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Parameter degeneracy and information loss in inverse flavor mapping for high-energy astrophysical neutrinos

arXiv: 2609.35132

Speaker: Zi-Qiang Chen (HIAS-UCAS)

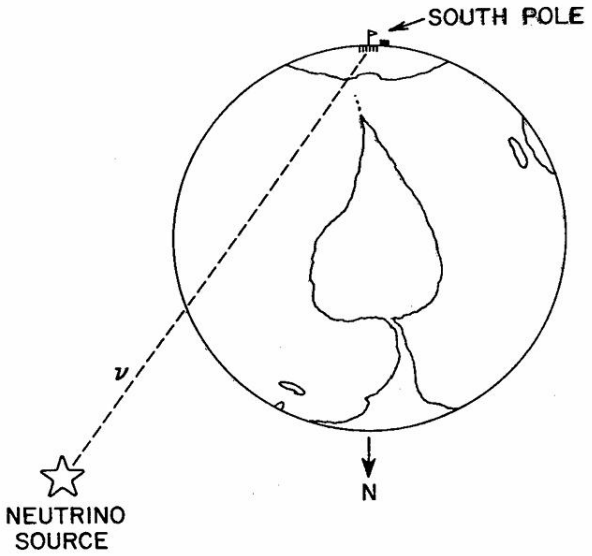
Collaborate with Zhi-zhong Xing and Ye-Ling Zhou

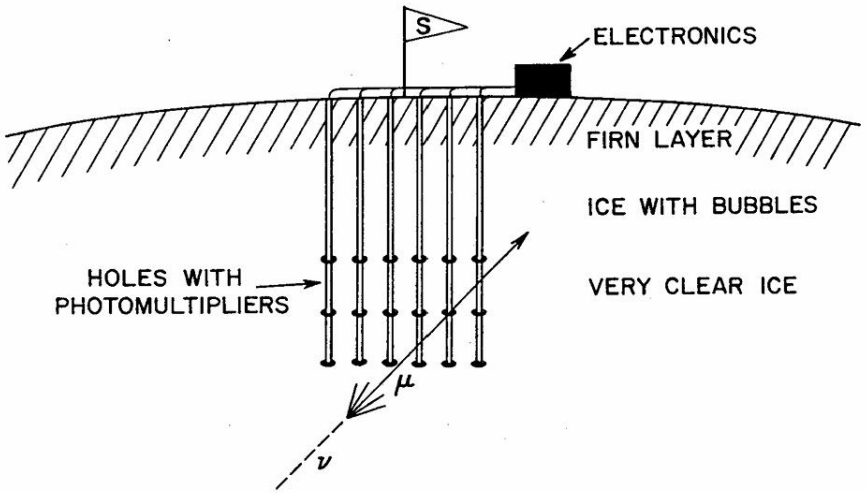
國科大杭州高等研究院
Hangzhou Institute for Advanced Study,UCAS

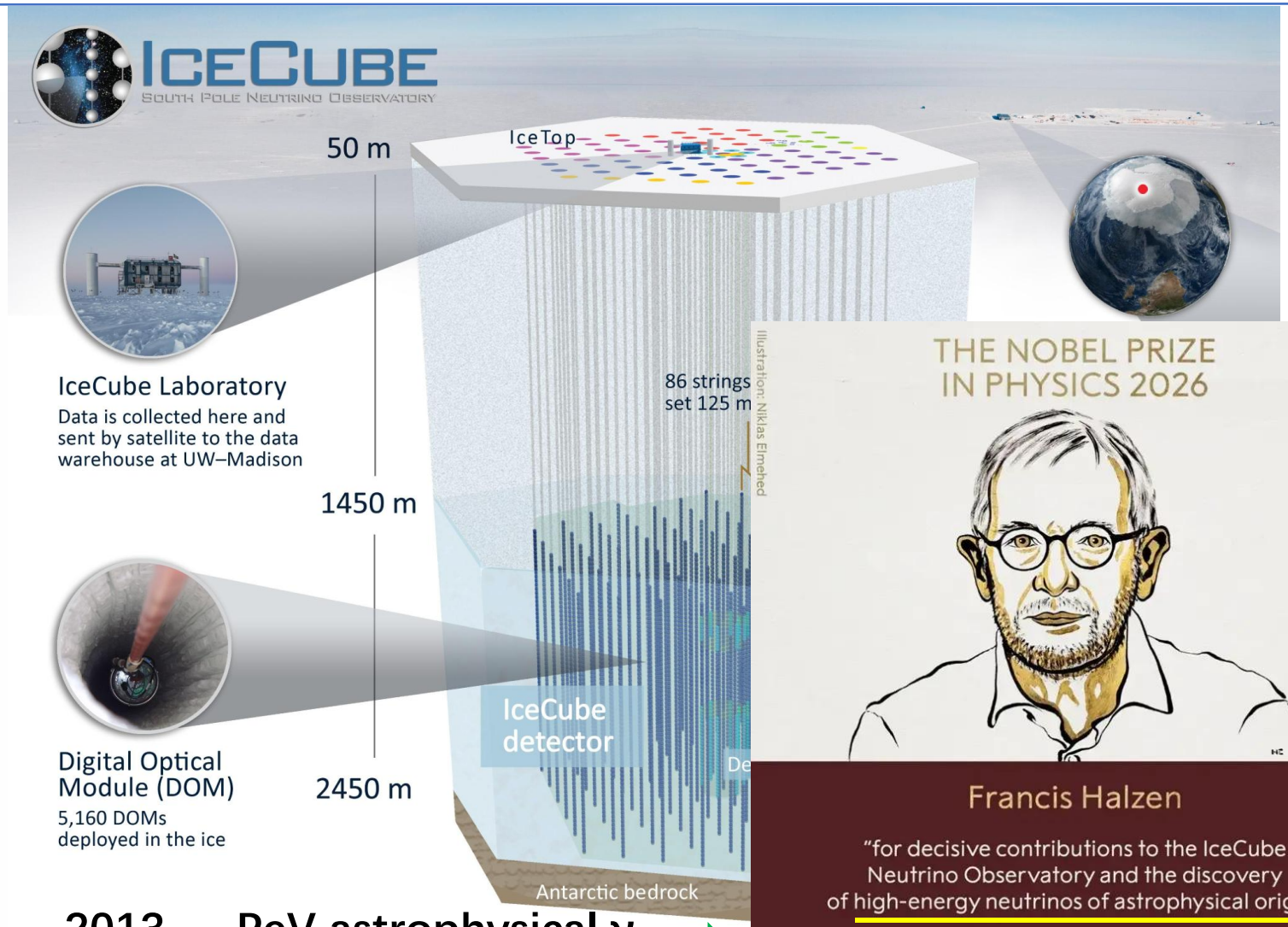
IceCube Observation

1988 — prototype

F. Halzen & J.G Learned







2013 — PeV astrophysical ν

- PRL 111, 021103
- Science 342, 1242856

How? What physical enevt?

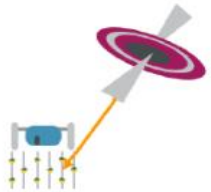
Messenger: ν flavor composition

- Specifically, IceCube interest:

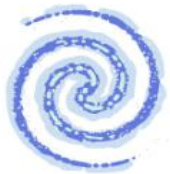
Astrophysical events produce ν flux 5 TeV-10 PeV



NGC 1068



TXS 0506+056



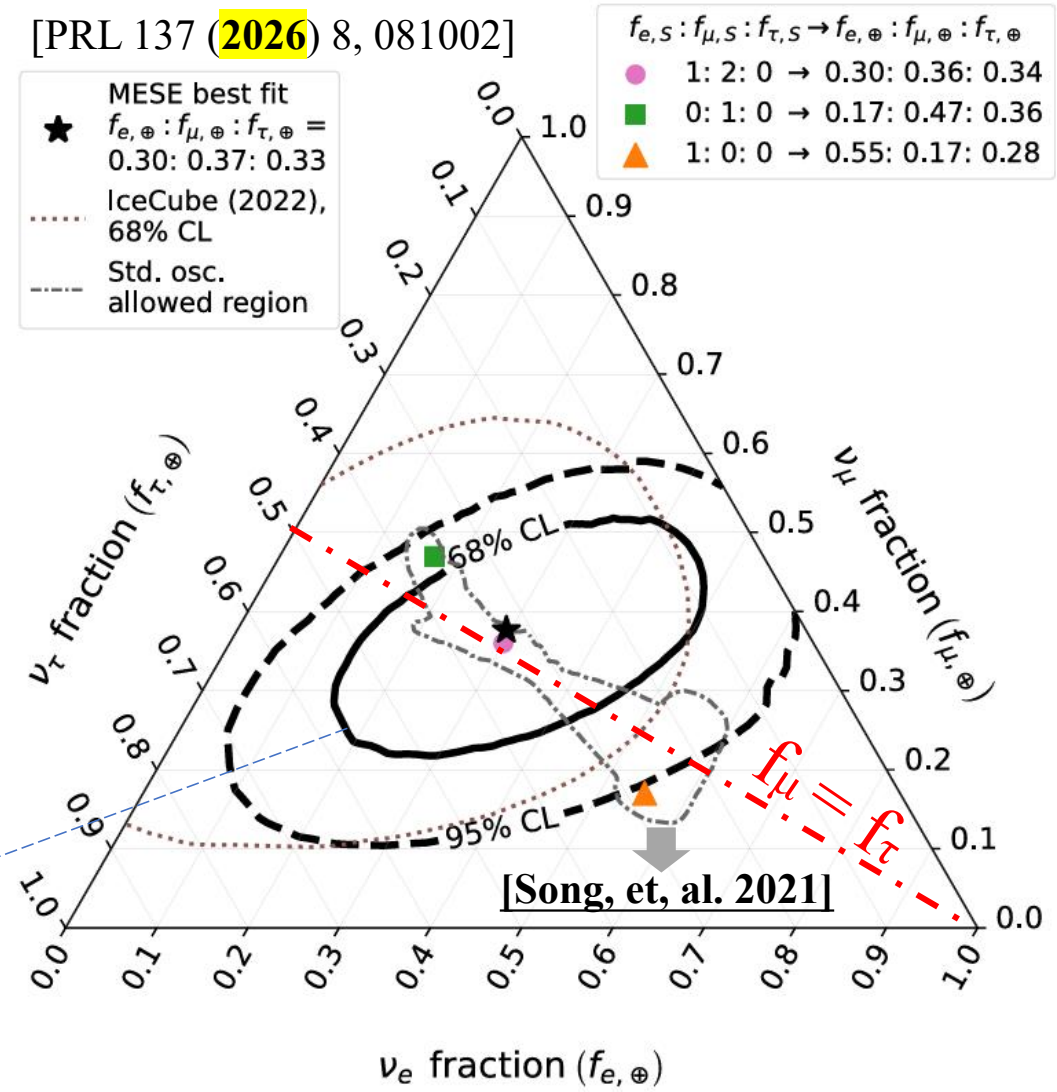
Milky Way

all-sky observation

New data countor support our source reconstruction

- 2026 IceCube flavor observation:

[PRL 137 (**2026**) 8, 081002]

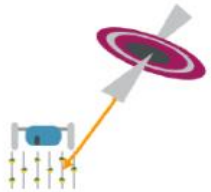


Messenger: ν flavor composition

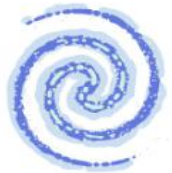
Multiple astrophysical events



NGC 1068



TXS 0506+056



Milky Way

High-energy neutrino production

1) pp or $p\gamma \to \pi^{\pm} + X$

For example:

2) $\pi^{\pm} \to \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$

3) $\mu^{\pm} \to e^{\pm} + \nu_{e}(\bar{\nu}_{e}) + \bar{\nu}_{\mu}(\nu_{\mu})$

The most natural (1: 2: 0) source

only 2) (0, 1, 0) & n decay (1, 0, 0) sources less favorable in IceCube

All possibilities, this work

Propagation



$$P(\nu_\alpha \to \nu_\beta) \equiv P_{\alpha\beta} = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2 + 2\sum_{i<j} \mathrm{Re}\left[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} e^{-i\Delta m^2_{ij} L/2E}\right]$$

$\Delta m^2_{ij} L/2E \gg 1$ averaged out

$$P_{\alpha\beta} = P_{\beta\alpha} = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$$

Normalized flavor composition:

$$\sum_\alpha f_\alpha = \sum_\alpha \eta_\alpha = 1$$

$$\begin{bmatrix} f_e \\ f_\mu \\ f_\tau \end{bmatrix} = \begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{\mu e} & P_{\mu\mu} & P_{\mu\tau} \\ P_{\tau e} & P_{\tau\mu} & P_{\tau\tau} \end{pmatrix} \begin{bmatrix} \eta_e \\ \eta_\mu \\ \eta_\tau \end{bmatrix}$$

telescope source

Solved through $\eta = P^{-1} f$,

(while det P=0 allowed)

e.g. μ-τ refl. symmetry

Motivation: To what extent we can know source ν-flavor component?

det P = 0 allowed: parameter degeneracy

- **Analytical det P:**

$$\det P = \begin{vmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{e\mu} & P_{\mu\mu} & P_{\mu\tau} \\ P_{e\tau} & P_{\mu\tau} & P_{\tau\tau} \end{vmatrix}$$

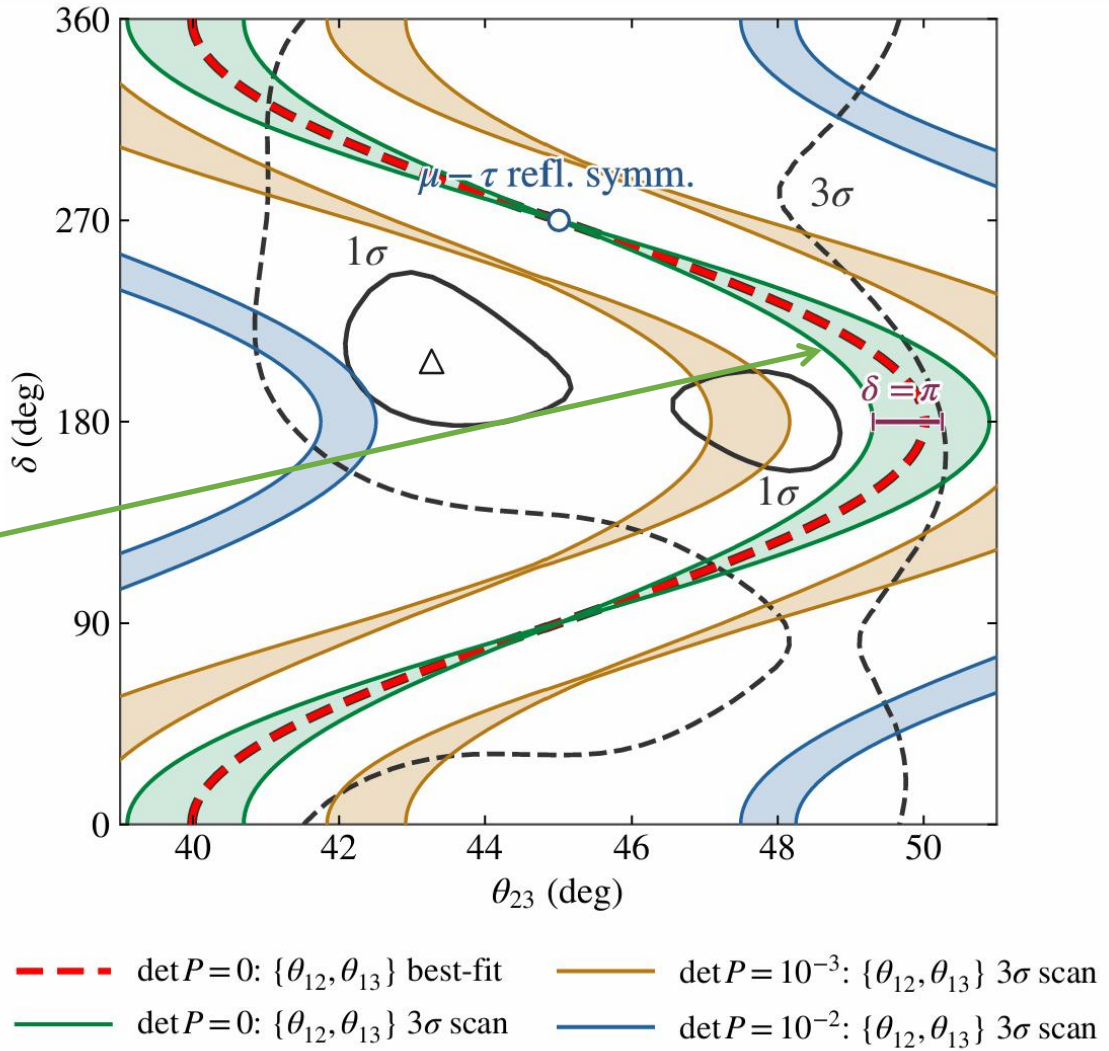
$$= \left[\cos 2\theta_{12}\cos 2\theta_{13}\cos 2\theta_{23} - \frac{1}{2}\sin 2\theta_{12}\sin\theta_{13}\left(1-3\sin^2\theta_{13}\right)\sin 2\theta_{23}\cos\delta\right]^2$$

- **Can be 0, if (at large region):**

$$\cot 2\theta_{23} = \frac{1}{2}\tan 2\theta_{12}\frac{\sin\theta_{13}\left(1-3\sin^2\theta_{13}\right)}{\cos 2\theta_{13}}\cos\delta$$

- **μ-τ reflection symmetry limit:**

$$\theta_{23} = \pi/4 \text{ and } \delta = 3\pi/2$$



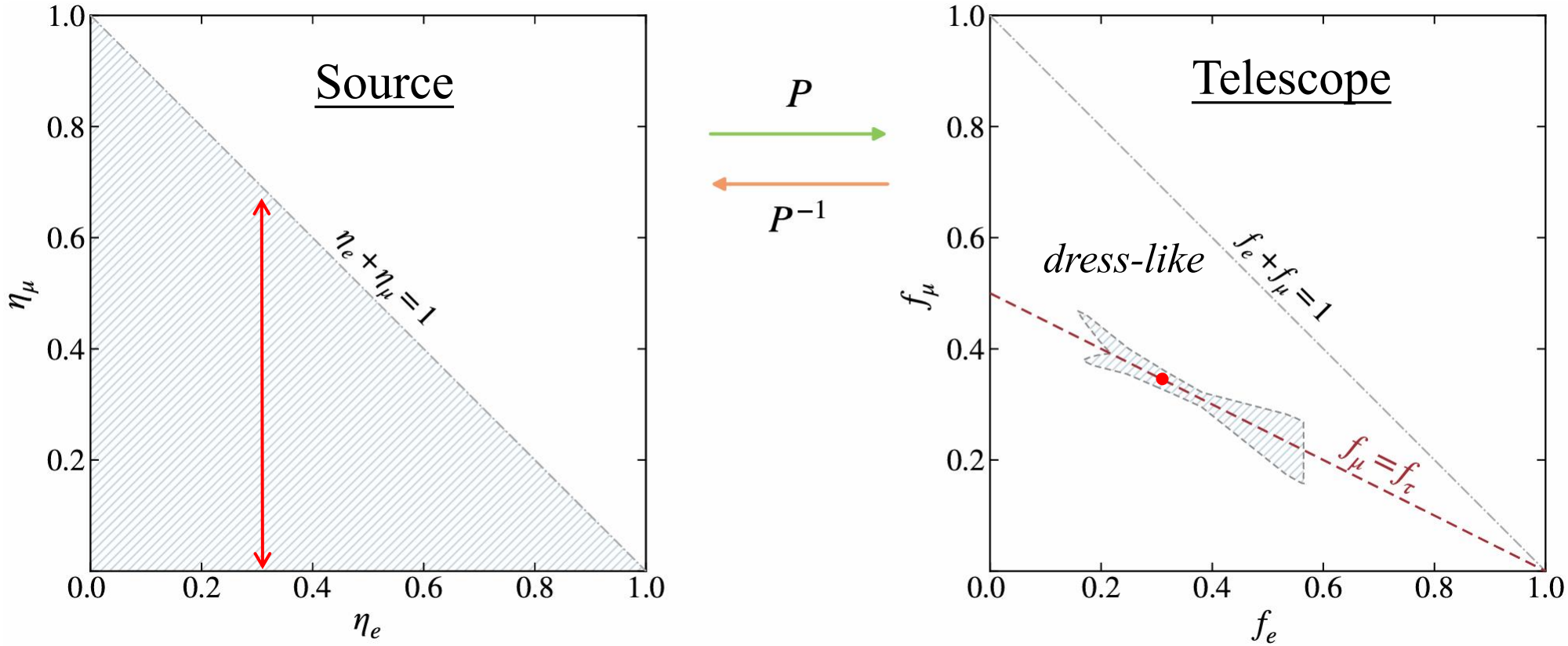
Future JUNO、DUNE/Hyper-K may give $(\det P)_{min}$ = ?

Currently, most det P < 0.01

=> strong parameter degeneracy

Mapping: source & telescope

- **Current result on e-μ pannel** (rare τ = 1-e-μ)**:**



- Forward: Shink to 3ν flavor allowed dress-like region (U_{PMNS} vary in NuFIT 6.1 3σ)
- Inverse: Source reconstruction: $\eta = P^{-1} f$ (failed if det P = 0)
- e.g. μ-τ refl. : $P_{\alpha\mu} \approx P_{\alpha\tau}$, surpressed to $f_\mu = f_\tau$ line

Information loss: $\eta_{\mu,\tau}$ can't be fully restored from $f_{\mu,\tau}$

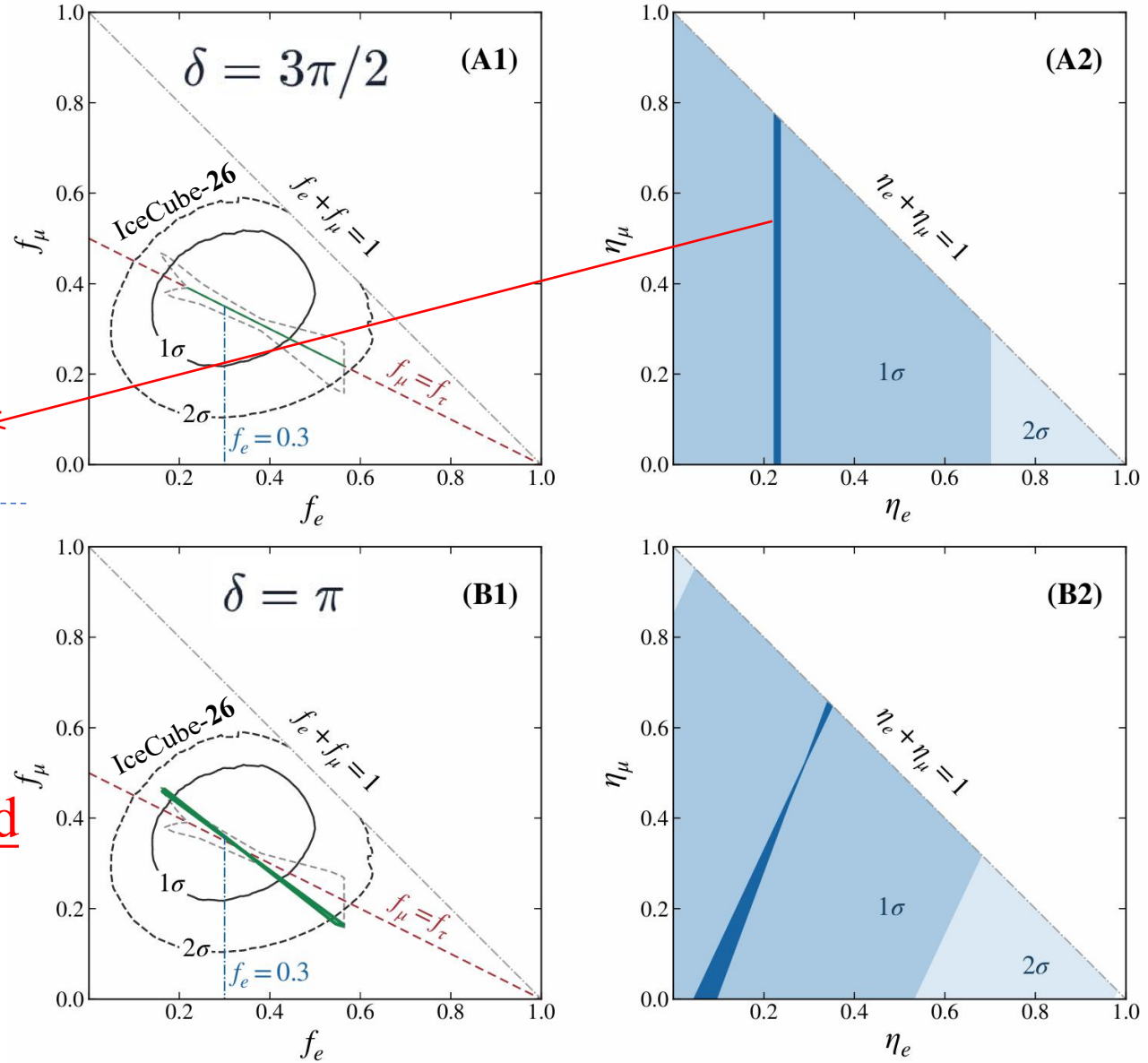
det P=0 : η reconstruction

- **Linear relation for f and η:**

$$f_\mu = \frac{f_e(P_{\mu\mu} - P_{\mu\tau}) + P_{e\mu}P_{\mu\tau} - P_{e\tau}P_{\mu\mu}}{P_{e\mu} - P_{e\tau}} = \frac{1 - f_e}{2} + \frac{(3f_e - 1)\cos 2\theta_{23}}{2(1 - 2\tan^2\theta_{13})},$$

$$\eta_\mu = \frac{\eta_e(P_{e\tau} - P_{ee}) + f_e - P_{e\tau}}{P_{e\mu} - P_{e\tau}}$$

$$= \frac{1 - \eta_e}{2} - \frac{1 - 2\tan^2\theta_{13}}{2\cos 2\theta_{23}}\left[\eta_e - \frac{4f_e - \sin^2 2\theta_{12}\cos^4\theta_{13} - \sin^2 2\theta_{13}}{3\cos^2 2\theta_{12}\cos^4\theta_{13} + (1 - 3\sin^2\theta_{13})^2}\right].$$



Two benchmarks:

(A) μ-τ reflection symmetry limit:

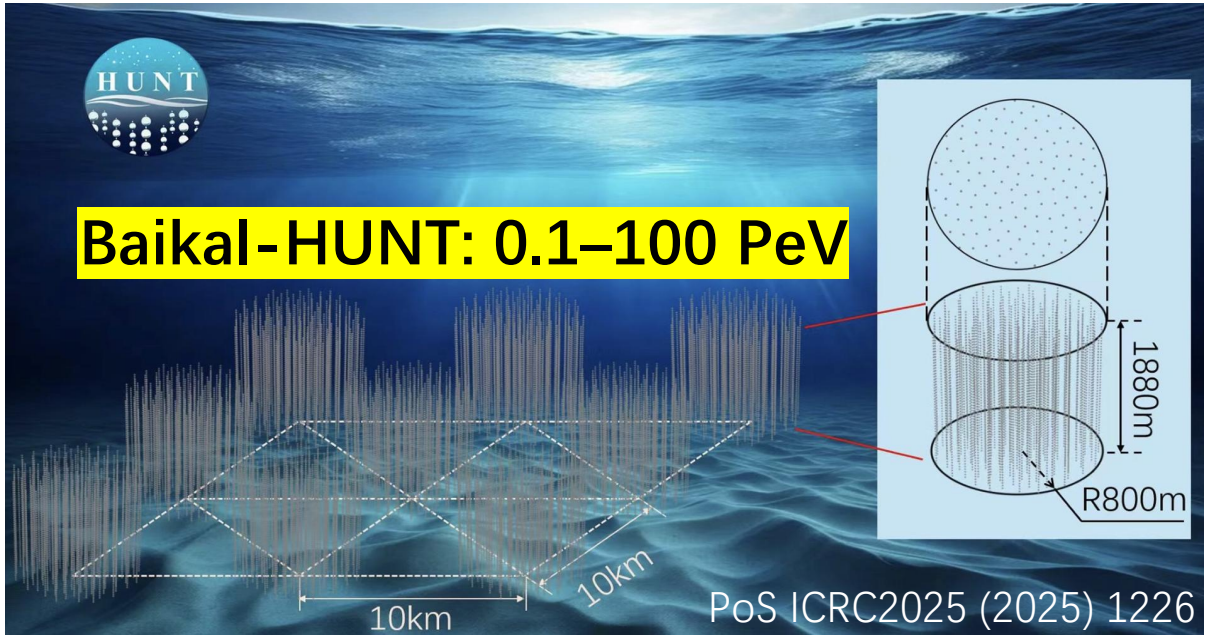
- fμ= fτ, information lost
- ηe solved, partition of μ-τ fully unconstrained

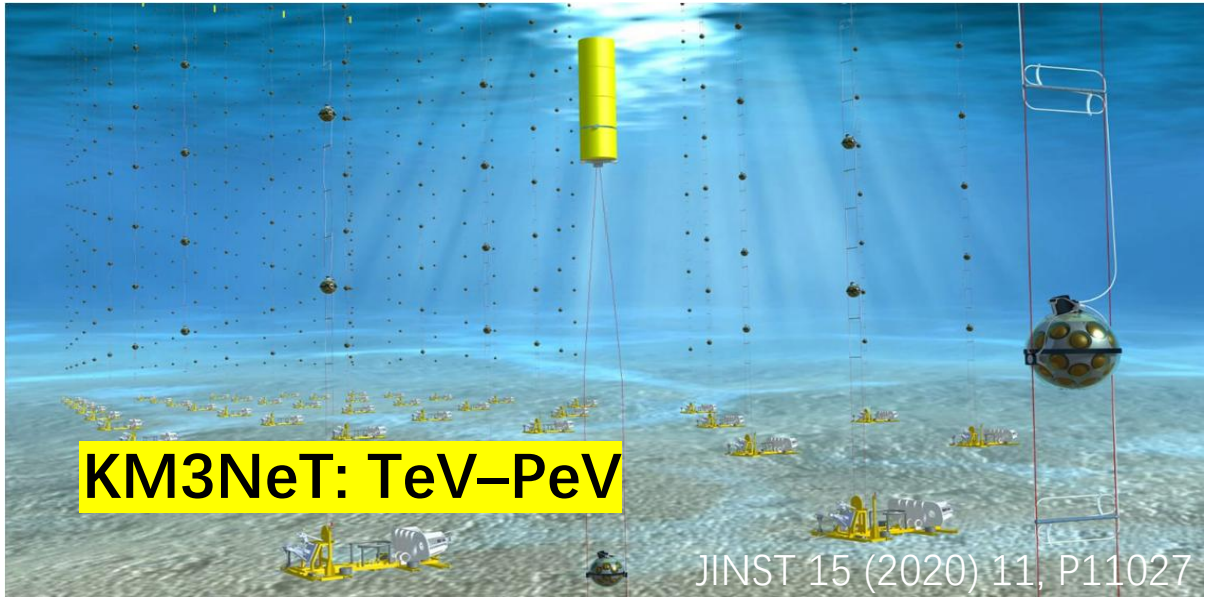
(B) other case of det P = 0

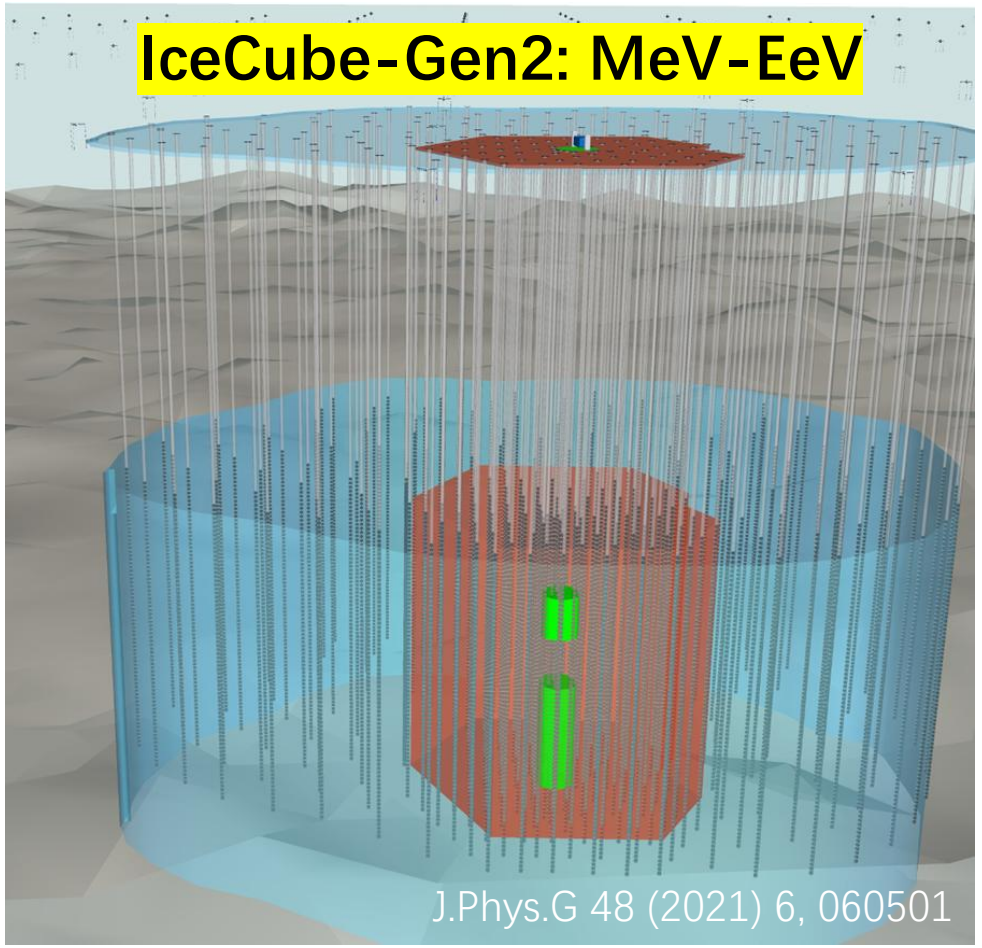
- 3σ ν data => linear band on telescope
- (ηe, ημ) ~ (0.08, 0) to (0.35, 0.65)

Call for future UHEν observation !

Future Observations







A new era in neutrino astronomy has begun!

Maarten de Jong (2015)

Summary

1) Current ν data still broadly allow det P = 0

   => Induce unavoidable parameter degeneracy on neutrino telescope

2) det P = 0: Source η can be reconstructed as linear correlation

3) In the μ-τ reflection limit: η_μ-η_τ fully unconstrained.

4) Future JUNO、DUNE/Hyper-K may give det P = ?

5) Waiting for IceCube-Gen2 to improve η reconstruction

Thanks for your listening!

Questions are welcomed

Analytical expression

$$
\begin{aligned}
P_{ee} &= c_{12}^4 c_{13}^4 + s_{12}^4 c_{13}^4 + s_{13}^4 \,, \\
P_{e\mu} &= c_{13}^2 s_{13}^2 s_{23}^2 + c_{12}^2 c_{13}^2 \left(s_{12}^2 c_{23}^2 + c_{12}^2 s_{13}^2 s_{23}^2 + 2 c_{12} s_{12} s_{13} c_{23} s_{23} \cos\delta_\nu\right) \\
&\quad + s_{12}^2 c_{13}^2 \left(c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2 c_{12} s_{12} s_{13} c_{23} s_{23} \cos\delta_\nu\right) \,, \\
P_{e\tau} &= c_{13}^2 s_{13}^2 c_{23}^2 + c_{12}^2 c_{13}^2 \left(s_{12}^2 s_{23}^2 + c_{12}^2 s_{13}^2 c_{23}^2 - 2 c_{12} s_{12} s_{13} c_{23} s_{23} \cos\delta_\nu\right) \\
&\quad + s_{12}^2 c_{13}^2 \left(c_{12}^2 s_{23}^2 + s_{12}^2 s_{13}^2 c_{23}^2 + 2 c_{12} s_{12} s_{13} c_{23} s_{23} \cos\delta_\nu\right) \,, \\
P_{\mu\mu} &= c_{13}^4 s_{23}^4 + \left(s_{12}^2 c_{23}^2 + c_{12}^2 s_{13}^2 s_{23}^2 + 2 c_{12} s_{12} s_{13} c_{23} s_{23} \cos\delta_\nu\right)^2 \\
&\quad + \left(c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2 c_{12} s_{12} s_{13} c_{23} s_{23} \cos\delta_\nu\right)^2 \,, \\
P_{\mu\tau} &= c_{13}^4 c_{23}^2 s_{23}^2 + \left(s_{12}^2 c_{23}^2 + c_{12}^2 s_{13}^2 s_{23}^2 + 2 c_{12} s_{12} s_{13} c_{23} s_{23} \cos\delta_\nu\right) \\
&\quad \times \left(s_{12}^2 s_{23}^2 + c_{12}^2 s_{13}^2 c_{23}^2 - 2 c_{12} s_{12} s_{13} c_{23} s_{23} \cos\delta_\nu\right) \\
&\quad + \left(c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2 c_{12} s_{12} s_{13} c_{23} s_{23} \cos\delta_\nu\right) \\
&\quad \times \left(c_{12}^2 s_{23}^2 + s_{12}^2 s_{13}^2 c_{23}^2 + 2 c_{12} s_{12} s_{13} c_{23} s_{23} \cos\delta_\nu\right) \,, \\
P_{\tau\tau} &= c_{13}^4 c_{23}^4 + \left(s_{12}^2 s_{23}^2 + c_{12}^2 s_{13}^2 c_{23}^2 - 2 c_{12} s_{12} s_{13} c_{23} s_{23} \cos\delta_\nu\right)^2 \\
&\quad + \left(c_{12}^2 s_{23}^2 + s_{12}^2 s_{13}^2 c_{23}^2 + 2 c_{12} s_{12} s_{13} c_{23} s_{23} \cos\delta_\nu\right)^2 \,.
\end{aligned}
$$

- <u>μ-τ refl. symmetry limit:</u>

$$\theta_{23} = \frac{\pi}{4} \,, \quad \delta_\nu = -\frac{\pi}{2} \pmod{2\pi} \,,$$

$$P_* = \frac{1}{4} \begin{pmatrix} 4P_{ee} & 2\left(1-P_{ee}\right) & 2\left(1-P_{ee}\right) \\ 2\left(1-P_{ee}\right) & 1+P_{ee} & 1+P_{ee} \\ 2\left(1-P_{ee}\right) & 1+P_{ee} & 1+P_{ee} \end{pmatrix}$$

<u>η_μ-η_τ contribution totally lost !</u>