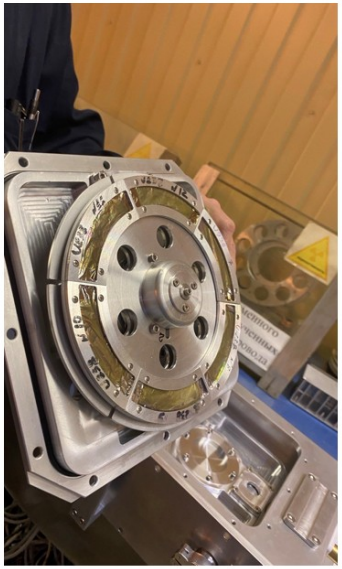
Stochastic Approach to Superheavy-Nuclei Fission within the Dinuclear System Concept

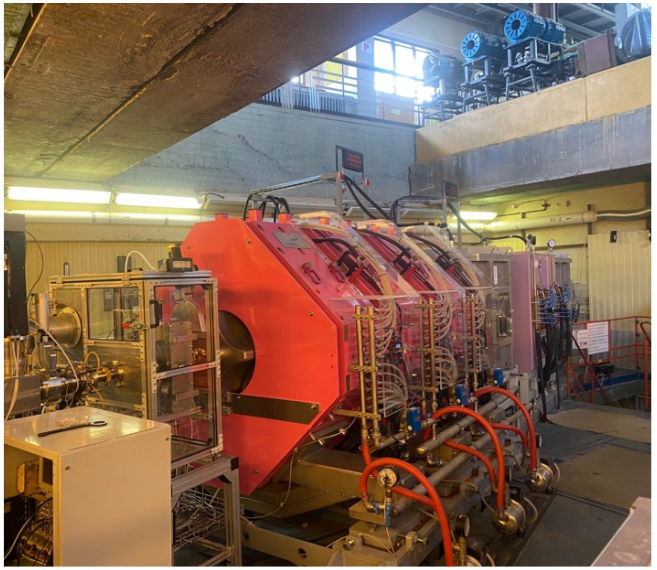
A. V. Andreev, A. Rahmatinejad, T. M. Shneidman, N. V. Antonenko
A. V. Isaev, R. S. Mukhin

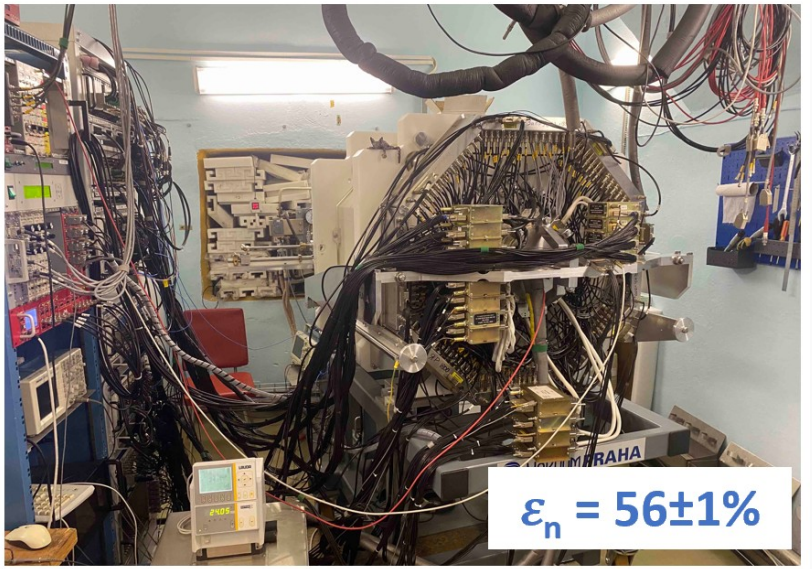
October 20, 2026

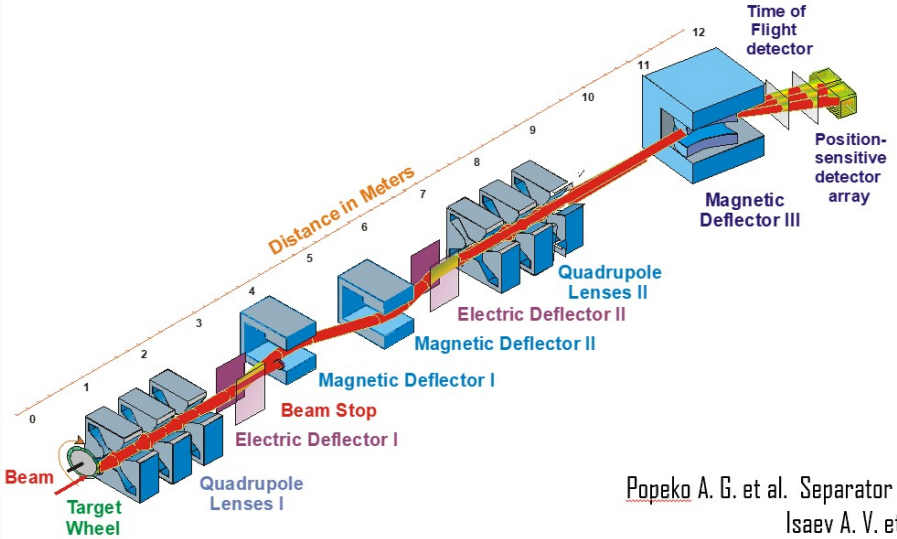
Joint Institute for Nuclear Research

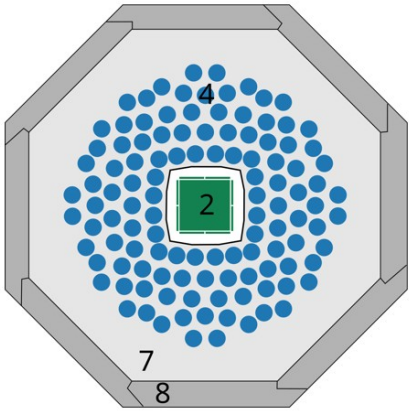
Separator SHELS and SFiNx Detector System

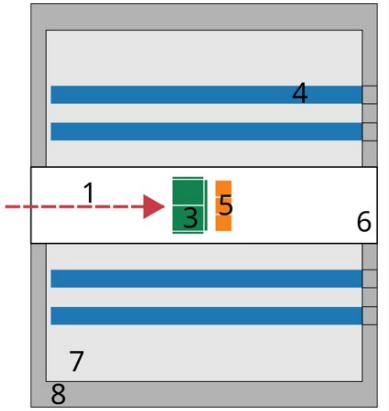






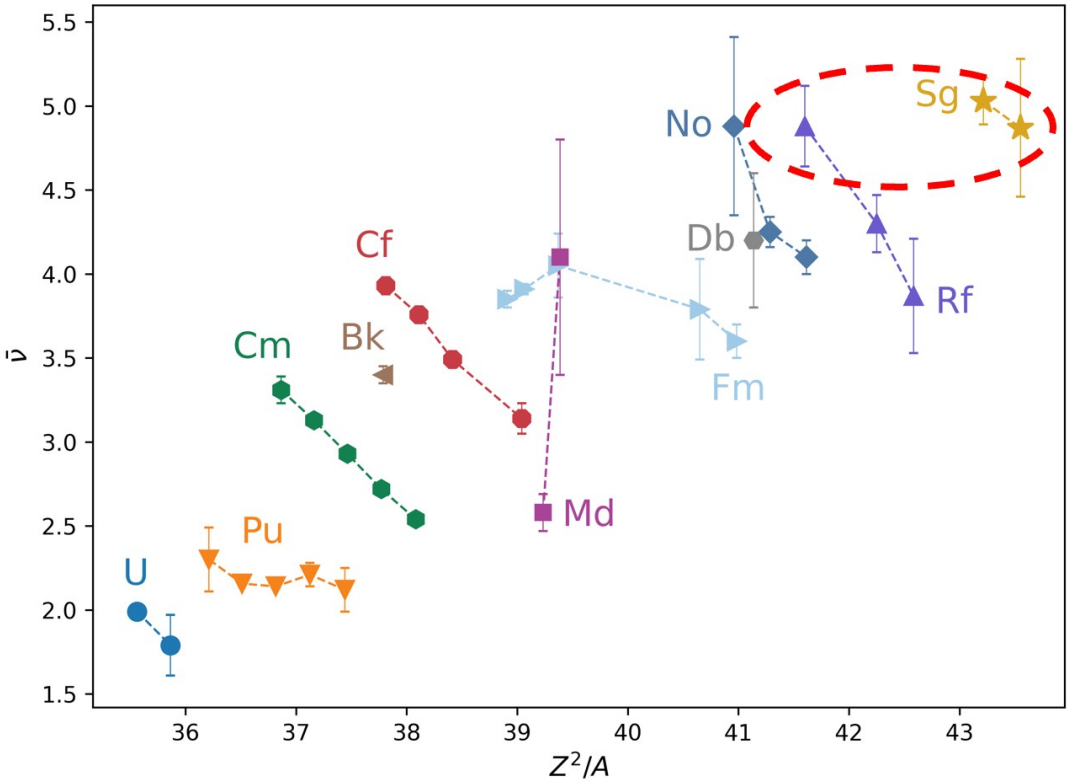






Popeko A. G. et al. Separator for Heavy ELement Spectroscopy – velocity filter SHELS, NIM B 376 (2016)
Isaev A. V. et al. The SFiNx detector system, PEPAN Letters 19 (2022)

Nu-bar Systematics



Average prompt neutron multiplicity vs. the parameter Z^2/A for SF of nuclides with Z = 92 – 106.

Reactions:

$^{207}Pb(^{52,54}Cr,1n)^{258,260}Sg$

$^{238}U(^{26}Mg,4n)^{260}Rf$

Statistics:

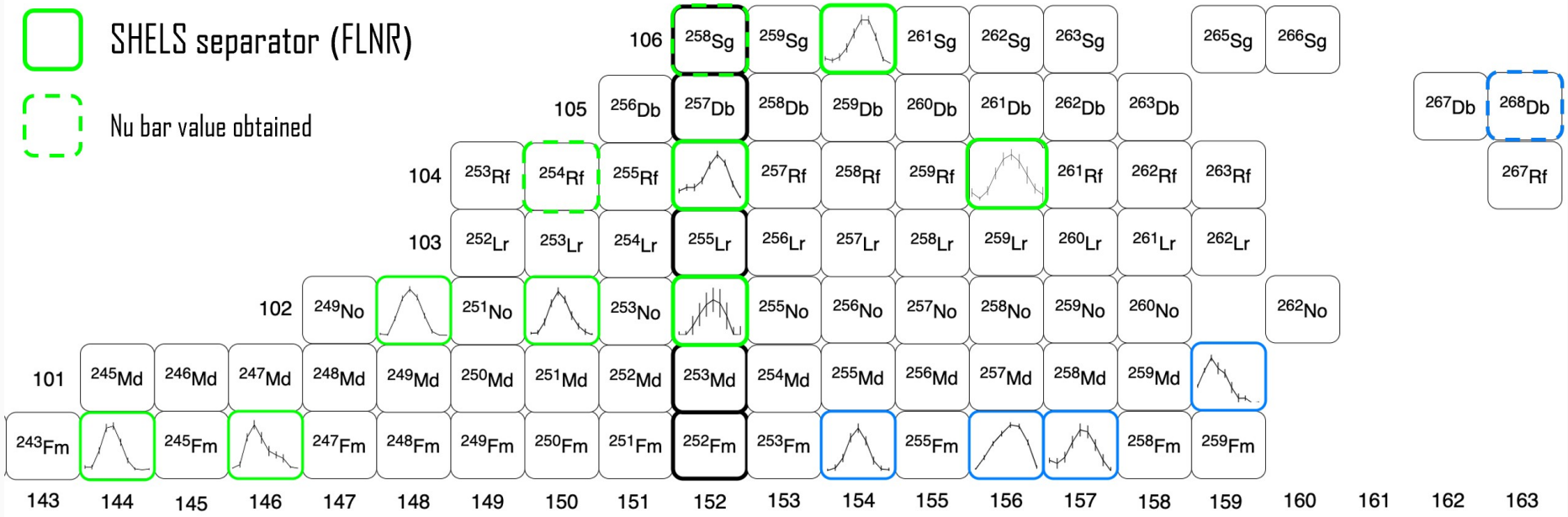
^{260}Sg – 231 SF

^{258}Sg – 9 SF

^{260}Rf – 53 SF

Isaev A. V., R. S. Mukhin, A.V. Andreev, A. Rahmatinejad, T.M. Shneidman et al.
Average number of prompt neutrons in the spontaneous fission of ^{260}Rf
Eur. Phys. J. A (2026) 62:155

Shapes of the Prompt Neutron Multiplicity Distributions



Isaev A. V. et al. The SFiNx detector system (Current Status), PEPAN Letters (2025) 22:2

Isotopes ^{260}Sg and ^{260}Rf

Reaction	^{54}Cr + ^{207}Pb
Targets PbS	345 and 455 μg/cm^2 1.5 μm Ti; ^{207}Pb > 99%
$E_{1/2}$, MeV	249 – 255
ε_n, %	56±1
$\sum_{SF}$	**231**
$\sum_n$	**645**
$\bar{\nu}$	5.03±0.14
P_n	received for the first time
$T_{1/2}$, ms	~7
b_α	~0.29

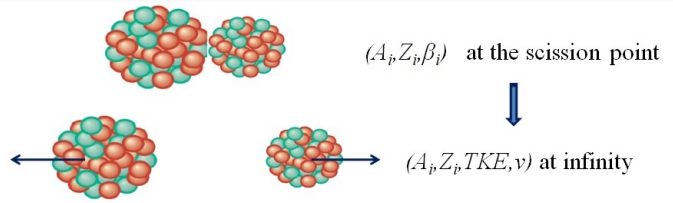
Reaction	^{26}Mg + ^{238}U
Target U_3O_8	287 μg/cm^2 1.5 μm Ti
$E_{1/2}$, MeV	128 – 134.5
σ_{max}, nb	~0.2
ε_n, %	57±1
$\sum_{SF}$	**53**
$\sum_n$	**147**
$\bar{\nu}$	4.88±0.24
$T_{1/2}$, ms	~17

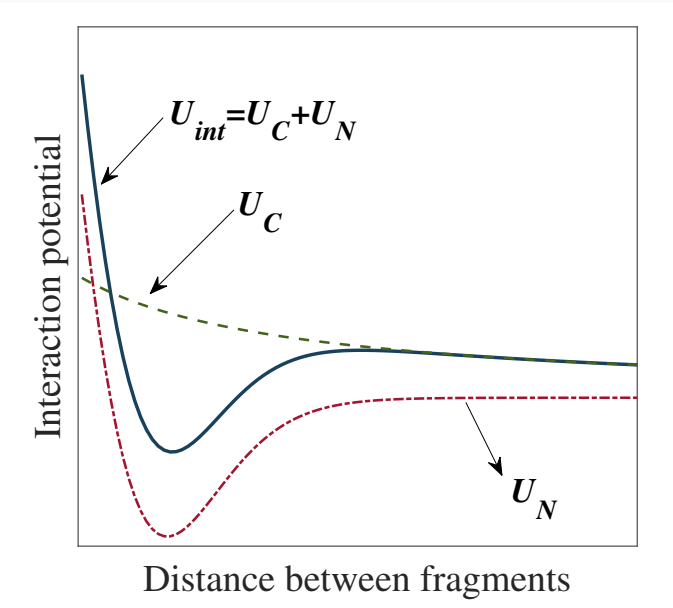
Isaev A. V., R. S. Mukhin, A.V. Andreev, A. Rahmatinejad, T.M. Shneidman et al. Average number of prompt neutrons in the spontaneous fission of ^{260}Rf, Eur. Phys. J. A (2026) 62:155

Isaev A. V., R. S. Mukhin, A.V. Andreev, A. Rahmatinejad, T.M. Shneidman et al. Prompt neutron emission in the spontaneous fission of ^{258,260}Sg, PRC Accepted, 10.1103/c323-g246

DNS approach for description of spontaneous fission

- After crossing the fission barrier, the fissioning nucleus is treated as a superposition of various dinuclear systems.
- DNS is a system of two fragments in touching configuration characterized by their charges, masses, and deformations.
- The potential energy of DNS as a function of relative distance coordinate *R* has a pocket in the vicinity of touching configuration.
- The decay of DNS competes with an evolution in mass/charge asymmetry coordinates and in deformation of the fragments.

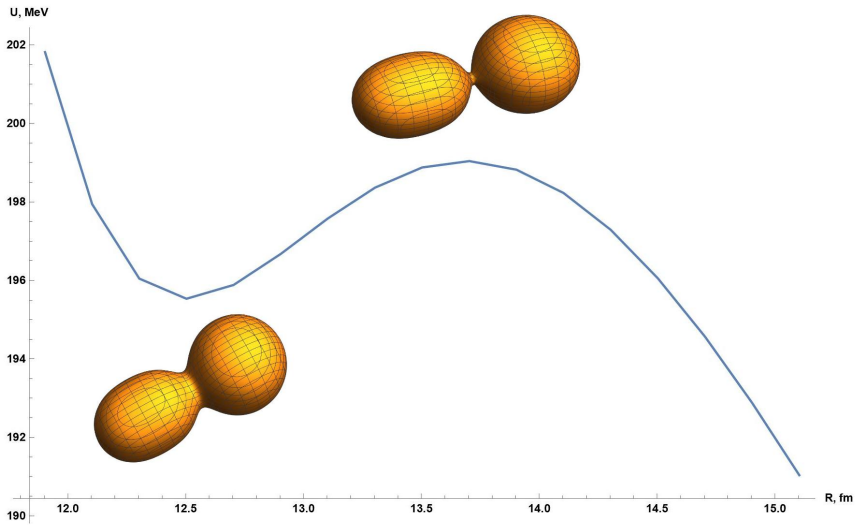




Density of dinuclear system: $^{232}\mathrm{Th} \longrightarrow {}^{132}\mathrm{Sn}(0.0) + {}^{100}\mathrm{Zr}(0.3)$

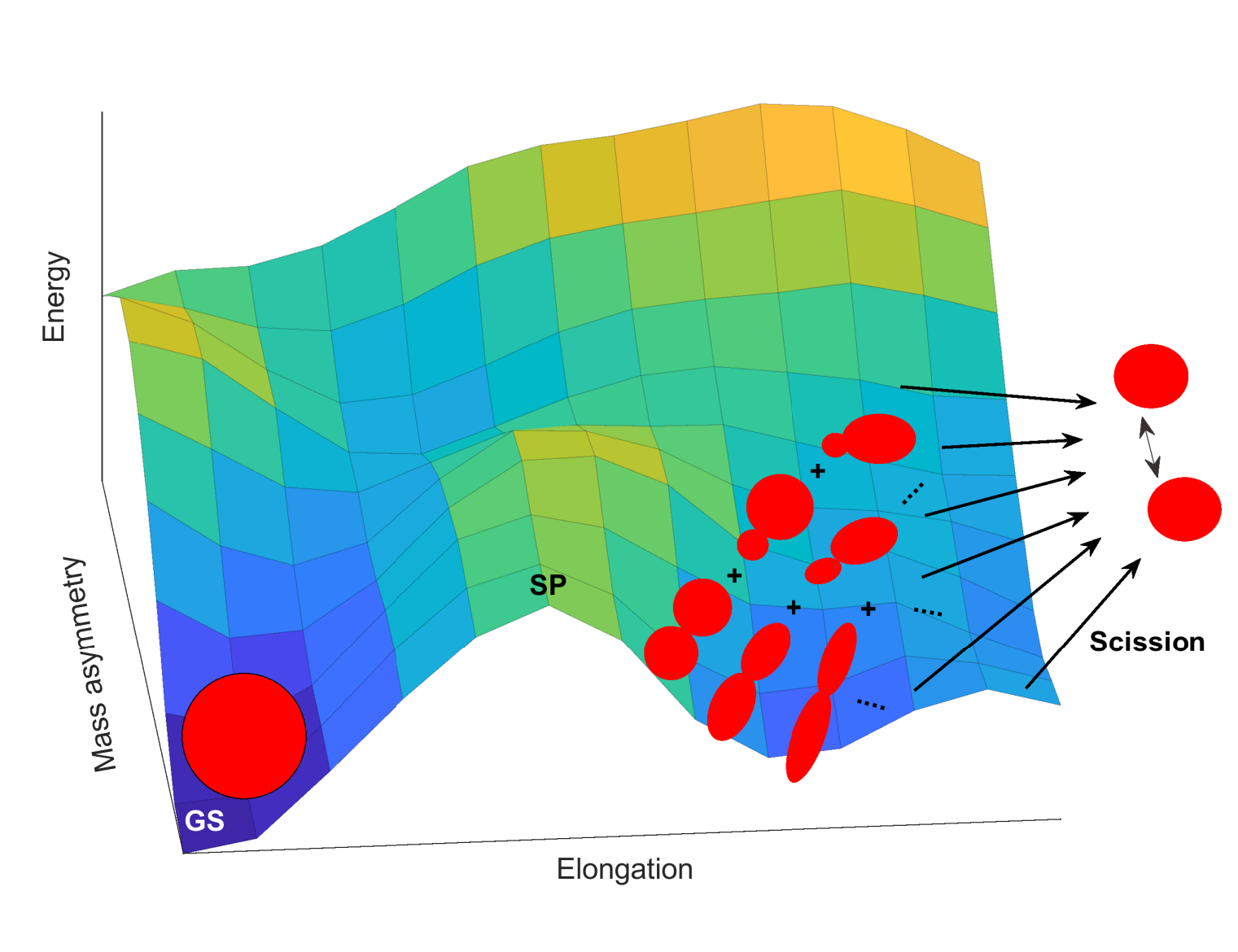
DNS density at the distance R between the fragments:

$$\rho(\mathbf{r}) = \rho_1(\mathbf{r}) + \rho_2(\mathbf{r} - \mathbf{R})$$



- The necked-in configuration of the fissioning nucleus can be viewed as a dinuclear system in a touching configuration.
- Near the top of the fission barrier, where the neck practically disappears, the nuclear interaction becomes weak $\longrightarrow$ the system accelerates rapidly toward scission.
- The fission observables are determined mainly by the DNS configuration in the pocket of the nucleus–nucleus potential (at touching).

Evolution of fissioning nucleus after crossing the fission barrier



Evolution of fissioning nucleus after crossing the fission barrier

The evolution of DNS distribution in time is described using the master equation:

$$n = (Z_1, A_1, \beta_1, Z_2, A_2, \beta_2).$$

$$\frac{dP(n)}{dt} = \sum_{n' \neq n} \Lambda(n'|n)P(n') - \sum_{n' \neq n} \Lambda(n|n')P(n) - \Lambda_d(n)P(n)$$

$P(n)$ – Probability that the system is in the state n.
$P_0(n) = P(n, t = 0)$ – Probabilities of initially-formed DNS.
$\Lambda(n|n')$ – Transition rates for switching from n to n'.
$\Lambda_d(n)$ – Decay rates.

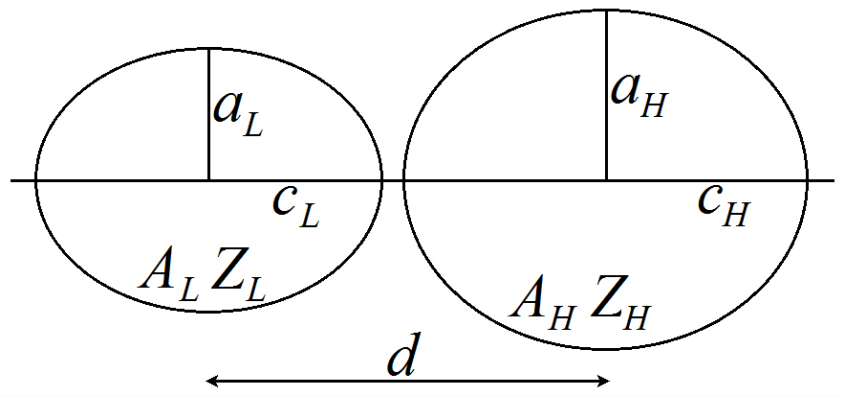
The distribution of primary fission fragments

$$P_f(n[Z_1, A_1, \beta_1, Z_2, A_2, \beta_2])$$

is obtained with Monte-Carlo technique.

Potential energy of the DNS

The contribution of different DNS is determined by its potential energy:
larger potential energy $\longrightarrow$ smaller excitation energy $\longrightarrow$ smaller probability



$$U_{sc}(\beta_L, \beta_H, d) = U_{LD}(\beta_{L,H}) + \delta U_{sh}(\beta_{L,H}) + V^C(\beta_{L,H}, d) + V^N(\beta_{L,H}, d)$$

- U_{LD}: Liquid-drop energy
- δU_{sh}: Shell correction
- V^C: Coulomb interaction
- V^N: Nuclear interaction

Excitation energy (spontaneous fission): $E^* = U_{comp} - U_{DNS}$

A.V. Andreev et al., EPJ A **22**, 51 (2004).

Nuclear interaction energy

Double folding potential: (G. G. Adamian *et al.*, IJMPE **5**, 191 (1996).)

$$V^N(d) = \int \rho_1(\mathbf{r}_1)\rho_2(\mathbf{d}-\mathbf{r}_2)F(\mathbf{r}_1-\mathbf{r}_2)\mathrm{d}\mathbf{r}_1\mathrm{d}\mathbf{r}_2,$$

with the density-dependent forces (Migdal forces):
A. B. Migdal, *Theory of finite Fermi Systems...* (Nauka, Moscow, 1982).

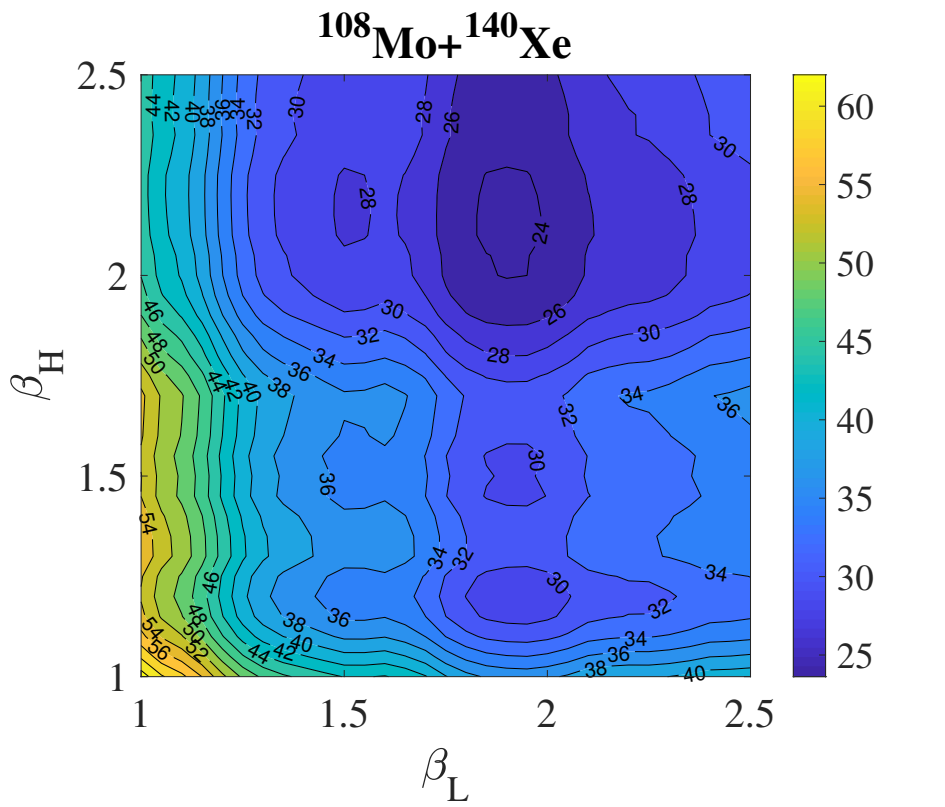
$$F(\mathbf{r}_1-\mathbf{r}_2) = C_0\left(F_{in}\frac{\rho_0(\mathbf{r}_1)}{\rho_{00}} + F_{ex}\left(1-\frac{\rho_0(\mathbf{r}_1)}{\rho_{00}}\right)\right)\delta(\mathbf{r}_1-\mathbf{r}_2),$$

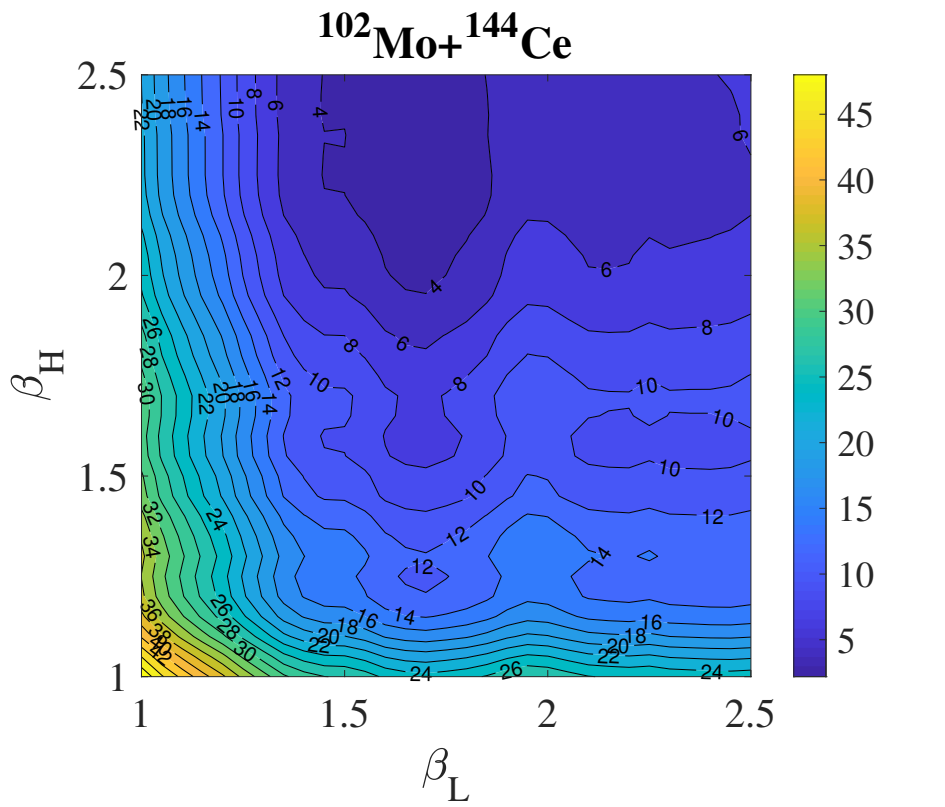
$F_{in,ex} = (f_{in,ex} + f'_{in,ex}\tau_1\cdot\tau_2) + (g_{in,ex} + g'_{in,ex}\tau_1\cdot\tau_2)\sigma_1\cdot\sigma_2$
$C_0 = 300\ \mathrm{MeV\ fm}^3$, $f_{in,ex} = 0.09(-2.59)$, $f'_{in,ex} = 0.42(0.54)$

$$\rho_0(\mathbf{r}) = \rho_1(\mathbf{r}) + \rho_2(\mathbf{r}).$$

Densities are in the form of Fermi distribution.

Potential energy of DNS as a function of the fragment deformations.





Complex structure of the PES (one or several local minima) is due to the shell correction of fragments.

Transition rates

The transition rates are expressed in terms of the microscopic transition probabilities and of the level densities of the final state.

$$\Lambda(n|n') = \lambda_{nn'}\rho(E^*_{n'}, n'), \quad \Lambda(n'|n) = \lambda_{n'n}\rho(E^*_n, n)$$

$$\lambda_{n'n} = \lambda_{nn'} = \lambda^{(i)}/\sqrt{\rho(n')\rho(n)}; \quad i = (m.a., \beta)$$

$$\Lambda_d(n) = \lambda_d\rho_{s.p.}(E^* - V_B, n)$$

$$\lambda_d = \begin{cases} 1/\sqrt{\rho_{s.p.}(n)\rho(n)}, & \text{if } (E^* > V_B) \\ 0, & \text{if } (E^* < V_B) \end{cases}$$

L.G. Moretto and J.S. Sventek, Phys. Lett. B **58**, 26 (1975).
G.G. Adamyan, A.K. Nasirov, N.V. Antonenko, R.V. Jolos , Fiz. Elem. Chastits At.Yadra **25**, 1379 (1994).

Level densities of DNS

The intrinsic level density (LD) of DNS:

$$\rho_{int}(E^*, n) = \int \rho_1(A_1, Z_1, \beta_1, \varepsilon)\rho_2(A_2, Z_2, \beta_2, E^* - \varepsilon)\mathrm{d}\varepsilon.$$

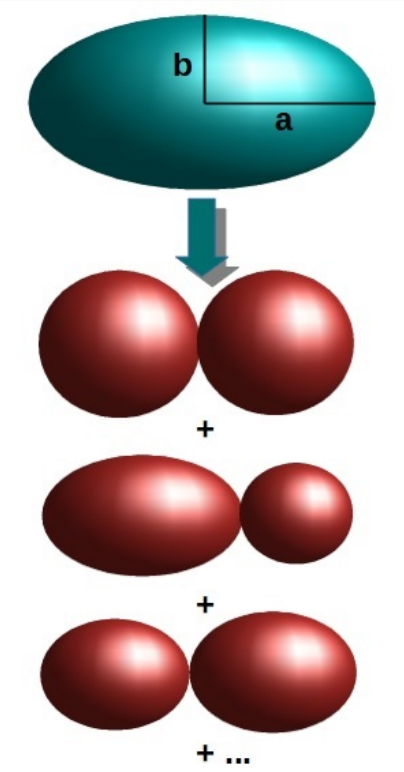
The LD of DNS fragments is calculated in superfluid formalism.

P. Decowski, et al., Nucl. Phys. A **110**, 129 (1968).
A. Bezbakh, et al., EPJA **52**, 353 (2015).

Choice of distribution of initial states

We choose the systems whose mass quadrupole moment lies in the interval of 10% around the value of quadrupole moment of ellipsoid with axis ratio $a : b$ and with restricted octupole moment.

$\beta^{in} = a/b \rightarrow Q_2^{ellips}$



$Q_2 \sim Q_2^{ellips} \pm 10\%$

- **Probability of each DNS**

$$P_0(n) \sim \rho(n, E^* = U_{comp} - U_{DNS})$$

- **Quadrupole moment of DNS**

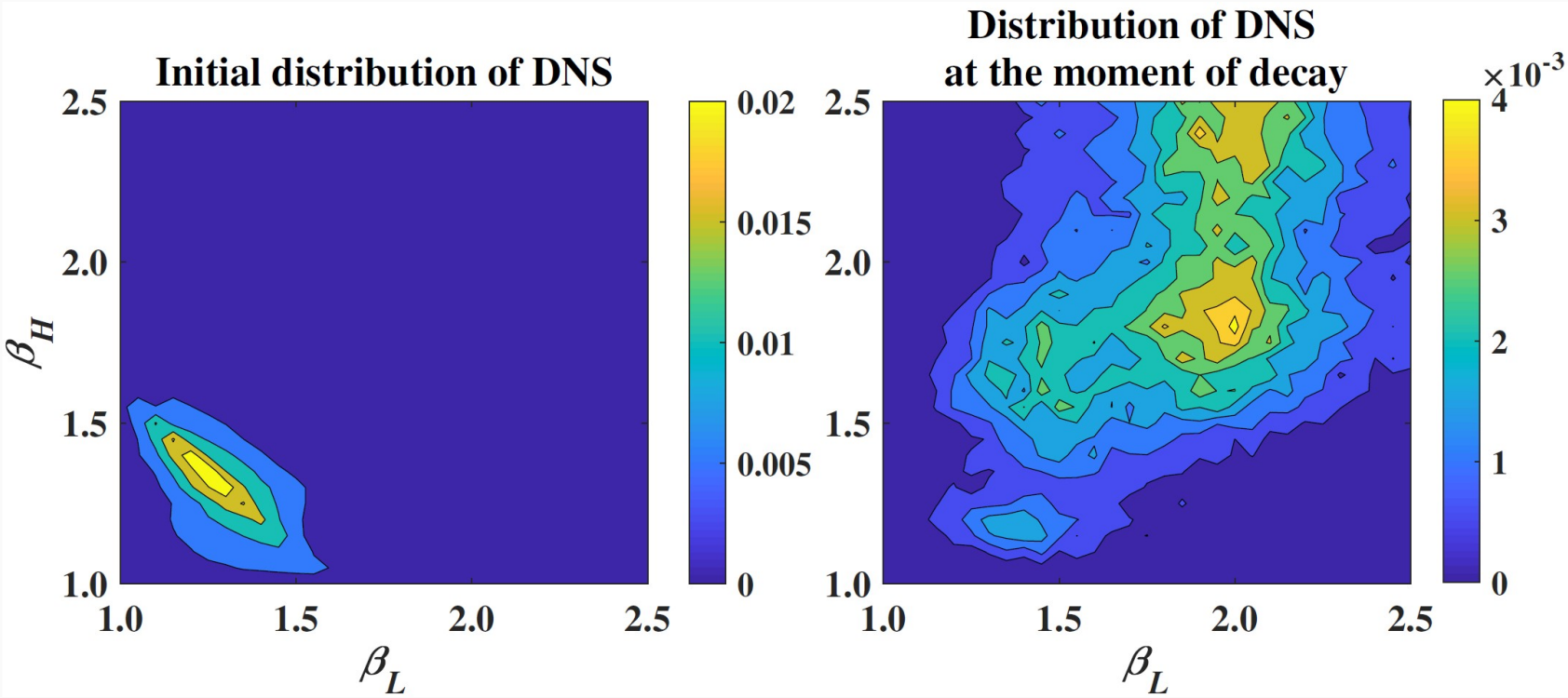
$$Q_2(n) = 2\frac{A_1 A_2}{A}R^2 + Q_2(A_1, \beta_1) + Q_2(A_2, \beta_2)$$

Various calculations show that nucleus can be presented as a DNS around $\beta^{in} \sim 3 : 1$.

S. Cwiok, W. Nazarewicz et al., PLB, **322**, 304 (1994).
T. M. Shneidman et al., NPA, **671**, 119 (2000).
A. V. Afanasjev et al., Phys. Scr. **93**, 034002 (2018).

Restriction on initial Q_3 of DNS: $\beta_{30}^{in} \leq \beta_{30}^{max}$.

Evolution of DNS distribution (sample: ^{256}Rf)



Evolution proceed from compact initial shapes
to larger values of fragment deformations.

Fission observable distributions: A, Z, TKE

The distribution of primary fission fragments:

$$P_f(n[Z_1, A_1, \beta_1, Z_2, A_2, \beta_2])$$

The mass $Y(A)$ and charge $Y(Z)$ distributions:

$$Y(A) = \sum_{n_j} P_f(n_j) \left[\delta_{A_1,A} + \delta_{A_2,A}\right]$$

$$Y(Z) = \sum_{n_j} P_f(n_j) \left[\delta_{Z_1,Z} + \delta_{Z_2,Z}\right]$$

TKE of primary fission fragments is defined as the interaction energy at the top of the barrier $R = R_b$:

$$TKE_n = V_C(n[Z_{1,2}, A_{1,2}, \beta_{1,2}, R_b]) + V_N(n[Z_{1,2}, A_{1,2}, \beta_{1,2}, R_b])$$

Then the TKE distribution: is calculated by summing over the probabilities $P_f(n)$ corresponding to the states with TKE within a given interval 2Δ:

$$Y(TKE) = \sum_j P_f(n_j)\Theta(|TKE_{n_j} - TKE| \leqslant \Delta)$$

Neutron multiplicity

After decay of DNS, fragments shrinks to their ground state releasing its deformation energy:

$$E_i^{def}(n) = \left[U_i^{LD}(Z_i, A_i, \beta_i) + E_i^{sh}(Z_i, A_i, \beta_i)\right] - \left[U_i^{LD}(Z_i, A_i, \beta_i^{gs}) + E_i^{sh}(Z_i, A_i, \beta_i^{gs})\right]$$

Excitation energies of fragments at scission:

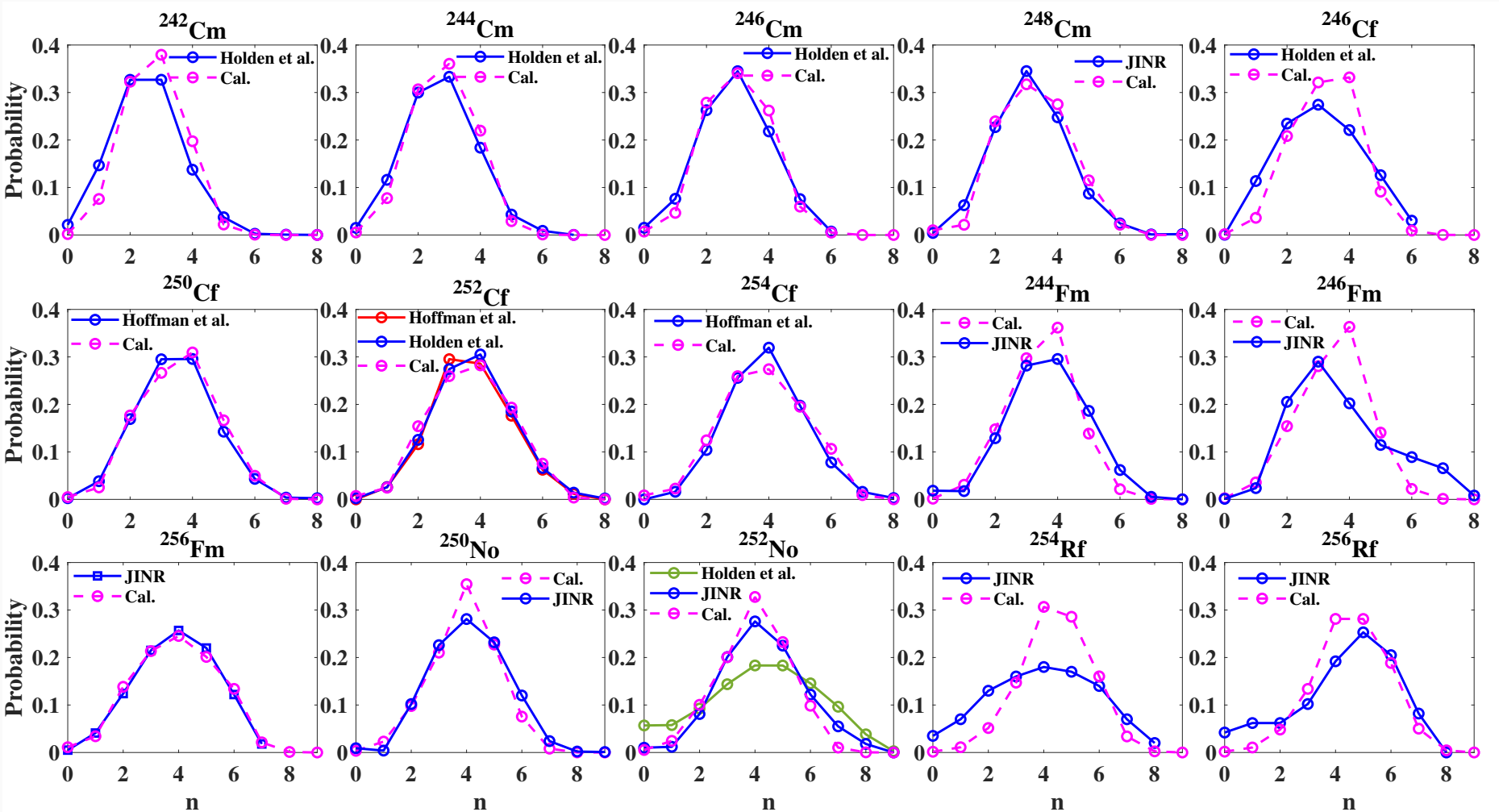
$$\left(\frac{1}{\rho_1(\varepsilon)}\frac{d\rho_1(\varepsilon)}{d\varepsilon}\right)\Bigg|_{\varepsilon=E_1^*} = \left(\frac{1}{\rho_2(\varepsilon)}\frac{d\rho_2(\varepsilon)}{d\varepsilon}\right)\Bigg|_{\varepsilon=E_2^*} = T^{-1}.$$

Neutron multiplicity:

$$Y(\nu) = \sum_j \left[\sum_{x=1}^{\nu} F_1(x, E_1^{def}(n_j) + E_1^*(n_j)) F_2(\nu - x, E_2^{def}(n_j) + E_2^*(n_j))\right] P_f(n_j).$$

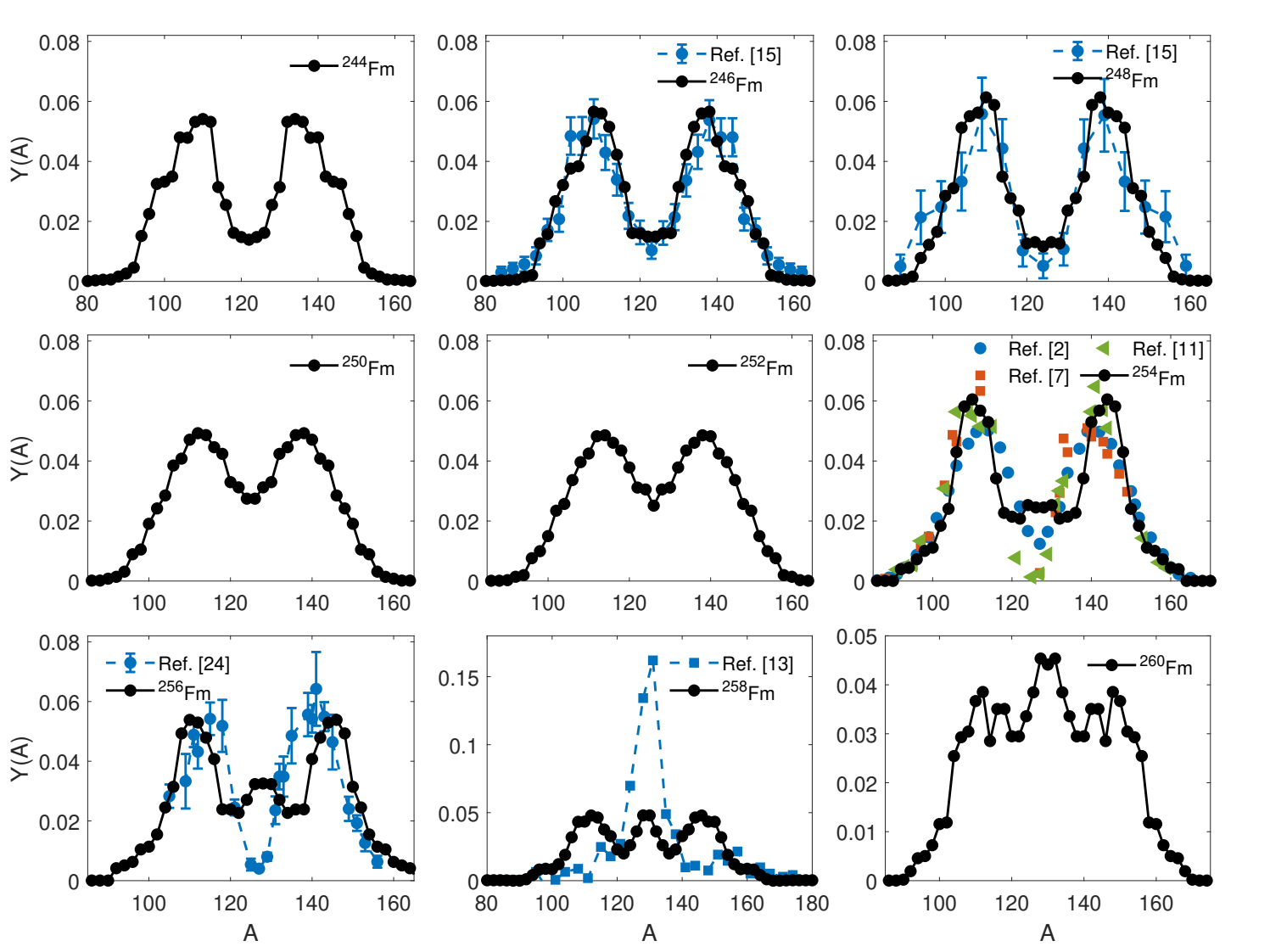
$F_{1,2}(x, E_{1,2})$ – probabilities to emit exactly x neutrons obtained using Monte-Carlo simulation. AR et al., PLB 844 (2023) 138099.

Neutron multiplicities of $Z \sim 100$ nuclei



$\beta^{in} \sim 3.2 : 1$

Mass distribution in SF of even $^{244-260}$Fm isotopes



A. Rahmatinejad et al., IJMPE 33 (2024) 2441018.

Half-lives of Fermium isotopes

Isotope	β_2 of SP	Fission barrier height (MeV)	Exp. SF half-lives
^{244}Fm	0.44	6.07	3.2165 ms
^{246}Fm	0.44	6.61	22.6471 s
^{248}Fm	0.44	6.89	9.5833 h
^{250}Fm	0.45	6.99	301.9292 day
^{252}Fm	0.45	6.98	126.0166 year
^{254}Fm	0.45	6.21	228.0417 day
^{256}Fm	0.45	5.40	2.8491 h
^{258}Fm	0.41	4.82	370 μs
^{260}Fm	0.57	4.55	-
^{262}Fm	0.57	4.74	-

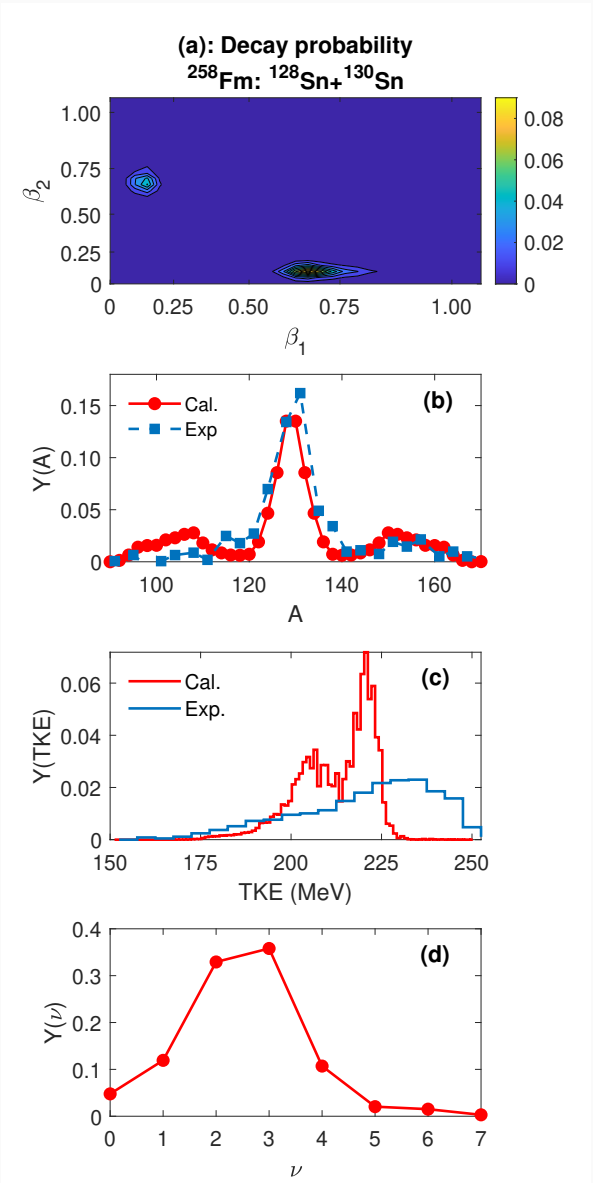
Fission barriers: P. Jachimowicz, et al., ADNDT **138**, 101393, (2021).

Half lives: www.nndc.bnl.gov/ensdf.

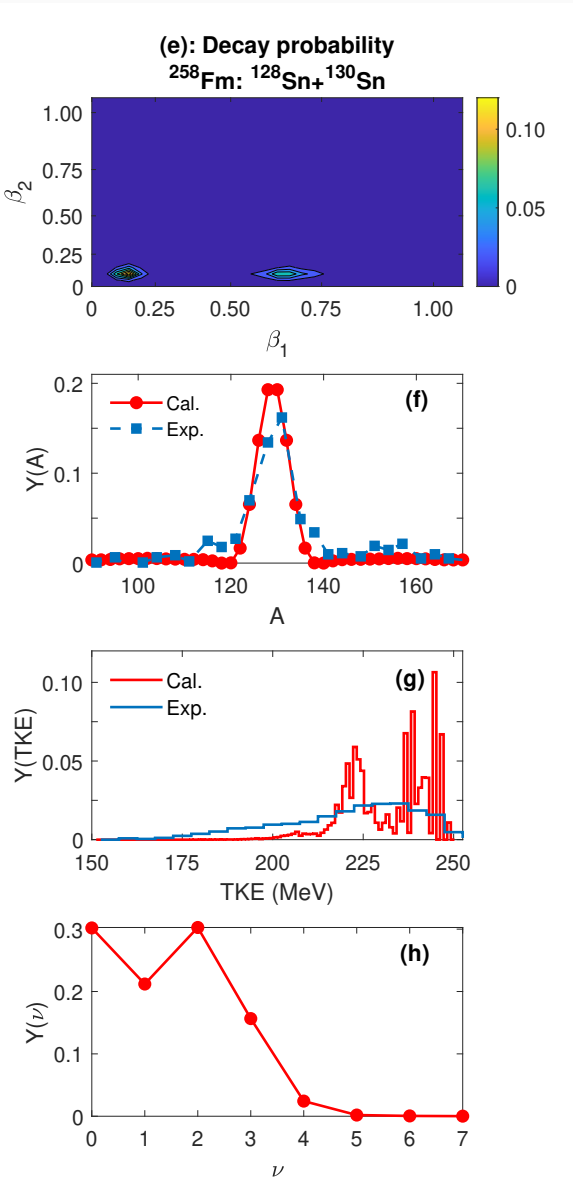
Super-short fission mode: M. Albertsson, et al., PRC **104**, 064616 (2021).

Compact SF of ^{258}Fm

$\beta^{in} = 3.0$

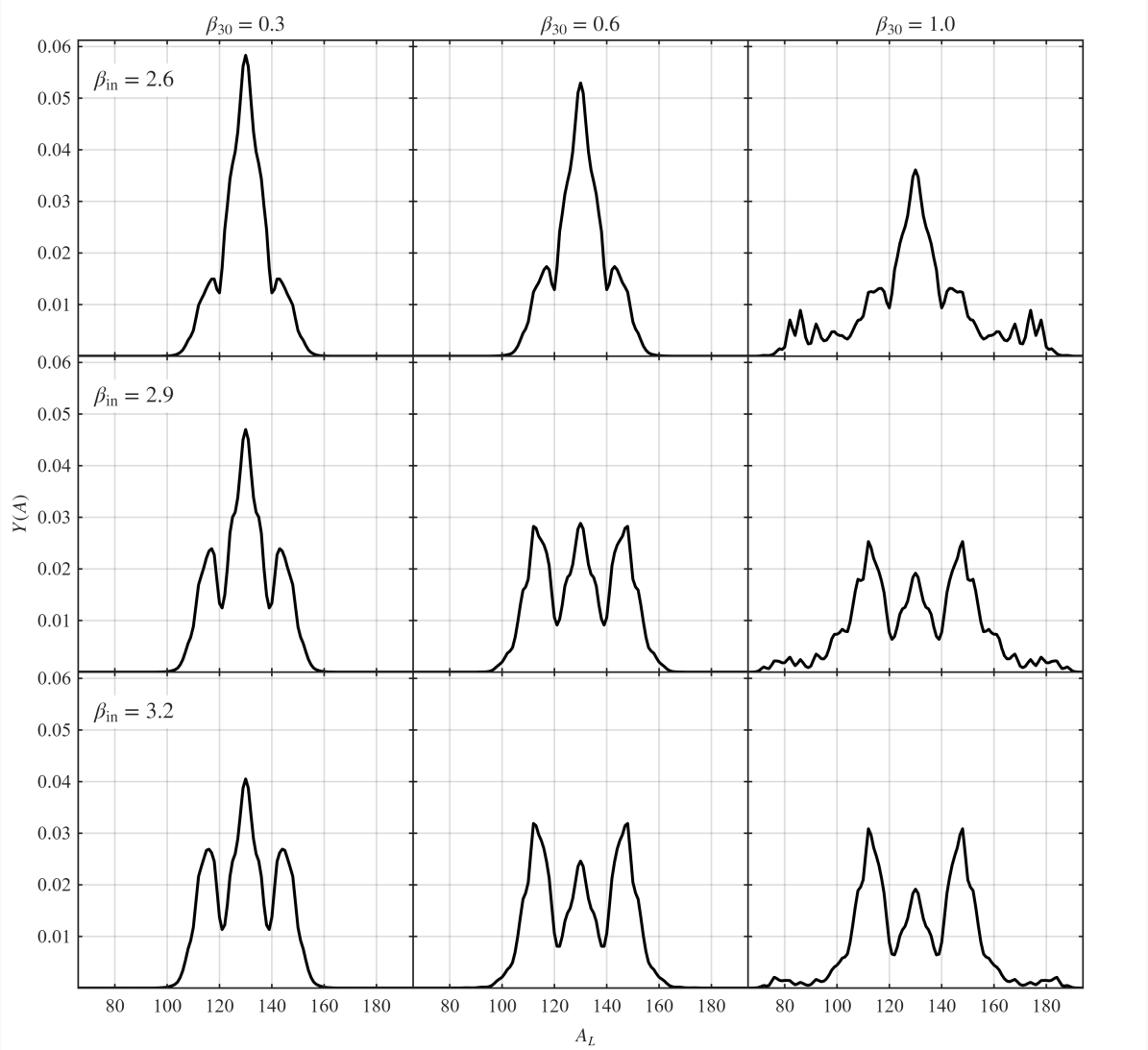


$\beta^{in} = 2.75$



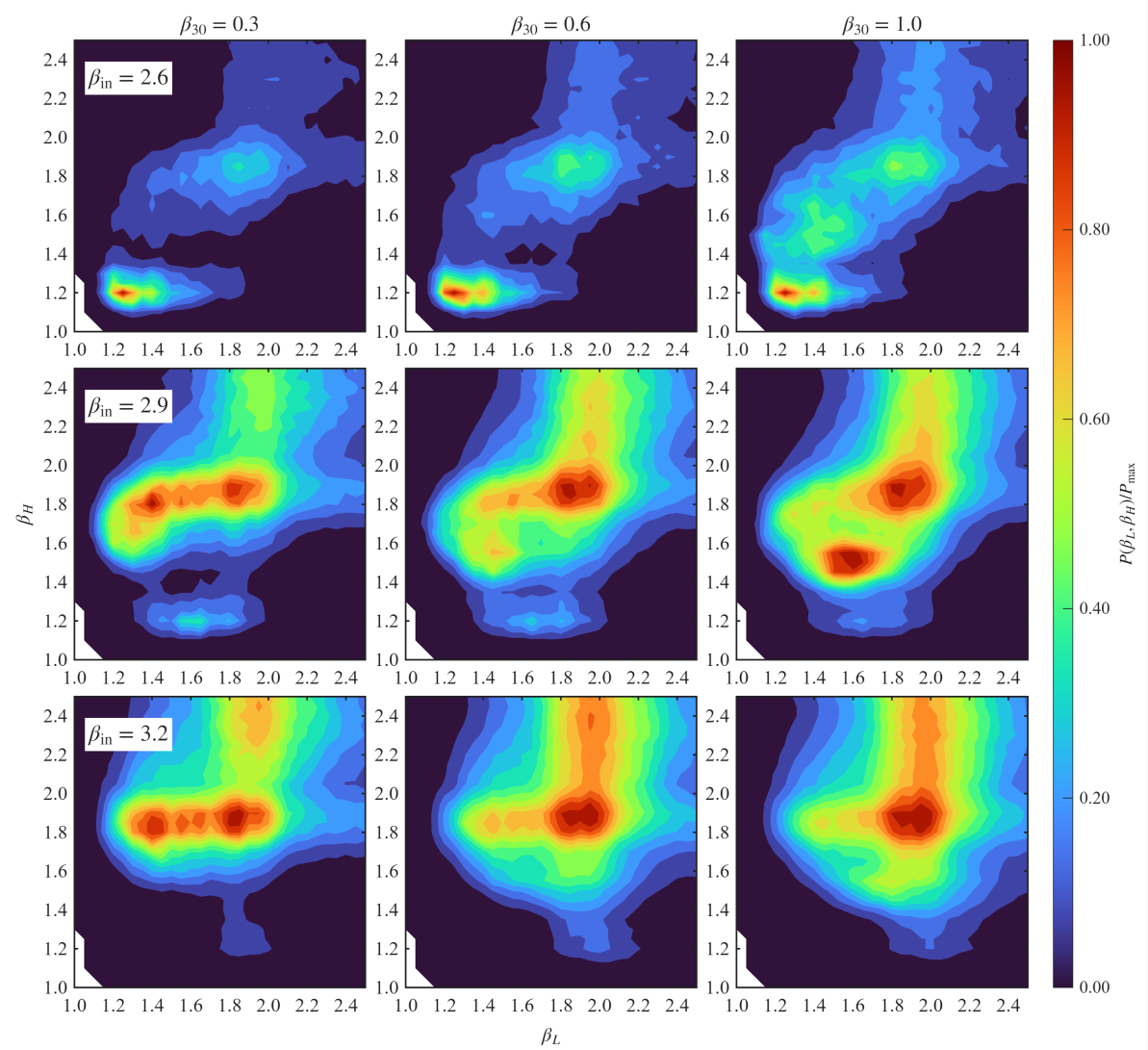
Exp. data:
D. C. Hoffman et. al, PRC, **21**, 972 (1980).
E. K. Hulet et. al, PRC **40**, 770 (1989).

Fission modes in ^{260}Sg - mass distribution



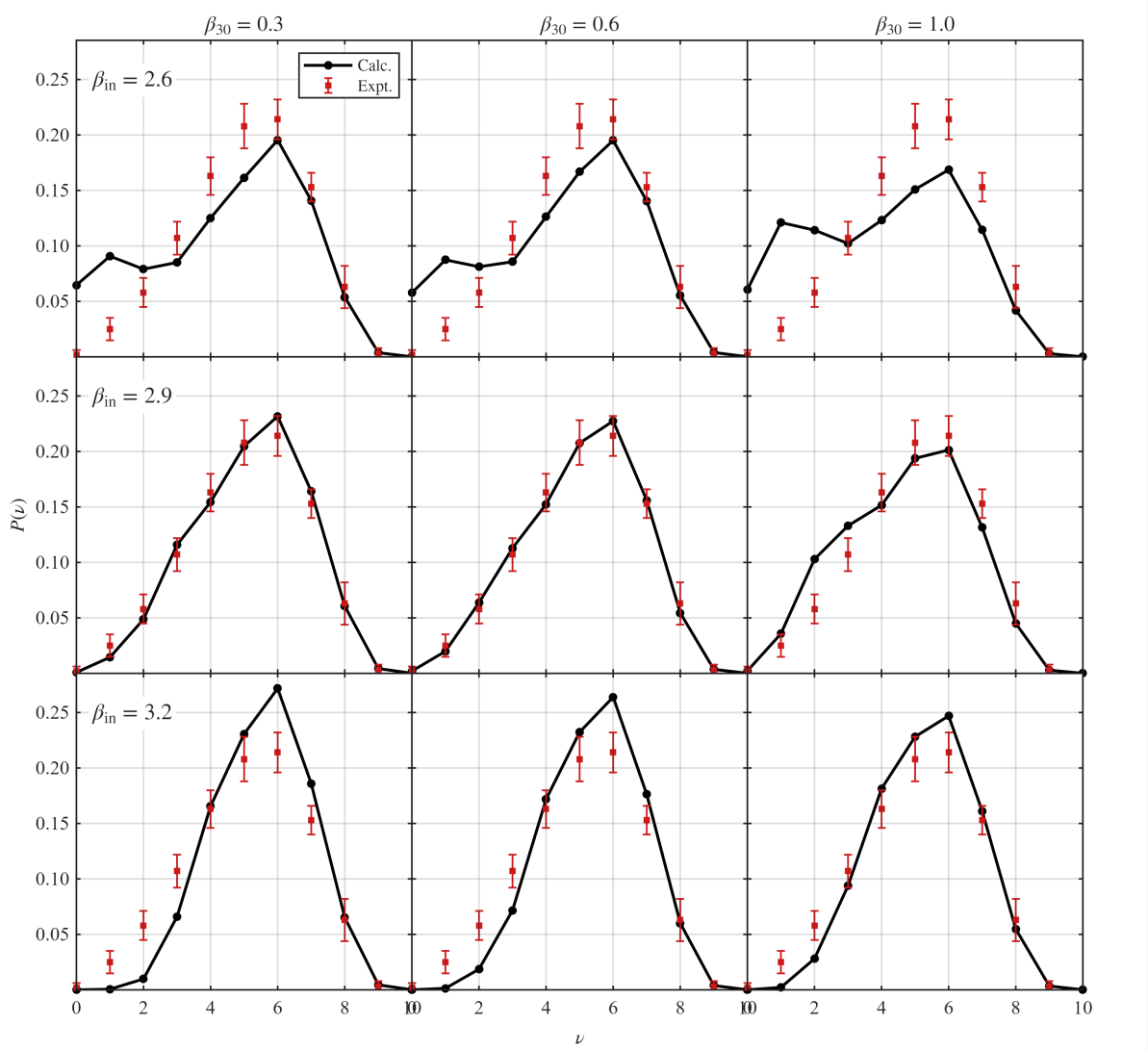
Appearance of symmetric and very asymmetric fission modes with decrease of β^{in} and increase of $\beta_{30}^{in,max}$ is revealed.

Fission modes in ^{260}Sg - decay probability



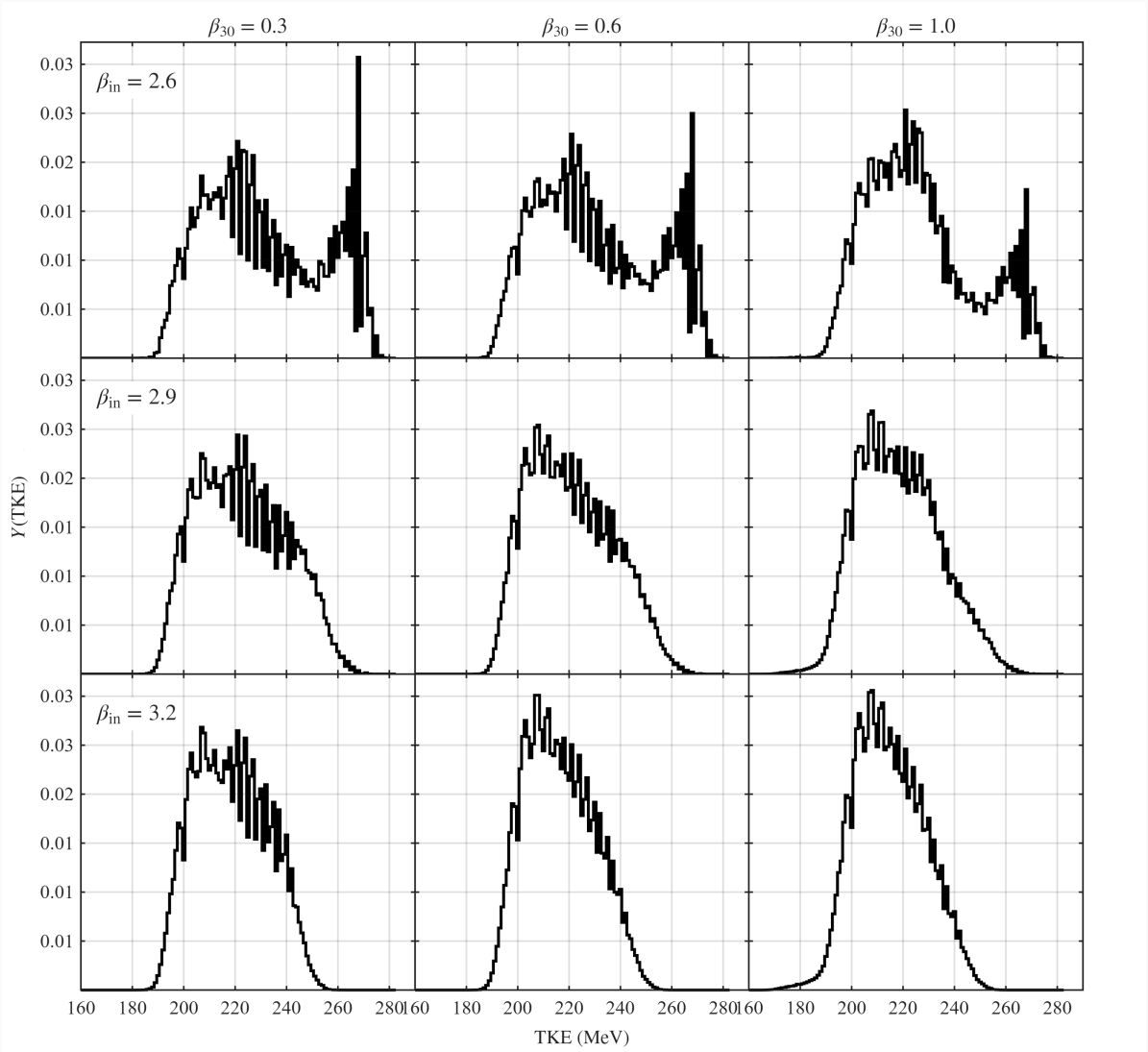
Decay probability maps demonstrate compactness of the symmetric and very asymmetric modes.

Neutron multiplicity distribution in fission of ^{260}Sg



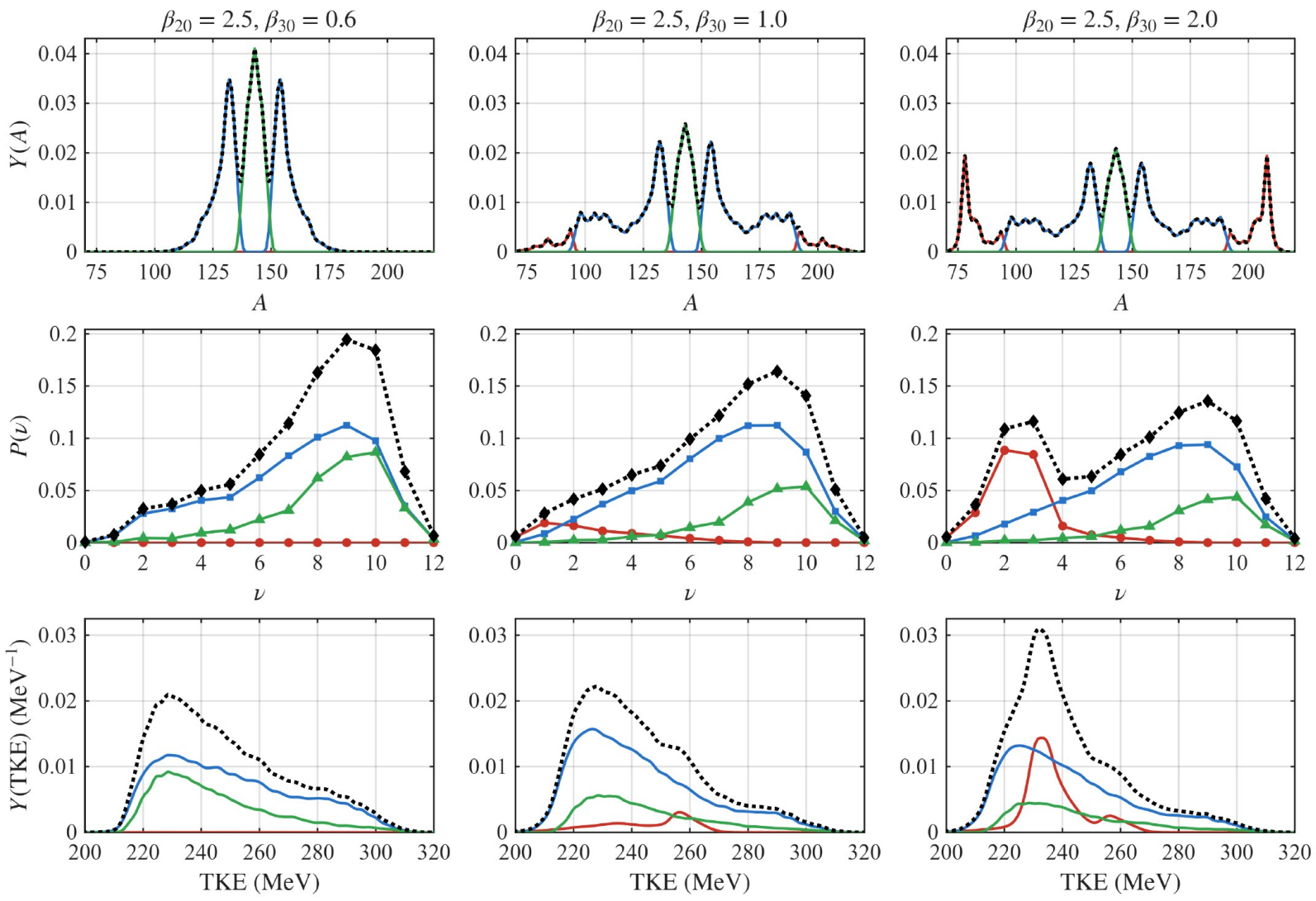
Compact symmetric and asymmetric modes may lead to enhanced yield of low neutron multiplicities. However, in ^{260}Sg this scenario does not realize.

Total kinetic energy distribution in fission of ^{260}Sg



On TKE distribution compact modes may produce enhanced yield of events with large TKE values.

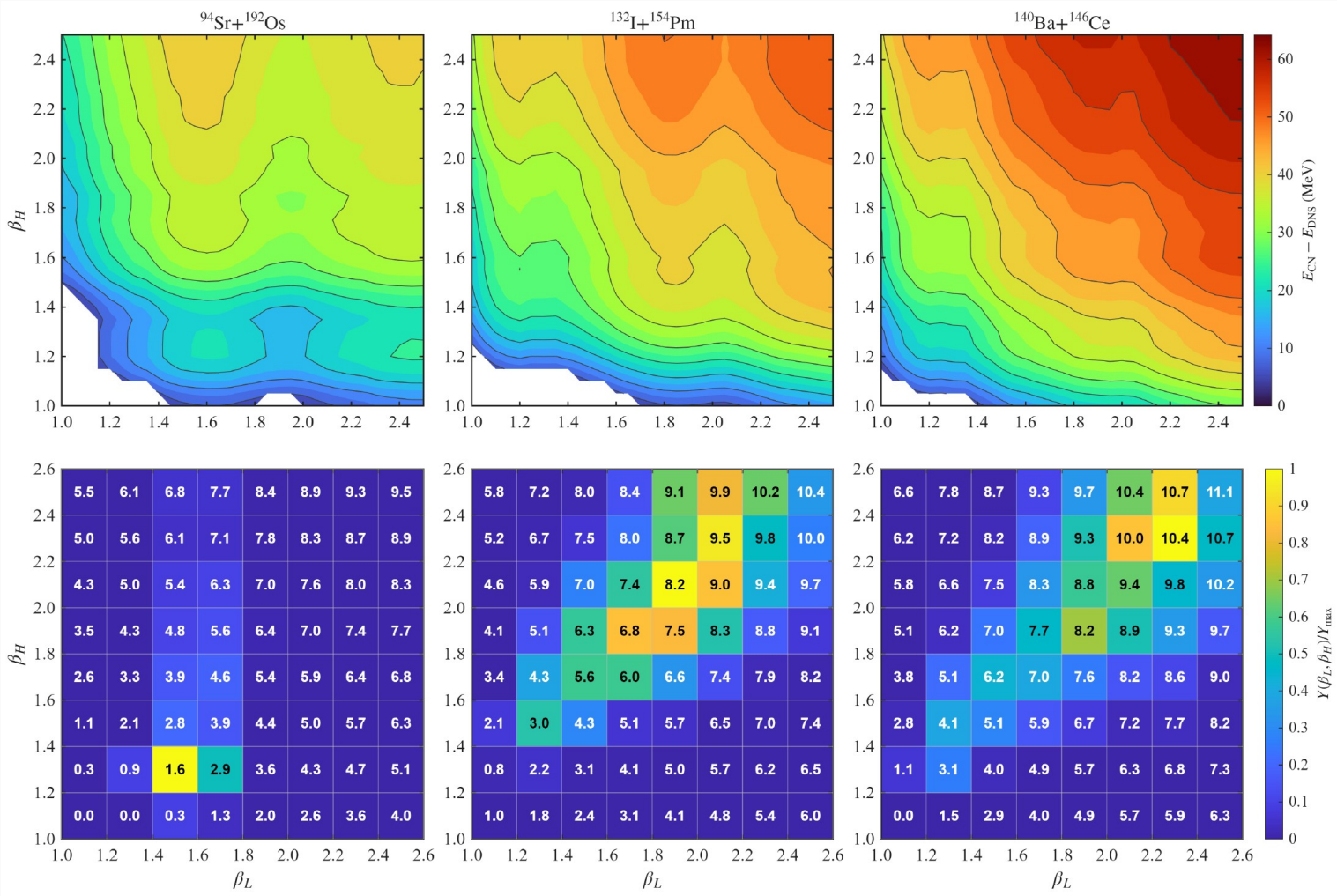
Fission modes in ^{286}Fl



Existence of very asymmetric compact mode in superheavy nuclei can be more prominent.

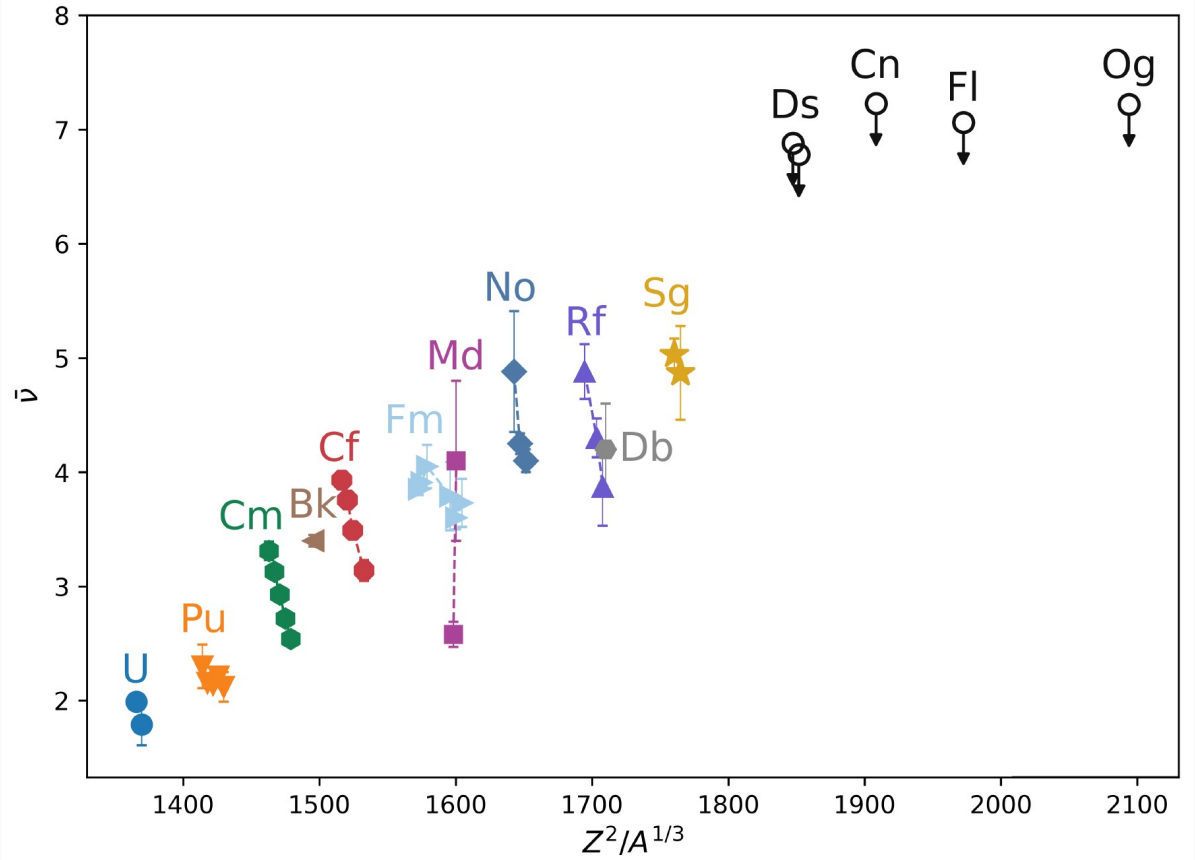
Decay properties of ^{286}Fl

$$\beta^{in} = 2.5,\ \beta^{in}_{30} \leq 1.0$$



Possible very asymmetric mode will lead to enhanced yield of events with small neutron multiplicity and decrease of the average neutron multiplicity.

Average neutron multiplicities for SHE



Predictions for superheavy nuclei are made with $\beta_{30}^{in} \leq 0.6$. Allowance of larger initial asymmetry may result in smaller average neutron multiplicity.

Measurements of neutron multiplicity can be a sensitive indicator of the fission mechanism in superheavy region.

Summary

- The model was presented to describe the evolution of a fissioning nucleus after tunneling through the fission barrier as a stochastic walk among various dinuclear configurations.
- The calculations of fission observables were compared with existing experimental data for heavy actinides and predictions for superheavy nuclei were made.
- Possible existence of very asymmetric fission mode in superheavy nuclei were discussed.

Outlook

- Definition of initial conditions for the presented Monte-Carlo calculations of DNS evolution by combination with cluster fission model of Rogov *et.al.* looks promising.

I.S. Rogov, G.G. Adamian, N.V. Antonenko, PRC 104, 034618 (2021)