

For ICFP 2026 @ Beijing

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Solving the Stong CP Problem with Heavy Axions

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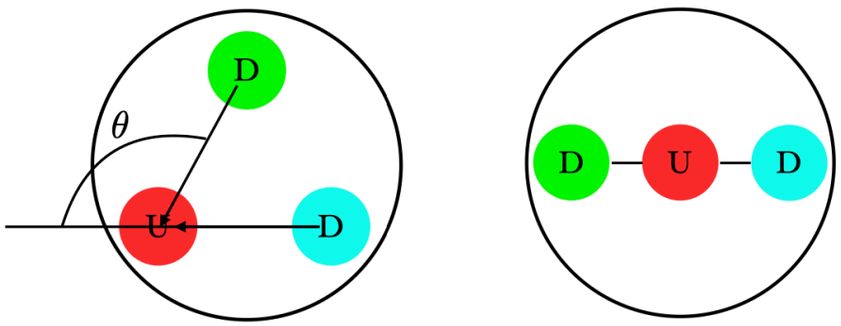
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Based on: 2611.xxxx, with D. Lin, T. Ma, V. Risi (ICTP-AP), and G. Cacciapaglia (LPTHE)

Strong CP & Axion

- Strong CP problem

$$\mathscr{L} \supset \frac{\bar{\theta}}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu a}$$



[Anson Hook, 1812.02669]

neutron eDM

$$\bar{\theta} \lesssim 10^{-10}$$

Why such tiny?

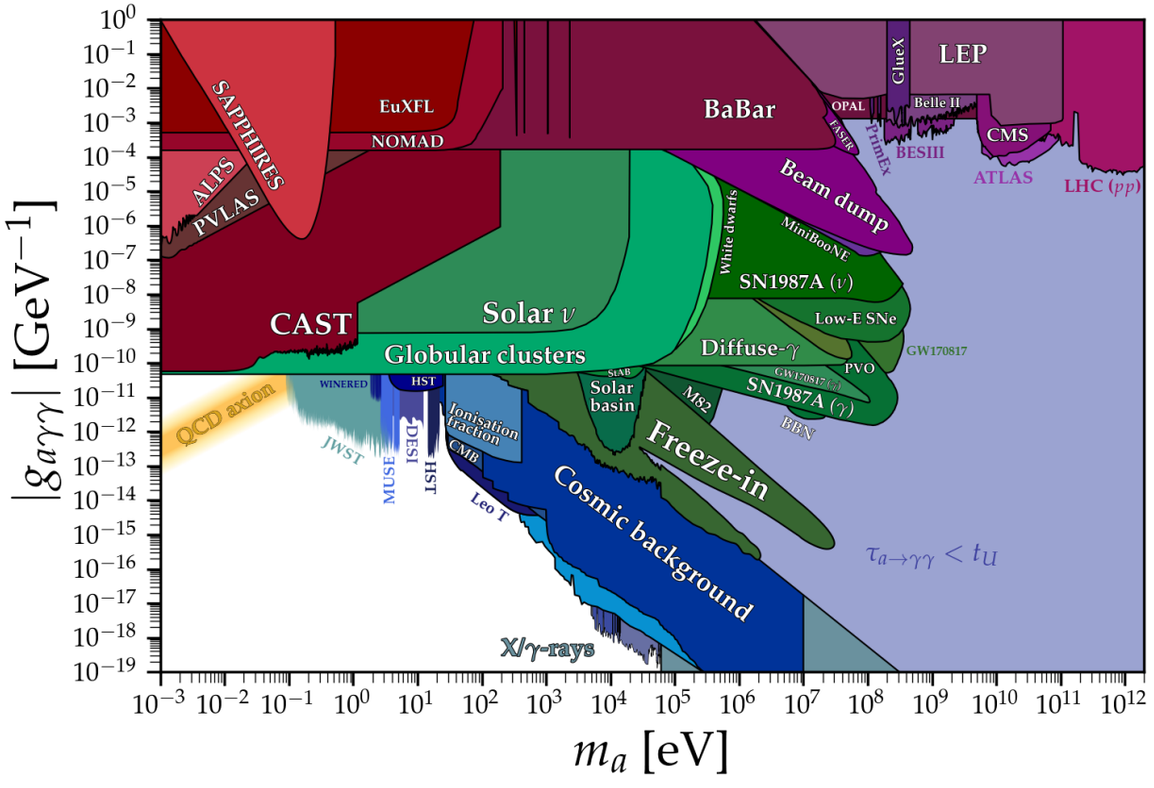
- Axion $\mathscr{L} \sim (\theta + \frac{a}{f_a}) F\tilde{F}$
  - Dynamically relaxing

$$\langle a \rangle = -f_a \bar{\theta}$$

  - QCD axion

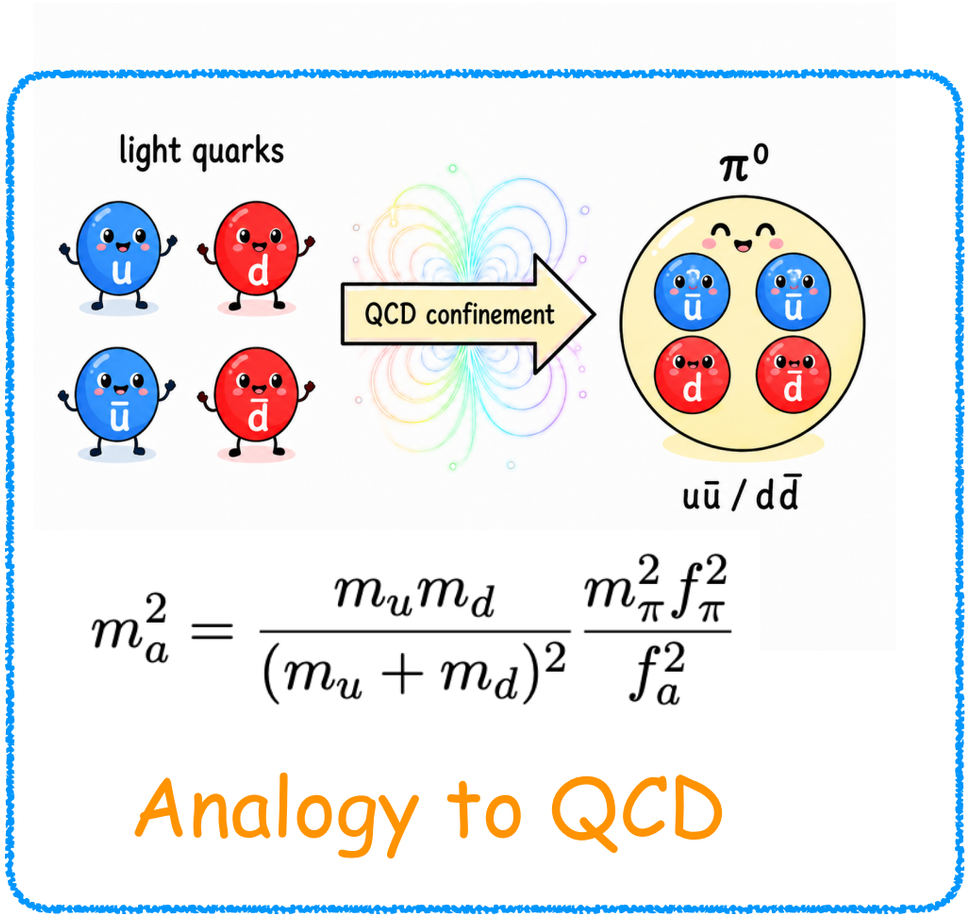
Very light

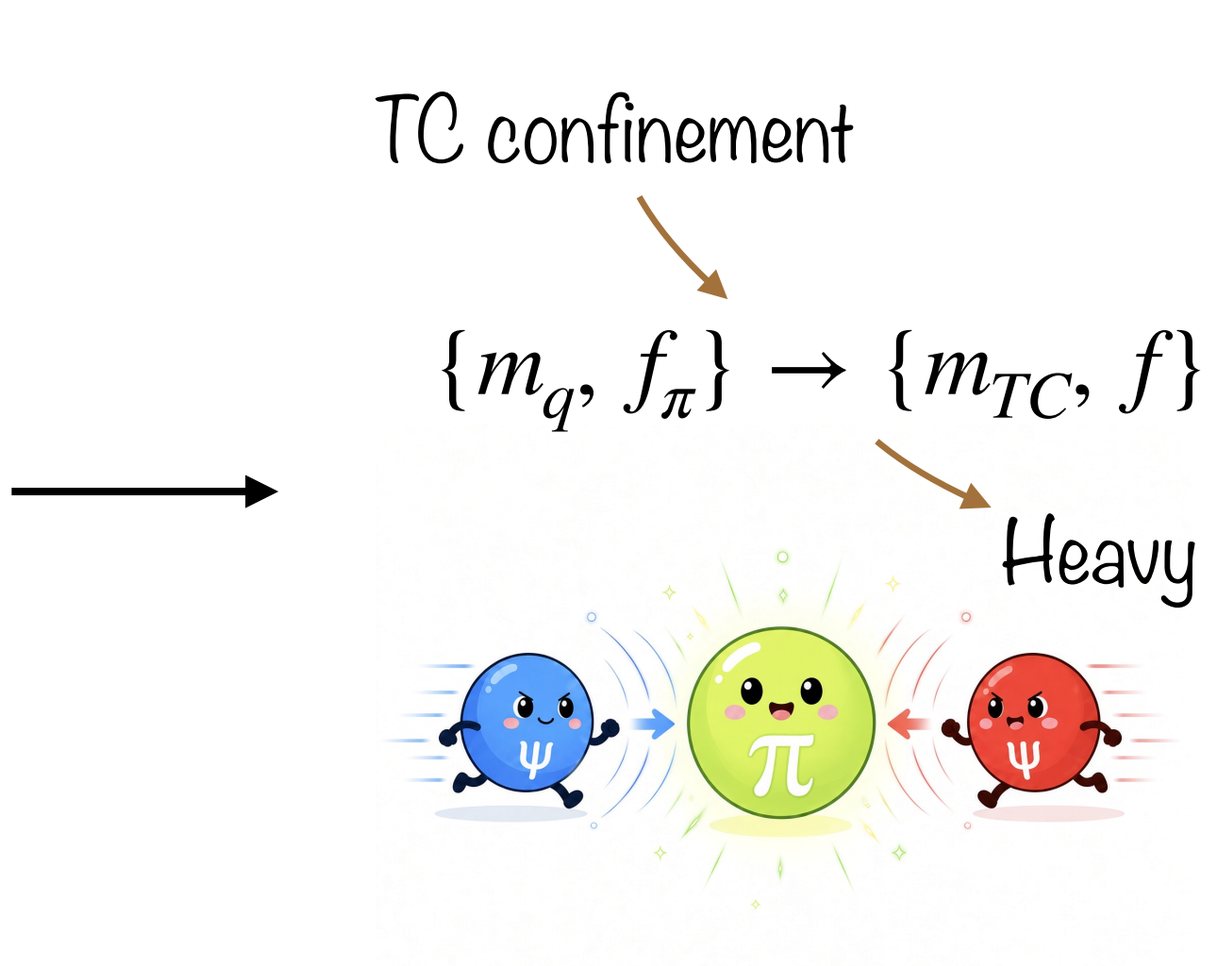
[PDG, 2025]



A Heavy Axion

- What if heavier axions
  - Feasibility @ exp facilities
  - to be high quality
- How to be heavy? Confinement? Small instanton?
  - A TC sector



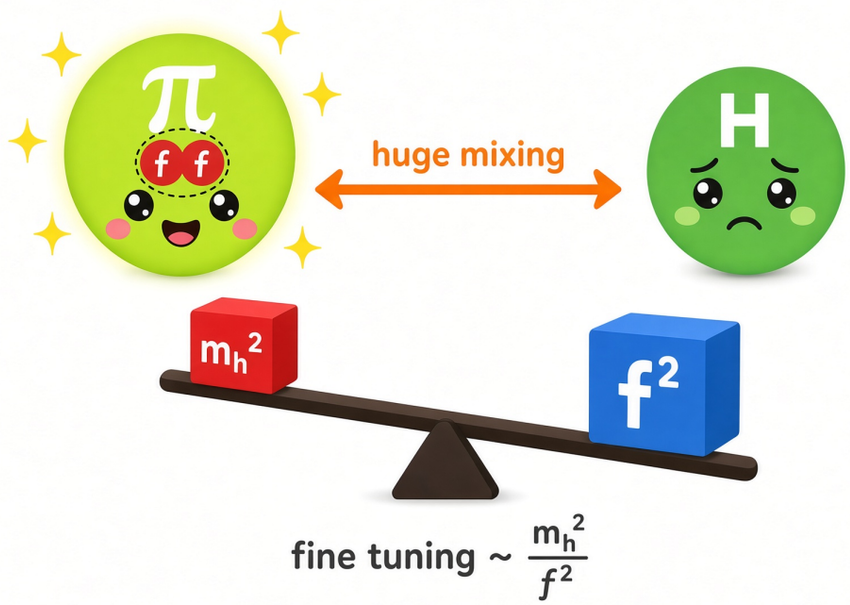


Confinement

- Grand Color [Valenti et. al., 2206.04077]
  - To extend QCD to SU(N) $\Psi = \begin{pmatrix} q \\ f \end{pmatrix}$ (SM quark / TC fermion)
  - TC to confine properly w heavy axion
  - Identical Yukawa structure $\mathscr{L} \supset y \Psi_S H \Psi_D$

to share top coupling: $y_t \; f_3 f_3 h$

  - Large scalar mixing
  - Fine tuning $\mathscr{T} \sim \frac{m_h^2}{f^2}$



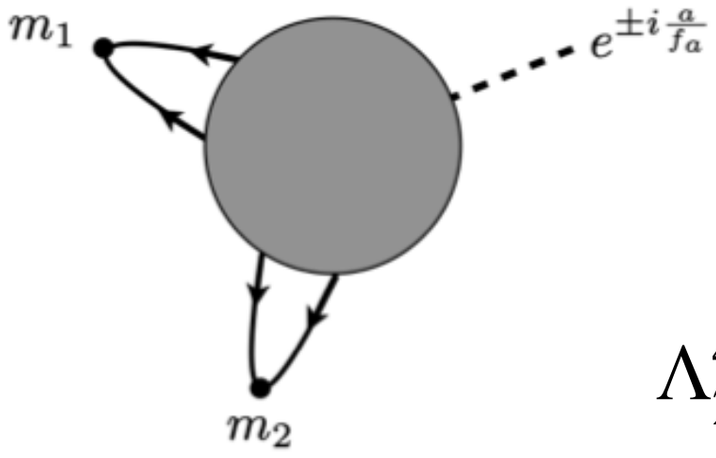
To reproduce SM Higgs

Heavy axion vs. small tuning: how?

Small Instanton

- To separate SM & TC fermions
- A TC gauge $Sp(2k)$?
- Small instanton

Itself as mass source!



[Csaki et. al., 2311.09285]

$$V_a \supset \Lambda_{2k}^4 \cos(\bar{\theta}_{2k} + \frac{a}{f_a})$$

$$\Lambda_{2k}^4 \sim \kappa_{sp} \int_0^{v_\Sigma^{-1}} \frac{2d\rho}{\rho^5} (v_\Sigma \rho)^{b_{2k}} (\frac{2\pi}{\alpha_{2k}})^{2(k+1)} e^{-\frac{2\pi}{\alpha_{2k}}} \prod_i (m_i \rho)$$

Fermion zero-mode suppression

Small Instanton

- Suppressed by TC mass $\Lambda_{2k} \sim (\frac{m_{TC}}{v_\Sigma})^{N_f/4}\ v_\Sigma\ F[\alpha;\ k]$

- A heavy axion: large TC mass
- TC confinement: no too heavy TC fermions

EW structure

- Confinement & Small ins to be heavy, but ...

What Wanted

- TC, QCD separated
- Small mixing y tuning
- Large mass for instanton
- Small mass to confine



既要又要，真难伺候

Wants both, so hard to please

Model

- $SU(2k+3) \times Sp(2k)' \times U(1)'_Y \times SU(2)_L \times Z_4$

Diagonally

W 2 axions

$SU(3)_c \times Sp(2k)_{TC} \times U(1)_Y \times SU(2)_L$

to confine

$$\mathscr{L} \supset (\theta_1 + \frac{a_1}{f_{a1}}) F_{su}\tilde{F}_{su} + (\theta_2 + \frac{a_2}{f_{a2}}) F_{sp}\tilde{F}_{sp}$$

- Particle content

Field	$SU(2k+3)$	$Sp(2k)'$	$SU(2)_L$	$U(1)'_Y$	Z_4
Ψ_1	$\overline{2k+3}$	1	1	$-\frac{1}{2} - \frac{1}{2(2k+3)}$	1
Ψ_2	$\overline{2k+3}$	1	1	$\frac{1}{2} - \frac{1}{2(2k+3)}$	1
Ψ_3	$2k+3$	1	2	$\frac{1}{2(2k+3)}$	1
L	1	1	2	$-\frac{1}{2}$	1
e^c_R	1	1	1	1	1
$\psi^{(\prime)}_{12}$	1	$2k$	2	0	$\mp i$
$\psi^{(\prime)}_3$	1	$2k$	1	$\frac{1}{2}$	$\pm i$
$\psi^{(\prime)}_4$	1	$2k$	1	$-\frac{1}{2}$	$\pm i$
Σ	$2k+3$	$2k$	1	$\frac{1}{2(2k+3)}$	$-i$
H	1	1	2	$\frac{1}{2}$	1

- 3 generations
- ψ & ψ' as TC fermion
- Z_4 w only mixing mass

$$\mathscr{L}_m \supset M_{12}\psi_{12}\psi'_{12} + M_{34}\psi_3\psi'_4 + M_{43}\psi_4\psi'_3 + \text{h.c.}$$

- Anomaly free

Interaction

- Bi-fundamental

$$-\mathcal{L}_\Sigma = Y_1 \Psi_1 \Sigma \psi_3 + Y_2 \Psi_2 \Sigma \psi_4 + Y_3 \Psi_3 \Sigma^\dagger \psi_{12} + \text{h.c.}.$$

- SM Yukawa

$$-\mathcal{L}_Y = y_u \Psi_3 H \Psi_1 + y_d \Psi_3 \widetilde{H} \Psi_2 + \text{h.c.},$$

- Small instanton

Large mass for instanton ✔

- To close the fermion loop w mixing mass

$$\mathscr{L}_m \supset M_{12}\psi_{12}\psi'_{12} + M_{34}\psi_3\psi'_4 + M_{43}\psi_4\psi'_3 + \text{h.c.}$$

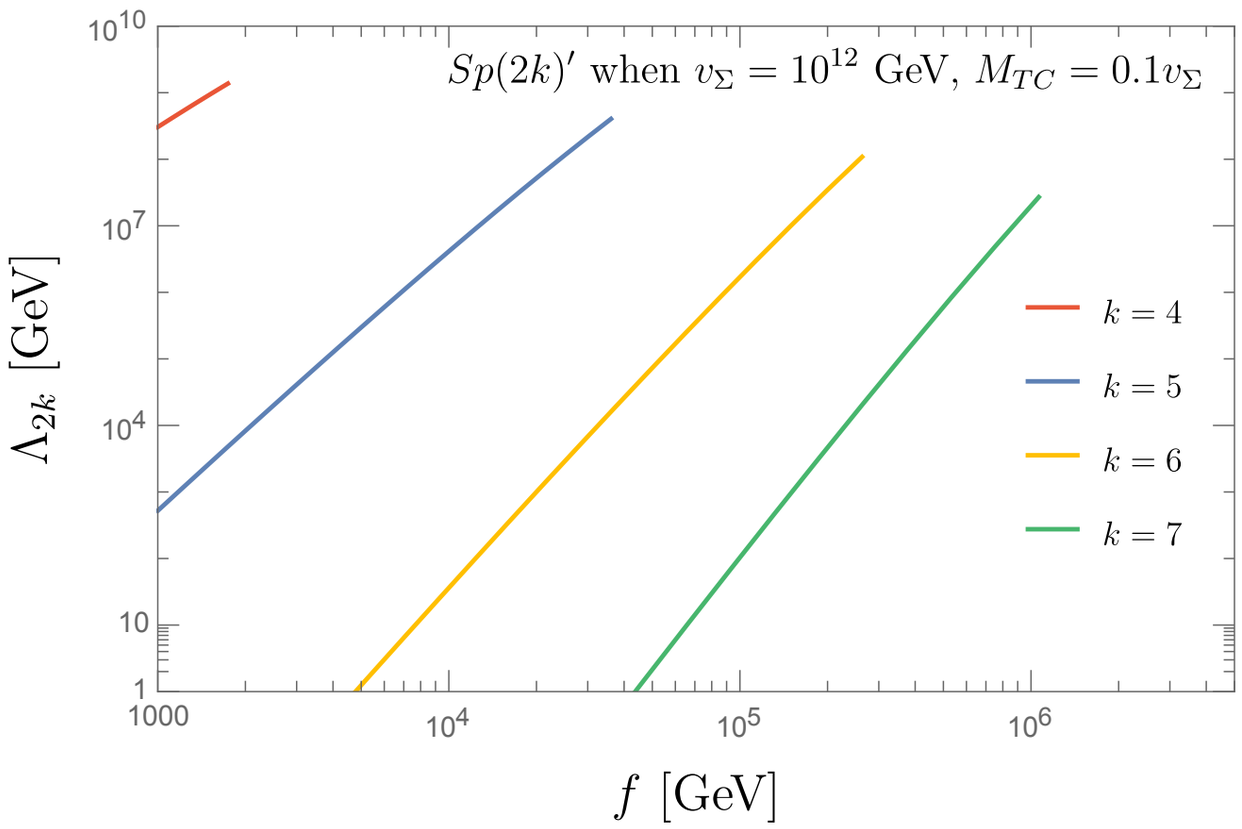
- Instanton scale — SSB scale v_Σ
- Mode decouple, when $M_i \gg v_\Sigma$ — Only Gen 3: $M_3 = 0.01 v_\Sigma$
- $\alpha_2(v_\Sigma)$ matters — RG running

RG Running & Small Instanton

- Running & matching
  - TC to confine at $4\pi f$
  - $\alpha_2(v_\Sigma\ ,\ f)$
- Axion potential

$$\alpha_s(v_\Sigma) = \alpha_1(v_\Sigma)\ ,$$
$$\frac{1}{\alpha_1(v_\Sigma)} + \frac{1}{\alpha_2(v_\Sigma)} = \frac{1}{\alpha_{TC}(v_\Sigma)}\ .$$

  - Large Λ_{2k}
  - Terminated at f_{max}



- Dilute gas approx

$$\left(\frac{2\pi}{\alpha_2}\right)^{2(k+1)} e^{-\frac{2\pi}{\alpha_2}} \lesssim \frac{1}{3}$$

Confinement

- The $\Psi_{TC} - \psi - \psi'$ system
- Mass matrix

$$\mathcal{M}_q(v) = \begin{pmatrix} m_q & Y_3 v_\Sigma & 0 \\ Y_q v_\Sigma & 0 & M_{34} \\ 0 & M_{12} & 0 \end{pmatrix}, \qquad q = u, d.$$

To give a light eigenstate **Light mass to confine** ✓

- First 2 generations: $\Psi - \psi - \psi'$ Decoupled

Identical to quark mass: $M_{TCi} = m_{q_i}$, $(i = 1, 2)$

- 3rd generation: $\Psi - \psi + \psi'$

$$M_{TC3} \sim \frac{M_3^2}{M_3^2 + Y^2 v_\Sigma^2} m_{q_3}$$

- Yukawa via fermion mixing $\psi'\psi' h \to \theta_f^2 tth$

$$y = y_t \frac{M_3^2}{M_3^2 + Y^2 v_\Sigma^2}$$

Suppressed scalar mixing ✓

Confinement

- A benchmark

  Bottom partner around d mass $m_{TC3} = \{\frac{m_t}{m_b} m_d \ , \ m_d\}$

- Fine tuning $\mathcal{T} \sim \frac{m_h^2}{f^2} \cdot \frac{m_b}{m_d}$ Less tuned

- Confinement

  - Dashen alignment $\mu \sim (\frac{1}{m_u} + \frac{2}{m_d})^{-1}$

  - Axion mass $m_{aCF}^2 \sim \frac{4\pi f^3 \mu}{f_a^2}$

  - Heavy Axion needs large f

Two Axions

- Mass matrix

$$M_a^2 = \begin{pmatrix} M_{CF}^2 & M_{CF}^2 \\ M_{CF}^2 & M_{CF}^2 + M_{ins}^2 \end{pmatrix}$$

$$M_{CF}^2 \sim \frac{4\pi f^3 \mu}{f_a^2}$$

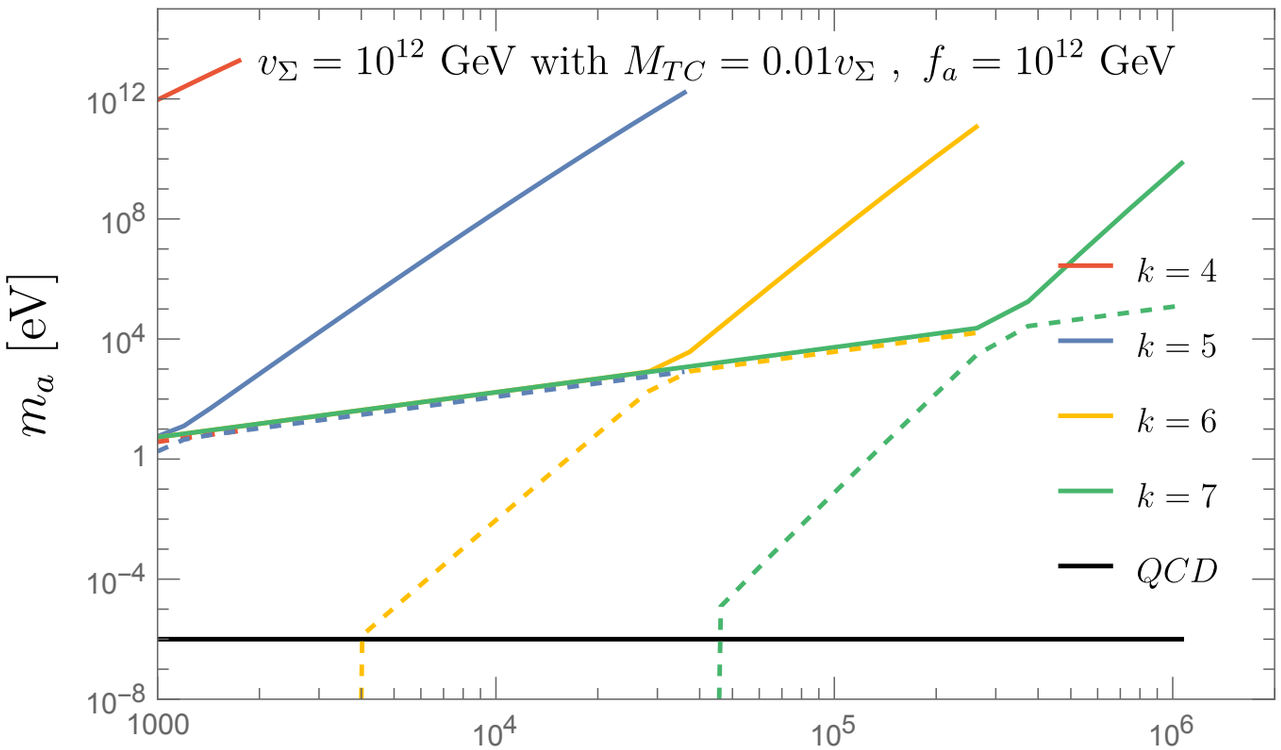
Confinement

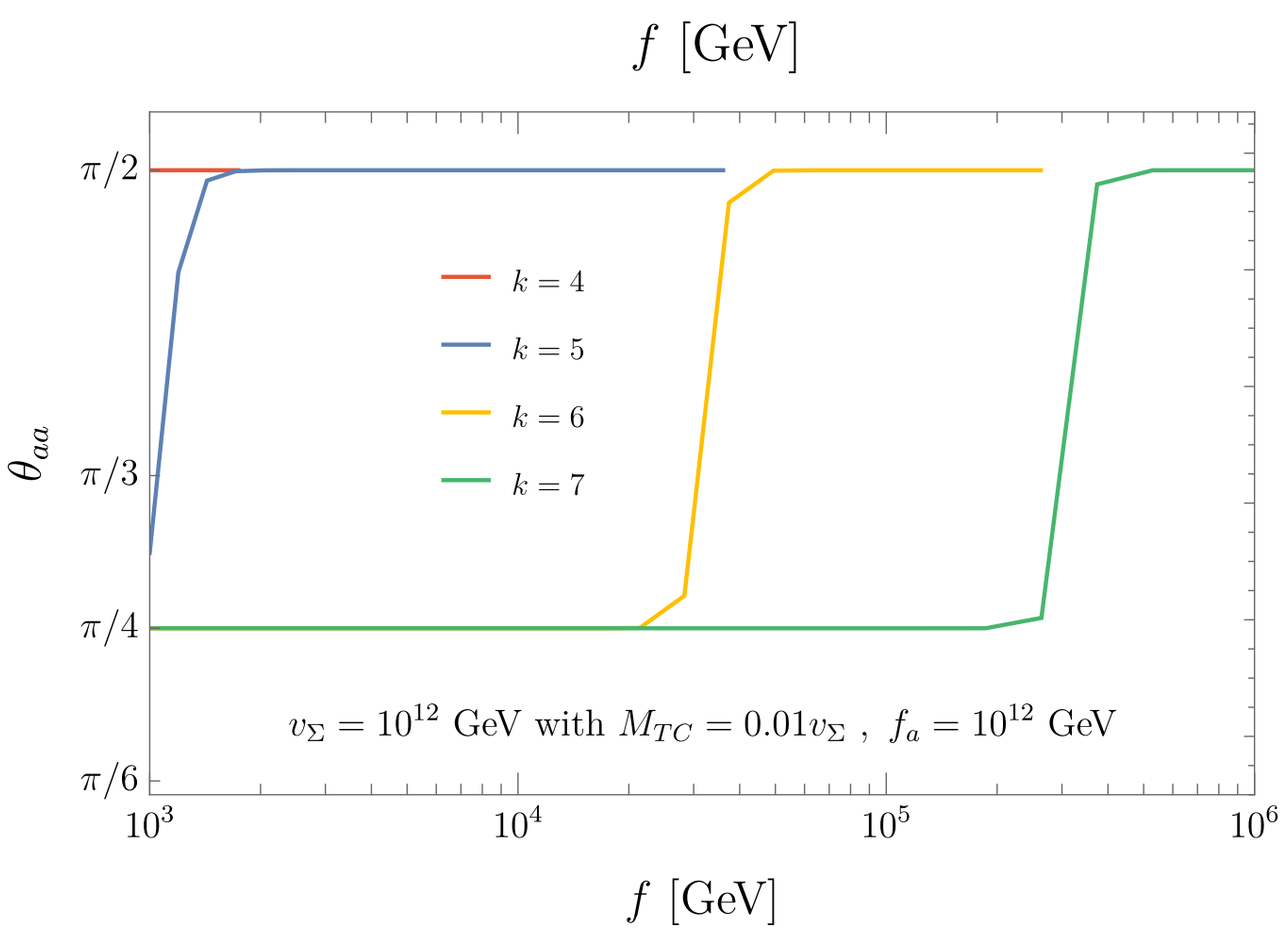
$$M_{ins}^2 = \frac{\Lambda_{2k}^4}{f_a^2}$$

Small instanton

- Eigenstate and mixing angle

Axion Masses





- Solid vs. Dashed

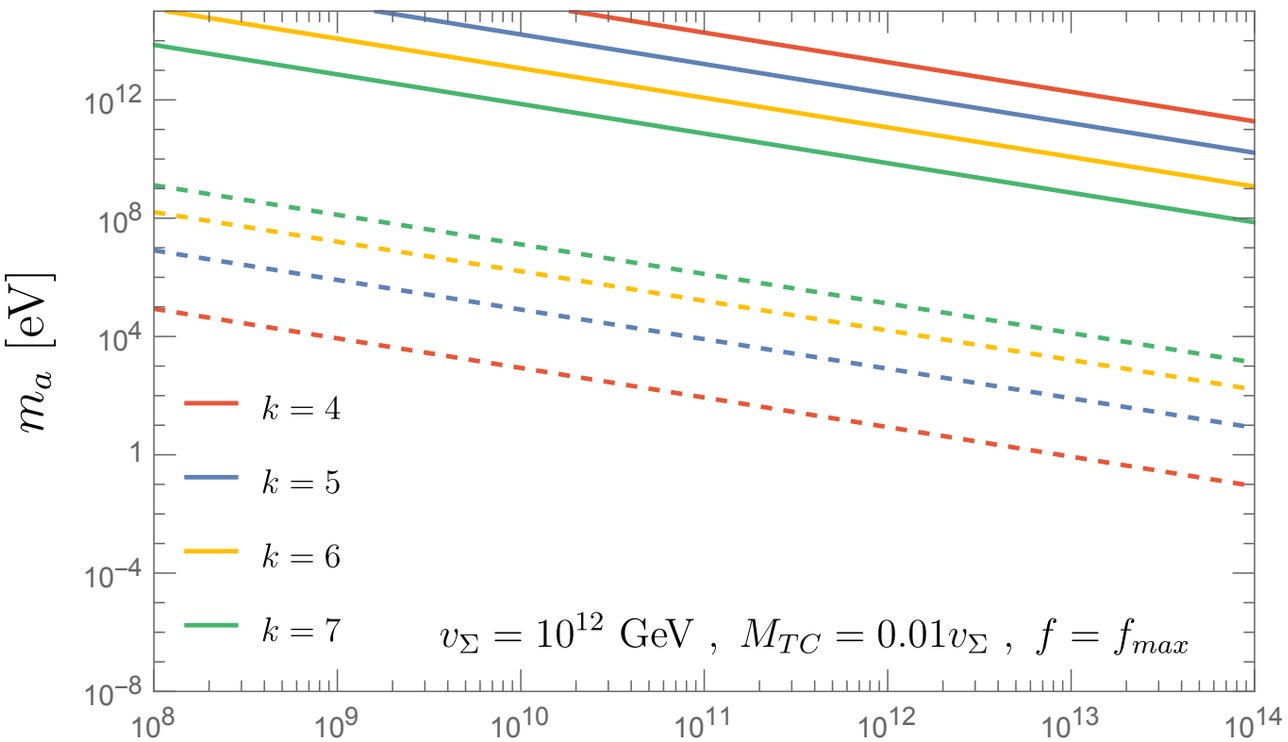
  heavy vs. light

- $\theta_{aa} = \text{Arg}\{a_H , a_{su}\}$
  - Low f: $\pm$ direction
  - High f: small instanton

    dominantly heavy

- Heavier than m_a^{QCD}

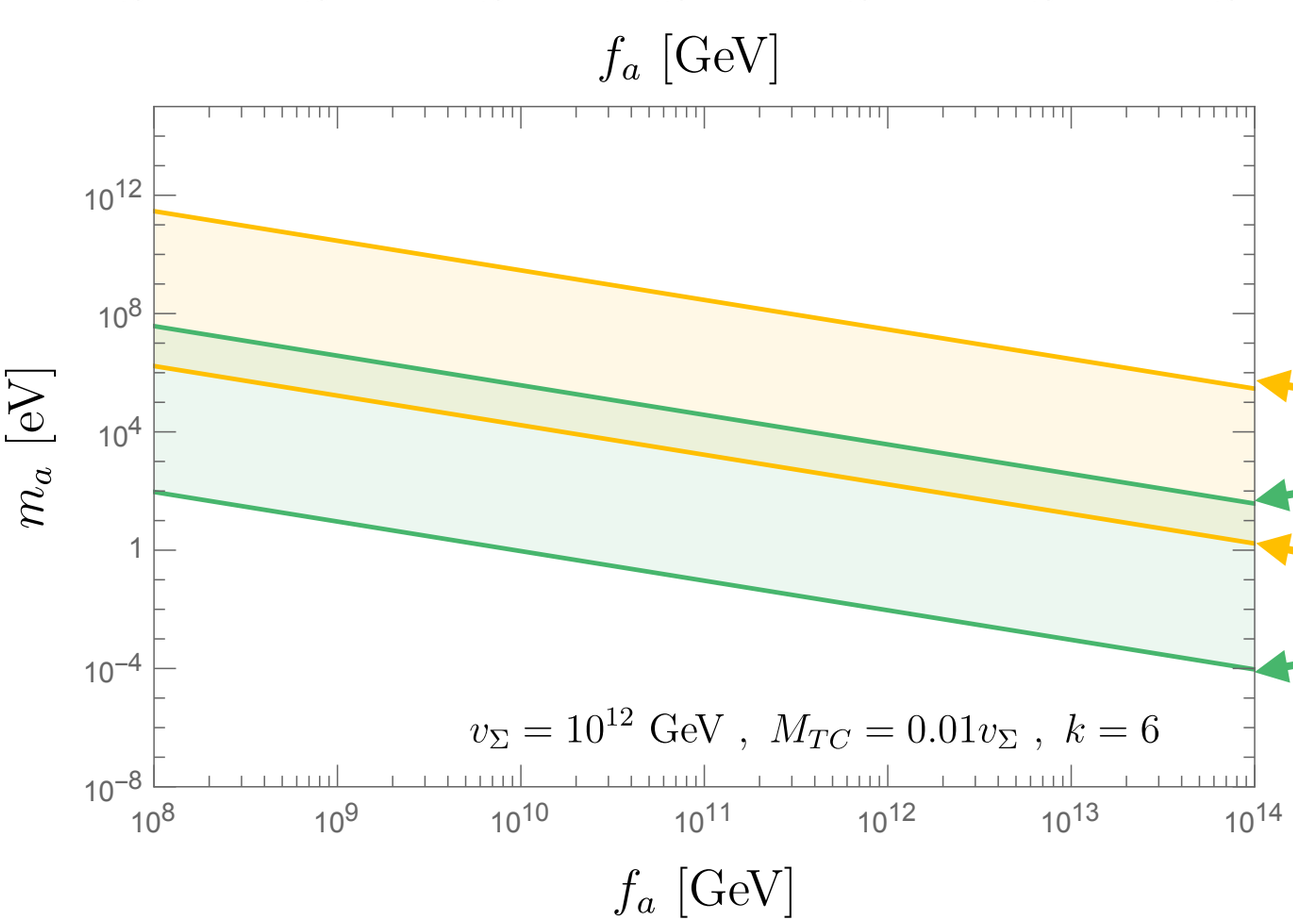
Axion Masses



2 axions

A larger group, the closer masses

both heavy



- k=6 for illustration

$f = 10^5$ GeV

$f = 10^4$ GeV

Summary

- Heavy axions with
  - Less tuning
  - Proper confinement

Doable!

  - Unsuppressed instanton effect

- Much heavier than QCD axion

More pheno to be updated...