

Canonical inflation

Goldstone loops and dilation symmetry in non-slow-roll inflation

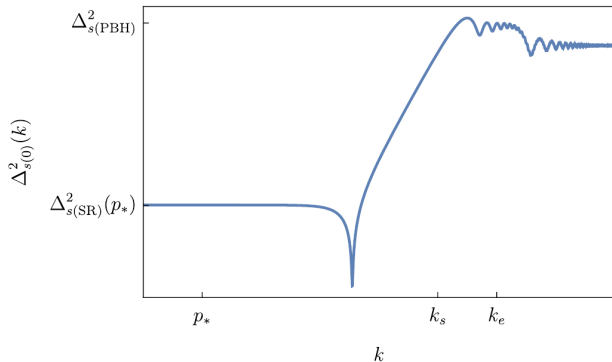
Nonlinear clock maps and momentum boundaries

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Short modes and CMB-scale loops



Scale separation

$$p \ll k$$

$$\mathcal{P}_L \simeq 2.1 \times 10^{-9}$$

$$\mathcal{P}_S^{\text{peak}} \sim 10^{-2}$$

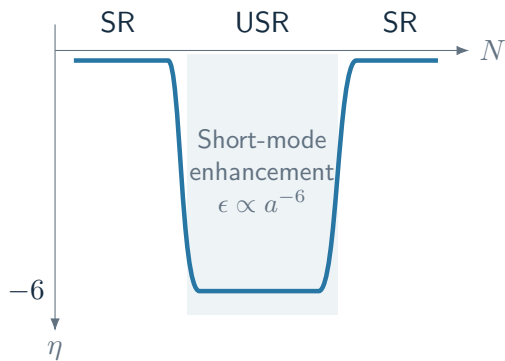
Nonlinear mode coupling

Enhanced short modes



Large-scale backreaction

Transient USR and superhorizon modes



$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = \frac{\dot{\epsilon}}{H\epsilon}$$

Superhorizon solutions

$$\zeta = C_{\text{ad}} + C_2 \int^t \frac{dt'}{a^3(t')\epsilon(t')}$$

$$\underbrace{\zeta_{\text{ad}} = \text{const}}_{\text{Adiabatic branch}}$$

$$\zeta_2 \propto \begin{cases} a^{-3}, & \text{SR: decaying} \\ a^3, & \text{USR: growing} \end{cases}$$

Large-scale structure of loop corrections

$$p \ll k$$

$$\mathcal{P}_\zeta^{\text{loop}}(p) = c \mathcal{P}_\zeta^{(0)}(p) \int d \ln k \mathcal{P}_\zeta^{(0)}(k) + \text{volume-suppressed terms}$$

Nonlinear feedback



Short modes $\longrightarrow$ Long-mode power

Central question

Does the first term survive?

Full graph sum Renormalization

The initial estimate and the ensuing debate

$$\text{SR } (\eta \simeq 0) \xrightarrow{\tau_s} \text{USR } (\eta \simeq -6) \xrightarrow{\tau_e} \text{SR } (\eta \simeq 0)$$

$$p \ll k_s, \quad k_s \leq k \leq k_e, \quad \eta' \simeq \Delta\eta \delta(\tau - \tau_e), \quad \Delta\eta = 6$$

$$H_3^{(\eta')} = -\frac{M_{\text{Pl}}^2}{2} \int d^3x a^2 \epsilon \eta' \zeta' \zeta^2$$

$$\boxed{\frac{\delta \mathcal{P}_L^{(\eta' \eta')}}{\mathcal{P}_L^{(0)}} \simeq \frac{(\Delta\eta)^2}{4} \mathcal{P}_{\text{PBH}} \left[1.1 + \ln \frac{k_e}{k_s} \right]}$$

Transition profile

Sharp / smooth matching

Background response

Tadpoles / linked CT

Action completeness

Gauge maps / quartic / boundary terms

Symmetry & renormalization

Soft relations / Ward / finite subtraction

Goldstone EFT: expansion and graphs

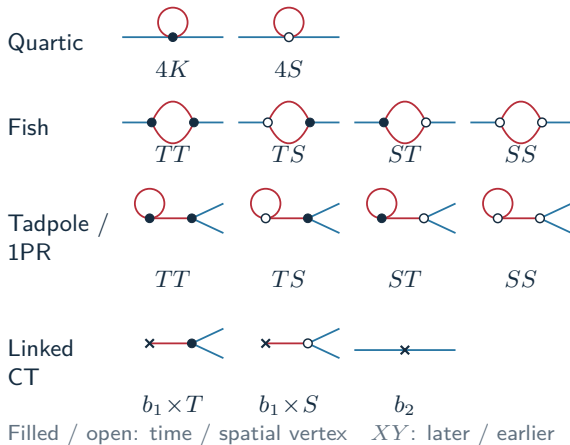
Canonical clock coefficient

$$A = -M_{\text{Pl}}^2 \dot{H} = \frac{\dot{\phi}^2}{2}$$

$$A(t + \pi) = A + \dot{A}\pi + \frac{1}{2}\ddot{A}\pi^2 + \dots$$

$$\mathcal{L}_{\text{dec}} = a^3 A(t + \pi) \times \left[\dot{\pi}^2 - \frac{(\partial\pi)^2}{a^2} \right]$$

Quartic K : Legendre contact



USR loops: graphs and regularization

Reported graph sum



Fish + quartic

Legendre contact included

Missing completion



1PR tadpoles + linked CT

UV renormalization deferred

Strong cutoff sensitivity

$$\Delta \mathcal{P} \propto \Delta N e^{6\Delta N} \mathcal{P}_{\text{CMB}}^2$$

Coefficient of this enhancement

Time lower bound

Coefficient

$$-1/q \quad \frac{6}{h}(h^2 + 24h + 180)$$

$$\tau_i \quad 6h + 54$$

$$h = -6 : \quad -72 \longleftrightarrow 18$$

h : exit sharpness ΔN : USR duration

Mode functions and dimensional regularization

Near-dS approximation

$$\epsilon \ll \eta \ll 1, \quad \frac{Z''}{Z} \simeq \frac{2}{\tau^2}$$

$$\pi_k^{(3)}(\tau) = \frac{(1 + i c_s k \tau) e^{-i c_s k \tau}}{2 M_{\text{Pl}} \sqrt{\epsilon c_s k^3}}$$

Analytic modes $\rightarrow$ continuation in d

Dimensional continuation of the modes

$$d = 3 + \delta : \quad \pi_k^{(d)} = \pi_k^{(3)} + \delta \pi_{k,1} + \cdots, \quad \underbrace{\frac{1}{\delta}}_{\text{UV pole}} \times \underbrace{\delta \pi_{k,1}}_{\text{Mode correction}} = \text{finite}$$

Generic non-attractor transition

$$w_k'' + \left(c_s^2 k^2 - \frac{Z''}{Z} \right) w_k = 0$$

Canonical mode: $w_k = Z \pi_k$

No generic closed-form modes

$\Rightarrow$ **Difficult direct dim-reg**

Feature hierarchy and diagram truncation

$$H_4 = \frac{C}{2} \int \pi^2 \left[(\alpha^2 - \dot{\alpha}) \dot{\pi}^2 + (\alpha^2 + \dot{\alpha}) \frac{(\partial \pi)^2}{a^2} \right]$$

$$\eta_2 = \frac{\dot{\eta}}{H\eta}, \quad \boxed{|\eta_2| \gg |\eta|}$$

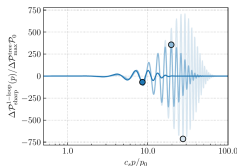
$$H_4 \xrightarrow{|\dot{\alpha}| \gg \alpha^2} -\frac{C\dot{\alpha}}{2} \int \pi^2 \left[\dot{\pi}^2 - \frac{(\partial \pi)^2}{a^2} \right]$$



$\mathcal{O}(\eta\eta_2)$: retained



$\mathcal{O}(\eta^2)$: subleading



USR: $\eta \simeq -6$, $\eta_2 \simeq 0$

Outside the feature hierarchy

Flat-gauge loops and the boundary

General modes

Leading large-scale limit

leading ϵ

$$\mathcal{L}_k = \partial_t^2 + 3H\partial_t + \frac{k^2}{a^2} + V^{(2)}, \quad \mathcal{L}_k v_k = 0$$

$$\mathcal{L}_k(\dot{v}_k + H\partial_{\ln k} v_k) = -V^{(3)}\dot{\phi} v_k$$



Feynman diagrams representing the boundary operator $\mathcal{B}_\phi$. The diagrams show a blue line with a red loop (self-energy) and a blue line with a red loop (tadpole). The sum of these diagrams is equal to $\mathcal{B}_\phi$.

$$\mathcal{B}_\phi(t) = \frac{H}{2\pi^2} \int dk \partial_k \left[k^3 |v_k(t)|^2 \right] = \frac{H}{2\pi^2} \left[k^3 |v_k(t)|^2 \right]_{\text{IR}}^{\text{UV}}$$

All leading large-scale loops $\rightarrow$ one momentum-boundary integral

Dilation symmetry in the flat variables

Dilation + time shift

$$\tilde{\mathbf{x}} = (1 - \lambda)\mathbf{x}, \quad \tilde{t} = t + \frac{\lambda}{H(t)}$$

$$\boxed{\delta\tilde{\phi}(\tilde{t}, \tilde{\mathbf{x}}) = \delta\phi(t, \mathbf{x}) - \lambda \frac{\dot{\phi}}{H}}$$

Same-coordinate generator

$$\delta_D \delta\phi = \mathbf{x} \cdot \nabla \delta\phi - H^{-1} \partial_t \delta\phi - \frac{\dot{\phi}}{H}$$

Spatial response + time response

Background–perturbation split

$$\boxed{\bar{\phi}(\tilde{t}) + \delta\tilde{\phi}(\tilde{t}, \tilde{\mathbf{x}}) \simeq \bar{\phi}(t) + \delta\phi(t, \mathbf{x})}$$

$$a^3(t) dt d^3x \simeq a^3(\tilde{t}) d\tilde{t} d^3\tilde{x}, \quad S_{\text{flat}}[\delta\tilde{\phi}] \simeq S_{\text{flat}}[\delta\phi]$$

Leading ϵ Tree action Transformed integration domain

arXiv:2605.13325, Eq. (10), App. C, Eqs. (C1)–(C2); this work, Sec. 3.2.

Consistency relations and mode identities

Separated-point Ward identity

Adiabatic soft leg leading ϵ

$$\epsilon_0^{(\phi)} \frac{\dot{\phi}_i}{H_i} \left[\langle \delta\phi_{\mathbf{0}}(t_i) \delta\phi_{\mathbf{k}}(t) \delta\phi_{-\mathbf{k}}(t) \rangle' + \text{c.c.} \right] = (3 + \partial_{\ln k} + H^{-1} \partial_t) P_S^{(\phi)}(k, t)$$

$\epsilon_0^{(\phi)}$: initial Gaussian kernel $P_S^{(\phi)} = |v_k|^2$ $H_i = H(t_i)$

Equivalent mode-function identity

$$\dot{P}_S^{(\phi)} + H(3 + \partial_{\ln k}) P_S^{(\phi)} = 2 \int_{t_i}^t dt' a^3(t') V^{(3)}(t') \dot{\phi}(t') \text{Im}[v_k^2(t) v_k^{*2}(t')]$$

Tree-level soft three-point function $\longleftrightarrow$ Retarded history

Separated Ward $\Longleftrightarrow$ mode-function response

Action equivalence and loop-level matching

Known at the action level

$$\delta\phi = \bar{\phi}(t + \pi) - \bar{\phi}(t) = \Pi\pi + \frac{\dot{\Pi}}{2}\pi^2 + \frac{\ddot{\Pi}}{6}\pi^3 + \dots, \quad \Pi = \dot{\bar{\phi}}$$

$$\boxed{S_{\text{flat}}[\delta\phi(\pi)] = S_{\text{E}}[\pi]}$$

Fixed FRW Potential vertices $\longleftrightarrow$ derivative vertices + time boundaries

Loop diagrams: no term-by-term match

$$\begin{aligned} \langle \delta\phi_{\mathbf{x}} \delta\phi_{\mathbf{y}} \rangle &= \Pi^2 \langle \pi_{\mathbf{x}} \pi_{\mathbf{y}} \rangle \\ &+ \frac{\Pi\dot{\Pi}}{2} [\langle \pi_{\mathbf{x}}^2 \pi_{\mathbf{y}} \rangle + \langle \pi_{\mathbf{x}} \pi_{\mathbf{y}}^2 \rangle] + \dots \end{aligned}$$

Full graph sum

Nonlinear external operators

Linked counterterms

The same dilation in three variables

Same flat slices: nonlinear field redefinition

$$\bar{\phi}(t + \pi) = \bar{\phi}(t) + \delta\phi$$

$$\delta\tilde{\phi}(\tilde{t}, \tilde{\mathbf{x}}) = \delta\phi(t, \mathbf{x}) - \lambda \frac{\Pi(t)}{H(t)} \iff \tilde{\pi}(\tilde{t}, \tilde{\mathbf{x}}) = \pi(t, \mathbf{x}) - \frac{\lambda}{H(t)}$$

Change slicing: invariant clock

$$t = t_1 - \pi_1(t_1, \mathbf{x}), \quad \pi_1(t_1, \mathbf{x}) = \pi(t, \mathbf{x})$$

$$\pi_1(t, \mathbf{x}) = \pi(t, \mathbf{x}) - \pi\dot{\pi} + \mathcal{O}(\pi^3) \quad \pi_1^{(1)} = \pi^{(1)}$$

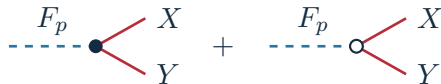
Fixed t_1 : comoving dilation

$$\boxed{\tilde{\pi}_1(t_1, \tilde{\mathbf{x}}) \simeq \pi_1(t_1, \mathbf{x}) - \frac{\lambda}{H(t_1)}} \implies \tilde{\zeta} \simeq \zeta + \lambda$$

Uniform-inflaton clock $\zeta \simeq -H(t_1)\pi_1$: long-wavelength geometry Leading ϵ

This work, Sec. 3.2, Eqs. (3.12)–(3.17), App. A. Curvature relation: long-wavelength limit.

Merging time and spatial vertices



Earlier vertex $t_2 < t_1$ External $F_p \simeq P_p^{(0)}$ Two hard kernels X, Y

$$C = a^3 A, \quad \alpha = \dot{A}/A, \quad g_T = -C\alpha, \quad g_S = C\alpha/a^2, \quad \omega_k^2 = k^2/a^2$$

Free equation + product rule

$$\partial_t(C\dot{X}) + C\omega_k^2 X = 0 \quad (X, Y), \quad \partial_t[C\partial_t(XY)] = 2C(\dot{X}\dot{Y} - \omega_k^2 XY)$$

$$\boxed{\underbrace{2g_T \dot{X}\dot{Y}}_T + \underbrace{2g_S k^2 XY}_S = -\alpha \partial_t[C\partial_t(XY)]}$$

A total derivative of the hard-kernel product; $\alpha(t)$ stays outside

This work, Sec. 5.2, vertex identity; App. C, product rule. Earlier vertex in the leading statistical soft projection.

Integrating the earlier fish vertex

Integrate t_2 by parts at fixed t_1

$$t_i < t_2 < t_1 < t, \quad h_k = C_2 \partial_{t_2} (F_k R_k), \quad Q_k = -\alpha_2 \partial_{t_2} h_k$$

$$\frac{\delta P_{\text{fish}}^{\text{IR}}}{2P_p^{(0)}} = \int_{t_i}^t dt_1 \int \frac{d^3 k}{(2\pi)^3} \alpha_1 \left\{ \underbrace{[\alpha_2 \partial_{t_1} (C_1 \partial_{t_1} (R_0 h_k))]_{t_2=t_i}^{t_2=t_1^-}}_{\text{Time endpoints}} - \underbrace{\int_{t_i}^{t_1} dt_2 \dot{\alpha}_2 \partial_{t_1} (C_1 \partial_{t_1} (R_0 h_k))}_{\text{Remaining history}} \right\}.$$

$R_0 = R_0(t, t_1)$ Initial endpoint removed by adiabatic switching

Keep derivatives on R_0 ; apply ∂_{t_1} before $t_2 \rightarrow t_1^-$

Quartic loops and the fish endpoint



$4K + 4S$

$$\lambda_K = \frac{C}{2}(\alpha^2 - \dot{\alpha}), \quad \lambda_S = \frac{C}{2a^2}(\alpha^2 + \dot{\alpha})$$

Kinetic quartic: Legendre contact included

Complete leading-soft quartic result

$$\frac{\delta P_4^{\text{IR}}}{2P_p^{(0)}} = \int_{t_i}^t dt_1 \int \frac{d^3 k}{(2\pi)^3} \left[C_1 R_0 \alpha_1^2 (L_{11} + \omega_k^2 L_{00}) \right. \\ \left. - C_1 R_0 \dot{\alpha}_1 (L_{11} - \omega_k^2 L_{00}) + (\alpha_1^2 - \dot{\alpha}_1) L_{01} \right].$$

$$L_{00} = |u_k|^2, \quad L_{01} = \text{Re}(u_k \dot{u}_k^*), \quad L_{11} = |\dot{u}_k|^2$$

$$\frac{\delta P_{\text{fish, upper}}^{\text{IR}}}{2P_p^{(0)}} = - \int_{t_i}^t dt_1 \int \frac{d^3 k}{(2\pi)^3} C_1 R_0 \alpha_1^2 (L_{11} + \omega_k^2 L_{00})$$

First quartic term + fish endpoint = 0

Remaining quartic terms retained

The reduced fish + quartic result

Insert the general-mode identity

$$Y_k = L_{01} + \frac{H}{2}(\partial_{\ln k} + 3)L_{00} + \frac{\alpha}{2}L_{00}$$

One remaining time integral

$$\frac{\delta P_{\text{fish}+4}^{\text{IR}}}{2P_p^{(0)}} = \int_{t_i}^t dt_1 \int \frac{d^3 k}{(2\pi)^3} \left[-\frac{\alpha_1}{2} \mathcal{B}_k + C_1 R_0(t, t_1) \dot{\alpha}_1 H_1 \partial_{\ln k} L_{01} \right. \\ \left. - \dot{\alpha}_1 L_{01} + \alpha_1^2 L_{01} \right],$$

$$\mathcal{B}_k = L_{11} - \omega_k^2 L_{00} + H_1 \partial_{\ln k} L_{01}$$

Local residual: $+\alpha^2 J$

$$J = \int \frac{d^3 k}{(2\pi)^3} L_{01}$$

Next: tadpole + b_1 insertion Remaining terms require the linked b_2 counterterm

This work, Sec. 5.2, Eqs. (5.20)–(5.23); App. E. Fish + quartic sum after the general-mode identity.

Tadpole contacts and linked counterterms



1PR tadpoles



$b_1 \times T$



$b_1 \times S$

$$\langle \pi \rangle_R = 0,$$

$$\boxed{b_2 = \frac{1}{2} \dot{b}_1}$$

$$J = \int \frac{d^3 k}{(2\pi)^3} L_{01}, \quad \pi_{\text{tad}} + \pi_{\text{ct}} = 0$$

Differentiated bridge: fixed integration domain

$$\boxed{(\dot{\pi})_{\text{tad}} = \partial_t \pi_{\text{tad}} + \alpha J}$$

$$\frac{\delta P_{\text{tad}} + \delta P_{b_1}}{2P_p^{(0)}} = - \int dt_1 \alpha_1^2 J_1$$

$$\underbrace{-\alpha^2 J}_{\text{Bridge contact}} + \underbrace{\alpha^2 J}_{\text{Mixed-quartic residual}} = 0$$

The remaining momentum boundary

$$\delta P_{\text{loop+ct}}^{\text{IR}} = 2P_p^{(0)} \int_{t_i}^t dt_1 R_0(t, t_1) \mathcal{W}(t_1)$$

$$\mathcal{W} = \partial_t (C \alpha H K_{\partial k}) + C H \dot{\alpha} K_{\partial k} + \mathcal{O}(\dot{H})$$

$$K_{\partial k} = \frac{1}{2\pi^2} \int dk \partial_k [k^3 \text{Re}(u_k \dot{u}_k^*)]$$

Non-boundary cancellation

General internal modes

Full graph sum + linked CT

UV boundary: scheme dependent

$$k^3 L_{01} \sim -\frac{2H + \alpha}{8a^2 A} k^2$$

Regulator + finite conditions

Coincidence and composite operators

Separated points $1 \neq 2$

$$-\epsilon_0 \frac{\dot{\hat{\phi}}_i}{H} [\langle \delta \hat{\phi}_{i0} \delta \hat{\phi}_1 \delta \hat{\phi}_2 \rangle + \text{c.c.}] = \langle \delta \delta \hat{\phi}_1 \delta \hat{\phi}_2 \rangle + 1 \leftrightarrow 2$$

$$\delta \delta \hat{\phi} = (\mathbf{x} \cdot \nabla - H^{-1} \partial_t) \delta \hat{\phi} - \dot{\hat{\phi}}/H$$

$\delta \phi_{i0}$: initial soft mode ϵ_0 : initial Gaussian kernel

Renormalized composite operator

$$[\delta \phi^2]_R^{(\mu_1, \mu_2)} = \lim_{y \rightarrow x} [\delta \phi(x) \delta \phi(y) - G_{\text{div}}^{(\mu_1)} - c_{\text{div}}^{(\mu_2)} \delta \phi(x)]$$

μ_1 : variance subtraction μ_2 : soft-response subtraction

$r \partial_r \ln r = 1$: finite local contact

Coincidence limit $\nrightarrow$ fixed subtraction scheme

Ward constraints on renormalization

Ward identity on the renormalized composite

$$-\epsilon_0 \frac{\dot{\phi}_i}{H} [\langle \delta \hat{\phi}_{i0} [\delta \hat{\phi}^2]_R \rangle + \text{c.c.}] = \langle [\delta \delta \hat{\phi} \delta \hat{\phi} + \delta \hat{\phi} \delta \delta \hat{\phi}]_R \rangle$$

Renormalized extension of App. C, Eq. (50) Local mixing and contact terms included

Hidden quadratic divergence

$$\mathcal{B}_\phi^{\text{UV}} \simeq \frac{H \Lambda^2}{4\pi^2 a^2}$$

$\Lambda = k_{\text{UV}}$, comoving

Boundary value from Ward consistency

$$\mathcal{B}_{\phi,R} = \text{Im } D_{2,R} - \partial_t D_{1,R}$$

$D_{1,R} = \langle [\delta \phi^2]_R \rangle$ $\text{Im } D_{2,R}$: soft response

$$\dot{A}_1 = 2\dot{\phi} A_2$$

Linked counterterms

Local mixing

Finite conditions

Conclusions

Nonlinear clock + slicing

$$\delta\phi \longleftrightarrow \pi \longleftrightarrow \zeta$$

General response identity

$$\text{Fish} + \text{quartic} + \text{tadpoles} + \text{CT} \longrightarrow K_{\partial k}$$

Renormalized composites

Ward consistency + local mixing + finite conditions

Non-boundary cancellation + Scheme-dependent boundary

Initial data in the mode identity

$$X_k = \dot{u}_k + H(\partial_{\ln k} + \frac{3}{2})u_k, \quad \mathcal{E}_k X_k = -\dot{\alpha} \dot{u}_k$$

Initial scaling BD state

$$X_k(t_i) = \dot{X}_k(t_i) = 0$$

General finite initial data

$$Y_k = L_{01} + \frac{H}{2}(\partial_{\ln k} + 3)L_{00} + \frac{\alpha - \alpha_i}{2}L_{00} - \text{Re}[u_k X_{k,\text{hom}}^*]$$

$$\text{Adiabatic external mode} \quad \not\Rightarrow \quad X_{k,\text{hom}} = 0$$

Moving cutoff surfaces

$$I(t) = \frac{1}{2\pi^2} \int_{k_{\text{IR}}(t)}^{k_{\text{UV}}(t)} dk k^2 f(k, t)$$

$$\begin{aligned} \dot{I} &= \frac{1}{2\pi^2} \int_{k_{\text{IR}}}^{k_{\text{UV}}} dk k^2 \partial_t f \\ &\quad + \frac{1}{2\pi^2} \left[\dot{k}_{\text{UV}} k_{\text{UV}}^2 f(k_{\text{UV}}, t) - \dot{k}_{\text{IR}} k_{\text{IR}}^2 f(k_{\text{IR}}, t) \right] \end{aligned}$$

$$k_{\text{UV}} = a(t) \Lambda_{\text{phys}} \quad \Longrightarrow \quad \dot{k}_{\text{UV}} = H k_{\text{UV}}$$

Fixed physical cutoff

Moving comoving surface

Extra boundary term

Scope of the EFT comparisons

Work	Propagation / hierarchy	Graph completion	Comparison
Firouzjahi	Sharp USR sectors	H_3, H_4 retained	General identity + CT closure
Braglia–Pinol	$\epsilon \ll \eta \ll 1$, dS modes	Contact + tadpole + CT	Freezing, finite remainder
Braglia– Céspedes–Pinol	$ \eta_2 \gg \eta $	Leading quartic	Fish: higher order
This work	General $A(t), u_k$	All same-order graphs	Explicit $K_{\partial k}$ boundary

Nonlinear maps and one-point conditions

$$\delta\phi = \Pi\pi + \frac{\dot{\Pi}}{2}\pi^2 + \dots$$

$$\langle\delta\phi\rangle_{1L} = \Pi\langle\pi\rangle_{1L} + \frac{\dot{\Pi}}{2}\langle\pi^2\rangle_0$$

$$\boxed{\langle\pi\rangle_R = 0 \quad \not\Rightarrow \quad \langle\delta\phi\rangle_R = 0}$$

Illustrative derivative subtraction:

$$[\partial_t\delta\phi^2]_R - \partial_t[\delta\phi^2]_R = \dot{\mathcal{A}}_{\text{comp}} \ln(\mu_1/\mu_2)$$

$\mathcal{A}_{\text{comp}} = z''/(8\pi^2 a^2 z)$, $z = a\Pi/H$; primes: conformal time.

Nonlinear operator conditions

Composite subtraction

Finite matching

This work, Discussion, Eq. (6.1); latest composite-operator analysis (work in progress).