Microscopic analysis of scissors mode in ^{254}No

V.O. Nesterenko and M.A. Mardyban

Laboratory of Theoretical Physics, Joint Institute for Nuclear Research,
Dubna, Moscow region, Russia

14th ITP/CAS-BLTP/JINR, Joint Workshop on Physics of Strong Interacting Systems
(18-23.10.26, Nanjing, China)

Introduction

** Nobelium isotopes $^{250-262}$No provide a benchmark test for theoretical models pretending to describe superheavy nuclei.

** Nobelium isotopes are the heaviest nuclei for which there is some (though yet poor) spectroscopic information ($^{250-256}$No)

**Isotopes ^{252,254}No are most studied.

** DFT models yet give very different predictions for No features (pairing, s-p spectra, K-isomers, …).

**The topic is very hot.

** Recent and planned experiments for spectroscopy of No isotopes

$$\underbrace{No^{102}, Lr^{103}, Rf^{104}, Db^{105}, Sg^{106}, Nh^{113}, Fl^{114}, Mc^{115}, Lv^{116}, Ts^{117}, Og^{118}}_{\text{Superheavy elements}}$$

Basic reviews on low-energy spectra (mainly for K-isomers and g.s. rotational bands):

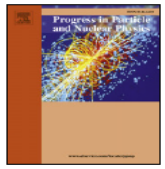
2008:

Progress in Particle and Nuclear Physics 61 (2008) 674–720

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp



Review

In-beam and decay spectroscopy of transfermium nuclei

R.-D. Herzberg [a,*], P.T. Greenlees [b]

arXiv: 2309.1046802 [nucl-ex]

K Isomers in Transuranium Nuclei

Fritz Peter Heßberger[1,2,*]

[1]GSI - Helmholtzzentrum für Schwerionenforschung GmbH, Planckstraße 1, 64291 Darmstadt, Germany
[2]Helmholtz Institut Mainz, Staudingerweg 18, 55128 Mainz, Germany

arXiv: 2501.04053v7 [nucl-ex]

Decay spectroscopy of heavy and superheavy nuclei

Dieter Ackermann[a,1]

[a]*Grand Accélérateur National d'Ions Lourds – GANIL, CEA/DRF-CNRS/IN2P3, 55027, F-14076 Caen, France*

Despite a huge experimental and theoretical effort for No isotopes, some open problems are yet disputed.
New experimental data appear.

PHYSICAL REVIEW C **111**, 064322 (2025)

Low-energy spectra of nobelium isotopes: Skyrme random-phase-approximation analysis

V. O. Nesterenko,[1,2] M. A. Mardyban,[1,2] A. Repko,[3] R. V. Jolos,[1,2] P.-G. Reinhard,[4] and Alan A. Dzhioev[1]
[1]*Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia*
[2]*Dubna State University, 141982 Dubna, Moscow region, Russia*
[3]*Institute of Physics, Slovak Academy of Sciences, 84511 Bratislava, Slovakia*
[4]*Institute for Theoretical Physics II, University of Erlangen, D-91058, Erlangen, Germany*

- systematic study of low-energy spectra in $^{250-262}$No
- detailed analysis for ^{252,254}No
- interpretation of K-isomers $K^{\pi} = 8^{-}, 2^{-}, 3^{+}, 4^{+}$
- origin of recently observed states $K^{\pi} = 0^{+}$ in ^{254}No

experiment Dubna-Strasbourg-GANIL: M. Forge et al., J. Phys.: Conf. Ser. 2586, 012083 (2023).

pairing vibrations vs superdeformed gs (Dao, Nowacki)

M1 scissors mode in ^{254}No



Low-energy (~2.5 MeV) collective mode

Characterized by strong orbital M1 transitions

Should exist in all deformed nuclei

Already observed in light, medium and heavy (U, Th) nuclei

What about superheavy nuclei and nearby?

Predicted:

N. Lo Iudice and F. Palumbo, Phys. Rev. Lett. 41, 1532 (1978)

First observed:

D. Bohle et al, Phys. Rev. Lett. 137, 27 (1984)

Review:

K. Heide, P. von Neumann-Cosel and A. Richter, Rev. Mod. Phys. 82, 2366 (2010) ,

V.O. Nesterenko, M.A. Mardyban and A.Repko, arXiv:2609.21489v1 [nucl-th], subm. to PRC.

Why SM in heavy nuclei is important?

Impact of SM to strength functions is relevant for description of stellar nucleosynthesis processes, like the rapid neutron-capture process (r-process), as it increases the neutron-capture probability.

Both centroid energy and integral M1 strength contribute to SM phenomenological formulas (ie.g. those based on two-rotor model).

SM estimation formulas should be tested in the wide mass region and especially for heavy nuclei.

Physics Letters B 834 (2022) 137479

Contents lists available at ScienceDirect

Physics Letters B

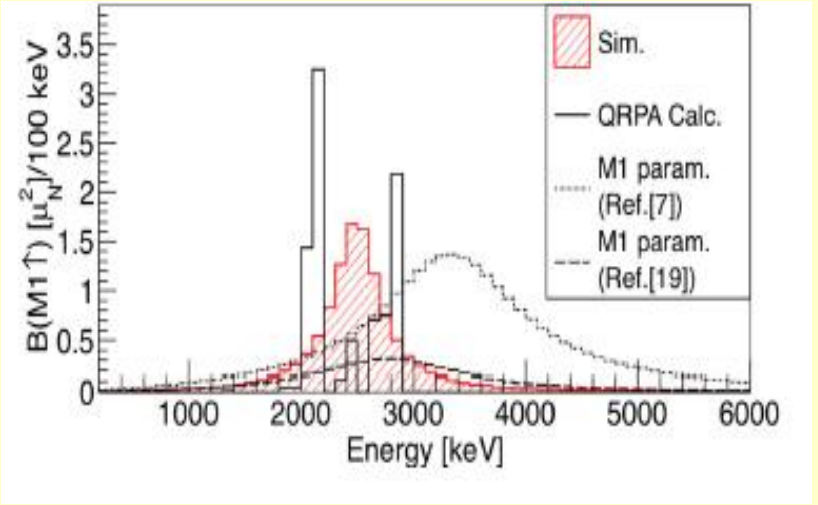
www.elsevier.com/locate/physletb

Experimental observation of the $M1$ scissors mode in ^{254}No

F.L. Bello Garrote [a,*], A. Lopez-Martens [b,d,*], A.C. Larsen [a,*], I. Deloncle [b,c], S. Péru [c,d], F. Zeiser [a], P.T. Greenlees [e], B.V. Kheswa [a,1], K. Auranen [e], D.L. Bleuel [f], D.M. Cox [e,2], L. Crespo Campo [a], F. Giacoppo [g,h], A. Görgen [a], T. Grahn [e], M. Guttormsen [a], T.W. Hagen [a], L. Harkness-Brennan [i], K. Hauschild [b,d], G. Henning [j], R.-D. Herzberg [i], R. Julin [e], S. Juutinen [e], T.A. Laplace [k], M. Leino [e], J.E. Midtbø [a], V. Modamio [a], J. Pakarinen [e], P. Papadakis [e,3], J. Partanen [e,4], T. Renstrøm [a], K. Rezynkina [l,5], M. Sandzelius [e], J. Sarén [e], C. Scholey [e,6], S. Siem [a], J. Sorri [e,7], S. Stolze [e], J. Uusitalo [e]

Experiment:

- accelerator Laboratory of the Univ. of Jyväskylä.
- Reaction $^{208}Pb(^{48}Ca,2n\gamma)^{254}No$
- Very complicated experiment



Estimations:

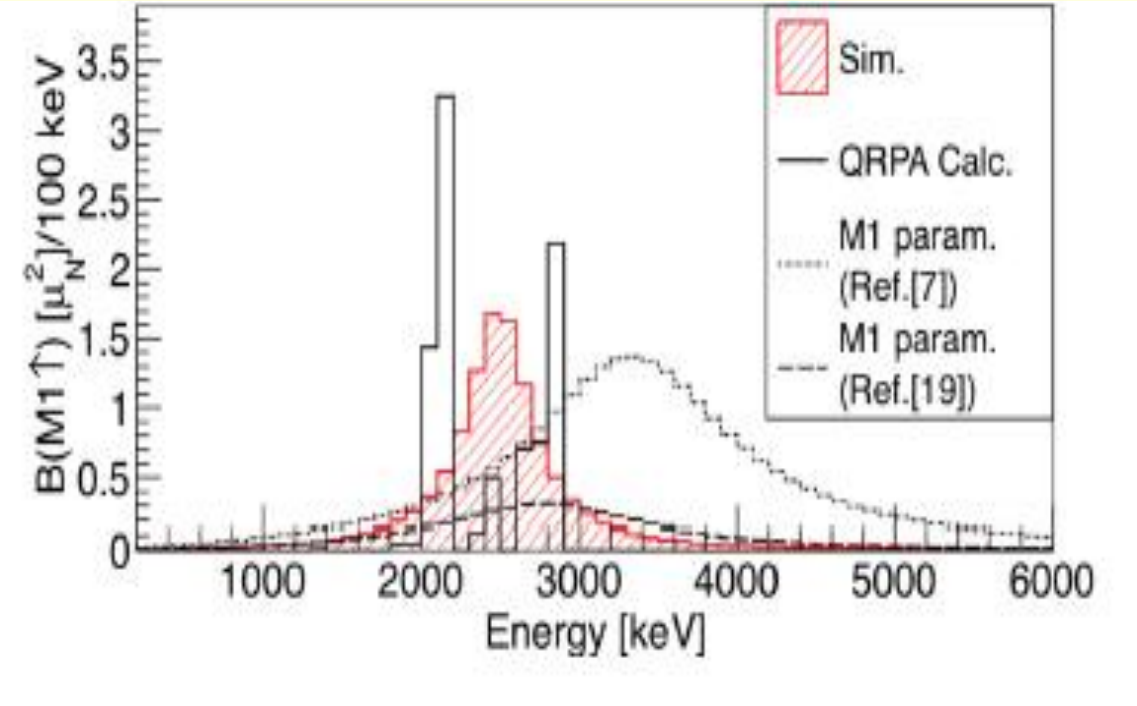
- Gogny QRPA
- empirical formulas

Results:

- first observation of M1 scissor mode (SM) in so heavy nucleus
- SM energy: ~ 2.5 MeV
- SM strength: $\sum B(M1\uparrow) = 11.8\,(19)\mu_N^2$

Experimental data and theoretical estimations

F.I. Bello Garotte et al, Phys. Lett. B 834, 137479 (2022).



Results:

- SM energy: ~ 2.5 MeV
- SM strength: 11.8 (19) μ_N^2

Sum rule (rigid MoI):

- SM energy: ~ 2.1 MeV
- SM strength: 12.1 μ_N^2

WFM:

- SM energy: ~ 2.6 MeV
- SM strength: 13.4

E.B. Balbutsev and I.V. Molodtsova, Eur. Phys. J. A 60, 185 (2024).

D1M Gogny QRPA: ad-hoc energy downshift

Empirical estimations:

- GLO: overestimation of the energy centroid and strength
- SMLO: underestimation of strength
- EGLO: does not converge

} RAINIER

E1 + M1 Lorentzians

A further microscopic analysis is necessary

The model:

fully self-consistent Skyrme QRPA: SG2, SG2+J^2, SLy4, SLy5

2d QRPA (codes of A. Repko):

- fully self-consistent matrix RPA,
- ph- and pp-channels,
- removal of spurious admixtures

A. Repko, J. Kvasil, VON., and P.-G. Reinhard, arXiv:1510.01248[nucl-th]
A. Repko , J. Kvasil, VON, W. Kleinig, P.-G. Reinhard EPJA, 53, 221 (2017)

A. Repko, J. Kvasil, VON, PRC, 99, 044307 (2019)
J. Kvasil, A. Repko, VON, EPJA, 55, 213 (2019)

Mean field (SKYAX)

- deformation from the minimum of the total energy
- 2D mesh in cylindrical coordinates
- calculation box: 3R, mesh step 0.4 fm
- sp levels up to +55 MeV

$$\beta = \frac{4\pi}{3}\frac{Q_{20}}{AR^2}$$

2qp basis (SLy5): 2qp states until ~ 100 MeV, EWSR ~100%

Pairing: volume monopole pairing, BCS

G. Colo, H. Sagawa, S. Fracasso, P.F. Bortignon, PLB, 646, 227 (2007) .

Inclusion of central-exchange J^2 term, which can be important for description of M1 excitations

Perturbative (SG2 vs SG2+J^2) and consistent (SLy4 vs SLy5) inclusion of central exchange J^2 term.

Can be important for description of M1 excitations

$$
\begin{aligned}
\mathcal{H}_{\rm Sk} = & \frac{b_0}{2}\rho^2 - \frac{b'_0}{2}\sum \rho_q^2 + \frac{b_3}{3}\rho^{\alpha+2} - \frac{b'_3}{3}\rho^{\alpha}\sum \rho_q^2 \\
& + b_1(\rho\tau - \mathbf{j}^2) - b'_1\sum(\rho_q\tau_q - \mathbf{j}_q^2) \\
& - \frac{b_2}{2}\rho\Delta\rho + \frac{b'_2}{2}\sum \rho_q\Delta\rho_q \\
& - b_4(\rho\nabla\cdot\mathbf{J} + (\nabla\times\mathbf{j})\cdot\mathbf{s}) \\
& - b'_4\sum(\rho_q\nabla\cdot\mathbf{J}_q + (\nabla\times\mathbf{j}_q)\cdot\mathbf{s}_q) \\
& + \frac{\tilde{b}_0}{2}\mathbf{s}^2 - \frac{\tilde{b}'_0}{2}\sum \mathbf{s}_q^2 + \frac{\tilde{b}_3}{3}\rho^{\alpha}\mathbf{s}^2 - \frac{\tilde{b}'_3}{3}\rho^{\alpha}\sum \mathbf{s}_q^2 \\
& - \frac{\tilde{b}_2}{2}\mathbf{s}\cdot\Delta\mathbf{s} + \frac{\tilde{b}'_2}{2}\sum \mathbf{s}_q\cdot\Delta\mathbf{s}_q \\
& + \tilde{b}_1(\mathbf{s}\cdot\mathbf{T} - \mathbf{J}^2) + \tilde{b}'_1\sum(\mathbf{s}_q\cdot\mathbf{T}_q - \mathbf{J}_q^2).
\end{aligned}
$$

spin-orbit terms

central-exchange term

G. Colo, H. Sagawa, S. Fracasso and P.F. Bortignon, Phys. Lett. B 646, 227 (2007).

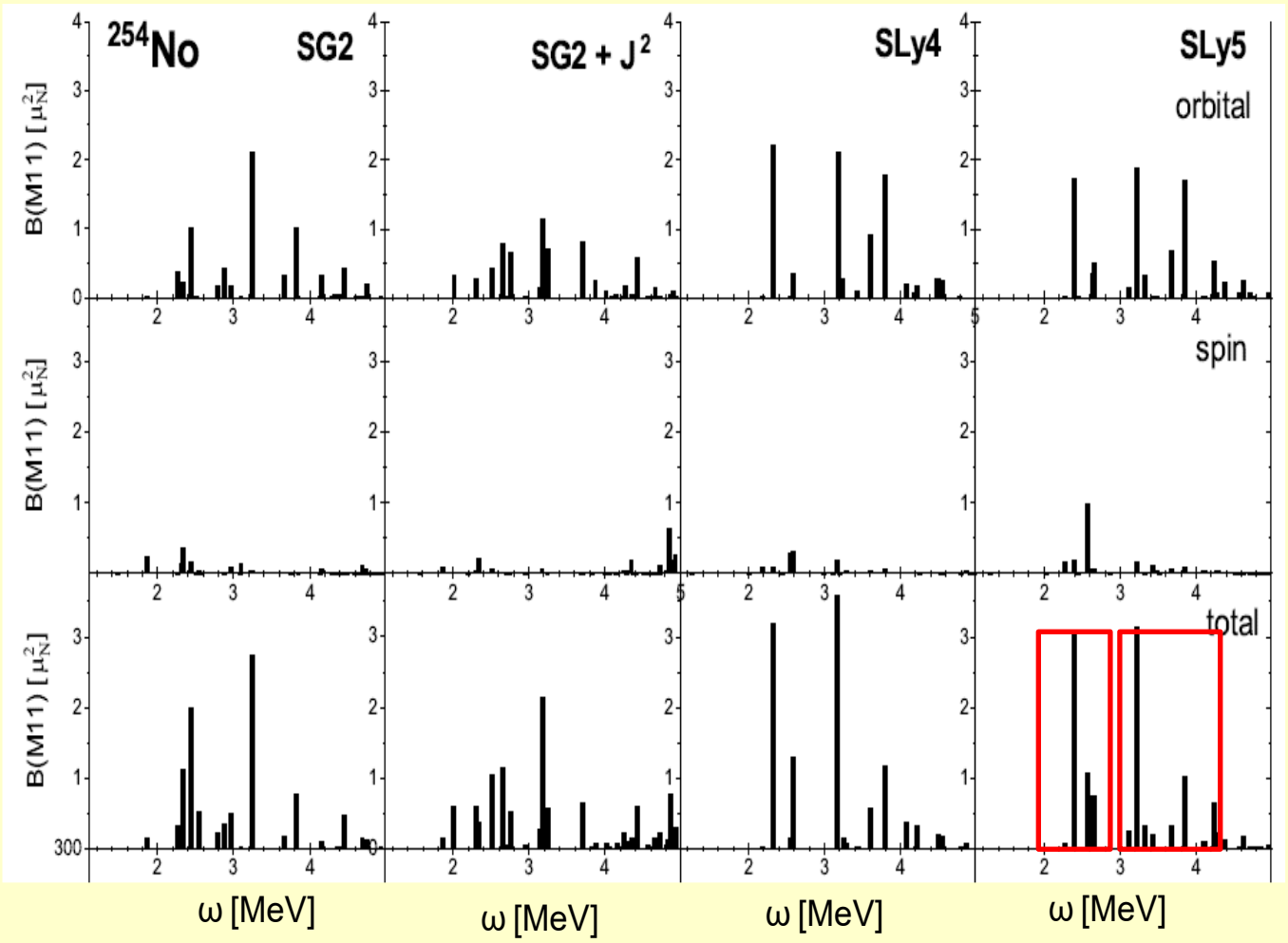
V.O. Nesterenko, P.I. Vishnevskiy, P.-G. Reinhard, A. Repko and J. Kvasil, EPJA 60, 28 (2024).

TABLE I. IS effective masses m_0/m, spin-orbit parameters b_4, and tensor parameters $\tilde{b}_1$ and $\tilde{b}'_1$ for SG2, SLy4 and SLy5.

force	m_0/m	b_4 MeV fm^5	$\tilde{b}_1$ MeV fm^5	$\tilde{b}'_1$ MeV fm^5
SG2	0.79	52.5	-9.96	-47.74
SLy4	0.70	61.5	47.37	-129.15
SLy5	0.70	63.0	48.87	-129.07

Orbital, spin, total B(M1) strengths.

V.O. Nesterenko, M.A. Mardyban and A.Repko, arXiv:2609.21489v1 [nucl-th]



Orbital strength dominates

Spin strength is small

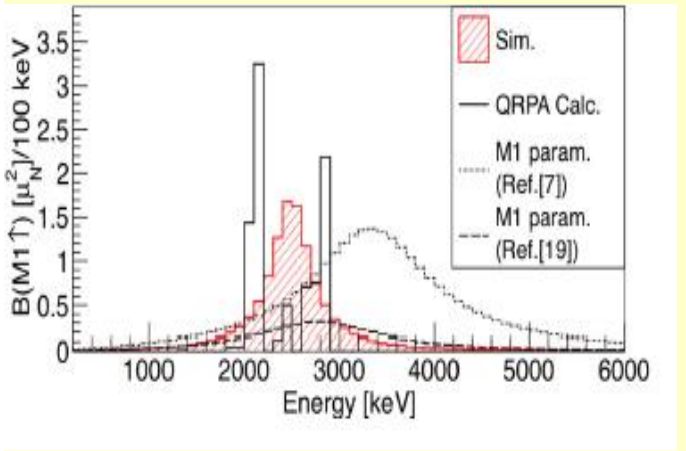
Strong constructive interference of spin and orbital contributions to the total strength.

Fine structure of SM: two groups of peaks

The impact of J^2 term is large in the perturbative case (SG2 vs SG2+J^2) and small in the consistent case (SLy4 vs SLy5)

Comparison with experiment and previous estimations:

Method	ΔE [MeV]	E_c	$\sum B_{orb}(M11)$	$\sum B_{spin}(M11)$	$\sum B_{tot}(M11)$
exp [12]	1.7 - 3.0	2.5	-	-	11.8
sum rules [12, 20]	-	2.1	12.1	-	-
QRPA, Gogny [12, 50]	1.8 - 2.9	2.4	9.2	-	-
GLO [12, 22]	1.0-6.0	3.4	-	-	28.2
SMLO [12, 23]	1.1 - 6.0	2.9	-	-	6.3
WFM [8]	-	2.6	-	-	13.4
SG2	1.2 - 3.0	2.5	2.82	1.04	5.22
	1.2 - 4.0	2.9	6.28	1.26	8.97
	1.2 - 5.0	3.8	7.47	1.59	10.19
SG2 + J^2	1.2 - 3.0	2.4	2.53	0.91	4.52
	1.2 - 4.0	2.8	5.71	1.21	8.30
	1.2 - 5.0	3.8	7.15	4.91	11.57
SLy4	1.2 - 3.0	2.4	2.61	0.84	4.71
	1.2 - 4.0	2.9	7.77	1.22	10.31
	1.2 - 5.0	3.1	8.87	1.40	11.65
SLy5	1.2 - 3.0	2.5	2.62	1.44	5.72
	1.2 - 4.0	3.0	7.36	1.92	11.03
	1.2 - 5.0	3.2	8.75	2.12	12.62



acceptable agreement with the experimental data:

$E \sim 2.5$ MeV

$$\sum B(M1\uparrow) = 11.8\,(19)\,\mu_N^2$$

Experiment has many subtle points in extraction of M1 strength

"should only be considered as as an estimation"

We predict for SM:

- a larger energy interval 1.2 – 5.0 MeV
- a higher centroid energy ~ 3 MeV
- fine structure: two groups of peaks

Nuclear transition current densities for QRPA states $|\nu\rangle$:

$$\delta \mathbf{j}_\nu(\mathbf{r}) = \langle \nu | \hat{\mathbf{j}} | 0 \rangle (\mathbf{r})$$

Convective nuclear current

$$\hat{\mathbf{j}}(\mathbf{r}) = -i \frac{e\hbar}{2m} \sum_{q=n,p} e^q_{\text{jeff}} \sum_{k \epsilon q} (\delta(\mathbf{r} - \mathbf{r}_k) \overrightarrow{\nabla}_k + \overleftarrow{\nabla}_k \delta(\mathbf{r} - \mathbf{r}_k))$$

Effective charges:

e^p_{jeff}=1, e^n_{jeff}=0 ← proton current

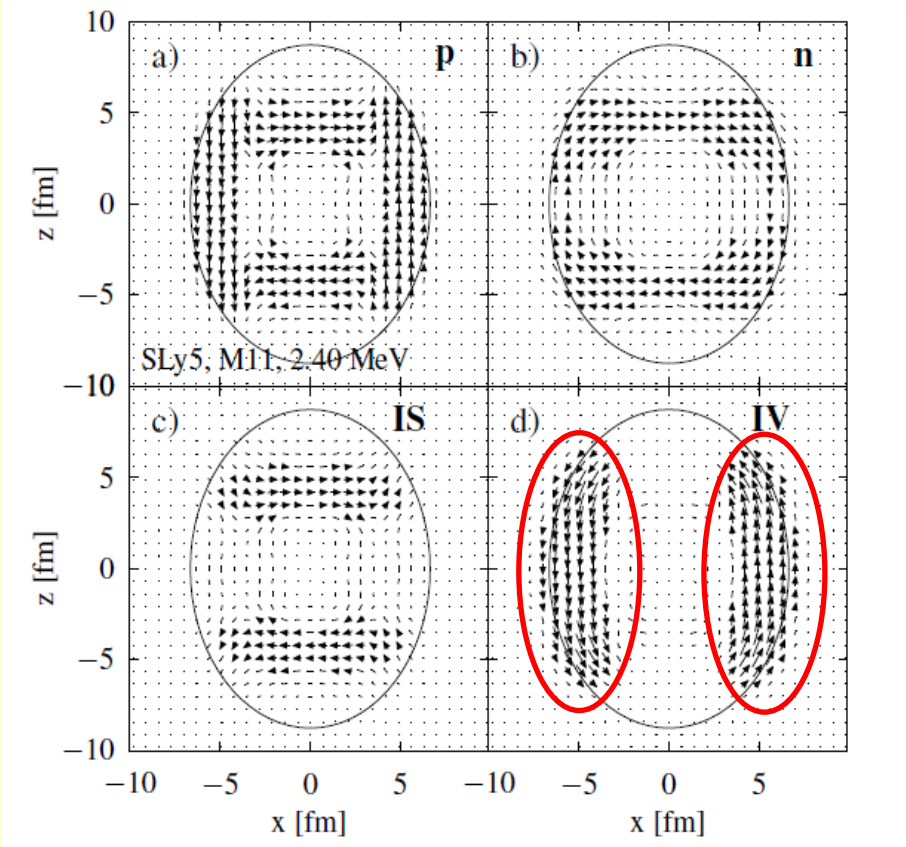
e^p_{jeff}=0, e^n_{jeff}=1 ← neutron current

e^p_{jeff}=e^n_{jeff}=1 ← IS current

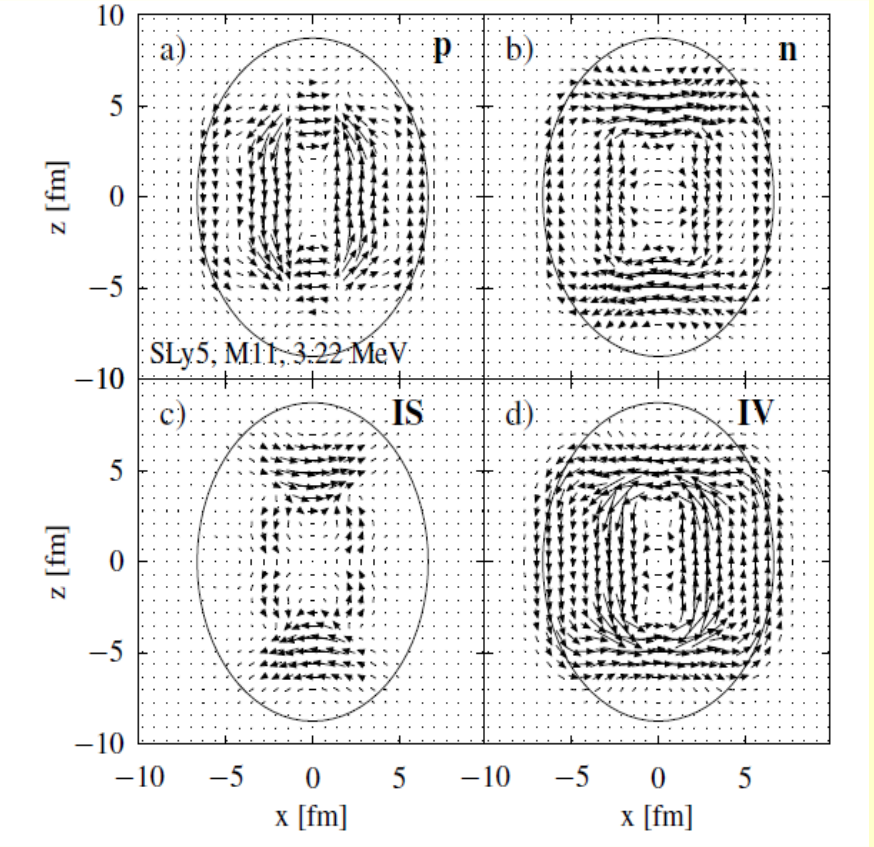
e^p_{jeff}=-e^n_{jeff}=1 ← IV current

SLy5 distributions of nuclear current for the states with the largest B(M1)

E=2.40 MeV



E=3.22 MeV



- strong IV flow, mainly in lateral (equator) regions
- different IV flow for states at 2.4 MeV and 3.2 MeV
- **IV flow does not support the scissors-like scheme (where the maximal currents must be at the poles)** →



So, predictions of two-rotor model should be taken with a precaution.

IV flows for ~2.4 MeV and ~3.2 MeV states: SG2, SG2+J^2, SLy4, SLy5

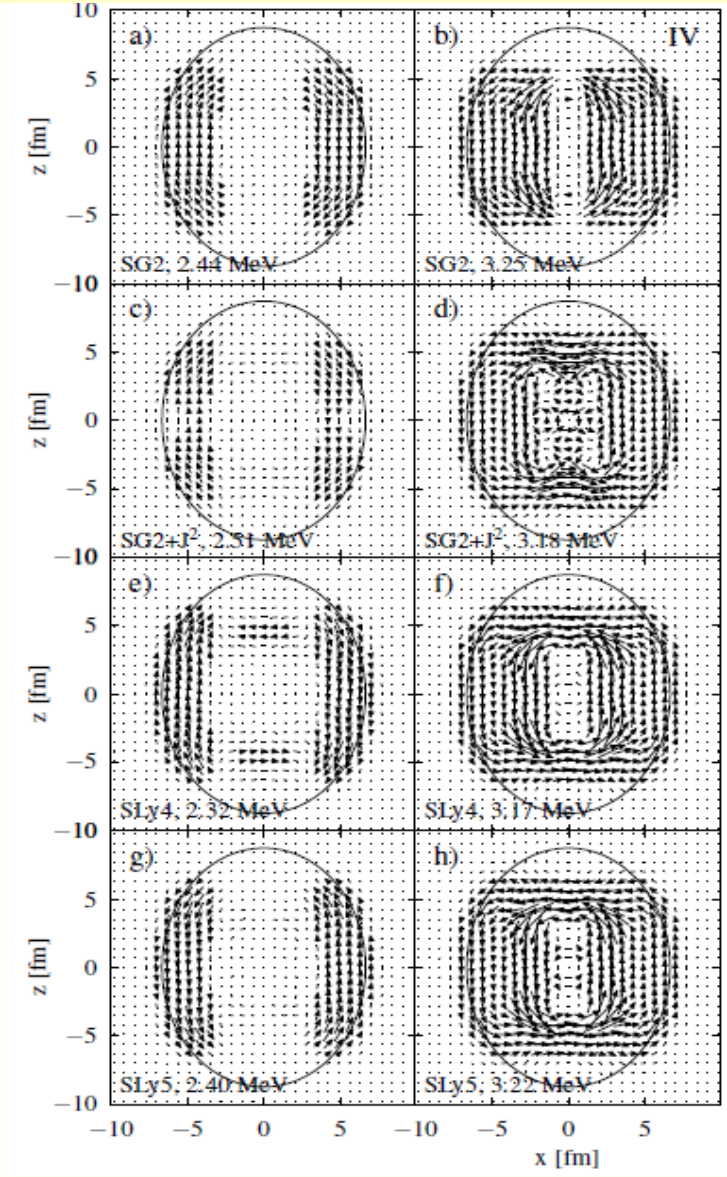
~2.4 MeV ~3.2 MeV

SG2

SG2+J^2

SLy4

SLy5



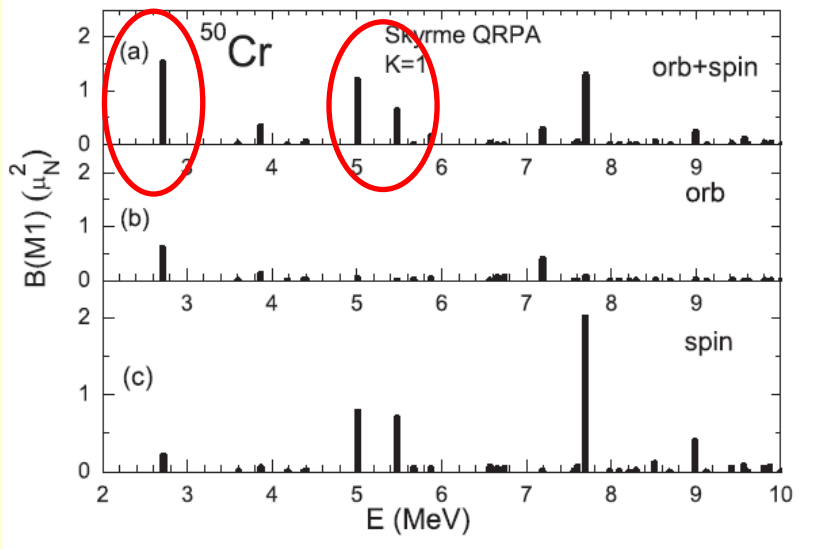
Similar results for the applied forces

Stability of our results to the choice of Skyrme force

SM fragmentation and lateral distributions of the nuclear current also takes place in light and medium and heavy deformed nuclei

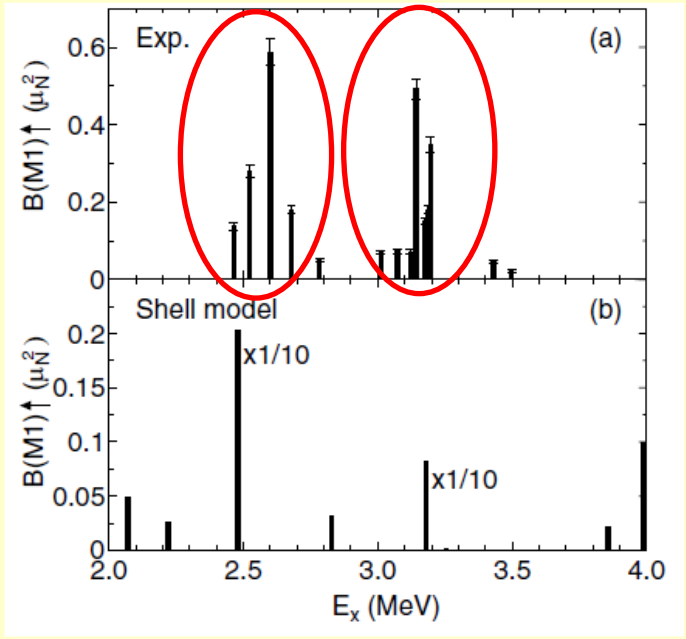
H. Pai et al,
PRC 93, 014318 (2016)

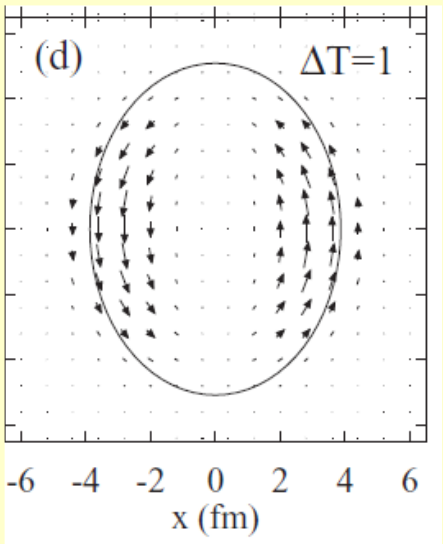
^{50}Cr



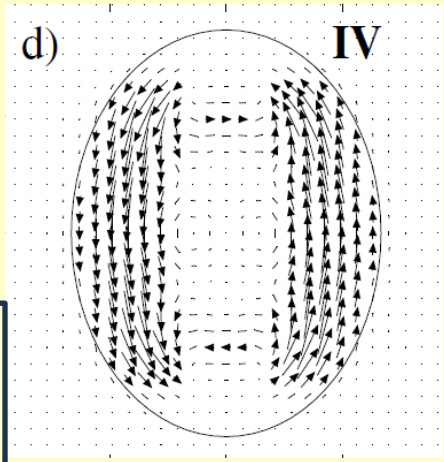
T. Shizuma et al,
PRC 113, 044325 (2026)

^{166}Er





SG2
2.7 MeV



SLy4
2.4 MeV

Similar pictures in
^{162,164}Dy, ^{156}Gd, ^{232}Th

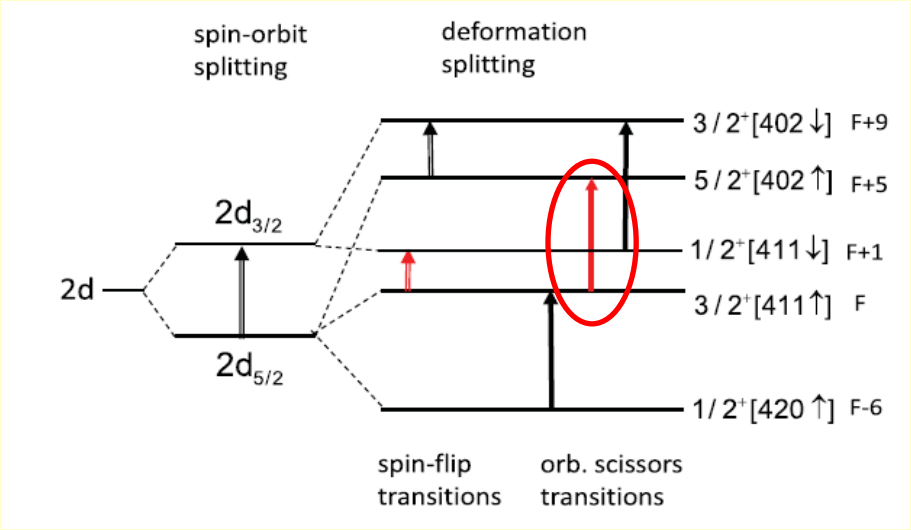
So,

1) fragmentation of M1 SM strength into distinct groups
2) specific lateral (non-scissors) distribution
   of the nuclear current

seem to be general phenomena.

Possible reasons of SM fine structure

Always: 1ph transitions between neighbouring s-p states
Arising due to the deformation splitting



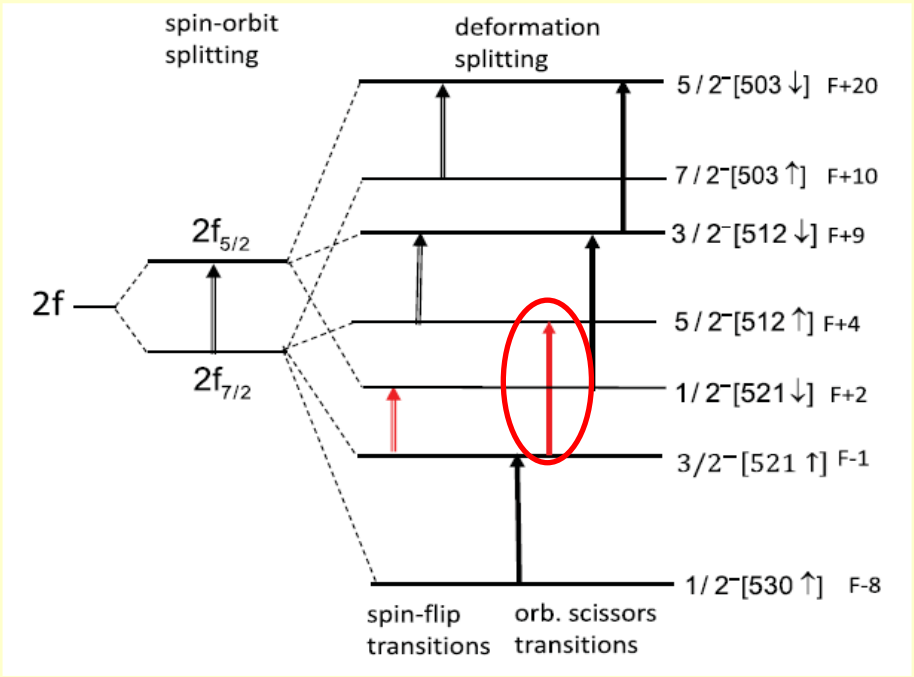
Fragment of proton s-p scheme near the Fermi level F

Strong M1 1ph transitions (between neighboring s-p states arising due to the deformation splitting)
in the proton and neutron s-p schemes already can lead to fragmentation of M1 SM strength into two fractions.

V. O. Nesterenko, P. I. Vishnevskiy, J. Kvasil, A. Repko and W. Kleinig, PRC, 103, 064313 (2021)

^{162}Dy

Fragment of neutron s-p scheme near the Fermi level F



However, the reasons of M1 fragmentation can be additionally influenced by other factors:

- strong (usually constructive) interference between orbital and spin fractions in M1 transitions

- high density of s-p states (e.g. in heavy nuclei)

  (e.g. in 254No two strong M1 transitions in the proton s-p scheme mainly produce the fragmentation)

- coupling with complex configurations

- triaxiality, shape coexistence

So, each nucleus can suggest its own set of fragmentation factors.

Conclusions for M1 “scissors” mode in ^{254}No

- Skyrme QRPA analysis of Jyväskylä experiment
- acceptable description of experimental data
- fine structure of SM, not only in ^{254}No
- deviation from the scissor scheme for IV nuclear currents !!!

fragmentation of M1 SM strength into distinct groups
and
lateral (non-scissors) distribution of the nuclear current
seem to be general phenomena.

Thank you for attention!