Quantum Geometry Effects in Quantum Field Theory: Hamiltonian constraint Generates Gravity-Matter Entanglement

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Outline

1. Motivation
2. Hamiltonian Formulation of GR coupling to scalar field
3. Quantum Geometry + Matter Entanglement
4. Physical Implications
5. Conclusion & Outlook

Purification of Hawking radiation

- The **Page curve** of Hawking radiation describes the expected entanglement entropy under **unitary** evaporation.
- Key insight: Purification requires entanglement between Hawking radiation and the **quantum degrees of freedom (of the geometry)** of the interior of black hole.

Central Challenge: How to introduce the entanglement physically?

- Gauss constraint $\rightarrow$ coherent intertwiner: Entanglement between the shapes of polytopes (G. Long, Q. Chen and J. Yang, PRD, 2024; J. Lewandowski and C. Zhang,PRD, 2022),

$$\mathrm{Ar}(\mathcal{B}) = \langle \Psi_\Gamma | \widehat{\mathrm{Ar}}(\mathcal{B}) | \Psi_\Gamma \rangle \approx 8\pi\beta\ell_{\mathrm{pl}}^2 \sum_{c=1}^{P} j_0^c, \tag{1}$$

$$E(\Gamma_a|\Gamma_b) \approx \frac{\mathrm{Ar}(\mathcal{B})}{4\ell_{\mathrm{pl}}^2} \frac{\left(\ln(2j_0^c+1) - \sum_{j_c} p_{j_c} \ln p_{j_c}\right)}{2\pi\beta j_0^c}. \tag{2}$$

Quantum field theory on curved spacetime

- Historically, quantum field theory on curved spacetime (QFTCS) sidesteps this issue by treating geometry as a fixed classical background $\mathfrak{g}$. The backreaction of the energy-momentum tensor $\hat{T}_{\mu\nu}$ of the quantum matter field on the classical geometry is ignored or effectively treated via the effective Einstein equation (A. Perez and D. Sudarsky, PRL,2026)

$$G_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle.$$

- However, from the perspective of quantum gravity, distinct matter states generically correspond to distinct geometric configurations, even if their classical limits coincide. In particular, these microscopic geometric distinctions may encode the microstates responsible for black hole entropy. Consequently, when considering the entanglement between geometry and matter, the QFTCS prescription—building all $|\phi\rangle$ on a single classical $\mathfrak{g}$—becomes ill-defined, as it conflates physically distinct quantum geometries.

Quantum field theory on quantum spacetime

- Quantum Hamiltonian constraint:

$$H = H^{\mathrm{g}} + H^{\phi} \approx 0, \tag{3}$$

$$H^{\mathrm{g}} = -2\frac{E^{\varphi}}{\partial_x E^x}\partial_x M + 2\frac{\sqrt{E^x}K_{\varphi}}{\partial_x E^x}H^{\mathrm{g}}_x, \tag{4}$$

$$H^{\phi} = \frac{1}{2}\left(\frac{\pi_{\phi}^2}{\sqrt{E^x}E^{\varphi}} + \frac{(E^x)^{3/2}}{E^{\varphi}}(\partial_x\phi)^2\right). \tag{5}$$

$\rightarrow$ solution: Entanglement between geometry and matter,

$$|\Psi\rangle = \sum_{\mathfrak{g},\phi} c_{\mathfrak{g},\phi}|\mathfrak{g}\rangle \otimes |\phi\rangle.$$

Spherically Symmetric Model: Phase Space Variables

Gravitational Sector $\mathcal{P}^{\mathrm{g}}$:

- Fields: $(K_x(x), E^x(x), K_\varphi(x), E^\varphi(x))$ defined on 1D manifold $\mathbb{X}$.
- Non-zero Poisson brackets:

$$\{K_x(x), E^x(x')\} = 2G\delta(x - x'), \quad \{K_\varphi(x), E^\varphi(x')\} = G\delta(x - x').$$

- The line element of spatial metric q_{ab} on can be expressed in terms of the canonical variables $(K_x, E^x, K_\varphi, E^\varphi)$ as

$$ds^2 = \frac{(E^\varphi)^2}{|E^x|}dx^2 + |E^x|(d\theta^2 + \sin^2\theta d\varphi^2). \tag{6}$$

Scalar Field Sector $\mathcal{P}^{\phi}$:

- Fields: $(\phi(x), \pi_\phi(x))$ on $\mathbb{X}$.
- Poisson bracket:

$$\{\phi(x), \pi_\phi(x')\} = \delta(x - x').$$

Total Phase Space: $\mathcal{P}^{\mathrm{g}} \times \mathcal{P}^{\phi}$.

First-Class Constraints System

This is a **totally constrained** Hamiltonian system.

1. **Diffeomorphism Constraint:**

$$H_x = H_x^{\mathrm{g}} + H_x^{\phi} \approx 0$$

2. **Hamiltonian Constraint:**

$$H = H^{\mathrm{g}} + H^{\phi} \approx 0 \tag{7}$$

Where:

$$H^{\mathrm{g}} = -2\frac{E^{\varphi}}{\partial_x E^x}\partial_x M + 2\frac{\sqrt{E^x}K_{\varphi}}{\partial_x E^x}H_x^{\mathrm{g}}, \quad H_x^{\mathrm{g}} = \frac{1}{2G}(-K_x\partial_x E^x + 2E^{\varphi}\partial_x K_{\varphi}),$$

$$H^{\phi} = \frac{1}{2}\left(\frac{\pi_{\phi}^2}{\sqrt{E^x}E^{\varphi}} + \frac{(E^x)^{3/2}}{E^{\varphi}}(\partial_x\phi)^2\right), \quad H_x^{\phi} = \pi_{\phi}\partial_x\phi$$

with $M = \frac{\sqrt{E^x}}{2G}\left(1 + K_{\varphi}^2 - \frac{(\partial_x E^x)^2}{4(E^{\varphi})^2}\right)$ (mass function).

Simplifying the Hamiltonian Constraint

- The constraint can be rewritten as:

$$H = -2\frac{E^{\varphi}}{\partial_x E^x}\tilde{H}$$

- Where:

$$\tilde{H} = \partial_x M - \frac{\sqrt{E^x}}{E^{\varphi}} K_{\varphi} H_x^{\mathrm{g}} - \frac{\partial_x E^x}{2E^{\varphi}} H^{\phi}$$

- **Key simplification:** $H = 0 \Leftrightarrow \tilde{H} = 0$.

Smeared Constraint

Consider finite interval $[x(v), x(v') = x(v) + \delta]$:

$$\tilde{H}^{\delta}(v, v') := \int_{x(v)}^{x(v')} dx\tilde{H} = (M_{v'} - M_v) - \tilde{H}_{\phi}^{\delta}(v, v') = 0 \tag{8}$$

This is the **fundamental constraint equation** we will solve.

Special Geometry: Near Apparent Horizon

Gauge Fixing & Special Conditions

We focus on a special class $\mathcal{S}_{v,v'}$ of geometries (near horizon geometries):

- Gauge: $E^x = x^2$, Diffeomorphism Constraint being solved;
- Condition (1): Apparent horizon (defined by the outging null expansion $(\frac{\partial_x |E^x|}{\sqrt{|E^x|}E^\varphi} - \frac{2K_\varphi}{\sqrt{|E^x|}} = 0)$) at $x = x_\mathrm{h} = x(v)$;
- Condition (2): $K_\varphi = 0$ and $E^\varphi > 0$ in region $x_\mathrm{h} \leq x \leq x_\mathrm{h} + \delta$, and $\int_{x_\mathrm{h}}^{x_\mathrm{h}+\delta} dx \frac{(E^\varphi)^2}{x^2} \to \infty$.

Consequences:

- Geometry in region $[x_\mathrm{h}, x_\mathrm{h} + \delta]$ determined solely by $E^\varphi(x)$;
- The matter term in Eq.(8) reads

$$\tilde{H}^\delta_\phi(v, v') := \int_{x(v)}^{x(v)+\delta} dx \Big(\frac{\sqrt{E^x}}{E^\varphi} K_\varphi H^\mathrm{g}_x + \frac{\partial_x E^x}{2E^\varphi} H^\phi\Big). \tag{9}$$

Modified Tortoise Coordinate

Why Modify the Standard Tortoise Coordinate?

- Standard tortoise: $\frac{\partial x}{\partial x^*_{\mathfrak{g}}} = \frac{x^2}{(E^{\varphi}(x))^2|_{\mathfrak{g}}}$;
- Satisfies $\lim_{x\to x_{\mathfrak{h}}} x^*_{\mathfrak{g}} \to -\infty, \quad \lim_{x\to x_{\mathfrak{h}}+\delta} x^*_{\mathfrak{g}} \to \delta^*_{\mathfrak{g}}, \quad 0 < \delta^*_{\mathfrak{g}} < +\infty$;
- **Problem:** Finite interval $\to$ cannot define standard plane-wave modes.

Modified Tortoise Coordinate

$$\check{x}^*_{\mathfrak{g}} := x^*_{\mathfrak{g}} + L - \delta^*_{\mathfrak{g}} \tag{10}$$

Then take $L \to \infty$ limit:

- Region $[-\infty, \delta^*_{\mathfrak{g}}]$ in $x^*_{\mathfrak{g}}$ maps to $[-\infty, +\infty]$ in $\check{x}^*_{\mathfrak{g}}$.
- Now we have an **infinite interval** $\to$ can define standard Fourier modes!

Construction of Fock States in Modified Coordinate

- **Step 1: Define Creation/Annihilation Operators**

$$a_k^{\mathfrak{g}} := \lim_{L\to\infty} \int_{-L}^{+L} d\check{x}_{\mathfrak{g}}^* e^{-ik\check{x}_{\mathfrak{g}}^*} \frac{1}{\sqrt{2\omega_k}} \left(x\omega_k \hat{\phi}(\check{x}_{\mathfrak{g}}^*) + i\frac{\hat{\pi}_\phi(\check{x}_{\mathfrak{g}}^*)}{x} \right)$$

Canonical commutation:

$$[\hat{\phi}(\check{x}_{\mathfrak{g}}^*), \hat{\pi}_\phi(\check{x}_{\mathfrak{g}}'^*)] = i\hbar\delta(\check{x}_{\mathfrak{g}}^* - \check{x}_{\mathfrak{g}}'^*) \Rightarrow [a_k^{\mathfrak{g}}, a_{k'}^{\mathfrak{g},\dagger}] \overset{L\to\infty}{=} 2\pi\hbar\delta(k-k').$$

- **Step 2: Define Field Operators in $\check{x}_{\mathfrak{g}}^*$**

$$\hat{\phi}(\check{x}_{\mathfrak{g}}^*) \overset{L\to\infty}{=} \int \frac{dk}{2\pi} \frac{1}{\sqrt{2\omega_k}} \left(\frac{e^{ik\check{x}_{\mathfrak{g}}^*}}{x} a_k^{\mathfrak{g}} + \frac{e^{-ik\check{x}_{\mathfrak{g}}^*}}{x} a_k^{\mathfrak{g},\dagger} \right)$$

$$\hat{\pi}_\phi(\check{x}_{\mathfrak{g}}^*) \overset{L\to\infty}{=} (-i) \int \frac{dk}{2\pi} \sqrt{\frac{\omega_k}{2}} x \left(e^{ik\check{x}_{\mathfrak{g}}^*} a_k^{\mathfrak{g}} - e^{-ik\check{x}_{\mathfrak{g}}^*} a_k^{\mathfrak{g},\dagger} \right)$$

where $\omega_k = |k|$, and $x = x(\check{x}_{\mathfrak{g}}^*)$.

Construction of Fock Space in Modified Coordinate

- The (Boulware-like) vacuum state $|0_{\mathrm{B}}\rangle_{\mathfrak{g}}$ in the region $-\infty < x_* < \delta^*_{\mathfrak{g}}$,

$$a^{\mathfrak{g}}_k |0_{\mathrm{B}}\rangle_{\mathfrak{g}} = 0, \quad \forall k. \tag{11}$$

- The Hilbert space $\mathcal{H}^{\phi}_{\mathfrak{g}}$ of the field ϕ spanned by acting on the vacuum with all possible combinations of $a^{\mathfrak{g},\dagger}_k$,

$$\mathcal{H}^{\phi}_{\mathfrak{g}} := \{|0_{\mathrm{B}}\rangle_{\mathfrak{g}}, a^{\mathfrak{g},\dagger}_k |0_{\mathrm{B}}\rangle_{\mathfrak{g}}, a^{\mathfrak{g},\dagger}_k a^{\mathfrak{g},\dagger}_{k'} |0_{\mathrm{B}}\rangle_{\mathfrak{g}}, a^{\mathfrak{g},\dagger}_k a^{\mathfrak{g},\dagger}_{k'} a^{\mathfrak{g},\dagger}_{k''} |0_{\mathrm{B}}\rangle_{\mathfrak{g}}, ...\}. \tag{12}$$

Solving the Constraint: Explicit State Construction

The Constraint on Fixed $\mathfrak{g}$:

$$M_{v'}(\mathfrak{g}) - M_v(\mathfrak{g}) = \frac{\langle\phi'|\widehat{\tilde{H}^{\delta}_{\phi}}(\mathfrak{g}; v, v')|\phi\rangle}{\langle\phi'|\phi\rangle}, \quad \forall\langle\phi'|; \tag{13}$$

Crucial Calculation: In the modified coordinate, the matter Hamiltonian simplifies:

$$\int_{x_h}^{x_h+\delta} dx \left(\frac{\partial_x E^x}{2E^{\varphi}}\right) \hat{H}^{\phi}(\mathfrak{g}) = \int \frac{dk}{2\pi} \omega_k a_k^{\mathfrak{g},\dagger} a_k^{\mathfrak{g}} \quad \text{(normal ordered)}$$

The Constraint on Fixed $\mathfrak{g}$:

The constraint becomes:

$$M_{v'}(\mathfrak{g}) - M_v(\mathfrak{g}) = \frac{\int \frac{dk}{2\pi} \omega_k \langle\phi'|a_k^{\mathfrak{g},\dagger} a_k^{\mathfrak{g}}|\phi\rangle}{\langle\phi'|\phi\rangle}, \quad \forall\langle\phi'|;$$

Explicit Form of Solutions $|\vec{k}_{\vec{n}}(\mathfrak{g})\rangle$

Solutions of the constraint:

$$|\vec{k}_{\vec{n}}(\mathfrak{g})\rangle := \left(\prod_k \frac{(a_k^{\mathfrak{g},\dagger})^{n_k}}{(\mathcal{N})^{n_k}\sqrt{n_k!}}\right)|0_{\mathrm{B}}\rangle_{\mathfrak{g}} \tag{14}$$

- $\vec{n} = (n_{k_1}, n_{k_2}, \dots)$: occupation numbers
- $\mathcal{N}$: normalization constant

Constraint Condition:

These states satisfy the constraint **iff**:

$$M_{v'}(\mathfrak{g}) - M_v(\mathfrak{g}) = \sum_k n_k \omega_k =: \omega_{\vec{k}_{\vec{n}}(\mathfrak{g})} \tag{15}$$

$\Rightarrow$ The **mass difference** equals the **total field energy** in the region.

The Fundamental Inconsistency Revealed

Consider two physical states $|\vec{k}_{\vec{n}}(\mathfrak{g})\rangle$ and $|\vec{k}'_{\vec{n}}(\mathfrak{g}')\rangle$:

$$|\vec{k}_{\vec{n}}(\mathfrak{g})\rangle + |\vec{k}'_{\vec{n}}(\mathfrak{g}')\rangle \quad \text{is ill-Defined!!!} \tag{16}$$

Why this superposition is ill-Defined?

1. **Underlying superposition of classical geometry:**
   - Each state is tied to its background $\mathfrak{g}$ via the Hamiltonian constraint;
   - Superposing states $\Rightarrow$ superposing classical geometries.
2. **Distinct, possibly inequivalent Fock spaces:**
   - $|\vec{k}_{\vec{n}}(\mathfrak{g})\rangle \in \mathcal{H}^{\phi}_{\mathfrak{g}}$, $|\vec{k}'_{\vec{n}}(\mathfrak{g}')\rangle \in \mathcal{H}^{\phi}_{\mathfrak{g}'}$
   - For generic $\mathfrak{g} \neq \mathfrak{g}'$: $\mathcal{H}^{\phi}_{\mathfrak{g}} \ncong \mathcal{H}^{\phi}_{\mathfrak{g}'}$ (unitarily inequivalent)
   - $\Rightarrow$ No mathematical meaning to $|\vec{k}_{\vec{n}}(\mathfrak{g})\rangle + |\vec{k}'_{\vec{n}}(\mathfrak{g}')\rangle$

This reveals the need for a quantum theory of geometry!

Step 1: Quantum Geometry via LQG Coherent States

- Geometry sector: LQG quantization of canonical pair (K_φ, E^φ)
  $\rightarrow$ Hilbert space $\mathcal{H}_{\mathrm{g}}^{\gamma}$ spanned by

$$T_{\gamma,\vec{\mu}}(K_\varphi) = \prod_{v\in\gamma} \exp\left(\mathbf{i}\mu_v K_\varphi(v)\right). \tag{17}$$

- Continuum limit:

$$\mathcal{H}_{\mathrm{g}} \underset{\mu_\gamma\to 0}{:=} \mathcal{H}_{\mathrm{g}}^{\gamma}. \tag{18}$$

- Use semiclassical **coherent states** $|\mathfrak{g}\rangle$:
  - Peaked on classical phase space point $\mathfrak{g}$.
  - Overlap: $\langle\mathfrak{g}|\mathfrak{g}'\rangle \sim \exp\left(-|\mathfrak{g}-\mathfrak{g}'|^2/t\right)$.
  - Satisfy Ehrenfest property: $\langle\mathfrak{g}|\hat{O}|\mathfrak{g}'\rangle \approx O(\mathfrak{g}')\langle\mathfrak{g}|\mathfrak{g}'\rangle$.

Step 2: A Common Fock Space for Matter

- Problem: For two coherent states $|\mathfrak{g}\rangle$ and $|\mathfrak{g}'\rangle$, the Fock spaces $\mathcal{H}^{\phi}_{\mathfrak{g}}$ and $\mathcal{H}^{\phi}_{\mathfrak{g}'}$ are generally **inequivalent**.
- Notice that $\mathcal{H}^{\phi}_{\mathfrak{g}}$ and $\mathcal{H}^{\phi}_{\mathfrak{g}'}$ are generated by $(a^{\mathfrak{g}}_k, a^{\mathfrak{g},\dagger}_k)$ and $(a^{\mathfrak{g}'}_k, a^{\mathfrak{g}',\dagger}_k)$ respectively, and thus one has the Bogoliubov transformation mapping the annihilation operators $a^{\mathfrak{g}'}_k$ to

$$a^{\mathfrak{g}}_k = \int \frac{dj}{2\pi}(\alpha_{kj}(\mathfrak{g}',\mathfrak{g})a^{\mathfrak{g}'}_j + \beta_{kj}(\mathfrak{g}',\mathfrak{g})a^{\mathfrak{g}',\dagger}_j), \tag{19}$$

where

$$\begin{aligned}\beta_{kk'}(\mathfrak{g}',\mathfrak{g}) \;=\; & \frac{1}{2}\sqrt{\frac{\omega_k}{\omega_{k'}}}\Big(\lim_{L\to\infty}\int_{-L}^{+L} d\check{x}^*_{\mathfrak{g}} e^{-\mathbf{i}k\check{x}^*_{\mathfrak{g}}} e^{-\mathbf{i}k'\check{x}^*_{\mathfrak{g}'}}\Big) \\ & -\frac{1}{2}\sqrt{\frac{\omega_{k'}}{\omega_k}}\Big(\lim_{L\to\infty}\int_{-L}^{+L} d\check{x}^*_{\mathfrak{g}'} e^{-\mathbf{i}k\check{x}^*_{\mathfrak{g}}} e^{-\mathbf{i}k'\check{x}^*_{\mathfrak{g}'}}\Big).\end{aligned} \tag{20}$$

Step 2: A Common Fock Space for Matter

- Considering a subset of geometries $\mathcal{U}_{v,v'} \subset \mathcal{S}_{v,v'}$. For any $\mathfrak{g}, \mathfrak{g}' \in \mathcal{U}_{v,v'}$, the Bogoliubov transformation between their Fock representations must satisfy the **Shale–Stinespring criterion**:

$$\int dk\, dj\, |\beta_{kj}(\mathfrak{g}', \mathfrak{g})|^2 < \infty$$

- $\Rightarrow$ There exists a **unitary equivalence** $\mathcal{H}^{\phi}_{\mathfrak{g}} \cong \mathcal{H}^{\phi}_{\mathfrak{g}'}$.
- We can then define a **common Fock space** $\mathcal{H}^{\phi} \equiv \mathcal{H}^{\phi}_{\mathfrak{g}} \cong \mathcal{H}^{\phi}_{\mathfrak{g}'}, \quad \forall\, \mathfrak{g}, \mathfrak{g}' \in \mathcal{U}_{v,v'}$.

Tensor product of state spaces:

Well-defined tensor product space: $\widetilde{\mathcal{H}}_{\mathrm{g}} \otimes \mathcal{H}^{\phi}$, where

$$\widetilde{\mathcal{H}}_{\mathrm{g}} = \{|\mathfrak{g}\rangle \in \mathcal{H}_{\mathrm{g}} \mid \mathfrak{g} \in \mathcal{U}_{v,v'}\}.$$

Step 3: Imposing the Quantum Constraint Weakly

- The full quantum Hamiltonian constraint:

$$\langle \hat{M}_{v'} \rangle - \langle \hat{M}_v \rangle = \langle \widehat{\tilde{H}^{\delta}_{\phi}}(v, v') \rangle \tag{21}$$

- For product states $|\mathfrak{g}, \phi\rangle = |\mathfrak{g}\rangle \otimes |\phi\rangle$, using coherent state properties:

$$M_{v'}(\mathfrak{g}) - M_v(\mathfrak{g}) \approx \frac{\langle \phi' | \widehat{\tilde{H}^{\delta}_{\phi}}(\mathfrak{g}; v, v') | \phi \rangle}{\langle \phi' | \phi \rangle} \tag{22}$$

Weak Solutions:

The constraint is solved (to leading order) by states of the form:

$$|\mathfrak{g}, \phi\rangle = |\mathfrak{g}(\vec{k}_{\vec{n}})\rangle \otimes |\vec{k}_{\vec{n}}\rangle \tag{23}$$

with $M_{v'}(\mathfrak{g}(\vec{k}_{\vec{n}})) - M_v(\mathfrak{g}(\vec{k}_{\vec{n}})) = \omega_{\vec{k}_{\vec{n}}}$.

The Space of Physical States

- Weak solutions span a subspace:

$$(\widetilde{\mathcal{H}}_{\mathrm{g}} \otimes \mathcal{H}^{\phi})^{\mathrm{sol.}}_{v,v'} = \{|\mathfrak{g}(\vec{k}_{\vec{n}})\rangle \otimes |\vec{k}_{\vec{n}}\rangle \mid M_{v'}(\mathfrak{g}(\vec{k}_{\vec{n}})) - M_{v}(\mathfrak{g}(\vec{k}_{\vec{n}})) = \omega_{\vec{k}_{\vec{n}}}\}$$

- General physical states are **linear combinations** of these product states:

$$c|\mathfrak{g}(\vec{k}_{\vec{n}})\rangle \otimes |\vec{k}_{\vec{n}}\rangle + c'|\mathfrak{g}(\vec{k}'_{\vec{n}})\rangle \otimes |\vec{k}'_{\vec{n}}\rangle + \ldots$$

- These are **inherently entangled** between geometry and matter. The entanglement is not put in by hand but **enforced by the Hamiltonian constraint of GR**.

Generalized Hartle-Hawking Vacuum on Quantum Geometry

- Extend our consideration to horizon: Boulware-like state $\rightarrow$ Hartle-Hawking vacuum.
- Standard (local) Hartle-Hawking vacuum on a fixed background $\mathfrak{g}$:

$$|0_{\mathrm{HH}}\rangle_{\mathfrak{g}} = \sum_{\vec{n}} \left(\prod_{k} (1 - e^{-2\pi \xi_{\mathfrak{g}}^{-1} \omega_k})^{1/2} e^{-\pi n_k \omega_k / \xi_{\mathfrak{g}}} \right) |\vec{k}_{\vec{n}}\rangle_{\mathfrak{g}}.$$

- Our prescription: Replace $|\vec{k}_{\vec{n}}\rangle_{\mathfrak{g}} \rightarrow |\mathfrak{g}(\vec{k}_{\vec{n}}), \vec{k}_{\vec{n}}\rangle$.

Generalized (local) Hartle-Hawking Vacuum

$$|0_{\mathrm{HHQG}}\rangle := \sum_{\vec{n}} \left(\prod_{k} (1 - e^{-2\pi \omega_k / \xi_{\mathfrak{g}(\vec{k}_{\vec{n}})}})^{1/2} e^{-\pi n_k \omega_k / \xi_{\mathfrak{g}(\vec{k}_{\vec{n}})}} \right) |\mathfrak{g}(\vec{k}_{\vec{n}}), \vec{k}_{\vec{n}}\rangle \quad (24)$$

- Quantum geometry and matter field are inherently **entangled**.

Generalized Hartle-Hawking Vacuum on Quantum Geometry

- To determine the parameter $\xi_{\mathfrak{g}(\vec{k}_{\vec{n}})}$ in Eq. (24) , let us require that the state $|0_{\mathrm{HHQG}}\rangle$ should get back to the local Hartle-Hawking vacuum $|0_{\mathrm{HH}}\rangle_{\mathfrak{g}_c}$ in the semiclassical limit of the quantum geometry; In other words, the parameter $\xi_{\mathfrak{g}(\vec{k}_{\vec{n}})}$ should be determined by the semiclassical consistency conditions

$$\lim_{\epsilon(\mathfrak{g}_c,\mathfrak{g}(\vec{k}_{\vec{n}}))\to 0} A_k^{\mathfrak{g}_c}|0_{\mathrm{HHQG}}\rangle = 0, \tag{25}$$

where $\epsilon(\mathfrak{g}_c,\mathfrak{g}(\vec{k}_{\vec{n}})) \equiv \sqrt{1-|\langle \mathfrak{g}_c|\mathfrak{g}(\vec{k}_{\vec{n}})\rangle|^2}$, $A_k^{\mathfrak{g}_c}, A_k^{\mathfrak{g}_c,\dagger}$ are defined on the classical geometry $\mathfrak{g}_c$, with $\mathfrak{g}_c$ given by the expectation values

$$\mathfrak{g}_c = \{\frac{\langle 0_{\mathrm{HHQG}}|\hat{E}^{\varphi}|0_{\mathrm{HHQG}}\rangle}{\langle 0_{\mathrm{HHQG}}|0_{\mathrm{HHQG}}\rangle}, \frac{\langle 0_{\mathrm{HHQG}}|\hat{K}_{\varphi}|0_{\mathrm{HHQG}}\rangle}{\langle 0_{\mathrm{HHQG}}|0_{\mathrm{HHQG}}\rangle}\}. \tag{26}$$

Generalized Hartle-Hawking Vacuum on Quantum Geometry

- Consider another type of generalized Hartle-Hawking vacuum state

$$|0_{\mathrm{HH}}, \mathfrak{g}_c\rangle := |0_{\mathrm{HH}}\rangle_{\mathfrak{g}_c} \otimes |\mathfrak{g}_c\rangle,$$

satisfying some semiclassical consistency conditions

$$A_k^{\mathfrak{g}_c} |0_{\mathrm{HH}}, \mathfrak{g}_c\rangle = 0. \tag{27}$$

- The state $|0_{\mathrm{HHQG}}\rangle$ weakly solves the quantum Hamiltonian constraints, and these constraints naturally encode the entanglements between field excitations and the underlying quantum geometry;
- The state $|0_{\mathrm{HH}}, \mathfrak{g}_c\rangle$ fails to satisfy the quantum Hamiltonian constraints and contains no geometry-matter entanglement.
- This distinction highlights a core advantage of the quantum gravity framework: physical constraints not only select permissible states but also determine their intrinsic entanglement structure.

Summary and Outlook

1. QFT on a fixed background geometry cannot handle the solution of Hamiltonian constraint of GR, and even the whole Einstein equation.
2. The entanglement between the geometry and matter emerges by solving the Hamiltonian constraint in the loop quantum gravity framework.
3. Our construction gives "instantaneous" states. Need a notion of **time evolution** to describe full dynamics (near the horizon at least).
4. Explore directly how the built-in entanglement in the states $|0_{\mathrm{HHQG}}\rangle$ can lead to unitary evolution and information recovery, providing a first-principles foundation for the Page curve.

Thank You!

Questions?