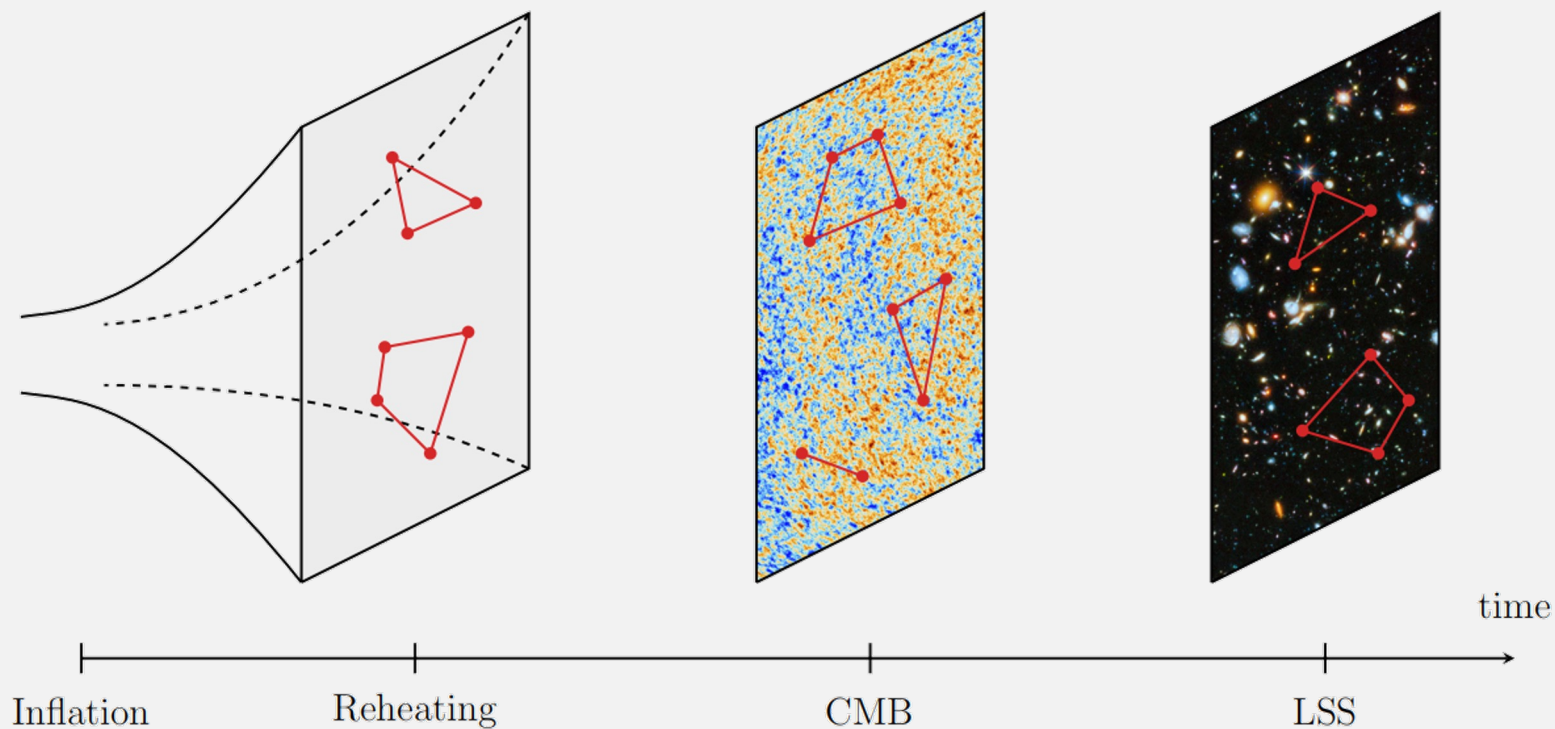


# Tracing Out Massive Fields in Cosmology

Guanhao Sun  
Chongqing University

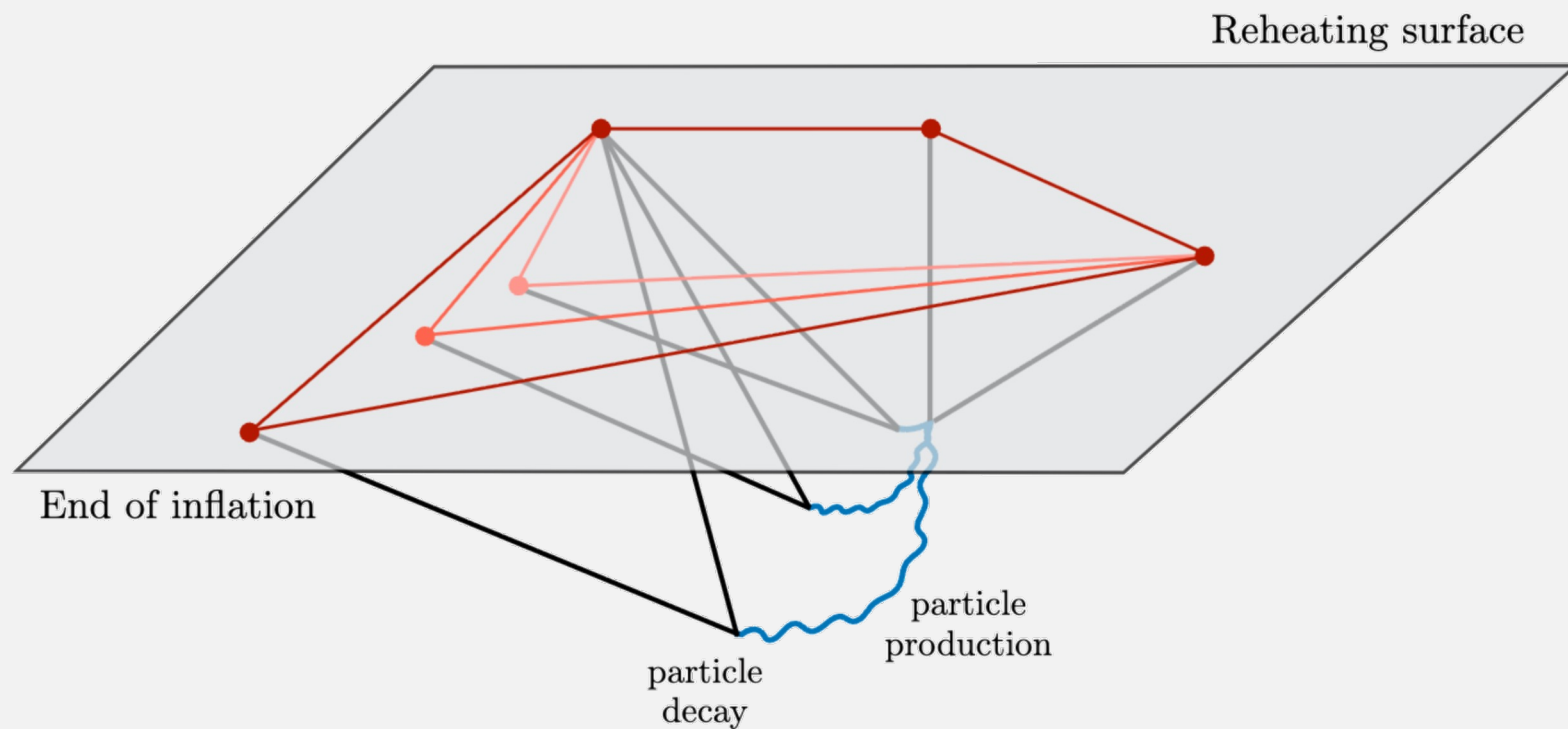
Based mainly on 2609.30370 with Sam S.C. Wong (CityU HK)  
and 2412.02739 with Daniel Green (UCSD)

# Correlation Functions



C. Duaso Pueyo, A Bootstrap Approach to Cosmology. PhD thesis, University of Amsterdam, 2022

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- EFT of inflation – most general action for SSB of time translation:

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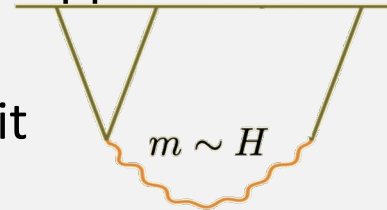
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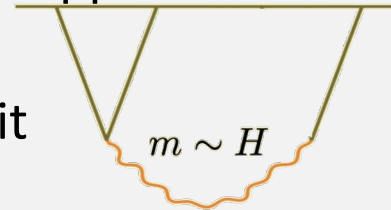
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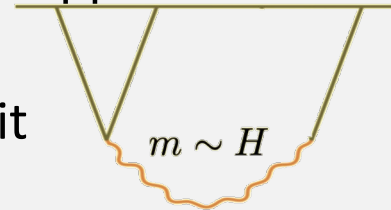
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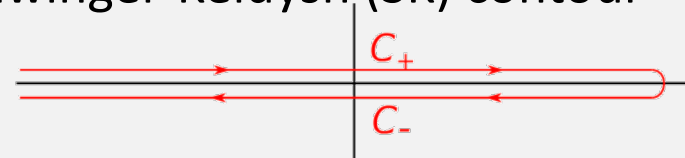
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- Open EFT of inflation – EFT on a Schwinger-Keldysh (SK) contour

- Bottom-up construction
- Includes dissipation and noise



Agui Salcedo et. al. 2404.15416  
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- Basis for understanding UV completion and more

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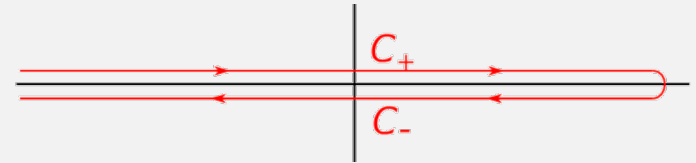
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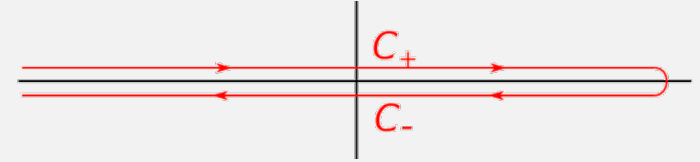
  
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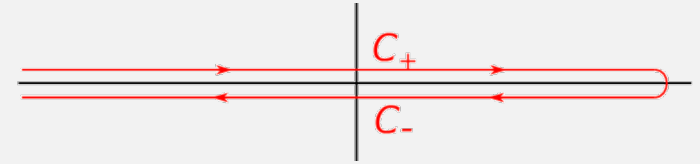
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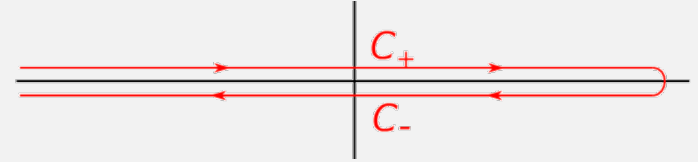


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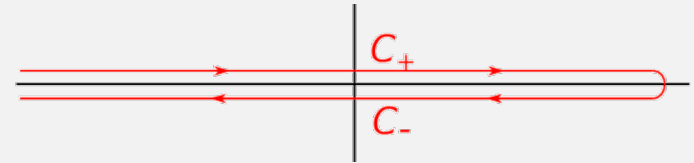
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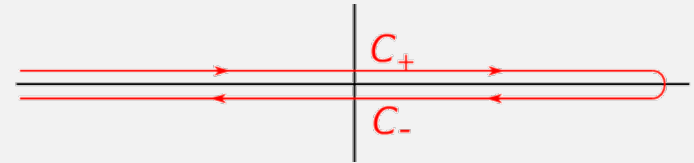
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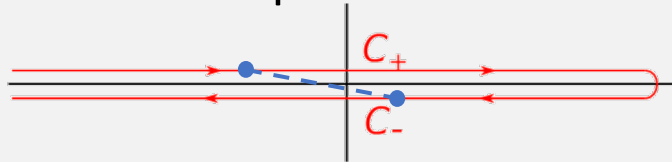
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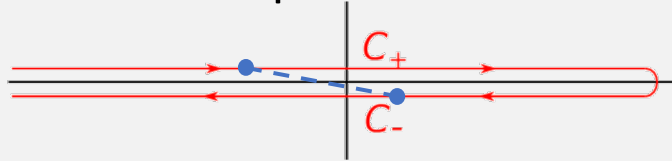


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- Their existence admits a non-perturbative proof through exact RG (see Green and GS 2412.02739 for details)

# General Procedure

- Start with the formal solution

$$\sigma_{k,+}(\tau) = \sigma_{k,-}^*(\tau) = \bar{\sigma}_k K_k(\tau) + \int_{\mathcal{C}_+} d\tau' a^4(\tau') G_k(\tau, \tau') J_{+,k}(\tau')$$

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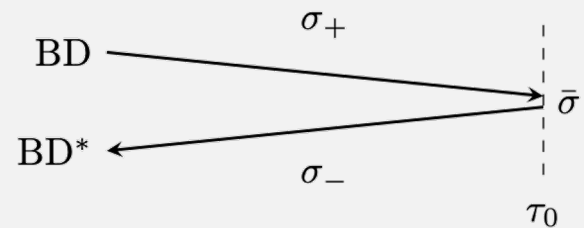
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- Here  $F$  is a generic function with zero constant term
- We write the formal solution to the heavy field

$$\sigma_{\pm, \mathbf{k}}^{(0)}(\tau) = \bar{\sigma}_{\mathbf{k}} K_{\pm, k}(\tau) + g \int_{\mathcal{C}_{\pm}} d\tau' a^2(\tau') G_{D, \pm, k}(\tau, \tau') F_{\pm, \mathbf{k}}(\tau') \quad .$$

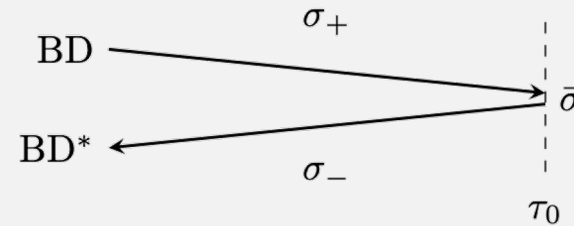
$$K_{+, k}(\tau) = \frac{u_k(\tau)}{u_k(\tau_0)} \quad , \quad G_{D, \pm, k}(\tau_0, \tau') = G_{D, \pm, k}(\tau, \tau_0) = 0$$

# The Boundary Value



- Now plug in  $\sigma$  and integrate over  $\bar{\sigma}$

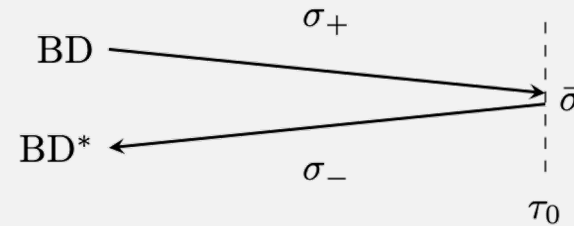
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- Now plug in  $\sigma$  and integrate over  $\bar{\sigma}$
- Obtain the “on-shell” action (zeroth order in  $\lambda$ )

$$S_{\sigma,\pm}^{(0),\text{os}} = \frac{1}{2} \int_{\mathbf{k}} \Omega_{\pm,k} \bar{\sigma}_{\mathbf{k}} \bar{\sigma}_{-\mathbf{k}} + g \int_{\mathbf{k}} \bar{\sigma}_{-\mathbf{k}} I_{\pm,\mathbf{k}} \\ + \frac{g^2}{2} \int_{\mathbf{k}} \int_{\mathcal{C}_{\pm}} d\tau \int_{\mathcal{C}_{\pm}} d\tau' a^2(\tau) a^2(\tau') F_{\pm,\mathbf{k}}(\tau) G_{D,\pm,k}(\tau, \tau') F_{\pm,-\mathbf{k}}(\tau') .$$

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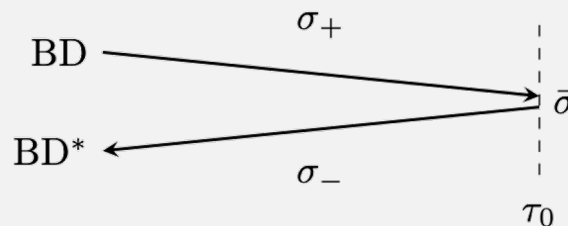
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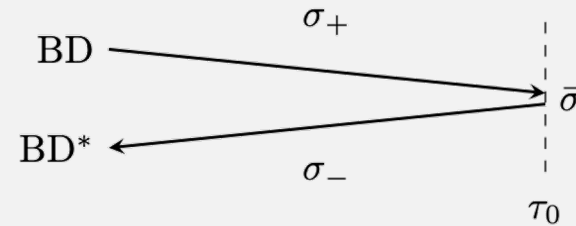
$$I_{\pm,\mathbf{k}}[F_{\pm}] \equiv \int_{\mathcal{C}_{\pm}} d\tau a^2(\tau) K_{\pm,k}(\tau) F_{\pm,\mathbf{k}}(\tau) \quad \Omega_{+,k} = a_0^2 K'_{+,k}(\tau_0)$$

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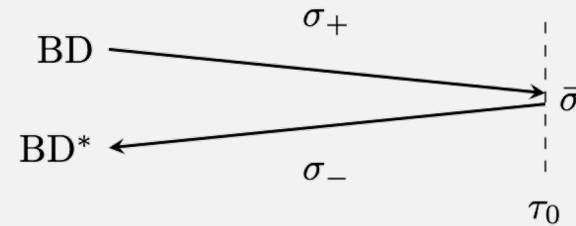
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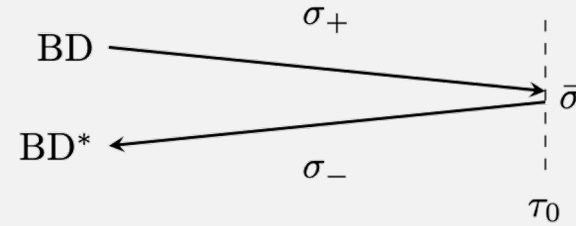
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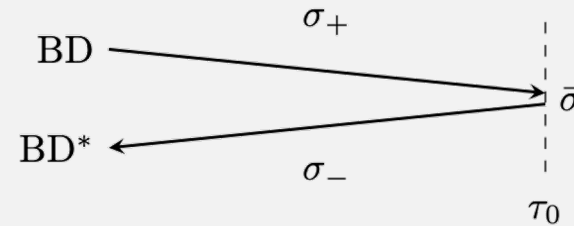
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- We can then complete the square for a Gaussian integral, with mean value given by

$$\bar{\sigma}_{*,\mathbf{k}} = -g \frac{\Delta I_{\mathbf{k}}}{\Delta\Omega_k} = igP_k \Delta I_{\mathbf{k}}$$

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- The boundary value admits a bulk and a boundary view

# Branch-Mixing terms

- To clearly separate out the branch-mixing terms, change  $G_D$  to  $G_F$ , check the full Schwinger-Keldysh action,

$$\begin{aligned} iS_{\text{IF}}^{(2)} = & \frac{ig^2}{2} \int_{\mathbf{k}} \left( \int_{\mathcal{C}_+} d\tau \int_{\mathcal{C}_+} d\tau' a^2(\tau) a^2(\tau') F_{+,\mathbf{k}}(\tau) G_{F,k}(\tau, \tau') F_{+,-\mathbf{k}}(\tau') \right. \\ & \left. - \int_{\mathcal{C}_-} d\tau \int_{\mathcal{C}_-} d\tau' a^2(\tau) a^2(\tau') F_{-,\mathbf{k}}(\tau) G_{F,k}^*(\tau, \tau') F_{-,-\mathbf{k}}(\tau') \right) \\ & + g^2 \int_{\mathbf{k}} P_k I_{+,\mathbf{k}} I_{-,-\mathbf{k}} . \end{aligned}$$

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- The last line contains all the branch-mixing terms

# The Reduced Density Matrix

- The proper object to study is then a reduced density matrix

$$\begin{aligned} \rho_{\text{red}}[\bar{\zeta}_+, \bar{\zeta}_-] \propto & \int_{\zeta_{\pm}(\tau_0)=\bar{\zeta}_{\pm}} \mathcal{D}\zeta_+ \mathcal{D}\zeta_- \exp \left\{ iS_{\zeta}[\zeta_+] - iS_{\zeta}[\zeta_-] \right. \\ & + \frac{ig^2}{2} \int_{\mathbf{k}} \int_{\mathcal{C}_+} d\tau \int_{\mathcal{C}_+} d\tau' a^2(\tau) a^2(\tau') F_{+,\mathbf{k}}(\tau) G_{F,k}(\tau, \tau') F_{+,-\mathbf{k}}(\tau') \\ & - \frac{ig^2}{2} \int_{\mathbf{k}} \int_{\mathcal{C}_-} d\tau \int_{\mathcal{C}_-} d\tau' a^2(\tau) a^2(\tau') F_{-,\mathbf{k}}(\tau) G_{F,k}^*(\tau, \tau') F_{-,-\mathbf{k}}(\tau') \\ & \left. + g^2 \int_{\mathbf{k}} P_k I_{+,\mathbf{k}} I_{-,-\mathbf{k}} + iS_{\text{IF}}^{(\lambda)} + \mathcal{O}(\lambda^2) \right\}. \end{aligned}$$

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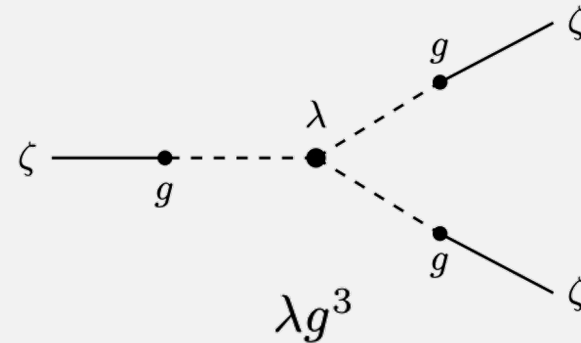
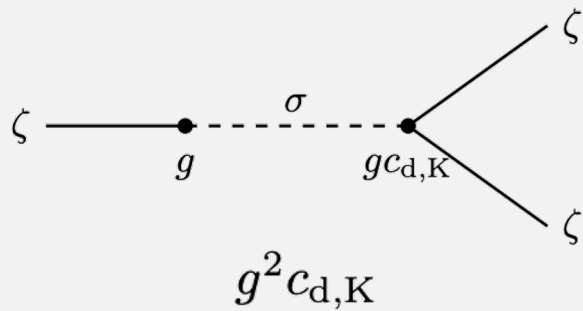
- Note the branch-mixing terms that cannot be reproduced from a pure state

# Cubic Results

- Take  $F_d[\zeta] \equiv \zeta' + 3c_d\zeta\zeta' = \partial_\tau \left( \zeta + \frac{3}{2}c_d\zeta^2 \right)$

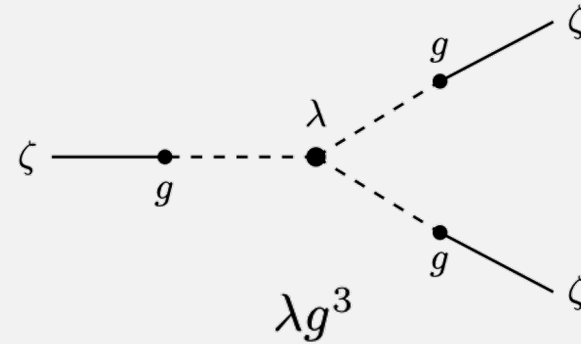
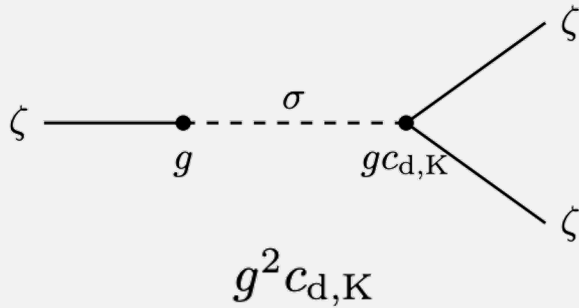
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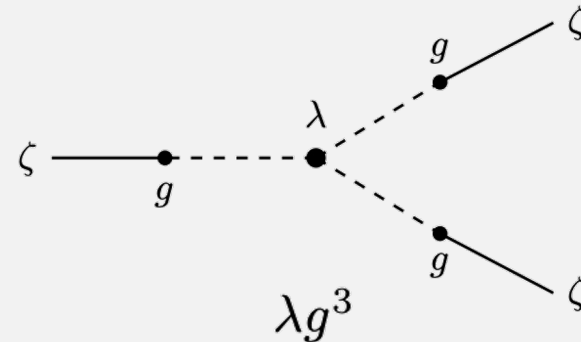
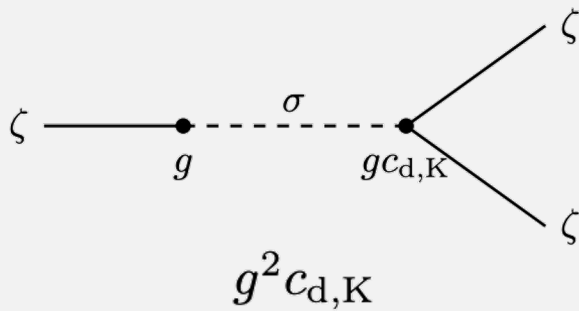


- Represent cubic interactions in the reduced density matrix using

$$\ln \rho_{\text{red}}|_{\bar{\zeta}^3} = -\frac{1}{3!} \int_{\mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3} (2\pi)^3 \delta^3 \left( \sum_{a=1}^3 \mathbf{k}_a \right) \sum_{s_1, s_2, s_3 = \pm} \Gamma_{s_1 s_2 s_3}^{(3)}(1, 2, 3) \prod_{a=1}^3 \bar{\zeta}_{s_a, \mathbf{k}_a}$$

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- The  $\Gamma$ 's can be calculated through convolutions of propagators

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- We can check several results. For example, when the heavy field is conformally coupled,  $m^2 = 2H^2$ , we can do the integrals explicitly, leading to

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- The behavior in the squeezed limit for a generic mass (of order Hubble) recovers previous cosmological collider calculations
- For very heavy mass, with adiabatic approximation, can obtain local EFT in heavy mass expansion which contains only in-out components at perturbative order

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- These are standard for open system EFT

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with constraints

$$\begin{aligned} \mathcal{D}_R(x,y)^* &= \mathcal{D}_R(x,y) , & \mathcal{D}_R(x,y) &= 0 \quad \text{for } \tau_x < \tau_y , \\ \mathcal{N}(x,y)^* &= \mathcal{N}(x,y) = \mathcal{N}(y,x) , \end{aligned}$$

$$\int_{x,y} f(x) \mathcal{N}(x,y) f(y) \geq 0 \quad \text{for every real test function } f .$$

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- For a generic kernel  $\mathcal{K}$ , the dilatation identity becomes

$$(\tau\partial_\tau + \tau'\partial_{\tau'} - k\partial_k - 3) \mathcal{K}_k(\tau, \tau') = 0$$

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- On future boundary  $\tau_0 \rightarrow 0$ ,

$$\begin{aligned}\delta_D \bar{\zeta} &= -\lambda (\Delta_\zeta + \mathbf{x} \cdot \nabla) \bar{\zeta} , \\ \delta_{K_b} \bar{\zeta} &= - \left[ 2(\mathbf{b} \cdot \mathbf{x}) (\Delta_\zeta + \mathbf{x} \cdot \nabla) - \mathbf{x}^2 \mathbf{b} \cdot \nabla \right] \bar{\zeta}\end{aligned}$$

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- The reduced density matrix then satisfies

$$\sum_{s=\pm} \int d^3x \, \delta_{\xi} \bar{\zeta}_s(\mathbf{x}) \frac{\delta}{\delta \bar{\zeta}_s(\mathbf{x})} \rho_{\text{red}}[\bar{\zeta}_+, \bar{\zeta}_-] = 0$$

# Symmetry Constraints

- On future boundary  $\tau_0 \rightarrow 0$ ,

$$\begin{aligned}\delta_D \bar{\zeta} &= -\lambda (\Delta_\zeta + \mathbf{x} \cdot \nabla) \bar{\zeta} , \\ \delta_{K_b} \bar{\zeta} &= -[2(\mathbf{b} \cdot \mathbf{x}) (\Delta_\zeta + \mathbf{x} \cdot \nabla) - \mathbf{x}^2 \mathbf{b} \cdot \nabla] \bar{\zeta}\end{aligned}$$

- The reduced density matrix then satisfies

$$\sum_{s=\pm} \int d^3x \delta_\xi \bar{\zeta}_s(\mathbf{x}) \frac{\delta}{\delta \bar{\zeta}_s(\mathbf{x})} \rho_{\text{red}}[\bar{\zeta}_+, \bar{\zeta}_-] = 0$$

- This leads to

$$\begin{aligned}& \left[ \sum_{a=1}^{N-1} \mathbf{k}_a \cdot \frac{\partial}{\partial \mathbf{k}_a} - 3 + N \Delta_\zeta \right] \Gamma_{s_1 \dots s_N}^{(N)} = 0 \\ & \sum_{a=1}^{N-1} \left[ -2 \Delta_\zeta \frac{\partial}{\partial k_a^i} + k_a^i \frac{\partial^2}{\partial k_a^j \partial k_a^j} - 2 k_a^j \frac{\partial^2}{\partial k_a^j \partial k_a^i} \right] \Gamma_{s_1 \dots s_N}^{(N)} = 0\end{aligned}$$

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- Studied SK and symmetry constraints, obtained Ward identities

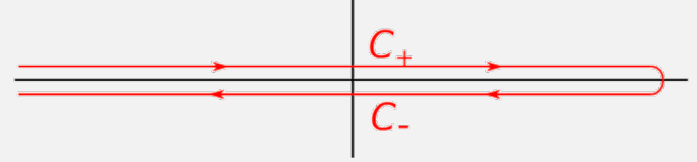
# Conclusion

- We established the procedure of tracing out a heavy field in cosmology
- Boundary value and branch-mixing terms are important
- The proper object to investigate is the reduced density matrix
- Checked conformal result, squeezed limit, and heavy mass expansion
- Studied SK and symmetry constraints, obtained Ward identities
- Future directions:
  - Realistic inflationary scenarios
  - Loops and non-trivial states/environments
  - Massive/massless spin fields
  - Beyond weak mixing
  - ...

Thank you!

Questions?

# In-Out vs. In-In



- In-in calculation:

$$\langle \mathcal{O}(t) \rangle = \left\langle \bar{\mathbf{T}} \left\{ e^{i \int_{-\infty}^t (1+i\epsilon) dt H_{\text{int}}(t)} \right\} \mathcal{O}^{\text{int}}(t) \mathbf{T} \left\{ e^{-i \int_{-\infty}^t (1-i\epsilon) dt H_{\text{int}}(t)} \right\} \right\rangle$$

- Expand in interacting Hamiltonian

$$\langle \mathcal{O}(t_0) \rangle = \sum_{N=0}^{\infty} i^N \int_{-\infty}^{t_0} dt_N \int_{-\infty}^{t_N} \cdots \int_{-\infty}^{t_2} dt_1 \langle [H_I(t_1), [H_I(t_2), \cdots [H_I(t_N), \mathcal{O}(t_0)] \cdots]] \rangle$$

- Alternatively,

$$\begin{aligned} \langle \mathcal{O}(t=0) \rangle &= \int \mathcal{D}\bar{\phi} \mathcal{D}\bar{\sigma} \langle 0 | \bar{\phi}, \bar{\sigma} \rangle \langle \bar{\phi}, \bar{\sigma} | \mathcal{O} | 0 \rangle \\ &= \int \mathcal{D}\bar{\phi} \mathcal{D}\bar{\sigma} \left( \int^{\bar{\phi}, \bar{\sigma}} \mathcal{D}\phi' \mathcal{D}\sigma' e^{iS[\phi', \sigma']} \right)^* \mathcal{O}[\bar{\phi}, \bar{\sigma}] \left( \int^{\bar{\phi}, \bar{\sigma}} \mathcal{D}\phi \mathcal{D}\sigma e^{iS[\phi, \sigma]} \right) \\ &= \int \mathcal{D}\bar{\phi} \mathcal{D}\bar{\sigma} |\Psi[\bar{\phi}, \bar{\sigma}]|^2 \mathcal{O}[\bar{\phi}, \bar{\sigma}] . \end{aligned}$$

# Cubic Results

- For very heavy mass, under heavy mass expansion, with adiabatic approximation, the branch-mixing terms are

$$\Delta \ln \rho_{\partial} \simeq \frac{g^2}{4a_0} \int_{\mathbf{k}} \frac{(F_+ + F_-)_{\mathbf{k}}(F_+ + F_-)_{-\mathbf{k}}}{\omega_k^3} \Big|_{\tau_0}$$

- At late time, this decays to zero as we take  $\tau_0 \rightarrow 0^-$  (end of inflation) since  $\zeta' \sim \mathcal{O}(\tau_0)$
- We then safely use the usual EFT heavy mass expansion

$$\sigma_{\text{loc}}^{(0)} = \frac{g}{m^2} \mathcal{F} + \frac{g}{m^4} \square \mathcal{F} + \dots$$

for  $\mathcal{F} = a^{-2} F$

- Perturbatively, we obtain a local EFT in heavy mass expansion

# Symmetry Constraints

- Our example  $F_d[\zeta] \equiv \zeta' + 3c_d\zeta\zeta' = \partial_\tau \left( \zeta + \frac{3}{2}c_d\zeta^2 \right)$  breaks the dS symmetries
- With a slightly different prescription  $F_d[\zeta] \equiv a\zeta' + 3ac_d\zeta\zeta'$ , the dilatation Ward identity is satisfied
- Interestingly, in this case, a single term like  $\Gamma_{+-}$  would contain large logs that diverge at  $\tau_0 \rightarrow 0$ , but in the full result

$$\Gamma^{(2)} = \Gamma_{++}^{(2)} + \Gamma_{--}^{(2)} + \Gamma_{-+}^{(2)} + \Gamma_{+-}^{(2)}$$

these large logs cancel

- One can derive Ward identities for theories with symmetry breaking, with exact form depending on the symmetry breaking pattern

# Use of AI

- We used ChatGPT to assist with calculations and manuscript preparation
- Calculations and texts are checked and reviewed
- Everything was done through work mode in Codex

This is the final version at the moment. Please check. Let me know of any inconsistencies and quick fixes. Make it a list so that I can quickly find and fix them.

A few optional cleanups:

- **Title page:** names and affiliations are absent; restore them unless intentionally anonymous.