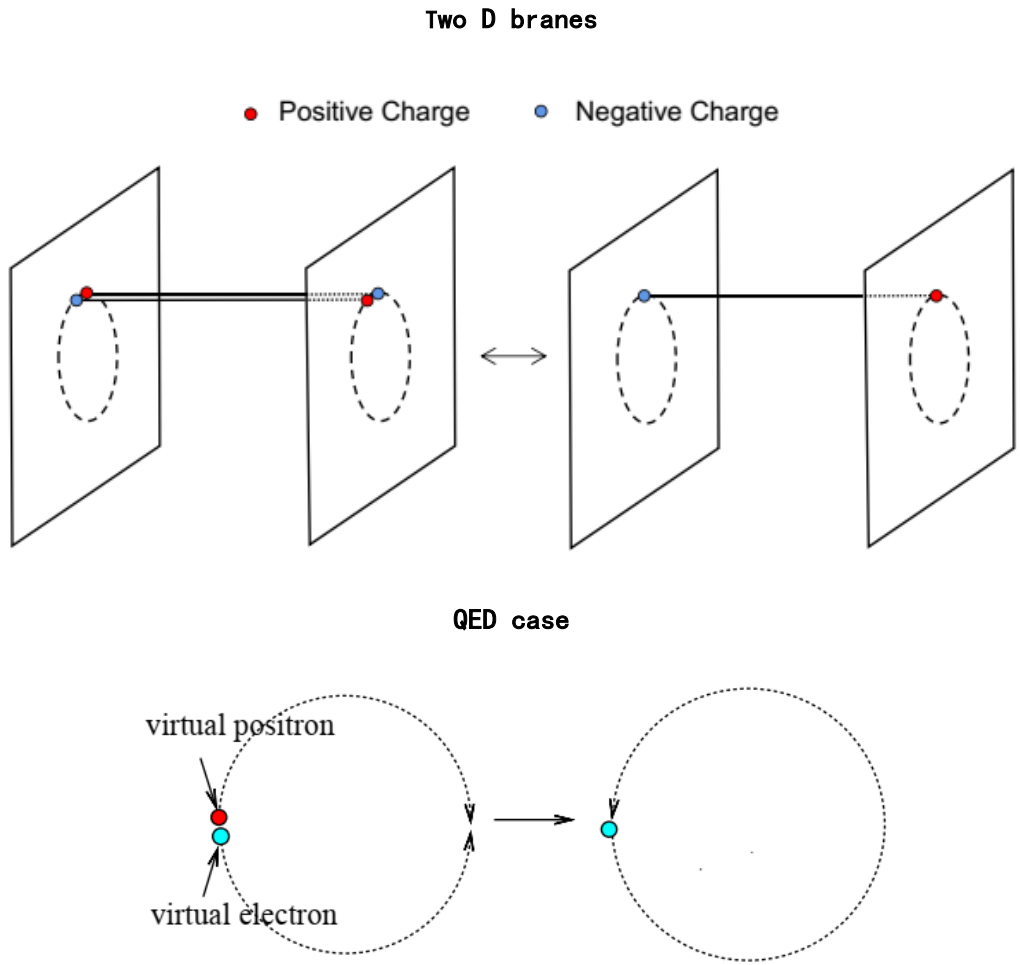
Two D-brane system

The simplest for pair production is to consider a system consisting of two Dp branes with $1 \leq p \leq 6$ (or $2 \leq d = 1 + p \leq 7$), placed parallel at a separation.



The analogous QED vacuum

This is analogous to the QED vacuum if one looks from an observer on the visible brane.



The OS one-loop amplitude

Need to compute the OS one-loop annulus amplitude in the most general allowed constant EM background on the visible brane (Jia et al'20).

$$\frac{\Gamma_{p,p}}{V_{1+p}} = \frac{2^2\sqrt{\det(\mathbb{I}+\hat{F})}}{(8\pi^2\alpha')^{\frac{p+1}{2}}}\int_0^\infty \frac{dt}{t^{\frac{p-3}{2}}}\prod_{\kappa=0}^{2}\frac{\sin\pi\hat{\nu}_\kappa}{\sinh\pi\hat{\nu}_\kappa t}\left[\sum_{\kappa=0}^{2}\cosh^2\pi\hat{\nu}_\kappa t - 2\prod_{\kappa=0}^{2}\cosh\pi\hat{\nu}_\kappa t - 1\right] \times e^{-\frac{y^2 t}{2\pi\alpha'}}\prod_{n=1}^{\infty} Z_n, \tag{2.1}$$

where $\hat{\nu}_0 = i\bar{\hat{\nu}}_0$ and

$$Z_n = \frac{\prod_{a=1}^{2}\left\{\left[1-2|z|^{2n}\cos\pi\bar{\hat{\nu}}_0 t\cosh\pi c_a t+|z|^{4n}\right]^2+4|z|^{4n}\sin^2\pi\bar{\hat{\nu}}_0 t\sinh^2\pi c_a t\right\}}{(1-|z|^{2n})^2[1-2|z|^{2n}\cos 2\pi\bar{\hat{\nu}}_0 t+|z|^{4n}]\prod_{a=1}^{2}[1-2|z|^{2n}\cosh 2\pi\hat{\nu}_a t+|z|^{4n}]}. \tag{2.2}$$

In the above, $|z| = e^{-\pi t}$, $c_a = \hat{\nu}_1 + (-)^a\hat{\nu}_2$, V_{1+p} the brane spacetime volume and y the brane separation.

The above amplitude has an infinite number of simple poles occurring at $t_k = k/\bar{\hat{\nu}}_0$ with $k = 1, 2, \cdots$,

- signaling the decay of the system via the production of open string pairs,
- therefore having an imaginary part of the amplitude.

Q. Jia, J. X. Lu, Z. Wu and X. Zhu, "On D-brane interaction & its related properties," NPB 953 (2020)114947.

The OS one-loop amplitude

This amplitude is completely determined by the dimensionless $\hat{F}_{\alpha\beta} \equiv 2\pi\alpha' e F_{\alpha\beta}$.

The parameters $\hat{\nu}_\kappa$ for $\kappa = 0, 1, \cdots [(p-1)/2]$ are determined via the relation $\lambda_\kappa + \lambda_\kappa^{-1} = 2\cos 2\pi\hat{\nu}_\kappa$ with both λ_κ and λ_κ^{-1} being pairwise eigenvalues of the following matrix

$$w_\alpha^{\ \beta} = \left[\left(\mathbb{I} - \hat{F}\right)\left(\mathbb{I} + \hat{F}\right)^{-1}\right]_\alpha^{\ \beta}. \tag{2.3}$$

Note one eigenvalue $\lambda = 1$ for even p ($d = 1 + p$), $\alpha, \beta = 0, 1, \cdots p$.

The parameters $\hat{\nu}_\kappa$ for $1 \le p \le 6$ are determined by the following respective equations:

p	Equation(s) for eigenvalue(s)
1	$2\cos 2\pi\hat{\nu}_0 = \mathrm{tr}w, \quad \hat{\nu}_1 = 0, \hat{\nu}_2 = 0.$
2	$2\cos 2\pi\hat{\nu}_0 = \mathrm{tr}w - 1, \ \lambda = 1, \quad \hat{\nu}_1 = 0, \hat{\nu}_2 = 0.$
3	$2\sum_{\kappa=0}^{1}\cos 2\pi\hat{\nu}_\kappa = \mathrm{tr}w, \ 4\sum_{\kappa=0}^{1}\cos^2 2\pi\hat{\nu}_\kappa = 4 + \mathrm{tr}w^2, \quad \hat{\nu}_2 = 0.$
4	$2\sum_{\kappa=0}^{1}\cos 2\pi\hat{\nu}_\kappa = \mathrm{tr}w - 1, \ 4\sum_{\kappa=0}^{1}\cos^2 2\pi\hat{\nu}_\kappa = 3 + \mathrm{tr}w^2, \ \lambda = 1, \ \hat{\nu}_2 = 0.$
5	$2\sum_{\kappa=0}^{2}\cos 2\pi\hat{\nu}_\kappa = \mathrm{tr}w, \ 4\sum_{\kappa=0}^{2}\cos^2 2\pi\hat{\nu}_\kappa = 6 + \mathrm{tr}w^2$ $\& \ 8\sum_{\alpha=0}^{2}\cos 2\pi\hat{\nu}_\kappa\left(\cos^2 2\pi\hat{\nu}_\kappa - \frac{3}{4}\right) = \mathrm{tr}w^3$
6	$2\sum_{\kappa=0}^{2}\cos 2\pi\hat{\nu}_\kappa = \mathrm{tr}w - 1, \quad 4\sum_{\kappa=0}^{2}\cos^2 2\pi\hat{\nu}_\kappa = 5 + \mathrm{tr}w^2,$ $8\sum_{\kappa=0}^{2}\cos 2\pi\hat{\nu}_\kappa\left(\cos^2 2\pi\hat{\nu}_\kappa - \frac{3}{4}\right) = \mathrm{tr}w^3 - 1, \quad \lambda = 1$

The OS pair production rate

Note in general $\det(\mathbb{I}+\hat{F}) = (1-\hat{\alpha})(1-\hat{\beta})(1-\hat{\gamma})$ with

$$\hat{\alpha}+\hat{\beta}+\hat{\gamma} = \frac{1}{2}\mathrm{tr}\hat{F}^2, \quad \hat{\alpha}(\hat{\beta}+\hat{\gamma})+\hat{\beta}\hat{\gamma} = -\frac{1}{4}\left[\mathrm{tr}\hat{F}^4 - \frac{1}{2}(\mathrm{tr}\hat{F}^2)^2\right],$$
$$\hat{\alpha}\hat{\beta}\hat{\gamma} = \frac{1}{3!}\left[\mathrm{tr}\hat{F}^6 - \frac{3}{4}\mathrm{tr}\hat{F}^4\mathrm{tr}\hat{F}^2 + \frac{1}{8}(\mathrm{tr}\hat{F}^2)^3\right] \geq 0. \tag{2.4}$$

One can prove in general by math induction ($\mathrm{tr}\hat{F}^k \equiv (\hat{F}^k)_\alpha{}^\alpha$)

$$\mathrm{tr}\hat{F}^{2n} = 2(\hat{\alpha}^n+\hat{\beta}^n+\hat{\gamma}^n) \quad \Longrightarrow \tag{2.5}$$

$$\begin{aligned}
\mathrm{tr}(\mathbb{I}+\hat{F})^{-1} &= \sum_{n=0}^{\infty}\mathrm{tr}\hat{F}^{2n}, \quad \mathrm{tr}(\mathbb{I}+\hat{F})^{-2} = \sum_{n=0}^{\infty}(2n+1)\mathrm{tr}\hat{F}^{2n}, \\
\mathrm{tr}(\mathbb{I}+\hat{F})^{-3} &= \sum_{n=0}^{\infty}(2n+1)(n+1)\mathrm{tr}\hat{F}^{2n},
\end{aligned} \tag{2.6}$$

noting $\mathrm{tr}\hat{F}^{2n+1} = 0$ due to $\hat{F}_{\alpha\beta} = -\hat{F}_{\beta\alpha}$. So we can have $\mathrm{tr}w$, $\mathrm{tr}w^2$ and $\mathrm{tr}w^3$ given the following

$$\begin{aligned}
w &= 2(\mathbb{I}+\hat{F})^{-1} - \mathbb{I}, \quad w^2 = 4(\mathbb{I}+\hat{F})^{-2} - 4(\mathbb{I}+\hat{F})^{-1} + \mathbb{I}, \\
w^3 &= 8(\mathbb{I}+\hat{F})^{-3} - 12(\mathbb{I}+\hat{F})^{-2} + 6(\mathbb{I}+\hat{F})^{-1} - \mathbb{I}.
\end{aligned} \tag{2.7}$$

Hence we can determine the parameters $\hat{\nu}_\kappa$ given the equations list in the previous table which can be simplified for $p = 5$ or 6 as

The OS pair production rate

$$\begin{aligned}
\cosh^2\pi\bar{\hat{\nu}}_0+\sum_{a=1}^{2}\cos^2\pi\hat{\nu}_a &= \frac{1}{1-\hat{\alpha}}+\frac{1}{1-\hat{\beta}}+\frac{1}{1-\hat{\gamma}},\\
\cosh^4\pi\bar{\hat{\nu}}_0+\sum_{a=1}^{2}\cos^4\pi\hat{\nu}_a &= \frac{1}{(1-\hat{\alpha})^2}+\frac{1}{(1-\hat{\beta})^2}+\frac{1}{(1-\hat{\gamma})^2},\\
\cosh^6\pi\bar{\hat{\nu}}_0+\sum_{a=1}^{2}\cos^6\pi\hat{\nu}_a &= \frac{1}{(1-\hat{\alpha})^3}+\frac{1}{(1-\hat{\beta})^3}+\frac{1}{(1-\hat{\gamma})^3},
\end{aligned}\tag{2.8}$$

which have the solutions, by the choice of $\hat{\alpha}>0$ and $\hat{\beta},\hat{\gamma}\le 0$ without loss of generality,

$$\sinh\pi\bar{\hat{\nu}}_0=\sqrt{\frac{\hat{\alpha}}{1-\hat{\alpha}}},\quad \sin\pi\hat{\nu}_1=\sqrt{\frac{-\hat{\beta}}{1-\hat{\beta}}},\quad \sin\pi\hat{\nu}_2=\sqrt{\frac{-\hat{\gamma}}{1-\hat{\gamma}}}.\tag{2.9}$$

The non-perturbative open string pair production rate for $1\le p\le 6$ can be obtained from the imaginary part of the amplitude as

$$\mathcal{W}^{(\mathrm{String})}_{p,\,p}=\frac{2^3\sqrt{\det(\mathbb{I}+\hat{F})}\sinh\pi\bar{\hat{\nu}}_0\prod_{a=1}^{2}\sin\pi\hat{\nu}_a}{\bar{\hat{\nu}}_0(8\pi^2\alpha')^{\frac{p+1}{2}}}\bar{\hat{\nu}}_0^{\frac{p-3}{2}}e^{-\frac{y^2}{2\pi\bar{\hat{\nu}}_0\alpha'}}\frac{\left[\cosh\frac{\pi\hat{\nu}_1}{\bar{\hat{\nu}}_0}+\cosh\frac{\pi\hat{\nu}_2}{\bar{\hat{\nu}}_0}\right]^2}{\sinh\frac{\pi\hat{\nu}_1}{\bar{\hat{\nu}}_0}\sinh\frac{\pi\hat{\nu}_2}{\bar{\hat{\nu}}_0}}B_1,\tag{2.10}$$

where, noting $|z_1|=e^{-\pi/\bar{\hat{\nu}}_0}$,

$$B_1=\prod_{n=1}^{\infty}\frac{[1+2|z_1|^{2n}\cosh\frac{\pi(\hat{\nu}_1+\hat{\nu}_2)}{\bar{\hat{\nu}}_0}+|z_1|^{4n}]^2[1+2|z_1|^{2n}\cosh\frac{\pi(\hat{\nu}_1-\hat{\nu}_2)}{\bar{\hat{\nu}}_0}+|z_1|^{4n}]^2}{(1-|z_1|^{2n})^4[1-2|z_1|^{2n}\cosh\frac{2\pi\hat{\nu}_1}{\bar{\hat{\nu}}_0}+|z_1|^{4n}][1-2|z_1|^{2n}\cosh\frac{2\pi\hat{\nu}_2}{\bar{\hat{\nu}}_0}+|z_1|^{4n}]}.\tag{2.11}$$

OS rate $\to$ QED rate

Note the above rate is for $p = 5\,\text{or}\,6$, the rate for $p = 3\,\text{or}\,4$ is obtained by taking $\hat{\nu}_2 \to 0$ and the rate for $p = 1\,\text{or}\,2$ is obtained by taking both $\hat{\nu}_2, \hat{\nu}_1 \to 0$.

Now take the so-called field theory limit: $|\hat{F}_{\alpha\beta}| = 2\pi\alpha' e|F_{\alpha\beta}| \ll 1$

$$\pi\bar{\hat{\nu}}_0 \approx \sqrt{\hat{\alpha}} \ll 1, \quad \pi\hat{\nu}_1 \approx \sqrt{-\hat{\beta}} \ll 1, \quad \pi\hat{\nu}_2 \approx \sqrt{-\hat{\gamma}} \ll 1, \tag{2.12}$$

and also set

$$\hat{\alpha} = (2\pi\alpha' e)^2\alpha, \quad \hat{\beta} = (2\pi\alpha' e)^2\beta, \quad \hat{\gamma} = (2\pi\alpha' e)^2\gamma, \tag{2.13}$$

In this limit, $|z_1| = e^{-\pi/\bar{\hat{\nu}}_0} \to 0$ due to $\bar{\hat{\nu}}_0 \ll 1$, therefore $B_1 \to 1$ from (2.11), the stringy rate (2.10) becomes

$$\mathcal{W}^{(\text{String})}_{p,\,p} = \frac{\left(e\sqrt{\alpha}\right)^{\frac{p-3}{2}}\, e^2\sqrt{|\beta||\gamma|}}{(2\pi)^{p-2}} \frac{\left[\cosh\pi\sqrt{\frac{|\beta|}{\alpha}} + \cosh\pi\sqrt{\frac{|\gamma|}{\alpha}}\right]^2}{\sinh\pi\sqrt{\frac{|\beta|}{\alpha}}\sinh\pi\sqrt{\frac{|\gamma|}{\alpha}}} e^{-\frac{\pi m^2}{e\sqrt{\alpha}}}, \tag{2.14}$$

with $m = \frac{y}{2\pi\alpha'}$, the mass for each mode, and

$$\begin{aligned} &\alpha\beta\gamma = \frac{1}{3!}[\text{tr}F^6 - \frac{3}{4}\text{tr}F^4\text{tr}F^2 + \frac{1}{8}(\text{tr}F^2)^3] \geq 0, \\ &\alpha\beta + \alpha\gamma + \beta\gamma = -\frac{1}{4}[\text{tr}F^4 - \frac{1}{2}(\text{tr}F^2)^2],\ \alpha + \beta + \gamma = \frac{1}{2}\text{tr}F^2. \end{aligned} \tag{2.15}$$

Stringy rate & QED rates

For a pure electric field, we have from (2.15) $\alpha = E^2$, $\beta = \gamma = 0$ and the rate from (2.14) as

$$\mathcal{W}^{(\mathrm{String})}_{p,\,p} = 16\,\frac{(eE)^{\frac{1+p}{2}}}{(2\pi)^p}\; e^{-\frac{\pi m^2}{eE}} = 16\,\mathcal{W}^{(\mathrm{QED})}_{\mathrm{scalar}}\,, \tag{2.16}$$

where $E = |\vec{E}|$ with $E_a = F_{0a}$ and all other $F_{ab} = 0$ with $a,\, b = 1, 2, \cdots p$.

As an illustration for $p = 3$, taking $\gamma \to 0$ and p = 3 in (2.14), we have

$$\mathcal{W}^{(\mathrm{String})}_{3,3} = \frac{2\,e^2\sqrt{\alpha|\beta|}}{(2\pi)^2}\;\frac{\left[1+\cosh\pi\sqrt{\frac{|\beta|}{\alpha}}\right]^2}{\sinh\pi\sqrt{\frac{|\beta|}{\alpha}}}\; e^{-\frac{\pi m^2}{e\sqrt{\alpha}}}\,, \tag{2.17}$$

and from (2.15)

$$\alpha = \frac{1}{2}\left[\vec{E}^2 - \vec{B}^2 + \sqrt{\left(\vec{E}^2 - \vec{B}^2\right)^2 + 4(\vec{E}\cdot\vec{B})^2}\right],\; \beta = \frac{1}{2}\left[\vec{E}^2 - \vec{B}^2 - \sqrt{\left(\vec{E}^2 - \vec{B}^2\right)^2 + 4(\vec{E}\cdot\vec{B})^2}\right], \tag{2.18}$$

noting $\mathrm{tr}F^2 = 2(\vec{E}^2 - \vec{B}^2)$ and $\mathrm{tr}F^4 - (\mathrm{tr}F^2)^2/4 = (\vec{E}^2 - \vec{B}^2)^2 + 4(\vec{E}\cdot\vec{B})^2$ where $\vec{E}$ and $\vec{B}$ are the usual electric and magnetic vector fields.

The general relations

The lowest open string modes consist of $8_{\rm B}$ and $8_{\rm F}$ charged massive DOFs, therefore giving total 16 pairs contributing to the above rate.

Given the DOF counting for a massive scalar, a massive spinor and a massive vector in dimension d

$$d_{\rm scalar} = 1, \quad d_{\rm spinor} = 2^{[\frac{d}{2}]-1}, \quad d_{\rm vec.} = d-1. \tag{2.19}$$

From the Dp brane worldvolume, the $8_{\rm B}$ and $8_{\rm F}$ DOFs will give in each d as

$d=1+p$ \ Fields	No of Scalars	No of Spinors	No of Vectors
7	2	2	1
6	3	2	1
5	4	4	1
4	5	4	1
3	8	8	0
2	8	8	0

We then have the following identity in each d = 1 + p as

d	Stringy rate and QED rate relation	Pure electric relation
7	$\mathcal{W}^{\rm String}_{6,6} = 2\,\mathcal{W}^{\rm QED}_{\rm scalar} + 2\,\mathcal{W}^{\rm QED}_{\rm spinor} + \mathcal{W}^{\rm QED}_{\rm vec.}$	$\mathcal{W}^{\rm QED}_{\rm spinor} = 4\,\mathcal{W}^{\rm QED}_{\rm scalar}$, $\mathcal{W}^{\rm QED}_{\rm vec.} = 6\,\mathcal{W}^{\rm QED}_{\rm scalar}$
6	$\mathcal{W}^{\rm String}_{5,5} = 3\,\mathcal{W}^{\rm QED}_{\rm scalar} + 2\,\mathcal{W}^{\rm QED}_{\rm spinor} + \mathcal{W}^{\rm QED}_{\rm vec.}$	$\mathcal{W}^{\rm QED}_{\rm spinor} = 4\,\mathcal{W}^{\rm QED}_{\rm scalar}$, $\mathcal{W}^{\rm QED}_{\rm vec.} = 5\,\mathcal{W}^{\rm QED}_{\rm scalar}$
5	$\mathcal{W}^{\rm String}_{4,4} = 4\,\mathcal{W}^{\rm QED}_{\rm scalar} + 4\,\mathcal{W}^{\rm QED}_{\rm spinor} + \mathcal{W}^{\rm QED}_{\rm vec.}$	$\mathcal{W}^{\rm QED}_{\rm spinor} = 2\,\mathcal{W}^{\rm QED}_{\rm scalar}$, $\mathcal{W}^{\rm QED}_{\rm vec.} = 4\,\mathcal{W}^{\rm QED}_{\rm scalar}$
4	$\mathcal{W}^{\rm String}_{3,3} = 5\,\mathcal{W}^{\rm QED}_{\rm scalar} + 4\,\mathcal{W}^{\rm QED}_{\rm spinor} + \mathcal{W}^{\rm QED}_{\rm vec.}$	$\mathcal{W}^{\rm QED}_{\rm spinor} = 2\,\mathcal{W}^{\rm QED}_{\rm scalar}$, $\mathcal{W}^{\rm QED}_{\rm vec.} = 3\,\mathcal{W}^{\rm QED}_{\rm scalar}$
3	$\mathcal{W}^{\rm String}_{2,2} = 8\,\mathcal{W}^{\rm QED}_{\rm scalar} + 8\,\mathcal{W}^{\rm QED}_{\rm spinor}$	$\mathcal{W}^{\rm QED}_{\rm spinor} = \mathcal{W}^{\rm QED}_{\rm scalar}$
2	$\mathcal{W}^{\rm String}_{1,1} = 8\,\mathcal{W}^{\rm QED}_{\rm scalar} + 8\,\mathcal{W}^{\rm QED}_{\rm spinor}$	$\mathcal{W}^{\rm QED}_{\rm spinor} = \mathcal{W}^{\rm QED}_{\rm scalar}$

An explicit p = 3 Example

For the most familiar example of $d = 1 + p = 4$ case. Now we have 5 scalar pairs, 4 spinor pairs and one vector pair, all massive, so

$$\mathcal{W}_{3,\,3}^{(\mathrm{String})} = 5\mathcal{W}_{\mathrm{scalar}}^{(\mathrm{QED})} + 4\mathcal{W}_{\mathrm{spinor}}^{(\mathrm{QED})} + \mathcal{W}_{\mathrm{vector}}^{(\mathrm{QED})}. \tag{3.1}$$

For a pure electric field,

$$\begin{aligned} \mathcal{W}_{p,\,p}^{(\mathrm{String})} &= 16\frac{(eE)^2}{(2\pi)^3}\, e^{-\frac{\pi m^2}{eE}} = 16\,\mathcal{W}_{\mathrm{scalar}}^{(\mathrm{QED})}, \\ \mathcal{W}_{\mathrm{spinor}}^{(\mathrm{QED})} &= 2\,\mathcal{W}_{\mathrm{scalar}}^{(\mathrm{QED})}, \quad \mathcal{W}_{\mathrm{vector}}^{(\mathrm{QED})} = 3\,\mathcal{W}_{\mathrm{scalar}}^{(\mathrm{QED})}, \end{aligned} \tag{3.2}$$

where $E = |\vec{E}|$.

We have from (3.3),

$$\begin{aligned} \mathcal{W}_{3,3}^{(\mathrm{String})} &= \frac{2\,e^2\sqrt{\alpha|\beta|}}{(2\pi)^2}\,\frac{\left[1+\cosh\pi\sqrt{\frac{|\beta|}{\alpha}}\right]^2}{\sinh\pi\sqrt{\frac{|\beta|}{\alpha}}}\, e^{-\frac{\pi m^2}{e\sqrt{\alpha}}} \\ &= \frac{e^2\sqrt{\alpha|\beta|}}{(2\pi)^2}\, e^{-\frac{\pi m^2}{e\sqrt{\alpha}}}\,\frac{\frac{5}{2}+4\cosh\pi\sqrt{\frac{|\beta|}{\alpha}}+\cosh 2\pi\sqrt{\frac{|\beta|}{\alpha}}+\frac{1}{2}}{\sinh\pi\sqrt{\frac{|\beta|}{\alpha}}}. \end{aligned} \tag{3.3}$$



An explicit LI Example

$$\mathrm{W}^{(\mathrm{QED})}_{\mathrm{vec.}} = \frac{e^2\sqrt{\alpha|\beta|}}{2(2\pi)^2}\frac{1+2\cosh 2\pi\sqrt{\frac{|\beta|}{\alpha}}}{\sinh\pi\sqrt{\frac{|\beta|}{\alpha}}}\, e^{-\frac{\pi m^2_{\mathrm{vec.}}}{e\sqrt{\alpha}}},$$

$$\mathcal{W}^{(\mathrm{QED})}_{\mathrm{scalar}} = \frac{e^2\sqrt{\alpha|\beta|}}{2(2\pi)^2}\,\mathrm{csch}\pi\sqrt{\frac{|\beta|}{\alpha}}\, e^{-\frac{\pi m^2_{\mathrm{scalar}}}{e\sqrt{\alpha}}},\ \mathcal{W}^{(\mathrm{QED})}_{\mathrm{spinor}} = \frac{e^2\sqrt{\alpha|\beta|}}{(2\pi)^2}\coth\pi\sqrt{\frac{|\beta|}{\alpha}}\, e^{-\frac{\pi m^2_{\mathrm{spinor}}}{e\sqrt{\alpha}}}, \quad (3.4)$$

where

$$\alpha = \frac{1}{2}\left[\vec{E}^2-\vec{B}^2+\sqrt{\left(\vec{E}^2-\vec{B}^2\right)^2+4(\vec{E}\cdot\vec{B})^2}\right],\ \beta = \frac{1}{2}\left[\vec{E}^2-\vec{B}^2-\sqrt{\left(\vec{E}^2-\vec{B}^2\right)^2+4(\vec{E}\cdot\vec{B})^2}\right]. \quad (3.5)$$

Now take $\vec{E}\cdot\vec{B}=\pm EB$, the above rates agree with the known rates for a scalar pair and a spinor pair Nikishov'70 and for a vector pair Kruhlov'01, a non-trivial test!

The $p=1$ or 2 case: By the same token, for p = 1, we have the explicit Lorentz invariant scalar rate and spinor rate as

$$\mathcal{W}^{(\mathrm{QED})}_{\mathrm{scalar}} = \frac{eE}{2\pi}\, e^{-\frac{\pi m^2_{\mathrm{scalar}}}{eE}},\ \mathcal{W}^{(\mathrm{QED})}_{\mathrm{spinor}} = \frac{eE}{2\pi}\, e^{-\frac{\pi m^2_{\mathrm{spinor}}}{eE}}, \quad (3.6)$$

where $E=|F_{01}|$, being Lorentz invariant by itself.

Explicit LI QED rates

For p = 2,

$$\mathcal{W}^{(\mathrm{QED})}_{\mathrm{scalar}} = \frac{\left(e\sqrt{\alpha}\right)^{\frac{3}{2}}}{(2\pi)^2}\, e^{-\frac{\pi m^2_{\mathrm{scalar}}}{e\sqrt{\alpha}}}, \quad \mathcal{W}^{(\mathrm{QED})}_{\mathrm{spinor}} = \frac{\left(e\sqrt{\alpha}\right)^{\frac{3}{2}}}{(2\pi)^2}\, e^{-\frac{\pi m^2_{\mathrm{spinor}}}{e\sqrt{\alpha}}}, \tag{3.7}$$

where the Lorentz invariant $\alpha = F^2/2 = E_1^2 + E_2^2 - (F_{12})^2 > 0$ with $E_a = F_{0a}$.

The p = 3 or 4 case: For p = 4,

$$\begin{aligned}
\mathcal{W}^{(\mathrm{QED})}_{\mathrm{vec.}} &= \frac{\left[e\sqrt{\alpha}\right]^{\frac{3}{2}}\, e|\beta|^{\frac{1}{2}}}{(2\pi)^3}\, \frac{\cosh 2\pi\sqrt{\frac{|\beta|}{\alpha}} + 1}{\sinh \pi\sqrt{\frac{|\beta|}{\alpha}}}\, e^{-\frac{\pi m^2_{\mathrm{vec.}}}{e\sqrt{\alpha}}}, \\
\mathcal{W}^{(\mathrm{QED})}_{\mathrm{spinor}} &= \frac{\left[e\sqrt{\alpha}\right]^{\frac{3}{2}}\, e|\beta|^{\frac{1}{2}}}{(2\pi)^3}\, \coth \pi\sqrt{\frac{|\beta|}{\alpha}}\, e^{-\frac{\pi m^2_{\mathrm{spinor}}}{e\sqrt{\alpha}}}, \\
\mathcal{W}^{(\mathrm{QED})}_{\mathrm{scalar}} &= \frac{\left[e\sqrt{\alpha}\right]^{\frac{3}{2}}\, e|\beta|^{\frac{1}{2}}}{2(2\pi)^3}\, \mathrm{csch}\pi\sqrt{\frac{|\beta|}{\alpha}}\, e^{-\frac{\pi m^2_{\mathrm{scalar}}}{e\sqrt{\alpha}}},
\end{aligned} \tag{3.8}$$

where

$$\alpha = \frac{1}{2}\left[\frac{1}{2}\mathrm{tr}F^2 + \sqrt{\mathrm{tr}F^4 - \frac{1}{4}(\mathrm{tr}F^2)^2}\right], \quad \beta = \frac{1}{2}\left[\frac{1}{2}\mathrm{tr}F^2 - \sqrt{\mathrm{tr}F^4 - \frac{1}{4}(\mathrm{tr}F^2)^2}\right]. \tag{3.9}$$

Explicit LI QED rates

p = 5 or 6 case: For p = 5,

$$\mathcal{W}^{(\mathrm{QED})}_{\mathrm{scalar}} = \frac{e^3\sqrt{\alpha\beta\gamma}}{4(2\pi)^3}\,\mathrm{csch}\pi\sqrt{\frac{|\beta|}{\alpha}}\,\mathrm{csch}\pi\sqrt{\frac{|\gamma|}{\alpha}}\,e^{-\frac{\pi m^2_{\mathrm{scalar}}}{e\sqrt{\alpha}}},$$

$$\mathcal{W}^{(\mathrm{QED})}_{\mathrm{spinor}} = \frac{e^3\sqrt{\alpha\beta\gamma}}{(2\pi)^3}\,\coth\pi\sqrt{\frac{|\beta|}{\alpha}}\,\coth\pi\sqrt{\frac{|\gamma|}{\alpha}}\,e^{-\frac{\pi m^2_{\mathrm{spinor}}}{e\sqrt{\alpha}}},$$

$$\mathcal{W}^{(\mathrm{QED})}_{\mathrm{vec.}} = \frac{e^3\sqrt{\alpha\beta\gamma}}{4(2\pi)^3}\,\frac{1+2\left[\cosh 2\pi\sqrt{\frac{|\beta|}{\alpha}}+\cosh 2\pi\sqrt{\frac{|\gamma|}{\alpha}}\right]}{\sinh\pi\sqrt{\frac{|\beta|}{\alpha}}\,\sinh\pi\sqrt{\frac{|\gamma|}{\alpha}}}\,e^{-\frac{\pi m^2_{\mathrm{vec.}}}{e\sqrt{\alpha}}}. \tag{3.10}$$

For p = 6,

$$\mathcal{W}^{(\mathrm{QED})}_{\mathrm{scalar}} = \frac{(e\sqrt{\alpha})^{\frac{3}{2}}\,e^2\sqrt{\beta\gamma}}{4(2\pi)^4}\,\mathrm{csch}\pi\sqrt{\frac{|\beta|}{\alpha}}\,\mathrm{csch}\pi\sqrt{\frac{|\gamma|}{\alpha}}\,e^{-\frac{\pi m^2_{\mathrm{scalar}}}{e\sqrt{\alpha}}},$$

$$\mathcal{W}^{(\mathrm{QED})}_{\mathrm{spinor}} = \frac{(e\sqrt{\alpha})^{\frac{3}{2}}\,e^2\sqrt{\beta\gamma}}{(2\pi)^4}\,\coth\pi\sqrt{\frac{|\beta|}{\alpha}}\,\coth\pi\sqrt{\frac{|\gamma|}{\alpha}}\,e^{-\frac{\pi m^2_{\mathrm{spinor}}}{e\sqrt{\alpha}}},$$

$$\mathcal{W}^{(\mathrm{QED})}_{\mathrm{vec.}} = \frac{(e\sqrt{\alpha})^{\frac{3}{2}}\,e^2\sqrt{\beta\gamma}}{2(2\pi)^4}\,\frac{1+\cosh 2\pi\sqrt{\frac{|\beta|}{\alpha}}+\cosh 2\pi\sqrt{\frac{|\gamma|}{\alpha}}}{\sinh\pi\sqrt{\frac{|\beta|}{\alpha}}\,\sinh\pi\sqrt{\frac{|\gamma|}{\alpha}}}\,e^{-\frac{\pi m^2_{\mathrm{vec.}}}{e\sqrt{\alpha}}}. \tag{3.11}$$

For both p = 5 and 6, the Lorentz invariants α, β and γ are given by (2.15), i.e.,

$$\alpha\beta\gamma = \frac{1}{3!}[\mathrm{tr}F^6 - \frac{3}{4}\mathrm{tr}F^4\mathrm{tr}F^2 + \frac{1}{8}(\mathrm{tr}F^2)^3] \geq 0, \alpha\beta+\alpha\gamma+\beta\gamma = -\frac{1}{4}[\mathrm{tr}F^4 - \frac{1}{2}(\mathrm{tr}F^2)^2],\ \alpha+\beta+\gamma = \frac{1}{2}\mathrm{tr}F^2. \tag{3.12}$$

Conclusion and discussion

- We have derived the explicit Lorentz invariant QED pair production rates, respectively, for a scalar pair, a spinor pair and a vector pair for the most general allowed constant EM field in diverse dimensions from superstring theories;
- The explicit Lorentz invariant scalar rate and the spinor rate for $d = 3$, the vector rate for $d = 4$ and all other rates for $d \geq 5$ are new and have not been given before;
- For $d \geq 5$, these results cannot be obtained from QED since the underlying QFT is not renormalizable and the one-loop computations cannot be obtained;
- D-branes say that a second brane is needed to have such rates while the charged particles originate from the open strings along the extra dimensions.

THANK YOU!

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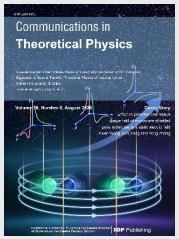
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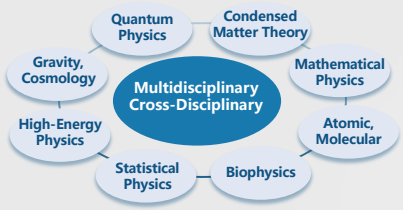
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