ICFP 2026

Multi-Band Constraints on Cosmic Strings

Unifying harmonic and burst spectra

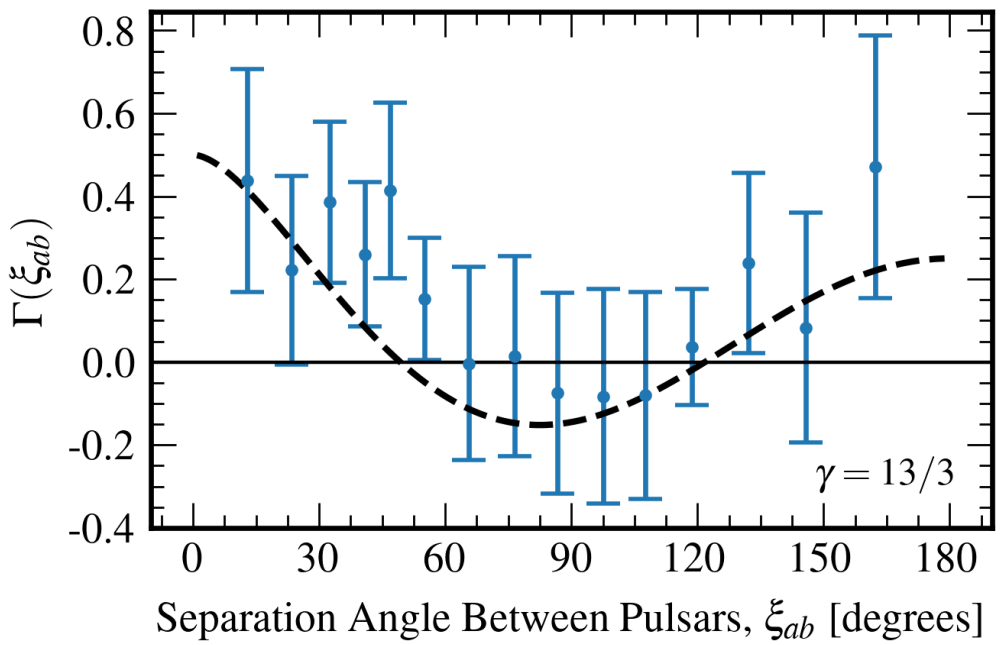
arXiv:2610.02590

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The gravitational-wave background searches across bands

NANOGrav 15-year

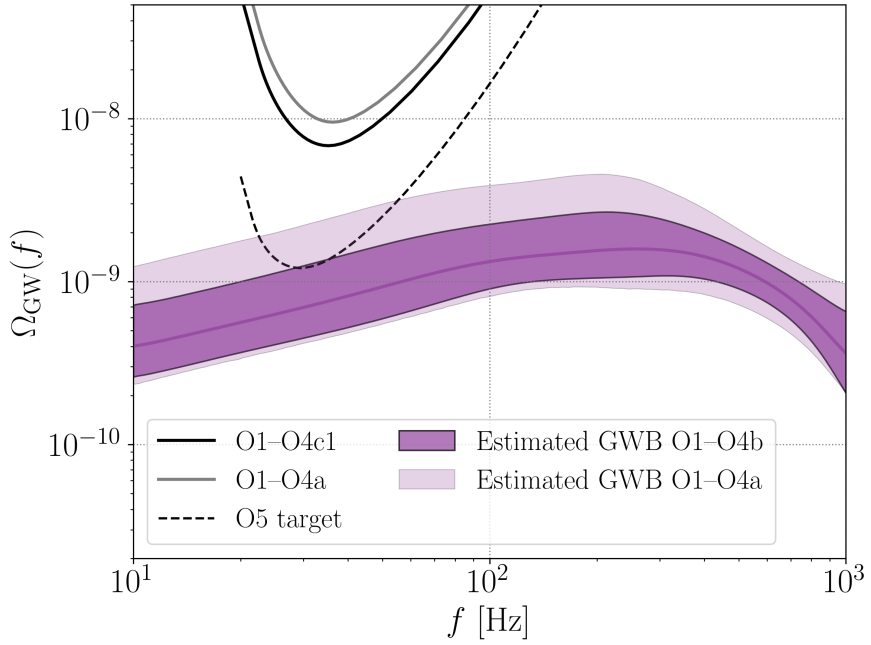


Dashed curve: Hellings–Downs prediction

Evidence for a correlated GWB

NANOGrav Collaboration, arXiv:2306.16213

LVK O1–O4c1



Black curve: O1–O4c1 2-sigma PI sensitivity

$$\Omega_{GW}(25\,\mathrm{Hz}) < 2.8 \times 10^{-9}$$

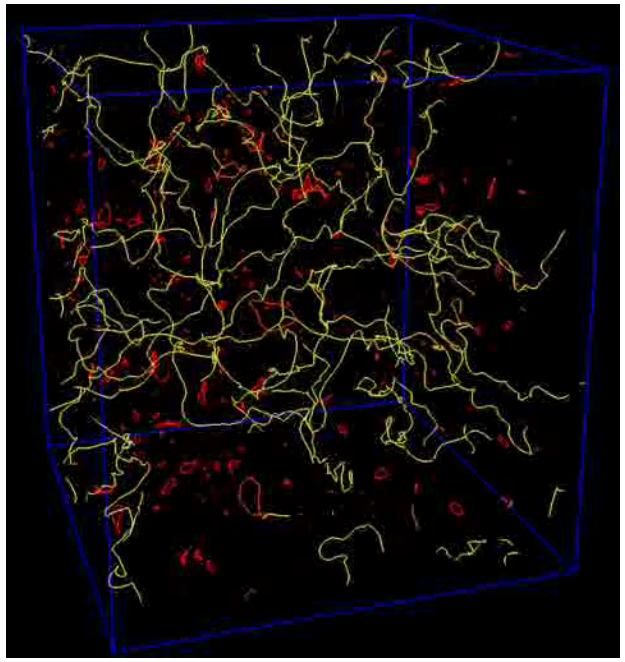
95% upper limit; flat spectrum; log-uniform prior

LVK Collaboration, arXiv:2608.23477

What are cosmic strings?

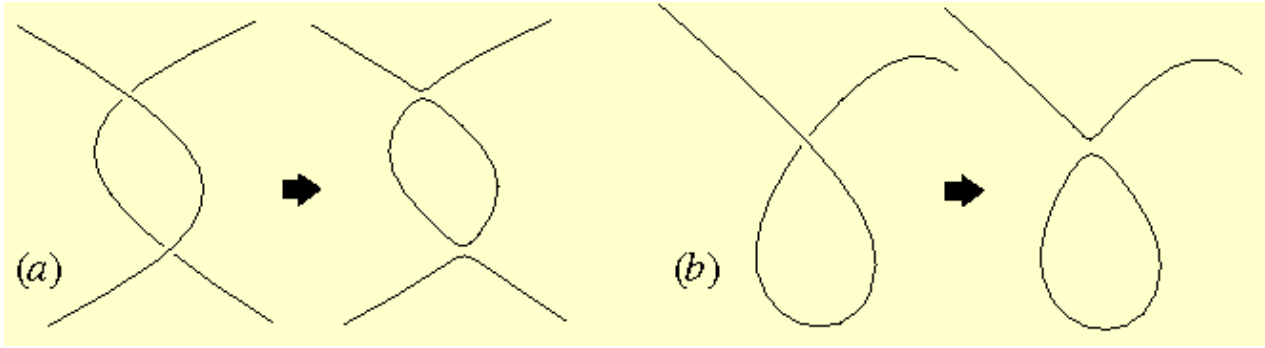
One-dimensional defects that may form during symmetry breaking in the early Universe.

A string network



Illustrations: Cambridge Cosmology Group / LIGO

The network produces loops



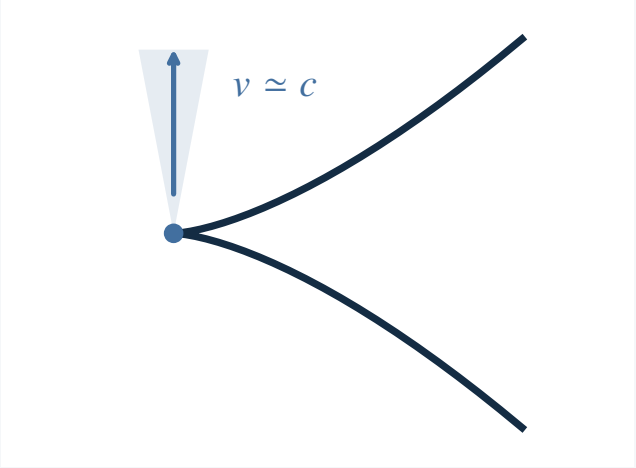
Loops oscillate and emit gravitational waves.

String tension: $G\mu$

Their accumulated radiation forms a broad-band gravitational-wave background.

Cusps and kinks radiate gravitational waves

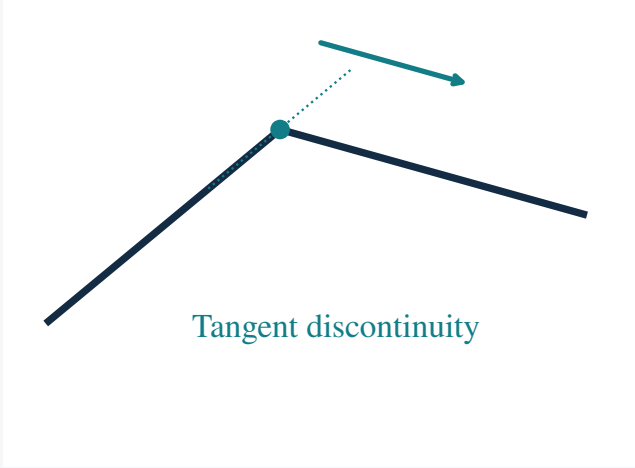
Cusp



A point briefly moves
at nearly the speed of light.

$$|\widetilde{h}_c(f)| \propto f^{-4/3}$$

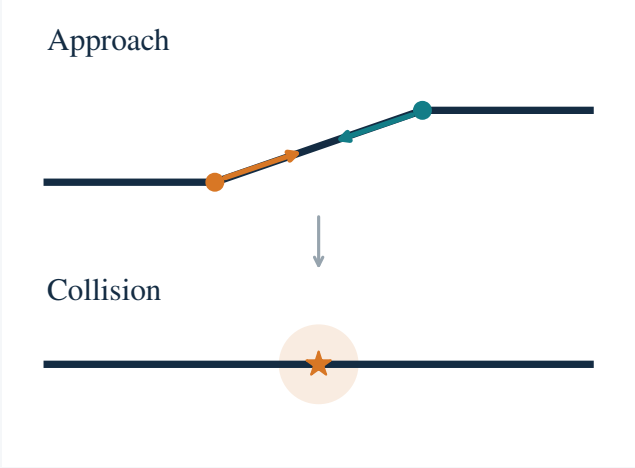
Kink



A sharp bend propagates
along the string.

$$|\widetilde{h}_k(f)| \propto f^{-5/3}$$

Kink–kink collision



Two oppositely moving
kinks meet.

$$|\widetilde{h}_{kk}(f)| \propto f^{-2}$$

Bursts from these features dominate the gravitational-wave emission from cosmic-string loops.

Two methods for calculating the spectrum

Harmonic expansion

Vachaspati & Vilenkin (1985)
Phys. Rev. D 31, 3052

Blanco-Pillado & Olum (2017)
arXiv:1709.02693

$$\Omega_{\rm H}^{s}(f) = \frac{8\pi(G\mu)^2}{3H_0^2} \sum_{j=1}^{\infty} P_{\rm H}^{s} \frac{2j}{f} \int dz \, \frac{n(2j/[f(1+z)], t(z))}{H(z)(1+z)^6}$$

$P_{\rm H}^{s} = A_{\rm H}^{s} j^{-q_s}$: dimensionless power in harmonic j

$n(l,t)$: loop number density

Burst superposition

Damour & Vilenkin (2000)
arXiv:gr-qc/0004075

Siemens et al. (2007)
arXiv:astro-ph/0610920

$$\Omega_{\rm B}^{s}(f) = \frac{4\pi^2 f^3}{3H_0^2} \int dz \, dl \, |h_s(f,z,l)|^2 \frac{d^2R_s}{dz \, dl}$$

$h_s(f,z,l)$: Fourier-domain burst strain

$\frac{d^2R_s}{dz\,dl}$: observed burst-event rate

A multi-band constraint needs a common physical description.

When do these calculations agree, and which spectrum should we use across bands?

What did earlier comparisons establish?

Continuum approximation



Replace the discrete spectrum by a continuous one.

Binétruy et al. (2012) · arXiv:1201.0983

Ringeval & Suyama (2017) arXiv:1709.03845

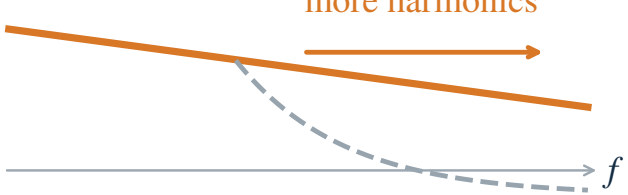
Plateau consistency



Compare the two calculations on the radiation-era plateau.

Auclair et al. (2020) arXiv:1909.00819

High-mode convergence



Recover the high-frequency tail by including more harmonics.

Cui, Lewicki & Morrissey (2020) arXiv:1912.08832

A fundamental kernel connects the two descriptions

$$\mathcal{K}(f) = \frac{2}{f}\int dz\, \frac{n(2/[f(1+z)], t(z))}{H(z)(1+z)^6}$$

Harmonic

Discrete normalized average

$$\langle \mathcal{K} \rangle_H \equiv \frac{\sum_{j=1}^{\infty} j^{-q_s}\mathcal{K}(f/j)}{\sum_{j=1}^{\infty} j^{-q_s}}$$

$$\Omega_H^s(f) = C\Gamma_s \langle \mathcal{K} \rangle_H$$

Burst

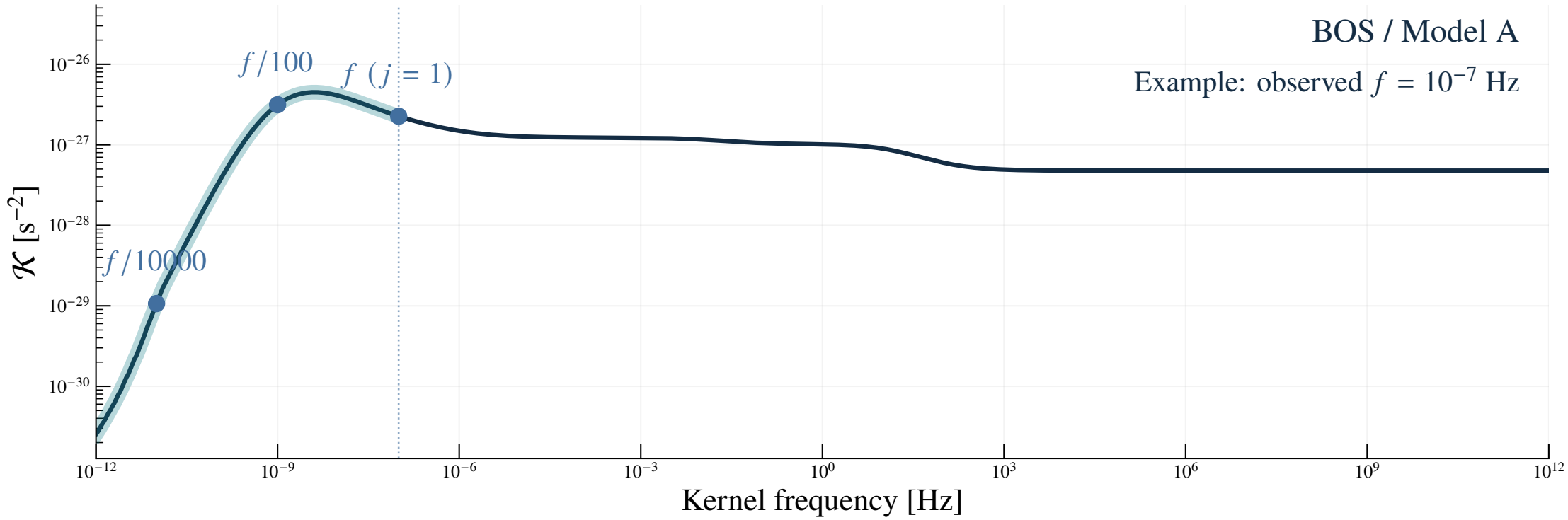
Continuous normalized average

$$\langle \mathcal{K} \rangle_B \equiv \frac{\int_{\delta_s}^{\infty} dx\, x^{-q_s}\mathcal{K}(f/x)}{\int_{\delta_s}^{\infty} dx\, x^{-q_s}}$$

$$\Omega_B^s(f) = C\Gamma_s \langle \mathcal{K} \rangle_B$$

$$C = \frac{8\pi(G\mu)^2}{3H_0^2}, \qquad \Gamma_s : \text{ power in radiation channel } s$$

One kernel determines the spectrum at every frequency

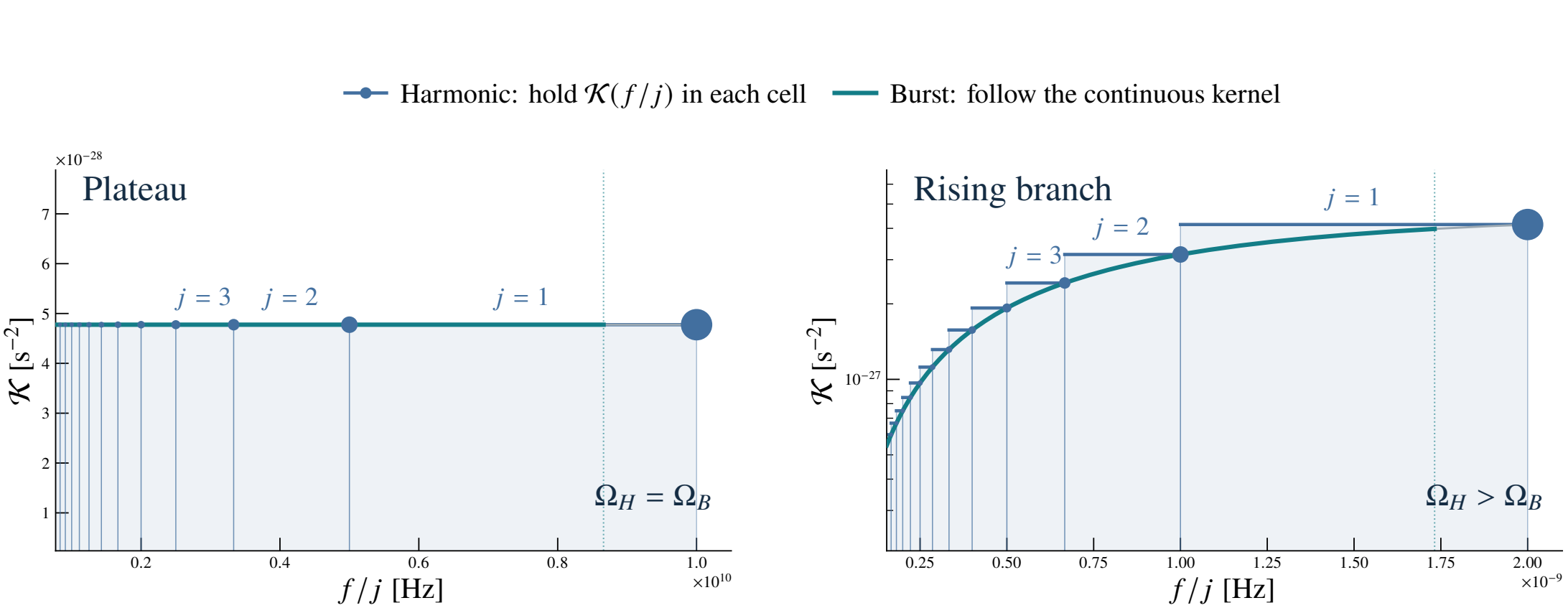


Continuous range: $0 < f/x \leq f/\delta_s$

Harmonic: weighted discrete samples

Burst: a weighted continuous integral

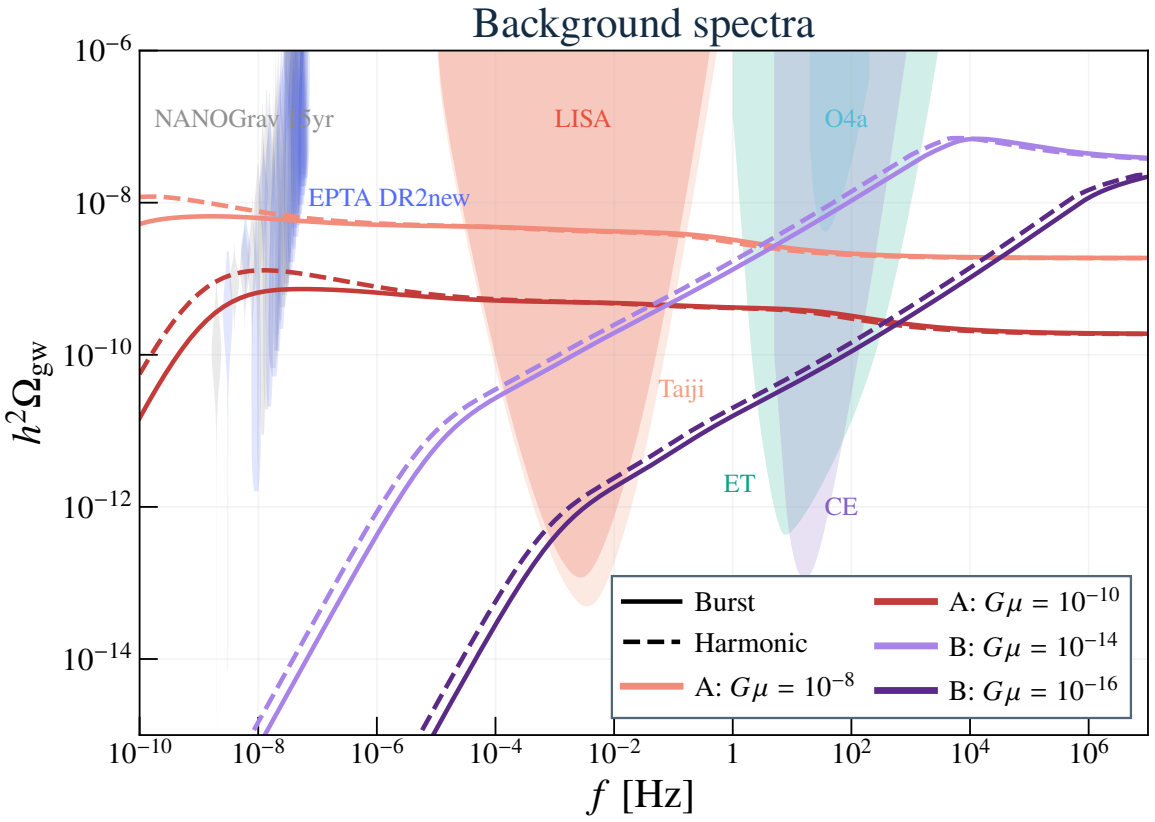
Why the spectra agree on the plateau and differ below it

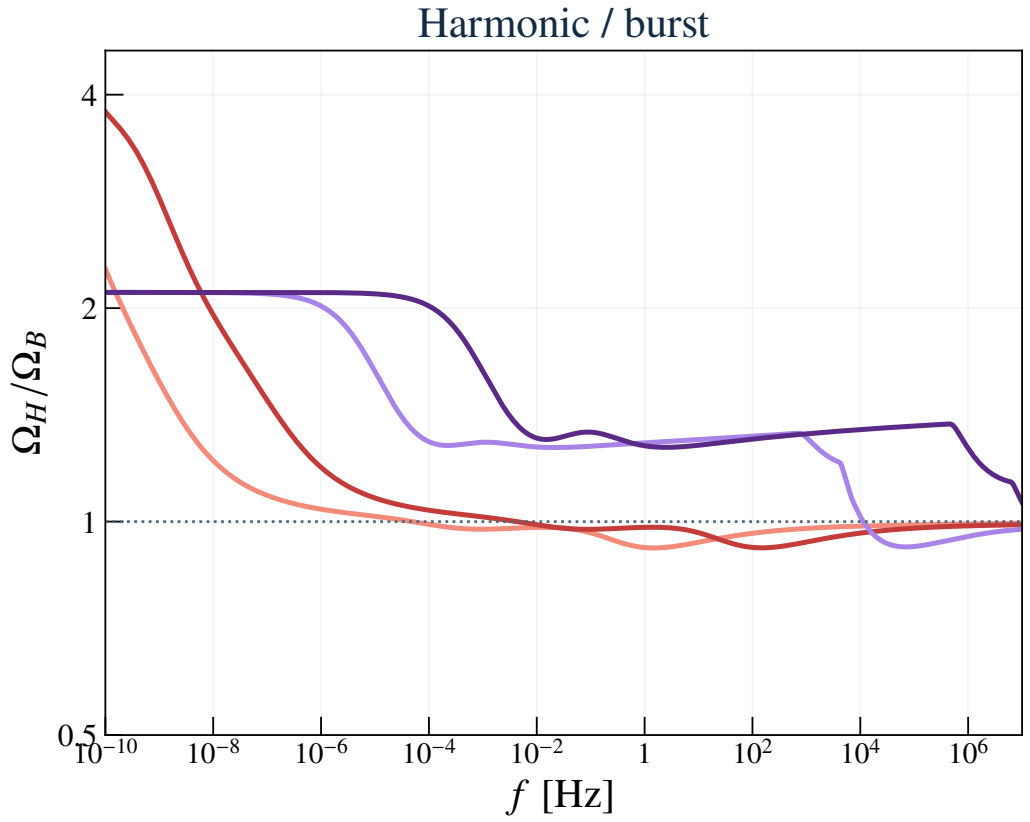


Normalized weights: $p_j = j^{-q_s}/\zeta(q_s)$, $p(x) = x^{-q_s}/\int_{\delta_s}^{\infty} u^{-q_s}\,du$

A constant kernel gives the same average;
a rising kernel makes the way we sample it important.

Cosmic-string spectra for Models A and B

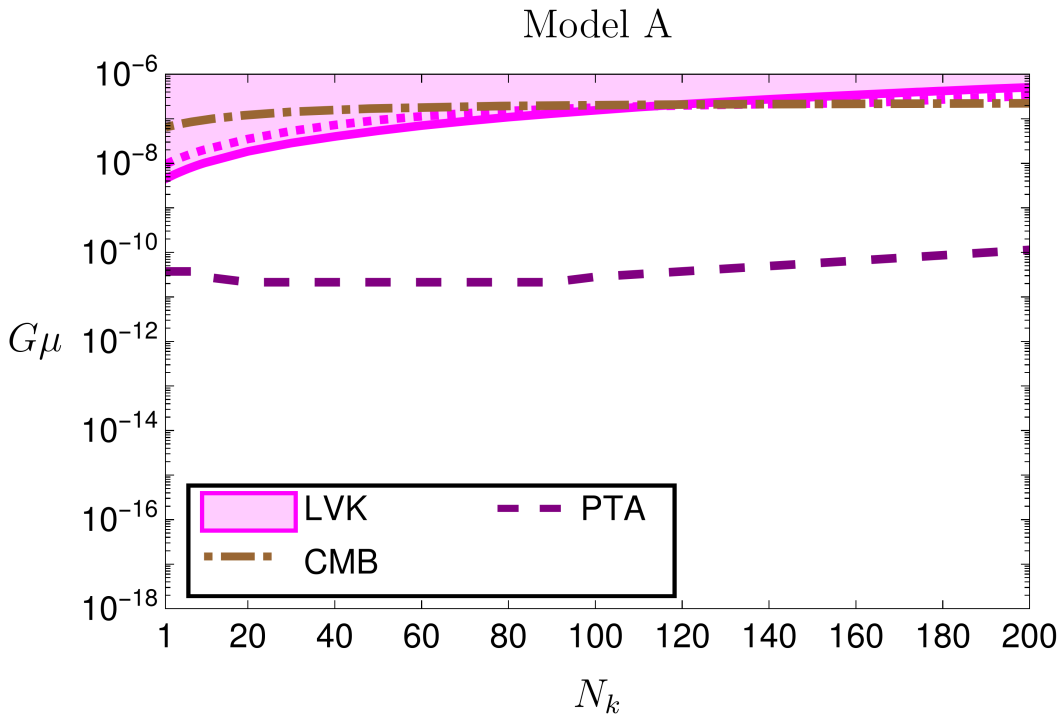


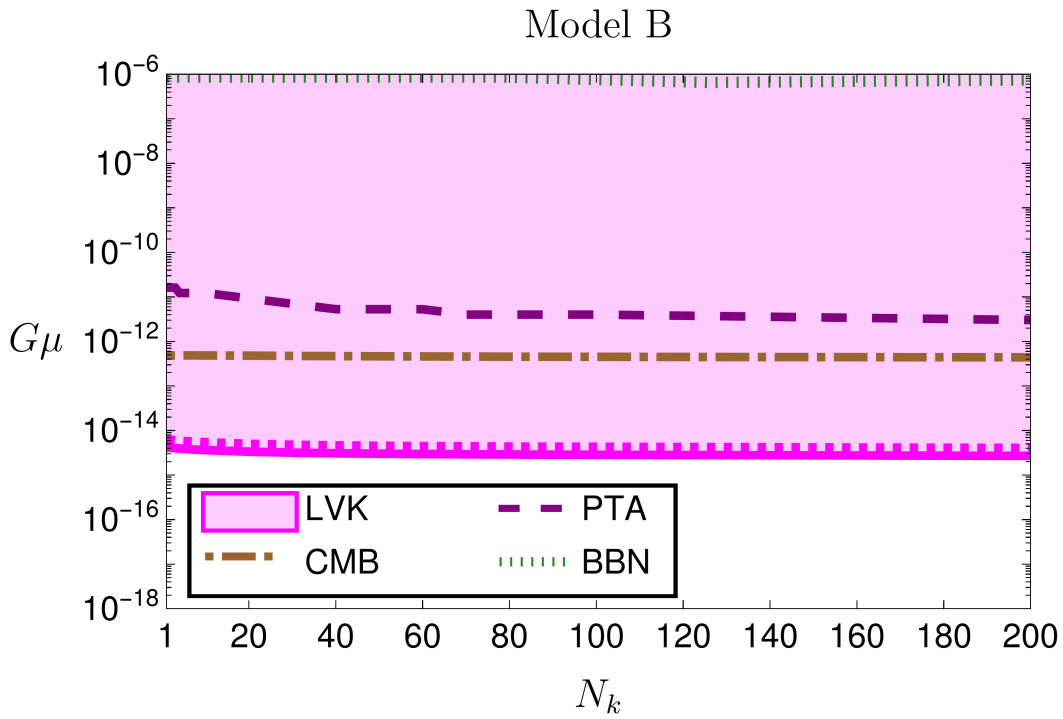


Previous O4a constraints: fixed cusp and kink choices

LVK Collaboration, arXiv:2510.26848

$$N_c = 1, \qquad N_k = 1, \ldots, 200$$





Discrete fixed- (N_c, N_k) **comparisons**

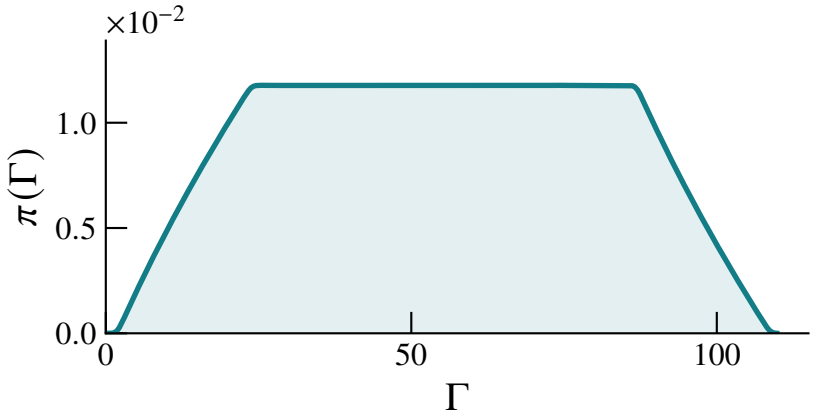
Cosmic-string-only analysis

Outdated PTA constraints

Cosmic-string priors and the multi-band analysis

Parameter	Physical meaning	Prior
$\log_{10} G\mu$	String tension	$U(-18, -6)$
N_c	Mean cusp number	$U(0, 3)$
N_k	Mean kink number	$U(1, 10)$

Induced prior



LVK O1–O4a

Cosmic strings + CBC

Cross-correlation spectra

NG15yr / EPTA DR2new

Cosmic strings + SMBHB

HD-correlated free spectra

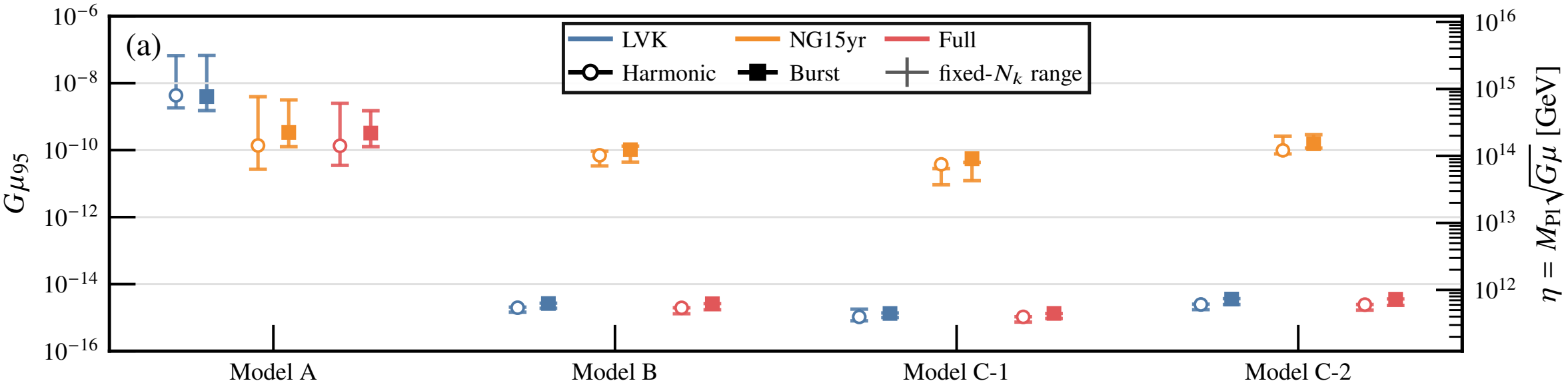
Joint

Shared string parameters

Both detector bands

$$\ln \mathcal{L}_{\text{joint}} = \ln \mathcal{L}_{\text{LVK}} + \ln \mathcal{L}_{\text{PTA}}$$

Constraints for four loop distribution models



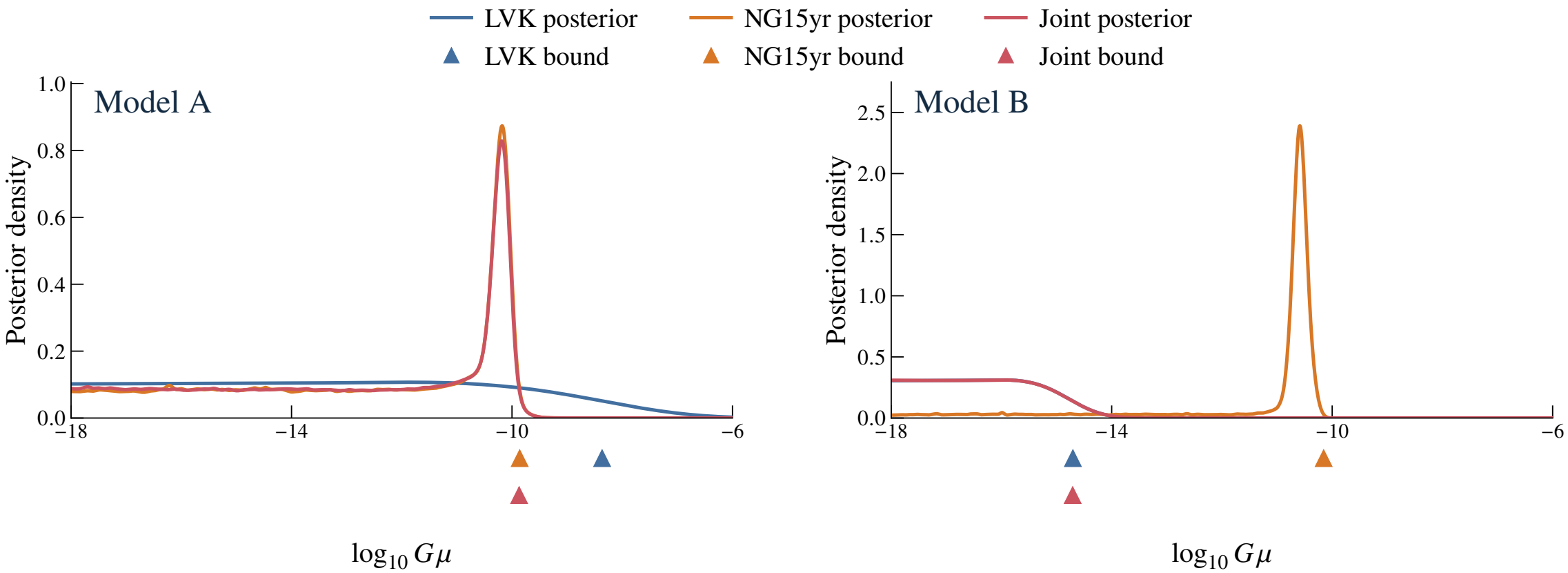
Model A: PTA dominated

Models B / C-1 / C-2: LVK dominated

Harmonic gives tighter LVK bounds than burst

Model B **25%** Model C-1 **21%** Model C-2 **31%**

Which band controls the joint tension bound?

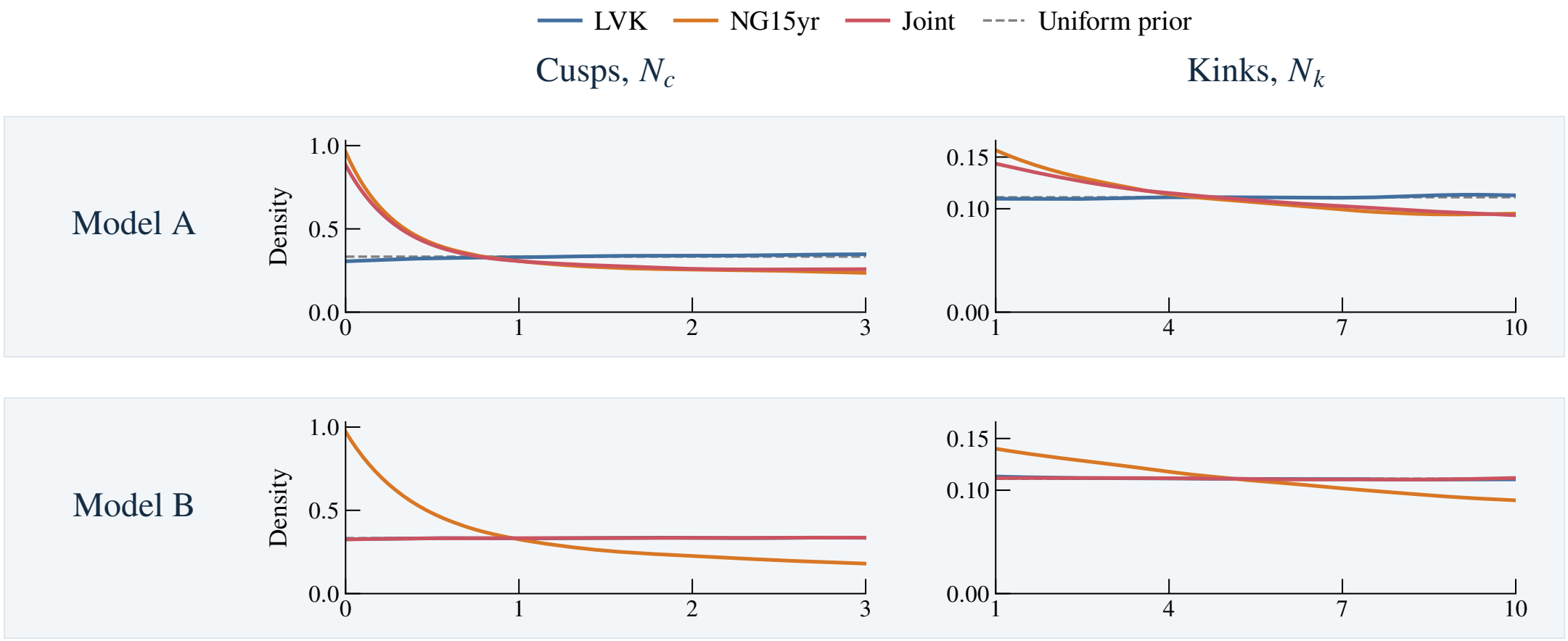


Model A: Joint follows PTA

Model B: Joint follows LVK

The LVK bound lies far below the PTA posterior peak.

What do the data tell us about radiation uncertainty?

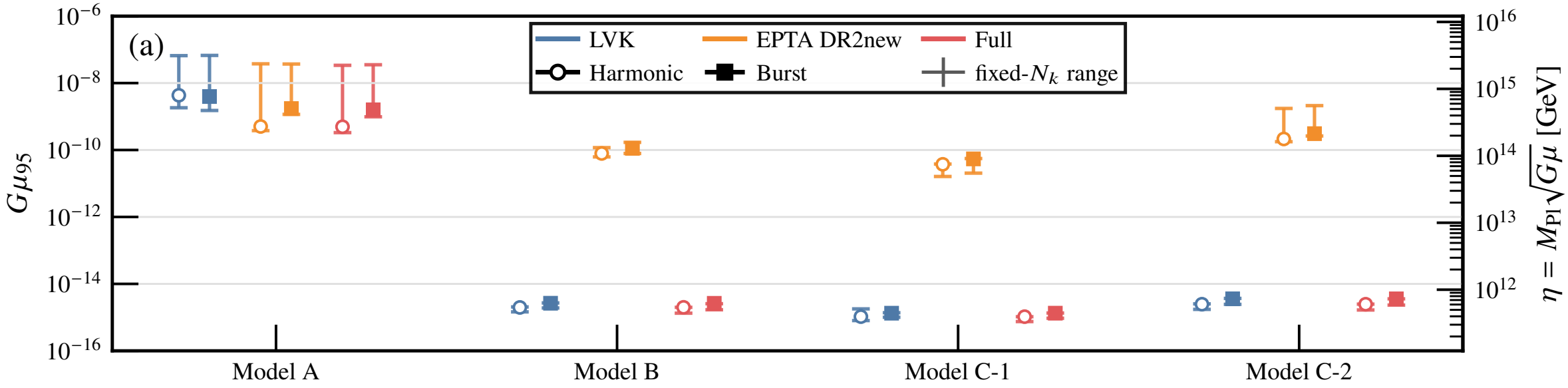


LVK: **prior-dominated** posteriors for $\{N_c, N_k\}$.

PTA: large $\{N_c, N_k\}$ are **disfavored**.

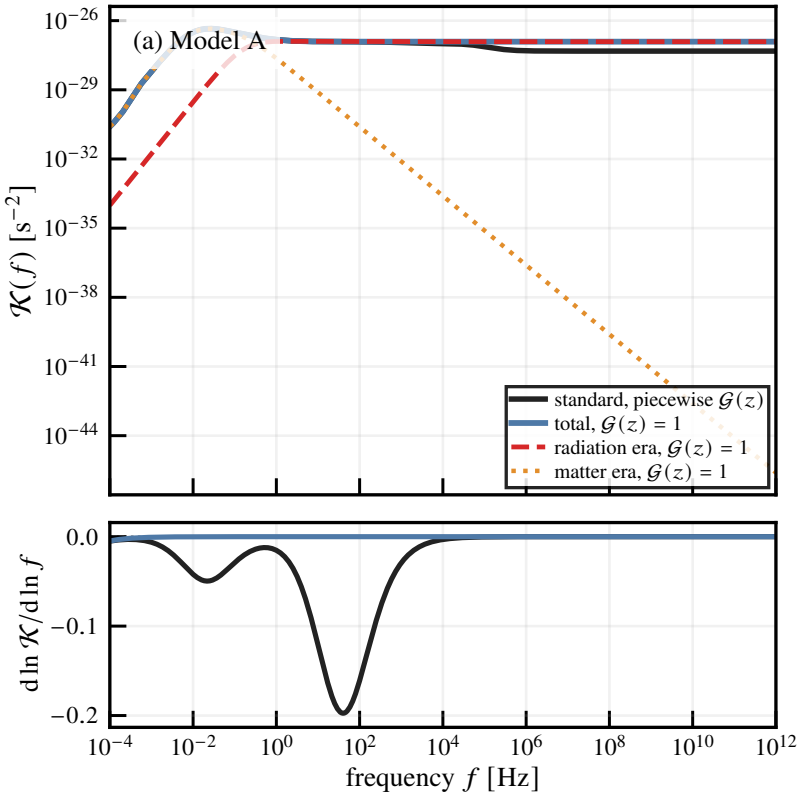
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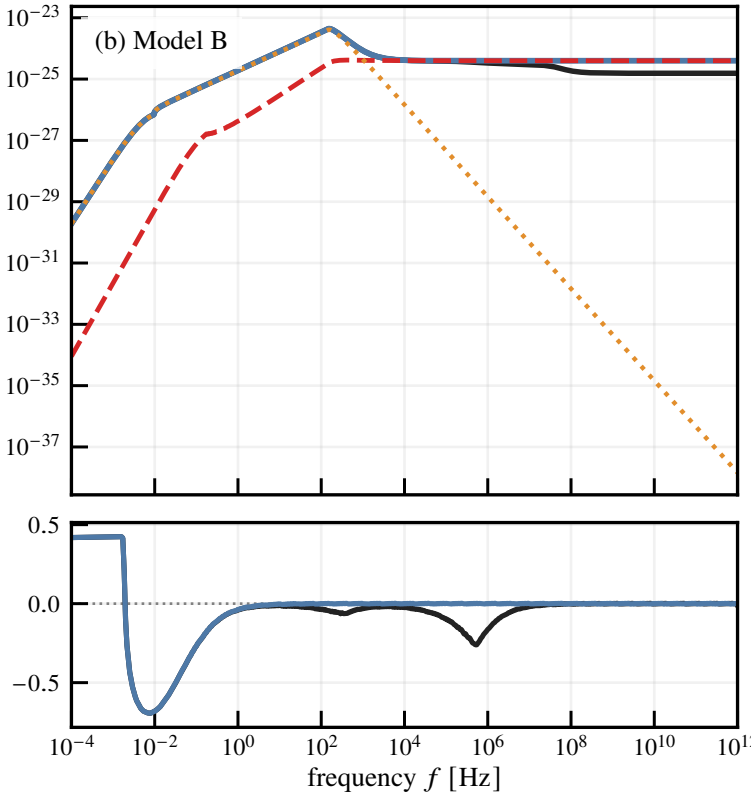
EPTA gives similar results



The same pattern: Model A is PTA dominated; Models B, C-1 and C-2 are LVK dominated.

Relativistic species leave steps in the kernel





Radiation-era dominance does not guarantee a perfectly flat kernel.

A downward step can reverse the small harmonic–burst offset.