



Space-like pion off-shell form factors in the Bethe-Salpeter approach

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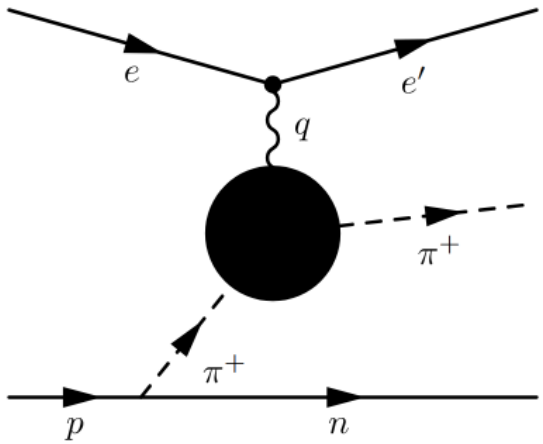
Introduction and Motivation

The pion, beeing the simplest bound system of a quark and an antiquark and having relatively low mass, occupies a special place among the mesons and play a key role in the description of nuclear dynamical processes.
There are many models for describing the pion which decribe its static propeties good.

Among the dynamic characteristics, the electromagnetic (EM) form factor of the pion defined on the mass shell characterizes the spatial distribution of charge inside the pion as a function of the squared four-momentum transfer. At low Q^2, this form factor can be directly accessed through elastic scattering experiments of pions on electrons. However, at intermediate and high values of the squared momentum transfer Q^2, direct extraction becomes difficult.

Introduction and Motivation

An experimental alternative for accessing the form factor in these regions is provided by the exclusive Sullivan process. This method involves analyzing the cross sections of the electromagnetic reaction ${}^1H(e, e'\pi^+)n$. However, since the pion in the intermediate state is virtual, the form factor is half-off-shell.



Introduction and Motivation

Recently, the off-shell pion form factor was extracted from Jefferson Lab data for pion electroproduction on the proton.

In appers [H.-M. Choi et al.Phys.Rev. D 100 (11) (2019) 116020, J. Leao et al. Phys. Rev. D 110 (7) (2024) 074035] the space-like off-shell pion form factors are investigated in the light front quark models within the relativistic impulse approximation without taking into account the interaction current.

In current work the importance of the interaction current contribution is investigated.

Bethe-Salpeter approach with a separable kernel

The Bethe-Salpeter equation for the vertex function of the pion is written as follows

$$\Gamma_{\alpha\beta}(k;p) = i\int \frac{d^4k''}{(2\pi)^4} V_{\alpha\beta:\epsilon\lambda}(k,k'';p) S_{\lambda\eta}(k''+p/2)\Gamma_{\eta\zeta}(k'';p) S_{\zeta\epsilon}(k''-p/2). \quad (1)$$

A separable interaction kernel of rank one is considered in the following form

$$V_{\alpha\beta:\delta\gamma}(k',k;p) = \Delta_{\alpha\beta}(k';p)\overline{\Delta}_{\delta\gamma}(k;p).$$

with $\Delta_{\alpha\beta}(k;p) = f(k^2, k\cdot p)\,\Omega_{\alpha\beta}$,
For pion: $\Omega_{\alpha\beta} = \gamma^5_{\alpha\beta}$ and the following types of radial part are considered:

$$f(k^2) = \begin{cases} (k^2 - \Lambda^2 + i\epsilon)^{-1} - \text{monopole} \\ (k^2 - \Lambda^2 + i\epsilon)^{-2} - \text{dipole} \end{cases}$$

the solution for the vertex function $\pi q\bar{q}$ is:

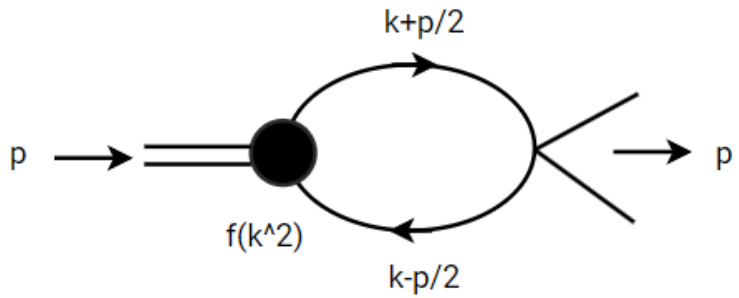
$$\Gamma(k;p) \equiv \Gamma(k) = N\gamma^5 f(k^2)\chi_c, \quad (2)$$

where N is the normalization constant and $\chi_c = (1/\sqrt{N_c})I_{c'c}$ is normalized wave function for the color singlet.

Pion constants

The following low-energy pion constants are considered to fix parameters of the model:

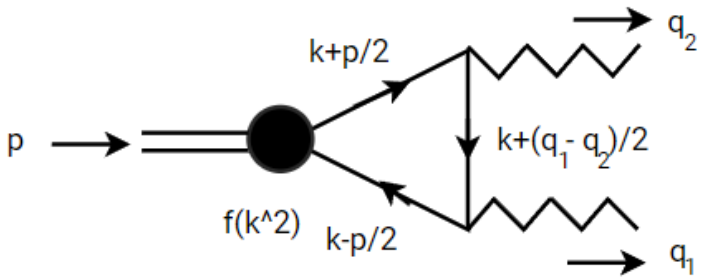
– the pion weak decay constant f_π:



– the decay width of the two-photon decay of the pion:

$$\Gamma_{\pi^0\to\gamma\gamma} = \frac{\pi\alpha^2}{4} m_\pi^3 G^2_{\pi\gamma}(m_\pi^2, 0, 0),$$

where amplitude $G(p^2, q_1^2, q_2^2)$ is:



Pion constants

– the radius of the junction $\gamma^*\pi^0 \to \gamma$:

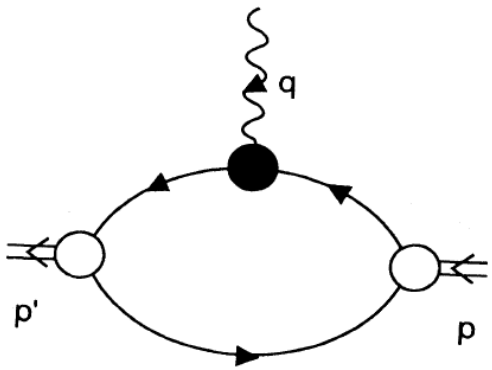
$$< r^2_{\pi\gamma} >= -6\frac{dF_{\pi\gamma}(Q^2)}{dQ^2}/F_{\pi\gamma}(Q^2)\Big|_{Q^2=0},$$

where the transition form factor is $F_{\pi\gamma}(Q^2) = G_{\pi\gamma}(m_\pi^2, Q^2, 0)$.

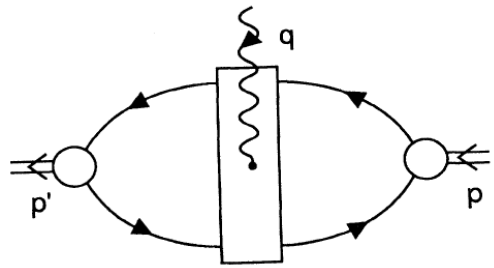
– the charge radius of the pion:

$$< r^2_{\pi} >= -6\frac{d}{dQ^2}F_{\pi}(Q^2)\Big|_{Q^2=0},$$

where $F_\pi(Q^2)$ is pion elastic EM form factor.



relativistic impulse approximation (RIA) $(a) + (\bar{b})$



interaction current (INT)

Pion constants

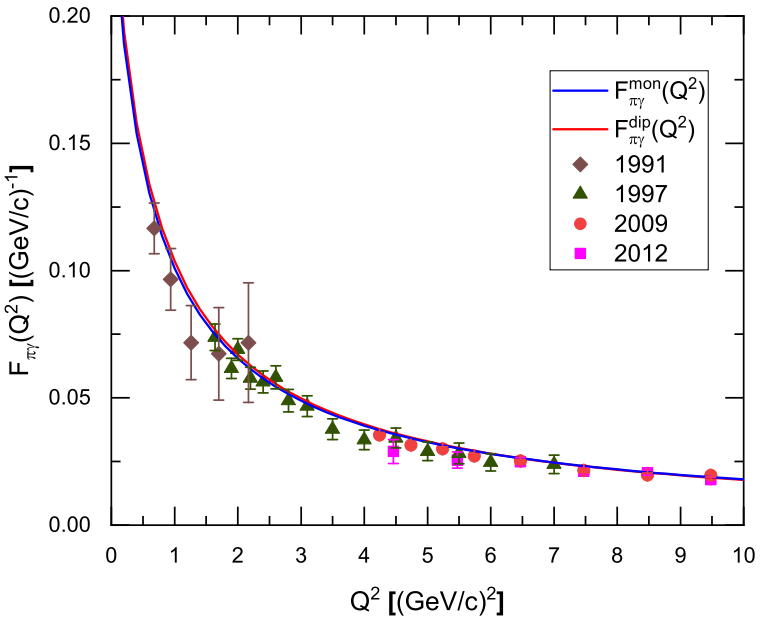
Model parameters: $\Lambda,\ m_q,\ m_\pi = 0.14$ GeV.

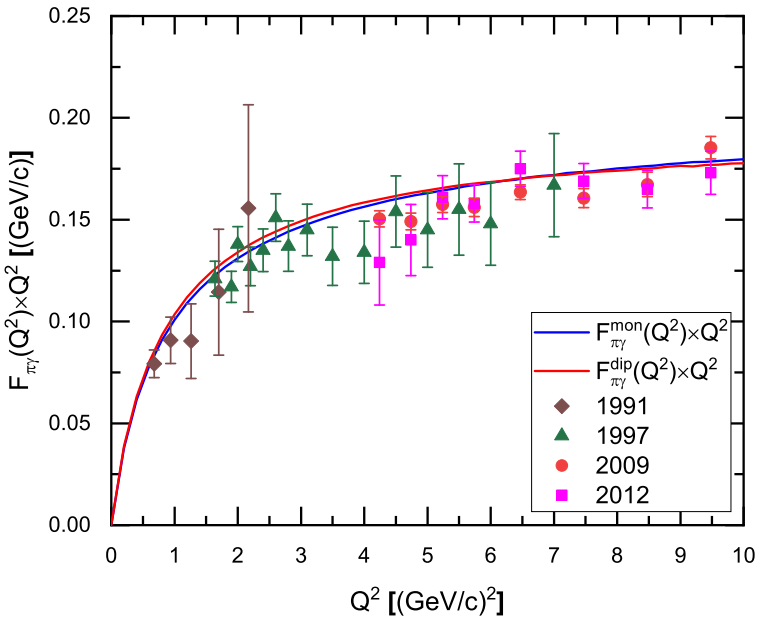
model	m_q (MeV)	Λ (MeV)	f_π (MeV)	$r_{\pi\gamma}$ (fm)	$< r_\pi^2 >$ (fm^2)	$\Gamma_{\pi^0 \to \gamma\gamma}$ (eV)
monopole	260	550	143.363(1)	0.640(1)	0.459(1)	7.25(1)
dipole	245	1100	135.962(1)	0.652(1)	0.438(1)	7.74(1)
[1]	267	400	132	0.655	0.533	—
[2]	300	500	153.7	—	0.41	—
[3]	220	600	130.67	—	0.542	—
Exp. [4]			130.410(2)	0.659(4)	0.434(8)	7.82(1)

[1]—Anikin, I.V., Ivanov, M.A., Kulimanova, N.B., Lyubovitskij, V.E. Z. Phys. C 65, 681–690 (1995)
[2]—Ito, H., Buck, W.W., Gross, F. Phys. Rev. C 45, 1918–1934 (1992)
[3]—Leao, J., Melo, J.P.B.C., Frederico, T., Choi, H.-M., Ji, C.-R. Phys. Rev. D 110(7), 074035 (2024)
[4]—Review of Particle Physics: Particle Data Group / M Tanabashi, PD Grp, K Hagiwara et al. // Physical Review D. — 2018. — 08. — Vol. 98.

Pion transition form factor

Comparison of the $F_{\pi\gamma}(Q^2)$ (left panel) and $F_{\pi\gamma}(Q^2) \times Q^2$ (right panel) with the experimental data.





The Ward-Takahashi identity

The off-shell photon-pion vertex can be represented in the following form:

$$G_\mu(p,p') = (p'+p)_\mu F_1(q^2,p^2,p'^2) + q_\mu F_2(q^2,p^2,p'^2), \tag{3}$$

where $p, p', q = p' - p$ are 4-momenta.
The Ward-Takahashi identity (WTI) is satisfied by this vertex on the off-shell

$$q^\mu G_\mu(p,p') = \Delta^{-1}(p') - \Delta^{-1}(p), \tag{4}$$

where $\Delta(p)$ is renormalized pion propagator. In the case when the initial pion is off-shell, and the final pion is on the mass shell ($p^2 = t; p'^2 = m_\pi^2$), it follows from above equations that

$$F_2(Q^2,t) = \frac{(t-m_\pi^2)[F_1(0,t) - F_1(Q^2,t)]}{Q^2} \tag{5}$$

The g form factor and pion constant

Since $F_2(Q^2, t = m_\pi^2) = 0$, then

$$g(Q^2, t) = \frac{F_2(Q^2, t)}{t - m_\pi^2};$$

$$g(Q^2 = 0, m_\pi) = -\frac{\partial}{\partial Q^2} F_1(Q^2 = 0, m_\pi) = \frac{1}{6}\langle r_\pi^2 \rangle.$$

Model	$g(0, m_\pi^2)$ [GeV^2]
monopole	1.97(2)
dipole	1.87(5)
[3]	2.32
Exp.	1.86(5)

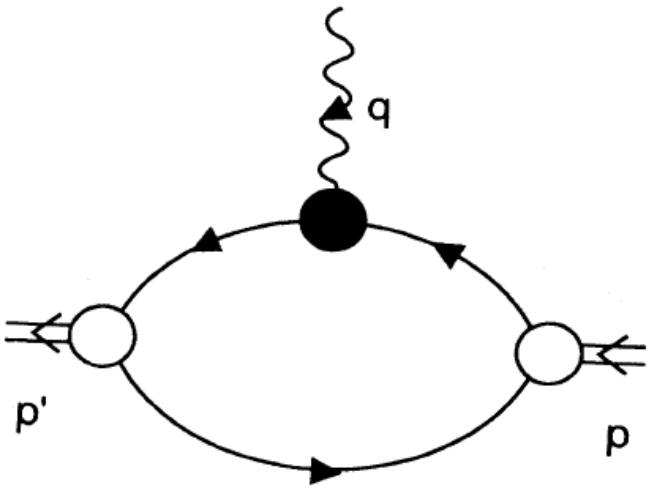
[3]—Leao, J., Melo, J.P.B.C., Frederico, T., Choi, H.-M., Ji, C.-R. Phys. Rev. D 110(7), 074035 (2024)

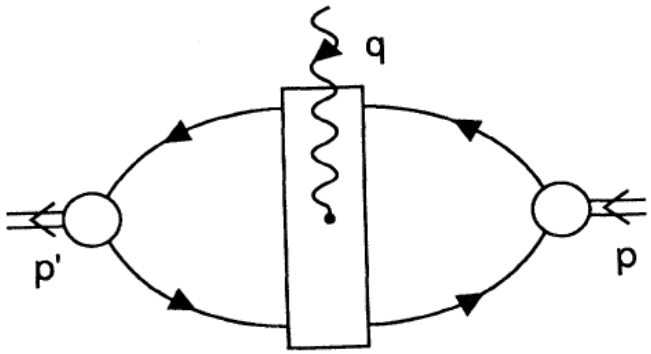
The pion elastic pion form factors

To construct the EM current for pion elastic scattering the minimal substitution in the BS equation used: $p_i^\mu \to p_i^\mu + ie_i A^\mu(x_i)$

from quark propagators $\hat{k}_i \to \hat{k}_i + ie_i\hat{A}$: relativistic impulse approximation (RIA) $(a) + (\bar{b})$

from interaction kernel $f(k^2) \to f(k^2) + ie_i J^{int}_\mu A^\mu$: interaction curent (INT)





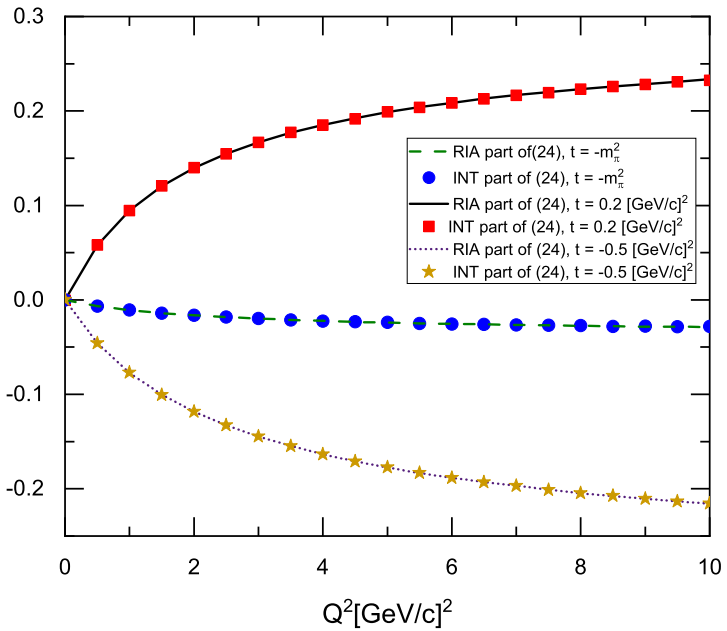
$$F_{(1,2)}(Q^2,t) = F^{\rm RIA}_{(1,2)}(Q^2,t) + F^{\rm int}_{(1,2)}(Q^2,t)$$

$$\int d^4k\ k^\mu\ \tilde{F}(k,Q^2,t) \to (p'+p)^\mu \int d^4k\ \tilde{F}_1(k,Q^2,t)\ +\ q^\mu \int d^4k\ \tilde{F}_2(k,Q^2,t).$$

Numerical verification of the Ward-Takahashi identity at different t

The WTI can be rewtitten in the following form:

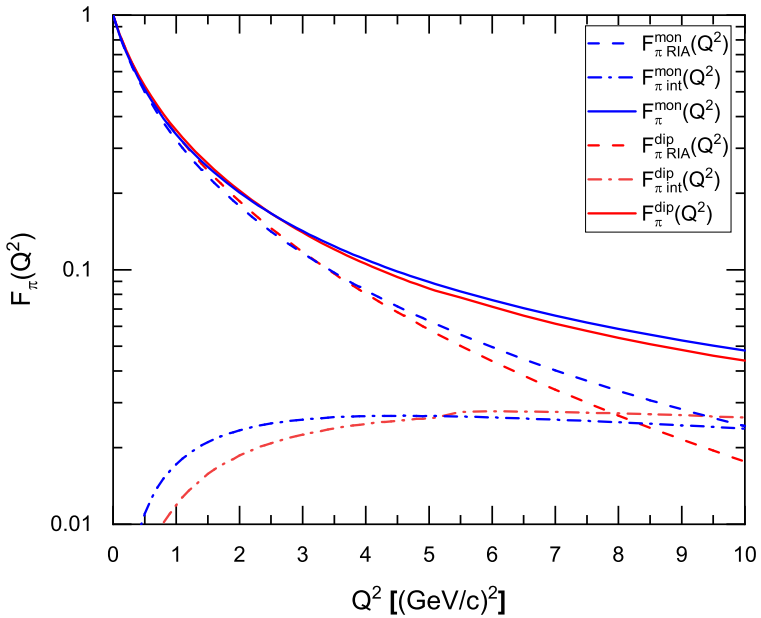
$$(t-m_\pi^2)[F_1^{\mathrm{RIA}}(0)-F_1^{\mathrm{RIA}}(Q^2)]-Q^2F_2^{\mathrm{RIA}}(Q^2)= Q^2F_2^{\mathrm{INT}}(Q^2)-(t-m_\pi^2)[F_1^{\mathrm{INT}}(0)-F_1^{\mathrm{INT}}(Q^2)].$$



The equality holds only if the INT part is taken into account.

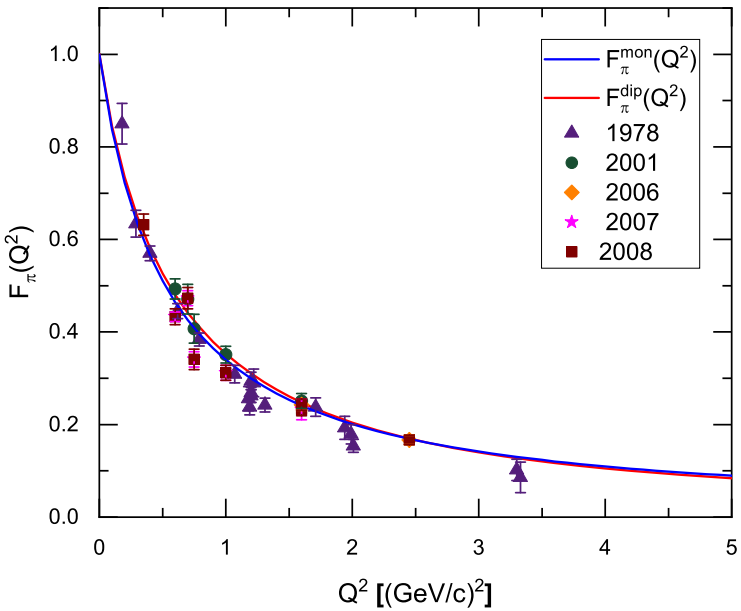
On-mass-shell form factor $F_\pi(Q^2) = F_1(Q^2, m_\pi^2)$

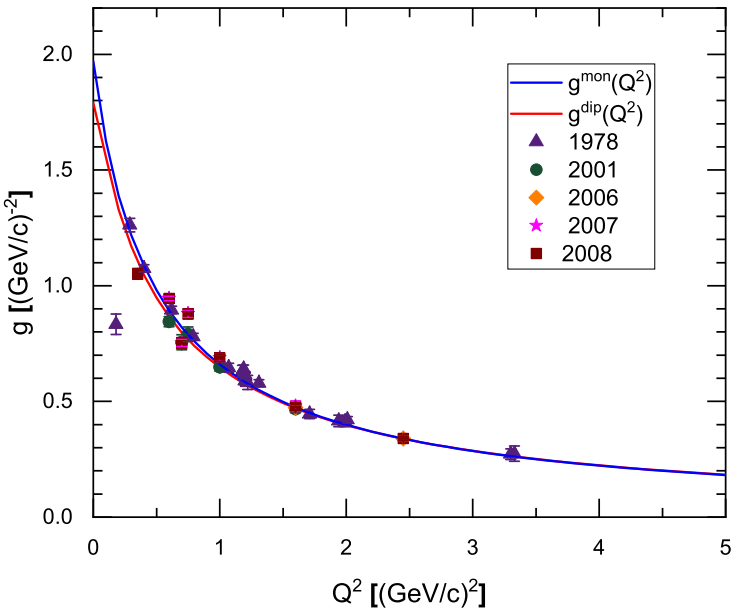
Comparison of the contribution of the relativistic impulse approximation and interaction current to the on-mass-shell form factor $F_\pi(Q^2) = F_1(Q^2, m_\pi^2)$



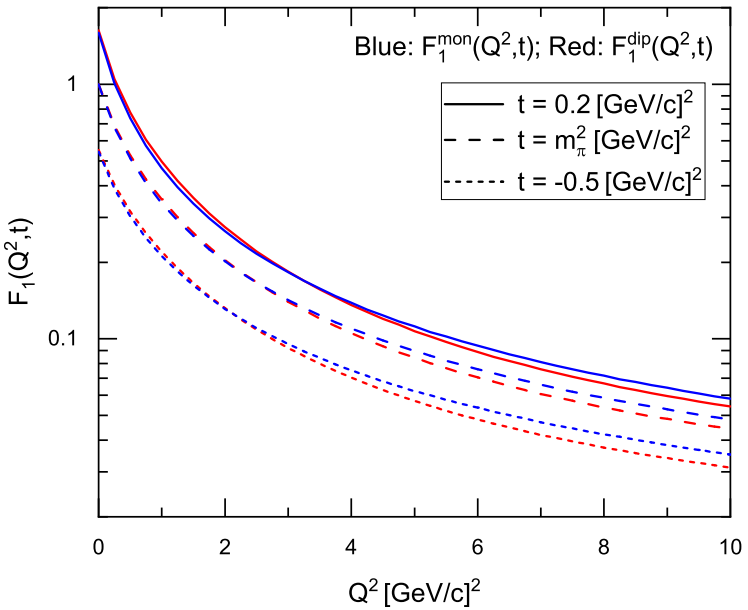
On-mass-shell form factor $F_1(Q^2, m_\pi^2)$ and $g(Q^2, m_\pi^2)$

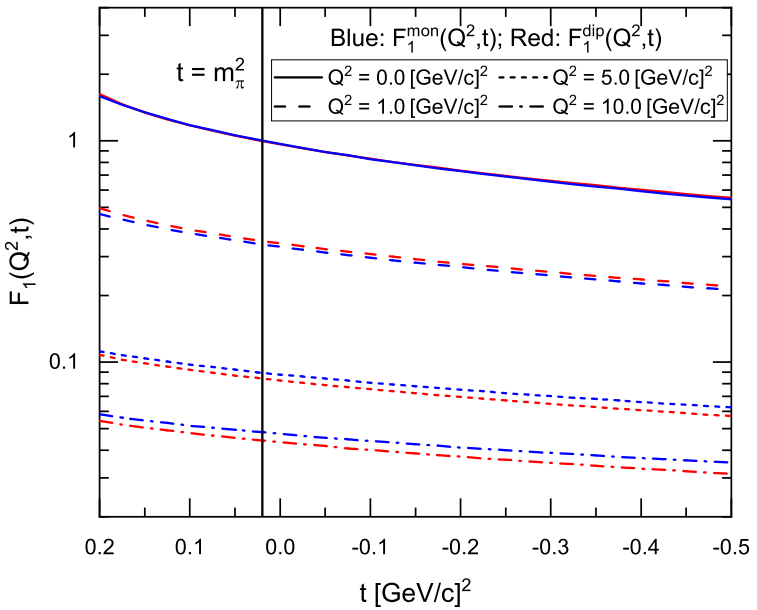
Comparison of the $F_1(Q^2, m_\pi^2)$ (left panel) and $g(Q^2, m_\pi^2)$ (right panel) with the experimental data





Off-shell form factor $F_1(Q^2, t)$ as a function of Q^2 at fixed t (left panel) and of t at fixed Q^2 (right panel)





Longitudinal cross section $\sigma_{\rm L}$ for pion electroproduction

The longitudinal differential cross-section can be expressed in the Born-term model formula from the pion-pole contribution:

$$\frac{d\sigma_{\rm L}}{dt} \sim (eG_{\pi NN})^2 \frac{-tQ^2}{t-m_\pi^2} F_1^2(Q^2,t)$$

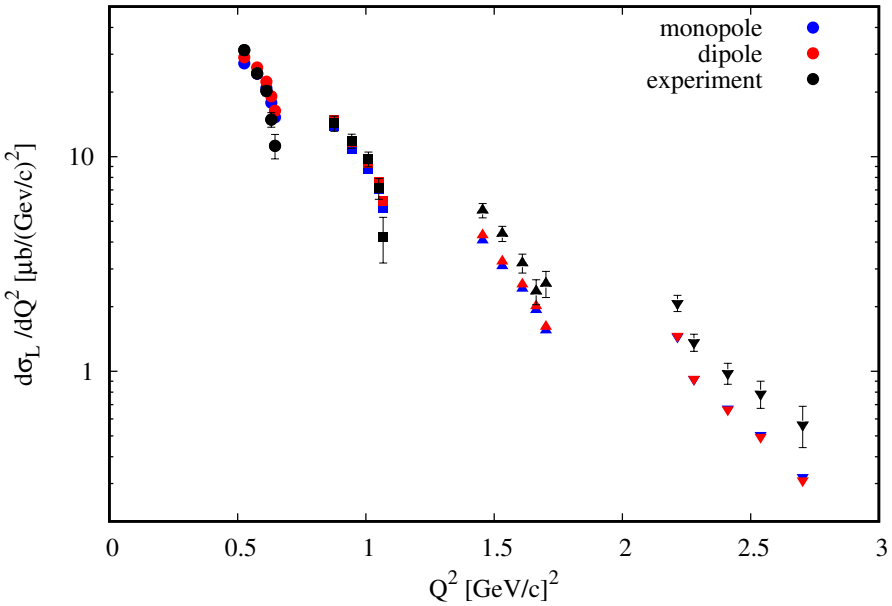
where the form factor $G_{\pi NN}$ has a monopole parametrization:

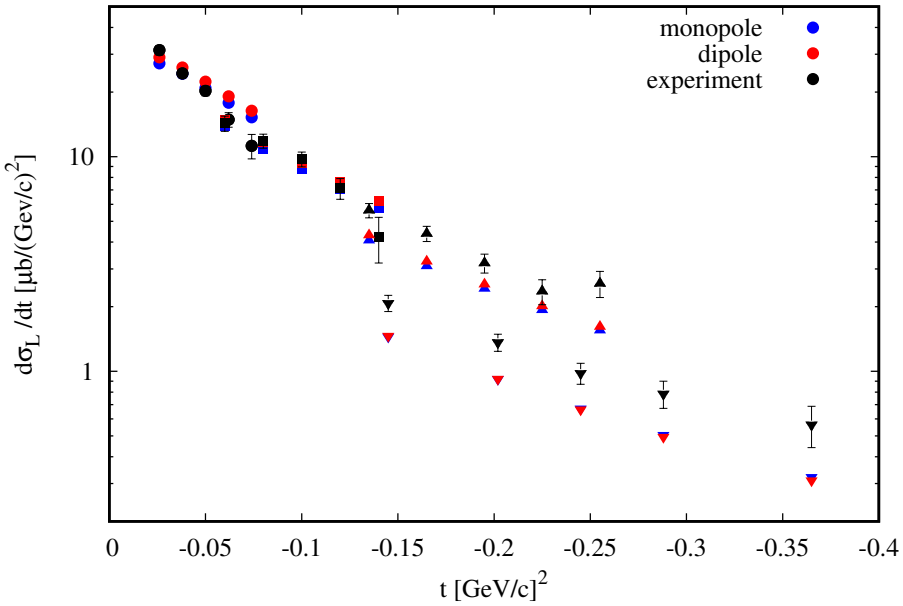
$$G_{\pi NN}(t) = G_{\pi NN}(m_\pi^2) \left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - t} \right),$$

with $G_{\pi NN}(m_\pi^2) = 13.4$ GeV and $\Lambda_\pi^2 = 0.8$ GeV.

Longitudinal cross section σ_{L}

Comparison of the longitudinal cross section $d\sigma_{\mathrm{L}}/dt$ as a function of Q^2 (left panel) and t (right panel) wiith with the experimental data.





Set I: $< Q^2 >= 0.60\ (\mathrm{GeV/c})^2, W = 1.95\ \mathrm{GeV}$
Set III: $< Q^2 >= 1.00\ (\mathrm{GeV/c})^2, W = 1.95\ \mathrm{GeV}$
Set IV: $< Q^2 >= 1.60\ (\mathrm{GeV/c})^2, W = 1.95\ \mathrm{GeV}$
Set VI: $< Q^2 >= 2.45\ (\mathrm{GeV/c})^2, W = 2.22\ \mathrm{GeV}$
W is the invariant mass of the photon-nucleon system.

Conclusions

- Using the Bethe-Salpeter model with a separable kernel, the pion constants and form factors on the on-shell and off-shell surfaces for two types of kernel functions are calculated and compared with experimental data.
- The Ward-Takahashi identity for a pion off-shell form factor has been numerically verified. In addition, in order for the identity to be fulfilled, it was necessary to take into account two contributions: the relativistic impulse approximation and the interaction current.
- The longitudinal differential cross section $d\sigma_L/dt$ of the exclusive Sullivan process was calculated and compared with experimental data.

Bondarenko, S.G., Slautin, M.K.: Pion in the Bethe-Salpeter Approach with Separable Kernel. Phys. Part. Nucl. Lett. 22(5), 1000–1004 (2025)

Bondarenko S., Slautin M. Space-like pion off-shell form factors in the Bethe-Salpeter approach // Memoirs of the Faculty of Physics. No.6, 2560102. 2025.

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