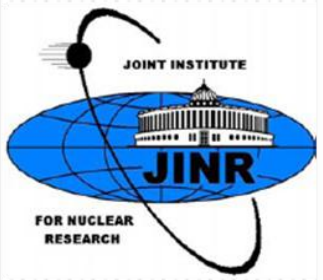
A microscopic shell-model counterpart of the particle-core-type models: the low-lying rotational bands in ^{163}Dy

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Many-particle nuclear system

Jacobi: $\{x_{is}, p_{jt} = -i\hbar\frac{\partial}{\partial x_{jt}}\}, \quad i,j = 1,2,3; s,t = 1,\ldots,m = A-1$

$$[x_{is}, p_{jt}] = i\hbar\delta_{ij}\delta_{st}I \quad \Rightarrow \quad HW(3m) = \{x_{is}, p_{jt}, I\}$$

$$HW(3m) = \{b^{\dagger}_{is}, b_{jt}, I\}$$

$$\begin{aligned} Q_{is,jt} &= x_{is}x_{jt}, \\ S_{is,jt} &= x_{is}p_{jt} + p_{is}x_{jt}, \\ L_{is,jt} &= x_{is}p_{jt} - x_{jt}p_{is}, \\ T_{is,jt} &= p_{is}p_{jt} \end{aligned}$$

Sp(6(A-1),R) DG of the whole nuclear system

U(3(A-1)) ⊂ Sp(6(A-1),R)

$$Sp(6(A-1),R) \equiv \{F_{is,jt} = b^{\dagger}_{is}b^{\dagger}_{jt}, G_{is,jt} = b_{is}b_{jt}, A_{is,jt} = \tfrac{1}{2}\left(b^{\dagger}_{is}b_{jt} + b_{jt}b^{\dagger}_{is}\right)\}$$

✓ Sp(6(A-1),R) – collective, s.p., cluster, etc., motions

Collective and intrinsic coordinates:

$Sp(6(A-1)) \supset G_{dyn} \otimes G_{kin} \Leftrightarrow \{x\} = \{x_D, x_K\}$ – dynamical (collective) and kinematical (intrinsic) variables

$$\Psi(x) = \sum_{\eta} \Theta_{\eta}(x_D)\Phi_{\eta}(x_K)$$

Different reductions of Sp(6(A-1),R)

✓ Sp(6(A-1),R) – collective, s.p., cluster, etc., motions

1) Symplectic Sp(6,R) model of Rowe and Rosensteel

$$Sp(6(A-1),R) \supset Sp(6,R) \otimes O(A-1)$$

G. Rosensteel and D. J. Rowe, Phys. Rev. Lett. 38, 10 (1977)

2) Symplectic symmetry approach to clustering (SSAC)

$$Sp(6(A-1),R) \supset Sp(6,R)_R \otimes Sp(6,R)_C \otimes O(A_1-1) \otimes \ldots \otimes O(A_k-1)$$

HGG, Commun. Theor. Phys. 77, 055301 (2025); Nucl. Phys. A 1062, 123162 (2025)

3) Microscopic shell-model counterpart of the particle-core-type models

$$Sp(6(A-1),R) \supset Sp(6,R)_{core} \otimes Sp(6,R)_{sp} \otimes O(A-2) \otimes O(1)$$

HGG, Nucl. Phys. A 1072, 123415 (2026)

Symplectic particle-core model (SPCM)

$\{\mathbf{x}_1, \ldots, \mathbf{x}_{A-1}\} \longrightarrow \{\mathbf{x}_1, \ldots, \mathbf{x}_{A-2}\}, \{\mathbf{x} \equiv \mathbf{x}_{s=A-1}\}$ – intrinsic states of the core and s.p.

SPCM:

$$Sp(6(A-1),R)$$

$$\supset Sp(6(A-2),R)_{core} \otimes Sp(6,R)_{sp}$$

$$\supset Sp(6,R)_{core} \otimes Sp(6,R)_{sp} \otimes O(A-2) \otimes O(1)$$
$$\langle\sigma^{core}\rangle \qquad \langle\sigma^{sp}\rangle \qquad \omega^{core} \qquad \omega^{sp}$$

$$\supset U(3)_{core} \otimes U(3)_{sp} \otimes S_{A-1} \otimes S_1$$
$$[E_1^{core}, E_2^{core}, E_3^{core}] \quad [\mathcal{N},0,0] \quad f_{A-1} \quad f_1$$

$$\supset U(3) \supset SO(3),$$
$$[E_1, E_2, E_3] \quad \kappa \quad L$$

Limiting case:

- scalar $Sp(6,R)_{sp}/SU(3)_{sp}$ IR's (no separation of the s.p. d.f.);

$\Rightarrow$ Sp(6,R) model /Rowe & Rosensteel/

HGG, Nucl. Phys. A 1072, 123415 (2026)

$$O(A-2) \otimes O(1) \supset S_{A-1} \otimes S_1$$

– proper permutational symmetry allowed by the Pauli principle

Basis:

$$|\Gamma[f]; (\lambda_{core}, \mu_{core}), (\mathcal{N},0); (\lambda,\mu); \omega^{core} f_{A-1}; \omega^{sp} f_1; \kappa L\rangle$$

Symplectic particle-core model (SPCM)

Core-subsystem:

$$b^{\dagger}_{is} = \sqrt{\frac{M\omega}{2\hbar}}\left(x_{is} - \frac{i}{M\omega}p_{is}\right),$$
$$b_{is} = \sqrt{\frac{M\omega}{2\hbar}}\left(x_{is} + \frac{i}{M\omega}p_{is}\right)$$

$(s,t = 1,2,\ldots,A-2)$

SP-subsystem:

$$b^{\dagger}_{i} = \sqrt{\frac{M\omega}{2\hbar}}\left(x_{i} - \frac{i}{M\omega}p_{i}\right),$$
$$b_{i} = \sqrt{\frac{M\omega}{2\hbar}}\left(x_{i} + \frac{i}{M\omega}p_{i}\right)$$

M – nucleon mass

$$Sp(6,R)_{core} \otimes Sp(6,R)_{sp} =$$

$$\{F^{core}_{ij} = \sum_{s=1}^{A-2} b^{\dagger}_{is}b^{\dagger}_{js},\ G^{core}_{ij} = \sum_{s=1}^{A-2} b_{is}b_{js},\ A^{core}_{ij} = \tfrac{1}{2}\sum_{s=1}^{A-2}\left(b^{\dagger}_{is}b_{js} + b_{js}b^{\dagger}_{is}\right),$$

$$F^{sp}_{ij} = b^{\dagger}_{i}b^{\dagger}_{j},\ G^{sp}_{ij} = b_{i}b_{j},\ A^{sp}_{ij} = \tfrac{1}{2}\left(b^{\dagger}_{i}b_{j} + b_{j}b^{\dagger}_{i}\right)\}$$

HGG, Nucl. Phys. A 1072, 123415 (2026)

Hamiltonian: $H = H_0 + V(F,G,A) + H_{res}$

Spin-isospin many-particle wave functions

$$Sp(8(A-1),R)_{ST}$$

$$\supset Sp(8(A-2),R)_{ST} \otimes Sp(8,R)_{st}$$

$$\supset \underset{\langle\sigma^{core}_{ST}\rangle}{Sp(8,R)_{ST}} \otimes \underset{\langle\sigma^{sp}_{st}\rangle}{Sp(8,R)_{st}} \otimes \underset{\omega^{core}_{ST}}{O(A-2)_{ST}} \otimes \underset{\omega^{sp}_{st}}{O(1)_{st}}$$

$$\supset \underset{\widetilde{f}_{A-1}}{S^{(\sigma\tau)}_{A-1}} \otimes \underset{\widetilde{f}_{1}}{S^{(\sigma\tau)}_{1}},$$

$$|\widetilde{\Gamma}[\widetilde{f}];\widetilde{S}M_{\widetilde{S}}\widetilde{T}M_{\widetilde{T}}\rangle = \sum_{\substack{SM_STM_T \\ sm_stm_t}} |\widetilde{\Gamma}_2[\widetilde{f}_{A-1}];SM_S;TM_T\rangle\ |\widetilde{\Gamma}_1[\widetilde{f}_1];sm_s;tm_t\rangle C^{[\widetilde{f}_{A-1}]\ \ [\widetilde{f}_1]\ \ [\widetilde{f}]}_{SM_STM_T\ \ sm_stm_t\ \ \widetilde{S}\widetilde{M}_S\widetilde{T}\widetilde{M}_T}$$

$$|\Gamma\widetilde{\Gamma};(\lambda_{core},\mu_{core}),(\mathcal{N},0);(\lambda,\mu);\omega^{core}(f_{A-1}\widetilde{f}_{A-1})a;\omega^{sp}(f_1\widetilde{f}_1)a;\kappa L(Ss)\widetilde{S};J\rangle$$

Application: ^{163}Dy system

Rotational bands: $K^\pi_J = 5/2^-,\ K^\pi_J = 1/2^-,\ K^\pi_J = 5/2^+$

Nilsson levels: $[523]5/2^-,\ [521]1/2^-,\ [642]5/2^+$

Pseudo-SU(3) scheme

^{162}Dy core: $\beta \approx 0.34$

10 protons in the pseudo-shell $\tilde{\mathcal{N}} = 3$ (plus 6 protons in the unique-parity $h_{11/2}$ orbit)

$SU_p(3)$ IRs: (λ_p,μ_p) = (10,4), (12,0), (7,7), …, (6,0), …, (0,0)

8 neutrons in the pseudo-shell $\tilde{\mathcal{N}} = 4$ (plus 6 neutrons in the unique-parity $i_{13/2}$ intruder)

$SU_n(3)$ IRs: (λ_n,μ_n) = (18,4), (20,0), (16,5), …, (14,0), …, (8,0), …, (2,0), (0,1)

Leading SU(3) IR: (28,8) triaxial

$\Rightarrow$ (32,0) axially-symmetric

$$\beta = \frac{3}{2}\frac{2\lambda+\mu}{N_0^{core}} = 0.30$$

(pseudo-)Nilsson ↔ SU(3) correspondence

Nilsson states	↔	SU(3) labels
$[\mathcal{N} n_z \Lambda]\Omega$		$(\lambda, \mu)\kappa$

Parity: $\pi_{sp} = (-1)^N = (-1)^{\widetilde{N}+1}$ $\quad \widetilde{N} = N - 1$

Model space: $Sp(6,R)_{core} \otimes Sp(6,R)_{sp}$ IR collective subspaces

$(\lambda_{core}, 0) \otimes (\widetilde{\mathcal{N}}, 0)$

^{163}Dy:

$K^\pi_J = 1/2^-$ $\quad K^\pi_J = 1/2^-$

Normal-parity: (32,0) ⊗ (4,0) = (36,0), (34,1), (32,2), (30,3), (28,4)

Unique-parity: (32,0) ⊗ (5,0) = (37,0), (35,1), (33,2), (31,3), (29,4), (27,5)

$K^\pi_J = 5/2^+$

(pseudo-)Nilsson ↔ SU(3) correspondence

Nilsson states	↔	SU(3) labels
$[\mathcal{N} n_z \Lambda]\Omega$		$(\lambda, \mu)\kappa$

^{163}Dy:

$K^\pi_J = 5/2^-$: $[523]5/2^- \rightarrow \widetilde{[422]}5/2 \leftrightarrow (32,2)2$

$K^\pi_J = 1/2^-$: $[521]1/2^- \rightarrow \widetilde{[420]}1/2 \leftrightarrow (32,2)0$

$K^\pi_J = 1/2^-$ $K^\pi_J = 1/2^-$

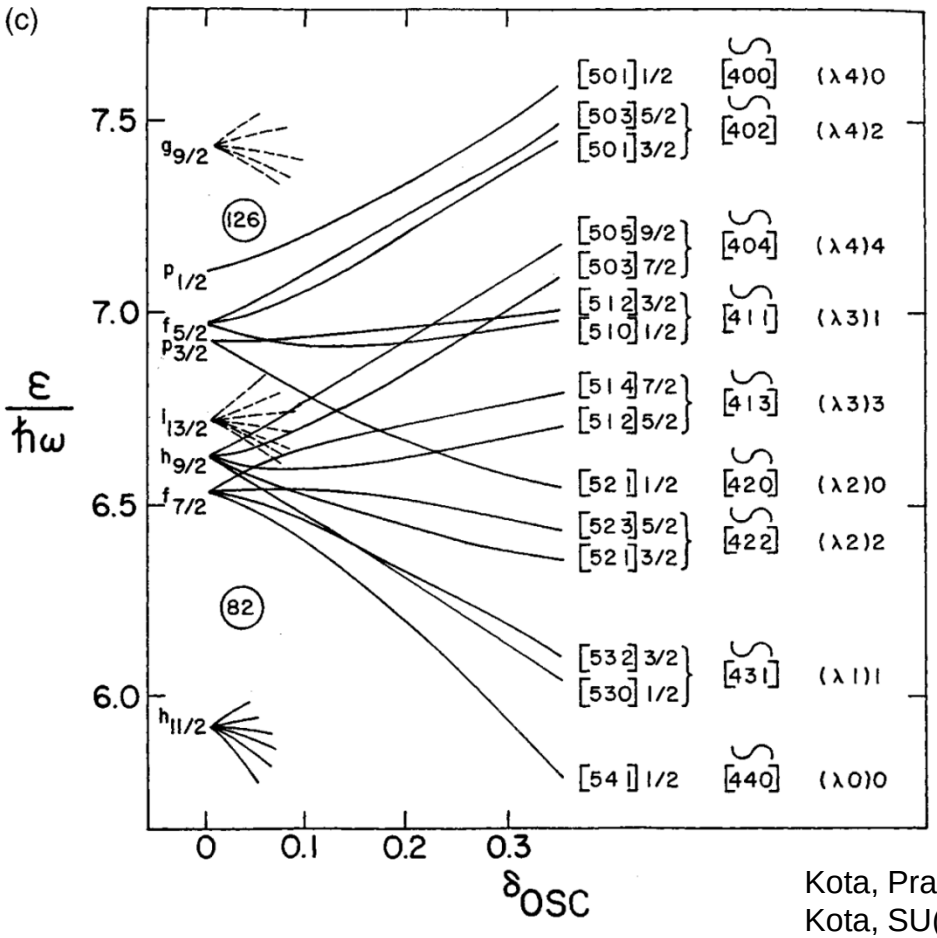
Normal-parity: (32,0) ⊗ (4,0) = (36,0), (34,1), (32,2), (30,3), (28,4)

Unique-parity: (32,0) ⊗ (5,0) = (37,0), (35,1), (33,2), (31,3), (29,4), (27,5)

$K^\pi_J = 5/2^+$

(pseudo-)Nilsson ↔ SU(3) correspondence

Nilsson states	↔	SU(3) labels
$[\mathcal{N} n_z \Lambda]\Omega$		$(\lambda, \mu)\kappa$



Kota, Pramanik, EPJA3,234(1998),
Kota, SU(3) Symmetry in Atomic Nuclei (2020)

Many-body particle-core SM subspaces

IR subspaces of $Sp(6,R)_{core} \otimes Sp(6,R)_{sp}$:

N	$\hbar\omega$	$SU(3)$ $IR's$ $\varrho(\lambda,\mu)$
⋮	⋮	...
N_0+4	4	$(40,0), 2(38,1), 4(36,2), 5(34,3), 6(32,4),$ $4(35,1), 5(30,5), 5(33,2), 6(31,3), 4(28,6),$ $2(34,0), 5(29,4), 5(32,1), 6(30,2), 2(26,7),$ $3(27,5), 4(28,3), (31,0), 2(29,1), (24,8),$ $(25,6), 2(26,4), (27,2), (28,0), 2(34,0)$
N_0+2	2	$(38,0), 2(36,1), 3(34,2), 3(32,3), 3(30,4),$ $(35,0), 2(33,1), 2(28,5), 2(31,2), 2(29,3),$ $(26,6), (32,0), (27,4), (30,1), (28,2)$
N_0	0	$(36,0), (34,1), (32,2), (30,3), (28,4)$

0p-0h (32,0) ⊗ (4,0) IR
of $Sp(6,R)_{core} \otimes Sp(6,R)_{sp}$

$K^\pi_J = 1/2^-$ and $K^\pi_J = 5/2^-$ bands in ^{163}Dy

$B(E2; 9/2^- \rightarrow 5/2^-) = 89$ W.u. (Exp: 111(20))

IR subspaces of $Sp(6,R)_{core} \otimes Sp(6,R)_{sp}$:

N	$\hbar\omega$	$SU(3)$ $IR's$ $\varrho(\lambda,\mu)$
⋮	⋮	...
N_0+4	5	$(41,0), 2(39,1), 4(37,2), 5(35,3), 6(33,4),$ $(31,5), 4(36,1), 5(31,5), 3(34,2), 3(29,6),$ $6(32,3), 5(30,4), 4(33,1), 5(28,5), 6(31,2),$ $6(29,3), (32,0), 4(27,4), 2(30,1), 2(28,2),$ $(23,9), (26,6), (24,7), (25,5), (26,3),$ $(29,0), (27,1), (25,5)$
N_0+2	3	$(39,0), 2(37,1), 3(35,2), 3(33,3), 2(31,4),$ $3(29,5), (36,0), 2(34,1), 2(32,2), 2(27,6),$ $2(30,3), 2(28,4), (33,0), (25,7), (31,1),$ $(26,5), (29,2), (27,3)$
N_0	1	$(37,0), (35,1), (33,2), (31,3), (29,4), (27,5)$

1p-1h (32,0) ⊗ (5,0) IR
of $Sp(6,R)_{core} \otimes Sp(6,R)_{sp}$

$K^\pi_J = 5/2^+$ band in ^{163}Dy

Application

Hamiltonian:

$$H = H_{DS} + H_{res} + H_{vmix},$$

$$H_{DS} = H_0 + BC_2[SU(3)] + C(C_2[SU(3)])^2 + \xi C_2[SU(3)_{sp}],$$

$$H_{res} = aK_J^2 + \frac{1}{2\mathcal{J}} C_2[SU(2)_J],$$

$$H_{vmix} = v_{mix}\left(A_2^{core} \cdot F_2^{core} + h.c.\right)$$

Basis: $|\Gamma\widetilde{\Gamma}; (\lambda_{core}, \mu_{core}), (\mathcal{N}, 0); (\lambda, \mu); \omega^{core}(f_{A-1}\widetilde{f}_{A-1})a; \omega^{sp}(f_1\widetilde{f}_1)a; \kappa L(Ss)\widetilde{S}; J\rangle$

Stretched SU(3) states: (λ_0+2k, μ_0), k = 0, 1, 2, …

E2 TO: $T_{2M}^{E2} = \left[e_{eff}\left(\frac{Z - Z_{sp}}{A - A_{sp} - 1}\right) Q_{2M}^{core} + e_{eff} q_{2M}^{sp}\right]$

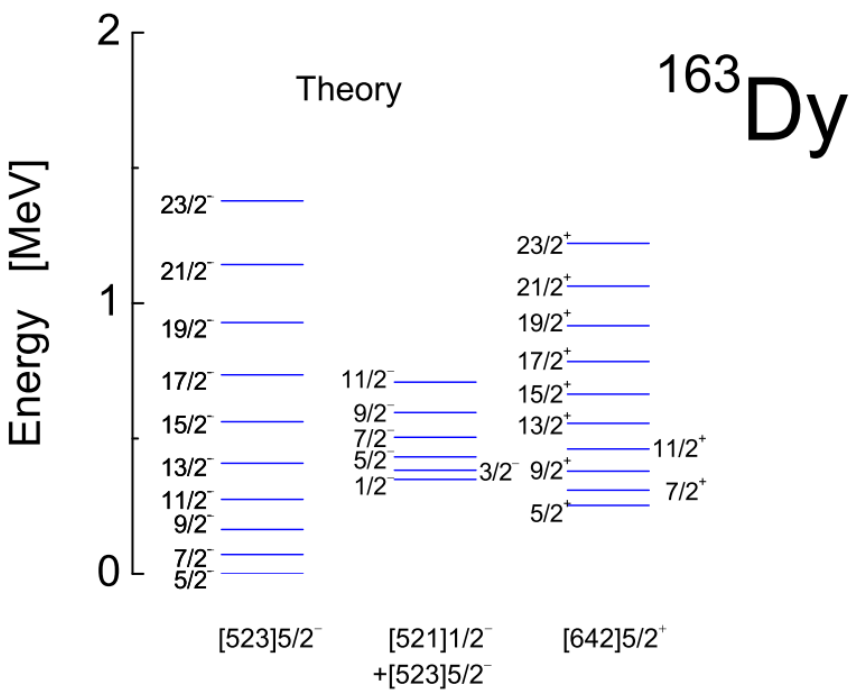
$Sp(6,R)_{core}$ $Sp(6,R)_{sp}$ /zero contribution/

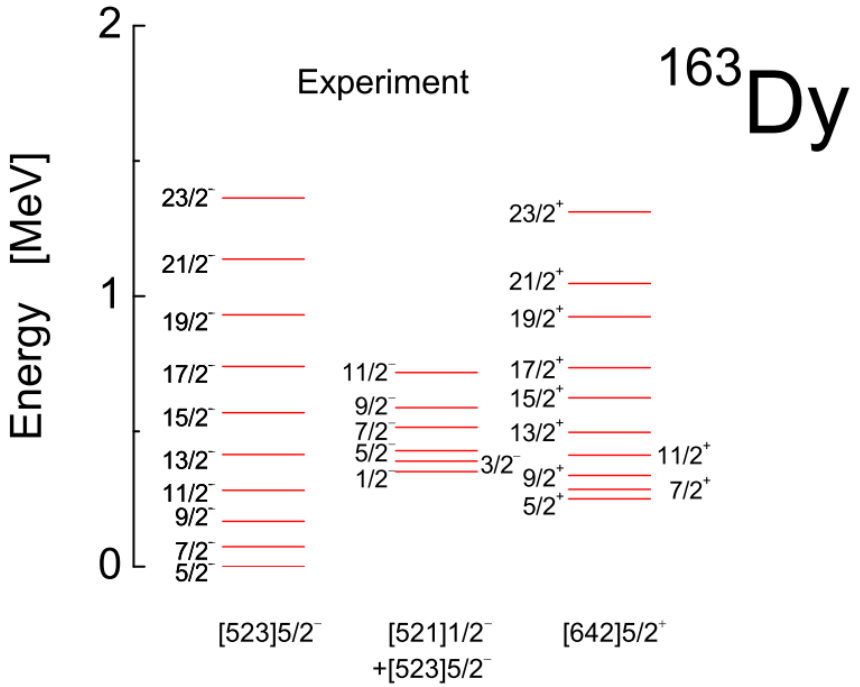
$$Q_{2M}^{core} = \sqrt{3}\left[A_{2M}^{core} + \tfrac{1}{2}\left(F_{2M}^{core} + G_{2M}^{core}\right)\right] \qquad q_{2M}^{sp} = \sqrt{3}\left[A_{2M}^{sp} + \tfrac{1}{2}\left(F_{2M}^{sp} + G_{2M}^{sp}\right)\right]$$

No effective charge: $e_{eff} = e$

HGG, Nucl. Phys. A 1072, 123415 (2026)

Energies





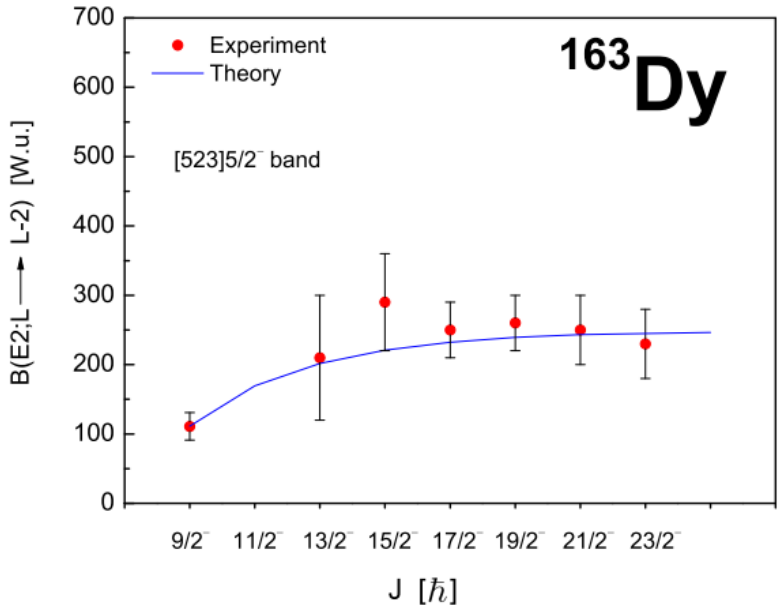
$B = -0.234$, $C = 0.000046$, $a = -0.176$, $v_{mix} = -0.0000078$ (in MeV),

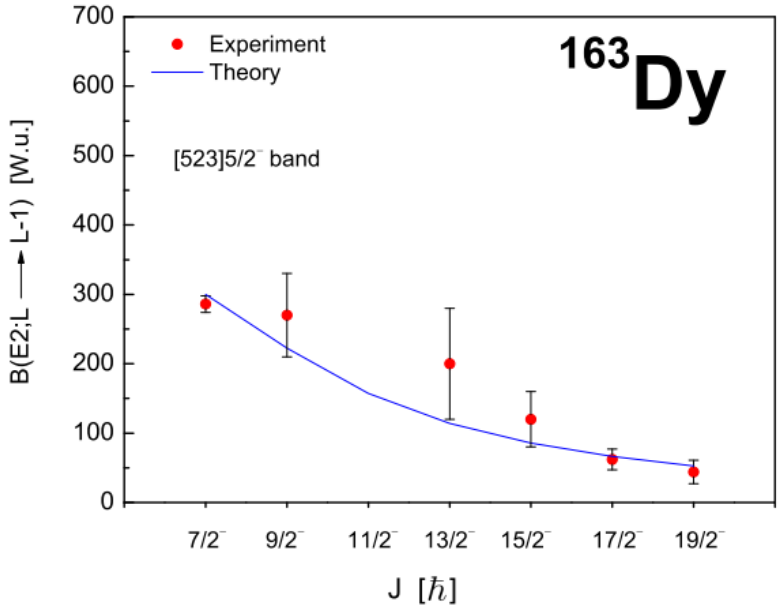
$\mathcal{J} = 49$, and $\alpha_{5/2^+} = 1.75$ (in MeV^{-1})

$$\mathcal{J} = \mathcal{J}_0(1 + \alpha_{5/2^+} E_{5/2^+})$$

✓ Regular rotational bands;

Transition probabilities





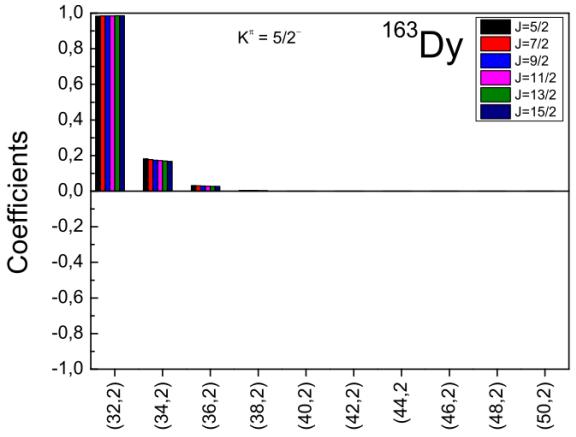
Interband:

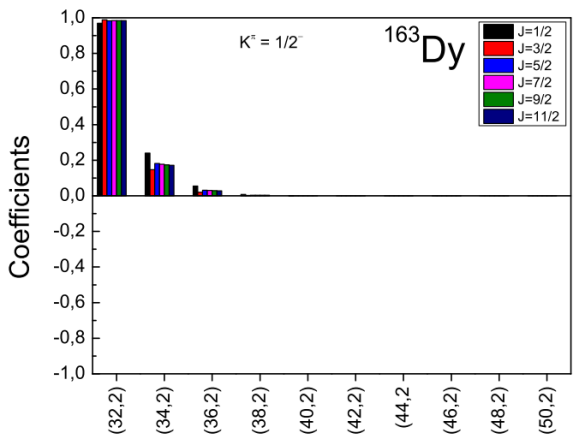
$i(K^\pi = 1/2^-)$	$f(K^\pi = 5/2^-)$	Th	Exp
$1/2^-$	$5/2^-$	8.2	7.4(15)
$5/2^-$	$9/2^-$	2.4	5.1(24)
$5/2^-$	$7/2^-$	5.6	6(3)
$3/2^-$	$7/2^-$	8.4	7(3)

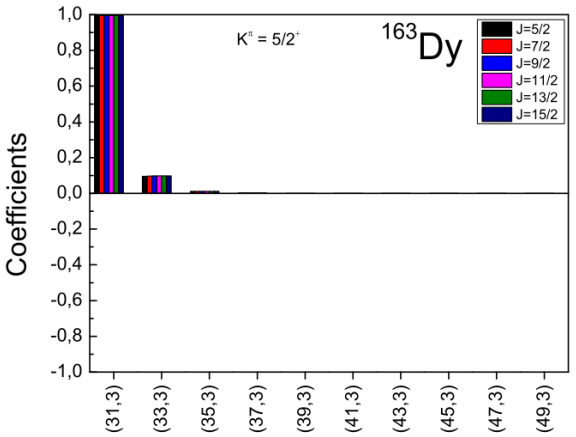
✓ No effective charge is used!

✓ SU(3) limit underestimates the observed B(E2) transition strengths;

Wave functions







- ✓ Slightly broken SU(3) symmetry; (simple structure)
- ✓ Small admixtures from higher SM configurations ($\hbar\omega$ spaces);
- ✓ Predominant bandhead (LW) SU(3) contribution;
- ✓ Approximate SU(3) QDS;

C. Bahri and D. Rowe, Nucl. Phys. A 662, 125 (2000);
D. J. Rowe, in Computational and Group-Theoretical Methods in Nuclear Physics, edited by J. Escher, O. Castanos, J. Hirsch, S. Pittel, and G. Stoitcheva (World Scientific, Singapore, 2004), pp. 165 − 173.

Thank You