# Exotic Nuclear Shapes: From Structure to Dynamics

**Pengwei Zhao** 赵鹏巍

School of Physics, Peking University

# Outline

- Nonaxial, Reflection-Asymmetric Nuclear Shapes
- Identification of Tetrahedral Nuclear Shapes
- Nonaxial Effects in Nuclear Fission Dynamics
- Summary

# Nuclear Shapes

**Nuclear deformation has played a central role in understanding nuclear structure and dynamics.**

- ✓ Nuclear spectroscopy
- ✓ Nuclear fission
- ✓ Hyperfine spectroscopy
- ✓ Nuclear reaction
- ✓ Relativistic heavy ion collisions
- ✓ …

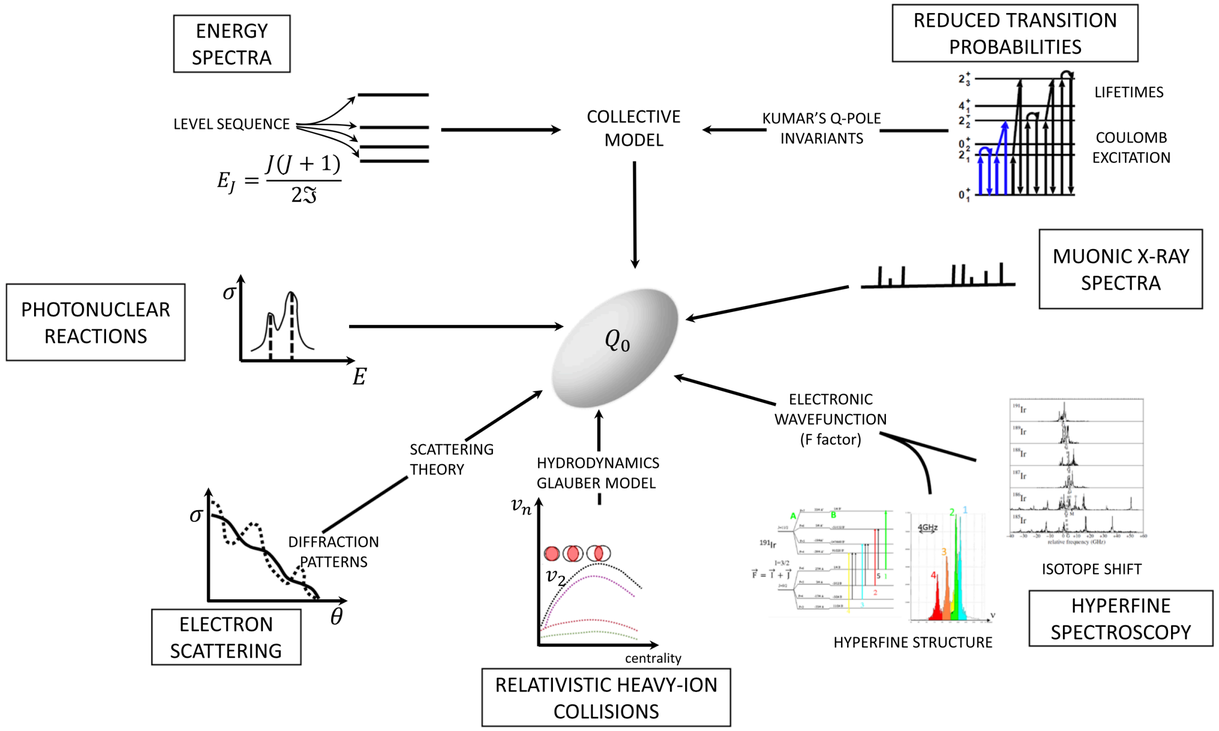
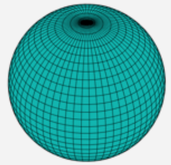
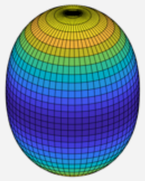
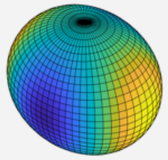
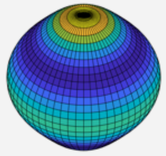
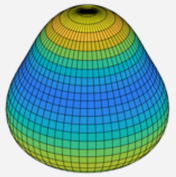
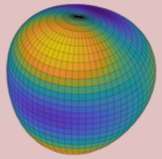
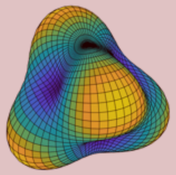
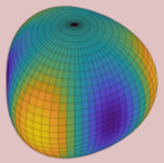



Figure from Verney, Eur. Phys. J. A (2025) 61:82

# How to describe nuclear shapes?

## Nuclear surface expansion

$$R(\theta,\varphi) = R_0\left[1 + \sum_{\lambda=1}^{\infty}\sum_{\mu=-\lambda}^{\lambda}\alpha_{\lambda\mu}Y^*_{\lambda\mu}(\theta,\varphi)\right]$$

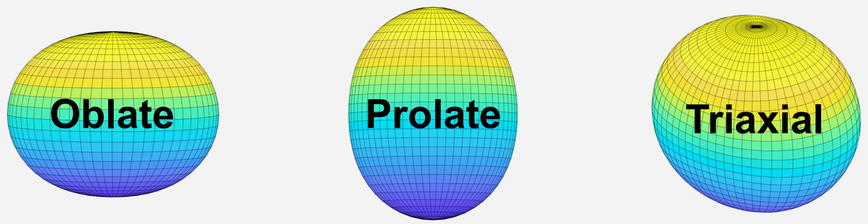
| $Y_{\lambda\mu}=0$ | $Y_{20}\neq 0$ | $Y_{22}\neq 0$ | $Y_{40}\neq 0$ |
|---|---|---|---|
|  |  |  |  |
| $Y_{30}\neq 0$ | $Y_{31}\neq 0$ | $Y_{32}\neq 0$ | $Y_{33}\neq 0$ |
|  |  |  |  |

**Nonaxial & Reflection-asymmetric**

## Quadrupole deformation

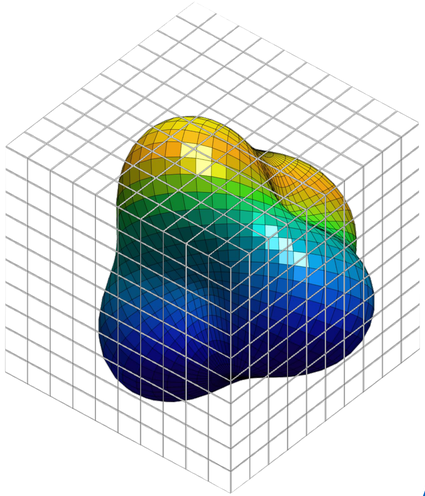
$$R(\theta,\varphi) = R_0\left[1 + \sum_{\mu=-2}^{2}\alpha_{2\mu}Y^*_{2\mu}(\theta,\varphi)\right]$$

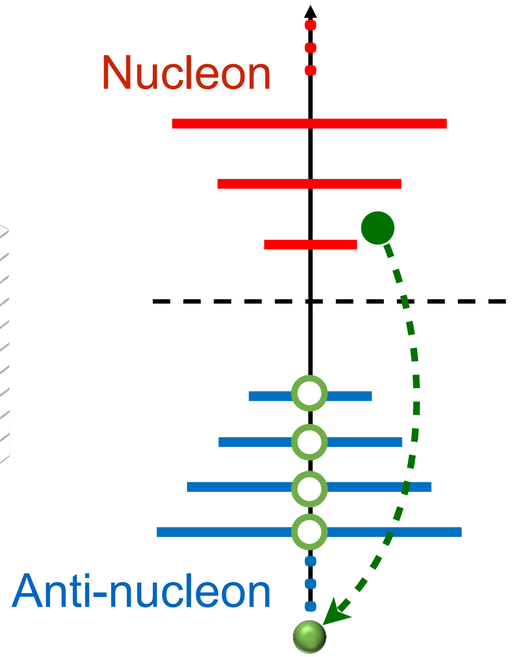
- ✓ Three Euler angles **5 - 3 = 2**
- ✓ **Two deformation parameters** $\beta$ and $\gamma$
- ✓ Defined in the **intrinsic frame**
- ✓ Diagonalize the inertia tensor



# Lattice RDFT

- ✓ **No spatial symmetry restriction**
- ✓ A long-term challenge due to **variational collapse problem**

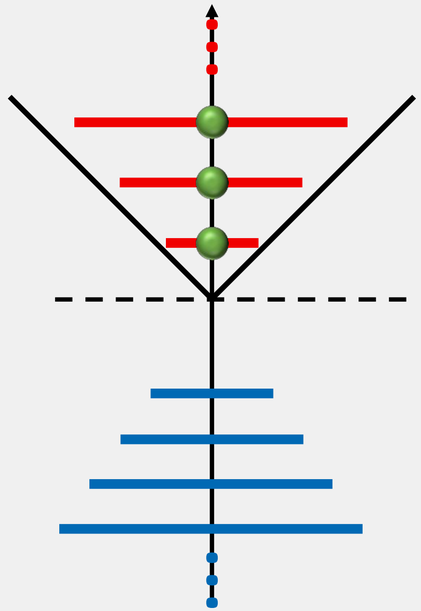





## PCG-F Method

$$\hat{P}_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\hat{F}(h_D) = \begin{pmatrix} 1 & 0 \\ 0 & \epsilon \end{pmatrix}$$



Li, Ren, **PWZ**, PRC 102, 044307 (2020)

Xu, Li, Ren, **PWZ**, PRC 109, 014311 (2024)

# Outline

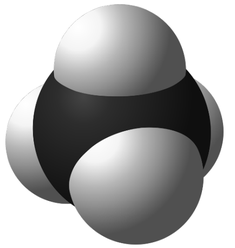
- Nonaxial, Reflection-Asymmetric Nuclear Shapes
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# Nuclear tetrahedral symmetry

✓ **Symmetry** is fundamental to understanding matter and interactions.

✓ **Tetrahedral symmetry** is widespread in molecules, clusters, and crystals.

✓ **Nuclear tetrahedral symmetry?**



NOVEMBER 1, 1938 PHYSICAL REVIEW VOLUME 54

**The Alpha-Particle Model of the Nucleus**

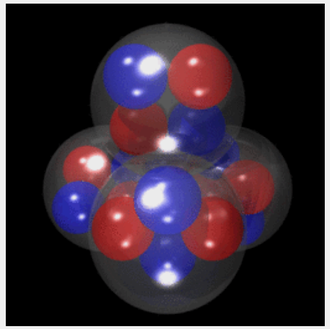
L. R. HAFSTAD
*Department of Terrestrial Magnetism, Carnegie Institution of Washington, Washington, D. C.*

AND

E. TELLER
*George Washington University, Washington, D. C.*

(Received August 26, 1938)

**Tetrahedral $4\alpha$ cluster**



Hafstad, Teller, Phys. Rev. 54, 681 (1938)

- Nonrelativistic density functional theory (DFT)
- Algebraic cluster model
- Lattice effective field theory
- Quantum molecular dynamics
- …

Girod, Schuck, PRL 111, 132503 (2013)

Bijker, Iachello, PRL 112, 152501 (2014)

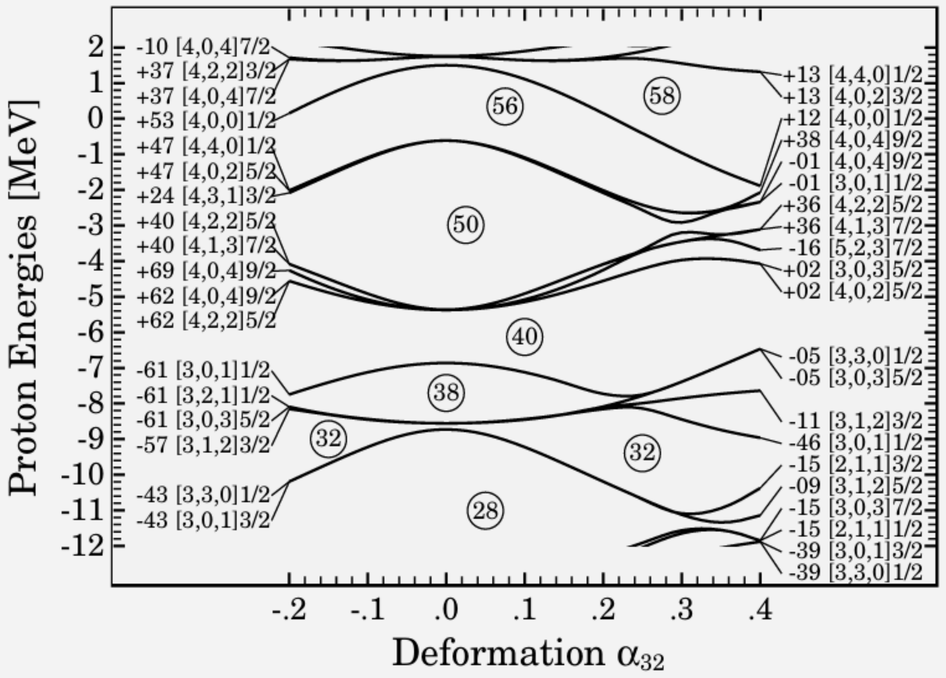
Epelbaum, Krebs, Lähde, Lee, Meißner, Rupak, PRL 112, 102501 (2014)

He, Ma, Cao, Cai, Zhang, PRL 113, 032506 (2014)

… …

# Predictions of Tetrahedral Nuclei

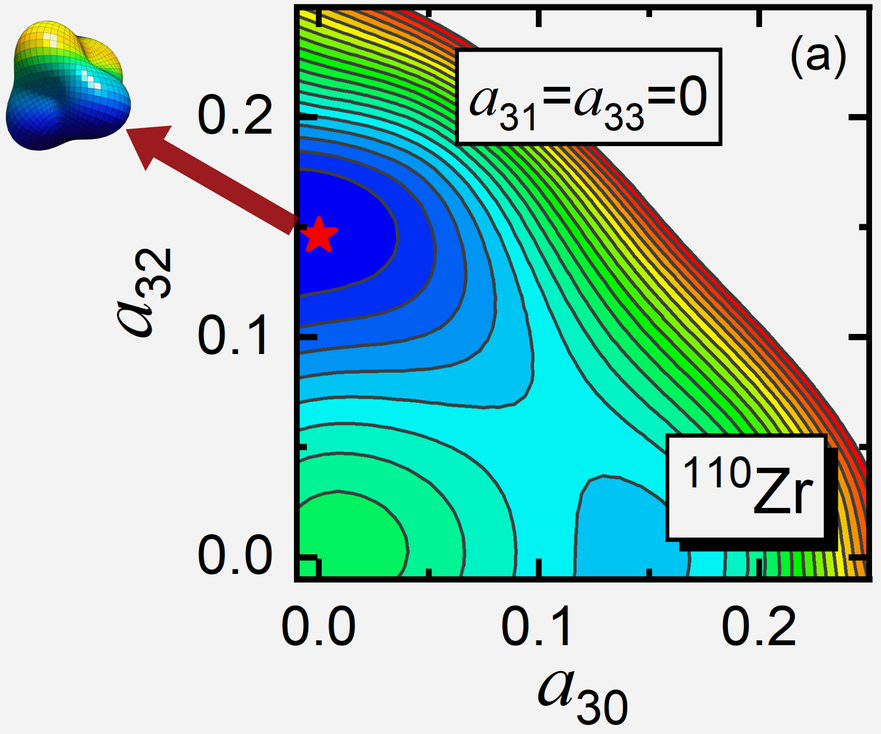
## Tetrahedral magic numbers




Hamamoto, Mottelson, Xie, Zhang, Z. Phys. D 21, 163 (1991)

Dudek, et al., PRL 88, 252502 (2002)

Dudek, et al., PRL 97, 072501 (2006)




Zhao, Lu, Zhao, Zhou, PRC 95, 014320 (2017)

Xu, Li, Ren, **PWZ**, PRC 109, 014311 (2024)

# Experimental evidence?

PRL **97,** 072501 (2006) PHYSICAL REVIEW LETTERS week ending 18 AUGUST 2006

**Island of Rare Earth Nuclei with Tetrahedral and Octahedral Symmetries: Possible Experimental Evidence**

J. Dudek,[1] D. Curien,[2] N. Dubray,[1] J. Dobaczewski,[3] V. Pangon,[1] P. Olbratowski,[3] and N. Schunck[4]
[1]*Institut Pluridisciplinaire Hubert Curien IN2P3-CNRS/Université Louis Pasteur, F-67037 Strasbourg Cedex 2, France*
[2]*Institut Pluridisciplinaire Hubert Curien IN2P3-CNRS, F-67037 Strasbourg Cedex 2, France*
[3]*Institute of Theoretical Physics, Warsaw University, Hoża 69, PL-00681 Warsaw, Poland*
[4]*Departamento de Fisica Teorica, Modulo C-XI, Universidad Autonoma de Madrid, Cantoblanco 28049 Madrid, Spain*
(Received 13 March 2006; published 15 August 2006)

- **Negative-parity** rotational band
- **Strong** electric **octupole** transition
- **Weak** electric **quadrupole** transition
- …

Dudek, et al., PRL 97, 072501 (2006)

PRL **104,** 222502 (2010) PHYSICAL REVIEW LETTERS week ending 4 JUNE 2010

**Ultrahigh-Resolution $\gamma$-Ray Spectroscopy of $^{156}$Gd: A Test of Tetrahedral Symmetry**

M. Jentschel,[1] W. Urban,[1,2] J. Krempel,[1] D. Tonev,[3] J. Dudek,[4] D. Curien,[4] B. Lauss,[5] G. de Angelis,[6] and P. Petkov[3]
[1]*Institut Laue-Langevin, 6 rue Jules Horowitz, BP 156, F-38042 Grenoble, France*
[2]*Faculty of Physics, University of Warsaw, ul. Hoża 69, PL-00-681 Warsaw, Poland*
[3]*Institute for Nuclear Research and Nuclear Energy, BAS, BG-1784 Sofia, Bulgaria*
[4]*Departement de Recherches Subatomiques, Institut Pluridisciplinaire Hubert Curien, DRS-IPHC, 23 rue du Loess, BP 28, F-67037 Strasbourg, France*
[5]*Paul Scherrer Institut, CH-5232 Villigen-PSI, Switzerland*
[6]*Laboratori Nazionali di Legnaro, I-35020 Legnaro, Italy*
(Received 26 February 2010; published 4 June 2010)

**But,** the observation **is incompatible with** a description based on **tetrahedral symmetry**.

Jentschel, et al., PRL 104, 222502 (2010)
Bark, et al., PRL 104, 022501 (2010)
Paul, et al., PRL 118, 032501 (2017)

**Alternative criteria and/or the search for new test candidates?**

# Triple mass differences

✳ Mass → Liquid model

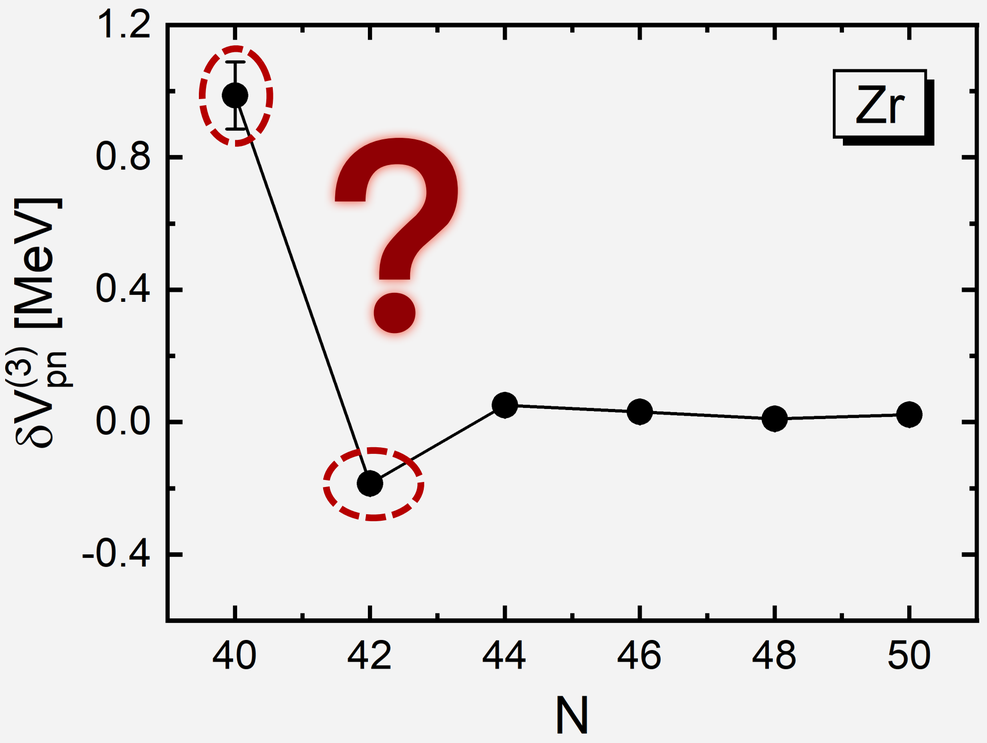
✳ Mass difference → Shell model

✳ **Double/Triple mass difference ?**

$$S_{2n}(Z,N) = E(Z,N) - E(Z,N-2)$$

$$\delta V_{pn}(Z,N) = -\frac{1}{4}[S_{2n}(Z,N) - S_{2n}(Z-2,N)]$$

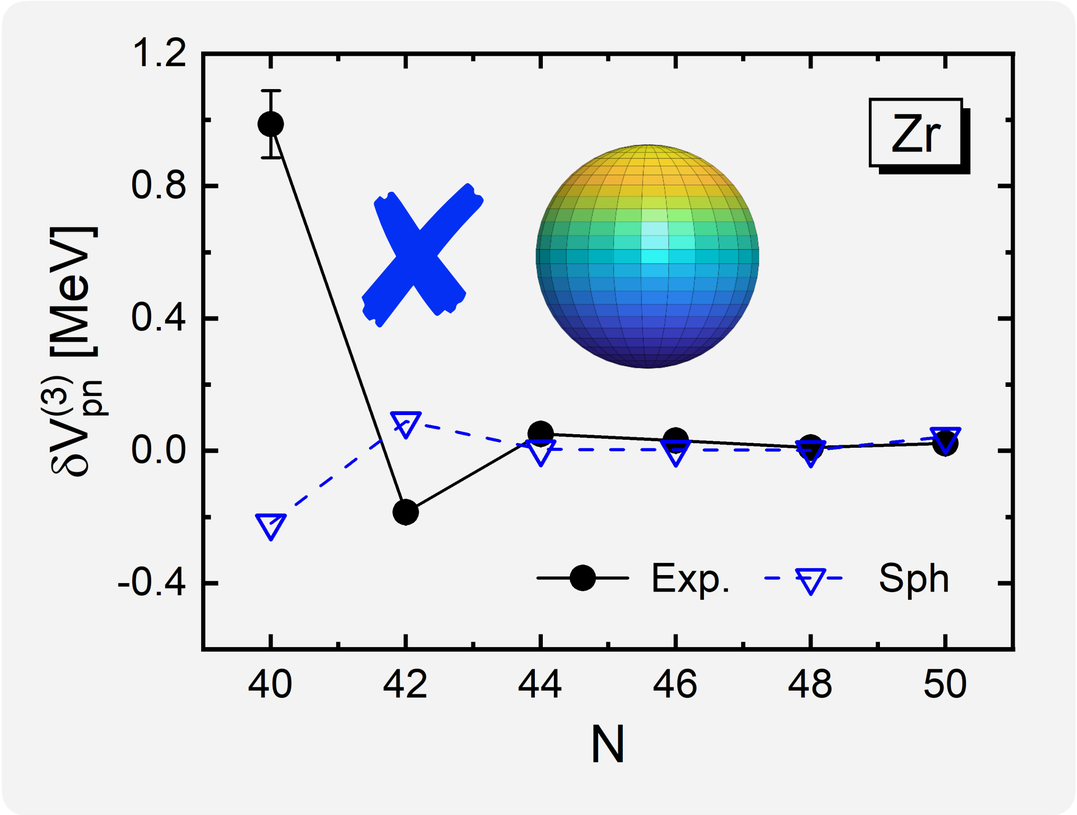
$$\delta V^{(3)}_{pn}(Z,N) = \delta V_{pn}(Z,N) - \delta V_{pn}(Z,N+2)$$

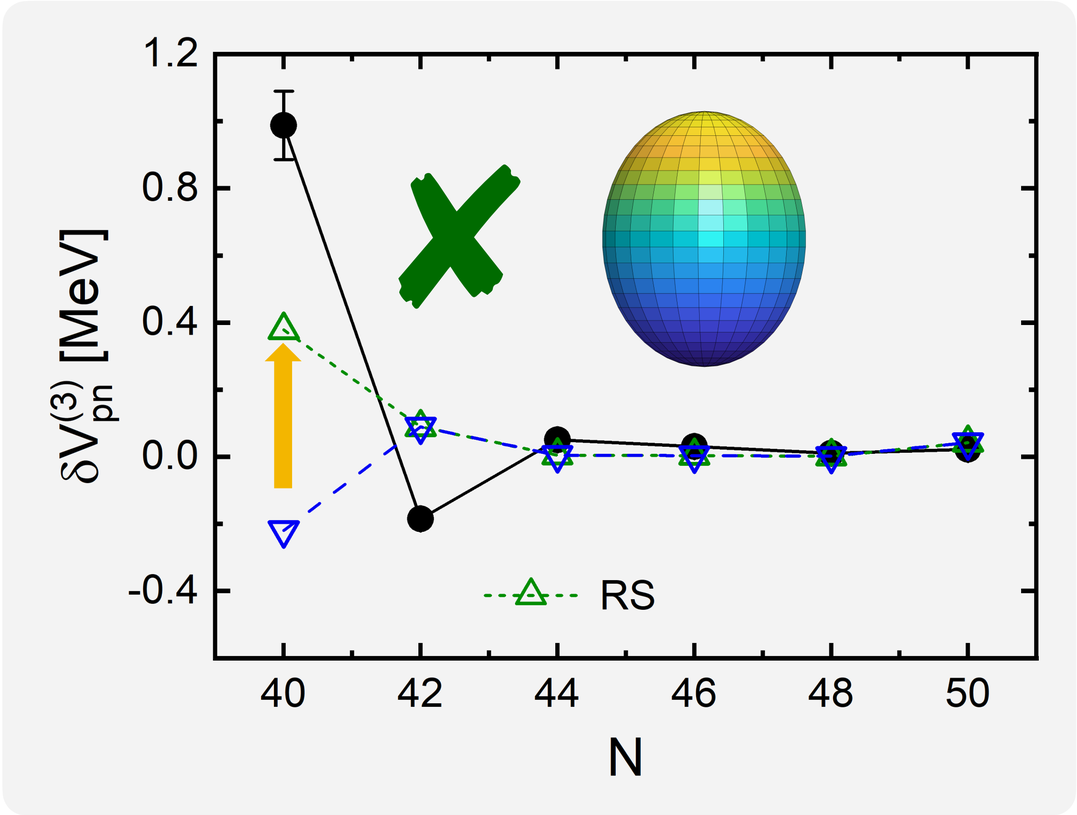



Wang, et al., Chin. Phys. C 45, 030003 (2021)

Hamaker, et al., Nat. Phys. 17, 1408 (2021)

# Quadrupole calculations



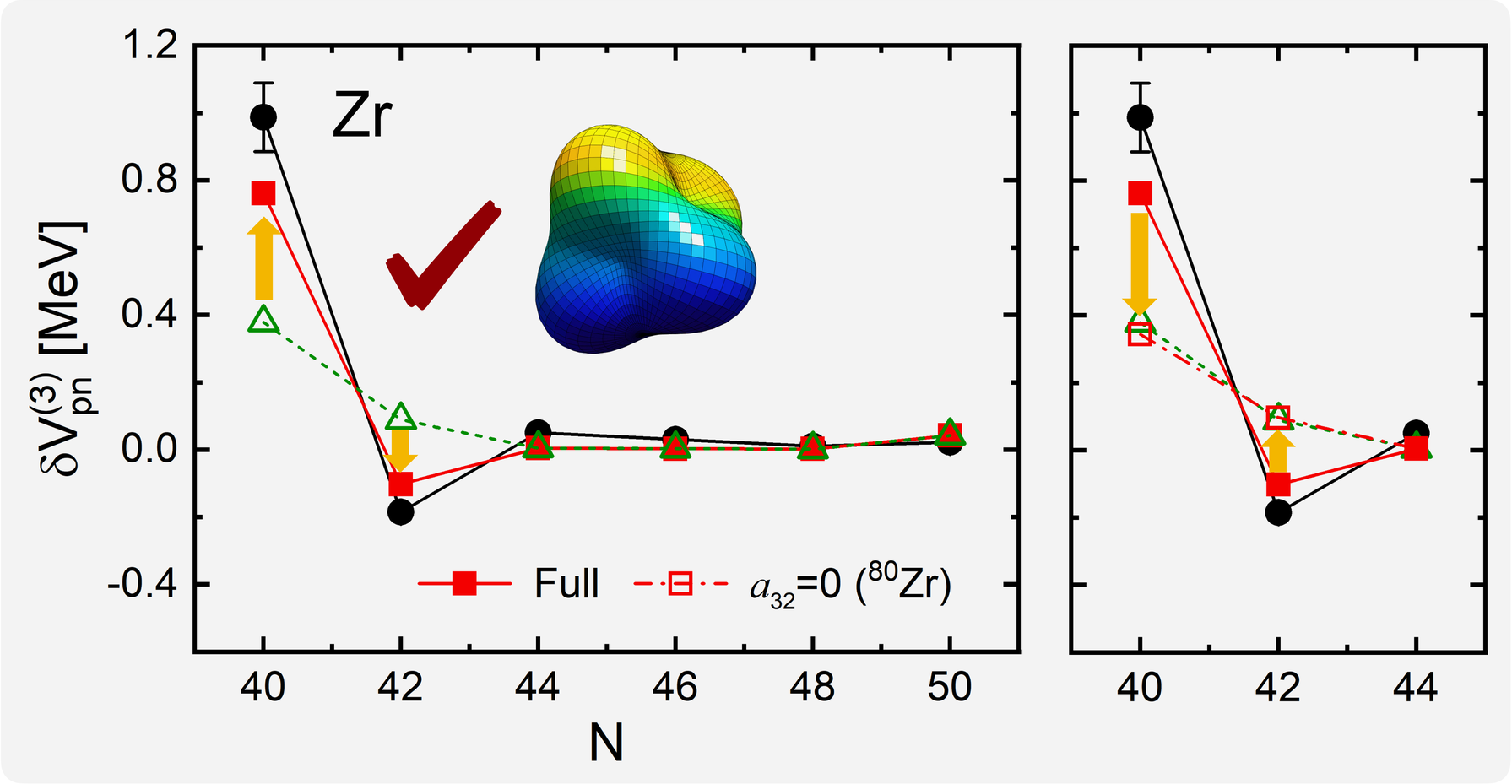



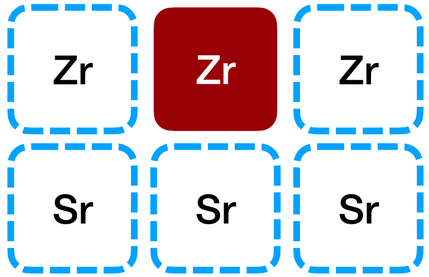

Calculations restricted to **quadrupole and triaxial shapes fail to reproduce** the localized enhancement.

Xu and **PWZ**, PRL 136, 232503 (2026)

# Octupole calculations

**Tetrahedral deformation of $^{80}$Zr is revealed from nuclear masses.**




$^{80}$**Zr** $a_{32} = 0.14$



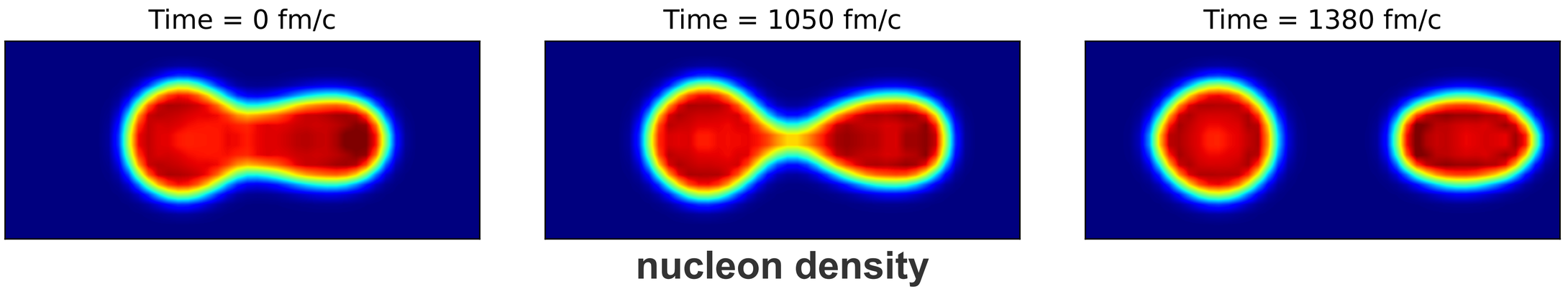

Xu and **PWZ**, PRL 136, 232503 (2026)

# Outline

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# Microscopic Dynamics of Nuclear Fission

**TDRDFT**：**Microscopic Self-Consistent Description of Fission Dynamics**




Ren, Vretenar, Nikšić, **PWZ**, Zhao, Meng, PRL 128, 172501 (2022)

# Angular Momentum of Fission Fragments

In 2021, experiment revealed the **angular momenta** of the two fission fragments are **nearly uncorrelated**.

Article

## Angular momentum generation in nuclear fission

https://doi.org/10.1038/s41586-021-03304-w

Received: 18 June 2020

Accepted: 9 December 2020

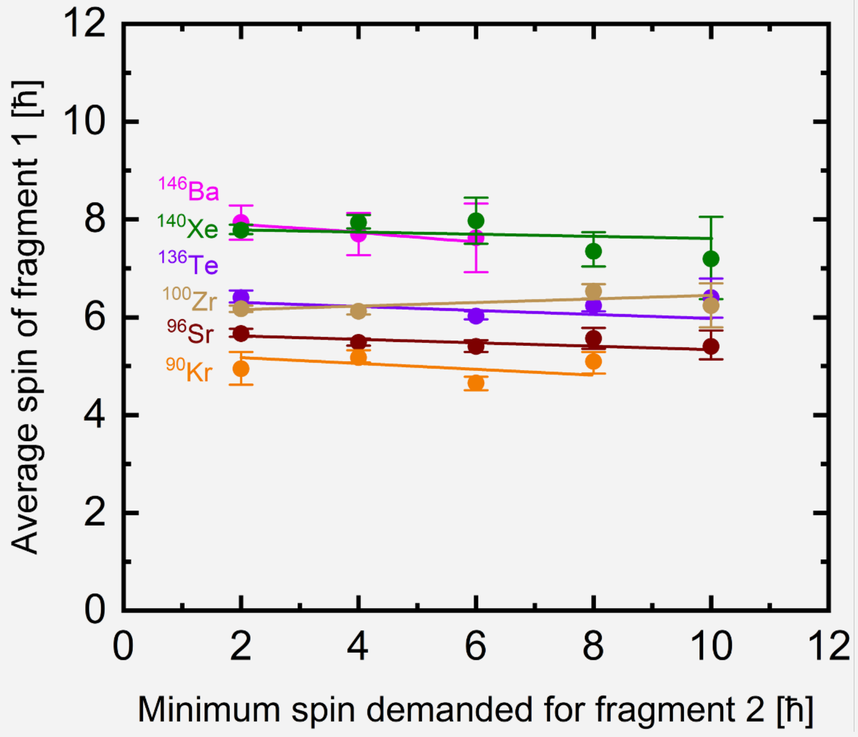
Published online: 24 February 2021

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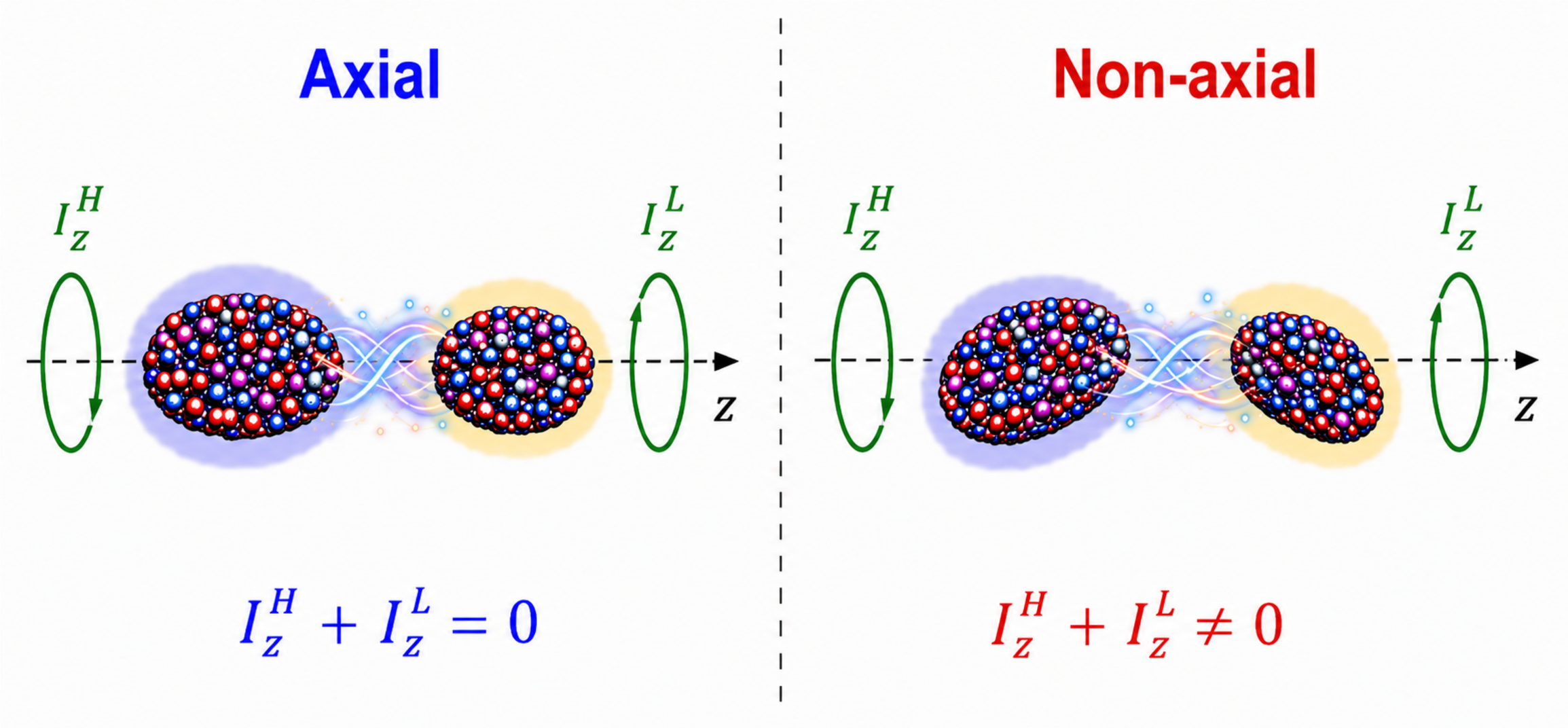
J. N. Wilson[1✉], D. Thisse[1], M. Lebois[1], N. Jovančević[1], D. Gjestvang[2], R. Canavan[3,4], M. Rudigier[3,5], D. Étasse[6], R-B. Gerst[7], L. Gaudefroy[8], E. Adamska[9], P. Adsley[1], A. Algora[10,11], M. Babo[1], K. Belvedere[3], J. Benito[12], G. Benzoni[13], A. Blazhev[7], A. Boso[4], S. Bottoni[13,14], M. Bunce[4], R. Chakma[1], N. Cieplicka-Oryńczak[15], S. Courtin[16], M. L. Cortés[17], P. Davies[18], C. Delafosse[1], M. Fallot[19], B. Fornal[15], L. Fraile[12], A. Gottardo[20], V. Guadilla[19], G. Häfner[1,7], K. Hauschild[1], M. Heine[16], C. Henrich[5], I. Homm[5], F. Ibrahim[1], Ł. W. Iskra[13,15], P. Ivanov[4], S. Jazrawi[3,4], A. Korgul[9], P. Koseoglou[5,21], T. Kröll[5], T. Kurtukian-Nieto[22], L. Le Meur[19], S. Leoni[13,14], J. Ljungvall[1], A. Lopez-Martens[1], R. Lozeva[1], I. Matea[1], K. Miernik[9], J. Nemer[1], S. Oberstedt[23], W. Paulsen[2], M. Piersa[9], Y. Popovitch[1], C. Porzio[13,14,24], L. Qi[1], D. Ralet[25], P. H. Regan[3,4], K. Rezynkina[26], V. Sánchez-Tembleque[12], S. Siem[2], C. Schmitt[16], P.-A. Söderström[5,27], C. Sürder[5], G. Tocabens[1], V. Vedia[12], D. Verney[1], N. Warr[7], B. Wasilewska[15], J. Wiederhold[5], M. Yavahchova[28], F. Zeiser[2] & S. Ziliani[13,14]

Wilson et al., Nature **590**, 566–570 (2021)

**Why? Existing microscopic theories could not explain …**

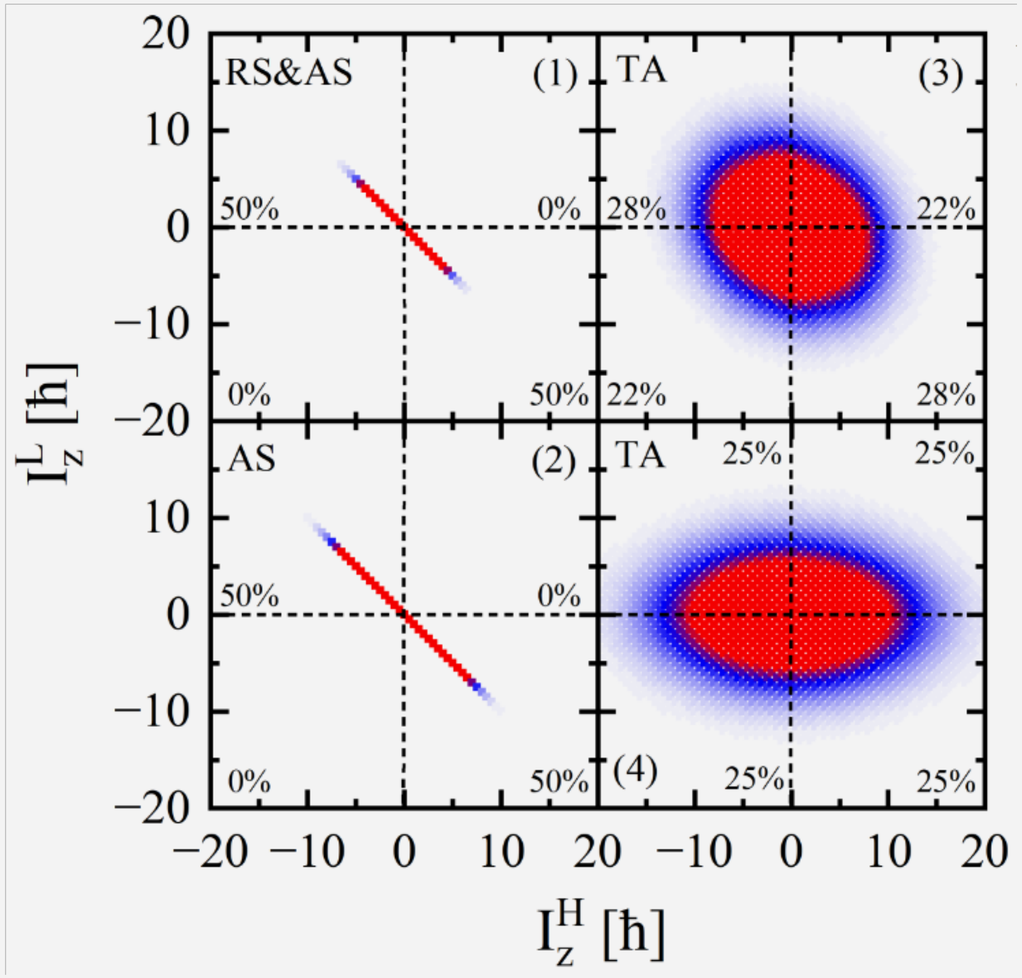


# Nonaxial effects in fission

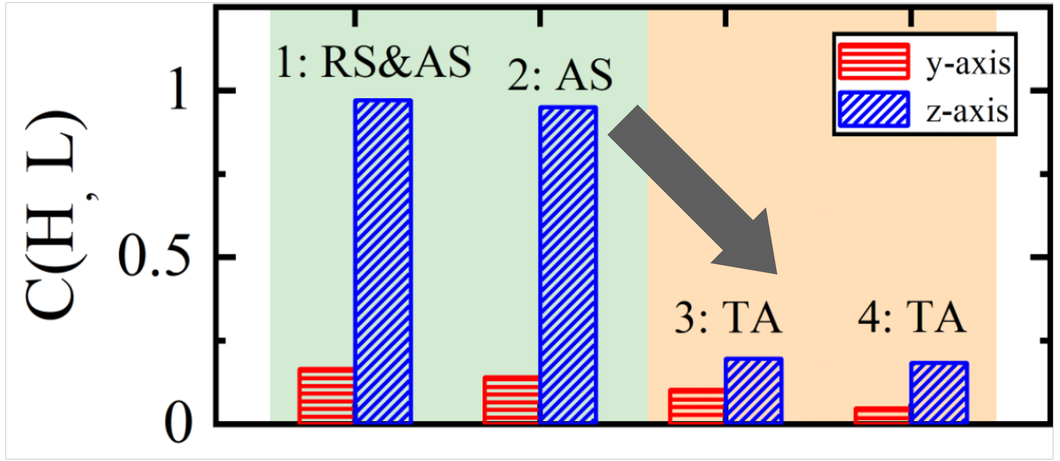



Li, Zhang, Vretenar, Nikšić, **PWZ**, Meng, PRL, 136, 252502 (2026)

# Spin correlations of the fission fragments




Mutual information analysis reveals that **axial-symmetry breaking reduces spin-spin correlations** along the fission axis.




Li, Zhang, Vretenar, Nikšić, **PWZ**, Meng, PRL, 136, 252502 (2026)

# Summary

Relativistic density functional theory (RDFT) has been solved in 3D lattice and extended to its time-dependent version for various nuclear structure and dynamics.

- Exotic Nuclear Shapes

  Nonaxial and reflection-asymmetric deformations leave distinct signatures in nuclear masses and fission dynamics

- Nuclear tetrahedral symmetry

  Triple mass differences provide an experimentally accessible probe

  Tetrahedral symmetry in Zr-80

- Nuclear fission dynamics

  Nonaxial deformation reduces the spin correlations between fission fragments

# Collaborations

- ✓ **Peking University:** Bo Li, Jie Meng, Yilong Yang
- ✓ **Beihang University:** Yakun Wang
- ✓ **Institute of Theoretical Physics, CAS:** Dandan Zhang
- ✓ **Fudan University:** Wanbing He, Yugang Ma, Zixiao Zhang
- ✓ **Fuzhou University:** Xinhui Wu
- ✓ **Central China Normal University:** Chen Ji
- ✓ **Nankai University:** Zhengxue Ren, Fangfang Xu
- ✓ **University of Zagreb:** D. Vretenar, T. Nikšić
- ✓ **Ruhr University Bochum, Germany:** E. Epelbaum
- ✓ **Technical University of Munich, Germany:** P. Ring
- ✓ **Kyoto University, Japan:** K. Hagino
- ✓ **University of Milan, Italy:** G. Colò
- ✓ … …

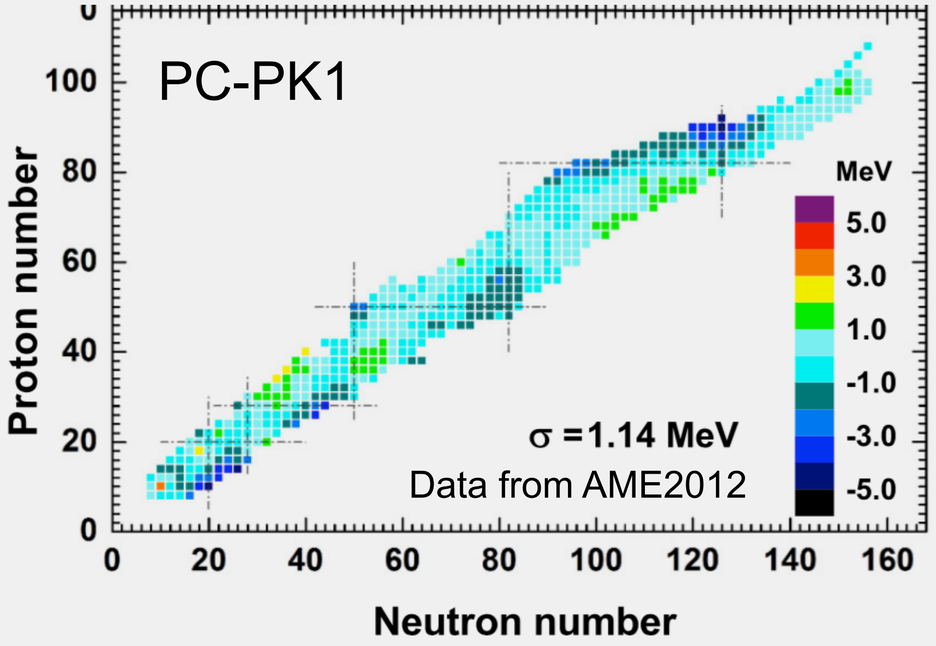
# Collaborations

- ✓ **Peking University:** Bo Li, Jie Meng, Yilong Yang
- ✓ **Beihang University:** Yakun Wang
- ✓ **Institute of Theoretical Physics, CAS:** Dandan Zhang
- ✓ **Fudan University:** Wanbing He, Yugang Ma, Zixiao Zhang
- ✓ **Fuzhou University:** Xinhui Wu
- ✓ **Central China Normal University:** Chen Ji
- ✓ **Nankai University:** Zhengxue Ren, Fangfang Xu
- ✓ **University of Zagreb:** D. Vretenar, T. Nikšić
- ✓ **Ruhr University Bochum, Germany:** E. Epelbaum
- ✓ **Technical University of Munich, Germany:** P. Ring
- ✓ **Kyoto University, Japan:** K. Hagino
- ✓ **University of Milan, Italy:** G. Colò
- ✓ … …

# Thank you!

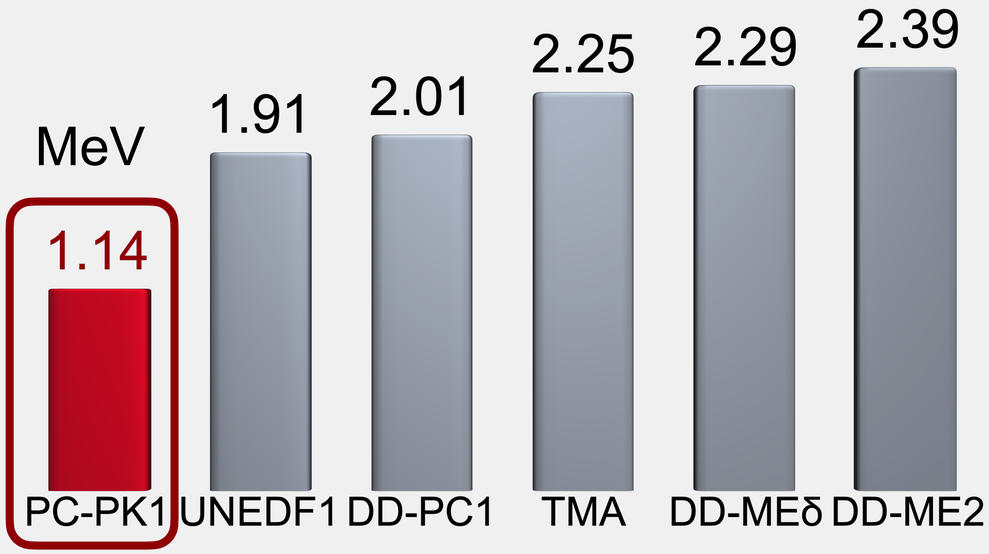
# Covariant density functional：PC-PK1

## Mass Deviations from Exp.




**PWZ**, Li, Yao, Meng, PRC 82, 054319 (2010)
Lu, Li, Li, Yao, Meng, PRC 91, 027304 (2015)

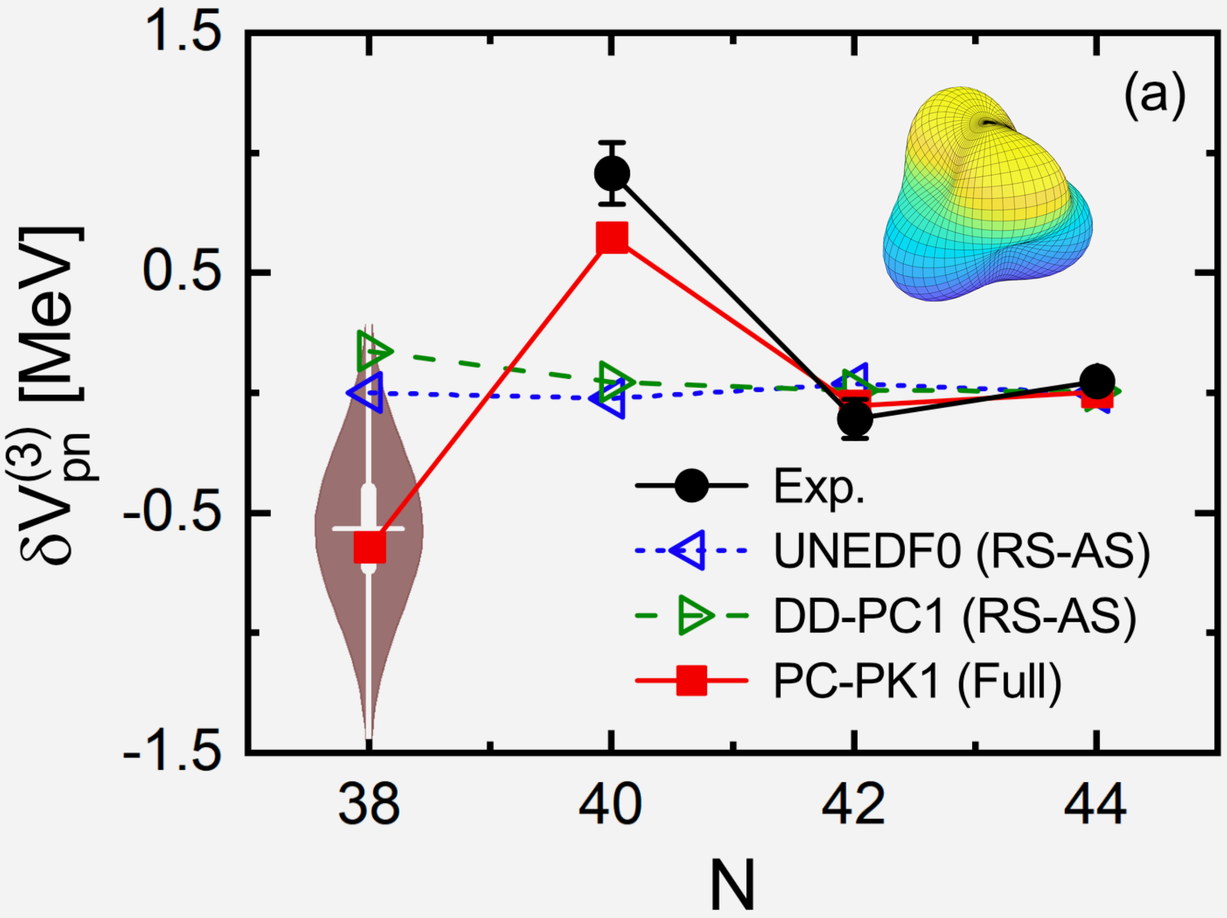
## RMS Deviation



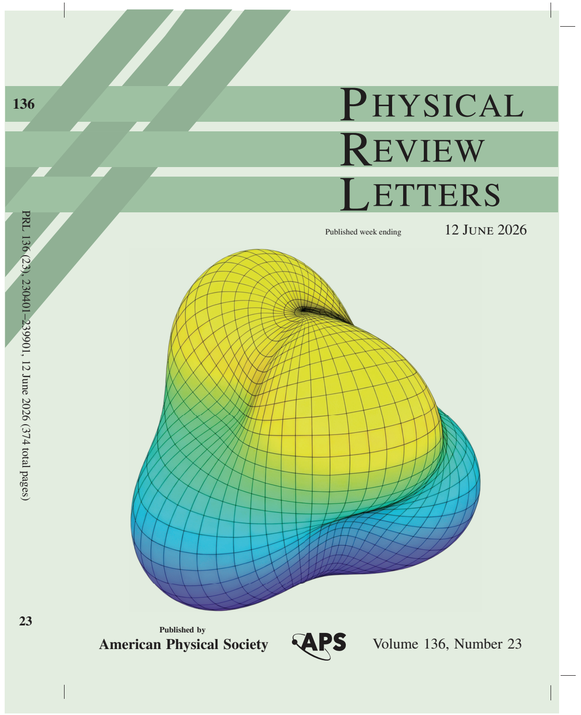

http://nuclearmap.jcnp.org
Yang, Wang, **PWZ**, Li, PRC 104, 054312 (2021)
Yang, **PWZ**, Li, PRC 107, 024308 (2023)

Among **the best density-functional description** for nuclear masses!

# Future experiment




Xu and **PWZ**, PRL 136, 232503 (2026)



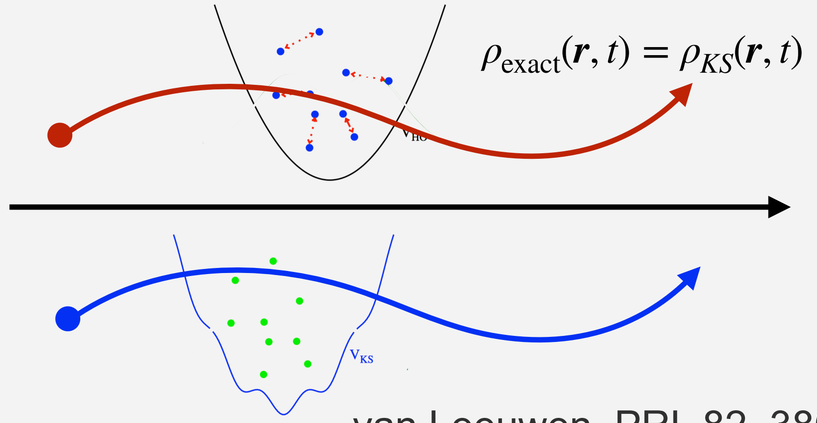
# Time-dependent RDFT

## Runge-Gross Theorem

There is a unique mapping between the time dependent external potential and the density, for many body systems evolving from a given initial state.

Runge and Gross, PRL 52, 997 (1984)

## Time-dependent Kohn-Sham DFT




van Leeuwen, PRL 82, 3863 (1999)

Ren, **PWZ**, Meng, PLB 801,135194 (2020)

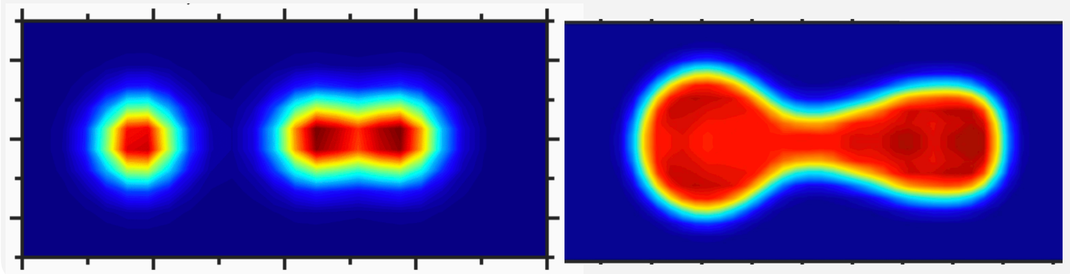
$$i\partial_t \begin{pmatrix} f \\ g \end{pmatrix} = \begin{pmatrix} m + V + S & \boldsymbol{\sigma}\cdot\boldsymbol{p} - \boldsymbol{\sigma}\cdot\boldsymbol{V} \\ \boldsymbol{\sigma}\cdot\boldsymbol{p} - \boldsymbol{\sigma}\cdot\boldsymbol{V} & -m + V - S \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix}$$

$$\rho(\boldsymbol{r},t) = \sum_i^N f_i^2 + g_i^2$$

# Application of TD-RDFT

## Applications

- Linear alpha-chain
- Nuclear fusion
- **Nuclear fission**
- Nuclear rotation and vibration
- **Multi-nucleon transfer reaction**
- …



## Fission

- Ternary fission
- Thermal effect
- Angular momentum
- …

## Multinucleon transfer reaction

- Cross section
- Angular momentum
- Deexcitation
- Orientation effects
- …

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