Processes of meson production in e+e- annihilation and tau decays within the NJL model

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in collaboration with prof. M.K. Volkov and A.A. Pivovarov

14th ITP/CAS – BLTP/JINR JOINT WORKSHOP ON PHYSICS OF STRONG INTERACTING SYSTEMS, NANJING

The goals and motivation

The importance of investigations is due to the need to study the physics of strong interactions at low energies

(a) processes of meson production in electron–positron collisions (< 2 GeV)
(b) mesonic τ decays (M_τ= 1.7 GeV)

For both classes of processes, contributions from intermediate scalar, pseudoscalar, vector and axial-vector mesons are considered.

The relevance of the processes under study is due to the fact that this area of research is included in the program of many modern experimental centers:



SLAC National Accelerator Laboratory



VEPP 2000



The Beijing Spectrometer III



The Belle experiment at KEK accelerator



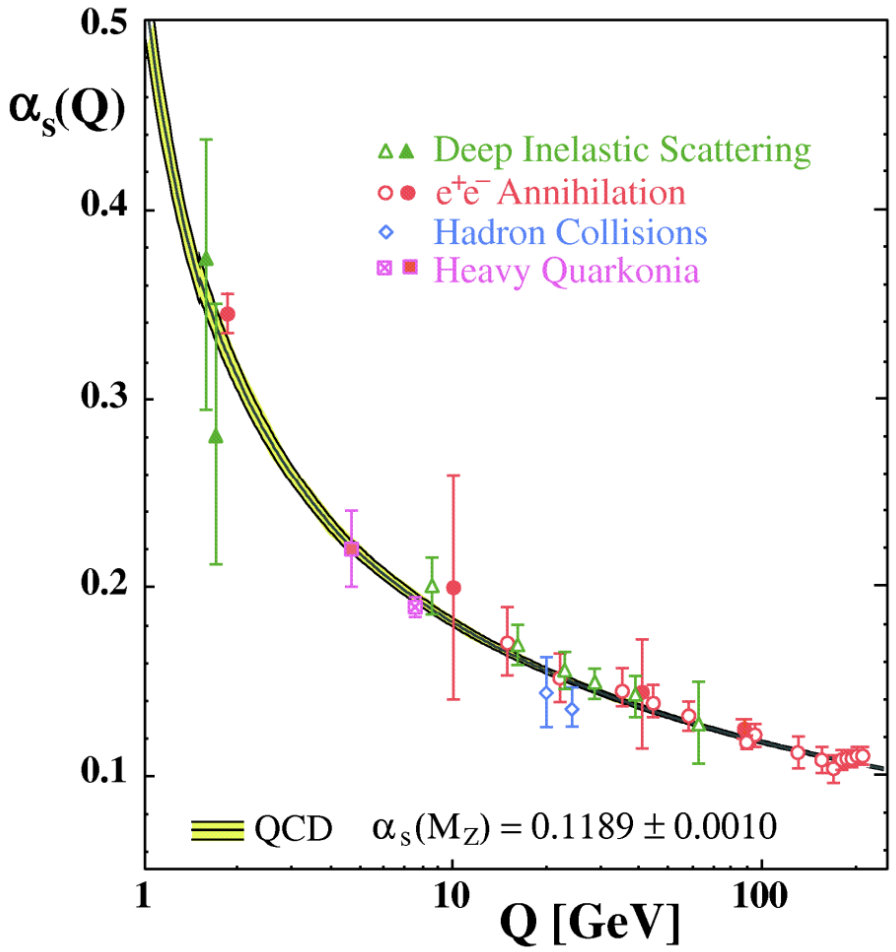
Compass Experiment at CERN

A.Y. Barniakov [Super Charm-Tau Factory], The **Super Charm-Tau Factory** in Novosibirsk, PoS LeptonPhoton2019 (2019), 062

X.R. Lyu et al. [STCF working group]. Physics program of the **Super Tau-Charm Factory**. Pos BEAUTY2020 060 (2021)

J. Elam et al. [REDTOP], arXiv:2203.07651 [hep-ex]

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[S. Bethke. Prog. Part. Nucl. Phys. 58 (2007), 351-386]

At energies E < 2 GeV QCD perturbation theory is not applicable
⇒ phenomenological models are required

Most phenomenological models involve a large number of free parameters to correctly describe experimental data.

Theoretical description of several classes of processes with meson interactions at energies up to 2 GeV

- verification of the NJL model on a new class of tasks
- definition of its applicability domain
- construction of theoretical predictions

- The model is based on effective four-fermion interactions. The model describes spontaneous breaking of the chiral symmetry.

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- The standard NJL model successfully describes the physical properties of scalar, pseudoscalar, vector and axial-vector meson nonets in the ground state, including the mass spectrum and their interactions at low energies using a limited number of parameters.

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- In the extended NJL model, the first radial excitations of mesons are introduced using a polynomial-type form factor of the lowest order in the transverse relative momentum of quarks in the meson.

- The chiral symmetry is preserved and there are no any additional parameters for interaction of the excited mesons.

Model input parameters

$m_u = m_d = 270$ MeV

$\Lambda = 1260\ MeV$

$m_s = 420$ MeV

$K = 13.0\ GeV^{-5}$

$G_1 = 4.743 \cdot 10^{-6} MeV^{-2}$,

$G_2 = 14.9 \cdot 10^{-6} MeV^{-2}$.

$\pi \to \mu\nu_\nu$ ($F_\pi = \frac{m_u}{g_\pi} = 92.4$) MeV

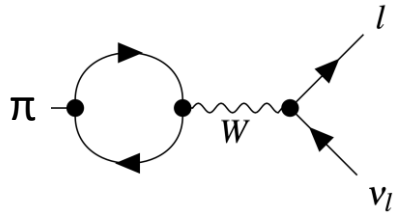
$\rho \to \pi\pi$ (g_ρ=6.0) MeV

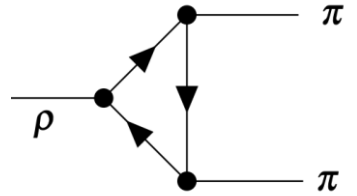
$M_K^{exp.} = 494\ MeV$

$M_\eta^{exp.} = 547\ MeV$

$M_\pi^{exp.} = 139\ MeV$

$M_\rho^{exp.} = 776\ MeV$





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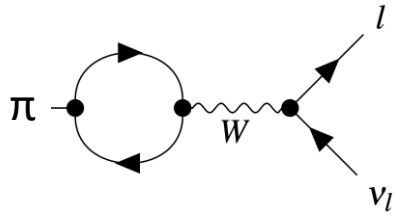
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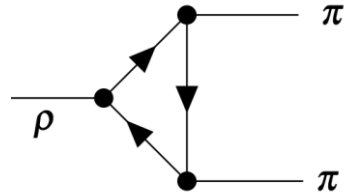
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$f_M(k^2) = 1 + d_M k^2$, $d_{uu} = 1.784\times10^{-2} GeV^{-2}$, $d_{us} = 1.761\times10^{-2} GeV^{-2}$, $d_{ss} = 1.737\times10^{-2} GeV^{-2}$.

M.K. Volkov and A.E. Radzhabov, Phys. Usp. 49 (2006)

The model masses of mesons

	GR	EXC	GR(Exp.) [1]	EXC(Exp.) [1]
M_σ	530	1330	400–1200	1200–1500
M_{f_0}	1070	1600	980±10	1712±5
M_{a_0}	830	1500	983.4±0.9	1474±19
$M_{K_0^*}$	960	1500	905±50 [5]	1429±12
M_π	140	1300	139.56995±0.00035	1300±100
M_K	490	1300	497.672±0.031	1460(?)
M_η	520	1280	547.30±0.12	1297.8±2.8
$M_{\eta'}$	910	1470	957.78±0.14	1440–1470
M_ρ	770	1470	770.0±0.8	1465±25
M_φ	1019	1682	1019.413±0.008	1680±20
M_{K^*}	887	1479	891.59±0.24	1412±12

η, $\eta'(958)$, $\eta(1295)$, $\eta(1475)$

But $\eta(1405)$ is missed, it is a pseudoscalar glueball candidate

M.K. Volkov and A.E. Radzhabov, Phys. Usp. 49 (2006)

$$\mathscr{L}_{int} = \bar{q}\Big[+\frac{1}{2}\gamma_\mu \sum_{i=\pm,0} \lambda_i^M (A_M V^i + B_M V'^i) + \frac{1}{2}\gamma_\mu\gamma_5 \sum_{i=\pm,0} \lambda_i^M (A_M A^i + B_M A'^i)$$
$$\sum_{i=\pm,0} \lambda_i^M (A_M S^i + B_M S'^i) + i\gamma_5 \sum_{i=\pm,0} \lambda_i^M (A_M P^i + B_M P'^i)$$
$$+i\gamma^5 \sum_{i=u,s} \lambda_i \left[A_\eta^i \eta + A_{\eta'}^i \eta' + A_{\hat{\eta}}^i \eta(1295) + A_{\hat{\eta}'}^i \eta(1475)\right]\Big] q,$$

$$A_M = \frac{1}{\sin(2\theta_M^0)} \left[g_M \sin(\theta_M + \theta_M^0) + g'_M f_M(k^2) \sin(\theta_M - \theta_M^0)\right],$$
$$B_M = \frac{-1}{\sin(2\theta_M^0)} \left[g_M \cos(\theta_M + \theta_M^0) + g'_M f_M(k^2) \cos(\theta_M + \theta_M^0)\right],$$

$$\begin{array}{lll}
\theta_\pi = 59.48^\circ & \theta_\pi^0 = 59.12^\circ, & \theta_K = 60.2^\circ, \\
\theta_K^0 = 57.13^\circ, & \theta_\rho = 81.8^\circ, & \theta_\rho^0 = 81.5^\circ \\
\theta_{K^*} = 84.7^\circ, & \theta_{K^*}^0 = 59.14^\circ, & \theta_\varphi = 68.4^\circ, \\
\theta_\varphi^0 = 57.13^\circ, & \theta_{a_0} = 72.0^\circ, & \theta_{a_0}^0 = 61.5^\circ, \\
\theta_{K_0^*} = 74.0^\circ & \theta_{K_0^*}^0 = 60.0^\circ. &
\end{array}$$

M.K. Volkov and V. L. Yudichev, Phys. Part. Nucl. 31 (2000), 282-311
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$$A_M^u = g_{\eta^u} a_{1M}^u + g'_{\eta^u} a_{2M}^u f_{uu}(k^2),$$
$$A_M^s = g_{\eta^s} a_{1M}^s + g'_{\eta^s} a_{2M}^s f_{ss}(k^2).$$

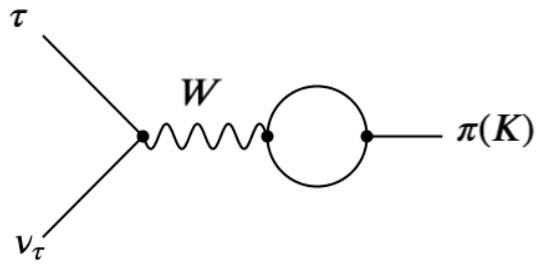
	η	$\eta(1295)$	η'	$\eta(1475)$
a_1^u	0.71	0.62	-0.32	0.56
a_2^u	0.11	-0.87	-0.48	-0.54
a_1^s	0.62	0.19	0.56	-0.67
a_2^s	0.06	-0.66	0.3	0.82

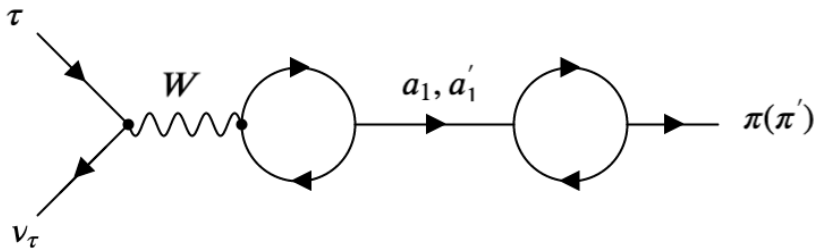
Table II: Mixing parameters of η mesons

$$g_\pi = g_{\eta^u} = \left(\frac{4}{Z_\pi} I_{20}\right)^{-1/2}, \quad g'_\pi = g'_{\eta^u} = \left(4 I_{20}^{f^2}\right)^{-1/2},$$
$$g_{\eta^s} = \left(\frac{4}{Z_{\eta^s}} I_{02}\right)^{-1/2}, \quad g'_{\eta^s} = \left(4 I_{02}^{f^2}\right)^{-1/2},$$
$$g_\rho = \left(\frac{2}{3} I_{20}\right)^{-1/2}, \quad g'_\rho = \left(\frac{2}{3} I_{20}^{f^2}\right)^{-1/2}.$$

$$I_{n_1 n_2}^{f^m} = -i \frac{N_c}{(2\pi)^4} \int \frac{f^m(k^2)}{(m_u^2 - k^2)^{n_1} (m_s^2 - k^2)^{n_2}} \Theta(\Lambda^2 - k^2) \mathrm{d}^4 k.$$

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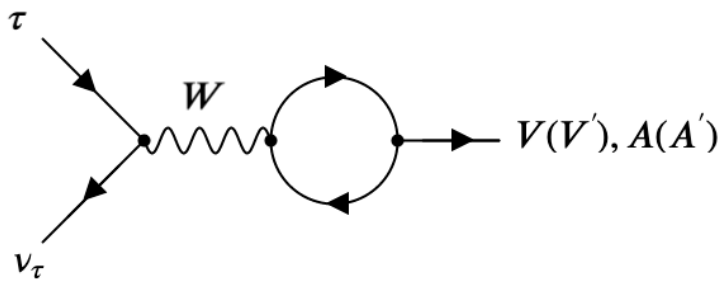




$$\mathcal{M}_{\tau\to\pi'\nu_\tau} = G_F V_{ud} F_{\pi'} L_\mu p_\mu,$$

$$F_{\pi'} = C_{\pi'} Z_\pi \frac{m_u}{g_\pi} - \frac{C_\rho}{g_\rho} 4 m_u I_2^{a_1\pi'} \frac{6m_u^2}{M_{a_1}^2 - M_{\pi'}^2 - i\Gamma_{a_1} M_{a_1}} \left(1 - \frac{M_{\pi'}^2}{M_{a_1}^2}\right)$$

$$- \frac{C_{\rho'}}{g_\rho} 4 m_u I_2^{a_1'\pi'} \frac{6m_u^2}{M_{a_1'}^2 - M_{\pi'}^2 - i\Gamma_{a_1'} M_{a_1'}} \left(1 - \frac{M_{\pi'}^2}{M_{a_1'}^2}\right),$$

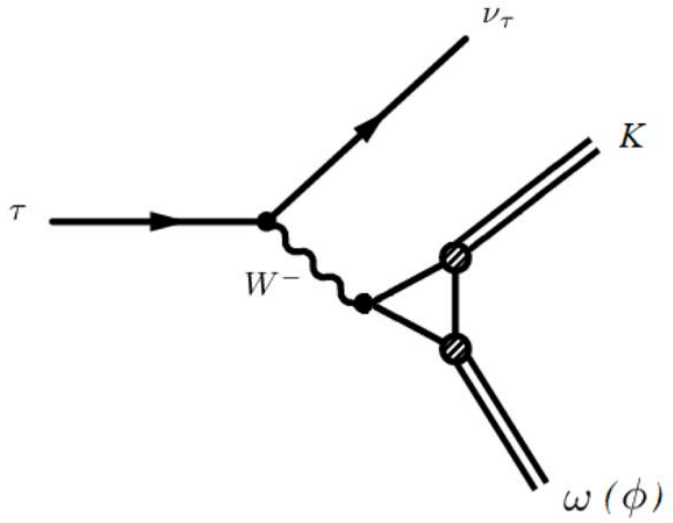


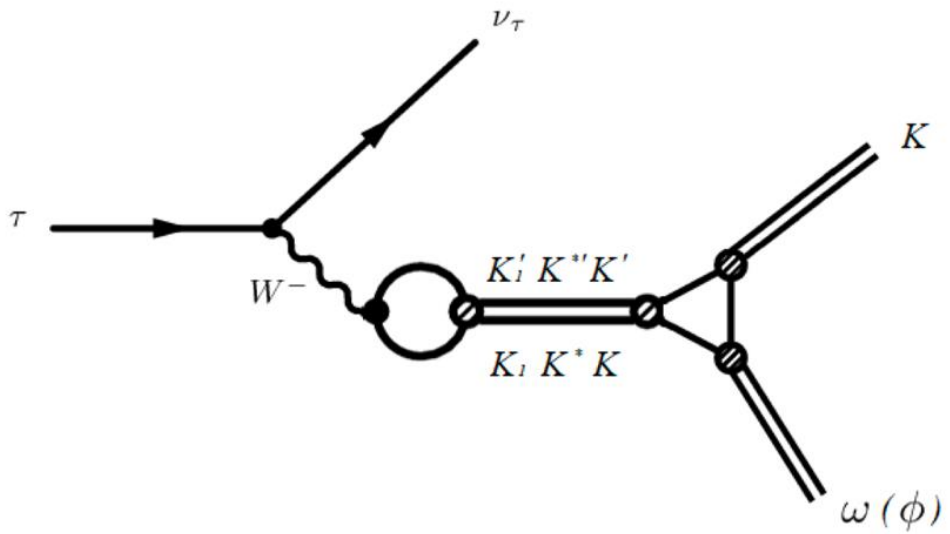
$$\mathcal{M}_{\tau\to\rho\nu_\tau} = G_F L_\mu V_{ud} \frac{C_\rho}{g_\rho} \left(g_{\mu\nu}p^2 - p_\mu p_\nu\right) e^\rho_\nu,$$

$$\mathcal{M}_{\tau\to K^*\nu_\tau} = G_F L_\mu V_{us} \frac{C_{K^*}}{g_{K^*}} \left(g_{\mu\nu}\left(p^2 - \frac{3}{2}(m_s - m_u)^2\right) - p_\mu p_\nu\right) e^{K^*}_\nu$$

Decay	NJL, MeV	Data, MeV
$\tau \to \rho\nu_\tau$	4.19×10^{-10}	–
$\tau \to \rho(1450)\nu_\tau$	4.79×10^{-11}	–
$\tau \to a_1(1260)\nu_\tau$	2.94×10^{-10}	–
$\tau \to a_1(1640)\nu_\tau$	6.96×10^{-11}	–
$\tau \to K^*(892)\nu_\tau$	2.07×10^{-11}	$(2.72 \pm 0.15)\times10^{-11}$
$\tau \to K^*(1410)\nu_\tau$	3.67×10^{-12}	$3.4\ (+3.17, -2.27)\times10^{-12}$
$\tau \to K_1(1270)\nu_\tau$	0.57×10^{-11}	$(1.06 \pm 0.24)\times10^{-11}$
$\tau \to K_1(1400)\nu_\tau$	2.40×10^{-12}	$(3.86 \pm 5.90)\times10^{-12}$
$\tau \to K_1(1650)\nu_\tau$	3.34×10^{-13}	–
$\tau \to K\nu_\tau$	1.59×10^{-11}	$(1.58 \pm 0.02)\times10^{-11}$
$\tau \to K'\nu_\tau$	1.42×10^{-13}	–
F_K	110.14	110.02 ± 0.12
$\tau \to \pi\nu_\tau$	2.466×10^{-10}	$(2.46 \pm 0.01)\times10^{-10}$
$\tau \to \pi'\nu_\tau$	1.04×10^{-13}	$< 4.31\times10^{-13}$

- Three body decays $\tau \to \omega K^- \nu_\tau, \tau \to \varphi K^- \nu_\tau, \tau \to K^*(892) K^- \nu_\tau, \tau \to K^*(892) \eta \nu_\tau$





$$
\begin{aligned}
K_{1A} &= K_1(1270)\sin\alpha + K_1(1400)\cos\alpha, \\
K_{1B} &= K_1(1270)\cos\alpha - K_1(1400)\sin\alpha.
\end{aligned}
$$

[M. Suzuki. Phys. Rev. D 47 (1993), 1252-1255] $\alpha = 45^\circ$

[M.K. Volkov. Phys. Part. Nucl. 24 (1993), 35-58] $\alpha = 57^\circ$

$$\mathcal{M} = -iG_F V_{us} L_\mu \{\mathcal{M}_c + \mathcal{M}_A + \mathcal{M}_V + \mathcal{M}_P + \mathcal{M}_{A'} + \mathcal{M}_{V'} + \mathcal{M}_{P'}\}^{\mu\nu} e^*_\nu(p_\omega),$$

$$\begin{aligned}
\mathcal{M}_c^{\mu\nu} &= (m_s + m_u) I_{11}^{K\omega} g^{\mu\nu} + i2m_u \left[I_{21}^{K\omega} + (m_s - m_u) m_u I_{31}^{K\omega}\right] e^{\mu\nu\lambda\delta} p_{K\lambda} p_{\omega\delta}, \\
\mathcal{M}_A^{\mu\nu} &= \frac{C_{K_1}}{g_{K_1}} (m_s + m_u) I_{11}^{K\omega K_1} \\
&\quad \times \left\{ \left[g^{\mu\nu} \left[q^2 - \frac{3}{2}(m_s + m_u)^2\right] - q^\mu q^\nu \left[1 - \frac{3}{2}\frac{(m_s + m_u)^2}{M^2_{K_{1(1270)}}}\right]\right] BW_{K_{1(1270)}} \sin^2\alpha \right. \\
&\quad \left. + \left[g^{\mu\nu} \left[q^2 - \frac{3}{2}(m_s + m_u)^2\right] - q^\mu q^\nu \left[1 - \frac{3}{2}\frac{(m_s + m_u)^2}{M^2_{K_{1(1400)}}}\right]\right] BW_{K_{1(1400)}} \cos^2\alpha \right\}, \\
\mathcal{M}_V^{\mu\nu} &= i2m_u \frac{C_{K^*}}{g_{K^*}} \left[I_{21}^{K\omega K^*} + (m_s - m_u) m_u I_{31}^{K\omega K^*}\right] \left[q^2 - \frac{3}{2}(m_s - m_u)^2\right] BW_{K^*} e^{\mu\nu\lambda\delta} p_{K\lambda} p_{\omega\delta}, \\
\mathcal{M}_P^{\mu\nu} &= 2(m_s + m_u) \frac{Z_K}{g_K} C_K I_{11}^{\omega K K} q^\mu q^\nu BW_K, \\
\mathcal{M}_{A'}^{\mu\nu} &= \frac{C'_{K_1}}{g_{K_1}} (m_s + m_u) I_{11}^{K\omega K'_1} \left\{ g^{\mu\nu} \left[q^2 - \frac{3}{2}(m_s + m_u)^2\right] - q^\mu q^\nu \left[1 - \frac{3}{2}\frac{(m_s + m_u)^2}{M^2_{K'_1}}\right]\right\} BW_{K'_1}, \\
\mathcal{M}_{V'}^{\mu\nu} &= i2m_u \frac{C'_{K^*}}{g_{K^*}} \left[I_{21}^{K\omega K^{*\prime}} + (m_s - m_u) m_u I_{31}^{K\omega K^{*\prime}}\right] \left[q^2 - \frac{3}{2}(m_s - m_u)^2\right] BW_{K^{*\prime}} e^{\mu\nu\lambda\delta} p_{K\lambda} p_{\omega\delta},
\end{aligned}$$

The constants C_M and C'_M appear in the quark loops of the W boson transition into the intermediate meson in the extended NJL model:

$$C_{K^*} = \frac{1}{\sin(2\theta^0_{K^*})}\left[\sin(\theta_{K^*} + \theta^0_{K^*}) + R_{K^*}\sin(\theta_{K^*} - \theta^0_{K^*})\right] = 0.90,$$
$$C_{K^{*\prime}} = \frac{-1}{\sin(2\theta^0_{K^*})}\left[\cos(\theta_{K^*} + \theta^0_{K^*} + R_{K^*}\cos(\theta_{K^*} - \theta^0_{K^*})\right] = 0.42.$$

The integrals arising in quark loops with meson vertices

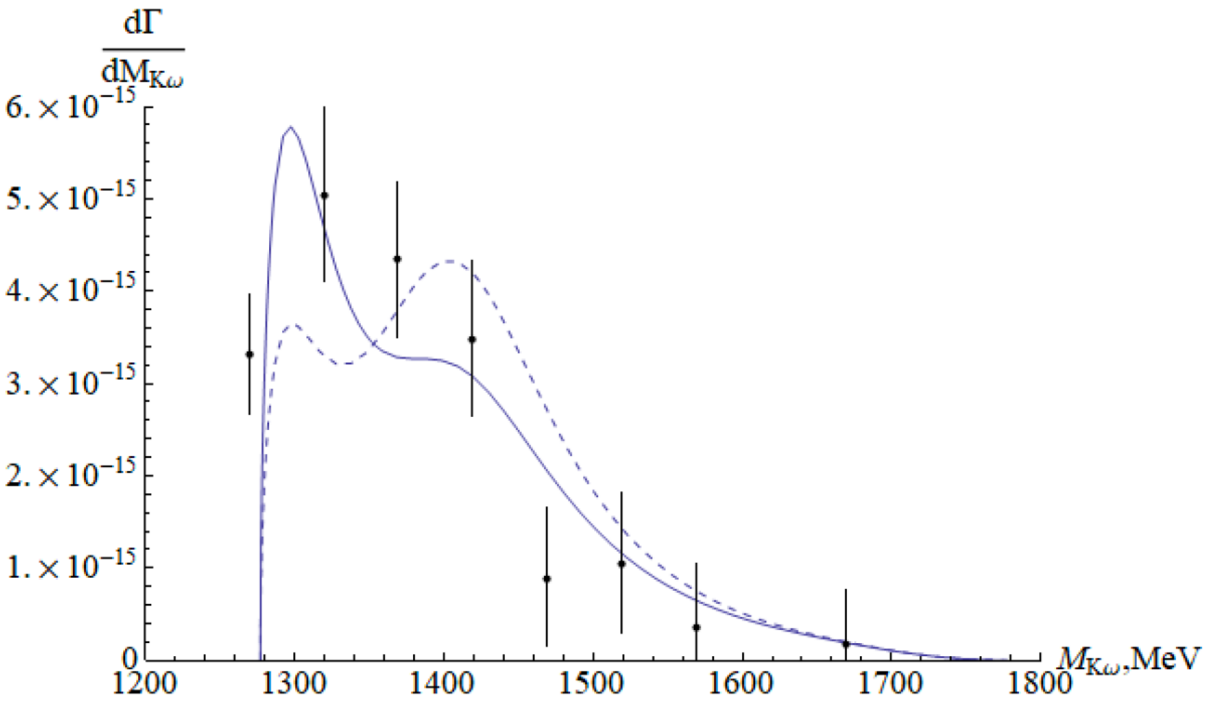
$$I^{M\ldots M'\ldots}_{n_1 n_2} = -i\frac{N_c}{(2\pi)^4}\int \frac{A_M \ldots B_M \ldots}{(m_u^2 - k^2)^{n_1}(m_s^2 - k^2)^{n_2}}\Theta(\Lambda^2 - k^2)\mathrm{d}^4k.$$

The intermediate mesons are described with the Breit-Wigner propagator

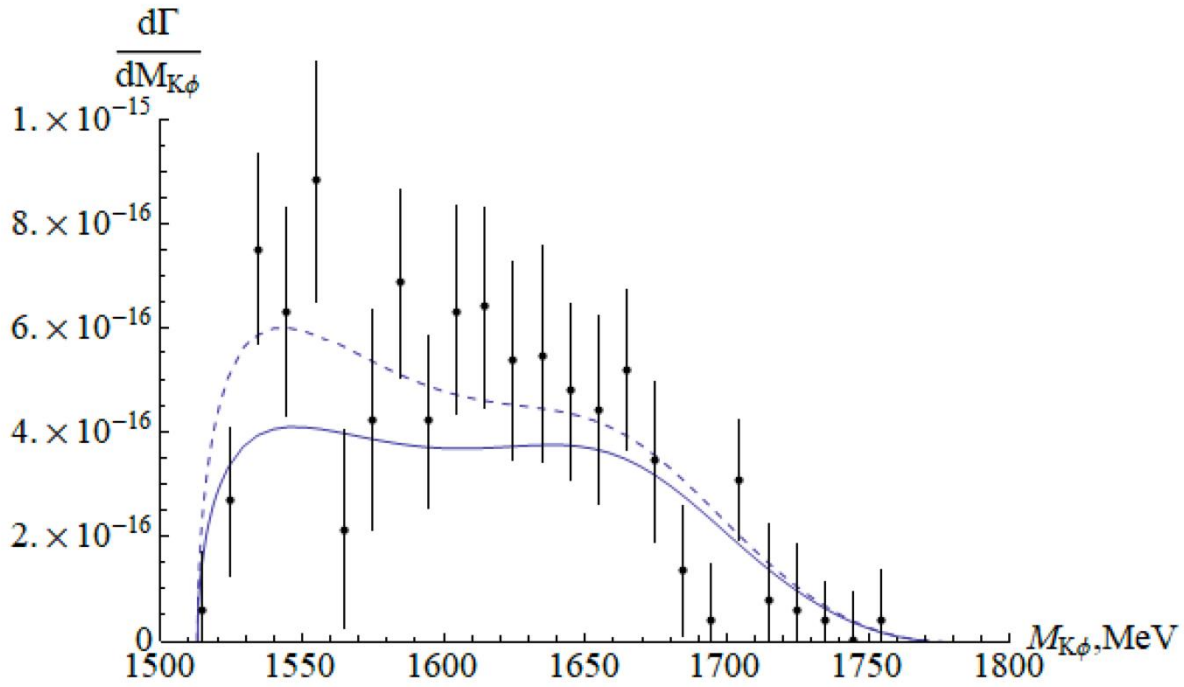
$$BW_M = \frac{1}{M_M^2 - q^2 - i\sqrt{q^2}\Gamma_M}.$$

Br (10^{-4})		
$\tau \to \omega(782) K \nu_\tau$		
	$\alpha = 57^\circ$	$\alpha = 45^\circ$
W_A	0.54	0.52
A	4.73	4.47
W_A + ***A***	3.08	3.27
W_V	0.66	0.64
V	1.99	1.94
W_V + ***V***	0.37	0.36
P	0.57	0.53
Ground	3.83	3.96
A'	5.4×10^{-3}	5.2×10^{-3}
V'	0.31	0.30
P'	8.6×10^{-4}	6.1×10^{-4}
Excited	0.32	0.31
Total	3.79	3.95
Data	4.1 ± 0.9	

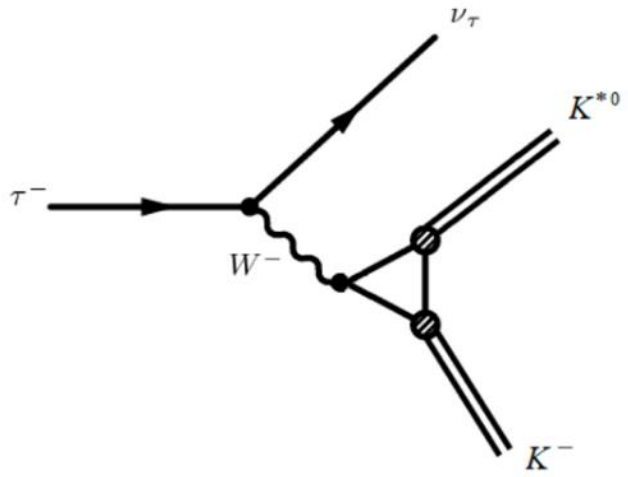
Br (10^{-4})		
$\tau \to \varphi(1020) K \nu_\tau$		
	$\alpha = 57^\circ$	$\alpha = 45^\circ$
W_A	2.02	1.97
A	7.49	8.59
W_A + ***A***	2.12	2.94
W_V	0.84	0.82
V	0.89	0.86
W_V + ***V***	2.6×10^{-3}	2.5×10^{-3}
P	0.66	0.61
Ground	2.57	3.34
A'	0.74	0.72
V'	18.1×10^{-3}	17.7×10^{-3}
P'	1.6×10^{-4}	1.2×10^{-4}
Excited	0.76	0.74
Total	3.15	4.04
Data	4.4 ± 1.6 4.05 ± 0.51 [Belle] 3.39 ± 0.48 [BaBar]	

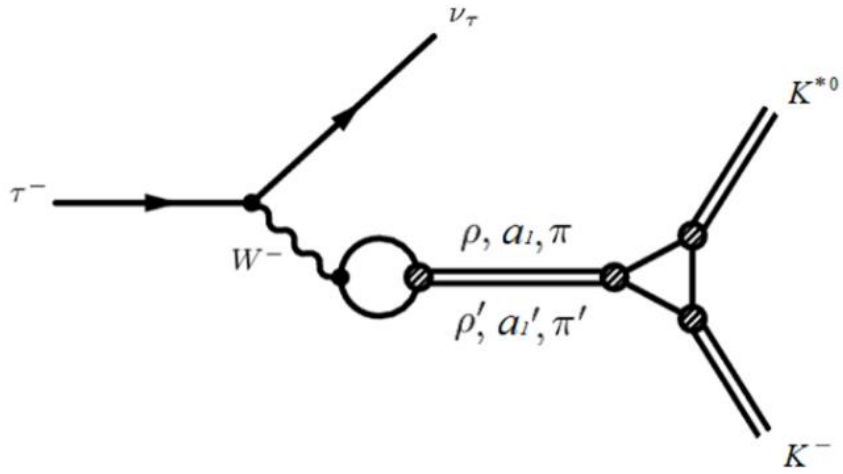


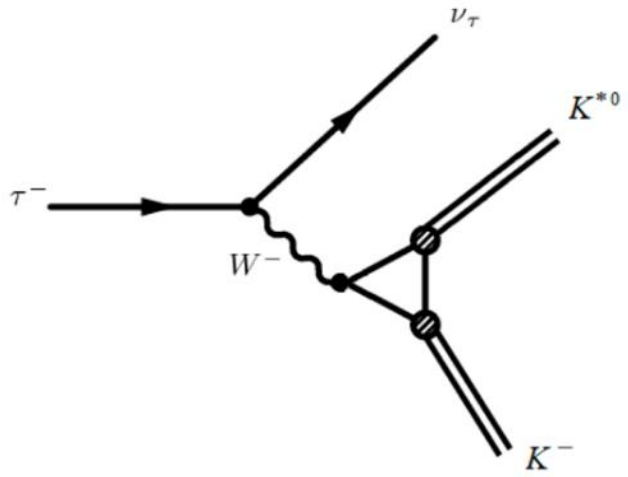
K.E. Arms et al.[CLEO]. Phys. Rev. Lett. 94 (2005), 241802

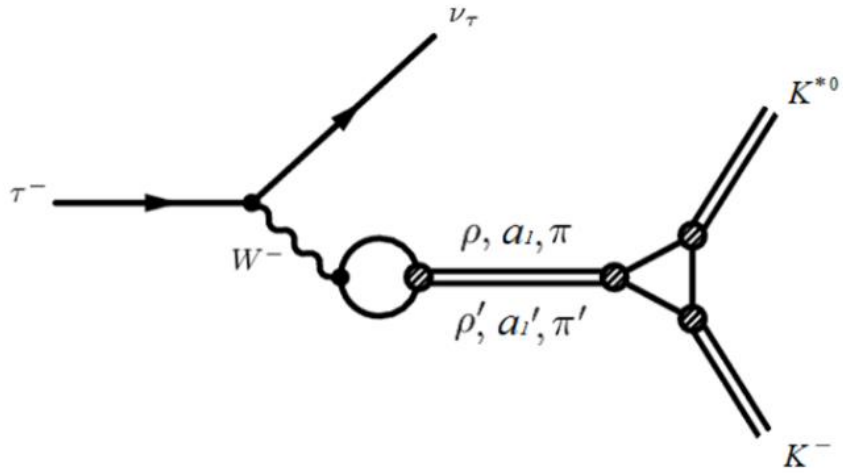


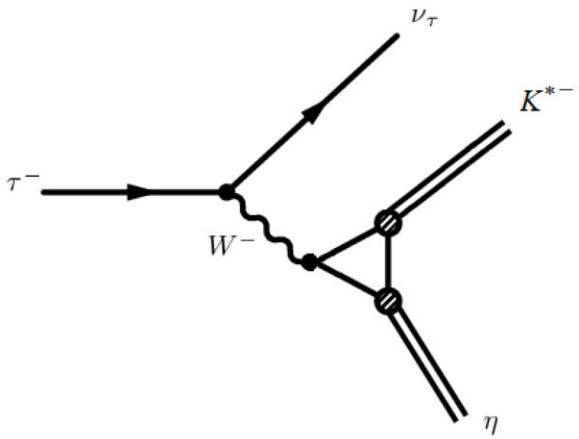
K. Inami et al. [Belle]. Phys. Lett. B 643 (2006), 5-10

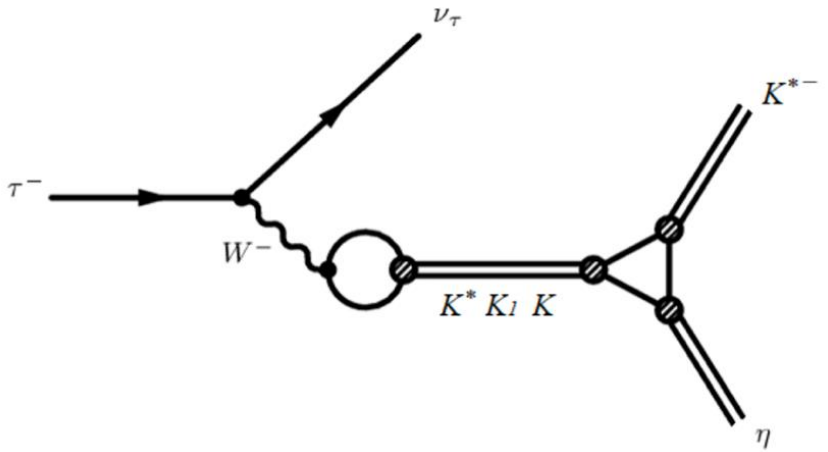

ν_τ
K^{*0}
τ^-
W^-
K^-


ν_τ
K^{*0}
τ^-
ρ, a_1, π
W^-
ρ', a_1', π'
K^-


ν_τ
K^{*0}
τ^-
W^-
K^-


ν_τ
K^{*0}
τ^-
W^-
ρ, a_1, π
ρ', a_1', π'
K^-


ν_τ
K^{*-}
τ^-
W^-
η

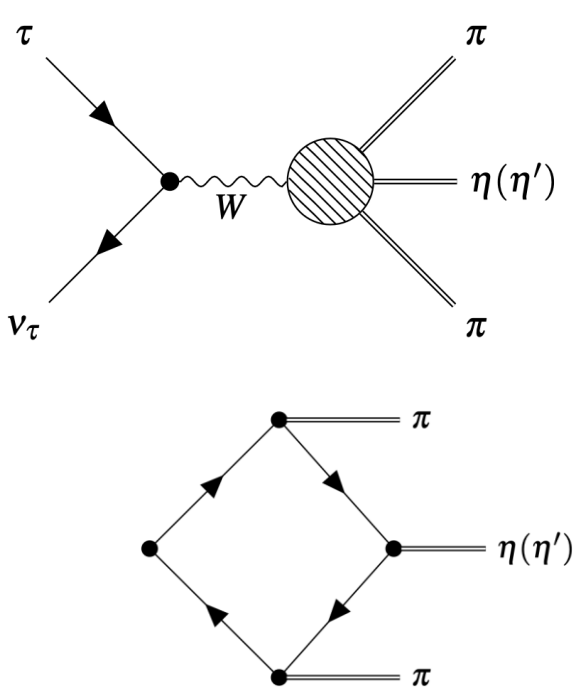

ν_τ
K^{*-}
τ^-
W^-
$K^* K_1 K$
η

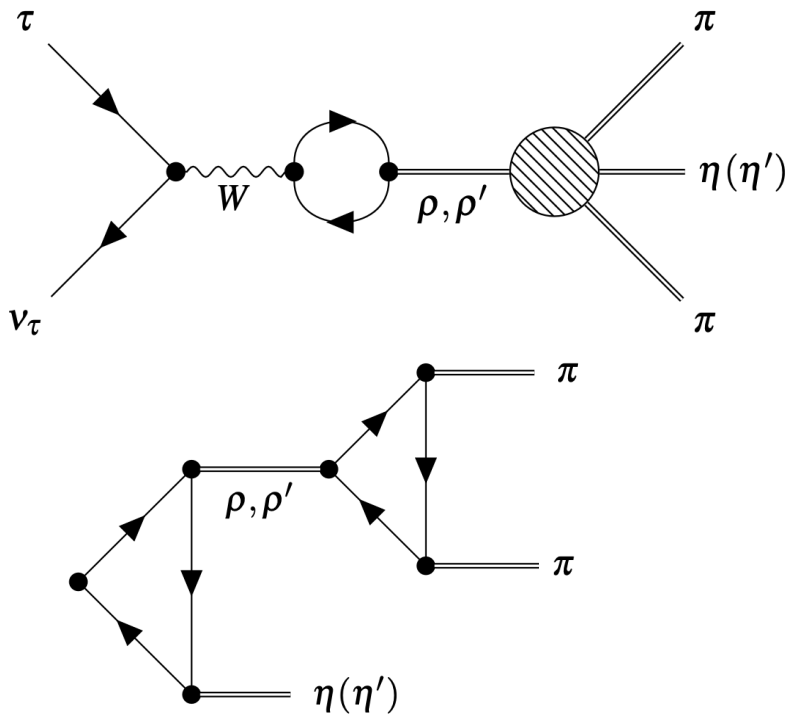
Br($\tau \to K^*(892)K^-\nu_\tau$) ($10^{-3}$)	
a_1	1.01
a'_1	0.000018
ρ	0.32
ρ'	0.24
π	0.09
π'	0.000035
Total	1.99
Experiment	2.1 ± 0.4

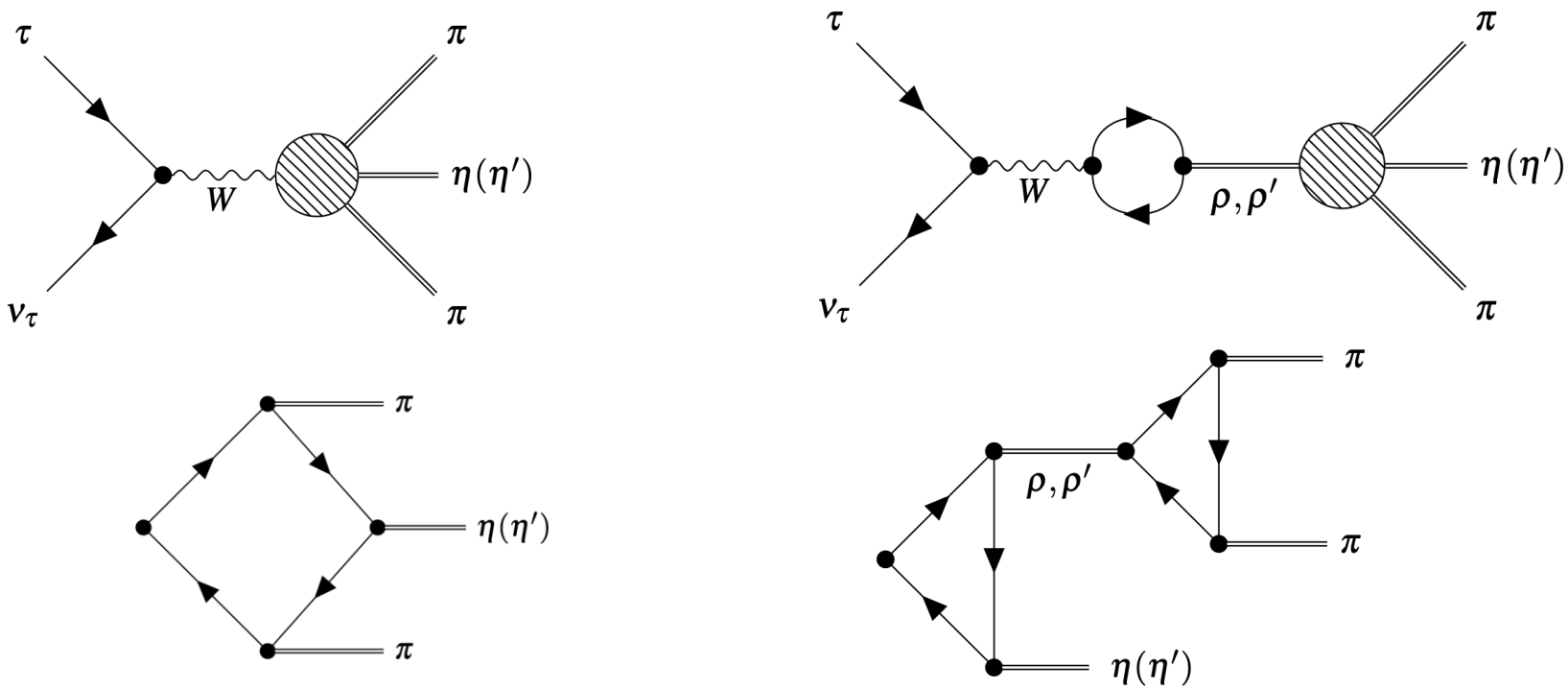
Br($\tau \to K^*(892)\eta\nu_\tau$) ($10^{-4}$)	
K_1	1.21
K^*	0.002
K	0.04
Total	1.23
Experiment	1.38 ± 0.15

$Br(\tau \to K^*(892)K^-\nu_\tau) = 3.92\times10^{-3}$[Li B.A. PRD. 1997. Vol. 55. P. 1436–1452]
$Br(\tau \to K^*(892)K^-\nu_\tau) = 4.93\times10^{-3}$ [Dai L.R. et al. EPJ A. 2019. vol. 55. P. 20]
$Br(\tau \to K^*(892)K^-\nu_\tau) = 1.50\times10^{-3}$[Guo Z.H. PRD. 2008. Vol. 78. P. 033004.]

$Br(\tau \to K^*(892)\eta\nu_\tau) = 1.01\times10^{-4}$
[Li B.A. PRD. 1997. Vol. 55. P. 1436–1452]


τ
π
η(η′)
W
ν_τ
π
π
π
η(η′)
π


τ
π
η(η′)
W
ρ, ρ′
ν_τ
π
π
π
ρ, ρ′
π
η(η′)

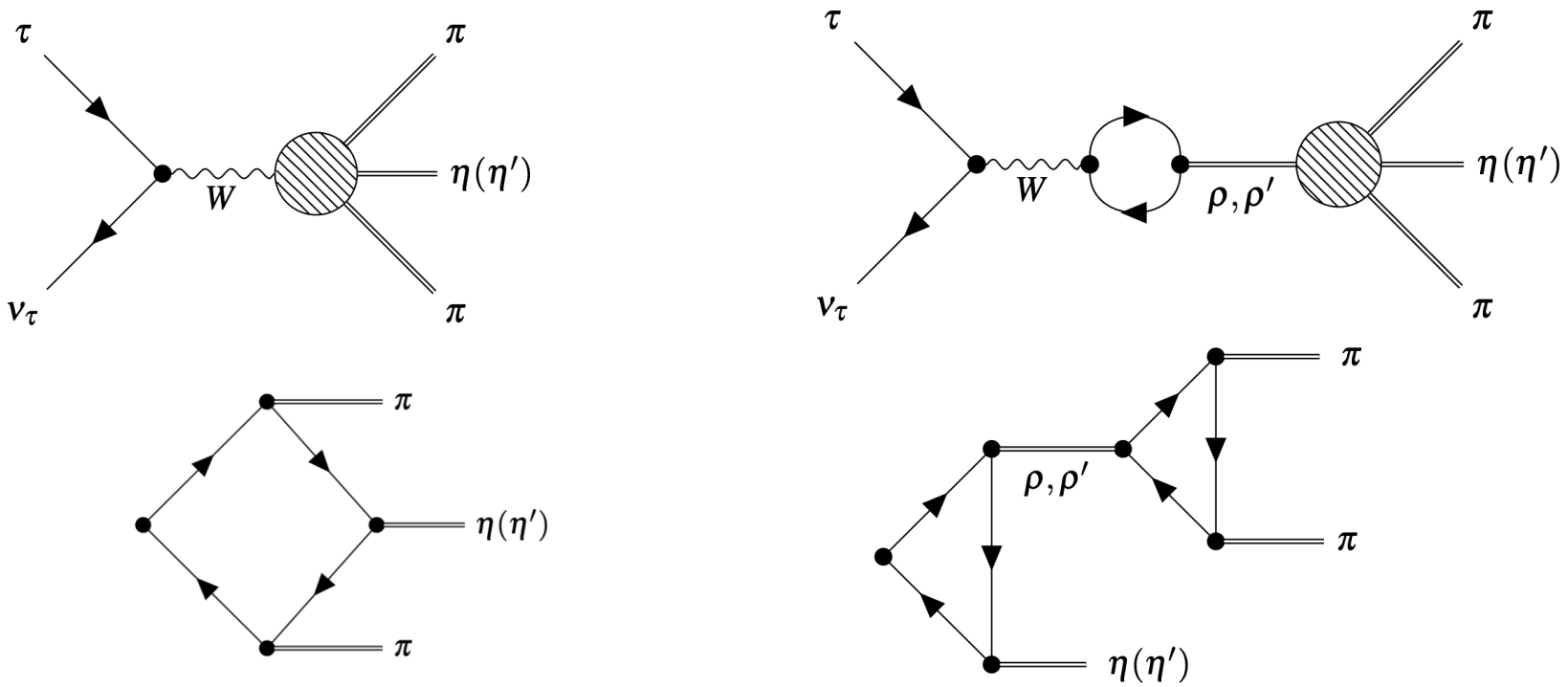


$$Br(\tau \to \pi\pi\eta\nu_\tau)_{NJL} = (1.84 \pm 0.27) \times 10^{-3}$$

$$Br(\tau \to \pi\pi\eta\nu_\tau)_{BaBar} = (1.63 \pm 0.08) \times 10^{-3}$$ [Phys. Rev. D 97, 052007 (2018)]

$$Br(\tau \to \pi\pi\eta\nu_\tau)_{CMD-3} = (1.68 \pm 0.17) \times 10^{-3}$$ [JHEP 01, 112 (2020)]

$$Br(\tau \to \pi\pi\eta\nu_\tau)_{PDG} = (1.39 \pm 0.07) \times 10^{-3}$$



$$Br(\tau \to \pi\pi\eta\nu_\tau)_{NJL} = (1.84 \pm 0.27) \times 10^{-3}$$

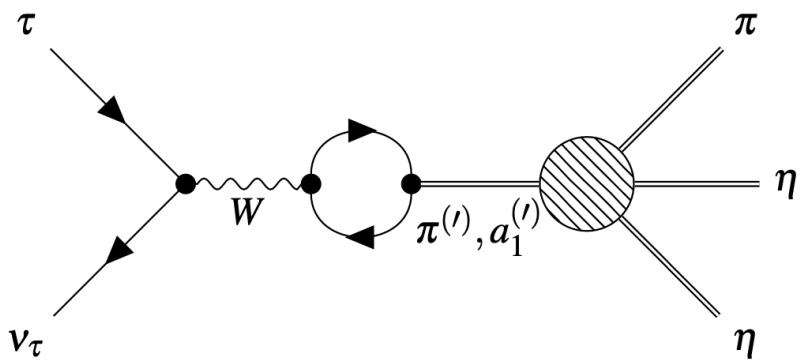
$$Br(\tau \to \pi\pi\eta\nu_\tau)_{BaBar} = (1.63 \pm 0.08) \times 10^{-3}$$ [Phys. Rev. D 97, 052007 (2018)]

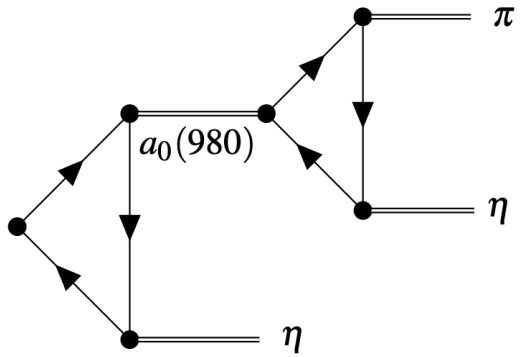
$$Br(\tau \to \pi\pi\eta\nu_\tau)_{CMD-3} = (1.68 \pm 0.17) \times 10^{-3}$$ [JHEP 01, 112 (2020)]

$$Br(\tau \to \pi\pi\eta\nu_\tau)_{PDG} = (1.39 \pm 0.07) \times 10^{-3}$$

$$Br(\tau \to \pi\pi\eta'\nu_\tau) = (0.32 \pm 0.04) \times 10^{-5}$$

$$Br(\tau \to \pi\pi\eta'\nu_\tau)_{PDG} < 1.2 \times 10^{-5}.$$





$$Br(\tau \to \pi\eta\eta\nu_\tau) = (6.0 \pm 0.9) \times 10^{-6},$$

$$Br(\tau \to \pi\eta\eta\nu_\tau)_{PDG} = < 7.4 \times 10^{-6}.$$

$$Br(\tau \to \pi\eta\eta'\nu_\tau) = (4.0 \pm 0.6) \times 10^{-7}.$$

Decay	NJL	Data
$\tau \to \omega\pi\nu_\tau$	1.85×10^{-2}	$(1.95 \pm 0.06) \times 10^{-2}$
$\tau \to \rho\eta\nu_\tau$	1.44×10^{-3}	–
$\tau \to K^*\pi\nu_\tau$	1.78×10^{-3}	$(2.2 \pm 0.5) \times 10^{-3}$
$\tau \to f_1\pi\nu_\tau$	3.98×10^{-4}	$(3.90 \pm 0.5) \times 10^{-4}$
$\tau \to a_1\pi\nu_\tau$	1.40×10^{-4}	–
$\tau \to K_1\pi\nu_\tau$	3.59×10^{-6}	–
$\tau \to K_1K\nu_\tau$	2.80×10^{-9}	–
$\tau \to f_0(500)\pi\nu_\tau$	5.06×10^{-2}	–
$\tau \to f_0(980)\pi\nu_\tau$	2.53×10^{-4}	–
$\tau \to f_0(500)K\nu_\tau$	9.70×10^{-5}	–
$\tau \to f_0(980)K\nu_\tau$	5.42×10^{-6}	–
$\tau \to a_0(980)\eta\nu_\tau$	4.30×10^{-5}	–
$\tau \to a_0(980)K\nu_\tau$	5.0×10^{-6}	–

Two meson τ decay branching fractions

Decay	NJL model	PDG	Belle	BaBar	Theor
$\tau \to \pi^-\pi^0\pi^0\nu_\tau$	9.15 ± 1.37	9.26 ± 0.10		–	5.77 [1]
$\tau \to \bar{K}^0\pi^-\pi^0\nu_\tau$	3.70 ± 0.55	3.82 ± 0.13	3.86 ± 0.14	–	5.77 [1]
$\tau \to K^-\pi^+\pi^-\nu_\tau$	3.27 ± 0.49	3.45 ± 0.07	3.30 ± 0.17	2.73 ± 0.09	5.67 [1]
$\tau \to K^-\pi^0\pi^0\nu_\tau$	0.68 ± 0.10	0.65 ± 0.22	–	–	1.08 [1]
$\tau \to K^-K^+K^-\nu_\tau$	0.020 ± 0.003	0.022 ± 0.008	–	–	0.0005 [1]
$\tau \to K^-K^0\bar{K}^0\nu_\tau$	0.019 ± 0.003	–	–	–	–
$\tau \to K^-\pi^0\eta\nu_\tau$	0.039 ± 0.006	0.048 ± 0.012	0.046 ± 0.011	–	0.008 [2]
$\tau \to \bar{K}^0\pi^-\eta\nu_\tau$	0.078 ± 0.012	0.094 ± 0.015	0.088 ± 0.014	–	–
$\tau \to K^-K^0\eta\nu_\tau$	0.0016 ± 0.0002	< 0.009	< 0.009	–	–
$\tau \to K^-\eta\eta\nu_\tau$	$(1.0 \pm 0.15)\times10^{-5}$	$< 3.0\times10^{-3}$	$< 3.0\times10^{-3}$	–	–

F. Takahashi et al. (Particle Data Group), Int. J. Mod. Phys. A 41, 2630011 (2026)
kman et al. [Particle Data Group], PTEP 2022 (2022), 083C01
B. Aubert et al. [BaBar], Phys. Rev. Lett. 100 (2008), 011801
M. J. Lee et al. [Belle], Phys. Rev. D 81 (2010), 113007
S. Ryu et al. [Belle], Phys. Rev. D 89 (2014) no.7, 072009
K. Inami et al. [Belle], Phys. Lett. B 672 (2009), 209-218

[1] R. Decker, E. Mirkes, R. Sauer and Z. Was, Z. Phys. C 58 (1993), 445-452
[2] A. Pich, Phys. Lett. B 196 (1987), 561-565

The amplitude includes the contact Feynman diagram with direct interaction of virtual photon with quarks and diagrams with intermediate vector meson:

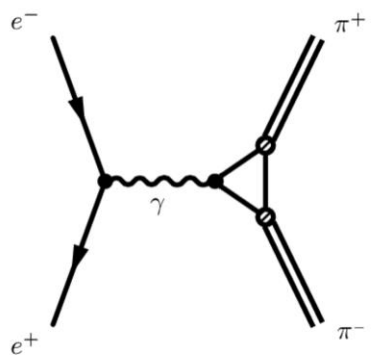
$$M(e^+e^- \to \pi\pi) = -\frac{4\pi\alpha_{em}}{s}\left[1 + \frac{s}{M_\rho^2 - s - i\sqrt{s}\Gamma_\rho} + \frac{s^2}{9}\frac{g_\rho^2\left[I_2(u) - I_2(d)\right]}{\left[M_\rho^2 - s - i\sqrt{s}\Gamma_\rho\right]\left[M_\omega^2 - s - i\sqrt{s}\Gamma_\omega\right]}\right]L_\mu^{em}(p_+ - p_-)^\mu,$$

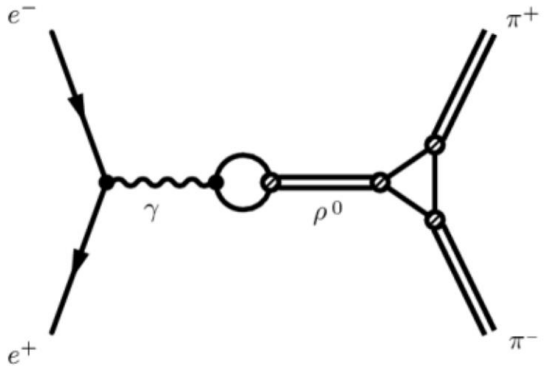
$s = (p_+ + p_-)^2$, $L_\mu^{em} = \bar{e}^-\gamma_\mu e^-$ is the electromagnetic lepton current.

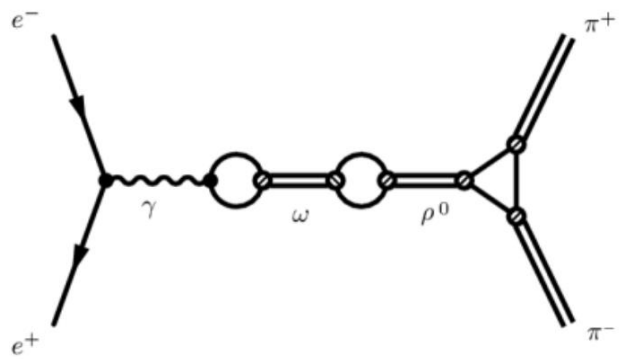
The amplitude includes the contact Feynman diagram with direct interaction of virtual photon with quarks and diagrams with intermediate vector meson:

$$M(e^+e^- \to \pi\pi) = -\frac{4\pi\alpha_{em}}{s}\left[1 + \frac{s}{M_\rho^2 - s - i\sqrt{s}\Gamma_\rho} + \frac{s^2}{9}\frac{g_\rho^2\left[I_2(u) - I_2(d)\right]}{\left[M_\rho^2 - s - i\sqrt{s}\Gamma_\rho\right]\left[M_\omega^2 - s - i\sqrt{s}\Gamma_\omega\right]}\right]L_\mu^{em}(p_+ - p_-)^\mu,$$

$s = (p_+ + p_-)^2$, $L_\mu^{em} = \bar{e}^-\gamma_\mu e^-$ is the electromagnetic lepton current.

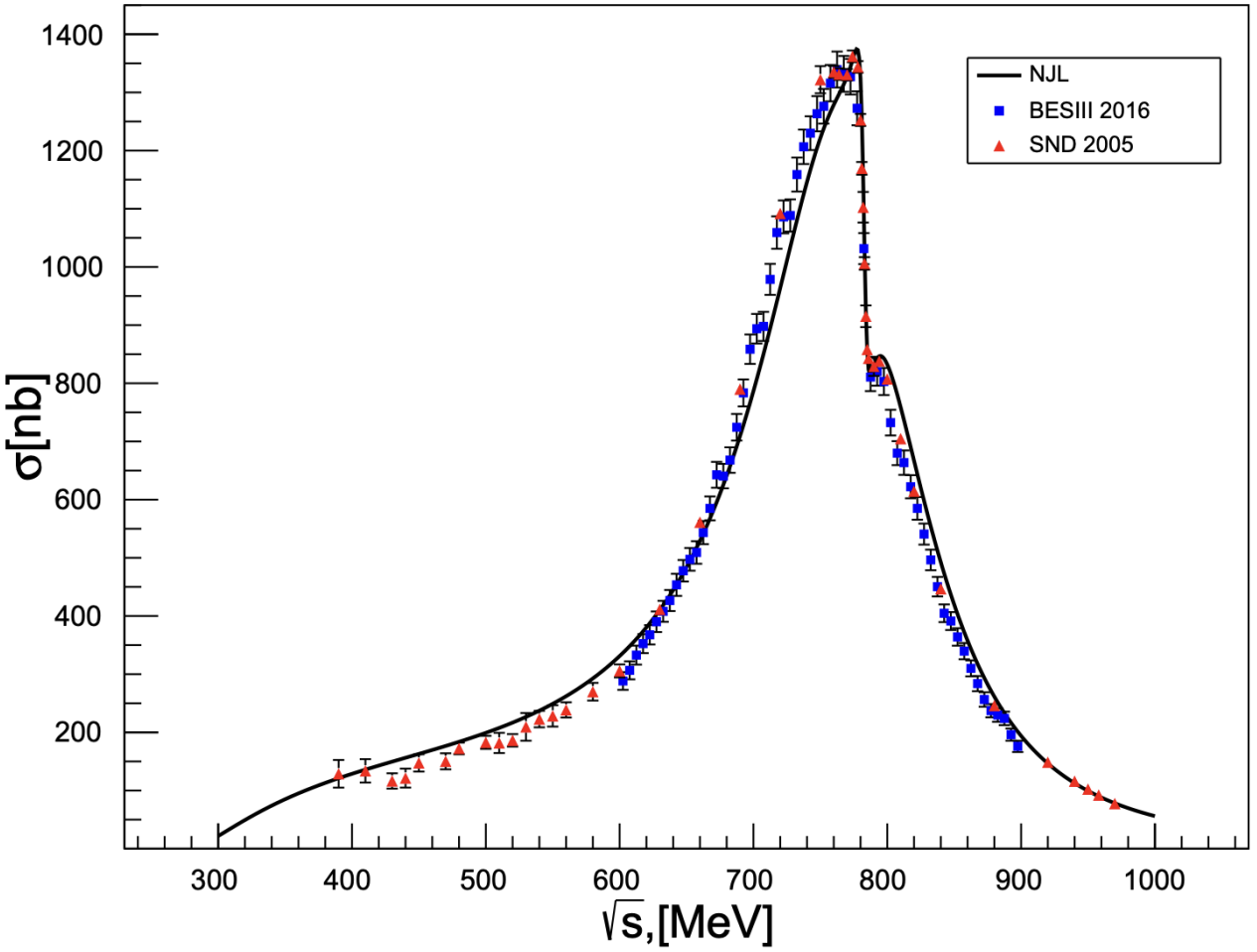






$I_2(u)$ and $I_2(d)$ are the loop integrals with the masses of u and d quarks appearing in $\omega \to \rho$ transition.
$m_d - m_u = 4$ MeV ($\omega \to \pi\pi$).

$$\Gamma_\rho(q^2) = \frac{g_\rho^2}{48\pi q^2}\left(q^2 - 4M_\pi^2\right)^{3/2}.$$



M.N. Achasov et al. [SND] J. Exp. Theor. Phys. 101 (2005) no.6, 1053-1070
M. Ablikim et al. [BESIII], Phys. Lett. B 753 (2016), 629-638

The hadron current obtained in the process $e^+e^- \to \pi^+\pi^-$ is associated with the decay $\tau^- \to \pi^-\pi^0\nu_\tau$ by the isospin transformation within Conserved Vector Current hypothesis. The cross section can be represented through the spectral hadron function $v^{\pi\pi}(s)$

$$\sigma(e^+e^- \to \pi^+\pi^-) = \left(\frac{4\pi^2\alpha_{em}^2}{s}\right) v^{\pi\pi}(s).$$

The appropriate differential width of the decay $\tau^- \to \pi^-\pi^0\nu_\tau$ takes the form

$$\frac{d\Gamma(\tau^- \to \pi^-\pi^0\nu_\tau)}{dq^2} = \frac{G_F^2 V_{ud}^2 S_{EW}^{\pi\pi}}{32\pi^2 M_\tau^3}\left(M_\tau^2 - q^2\right)^2\left(M_\tau^2 + 2q^2\right) v^{\pi\pi^0}(q^2),$$

where $v^{\pi\pi}(s) = v^{\pi\pi^0}(q^2), s = q^2$.

The hadron current obtained in the process $e^+e^- \to \pi^+\pi^-$ is associated with the decay $\tau^- \to \pi^-\pi^0\nu_\tau$ by the isospin transformation within Conserved Vector Current hypothesis. The cross section can be represented through the spectral hadron function $v^{\pi\pi}(s)$

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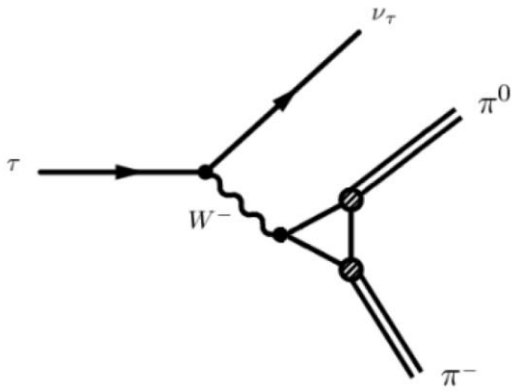
$$\frac{d\Gamma(\tau^- \to \pi^-\pi^0\nu_\tau)}{dq^2} = \frac{G_F^2 V_{ud}^2 S_{EW}^{\pi\pi}}{32\pi^2 M_\tau^3}\left(M_\tau^2 - q^2\right)^2\left(M_\tau^2 + 2q^2\right) v^{\pi\pi^0}(q^2),$$

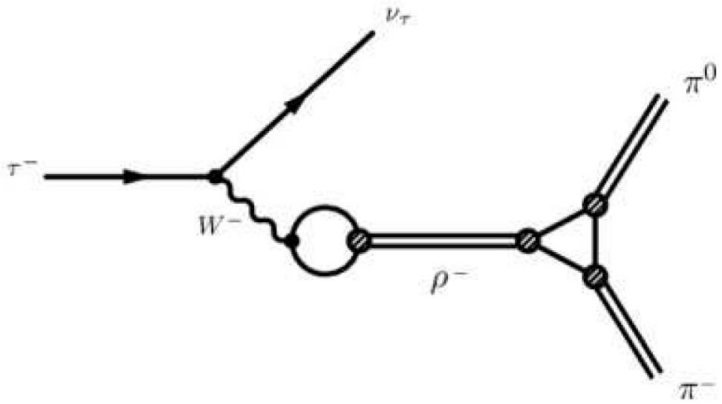
where $v^{\pi\pi}(s) = v^{\pi\pi^0}(q^2), s = q^2$.

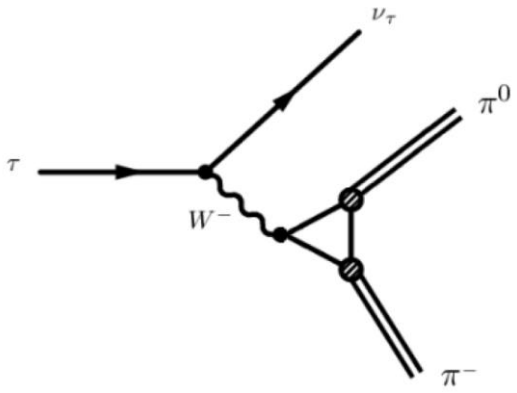
$$Br(\tau^- \to \pi^-\pi^0\nu_\tau) = (26.2 \pm 1.3)\%.$$

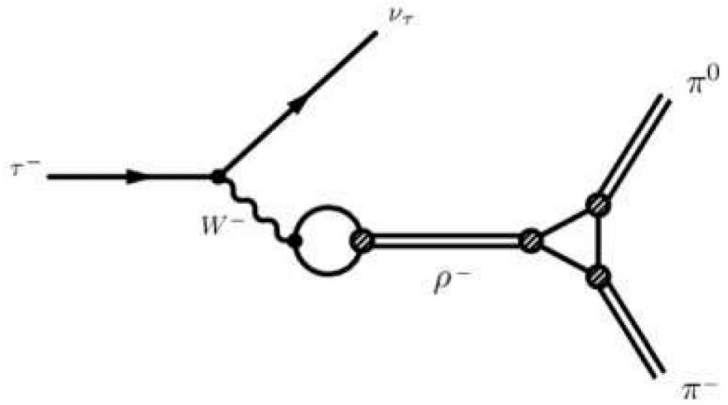
$$Br(\tau^- \to \pi^-\pi^0\nu_\tau)_{exp} = (25.5 \pm 0.1)\%$$

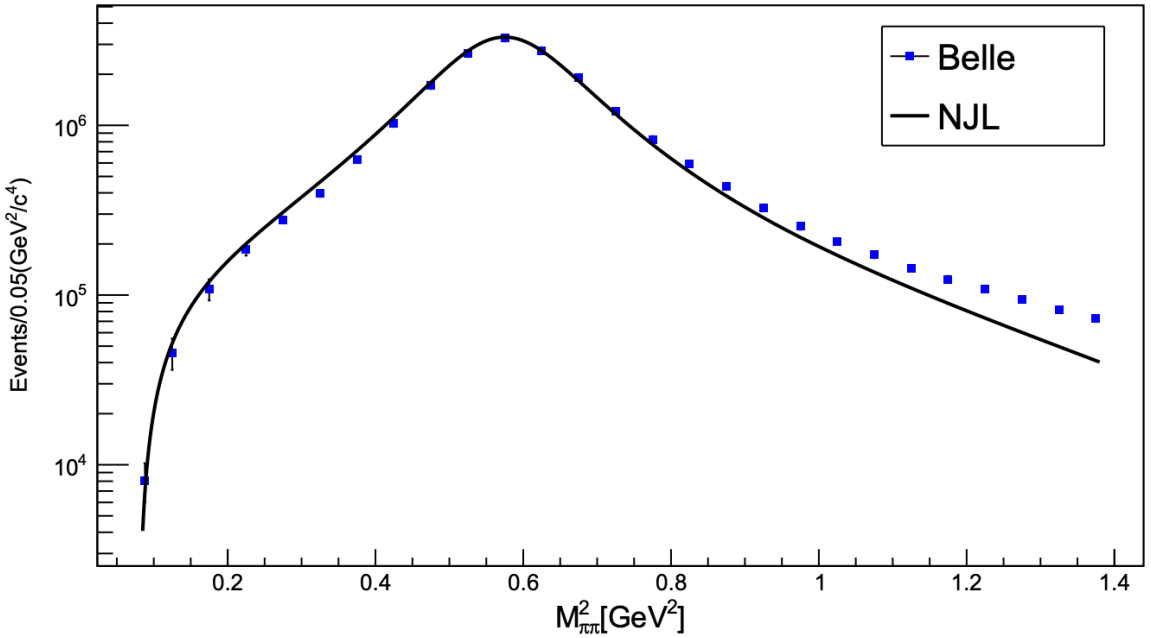
S. Navas et al. [Particle Data Group], Phys. Rev. D 110 (2024) no.3, 030001


ν_τ
π^0
τ
W^-
π^-


ν_τ
π^0
τ^-
W^-
ρ^-
π^-







The invariant mass distribution for the process $\tau \to \pi^0\pi^-\nu_\tau$

$$Br(\tau^- \to \pi^-\pi^0\nu_\tau) = (26.6 \pm 1.3)\%$$

$$Br(\tau^- \to \pi^-\pi^0\nu_\tau)_{CVC} = (26.2 \pm 1.3)\%$$

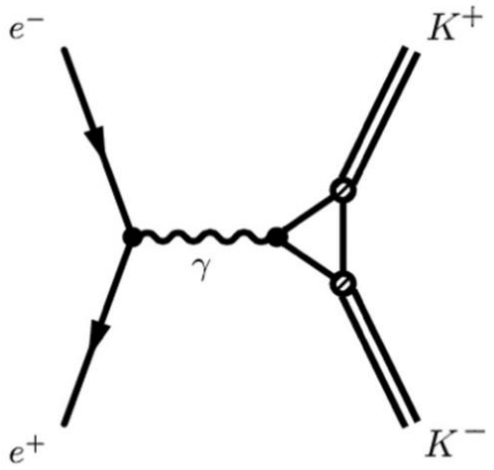
$$Br(\tau^- \to \pi^-\pi^0\nu_\tau)_{exp} = (25.5 \pm 0.1)\%$$

The amplitude includes the contact Feynman diagram with direct interaction of virtual photon with quarks and diagrams with intermediate vector meson:

$$\mathcal{M} = \frac{16\pi\alpha_{em}}{s} l^\mu \left\{ B_{(\gamma)} + B_{(\rho+\rho')} + B_{(\omega+\omega')} + e^{i\pi} B_{(\phi+\phi')} \right\}_{\mu\nu} (p_{K^+} - p_{K^-})^\nu,$$

M.N. Achasov et al. [SND collaboration], Phys. Rev. D 76 (2007), 072012

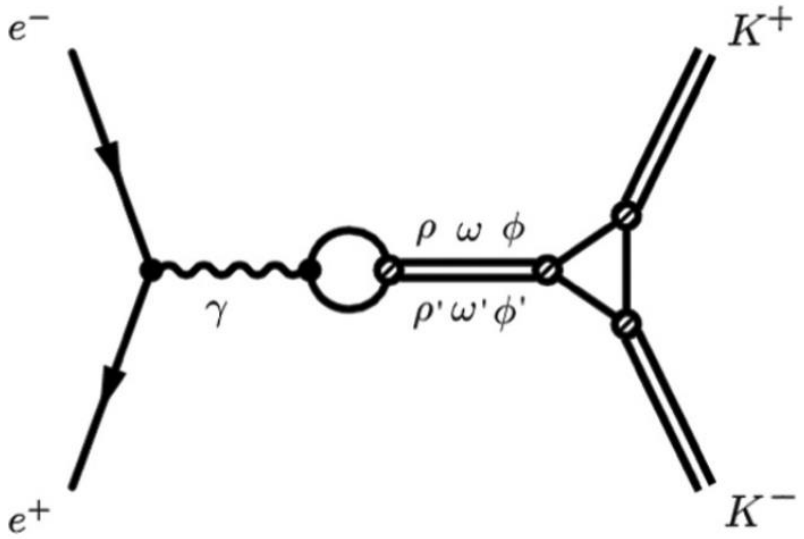
The contact diagram contribution reads $B_{(\gamma)\mu\nu} = g_{\mu\nu} I_2^{\gamma KK}$,



Contributions of intermediate vector mesons

$$B_{(V+V')\mu\nu} = r_V \left[\frac{C_V}{g_V} \frac{g_{\mu\nu}s - p_\mu p_\nu}{M_V^2 - s - i\sqrt{s}\Gamma_V(s)} I_2^{VKK} + \frac{C_{V'}}{g_V} \frac{g_{\mu\nu}s - p_\mu p_\nu}{M_{V'}^2 - s - i\sqrt{s}\Gamma_{V'}(s)} I_2^{V'KK} \right].$$

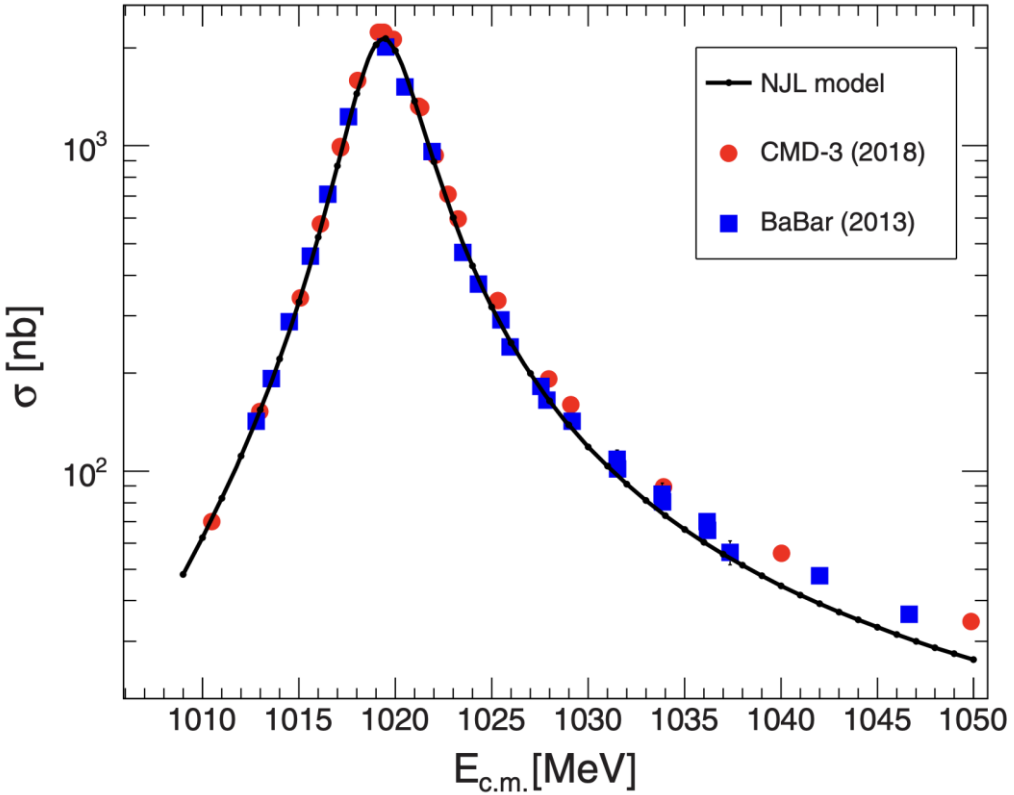
where $r_\rho = 1/2,\ r_\omega = 1/6,\ r_\phi = 1/3.$



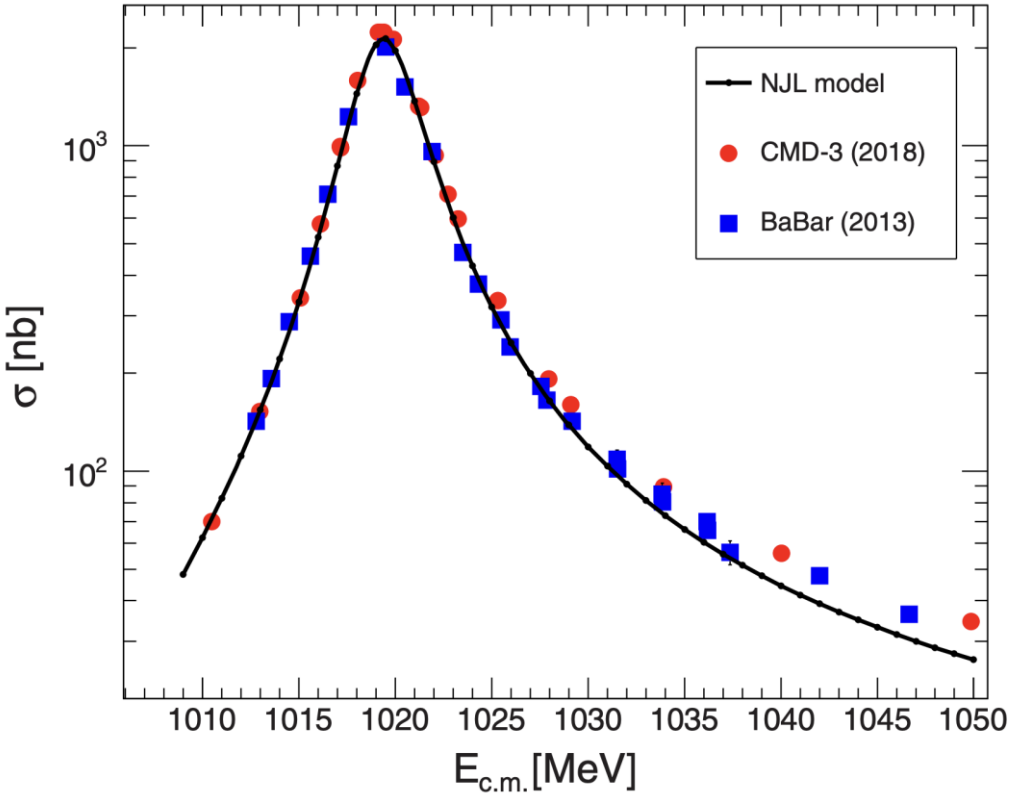
$$\Gamma_V(s) = \Gamma_V \frac{s}{M_V^2} \left(\frac{\beta(s, M_K)}{\beta(M_V^2, M_K)} \right)^3,$$

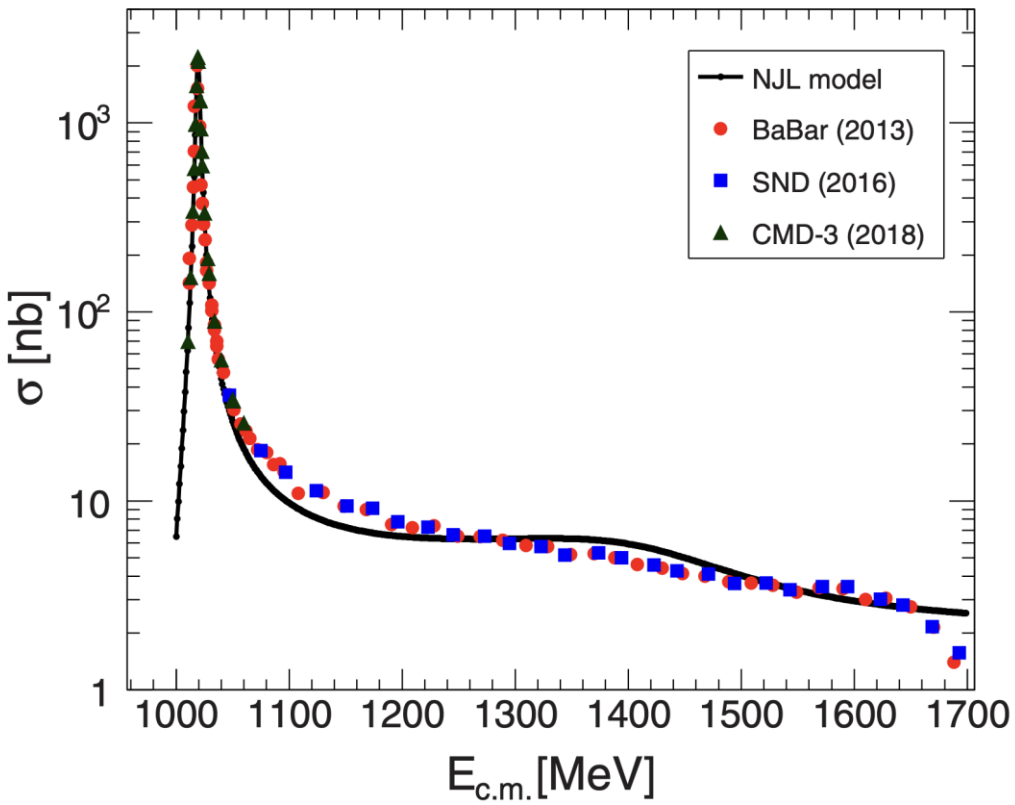
where $\beta(s, M_K) = \sqrt{1 - 4{M_K}^2/s}.$

J.P. Lees et al.[BaBar], Phys. Rev. D 88 2013 no.3, 032013



J.P. Lees et al. [BaBar], Phys. Rev. D 88 (2013) no.3, 032013
M.N. Achasov et al. [SND] Phys. Rev. D 94 (2016) no.11, 112006
E.A. Kozyrev et al. [CMD-3]Phys. Lett. B 779 (2018), 64-71



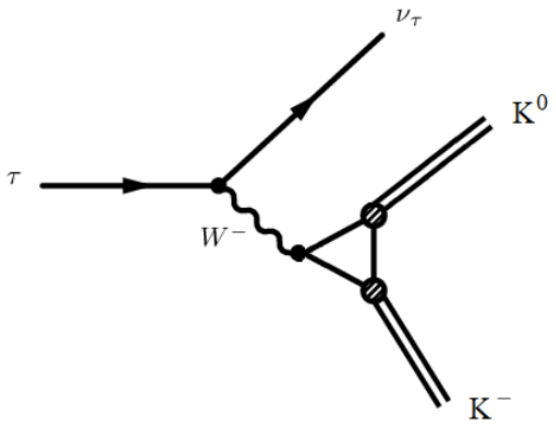


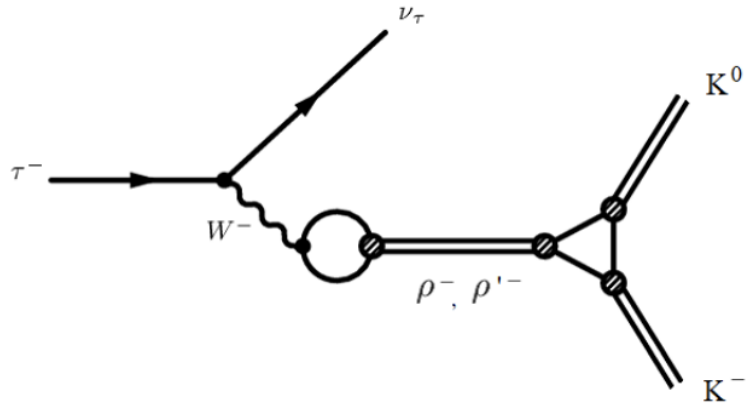
J.P. Lees et al. [BaBar], Phys. Rev. D 88 (2013) no.3, 032013
M.N. Achasov et al. [SND] Phys. Rev. D 94 (2016) no.11, 112006
E.A. Kozyrev et al. [CMD-3]Phys. Lett. B 779 (2018), 64-71

Comparisons of separate contributions with results of other models:

	Our result	[10]	[32]
N_ρ	0.44	0.598	0.598
N_ω	0.147	0.171	0.199
N_ϕ	0.34	0.283	0.339
$N_{\rho'}$	0.066	0.056	0.056
$N_{\omega'}$	0.022	0.016	0.019
$N_{\phi'}$	0.0005	0.005	0.006

[10] S.A. Ivashyn and A.Y. Korchin, Eur.Phys. J. C 49, 697 (2007).
[32] C. Bruch, A. Khodjamirian and J.H. Kuhn, Eur.Phys.J. C 39, 41 (2005).





$$Br(\tau \to K^- K^0 \nu_\tau) = 13.95 \times 10^{-4}.$$

$$Br(\tau \to K^- K^0 \nu_\tau)_{exp} = (14.86 \pm 0.34) \times 10^{-4}.$$

A good description of the following annihilation processes was achieved:

$$
\begin{aligned}
&e^+e^- \to [\pi, \pi(1300)]\gamma,\\
&e^+e^- \to [\eta, \eta', \eta(1295), \eta(1475)]\gamma,\\
&e^+e^- \to [f_1(1285), a_1(1260)]\gamma,\\
&e^+e^- \to \gamma\pi\pi,\\
&e^+e^- \to [\pi, \pi(1300)]\pi,\\
&e^+e^- \to \omega\pi, \phi\pi,\\
&e^+e^- \to \rho\eta,\\
&e^+e^- \to K^{\pm}\left[K^{*\mp}(892), K^{*\mp}(1410)\right],\\
&e^+e^- \to [\eta, \eta'][\phi(1020), \phi(1680)],\\
&e^+e^- \to K^+K^-,\\
&e^+e^- \to 2\pi[\eta, \eta'],\\
&e^+e^- \to K^+K^-[\pi, \eta],\\
&e^+e^- \to a_1\pi, K_1K.
\end{aligned}
$$

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$$e^+e^- \to \gamma\pi\pi,$$
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$$e^+e^- \to \omega\pi, \phi\pi,$$
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$$e^+e^- \to K^+K^-[\pi, \eta],$$
$$e^+e^- \to a_1\pi, K_1K.$$

Predictions were obtained for:

$$e^+e^- \to \pi(1300)\gamma,$$
$$e^+e^- \to [\eta(1295), \eta(1475)]\gamma,$$
$$e^+e^- \to [f_1(1285), a_1(1260)]\gamma,$$
$$e^+e^- \to \pi(1300)\pi,$$
$$e^+e^- \to K^{\pm}K^{*\mp}(1410),$$
$$e^+e^- \to \eta\phi(1680),$$
$$e^+e^- \to \eta'[\phi(1020), \phi(1680)],$$
$$e^+e^- \to 2\pi\eta',$$
$$e^+e^- \to a_1\pi, K_1K.$$

Whole classes of processes are systematically considered within the same model without changing parameter values

Theoretical predictions for future experiments were constructed

The extended NJL model successfully works for e+e- annihilation and τ lepton decays at energies up to ~2 GeV

The nearest plans concern processes with axial-vector currents

THANKS FOR YOUR ATTENTION !



Nurlan Kanat
E-mail: nurlan@thoer.jinr.ru

Very simple four-fermion (current x current) interactions with chiral symmetry

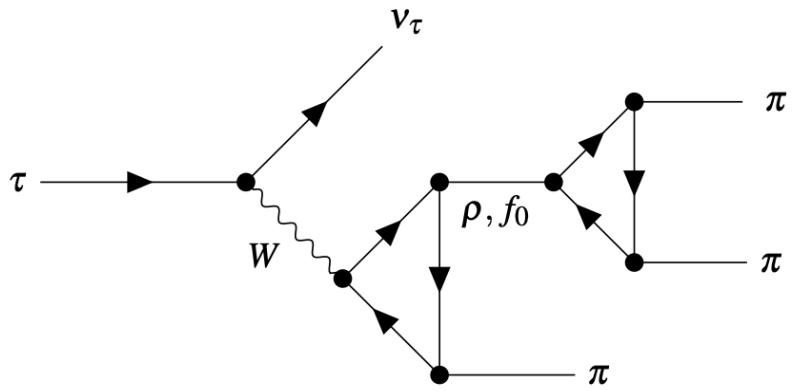
$$\begin{aligned}\mathcal{L}_q(\bar{q},q) &= \int \mathrm{d}^4x \Big\{\bar{q}(x)(i\hat{\partial}_x - m_0)q(x) \\ &+ \sum_{a=0}^{8}\sum_{i=1}^{2}\Big[\frac{G_1}{2}\left[j^a_{S,i}(x)j^a_{S,i}(x) + j^a_{P,i}(x)j^a_{P,i}(x)\right] \\ &- \frac{G_2}{2}\left[j^{a,\mu}_{V,i}(x)j^{a,\mu}_{V,i}(x) + j^{a,\mu}_{A,i}(x)j^{a,\mu}_{A,i}(x)\right]\Big]\Big\},\end{aligned}$$

Currents

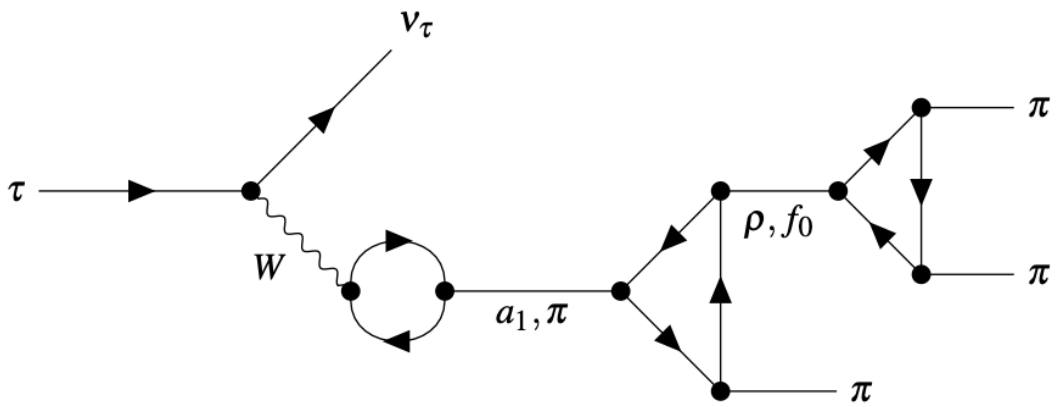
$$\begin{aligned}j^a_{S,i}(x) &= \int \mathrm{d}^4x_1 \int \mathrm{d}^4x_2 \bar{q}(x_1)F^a_{S,i}(x;x_1,x_2)q(x_2), \\ j^a_{P,i}(x) &= \int \mathrm{d}^4x_1 \int \mathrm{d}^4x_2 \bar{q}(x_1)F^a_{P,i}(x;x_1,x_2)q(x_2), \\ j^{a,\mu}_{V,i}(x) &= \int \mathrm{d}^4x_1 \int \mathrm{d}^4x_2 \bar{q}(x_1)F^{a,\mu}_{V,i}(x;x_1,x_2)q(x_2), \\ j^{a,\mu}_{A,i}(x) &= \int \mathrm{d}^4x_1 \int \mathrm{d}^4x_2 \bar{q}(x_1)F^{a,\mu}_{A,i}(x;x_1,x_2)q(x_2),\end{aligned}$$

$$F^a_{S,i}(k,P) = \lambda^a f^a_i(k_\perp), \quad F^a_{P,i}(k,P) = i\gamma^5\lambda^a f^a_i(k_\perp),$$

$$F^{a,\mu}_{V,i}(k,P) = \gamma^\mu\lambda^a f^a_i(k_\perp), \quad F^{a,\mu}_{A,i}(k,P) = \gamma^\mu\gamma^5\lambda^a f^a_i(k_\perp),$$



$\tau \to 3\pi\nu_\tau$ decay contact channel



: Diagram with intermediate mesons describing the decay $\tau \to 3\pi\nu_\tau$.

$$\mathscr{M}(\tau \to \pi^-\pi^0\pi^0\nu_\tau) = -iF_\pi G_F V_{ud} L_\mu \left\{\mathscr{M}_{a_1\rho} + \mathscr{M}_{\pi\rho} + \mathscr{M}_{a_1 f_0} + \mathscr{M}_{\pi f_0}\right\}^\mu,$$

$$\mathscr{M}^\mu_{a_1\rho} = Z_\pi g_\rho^2 \left[g^{\mu\nu} + \left((q^2 - 6m_u^2)g^{\mu\nu} - q^\mu q^\nu\right) BW_{a_1}\right] BW_\rho (p_{\pi^-} - p^{(1)}_{\pi^0})_\nu + \left(p^{(1)}_{\pi^0} \leftrightarrow p^{(2)}_{\pi^0}\right),$$

$$\mathscr{M}^\mu_{\pi\rho} = g_\rho^2 BW_\pi q^\mu \left(q + p^{(2)}_{\pi^0}\right)^\nu \left(p_{\pi^-} - p^{(1)}_{\pi^0}\right)_\nu BW_\rho + \left(p^{(1)}_{\pi^0} \leftrightarrow p^{(2)}_{\pi^0}\right),$$

$$\mathscr{M}^\mu_{a_1 f_0} = 4Z_\pi^2 g^2_{f_{0u}} \cos^2\bar{\phi} \left[g^{\mu\nu} + \left((q^2 - 6m_u^2)g^{\mu\nu} - q^\mu q^\nu\right) BW_{a_1}\right] BW_{f_0} (p_{\pi^-} - p^{(1)}_{\pi^0} - p^{(2)}_{\pi^0})_\nu,$$

$$\mathscr{M}^\mu_{\pi f_0} = 16Z_\pi^2 m_u g^2_{f_{0u}} \cos^2\bar{\phi} q^\mu BW_\pi BW_{f_0}.$$

$$BW_{meson} = \frac{1}{M^2_{meson} - q^2 - i\sqrt{q^2}\Gamma_{meson}},$$

Channels of the decay	B fractions (%)	CLEO II [38]
$\rho\pi$	60.0	68.11
$f_0(500)\pi$	12.8	16.18 ± 3.85

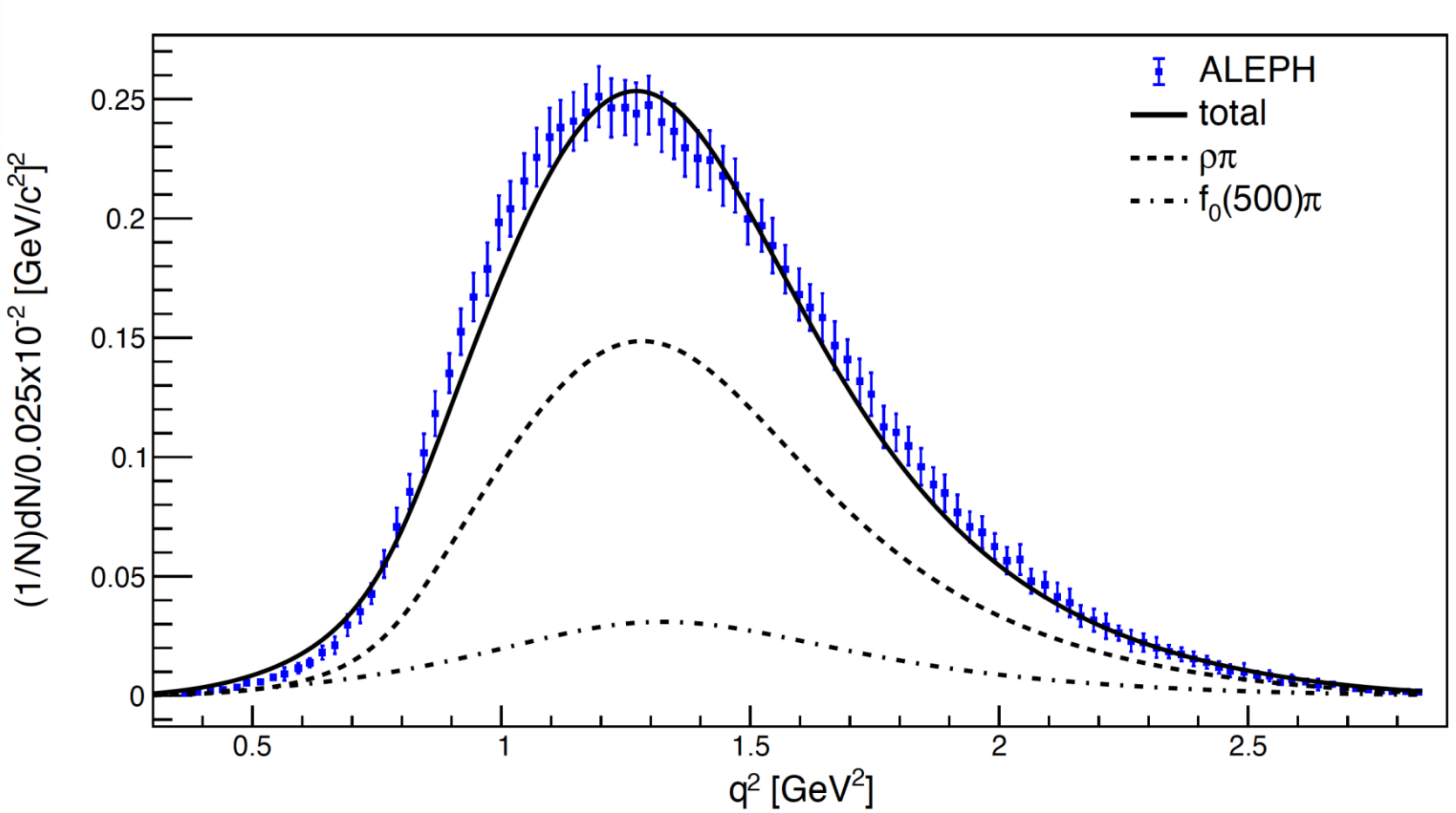
D.M. Asner et al. [CLEO], Phys. Rev. D 61 (2000), 012002

$$Br(\tau \to \pi^- \pi^0 \pi^0 \nu_\tau)_{NJL} = (9.15 \pm 1.37)\%$$

$$Br(\tau \to \pi^- \pi^0 \pi^0 \nu_\tau)_{exp} = (9.26 \pm 0.10)\%$$

S. Navas et al. [Particle Data Group], Phys. Rev. D 110, no.3, 030001 (2024)

M.K. Volkov, A.A. Pivovarov and K. Nurlan, Eur. Phys. J. A 61 (2025) no.3, 66



The theoretical predictions of the NJL model for the three pion invariant mass distribution and a comparison with the ALEPH data.

S. Schael et al. [ALEPH], Phys. Rept. 421 (2005), 191-284

M.K. Volkov, A.A. Pivovarov and K. Nurlan, Eur. Phys. J. A 61 (2025) no.3, 66