# Cumulative proton production in relativistic proton-deuteron collisions

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***Based on PRC 113, 064611 (2026) [arXiv:2509.22433]***
***and work in progress***

**14th ITP/CAS - BLTP/JINR Joint Workshop on Physics of Strong Interacting Systems**
**Nanjing, China, 18-23.10.2026**

## *Outline:*

- *Introduction: possible mechanisms of cumulative production.*

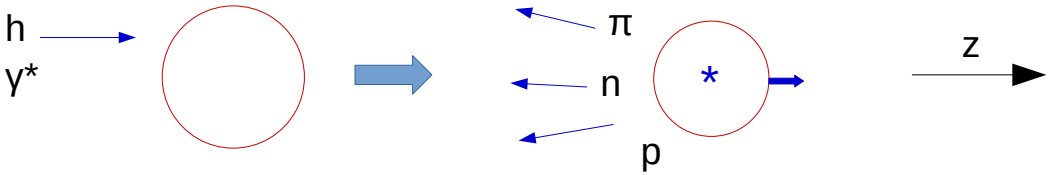
- *The process pd → ppn with backward proton production: one- and two-step rescattering amplitudes with intermediate nucleon and Δ(1232) resonance, generalized eikonal approximation (GEA).*

- *Comparison of energy and momentum spectra of cumulative protons in pd → ppn with experimental data at $\sqrt{s_{NN}} < 5$ GeV.*

- *Deuteron tensor analysing power in pd → ppn.*

- *Summary and outlook.*

# Introduction




Cumulative region is the kinematic region forbidden for production on free nucleon target.
At high energies in the target nucleus rest frame it corresponds to

$$x_F = \frac{\sqrt{m^2 + \boldsymbol{p}^2} - p^z}{m_N} > 1$$

where m is the mass of produced particle and the axis z is along the beam momentum.

For nucleons and light mesons (π,η,K,…) at small $p_t$ the cumulative region covers backward angles. (For heavy particles with $m > m_N$ and/or at large $p_t$ also forward angles are included.)

***Calculations in the impulse approximation (IA) typically describe cumulative production of slow particles but fail for energetic ones.***

Several scenarios are possible:

(I) The incoming particle may interact with a heavy object composed of several nucleons, called ***fluctuon*** that is a density fluctuation in a small volume inside nucleus.
***D. I. Blokhintsev, Sov. Phys. JETP 6, 995 (1958)***;
***V. V. Burov, V. K. Lukyanov, and A. I. Titov, Phys. Lett. B 67, 46 (1977).***
The compact multiquark clusters can also be regarded as fluctuons
***V. V. Burov, V. K. Lukyanov, and A. I. Titov, Z. Phys. A 318, 67 (1984).***

(II) Interaction with ***short-range-correlated NN pairs*** in the nucleus (spectator mechanism).
Light cone approach: ***L. L. Frankfurt and M. I. Strikman, Phys. Lett. B 69, 93 (1977); Phys. Lett. B 83, 407 (1979).***
Non-relativistic approach: ***T. Yukawa and S. Furui, Phys. Rev. C 20, 2316 (1979).***

(III) ***Multistep rescattering processes*** (limiting case - full thermalization).
Hadron cluster (fireball) model: ***M. I. Gorenstein and G. M. Zinovjev, Phys. Lett. B 67, 100 (1977);***
***D. V. Anchishkin, M. I. Gorenstein, and G. M. Zinovjev, Phys. Lett. B 108, 47 (1982).***
Collision rates calculation: ***P. Danielewicz, Phys. Rev. C 42, 1564 (1990).***
Heavy baryonic resonance model: ***A. Motornenko and M. I. Gorenstein, J. Phys. G 44, 025105 (2017).***
UrQMD model: ***O. Panova, A. Motornenko, M. I. Gorenstein, J. Steinheimer, and H. Stoecker, Phys. Rev. C 100, 054617 (2019).***
Hydrodynamic approach: ***A. T. D'yachenko, Phys. Part. Nucl. 55, 786 (2024).***

## *Deuteron target:*

The contribution of ***double scattering*** processes to the backward proton production in pd→ ppn:
Triangular graphs with pion exchange (coherent):
***N.S. Amelin and G.I. Lykasov, Sov. J. Nucl. Phys. 28, 648 (1978);***
Triangular graphs with nucleon and Δ exchange (incoherent):
***V. B. Kopeliovich and V. B. Radomanov, (1978), JINR-P2-11938;***
Nucleon and Δ exchange (coherent):
***Y. Haneishi and T. Fujita, PRC 33, 260 (1986);***
Elastic and inelastic screening and FSI (coherent):
***L. G. Dakhno and V. A. Nikonov, Nucl. Phys. A 491, 652 (1989).***

pd→dp:
one-nucleon exchange, single scattering, and intermediate Δ:
***O. Imambekov, Y. N. Uzikov, and L. V. Shevchenko, Z. Phys. A 332, 349 (1989);***
***Y. N. Uzikov, Phys. Part. Nucl. 29, 583 (1998);***
one-nucleon exchange, relativistic DWF from the Bethe–Salpeter equation:
***L. P. Kaptari, B. Kämpfer, S. M. Dorkin, and S. S. Semikh, PRC 57, 1097 (1998);***
***S. Bondarenko, S. Yurev, Phys. Part. Nucl. Lett. 20, 598 (2023)***

***Backward proton energy spectra and $T_{20}$ not yet fully understood***

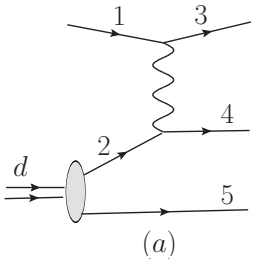
# The process pd → ppn with backward proton production

*AL, PRC 113, 064611 (2026)*

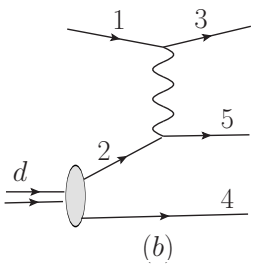
## *Single step (IA) amplitudes*

d - deuteron
1 - incoming proton
2 - struck nucleon
3 - forward proton
4 - ***backward proton***
5 – forward neutron

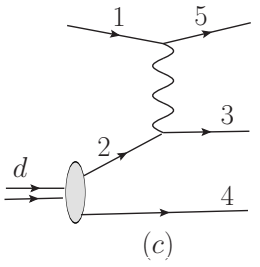
quasielastic knockout amplitude



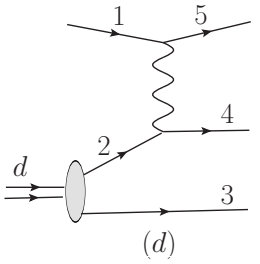

**spectator amplitude**




spectator amplitude with CEX




knockout amplitude with CEX




In the rest frame of d:

$$M_{\rm IA}^{(b)} = \left(\frac{2E_4 m_d}{p_2^0}\right)^{1/2} (2\pi)^{3/2} \sum_{\lambda_2,t_2} \phi_{\lambda_2\lambda_4}(\boldsymbol{p}_2)\underbrace{[\delta_{\frac{1}{2},t_2}\delta_{-\frac{1}{2},t_4} - \delta_{\frac{1}{2},t_4}\delta_{-\frac{1}{2},t_2}]}_{\propto \text{ isospin part of the DWF } (I=0)} M(3,5;1,2)$$

$$E_4 = \sqrt{\boldsymbol{p}_4^2 + m_N^2}, \quad p_2^0 = m_d - E_4, \quad M(3,5;1,2) = 2I_{NN}\sigma_{NN}(i+\rho_{NN})\mathrm{e}^{B_{NN}t/2}\delta_{\lambda_3\lambda_1}\delta_{t_3t_1}\delta_{\lambda_5\lambda_2}\delta_{t_5t_2}\ ,$$

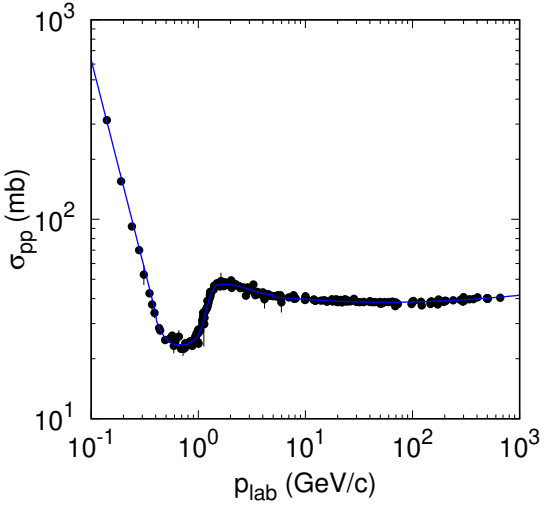
$I_{NN} = m_N\sqrt{s^2/4m_N^2 - s}$ — the Moeller flux factor

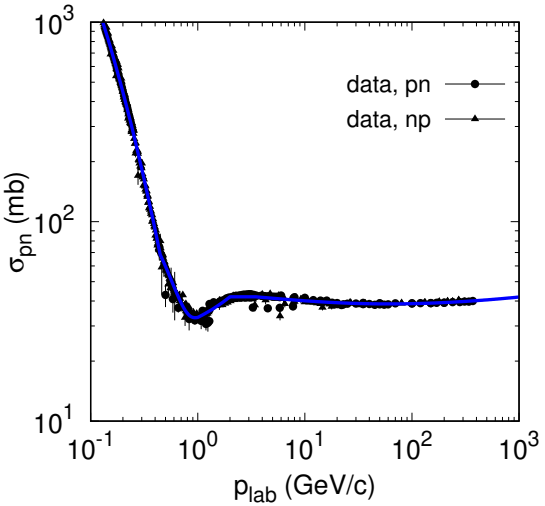
$s = (p_1+p_2)^2$, $t = (p_1-p_3)^2$

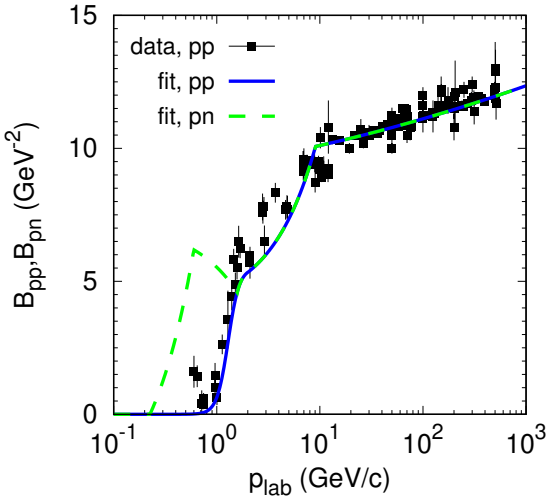
… Similar forms for the amplitudes a,c,d.
Parameters from ***J. Cugnon et al., NIM B 111, 215 (1996).***
Spin structure of CEX np→pn amplitudes
according to ***N.G. Kelkar, B.K. Jain, NPA 612, 457 (1997).***

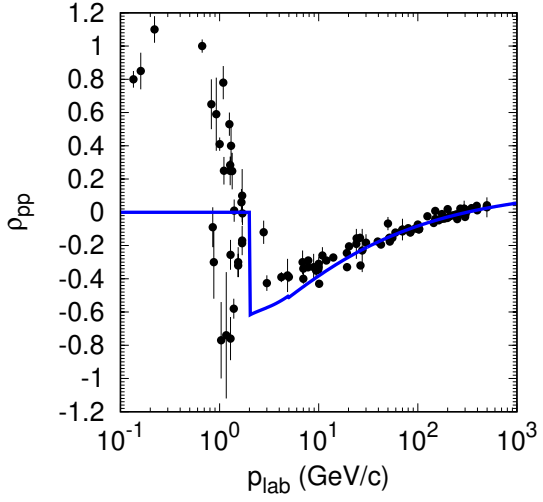
**Apply phenomenological parameterizations assuming scattering on a free nucleon:**

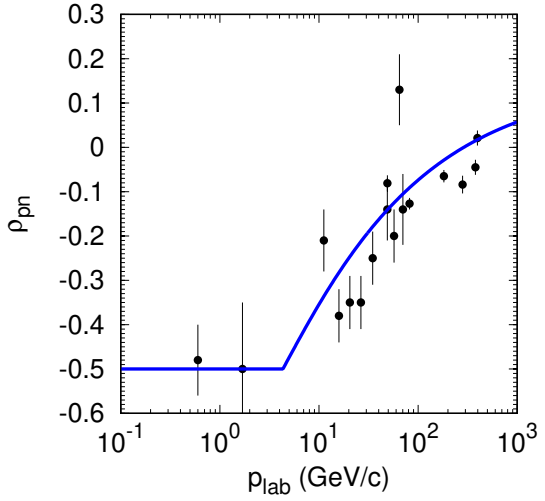
$$\sigma_{NN}(p_{\rm lab}), \rho_{NN}(p_{\rm lab}), B_{NN}(p_{\rm lab}), \quad p_{\rm lab} = \sqrt{s^2/4m_N^2 - s}$$



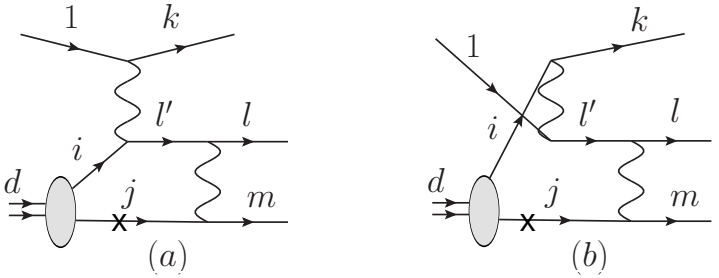




$\sigma_{pp}(\sigma_{pn})$ at $p_{lab}$ < 5 (3.5) GeV/c and
$B_{pp}$,$B_{pn}$ at $p_{lab}$ < 9.1 GeV/c
- fits from ***J. Cugnon et al., NIM B 111, 215 (1996).***
$\sigma_{pp}$,$\sigma_{pn}$,$\rho_{pp}$,$\rho_{pn}$ at high $p_{lab}$
- Regge-Gribov fits from ***C. Patrignani et al. (Particle Data Group), Chin. Phys. C40, 100001 (2016).***
$B_{pp}$,$B_{pn}$ at high $p_{lab}$
- “PYTHIA” parameterization from
***T. Falter et al., PRC70, 054609 (2004)***

## *Elastic rescattering amplitudes*




$k,l,m \in \{3,4,5\}$

$i,j \in \{2,5'\}$

2 - proton
3 - forward proton
4 - backward proton
5,5' - neutrons

$$M_{\rm el} = -\sum_{k,l,m\in\{3,4,5\}} (-1)^{\eta_{k,l,m}} \sum_{i,j\in\{2,5'\}} (-1)^{\eta_{i,j}} \int \frac{d^3p_j}{(2\pi)^{3/2}} \left(\frac{m_d}{2E_j(m_d-E_j)}\right)^{1/2} \phi(-\boldsymbol{p}_j)$$
$$\times \frac{M(l,m;l',j)}{p_{l'}^2 - m_N^2 + i0}[M(k,l';1,i)(1-\delta_{k4}) - M(k,l';i,1)(1-\delta_{l4})]\delta_{t_k+t_{l'},t_1+t_i} \ ,$$

$\eta_{i,j}, \eta_{k,l,m}$ – parities of permutations: $\eta_{2,5'} = \eta_{3,4,5} = 0$

Eikonal form of the inverse propagator of the nucleon (GEA):

$$p_{l'}^2 - m_N^2 + i0 = (p_l + q')^2 - m_N^2 + i0 = 2p_l q' + q'^2 + i0 = 2|\boldsymbol{p}_l|(-q'^{\tilde{z}} + \Delta_l + i0) \ ,$$

$$q' = p_m - p_j, \ \boldsymbol{e}_{\tilde{z}} \uparrow\uparrow \boldsymbol{p}_l$$

$$\Delta_l \equiv \frac{E_l q'^0}{|\boldsymbol{p}_l|} + \frac{q'^2}{2|\boldsymbol{p}_l|} \simeq \frac{(E_l - m_N)(E_m - m_N)}{|\boldsymbol{p}_l|}$$

neglect Fermi motion
in the deuteron

DWF in the Paris potential model ***M. Lacombe et al., PLB 101, 139 (1981)***

$$\phi(\boldsymbol{p}) = \frac{1}{\sqrt{4\pi}} \sum_{n=1}^{n_{\max}} \frac{\phi_n^{\lambda_d}(\boldsymbol{p})}{p^2 + m_n^2} \ , \qquad \phi_n^{\lambda_d}(\boldsymbol{p}) = \left(\frac{2}{\pi}\right)^{1/2} \left(c_n + \frac{d_n}{\sqrt{8}} S(\boldsymbol{p})\right) \chi^{\lambda_d} \ , \qquad S(\boldsymbol{p}) = \frac{3(\boldsymbol{\sigma}_p \boldsymbol{p})(\boldsymbol{\sigma}_n \boldsymbol{p})}{p^2} - \boldsymbol{\sigma}_p \boldsymbol{\sigma}_n$$

$n_{\max} = 13$

spin tensor operator

spin WF of the S=1 pn state with spin projection $\lambda_d$=0,±1

- neglect the dependence of the elastic rescattering amplitudes on the longitudinal (along $\tilde{z}$) component of the momentum of intermediate spectator j and perform contour integration over $p_j^{\tilde{z}}$ closing contour in the upper part of the complex plane :

$$\int \frac{dp_j^{\tilde{z}}}{2\pi} \frac{1}{(p_j^{\tilde{z}} - p_m^{\tilde{z}} + \Delta_l + i0)[(p_j^{\tilde{z}})^2 + m_{nt}^2]} = \frac{1}{2m_{nt}(im_{nt} - p_m^{\tilde{z}} + \Delta_l)} \ , \qquad m_{nt} \equiv \sqrt{\boldsymbol{p}_{jt}^2 + m_n^2}$$

$$M_{\rm el} = -\frac{1}{8\sqrt{2}\pi} \sum_{k,l,m \in \{3,4,5\}} (-1)^{\eta_{k,l,m}} |\boldsymbol{p}_l|^{-1} \sum_{i,j \in \{2,5'\}} (-1)^{\eta_{i,j}} \int d^2 q_t' \left(\frac{m_d}{2E_j(m_d - E_j)}\right)^{1/2}$$

$$\times M(l,m;l',j)[M(k,l';1,i)(1-\delta_{k4}) - M(k,l';i,1)(1-\delta_{l4})]\delta_{t_k + t_{l'}, t_1 + t_i} \sum_{n=1}^{n_{\max}} \frac{\phi_n^{\lambda_d}(-\boldsymbol{p}_j)}{m_{nt}(im_{nt} - p_m^{\tilde{z}} + \Delta_l)} \ ,$$

$$\boldsymbol{p}_j \equiv \hat{A}(\boldsymbol{p}_{jt}, im_{nt}),$$

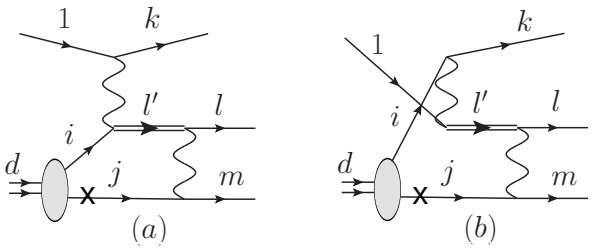
$$\boldsymbol{p}_{jt} = \boldsymbol{p}_{mt} - \boldsymbol{q}_t', \quad \boldsymbol{q}_t' \perp \boldsymbol{p}_l$$

$\hat{A}$ - rotation matrix to the original coord. frame with z (spin quantization axis) along $\mathbf{p}_1$

# *Inelastic rescattering amplitudes*

- include only Δ(1232) intermediate state

- OPEM with couplings and vertex formfactors from ***V. Dmitriev et al, NPA 459, 503 (1986)***




$i,j \in \{2,5'\}$
$k,l,m \in \{3,4,5\}$

2 - proton
3 - outgoing forward proton
4 - outgoing backward proton
5,5' - neutrons

$$M_{\rm inel} = \sum_{i,j\in\{2,5'\}} \sum_{k,l,m\in\{3,4,5\}} (-1)^{\eta_{i,j}} (-1)^{\eta_{k,l,m}} \int \frac{d^3p_j}{(2\pi)^{3/2}} \left(\frac{m_d}{2E_j(m_d-E_j)}\right)^{1/2} \phi(-\boldsymbol{p}_j)$$
$$\times\left[(1-\delta_{k4})M(k,l,m;1,i,j) - (1-\delta_{l4})M(k,l,m;i,1,j)\right] ,$$

$$M(k,l,m;1,i,j) = -\left(\frac{f_{\pi NN} f_{\pi N\Delta} F(q^2) F(q'^2)}{m_\pi^2}\right)^2 \frac{\bar u(p_k,\lambda_k) \not q \gamma^5 u(p_1,\lambda_1)\, \bar u(p_m,\lambda_m) \not q' \gamma^5 u(p_j,\lambda_j)}{(q^2-m_\pi^2+i0)(q'^2-m_\pi^2+i0)}$$
$$\times \bar u(p_l,\lambda_l) G^{\mu\nu}(p_{l'}) q'_\mu q_\nu u(p_i,\lambda_i)\, \mathcal{I}_{\rm iso} ,$$

$q = p_1 - p_k,\ q' = p_m - p_j,$

$$G^{\mu\nu}(p) = -\frac{(\not p + m_\Delta)}{p^2 - m_\Delta^2 + i\Gamma_\Delta m_\Delta}\left(g^{\mu\nu} - \frac{\gamma^\mu\gamma^\nu}{3} - \frac{2p^\mu p^\nu}{3m_\Delta^2} + \frac{p^\mu\gamma^\nu - p^\nu\gamma^\mu}{3m_\Delta}\right)$$

- Rarita-Schwinger propagator of the Δ resonance, $\Gamma_\Delta = 0.118$ GeV,

$$\mathcal{I}_{\rm iso} = (\boldsymbol{T}^\dagger_{t_l t_{l'}} \cdot \boldsymbol{\tau}_{t_m t_j})(\boldsymbol{\tau}_{t_k t_1} \cdot \boldsymbol{T}_{t_{l'} t_i}) .$$

$$\boldsymbol{T}_{t_\Delta t_N} = \sum_{l=0,\pm1} < \frac{3}{2} t_\Delta | \frac{1}{2} t_N; 1l > \boldsymbol{t}^{(l)*}$$

- isospin 1/2 → 3/2 transition operator, $\boldsymbol{t}^{(0)} = (0,0,1), \boldsymbol{t}^{(\pm1)} = \mp\frac{1}{\sqrt{2}}(1,\pm i,0)$

***G. E. Brown and W. Weise, Phys. Rept. 22, 279 (1975)***

## *The differential cross section of backward proton production*

$$d\sigma = \frac{(2\pi)^4 \overline{|M|^2}}{4 I_{1d}} d\Phi \ ,$$

$$I_{1d} = [(p_1 p_d)^2 - m^2 m_d^2]^{1/2}$$ - flux factor,

$$d\Phi = \delta^{(4)}(p_1 + p_d - p_3 - p_4 - p_5) \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} \frac{d^3 p_5}{(2\pi)^3 2E_5}$$

- the invariant three-body phase space volume element,

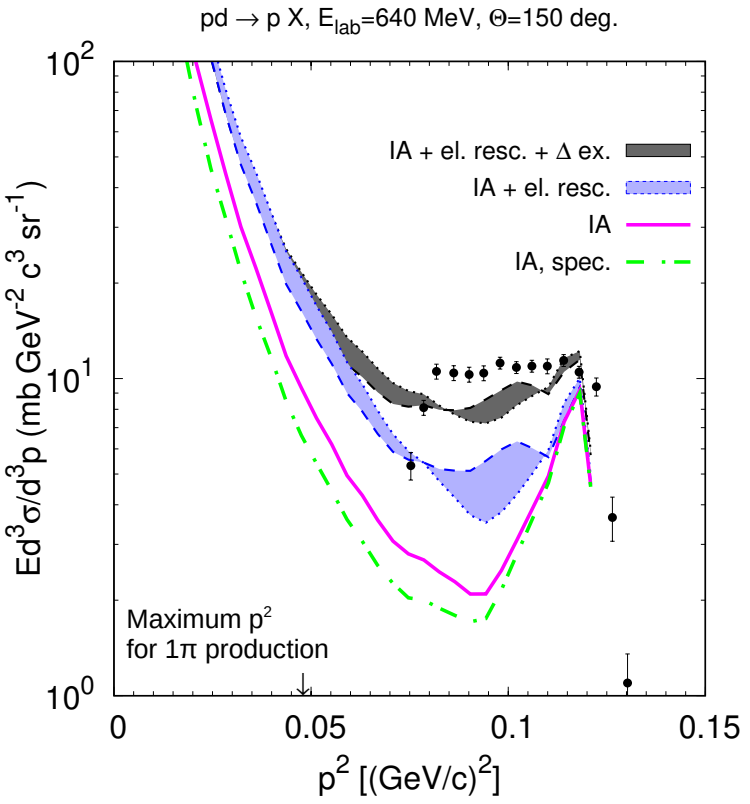
$$E\frac{d\sigma}{d^3 p} = E\frac{(2\pi)^4}{4 I_{1d}} \underbrace{\langle \overline{|M|^2} \delta^{(3)}(\boldsymbol{p} - \boldsymbol{p}_4) \rangle}_{\text{Calculated by Monte-Carlo}} \Phi \ , \quad \Phi = \int d\Phi$$

- *IA strongly underestimates backward proton production except at the kinematic limit.*

- *Elastic rescattering increases the cross section by 2-3 times.*

- *Intermediate Δ gives further increase by ~50%.*

- *On-shell setting for NN→ NN and NN ↔ NΔ amplitudes (dashed lines) gives a more pronounced enhancement as compared to the quasi-free setting (dotted lines).*



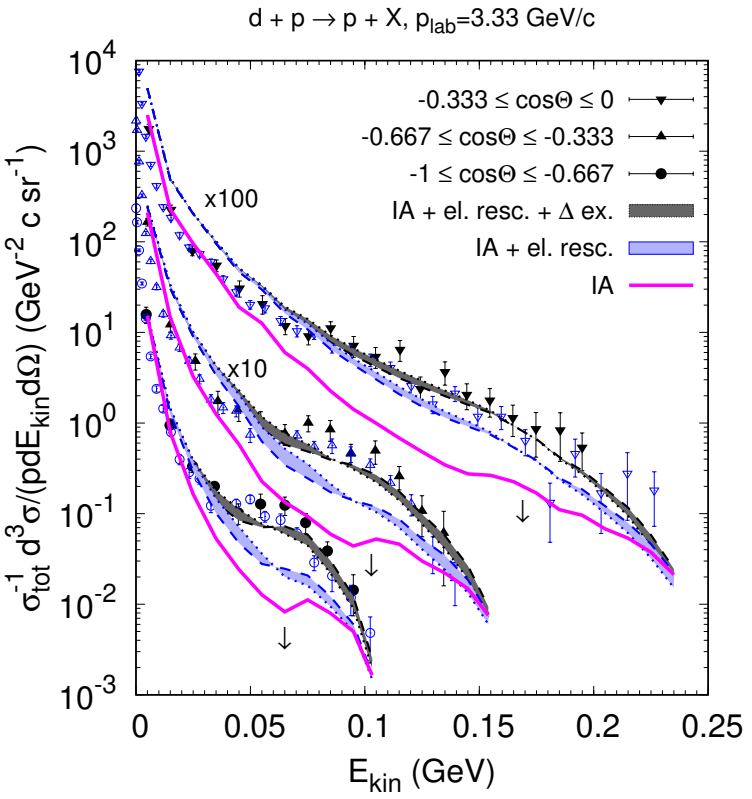

*AL, PRC 113, 064611 (2026)*

Data (JINR): ***J. Ero et al, NPA 367, 419 (1981)***

Corresponds to the incident proton energy 0.97 GeV in the deuteron r.f.

- ***IA describes low-energy part of the proton spectra in the backward hemisphere in the deuteron rest frame, but totally fails for high-energy part.***

- ***Elastic and inelastic intermediate states substantially increase cross section at large $E_{kin}$ but underestimation by a factor of 2 still remains.***




$E_{kin}$ – proton energy in the deuteron r.f.

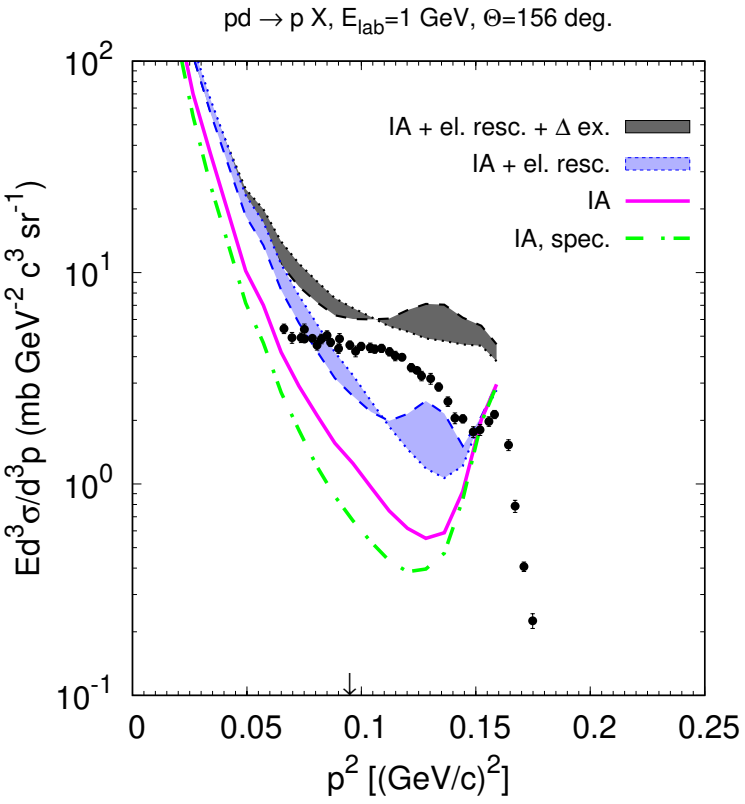
Θ – polar angle of proton momentum relative to the incident proton momentum in the deuteron r.f.

Data (JINR):
***B.S. Aladashvili et al., Yad. Fiz. 27, 704 (1978);***
***V.V. Glagolev et al., Z. Phys. A 357, 107 (1997)***

***AL, PRC 113, 064611 (2026)***

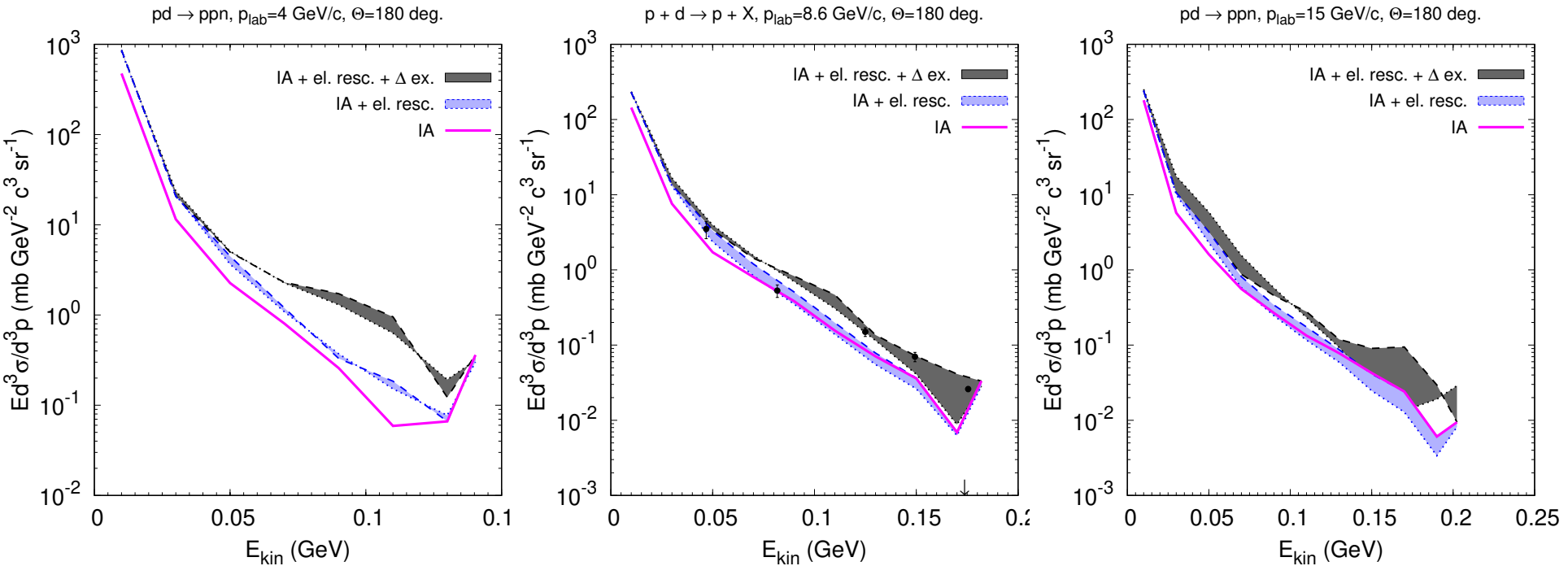
***- strong enhancement due to intermediate Δ***

***- overestimation at kinematic limit***




Data (Gatchina):
***M. N. Andronenko et al., JETP Lett. 37, 530 (1983)***

***AL, PRC 113, 064611 (2026)***




***- the sensitivity to the choice of kinematics of the NN ↔ NΔ amplitudes grows with beam momentum due to the sharp momentum-transfer dependence of the pion propagators***

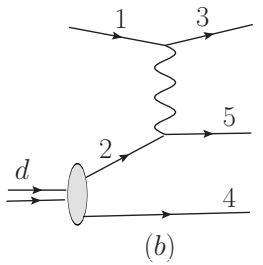
***- the data at 8.6 GeV/c are best described with on-shell kinematics for intermediate Δ***

Data at 8.6 GeV/c (JINR):
***A. M. Baldin et al., 1977, JINR-P1-11168***

***AL, PRC 113, 064611 (2026)***

# Deuteron tensor analysing power

$$T_{20} = \frac{A_{zz}}{\sqrt{2}} = \frac{\sigma_z(+1) + \sigma_z(-1) - 2\sigma_z(0)}{\sqrt{2}[\sigma_z(+1) + \sigma_z(-1) + \sigma_z(0)]}$$




d - deuteron
1 - incoming proton
2 - struck nucleon
3 - forward proton
4 - ***backward proton***
5 – forward neutron

$\sigma_z(\lambda)$ - the reaction cross section for the projection of the deuteron spin λ=±1,0 on the axis z (along beam proton momentum in the deuteron rest frame)

In the kinematics of cumulative proton production, if only the IA spectator diagram is included:

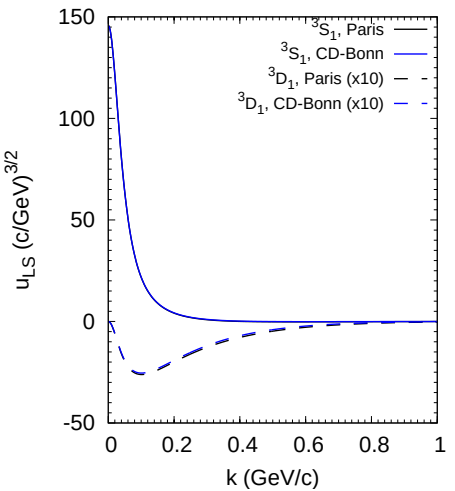
$$T_{20}^{\mathrm{IA,spec.}} = \frac{\sqrt{2}(\sqrt{2}u(k)w(k) - w^2(k)/2)}{u^2(k) + w^2(k)}$$

$u(k)$ and $w(k)$ - S- and D-wave components of the DWF,

***k*** - internal momentum defined in the light cone formalism,
c.f. ***L. L. Frankfurt and M. I. Strikman, NPB 148, 107 (1979)***

$$\alpha = \frac{2(E_4 - p_4^z)}{m_d} = 1 + \frac{k^z}{\sqrt{m_N^2 + \boldsymbol{k}^2}}\ , \quad \boldsymbol{k}_t = -\boldsymbol{p}_{4,t}$$

(In the non-relativistic limit $\boldsymbol{k} \rightarrow -\boldsymbol{p}_4$)

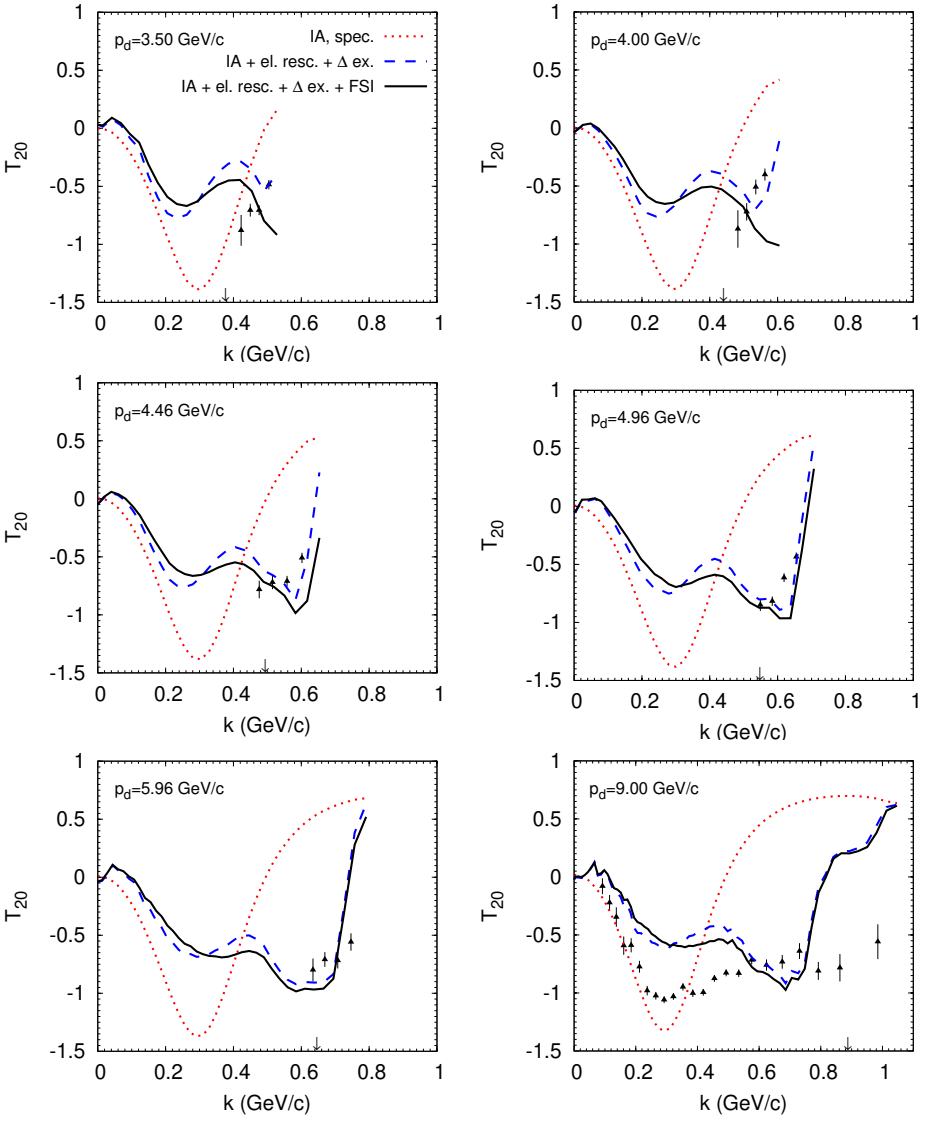


- *Only spectator diagrams not enough.*

- *Rescattering diagrams improve agreement with data (except at low $p_d$).*

- *Rescattering not important at the kinematical limit (largest k). Thus, non-distorted high-momentum DWF can be probed.*

- *The data at 9 GeV/c indicate suppression of S-wave at large k ($T_{20} \approx -1/\sqrt{2}$).*




Data (JINR):
***L. S. Azhgirey et al., PLB 391, 22 (1997);***
***Phys. At. Nucl. 61, 432 (1998); PLB 387, 37 (1996)***

## Summary

- *Proton production in the exclusive pd → ppn process in the backward hemisphere in the deuteron rest frame was studied on the base of the GEA with inclusion of single- (i.e. IA) and double-rescattering amplitudes with nucleon (elastic) and Δ-resonance (inelastic) intermediate states.*

- *IA strongly underestimates experimental cross sections. The results of calculations with full inclusion of the elastic and Δ-resonance intermediate states are in a reasonable agreement with the JINR data at $p_{lab}$ = 1.27 GeV/c ($E_{lab}$ = 640 MeV) and at $p_{lab}$ = 8.6 GeV/c. At the same time, the full calculation underestimates JINR data at $p_{lab}$ = 1.67 GeV/c (deuteron beam momentum 3.33 GeV/c) and overestimates Gatchina data at $p_{lab}$ = 1.70 GeV/c ($E_{lab}$ = 1 GeV).*

- *Two-step processes improve agreement with data on $T_{20}$. Discrepancies at small deuteron momenta still remain. $T_{20}$ at large k (kinematic limit) is sensitive to the DWF.*

## Next step

- *$A_{yy}$ in p(d,p)X process at 9 GeV/c for large transverse momenta of the proton*
  *V.P. Ladygin et al., PLB 629, 60 (2005)*

# Outlook

- *Cumulative processes at NICA and HIAF*

- *Pion production in pd collisions (important for small k).*

- *Improvements for pd reactions:*
  *propagators beyond GEA, two-step rescattering within LC formalism ...*

- *Model generalization for $p^3$He and $p^4$He reactions.*

- *The studies of SRCs in pA and AA reactions with heavy nuclei*
  *AL, Yu.N. Uzikov, PRC 109, 064601 (2024);*
  *Pengcheng Li et al, arXiv:2609.25818*

*Thank you for your attention !*