WEIZMANN INSTITUTE OF SCIENCE

第十五届新物理研讨会

Unearthing Primordial Dark-Matter Velocities from the Matter Power Spectrum

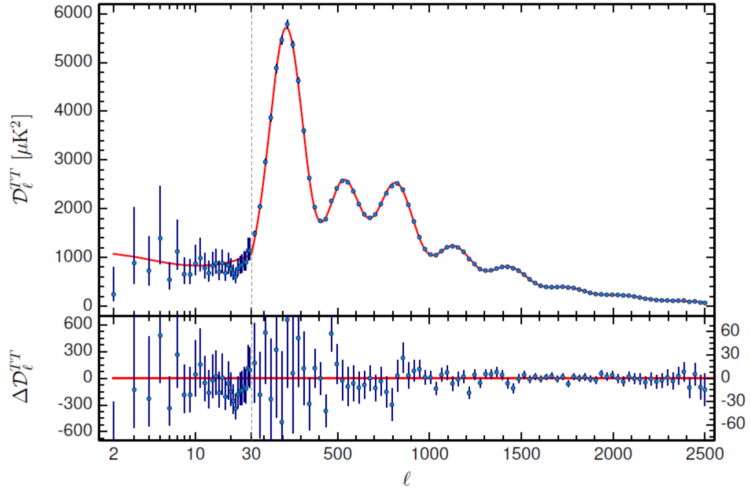
黄飞

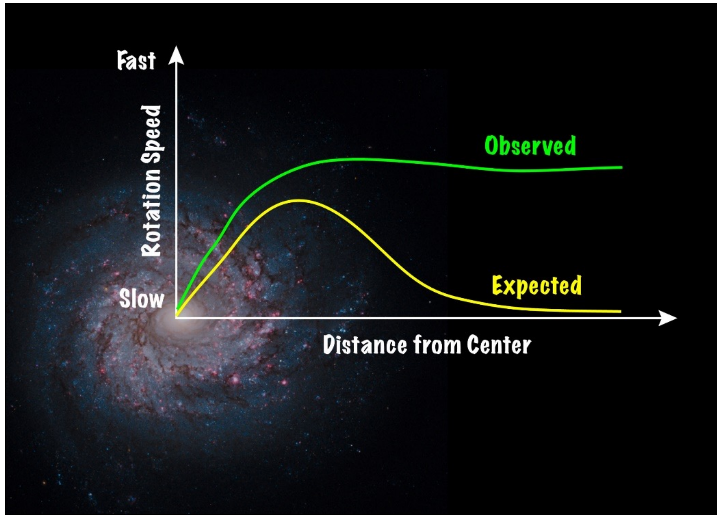
Based on work done in collaboration with

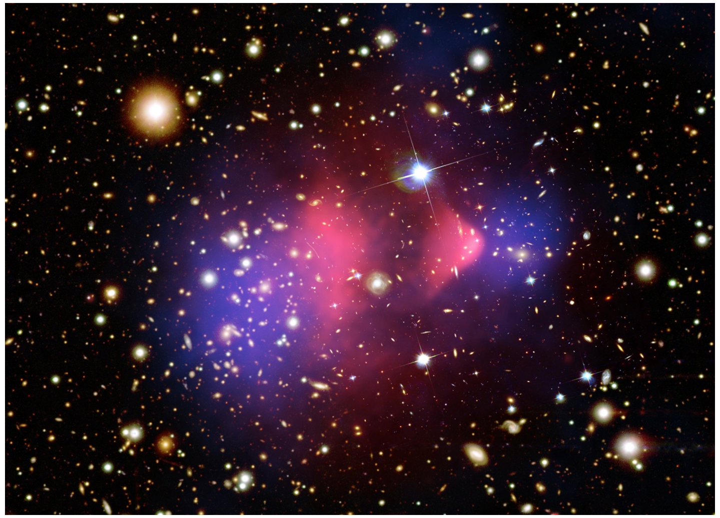
K. Dienes, Kost, S. Su, B. Thomas [arXiv:2001.02193]
K. Dienes, J. Kost, K. Manogue, B. Thomas [arXiv:2101.10337]
Y. Du, H. Li, Y. Li, J. Yu [arXiv:2111.01267]
K. Dienes, J. Kost, B. Thomas, H. Yu [arXiv:2112.09105]
Y. Li, J. Yu [arXiv:2306.00065]
K. Dienes, J. Howard, Y. Li, B. Thomas [arXic:2606.13527]

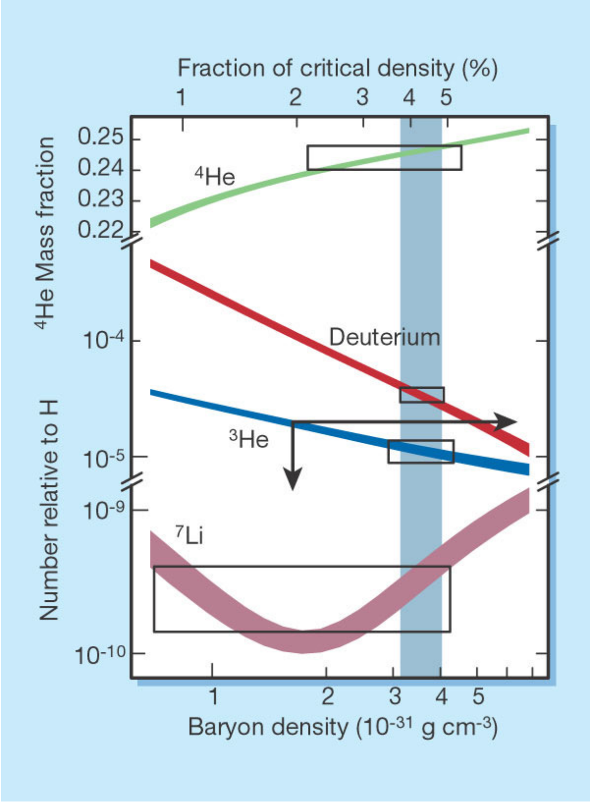
Aug 28, 2026

The existence of dark matter is supported by observations across many different scales

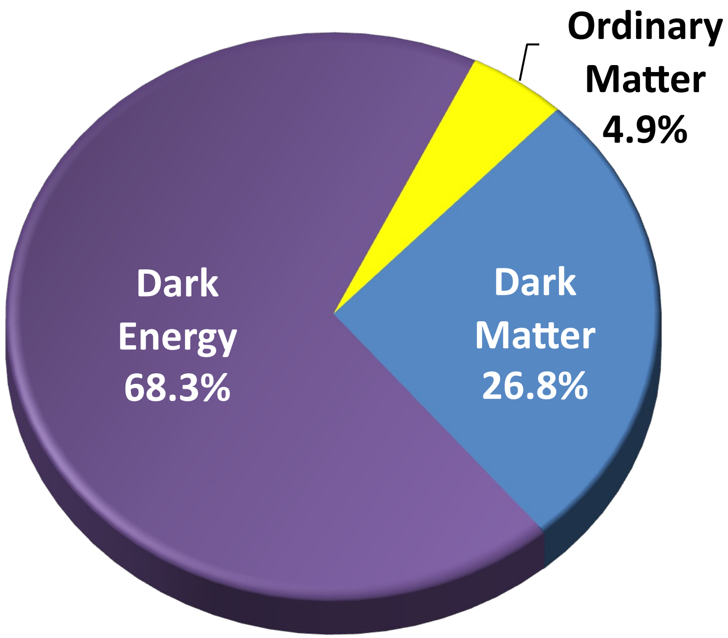








What have we learnt?



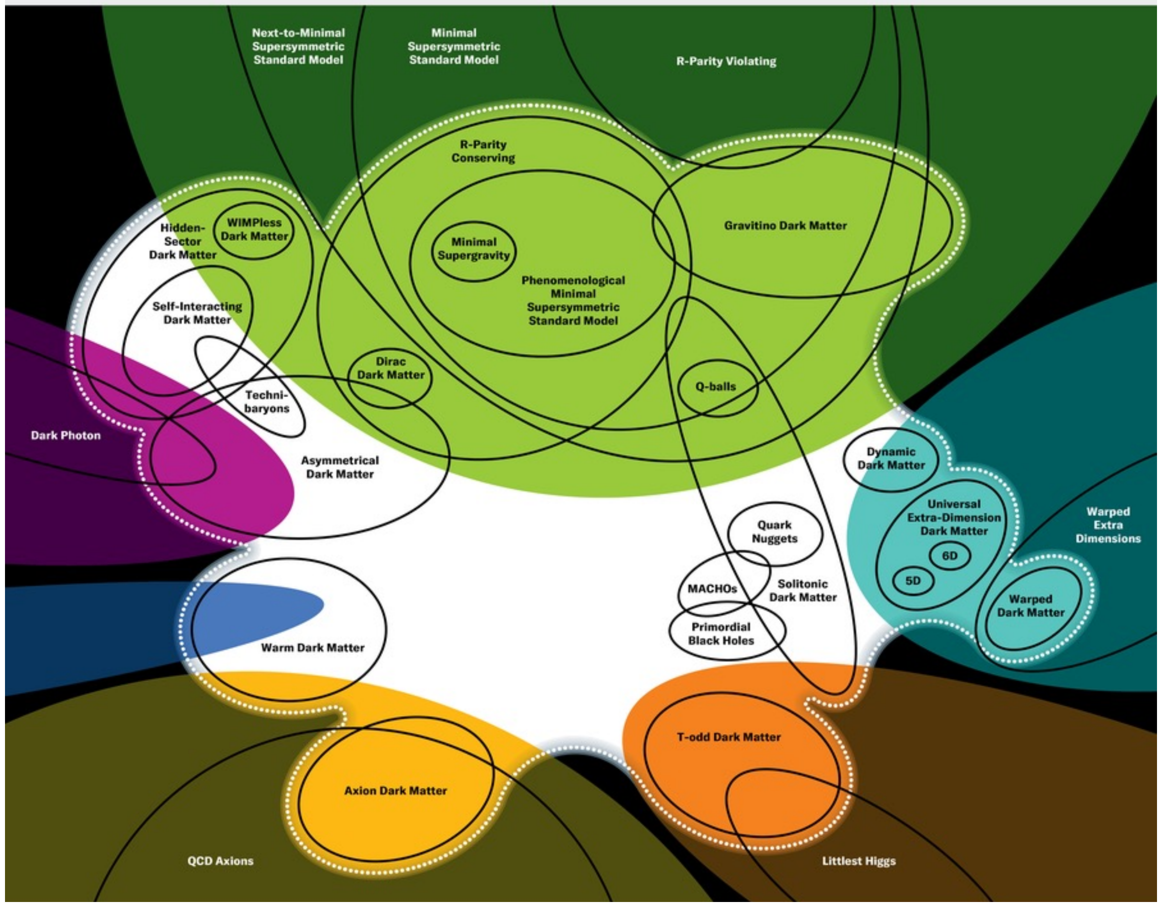
- **Abundance:** $\sim$26% of the universe
- **Cold**: Non-relativistic, massive
- **Dark**: Negligible nongravitational interaction with Standard Model fields
- **Nonbaryonic**: Baryonic matter is simply not enough
- **BSM**: Not Standard Model particle

We still don't know:

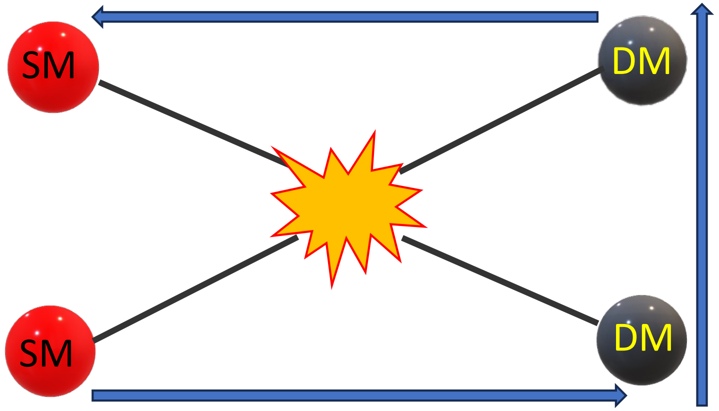
- **Particle properties**: mass, spin, fundamental or composite, single-component or multi-component, etc.
- **Production mechanism**: freeze-out, freeze-in, decays, misalignment?

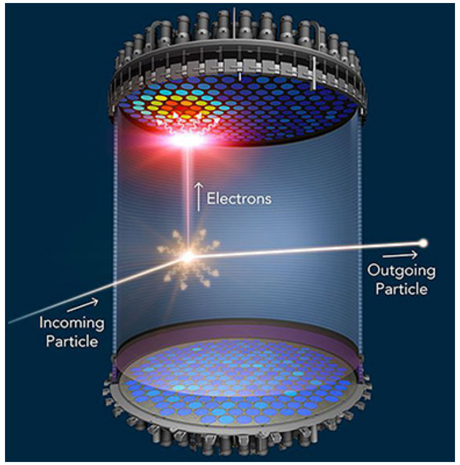
Where do we stand?

Since DM is beyond our current knowledge of fundamental particle physics, there has been massive theoretical and experimental efforts to understand DM over the past several decades



by T. Tait





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*Thus far, these experiments have **only set bounds** on the dark matter parameters, without a positive detection.*

A potential problem

- Evidence of dark matter all come from DM’s gravitational effects
- All of these methods for probing the nature of dark matter assume that dark matter interacts with normal matter in some way beyond gravity.

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A "Nightmare" Scenario

If the dark matter only interacts with normal matter through gravity – or only through gravity and other ***<u>extremely</u>*** weak forces – these experiments will ***<u>never</u>*** see a signal.

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A ”Nightmare” Scenario

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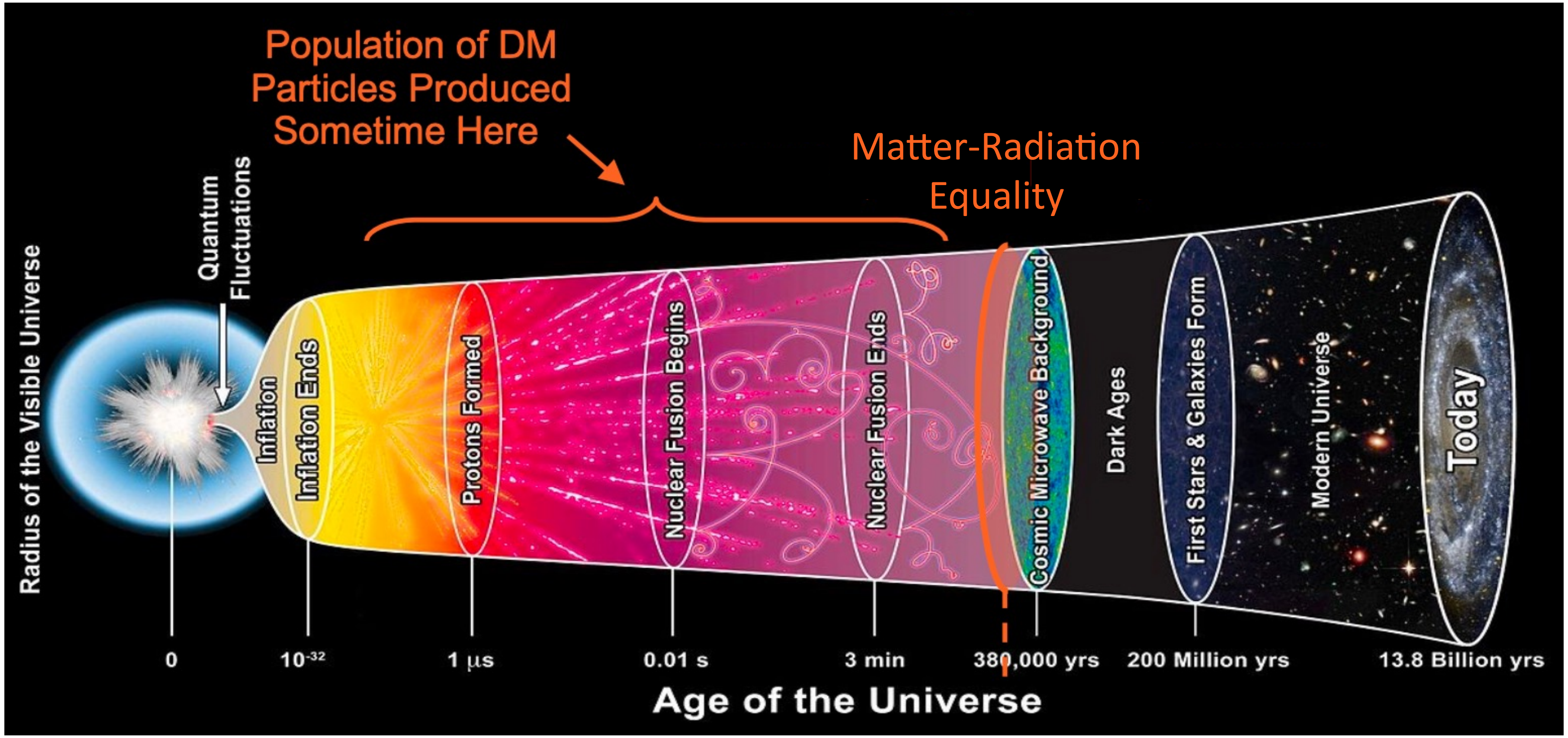
Fortunately, there are ways in which we can potentially use cosmology as a tool to learn about the nature of dark matter – even if it interacts with regular matter only gravitationally.

Cosmic timeline

To learn about dark matter using cosmology, let us look at the cosmological timeline and understand the role DM plays in our universe's history.



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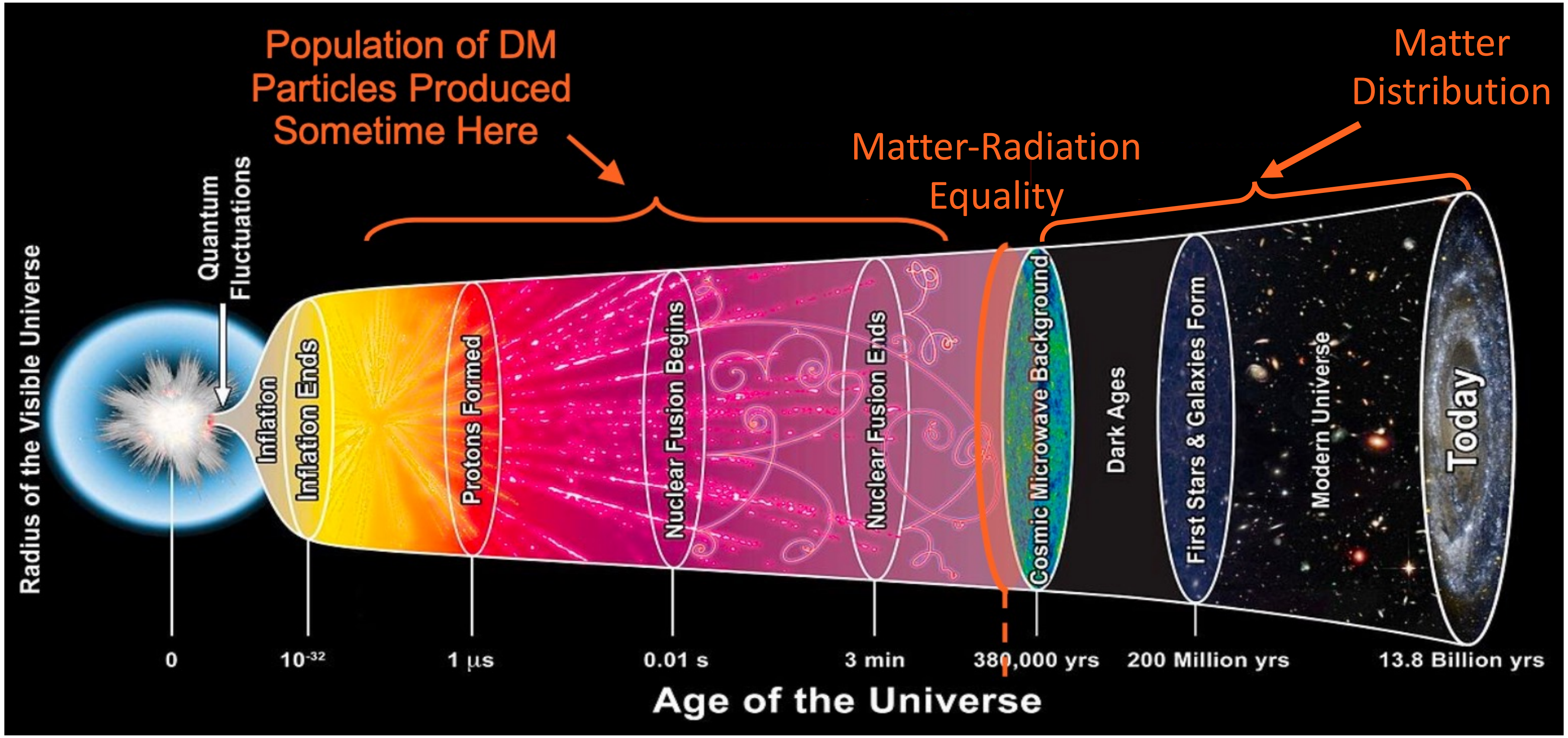

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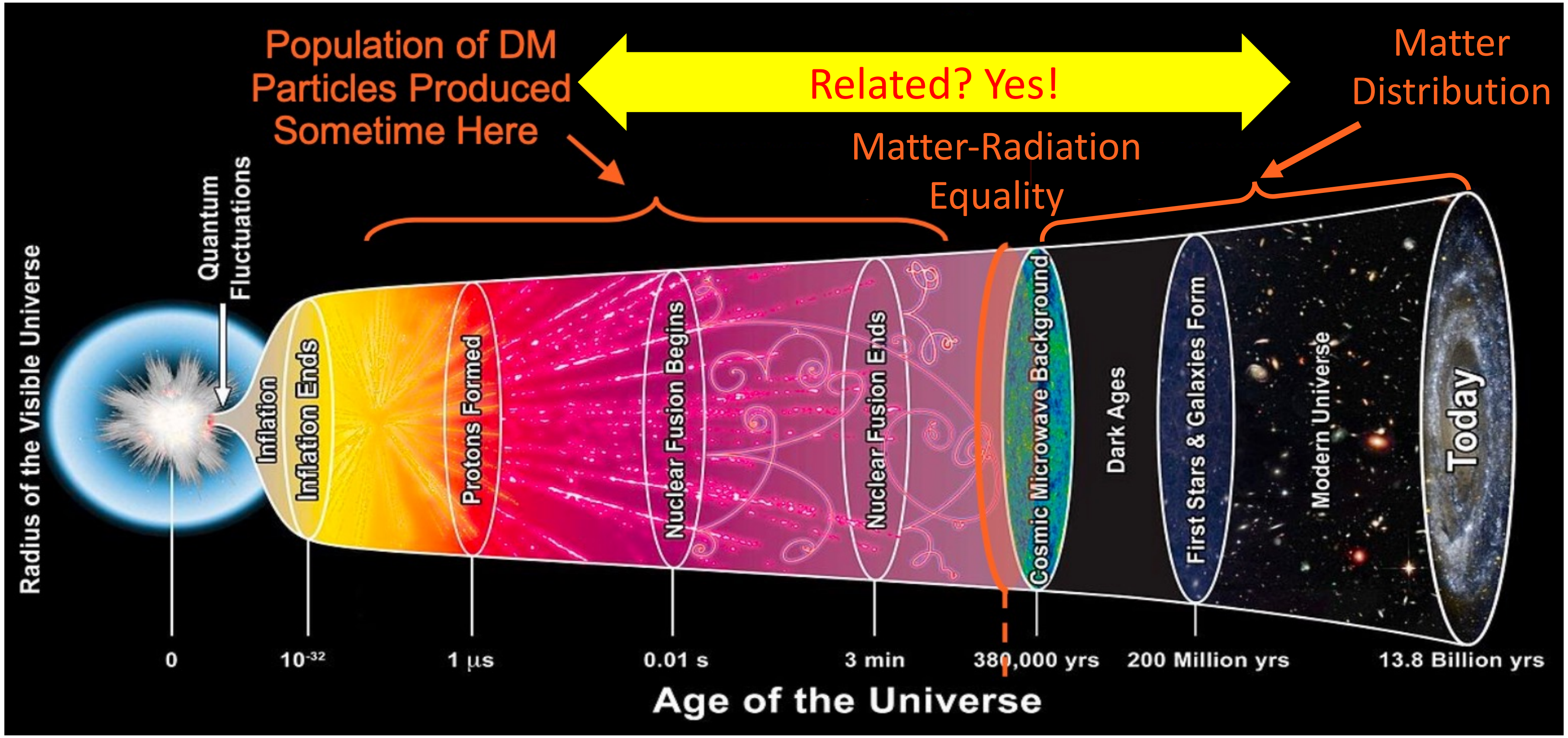

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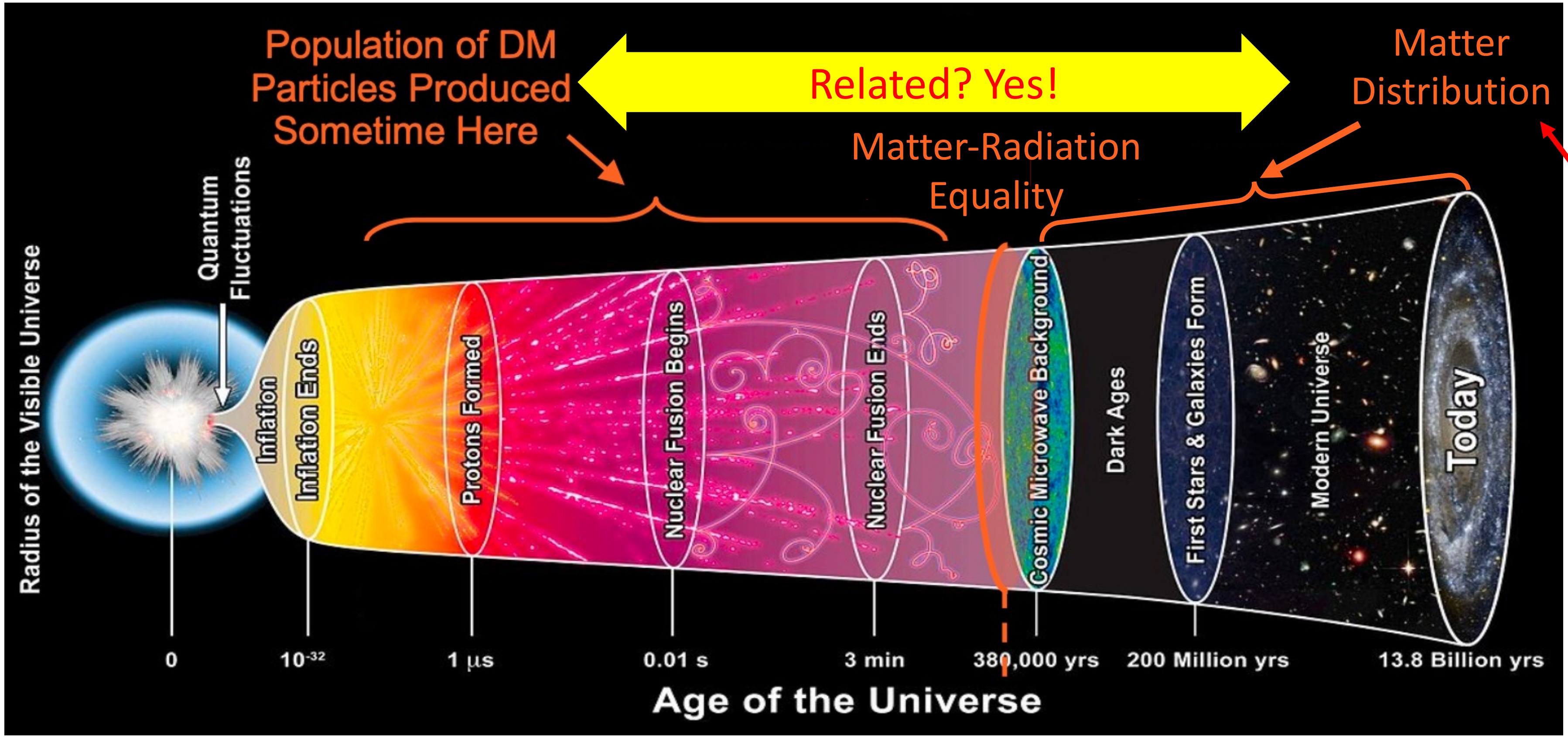

originated from DM density fluctuations, whose evolution can be affected by the thermal history

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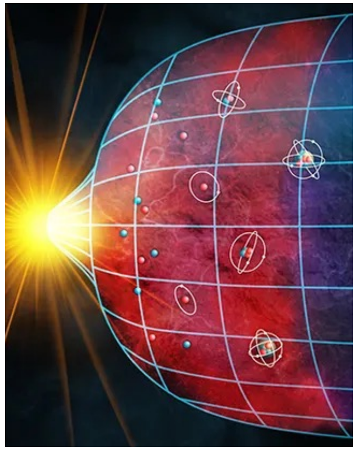
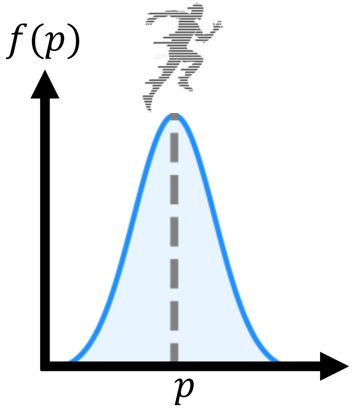


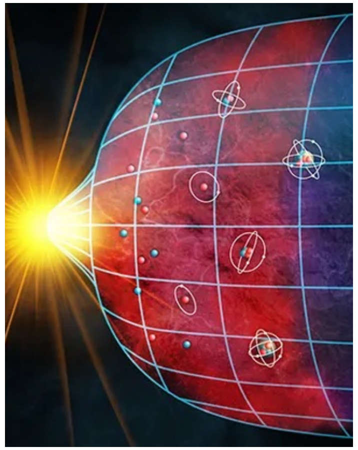
DM Thermal History



(primordial) DM Phase-Space Distribution

cosmic structure

Cosmic timeline



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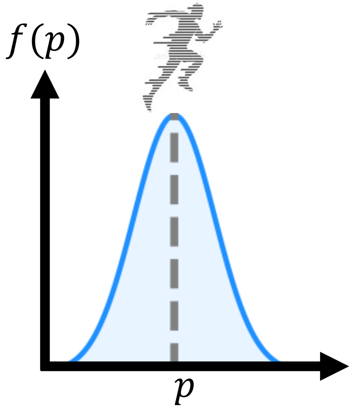
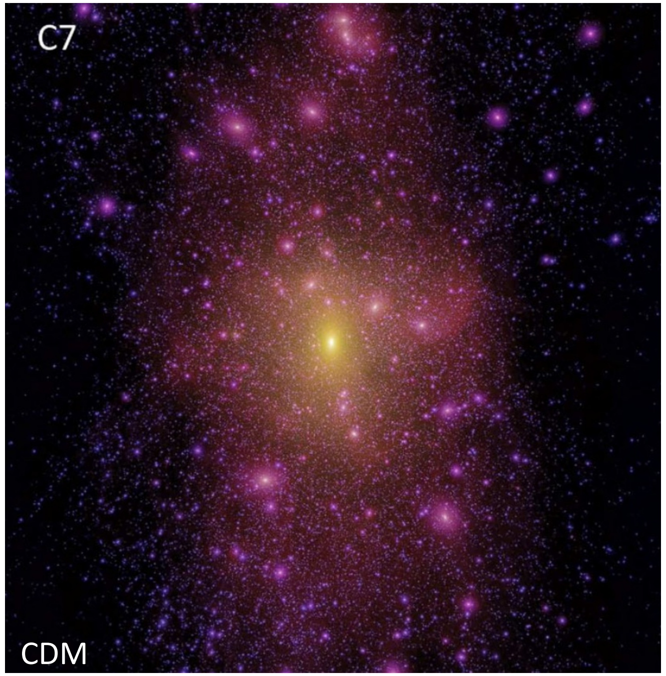
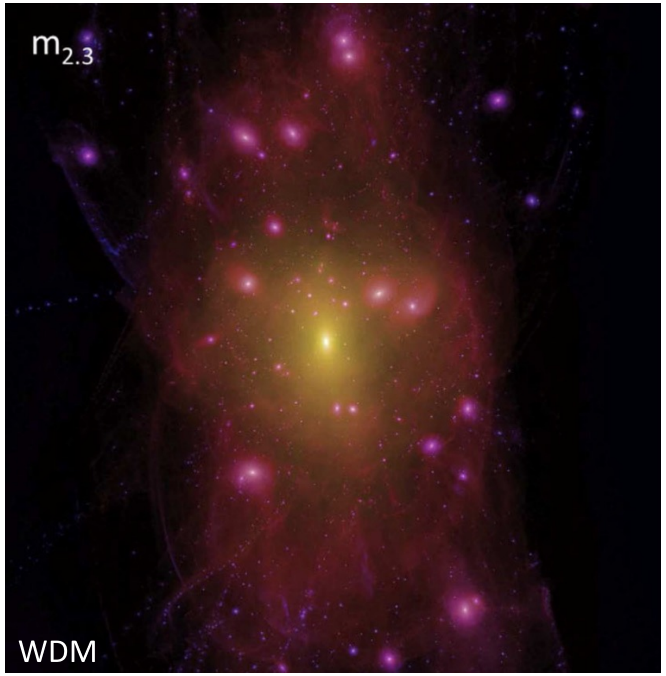


(primordial) DM Phase-Space Distribution

cosmic structure

Historically, this logic has been used to exclude HDM like SM neutrinos, and place severe constraints on scenarios like WDM.





Lovel et al MNRAS 2014

Phase-Space Distribution

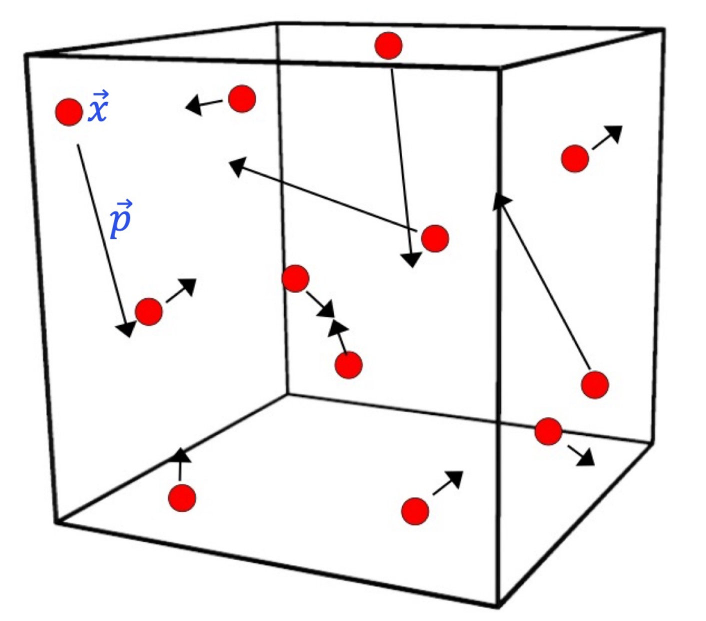
DM velocities are described by the PSD

$$f(\vec{x},\vec{p},t)$$

It is the ***central*** quantity that links the thermal history of DM and the present-day spatial distribution of matter

Assuming spatial homogeneity and isotropy

$$f(\vec{x},\vec{p},t) \rightarrow f(p,t)$$



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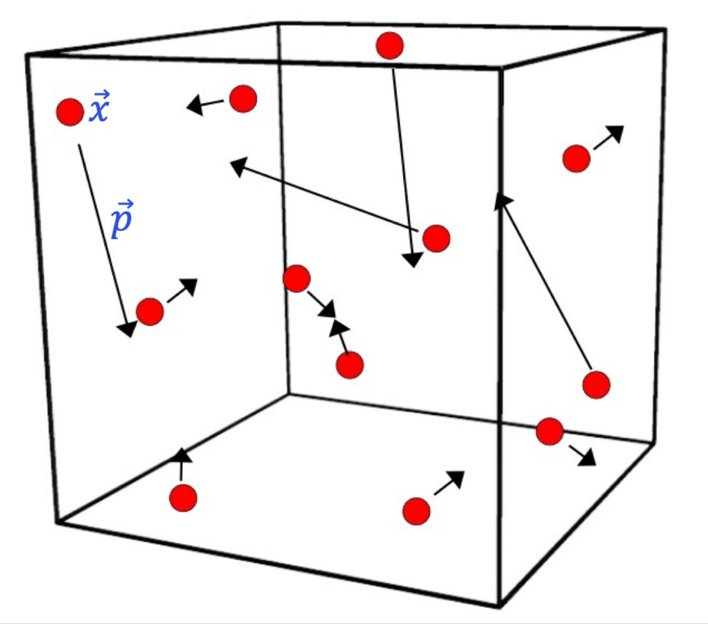

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Assuming spatial homogeneity and isotropy

$$f(\vec{x}, \vec{p}, t) \;\to f(p, t)$$

In general, $f(p, t)$ could take any reasonable functional form. Evolution in time is governed by the Boltzmann equation:

$$\frac{\partial f}{\partial t} = Hp\frac{\partial f}{\partial p} + C[f] \qquad H(t) \equiv \frac{\dot{a}}{a}$$

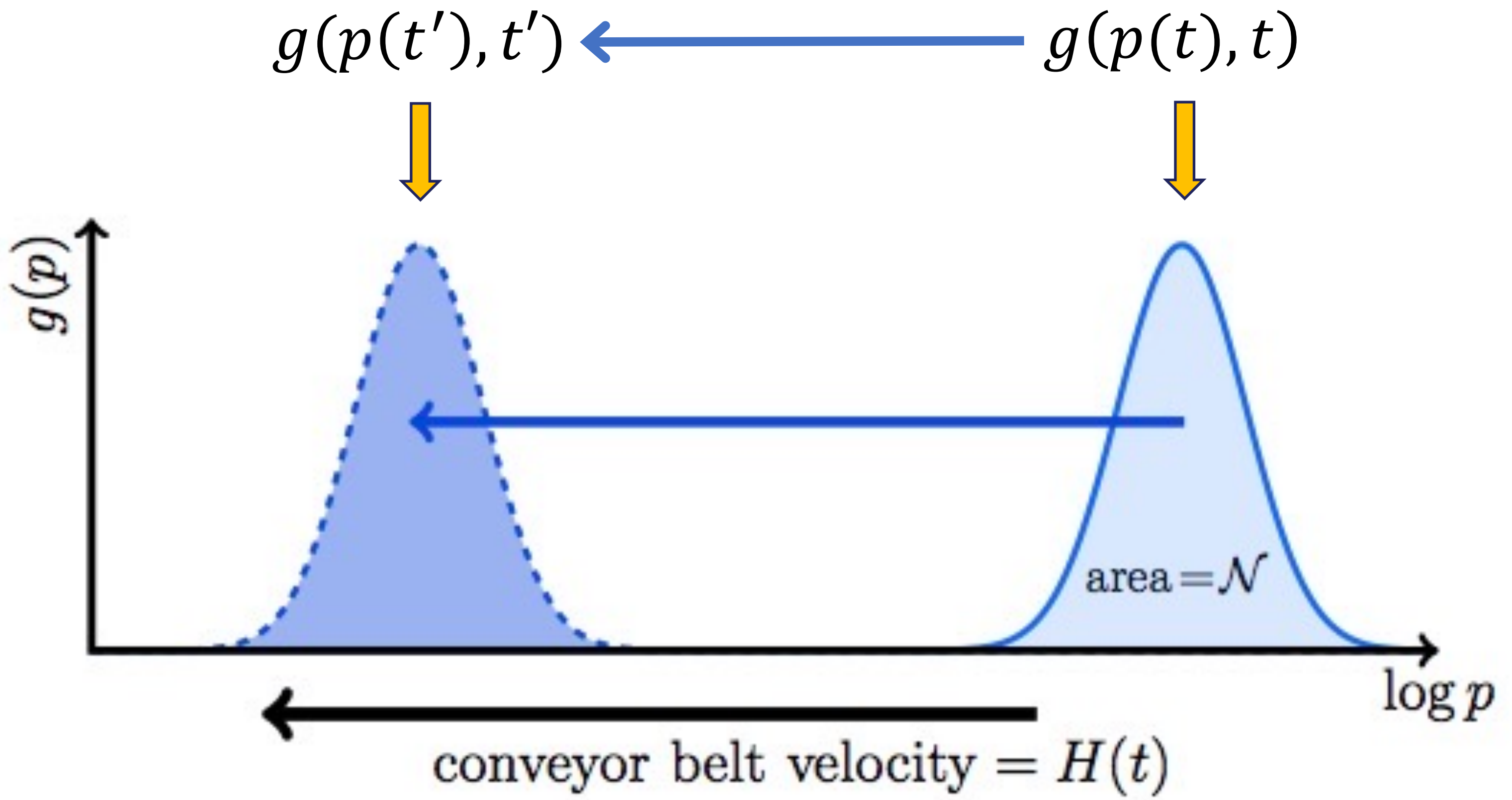
basically, two effects: (1) cosmological redshift and (2) particle interactions

Phase-Space Distribution

Instead of using $f(p,t)$ vs p, we shall look at

$$\boxed{g(p,t) \equiv a^3(t)p^3 f(p,t)}$$

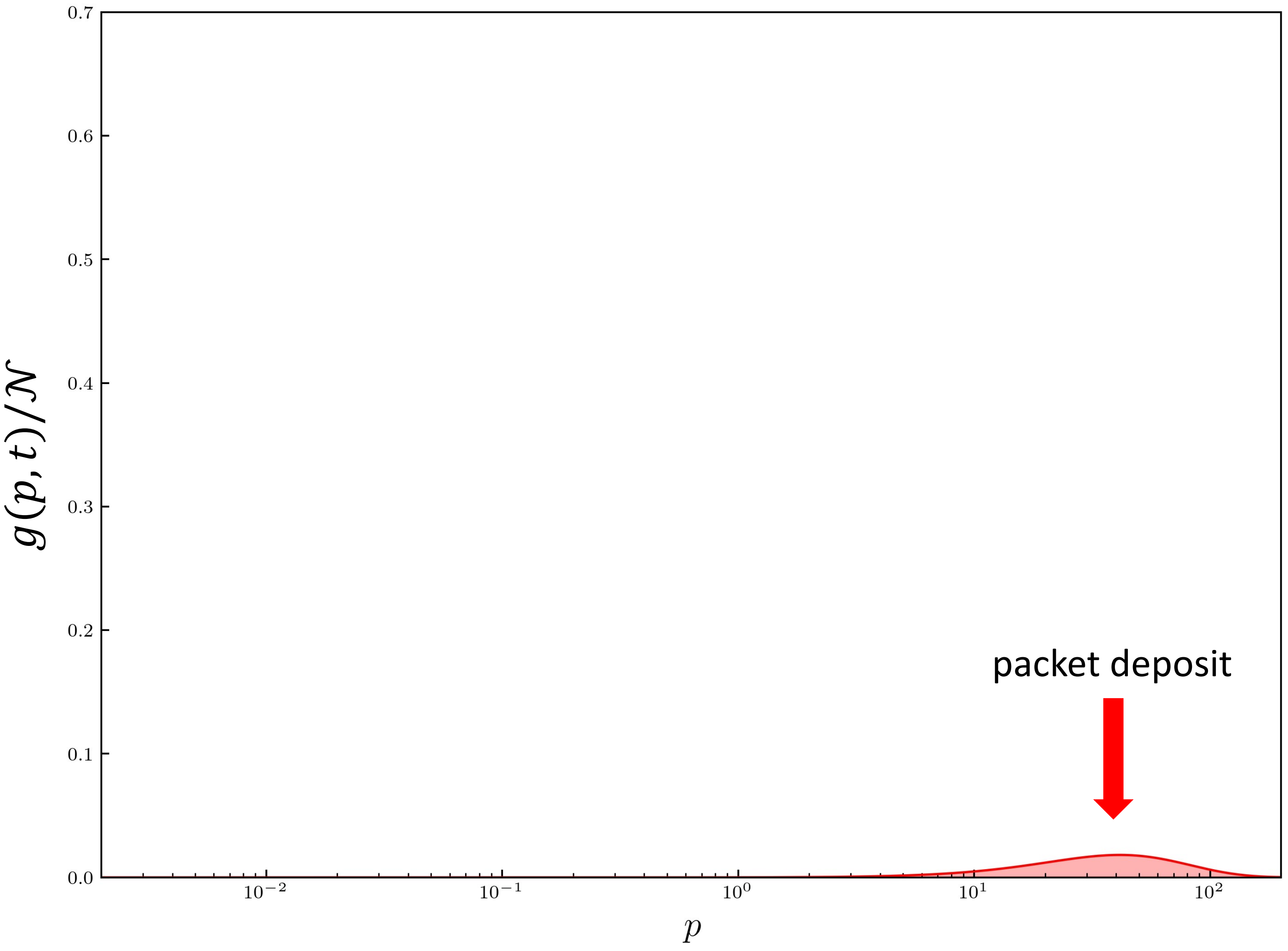
and plot $g(p,t)$ against $\log p$



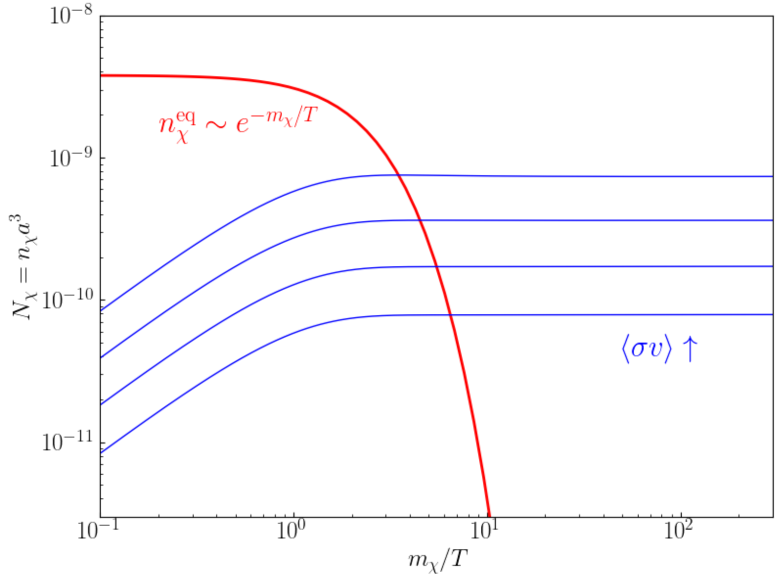
Advantages:

- once DM production complete, ***area (comoving number density) is fixed*** under time evolution
- redshift: $g(p,t)$ ***slides rigidly*** to smaller $\log p$, as if carried along a ***"cosmological conveyor belt"***, its ***shape stays invariant***

Example: freeze-in



comoving number density



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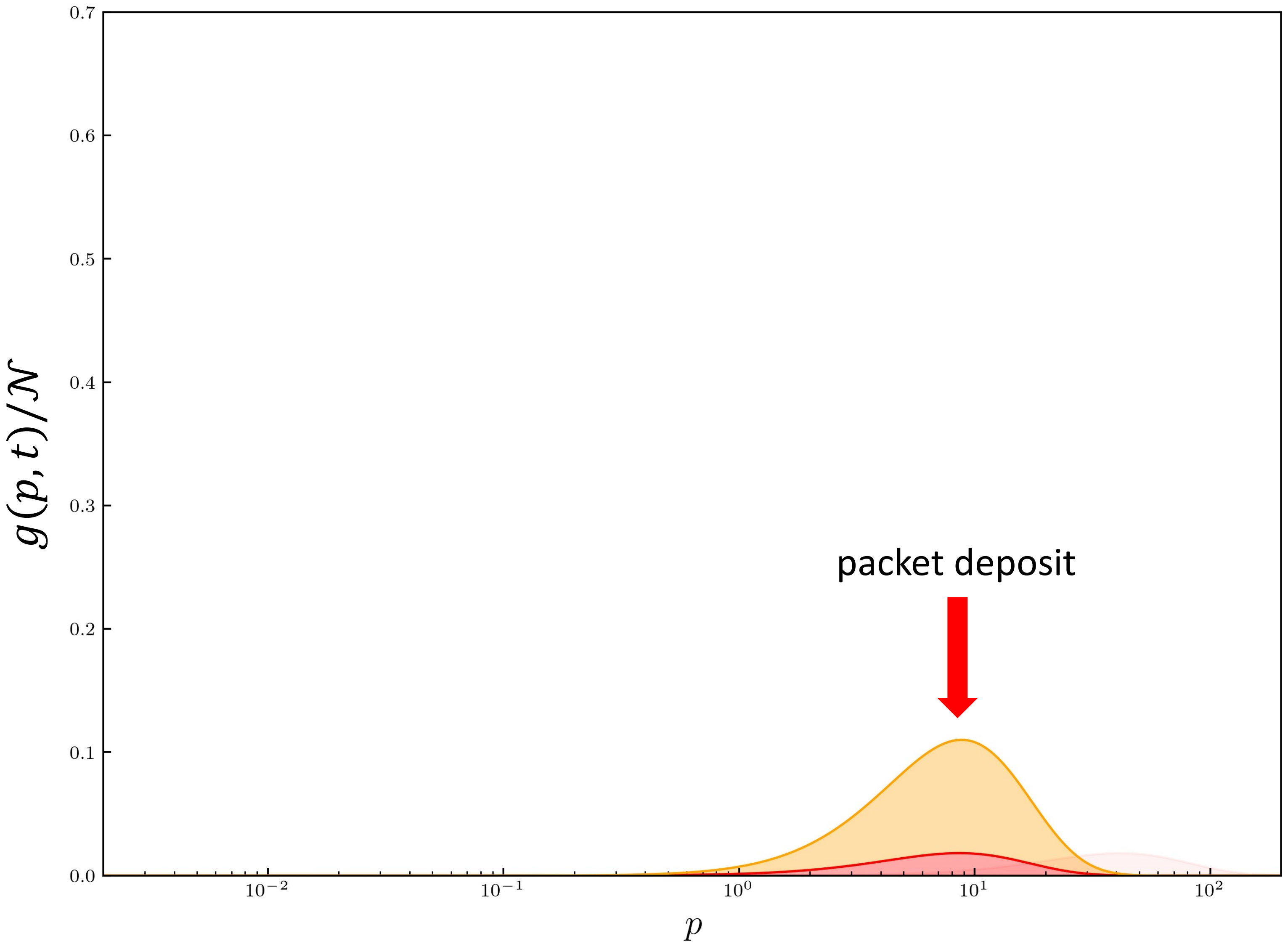

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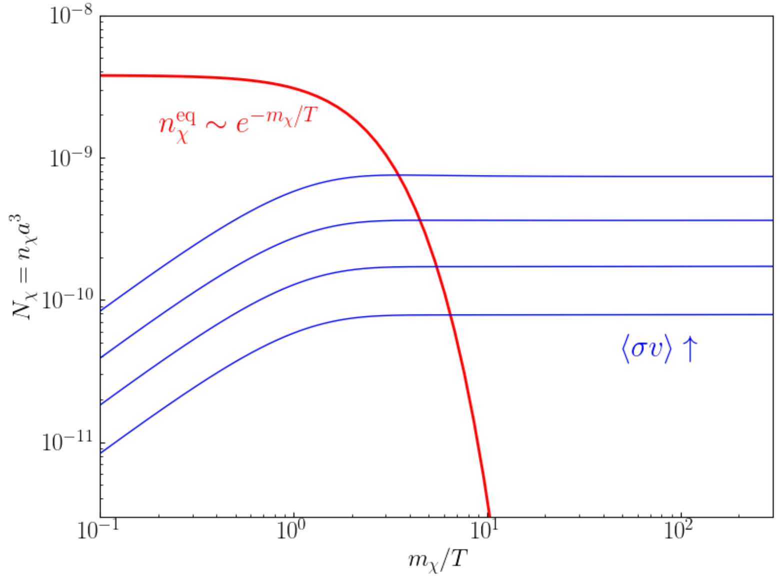

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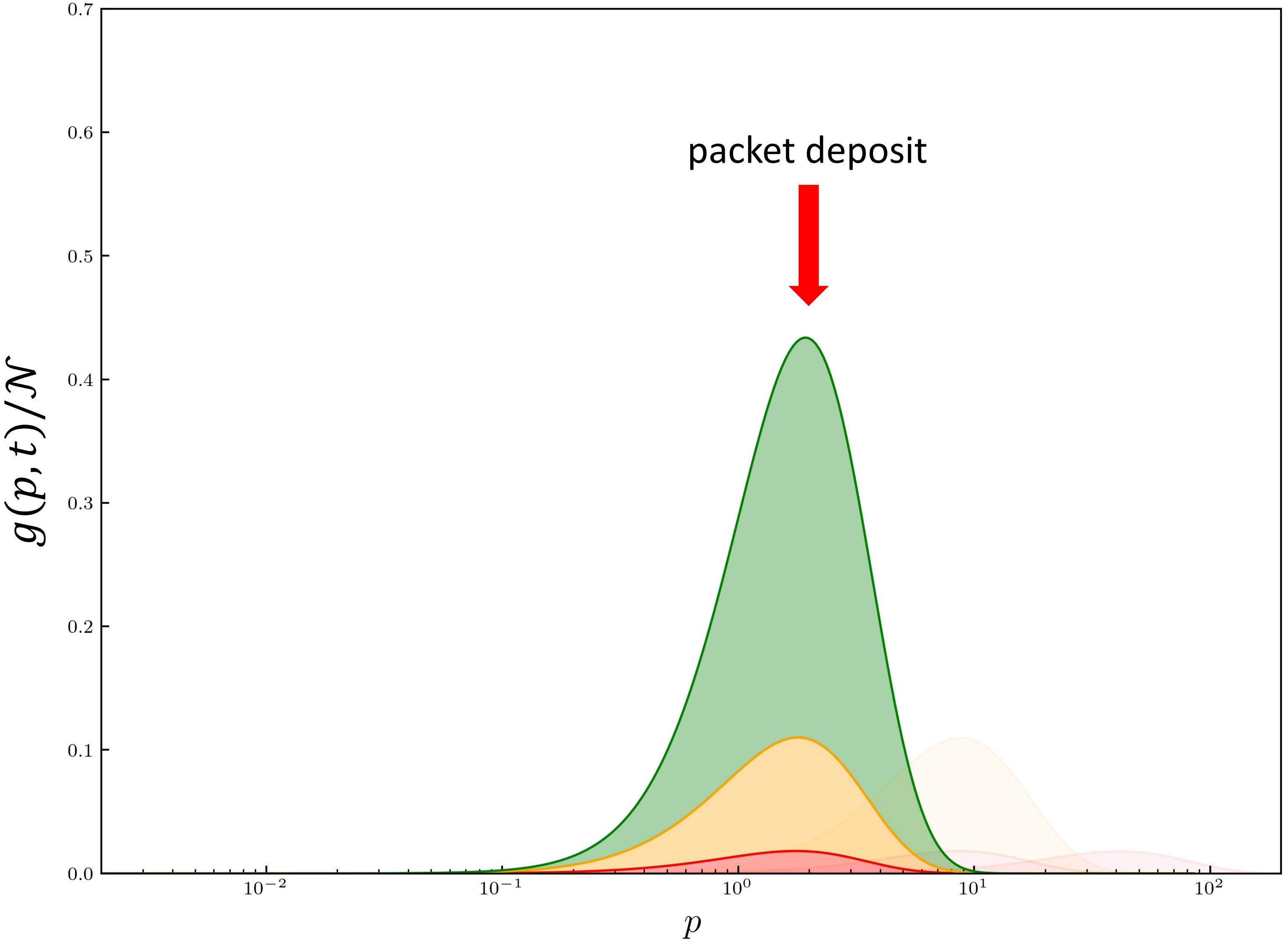

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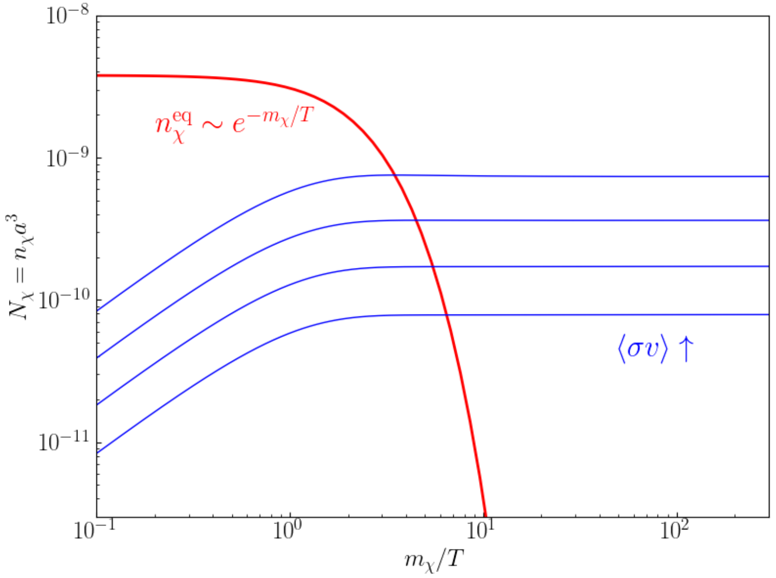

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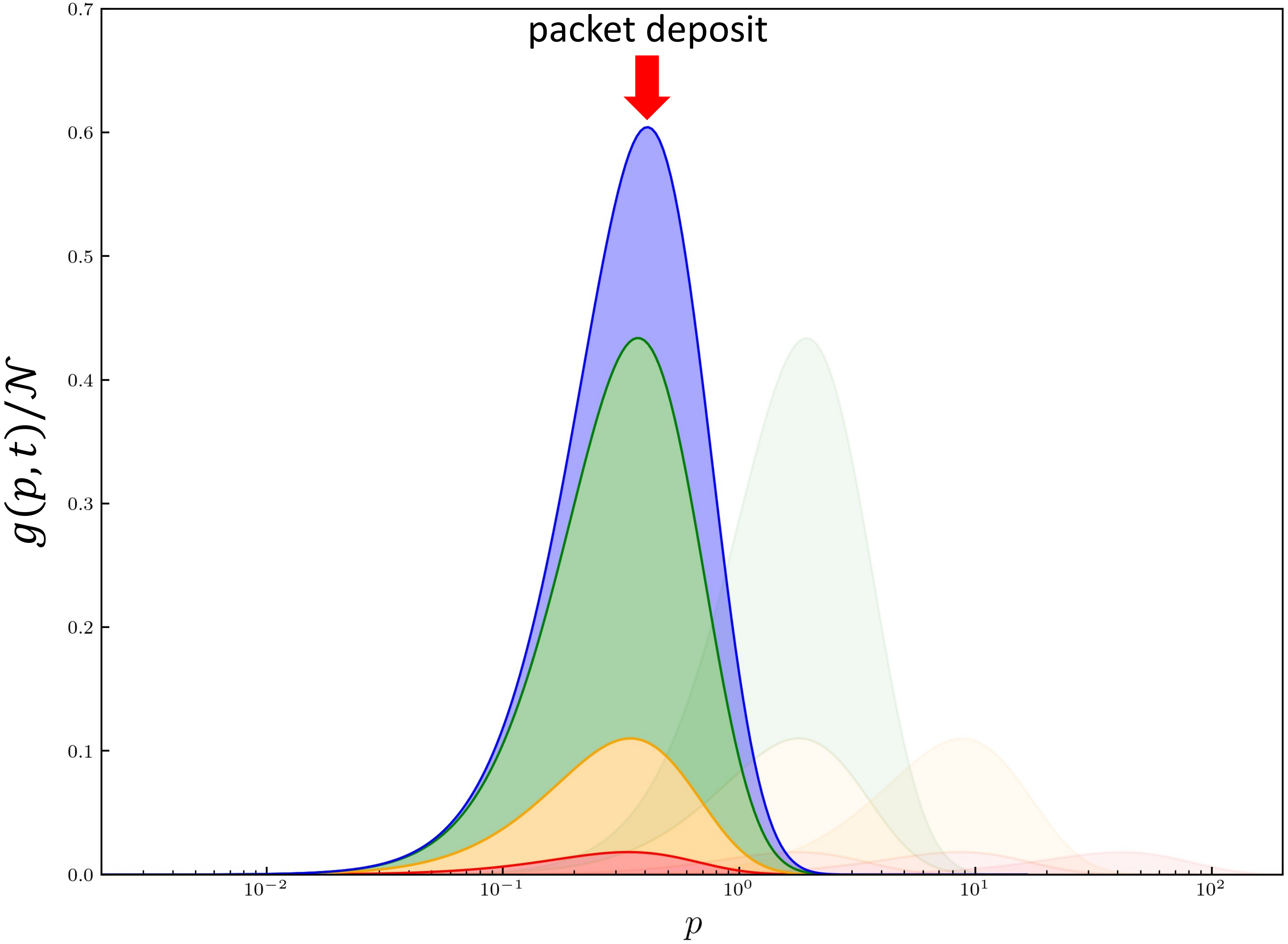

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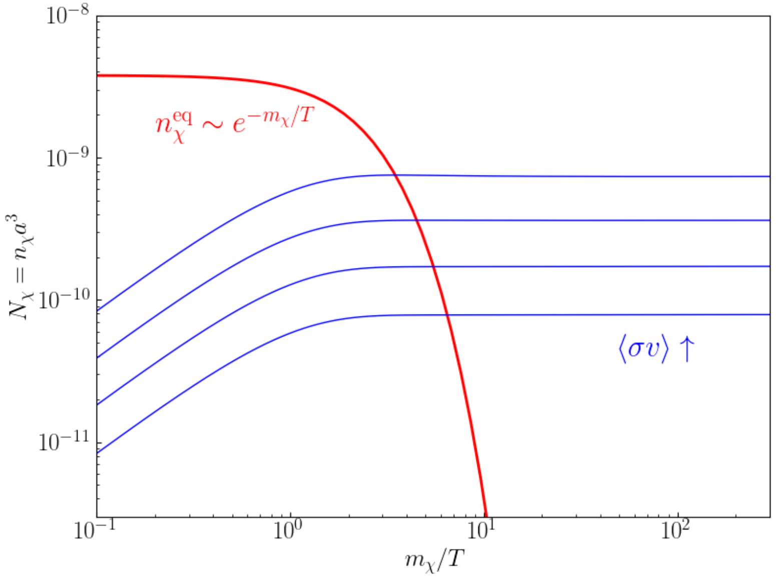

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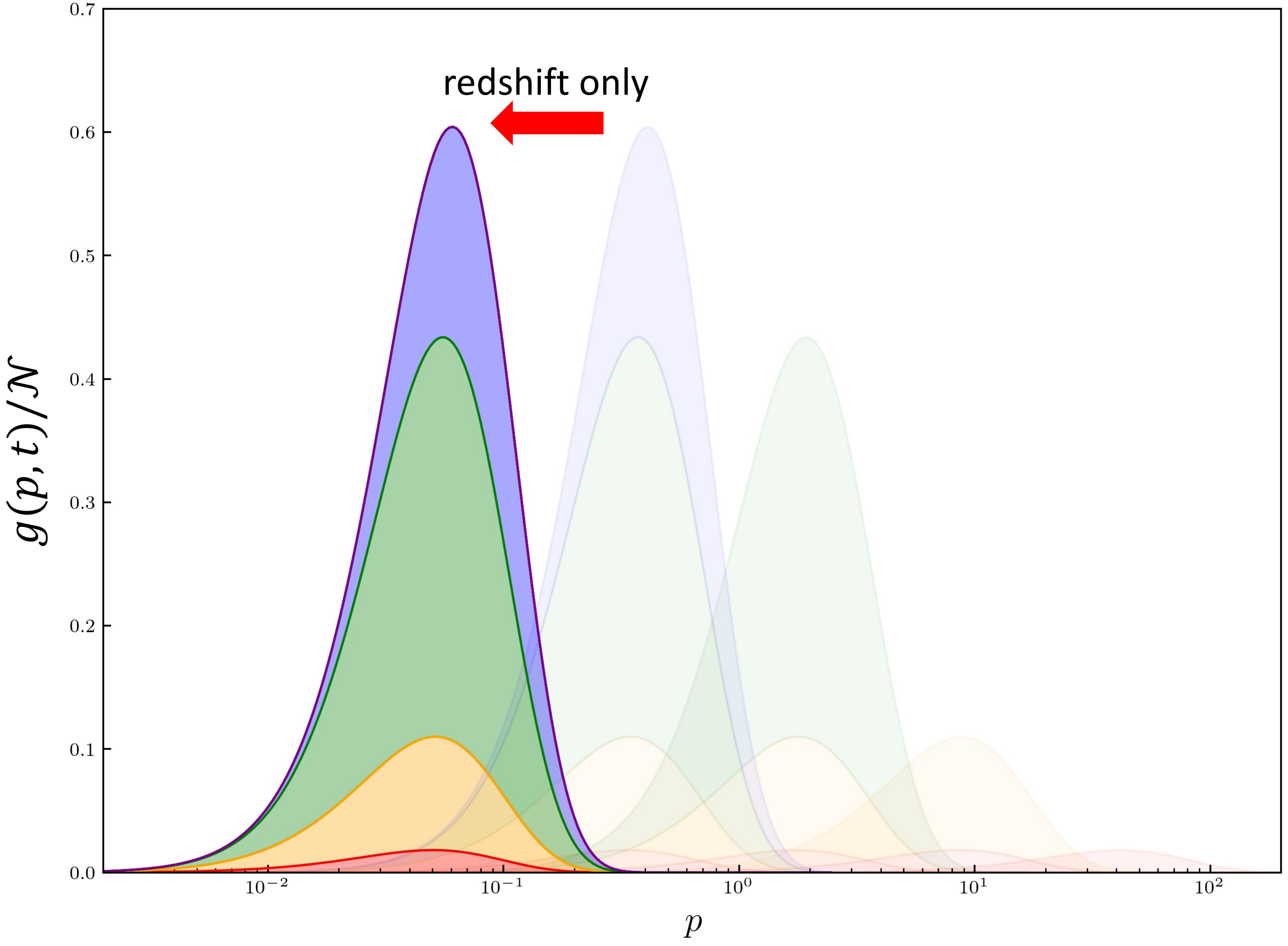

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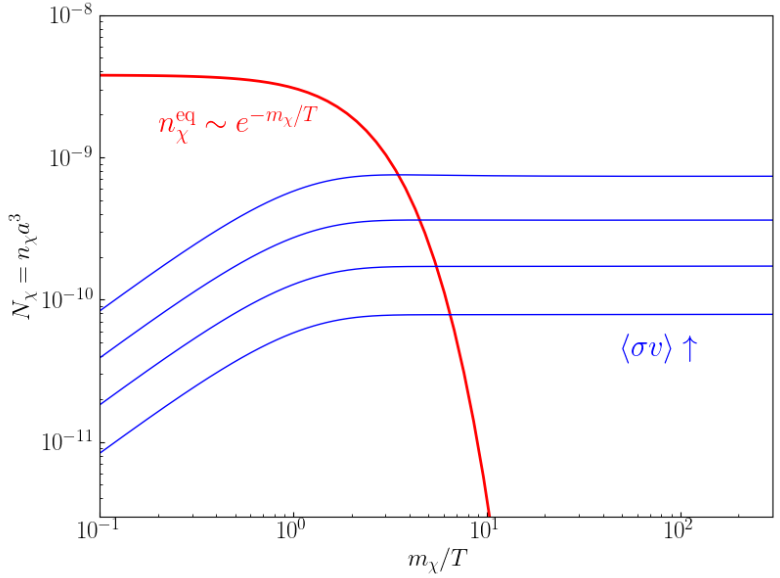

Example: freeze-out

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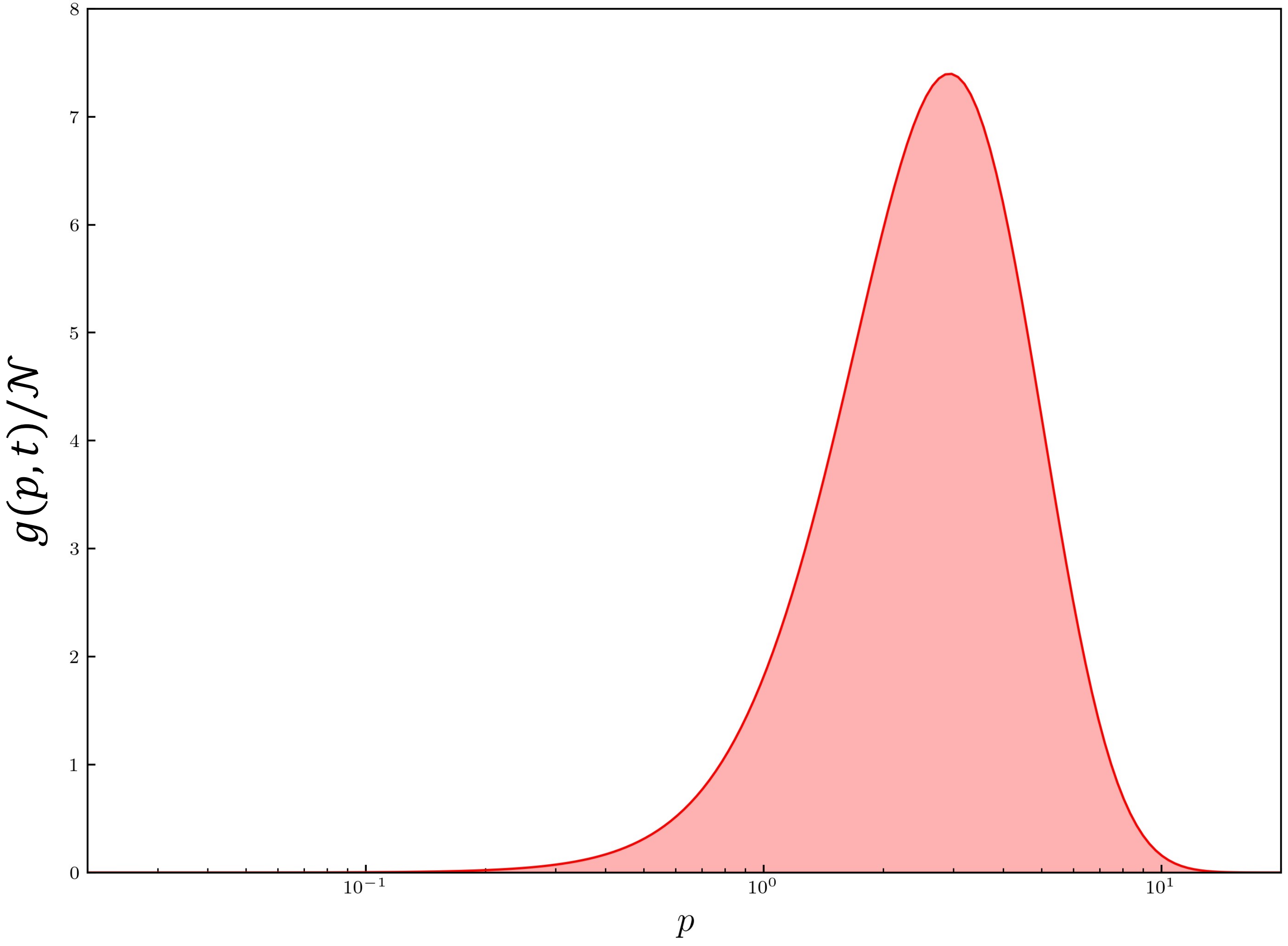

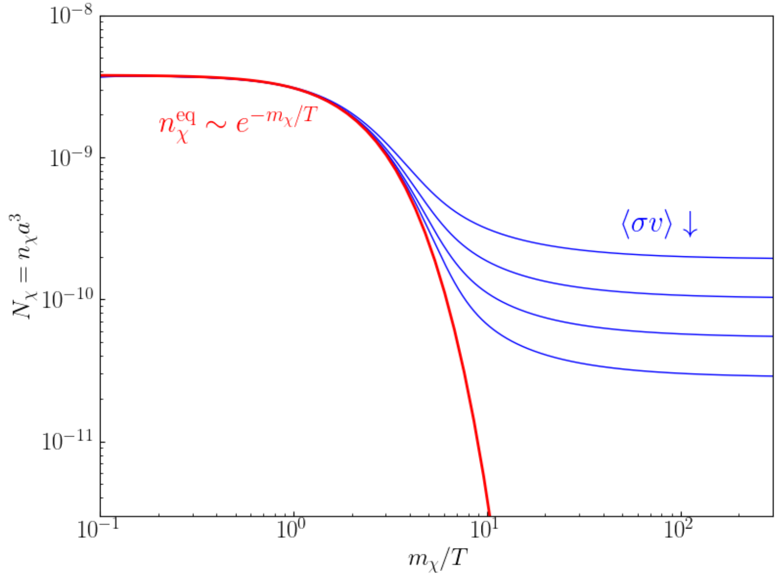

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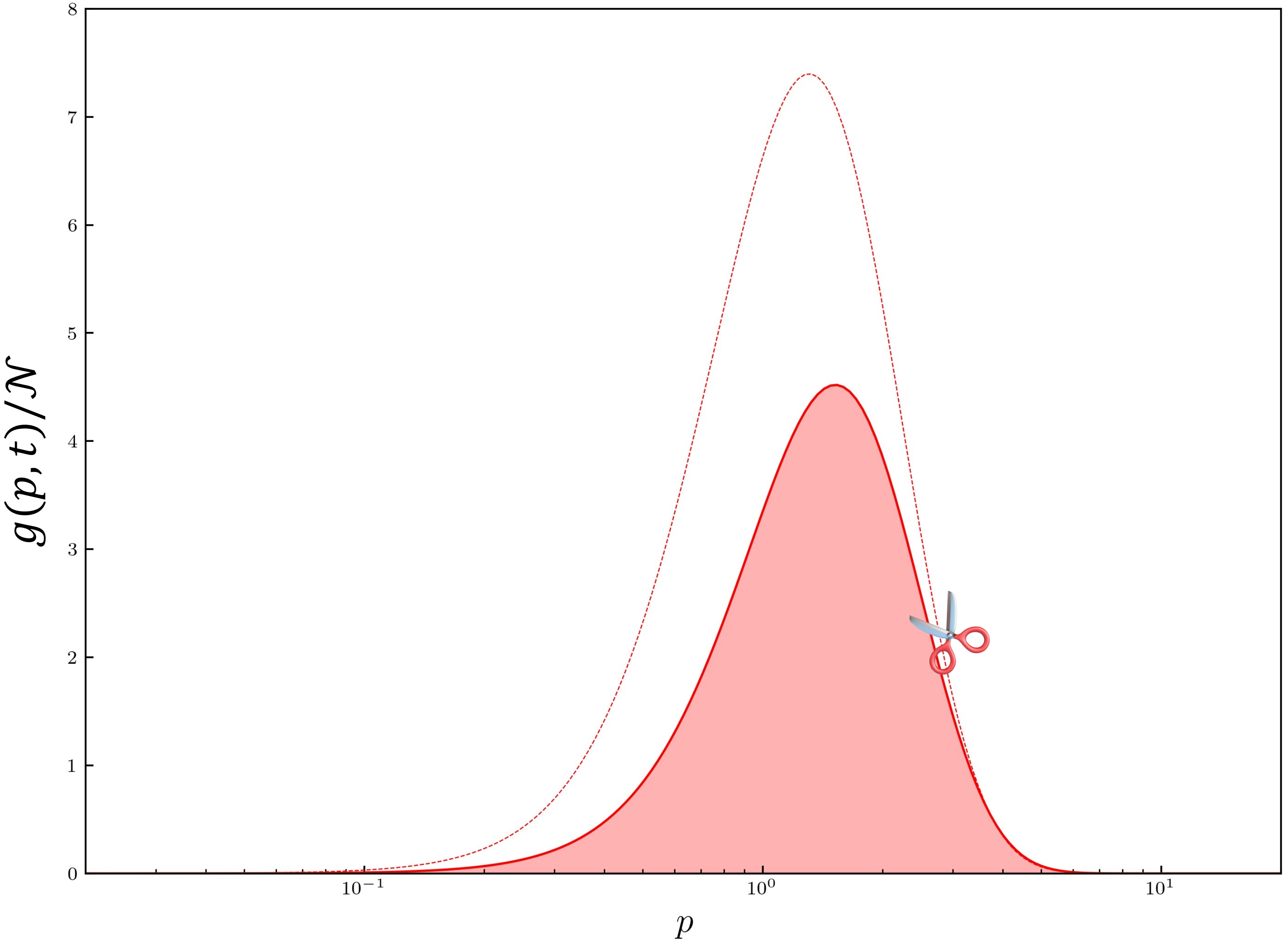

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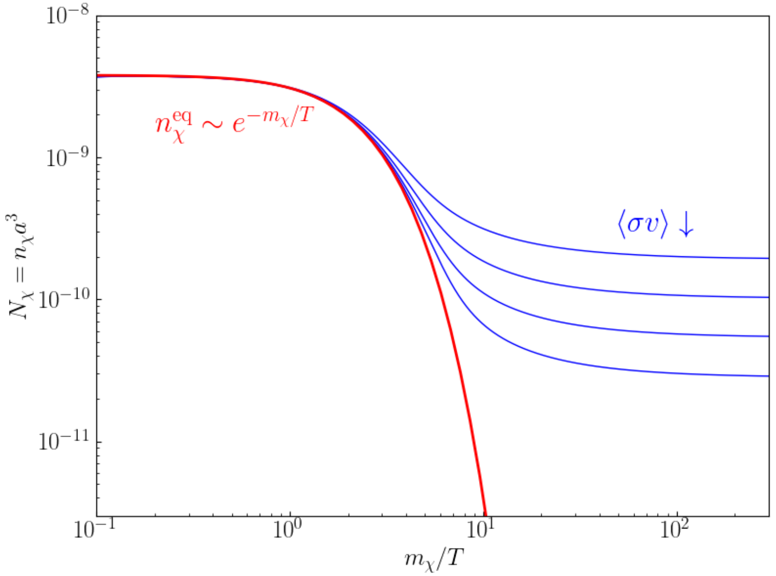

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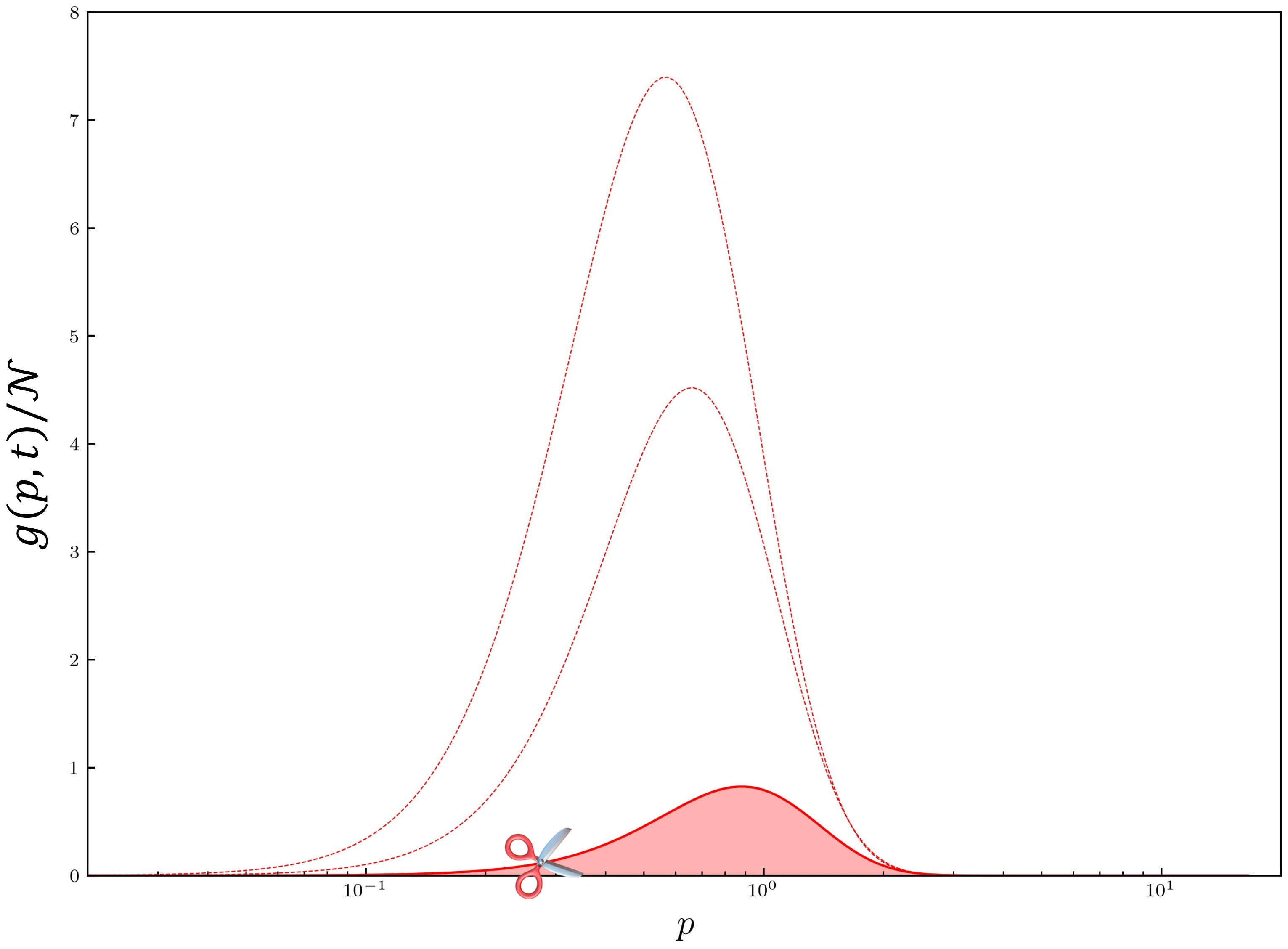

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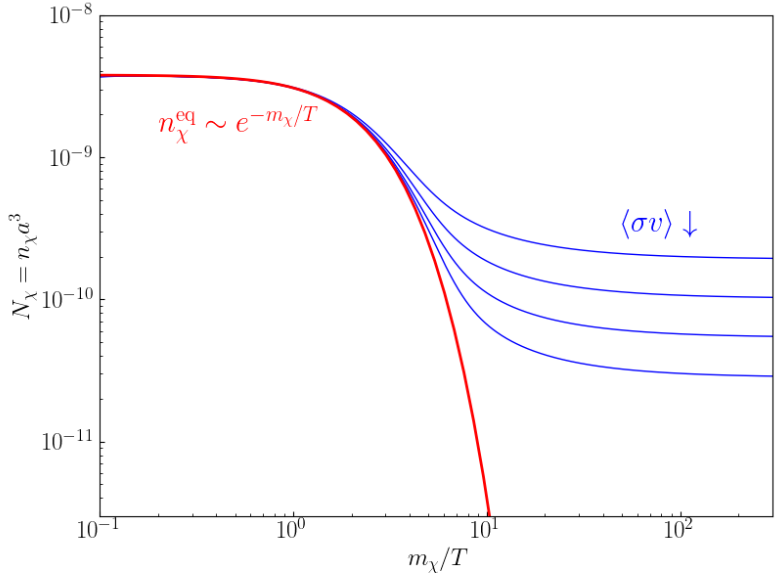

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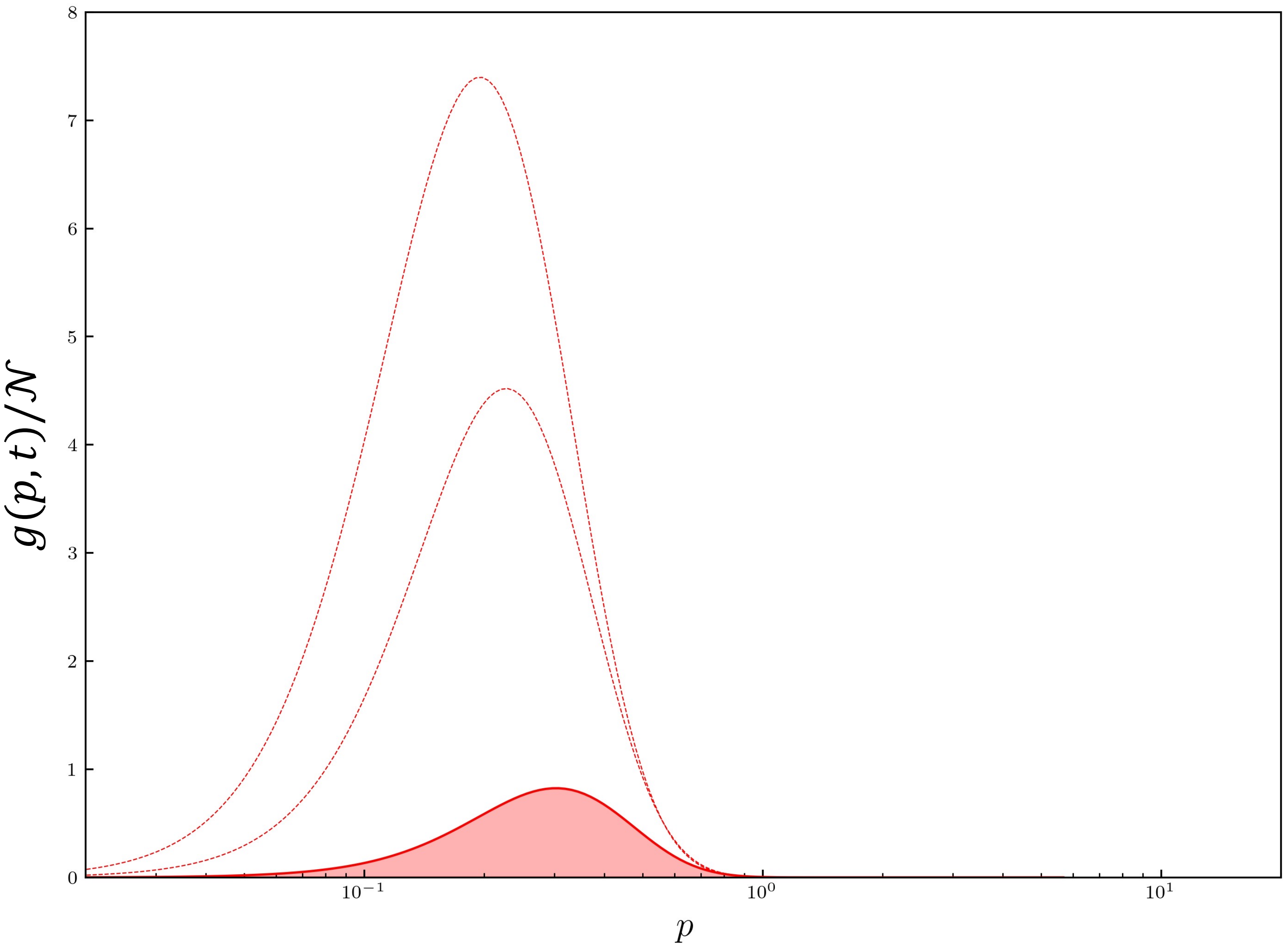

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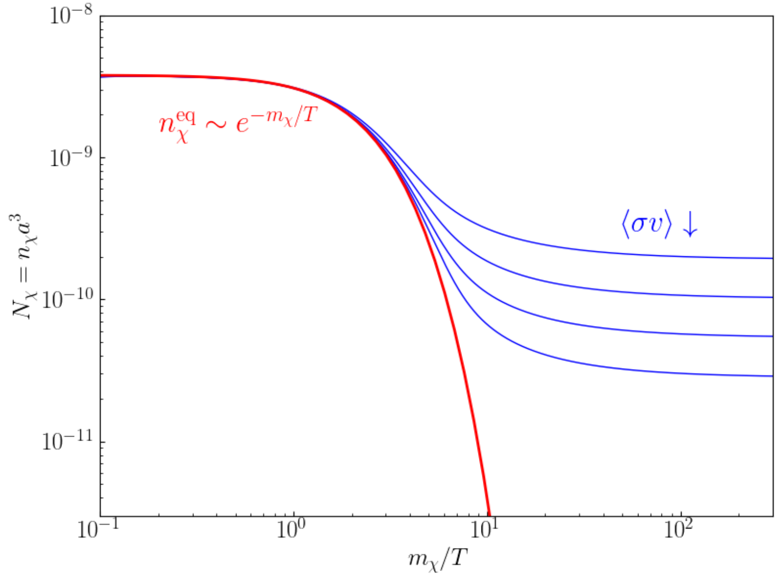

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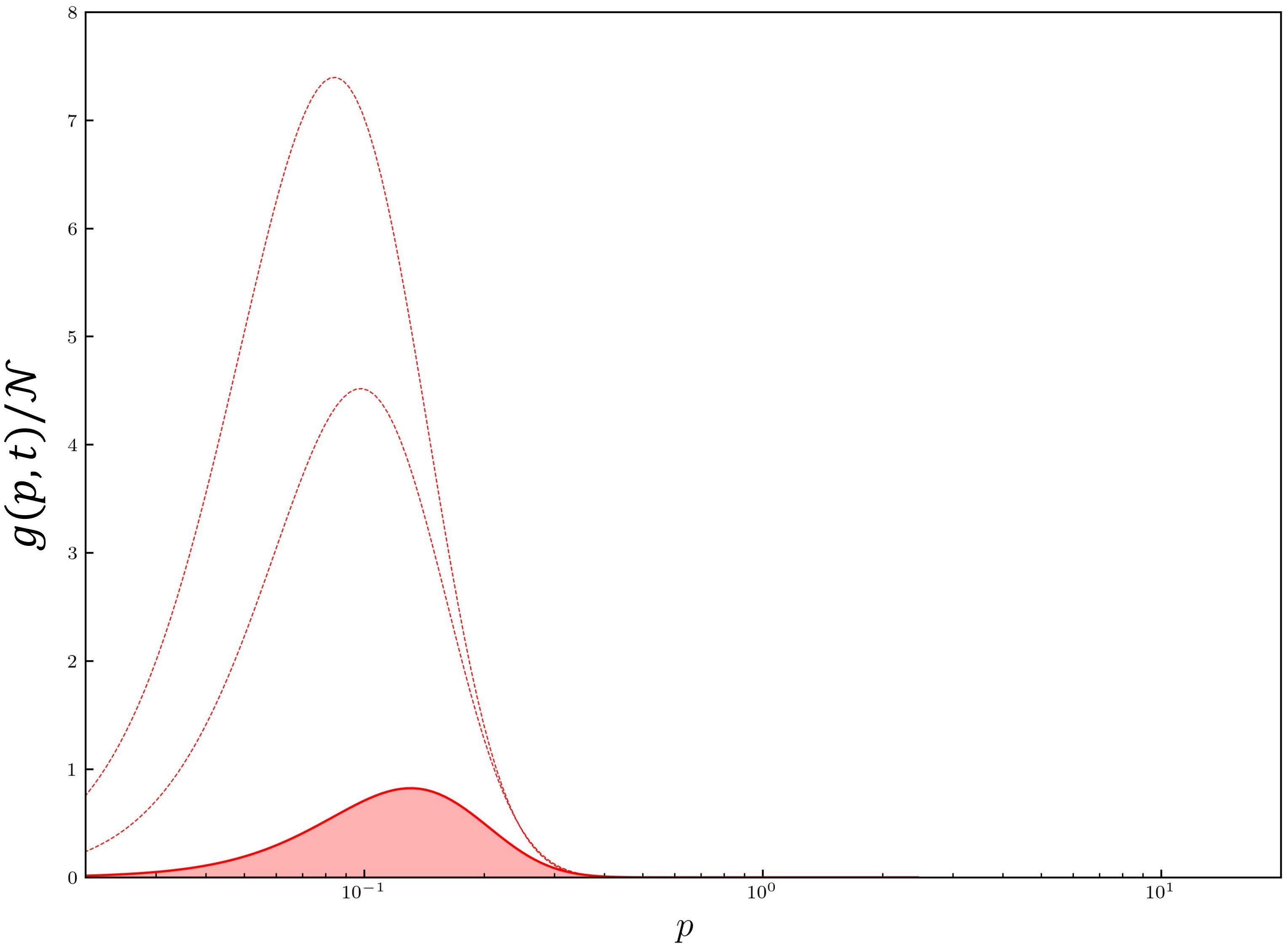

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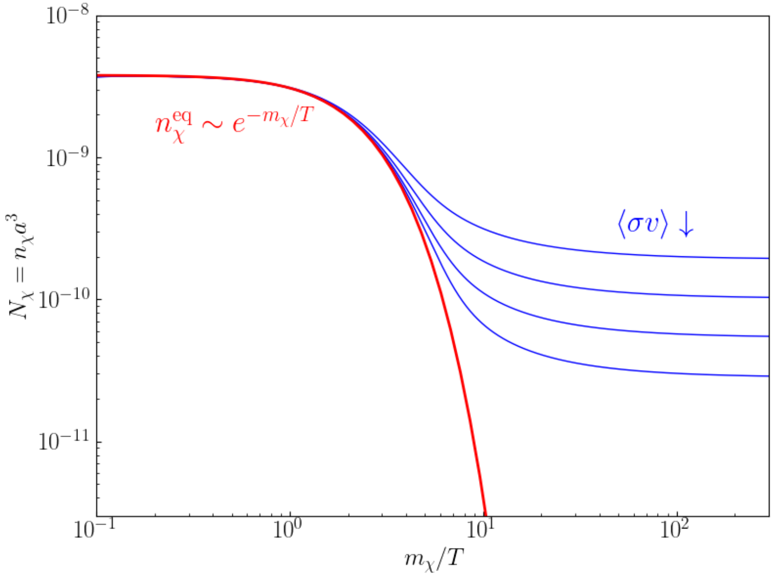
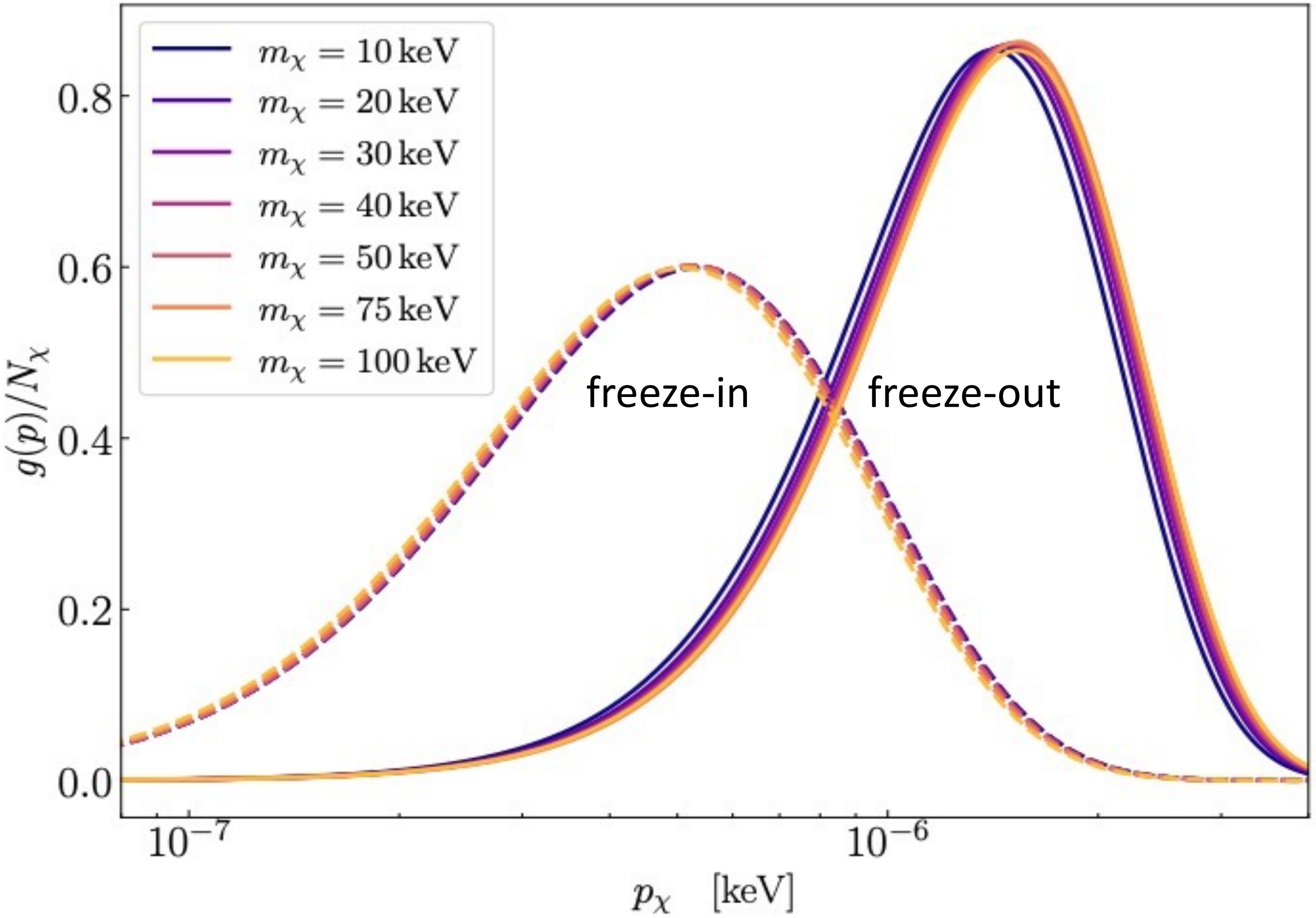


Example: freeze-in and freeze-out



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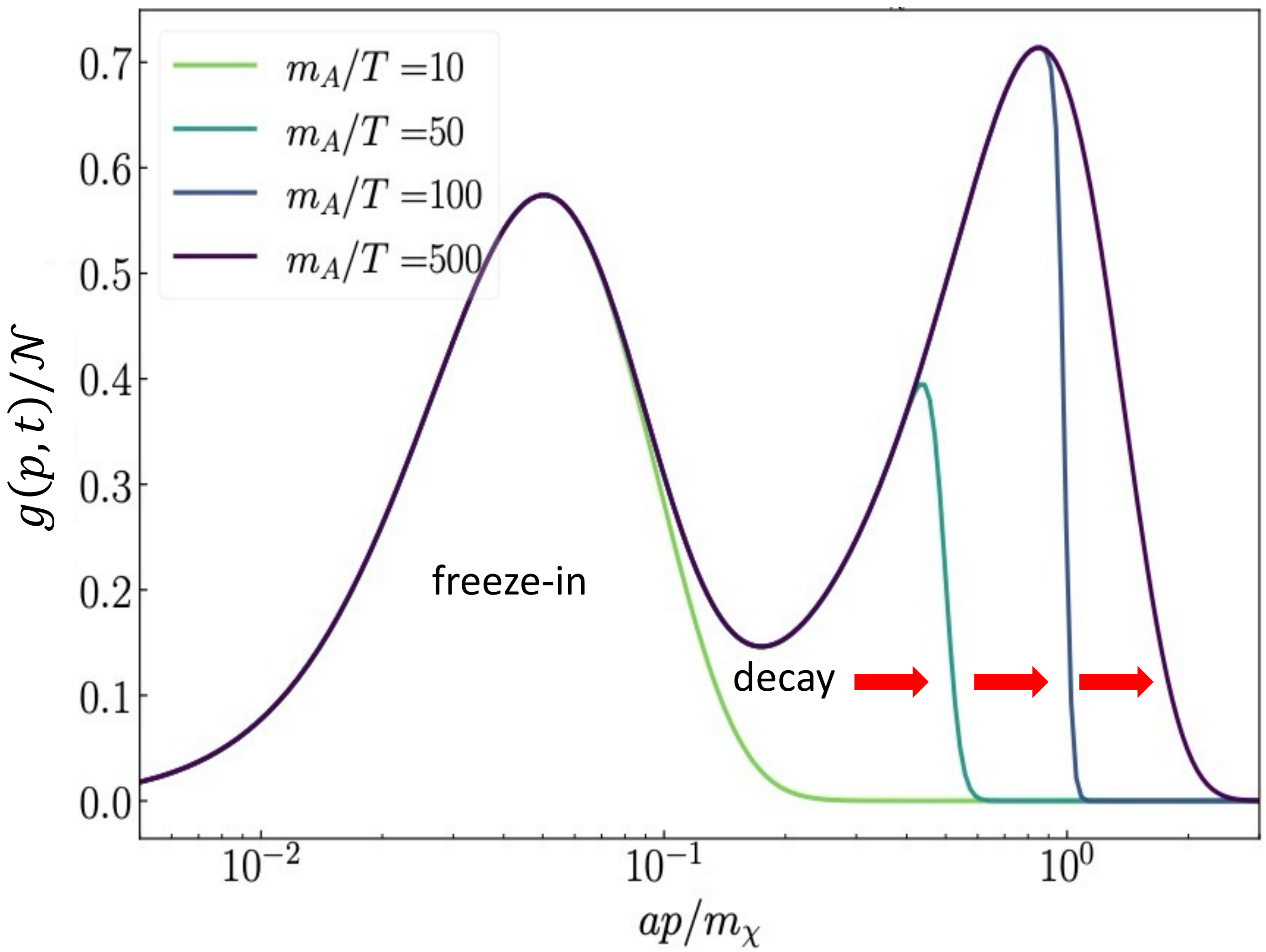
JCAP 01 (2024) 023

Example: freeze-in + out-of-eq decay



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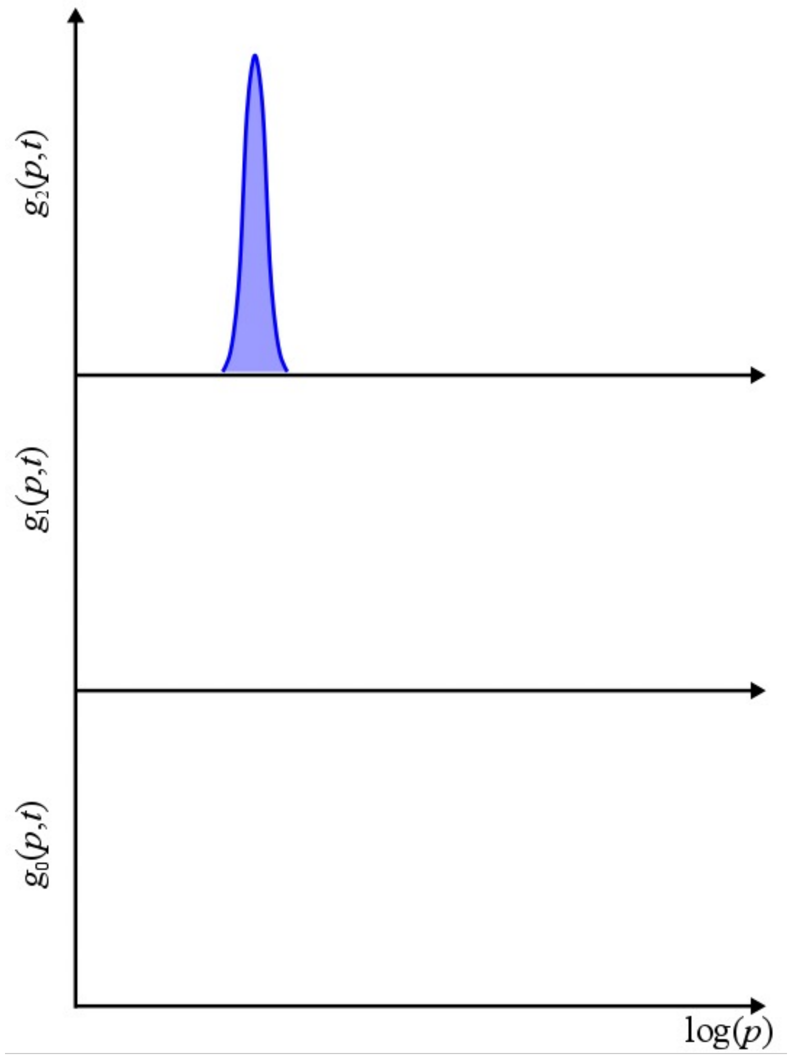
Example: intra-ensemble decays in multi-state system



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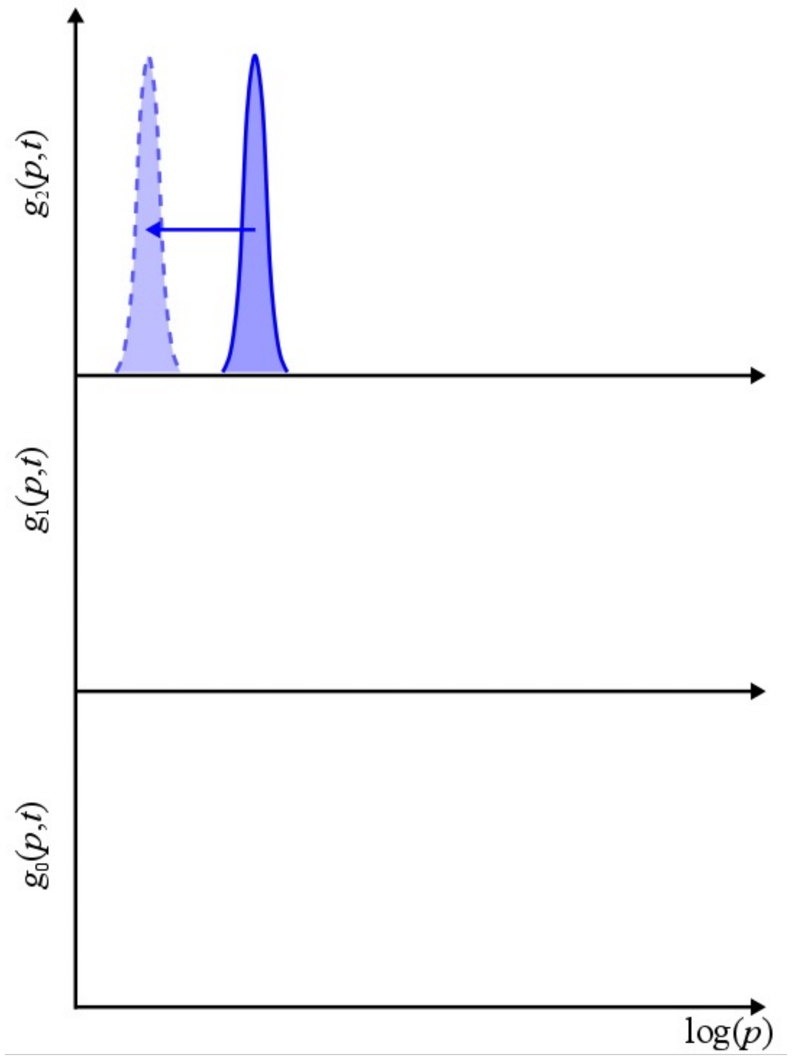

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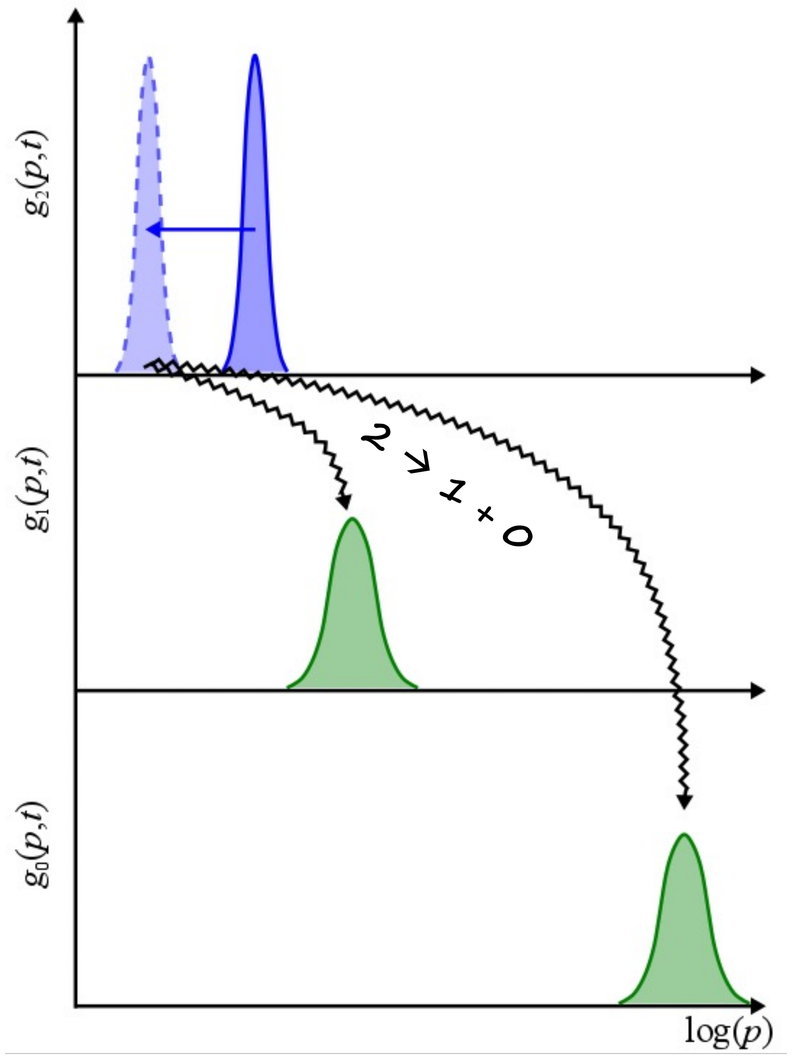

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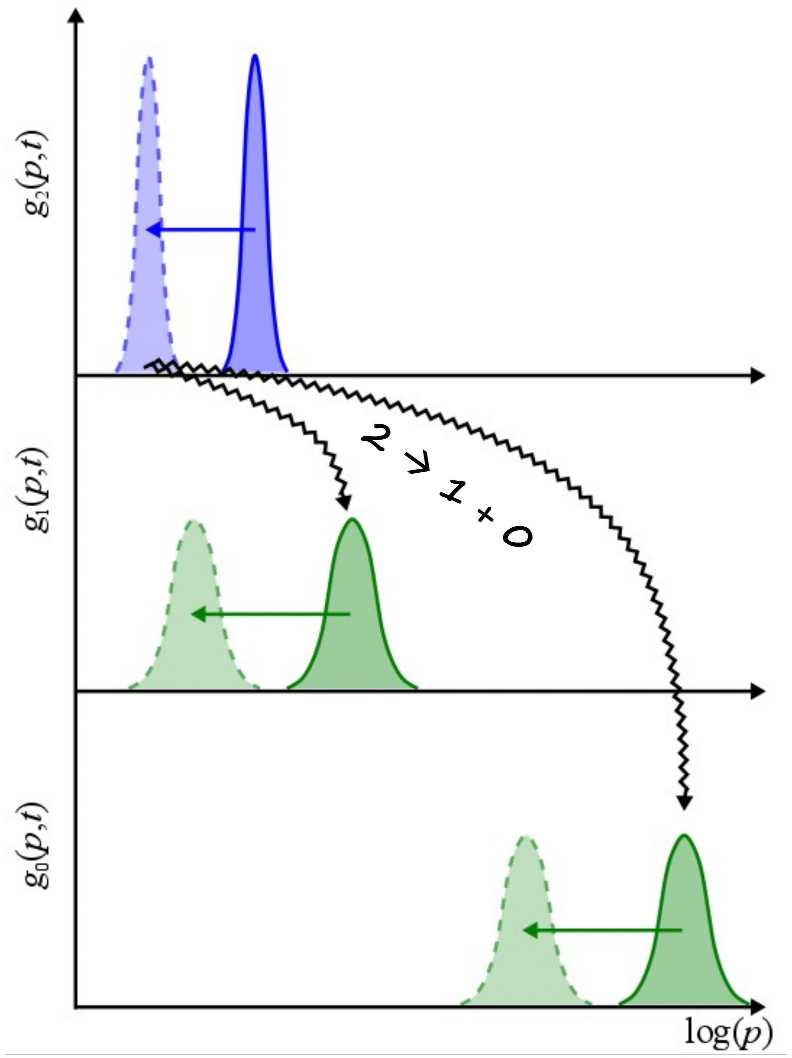

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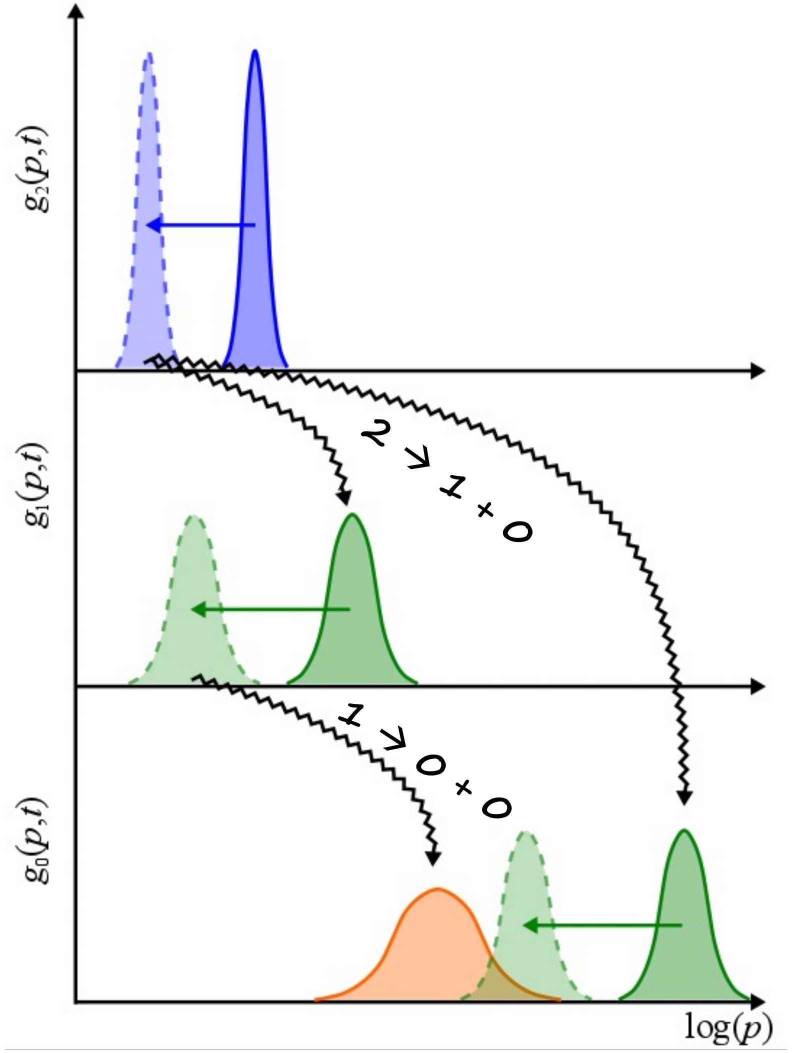

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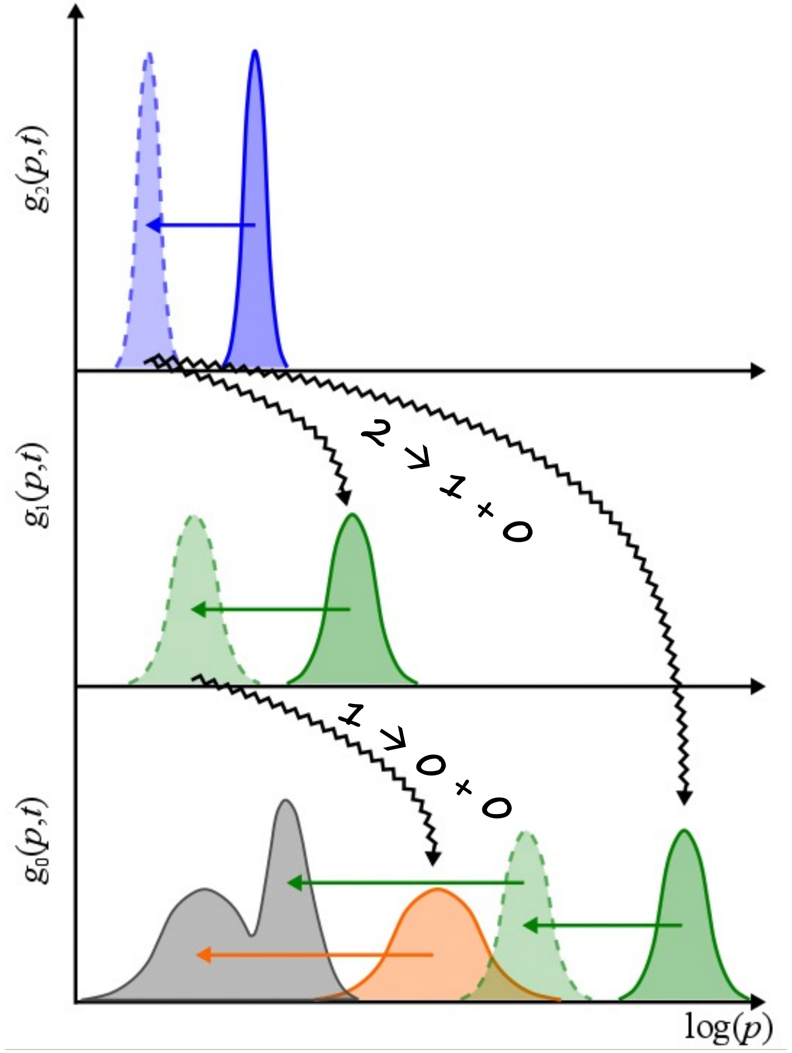

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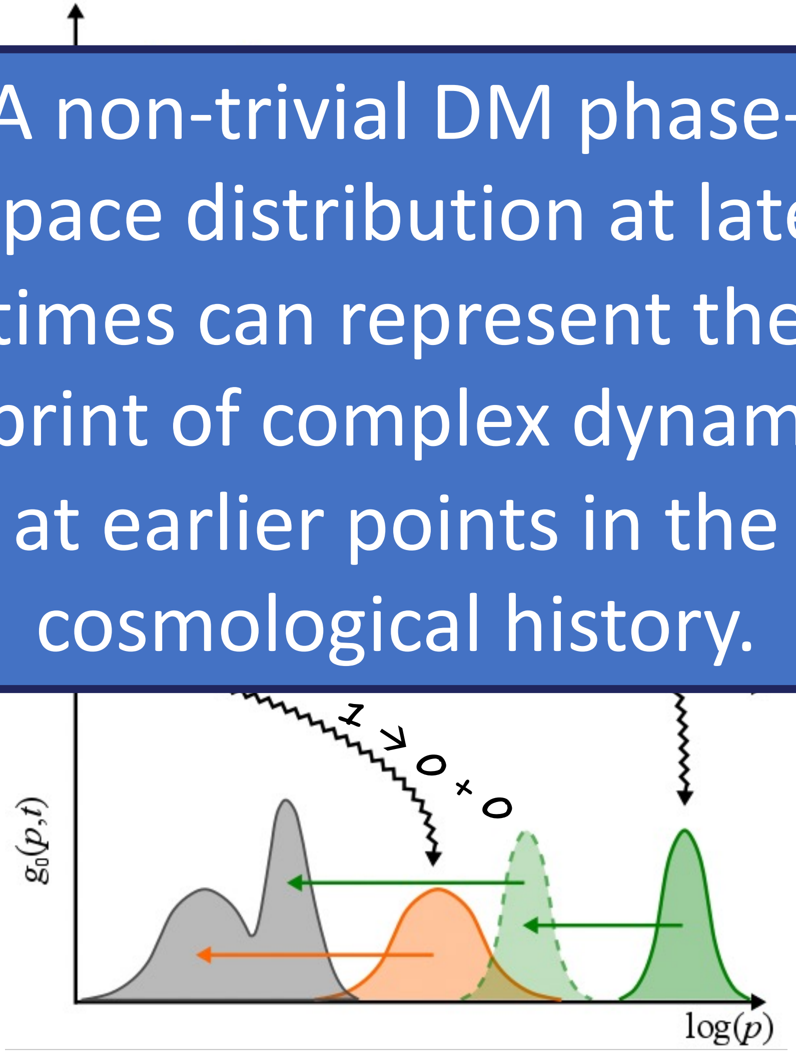
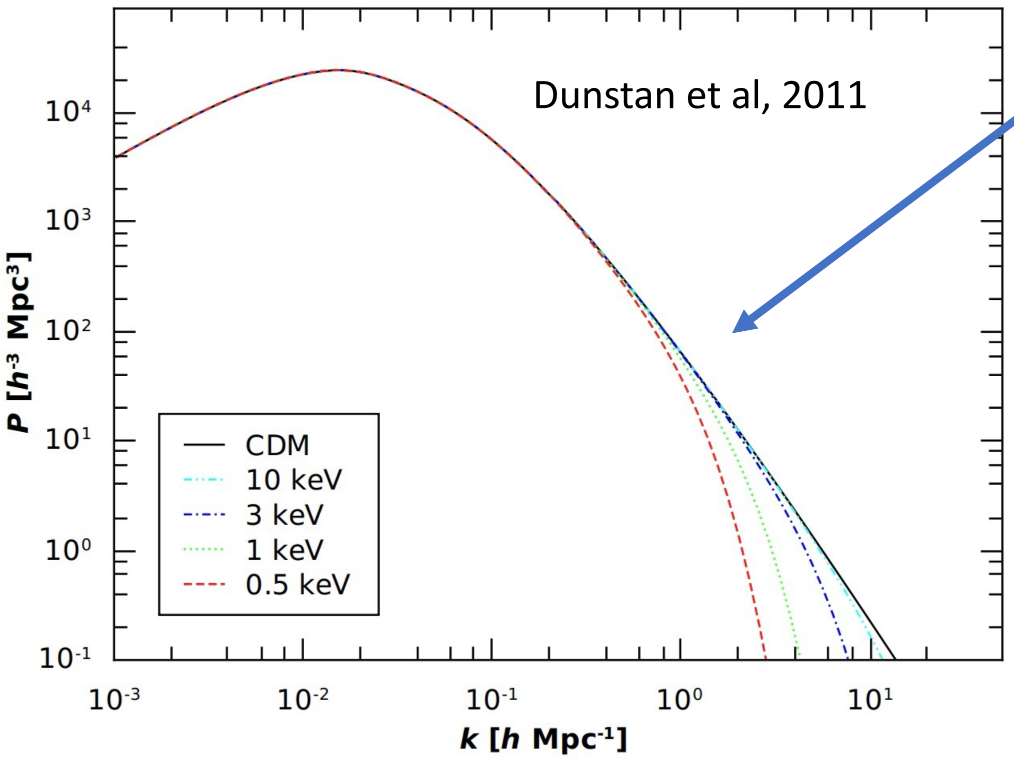


A non-trivial DM phase-space distribution at late times can represent the imprint of complex dynamics at earlier points in the cosmological history.

Effects of DM PSD

Formation of structure (clusters, galaxies ...) is sensitive to the velocities of DM!

Non-negligible velocities leads to suppression in the matter power spectrum at distance scales below the free-streaming horizon.



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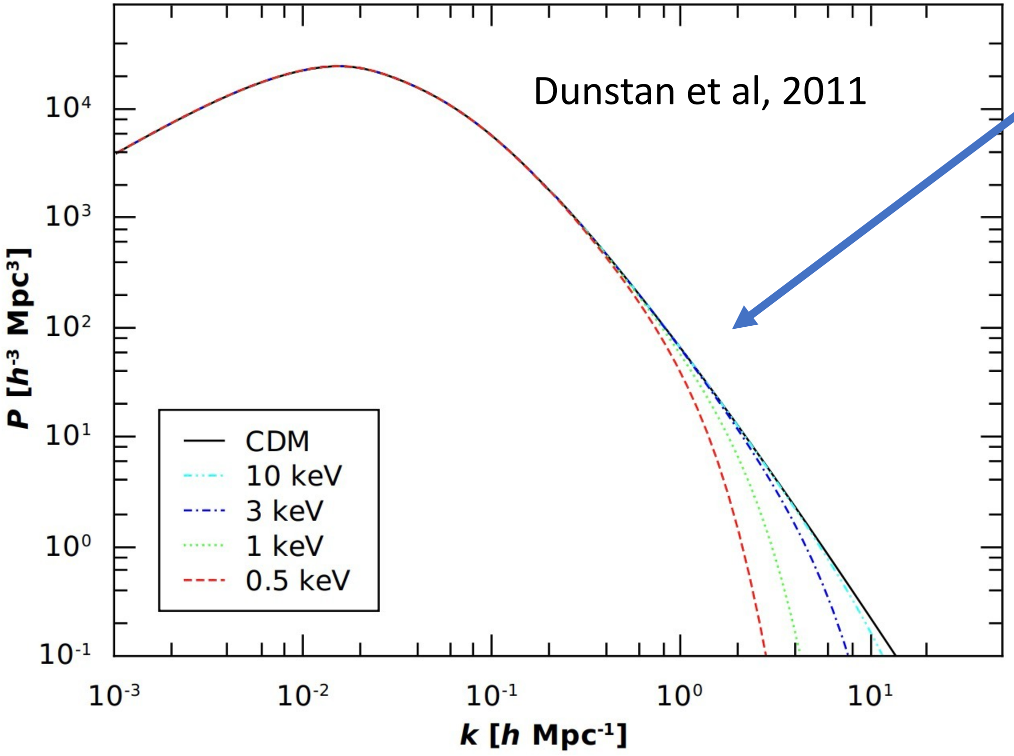

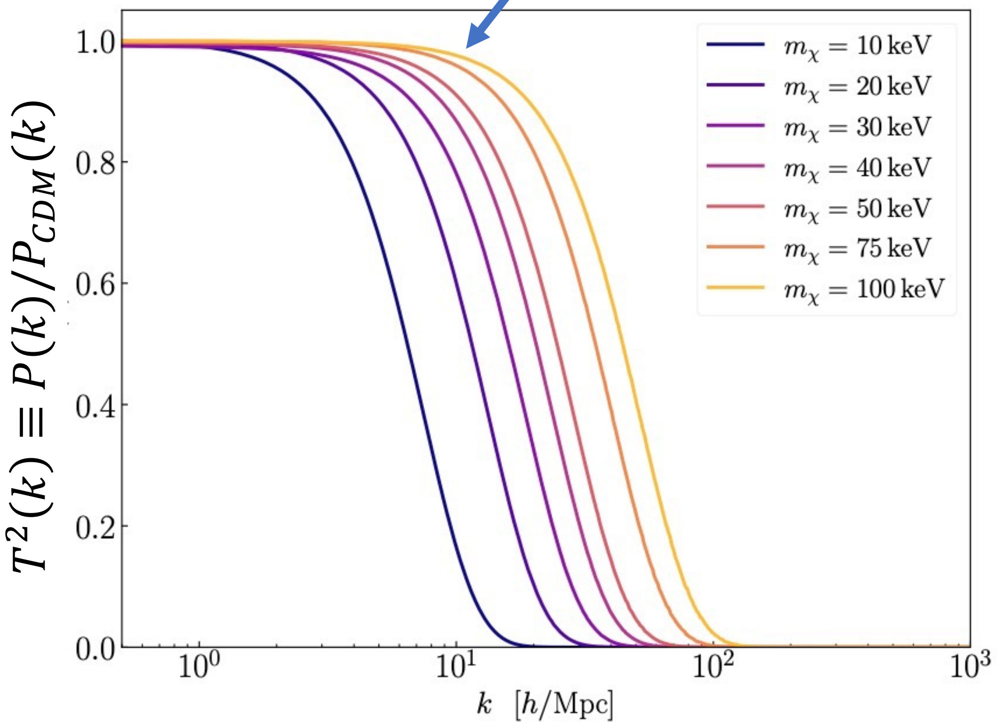

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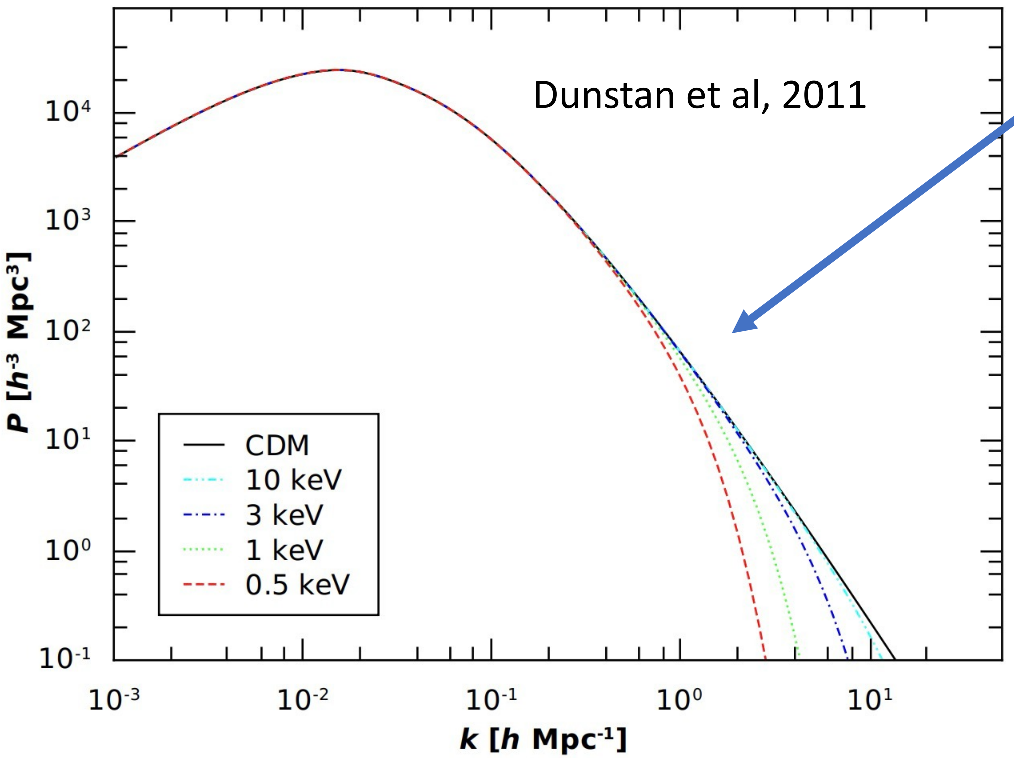
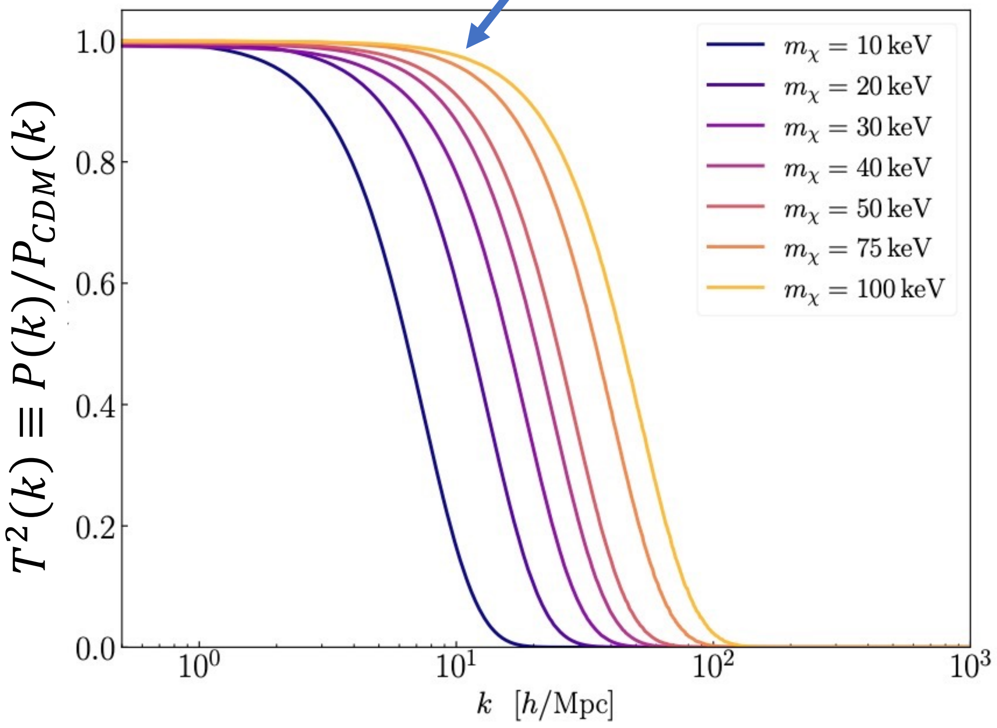


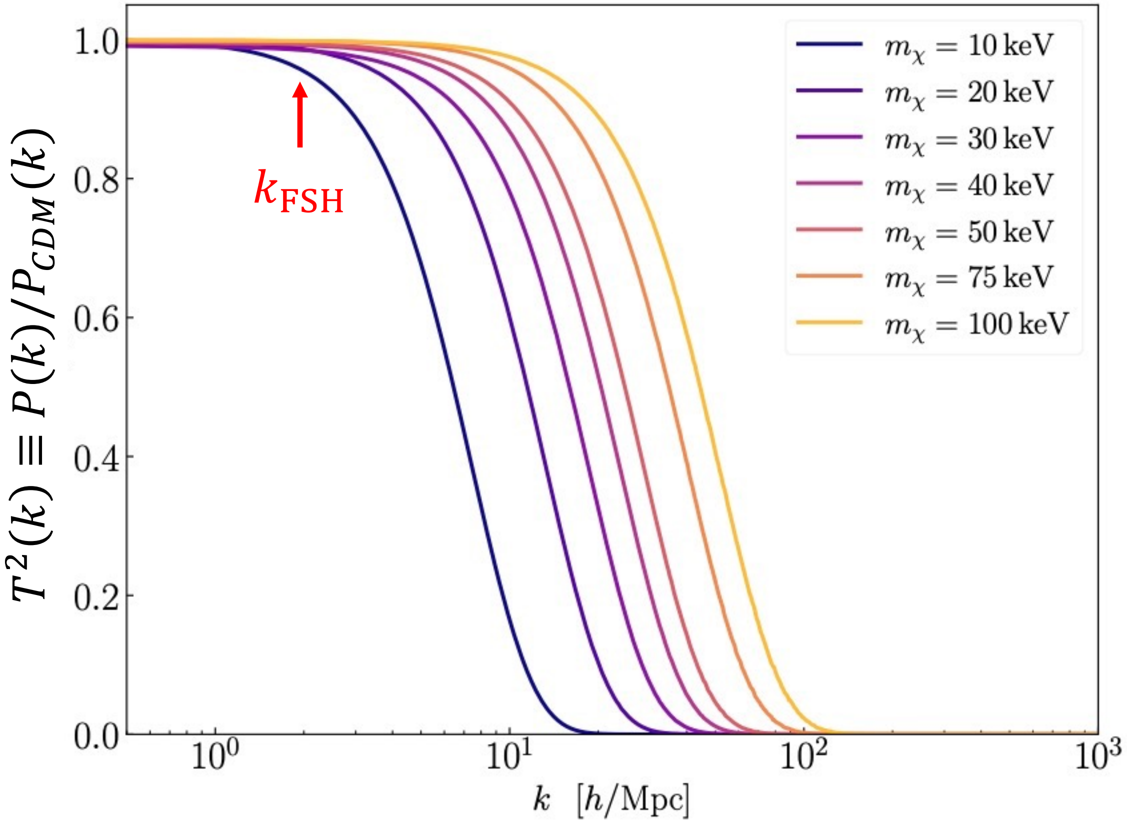


For free streaming, any deviation from CDM prediction may reveal information about DM velocities. The departure from CDM prediction can be expressed in terms of the squared transfer function $T^2(k)$

Effects of DM PSD

To study the impact of non-negligible velocities on $P(k)$ or $T^2(k)$, a standard approach is to define a single "free-streaming horizon" as a benchmark scale below which structure is suppressed

$$k_{\rm FSH} \equiv \left[\int_{t_{\rm prod}}^{t_{\rm now}} dt\, \frac{\langle v(t)\rangle}{a(t)}\right]^{-1}$$



Effects of DM PSD

To study the impact of non-negligible velocities on $P(k)$ or $T^2(k)$, a standard approach is to define a single "free-streaming horizon" as a benchmark scale below which structure is suppressed

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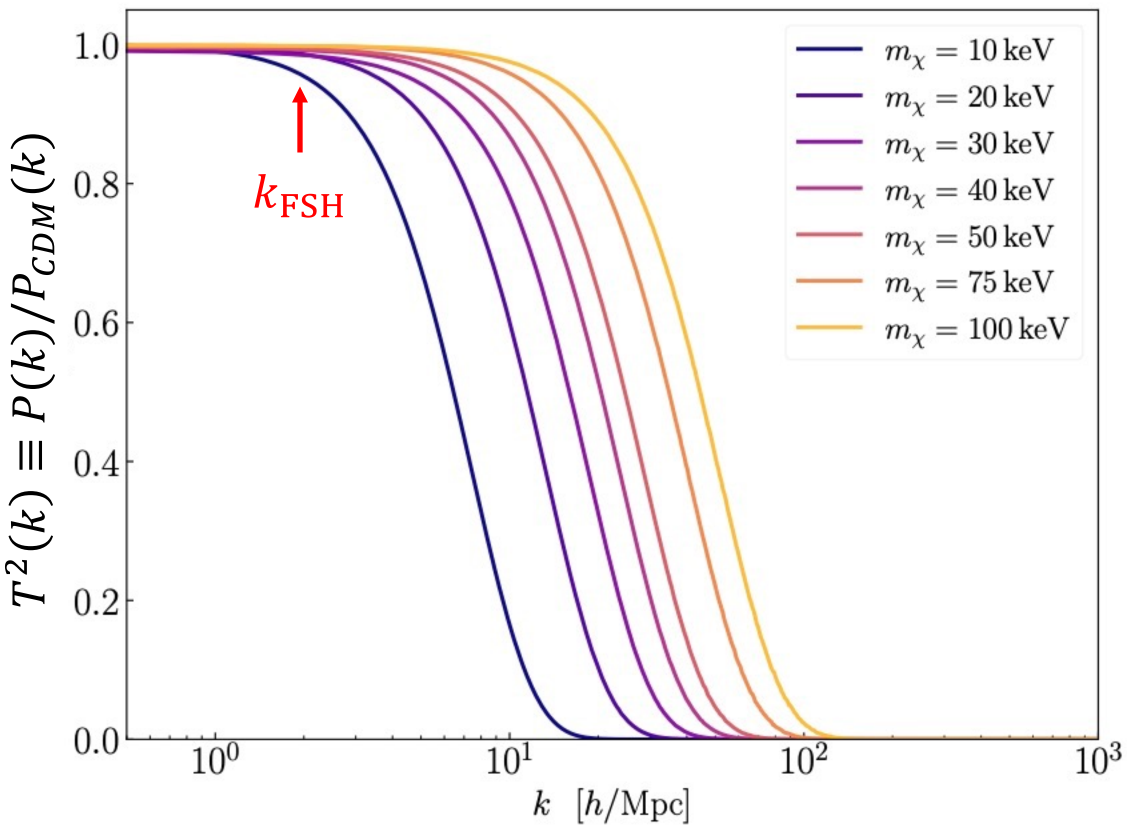

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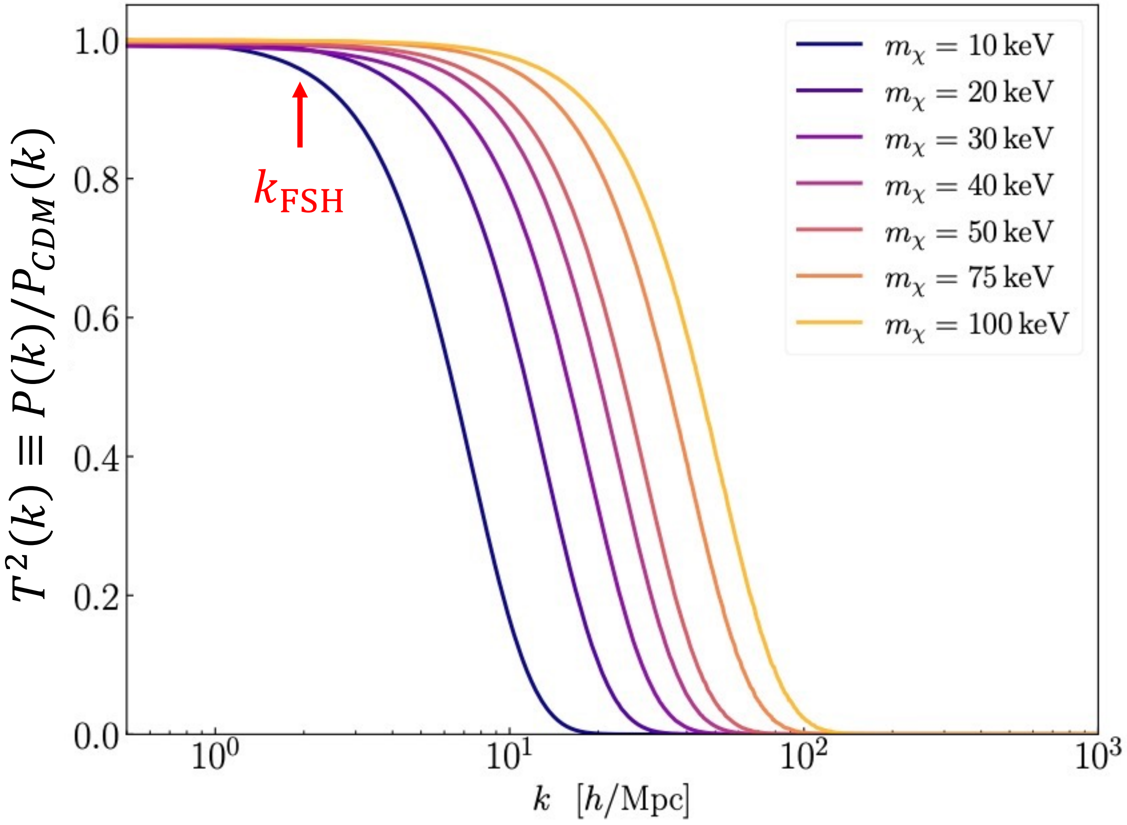

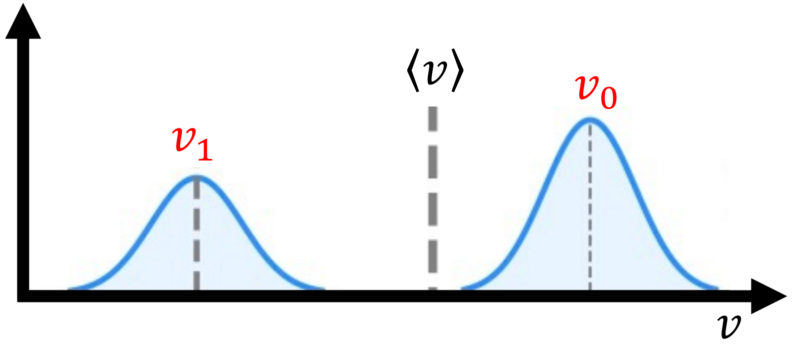

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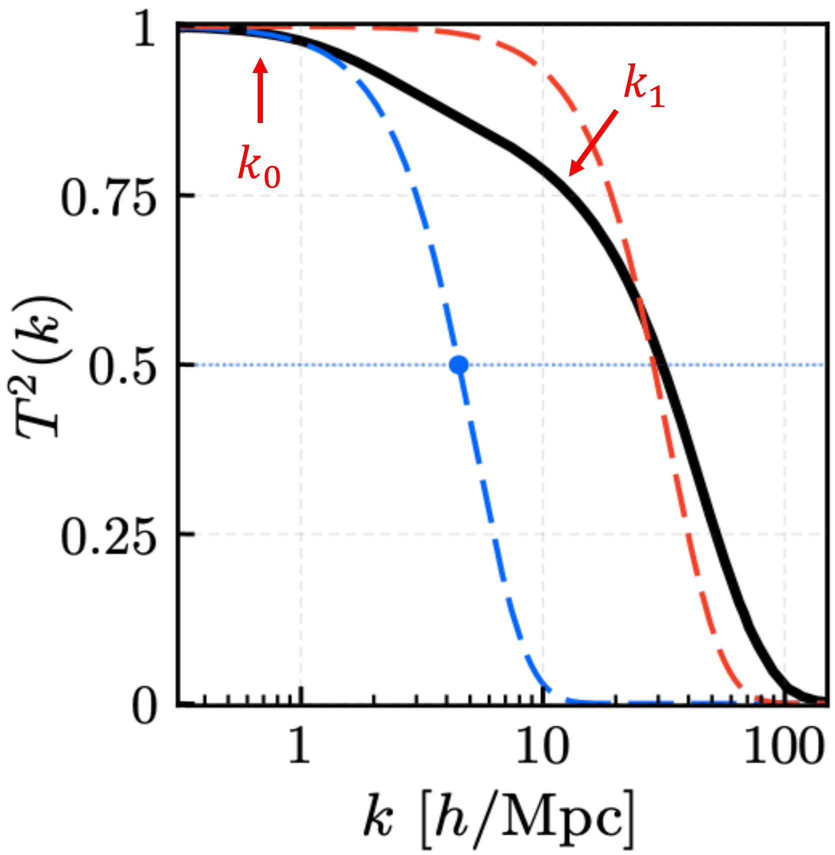

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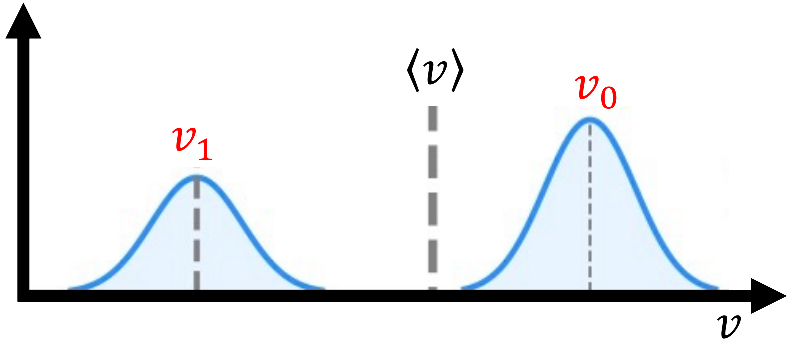

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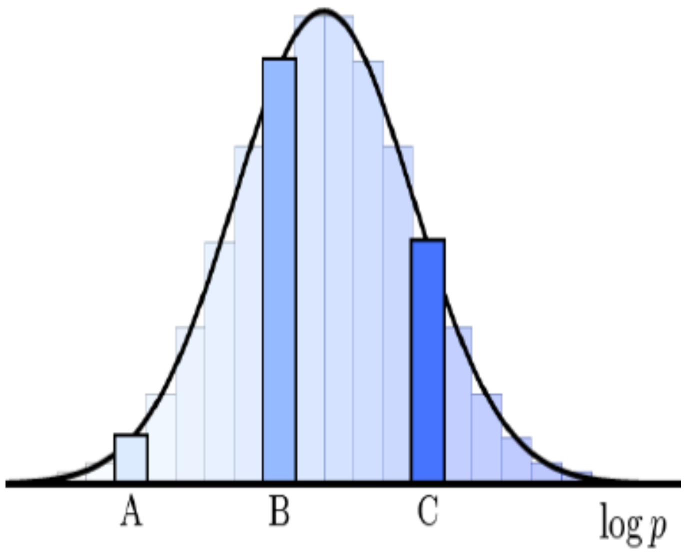
PSD in wavenumber space

We ***decompose*** our dark-matter packet into momentum slices, relating ***each slice*** of momentum p to a corresponding value $k_{\rm hor}(p)$.



$$k_{\rm hor}(p) \equiv \xi \left[\int_{t_{\rm prod}}^{t_{\rm now}} dt\, \frac{v(t)}{a(t)} \right]^{-1}$$

maximum length scale

Normally, $k_{\rm hor}$ would be interpreted as defining the ***minimum*** value of k which can be affected by dark matter in that momentum slice.

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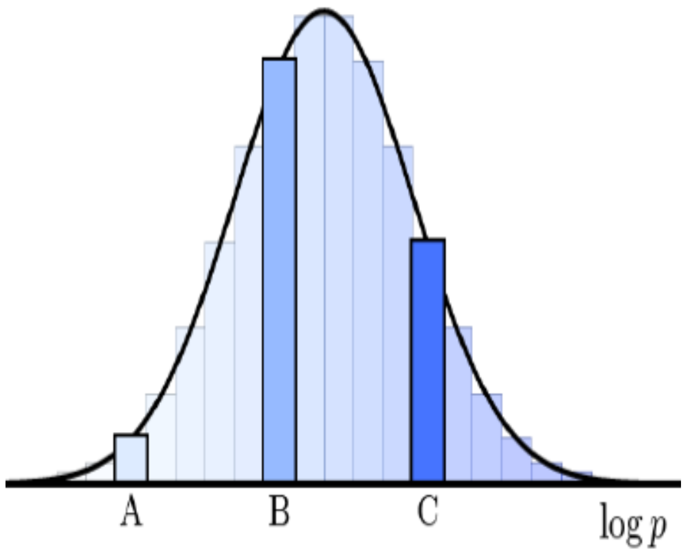

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However, we shall instead take the defining relation for $k_{\rm hor}(p)$ as defining a ***mapping*** between the p-variable of $g(p)$ and the k-variable of $P(k)$.

$$p \to k$$

PSD in wavenumber space

In other words, we ***identify*** $k_{\mathrm{hor}}(p)$ with k and thereby consider $g(p)$ as having a corresponding profile in k-space:

$$g(p) \to \tilde{g}(k)$$

It then follows

$$N(t) \sim \int d\log p \;\; g(p) = \int d\log k \;\; \tilde{g}(k)$$

Thus $\tilde{g}(k)$ describes a **dark-matter distribution in k-space**!

Moreover, because this $\tilde{g}(k)$ lives in the same space as $P(k)$, these two functions can even be plotted together along the same axis!

Now it makes sense to ask:
Can we discover/conjecture any relation between *these* two functions?

Examine the relations

<u>Vary height/area with width fixed</u>

$\Omega_{DM} = \Omega + \Omega_{CDM} \approx 0.26$

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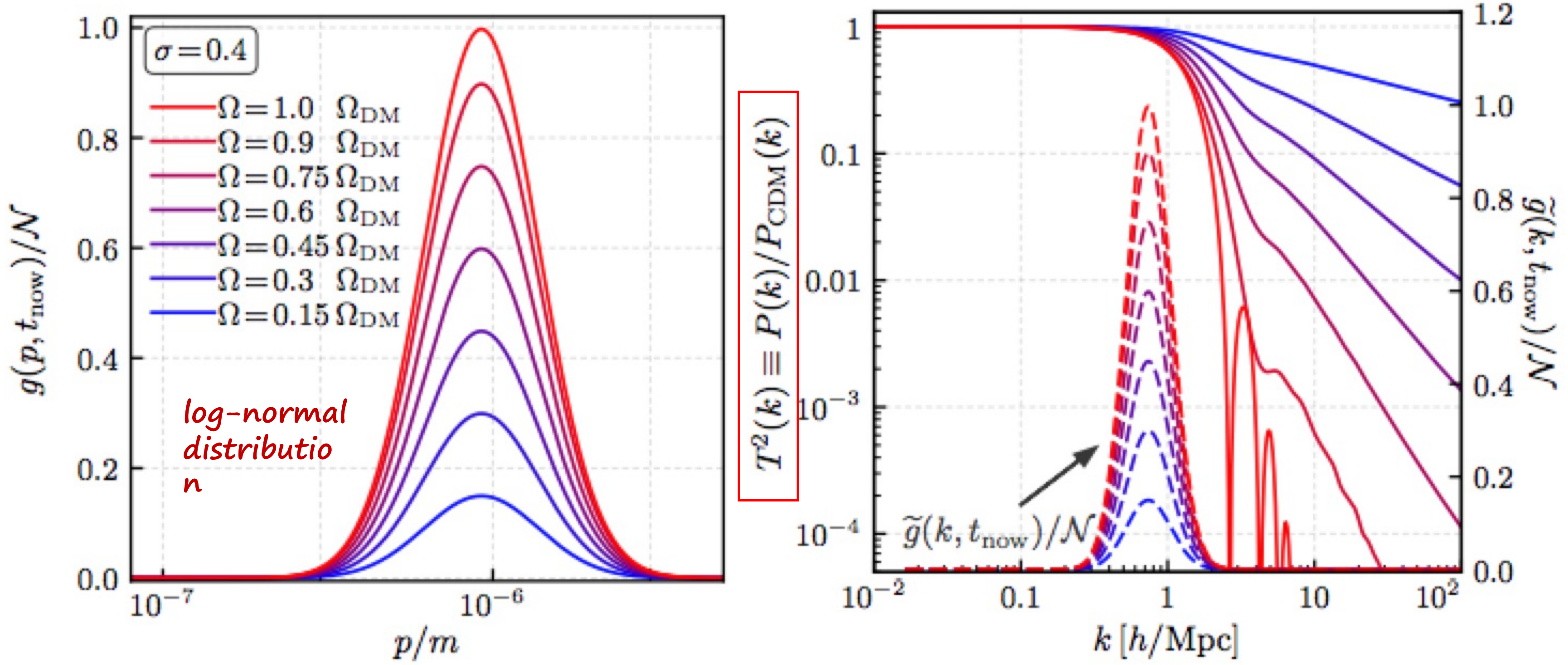

- No power suppression until we approach where $\tilde{g}(k)$ is concentrated
- Main observation: Larger packet → Stronger <u>suppression</u> and steeper <u>slope</u>

Examine the relations

<u>Vary width with average/area fixed</u>

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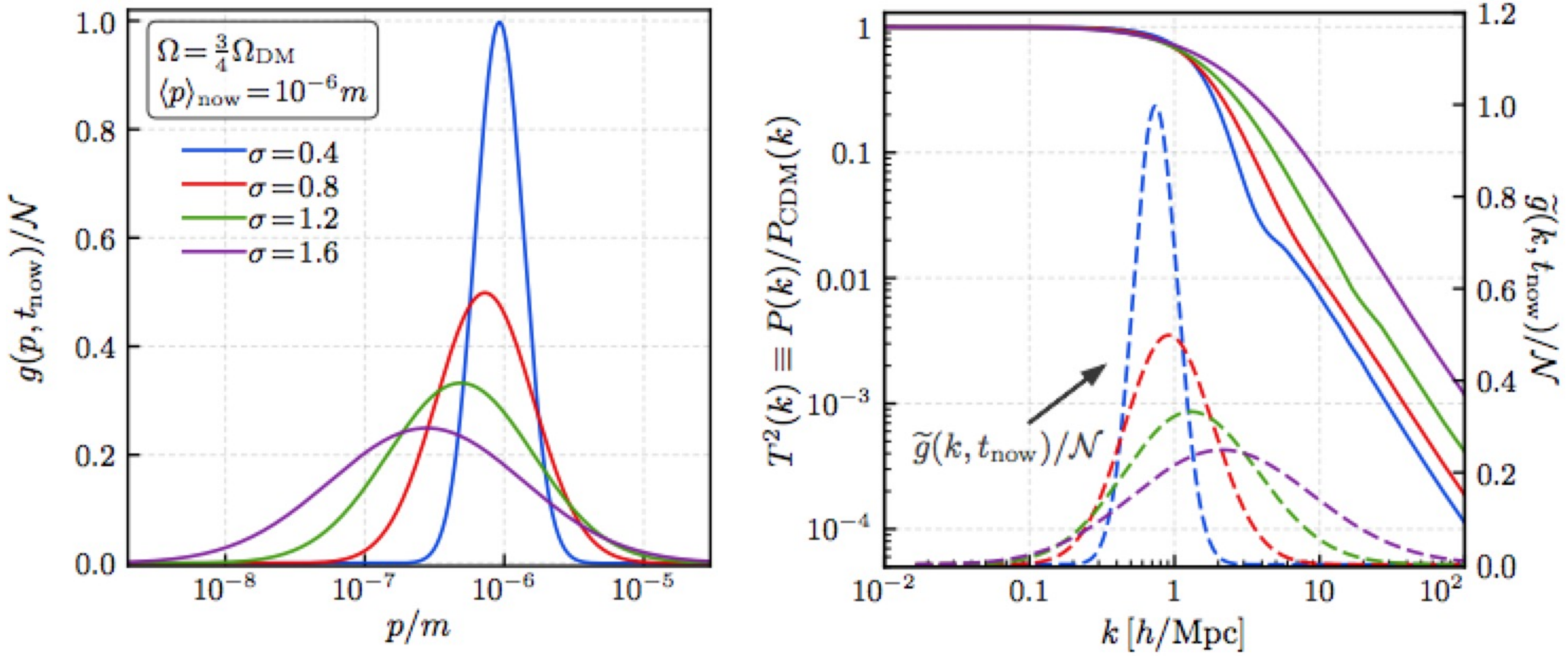

- The amount of <u>suppression</u> differs, but the ***<u>slope</u>*** at large k is essentially *<u>unaffected</u>* by widths! This suggests the ***<u>accumulative abundance is correlated with the slope</u>***, *NOT* with the net suppression.

Examine the relations

<u>Vary relative sizes of *two* packets</u>
(two packets together carry the total DM abundance)

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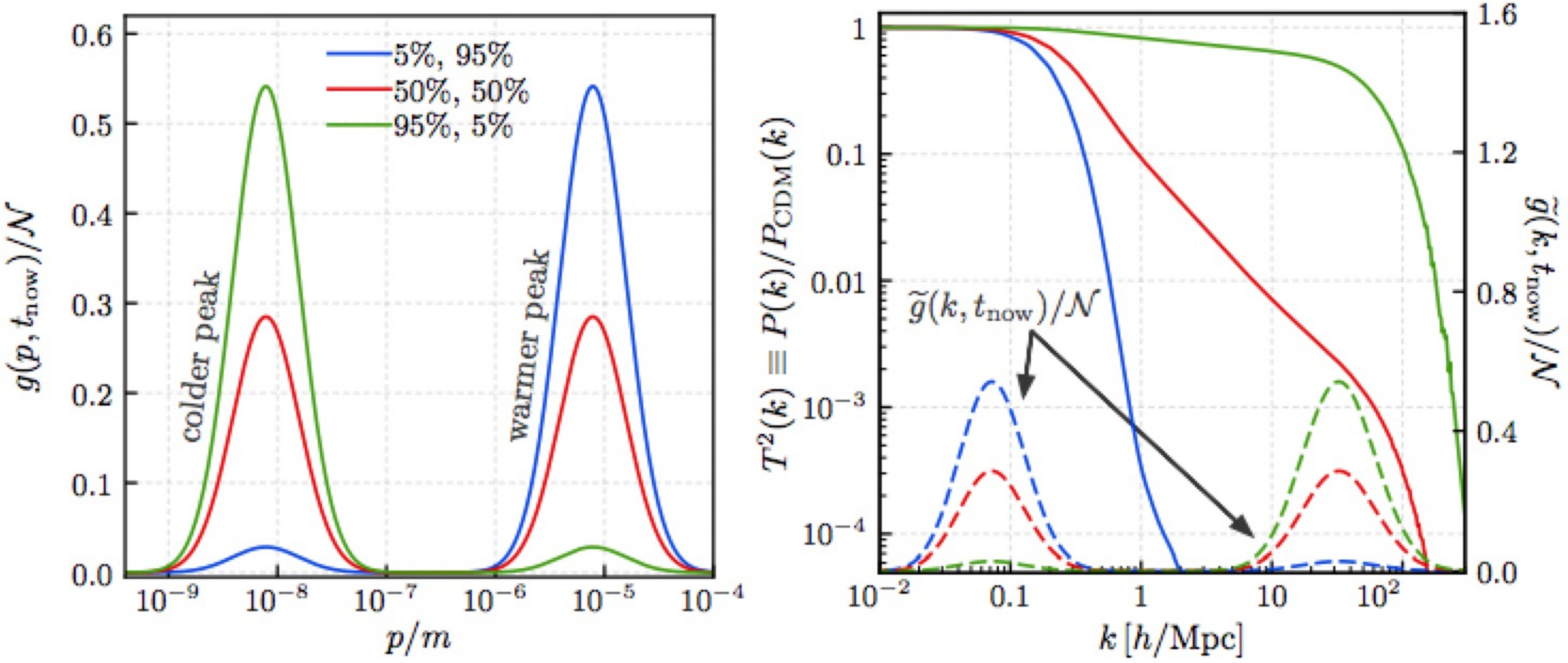

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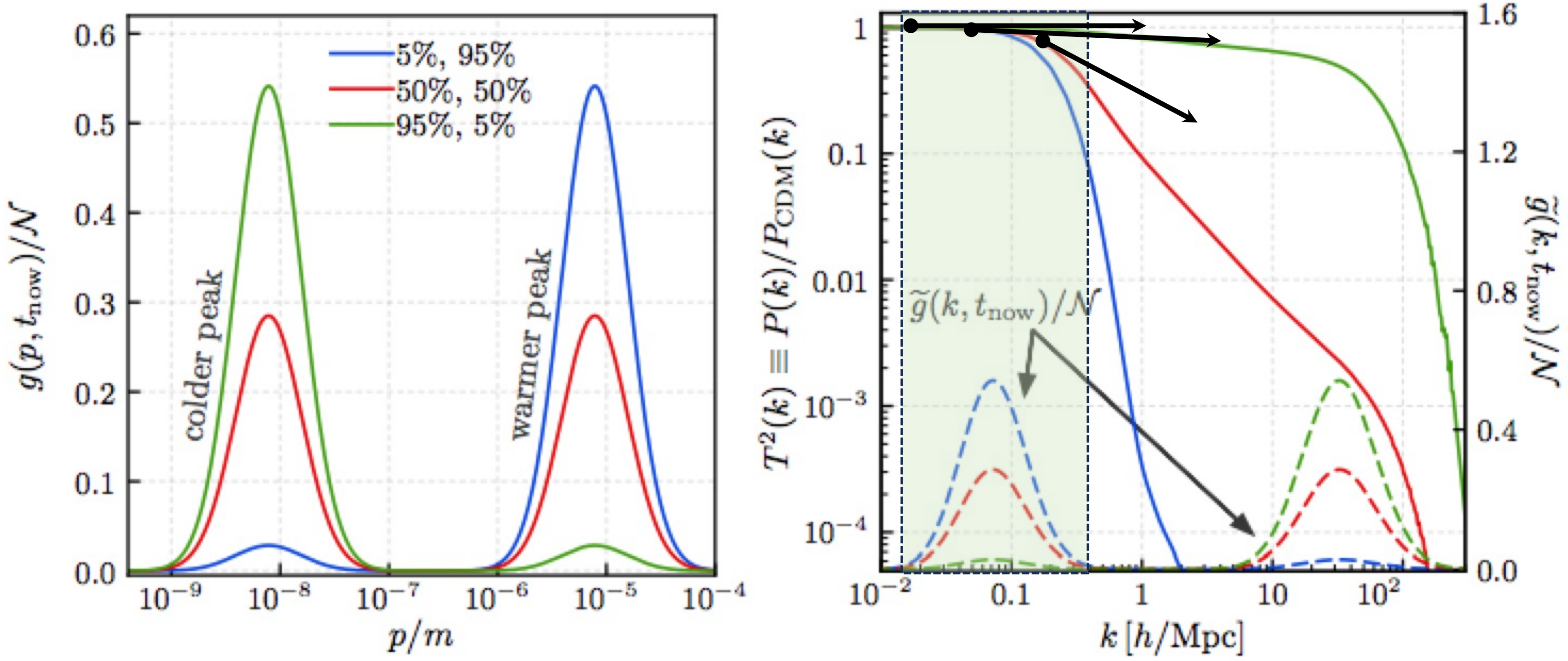

As we sweep from left to right in the k-space,

- *within* a peak: accumulative abundance increases → slope increases!

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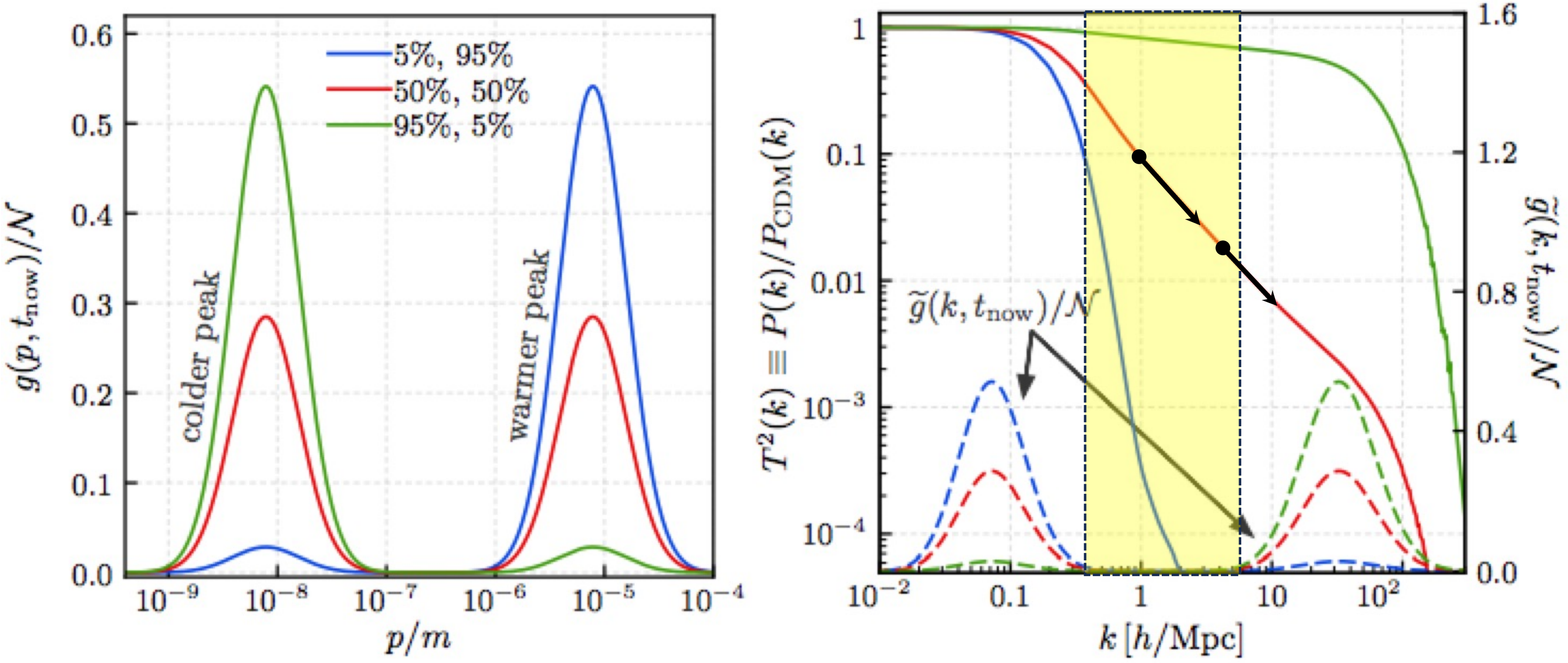

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- *between* peaks: no change in accumulation of abundance → slope approximately constant!

Still find **accumulative abundance → slope!**

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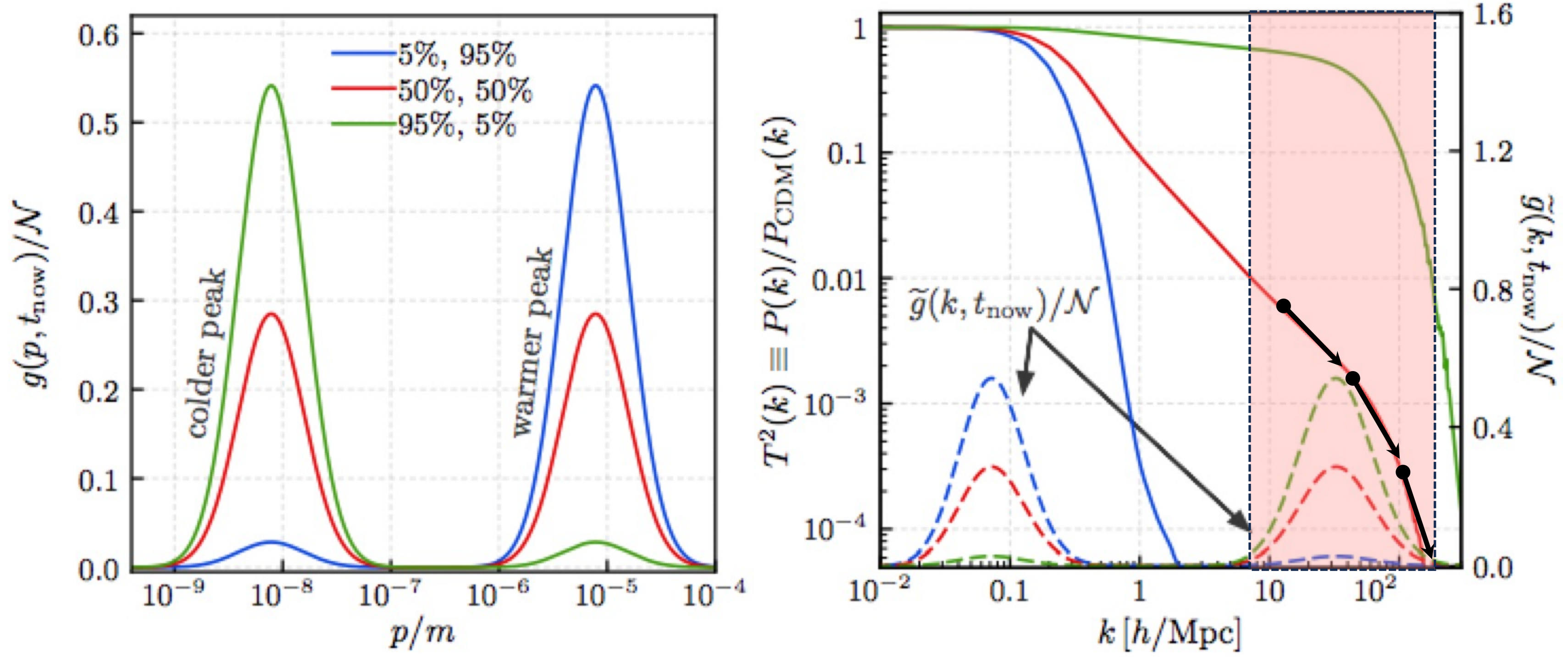

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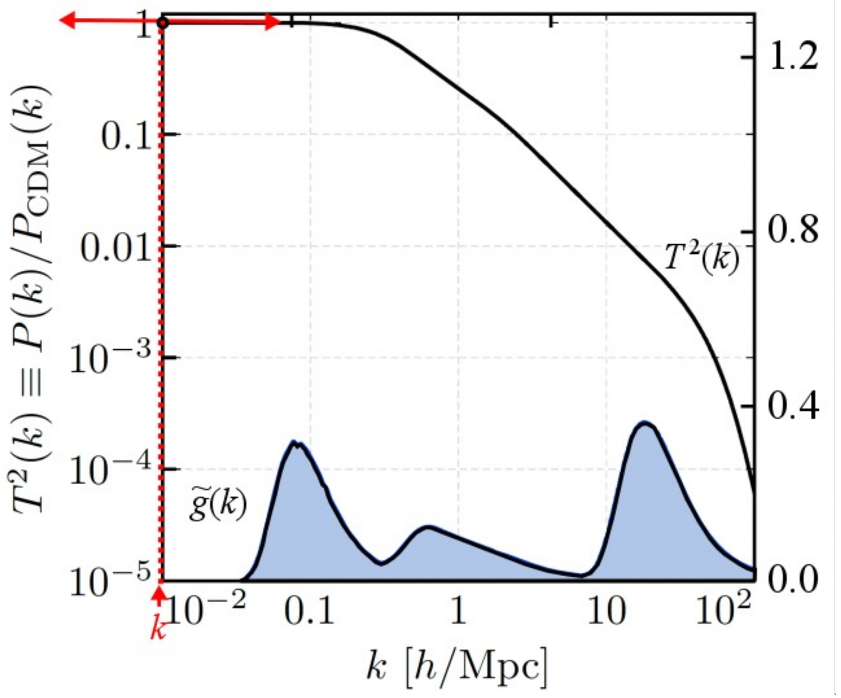
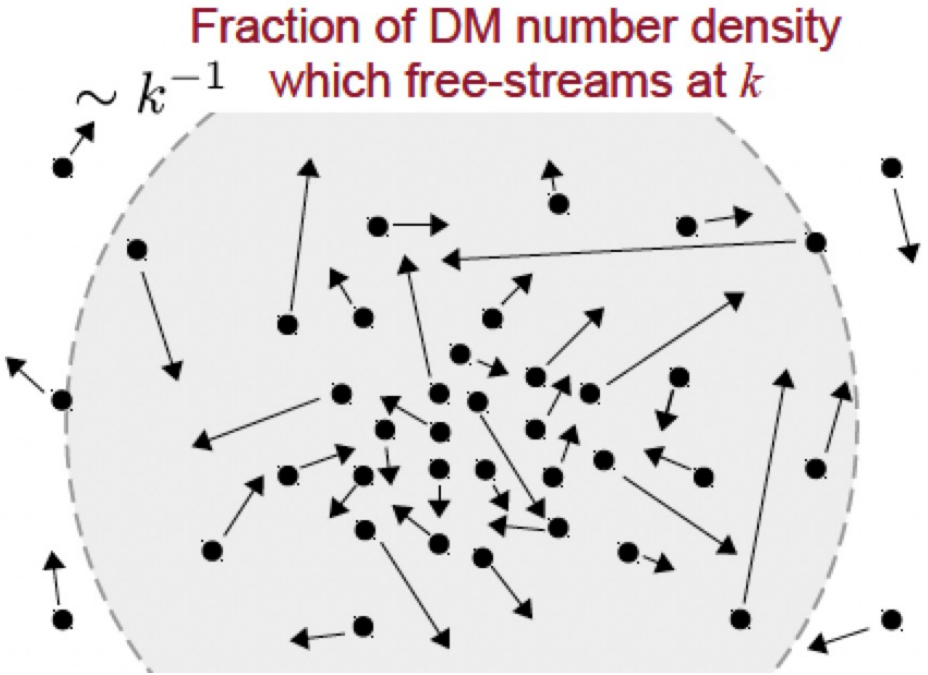
Relating $\tilde{g}(k)$ and $T^2(k)$

Based on our observation, our claim is that the ***slope*** of the transfer function at a particular scale k is related to the amount of DM particles that is able to freestream a distance larger than $\sim 1/k$,

Motivated by this observation, we define the ***hot-fraction function***,
i.e., the fraction of DM particles that is effectively "hot" relative to the scale k

$$F(k) \equiv \frac{\int_{-\infty}^{\log k} \tilde{g}(k')\, d\log k'}{\int_{-\infty}^{\infty} \tilde{g}(k')\, d\log k'}$$





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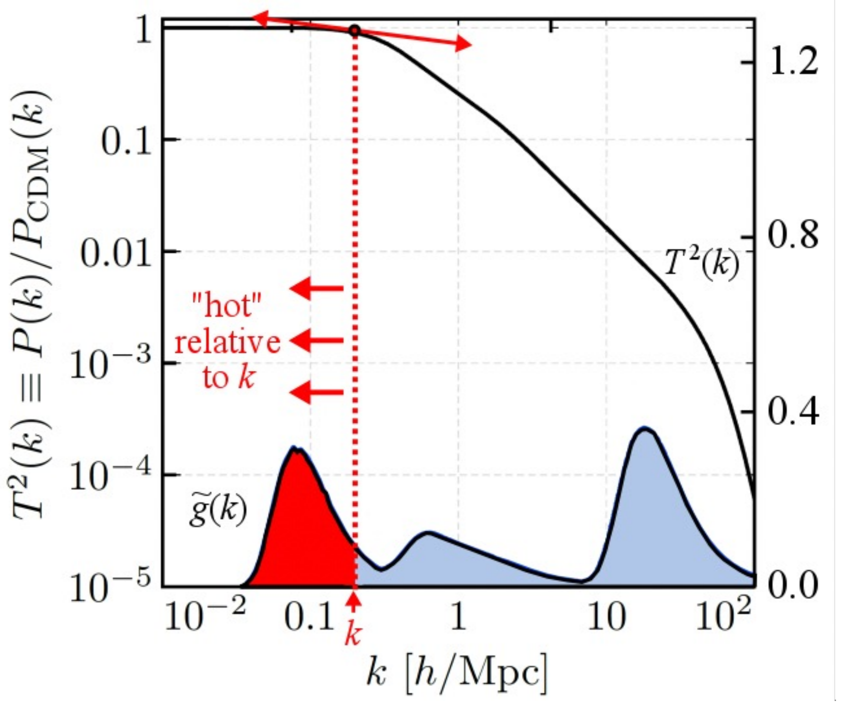
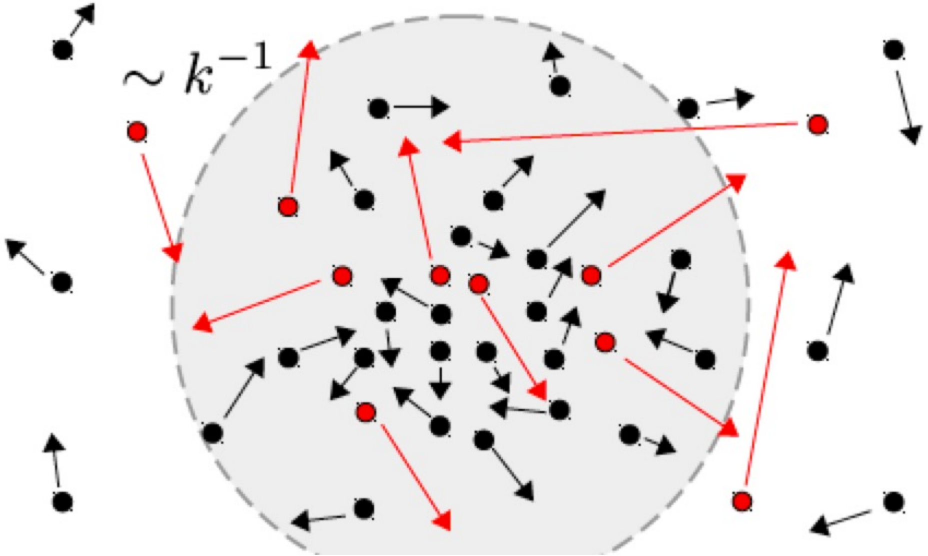


Fraction of DM number density which free-streams at k



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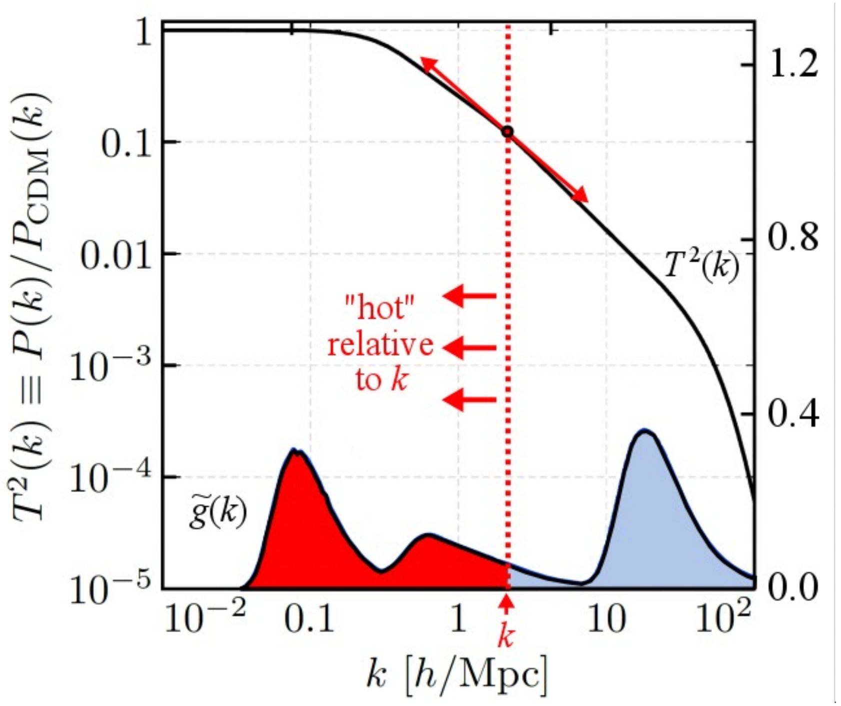

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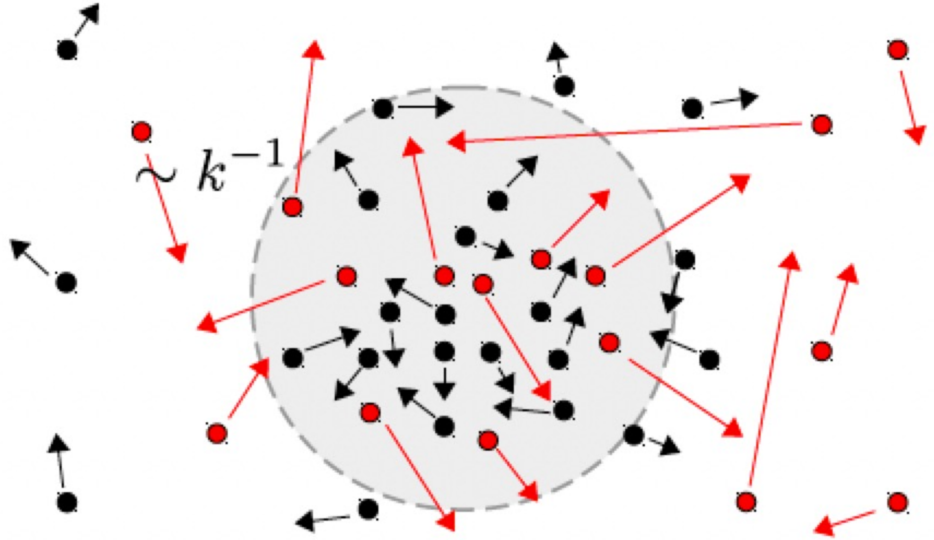

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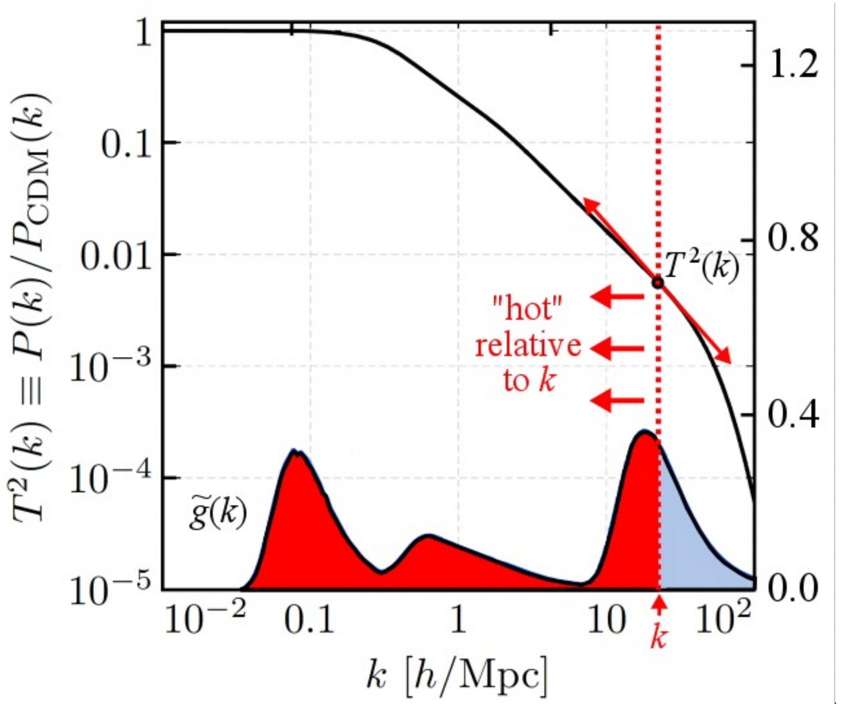

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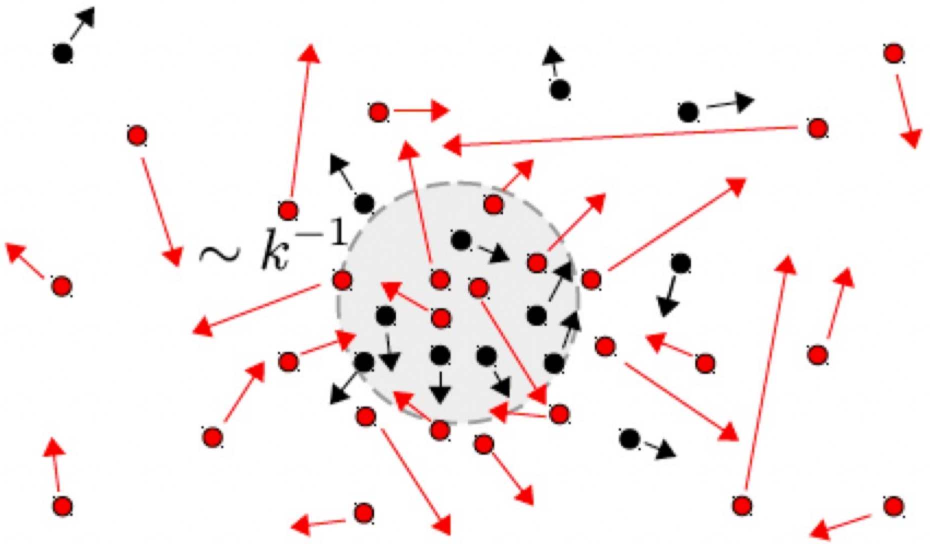

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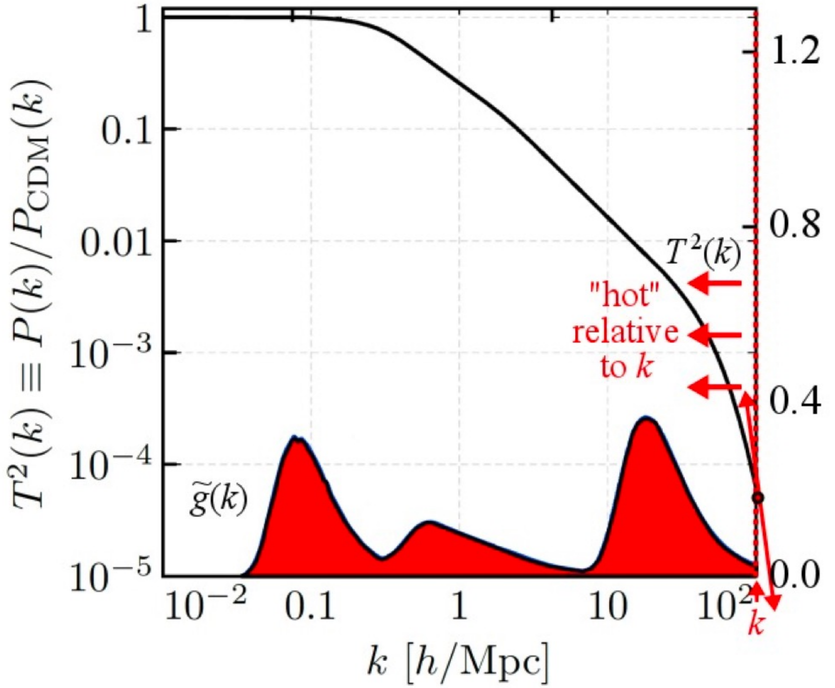

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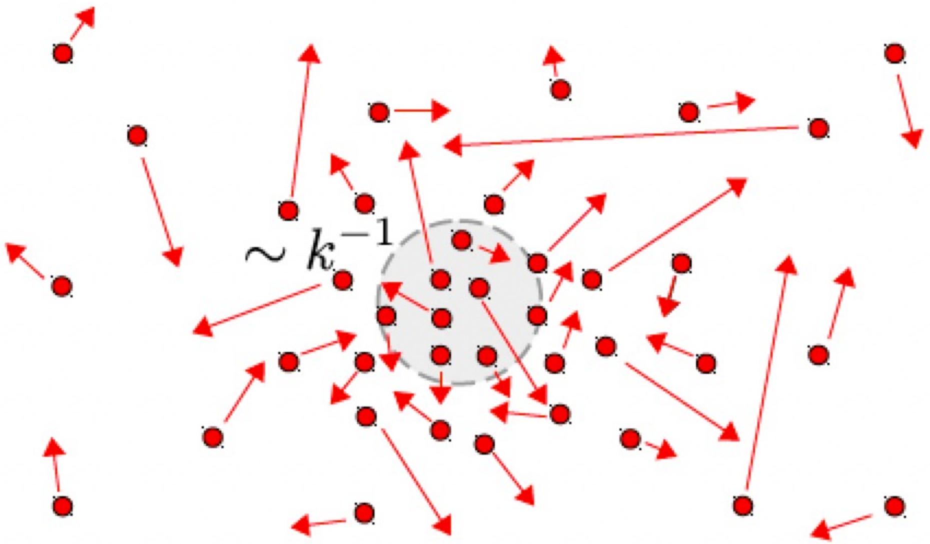


Relating $\tilde{g}(k)$ *and* $T^2(k)$

Our claim can then be formulated as

some as-yet
unknown function

$$F(k) \;\approx\; \eta\left(\left|\frac{d\log T^2}{d\log k}\right|\right)$$

logarithmic slope of
transfer function

Taking derivative of both sides

$$\frac{\tilde{g}(k)}{\mathcal{N}} \;\approx\; \eta'\left(\left|\frac{d\log T^2}{d\log k}\right|\right)\left|\frac{d^2\log T^2}{(d\log k)^2}\right|$$

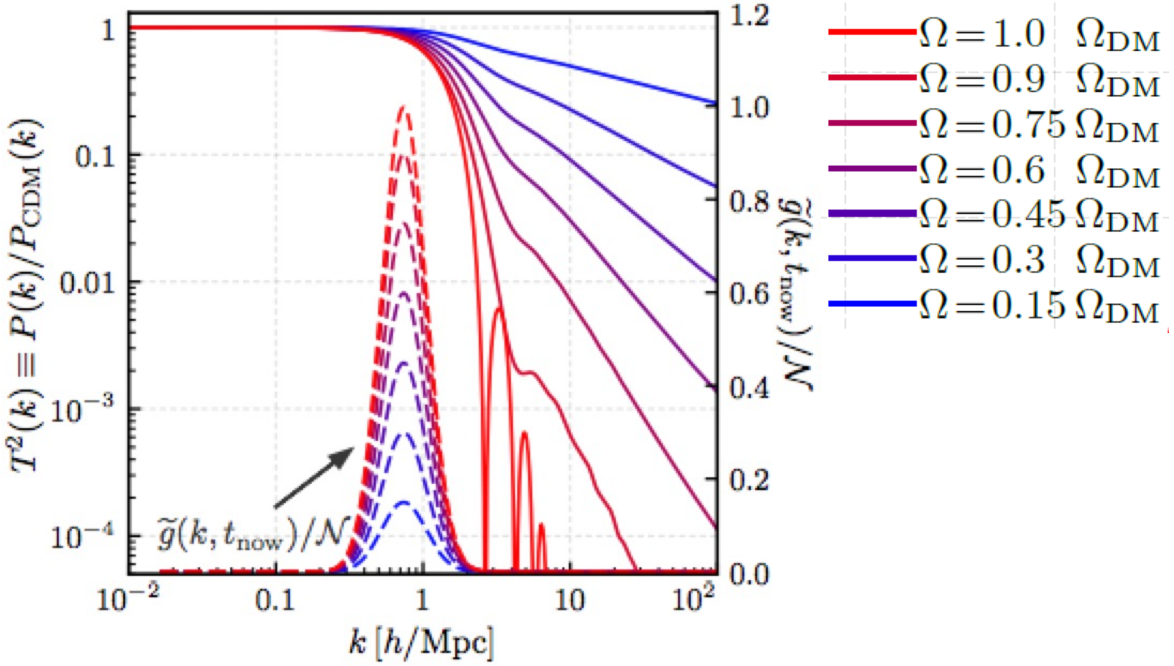
Normalized DM PSD

first derivative
of T^2

second derivative
of T^2

Once η is known, we will be able to resurrect the phase-space distribution!

Relating $\tilde{g}(k)$ *and* $T^2(k)$



$$\left|\frac{d\log T^2}{d\log k}\right| \approx [F(k)]^2 + \frac{3}{2}F(k)$$

relation holds to very high precision!

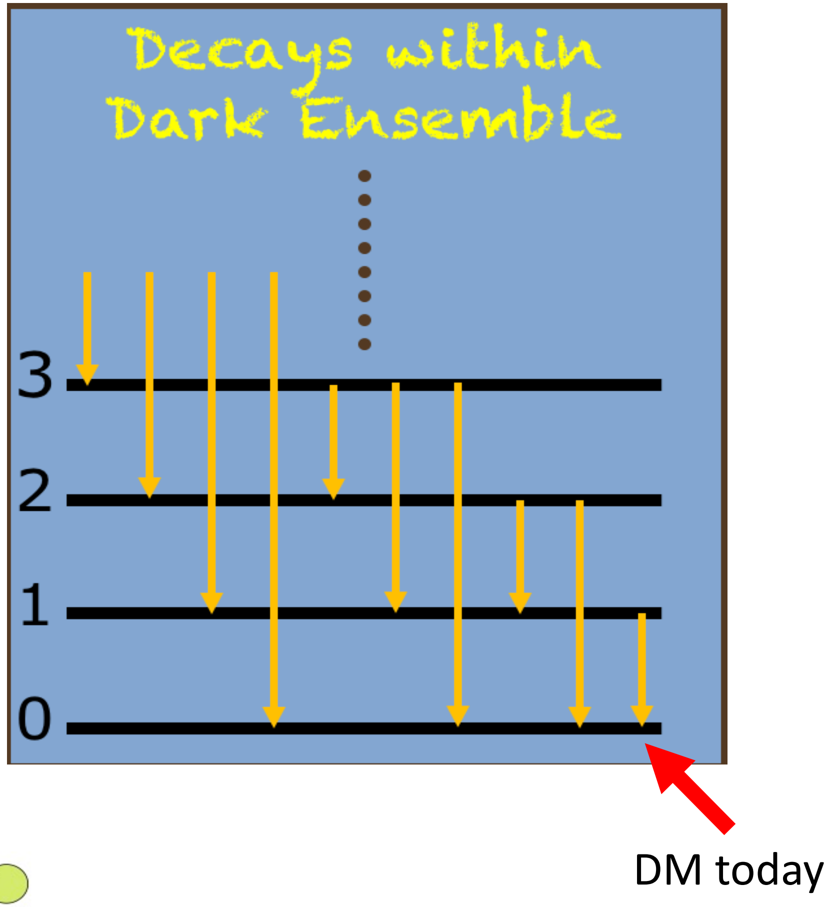
Our final conjecture:

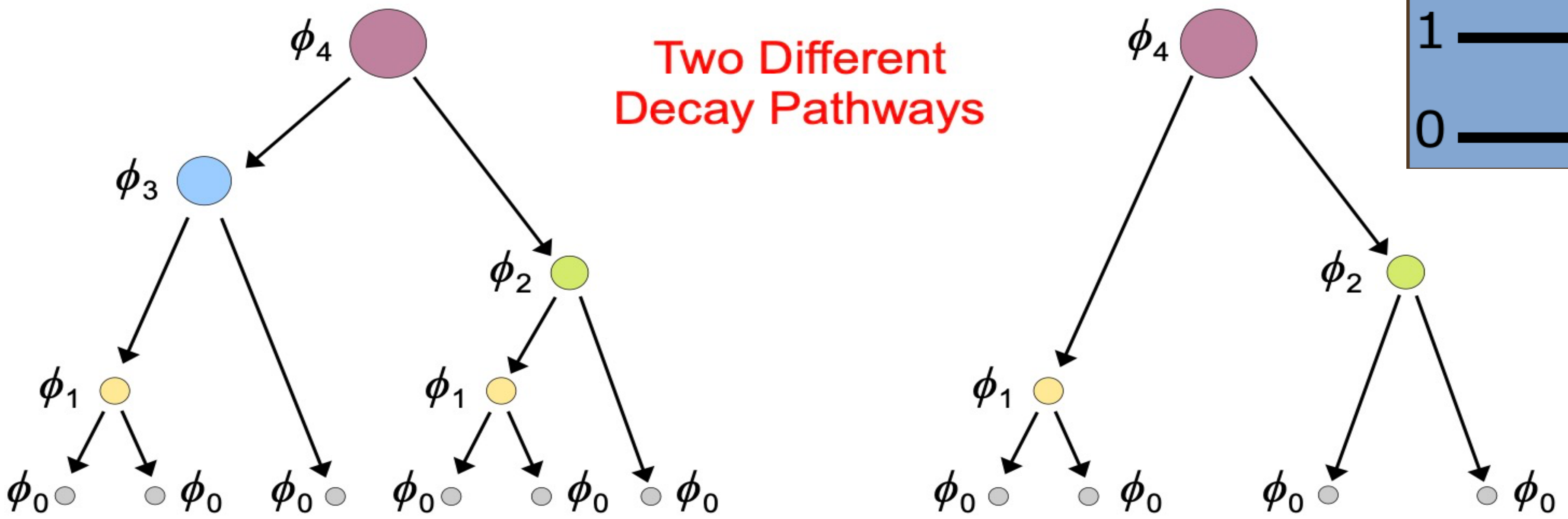
$$\frac{\tilde{g}(k)}{\mathcal{N}} \approx \frac{1}{2}\left(\frac{9}{16} + \left|\frac{d\log T^2}{d\log k}\right|\right)^{-1/2} \left|\frac{d^2\log T^2}{(d\log k)^2}\right|$$

This allows us to "resurrect" $\tilde{g}(k)$ ***from the transfer function*** $T^2(k)$***!***

Let's now see how these ideas play out in practice!

As an example, we study a model in which the dark sector contains *many* components with many different masses and thus many possible decay chains.





Linear regime: $g(p) \to P(k)$

Squared transfer function obtained by feeding $g(p)$ to the **CLASS** code

$\tilde{g}(k)$ obtained by mapping p to k_{hor}

Rainbow colors correspond to ***hot-fraction function*** $F(k)$:

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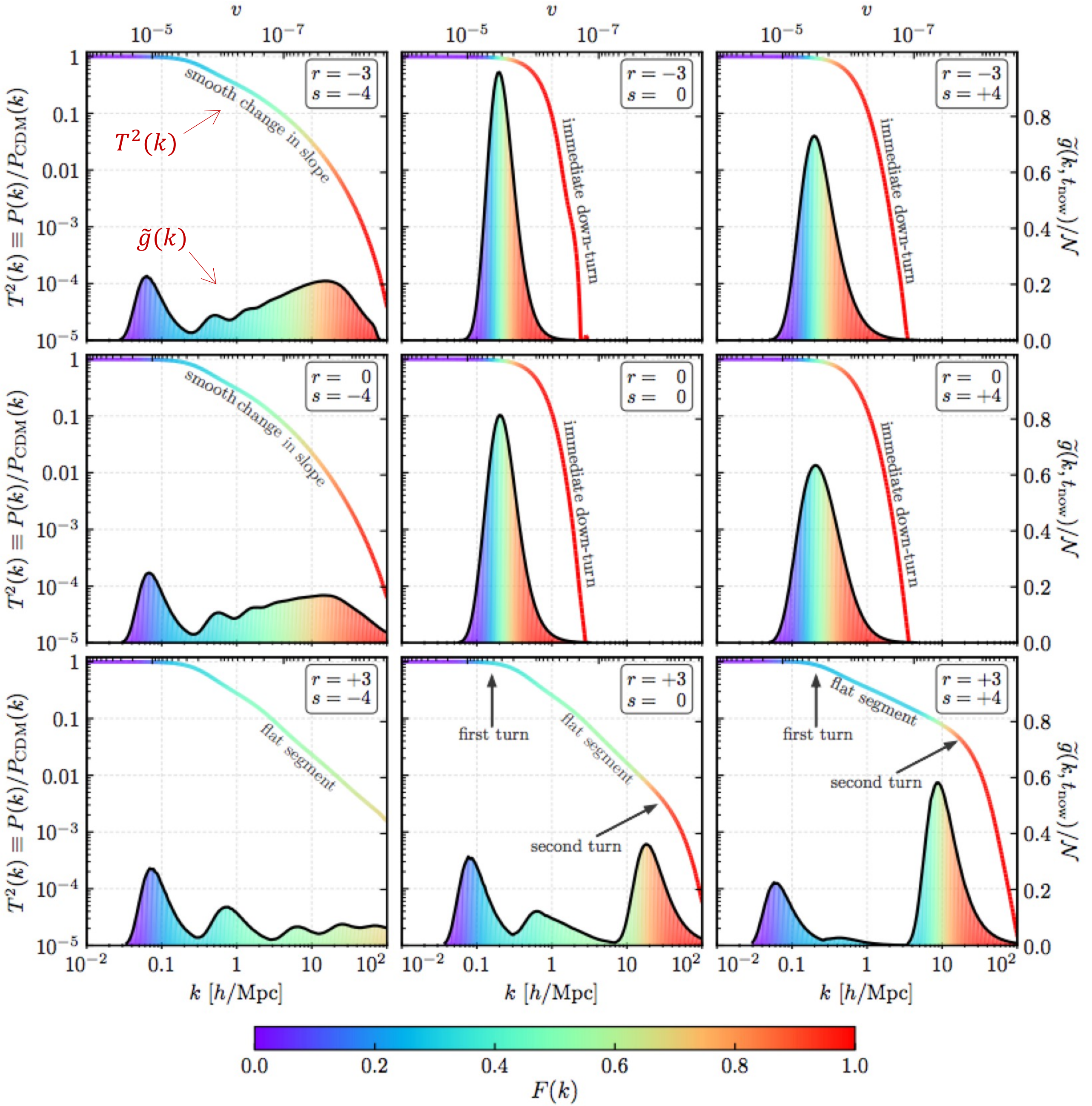

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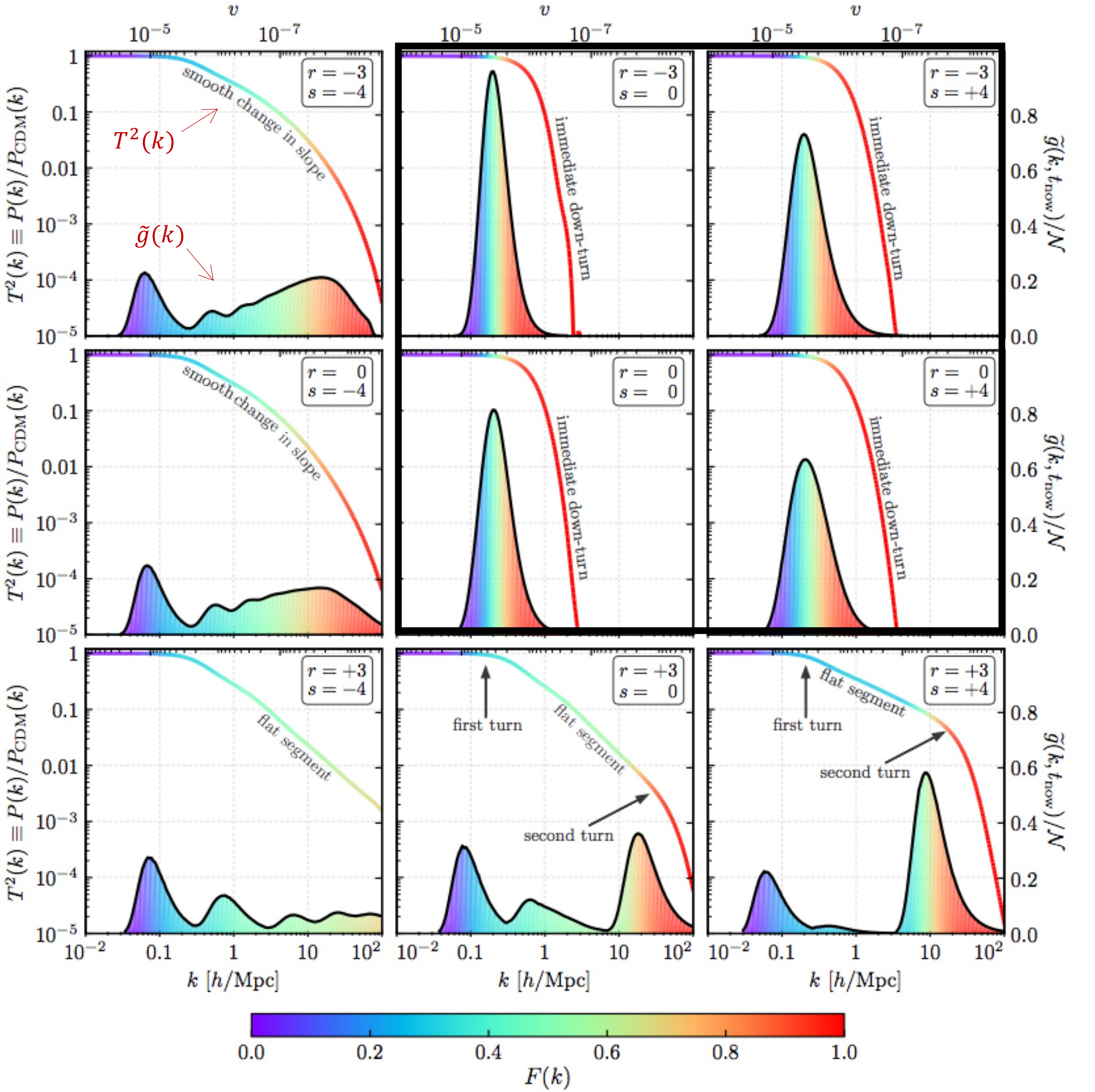

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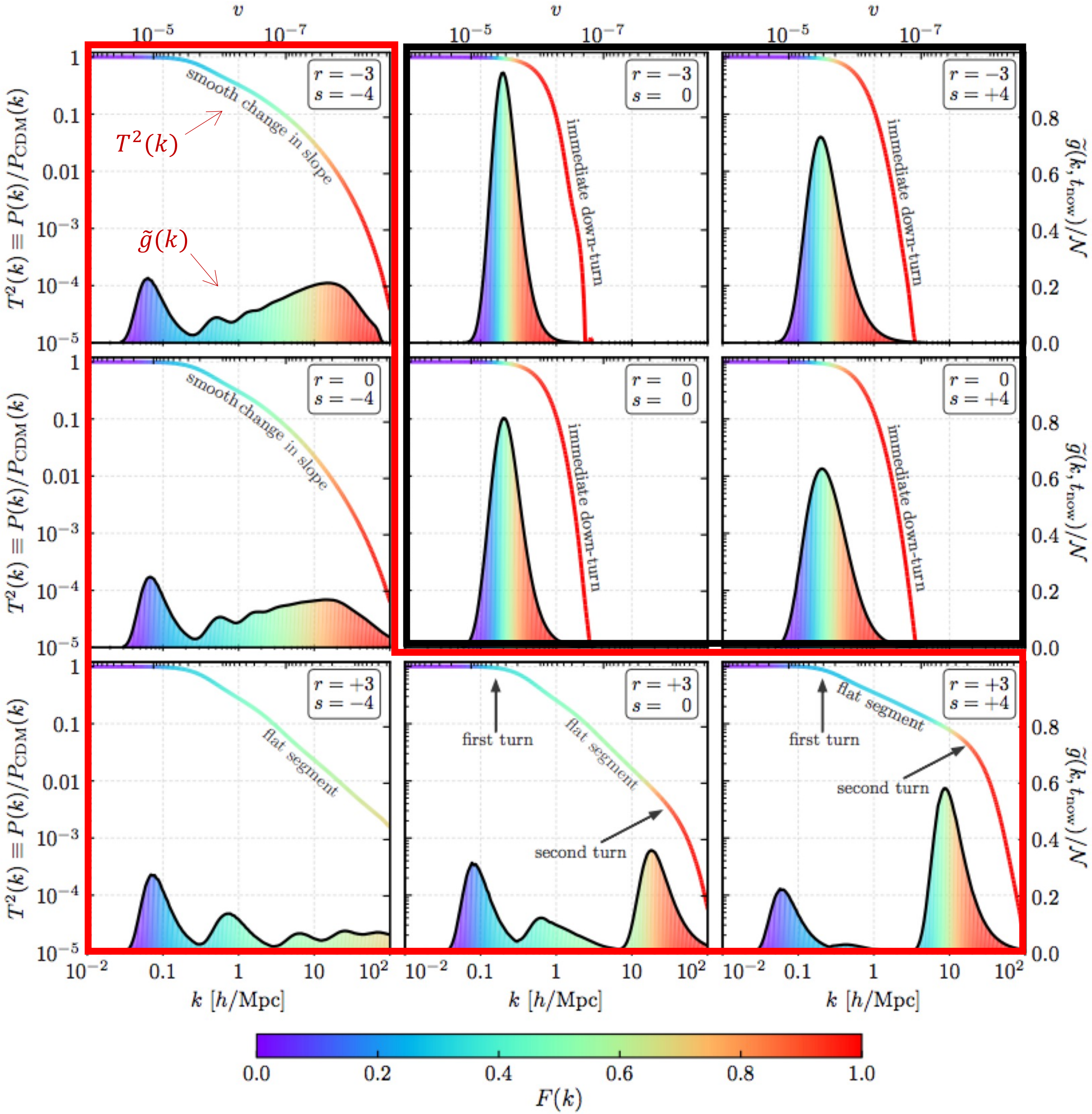

Linear regime: Reconstruction Conjecture

To what extent can we "***resurrect***" the DM phase-space distribution from the transfer function?

Recall our conjecture...

$$\frac{\tilde{g}(k)}{\mathcal{N}} \approx \frac{1}{2}\left(\frac{9}{16} + \left|\frac{d\log T^2}{d\log k}\right|\right)^{-1/2} \left|\frac{d^2\log T^2}{(d\log k)^2}\right|$$

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K. Dienes, **FH**, J. Kost, S. Su, B. Thomas, arXiv:2001.02193

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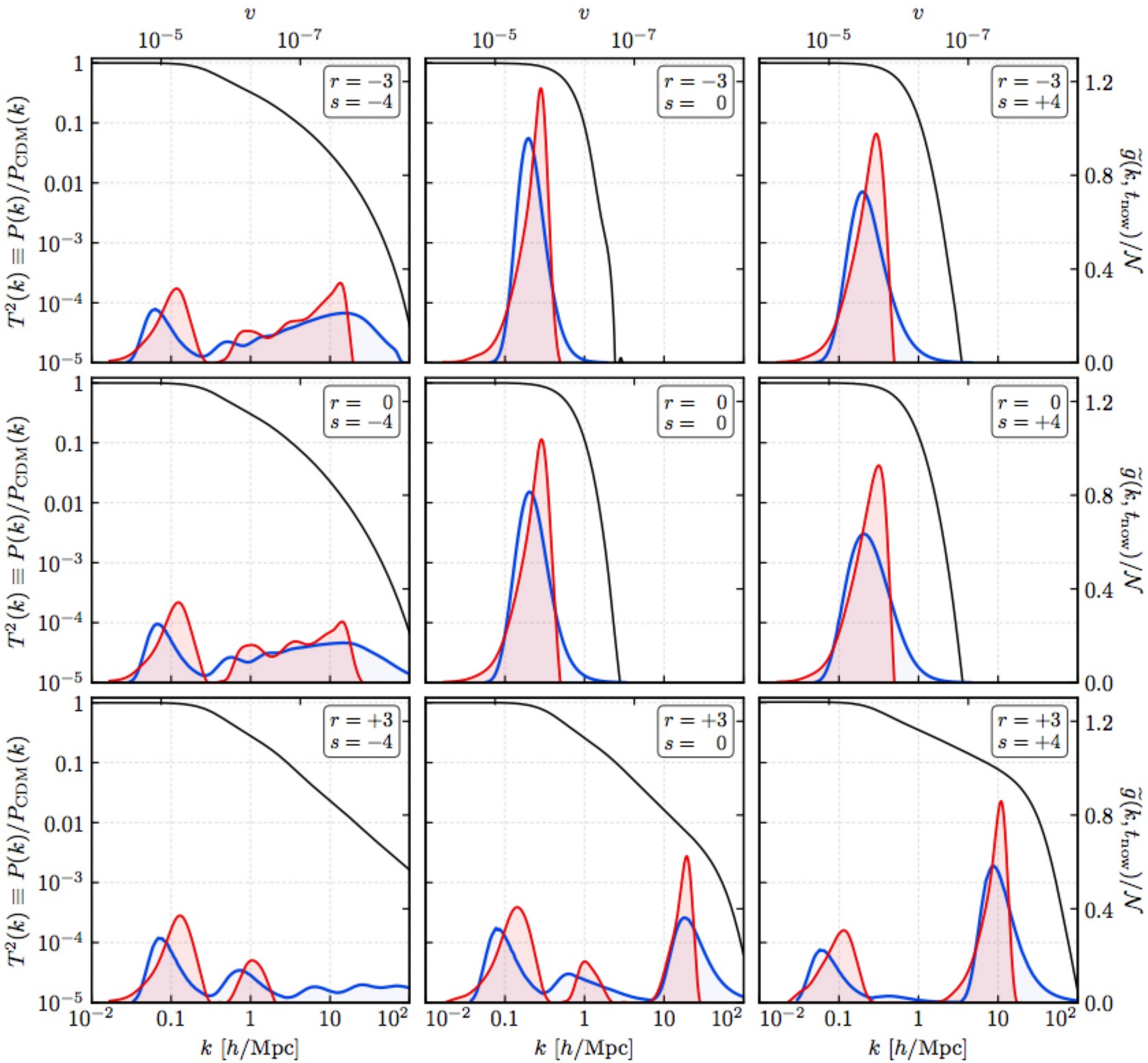
Blue: original DM distribution in k-space
Red: reconstruction directly from $T^2(k)$

Archaeological reconstruction is surprisingly accurate for a **variety** of possible DM distributions.
Able to resurrect the **salient features** of the original distribution!



Machine learning approach

+

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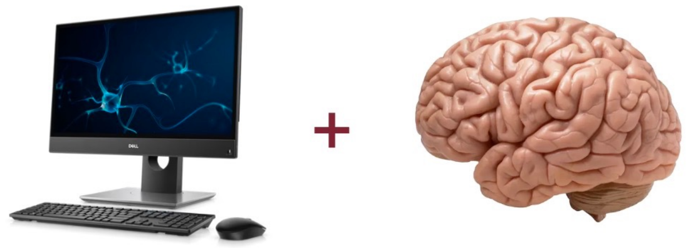
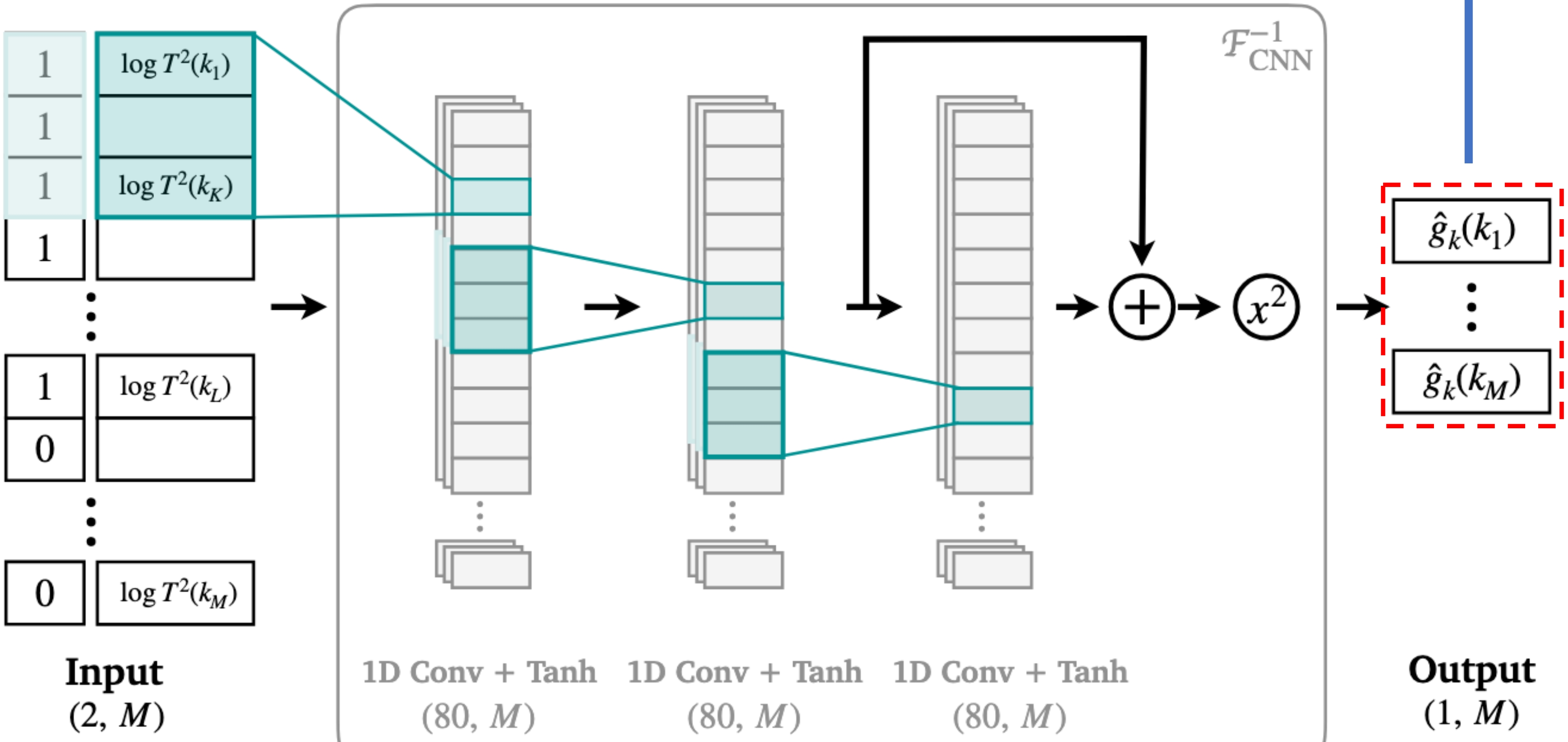


$$\mathcal{L}_{\mathrm{MSE}} = \frac{\sum_{i=1}^{M} m_i \left|\hat{g}_k(k_i) - g_k(k_i)\right|^2}{\sum_{i=1}^{M} m_i}$$



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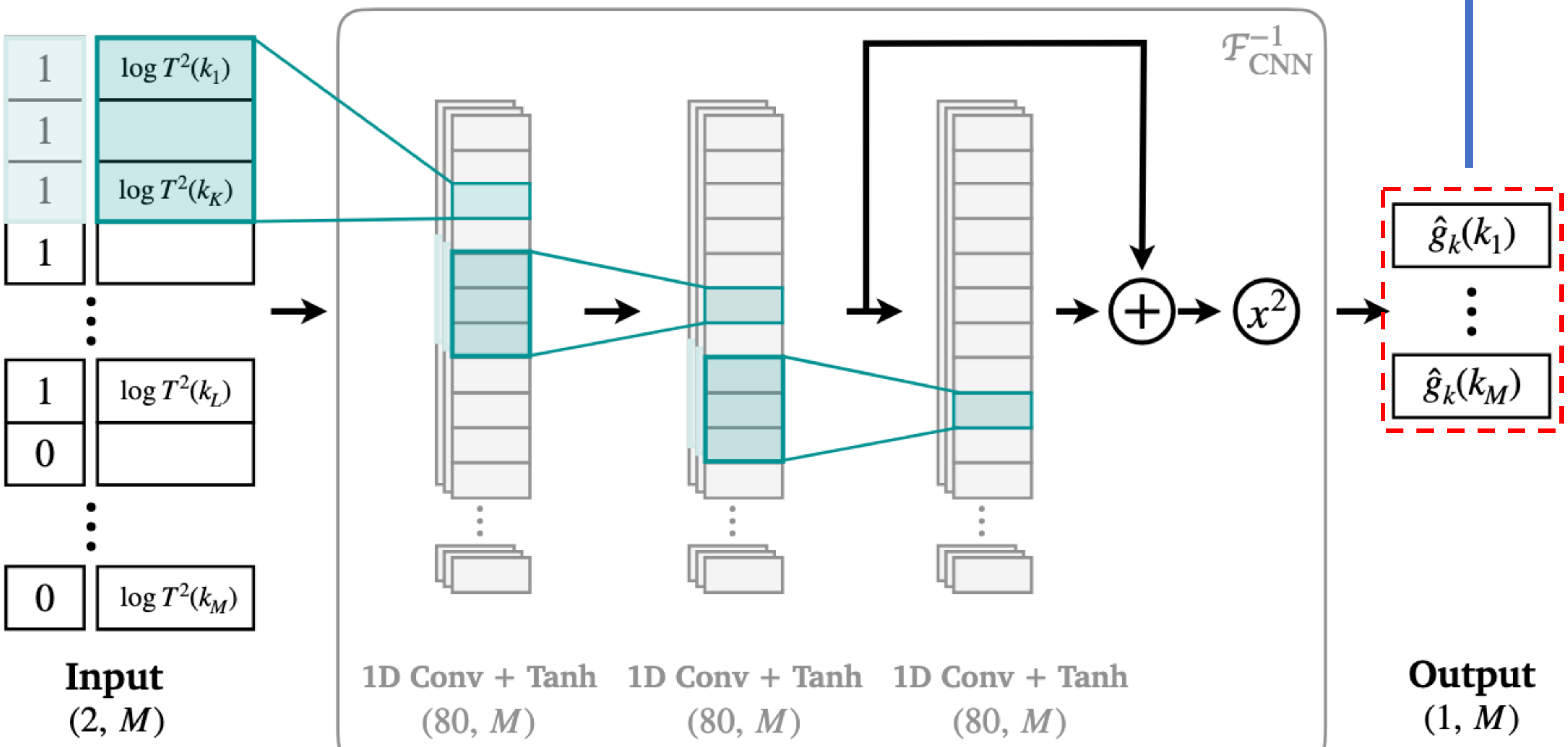

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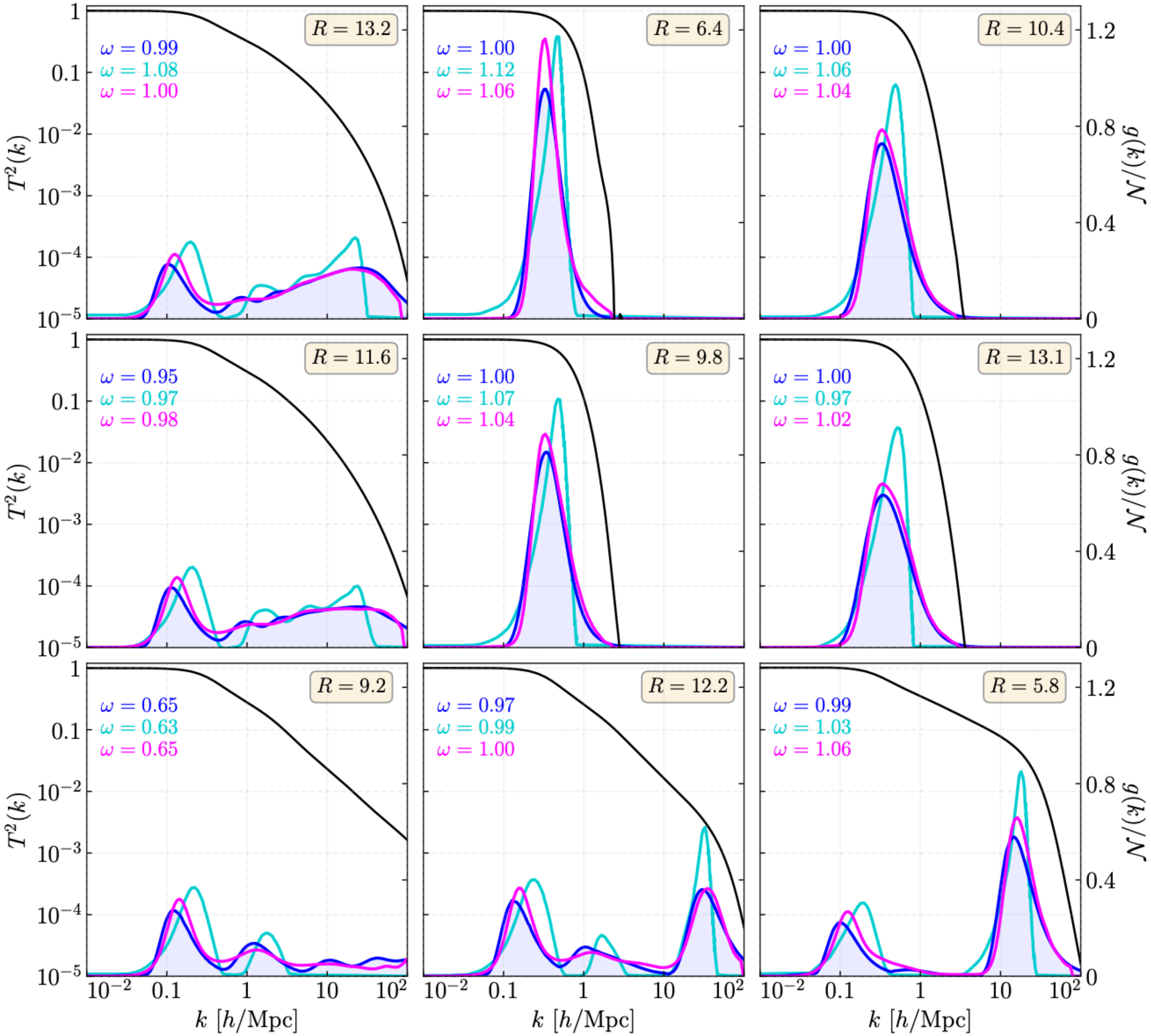

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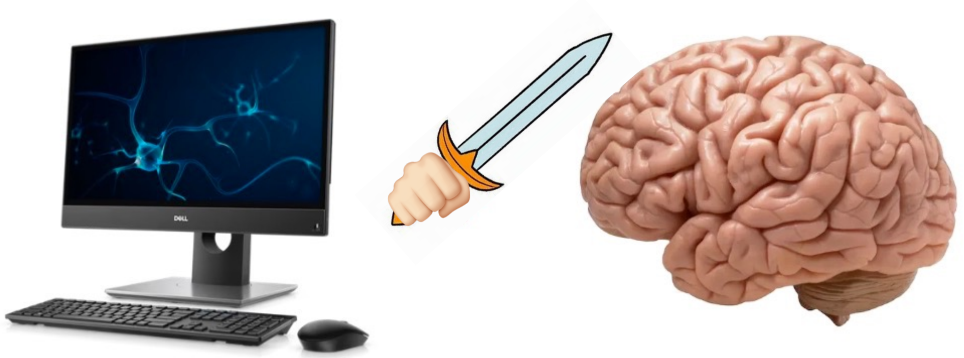
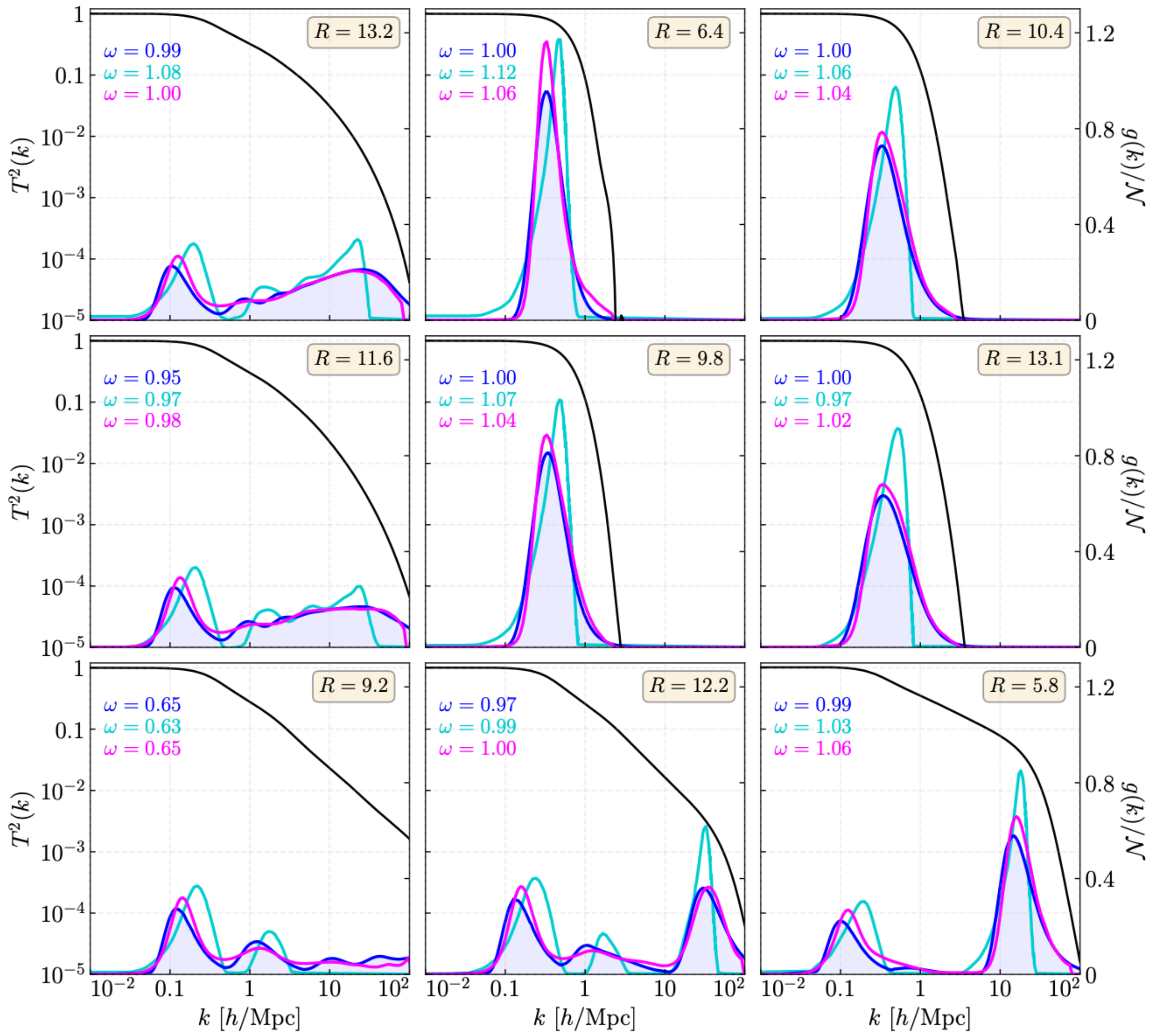


ML approach reconstructs the DM PSD with much higher accuracy!

Machine learning does it and does it better!



Conclusion

early-universe dynamics

dark-matter phase-space distribution *f(p)*

linear matter power spectrum *P(k)*

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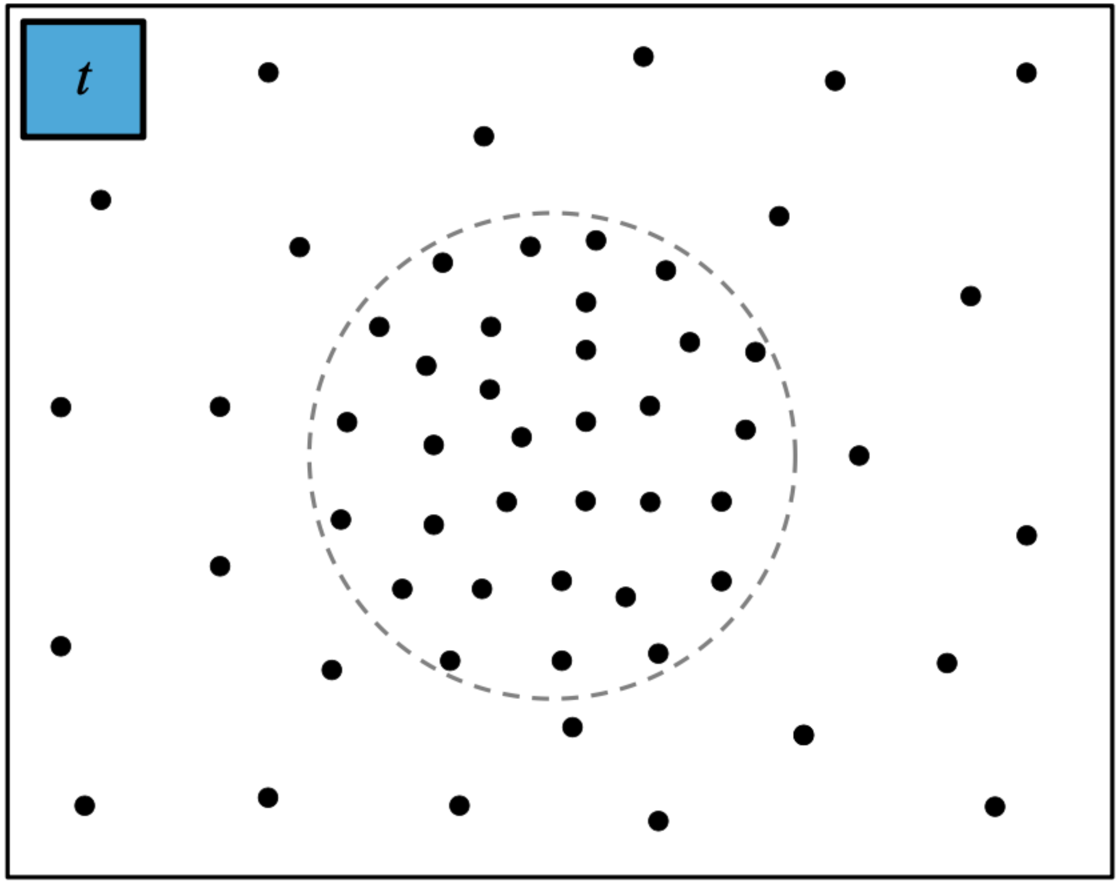
Machine Learning

reconstructed *f(p)*

Thermal history

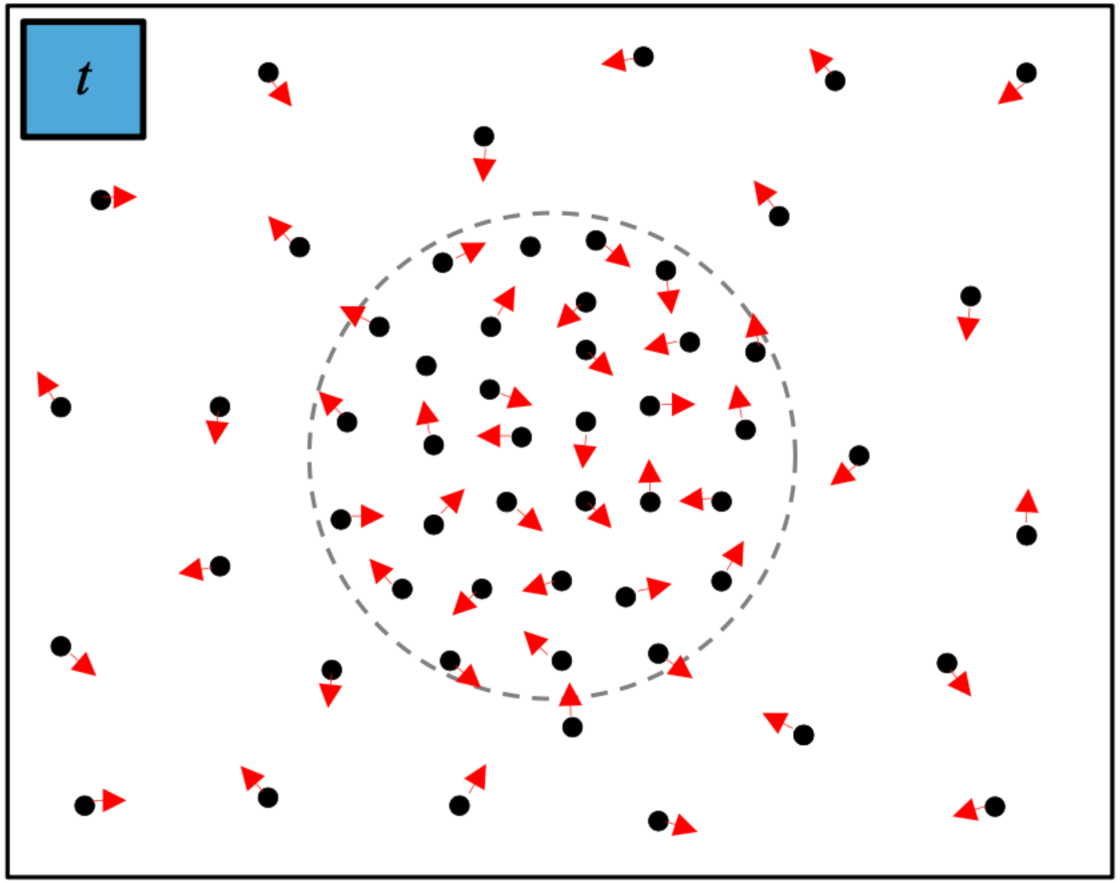
- Let's look at a region with initial overdensity



backup

Thermal history

- Let’s look at a region with initial overdensity
- Suppose DM particles have ***relatively small*** velocities



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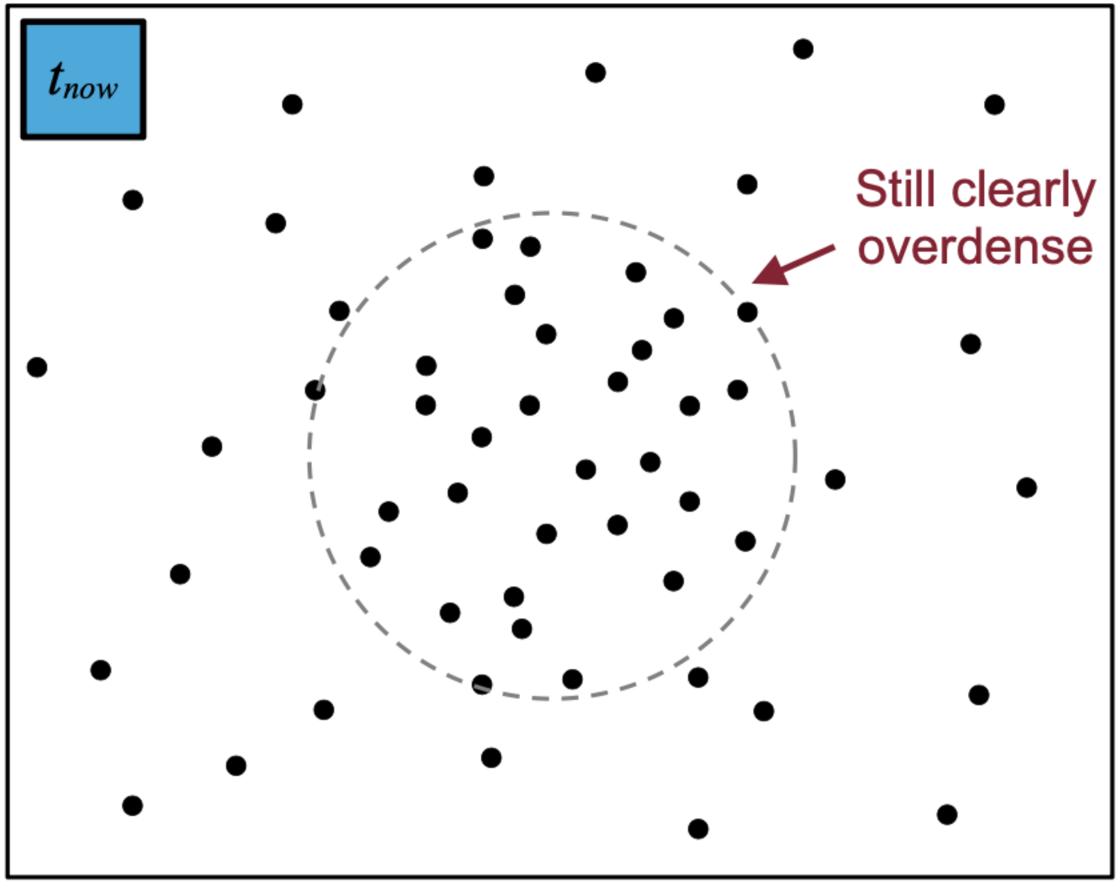
Thermal history

- Let's look at a region with initial overdensity
- Suppose DM particles have ***relatively small*** velocities
- After a while, initial velocities are redshifted away, the overdense region ***remains overdense***



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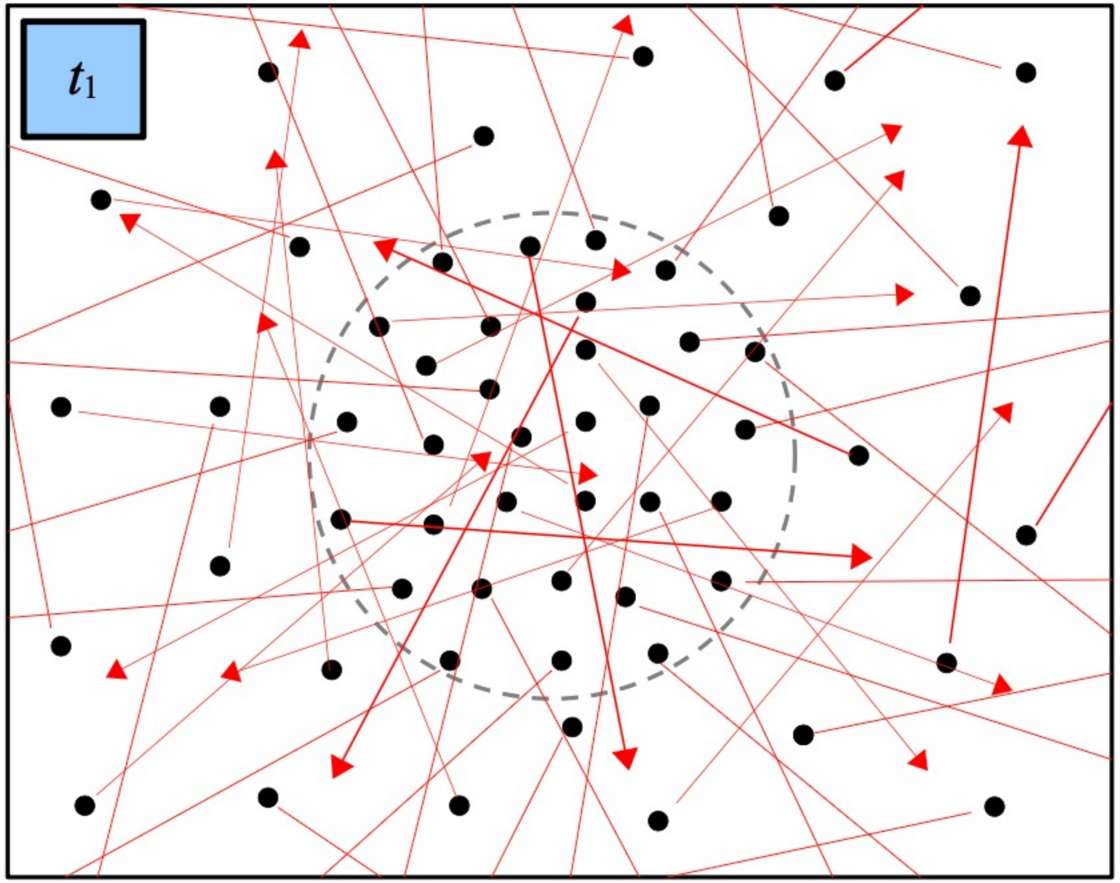

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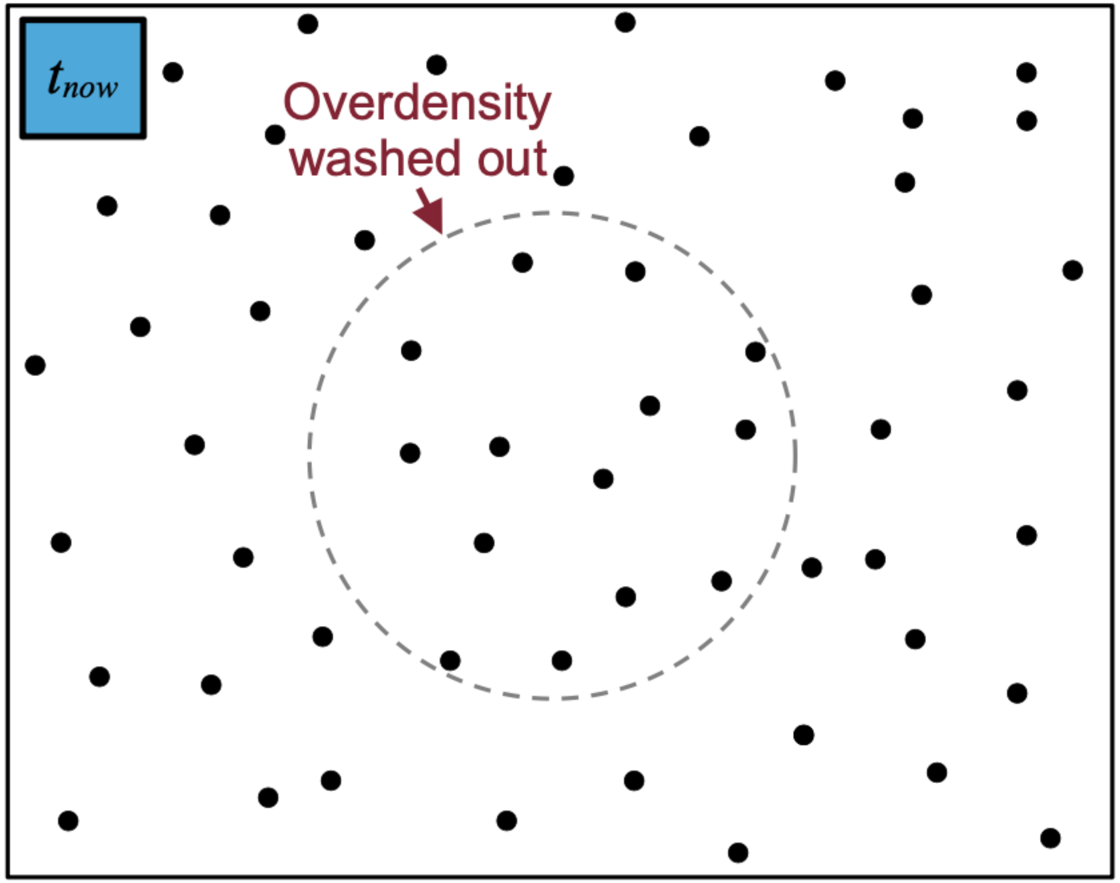

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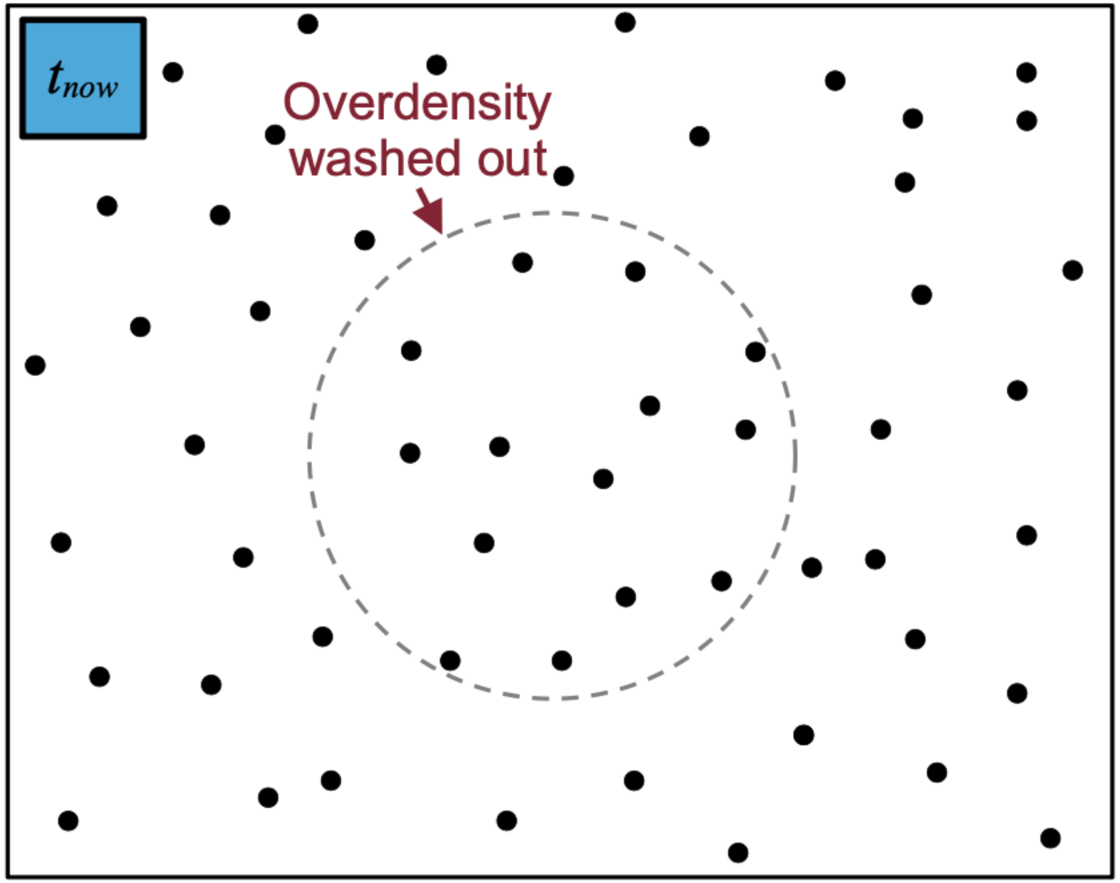
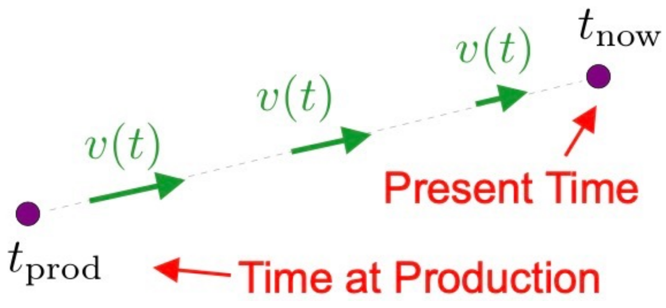
- After a while, initial velocities are still redshifted away, but the overdense region has been ***washed out***
- The extent to which these particles can wash out overdensities depend on their ***particle horizon:***

$$d_{hor} \equiv \int_{t_{prod}}^{t_{now}} \frac{dt}{a(t)}\, v(t)$$

- Particles with particle horizon d_{hor} can wash out overdense regions whose size is smaller than d_{hor}.





backup

Cosmological redshift

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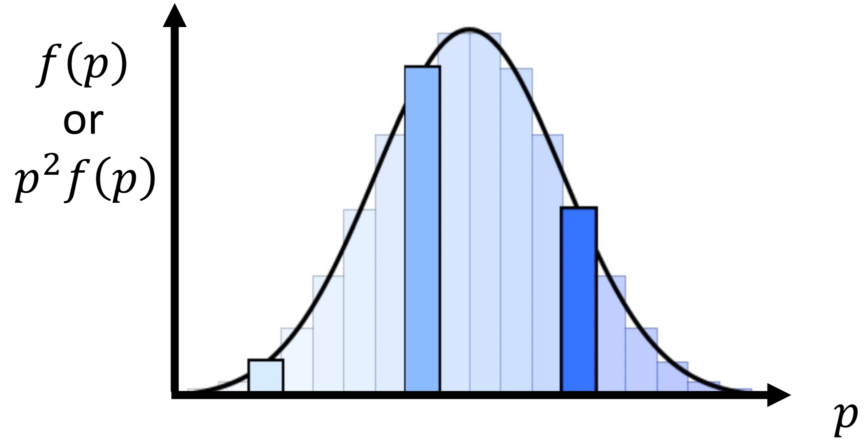

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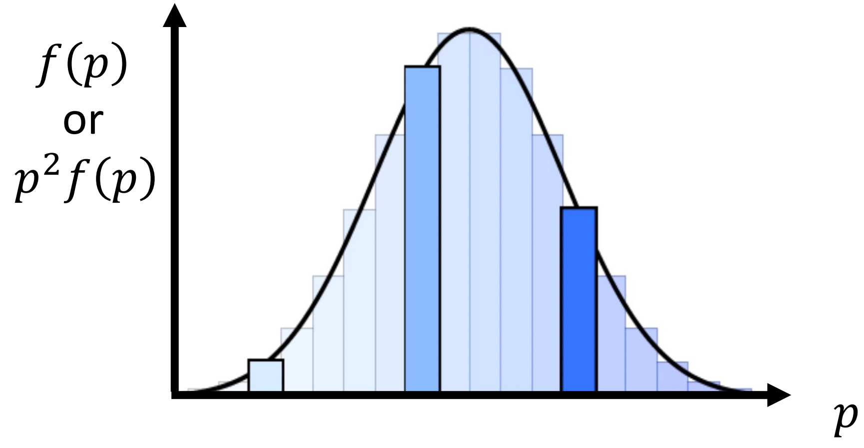
$$p \sim a^{-1} \Rightarrow \Delta p \sim a^{-1}$$



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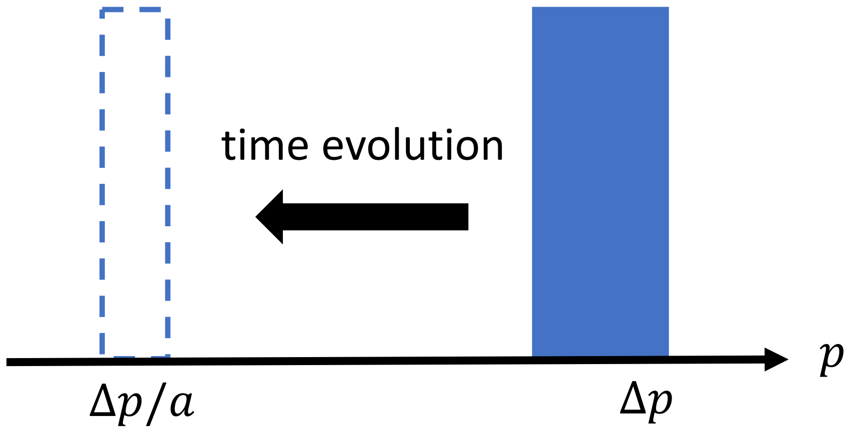

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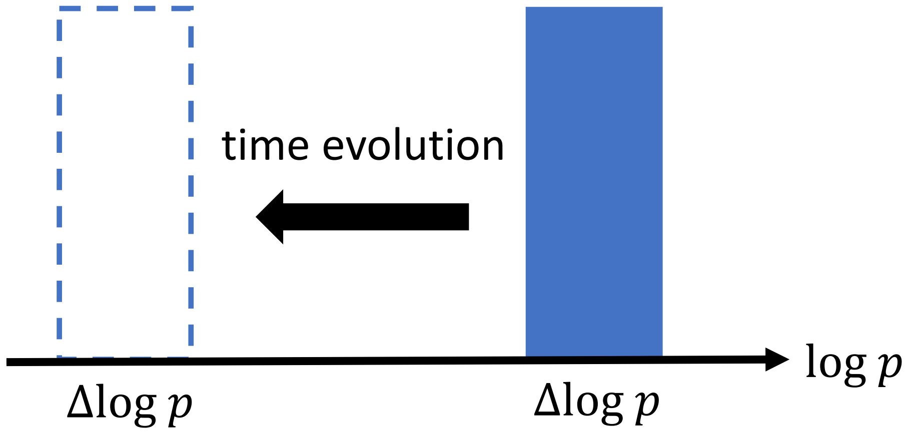
$$\Delta \log p = \log\left(\frac{p + \Delta p}{p}\right) = const.$$



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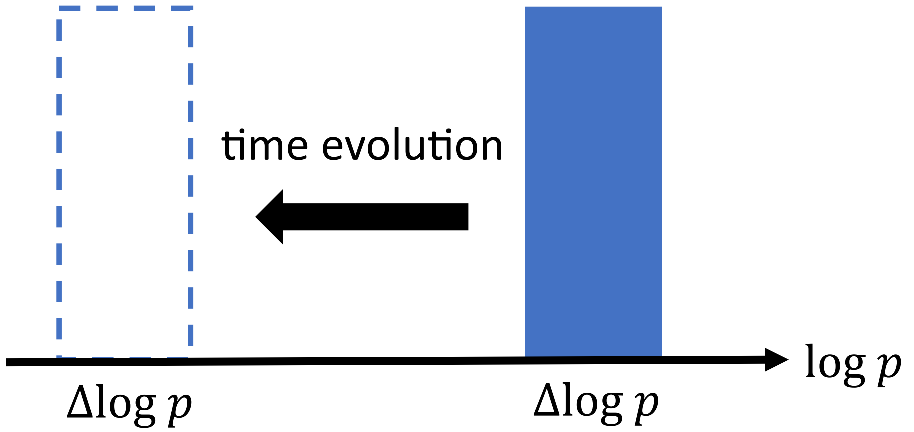

This motivates us to look at the distribution w.r.t. $\log p$

physical number density $n(t) \sim \int d^3p\, f(p,t) \sim \int dp\, p^2 f(p,t) \sim \int d\log p\;\; p^3 f(p,t)$

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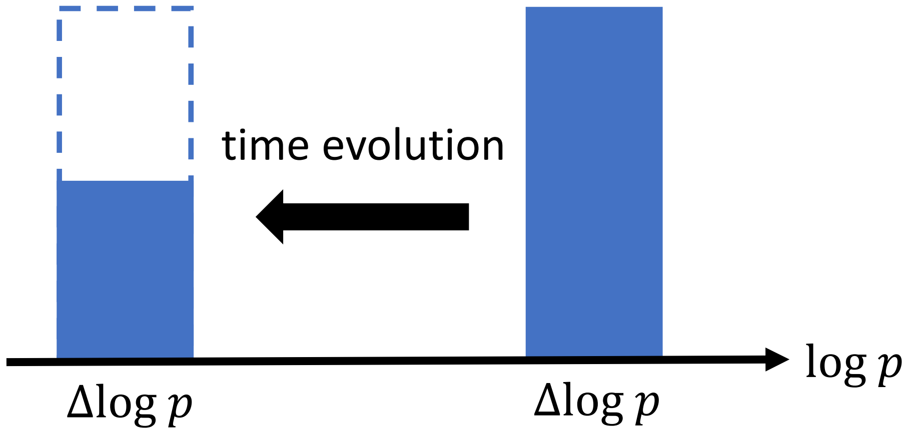

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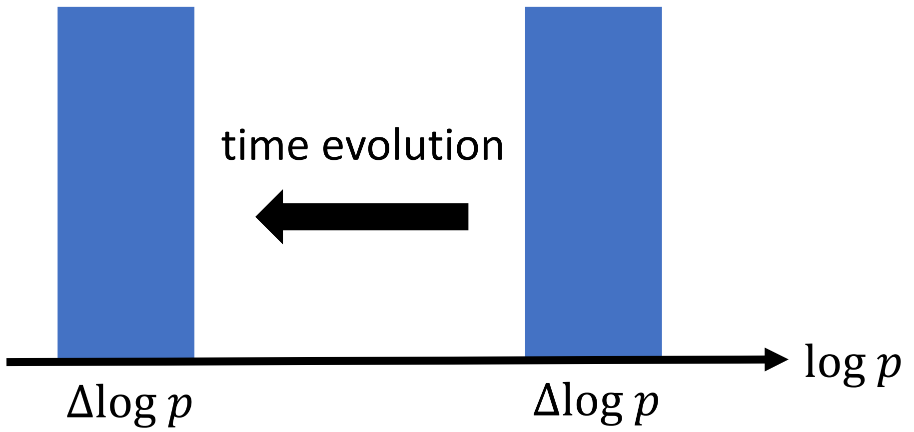

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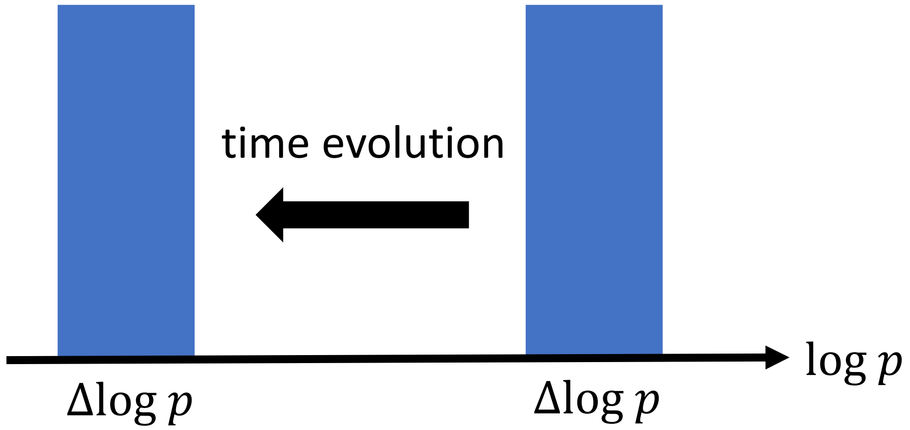

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We therefore define: $\boxed{g(p,t) \equiv a^3(t) p^3 f(p,t)}$

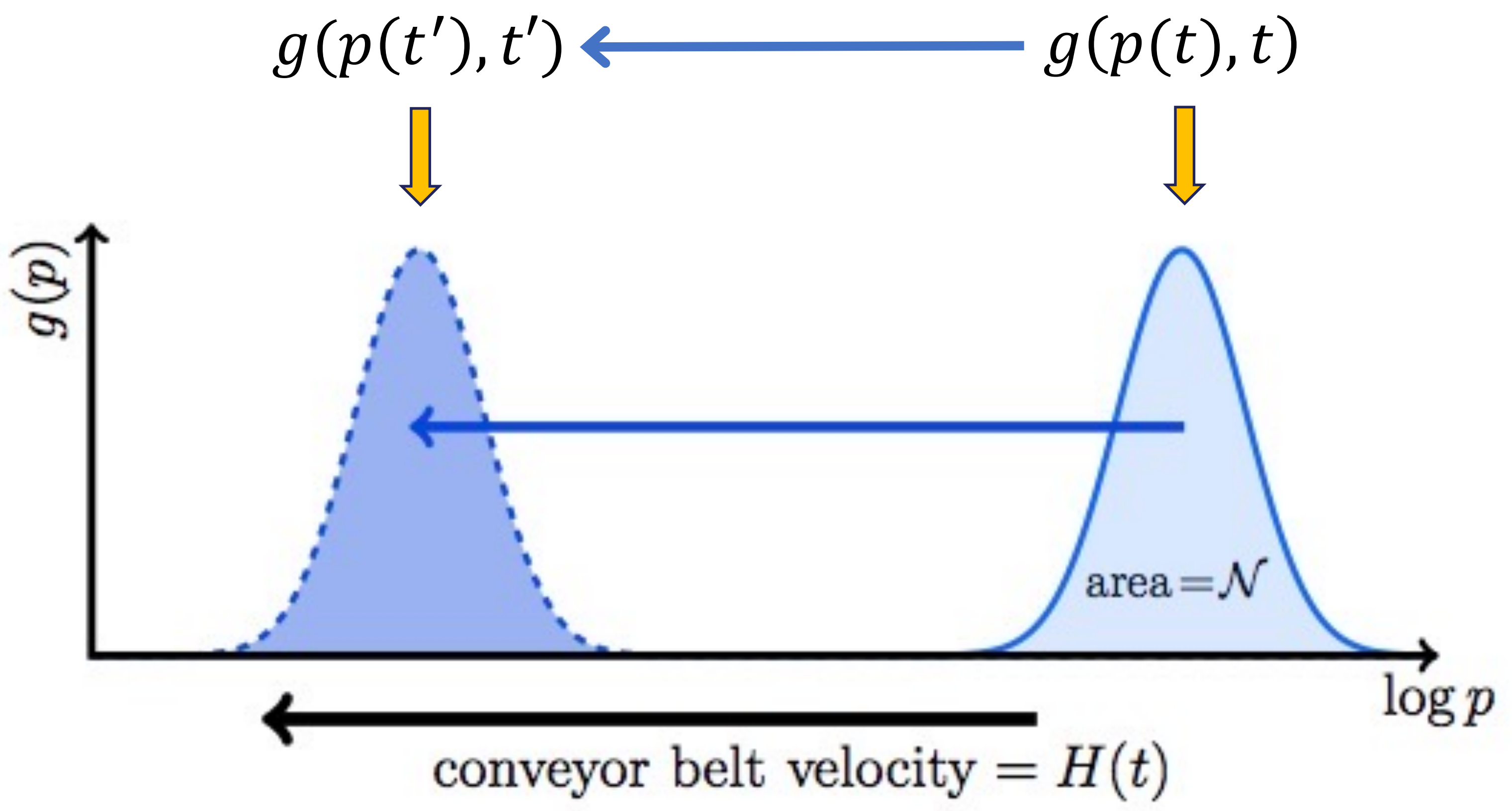
backup

Cosmological conveyor belt

If we plot $g(p,t)$ against $\log p$:

- once DM production complete, ***area (comoving number density) is fixed*** under time evolution

- redshift: $g(p,t)$ ***slides rigidly*** to smaller $\log p$, as if carried along a ***"cosmological conveyor belt"***, its ***shape stays invariant***



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Similar to dropping groceries on a conveyor belt,
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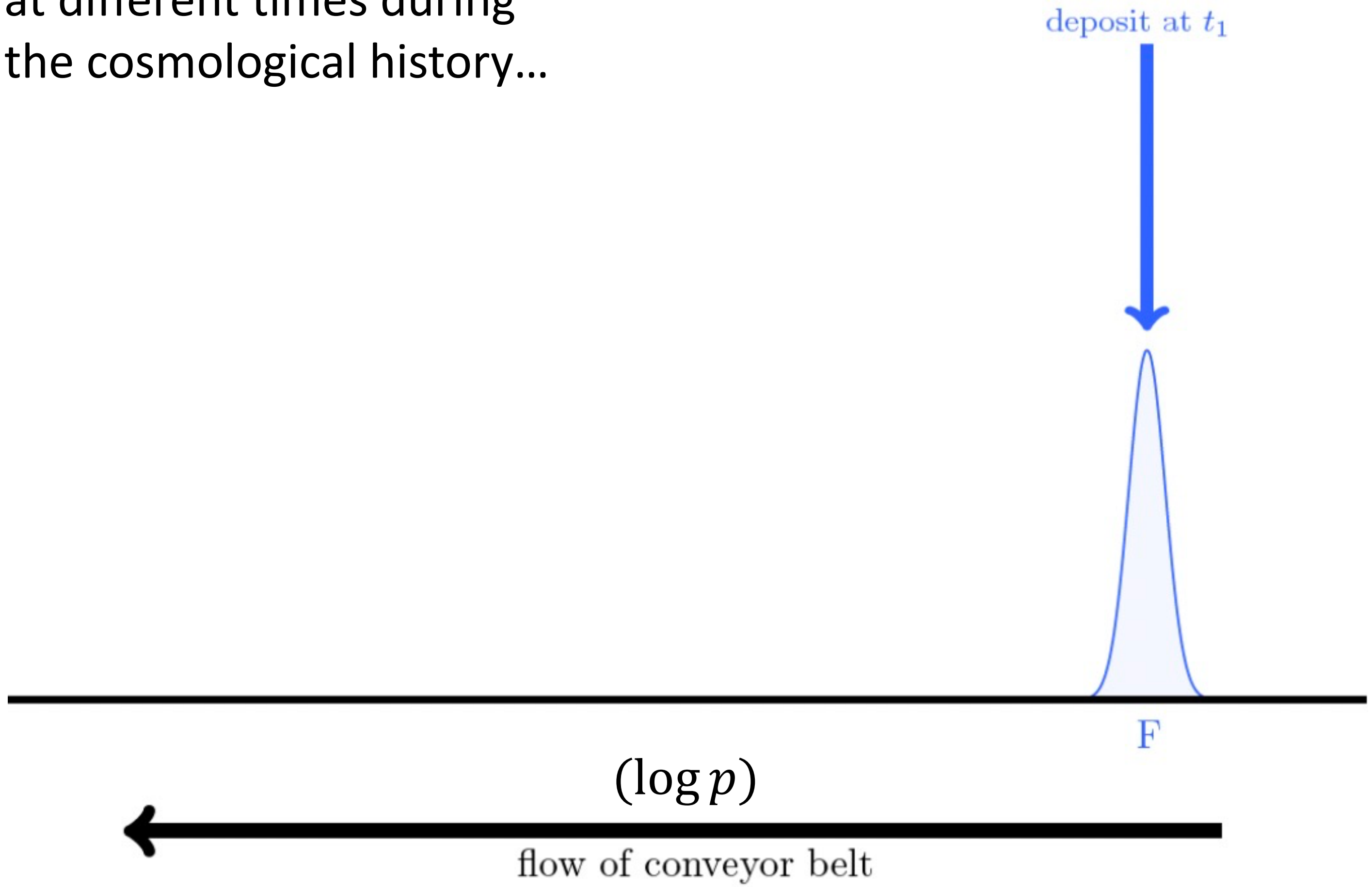

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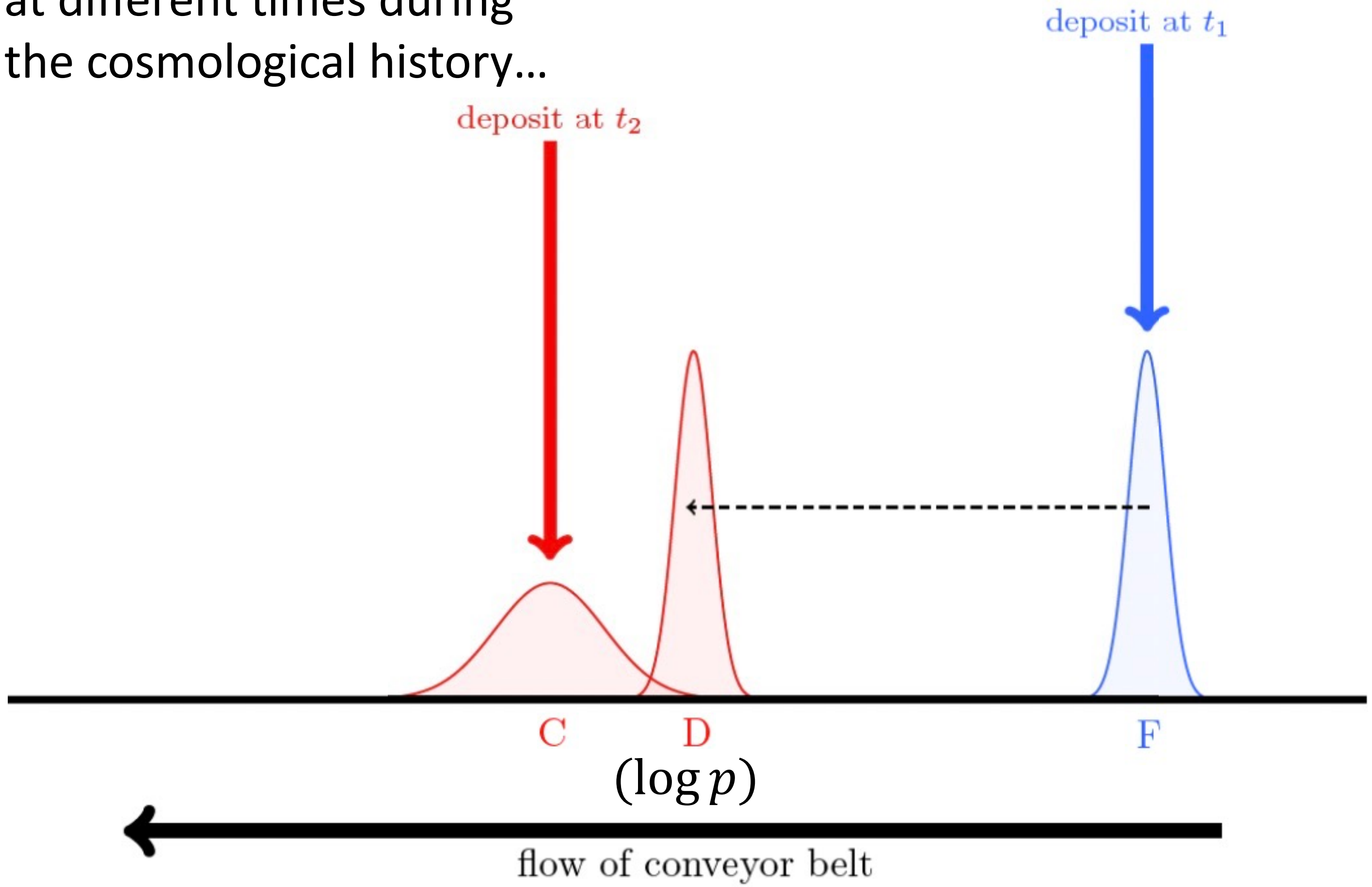

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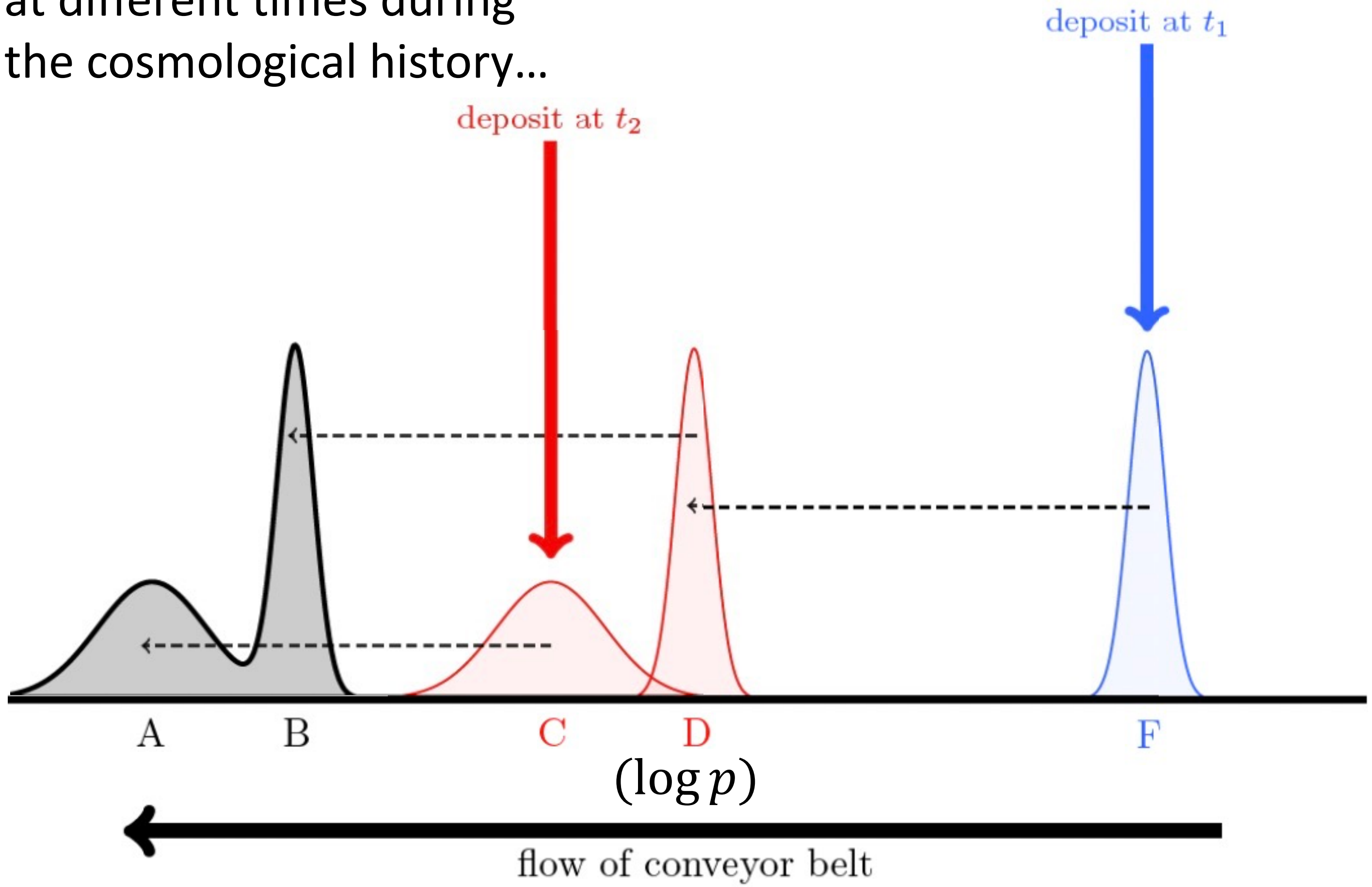


***Nontrivial, multi-modal** distribution can result at present time!*

backup

Toy Model: Final Phase-Space Distribution

$g(p)$ is phase-space distribution w.r.t. $\log p$

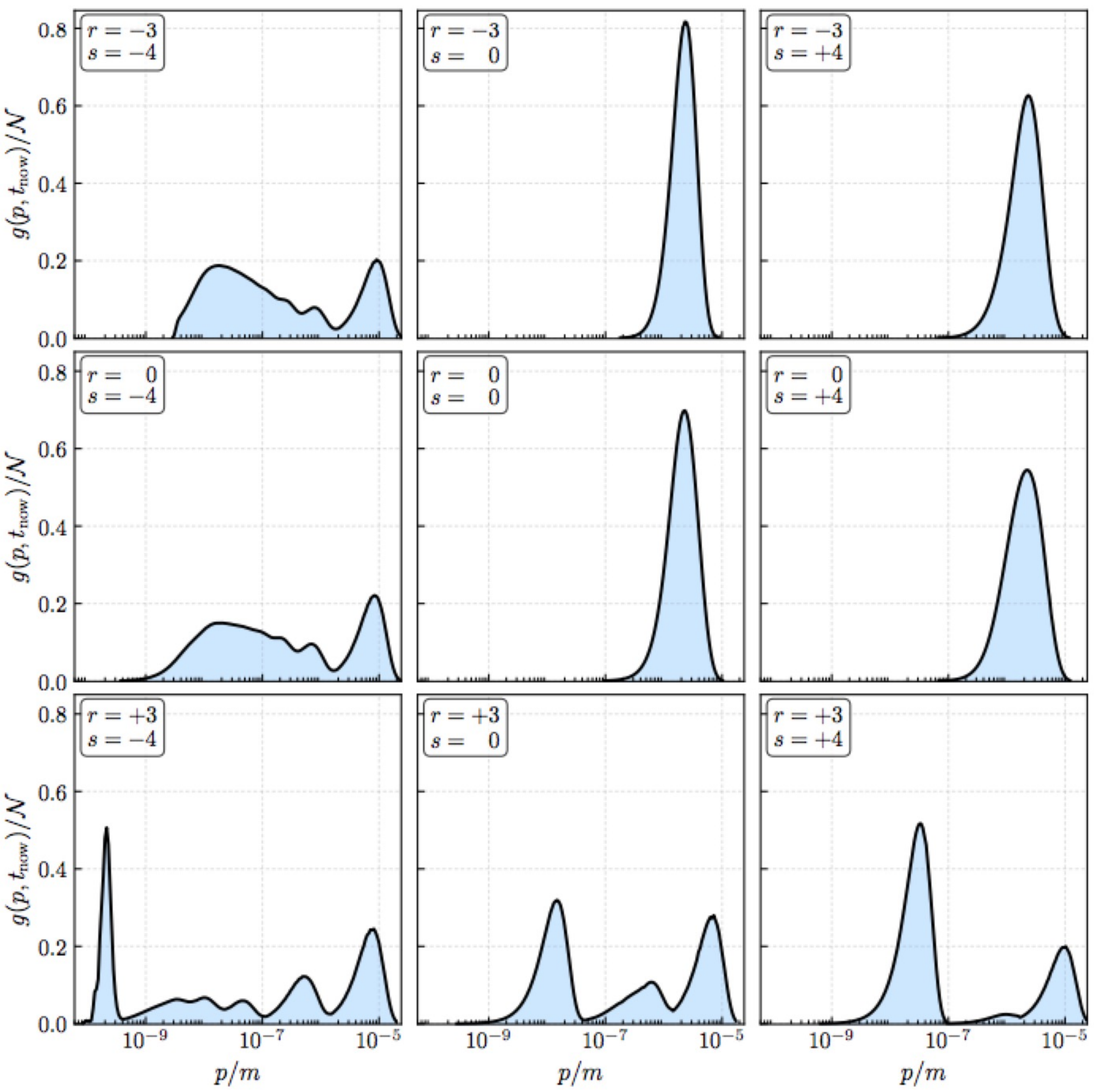
$$N(t) \;=\; \frac{g_{\rm int}}{2\pi^2}\int_{-\infty}^{\infty} d\log p\; g(p,t) \;\equiv\; \frac{g_{\rm int}}{2\pi^2}\mathcal{N}(t)$$

Numerically solve the Boltzmann equation

$$\frac{\partial f_i}{\partial t} = Hp\frac{\partial f_i}{\partial p} + C[\{f_1, f_2, \dots\}]$$

assuming only the heaviest state is populated initially.

A rich variety of distributions emerges!



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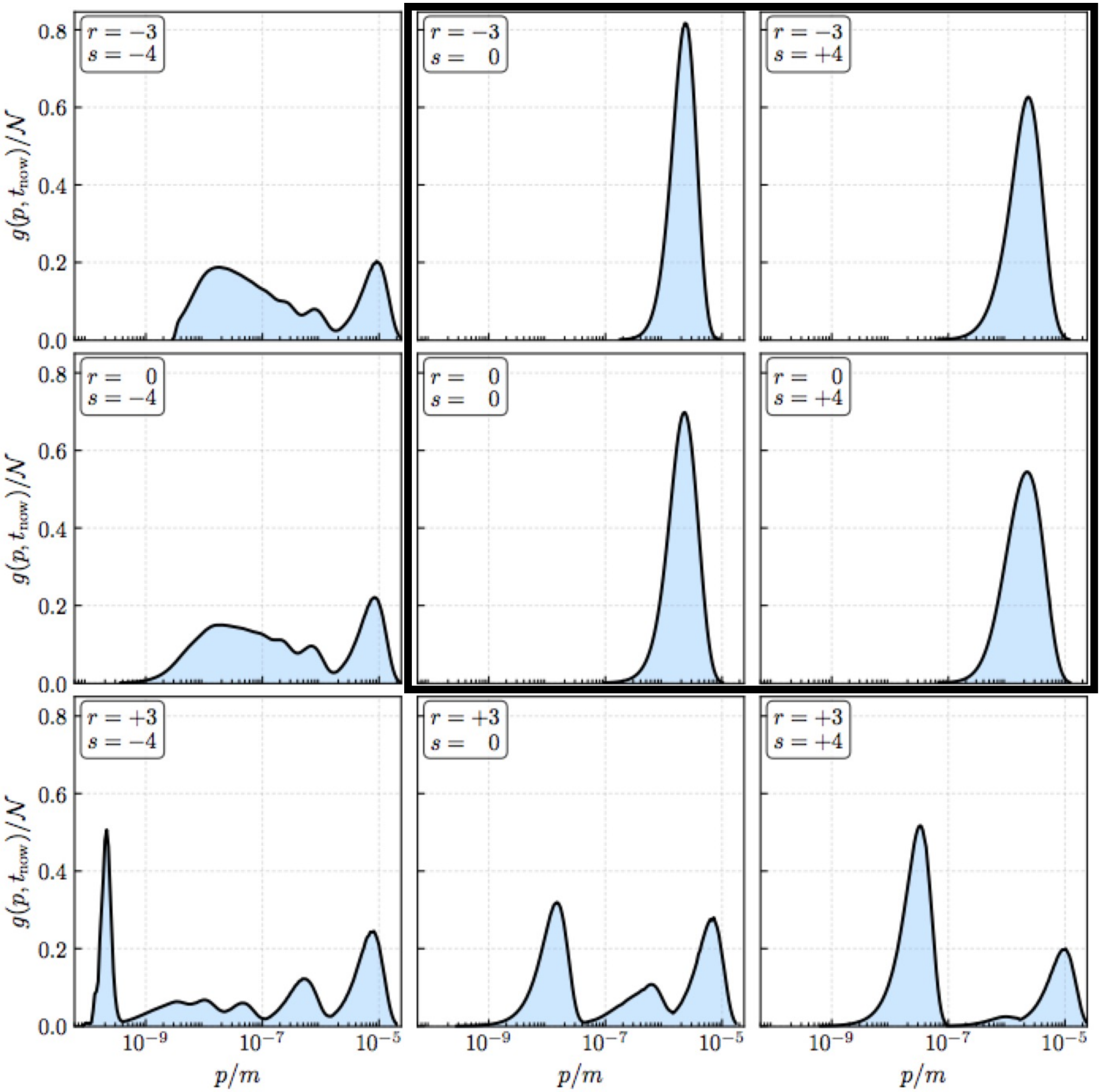

unimodal distributions

backup

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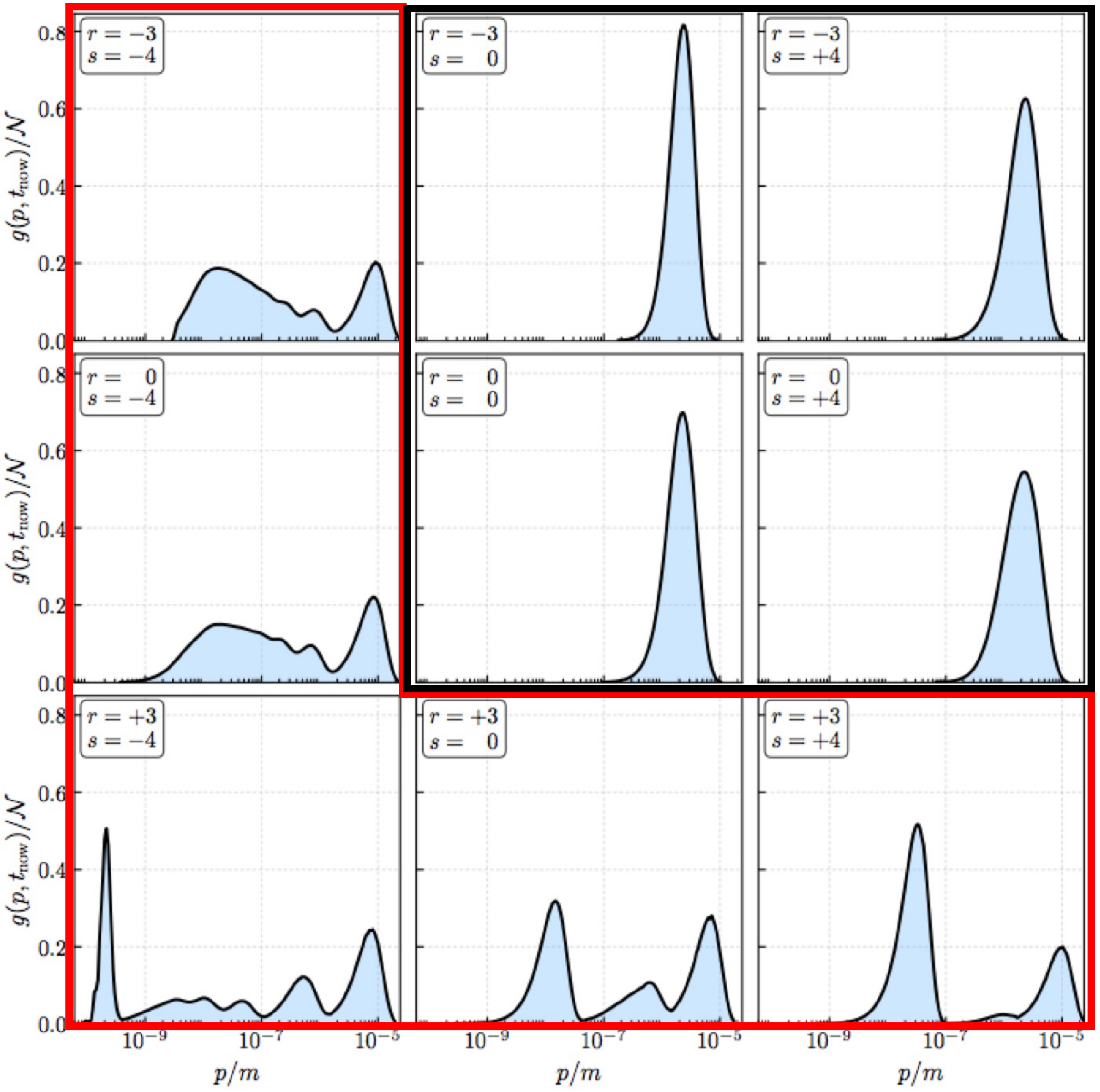


unimodal distributions

multi-modal distributions

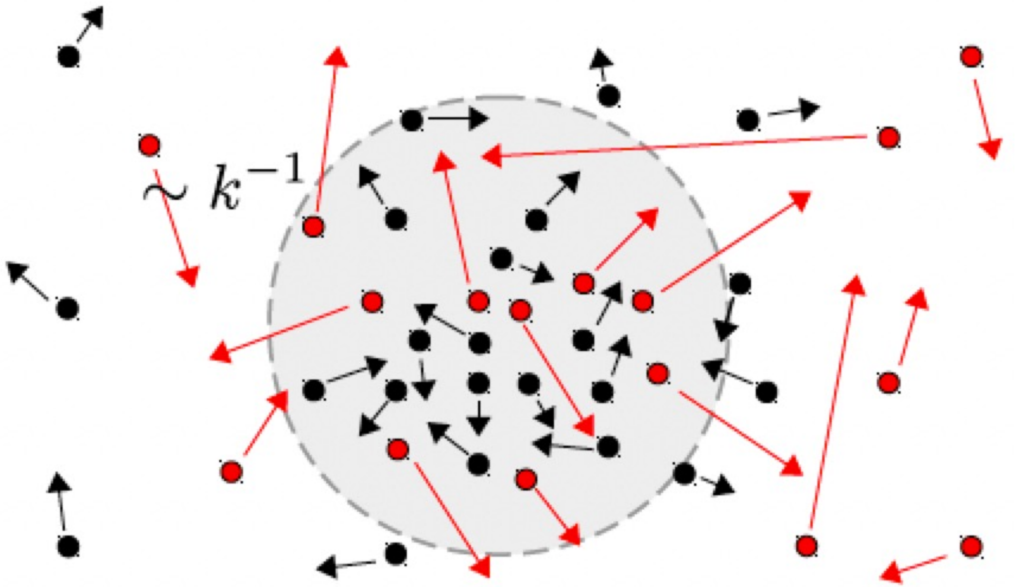
backup

Linear regime

$$g(v) \overset{\textbf{CLASS}}{\rightleftarrows} P(k)$$

Inversion possible? √

Fraction of DM number density which free-streams at k



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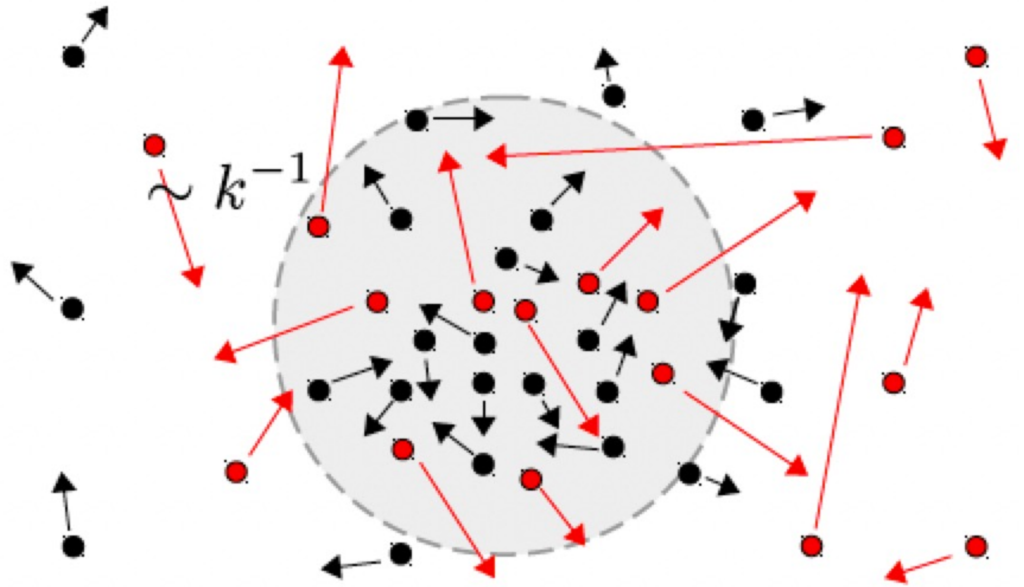

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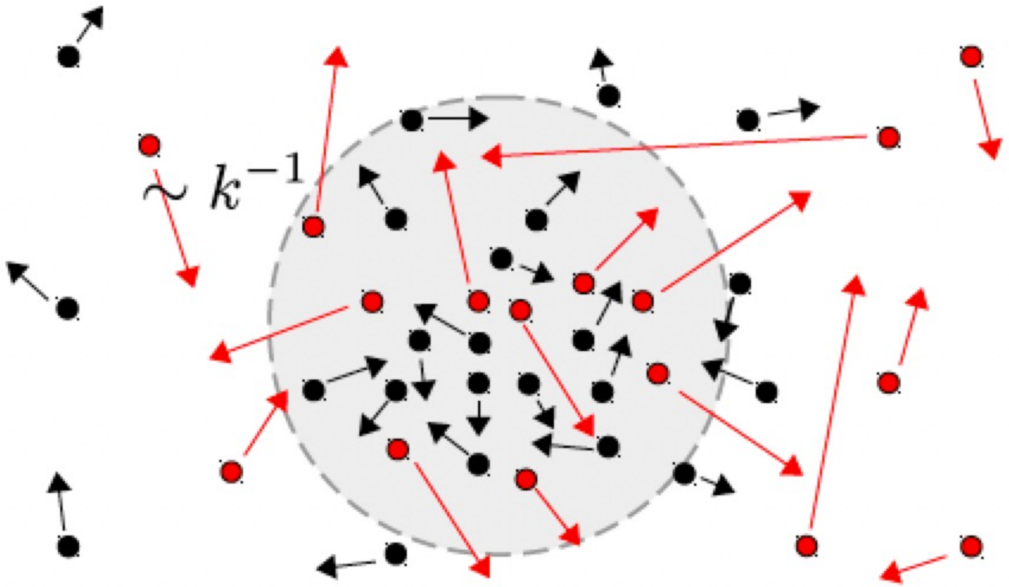

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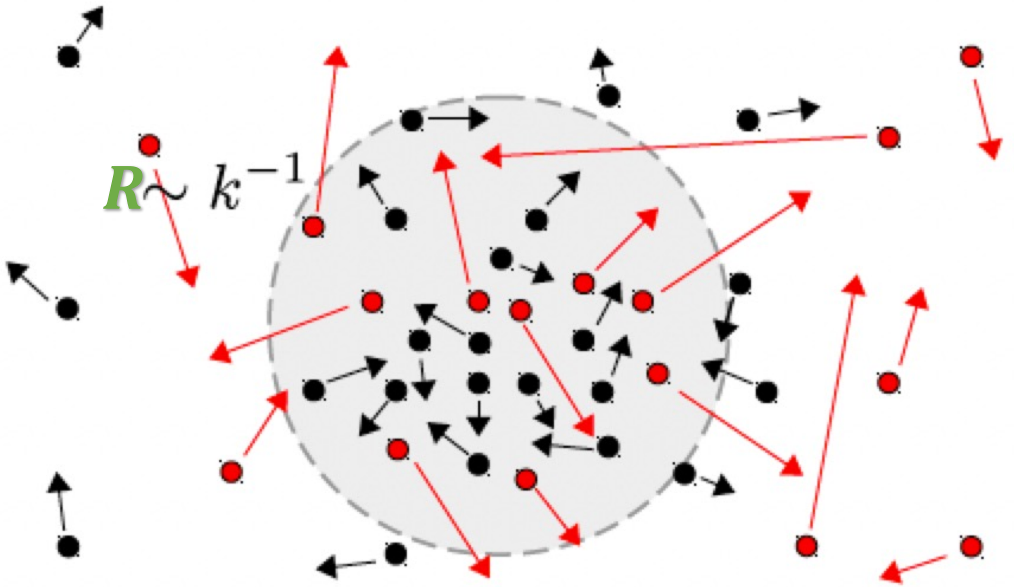

Halos of mass $M \sim \bar{\rho} \times R^3$

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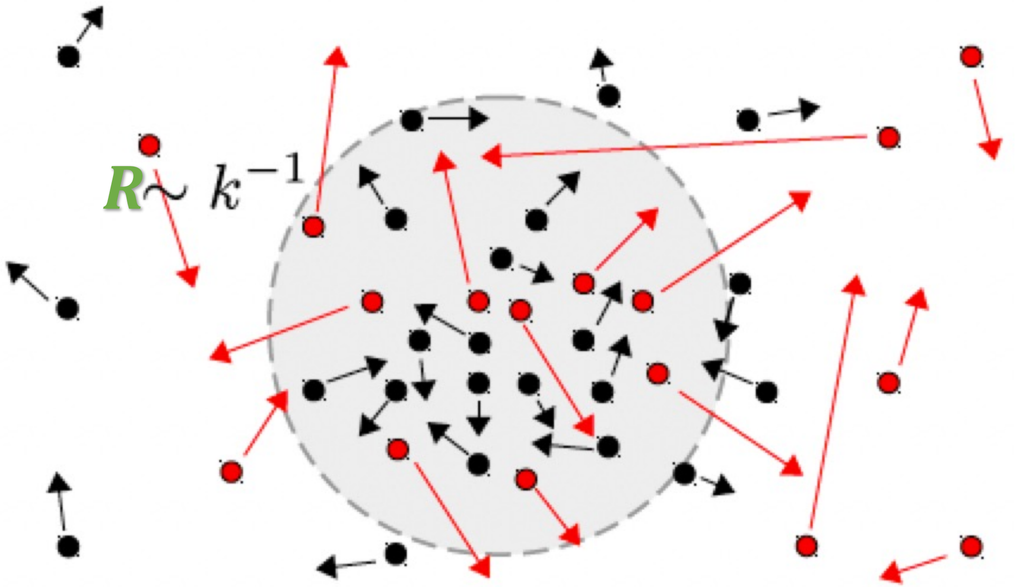

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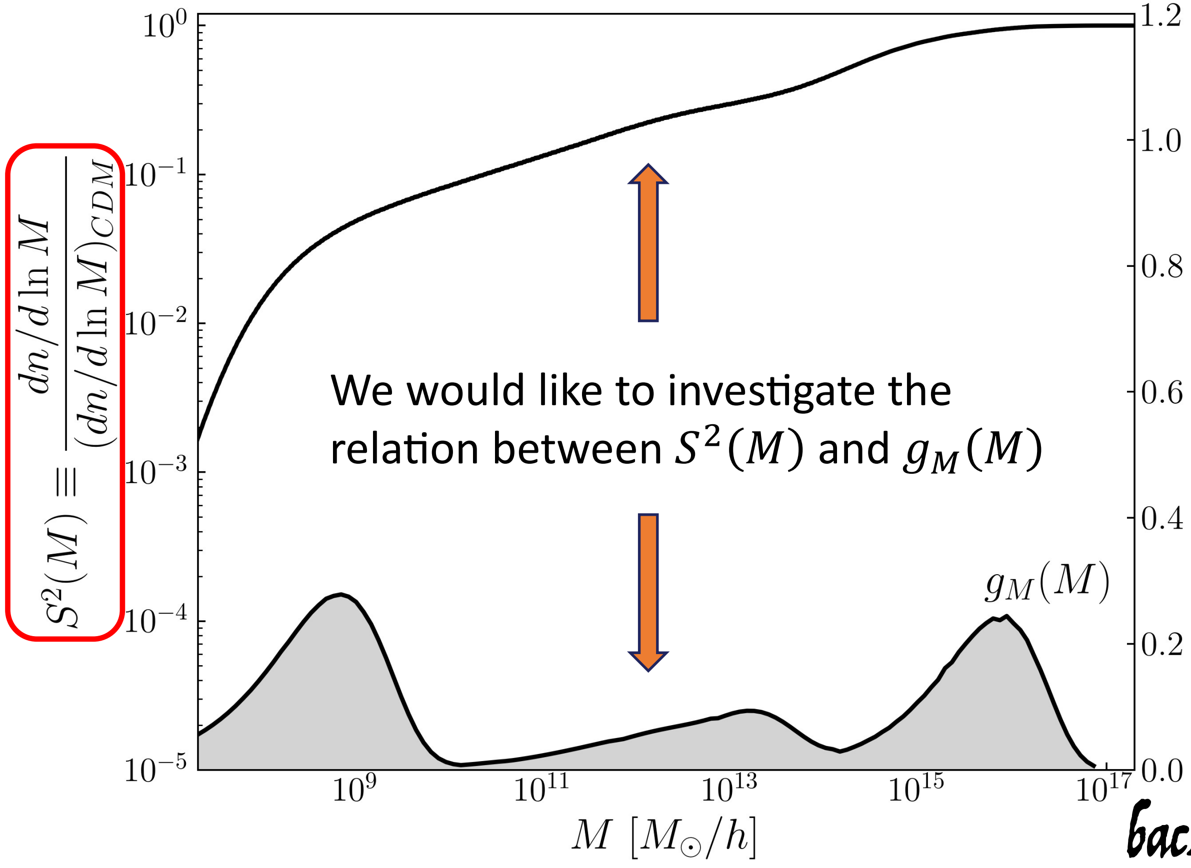
Halo mass function and distribution in *M-space*

Analogous to the transfer function, we define the ***structure-suppression function***

$$S^2(M) \equiv \frac{dn/d\log M}{(dn/d\log M)_{CDM}}$$

$p \rightarrow d_{hor}(p) \rightarrow \bar{\rho} \times d_{hor}^3 \rightarrow M(k)$

$\Rightarrow g(p) \rightarrow g_M(M)$



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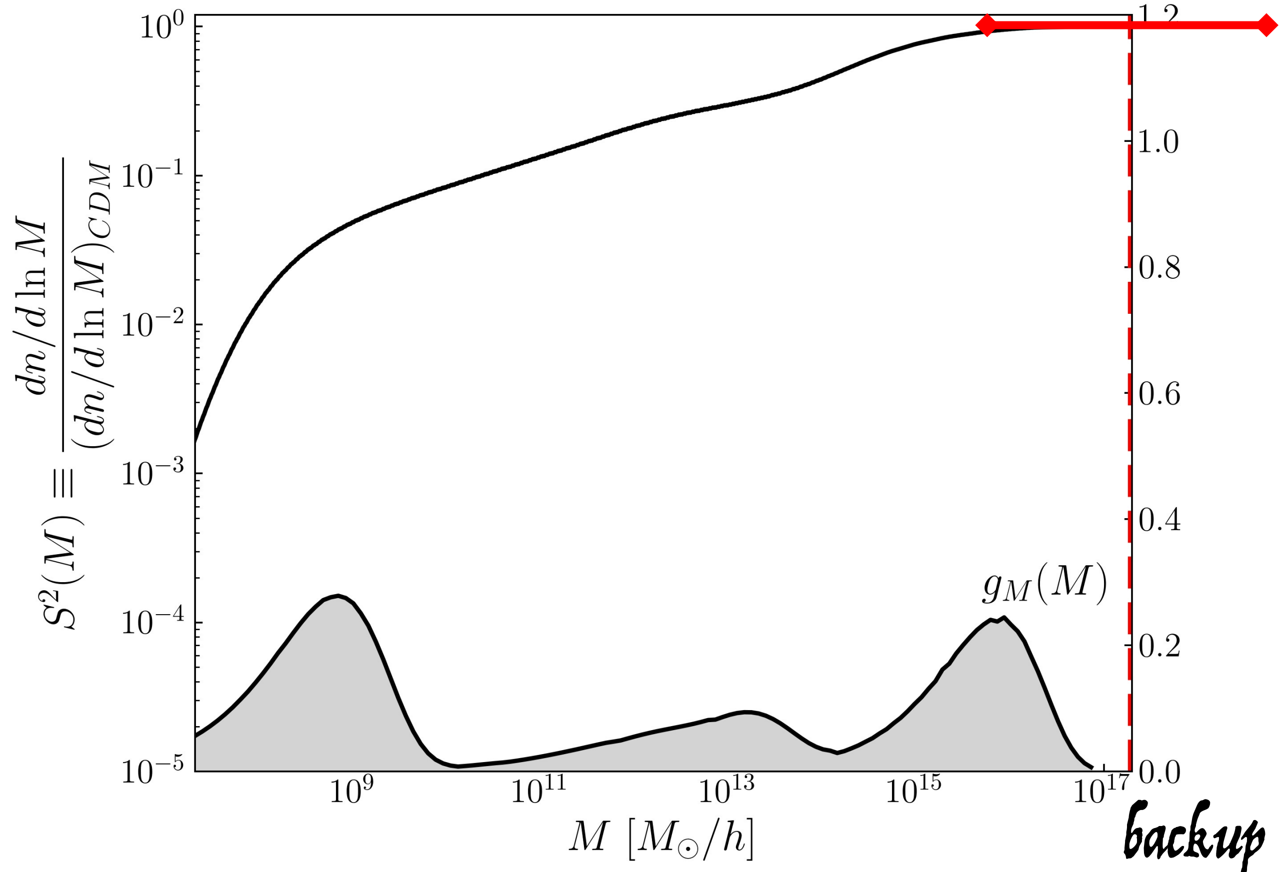

Hot-fraction function in the M-space:

$$F(M) \equiv \frac{\int_{\log M}^{\infty} d\log M'\, g_M(M')}{\int_{-\infty}^{\infty} d\log M'\, g_M(M')}$$

fraction of DM with velocities large enough to escape a region that would collapse into a halo of mass M

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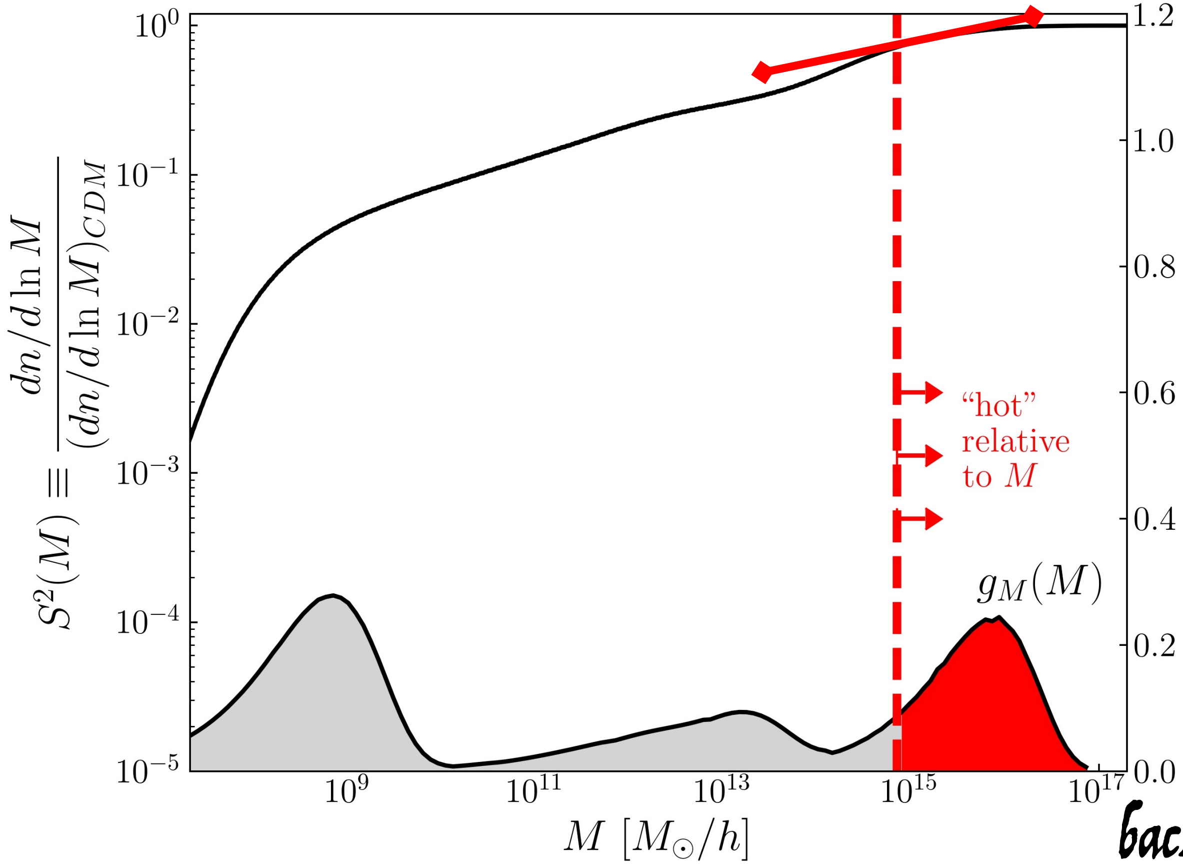

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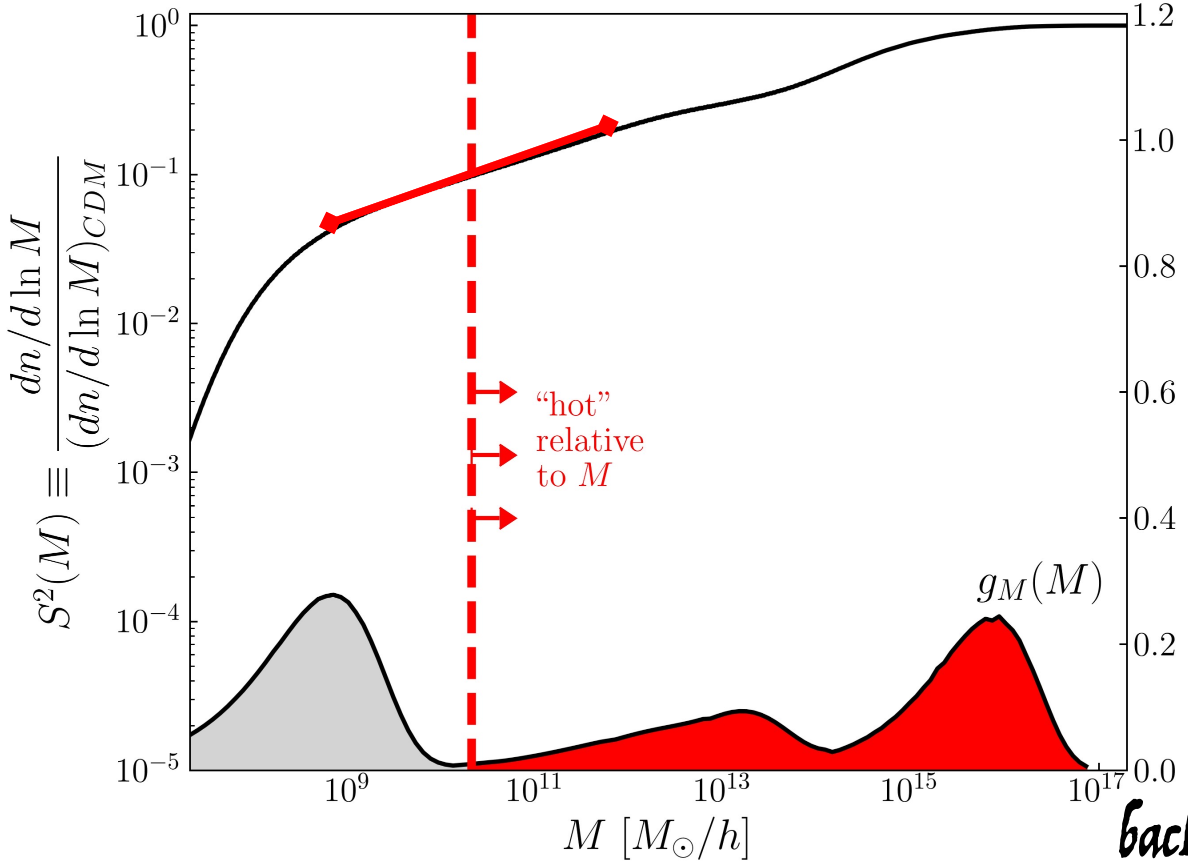

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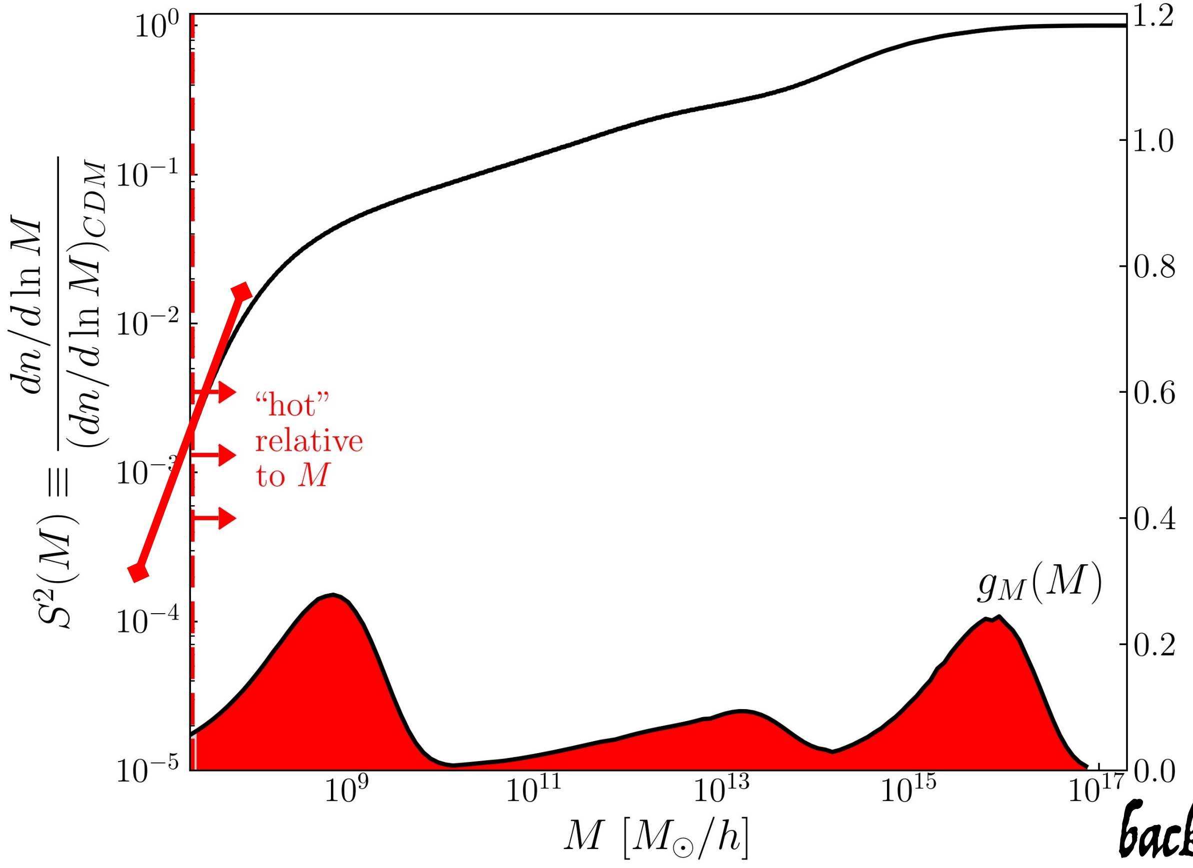

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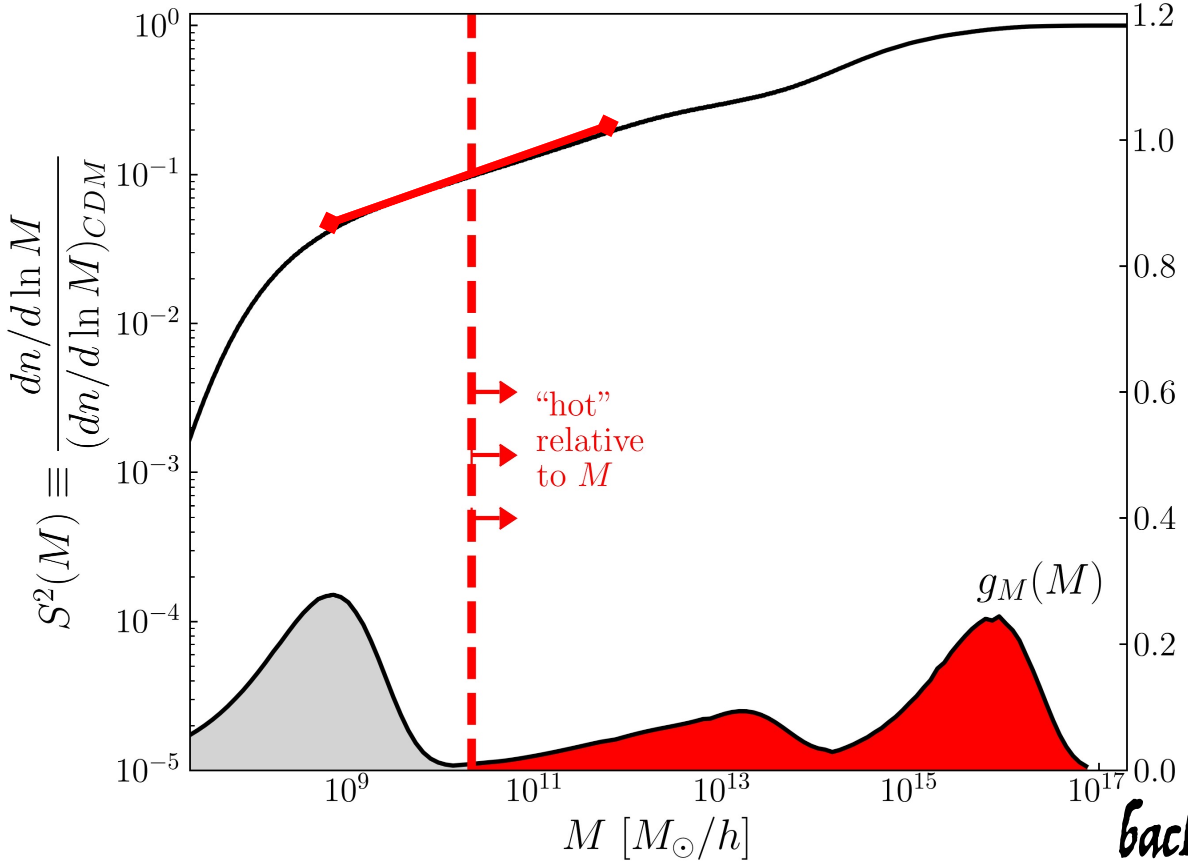
Once again, we find the **logarithmic slope** of $S^2(M)$ is ***directly*** related to $F(M)$!

$$\frac{d\log S^2(M)}{d\log M} \approx \frac{7}{10} F^2(M)$$



backup

Non-linear regime: Reconstruction Conjecture

With $$\frac{d\log S^2(M)}{d\log M} \approx \frac{7}{10} F^2(M)$$

and $$\frac{dF}{d\log M} = \frac{g_M(M)}{\mathcal{N}}$$

take derivatives on both sides and find

$$\boxed{\frac{g_M(M)}{\mathcal{N}} \approx \sqrt{\frac{5}{14}} \left(\frac{d\log S^2(M)}{d\log M}\right)^{-\frac{1}{2}} \left|\frac{d^2\log S^2(M)}{(d\log M)^2}\right|}$$

This allows us to ***reconstruct*** the DM phase-space distribution ***directly*** from $S^2(M)$!

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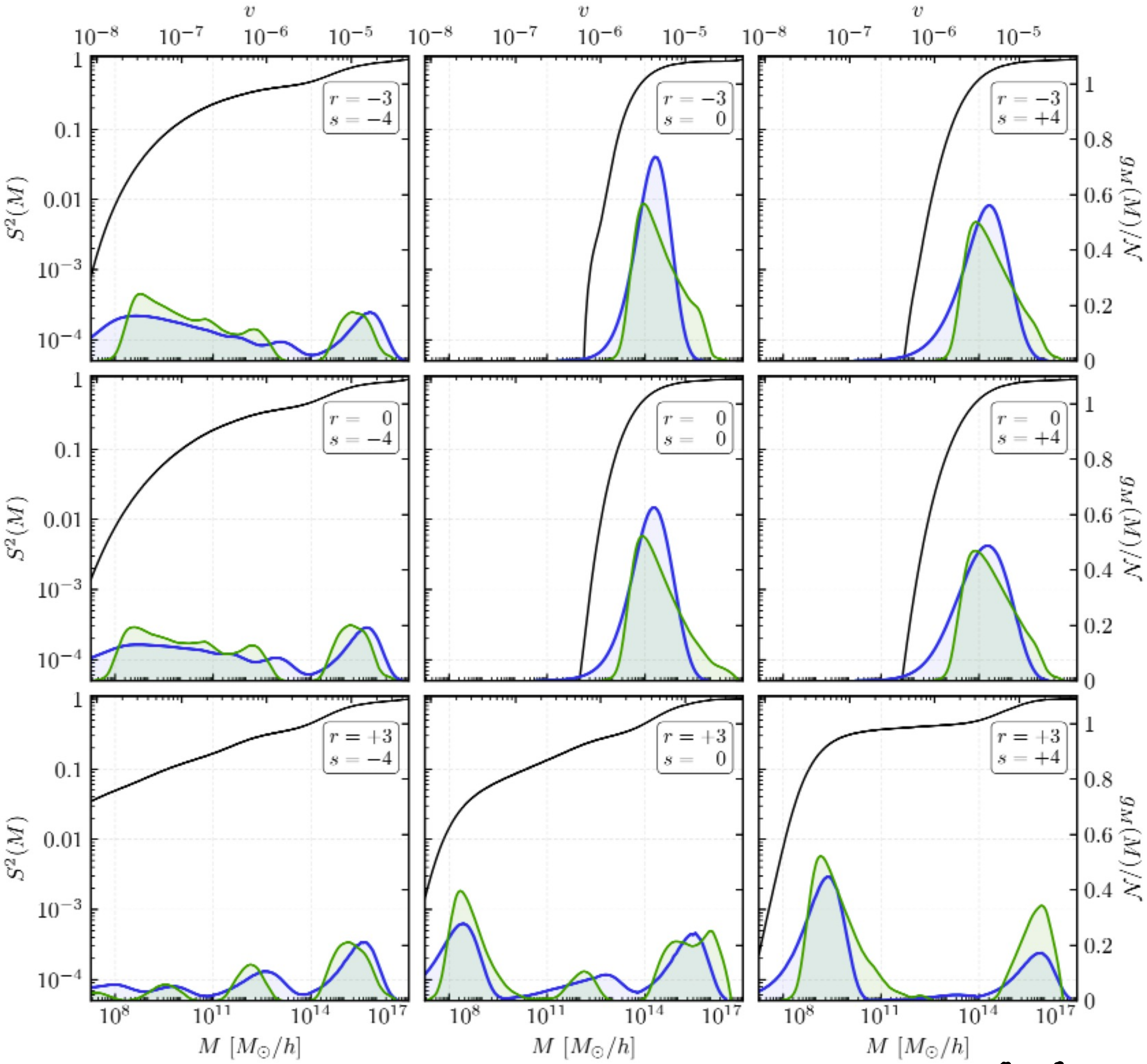
This allows us to ***reconstruct*** the DM phase-space distribution ***directly*** from $S^2(M)$!

Blue: original DM distribution in M-space
Green: reconstruction directly from $S^2(M)$

Once again, able to resurrect the **salient features** of the original distribution!

K. Dienes, **FH**, J. Kost, K. Manogue, B. Thomas, arXiv:2101.10337



backup

Conclusion

early-universe dynamics

dark-matter phase-space distribution *f(p)*

linear matter power spectrum *P(k)*

Density perturbations eventually go nonlinear

nonlinear regime

reconstructed *f(p)*

⋮

backup