Description of the correlations between fission observables in spontaneous fission of ^{252}Cf

T. M. Shneidman
A. Rahmatinejad, N. V. Ryzhkov, G. G. Adamian, N. V. Antonenko

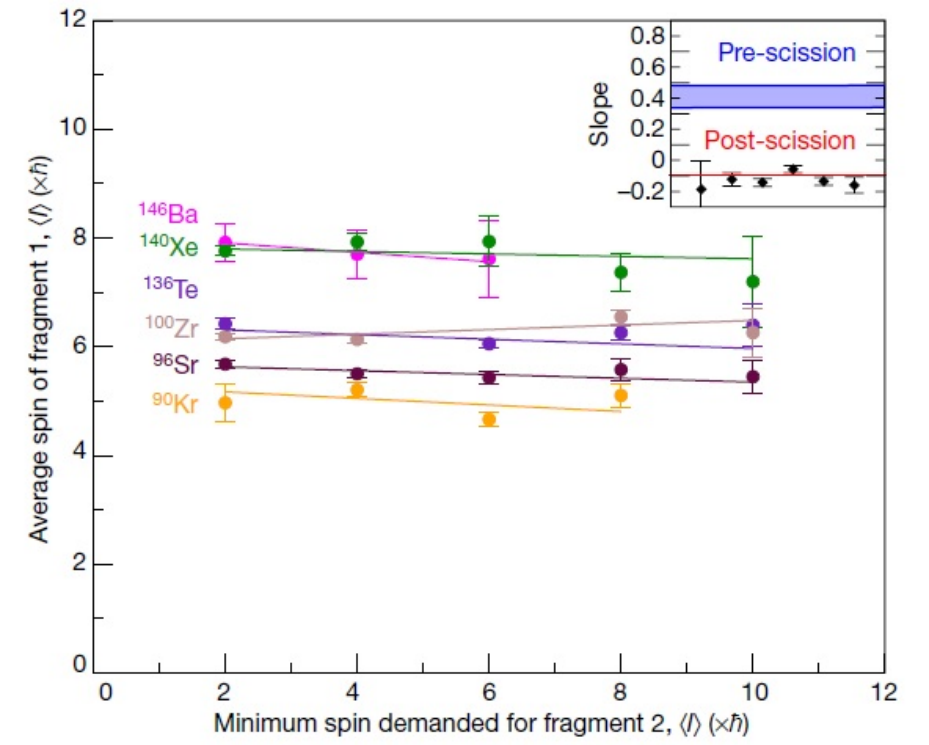
October, 2026

Joint Institute for Nuclear Research



Correlation between fragment spins

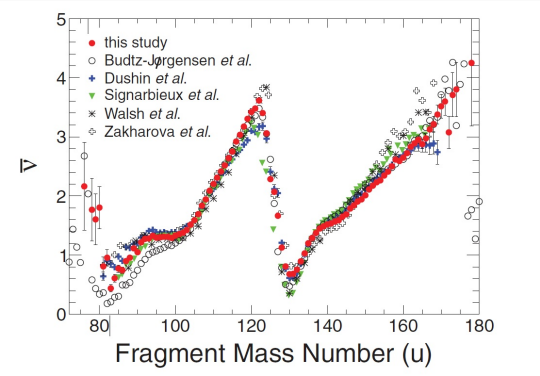
Recent experiments have shown no significant correlation between the spins of fragment partners.



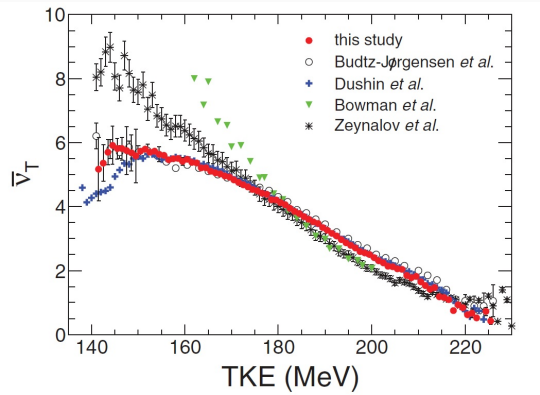
J. N. Wilson et. al, Nature **590**, 566–570 (2021).

Correlation between fission observables

Average number of emitted neutrons vs mass

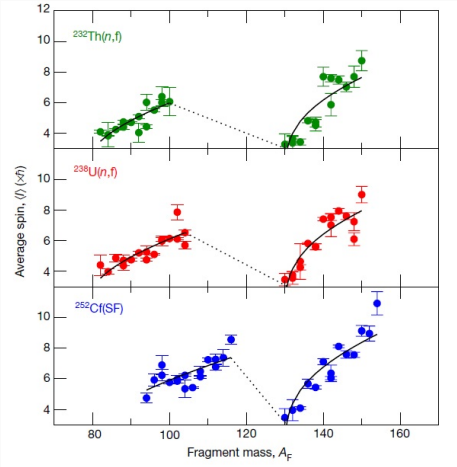


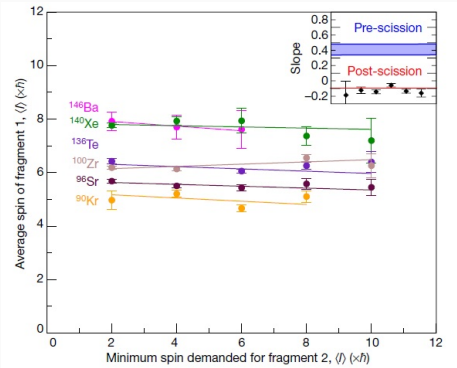
Total kinetic energy vs number of emitted neutrons



A. Göök et al., Phys. Rev. C **90**, 064611 (2014).

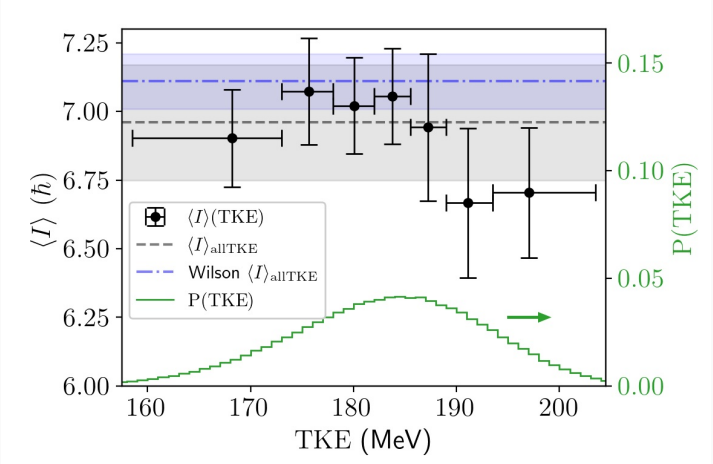
Average angular momentum vs mass





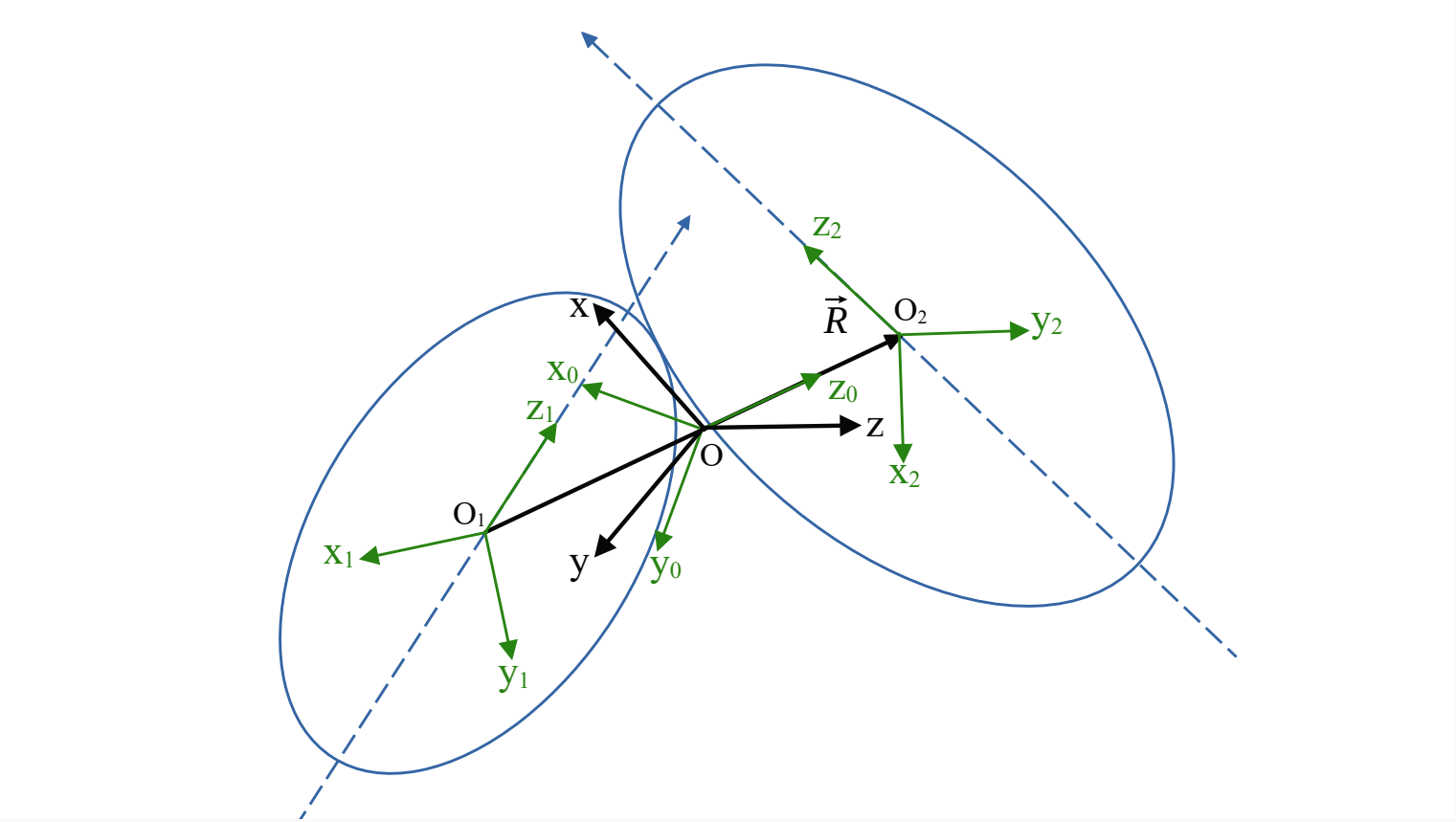
J. N. Wilson et. al, Nature **590**, 566–570 (2021).

Average angular momentum vs total kinetic energy



N. P. Giha *et al.*, Phys. Rev. C, **111** 014605 (2025).

Angular motion at scission



$\Omega_{1,2} = (\phi_{1,2}, \theta_{1,2}, 0)$ – orientation of fragments with respect to the lab. system

$\tilde{\Omega}_{1,2} = (\varepsilon_{1,2}, \gamma_{1,2}, 0)$ – orientation of fragments with respect to the body-fixed system

$\mathbf{R} = (R, \theta_R, \phi_R)$ – vector connecting centers of the fragments.

Hamiltonian

$$H = T + V$$

$$T = \frac{\hbar^2 \hat{l}_0^2}{2\mu R_m^2} + \frac{\hbar^2 \hat{l}_1^2}{2\Im_1} + \frac{\hbar^2 \hat{l}_2^2}{2\Im_2}$$

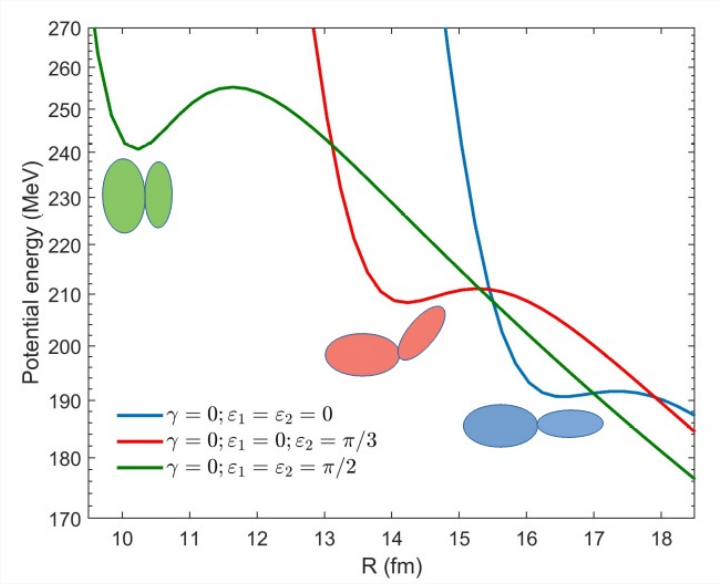
$$\hat{l}_i^2 = -\left(\frac{1}{sin\theta_i} \frac{\partial}{\partial \theta_i} sin\theta_i \frac{\partial}{\partial \theta_i} + \frac{1}{sin^2\theta_i} \frac{\partial^2}{\partial \phi_i^2} \right), \quad i = 0, 1, 2$$

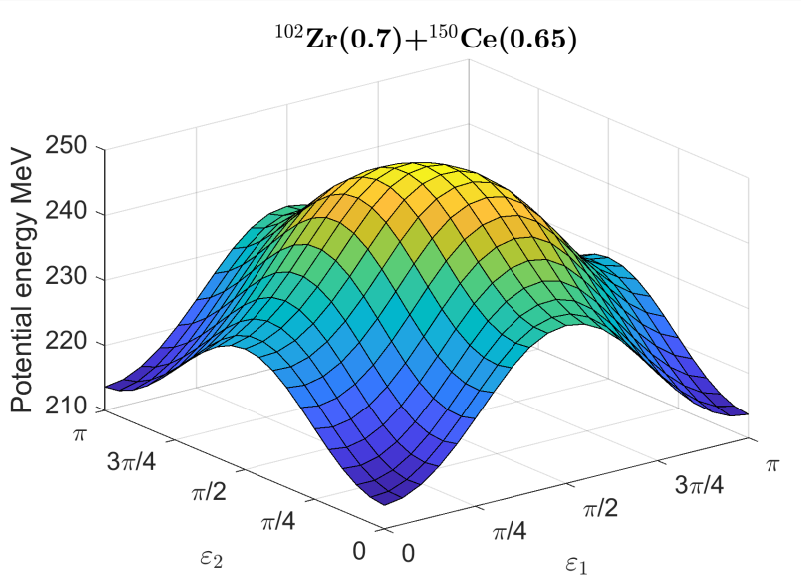
$\Im_{1,2}$: moments of inertia of fragments.

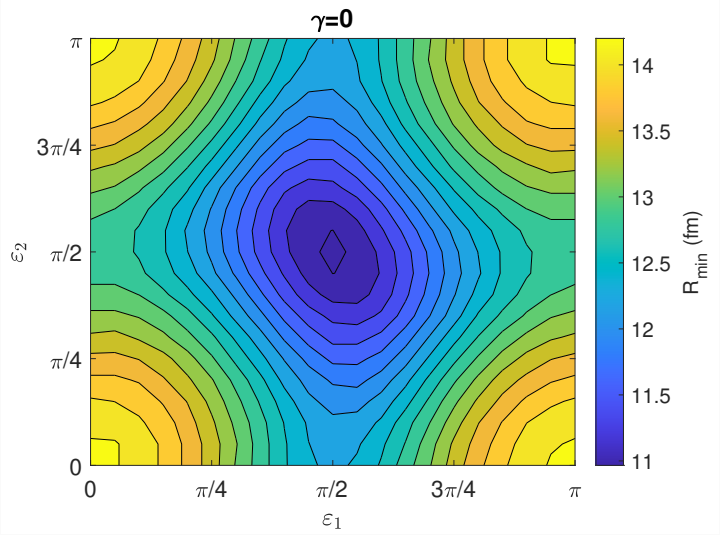
$\Im_R = \mu R_m^2$: moment of inertia of relative motion.

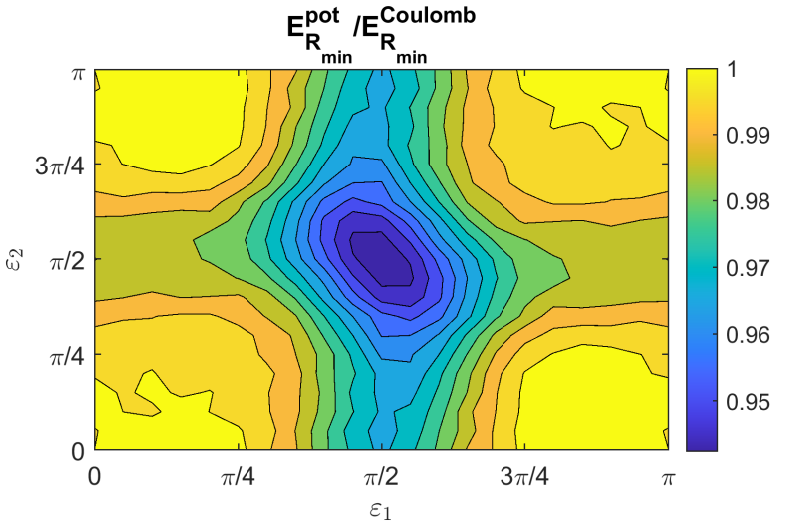
$$V(\Omega_1, \Omega_2, \Omega_0) = \sum_{\lambda_0, \lambda_1, \lambda_2} V_{\lambda_0, \lambda_1, \lambda_2} \left[Y_{\lambda_0}(\Omega_0) \times [Y_{\lambda_1}(\Omega_1) \times Y_{\lambda_2}(\Omega_2)]_{\lambda_0} \right]_{(00)}$$











$$V(\tilde{\Omega}_1, \tilde{\Omega}_2) = V_{coul}(R_m(\tilde{\Omega}_1, \tilde{\Omega}_2), \tilde{\Omega}_1, \tilde{\Omega}_2) + V_N(R_m(0,0), 0, 0)$$

Basis

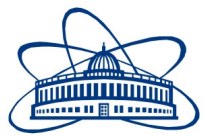
For a given total angular momentum I and its projection M, Hamiltonian is diagonalized on the set of tripolar spherical functions.

$$[i_0 \times [i_1 \times i_2]_{i_{12}}]_{(I,M)} \equiv \left[Y_{i_0}(\Omega_0) \times [Y_{i_1}(\Omega_1) \times Y_{i_2}(\Omega_2)]_{i_{12}}\right]_{(I,M)}$$
$$= \sum_{m_0 m_1 m_2 m_{12}} C^{IM}_{i_0 m_0, i_{12} m_{12}} C^{i_{12} m_{12}}_{i_1 m_1, i_2 m_2} Y_{i_0 m_0}(\Omega_R) Y_{i_1 m_1}(\Omega_1) Y_{i_2 m_2}(\Omega_2)$$

$$\psi^n_{IM} = \sum_{i_0 i_1 i_2 i_{12}} a^n_{i_0 i_1 i_2 i_{12}} \left[i_0 \times [i_1 \times i_2]_{i_{12}}\right]_{(IM)}$$

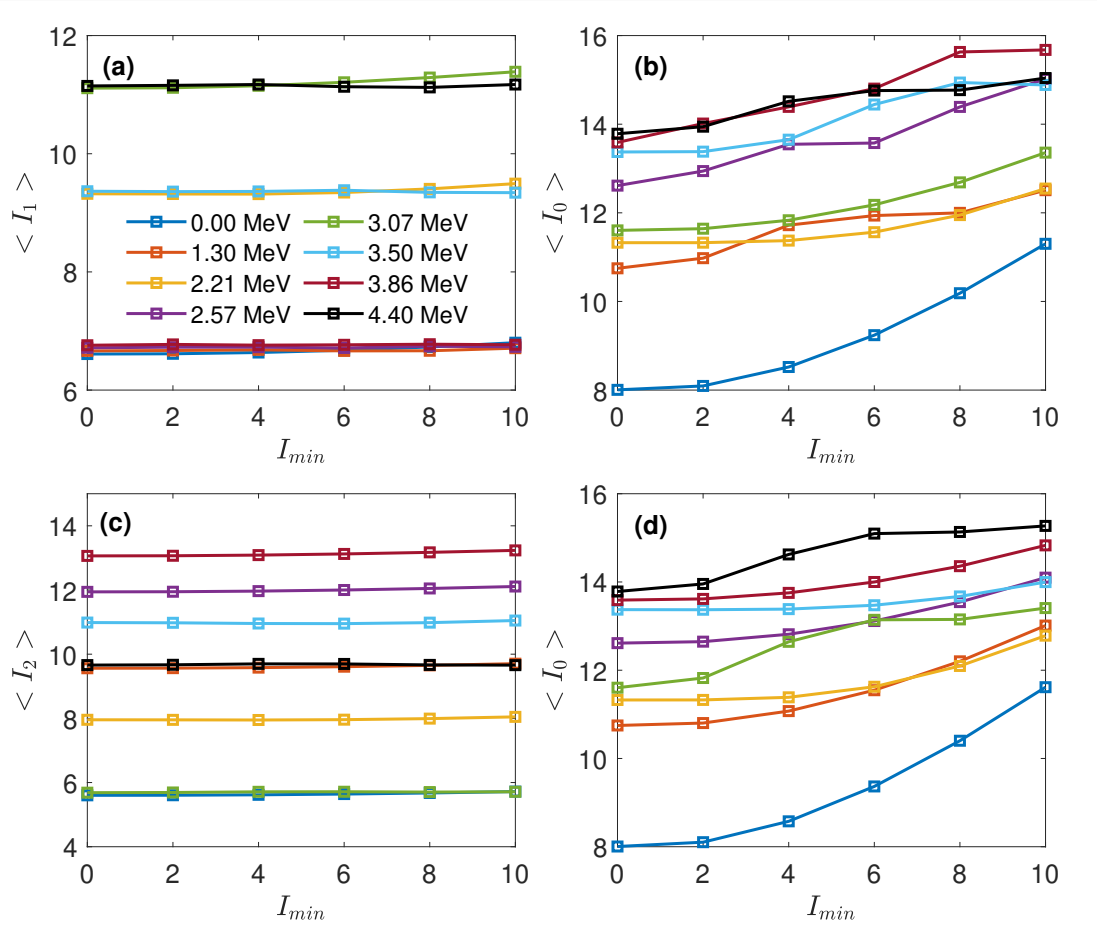
$\mathcal{P}(i_0 i_1 i_2) = \sum_{i_{12}} |a_{i_0 i_1 i_2 i_{12}}|^2$: Probability that angular momentum of first fragment is i_1, of second fragment is i_2, and relative angular momentum is i_0.

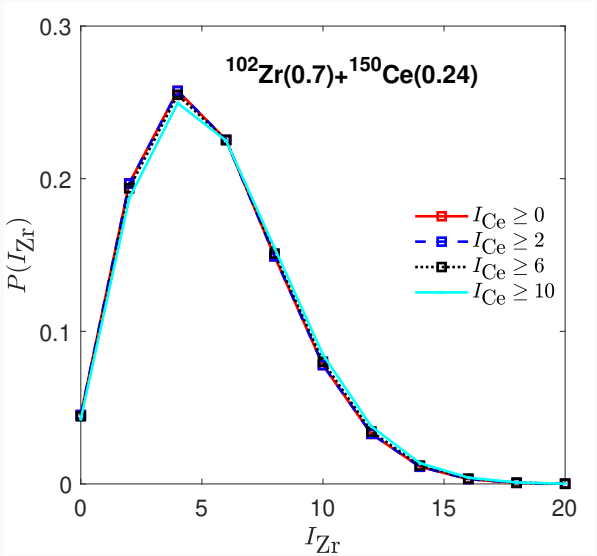
$$\langle I_i \rangle_n = \left[\sum_{I_0 I_1 I_2} I_i(I_i+1)\mathcal{P}(I_0 I_1 I_2)\right]^{1/2} \qquad i = 0, 1, 2$$



Correlation between fragments angular momenta

$$\langle I_1 \rangle_n = \left[\sum_{i_0, i_1, i_2 \geq I_{min}} |a^{(n)}_{i_0 i_1 i_2}|^2 i_1(i_1+1) \right]^{1/2}$$

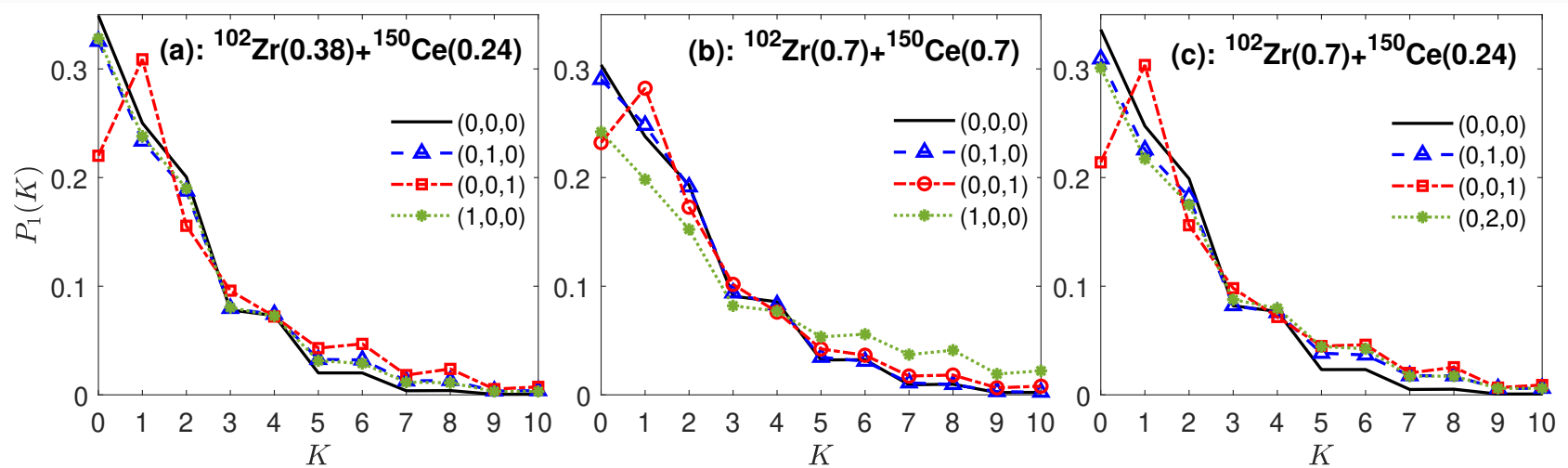




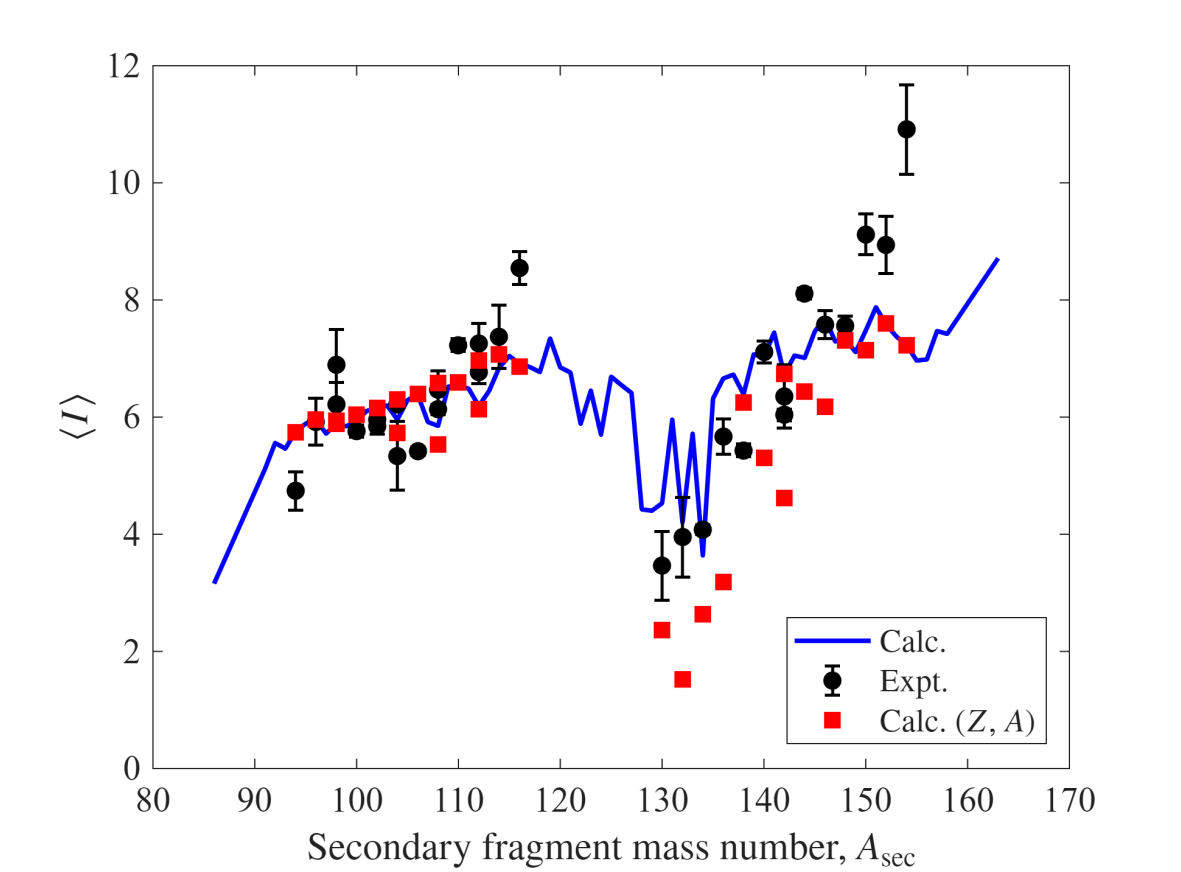
$\mathbf{I_0 + I_1 + I_2 = 0}, \quad \Im_0 \gg \Im_{1,2}$

K-distributions

$$P_n(K) = 2\sum_{i_0 i_1 i_2} |a^{(n)}_{i_0 i_1 i_2}|^2 |C^{i_0 0}_{i_1 K i_2 - K}|^2$$

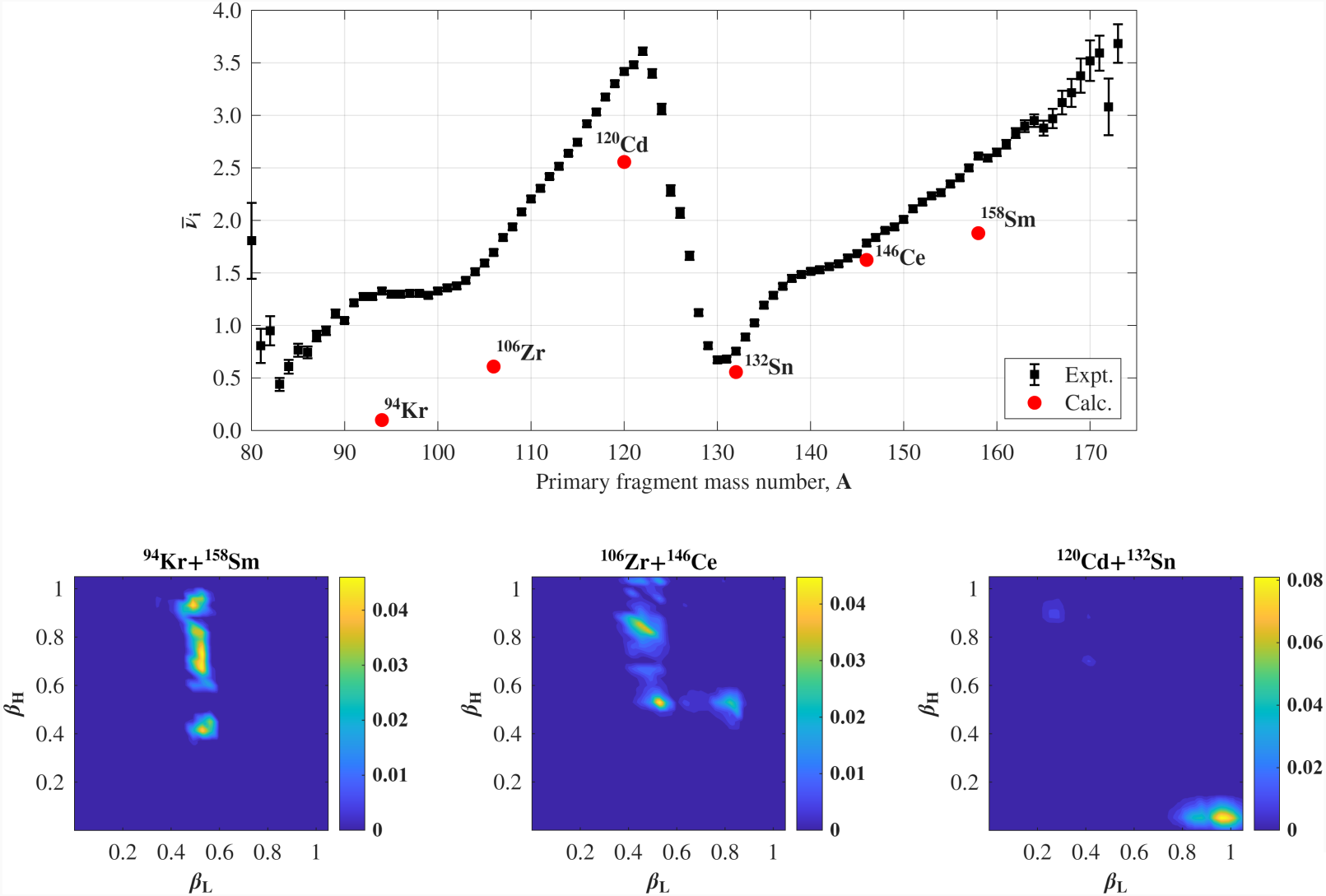


Angular momentum vs fragment mass

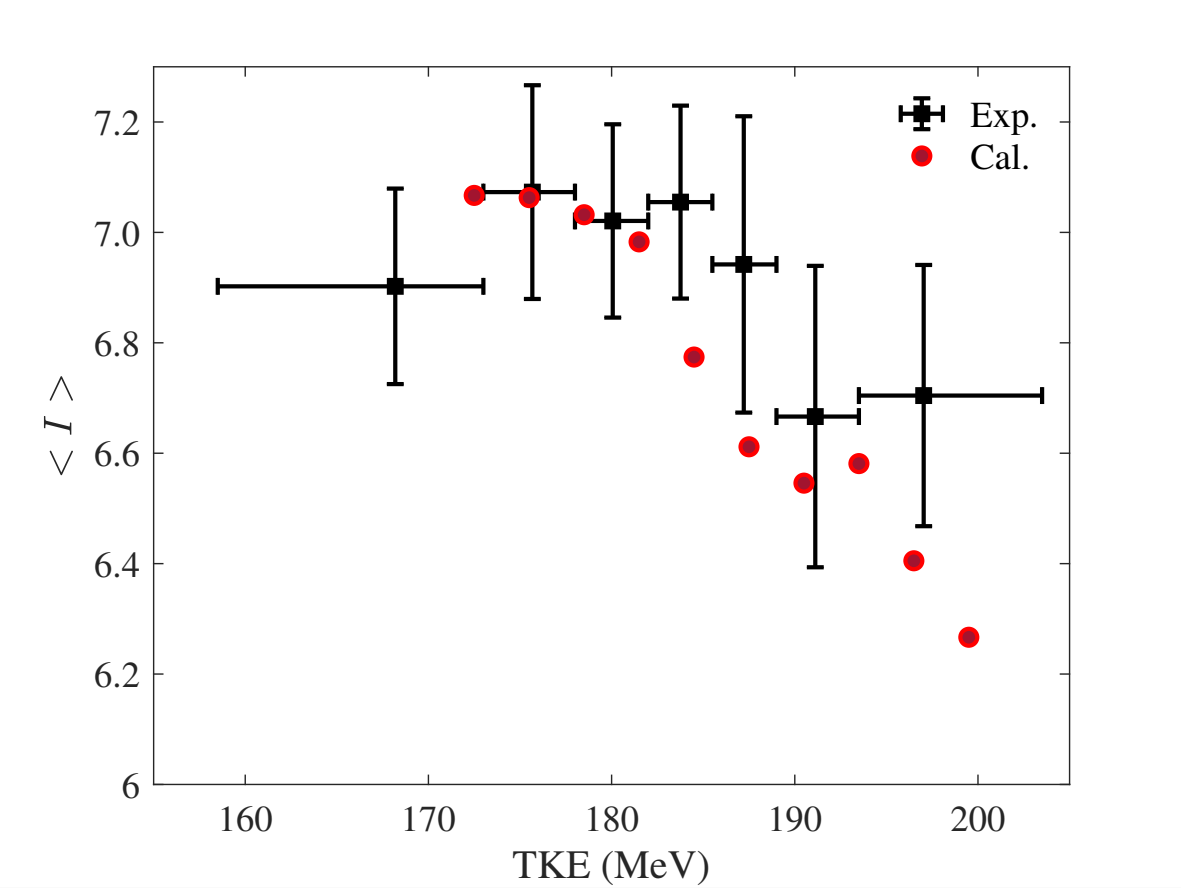


Expt. data from: J. N. Wilson et. al, Nature **590**, 566–570 (2021).

Decay-distributions of various DNS from SF of ^{252}Cf



Spin vs TKE: ^{144}Ba fragment of ^{252}Cf SF: PRC 112, 044610 (2025).



Average spins $\langle I \rangle$ of post-scission ^{144}Ba (red circles) as a function of total kinetic energy (TKE).

Exp. data (black squares) from N. P. Giha *et al.*, PRC **111** 014605 (2025).

Angular vibrations at scission

Kinetic energy ($\mathbf{I_0} + \mathbf{I_1} + \mathbf{I_2} = \mathbf{0}$):

$$T = \frac{\hbar^2(\hat{I}_1 + \hat{I}_2)^2}{2\mu R_m^2} + \frac{\hbar^2 \hat{I}_1^2}{2\Im_1} + \frac{\hbar^2 \hat{I}_2^2}{2\Im_2} = \frac{\hbar^2 \hat{I}_1^2}{2\Im_{b,1}} + \frac{\hbar^2 \hat{I}_2^2}{2\Im_{b,2}} + \frac{\hbar^2 \hat{I}_1 \hat{I}_2}{\mu R_m^2};$$

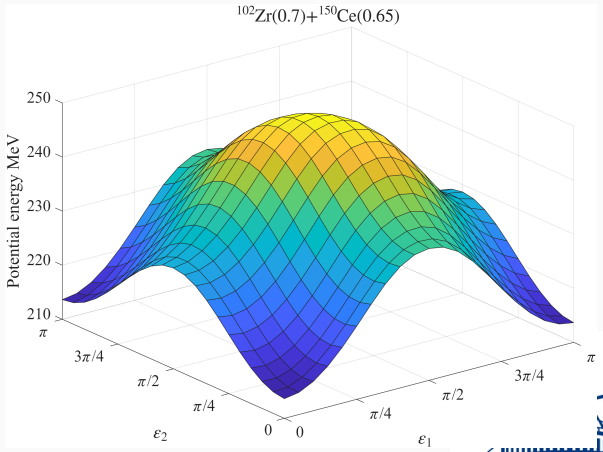
$$\Im_{bi} = \frac{\Im_i \mu R_m^2}{\Im_i + \mu R_m^2} \ll \mu R_m^2$$

For large enough deformations ($\beta_i \geqslant 0.15$) Potential energy:

$$V(\tilde{\Omega}_1, \tilde{\Omega}_2) \approx \frac{C_1}{2} \sin^2 \varepsilon_1 + \frac{C_2}{2} \sin^2 \varepsilon_2 + C_{12} \sin \varepsilon_1 \sin \varepsilon_2 \cos(\gamma_1 - \gamma_2)$$

$$V(\varepsilon_1', \varepsilon_2') \approx \frac{C_1}{2} \varepsilon_1'^2 + \frac{C_2}{2} \varepsilon_2'^2$$

$$(\varepsilon_i' = \varepsilon_i, \pi - \varepsilon_i)$$



Vibrations of two independent oscillators

Hamiltonian:

$$\hat{H} = \hat{H}_1 + \hat{H}_2 + \hat{H}_{coupl}$$

$$\hat{H}_i = -\frac{\hbar^2}{2\Im_{bi}}\left[\frac{1}{\varepsilon'_i}\frac{\partial}{\partial\varepsilon'_i}\varepsilon'_i\frac{\partial}{\partial\varepsilon'_i} + \frac{1}{\varepsilon'^2_i}\frac{\partial^2}{\partial\gamma_i^2}\right] + \frac{C_i}{2}\varepsilon'^2_i, \quad (i = 1, 2)$$

$$\hat{H}_{coupl} = \frac{\hbar^2\hat{\mathbf{I}}_1\hat{\mathbf{I}}_2}{\mu R_m^2} + C_{12}\varepsilon'^2_1\varepsilon'^2_2\cos(\gamma_1 - \gamma_2)$$

The eigenstates and eigenvalues of Hamiltonians $\hat{H}_{1,2}$

$$E_{n_iK_i} = \hbar\omega_i(2n_i + |K_i| + 1)$$

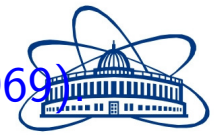
$$\phi_{n_iK_i}(\varepsilon'_i, \gamma_i) = N_{n_iK_i}\left(\frac{\varepsilon'^{K_i}_i}{\varepsilon_{0i}^{K_i+1}}\right)\exp\left(-\frac{\varepsilon'^2_i}{2\varepsilon^2_{0i}}\right)L_{n_i}^{(|K_i|)}\left(\frac{\varepsilon'^2_i}{\varepsilon^2_{0i}}\right)\exp\left(-iK_i\gamma_i\right)$$

$$\omega_i = (C_i/\Im_{bi})^{1/2}, \qquad \varepsilon^2_{0i} = \hbar/(C_i\Im_{bi})^{1/2}$$

Expansion in terms of spherical harmonics

$$\Phi_{n_iK_i}(\varepsilon_i, \gamma_i) = \sum_{I_i} a_{n_iK_i}(I)\,Y_{I_i,K_i}(\varepsilon_i, \gamma_i)$$

J. O. Rasmussen, W. Nörenberg, H. J. Mang, Nucl. Phys. A **136**, 465 (1969)



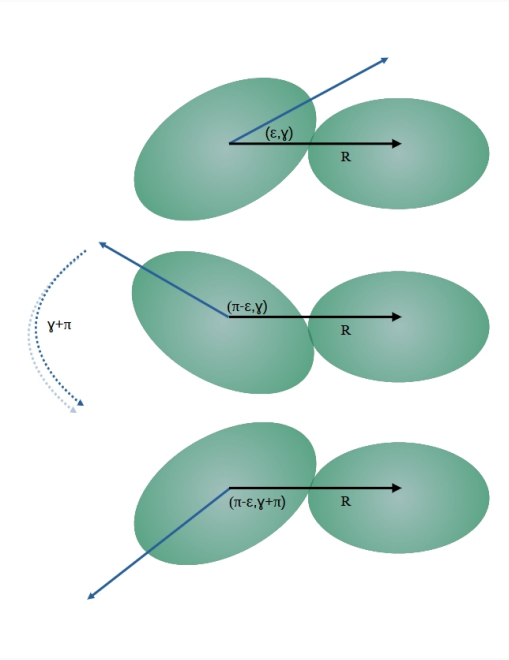
Symmetries

- $\Phi_{n_i K_i}(\varepsilon_i, \gamma_i) \propto exp(-iK_i\gamma_i) \rightarrow \quad K_1 = -K_2 = K$

- $(\varepsilon_i, \gamma_i) \longrightarrow (\pi - \varepsilon_i, \gamma_i + \pi)$

  Symmetrized solution:

$$\Phi^{\pm}_{n_i K_i}(\varepsilon_i, \gamma_i) \sim \phi_{n_i K_i}(\varepsilon_i, \gamma_i) \pm \phi_{n_i K_i}(\pi - \varepsilon_i, \gamma_i + \pi)$$



- $Y_{l_i K_i}(\pi - \varepsilon_i, \gamma_i + \pi) = (-1)^{l_i} Y_{l_i K_i}(\varepsilon_i, \gamma_i)$

$$\Phi^{+}_{n_i K_i}(\varepsilon_i, \gamma_i) = \sum_{\text{even } l_i} a_{n_i l_i K_i} Y_{l_i, |K_i|}(\varepsilon_i, \gamma_i),$$

$$\Phi^{-}_{n_i K_i}(\varepsilon_i, \gamma_i) = \sum_{\text{odd } l_i} a_{n_i l_i K_i} Y_{l_i, |K_i|}(\varepsilon_i, \gamma_i).$$

$\phi^{-}_{n_i K_i}$ is absent for the case of quadrupole deformed fragments.



Total wave function

$$\Psi_{n_1,K_1,n_2,K_2} = \sum_{even l_1,l_2} a^{(1)}_{n_1K_1} a^{(2)}_{n_2K_2} Y_{l_1,K_1}(\varepsilon_1,\gamma_1) Y_{l_2,K_2}(\varepsilon_2,\gamma_2)$$

$$E_{n_1,n_2,K} = \hbar\omega_1(2n_1 + K + 1) + \hbar\omega_2(2n_2 + K + 1)$$

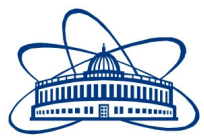
Probabilities of various angular momenta of fragments are independent on each other.

For the GS: $P^0_i(I) = |a^{(i)}(I)|^2 = 2\varepsilon^2_{0i}(2I+1)\exp\left\{-(I+\frac{1}{2})^2\varepsilon^2_{0i}\right\}$

$$\omega_i = (C_i/\Im_{bi})^{1/2}, \qquad \varepsilon^2_{0i} = \hbar/(C_i\Im_{bi})^{1/2}$$

Average angular momenta of the fragments:

$$\langle I^2_{i\perp}\rangle_0 = 1/\varepsilon^2_{0i}, \qquad \mathrm{Cov}(I_1, I_2) = 0$$



Mixing by kinetic and potential coupling

The coupling contains both kinetic and potential contributions:

$$\hat{H}_{\mathrm{coupl}} = \frac{\hbar^2}{I_R}\hat{I}_1 \cdot \hat{I}_2 + C_{12}\varepsilon_1\varepsilon_2 \cos(\gamma_1 - \gamma_2).$$

At first order, the ground state mixes only with the symmetric combination of $m = \pm 1$ states:

$$\boxed{\left\langle \hat{I}_1 \cdot \hat{I}_2 \right\rangle = \frac{I_{b1} I_{b2}}{I_{b1}\varepsilon_{01}^2 + I_{b2}\varepsilon_{02}^2} \left(\frac{C_{12}\varepsilon_{01}^2\varepsilon_{02}^2}{\hbar^2} - \frac{1}{I_R} \right)}$$

Potential dominated



$\langle I_1 \cdot I_2 \rangle > 0$

Wriggling-like correlation

Kinetic dominated



$\langle I_1 \cdot I_2 \rangle < 0$

Bending-like correlation

Physical interpretation

Wriggling-like and bending-like correlations indicate **weak preferences for parallel or antiparallel fragment spins**, rather than pure excited collective modes.

Opening angle between fragment spins

$$\boxed{\cos\theta_{l_1 l_2 l_0} = \frac{l_0(l_0+1) - l_1(l_1+1) - l_2(l_2+1)}{2\sqrt{l_1(l_1+1)l_2(l_2+1)}}.}$$

Discrete quantum-angle distribution

$$p(\theta) = \frac{1}{Q} \sum_{l_1, l_2 > 0} \sum_{l_0} P(l_1, l_2, l_0)\, \delta(\theta - \theta_{l_1 l_2 l_0}), \quad Q = \sum_{l_1, l_2 > 0, l_0} P.$$

Q excludes channels with $l_1 = 0$ or $l_2 = 0$, where the opening angle is undefined. This spectral angle is not a simultaneous measurement of three spin components.

For the uncoupled ground state:

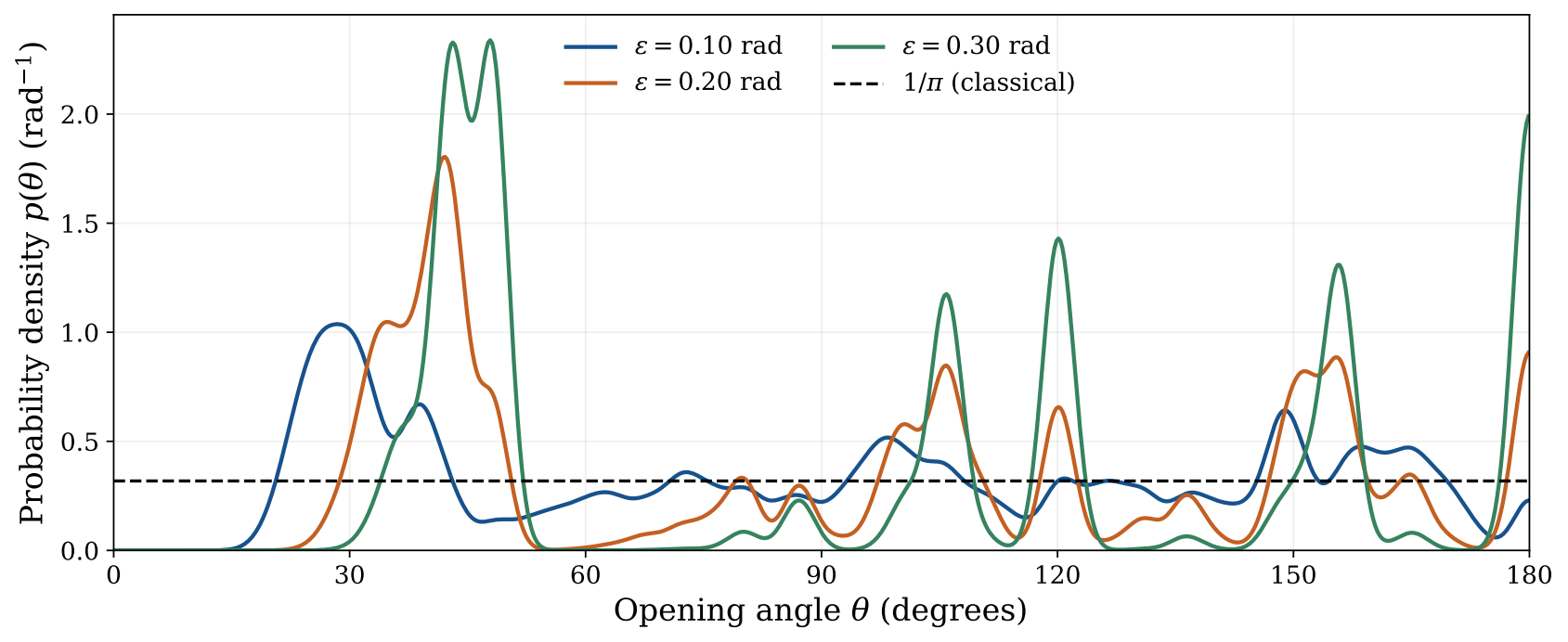
$$P_0(l_1, l_2, l_0) = P_1^{(0)}(l_1) P_2^{(0)}(l_2) \left| \langle l_1 0, l_2 0 \mid l_0 0 \rangle \right|^2 .$$



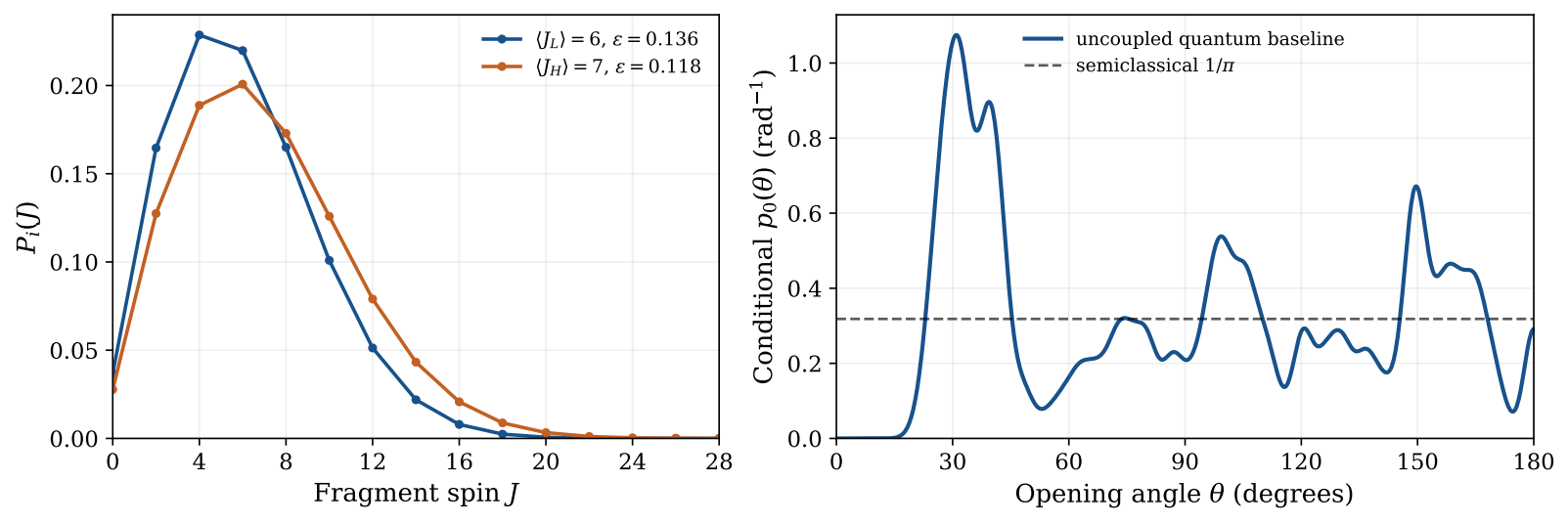
Opening angle distribution

Large-spin limit:

$$\boxed{p_0^{\mathrm{cl}}(\theta) = \frac{1}{\pi}, \qquad \langle\theta\rangle^{\mathrm{cl}} = \frac{\pi}{2}}$$



From measured fragment spins to opening angles



Illustrative example: $\langle J_L \rangle = 6$ and $\langle J_H \rangle = 7$ are representative values, not a specific mass split from Wilson *et al.* Gaussian smoothing (2.5°) is applied only to visualize the discrete opening-angle distribution.

Proposed experimental connection

Measured $\langle J_{L,H}(A) \rangle$ or $P_{L,H}(J) \longrightarrow$ Harmonic widths $\varepsilon_{L,H} \longrightarrow$ Predicted $P(\theta)$

Fragment-spin measurements provide a model-based prediction of the opening-angle distribution, without fitting opening-angle data.

Summary

- We propose a model based on the **DNS approach** to explain the observed near-absence of correlations between the angular momenta of FF.
- Combined with a description of primary FF distributions, the model successfully reproduces experimental trends:
  - The sawtooth dependence of fragment angular momentum on mass.
  - Correlations between TKE and fragment angular momentum.
- Near scission, the fragments undergo approximately **independent angular vibrations** around the pole–pole configuration, resulting in weak spin correlations.
- The harmonic description connects measured fragment-spin distributions with the **opening-angle distribution**, providing a new probe of **collective dynamics at scission**.

Thank You!

