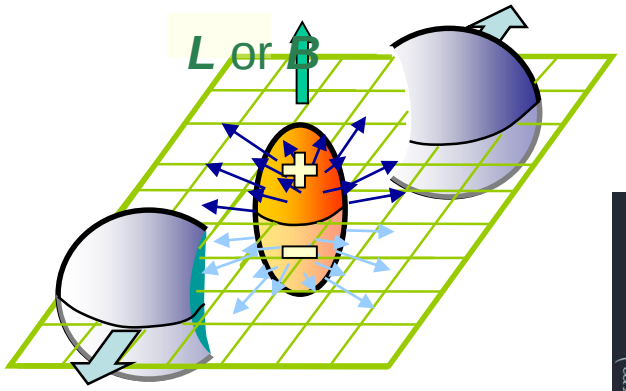
Properties and synthesis of magnetized heavy nuclei

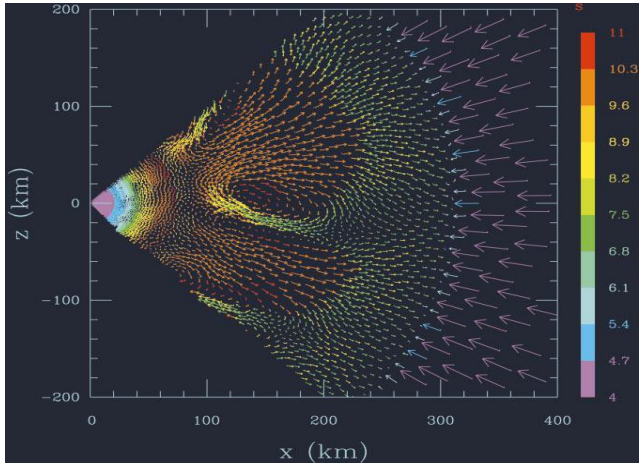
Nanjing_2026

V.N. Kondratyev,

Bogolubov Laboratory of Theoretical Physics, JINR, 141980, Dubna, Russia

Motivation



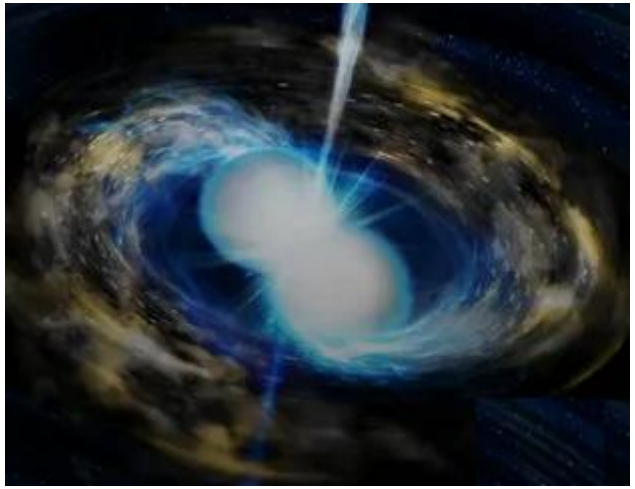


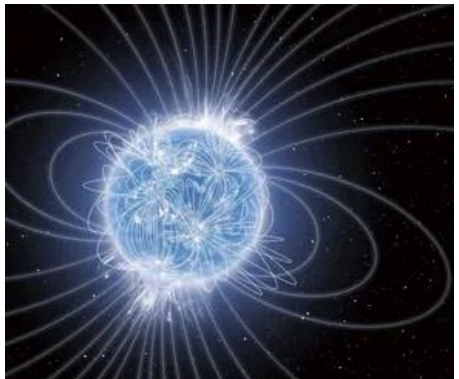
Heavy Ion collisions

Supernovae,

Neutron star mergers

Magnetrs





Magnetic field ~ 10^{18} Гс,
Energy ~ 10^{7} эВ,
Length ~ 10^{-14} м

Atomic nuclei in magnetic firld H

Hamiltonian $\mathbf{H}(\boldsymbol{p},\boldsymbol{r})$ $\quad \boldsymbol{p} \rightarrow \boldsymbol{p} - e/c\,\boldsymbol{A}, \; -\boldsymbol{m}_s\boldsymbol{H}$

m_s —spin magnetic moment

Magnetic Field vector potential $\boldsymbol{A}=\boldsymbol{H} \times \mathbf{r}/2$

Nonrelativistic limit $\quad \mathbf{H}_M \approx -M_z H$

μ_N - nucleon magneton $\quad M_z = \mu_N \Sigma_I \left(g_\alpha^o m_i + g_\alpha \sigma_i\right).$

Partition function $\quad \sum = \sum_i \exp\{-E_i/kT\},$

E_i — i-state energy

Free energy

$$Q = kT^2 \, \mathrm{d}ln(\sum)/\mathrm{d}\,T$$

Structure of magnetized nuclei

$$\hat{\mathrm{H}} = \hat{\mathrm{H}}^0 - \hat{\mathrm{H}}_\mathrm{M}, \quad \hat{\mathrm{H}}_\mathrm{M} = \sum_{i=1}^{A}((\hat{\tau}_3^i + 1/2)\hat{\mathbf{l}}_i + g(\hat{\tau}_3^i)\hat{\mathbf{s}}_i)\omega_\mathrm{L}$$

H^0 -- *Hamiltonian of nuclei without magnetic field* & H_M -- *interaction of nucleon magnetic moments with field,* $\omega_L = \mu_N H$, μ_N = nucleon magneton

$H_c \sim \delta_{so} / \mu_N \sim 10^{1.5}$ ТТл : $H < H_c$ Zeeman *linear response*

$$\Delta B_m = B_m(H) - B_m(0) = \sum_{\mathrm{occ}} \langle nljm_j|\hat{\mathrm{H}}_\mathrm{M}|nljm_j\rangle$$

$$\kappa = \sum_{i-\mathrm{v};m,s} |\langle l^i m, \frac{1}{2}s|j^i m_j^i\rangle|^2((\tau_3^i + 1/2)m + g(\tau_3^i)s)$$

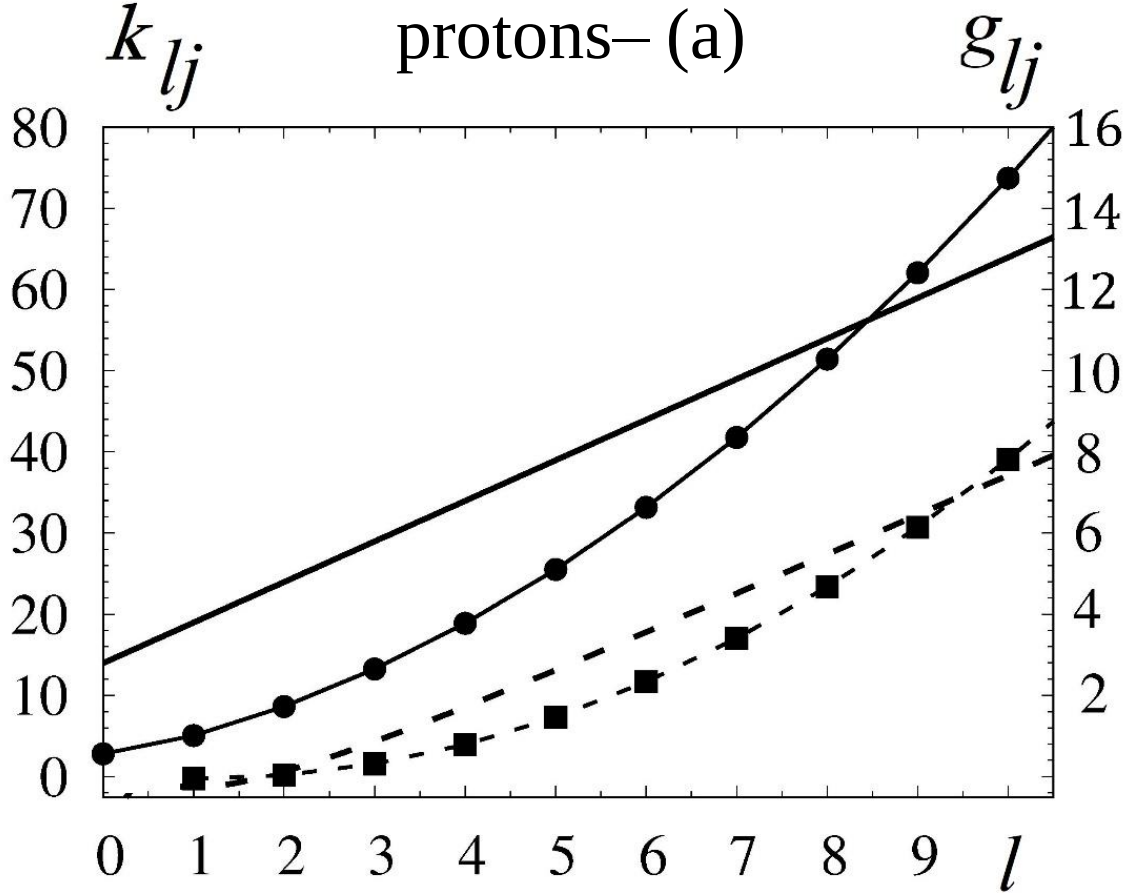
$$\Delta B_m = \kappa\omega_\mathrm{L},$$

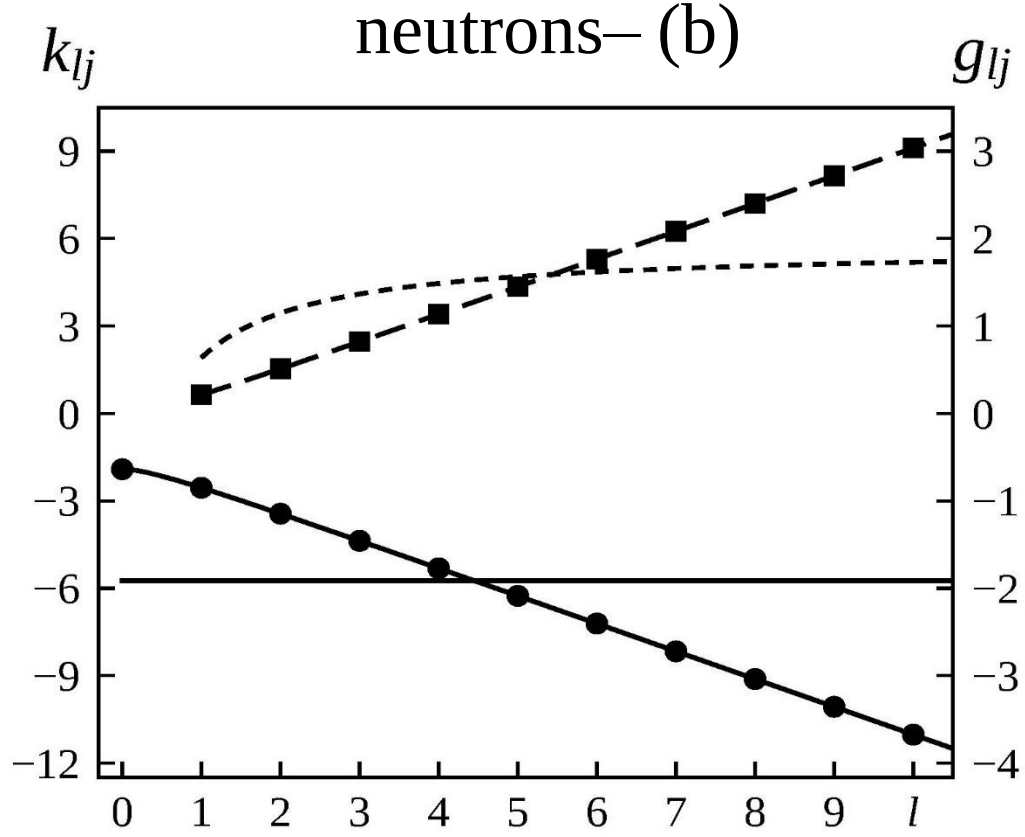
g–factors of proton $g(1/2) \approx 5.586$ & neutron $g(-1/2) \approx -3.826$

At H > 0.1 TT k is more reliable as compared to nuclear g–factors

Magnetic susceptibility maximum k_{ij} *half filled shells*

g_{ij} -- g-factor: —●— $j = l + 1/2$: – –■– – – $j = l - ½$

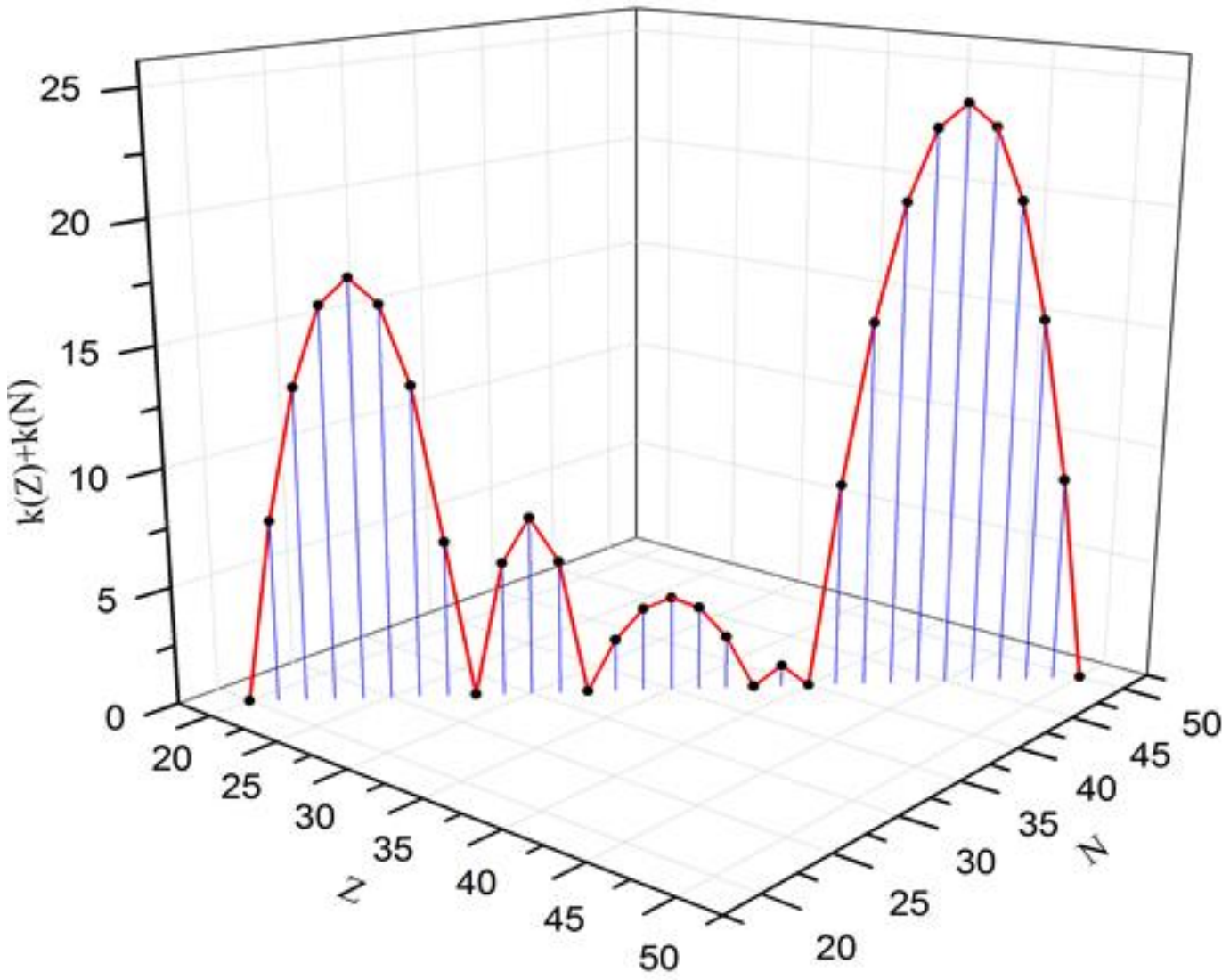




Таб.1

	Z	k(1/2)	N	K(-1/2)
j=3/2	20	0.00	20	0.00
j=7/2	21	5.793	21	1.913
	22	9.931	22	3.279
	23	12.414	23	4.099
	24	13.241	24	4.373
	25	12.414	25	4.099
	26	9.931	26	3.279
	27	5.793	27	0.862
	28	0.00	28	0.00
j=3/2	29	3.793	29	1.913
	30	5.057	30	2.551
	31	3.793	31	1.913
	32	0.00	32	0.00
j=5/2	33	0.862	33	1.366
	34	1.379	34	2.186
	35	1.55	35	2.459
	36	1.379	36	2.185
	37	0.862	37	1.366
	38	0.00	38	0.00
j=1/2	39	0.264	39	0.638
	40	0.00	40	0.00
j=9/2	41	6.793	41	1.913
	42	12.076	42	3.401
	43	15.85	43	4.464
	44	18.115	44	5.101
	45	18.869	45	5.314
	46	18.115	46	5.101
	47	15.85	47	4.464
	48	12.076	48	3.401
	49	6.793	49	1.913
	50	0.00	50	0.00

For symmetric nuclei



Nucleosynthesis

Nucleation

Nucleation : reaction $I(n,\gamma)O$

Cross section

$$\sigma = \pi\lambda^2 \sum_{m_\pi} \frac{T_n(m_\pi)\,T_\gamma(m_\pi)}{T_{\mathrm{tot}}(m_\pi)},$$

Hauser-Feshbach approach

$$T_i(m_\pi) = \int_{-\infty}^{E-S_{Ii}} \sum_{m_\pi^I} T_i(m_\pi, E_I, m_\pi^I)\,\mathcal{W}(E_I, m_\pi^I)\,dE_I,$$

Level density

$$\mathcal{W}(E_I, m_\pi^I) = \Sigma_\nu\ \delta(E_I - E_I^\nu(m_\pi^I))$$

transmission coefficient

Reaction $A(n,\gamma)A+1$ $\quad T_n >> T_\gamma$

VNK PRC69(2004)038801

$$\sigma_n \sim T_\gamma$$

$$T_{\mathrm{E1}}(E_\gamma) = \frac{8}{3}\frac{NZ}{A}\frac{e^2}{\hbar m_{\mathrm{N}} c^3}\frac{\Gamma_{\mathrm{GDR}} E_\gamma^4}{(E_\gamma^2 - E_{\mathrm{GDR}}^2)^2 + \Gamma_{\mathrm{GDR}}^2 E_\gamma^2}.$$

$$E_{\mathrm{M}} = (\mathcal{M}_J - \mathcal{M}_O)B \qquad \mathcal{M}_i = g_i m_i$$

Level density — **BSFG** модель (back shifted Fermi-gas)

$$\mathcal{W}(U) = \frac{\sqrt{\pi}}{12a^{1/4}} \frac{\exp(2\sqrt{aU})}{U^{5/4}},$$

$$U = E - \delta$$

$$a(U,Z,N) = \tilde{a}(A)\left[1 + C(Z,N) f(U)/U\right]$$

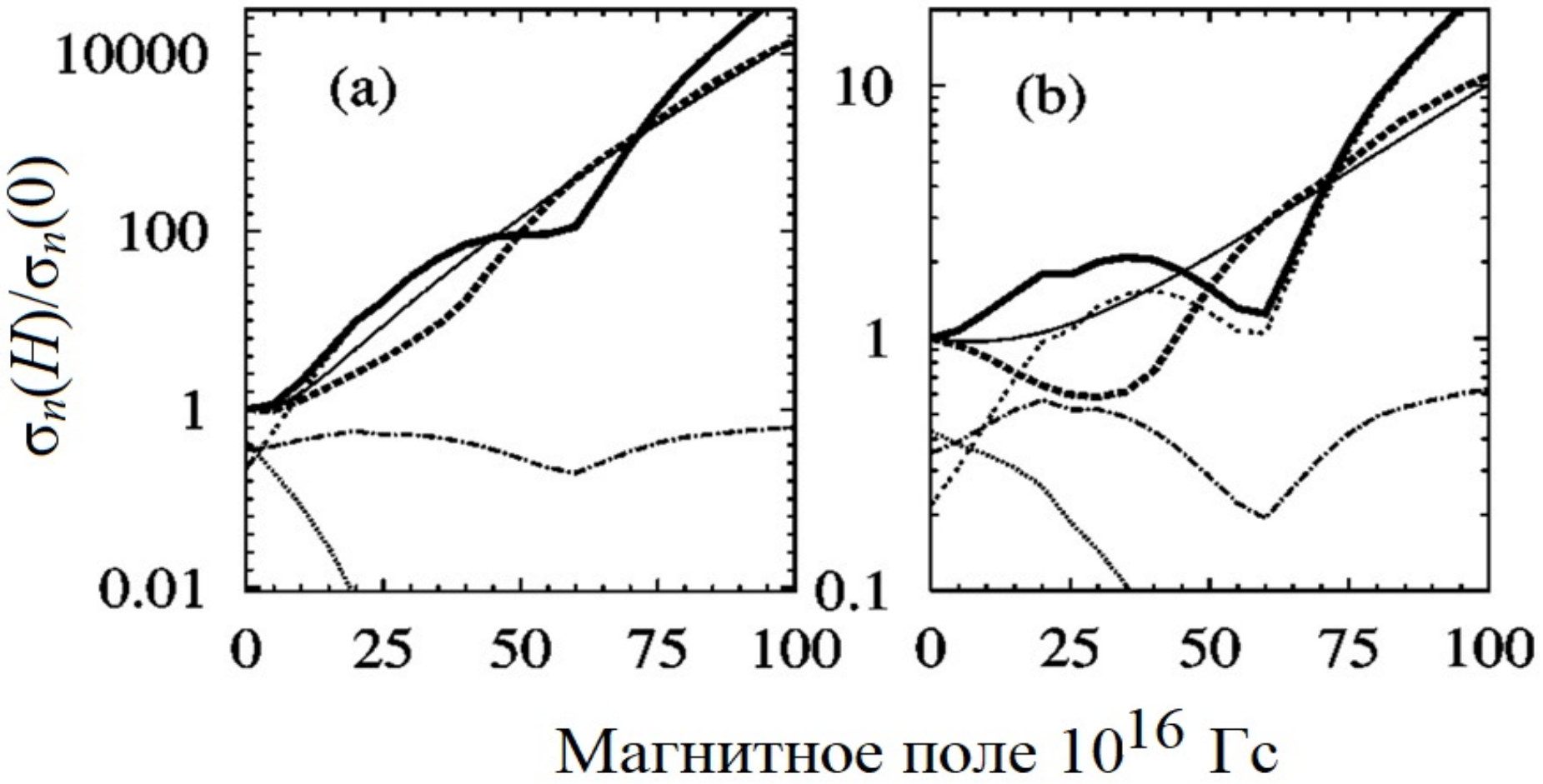
$$\tilde{a}(A) = \alpha A + \beta A^{2/3}$$

$C(Z,N)$ – оболочечная поправка

$$\mathcal{F}(m) = (\sqrt{\pi}\kappa)^{-1} \exp\left[-(m - m_F)^2/\kappa^2\right]$$

$$\kappa^2 \approx 0.225 A^{2/3} \sqrt{aU}.$$

Relative Cross section $A(n,\gamma)A+1$ *reaction*



Synthesis: I–Nuclear statistical equilibrium NSE: Saha Equality

VNK, Universe 7(2021)487

Yield Y_A of nucleus $A = {}^{A}_{N}Z$ $\quad Y_A = Y_p^Z \, Y_n^N \, \Sigma_A \, \Sigma_p^{-Z} \, \Sigma_n^{-N}$,

$\Sigma = \sum_i \exp\{-e_i/kT\}$, where $e_i -$ $i -$ stste nuclear particle energy

Relative yield in *magnetic field* $\boldsymbol{H}$

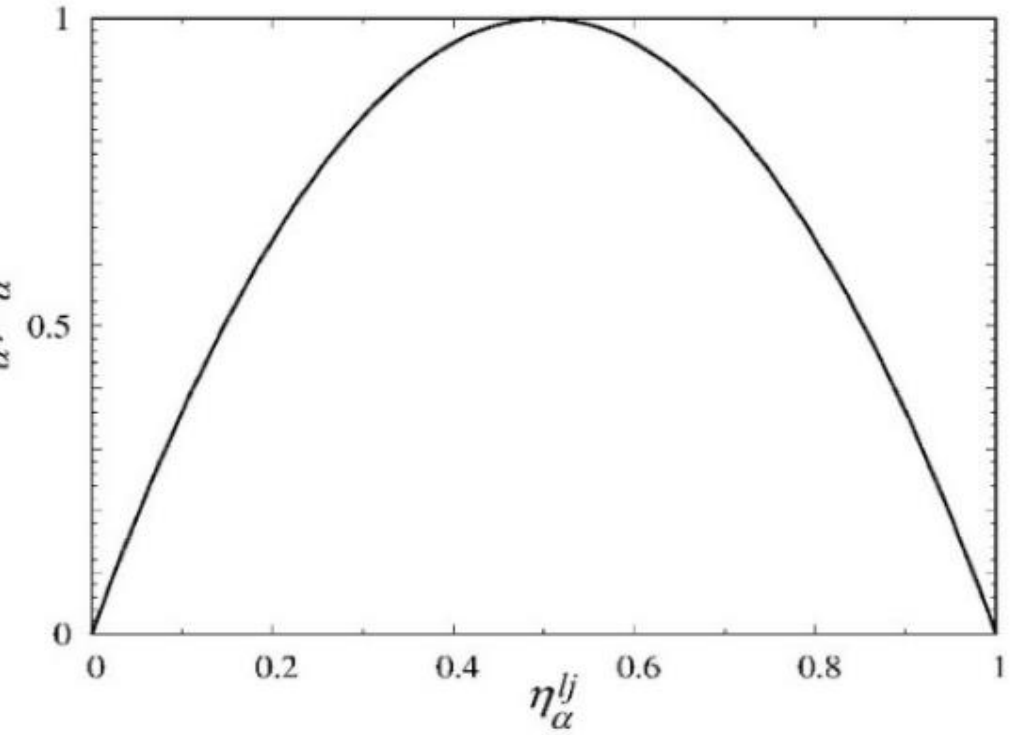
$$\mathrm{y} = \mathrm{Y}(H)/\mathrm{Y}(0) = \mathrm{S}_A \, \mathrm{S}_p^{-Z} \, \mathrm{S}_n^{-N} \qquad S_i(H) = \Sigma_i(H)/\Sigma_i(0)$$

$T \, \mathrm{d}ln(S(H))/\mathrm{d}\,T << 1$ Exponential approximation

$$y \approx \exp\{(\Delta Q_A - Z\,\Delta Q_p - N\,\Delta Q_n)/k\,T\}$$

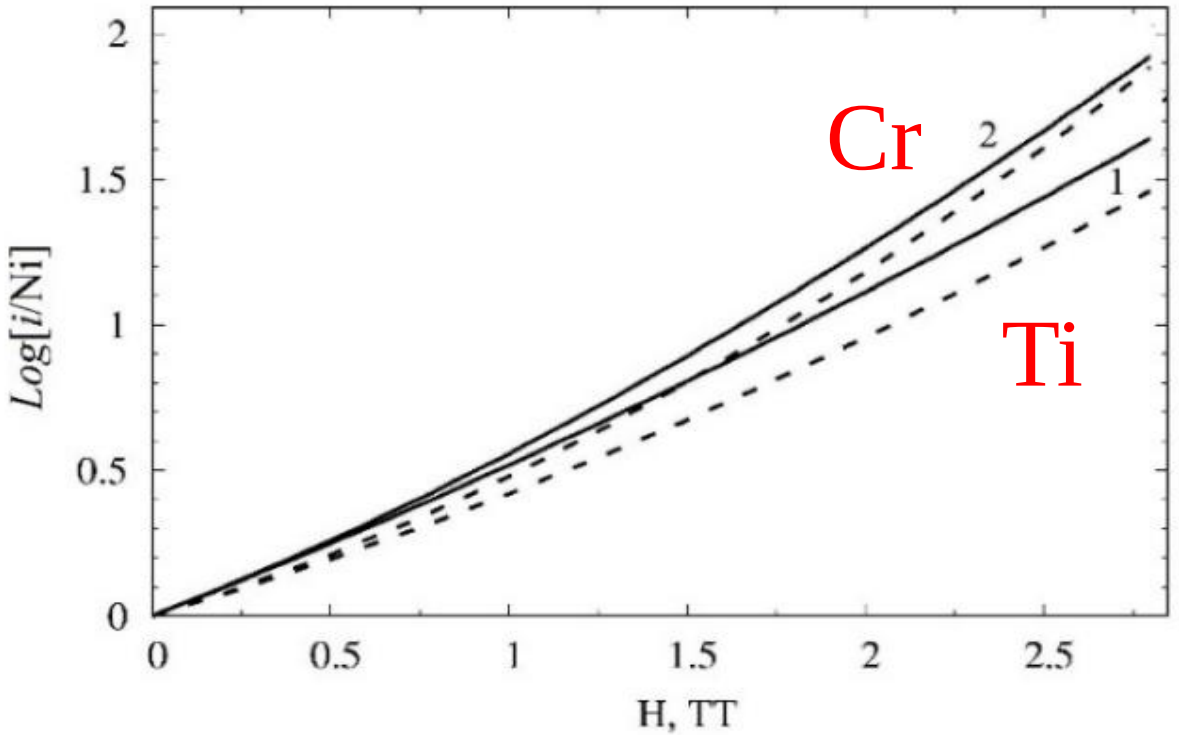
Change of free energy $\quad \Delta\, Q = kT^2 \, \mathrm{d}ln(S(H))/\mathrm{d}\,T$

Magnetic susceptibility *vs* shell filling

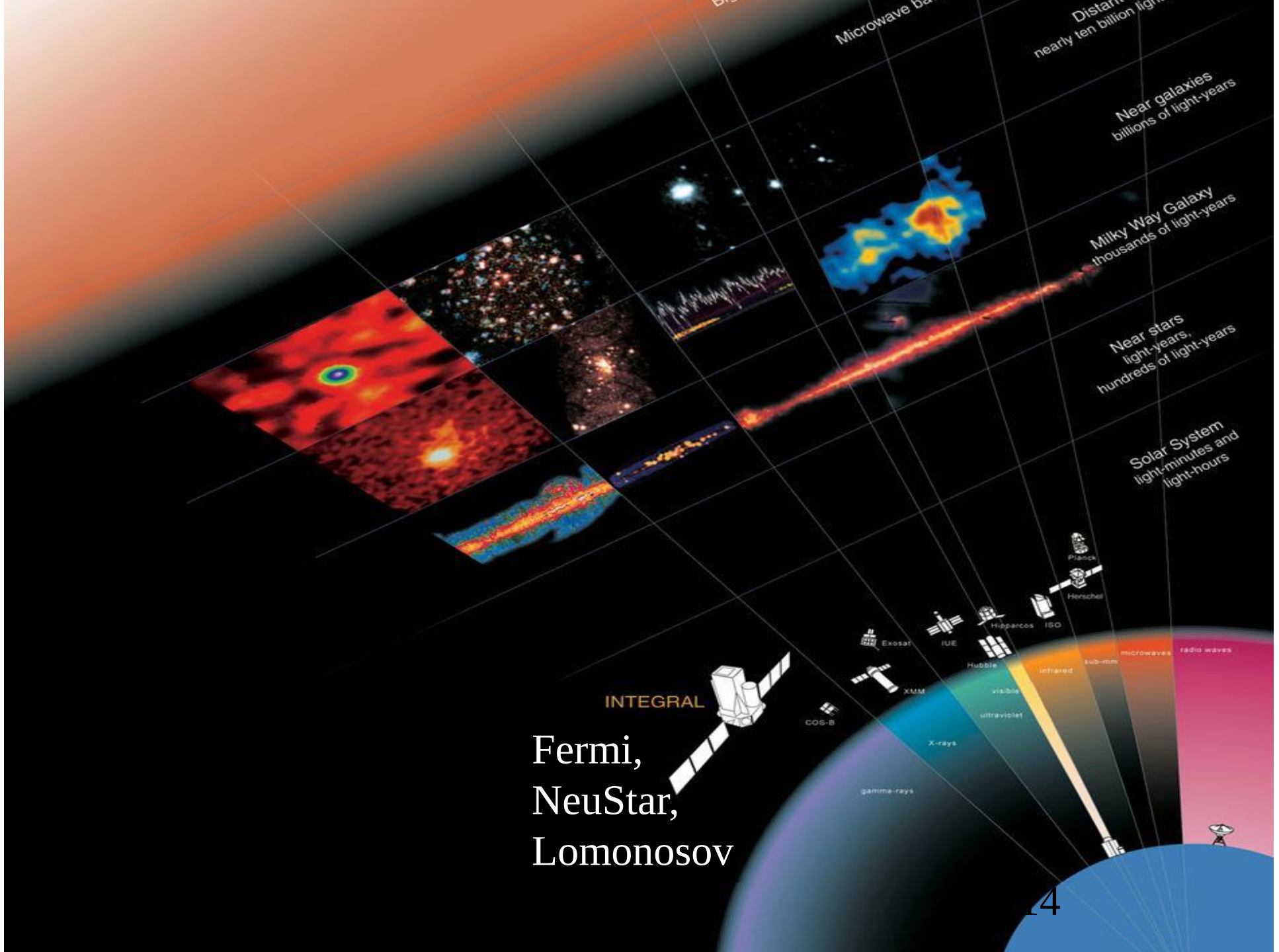


(a)

Relative yield Ti to Ni



(b)

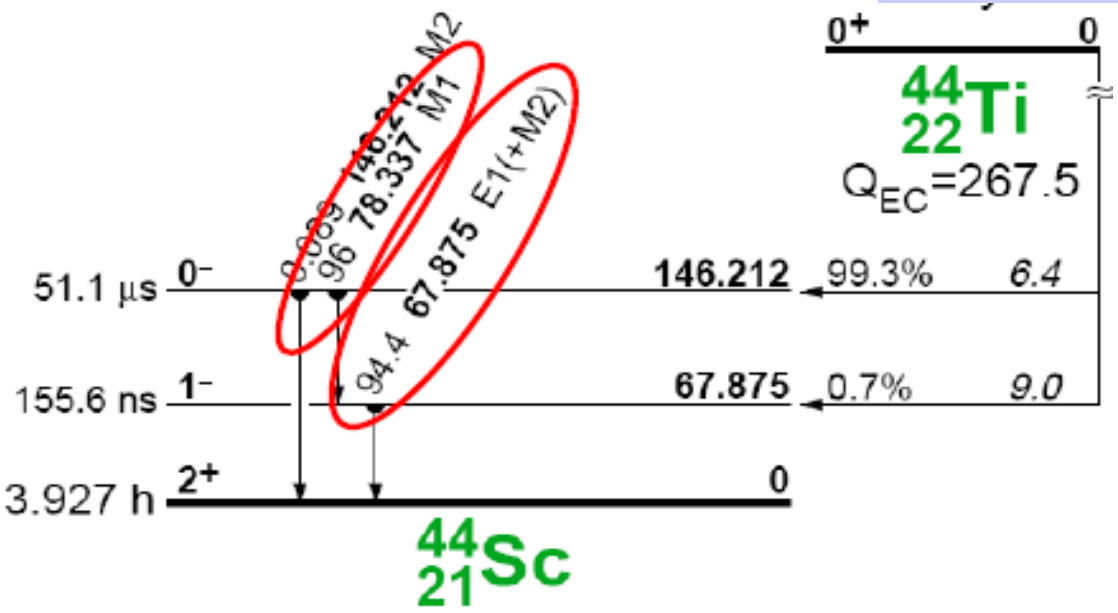


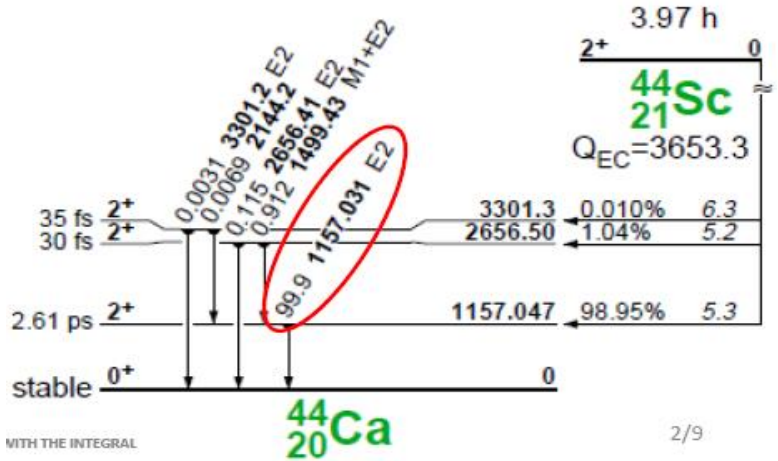
Fermi,
NeuStar,
Lomonosov

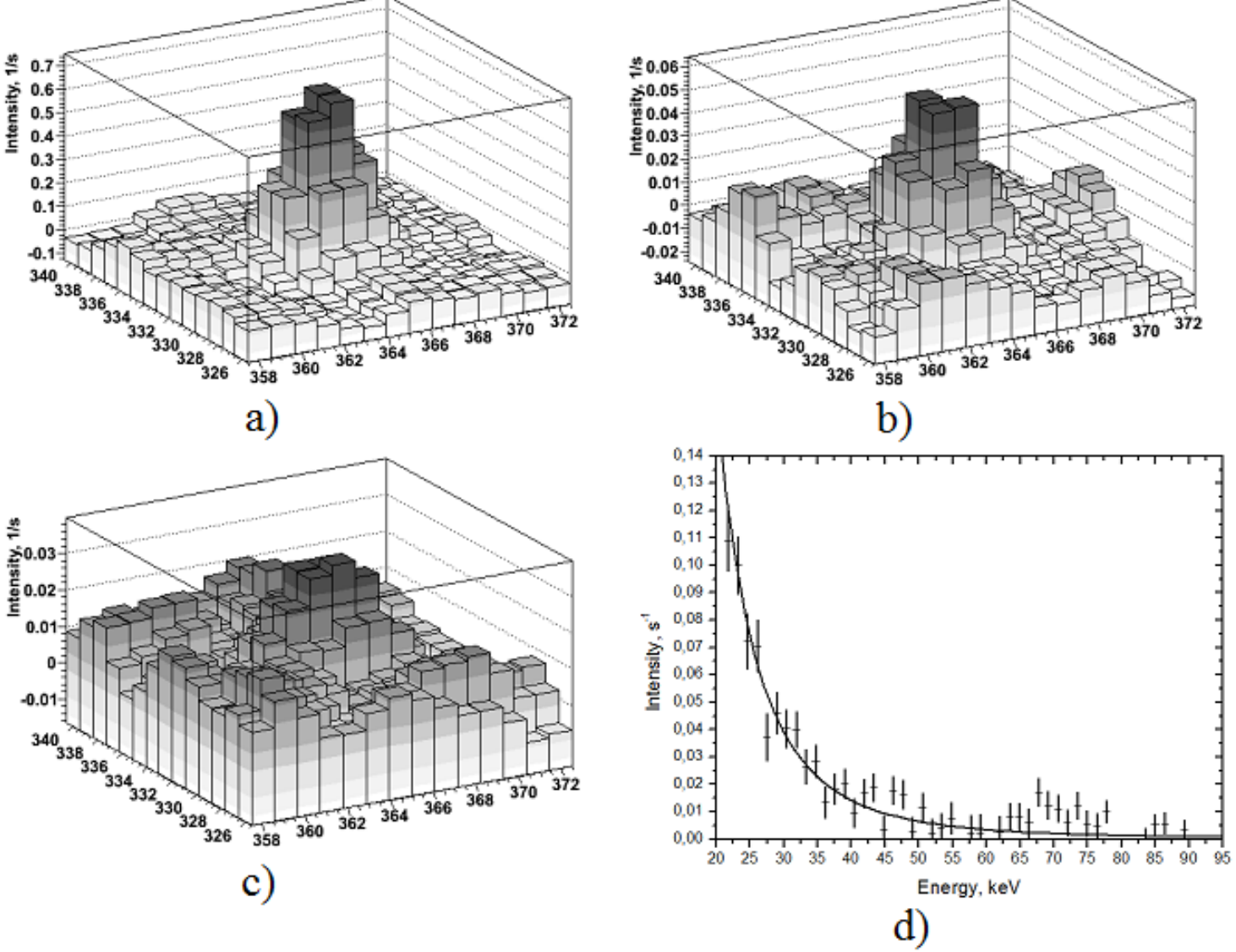
The scheme of the ^{44}Ti decay

Earth environment

59 y

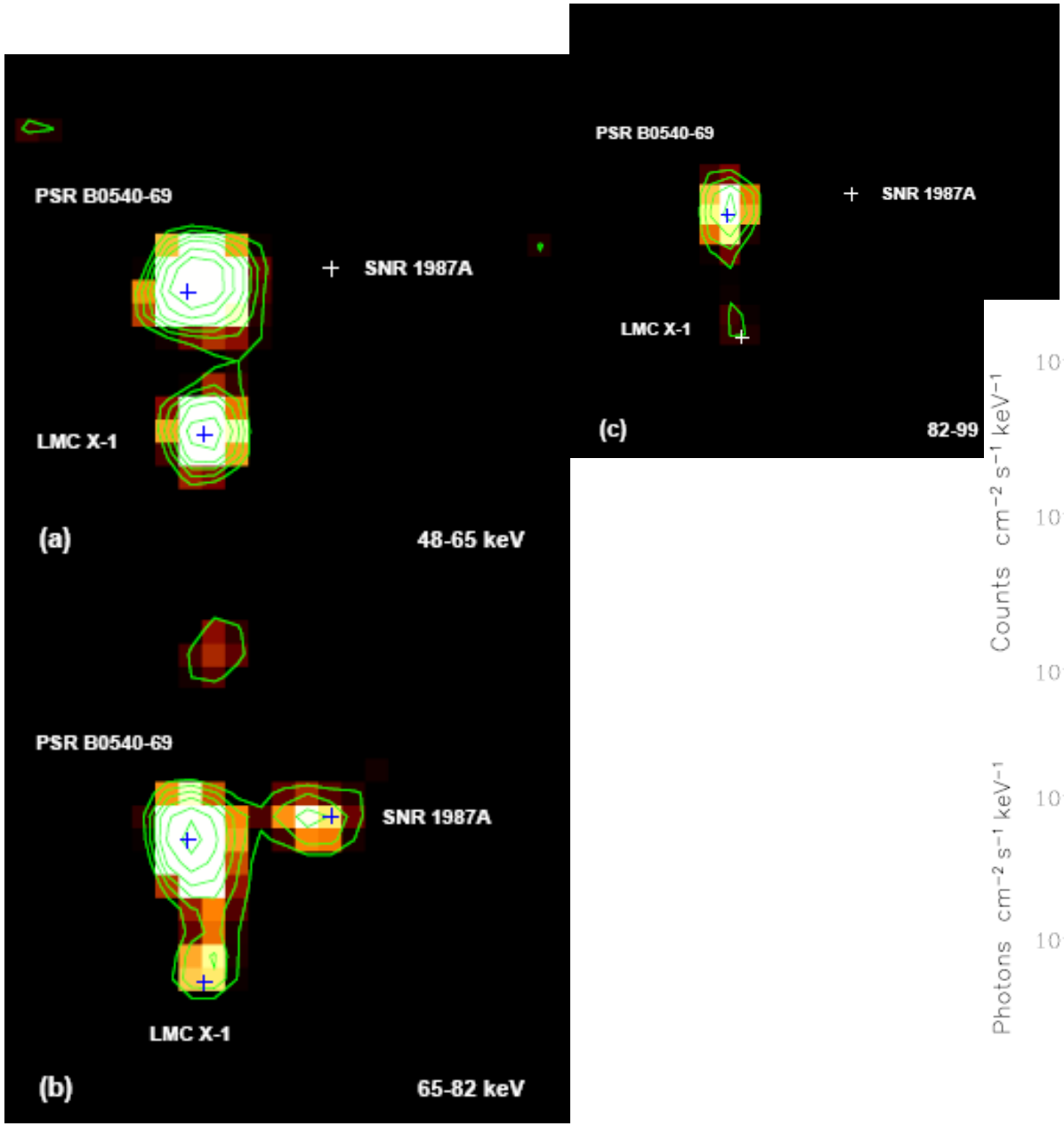




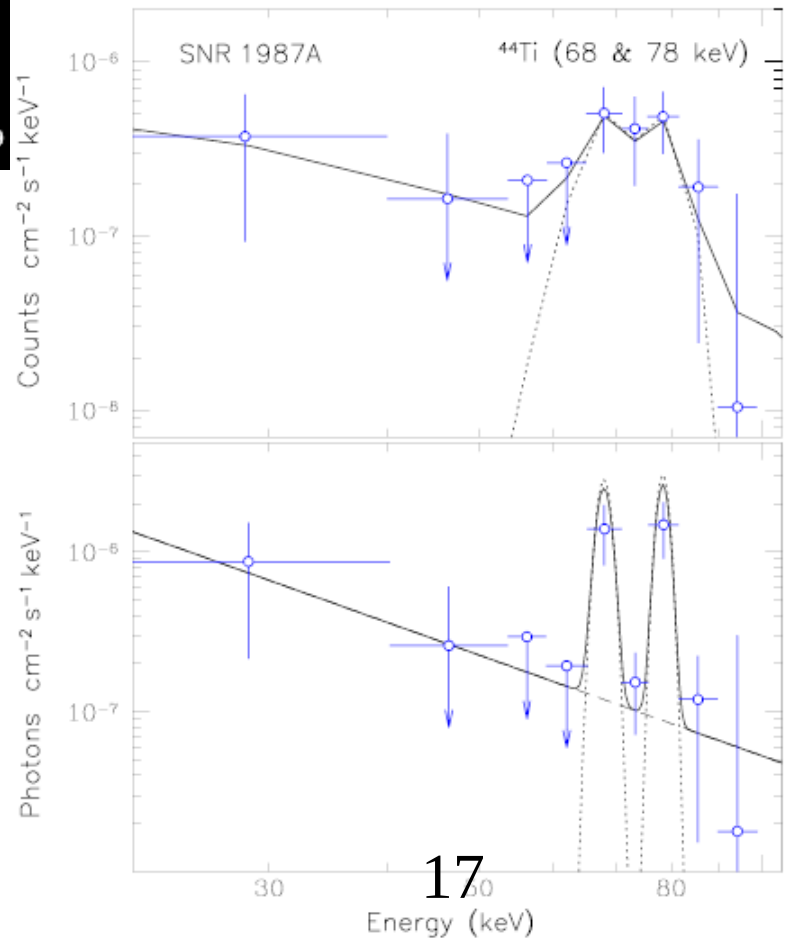


The direction (i.e.. pixel number) dependence of the registered gamma-ray flux at different energy ranges: **20–62 keV - a, 62–72 keV - b, 72–82 keV – c**; for the angle region containing the Cassiopeia A SN remnant. The right bottom panel (d) represents the spectrum from the Cassiopeia A in the energy range 20–95 keV, the solid line shows the fit with the power law energy *E* dependence, .

S.A.Grebenev et al *Nature*, *490*, *373-375 (2012)*.



$$(3.1 \pm 0.8) \times 10^{-4} \quad M_{\odot}$$



THEORY

& Observations

SN	M_{Ti}, 10^{-4} M_{Sun}
CAS A [9, 10]	$\left(3.3^{+0.9}_{-0.7}\right)$
SN1987A [11]	3.1 ± 0.8

SN1a Model

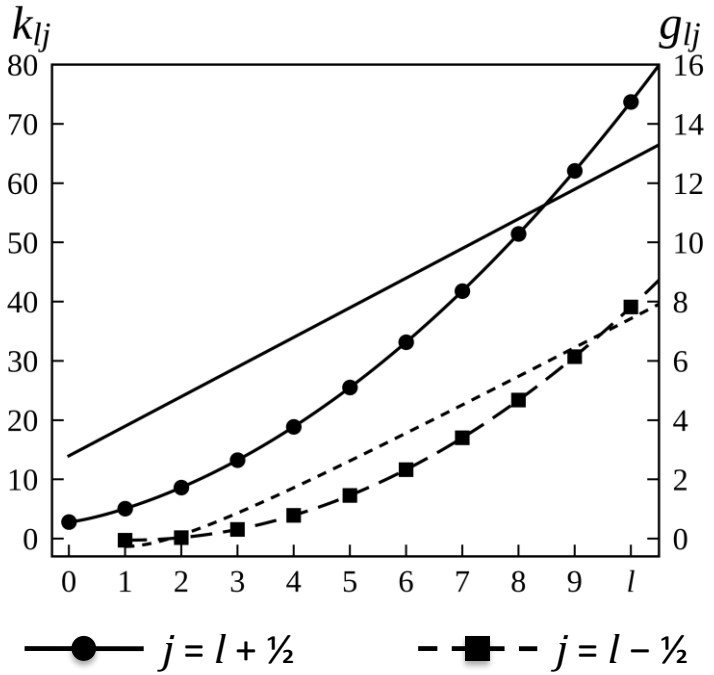
10^{-1}

Magnetic susceptibility

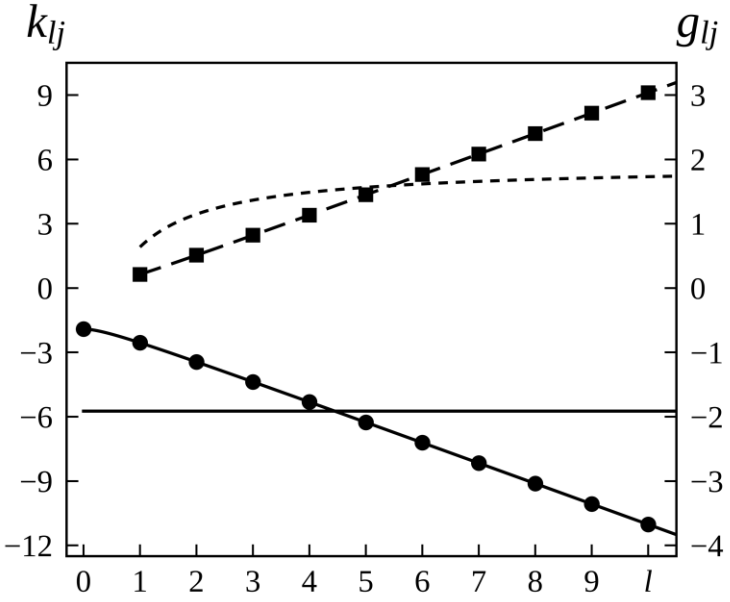
$$\Delta B_{\rm m} = \kappa\,\omega_{\rm L}, \qquad \kappa = \sum\nolimits_{\alpha=p,n} 4\,\eta_\alpha\,(1-\eta_\alpha)\,\left|k^\alpha_{lj}\right|, \qquad \eta_\alpha = N^\alpha_{lj}/(2j+1)$$

k_{lj}: maximum susceptibility of an lj shell, reached at half filling (η = ½). Closed shells: κ = 0.

(a) protons



(b) neutrons



g-factor g_{lj} (right axis)

$1f_{7/2}$ shell

Half-filled maxima (l = 3):
$|k_p|$ = 13.2, $|k_n|$ = 4.4

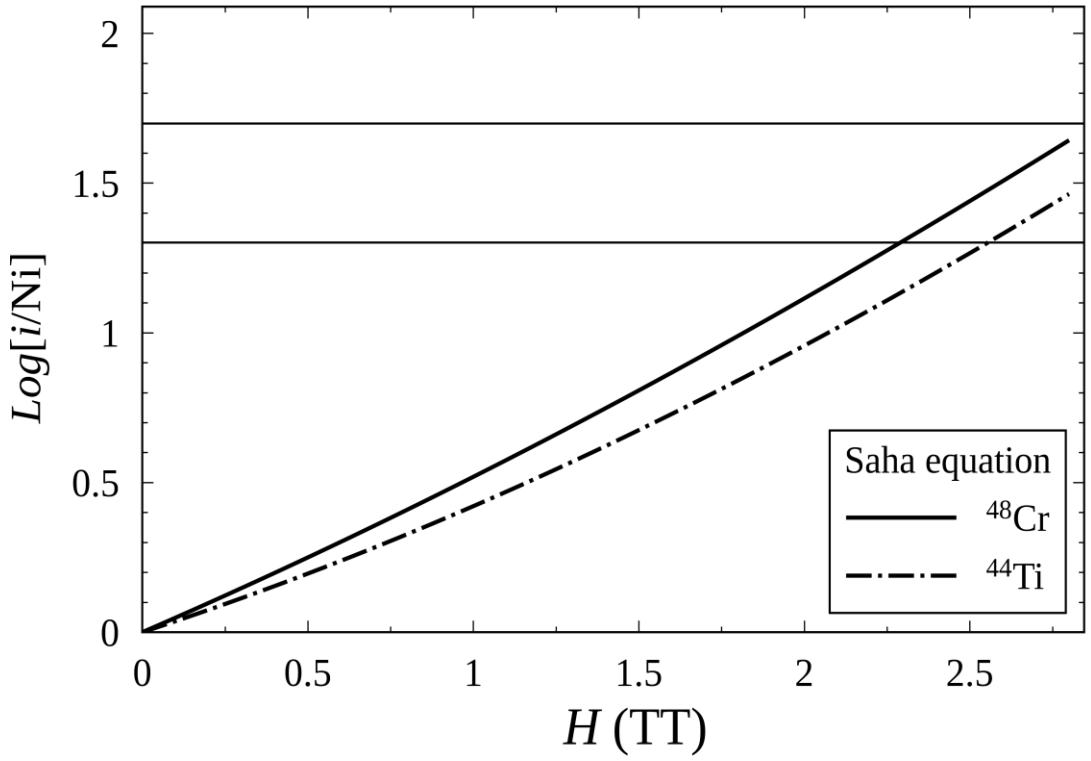
^{48}Cr	η = ½	κ = **17.6**
^{44}Ti	η = ¼	κ = **13.2**
^{56}Ni	η = 1	κ = **0**

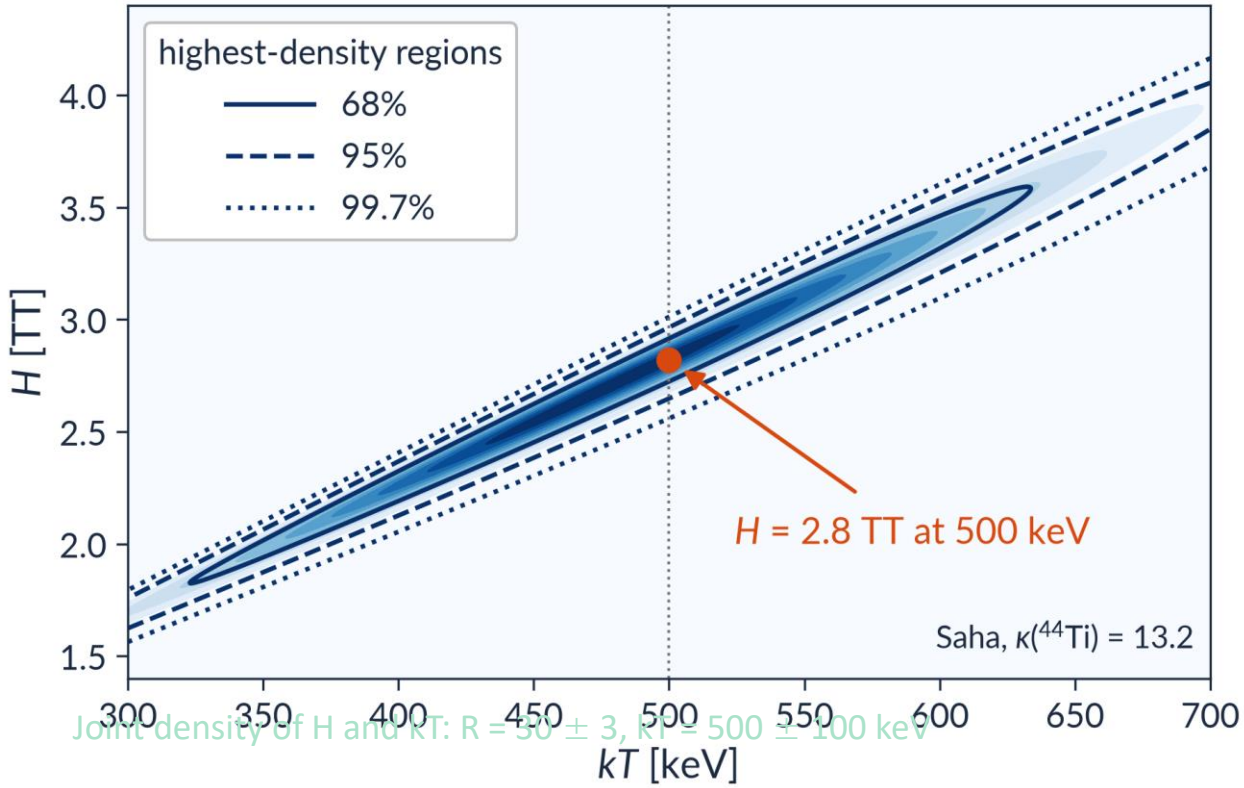
κ in units of μ_N

Extra binding $\kappa\omega_L$ for ^{44}Ti and ^{48}Cr, none for ^{56}Ni

V. N. Kondratyev, PLB 782 (2018) 167; MNRAS 480 (2018) 5380; Universe 7 (2021) 487

^{44}Ti and /^{56}Ni ratio and distribution of field H





H ≈ 2.8 TT at kT = 0.5 MeV; 68% range 2.3–3.4 TT for kT = 500 ± 100 keV

r–process. *(n, γ) ←→ (γ, n)* при ЯСР -- A + n ←→ (A+1) + γ

Saha equality.

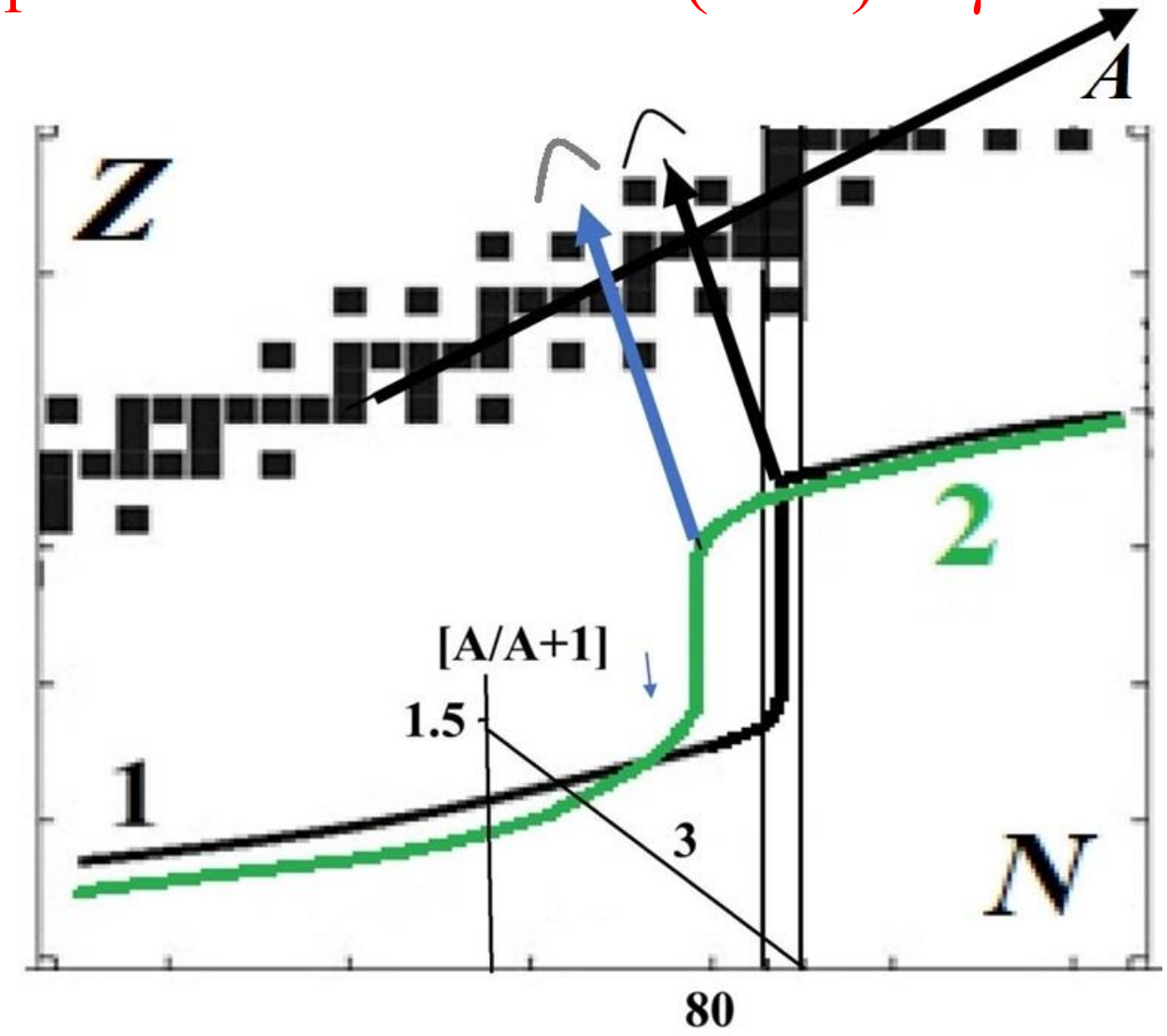
$$N_{A+1} = N_A N_n \frac{g_{A+1}}{g_A g_n}\left(\frac{2\pi\hbar^2}{\mu kT}\right)^{3/2} \exp\{B/kT\}$$

Magnetic change in nuclear volume

$$\frac{y_{A+1}}{y_A} \approx \frac{\exp\left(g_n^{lj}\, m\frac{\omega_L}{kT}\right)}{\cosh(g_n\ \omega_L/2\,kT)}$$

R-process path $\delta_n(H) \approx \delta_n(0) -$

$$- g_n^{lj}\, m\,\omega_L/j + kT\ ln\left(cosh\left(\frac{g_n\,\omega_L}{2kT}\right)\right)$$



SUMMARY

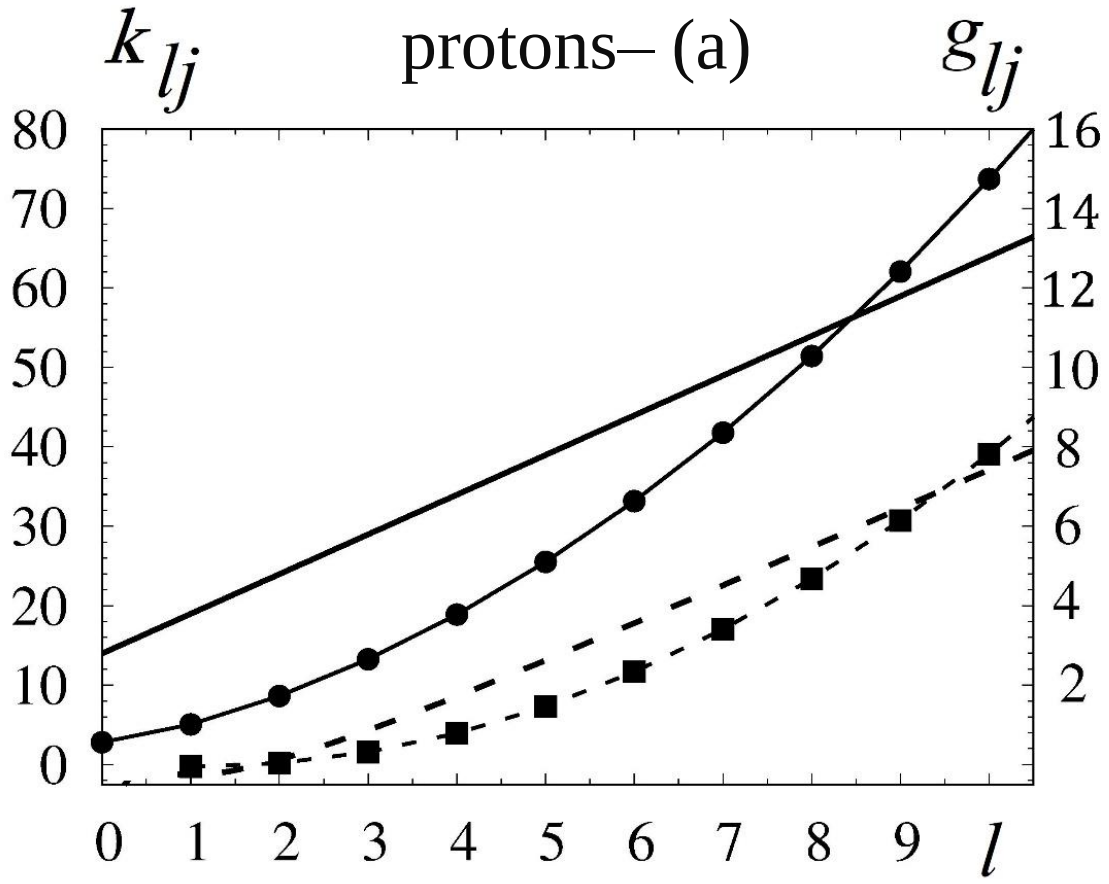
- Magnetism of Atomic Nuclei
  Thermodynamic formalism
- Magnetic fieldы less 10 Tera-Tesla --- linear Magnetic Response due to Zeeman splitting
- Pauli type Paramagnetic Response --- Neutrons
- Landau-type Orbital Magnetism --- Protons
- Magnetic fields of Tera-Tesla shift Nuclear Magics of Iron region towards Smaller Masses → new seed Ti-44, Ti-48
- Enhancement in Yield Smaller Masses of R-process nuclei

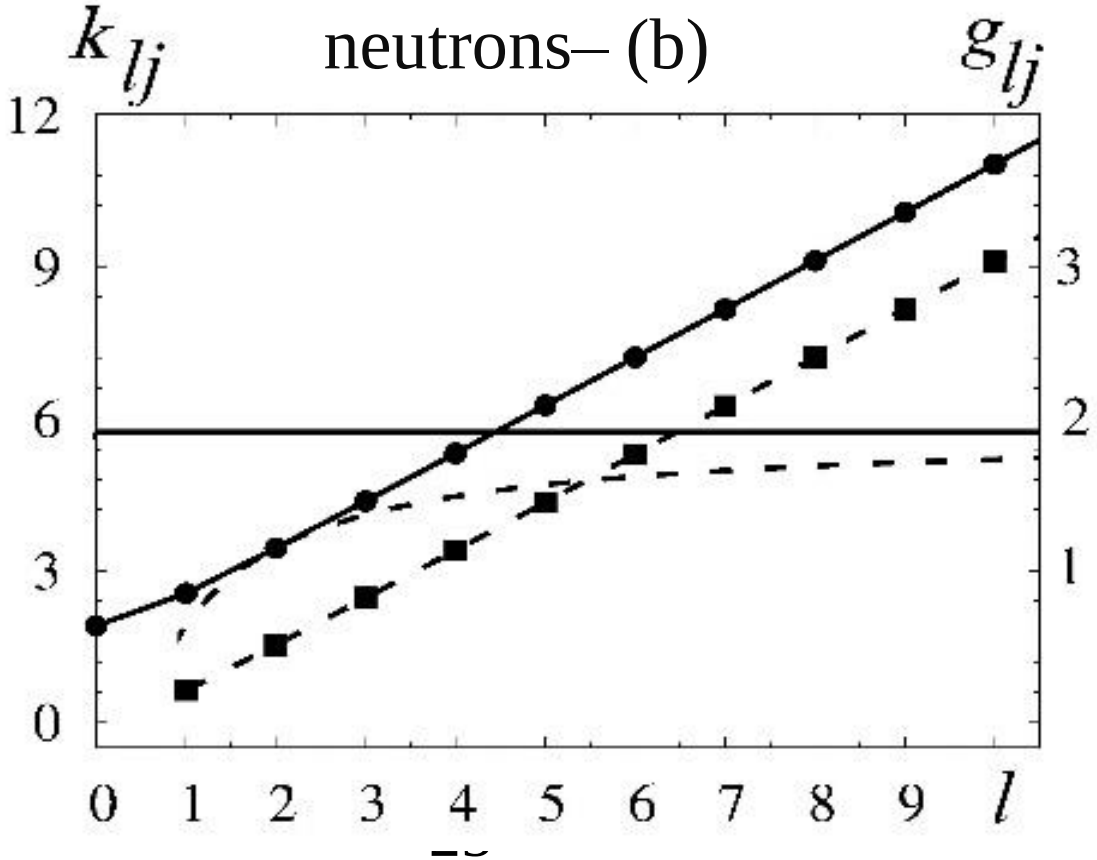
SUMMARY

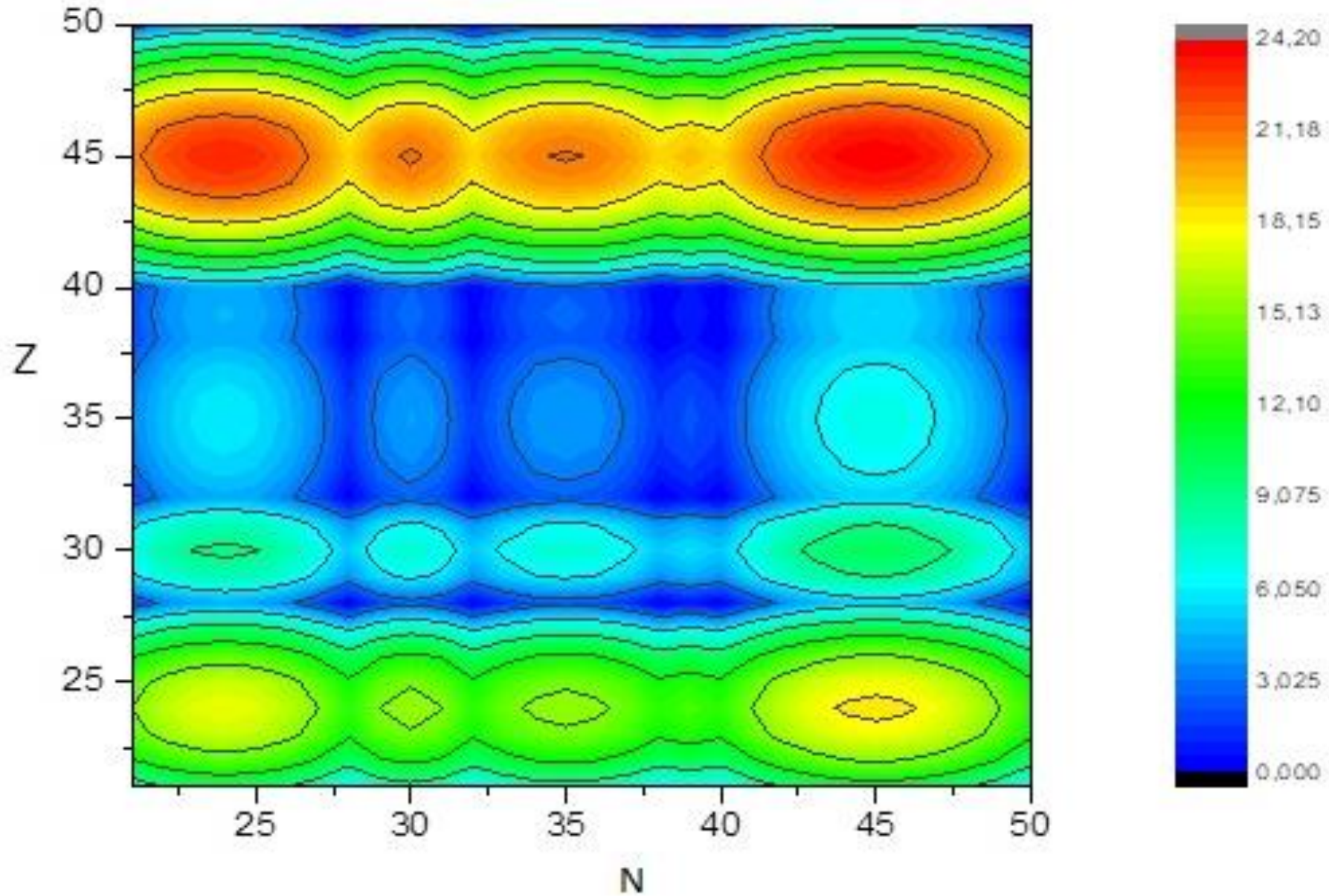
- We analyze the synthesis and decay of ^{n} nuclides in SNRs
- Obtained the spectra and flux for the specific lines of decay products

Absolute value magnetic susceptibility maximum k_{ij} *half filled shells*

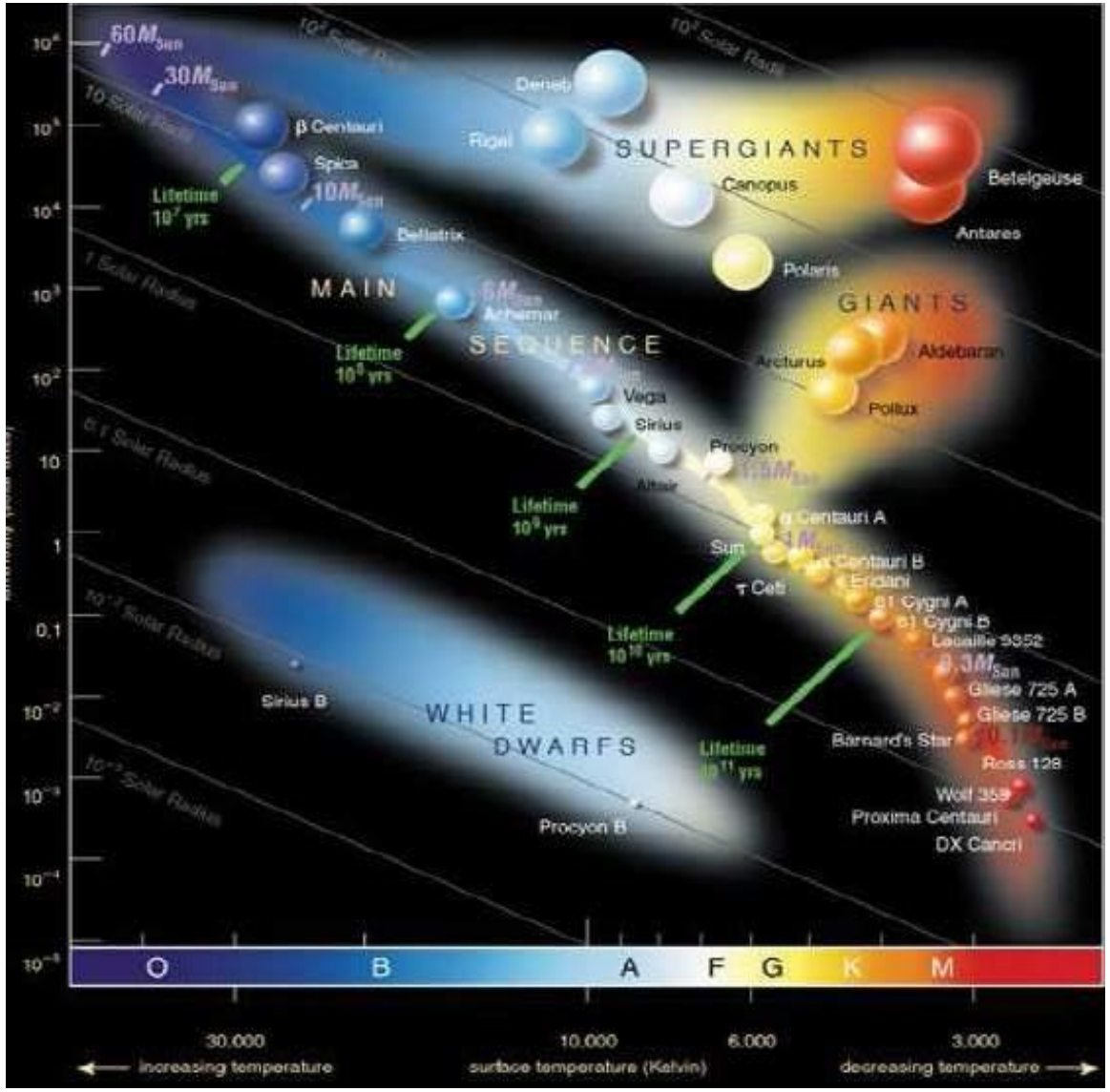
g_{ij} -- g-factor: —●— $j = l + 1/2$ – –■– – – $j = l - ½$






50
45
40
35
30
25
Z
25
30
35
40
45
50
N
24,20
21,18
18,15
15,13
12,10
9,075
6,050
3,025
0,000

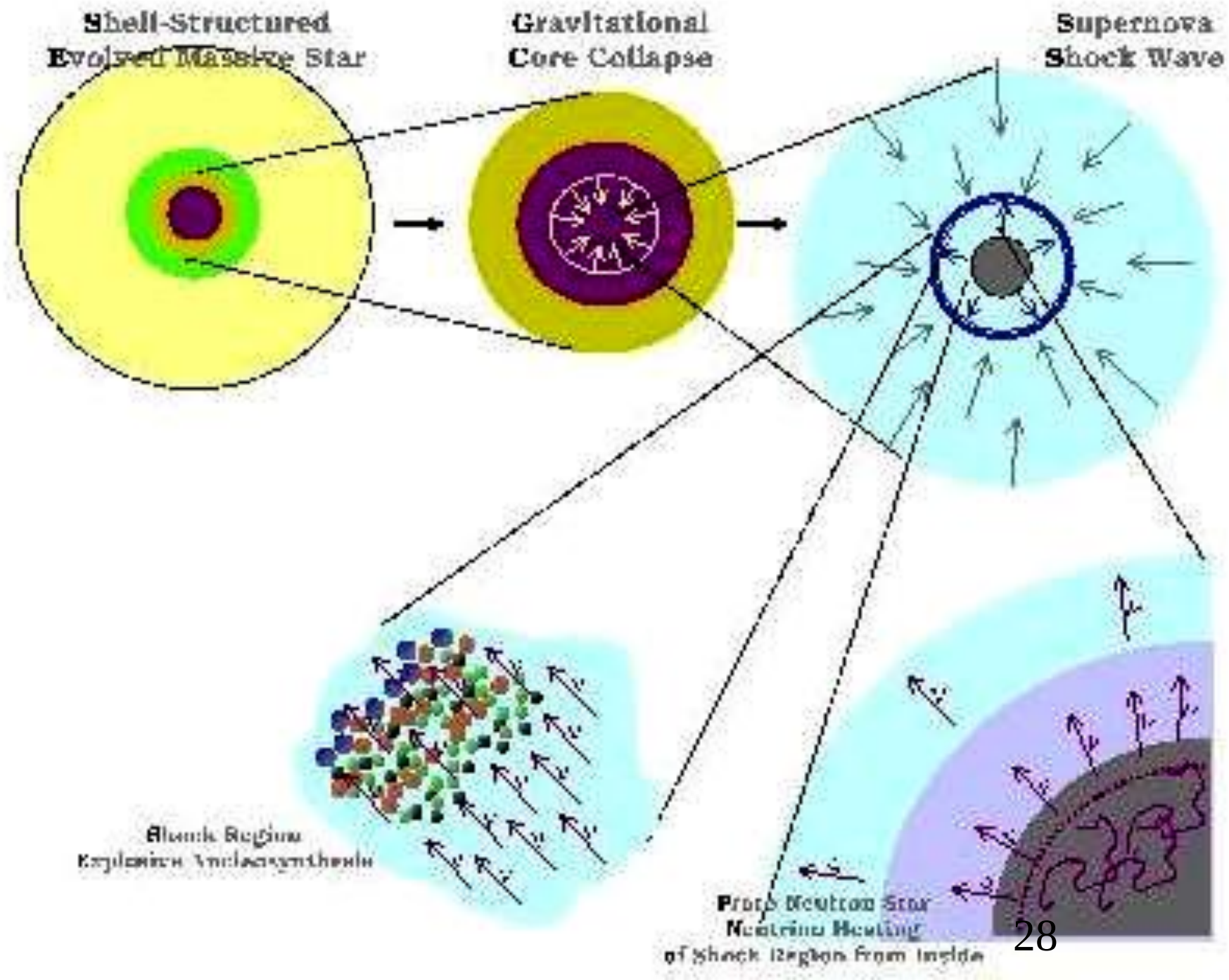
Hertzsprung- Russell (H-R) diagram



Stefan-Boltzmann Law for flux

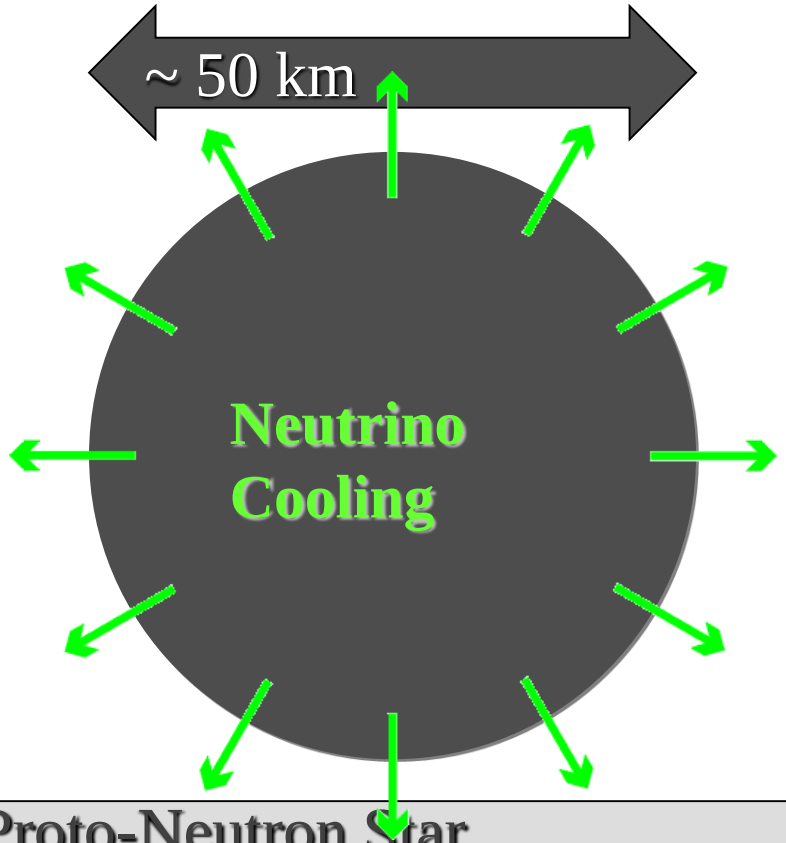
luminosity L of a star with *radius R* & *surface temperature T*

$L \sim (\text{Surface}) T^4 \sim R^2 T^4$


Shell-Structured
Evolved Massive Star
Gravitational
Core Collapse
Supernova
Shock Wave
Prompt Neutron Star
Neutrino Heating
of Shock Region from Inside

Stellar Collapse and Supernova Explosion

Newborn Neutron Star



Proto-Neutron Star

$\rho \approx \rho_{nuc} = 3 \times 10^{14}$ g cm^{-3}

$T \approx 10$ MeV

Gravitational binding energy

$E_b \approx 10^{53,5}$ erg $\approx$ 20% $M_{SUN}\, c^2$

This is distributed as

99% Neutrinos

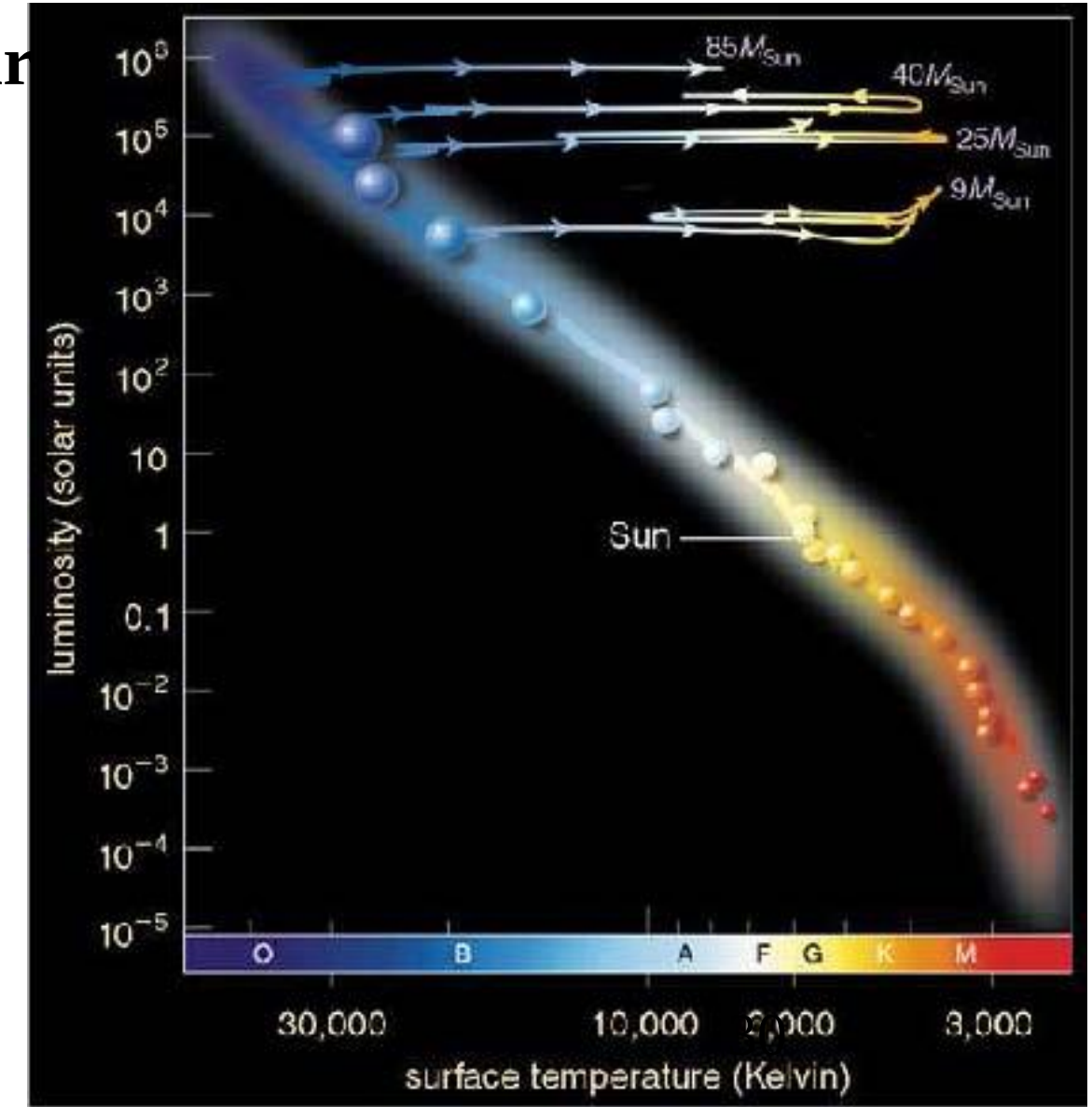
1% Kinetic energy of explosion
(1% of this into cosmic rays)

0.01% Photons, outshine host galaxy

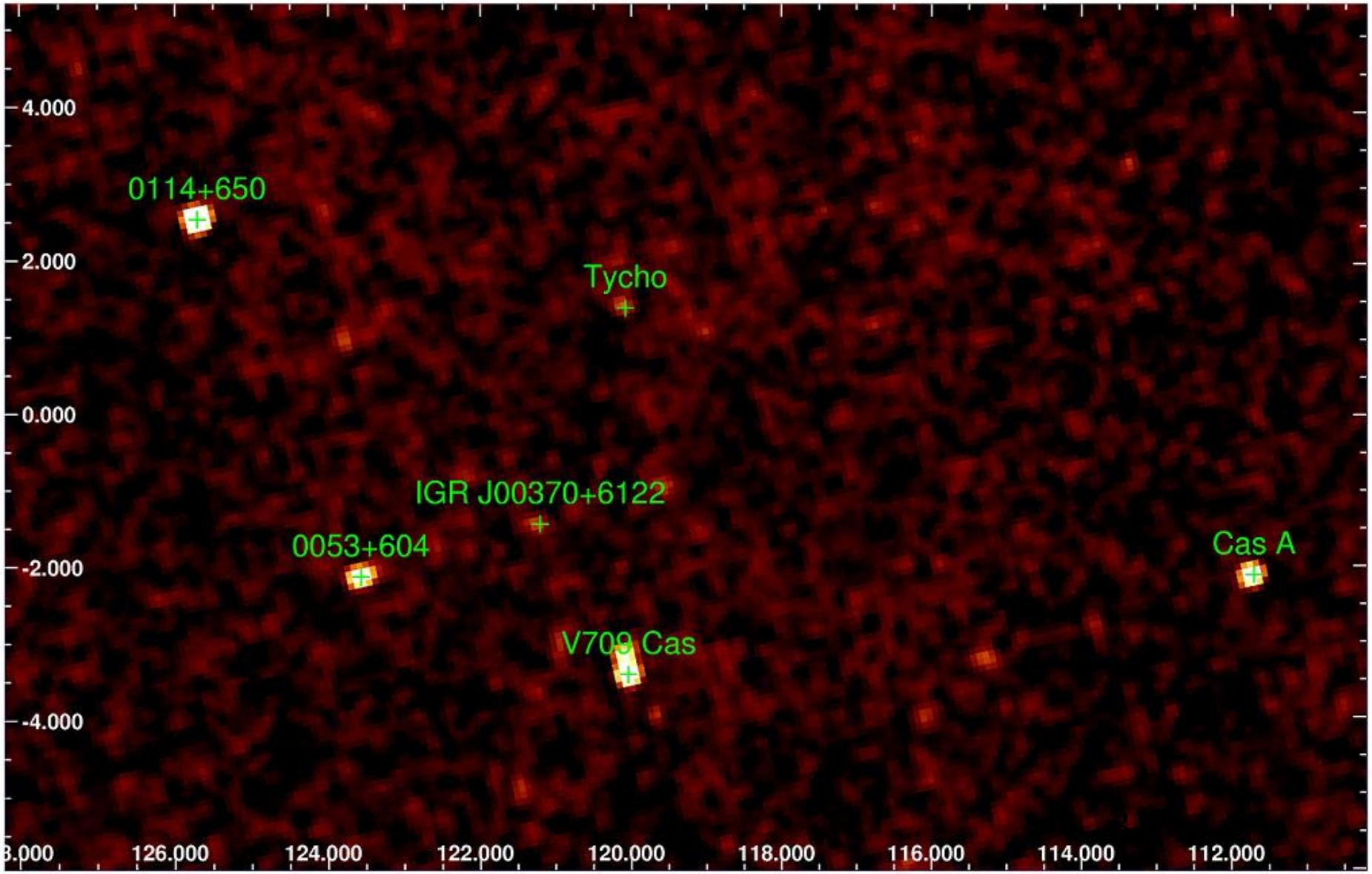
Core-collapse supernova

***high-mass(M>10M$_o$)*star**
***Evolution* on HR diagram**

explosive
nucleosynthesis
origin of
Heavy Nuclides



CAS A$(3.4^{+0.3}_{-0.1})$kpc, TYCHO$(2.2^{+/-0.3})$kpc



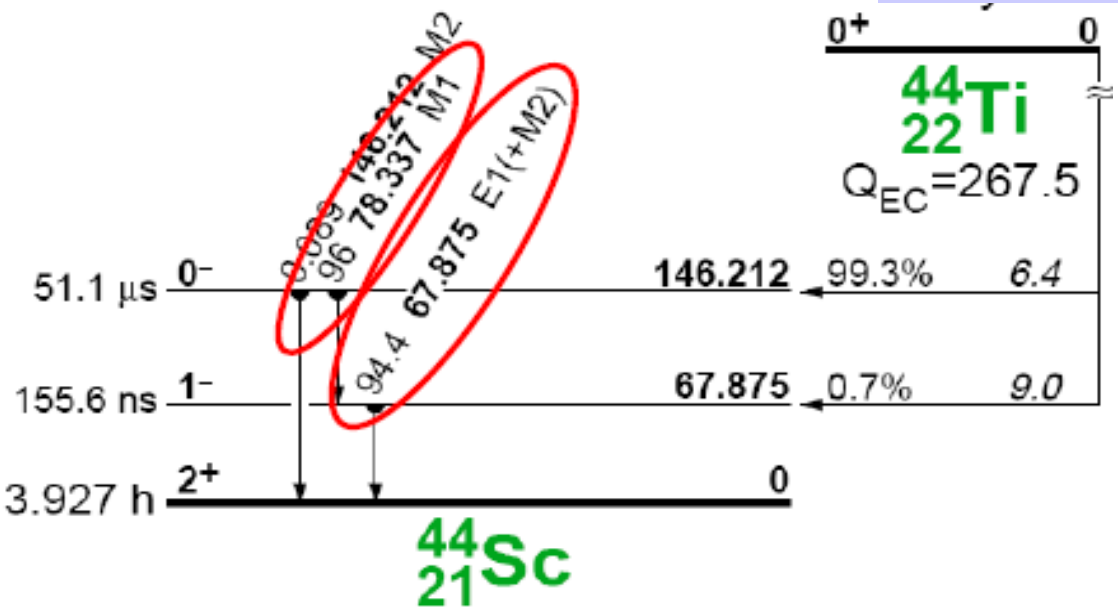
Investigated Supernovas

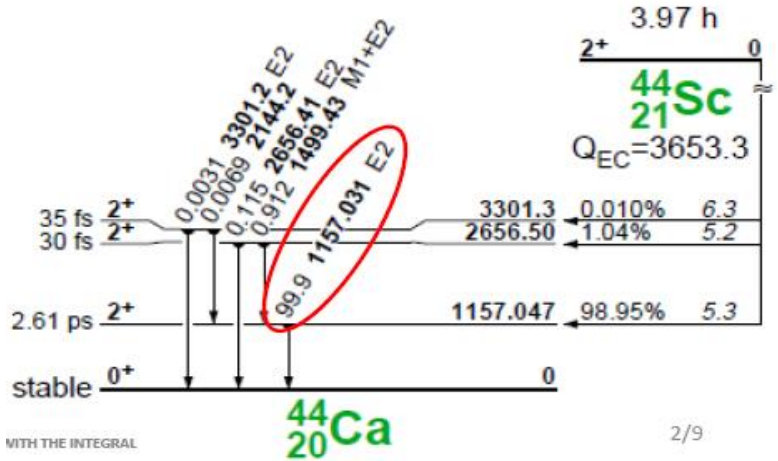
Name	Coordinate	Age
SN 1987A	- 279.7 -31.9,	**35 y**
Cas A	- 111.7 -2.1,	**330 y**
TYCHO	- 120.1 +1.4,	**440 y**
Vela Junior	- 266.3 -1.2	**?**

The scheme of the ^{44}Ti decay

Earth environment

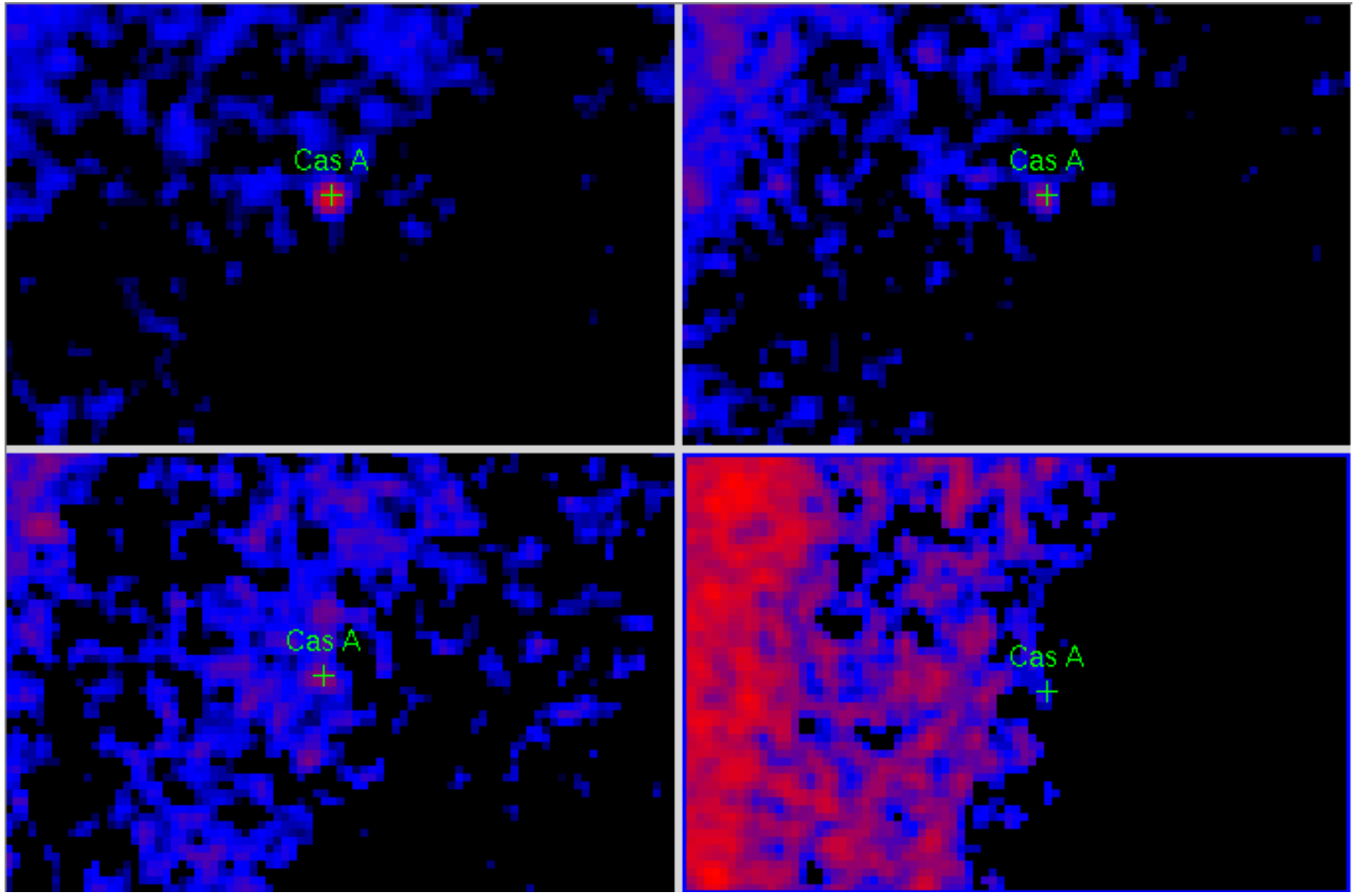
59 y

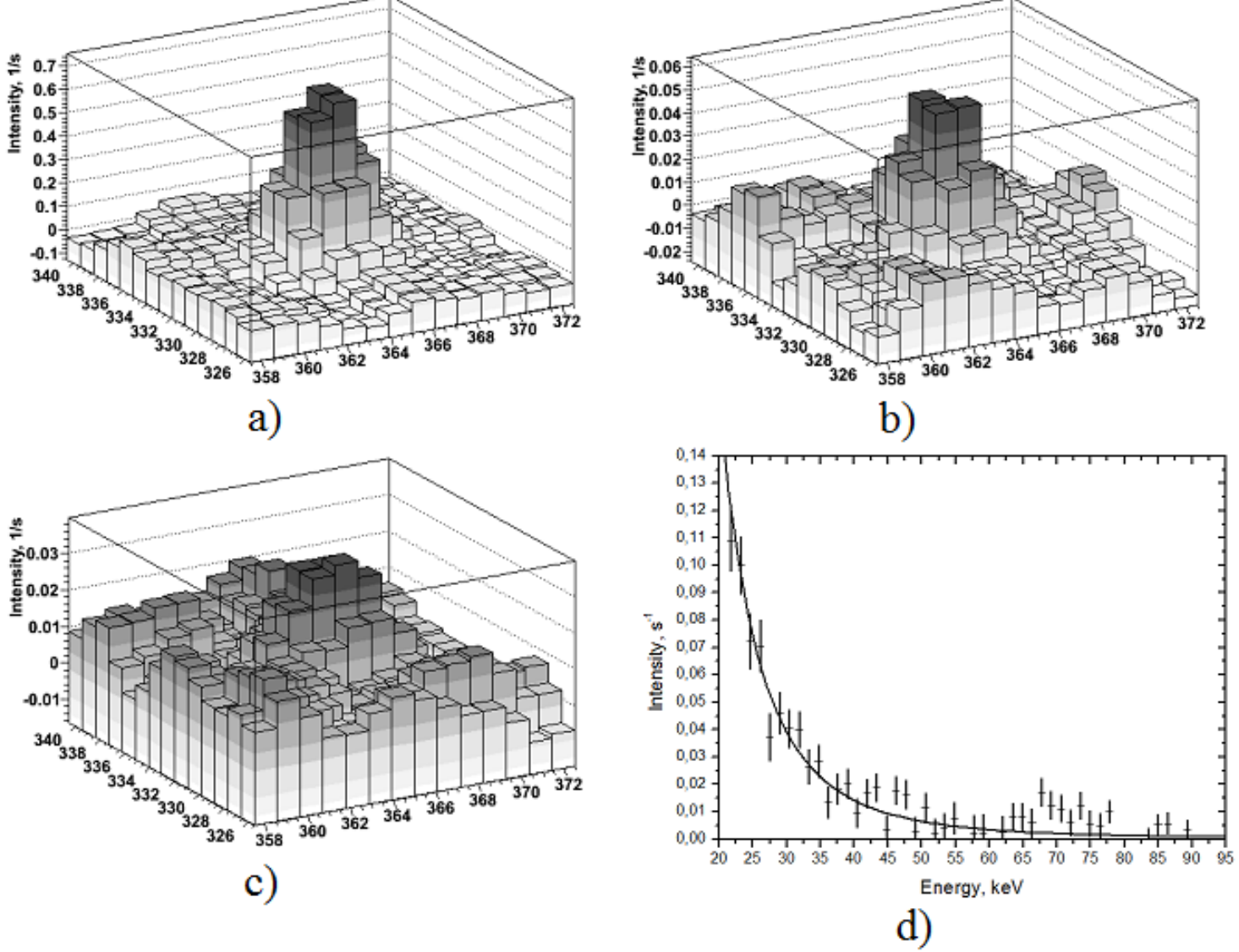




Cassiopeia A (3.4+0.3-0.1)kpc

Energy range (keV): **20-62-72-82-100**



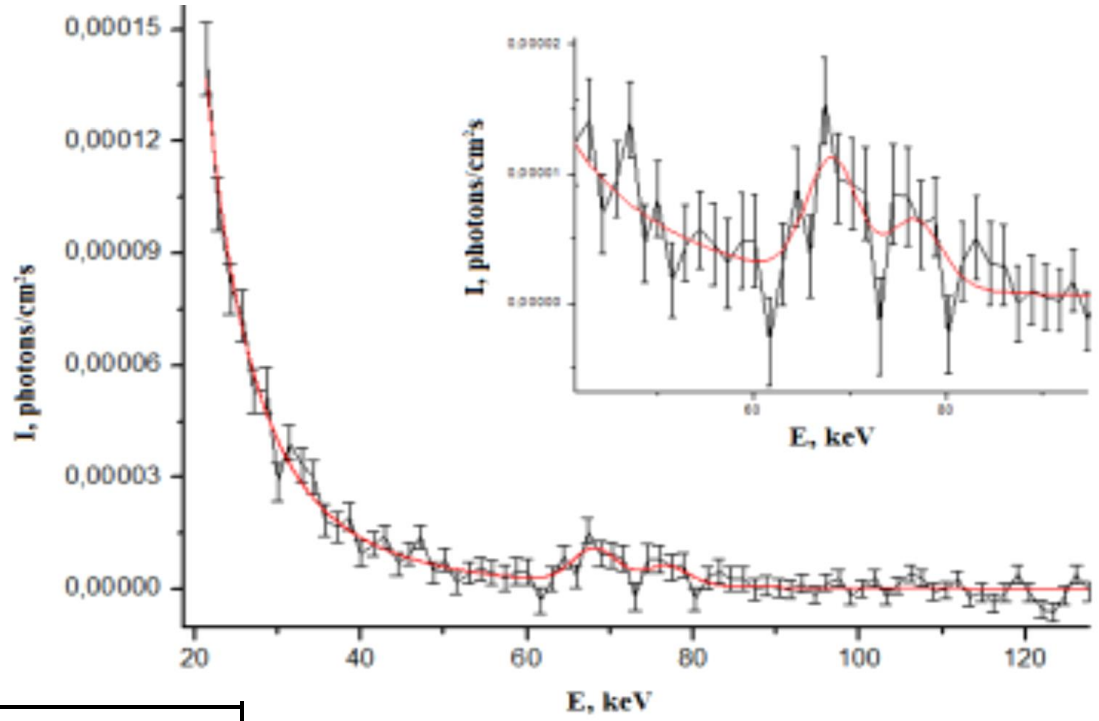


F The direction (i.e.. pixel number) dependence of the registered gamma-ray flux at different energy ranges: **20–62 keV - a, 62–72 keV - b, 72–82 keV – c**; for the angle region containing the Cassiopeia A SN remnant. The right bottom panel (d) represents the spectrum from the Cassiopeia A in the energy range 20–95 keV, the solid line shows the fit with the power law energy *E* dependence, .

Background estimate + Lines

$K \cdot E^a$

a=-3.64 +/- 0.09

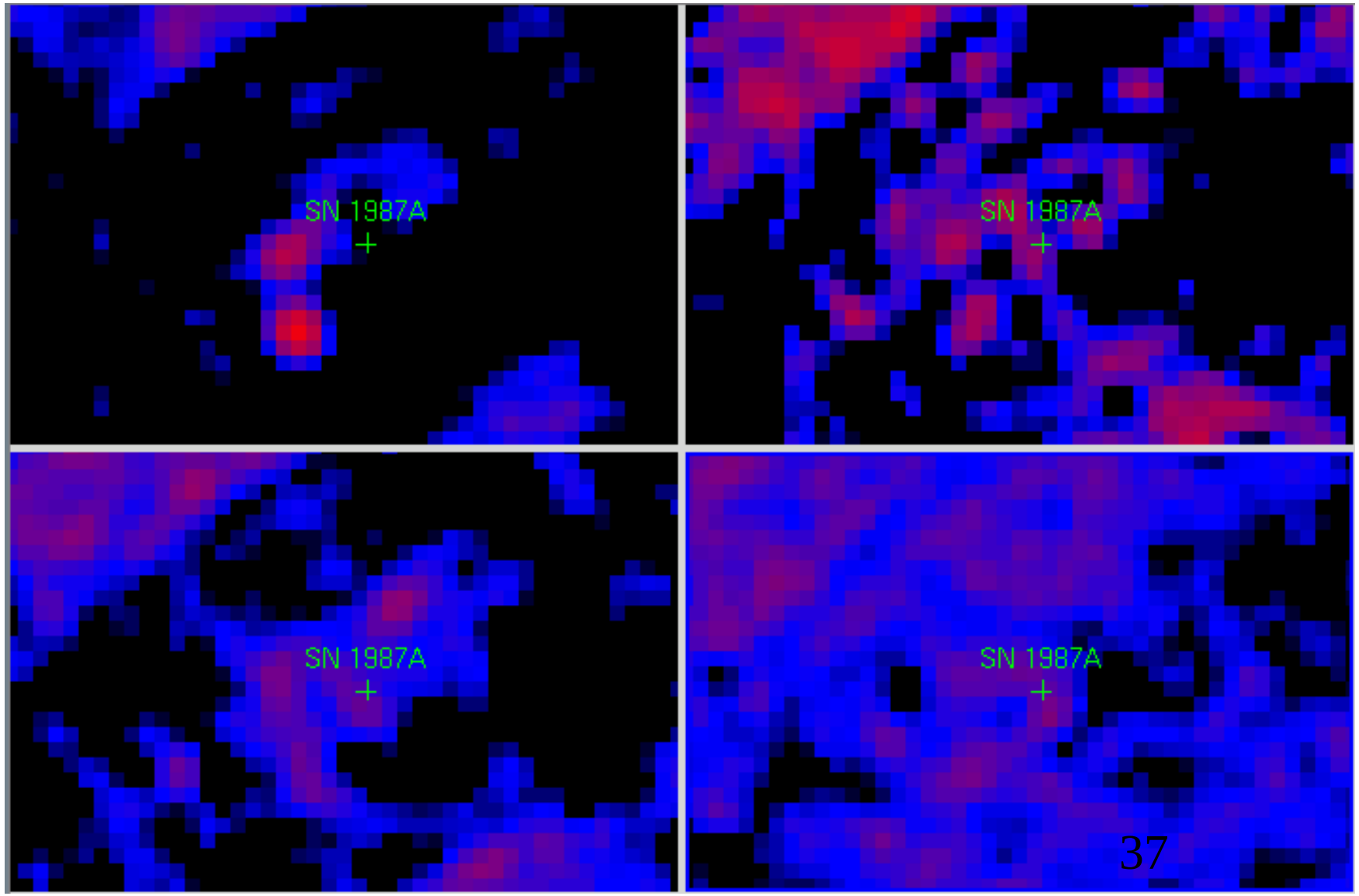


E, keV	$I \pm \Delta I$, photons/(cm^{2}s)
67.9	$(6.0 \pm 1.0) \cdot 10^{-5}$
78.3	$(4.0 \pm 1.0) \cdot 10^{-5}$

T= 1.5Ms

SN1987 A (50kpc)

Energy range (keV): **20-62-72-82-100**



Magnetic field estimates

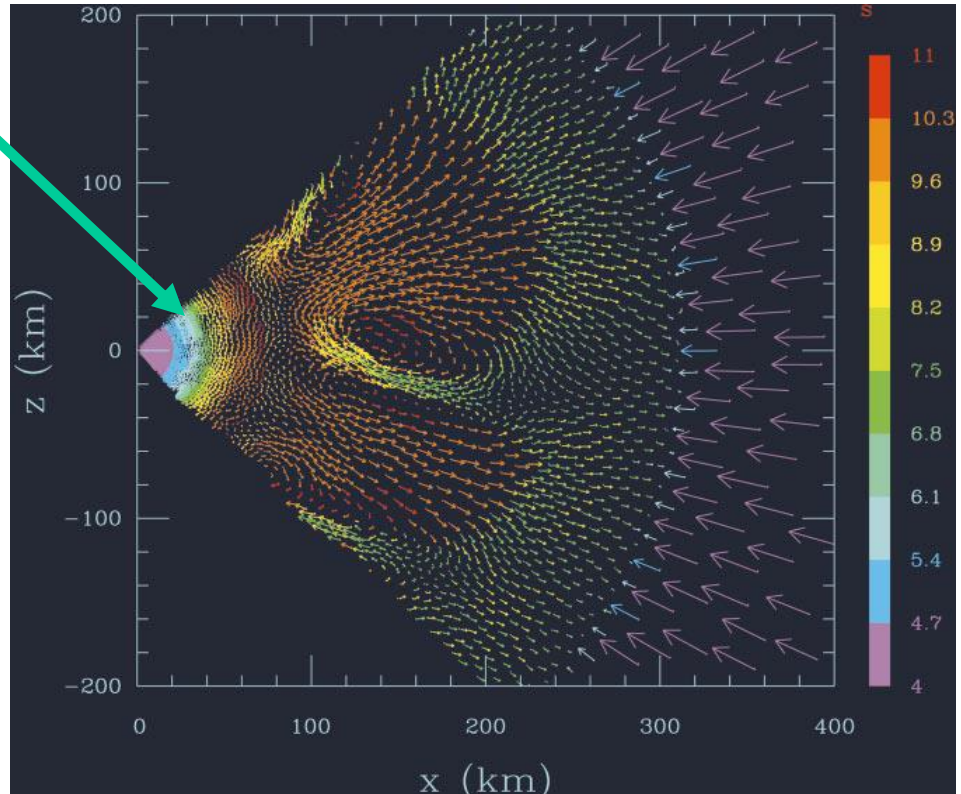
predominant energy component of shock wave E_S originates from the magnetic pressure

$$\langle B_\nu^2 \rangle R_\nu^2 \Delta R \sim 2 E_S \quad \sim 10^{51,5} \text{ ergs}$$

$R_\nu \sim$ 40Km; $\Delta R \sim$ 1Km

$B_\nu \sim 10^1 - 10^2$ TeraTesla

$$B(R) \sim B_\nu \quad R_\nu / R$$



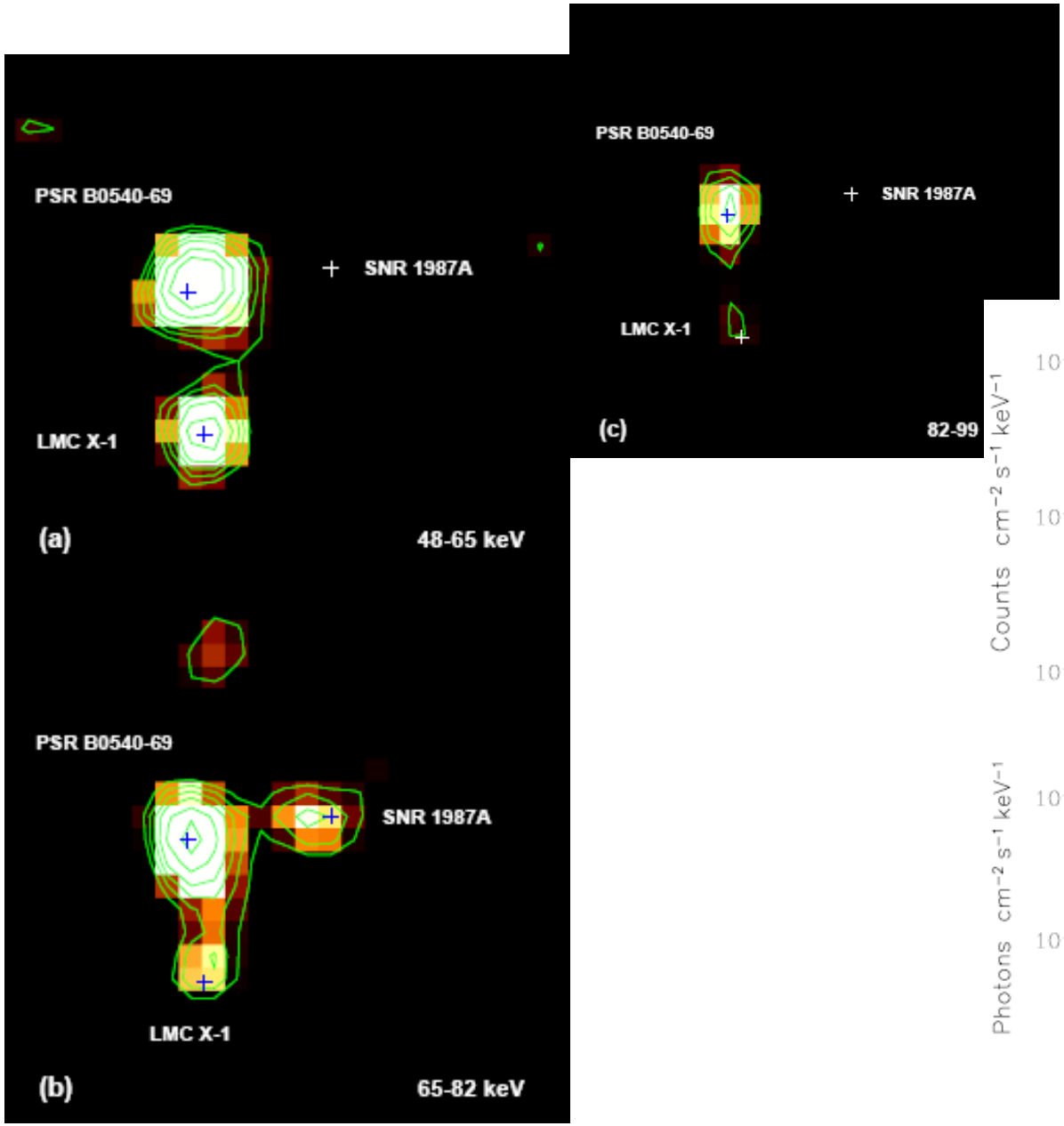
mass of ^{44}Ti synthesized at Cassiopeia A explosion [VNK et al *'Nucleus2004'; PhAN (2009)*]

$$m = \frac{4\pi R^2 \cdot T_{1/2} \cdot M \cdot I}{\ln 2 \cdot N_a \cdot p} \cdot 2^{-\frac{t}{T_{1/2}}}$$

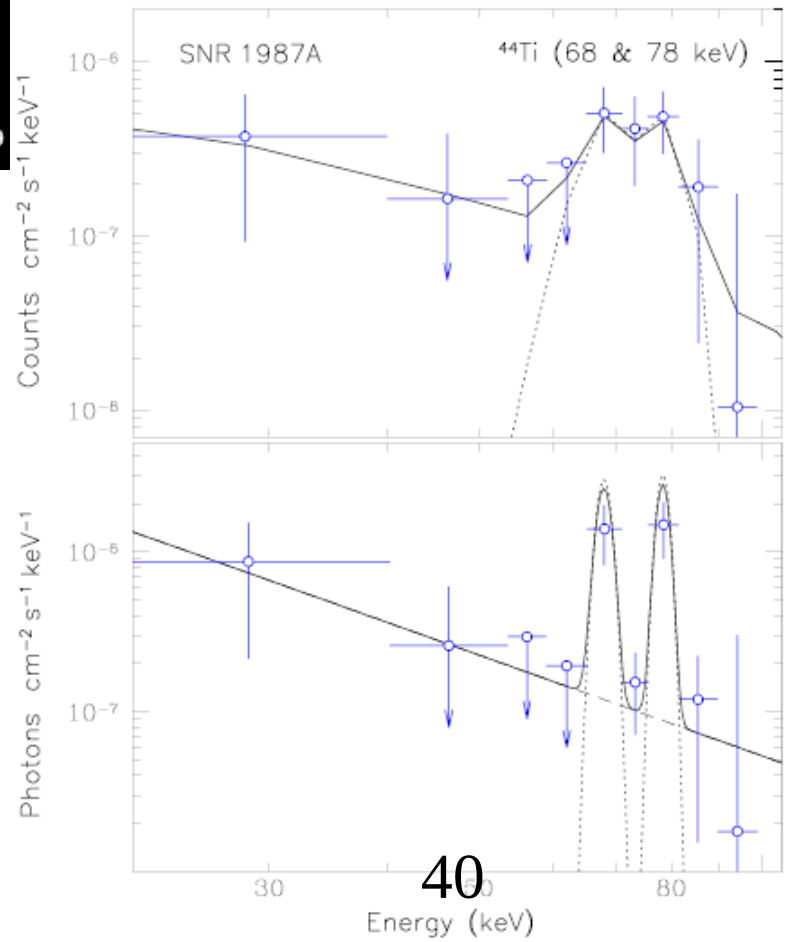
R–distance to the object, $T_{1/2}$–the element half-life, N_a–theAvogadro constant,M–mola rmass,I–γ-quanta flux,p-quantum yield, t–remnant age.

E, keV	I ± ΔI, 10^{-5} photons/cm^{2}s	m ± Δm, 10^{-4} $M_\odot$
67.9 keV	6.0 ± 1.0	4.0 ± 0.7
78.3 keV	4.0 ± 1.0	2.6 ± 0.7
1157.1 keV	5.1 ± 1.1	3.3 ± 0.7

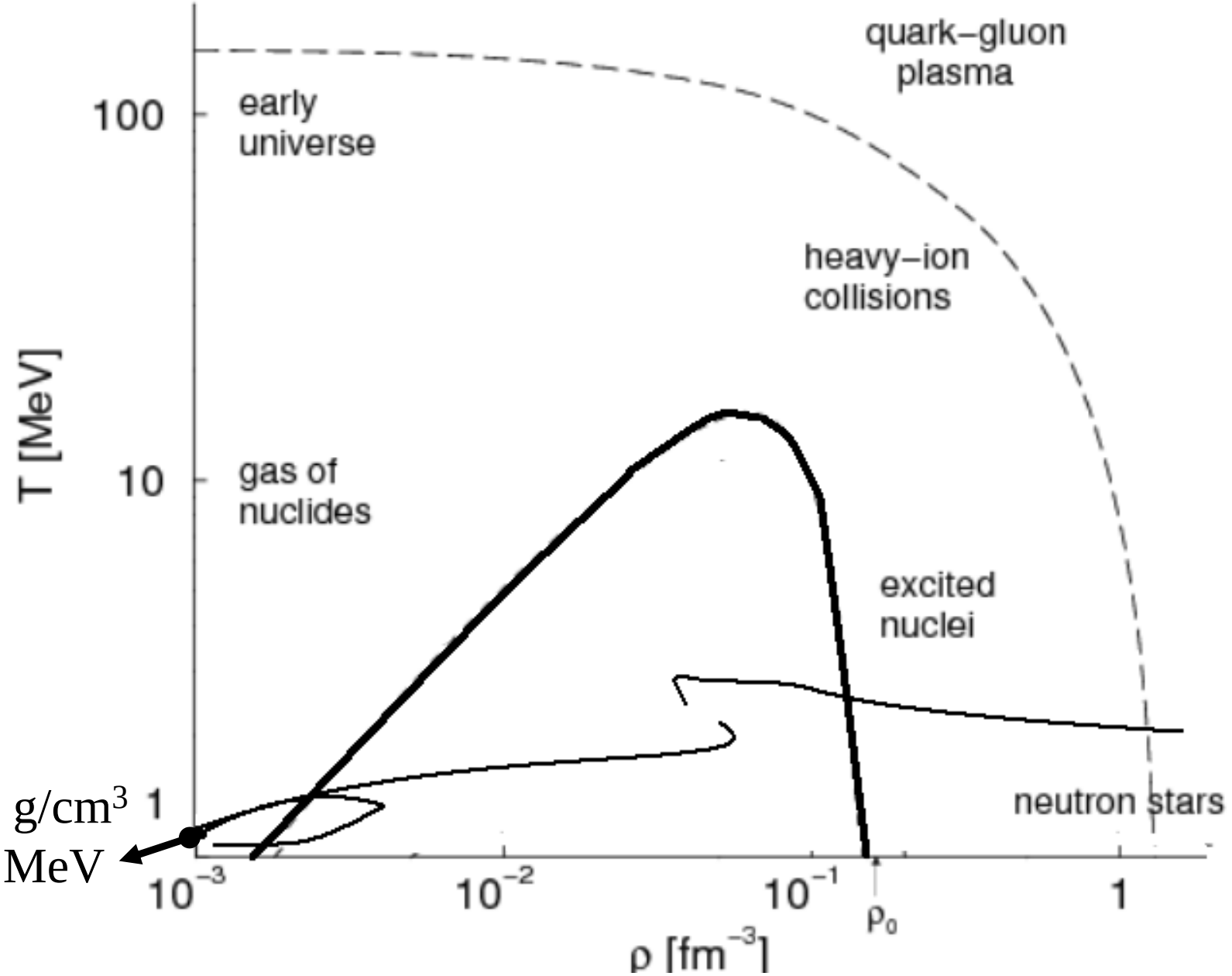
S.A.Grebenev et al *Nature*, *490*, *373-375 (2012)*.



$$(3.1 \pm 0.8) \times 10^{-4} \quad M_{\odot}$$



Symmetric nuclear matter: Phase diagram



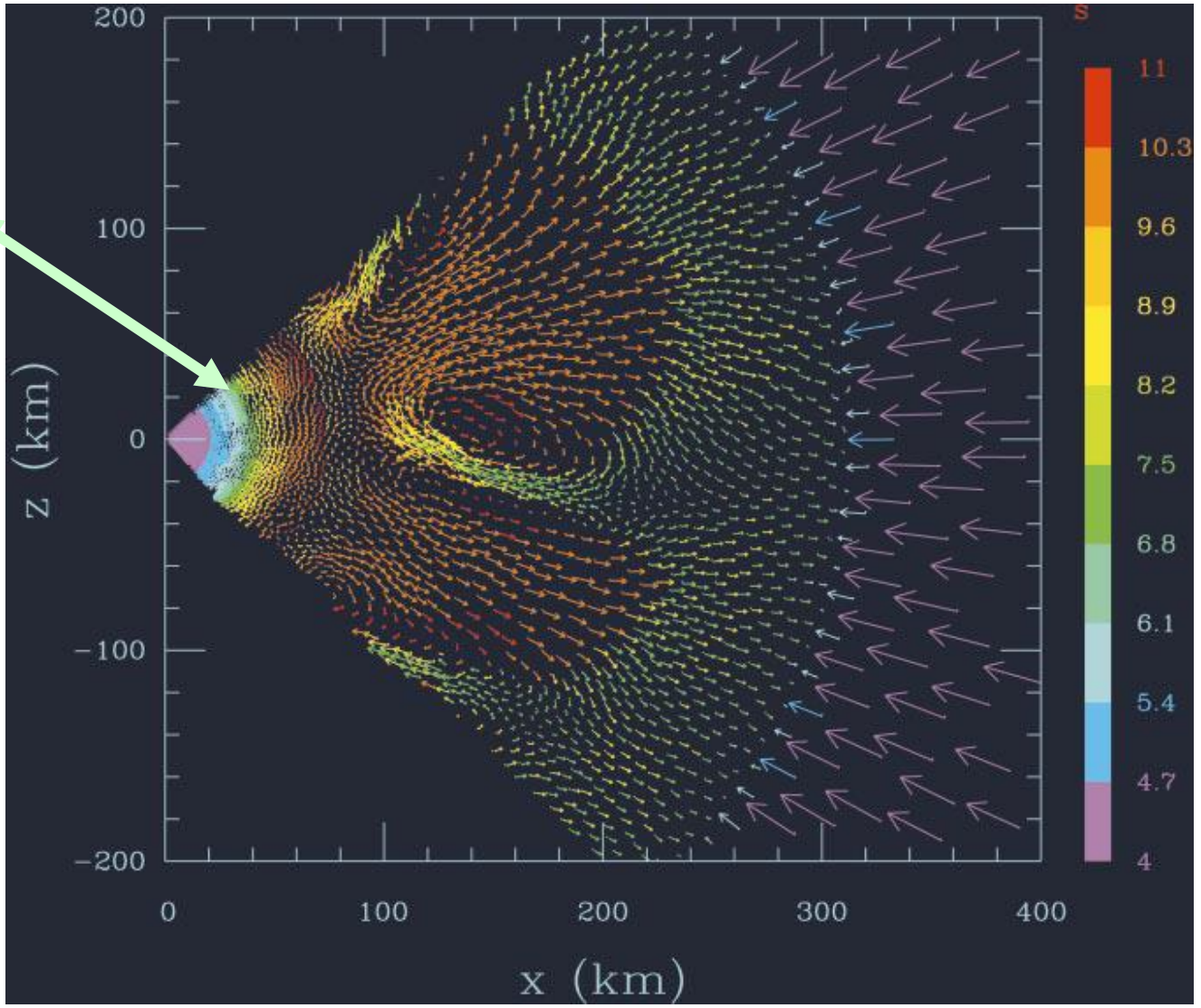
D=10^9 g/cm^3
T=0.1 MeV

explosion proceeds through convection processes

ν-sphere

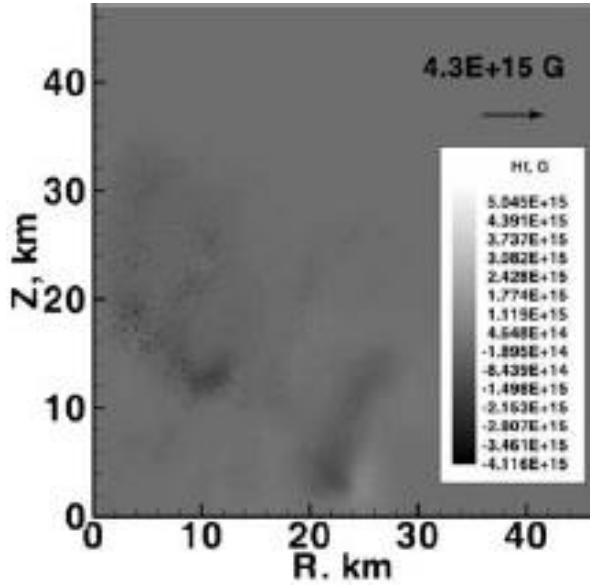
magneto-rotational instabilities & dynamo-action → amplifying

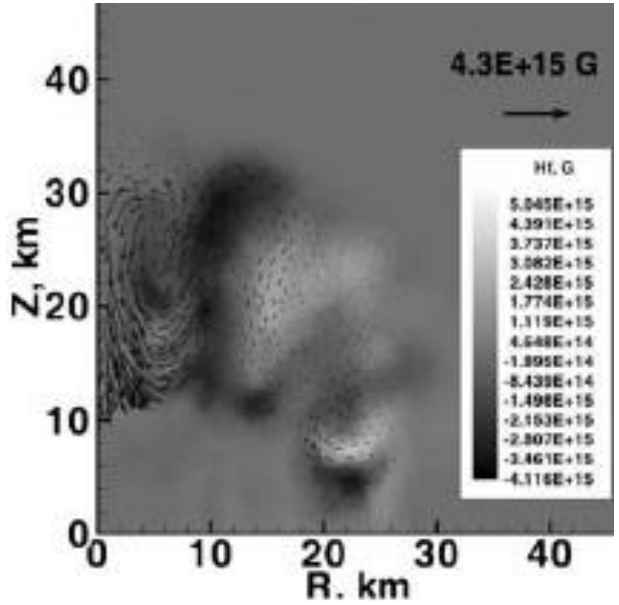
Magnetic fields up to strengths hundred *tera-tesla*

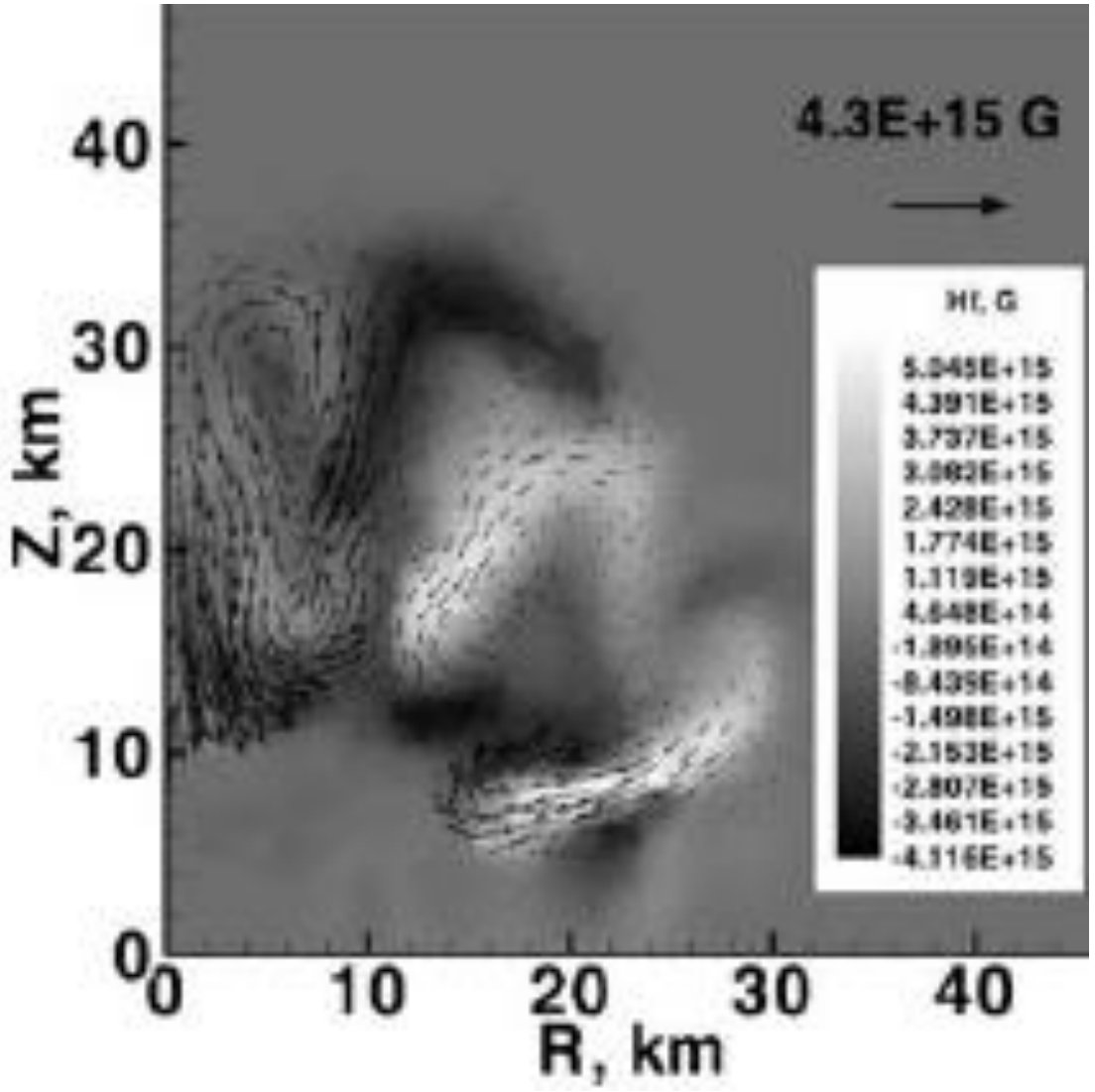


The magnetic field evaluation

(S.G.Moiseenko, G.S.Bisnovatyj-Kogan, N.V.Ardeljan, MNRAS 370 (2006) 501)







Magnetic field estimates

Magnetic and gravitational forces

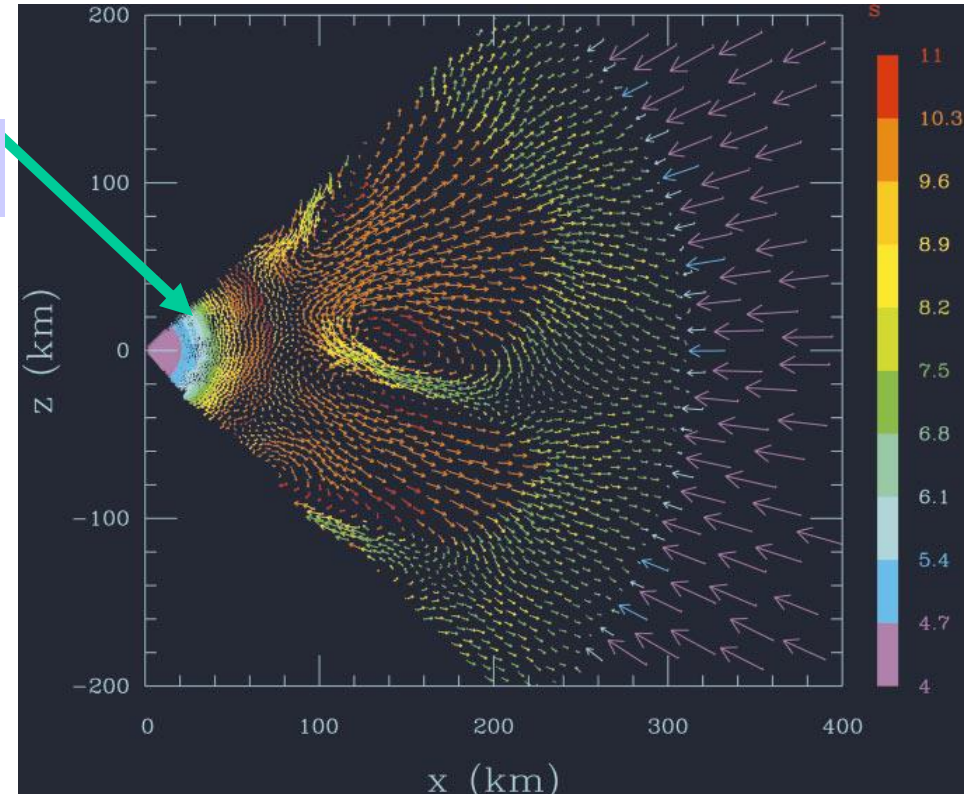
$$dB_{\square}^2/dR \sim 8\pi\, GM\, n(R)/R^2$$

$$4\pi R^2\, n(R) = dM/dR$$

$B \sim 10^{1.5}$ TeraTesla $(M/M_o)(10km/R)^2$

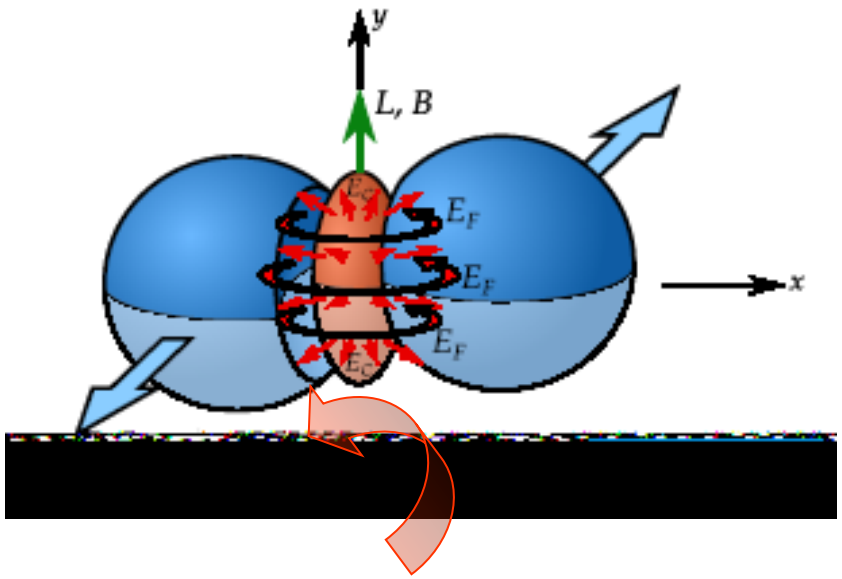
$R_\nu \sim$ 40Km; $\Delta R \sim$ 1Km

$B_\nu \sim 10^1 - 10^2$ TeraTesla

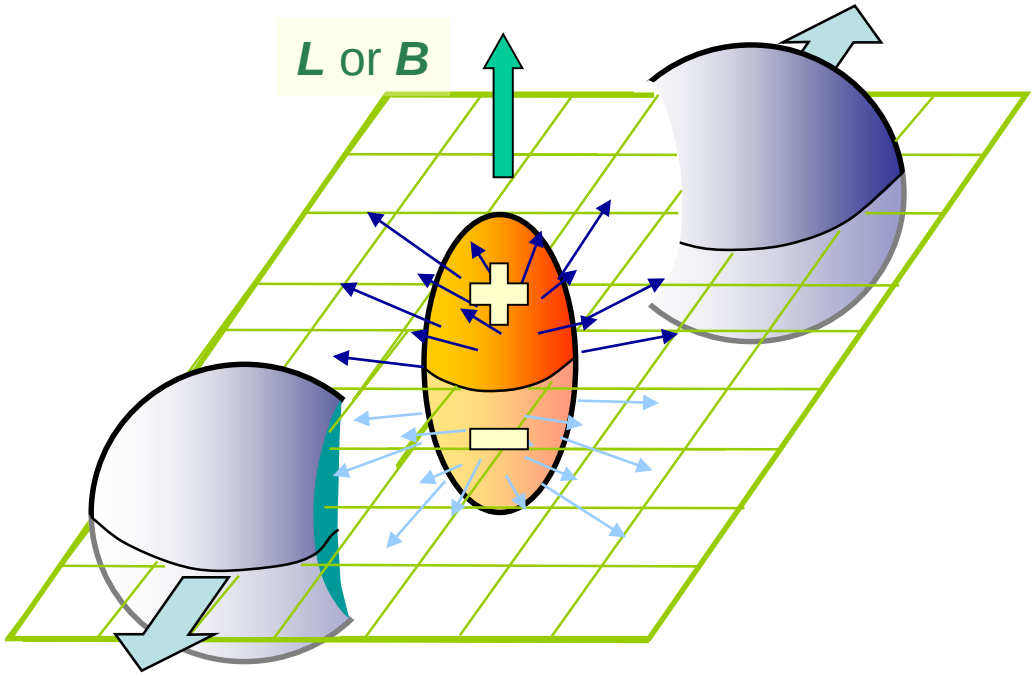


Magnetic fields in HIC

Magnetic field is induced through the axial anomaly



Non-zero angular momentum
(or equivalently magnetic field)
in heavy-ion collisions



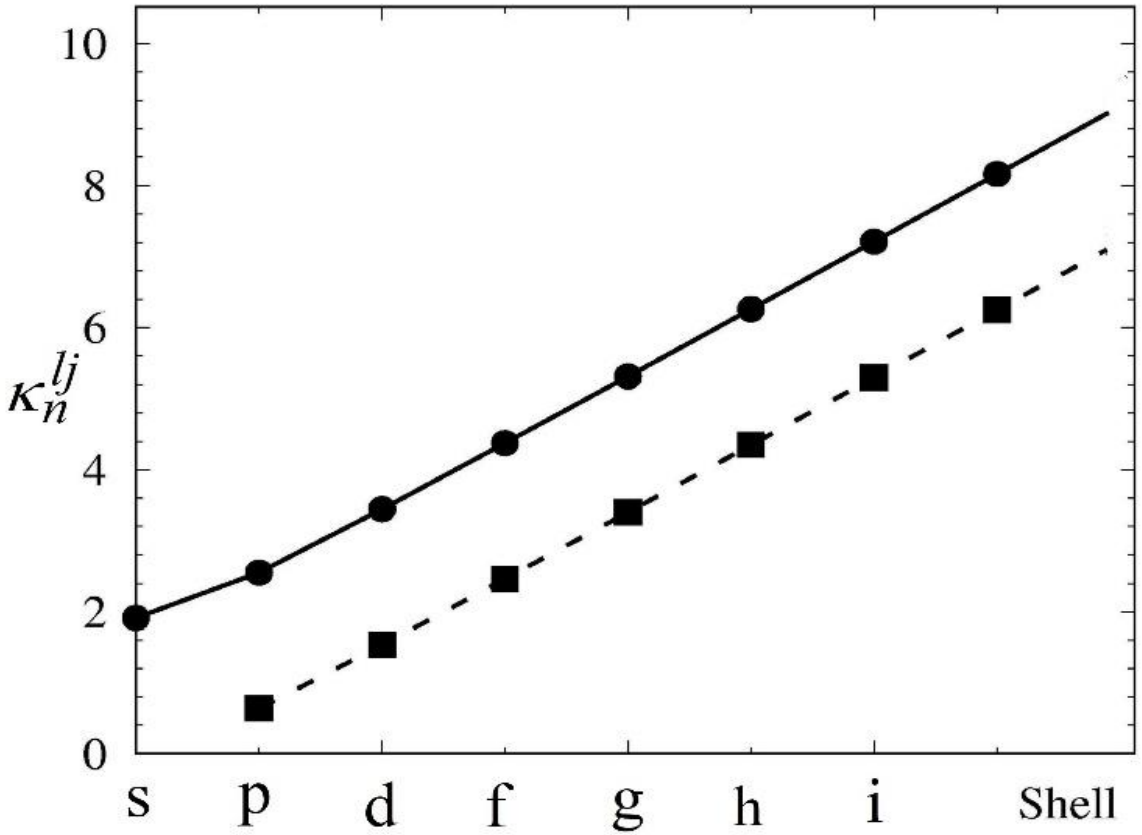
$B \sim 10^{1.5}\,\mathrm{TeraTesla}\,(E/E_o)^{1/2}$

Maximum magnetic susceptibility of a half-filled shells

Solid lines and circles -- j = l + 1/2, dashed lines and squares -- j = l – 1/2

protons – (a)

neutrons – (b)

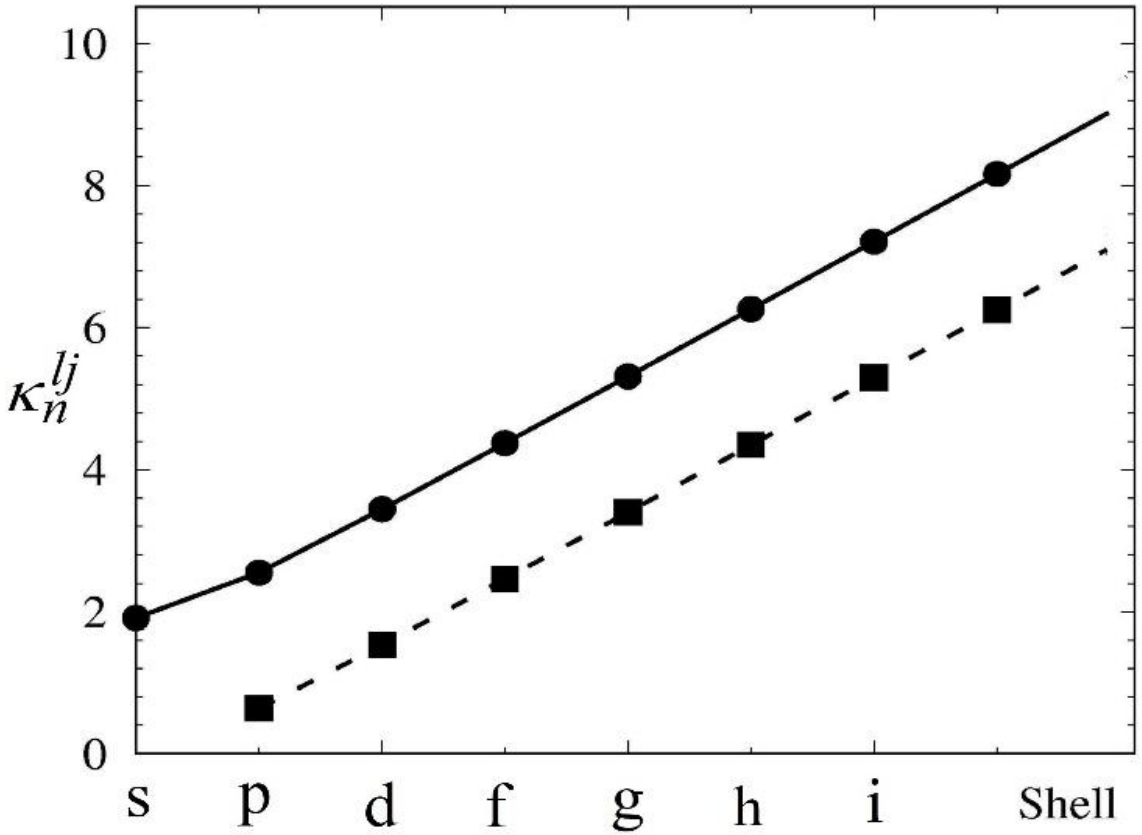


Maximum magnetic susceptibility of a half-filled shells

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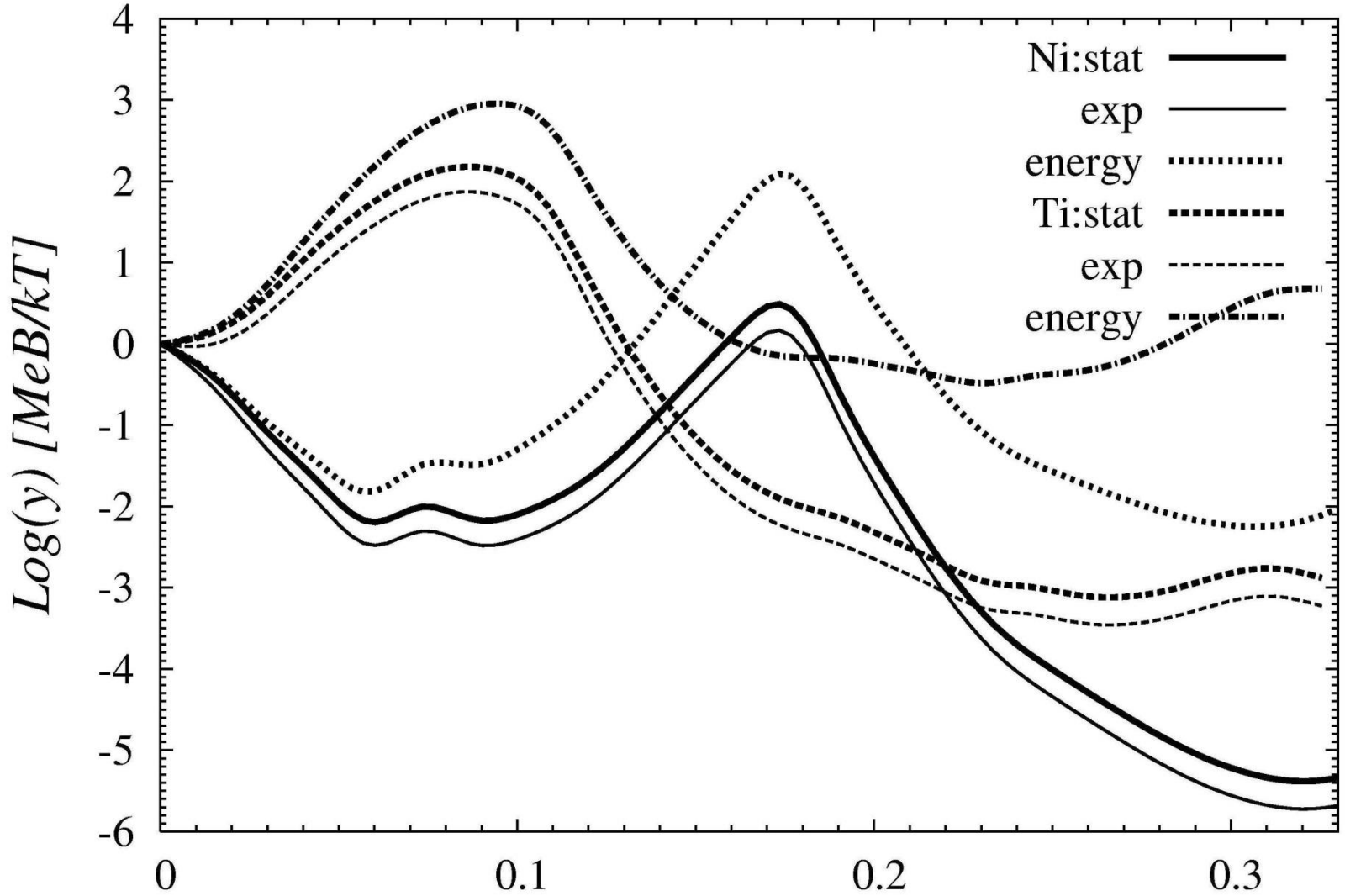
protons – (a)

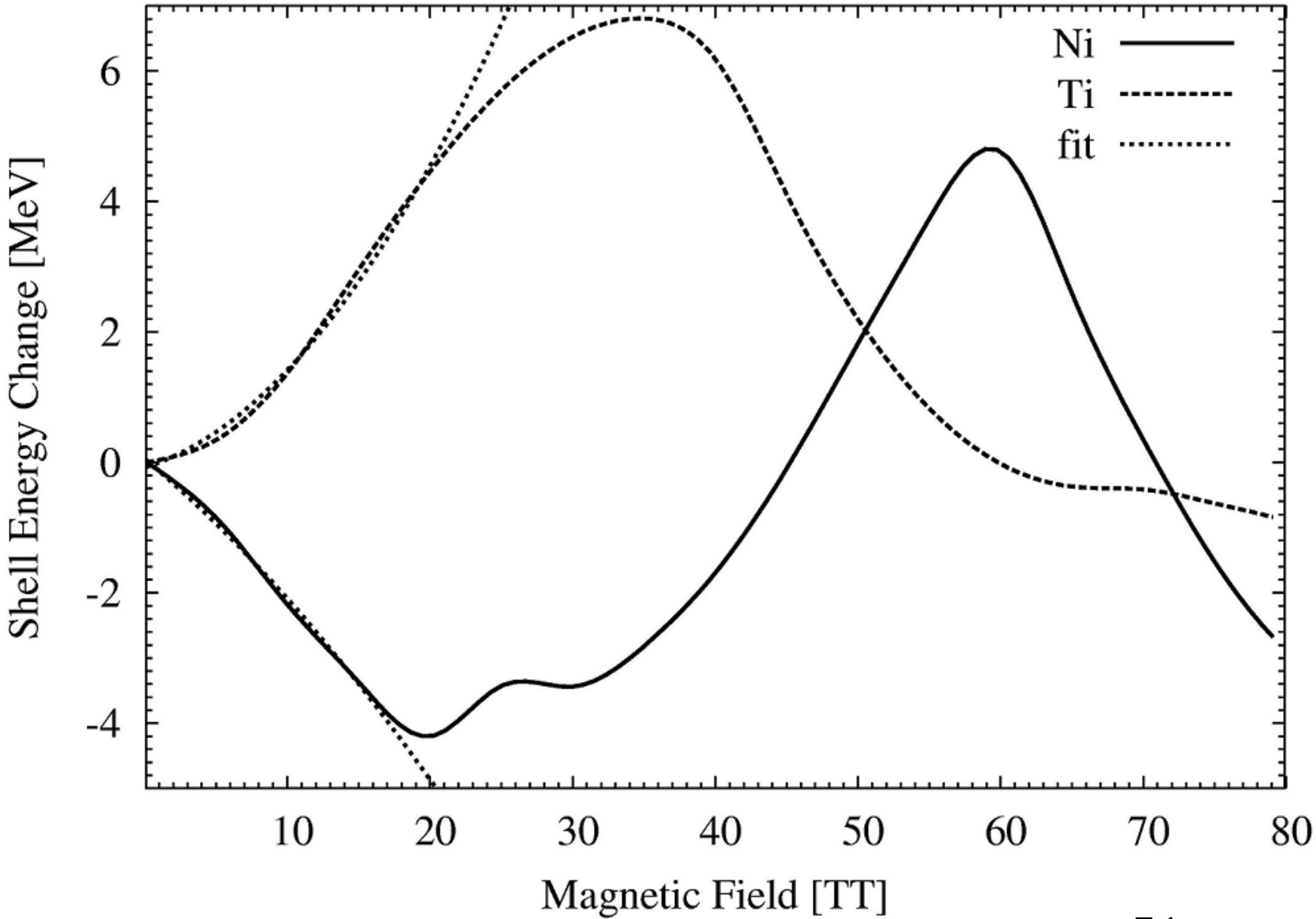
neutrons – (b)



relative yield $y=Y(H)/Y(0)$

^{56}Ni(solid) i ^{44}Ti(dashed line)




Ni
Ti
fit
Shell Energy Change [MeV]
6
4
2
0
-2
-4
10
20
30
40
50
60
70
80
Magnetic Field [TT]

1kpc = 3.08567758 × 10^19 метра