14th ITP/CAS-BLTP/JINR

Joint Workshop on Physics of Strong Interacting Systems

Meson-baryon interactions in covariant Chiral EFT

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XLR, et al., Eur. Phys. J C 80 (2020) 406, 81 (2021) 582

XLR, Phys. Lett. B 855 (2024) 138802

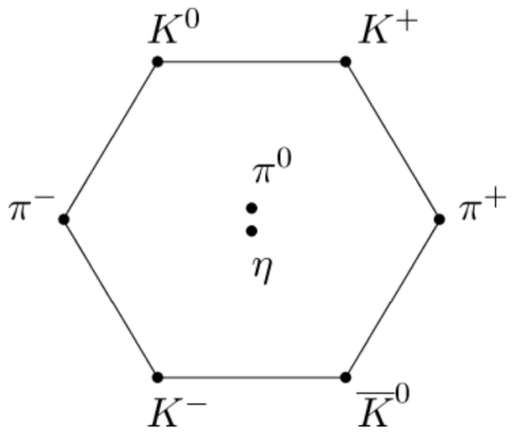
XLR, Phys. Rev. D 113, 114027 (2026)

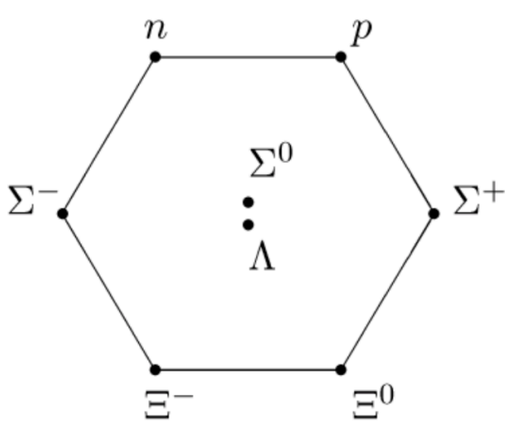
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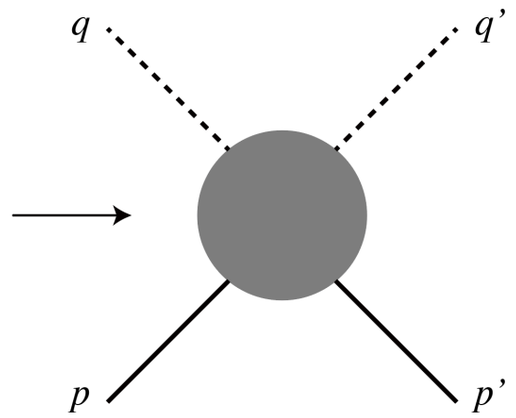
- Introduction
- Theoretical framework
- Results and discussion
- Summary

Meson-baryon scattering

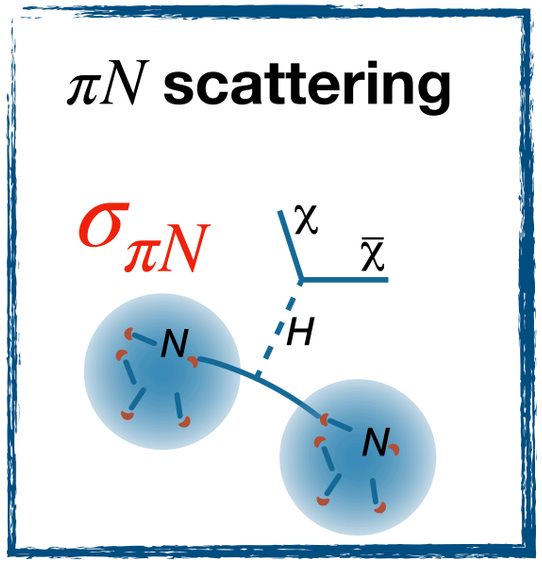
- **Lowest-lying** Meson-Baryon scattering

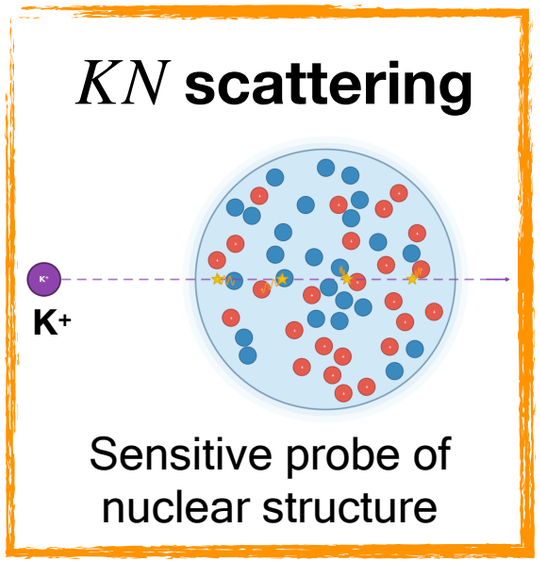


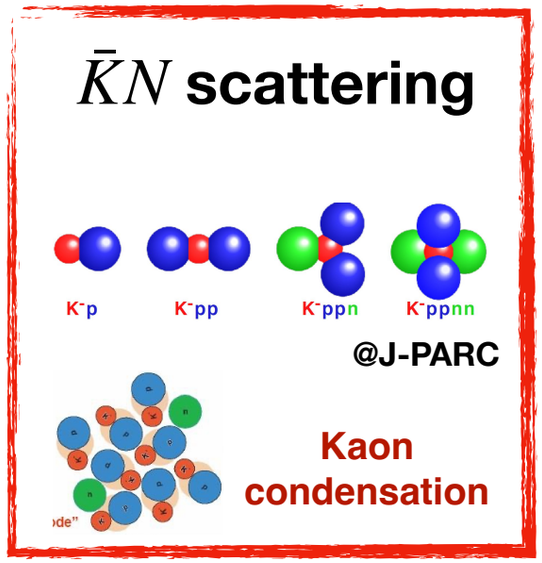


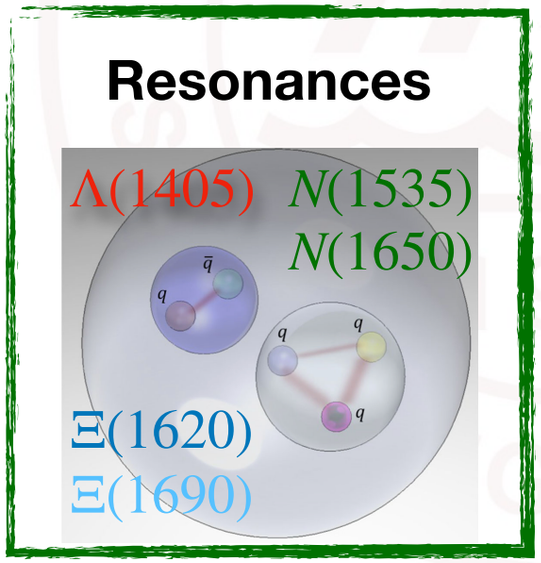


- **Interesting phenomena**





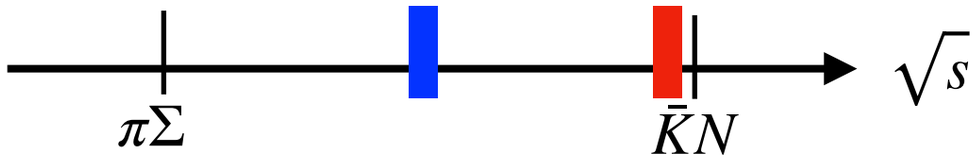


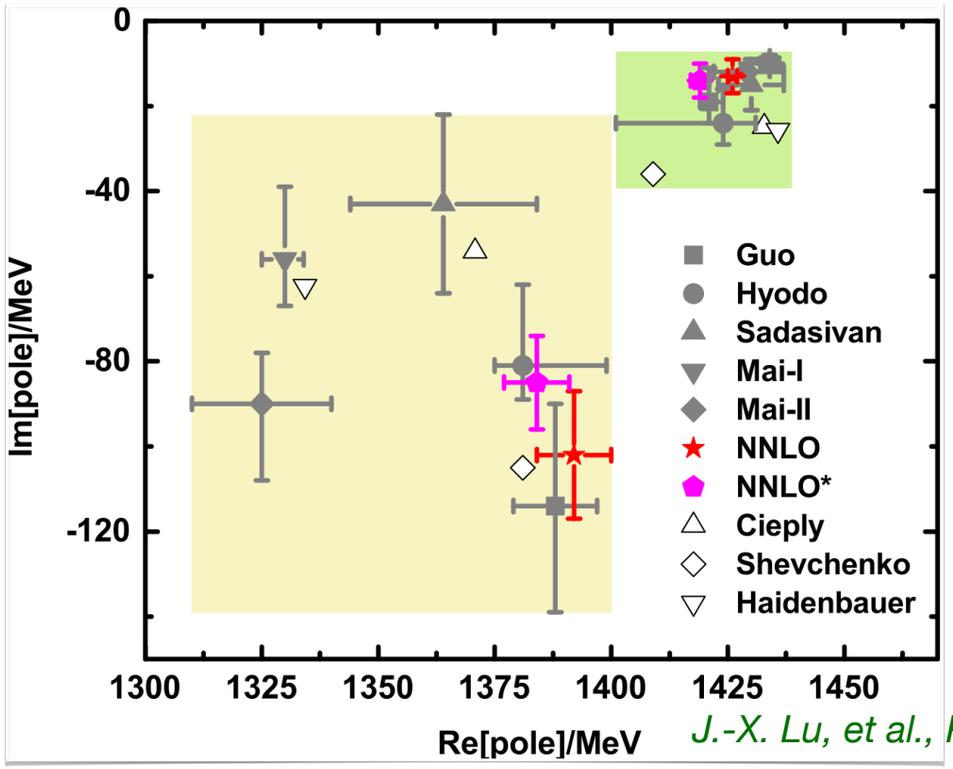


- **Deepen understanding of SU(3) dynamics in nonperturbative QCD**

Structure of $\Lambda(1405)$ resonance

- Double-pole predicted by **chiral unitary approach**





J.-X. Lu, et al., PRL130(2023)071902

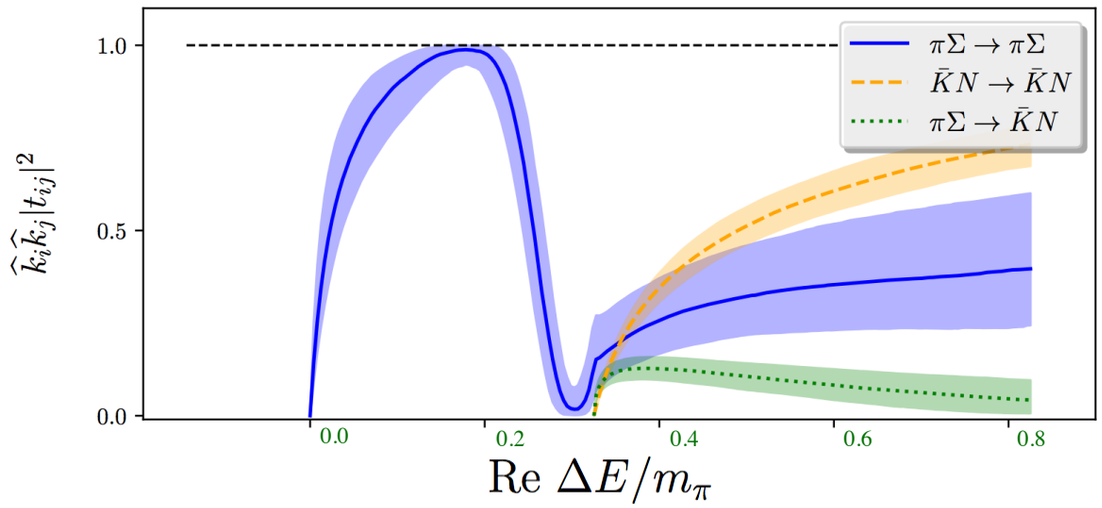
$\Lambda(1405)\ 1/2^-$ $I(J^P) = 0(\frac{1}{2}^-)$ Status: ****

$\Lambda(1380)\ 1/2^-$ $J^P = \frac{1}{2}^-$ Status: **

✓ Pole 1: Λ(1405) is around 1420 MeV

✓ Pole 2: Λ(1380) needs further studies to fix its position

- Double-pole structure verified by LQCD



$m_\pi \approx 200$ MeV, $m_K \approx 487$ MeV

Lower Pole : $E_1 = 1392(9)_{\text{stat}}(2)_{\text{model}}(16)_{\text{a}}$ MeV

Higher Pole : $E_2 = 1455(13)_{\text{stat}}(2)_{\text{model}}(17)_{\text{a}} - i11.5(4.4)_{\text{stat}}(4.0)_{\text{model}}(0.1)_{\text{a}}$ MeV

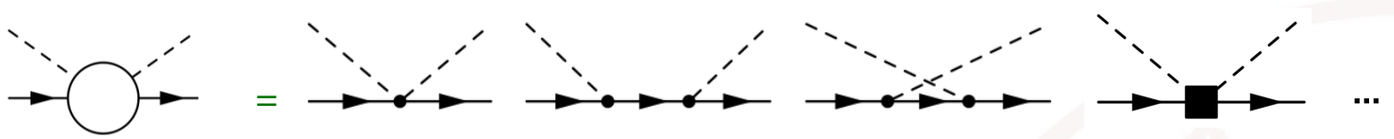
Baryon Scattering Coll., PRL132(2024)051901

Chiral Unitary approach

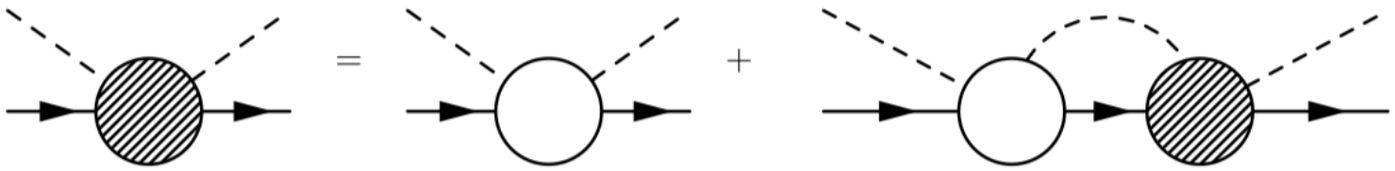
- **Chiral symmetry** of low-energy QCD + **Unitary Relation**

  *J.A.Oller et al.,PPNP45(2000)157-242; T.Hyodo et al.,PPNP*120 (2021)103868 …

- Interaction kernel V: calculate in ChPT order by order
  - Leading, next-to-leading order, …



- Scattering T-matrix: solve scattering equations



  - Lippmann-Schwinge equation or Bethe-Salpeter equation

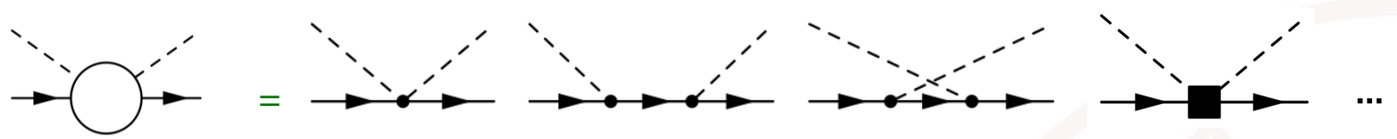
$$T(p',p) = V(p',p) + i\int \frac{d^{3(4)}k}{(2\pi)^{3(4)}} V(p',k)\, G(k)\, T(k,p)$$

Chiral Unitary approach

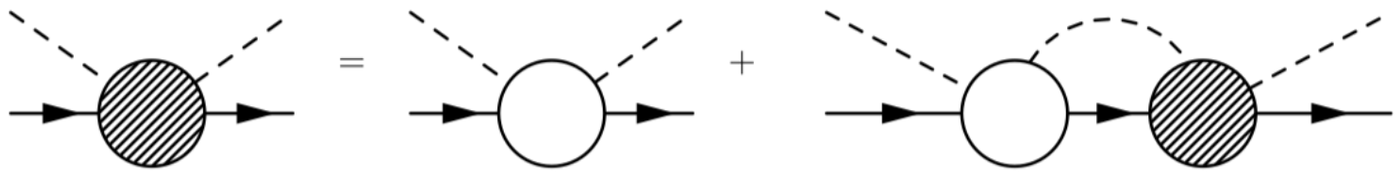
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  - Lippmann-Schwinge equation or Bethe-Salpeter equation

$$T(p',p) = V(p',p) + i\int \frac{d^{3(4)}k}{(2\pi)^{3(4)}} V(p',k)\, G(k)\, T(k,p)$$

  - On-shell factorization $\rightarrow V(p',p) + V(p',p)\left(i\int \frac{d^4k}{(2\pi)^4} G(k) \right) T(p',p)$

Neglecting off-shell effect

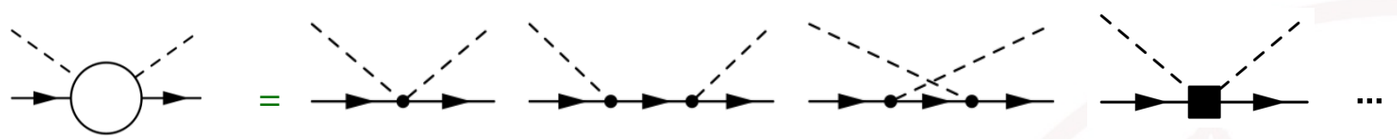
→ cause troubles in the study of three-body interaction?

Chiral Unitary approach

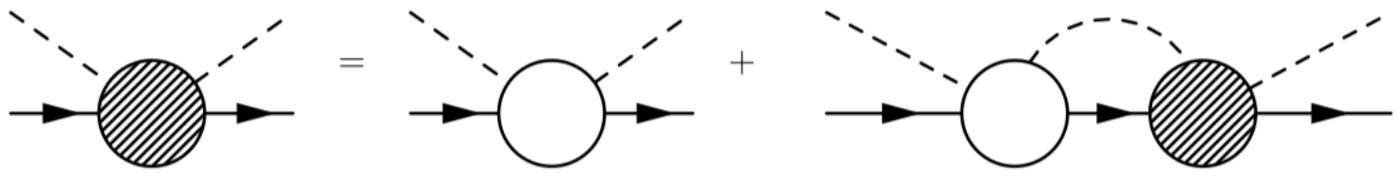
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- Scattering T-matrix: solve scattering equations



  - Lippmann-Schwinge equation or Bethe-Salpeter equation

$$T(p',p) = V(p',p) + i\int \frac{d^{3(4)}k}{(2\pi)^{3(4)}} V(p',k)\, G(k)\, T(k,p)$$

$$\rightarrow V(p',p) + V(p',p)\left(i\int \frac{d^4k}{(2\pi)^4} G(k) \right) T(p',p)$$

  - On-shell factorization

Neglecting off-shell effect

→ cause troubles in the study of three-body interaction?

  - Form factor with Finite cutoff / subtraction constant to solve scattering equations ⟶ **Cutoff / Model dependence**

In this work

- Facing the rapid progress of precision experiments, **a model-independent formalism would be needed** ALICE, AMADEUS, J-PARC, STAR...

- We tentatively propose **a renormalizable framework** of Chiral EFT for meson-baryon scattering by using **time-ordered perturbation theory** with covariant chiral Lagrangians

  - Leading order studies
    - ✓ pion-nucleon scattering

      XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C80 (2020) 406
    - ✓ $\bar{K}N$ scattering at LO and $\Lambda(1405)$ state

      XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C81 (2021) 582
    - ✓ Light-quark mass dependence of $\Lambda(1405)$

      XLR, Phys. Lett. B 855 (2024) 138802

  - Next-to-leading order studies
    - ✓ KN scattering

      XLR, Phys. Rev. D 113, 114027 (2026)
    - ✓ pion-nucleon scattering

      XLR, et al., in progress

Theoretical framework

Time-ordered perturbation theory

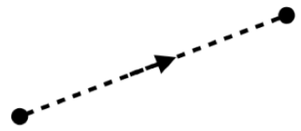
- Re-express the Feynman integral in a form that makes the connection with on-mass-shell (off-energy shell) state explicit.
- This form is called **TOPT (or old-fashioned perturbation theory)**

XLR, PoS(CD2021)007

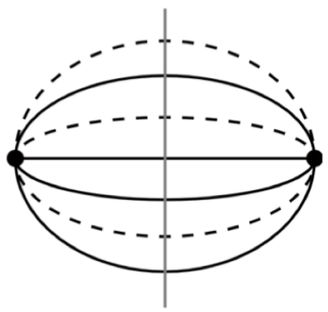
▸ External lines

Spin 0 boson (in, out)		$\mathbf{1}$
Spin 1/2 fermion (in, out)		$u(\mathbf{p}), \quad \bar{u}(\mathbf{p}')$

▸ Internal lines

Spin 0 (anti-)boson		$\frac{1}{2\,\epsilon_q} \qquad \epsilon_q \equiv \sqrt{\mathbf{q}^2 + M^2}$
Spin 1/2 fermion		$\frac{m}{\omega_p}\sum u(\mathbf{p})\bar{u}(\mathbf{p}) \qquad \omega_p \equiv \sqrt{\mathbf{p}^2 + m^2}$
anti-fermion		$\frac{m}{\omega_p}\sum u(\mathbf{p})\bar{u}(\mathbf{p}) - \gamma_0$

▸ Intermediate state

A set of lines between two vertices		$\frac{1}{E - \sum_i \omega_{p_i} - \sum_j \epsilon_{q_j} + i\epsilon}$

▸ Interaction vertices: **the standard Feynman rules**

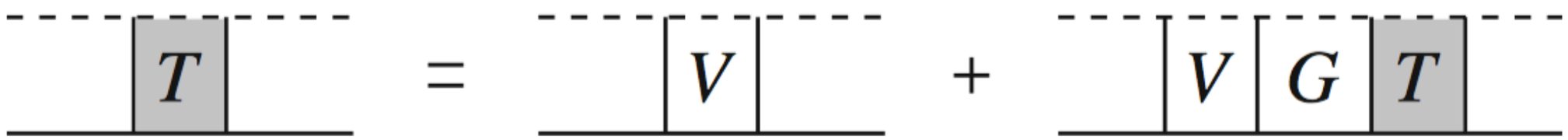
- Take care of zeroth components of integration momenta

✓ particle $p^0 \to \omega(p,m)$

✓ antiparticle $p^0 \to -\omega(p,m)$

Meson-baryon scatterings in TOPT

- Interaction kernel / potential V
  - **Define:** sum up the one-meson and one-baryon **irreducible diagrams**
  - **Power counting:** Q/Λ_χ systematic ordering of all graphs
- Scattering equation



  - Coupled-channel integral equation for T-matrix

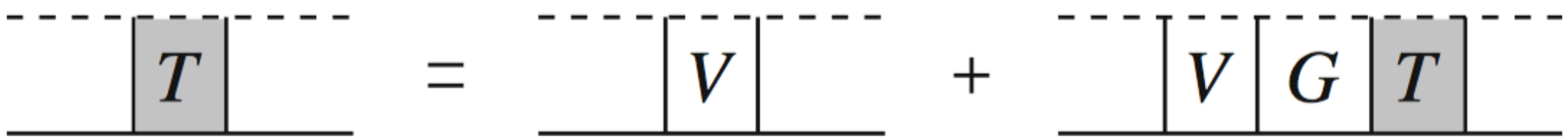
$$T_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) = V_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) + \sum_{MB}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3} V_{M_jB_j,MB}(\boldsymbol{p}',\boldsymbol{k};E)\, G_{MB}(E)\, T_{MB,M_iB_i}(\boldsymbol{k},\boldsymbol{p};E)$$

  - **Meson-baryon Green function in TOPT**

$$G_{MB}(E) = \frac{m}{2\omega(k,M)\,\omega(k,m)} \frac{1}{E-\omega(k,M)-\omega(k,m)+i\epsilon}$$

Meson-baryon scatterings in TOPT

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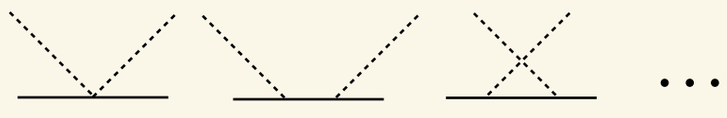
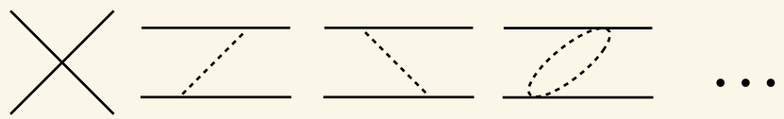
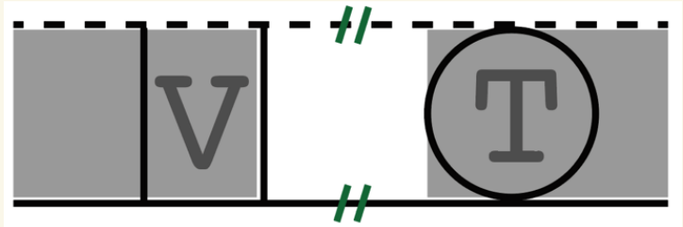
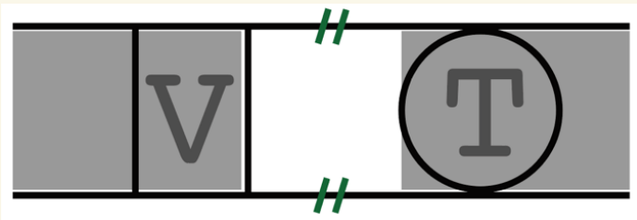
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  - **Meson-baryon Green function in TOPT**

$$G_{MB}(E) = \frac{m}{2\omega(k,M)\,\omega(k,m)} \frac{1}{E-\omega(k,M)-\omega(k,m)+i\epsilon}$$

Potential and scattering equation are obtained on an equal footing!

Baryon-baryon scatterings in TOPT

	Meson-baryon scattering	Baryon-baryon scattering
Potential TOPT diagrams		
Green function	 $G^{MB}(E) = \frac{m}{2\omega_M\,\omega_m}\frac{1}{E-\omega_M-\omega_m+i\epsilon}$	 $G^{BB}_{ij}(E) = \frac{m_i\,m_j}{\omega_{m_i}\,\omega_{m_j}}\frac{1}{E-\omega_{m_i}-\omega_{m_j}+i\epsilon}$

- Extend to the baryon-baryon scatterings
  - Formulate NN and YN interactions at leading order

    V. Baru, E. Epelbaum, J. Gegelia, XLR*, Phys. Lett. B 798,134987 (2019)

    XLR, E.Epelbaum, J.Gegelia, Phys. Rev. C 101,034001 (2020)
  - Extend NN interaction to **next-to-next-to-leading order**

    XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 106, 034001 (2022)

    XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 113, 034002(2026)

Leading order studies

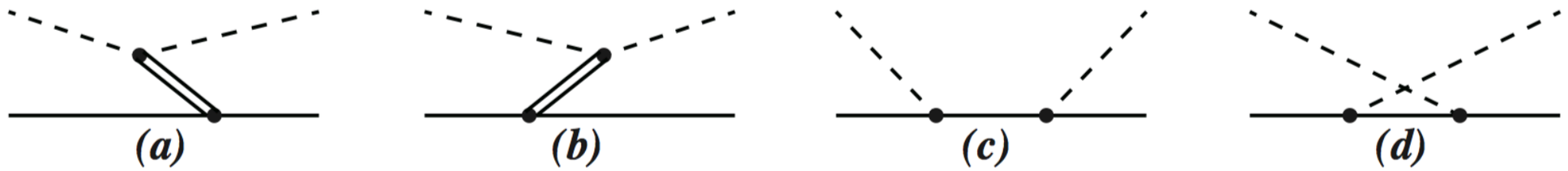
- XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C80 (2020) 406;
- XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C81 (2021) 582;
- XLR, Phys. Lett. B 855 (2024) 138802

Leading order potential

- Chiral effective Lagrangian

$$\mathcal{L}_{\rm LO} = \frac{F_0^2}{4}\left\langle u_\mu u^\mu + \chi_+\right\rangle + \left\langle \bar{B}\left(i\gamma_\mu\partial^\mu - m\right)B\right\rangle + \frac{D/F}{2}\left\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]_\pm\right\rangle$$
$$-\frac{1}{4}\left\langle V_{\mu\nu}V^{\mu\nu} - 2\mathring{M}_V^2\left(V_\mu - \frac{i}{g}\Gamma_\mu\right)\left(V^\mu - \frac{i}{g}\Gamma^\mu\right)\right\rangle + g\left\langle \bar{B}\gamma_\mu[V^\mu, B]\right\rangle$$

- Time ordered diagrams



- **Vector mesons included as explicit degrees of freedom**
  - ✓ VME potential instead of the Weinberg-Tomozawa term Weinberg:1968de, Borasoy:1995ds
  - ✓ Improve the ultraviolet behaviour without changing the low-energy physics

- LO potential in TOPT
  - Dirac spinor is decomposed as $u_B(p,s) = u_0 + [u(p) - u_0] \equiv (1,0)^\dagger\chi_s$+high order

$$V^{(a+b)}_{M_jB_j,M_iB_i} = -\frac{1}{32F_0^2}\sum_{V=K^*,\rho,\omega,\phi} C^V_{M_jB_j,M_iB_i}\frac{\mathring{M}_V^2}{\omega_V(q_1-q_2)}\left(\omega_{M_i}(q_1)+\omega_{M_j}(q_2)\right)$$
$$\times\left[\frac{1}{E-\omega_{B_i}(p_1)-\omega_V(q_1-q_2)-\omega_{M_j}(q_2)} + \frac{1}{E-\omega_{B_j}(p_2)-\omega_V(q_1-q_2)-\omega_{M_i}(q_1)}\right]$$

$$V^{(c)}_{M_jB_j,M_iB_i} = \frac{1}{4F_0^2}\sum_{B=N,\Lambda,\Sigma,\Xi} C^B_{M_jB_j,M_iB_i}\frac{m_B}{\omega_B(P)}\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{q}_2)(\boldsymbol{\sigma}\cdot\boldsymbol{q}_1)}{E-\omega_B(P)}.$$

$$V^{(d)}_{M_jB_j,M_iB_i} = \frac{1}{4F_0^2}\sum_{B=N,\Lambda,\Sigma,\Xi} \tilde{C}^B_{M_jB_j,M_iB_i}\frac{m_B}{\omega_B(K)}\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{q}_1)(\boldsymbol{\sigma}\cdot\boldsymbol{q}_2)}{E-\omega_{M_i}(q_1)-\omega_{M_j}(q_2)-\omega_B(K)}.$$

Subtractive renormalization

- LO potential: **one-baryon irreducible and reducible parts**

$$V_{\rm LO} = V_I \left(\quad \right) + V_R \left(\quad \right)$$



- LO T-matrix

$$T_{\rm LO} = V_{\rm LO} + V_{\rm LO}\, G\, T_{\rm LO} \Longrightarrow \begin{cases} T_{\rm LO} = T_I + (1 + T_I G)\, T_R\, (1 + G\, T_I) \\ T_I = V_I + V_I\, G\, T_I \\ T_R = V_R + V_R\, G\, (1 + T_I G)\, T_R \end{cases}$$

  - Irreducible part: $T_I \xrightarrow{\Lambda \sim \infty}$ Finite
  - Reducible part: $T_R \xrightarrow{\Lambda \sim \infty}$ Divergent
    - ✓ **Potential can be rewritten as separable form**

$$V_R(p', p; E) = \xi^T(p')\, C(E)\, \xi(p)$$

C(E): constant $\quad \xi^T(q) := (1, q)$

    - ✓ T_R can be rewritten as $\quad T_R(p', p; E) = \xi^T(p')\, \chi(E)\, \xi(p) \quad \chi(E) = \left[C^{-1} - \xi\, G\, \xi^T - \xi\, G\, T_I^S\, G\, \xi^T\right]^{-1}$

D.B.Kaplan, et al.,NPB478,629(1996); E. Epelbaum, et al.,EPJA51,71(2015)

    - ✓ Using **subtractive renormalization,** replacing Green function $G^{Rn} = G(E) - G(m_B)$

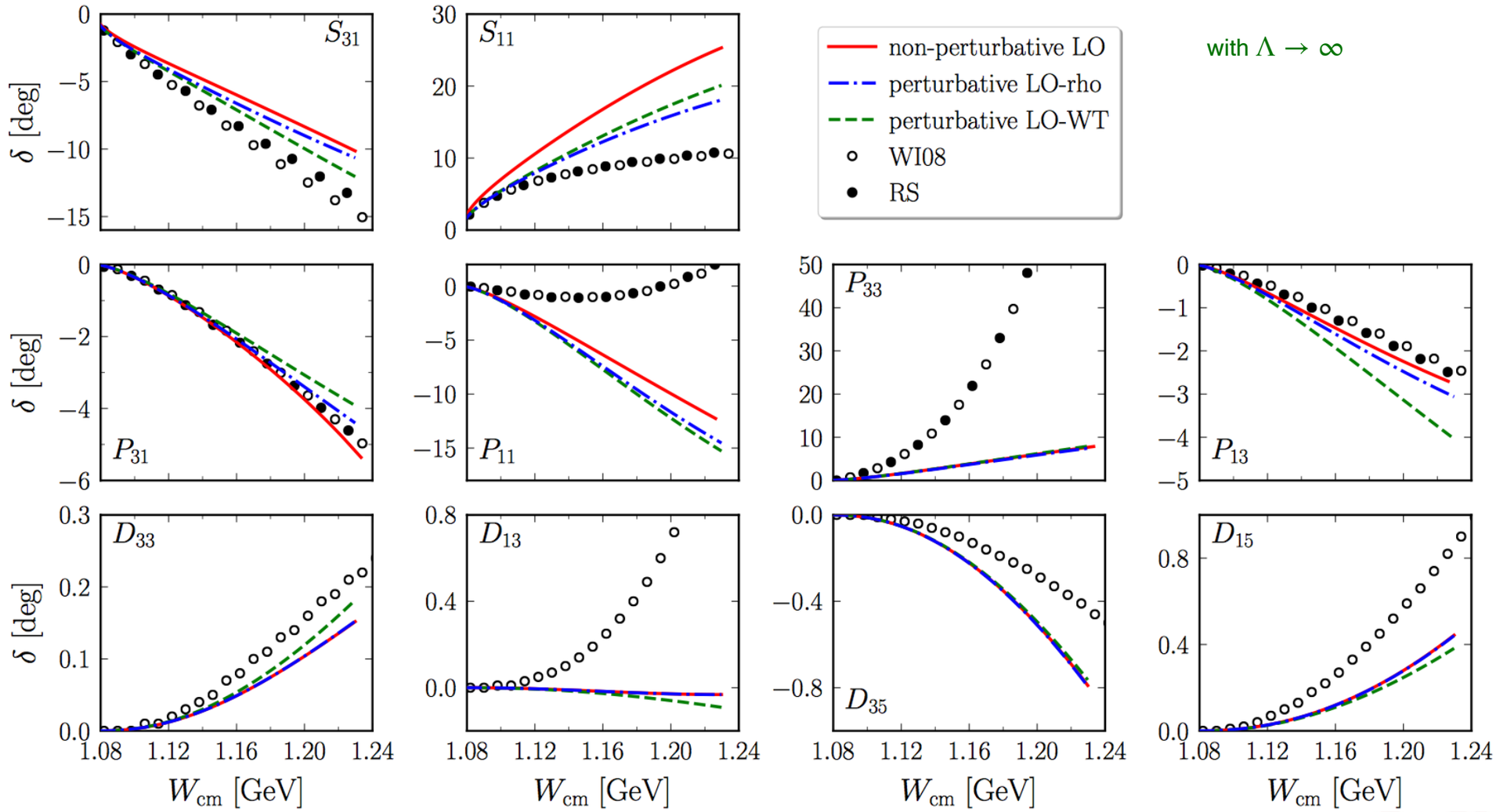
E. Epelbaum, et al., EPJA56(2020)152

Renormalized LO T-matrix

$$T_{\rm LO}^{Rn} = T_I + \left(\xi^T + T_I\, G^{Rn}\, \xi^T\right)\, \chi^{Rn}(E)\, \left(\xi + \xi\, G^{Rn}\, T_I\right)$$

Pion-Nucleon scattering

- Description phase shifts of pion-nucleon scattering



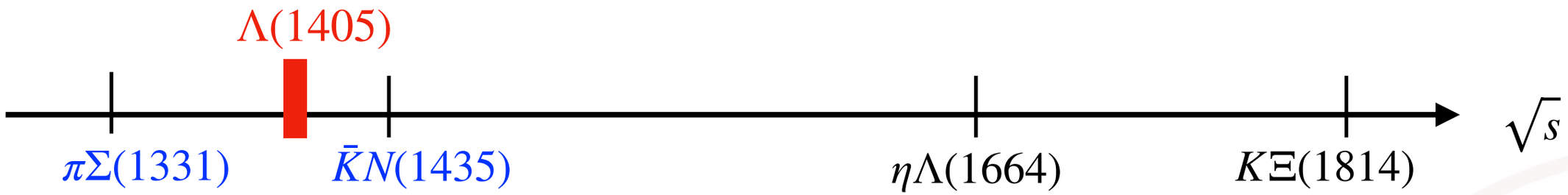
- Rho-meson-exchange contribution is similar as WT term
- Non-perturbative results are only slightly different from the ones of the perturbative approach

✓ **Non-perturbative treatment is valid**, since ChPT has good convergence in SU(2) sector

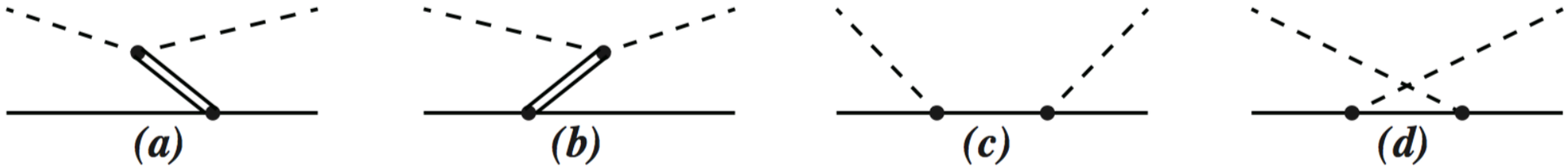
XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C80 (2020) 406

S=-1 meson-baryon scattering

- Four coupled channels $\bar{K}N, \pi\Sigma, \eta\Lambda, K\Xi$ in isospin limit



- Focus on the S-wave potential



- Born term (p-wave) does not contribute
- Crossed-Born term ~5% of VME contribution
- VME potential couplings

C^V	$\pi\Sigma$	$\bar{K}N$	$\eta\Lambda$	$K\Xi$
$\pi\Sigma$	$C^{\rho} = -16$	$C^{K^*} = 2\sqrt{6}$	0	$C^{K^*} = -2\sqrt{6}$
$\bar{K}N$	attractive	$C^{\{\rho,\omega,\phi\}} = \{-6, -2, -4\}$	$C^{K^*} = -6\sqrt{2}$	0
$\eta\Lambda$		attractive	0	$C^{K^*} = 6\sqrt{2}$
$K\Xi$				$C^{\{\rho,\omega,\phi\}} = \{-6, -2, -4\}$

S=-1 meson-baryon scattering

- P. W. scattering equation

$$T^{LJ}_{M_jB_j,M_iB_i}(p',p) = V^{LJ}_{M_jB_j,M_iB_i}(p',p) + \sum_{MB}\int \frac{dkk^2}{(2\pi)^3} V^{LJ}_{M_jB_j,MB}(p',k)\frac{1}{2\,\omega_M\omega_B}\frac{m_B}{E-\omega_M-\omega_B+i\epsilon} T^{LJ}_{MB,M_iB_i}(k,p)$$

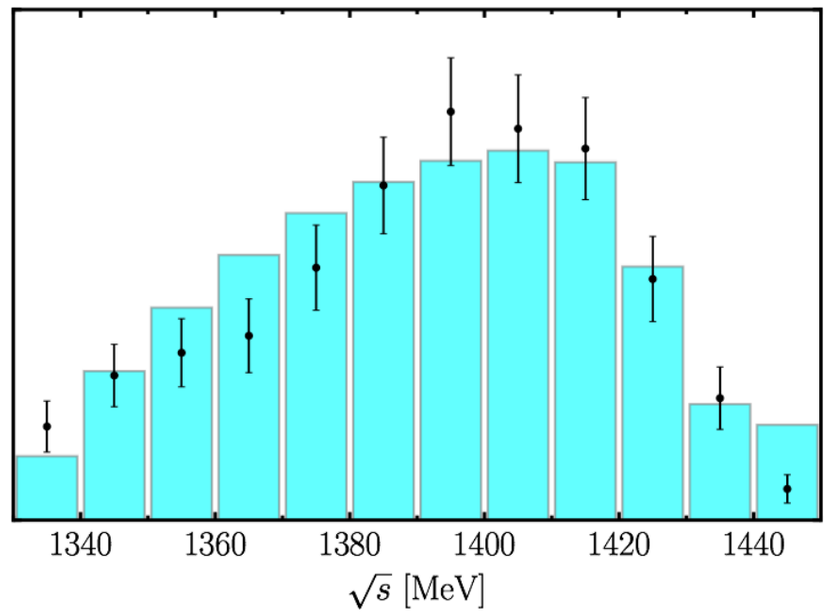
  - Take into account **the off-shell effects of potential**
  - Use **subtractive reormalization** to obtain the renormalized T-matrix
    - ➡ Cutoff-independent: $\Lambda \to \infty$

No free parameters needed to be fitted!

- Two pole positions of $\Lambda(1405)$

		lower pole	higher pole
This work	$F_0 = F_\pi$	$1337.7 - i\,79.1$	$1430.9 - i\,8.0$
(LO)	$F_0 = 103.4$	$1348.2 - i\,120.2$	$1436.3 - i\,0.7$
NLO	*Y. Ikeda,NPA(2012)*	$1381^{+18}_{-6} - i81^{+19}_{-8}$	$1424^{+7}_{-23} - i26^{+3}_{-14}$
	Z.-H.Guo,PRC(2013)-Fit II	$1388^{+9}_{-9} - i114^{+24}_{-25}$	$1421^{+3}_{-2} - i19^{+8}_{-5}$
	M.Mai,EPJA(2015)-sol-2	$1330^{+4}_{-5} - i56^{+17}_{-11}$	$1434^{+2}_{-2} - i10^{+2}_{-1}$
	M.Mai,EPJA(2015)-sol-4	$1325^{+15}_{-15} - i90^{+12}_{-18}$	$1429^{+8}_{-7} - i12^{+2}_{-3}$

πΣ invariant mass spectrum



XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C81 (2021) 582

Apply our framework to unphysical world

- BaSc results provide an ideal playground

PRL 132, 051901 (2024); PRD109,014511(2024)

  - First lattice study of $\Lambda(1405)$ pole positions

Virtual bound state

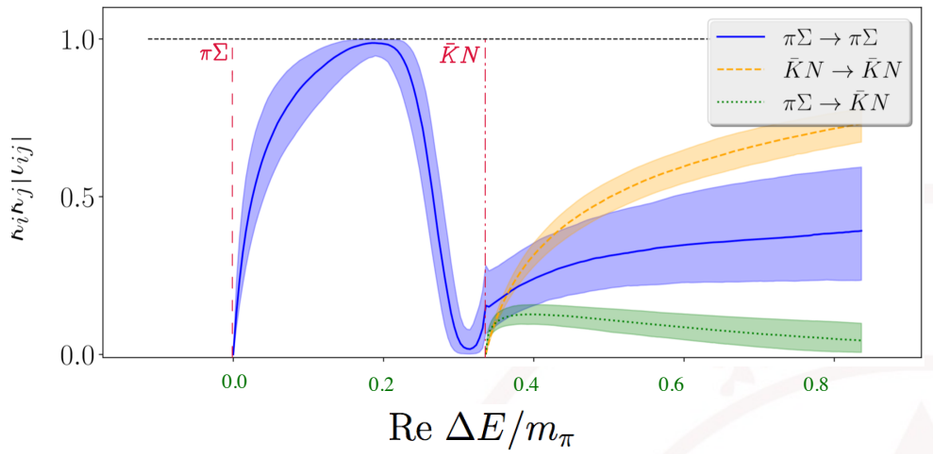
$$E_1 = 1392(9)_{\text{stat}}(2)_{\text{model}}(16)_{\text{a}}\text{MeV}$$

$$\left|\frac{c^{(1)}_{\pi\Sigma}}{c^{(1)}_{\bar{K}N}}\right| = 1.9(4)_{\text{stat}}(6)_{\text{model}}$$

Resonance

$$E_2 = \big[1455(13)_{\text{stat}}(2)_{\text{model}}(17)_{\text{a}} - i11.5(4.4)_{\text{stat}}(4.0)_{\text{model}}(0.1)_{\text{a}}\big]\text{MeV}$$

$$\left|\frac{c^{(2)}_{\pi\Sigma}}{c^{(2}_{\bar{K}N}}\right| = 0.53(9)_{\text{stat}}(10)_{\text{model}}$$



  - **Check/verify the predictive power of existing chiral unitary approaches**

Apply our framework to unphysical world

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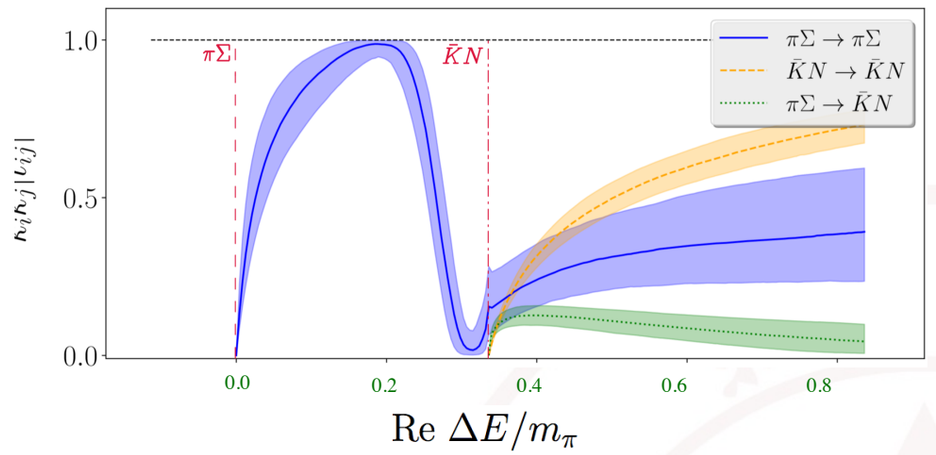
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  - **Check/verify the predictive power of existing chiral unitary approaches**

- Extend our calculation to the unphysical quark mass region

  - Use the **same** meson and baryon masses as the BaSc study
  - Focus on the $\pi\Sigma - \bar{K}N$ coupled channels
  - **Consistent with the BaSc results** $\Lambda \to \infty$, **no free parameters**

	BaSc [PRL2024]	This work			
$\Lambda(1405)$	z_R [MeV]	z_R [MeV]	$g_{\pi\Sigma}$	$g_{\bar{K}N}$	$\lvert g_{\pi\Sigma}\rvert/\lvert g_{\bar{K}N}\rvert$
Lower pole	**1392(18)**	1387.14	$0.021 + i1.87$	$0.017 + i1.55$	1.21
Higher pole	$\mathbf{1455(21) - i11.5(6.0)}$	$1469.86 - i4.71$	$0.038 + i0.98$	$1.51 - i1.22$	0.50

XLR, Phys. Lett. B 855 (2024) 138802

Next-to-leading order studies

- XLR, Phys. Rev. D 113, 114027 (2026)
- XLR, et al., In progress

Beyond leading order

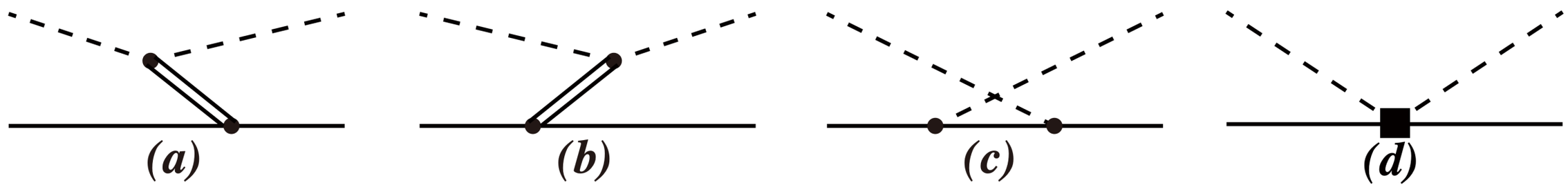
- **Maintain the scattering T-matrix renormalizable**
  - Take LO potential non-perturbatively
  - Include Higher order corrections perturbatively
- Up to next-to-leading order
  - Potential: $V = V_{\rm LO} + V_{\rm NLO}$
  - T-matrix: $T = T_{\rm LO} + T_{\rm NLO}$

$$T_{\rm LO} = V_{\rm LO} + V_{\rm LO} G T_{\rm LO} \qquad \text{(non-perturbative)}$$

$$T_{\rm NLO} = V_{\rm NLO} + T_{\rm LO} G V_{\rm NLO} + V_{\rm NLO} G T_{\rm LO} + T_{\rm LO} G V_{\rm NLO} G T_{\rm LO}$$

  - Use **the subtractive renormalization** to remove divergent terms and power-counting breaking terms

KN scattering up to NLO



- Chiral effective Lagrangians at NLO
  - For S-wave scattering

$$\mathscr{L}_{\rm NLO} = b_0\langle\chi_+\rangle\langle\bar{B}B\rangle + b_D\langle\bar{B}\{\chi_+,B\}\rangle + b_F\langle\bar{B}[\chi_+,B]\rangle$$
$$+ d_1\langle\bar{B}\{u_\mu,[u^\mu,B]\}\rangle + d_2\langle\bar{B}\left[u_\mu,[u^\mu,B]\right]\rangle$$
$$+ d_3\langle\bar{B}u_\mu\rangle\langle u^\mu B\rangle + d_4\langle\bar{B}B\rangle\langle u^\mu u_\mu\rangle,$$

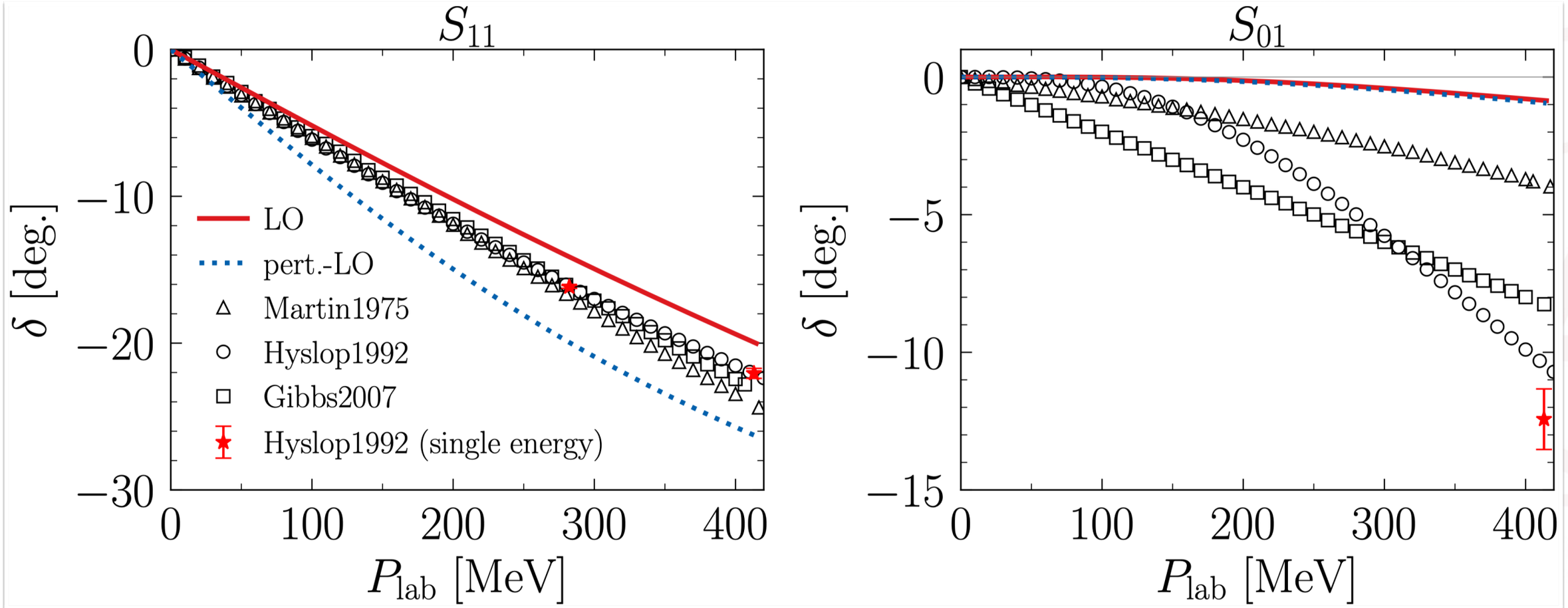
- Effective potential at NLO

$$V = V_{\rm LO} + V_{\rm NLO}$$
$$= V^{(a+b+c)}\big|_{u=u_0\sim(1,0)^\dagger} + V^{(a+b+c)}\big|_{u=u_1\sim\mathcal{O}(p)} + V^{(d)}\big|_{u=u_0\sim(1,0)^\dagger}$$

$$u_N(p,s) = u_0 + u_1 + \cdots = \left[\begin{pmatrix}1\\0\end{pmatrix} + \frac{1}{2m_N}\begin{pmatrix}0\\\boldsymbol{\sigma}\cdot\boldsymbol{p}\end{pmatrix} + \ldots\right]\chi_s$$

KN: Leading order results

- Prediction of s-wave phase shifts
  - $T_{\rm LO} = V_{\rm LO} + V_{\rm LO} G T_{\rm LO}$ vs. perturbative ($T_{\rm LO} = V_{\rm LO}$)

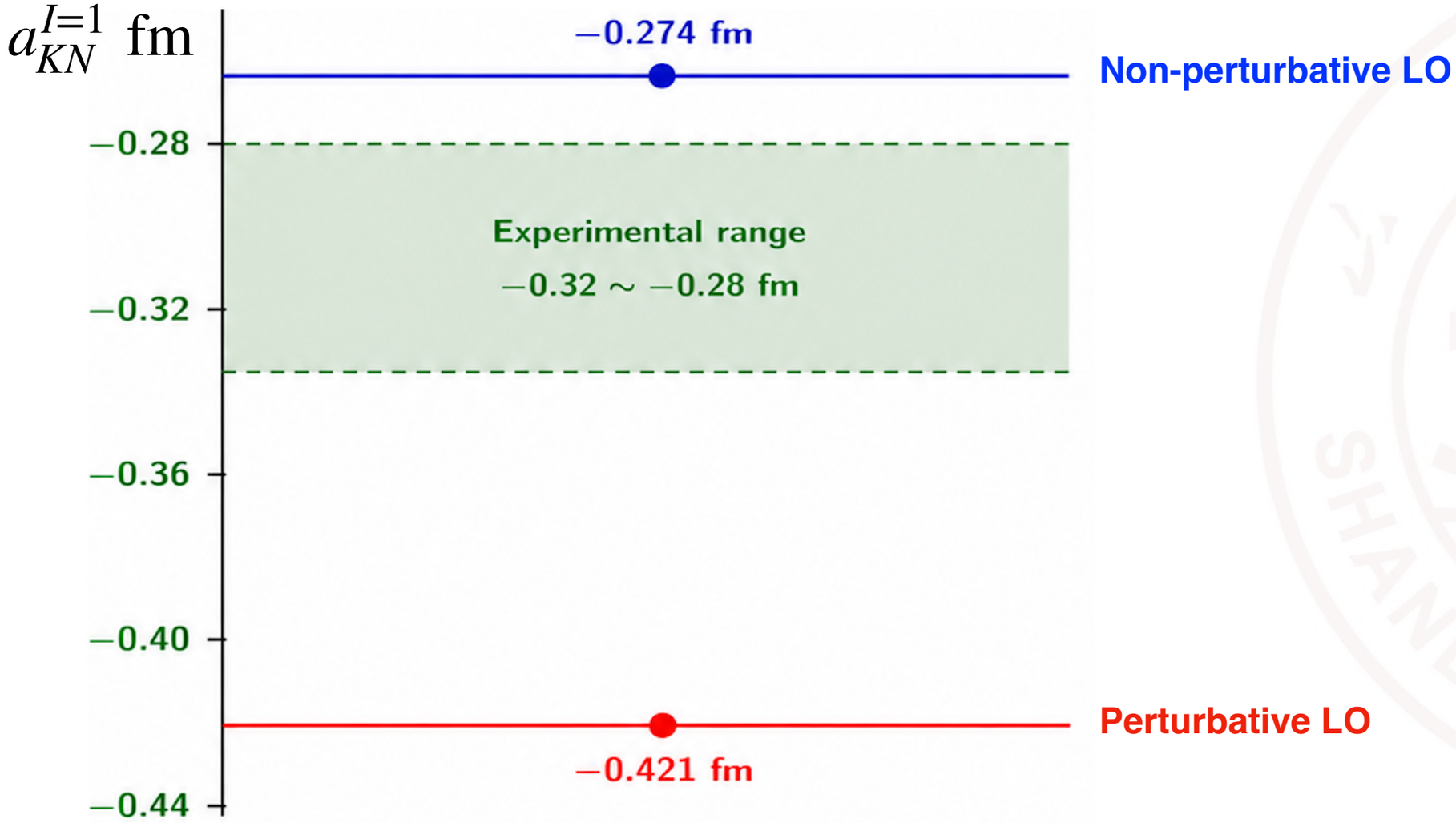


- **S11 channel**: non-pert. result better than the perturbative ones
- **S01 channel**: LO potential is very weak & PWAs are not consistent

KN: Leading order results

- Effective range expansion

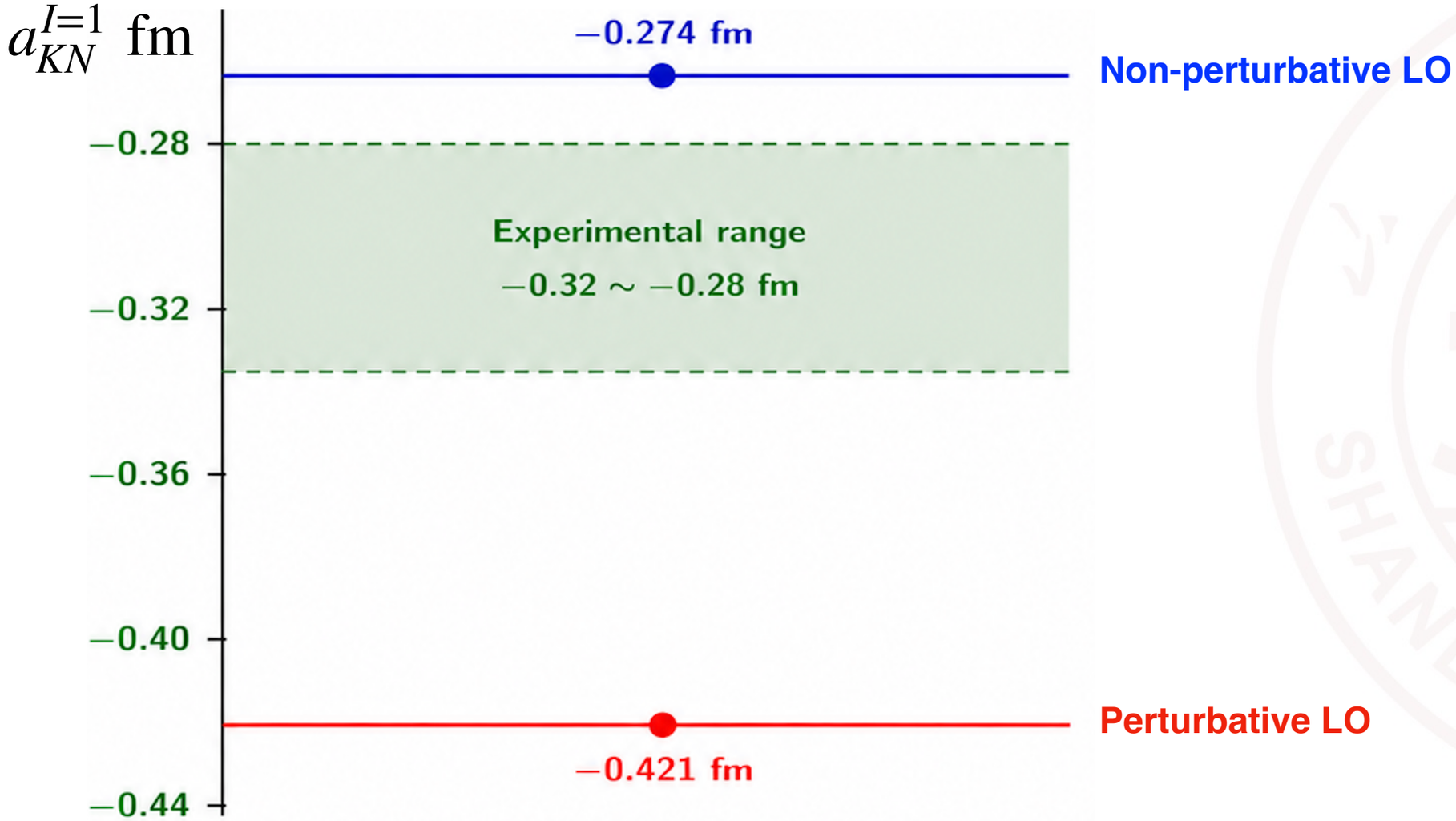
$$p_{\rm cm}\cot\delta(p_{\rm cm}) = \frac{1}{a} + \frac{1}{2}r(p_{\rm cm})\,p_{\rm cm}^2 + \mathscr{O}(p_{\rm cm}^4)$$



KN: Leading order results

- Effective range expansion

$$p_{\rm cm}\cot\delta(p_{\rm cm}) = \frac{1}{a} + \frac{1}{2}r(p_{\rm cm})\,p_{\rm cm}^2 + \mathscr{O}(p_{\rm cm}^4)$$



Non-perturbative treatment is essential, at least at lowest order, in the SU(3) sector of KN scattering

KN: Next-to-leading order results

- Four parameters at NLO

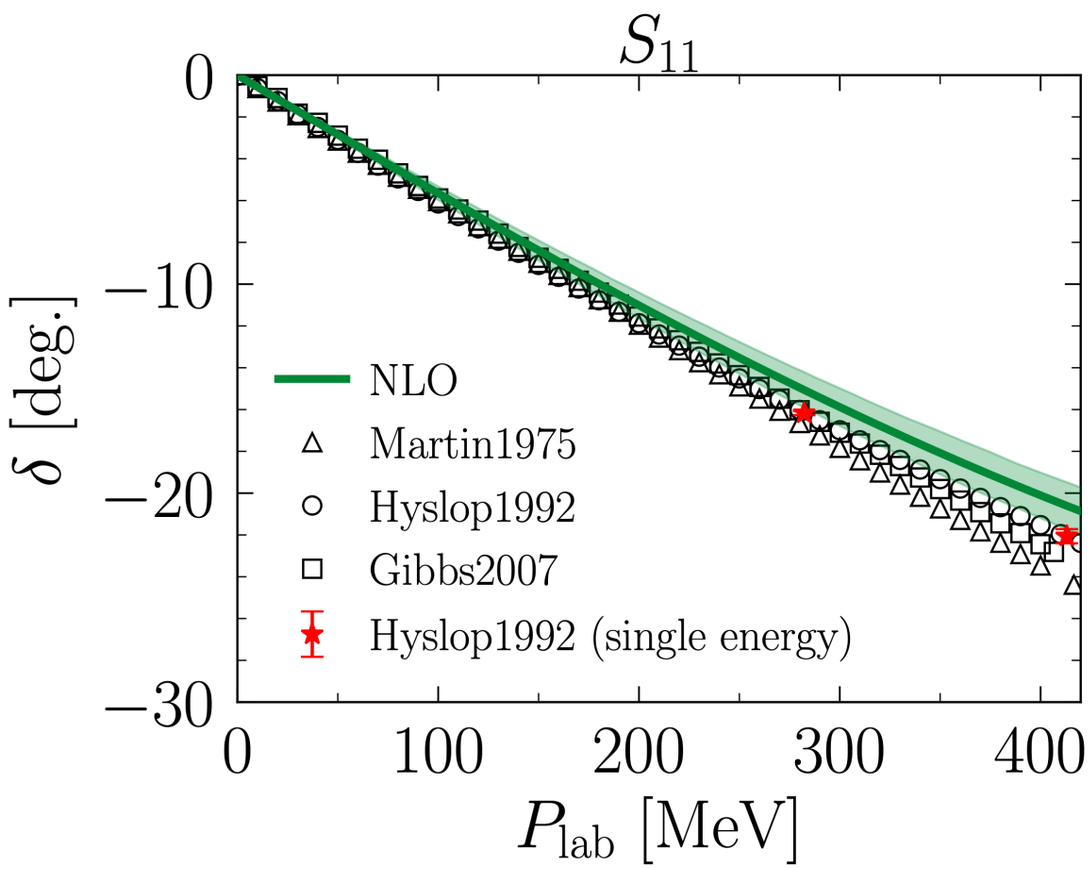
$$b^{I=0} = 4(b_0 - b_F), \qquad d^{I=0} = 2(2d_1 + d_3 - 2d_4), \longrightarrow \textbf{0.635(759) GeV}^{-1}$$

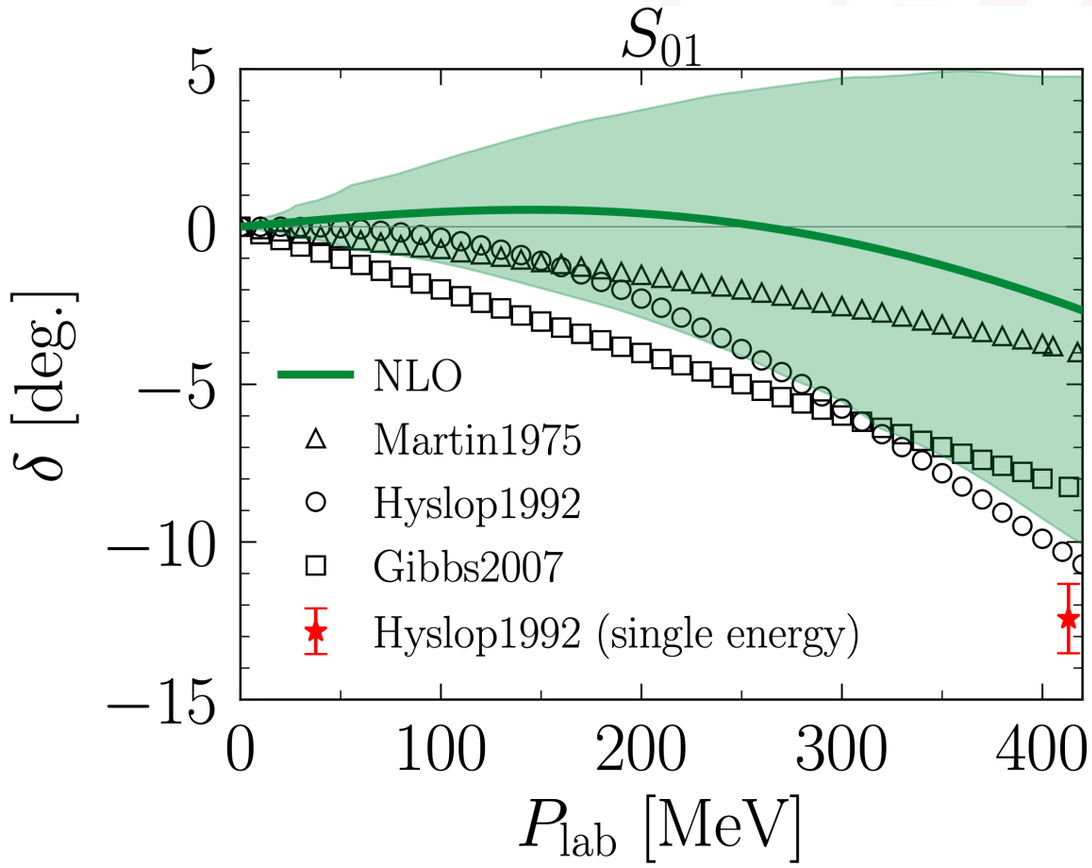
$$b^{I=1} = 4(b_0 + b_D), \qquad d^{I=1} = 2(2d_2 + d_3 + 2d_4), \longrightarrow \textbf{−1.486(360) GeV}^{-1}$$

Fixed by RQCD's $b_{0,D,F}$ Determined by S_{11} and S_{01} scattering lengths

$a_{KN}^{I=0,\,\mathrm{exp}} = 0.03 \pm 0.15\mathrm{fm}$ and $a_{KN}^{I=1,\,\mathrm{exp}} = -0.30 \pm 0.03\mathrm{fm}$

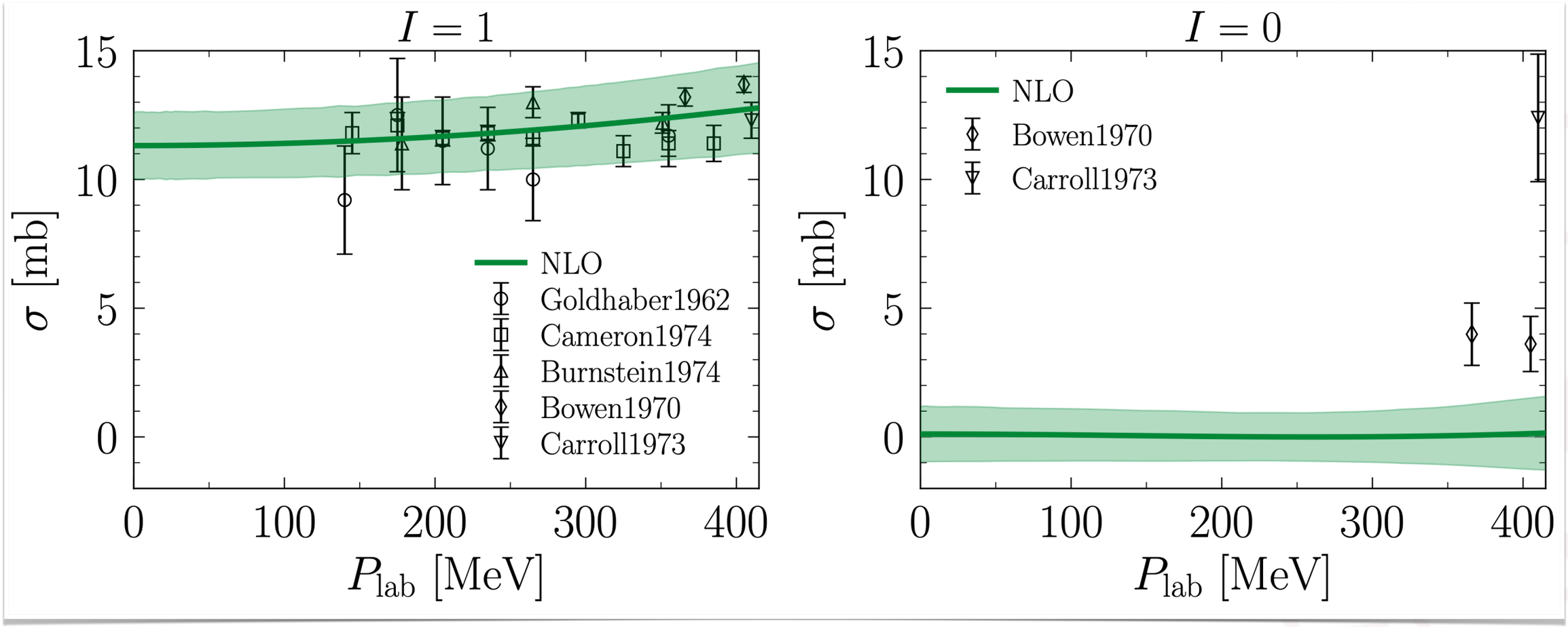
- Prediction of s-wave phase shifts





KN: Next-to-leading order results

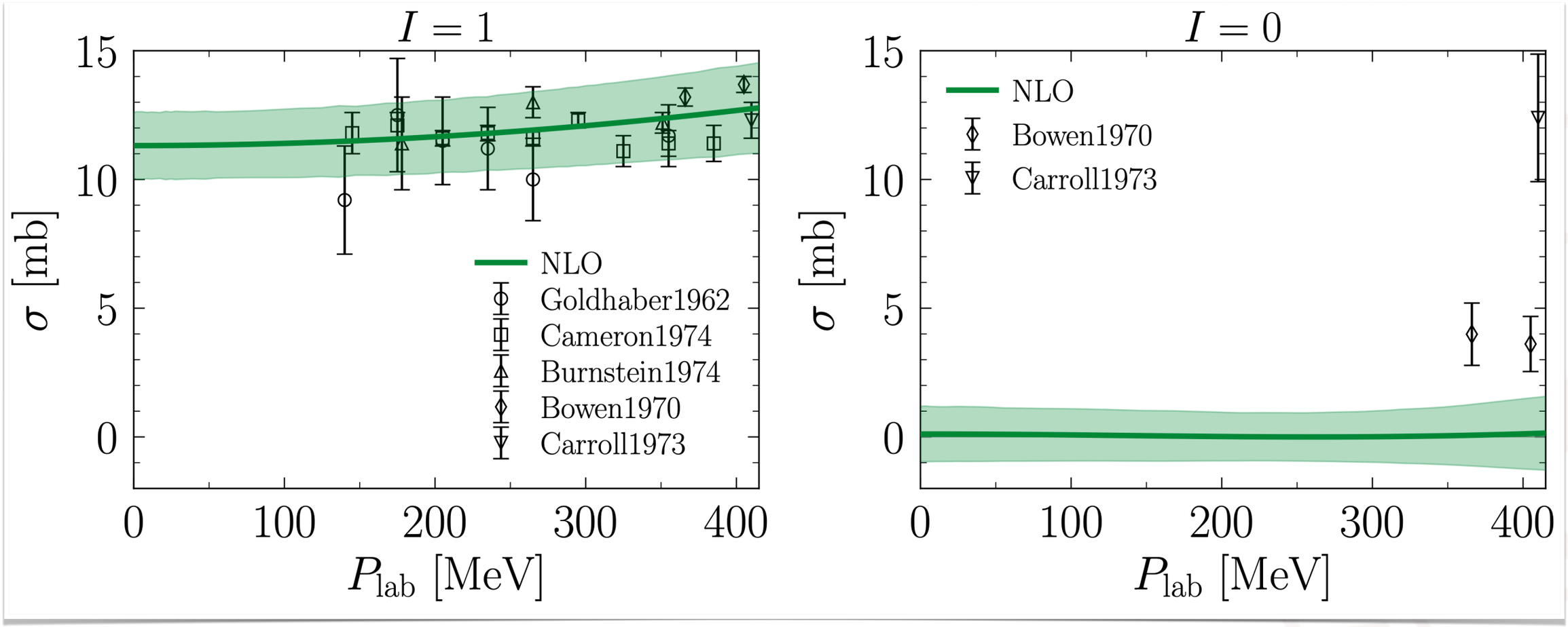
- S-wave contributions to the total cross sections



- **<u>I=1 channel</u>**: NLO s-wave result **agrees well** with experimental data and slowly increases with Plab
- **<u>I=0 channel</u>**: NLO s-wave contribution is too small to reproduce the observed total cross section

KN: Next-to-leading order results

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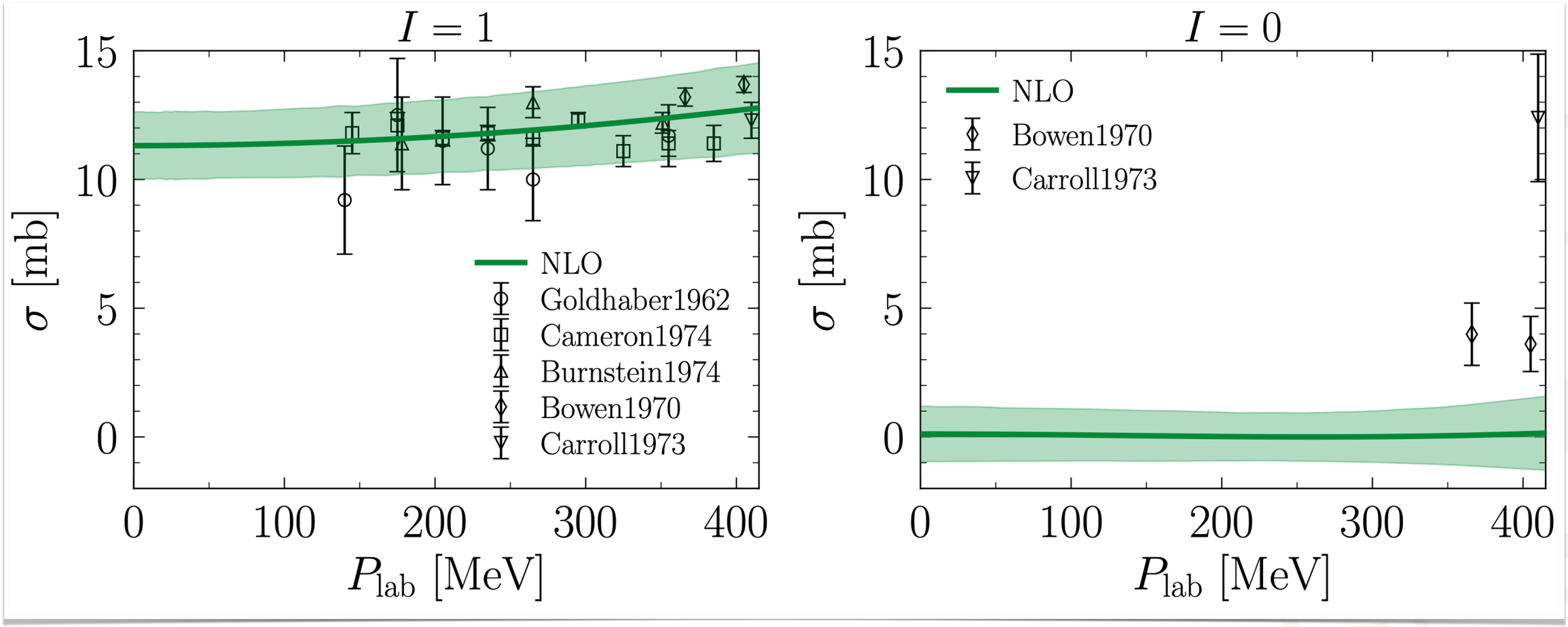


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✓ P-wave contribution dominant !

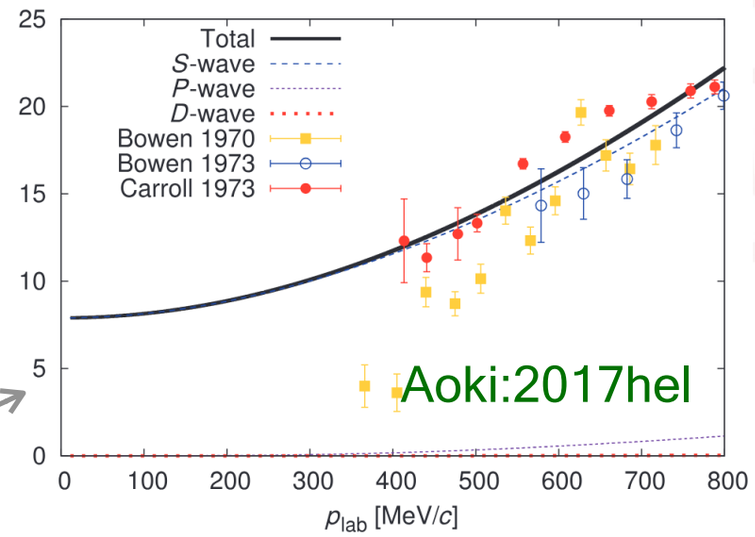
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Summary

- **A renormalized framework** for MB scattering is proposed
  - Time-ordered perturbation theory + Covariant chiral Lagrangians
  - Take into account the off-shell effects of potential
  - Employ the subtractive renormalization
    - ✓ **Achieve T-matrix cutoff-independent**
- Leading order studies
  - πN scattering; KN scattering with coupled channels
  - Obtain the **two-pole structure** of $\Lambda(1405)$
  - **Predictive power:** consistent with LQCD results
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  - Higher order corrections are perturbatively included
  - $KN, \pi N$ scatterings: improve the description of phase shifts

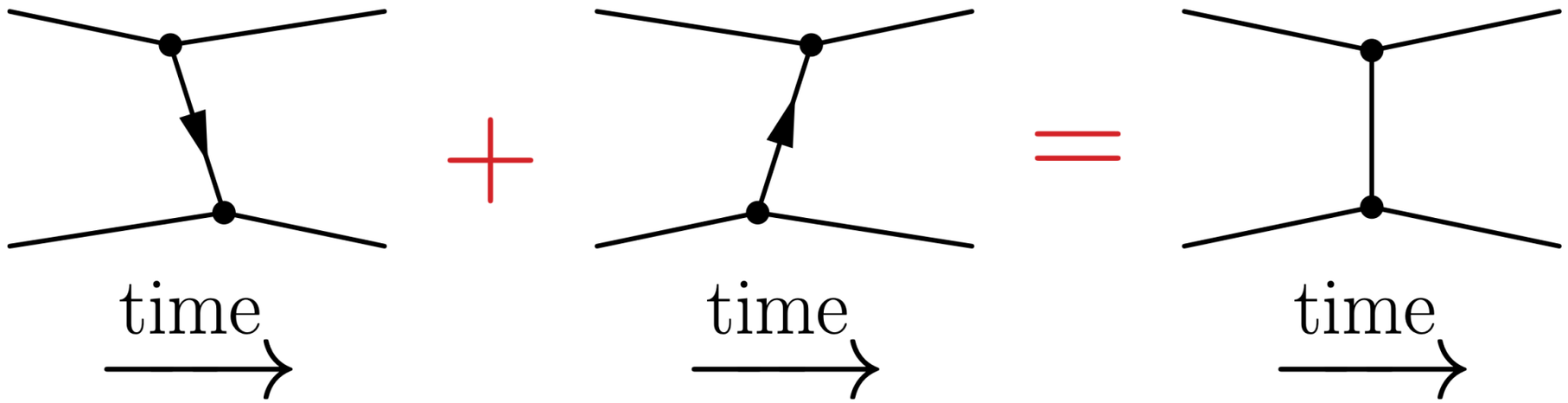
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  - Working on $\bar{K}N$ scattering/$\Lambda(1405)$ at NLO

Thank you for your attention!

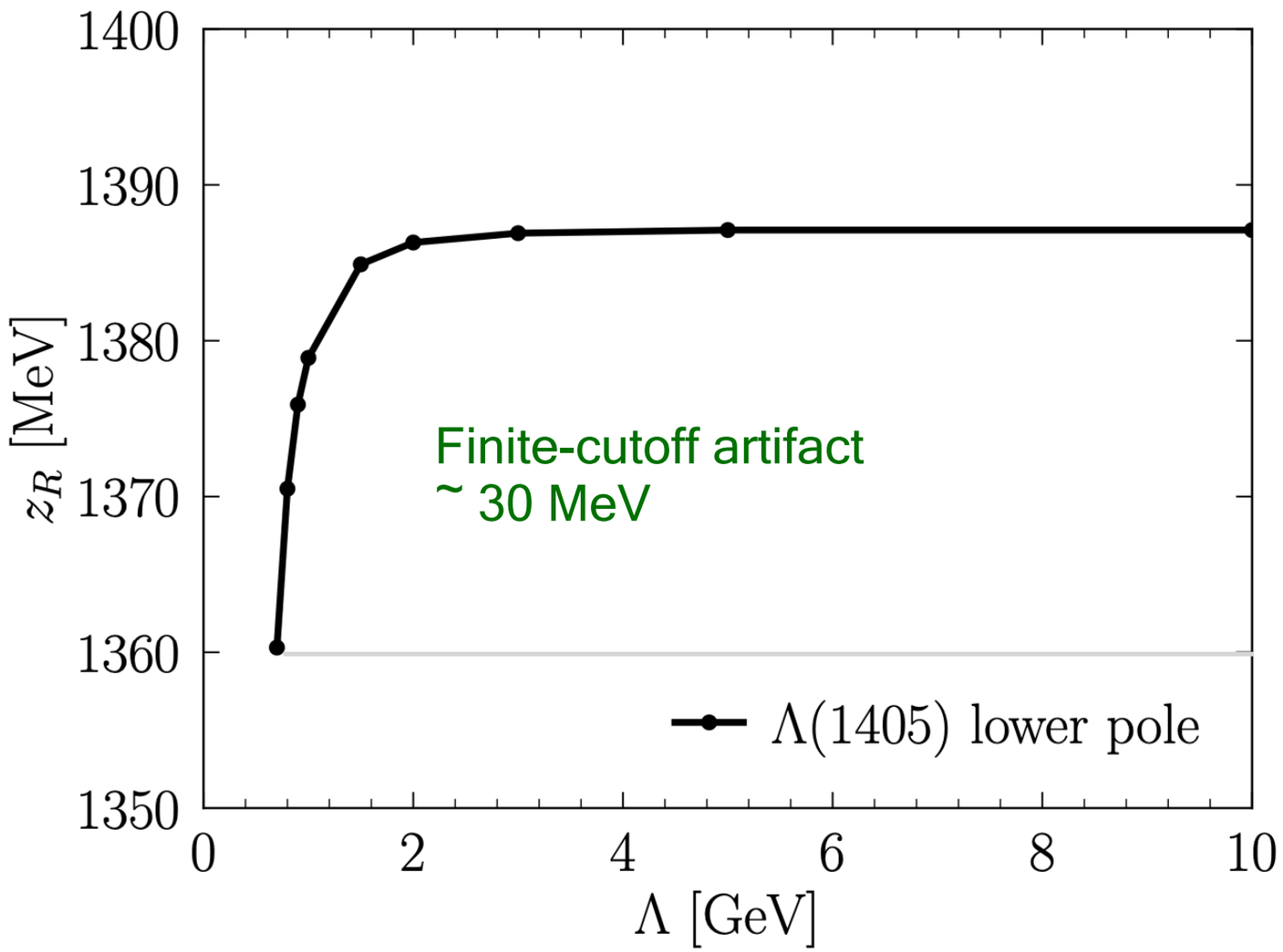
Back up

A Feynman diagram represents the sum of **all** time orderings



Finite cutoff artifact

- We extend the calculation to the unphysical quark mass region
  - Use the same meson and baryon masses as the BaSc study
    - ✓ $M_\pi = 203.7\ \text{MeV}, M_K = 486.4\ \text{MeV}, m_N = 979.8\ \text{MeV}, m_\Sigma = 1193.9\ \text{MeV}$
    - ✓ $F_0 = F_\pi = 93.2\ \text{MeV}$
  - Focus on the $\pi\Sigma - \bar{K}N$ coupled channels

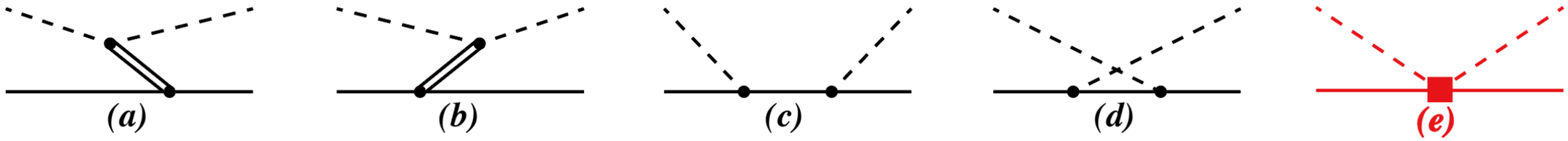


- $\Lambda \geq 3\ \text{GeV}$, cutoff-independent results achieved!

➡ Renormalized $\pi\Sigma - \bar{K}N$ amplitudes

➡ Fix $\Lambda = 10\ \text{GeV}$

πN scattering at NLO



- Chiral effective Lagrangian

$$\mathcal{L}_{\pi N}^{(2)} = \bar{\Psi}_N \left\{ c_1 \langle \chi_+ \rangle - \frac{c_2}{4m^2} \langle u^\mu u^\nu \rangle \left(D_\mu D_\nu + \text{ h.c. } \right) + \frac{c_3}{2} \langle u^\mu u_\mu \rangle - \frac{c_4}{4} \gamma^\mu \gamma^\nu \left[u_\mu, u_\nu \right] \right\} \Psi_N$$

  - Fix $c_1 = -0.74,\ c_2 = 1.81,\ c_3 = -3.61,\ c_4 = 2.17\ \text{GeV}^{-1}$

D. Siemens, et al., PLB770 (2017) 27-34

- NLO potential

$$V = V_{\text{LO}} + V_{\text{NLO}}$$
$$= V^{(a+b+c+d)}|_{u=u_0\sim(1,0)^\dagger} + V^{(a+b+c+d)}|_{u=u_1\sim\mathcal{O}(p)} + V^{(e)}|_{u=u_0\sim(1,0)^\dagger}$$

- Prediction for the πN phase shifts

