Renormalon-chain contributions to two-point correlators of nonlocal quark currents and light-meson distribution amplitudes

Nikolay Volchanskiy (BLTP JINR)

10/21/2026

第十四届强相互作用物理联合研讨会 ● BLTP JINR/ITP CAS ● 南京

Outline

Introduction: What are QCD composite vertices and renormalon chains?

The correlator of two composite operators

$(x, \underline{0})$ moment of the correlator and mesonic distribution amplitudes

Renormalon chains in light-meson distribution amplitudes (DA)

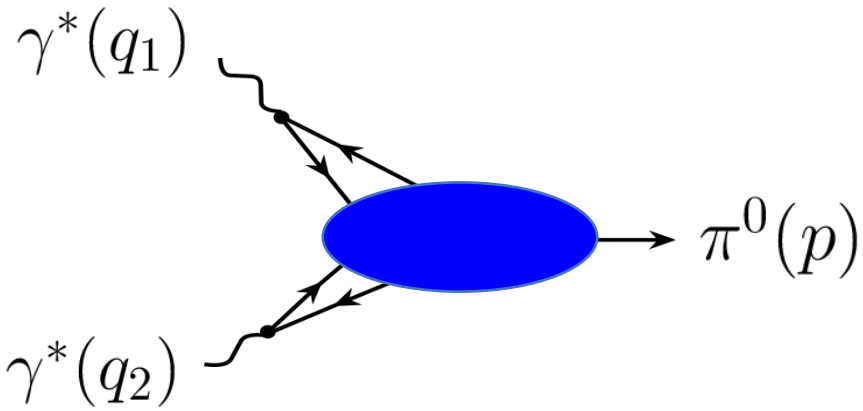
Summary

Nonlocal composite vertices in QCD

Q: In what physical setting do the **composite vertices** emerge?

A: When the **QCD FACTORIZATION** of large and small scales is observed in a hard process

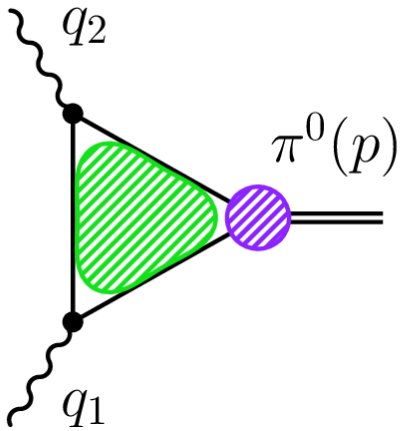
Example: $\gamma^*(q_1)\gamma^*(q_2) \to \pi^0(p)$
with $-q_1^2, -q_2^2 \gg 1$ GeV^2
and p^2 is at the hadron scale



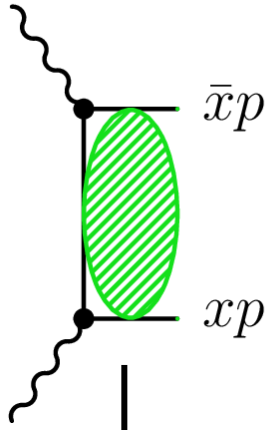
$$\langle 0|\mathrm{T}\left[V_\mu(q_1)V_\nu(q_2)\right]|\pi(p)\rangle \equiv i\int \mathrm{d}^4\mathrm{z}\, e^{iq_1\mathrm{z}}\langle 0|\mathrm{T}\left[j_\mu(\mathrm{z})j_\nu(0)\right]|\pi(p)\rangle$$
$$= e_{\mu\nu\lambda\sigma}q_1^\lambda q_2^\sigma \boldsymbol{F^{\gamma^*\gamma^*\pi}(q_1^2, q_2^2)}$$

transitional pion form factor

Nonlocal composite vertices in QCD



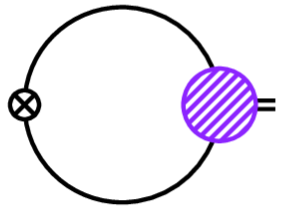
$=$



$\star$

$\int_0^1 \mathrm{d}x$

$\star$



$+ O\left(\dfrac{1}{q_1^2 + q_2^2}\right)$

perturbative coefficient function

distribution amplitude

$= \quad C(q_1^2, q_2^2; x) \qquad f_\pi \Phi_\pi(x)$

+ higher twists

hard momenta / small scales

soft momenta / large scales

Nonlocal composite vertices in QCD

Meson distribution amplitude (DA)

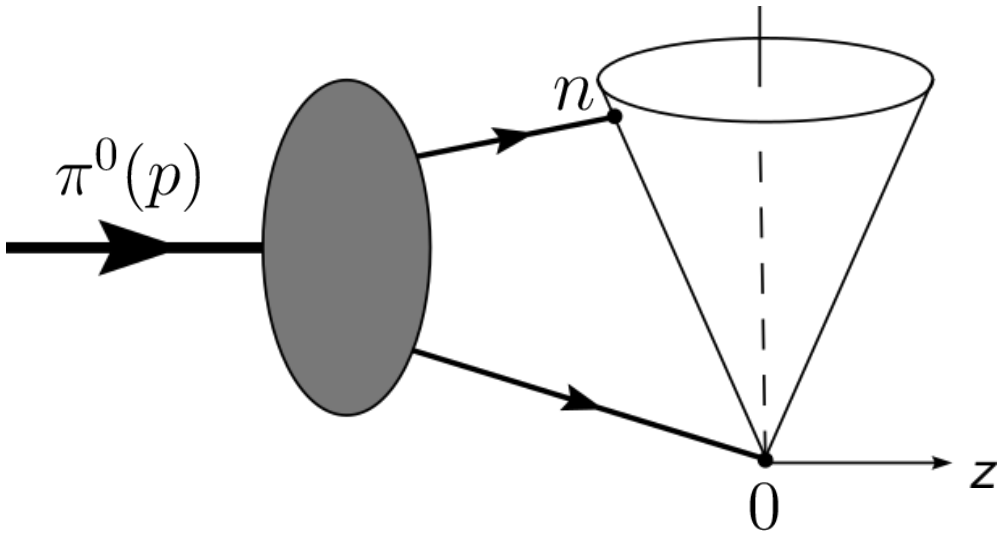
$$f_\pi \int_0^1 \mathrm{d}x e^{i(np)x} \mathbf{\Phi_\pi}(\boldsymbol{x}) = \langle 0 | \boxed{\bar{d}(n) \frac{\hat{\Gamma}}{(np)} [n,0]\, u(0)} | \pi(p) \rangle$$

Bilocal current (on the light cone)

Fourier transform

$$\boxed{\hat{O}(x,0)}$$

Composite operator



$$[n,0] = \mathtt{P} \exp\left[igt_a \int_0^n \mathrm{d}\mathtt{z}^\mu\, A_\mu^a(\mathtt{z}) \right]$$

Correlators of composite vertices in QCD

x  y $= \Pi(x,y;p^2) = \int \mathrm{d}^D \mathtt{z}\, e^{ip\mathtt{z}} \langle 0|\mathtt{T}\left[\hat{O}(x;0)\hat{O}(y;\mathtt{z})\right]|0\rangle$

The correlator of composite operators describes the perturbative content of DAs

QCD SR $\Phi_{\text{meson}}(x) \sim$ **Borel transform** $_{-p^2\to M^2}$ $\left[\int_0^1 \mathrm{d}y\, \Pi\left(x,y;p^2\right)\right]$

Feynman integrals in QCD after factorization:

$$f_1(x) \star \Pi(x,y;p^2) \star f_2(y) \qquad f_1(x) \star f_2(x) = \int_0^1 \mathrm{d}x f_1(x) f_2(x)$$

Correlators of composite vertices in QCD

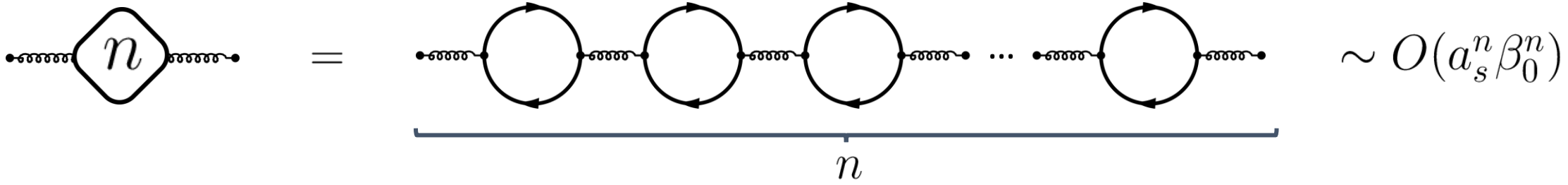
x  y $= \Pi(x,y;p^2) = \int \mathrm{d}^D\mathtt{z}\, e^{ip\mathtt{z}} \langle 0|\mathtt{T}\left[\hat{O}(x;0)\hat{O}(y;\mathtt{z})\right]|0\rangle$

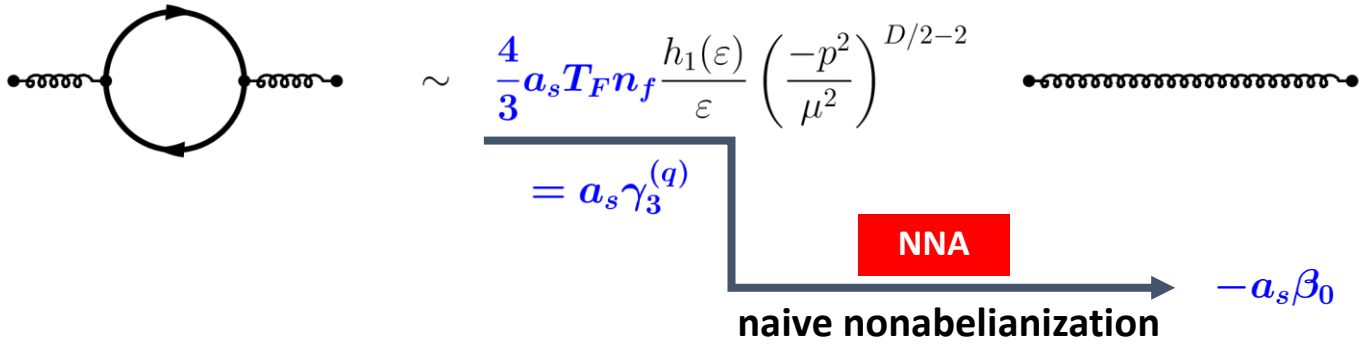
$$\dot{\Pi}(x,y;-Q^2) = \frac{\mathrm{d}}{\mathrm{d}\ln\left(\frac{Q^2}{\mu^2}\right)}\Pi(x,y;-Q^2)$$

The Adler function $D(Q^2) \sim \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y\, \dot{\Pi}(x,y;-Q^2)$

The R-ratio $R(s) = \dfrac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \sim \mathrm{Im} \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y\, \Pi(x,y;-s-i0)$

Renormalon-chain correlators





$$n_f = 3, \qquad \beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_F n_f = 9$$

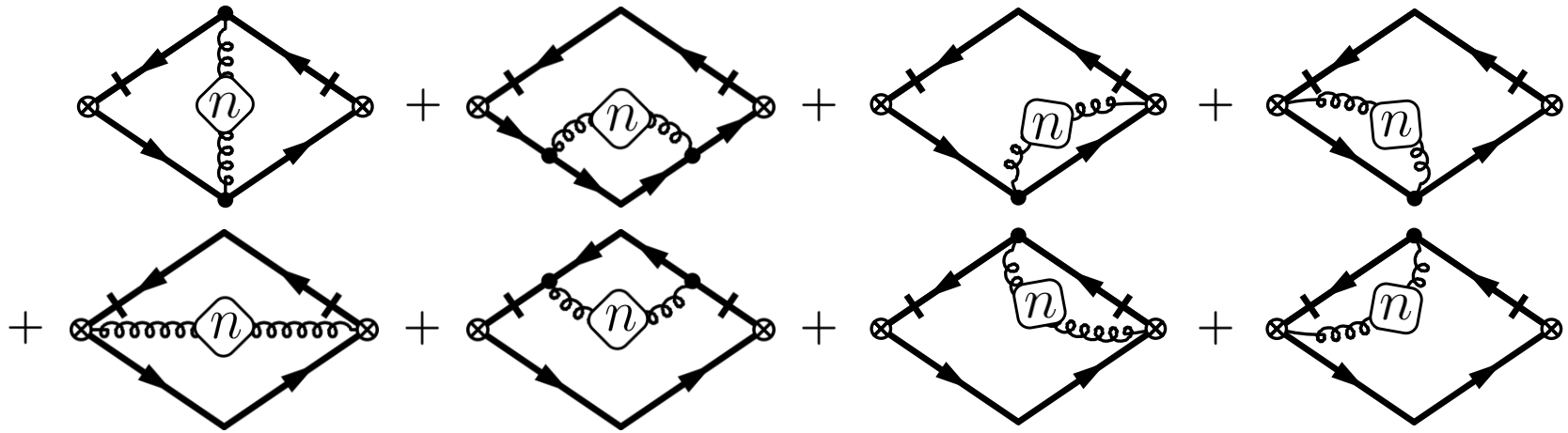
$$h_1(\varepsilon) = \frac{(1-\varepsilon)\Gamma(1+\varepsilon)\Gamma^3(1-\varepsilon)}{(1-2\varepsilon/3)(1-2\varepsilon)\Gamma(1-2\varepsilon)}$$

$$h_1(0) = 1, \qquad h_1'(0) = -C = -\frac{5}{3}$$

The correlator $\Pi_n(x,y)$

$$-i\frac{a_s}{\pi^2}N_cC_F\boldsymbol{A^n\Pi_n(x,y;L)} =$$

Bj variables



$$L=\ln\frac{-p^2}{\mu^2} \qquad a_s=\frac{\alpha_s}{4\pi}$$

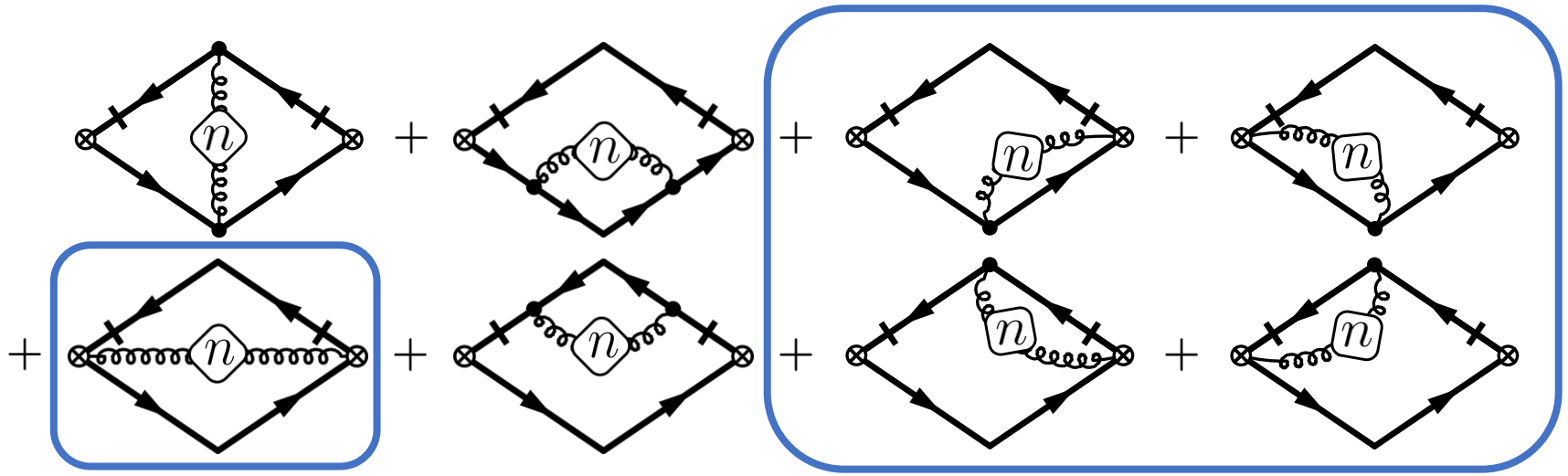
$$\boldsymbol{A=a_s\gamma_3^{(q)}=\frac{4}{3}a_sT_Fn_f} \quad \text{or} \quad \boldsymbol{-a_s\beta_0}$$

for large n_f **for large β_0**

The correlator $\Pi_n(x,y)$

$$-i\frac{a_s}{\pi^2}N_cC_F\boldsymbol{A^n}\boldsymbol{\Pi_n(x,y;L)} =$$

Bj variables



$$L = \ln\frac{-p^2}{\mu^2} \qquad a_s = \frac{\alpha_s}{4\pi}$$

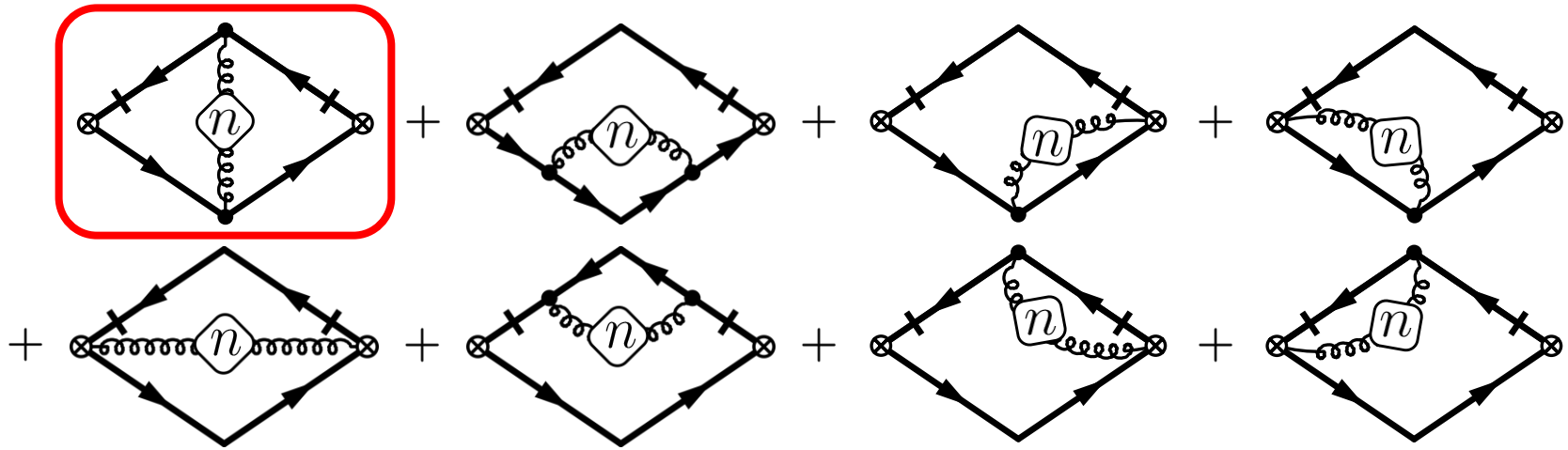
$$\boldsymbol{A = a_s\gamma_3^{(q)} = \frac{4}{3}a_sT_Fn_f} \quad \text{or} \quad \boldsymbol{-a_s\beta_0}$$

for large n_f **for large β_0**

The correlator $\Pi_n(x,y)$

$$-i\frac{a_s}{\pi^2}N_cC_F\boldsymbol{A^n\Pi_n(x,y;L)} =$$

Bj variables

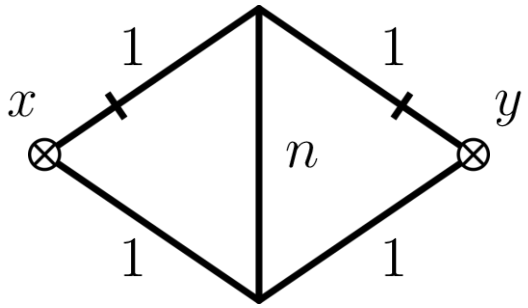


$$L=\ln\frac{-p^2}{\mu^2} \qquad a_s=\frac{\alpha_s}{4\pi}$$

$$\boldsymbol{A=a_s\gamma_3^{(q)}=\frac{4}{3}a_sT_Fn_f} \quad \text{or} \quad \boldsymbol{-a_s\beta_0}$$

for large n_f **for large β_0**

Two-loop master integral



$$= (-)^{n+1}\pi^D \left(-p^2\right)^{\omega/2} \frac{\hat{\mathbf{S}} f(n; \boldsymbol{z}; D)}{|\boldsymbol{x} - \boldsymbol{y}|^{-\omega/2}} \qquad \boldsymbol{z} = \frac{\bar{x}y}{x\bar{y}} \text{ is the conformal ratio}$$

This special case of the kite integral and its Mellin moments can be expressed in terms of generalized hypergeometric functions not higher than ${}_3\boldsymbol{f}_2$.

$$f(n; z; D) = \Gamma\begin{bmatrix} 2+\ddot{n},\ \dot{n},\ 1-\dot{n} \\ n,\ \lambda \end{bmatrix} z^{\lambda-1}\bar{z}^{2-\lambda}\theta(\bar{z}) \times \left[{}_3\boldsymbol{f}_2\left(\begin{matrix} 1,\ 1,\ \lambda \\ 1-\dot{n},\ \dot{n}+2 \end{matrix}\middle|\, \bar{z}\right) - \bar{z}^{\dot{n}}\, {}_2f_1\left(\begin{matrix} n,\ \dot{n}+1 \\ 2(\dot{n}+1) \end{matrix}\middle|\, \bar{z}\right)\right]$$

where $\lambda = \dfrac{D}{2} - 1, \qquad \dot{n} = n - \lambda, \qquad \ddot{n} = n - 2\lambda$

$$\hat{\mathbf{S}} f(x, y) = f(x, y) + f(\bar{x}, \bar{y}) \qquad x = 1 - x,\ y = 1 - y$$

$$\Gamma_r\begin{bmatrix} a_1, \ldots,\ a_p \\ b_1, \ldots,\ b_q \end{bmatrix} = \frac{\prod_{i=1}^{p}\Gamma(a_i + r)}{\prod_{i=1}^{q}\Gamma(b_i + r)}, \qquad \Gamma\begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} = \Gamma_0\begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix}$$

$${}_pf_q\left(\begin{matrix} \mathbf{a} \\ \mathbf{b} \end{matrix}\middle|\, z\right) = \Gamma\begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} {}_pF_q\left(\begin{matrix} \mathbf{a} \\ \mathbf{b} \end{matrix}\middle|\, z\right) = \sum_{r\geqslant 0}\Gamma_r\begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix}\frac{z^r}{r!}$$

Mikhailov, Volchanskiy, JHEP (2019) 202

$(x, \underline{0})$ moment of the correlator

$$\dot{\Pi}_n(x, \underline{0}; L) = \int_0^1 \mathrm{d}y\, \dot{\Pi}_n(x, y; L) = \dot{\Pi}'_n(x, \underline{0}; L) + \dot{\Pi}''_n(x, \underline{0}; L)$$

purely finite part

from counterterms to the nonlocal vertices

Exponential generating function:

$$\sum_{n\geqslant 0} \frac{A^n}{n!} \dot{\Pi}'_n(x, \underline{0}; L) = \hat{\mathbf{S}} \left\{ \frac{e^{A(L-5/3)} x^A}{A^2(1+A)(2+A)} \left[-\bar{x}(A+4x) + 2x\bar{x} \frac{(\pi A)^2 \cot(\pi A)}{x^A \sin(\pi A)} + Ax(2\bar{x}+A) \mathsf{B}_{\bar{x}}(A, 1-A) \right.\right.$$
$$\left.\left. + \frac{2x^2 \bar{x} A^2}{(1+A)^2} {}_3F_2 \left(\begin{matrix} 1,\ 1,\ 1+A \\ 2+A,\ 2+A \end{matrix} \middle| x \right) \right] \right\}$$

Ordinary generating function:

$$\sum_{n\geqslant 0} A^n \dot{\Pi}''_n(x, \underline{0}; L) = -\frac{1}{2A} \int_0^A \mathrm{d}s \int_0^1 \mathrm{d}y\, \frac{y\bar{y} V(x, y; s)_{+(x)}}{s h_1(s)}$$

Mellin moments $f(\underline{a}) = \int_0^1 \mathrm{d}x\, x^a f(x)$

generalized ERBL evolution kernel

$$V(x, y; \varepsilon) = 2\, \hat{\mathbf{S}} \left[\theta(y-x) \left(\frac{x}{y} \right)^{1-\varepsilon} \left(1 - \varepsilon + \frac{1}{y-x} \right) \right]$$

Generalized hypergeometric function

The epsilon-expansion of the hypergeometric function can be written in terms of HPLs as

$$ {}_pF_{p-1}\left(\left.\begin{matrix}\mathbf{a}\varepsilon \\ 1+\mathbf{b}\varepsilon\end{matrix}\right| z\right) = 1 + \hat{\omega}_0^{p-1} \frac{\varepsilon^p e_p^a}{1 - \sum_{n=1}^{p} \varepsilon^n \left[(e_n^a - e_n^b)\hat{\omega}_1 - e_n^b \hat{\omega}_0\right] \hat{\omega}_0^{n-1}} \hat{\omega}_1 1 $$

The elementary symmetric polynomials are defined as $e_n^a = \sum_{1 \leqslant j_1 < \cdots < j_n \leqslant p} \prod_{k=1}^{n} a_{j_k}$

The harmonic polylogarithms (HPLs) are $\mathrm{H}_{\mathbf{k}}(z) = \hat{\omega}_0^{|k_1|-1} \hat{\omega}_{\pm 1} \cdots \hat{\omega}_0^{|k_n|-1} \hat{\omega}_{\pm 1} 1$,

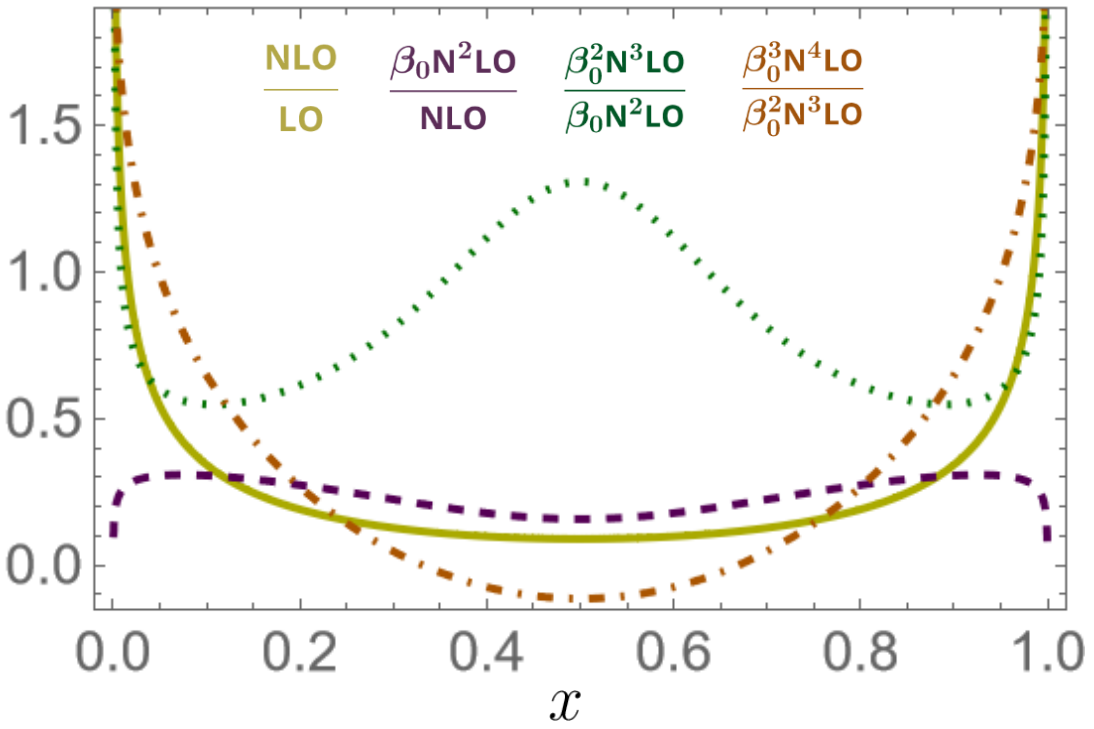
where $\hat{\omega}_0 = \int_0^z \frac{\mathrm{d}t}{t}$ and $\hat{\omega}_1 = \int_0^z \frac{\mathrm{d}t}{\bar{t}} = \int_0^z \frac{\mathrm{d}t}{1-t}$

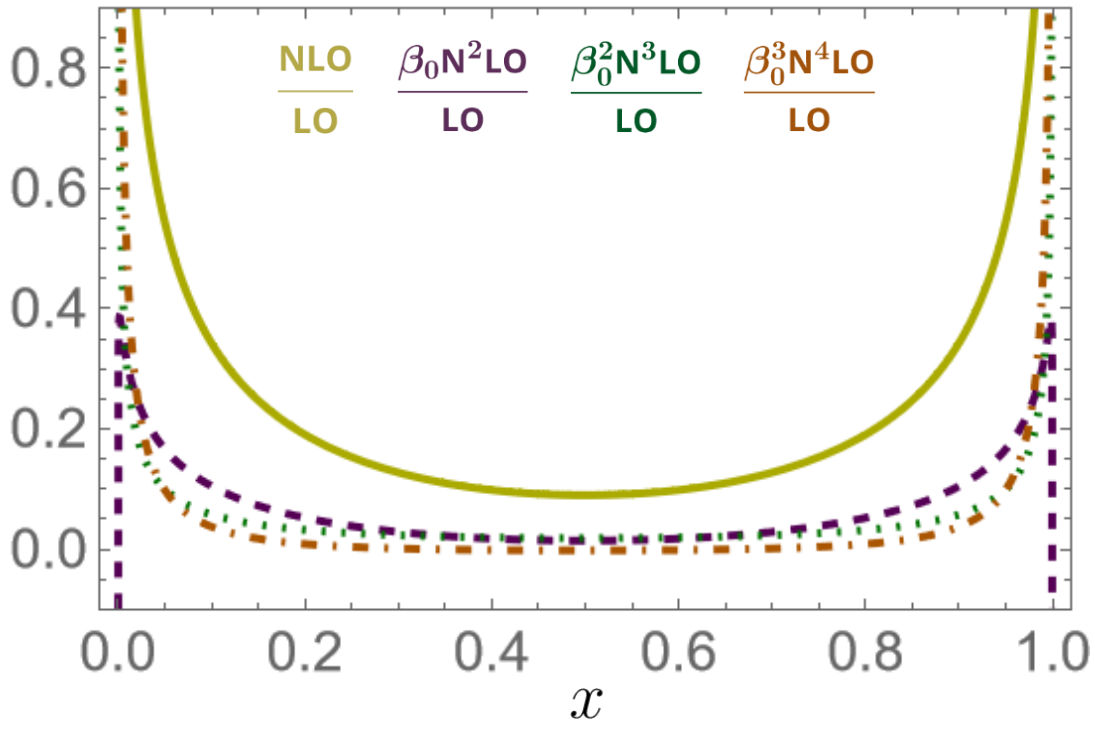
Remiddi, Vermaseren, IJMPA 15 (2000) 725

Kalmykov, Ward, Yost, JHEP 11 (2007) 009

Puhlfürst, Stieberger, NPB 902 (2015) 186

$(x, \underline{0})$ moment of the correlator

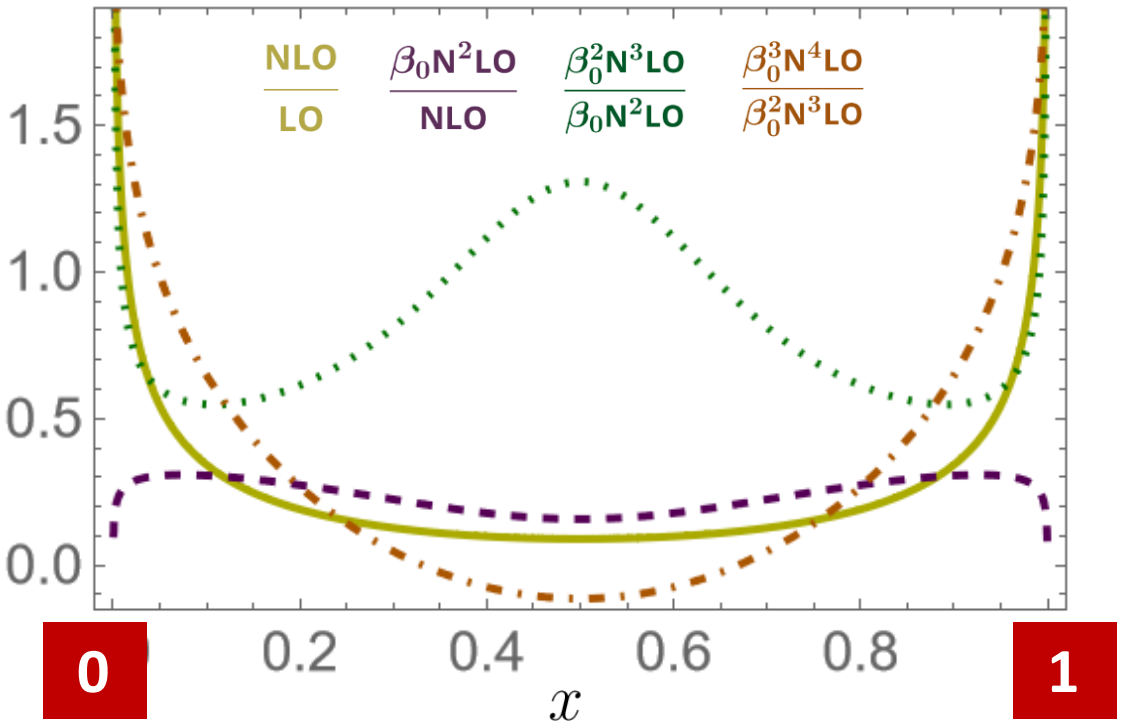


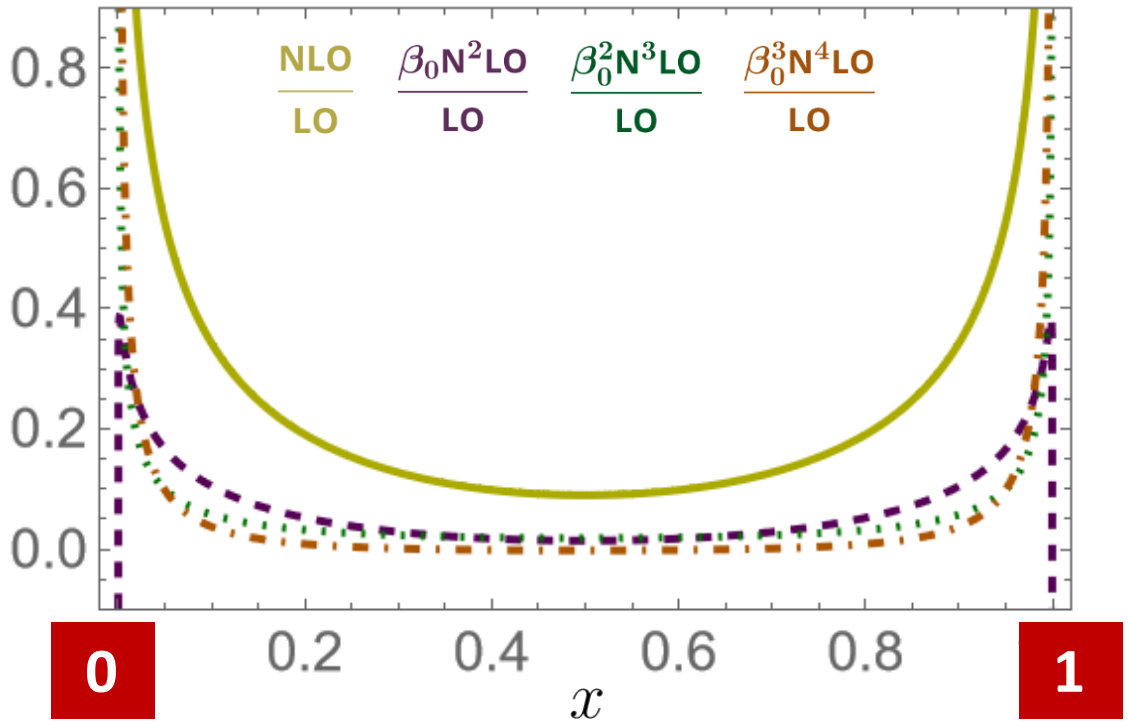


$$a_s = \frac{\alpha_s(\mu^2 = 1\ \text{GeV}^2)}{4\pi} = \frac{0.494}{4\pi} \qquad n_f = 3, \qquad \beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f = 9$$

Agaev et al, PRD 83 (2011) 054020

$(x, \underline{0})$ moment of the correlator





$$a_s = \frac{\alpha_s(\mu^2 = 1\ \text{GeV}^2)}{4\pi} = \frac{0.494}{4\pi} \qquad n_f = 3, \qquad \beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f = 9$$

Agaev et al, PRD 83 (2011) 054020

$(\underline{-1}, \underline{0})$ moment of the correlator

$$\dot{\Pi}_n(\underline{-1}, \underline{0}; L) = \int_0^1 \frac{\mathrm{d}x}{x}\, \dot{\Pi}_n(x, \underline{0}; L)$$

Exponential generating function:

trigamma function

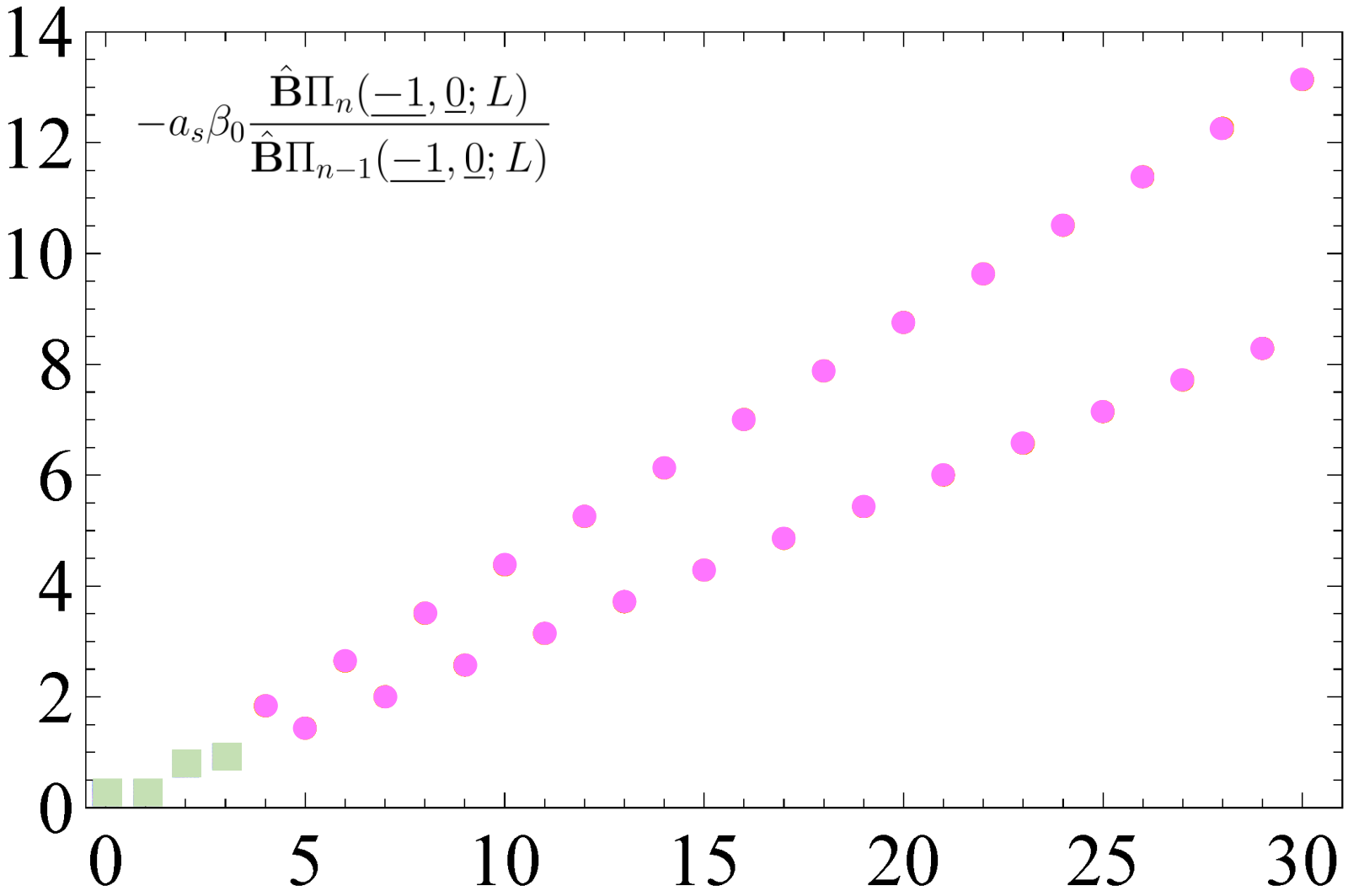
$$\sum_{n\geqslant 0} \frac{A^n}{n!} \dot{\Pi}'_n(\underline{-1}, \underline{0}; L) = \frac{e^{A(L-5/3)}}{2(1+A)(2+A)} \left[\psi_1\left(\frac{2-A}{2}\right) - \psi_1\left(\frac{1-A}{2}\right) \right]$$

Ordinary generating function:

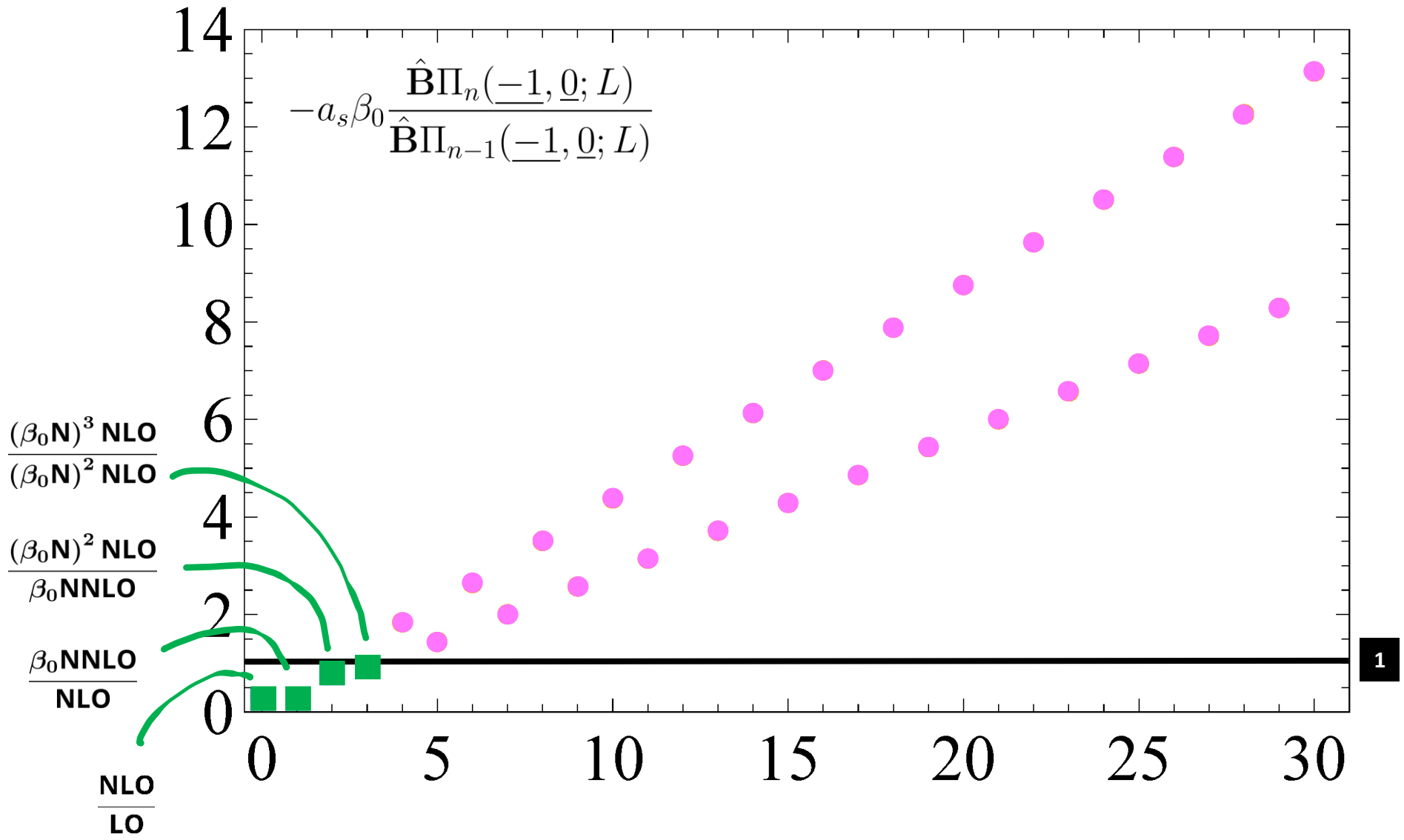
digamma function

$$\sum_{n\geqslant 0} A^n \dot{\Pi}''_n(\underline{-1}, \underline{0}; L) = -\frac{1}{A} \int_0^A \frac{\mathrm{d}a}{h_1(a)} \left\{ \frac{5+6a-5a^2}{(1-a^2)(4-a^2)} + \frac{(1+2a)[\gamma_{\mathsf{EM}} + \psi(1-a)]}{a(1+a)(2+a)} \right\}$$

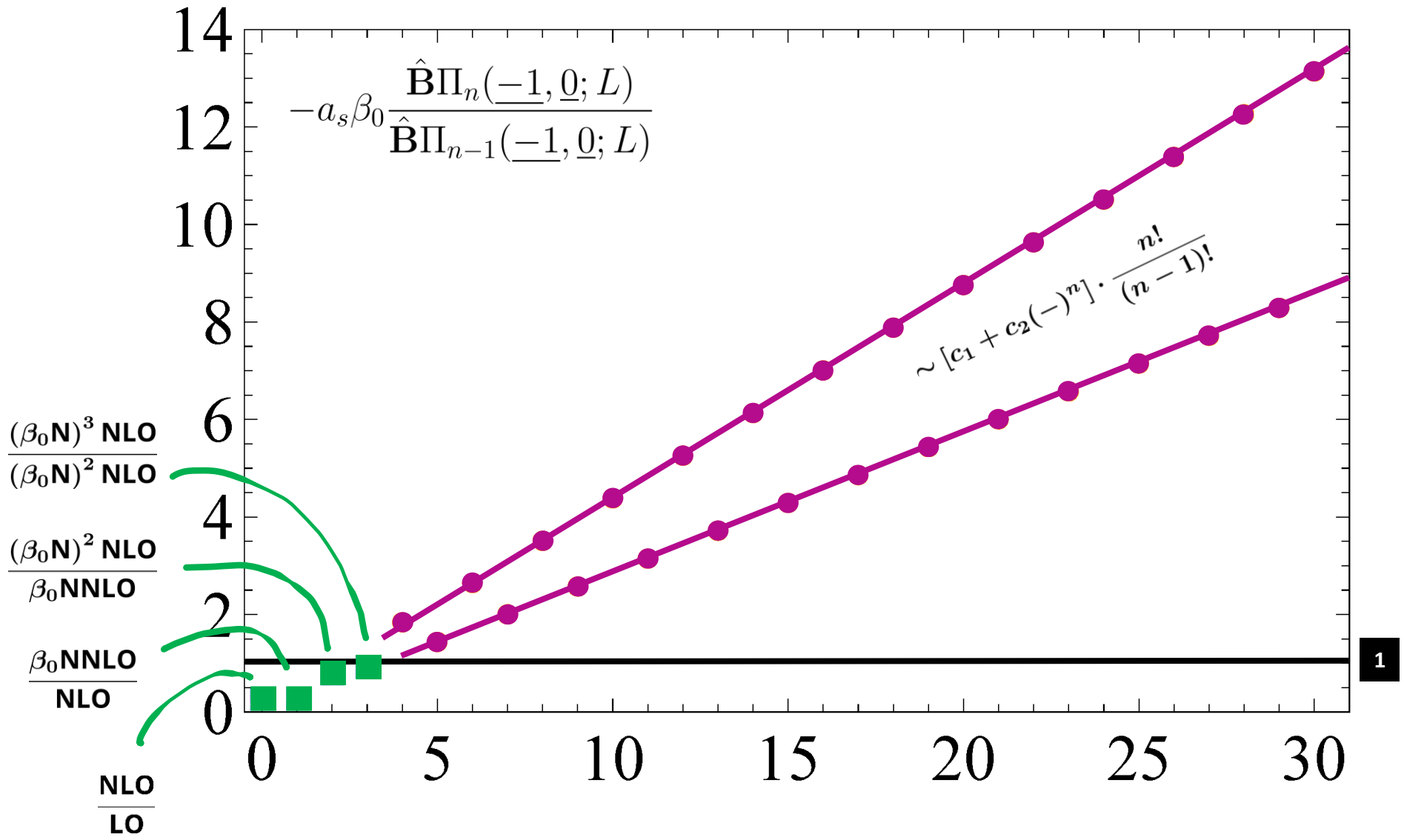
$(\underline{-1}, \underline{0})$ moment of the correlator



$(\underline{-1}, \underline{0})$ moment of the correlator



$(\underline{-1}, \underline{0})$ moment of the correlator



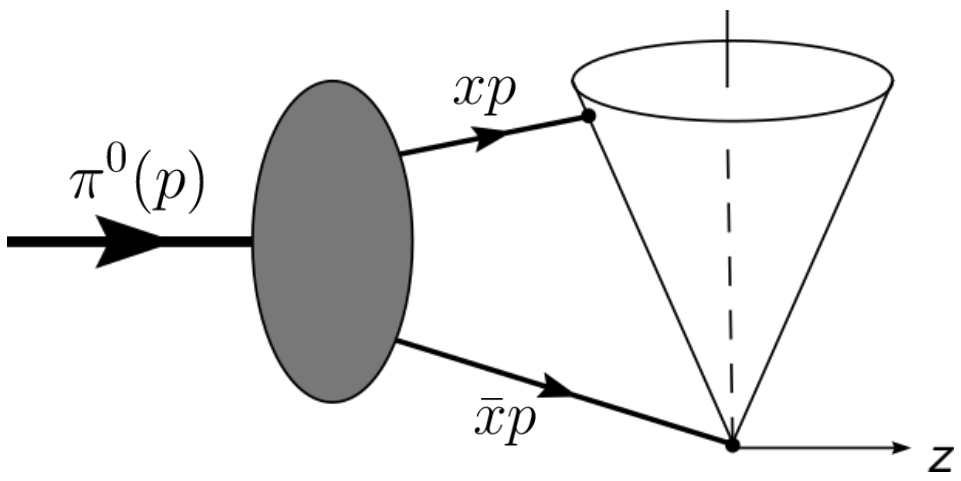
Renormalon chains in light-meson DAs

QCD SR

$$\varphi_{\text{meson}}(x) \sim \underset{P^2 \to M^2}{\textbf{Borel transform}} \left[\Pi\left(x, \underline{0}; L = \ln \frac{-p^2}{\mu^2} \right) \right]$$

φ_{meson}, meson $= \pi^0$ and ρ^0_L is a distribution amplitude (DA) of twist-2.

longitudinal polarization



DA behavior at endpoints $x = 0$ and 1 is crucially important for

the $\pi^{\pm}$ electromagnetic FF and **the π^0 transition FF**

$\gamma^*(q)\pi^{\pm} \to \pi^{\pm}$ $\qquad$ $\gamma^*(q)\gamma \to \pi^0$

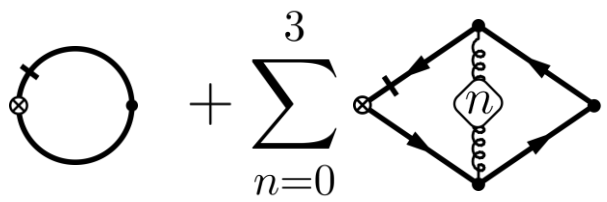
Renormalon chains in light-meson DAs

QCD SR

$$f^2_{\text{mes}}\varphi_{\text{mes}}(x)e^{-m^2_{\text{mes}}/M^2} = \Phi^{\text{PT}}_{\text{mes}}(x) + \Phi^{\text{NP}}_{\text{mes}}(x), \qquad \text{mes} = \rho_L \text{ or } \pi$$

QCD sum rules for DA is an interplay between **perturbative**

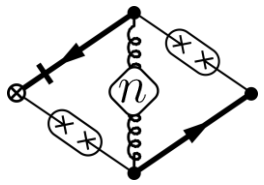
$$\Phi^{\text{PT}}(x) \sim \hat{\mathbf{B}}\Pi^{\text{PT}}(x) = \hat{\mathbf{B}}\Big[$$



$$\Big]$$

and **nonperturbative** contributions

$$\Phi^{\text{NP}}(x) \sim \hat{\mathbf{B}}\Pi^{\text{NP}}(x) = \underset{\pi}{\overset{\rho_L}{\pm}}\hat{\mathbf{B}}\sum_{n=0}^{3}$$



$$+ \text{ other terms (the same for both mesons)}$$

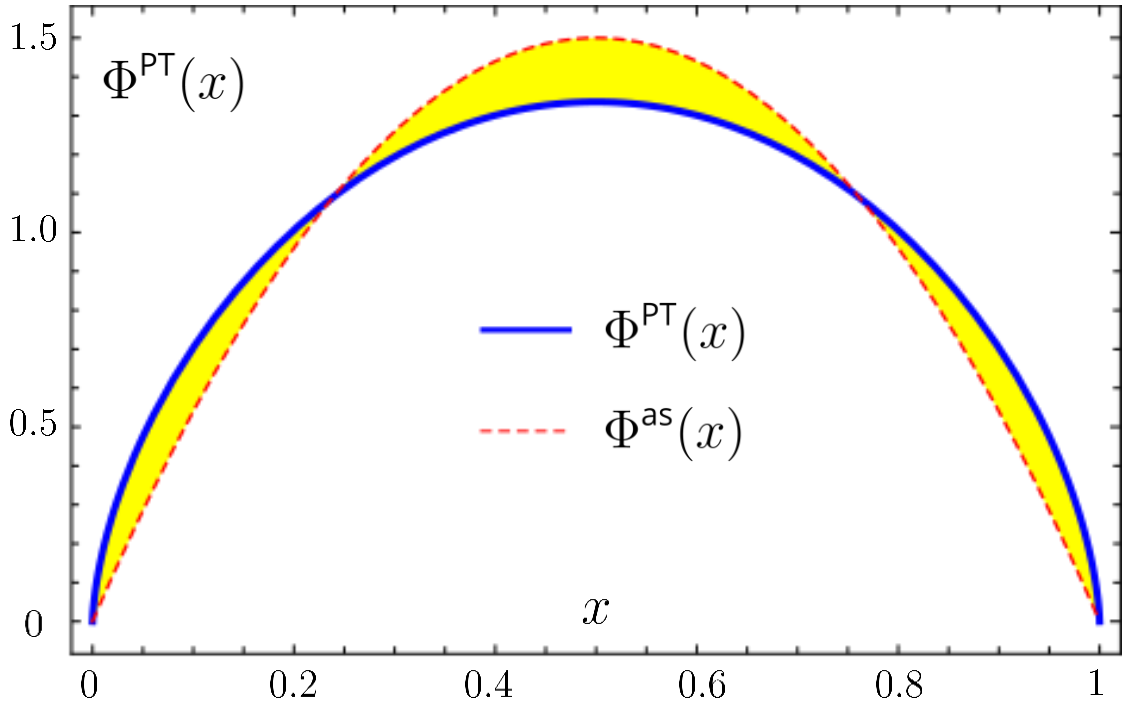
ρ_L π

dominant at endpoints **a few %**

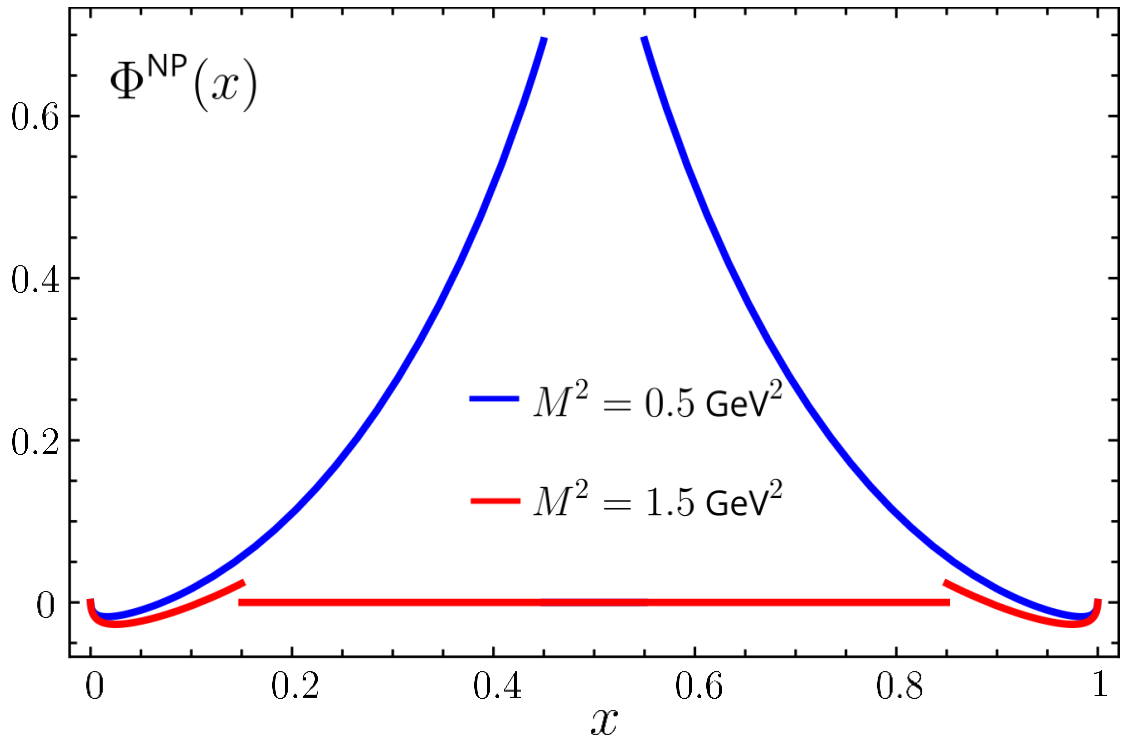
Renormalon chains in light-meson DAs

QCD SR

$$f_{\rm mes}^2 \varphi_{\rm mes}(x) e^{-m_{\rm mes}^2/M^2} = \Phi_{\rm mes}^{\rm PT}(x) \pm \Phi_{\rm mes}^{\rm NP}(x) \qquad \text{mes} = \rho\text{L} \text{ or } \pi$$



perturbative contributions

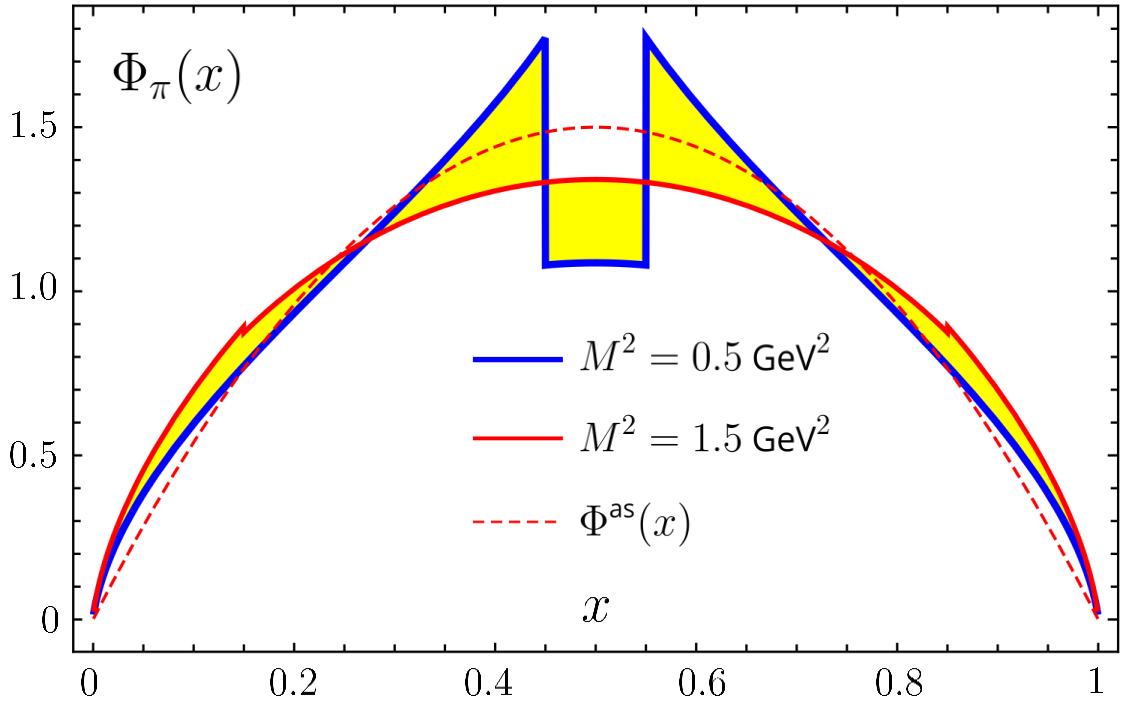


scalar nonlocal vacuum condensate

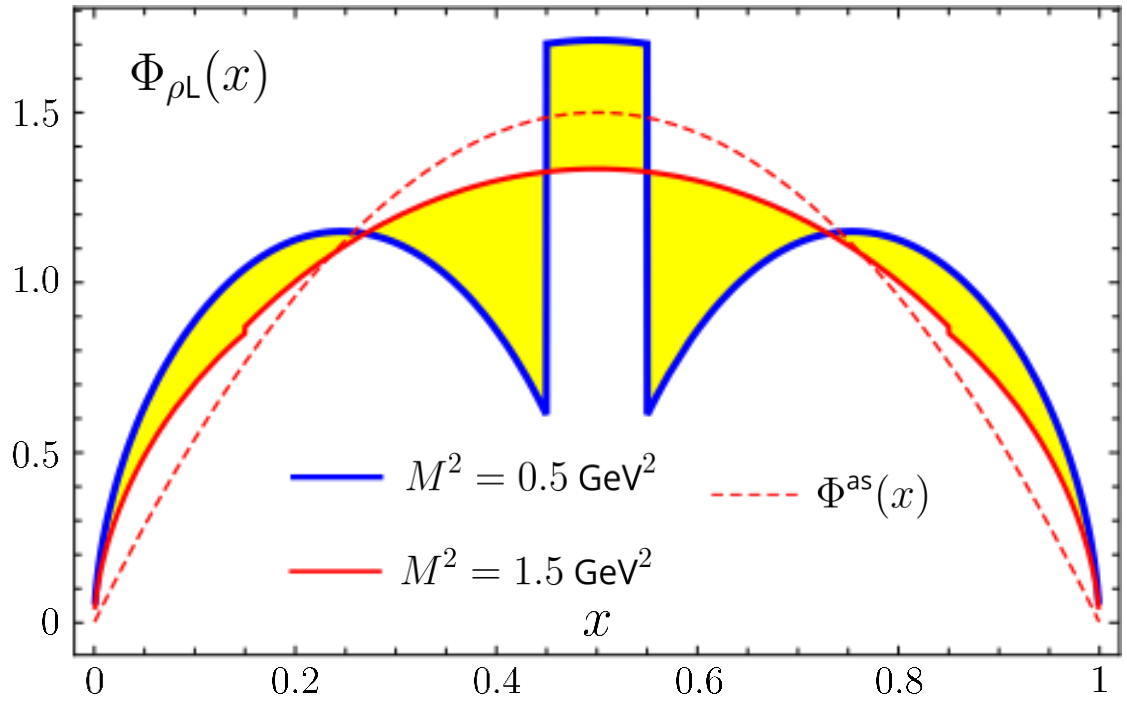
Renormalon chains in light-meson DAs

QCD SR

The r.h.s. of SR $\Phi_{\rm mes}(x) = \Phi^{\rm PT}_{\rm mes}(x) \pm \Phi^{\rm NP}_{\rm mes}(x)$ mes = ρL or π



pion



longitudinally polarized rho meson

Renormalon chains in pion DA

The **perturbative** part of the QCD sum rules increases the steepness of DA at the endpoints up to order $a_s(a_s\beta_0)^3$
(The asymptotic series should be truncated at this order.)

$$\int_0^1 \frac{\mathrm{d}x}{x}\,\Phi_\pi^{\mathsf{PT}}(x) \approx 3.5$$ **at tension with the experimental data available**

The **condensate** contributions compensate the perturbative effect, which eases the tension with the data:

$$\int_0^1 \frac{\mathrm{d}x}{x}\left[\Phi_\pi^{\mathsf{PT}}(x)+\Phi_\pi^{\mathsf{NP}}(x)\right] \approx 3.34$$ The slope is 6.2 ± 0.5 (asymptotic value -6)

The global fit to the experimental data is $\int_0^1 \frac{\mathrm{d}x}{x}\Phi_\pi(x) \approx 3.25 \pm 0.20$ **(CELLO, CLEO, BaBar, Belle in LCSR)**

Higher-order renormalon corrections do not spoil the agreement with the data
and prefer DA of a moderate width.

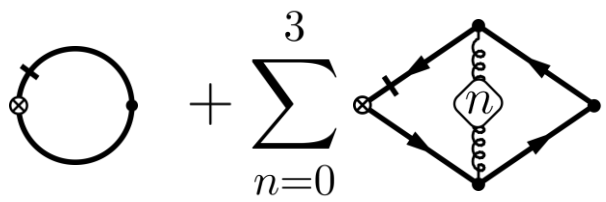
Renormalon chains in light-meson DAs

QCD SR

$$f^2_{\text{mes}}\Phi_{\text{mes}}(x)e^{-m^2_{\text{mes}}/M^2} = \Phi^{\text{PT}}_{\text{mes}}(x) + \Phi^{\text{NP}}_{\text{mes}}(x), \qquad \text{mes} = \rho_L \text{ or } \pi$$

QCD sum rules for DA is an interplay between **perturbative**

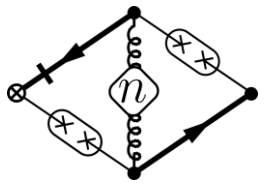
$$\Phi^{\text{PT}}(x) \sim \hat{\mathbf{B}}\Pi^{\text{PT}}(x) = \hat{\mathbf{B}}\Bigg[$$



$$\Bigg]$$

and **nonperturbative** contributions

$$\Phi^{\text{NP}}(x) \sim \hat{\mathbf{B}}\Pi^{\text{NP}}(x) = \underset{\pi}{\overset{\rho_L}{\pm}}\hat{\mathbf{B}}\sum_{n=0}^{3}$$



$$+ \text{ other terms (the same for both mesons)}$$

ρ_L (over the other terms)

π (over the other terms)

dominant at endpoints

a few %

Renormalon chains in light-meson DAs

QCD SR

$$\varphi_{\rho\mathsf{L}}(x) \approx \left[\varphi_\pi(x) - \frac{2}{f_\pi^2}\Phi_{\mathsf{sNLC}}(x,M) + \text{contributions of higher resonances}\right] \frac{f_\pi^2}{f_\rho^2} e^{m_\rho^2/M^2}$$

(negligible)

Mikhailov, Stefanis, PRD 104 (2021) 096013

BEFORE

NLO QCD SR

$$0.032(46) = a_2^{\rho\mathsf{L}} < a_2^\pi = 0.149^{+0.052}_{-0.043}$$

Mikhailov, Stefanis, PRD 104 (2021) 096013

lattice QCD

$$0.132(27) = a_2^{\rho\mathsf{L}} > a_2^\pi = 0.116(20)$$

Braun et al., JHEP (2017) 082 Bali et al., JHEP (2020) 037

AFTER

large-β_0 N^{4}LO QCD SR

$$a_2^{\rho\mathsf{L}} > a_2^\pi$$

Mikhailov, Volchanskiy, PRD 108 (2023) 096015

$\lambda_q^2 \approx 0.45$ GeV^2 $\mu^2 = 1$ GeV^2

Summary

We have evaluated correlators $\Pi(x,y;L)$ of two vector composite quark currents of order $\beta_0^n \mathrm{N}^{n+1}\mathrm{LO}$ in QCD, $n \geqslant 0$. The double-zeroth moment as well as some other fixed-order special cases agree with previous calculations in the literature. **Generating functions** for the correlator have been constructed. The correlator $\Pi_n(x,y)$ and $\Pi_n(x,\underline{0})$ at any fixed order $a_s^{n+1}\beta_0^n$ can be expressed in terms of **harmonic polylogarithms** of weight $n+2$.

We have estimated quantitative significance of the lower-order $a_s^{n+1}\beta_0^n$ contributions to the QCD sum rules for the light-meson distribution amplitudes (pion and longitudinal rho).

Higher-order renormalon corrections **do not spoil the agreement** with the data and prefer pion DA of a moderate width.

Higher-order renormalon corrections **significantly change** the endpoint behavior of longitudinal rho DA reconciling QCD SR with the lQCD results for the hierarchy of the 2nd Gegenbauer moment $a_2^{\rho\mathsf{L}} > a_2^{\pi} > 0$

Borel transform

QCD SR

$$\Phi_{\text{meson}}(x) \sim \underset{-p^2 \to M^2}{\textbf{Borel transform}} \left[\Pi\left(x, \underline{0}; L = \ln \frac{-p^2}{\mu^2} \right) \right]$$

$$\hat{\mathbb{B}}\left[f(t)\right](\mu) = \lim_{\substack{t=n\mu \\ n\to\infty}} \frac{(-t)^n}{\Gamma(n)} \frac{\mathrm{d}^n}{\mathrm{d}t^n} f(t),$$

$$\hat{\mathbb{B}}\left[t^{-a}\right](\mu) = \frac{\mu^{-a}}{\Gamma(a)}, \quad a > 0, \qquad \hat{\mathbb{B}}\left[e^{-at}\right](\mu) = \delta(1-\mu a), \quad a > 0.$$

$$\hat{\mathbb{B}}\left[\ln^m(t)\right](\mu) = m(-1)^m \left(\frac{d}{da}\right)^{m-1} \frac{e^{-al}}{\Gamma(1+a)}\bigg|_{a=0} = -m! \sum_{s=0}^{m-1} \frac{1}{s!} \left[\ln\left(\mu e^{\gamma_{\mathrm{E}}}\right)\right]^s \sum_{\forall \Pi} \prod_{i=1}^{N} \frac{(-\zeta_{p_i})^{r_i}}{p_i^{r_i} r_i!}$$

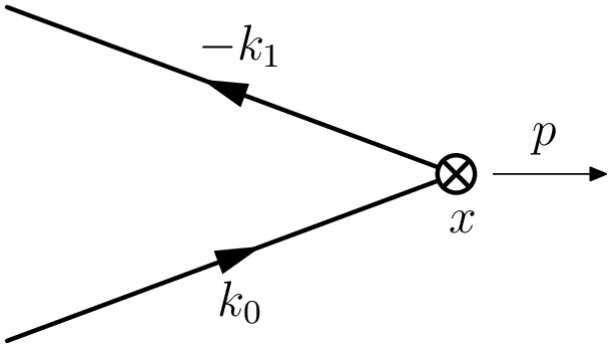
Here, $\Pi = (p_1^{r_1}, p_2^{r_2}, \ldots, p_N^{r_N})$ is a partition of $n \in \mathbb{N}$,

i.e. $n = \sum_{i=1}^{N} p_i r_i$: $1 < p_1 < p_2 < \cdots < p_N$ with $p_i, r_i \in \mathbb{N}$.

Feynman rules for composite vertices ⊗

(in a covariant gauge)

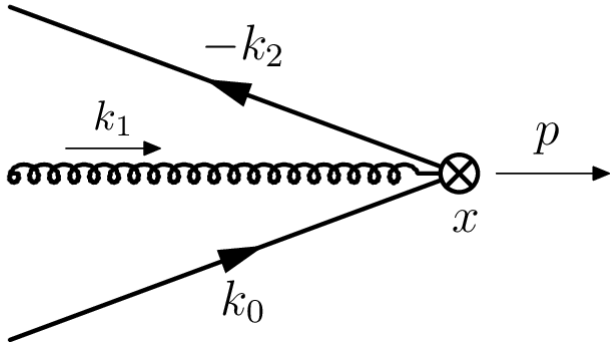
$$\hat{O}(x,0) \;=$$



$$= \Gamma\delta(x-\tilde{n}k_0) \qquad \Gamma = \not{\tilde{n}}\gamma_5, \quad \sigma_{\mu\nu}\tilde{n}^\nu, \ldots$$

$+$

$\mu,\, a$



$$= \frac{g}{2}\lambda_a\Gamma\tilde{n}_\mu\frac{1}{x_1}\left[\delta\left(x-x_0-x_1\right)-\delta\left(x-x_0\right)\right]$$

$+$

$\ldots$

Light-cone vector

$$\tilde{n}^2 = 0, \qquad \tilde{n}p = 1, \qquad p = \sum k_i$$

Mikhailov, Volchanskiy, JHEP (2020) 197

$(x, \underline{0})$ moment of the correlator

$$\dot{\Pi}_n(x, \underline{0}; L) = \frac{\mathrm{d}}{\mathrm{d}L}\Pi_n(x, \underline{0}; L) = (-1)^{n+1} n! \sum_{k=0}^{n} \frac{(-L)^k}{k!} \Pi_n^{k+1}(x, \underline{0})$$

$$\boldsymbol{\Pi_n^{n+1}(x, \underline{0})} = \frac{1}{2}\hat{\mathbf{S}}\,(x \ln x) + \delta_{0,n}\left[-\frac{1}{2}\hat{\mathbf{S}}\,(x \ln x) + \frac{1}{2}x\bar{x}\left(\frac{\pi^2}{3} - 5 - \boldsymbol{\ln^2 \frac{x}{\bar{x}}}\right)\right]$$

$$\boldsymbol{\Pi_n^{n}(x, \underline{0})} = \hat{\mathbf{S}}\Bigg\{x\bar{x}\left(-3\boldsymbol{\mathrm{Li}_3\, x} + \ln x\,\mathrm{Li}_2\, x + \frac{\pi^2}{6}\ln x\right) - \frac{x}{2}\left(\mathrm{Li}_2\, x - \frac{\pi^2}{6} - \frac{1}{2}\ln^2 x + \frac{19}{6}\ln x\right) + \delta_{1,n}\frac{1}{24}x\ln x(7 + 3\ln x)$$
$$+ \delta_{1,n}\frac{1}{2}x\bar{x}\left[\boldsymbol{\mathrm{Li}_3\, x} - \ln x\,\mathrm{Li}_2\, x + \frac{1}{6}\ln^3 x - \frac{1}{2}\ln x \ln^2 \bar{x} - \frac{5}{6}\ln^2 x - \frac{5}{3}\ln x \ln \bar{x} - \frac{5\pi^2}{36} + \frac{7}{12}\right]\Bigg\}$$

$$\boldsymbol{\Pi_n^{n-1}(x, \underline{0}) \sim \hat{\mathbf{S}}\,\mathrm{Li}_4\, x} \qquad \boldsymbol{\Pi_n^{n-2}(x, \underline{0}) \sim \hat{\mathbf{S}}\,\mathrm{H}_{3,2}(x)}$$

$$\boldsymbol{\Pi_n^{k+1}(x, \underline{0}) \sim \hat{\mathbf{S}}\,\mathrm{H}_\mu(x), \qquad \mu = m_1, \ldots m_r : \sum_{i=1}^{r} m_i = n - k + 2}$$

harmonic polylogarithm

REVIEWS: A. G. Grozin, Int. J. Mod. Phys. A 27 (2012) 1230018, arXiv:1206.2572
A. V. Kotikov, S. Teber, Phys. Part. Nucl. 50 (2019) 1, arXiv:1805.05109

