The 14th Joint Workshop on Physics of Strong Interacting Systems

18–23 Oct. 2026, Nanjing

Exploring novel cluster structures in light nuclei

From the Hoyle state to nuclear clustering and triple-α capture

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19 Oct. 2026 · Nanjing

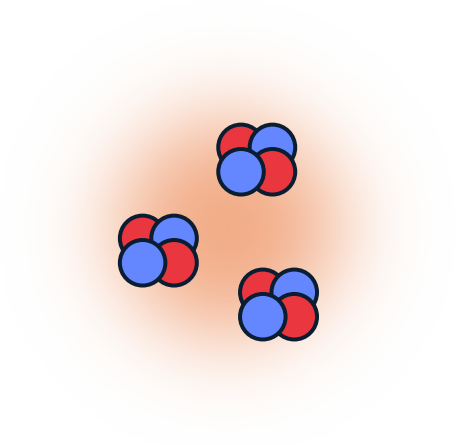
Outline

1 **The Hoyle state in ^{12}C**

structural characteristics and microscopic diagnostics

2 **Hoyle-like clustering in light nuclei**

multi-α systems and nuclei with valence nucleons

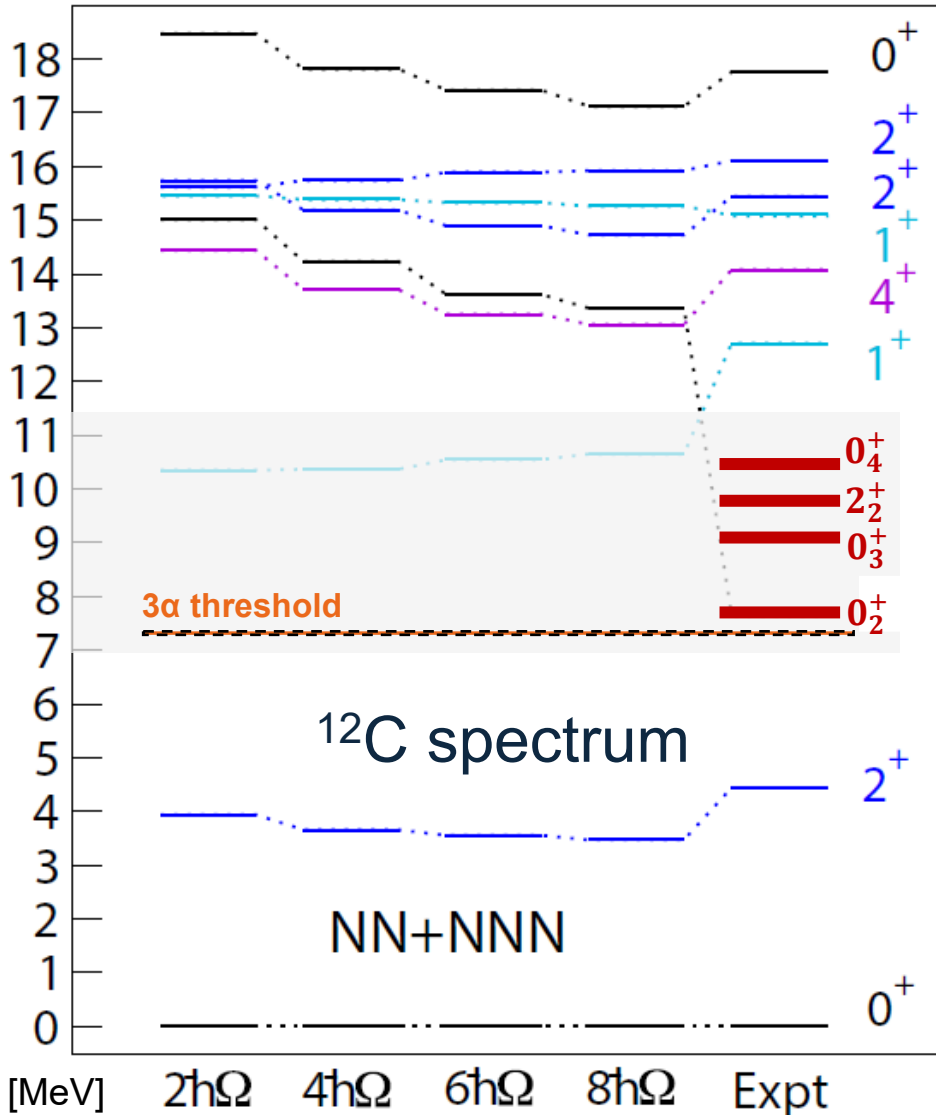


3 **Three-body capture and the triple-α reaction**

imaginary-time method and the IT-HH framework

4 **Summary and outlook**

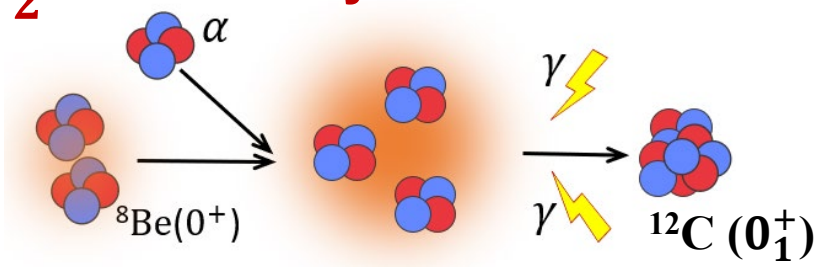
The Hoyle state in ^{12}C: the starting point



Recent no-core shell-model calculations

V.Somà,P.Navrátil, et al. PRC,101,014318 (2020)

0^+_2 Hoyle State



Hoyle state: spatially extended three-α correlations

*Rev. Mod. Phys. **89**, 011002 (2017)*

$0^+_{3,4}$

Broad 0^+ resonances with large α-decay widths

*Phys. Rev. C **84**, 054308 (2011)*

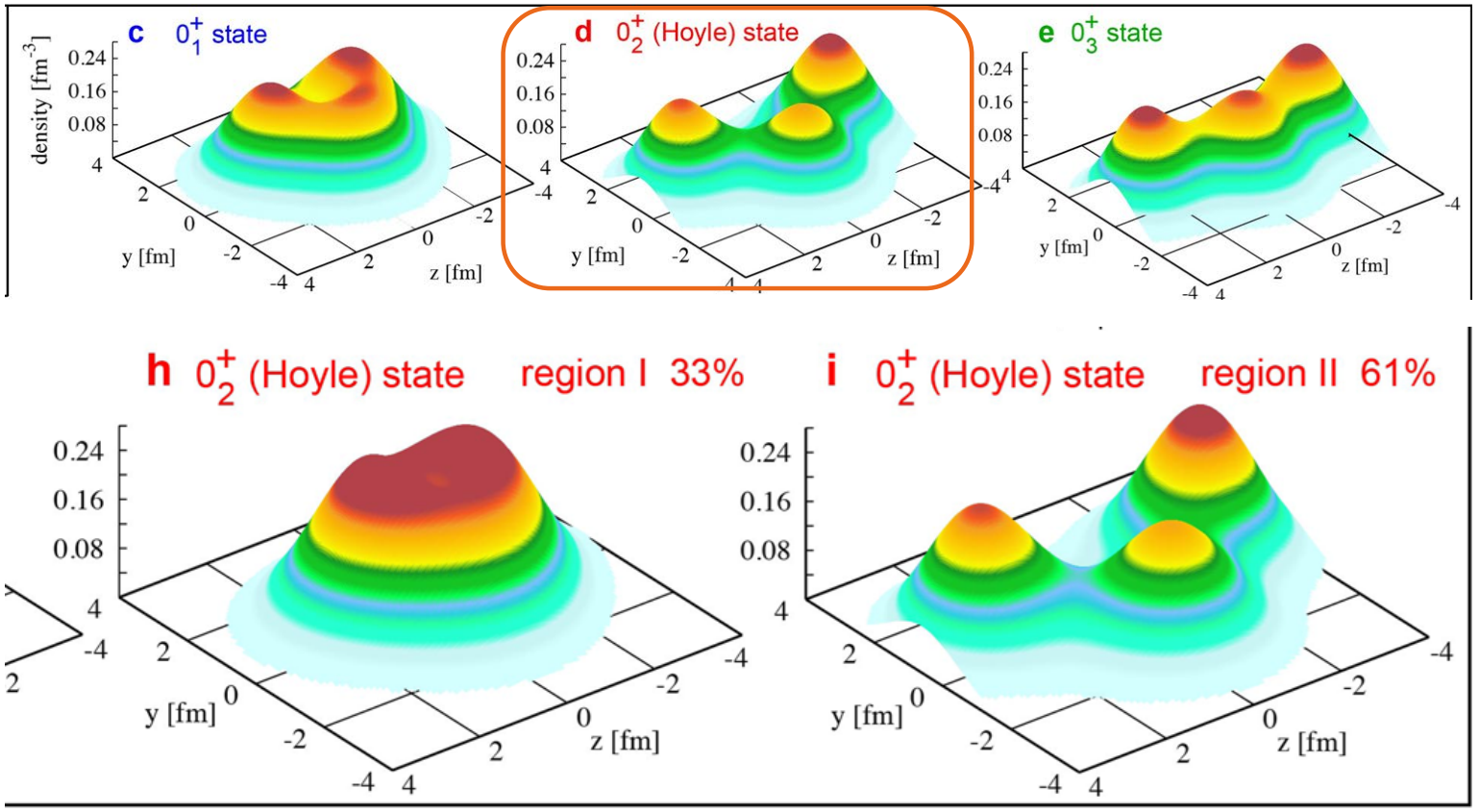
2^+_2

The 2_2^+ resonance provides a key Hoyle-band candidate

*Phys. Rev. Lett. **110**, 152502 (2013)*

Search for near-threshold novel clustering

Structural characteristics of the Hoyle state



T. Otsuka et al., Nat. Commun. 13, 2234 (2022)

Near the 3α threshold, spatially extended, and not represented by a unique rigid intrinsic geometry.

- **Near-threshold position.** The 0_2^+ state is ≈0.38 MeV above the 3α threshold.
- **Spatial extension.** Calculated radii and cluster-channel amplitudes are larger than for the ground state.
- **Nonunique intrinsic density.** Several configurations contribute after symmetry restoration and configuration mixing.
- **Consistent observables.** Radius, E0 strength, overlaps, and α-decay properties should support the same interpretation.

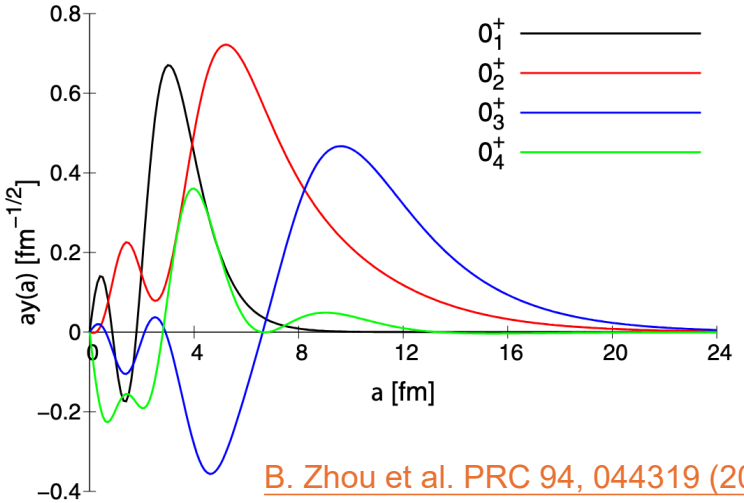
"Hoyle-like" means analogous structure, not identical wave functions.

Probing cluster structure: RWA and TCOA

RWA probes asymptotic channels; TCOA maps the full three-cluster configuration space.

Reduced-width amplitude (RWA)

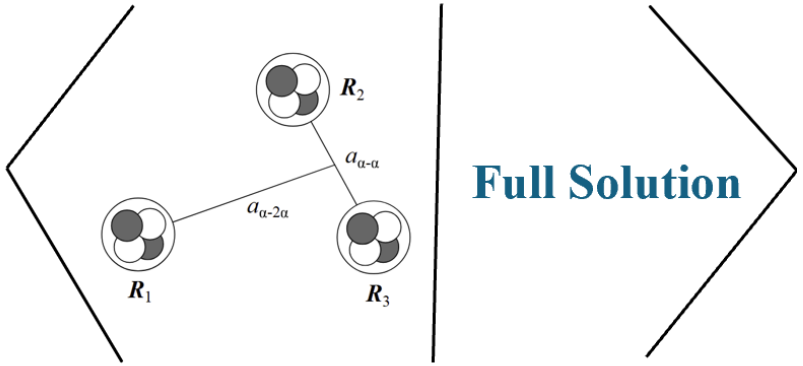
$$y_c^{J\pi}(a) = \sqrt{\frac{A!}{(1+\delta_{C_1C_2})C_1!C_2!}} \times \left\langle \frac{\delta(r-a)}{r^2} \left[Y_l(\hat{\boldsymbol{r}}) \otimes \left[\Phi_{C_1}^{j_1\pi_1} \otimes \Phi_{C_2}^{j_2\pi_2} \right]_{j_{12}} \right]_{JM} \middle| \Psi_M^{J\pi} \right\rangle$$



B. Zhou et al. PRC 94, 044319 (2016).

Two-cluster overlap amplitude (TCOA)

$$\mathcal{Y}_c^{J\pi}(a_1, a_{23}) = \sqrt{\frac{A!}{C_1!C_2!C_3!}} \left\langle \frac{\delta(r_1-a_1)\delta(r_{23}-a_{23})}{r_1^2 r_{23}^2} \right.$$
$$\times \left[[Y_{l_1}(\hat{r}_1) \otimes Y_{l_{23}}(\hat{r}_{23})]_L \otimes \left[\Phi_{C_1}^{j_1\pi_1} \otimes \left[\Phi_{C_2}^{j_2\pi_2} \otimes \Phi_{C_3}^{j_3\pi_3} \right]_{j_{23}} \right]_{j_{123}} \right]_{JM} \left| \Psi_M^{J\pi} \right\rangle$$

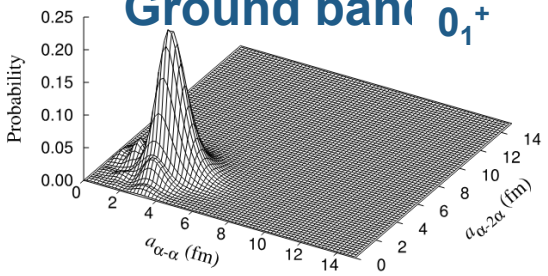


Deye Tao et al., NST 36, 56 (2025).

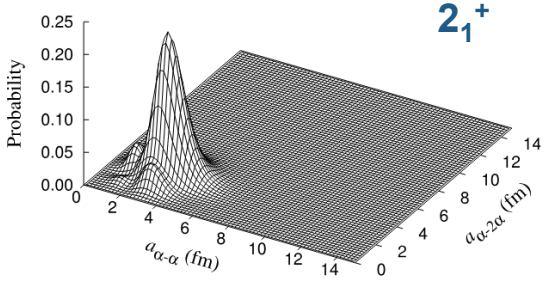
Visualization of rotational-band configurations

De-Ye Tao,et al, PLB 874(2026)

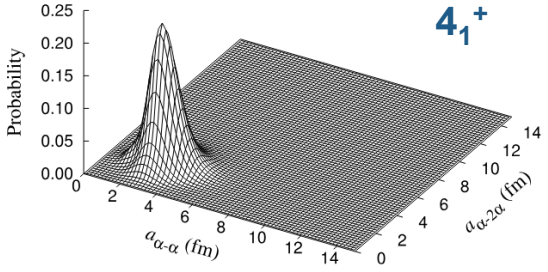
Ground band 0_1^+



2_1^+

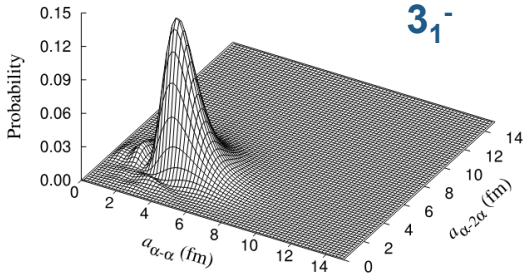


4_1^+

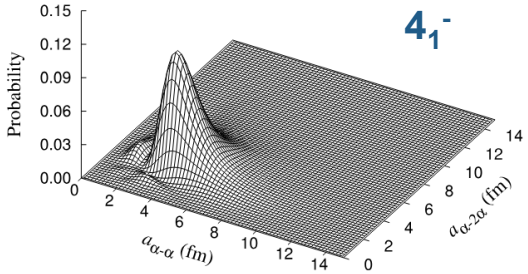


Kπ = 3⁻ band

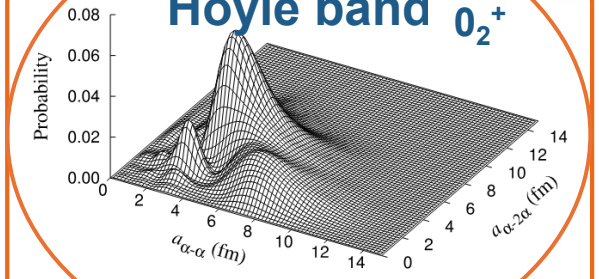
3_1^-



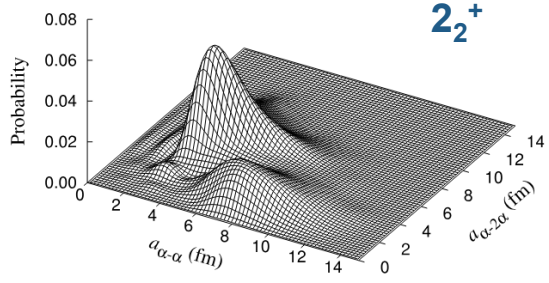
4_1^-



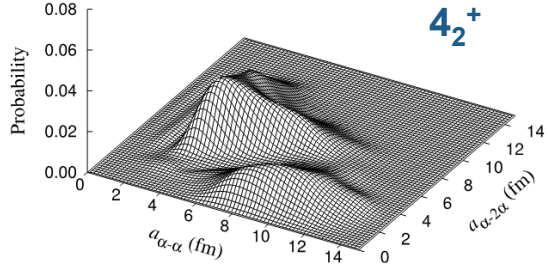
Hoyle band 0_2^+



2_2^+



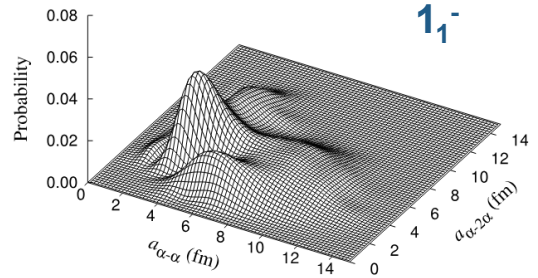
4_2^+



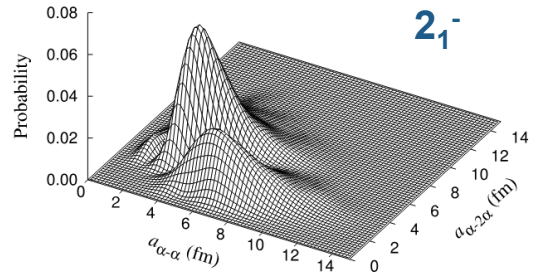
extended 3α motion

Kπ = 1⁻ band

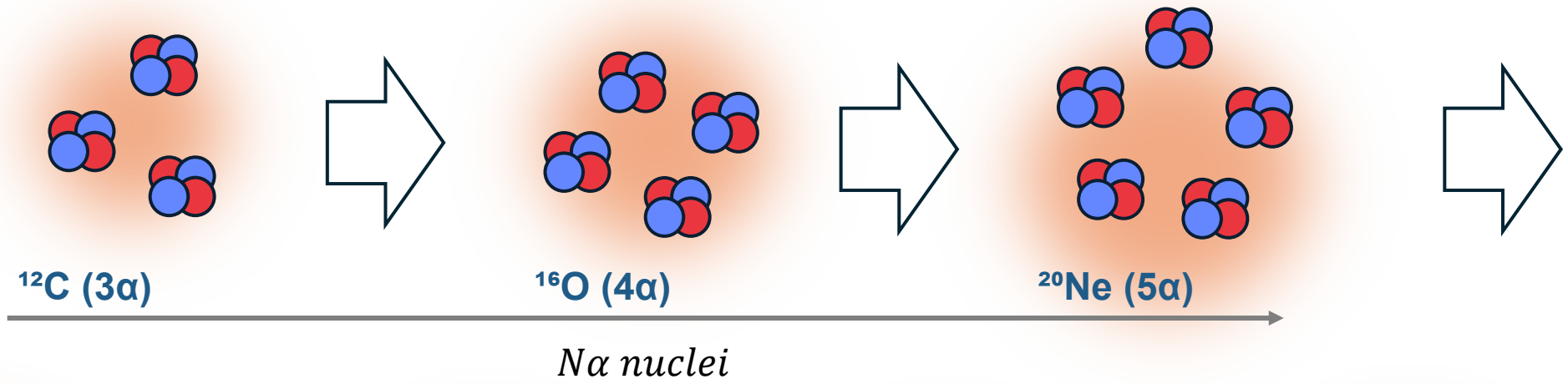
1_1^-

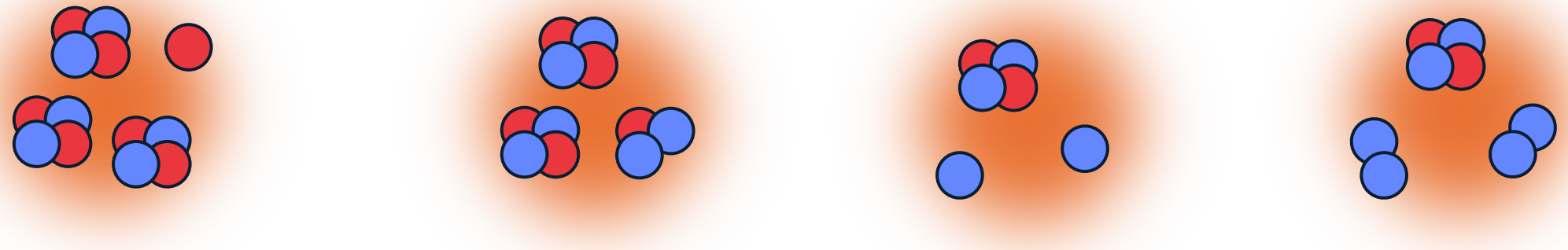


2_1^-



Toward Hoyle-like cluster states

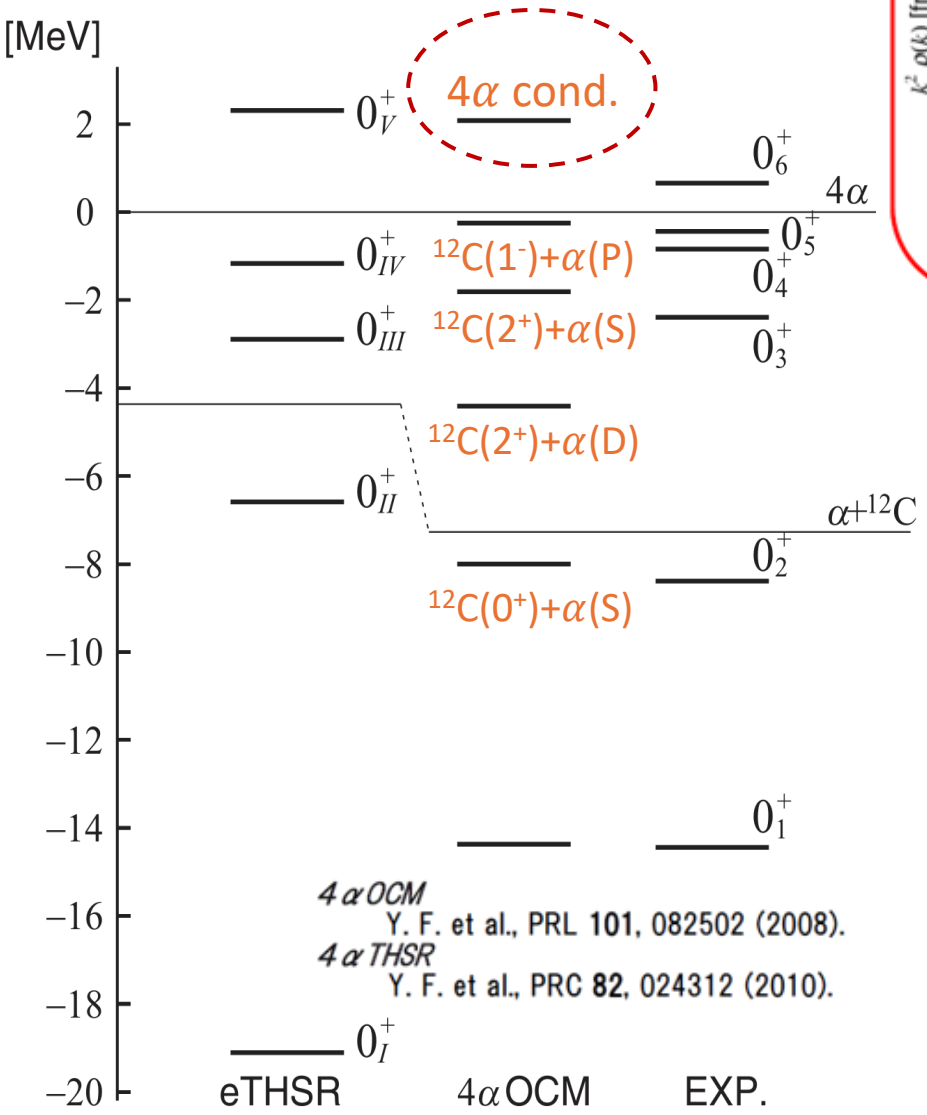


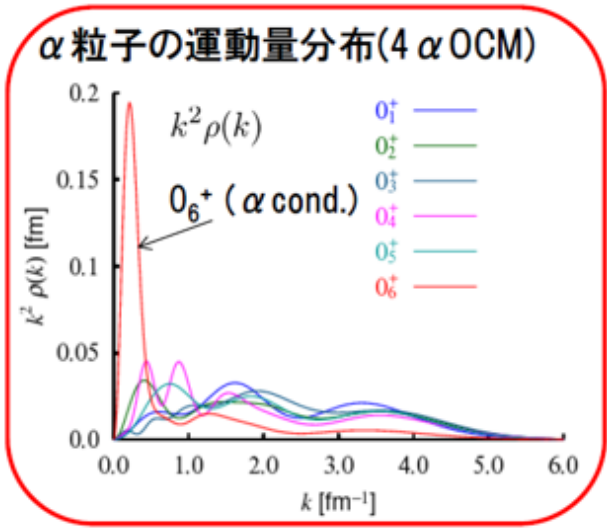


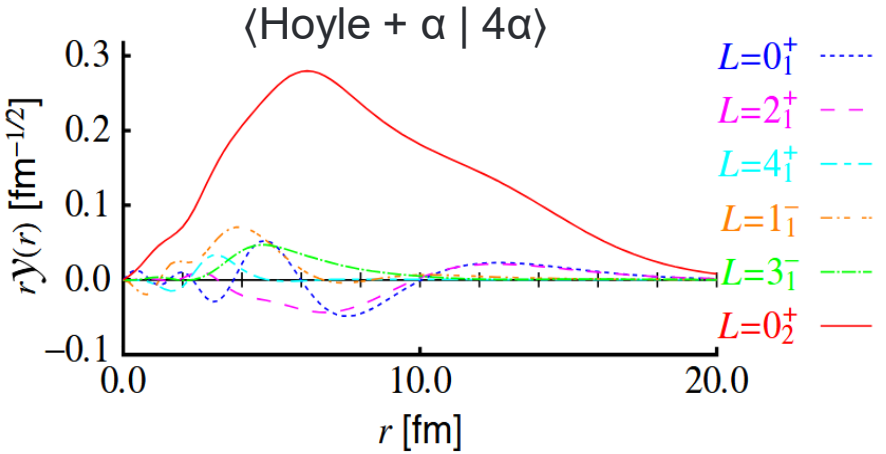
Two directions: increase the number of α clusters,
or couple valence nucleons to an α-cluster core.

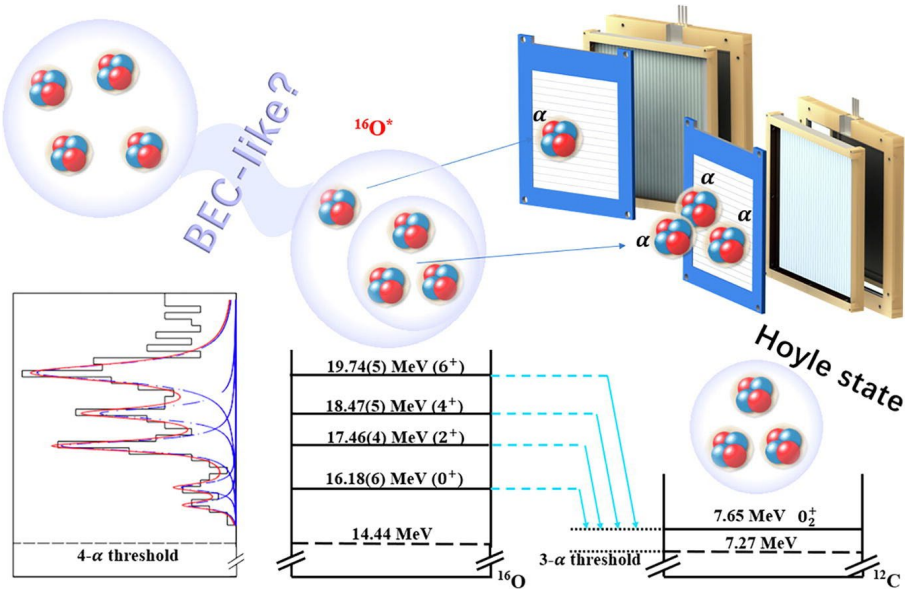
Four-α clustering in ^{16}O

Candidate 0_6^+: near the 4α threshold, with strong 4α and Hoyle+α components.

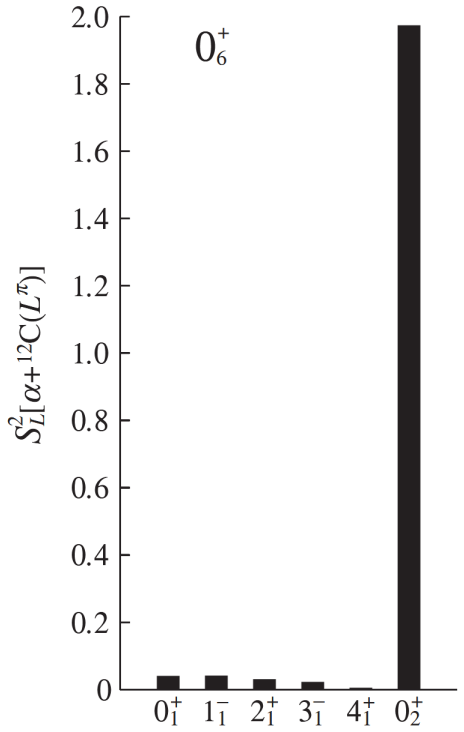






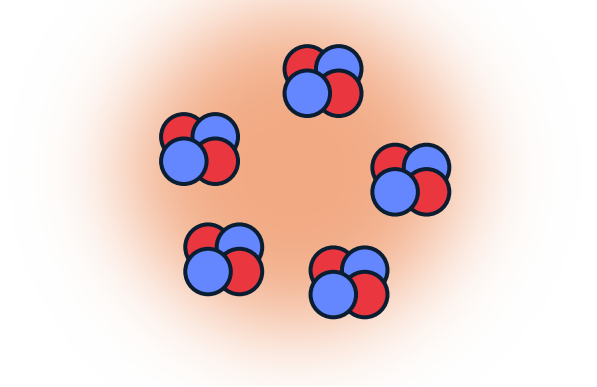


Jiahao Chen et al., Science Bulletin, 68 (2023)

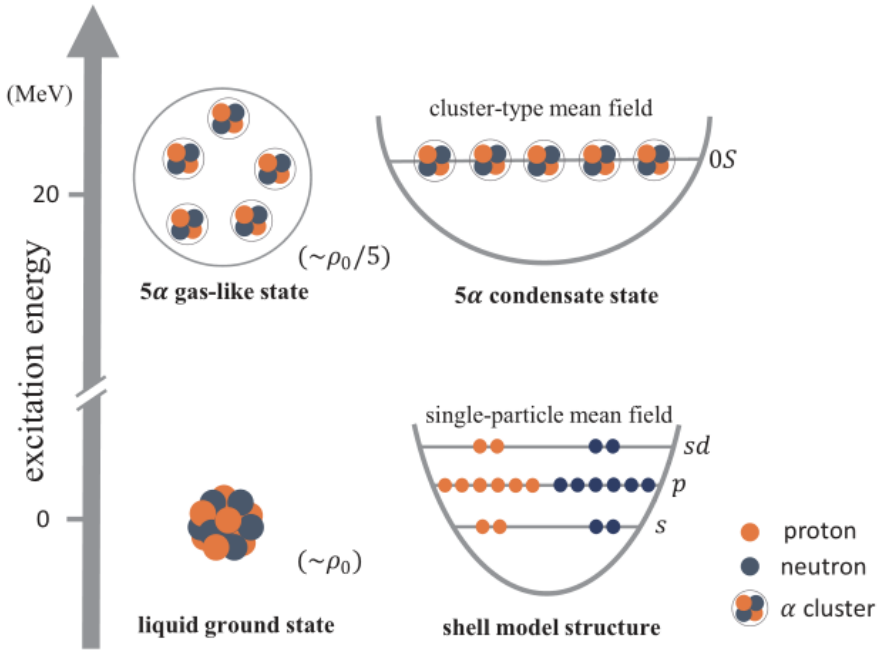


*Yamada et al.PRC **85**, 034315*

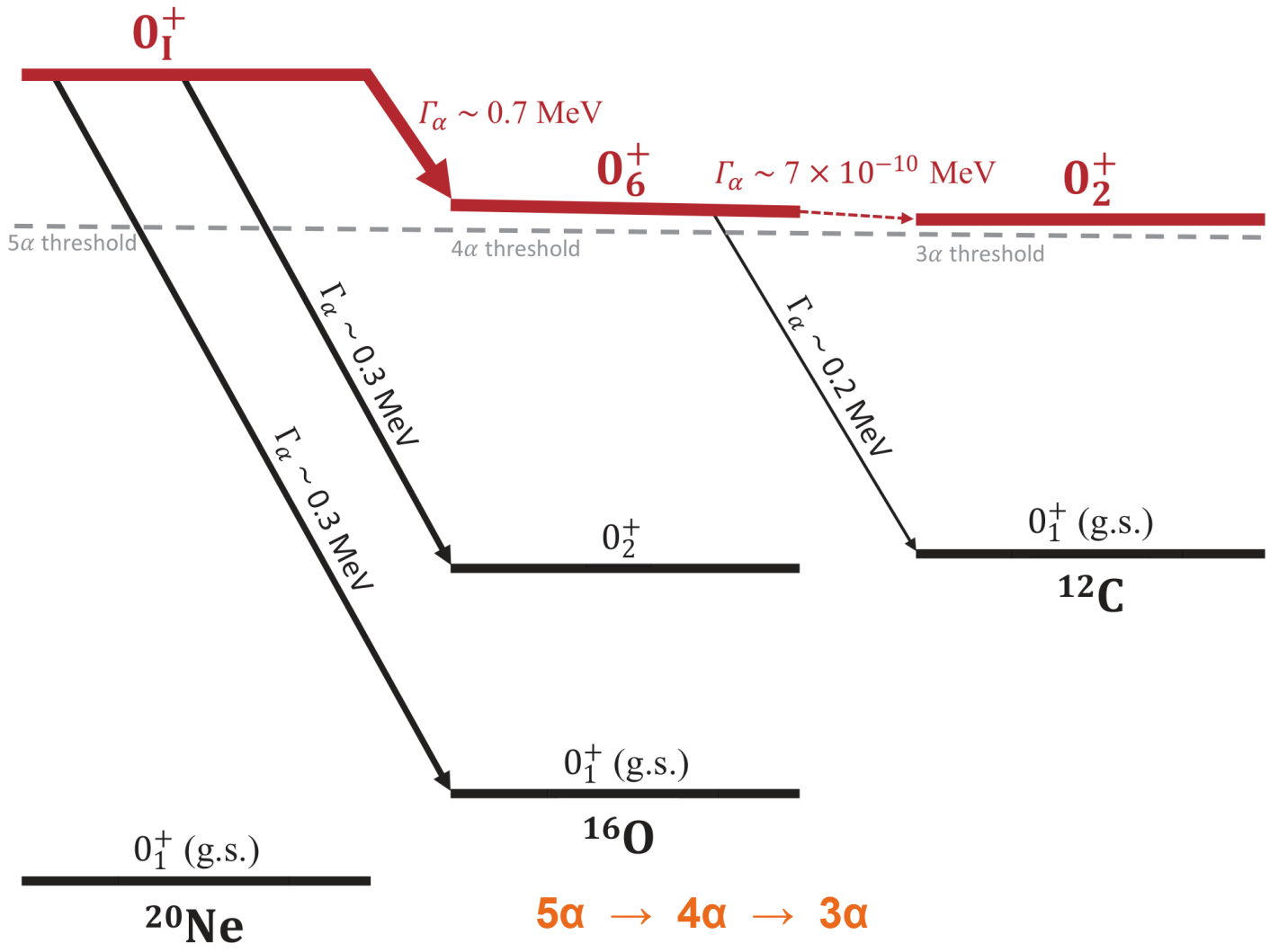
Decay links among multi-α cluster candidates



5α condensate candidate



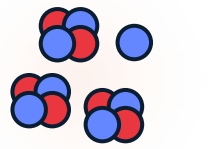
B. Zhou, et al., Nat. Commun. **14**, 8206 (2023).



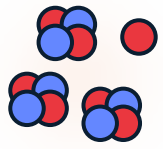
5α → 4α → 3α

The calculated α-decay cascade provides an experimental signature connecting the 5α, 4α, and 3α candidates.

Hoyle-analog candidate in ^{13}N



^{13}C(3α+n)



^{13}N(3α+p)

Hoyle-like 3α correlation can survive addition of a proton?

Experiment

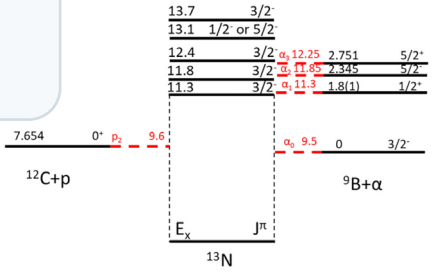
Beta-delayed charged-particle spectroscopy identified four α-decaying states at 11.3, 12.4, 13.1, and 13.7 MeV in the 3α+p channel.

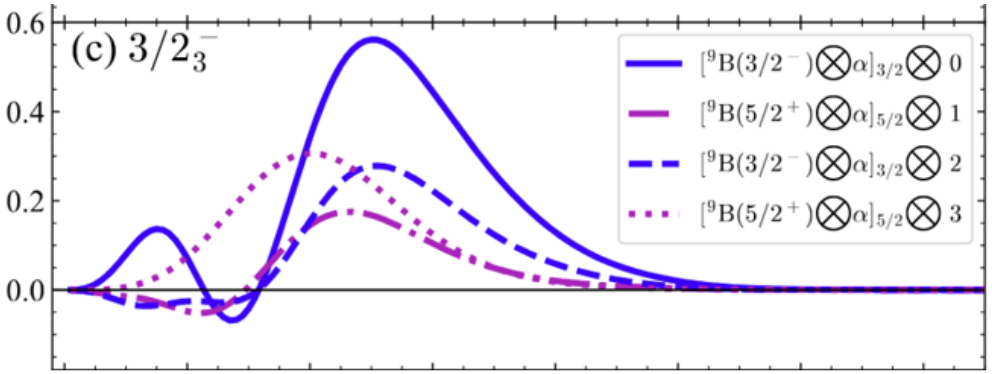
Experiment: Phys. Rev. C 109, 054308 (2024)

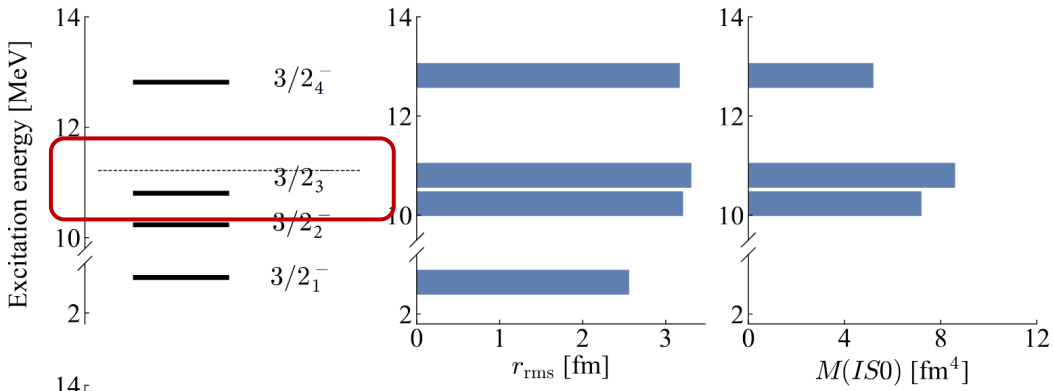
Theory

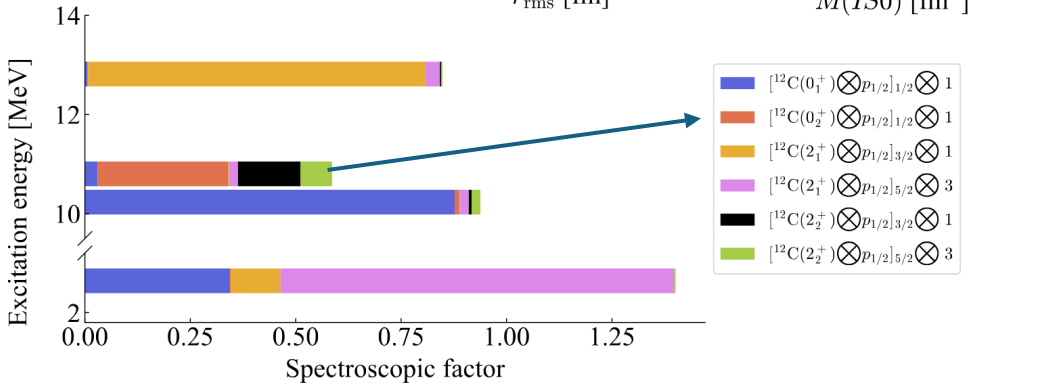
The calculated $3/2_3^-$ candidate near 11 MeV contains sizable $^{12}C(0_2^+)$+p and ^{9}B+α components, together with an extended cluster distribution.

The state retains a sizable Hoyle+p component, but it is a mixed-channel state rather than a one-to-one copy of the ^{12}C Hoyle state.







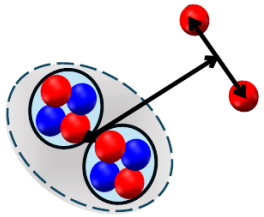


Y. Y. Cao et al., Phys. Rev. C 112, 034313 (2025)

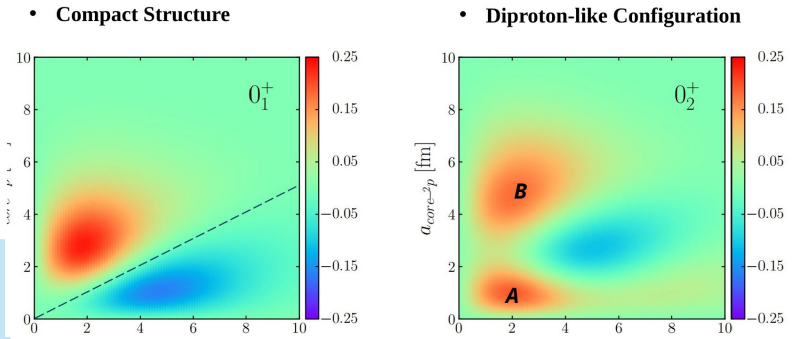
Hoyle-like correlations with valence nucleons

Beyond self-conjugate Nα nuclei, correlated valence nucleons can reorganize around an α-cluster core.

^{10}C: diproton-like correlations around a 2α core

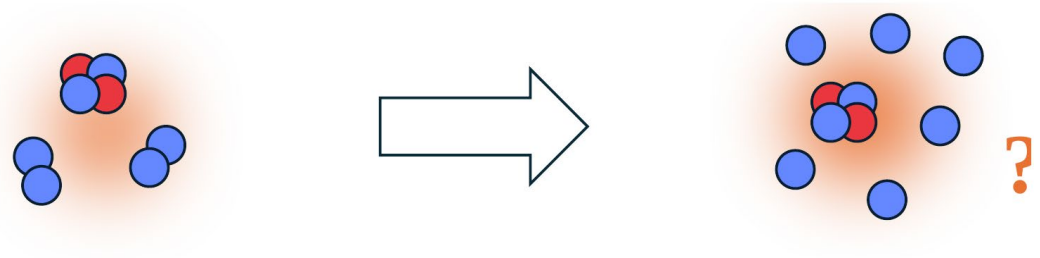


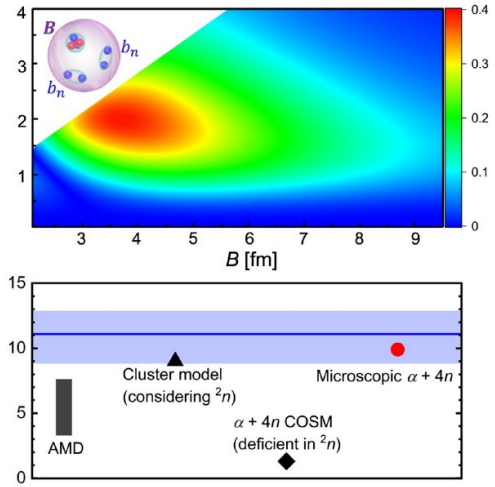
"diproton-like" configuration



The 0_2^+ wave function contains compact and spatially extended components. In the extended component, the proton pair remains correlated around the 2α core.

^{8}He: an exotic 0_2^+ cluster state





The observed 0_2^+ state motivates microscopic tests of α+4n configurations. The open question is whether the four neutrons form correlated pairs or a more collective multi-neutron distribution.

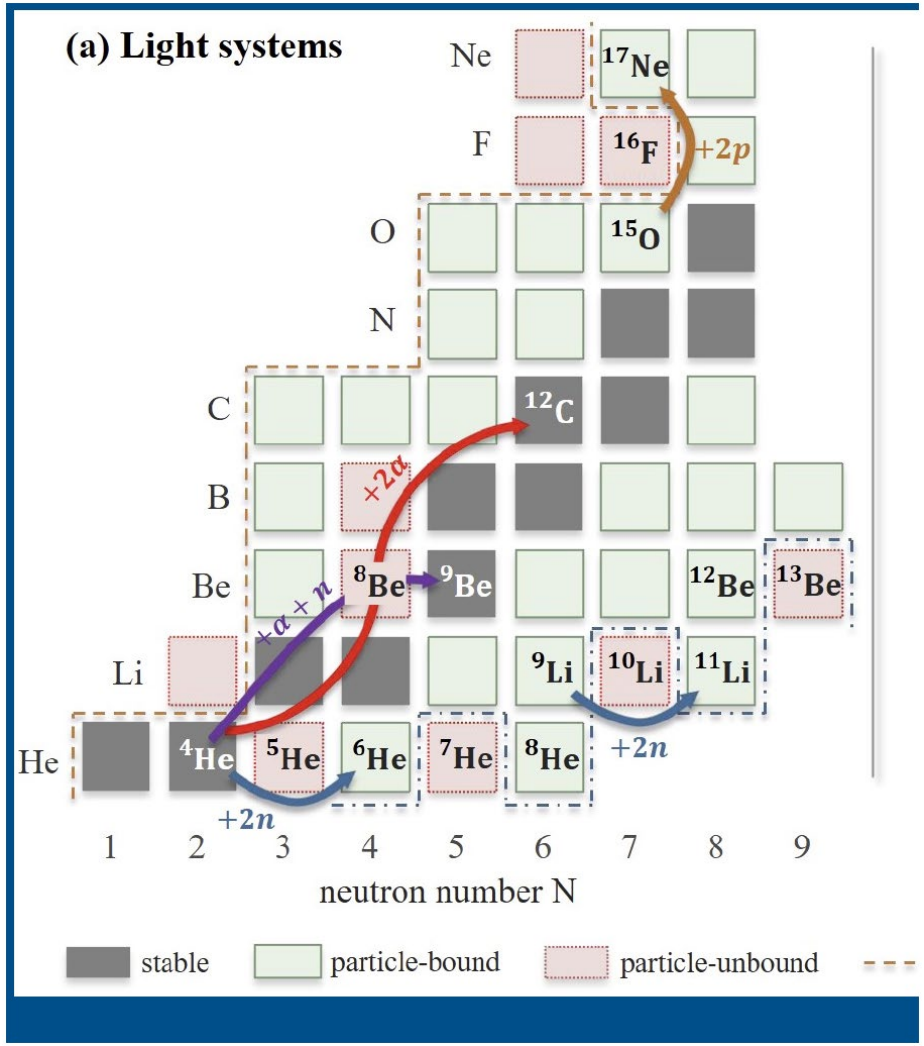
In both nuclei, the analogy concerns an extended cluster degree of freedom modified by valence nucleons.

L.-D. Song et al., Phys. Rev. C 113, 024316 (2026)

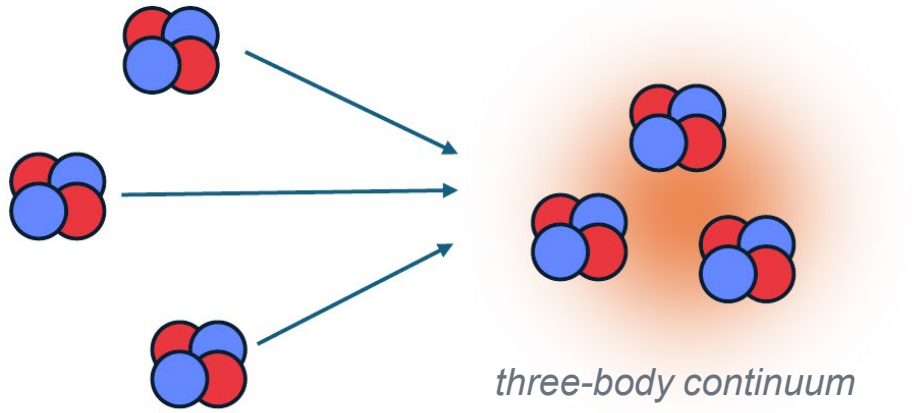
Z. H. Yang et al., Phys. Rev. Lett. 131, 242501 (2023)

Why three-body capture reactions?

Mass gaps and unbound intermediates



Triple-α capture



$$\alpha + \alpha \rightleftharpoons {}^{8}\mathrm{Be}, \qquad {}^{8}\mathrm{Be} + \alpha \rightarrow {}^{12}\mathrm{C}(0_2^{+}) \rightarrow {}^{12}\mathrm{C} + 2\gamma$$

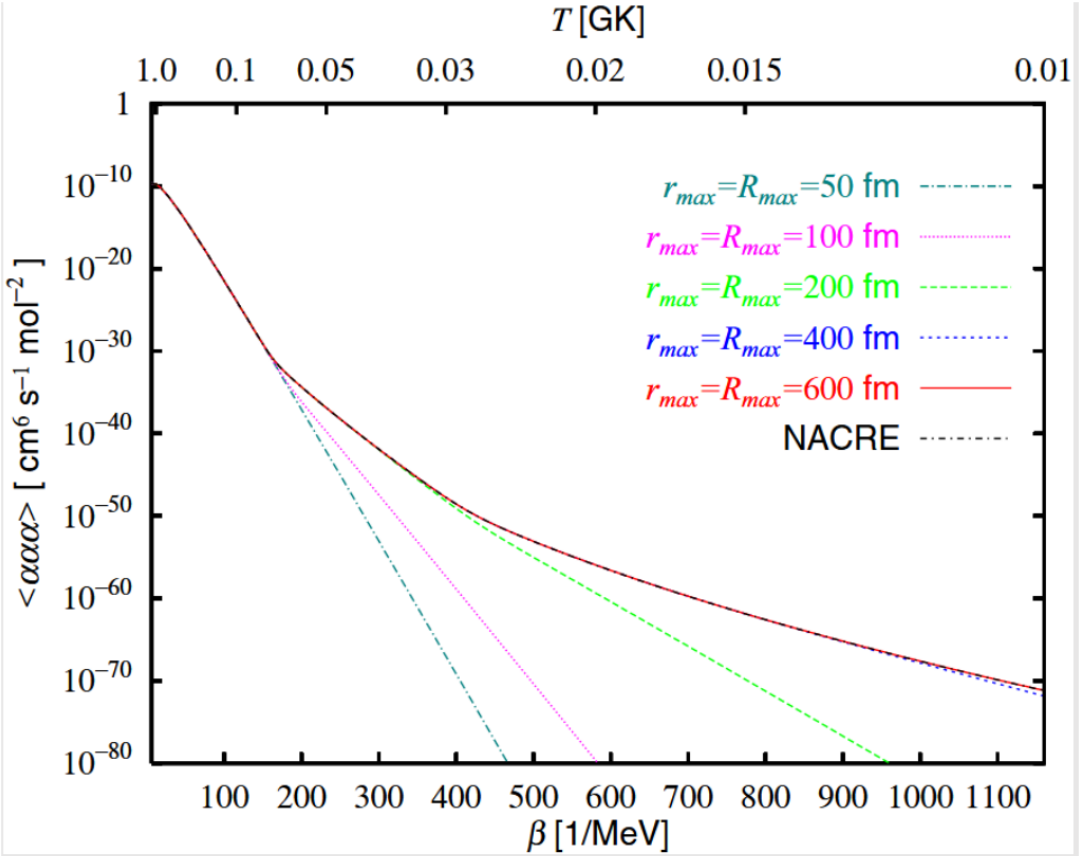
Because ^{8}Be is unbound, the reaction samples short-lived three-body continuum configurations rather than a sequence of two ordinary bound-state captures.

The Hoyle resonance dominates the usual helium-burning window, while nonresonant continuum contributions become increasingly important toward lower temperatures.

The same 3α dynamics that generates the Hoyle state also controls the stellar production of ^{12}C.

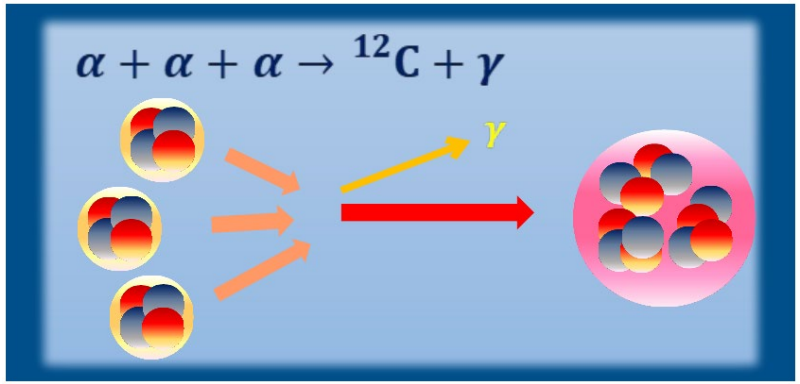
Resonant and nonresonant contributions to triple-α capture

Previous imaginary-time result (2015)

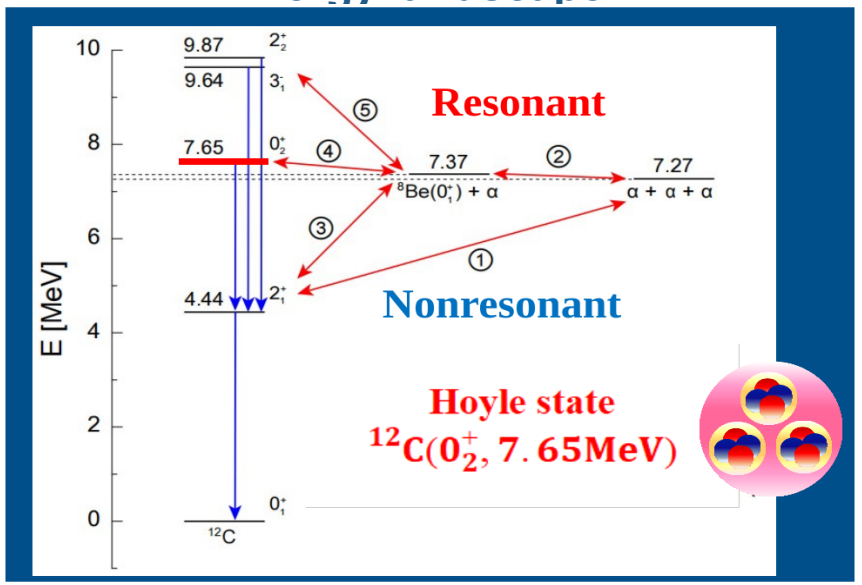


The direct low-temperature contribution converges only when very large three-body distances are included.

Resonant and nonresonant paths



Energy landscape



The Hoyle pole dominates the resonant rate; continuum contributions grow toward lower T.

T. Akahori et al., Phys. Rev. C 92, 022801(R) (2015)

L. Pei et al., Sci. Sin.-Phys. Mech. Astron. (2024)

Imaginary-time method: avoiding an explicit continuum sum

The direct thermal rate contains a sum over all three-body scattering eigenstates. Imaginary-time propagation replaces this sum by an operator function of H.

Direct formulation

$$\langle abc\rangle = (1+\Delta_{abc})\frac{1}{\omega_i}\left(\frac{4\pi^2\beta^2\hbar^4}{\mu}\right)^{3/2}\frac{8\pi(\lambda+1)}{\hbar\lambda[(2\lambda+1)!!]^2}$$
$$\times\sum_{M_f\mu}\sum_{i} e^{-\beta E_i}\left(\frac{E_i-E_f}{\hbar c}\right)^{2\lambda+1}\left|\langle\Phi_f|M_{\lambda\mu}|\Phi_i\rangle\right|^2,$$

An explicit sum over continuum states $|\Phi_i\rangle$ is required.

Imaginary-time formulation

$$\langle abc\rangle = (1+\Delta_{abc})\frac{1}{\omega_i}\left(\frac{4\pi^2\beta^2\hbar^4}{\mu}\right)^{3/2}\frac{8\pi(\lambda+1)}{\hbar\lambda[(2\lambda+1)!!]^2}$$
$$\times\langle\Phi_f|M_{\lambda\mu}e^{-\beta H}\left(\frac{H-E_f}{\hbar c}\right)^{2\lambda+1}\hat{P}M^{\dagger}_{\lambda\mu}|\Phi_f\rangle, \qquad (62)$$

The factor $e^{-\beta H}$ propagates a transition source generated from the final bound state.

A projector removes unwanted bound eigenstates. Only the final bound-state wave function is prepared explicitly; scattering information enters through the Hamiltonian and imaginary-time evolution.

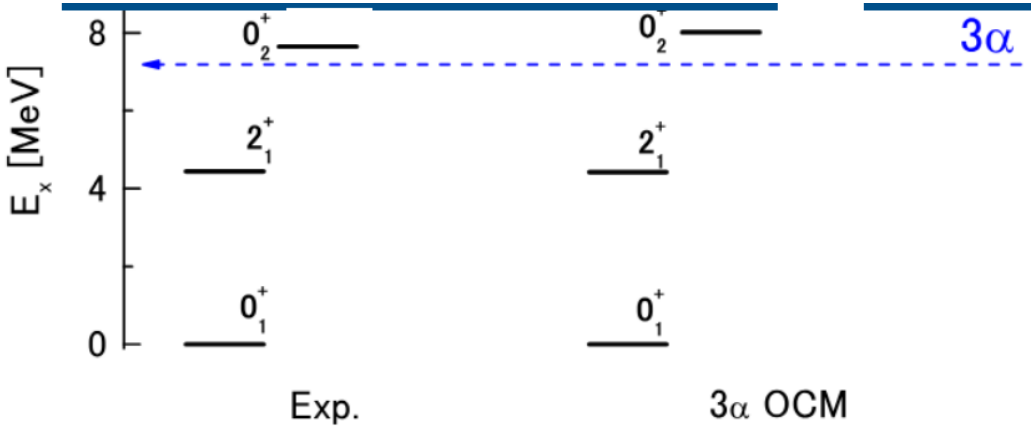
final bound state → transition source → $e^{-\beta H}$ propagation → thermally averaged capture rate

K. Yabana et al., Phys. Rev. C 85, 055803 (2012)

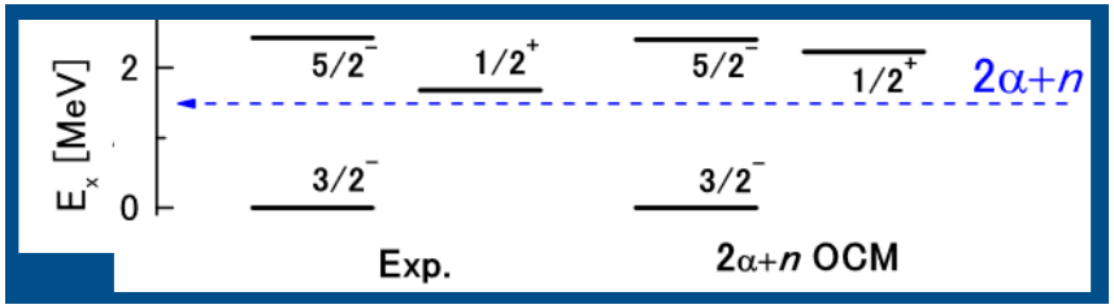
Earlier imaginary-time calculations in Jacobi coordinates

The imaginary-time formalism was implemented for both 3α and α+α+n systems in Jacobi coordinates.

3α OCM spectrum



α+α+n OCM spectrum



H = kinetic energy + pair interactions + phenomenological Pauli-blocking operator

Pauli treatment. Deep two-body potentials contain forbidden components, removed by a projection operator.

Simplified interactions. The 3α and α+α+n applications used restricted, partly angular-momentum-independent interactions.

Consistency issue. The structure state and the imaginary-time evolution were not always generated by the same Hamiltonian.

These calculations established feasibility, but systematic symmetry, coupled-channel, and convergence tests remain necessary.

Limitations of the existing implementations

These points concern particular calculations—not Jacobi coordinates as a representation.

Hamiltonian consistency

- Treatment of Pauli-forbidden components
- Different Hamiltonians for the structure state and IT evolution
- Sensitivity to phenomenological potential choices

Three-body dynamics

- Explicit exchange-symmetry checks for three identical α particles
- Systematic partial-wave and channel coupling
- Angular-momentum-dependent interactions and convergence control

IT-HH is introduced as a complementary basis: the rate formalism is unchanged, while permutation symmetry, channel coupling, and convergence can be tested more systematically.

Goal: one self-consistent three-body Hamiltonian for both the bound state and the imaginary-time evolution.

IT-HH framework: imaginary time in a hyperspherical basis

Imaginary-time method + **Hyperspherical-harmonic basis** → **IT-HH**

The rate formula is unchanged; the three-body dynamics is expanded in an HH basis.

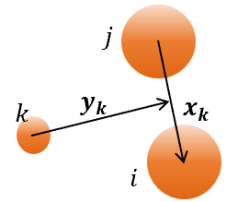
Unchanged imaginary-time rate formula

$$\langle abc\rangle = (1+\Delta_{abc})\frac{1}{\omega_i}\left(\frac{4\pi^2\beta^2\hbar^4}{\mu}\right)^{3/2}\frac{8\pi(\lambda+1)}{\hbar\lambda[(2\lambda+1)!!]^2}$$
$$\times\langle\Phi_f|M_{\lambda\mu}e^{-\beta H}\left(\frac{H-E_f}{\hbar c}\right)^{2\lambda+1}\hat{P}M^{\dagger}_{\lambda\mu}|\Phi_f\rangle,$$

Hamiltonian and wave function in the HH basis

$$H = T_\rho + \frac{\hat{K}^2}{\rho^2} + \sum_{i<j} V_{ij} + V_{3b}(\rho)$$

$$\Psi^{JM} = \rho^{-5/2}\sum_\gamma \chi_\gamma(\rho)\,\mathcal{Y}^{JM}_\gamma(\Omega)$$



What can be controlled systematically

- permutation symmetry for three identical α particles
- coupled channels and partial waves
- HH truncation and hyperradial convergence

Physical objective

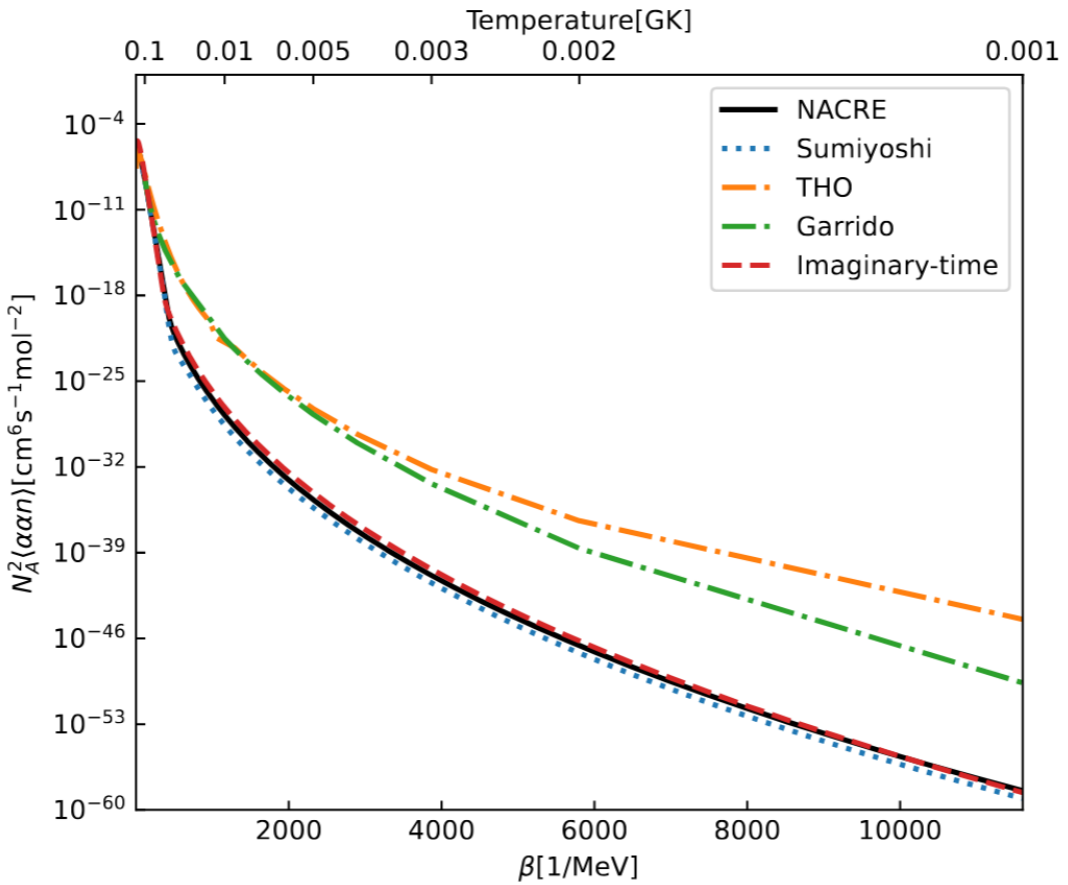
- one Hamiltonian for the bound state and IT evolution
- identify the origin of low-temperature rate differences
- quantify remaining model dependence

The basis change is useful only if it enables explicit symmetry checks and controlled convergence.

Benchmark: α + α + n radiative capture

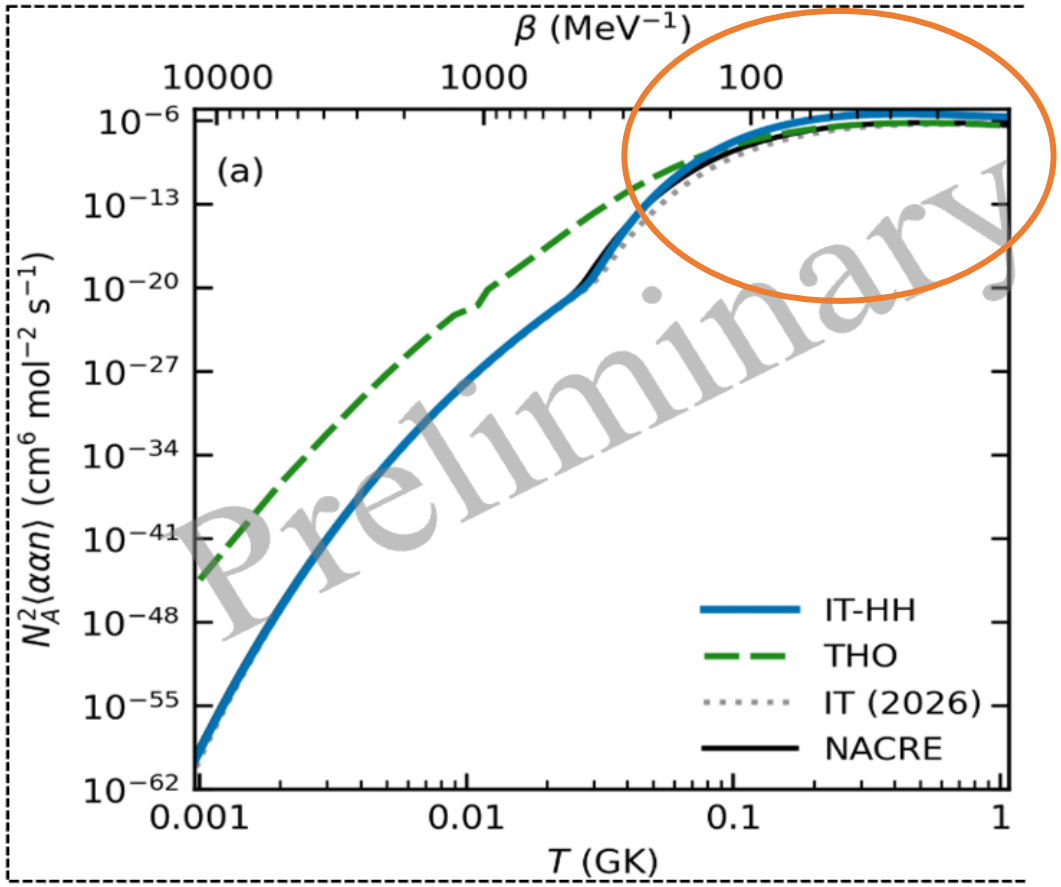
The α+α+n system tests the framework before the more demanding three-identical-α calculation.

Published IT-Jacobi result



L. Pei et al., Phys. Lett. B 873, 140196 (2026)

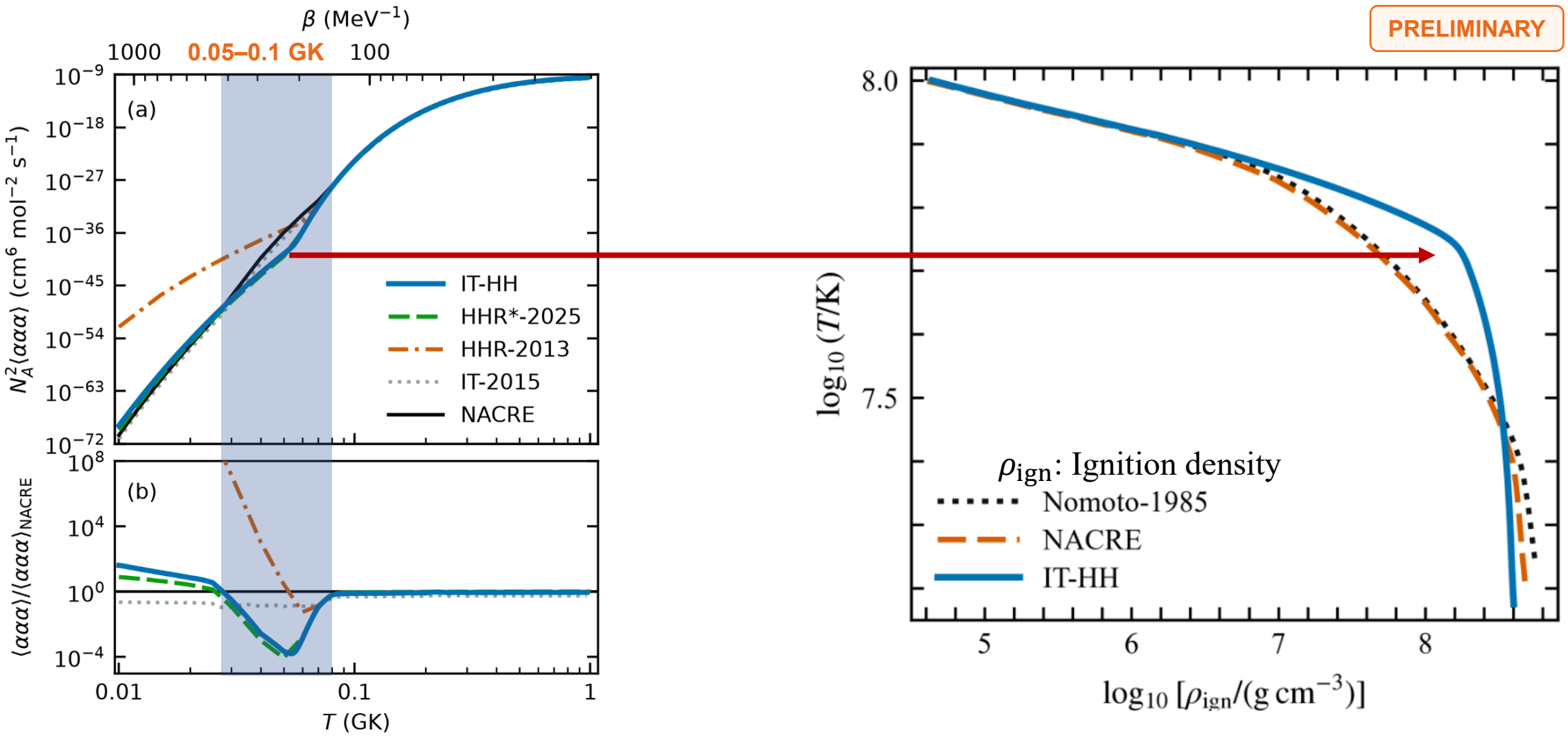
IT-HH recalculation



Agreement and differences help separate effects of the Hamiltonian and basis from the imaginary-time formalism itself.

Triple-α capture: preliminary IT-HH results

PRELIMINARY



Current trend: reduced rate at T ≈ 0.05–0.1 GK and enhanced rate below ≈ 0.03 GK.

Required checks: stable HH truncation, large-hyperradius propagation, and a consistent ^{12}C resonance spectrum.

Luanhong Pei, et al (in preparation)

Summary and outlook

- **Structural identification.** Hoyle-like clustering should be supported by several consistent observables; no single intrinsic geometry is sufficient.
- **Extension to other nuclei.** Candidates in ^{16}O, ^{20}Ne, ^{13}N, ^{10}C, and ^{8}He broaden the discussion to multi-α and valence-nucleon systems.
- **Reaction theory.** IT-HH offers a controlled route to three-body capture, while the low-temperature triple-α rate still requires convergence and astrophysical validation.

