Dark Matter and R–P scaling relations in neutron stars

Implications for Extracting the Nuclear Equation of State

Zhi-yao Li, Wei-zhou Jiang, Dian Sun

School of Physics, Southeast University, Nanjing, China

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Outline

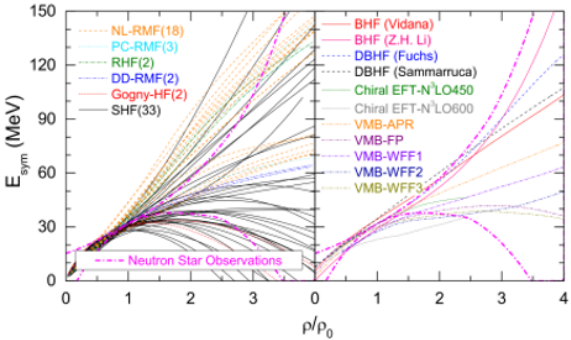
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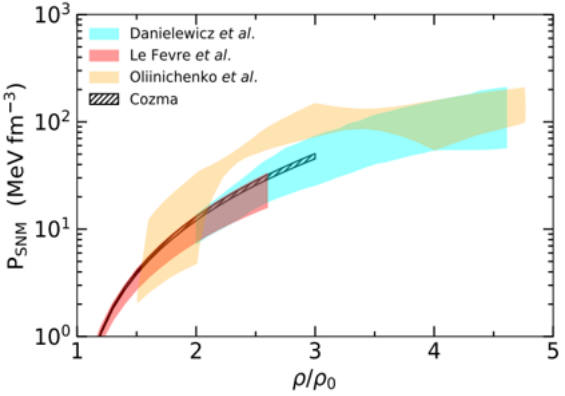
Nuclear Equation of State and Its Uncertainties

The nuclear EOS encodes symmetric nuclear matter EOS and symmetry energy.

Both carry substantial uncertainties:

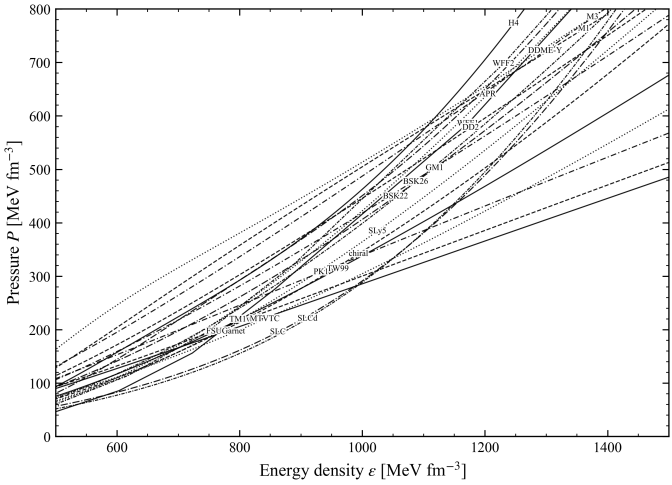


Courtesy of Dr. Chen



Courtesy of Dr. Wang

Equations of State for β-Equilibrated NS Matter



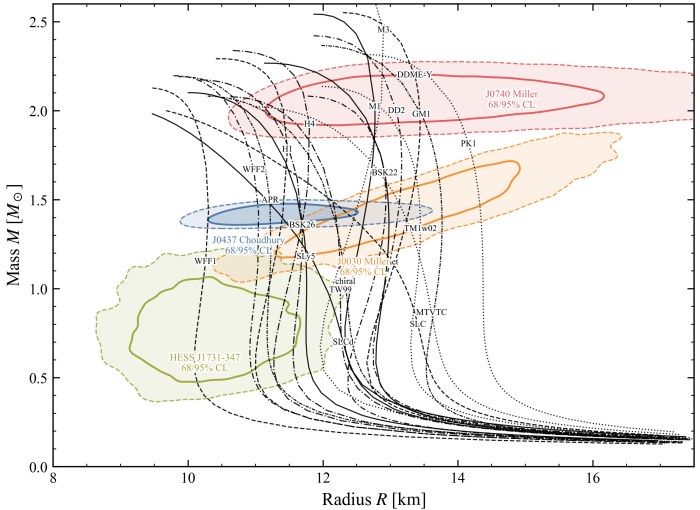
- Many-body methods: Brueckner, Green's functions, RMF, density functionals, chiral EFT.
- Compositions: $e + \mu +$ nucleons $+$ (hyperons...)

Pressure P versus energy density ε for the 21 nuclear EOSs.

Why Neutron Stars?

- Natural laboratories for dense matter physics.
- Core density up to several times $\rho_0 \simeq 0.16\ \mathrm{fm}^{-3}$.
- Observables (M, R, tidal deformability, cooling) encode the EOS.
- Radius is governed by the symmetry energy near ρ_0; maximum mass probes high-density stiffness.

See next page for the mass–radius trajectories.



Mass–radius trajectories of the 21 nuclear EOSs, with NICER and GW170817 constraints shown as shaded contours.

Analytic Stellar Models for the R–P Relation

Three classic analytic solutions are often used to derive the scaling relation:

1. Buchdahl solution [Astrophys. J. **147**, 310 (1967)]

Metric: $e^{-\lambda(r)} = \frac{1-2\beta}{1-\beta+u(r)}$, with $u(r) = \frac{\beta \sin(Ar)}{Ar}$.
Energy density: derived from the Buchdahl EOS $\varepsilon = 12\sqrt{p_* P} - 5P$.

2. Tolman VII solution [Phys. Rev. **55**, 364 (1939)]

Metric: $e^{-\lambda(r)} = 1 - \beta \left(\frac{r}{R}\right)^2 \left[5 - 3\left(\frac{r}{R}\right)^2\right]$.
Energy density: $\varepsilon(r) = \varepsilon_c \left[1 - \left(\frac{r}{R}\right)^2\right]$.

3. Finch–Skea solution [Class. Quantum Grav. **6**, 467 (1989)]

Metric: $e^{\lambda(r)} = 1 + \frac{r^2}{R^2}$ (Class I embedding).
Energy density: $\varepsilon(r) = \frac{3 + r^2/R^2}{8\pi R^2 (1 + r^2/R^2)^2}$.

Buchdahl-Lattimer–Prakash Scaling Relation

For normal neutron stars, at fixed mass,

$$R \propto P^{1/4},$$

where P is the nuclear-matter pressure near saturation density [Astrophys. J. **550**, 426 (2001)].

Analytic verification from the Buchdahl solution [Astrophys. J. **147**, 310 (1967)]:

$$\varepsilon = 12\sqrt{p_* P} - 5P,$$

$$\eta = \left.\frac{d\ln R}{d\ln P}\right|_{\varepsilon,M} = \frac{1}{2}\left(1 - \frac{5}{6}\sqrt{\frac{P}{p_*}}\right)\frac{(1-\beta)(1-2\beta)}{1-3\beta+3\beta^2},$$

with $\beta = GM/(Rc^2)$. For typical NS values, $\eta \approx 0.31$, close to $1/4$.

Dark Matter Admixed Neutron Stars (DANSs)

- DM interacts with normal matter (NM) only through gravity.
- Two-fluid TOV equations [Xiang et al. PRC **89**, 025803 (2014)] :

$$\frac{dP_i}{dr} = -\frac{(\varepsilon_i + P_i)\left[4\pi r^3 P + M\right]}{r\,(r-2M)}, \qquad \frac{dM_i}{dr} = 4\pi r^2 \varepsilon_i,$$
$$M = M_{\rm NM} + M_{\rm DM}, \qquad P = P_{\rm NM} + P_{\rm DM}.$$

- Each fluid has its own surface: $R_{\rm NM}$ and $R_{\rm DM}$.
- Observable radius: $R_{\rm NM}$.
- DM changes the common gravitational field and the NM distribution.

DM Equations of State

Fermionic DM: scalar attraction + vector repulsion

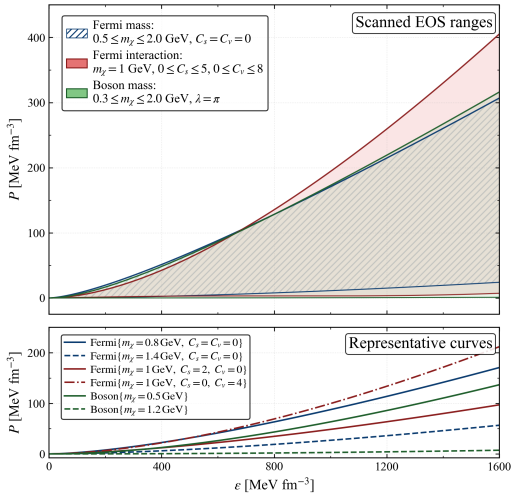
$$C_s = \frac{g_s}{m_s}, \qquad C_v = \frac{g_v}{m_v}, \qquad 500 \leq m_\chi \leq 2000 \ \mathrm{MeV}.$$

Bosonic DM: repulsive self-interacting scalar field (BEC)

$$P = \frac{m_\chi^4}{9\lambda}\left(\sqrt{1+\frac{3\lambda}{m_\chi^4}\varepsilon}-1\right)^2, \qquad 300 \leq m_\chi \leq 2000 \ \mathrm{MeV}.$$

- Fermionic: C_s (attraction), C_v (repulsion), $m_s = m_v = 1 \ \mathrm{GeV}$.
- Bosonic: $\lambda = \pi$ and 4π.

Scanned DM Equations of State



Pressure ranges covered by the fermionic mass scan, fermionic self-interaction scan, and bosonic mass scan.

Two-Fluid Extension of the Buchdahl Solution

Treat the two fluids as an effective isotropic fluid:

$$\varepsilon_t = 12\sqrt{p_{*,t}(P_{\rm NM} + P_{\rm DM})} - 5(P_{\rm NM} + P_{\rm DM}).$$

Total-fluid boundary:

$$R_t = (1-\beta_t)\left[\frac{\pi}{288 p_{*,t}(1-2\beta_t)}\right]^{1/2}, \qquad \beta_t = \frac{M}{R_t}.$$

Define

$$\gamma = \frac{R_{\rm NM}}{R_t}, \qquad \beta = \frac{M}{R_{\rm NM}}, \qquad \beta_t = \gamma\beta.$$

Scaling exponent for the visible NM component:

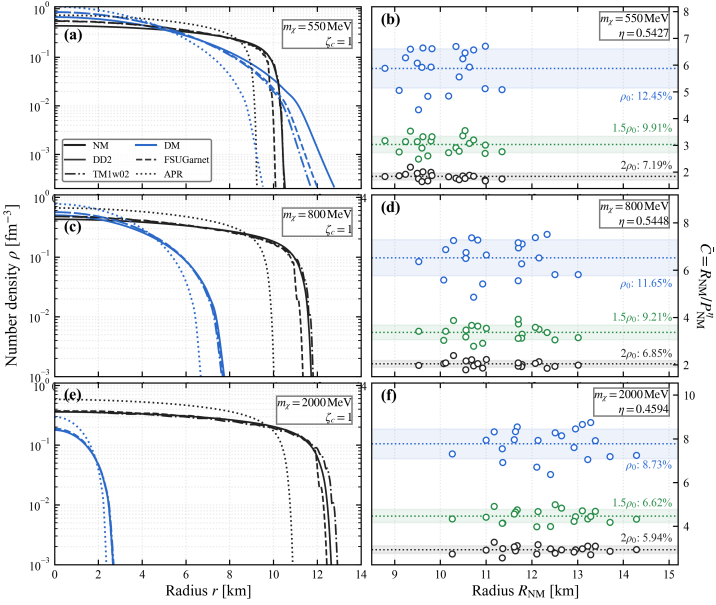
$$\eta = \frac{1}{2}\frac{P_{\rm NM}}{P_{\rm NM}+P_{\rm DM}}\left(1 - \frac{5}{6}\sqrt{\frac{P_t}{p_{*,t}}}\right)\frac{(1-\gamma\beta)(1-2\gamma\beta)}{1-3\gamma\beta+3\gamma^2\beta^2}.$$

Numerical Procedure

- Solve two-fluid TOV for 21 nuclear EOSs and selected DM models.
- Fit $R_{\rm NM} = \bar{C} P_{\rm NM}^{\eta}$ at ρ_0, $1.5\rho_0$, $2\rho_0$.
- For each trial η, define

$$C_i(\rho_k, \eta) = \left. \frac{R_{{\rm NM},i}}{P_{{\rm NM},i}^{\eta}} \right|_{\rho_k}, \qquad C_i = a_k R_{{\rm NM},i} + b_k.$$

- Choose η that minimizes $\sum_k a_k^2(\eta)$.
- $\bar{C}(\rho_k)$ is the sample mean of C_i.



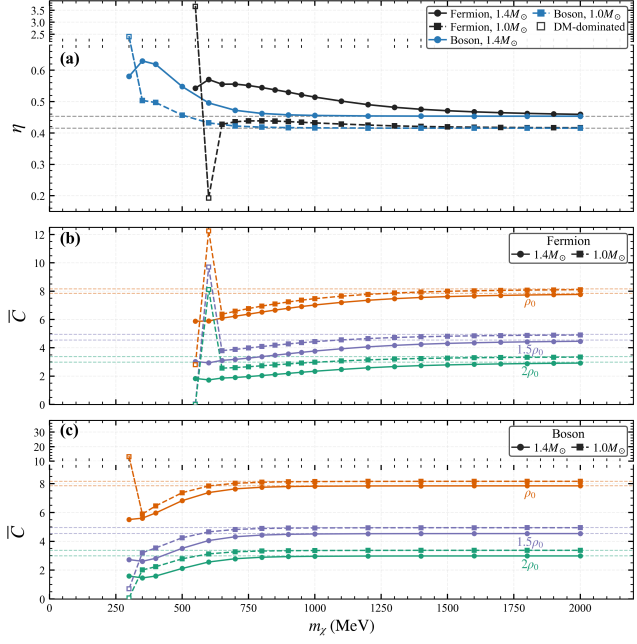
DM Mass and Radial Distribution

$m_\chi = 550$ MeV: extended DM halo;
$m_\chi = 800$ MeV: transition region;
$m_\chi = 2000$ MeV: compact dark core.

Results: R–P Scaling Fit

- $1.4M_\odot$ DANSs: $\eta = 0.5427,\ 0.5448,\ 0.4594$ for $m_\chi = 550,\ 800,\ 2000\ \mathrm{MeV}$.
- Scatter at $2\rho_0$ is notably smaller than at ρ_0.
- Reason: at higher density, the symmetry-energy slope contributes less relative to symmetric nuclear matter.

$$P(\rho) = \left(\frac{\rho}{\rho_0}\right)^2 \left[\frac{K_0}{9}(\rho - \rho_0) - \frac{J_0}{54\rho_0}(\rho - \rho_0)^2\right] + \frac{\rho}{3}L(\rho)\alpha^2 + \rho E_{\mathrm{sym}}(\rho)x_p\alpha.$$



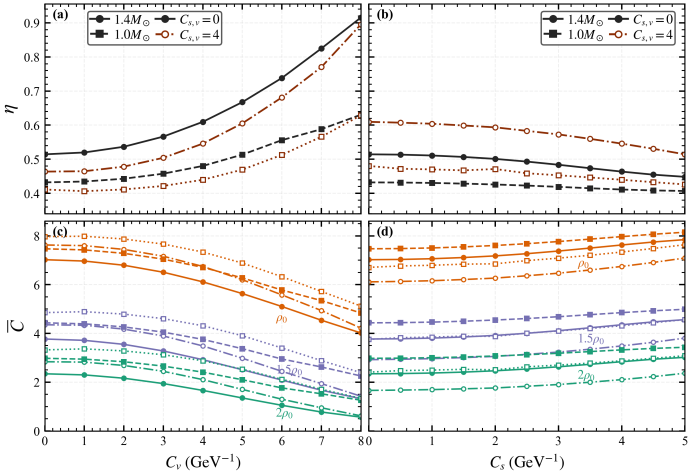
Fermionic vs. Bosonic DM: Scaling Parameters

Scaling exponent η and coefficients $\bar{C}$ as functions of the DM particle mass for $1.0M_\odot$ and $1.4M_\odot$ DANSs.

Results: DM Content (Fermionic DANSs)

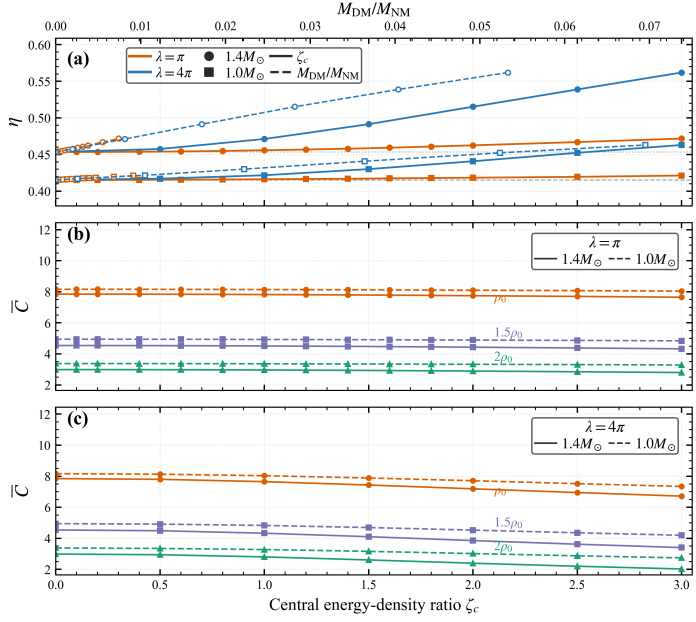
ζ_c	$R_{\rm NM}/R_{\rm DM}$ (km)	$M_{\rm DM}/M_{\rm NM}$	$\bar{C}(2\rho_0)$	η
0.0 (No DM)	13.07/0.00	0.000	3.375	0.4153
0.1	13.04/4.19	0.002	3.365	0.4157
0.2	12.98/5.06	0.007	3.341	0.4166
0.3	12.90/5.59	0.015	3.311	0.4176
0.4	12.81/5.95	0.025	3.273	0.4190
0.5	12.72/6.22	0.036	3.230	0.4206
0.6	12.62/6.43	0.048	3.184	0.4225
0.7	12.51/6.58	0.060	3.131	0.4249
0.8	12.41/6.70	0.074	3.077	0.4275
0.9	12.30/6.80	0.087	3.026	0.4299
1.0	12.19/6.87	0.101	2.978	0.4320
1.2	11.98/6.97	0.130	2.886	0.4363
1.4	11.78/7.02	0.159	2.820	0.4381
1.6	11.60/7.03	0.188	2.746	0.4414
1.8	11.45/7.02	0.216	2.648	0.4481
2.0	11.32/6.99	0.244	2.551	0.4554
2.5	11.01/6.86	0.310	2.392	0.4657
3.0	10.66/6.74	0.371	2.395	0.4565

$m_\chi = 1\ \mathrm{GeV}$, no DM self-interaction.



Fermionic DM Self-Interaction

Vector repulsion C_v raises η and lowers $\bar{C}$; scalar attraction C_s has the opposite effect. Attraction and repulsion partially cancel.



Bosonic DM Content and Self-Interaction

BDM scaling exponent increases monotonically with ζ_c. Stronger repulsion ($\lambda = 4\pi$) enhances the change in $M_{\rm DM}/M_{\rm NM}$ and shifts η, $\bar{C}$.

Typical DM Signals in the R–P Relation

- Systematic shifts in η and $\bar{C}$ relative to the pure-nuclear baseline.
- Non-monotonic behaviour: maximum in η, minimum in $\bar{C}$.
- DM-dominated, radially extended configurations:
  - Fermionic: $m_\chi \sim 550$–$600\ \mathrm{MeV}$.
  - Bosonic: $m_\chi \sim 300\ \mathrm{MeV}$.
  - R_{NM} can reach tens or even $\sim 100\ \mathrm{km}$.
  - Central NM density falls below the crust–core transition; uniform-matter scaling breaks down.
- Strong repulsion can smear out nuclear-EOS differences and decouple the scaling relation.

Impact on Extracting the Nuclear EOS

- If DM is ignored, the pure-nuclear $R \propto P^{1/4}$ relation can bias the extracted EOS parameters.
- DM shifts R_{NM}, η, and $\bar{C}$.
- Degeneracy:
  - Soft EOS + DM can mimic a stiff EOS.
  - Stiff EOS + DM can mimic a soft EOS.
- Therefore:
  - Combine multiple messengers (NICER, GW170817, mass measurements).
  - Low-mass, large-radius DANSs are promising discriminators.
  - Tighter laboratory constraints on nuclear pressure help break the degeneracy.

Summary and Outlook

- Extended the Buchdahl solution to a two-fluid system and derived the analytic R–P scaling exponent for DANSs.
- Numerically fitted $R_{\rm NM} = \bar{C} P_{\rm NM}^{\eta}$ using 21 nuclear EOSs for $1.0 M_\odot$ and $1.4 M_\odot$ DANSs.
- DM mass, content, and self-interaction all shift η and $\bar{C}$.
- Fermionic and bosonic DM show different mass dependence.
- Low-mass DM can produce DM-dominated, extended configurations where the conventional scaling relation fails.
- Implication: DM can bias the extraction of the nuclear EOS from radius measurements; multi-messenger analyses are essential.

Thank you for your attention!

arXiv:2610.06934 [nucl-th]

Backup: Key Formulas

- Buchdahl EOS: $\varepsilon = 12\sqrt{p_* P} - 5P$.
- Single-fluid scaling exponent: $\eta = \frac{1}{2}\left(1 - \frac{5}{6}\sqrt{P/p_*}\right)\frac{(1-\beta)(1-2\beta)}{1-3\beta+3\beta^2}$.
- Two-fluid scaling exponent: $\eta = \frac{1}{2}\frac{P_{\rm NM}}{P_t}\left(1 - \frac{5}{6}\sqrt{P_t/p_{*,t}}\right)\frac{(1-\gamma\beta)(1-2\gamma\beta)}{1-3\gamma\beta+3\gamma^2\beta^2}$.
- Fitting relation: $R_{\rm NM} = \bar{C} P_{\rm NM}^{\eta}$.