Nuclear Contacts and Their Scaling with Nuclear Radii

Tongqi Liang（梁桐启）

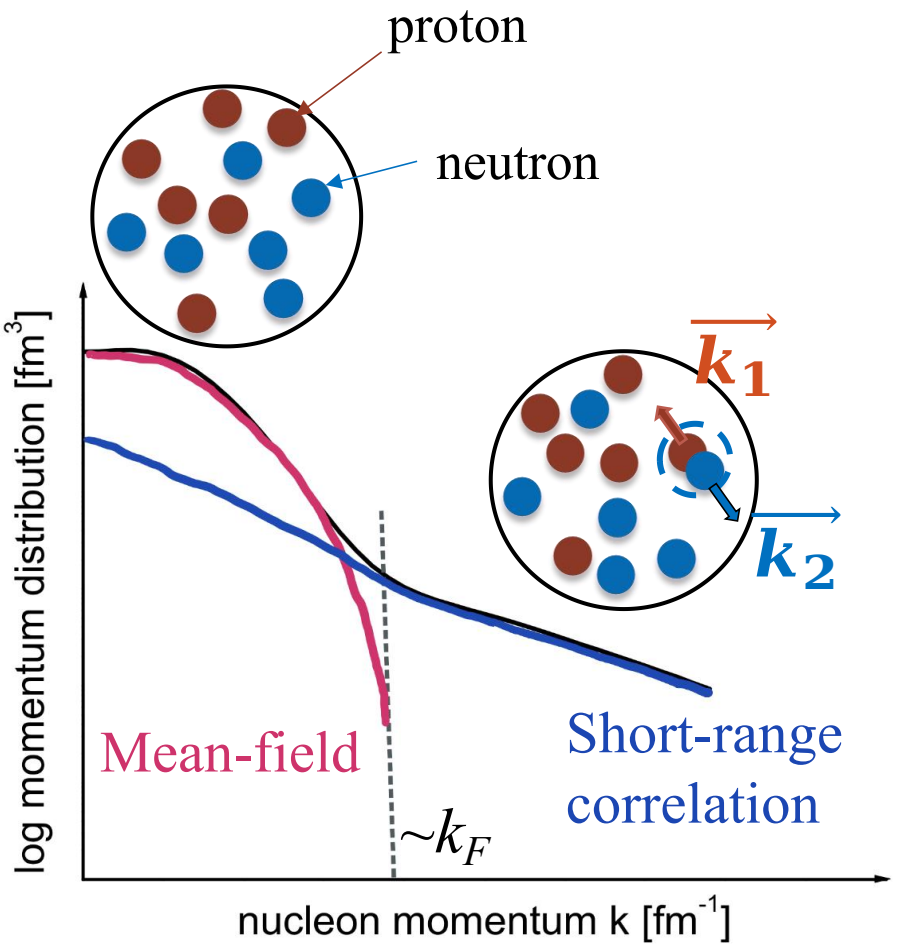
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Outline

- Introduction: Short-range correlations (SRCs)
- Nuclear contacts and generalized contact formalism (GCF)
- Calculations of nuclear contacts
  - Sn isotopes, Pb isotopes
- Nuclear contacts and nuclear radius
- Experimental nucleon momentum distributions
- Summary

Short-range correlations (SRCs)



Nucleon momentum distribution.

SRC pairs

- Short separation distance
- Low center-of-mass momentum
- High relative momentum
- np dominance
- ……

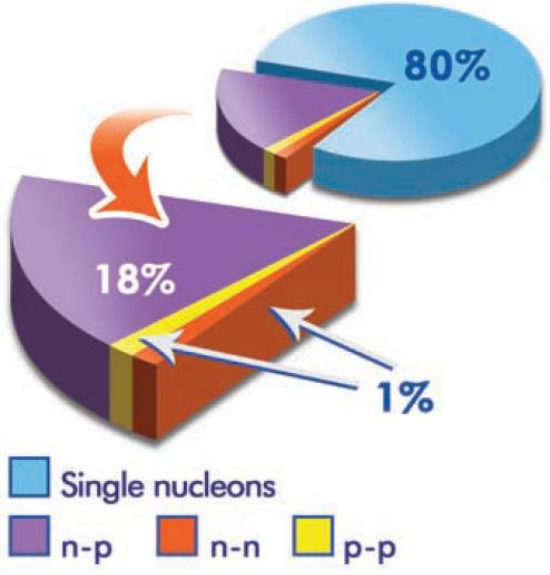
Important for understanding:

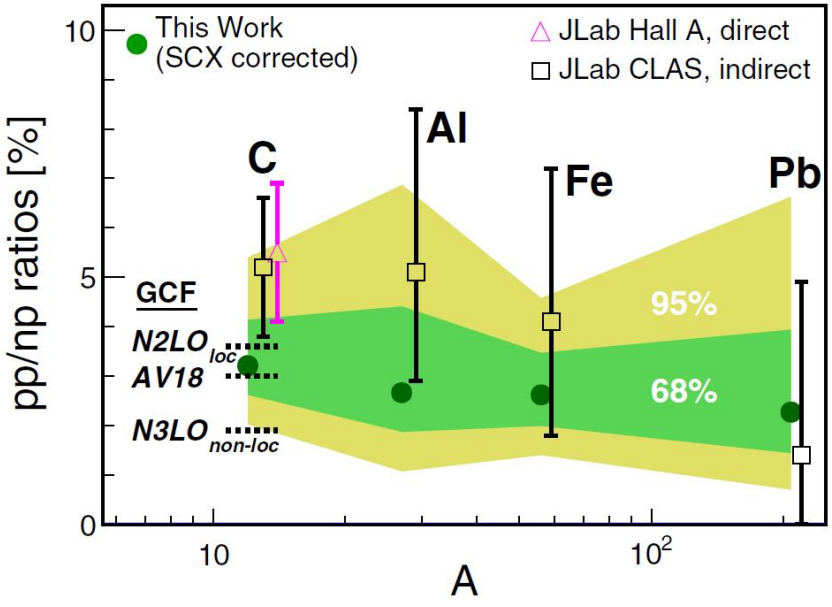
- ✓ *NN* interactions
- ✓ EMC effect
- ✓ Neutron-star properties

… ...

Short-range correlations (SRCs)

- Inverse kinematics (p,2p) scattering Nature Phys. 17, 693 (2021)
- Quasi-elastic electron scattering (e,e'p) Nature 609, 4 (2022)
- Gamma-ray emission in heavy-ion collisions Phys. Rev. Res. 7, 043174 (2025)



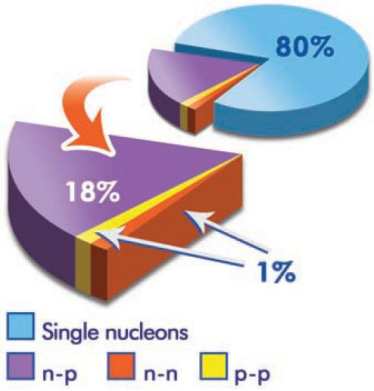


- For ^{12}C, 20% of nucleons form nucleon pairs, np pairs dominate Science 320, 1476 (2008).
- Almost the same pp/np ratio for C, Al, Fe, and Pb PRL 122, 172501 (2019).

Nuclear contacts

SRC-pair abundances

- High-momentum-tail fractions; Inclusive cross-section ratios; Exclusive electron scattering



Investigations of SRC-pair abundances are currently limited to light and stable nuclei.

Generalized contact formalism (GCF)

R. Weiss *et al.*, PRC 92, 054311 (2015)

$$\Psi \xrightarrow[r_{ij}\to 0]{} \sum_{\alpha} \varphi_{\alpha}(\mathbf{r}_{ij}) A_{ij}^{\alpha}(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k\neq i,j}).$$

spin-zero pp, nn, and np pairs;
spin-one np pairs (deuteron channel)

φ_α :asymptotic two-body wave function

Pair density: at short separation distance

$$\rho_{pp/nn}(r) = C_{pp/nn}^{\alpha} \mid \varphi_{pp/nn}^{\alpha}(r) \mid^2 .$$

at large relative momentum

$$n_{pp/nn}(k) = \tilde{C}_{pp/nn}^{\alpha} \mid \tilde{\varphi}_{pp/nn}^{\alpha}(k) \mid^2 .$$

Nuclear contacts: $C_{pp/nn}^{\alpha}$ $\tilde{C}_{pp/nn}^{\alpha}$

quantifying the abundance of pairs in channel α.

Nuclear contacts

Small separation, r

$$\rho_{pp/nn}(r) = C^{a}_{pp/nn} \mid \varphi^{a}_{pp/nn}(r) \mid^2 .$$

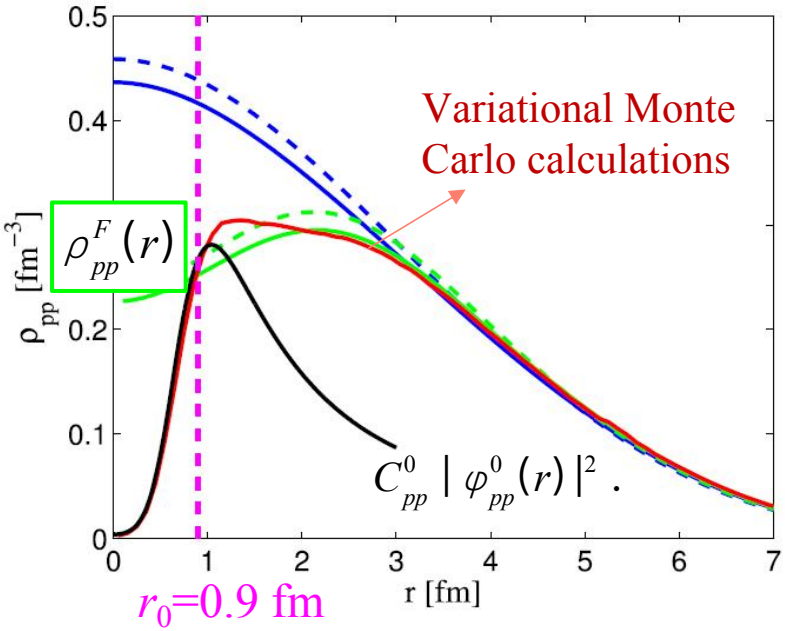
Large separation, r

$$\rho^{UC}_{NN}(\mathbf{r}) \equiv \int d\mathbf{R} \rho_N(\mathbf{R} + \mathbf{r}/2) \rho_N(\mathbf{R} - \mathbf{r}/2),$$

ρ_N : one-body density

Accounting for the fermionic nature of the pp/nn pairs

$$\rho^{F}_{pp}(r) = N \rho^{UC}_{pp}(r) \left[1 - \frac{3}{2} j_1(k^P_F r) \left(\frac{k^P_F r}{2} \right)^2 \right].$$



The *pp* pair density for ^{40}Ca.

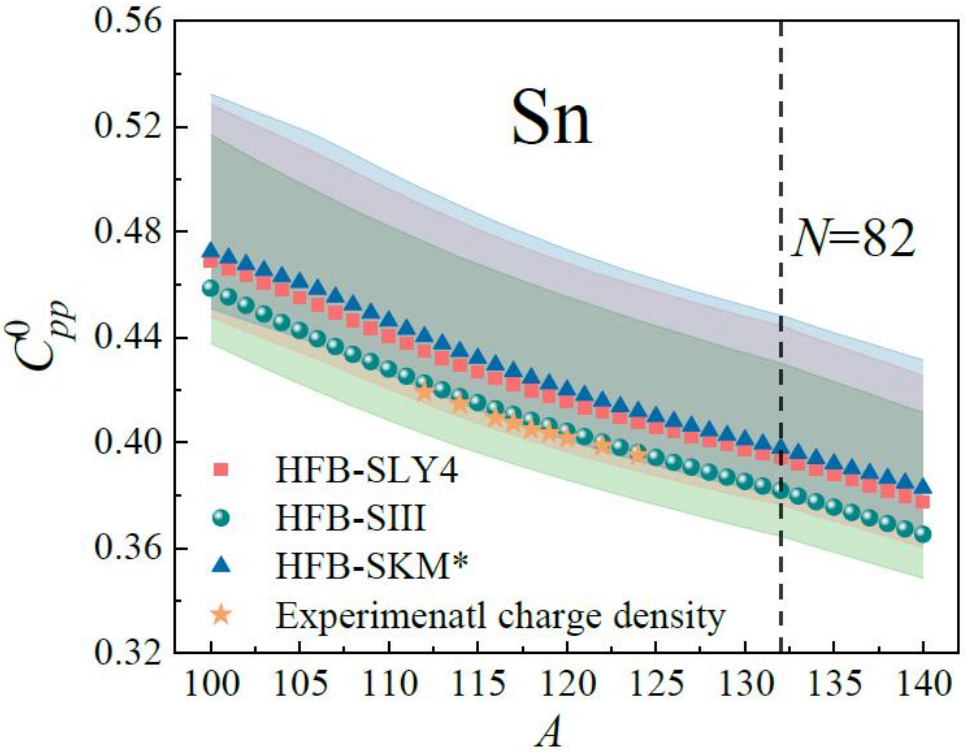
- At r_0=0.9 fm, $\rho^{F}_{pp}(r) \approx C^{0}_{pp} \mid \varphi^{0}_{pp}(r) \mid^2$,
- Possible extraction of nuclear contacts for heavy nuclei and unstable nuclei.

R. Weiss *et al.*, Phys. Lett. B **780,** 211 (2018).
R. Weiss *et al.*, Phys. Lett. B **790,** 484 (2019).

Nuclear contacts of unstable nuclei

point-proton density and GCF → *pp* contacts

(Skyrme Hartree-Fock-Bogoliubov, HFB)



The *pp* contact C^0_{pp} values as a function of *A* for Sn isotopes.

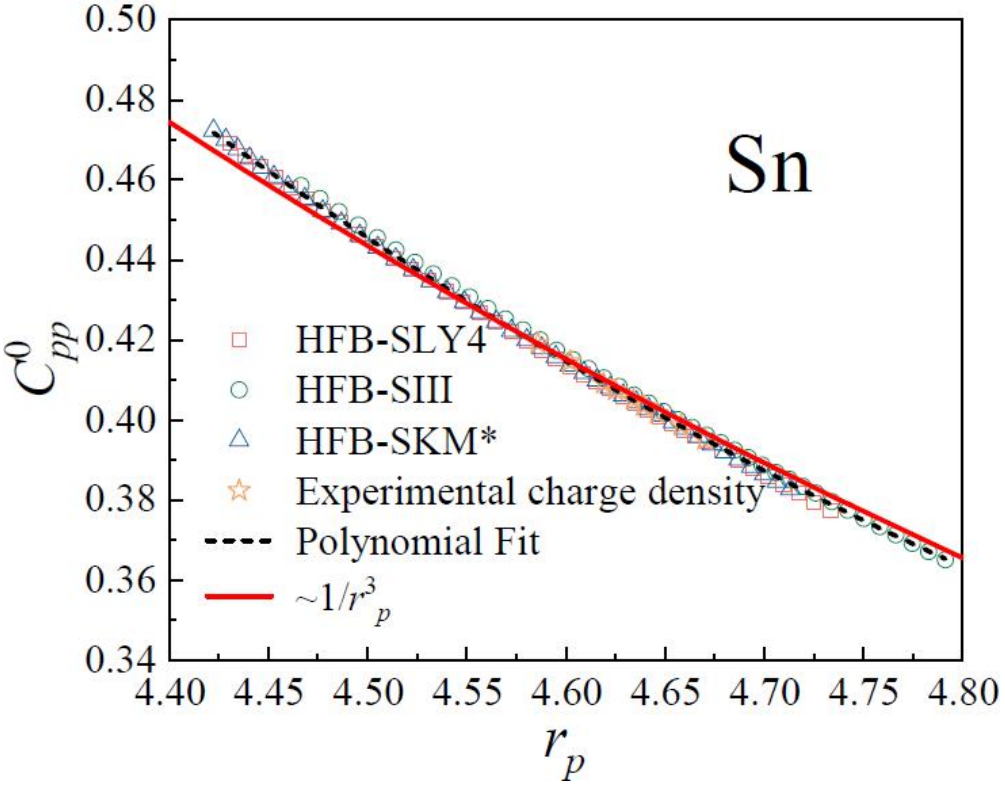
- Experimental charge density and Skyrme HFB point-proton density (SLY4, SIII, SKM*) are used to evaluate the *pp* contact C^0_{pp}.
- The *pp* contact C^0_{pp} generally decreases with increasing *A*.
- The correlation between C^0_{pp} and *A* is model-dependent.

T. Liang et al., Phys. Lett. B **857** 138965 (2024).

Nuclear contacts and nuclear radii

point-nucleon density

RMS radius ⟷ correlation? ⟷ $C^{a}_{NN} = \dfrac{\rho^{F}_{NN}(\boldsymbol{r}_0)}{\left|\varphi^{a}_{NN}(\boldsymbol{r}_0)\right|^2}$.



- A correlation between C^0_{pp} and r_p that is largely insensitive to the model choice.
- A similar correlation is obtained for both r_p and r_C.
- C^0_{pp} is approximately proportional to $1/r^3_p$.

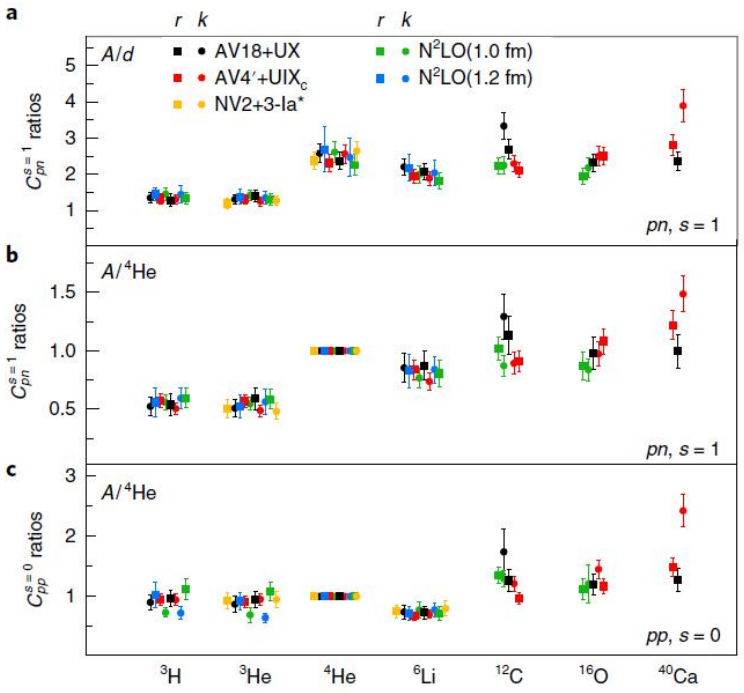
T. Liang *et al.*, Phys. Lett. B **857** 138965 (2024).

Nuclear contacts and nuclear radii

Quantum Monte Carlo calculations:

R. Cruz-Torres et. al., Nature Phys. 17, 306 (2021)

- The calculated nuclear contacts show weak dependence on the choice of NN interaction.
- Nuclear contact (the abundance of SRC pairs) is a long-range property.



Generalized Levinger model

R. Yankovich et. al., Phys. Rev. C, 111 L051304 (2025).

$$C^{A}_{0,N_1N_2} \sim \rho_{N_1}\rho_{N_2}\Omega_{N_1N_2}$$

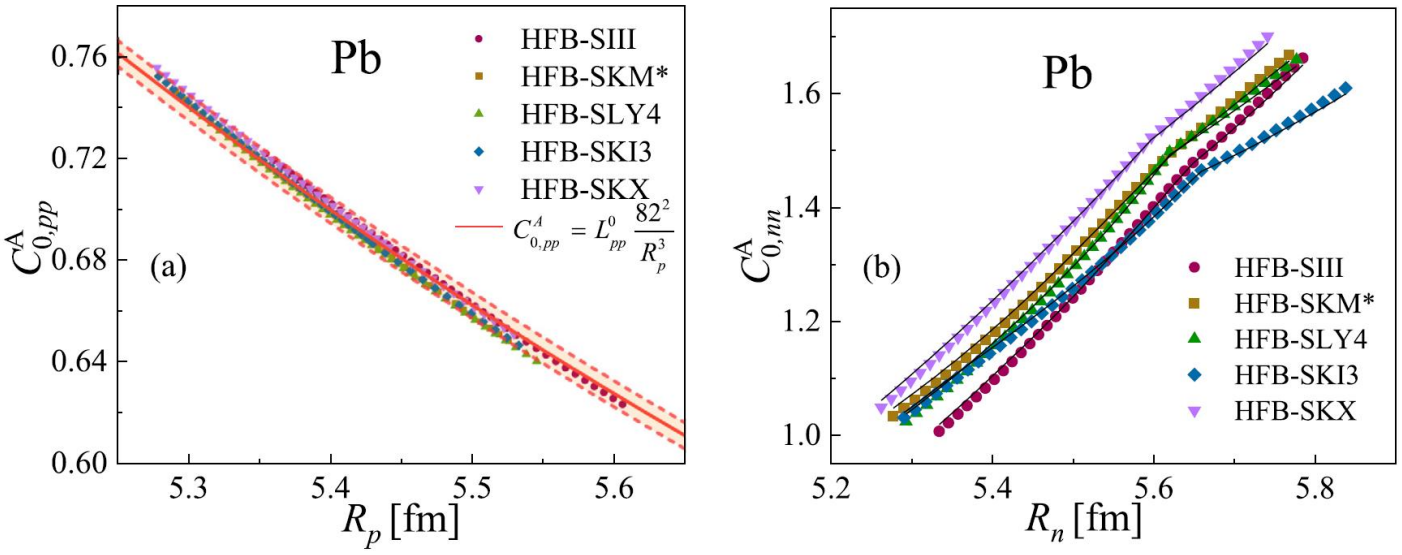
$$\Omega_{p(n)} \sim r^3_{p(n)}, \quad \rho_{p(n)} \sim Z(N)/r^3_{p(n)}$$

$$C^{A}_{0,pp} = L^0_{pp}\frac{Z^2}{R_p^3},$$

$$C^{A}_{0,nn} = L^0_{nn}\frac{N^2}{R_n^3}.$$

L^0_{pp}, L^0_{nn}: The generalized Levinger constants connect nuclear contacts with RMS radii.

Nuclear contacts and nuclear radii



Nuclear contacts for Pb isotopes versus RMS radii of nucleon density distributions for *pp* pairs and *nn* pairs. Lines are the results of the generalized Levinger model.

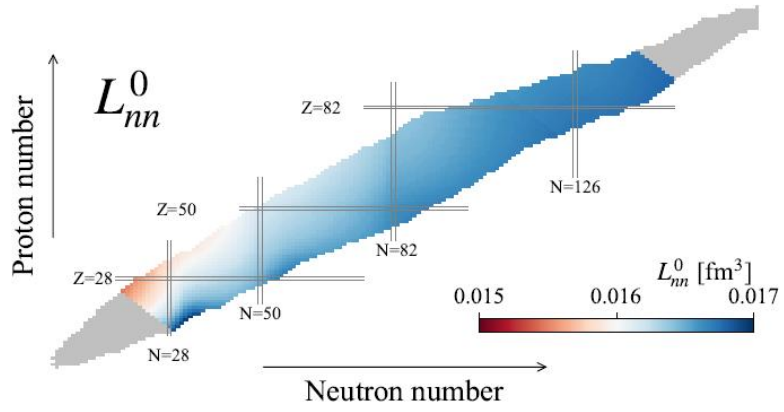
$$C^A_{0,pp} = L^0_{pp} \frac{Z^2}{R_p^3},$$

$$C^A_{0,nn} = L^0_{nn} \frac{N^2}{R_n^3}.$$

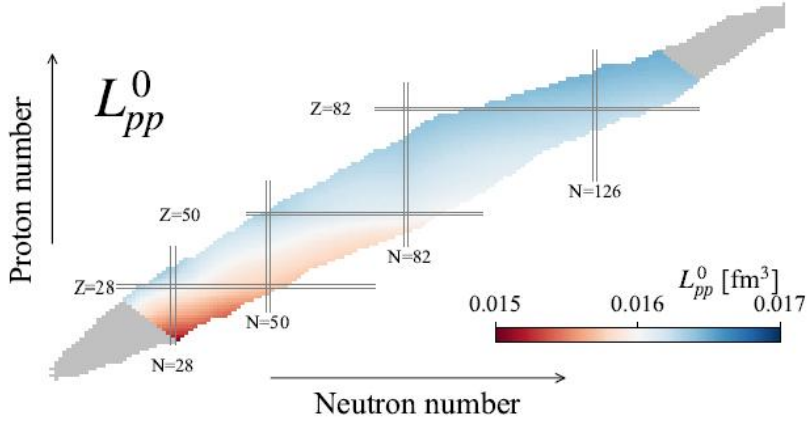
- The generalized Levinger model can well describe the correlation between nuclear contacts and RMS radii.
- Model-independent correlations between nuclear contacts and RMS radii.

T. Liang et al., Phys. Lett. B **863** 139351 (2025).

Generalized Levinger constant



Generalized Levinger constant for *nn* pairs L^0_{nn}



Generalized Levinger constant for *pp* pairs L^0_{pp}

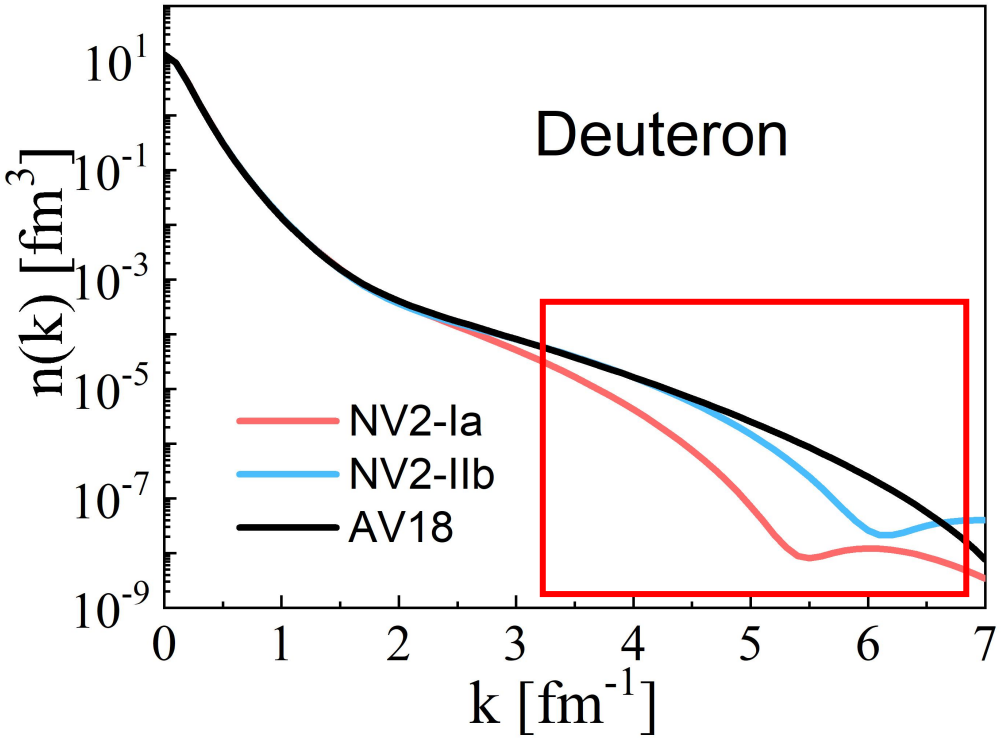
T. Liang et al., Phys. Lett. B 863 139351 (2025).

L^0_{nn} : 0.0155 to 0.0170 fm^3

L^0_{pp} : 0.0151 to 0.0165 fm^3

- The values of L^0_{nn} *and* L^0_{pp} remain nearly unchanged.
- Universal relationship between nuclear contacts and nuclear radii.
- Nuclear contact is a long-range property.

Experimental nucleon momentum distributions



Deuteron nucleon momentum distributions $n(k)$ calculated with various NN interactions

The high-momentum region remains poorly constrained experimentally.

https://www.phy.anl.gov/theory/research/momenta/

Plane Wave Impulse Approximation, PWIA

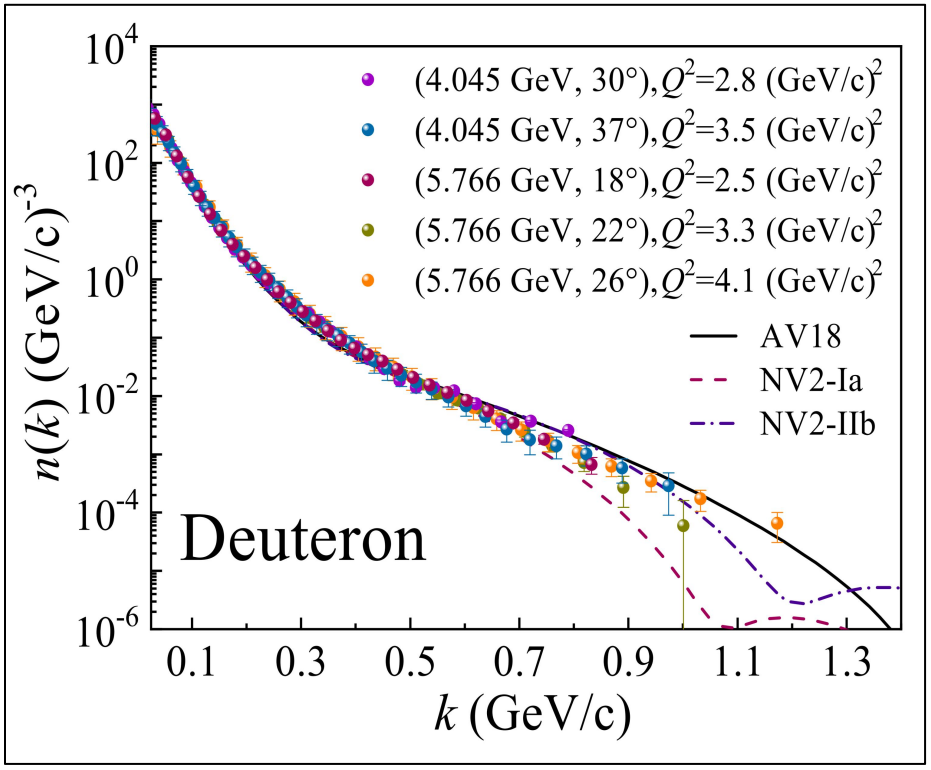
Inclusive cross sections $\frac{d^2\sigma}{d\omega d\Omega}$

↓ large Q^2

Scaling functions F(y)

↓

Nucleon momentum distributions $n(k)$

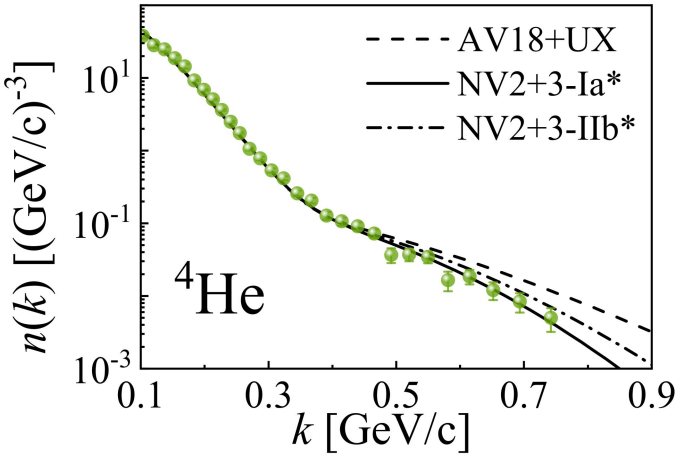


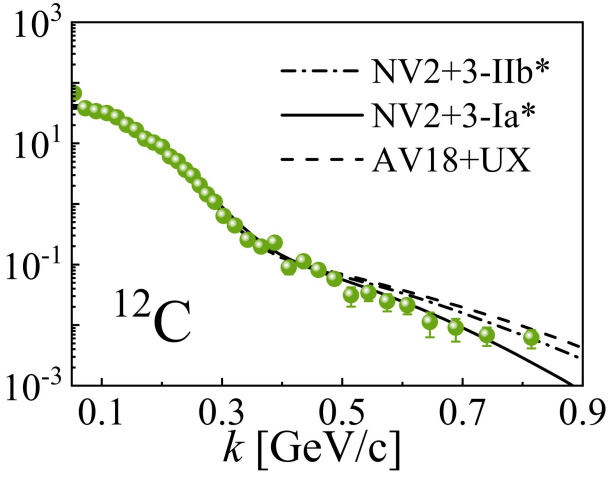
For A>2, excitation energy of the recoil A-1 system affects the extraction of nucleon momentum distributions.

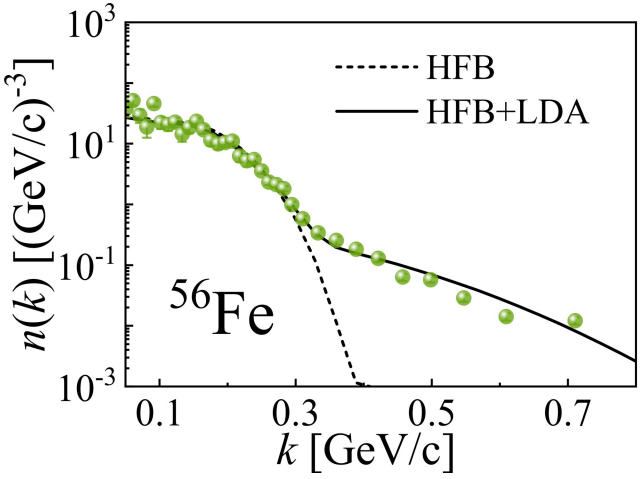
T. Liang, D. Bai, Z. Ren, Phys. Rev. C 109, 054313 (2024).

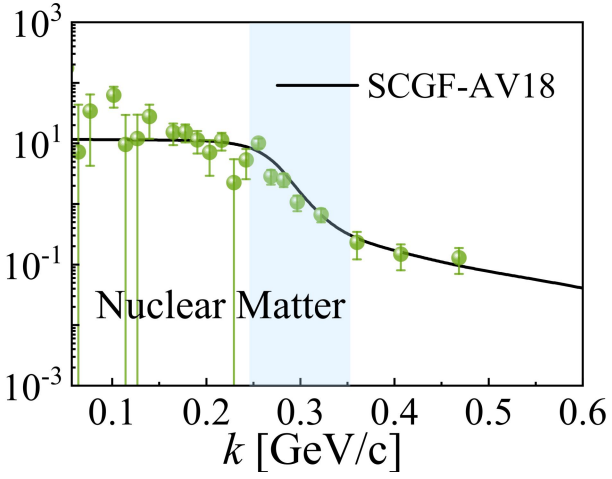
Excitation energy of the recoil *A*-1 system

$$E^{*\mathrm{RFG}}_{A-1} = \begin{cases} \sqrt{k_F^2 + m_N^2} - \sqrt{\mathbf{k}^2 + m_N^2}, & k < k_F, \\ \sqrt{\mathbf{k}^2 + m_N^2} - \sqrt{k_F^2 + m_N^2} - E^{\mathrm{R}}_{A-1}, & k > k_F, \end{cases}$$









T. Liang, D. Bai, Z. Ren, Phys. Rev. C 113, 064319 (2026)

Summary

Nuclear radii and nuclear contacts are connected through the generalized Levinger constant:

$$C_{0,pp}^{A} = L_{pp}^{0} \frac{Z^2}{R_p^3},$$

$$C_{0,nn}^{A} = L_{nn}^{0} \frac{N^2}{R_n^3}.$$

- Providing evidence that spin-zero SRC pairs is a long-range property.
- These relationships require further experimental validation.
- New extractions of nucleon momentum distributions, including recoil excitation effects for $A > 2$.

Thanks!