

# Solitons in Warm, Noisy and Spinning Wave Dark Matter

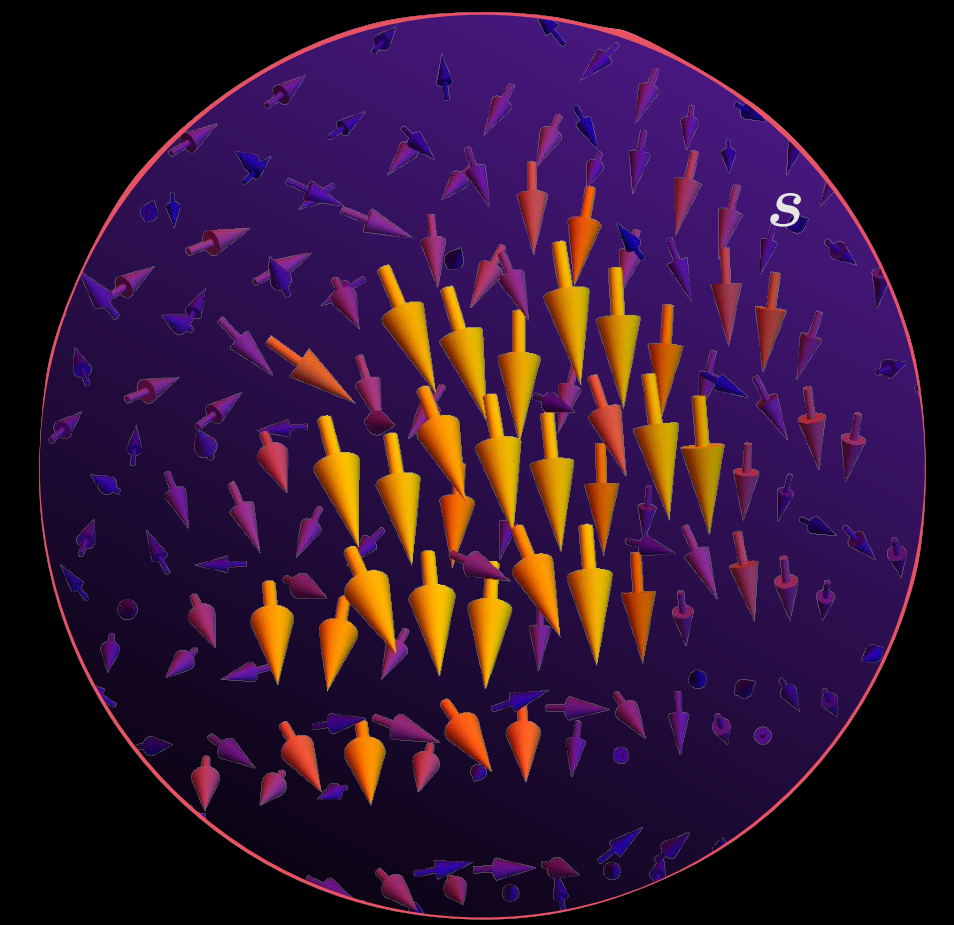
Mustafa A. Amin



# ③ ② Solitons in Warm, Noisy and Spinning Wave Dark Matter ①

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# Spinning Wave Dark Matter ①

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RICE



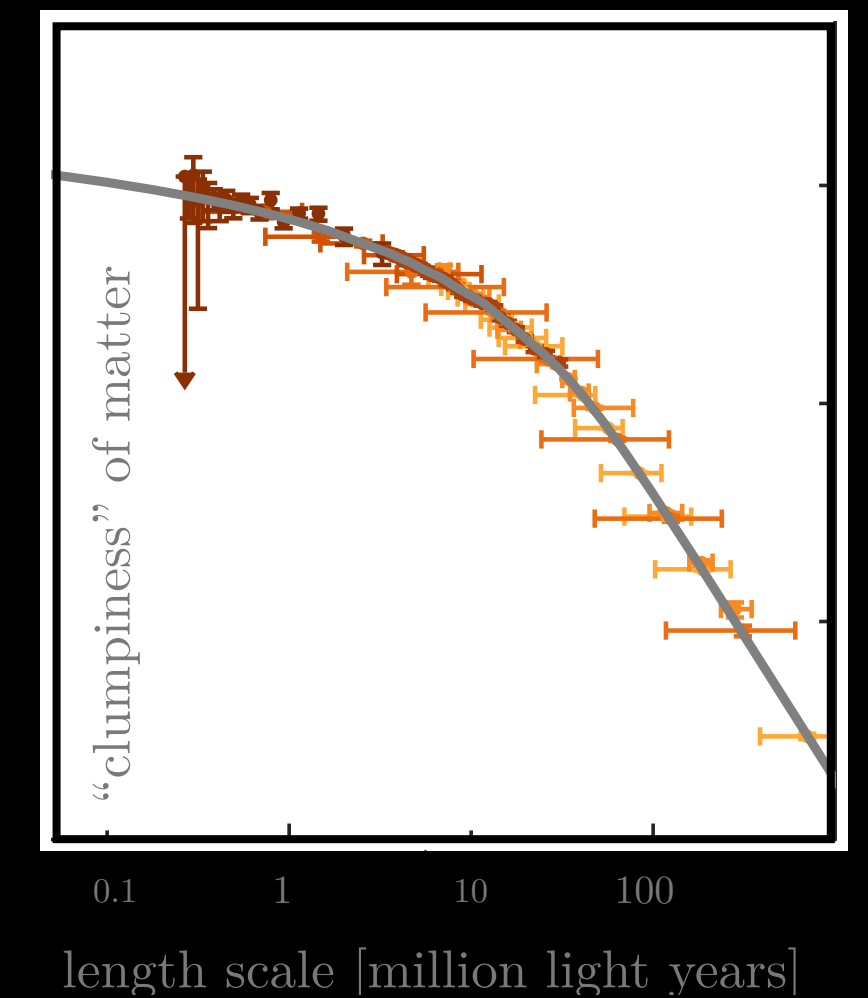
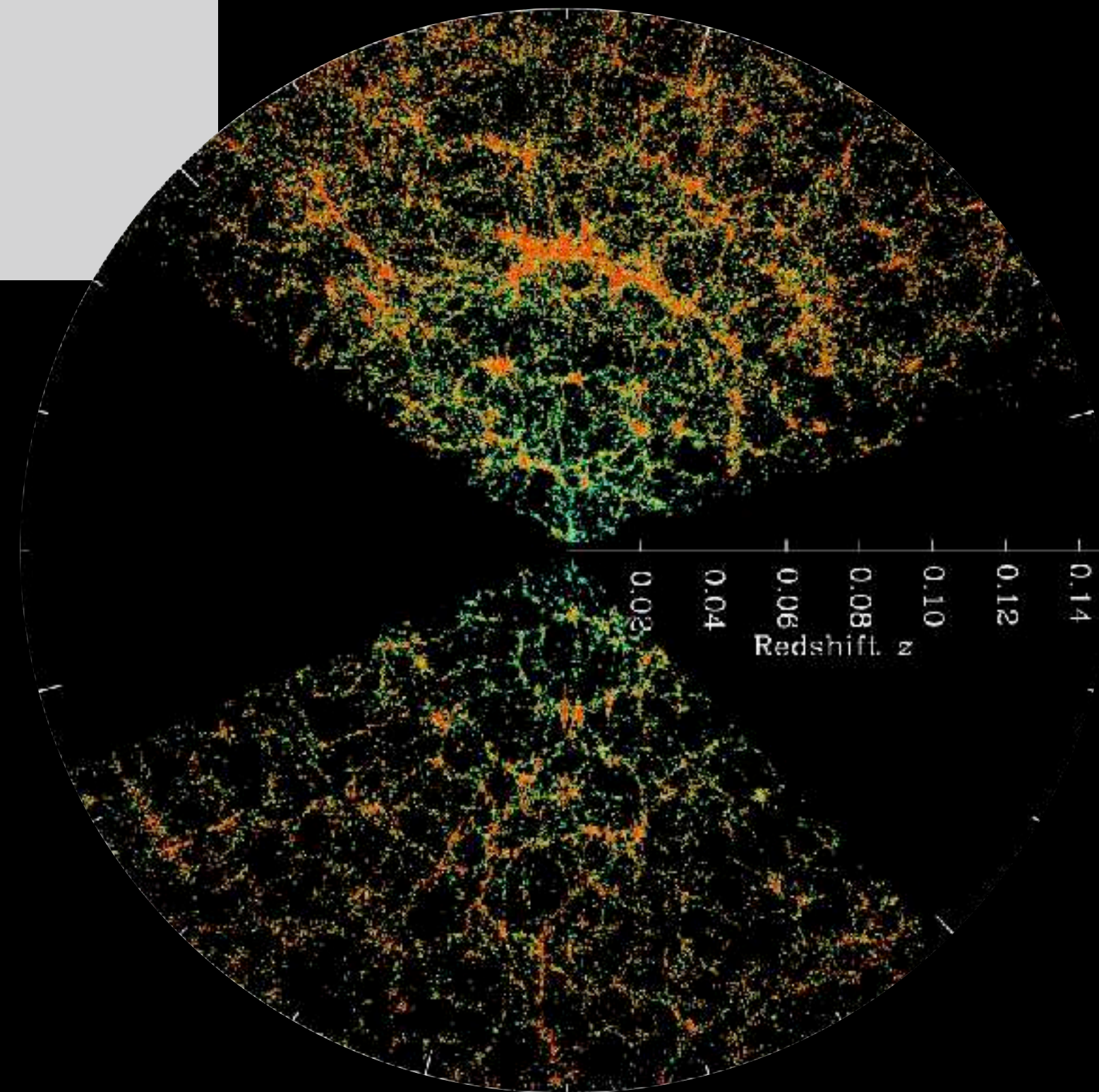
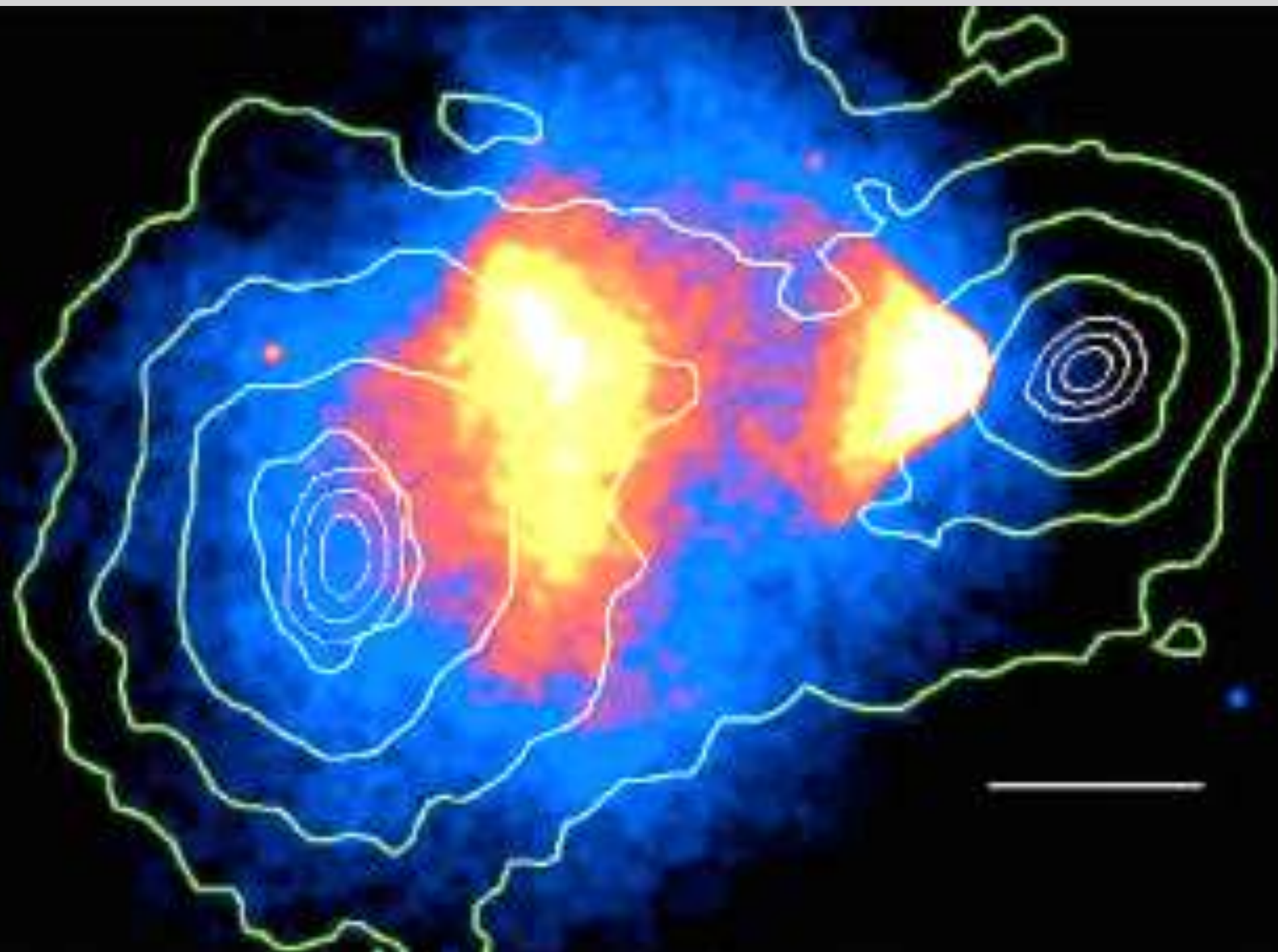
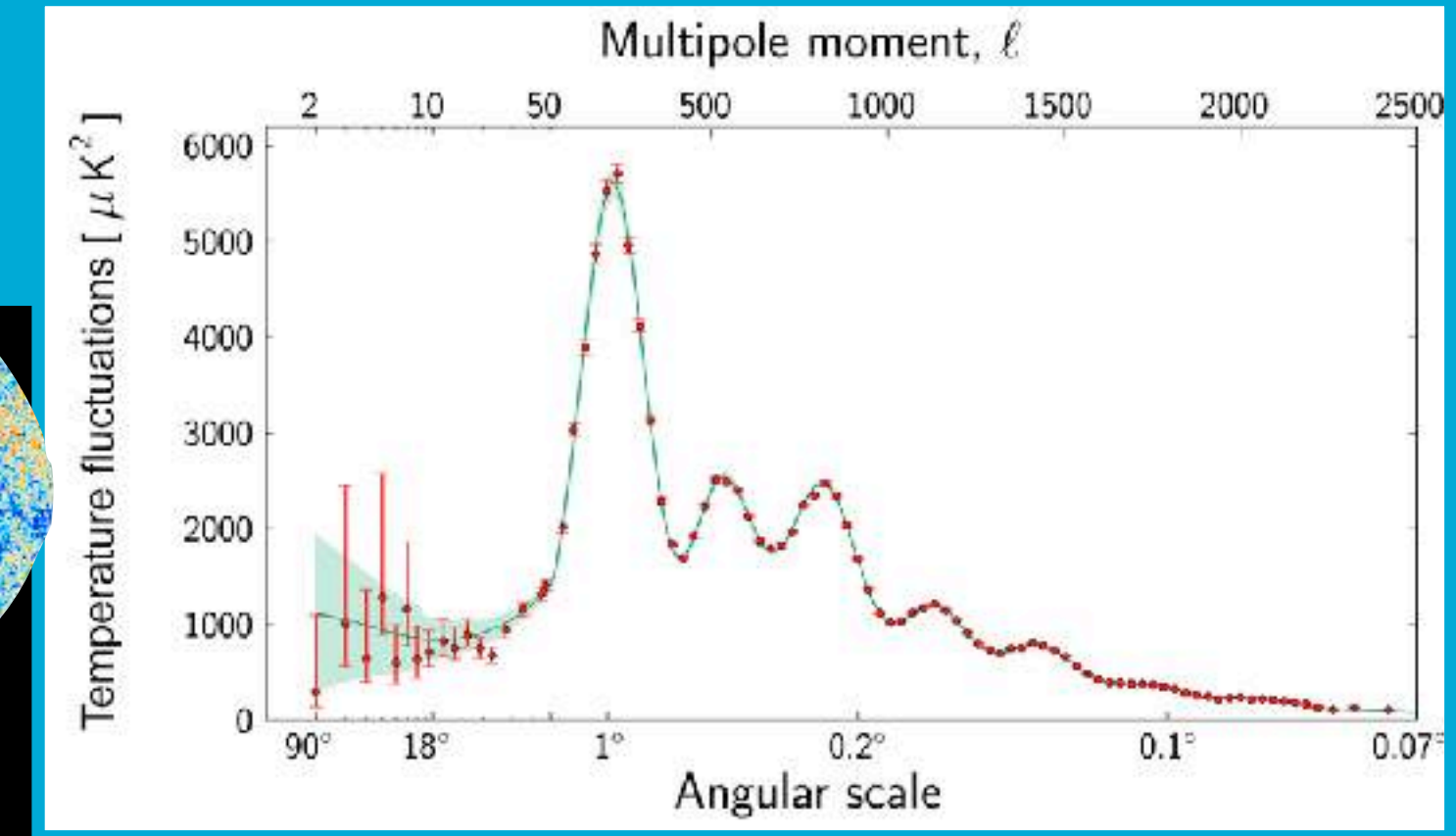
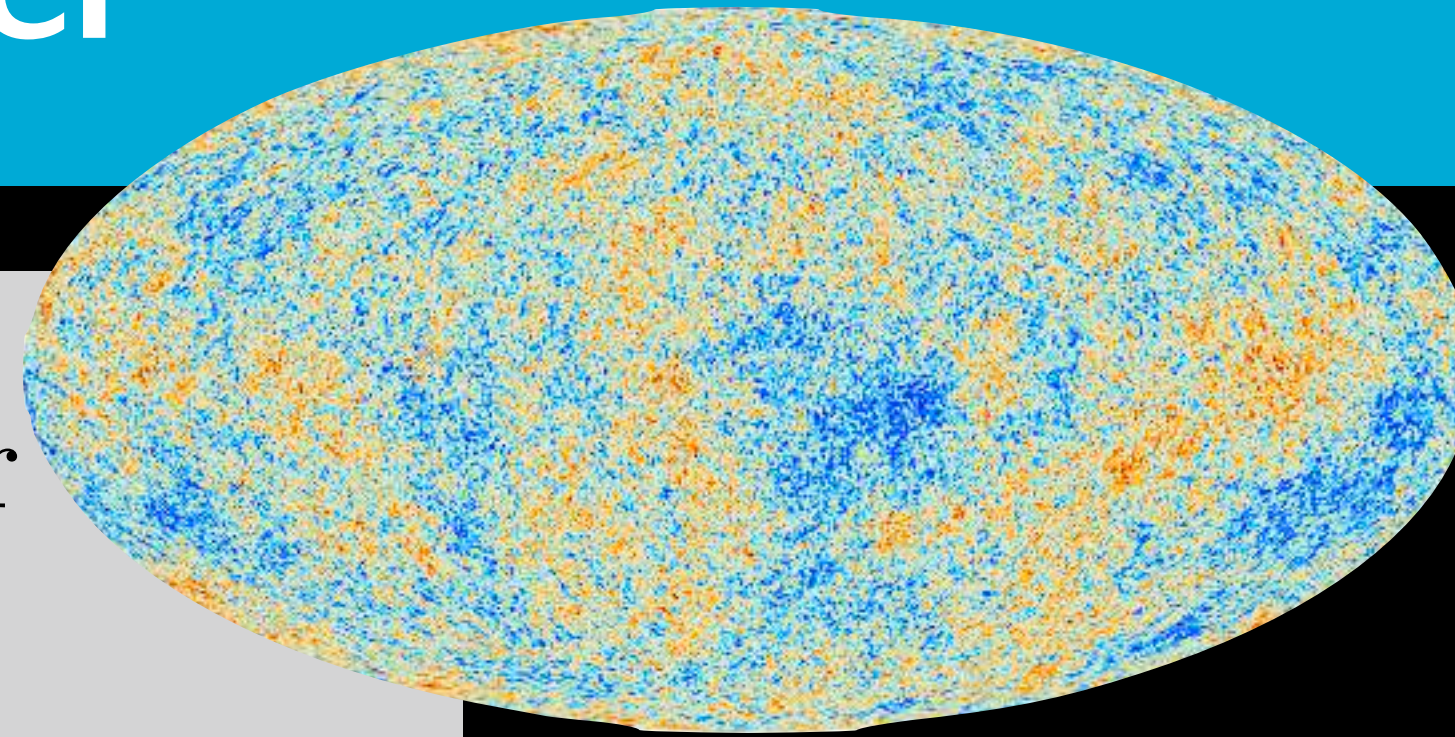


**motivation**

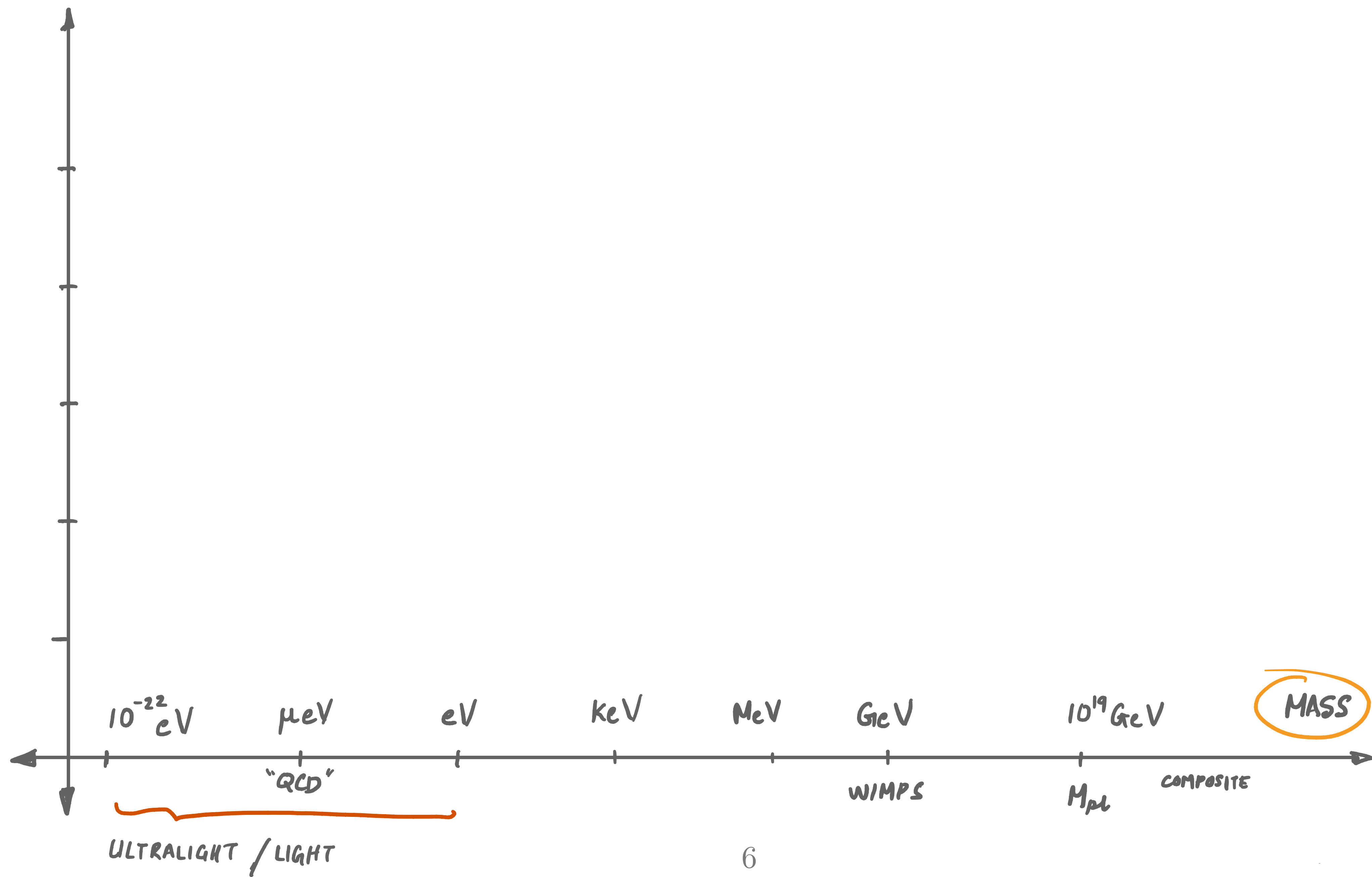


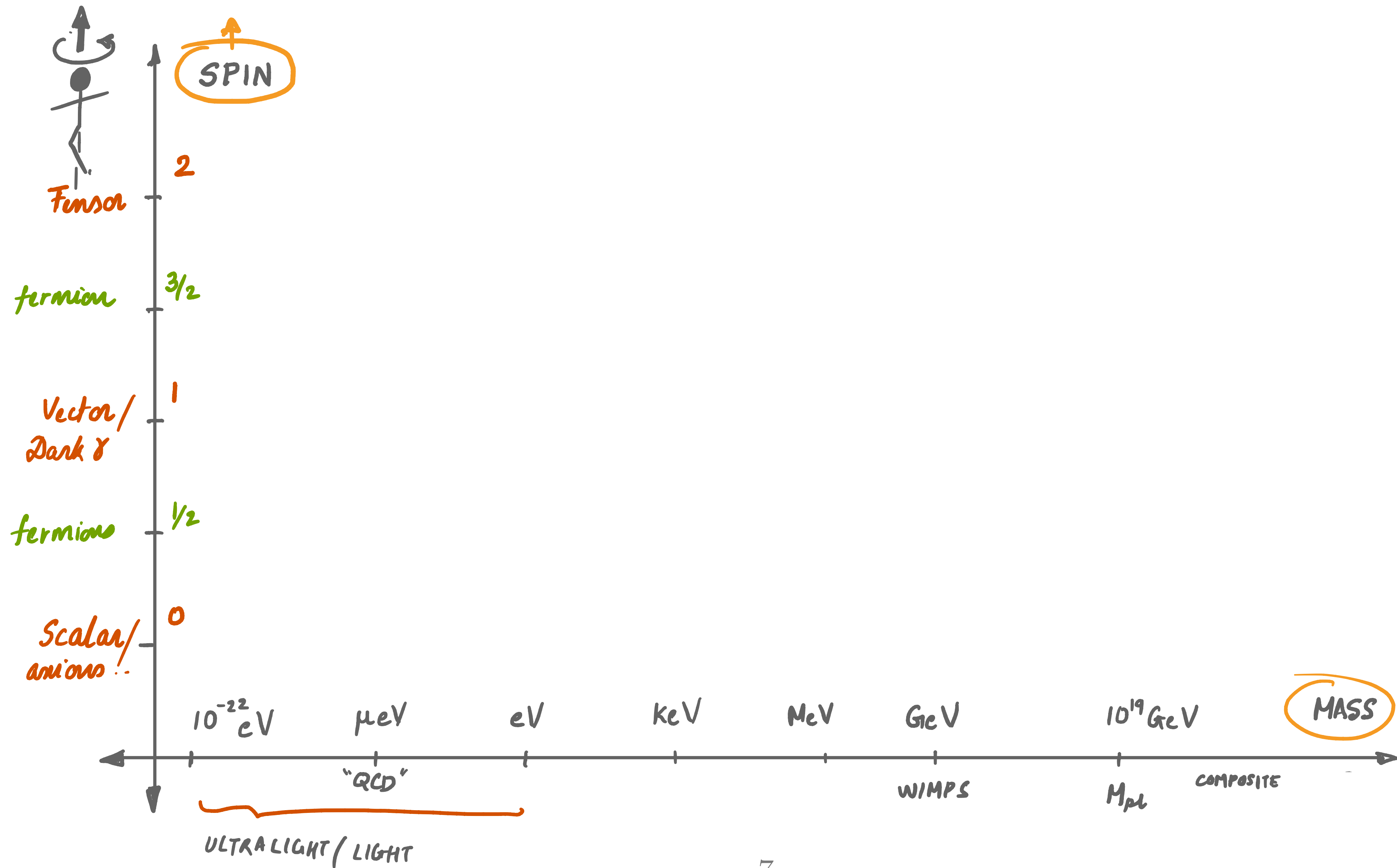
# evidence for dark matter

- dark matter exists-85% of matter
- gravitational interactions ✓
- mass, spin ?

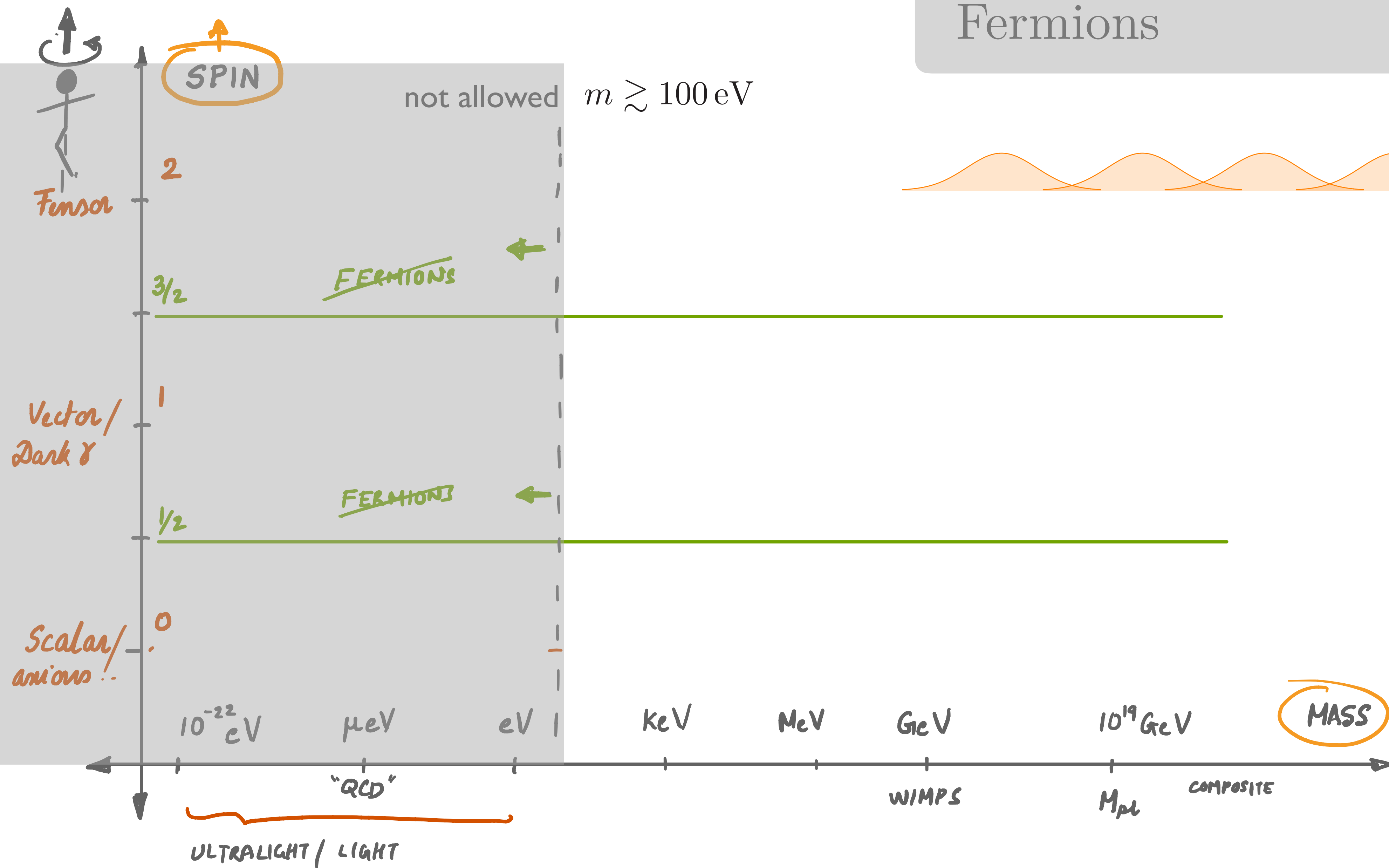






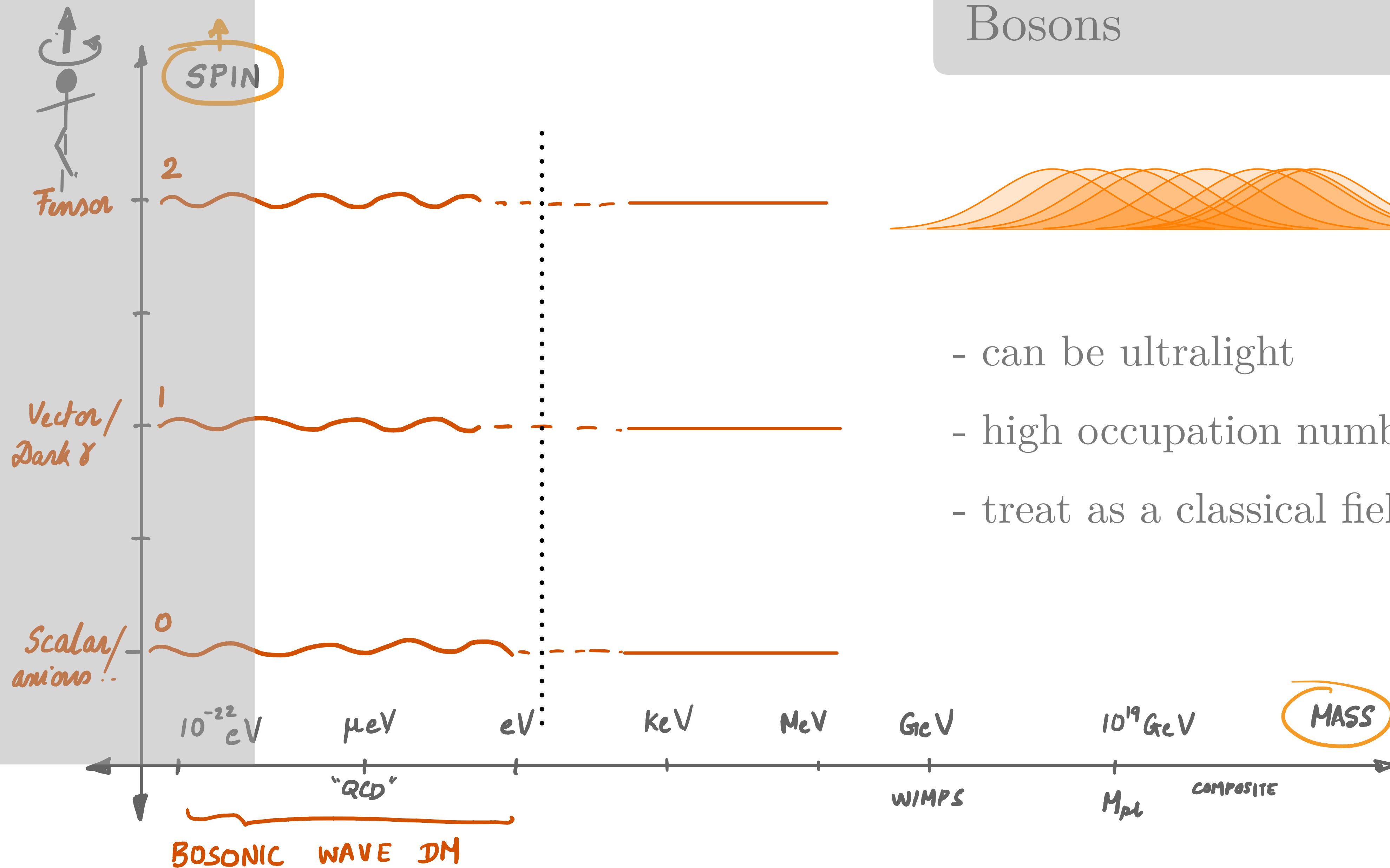


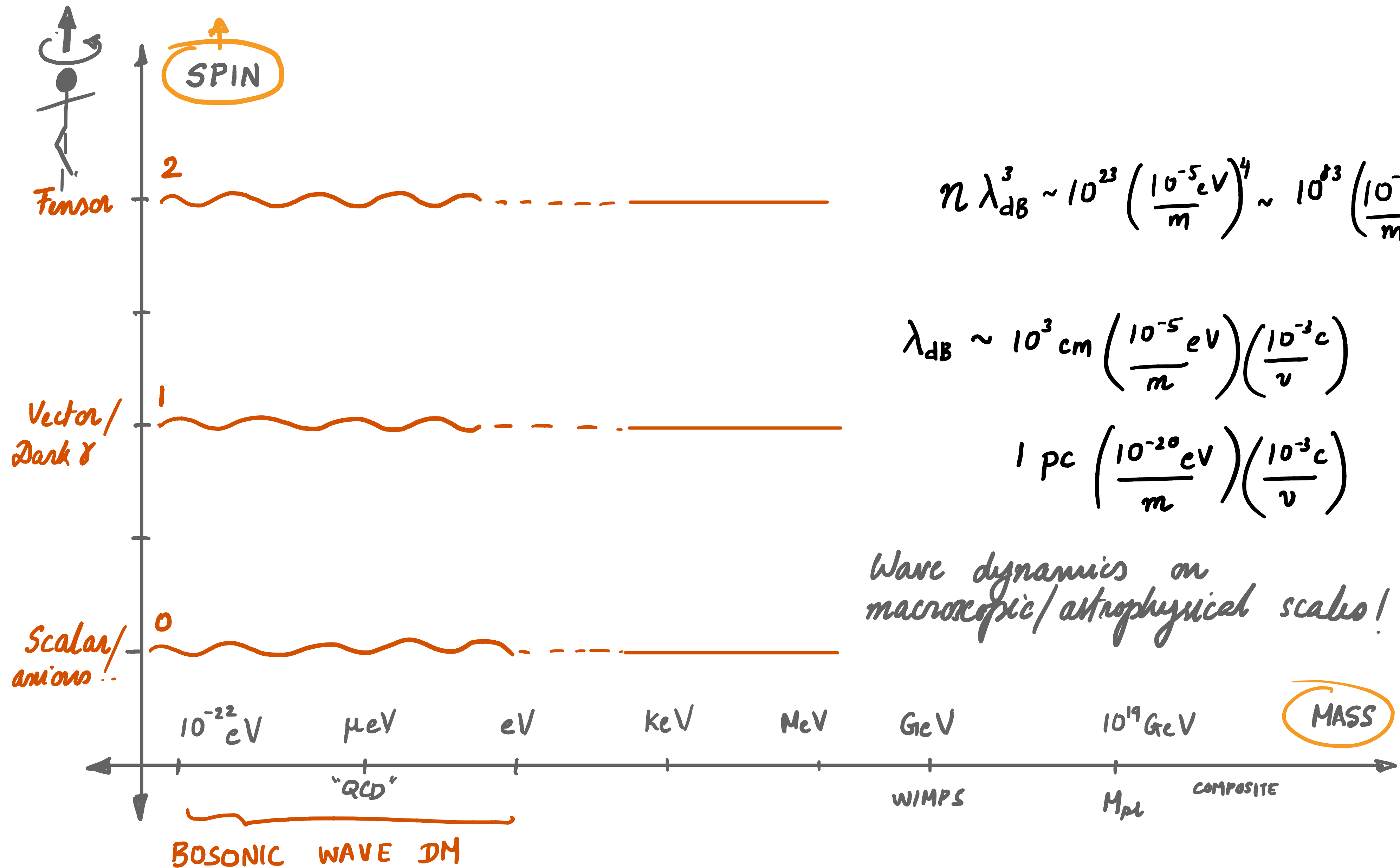
# Fermions





not allowed





# how spin can effect small scale structure in wave DM

wave interference

solitons+halo profiles

Bose effects

interactions

initial conditions+growth of structure



# spin-0 (scalar) and spin-1(vector) models

# scalar = spin-0 wave dark matter

\* assuming classical fields

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \nabla^\mu \varphi \nabla_\mu \varphi - \frac{1}{2} m^2 \varphi^2 + \frac{1}{8\pi G} R + \dots \right] + \text{non-grav, interactions}$$

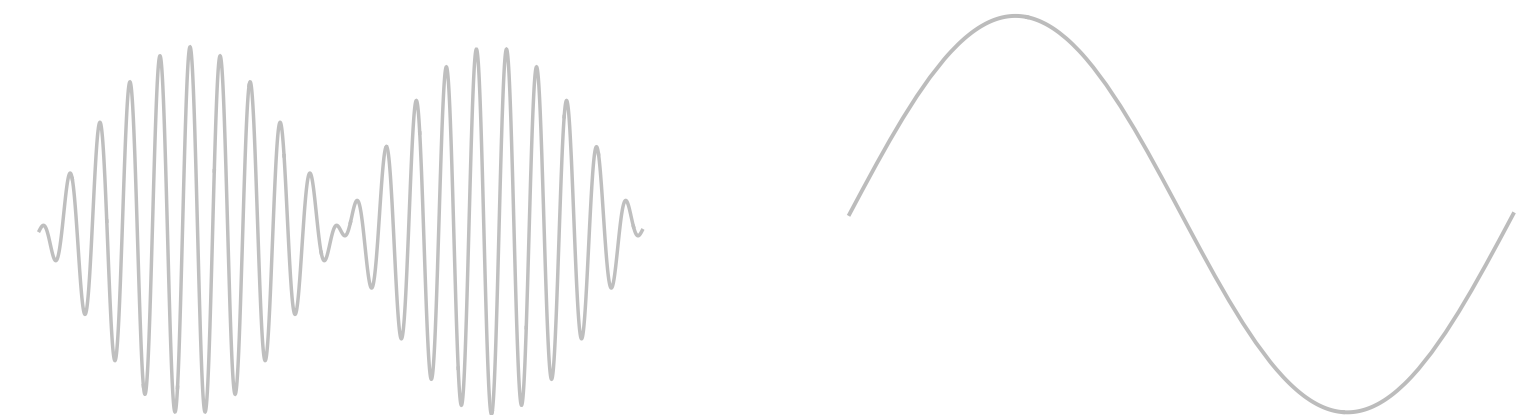
$$\nabla^\mu \nabla_\mu \varphi + m^2 \varphi = 0 \quad G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

# scalar = spin-0 wave dark matter

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \nabla^\mu \varphi \nabla_\mu \varphi - \frac{1}{2} m^2 \varphi^2 + \frac{1}{8\pi G} R + \dots \right] + \text{non-grav, interactions}$$

non-relativistic limit

$$\varphi(t, \mathbf{x}) \equiv \frac{1}{\sqrt{2m}} \Re \left[ \Psi(t, \mathbf{x}) e^{-imt} \right]$$



split in “fast” and “slow” parts

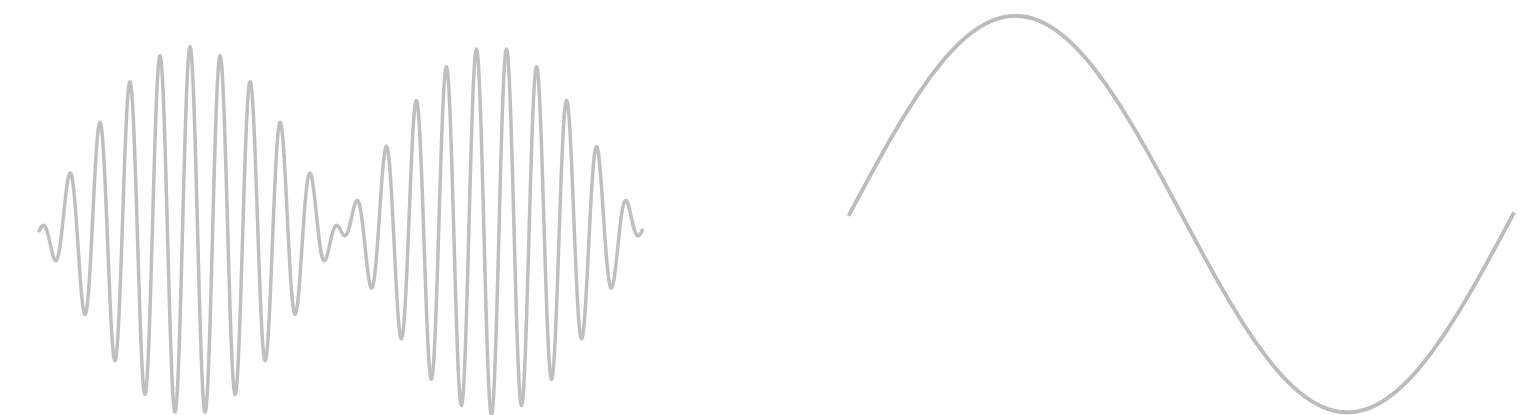


# scalar = spin-0 wave dark matter

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \nabla^\mu \varphi \nabla_\mu \varphi - \frac{1}{2} m^2 \varphi^2 + \frac{1}{8\pi G} R + \dots \right] + \text{non-grav, interactions}$$

non-relativistic limit

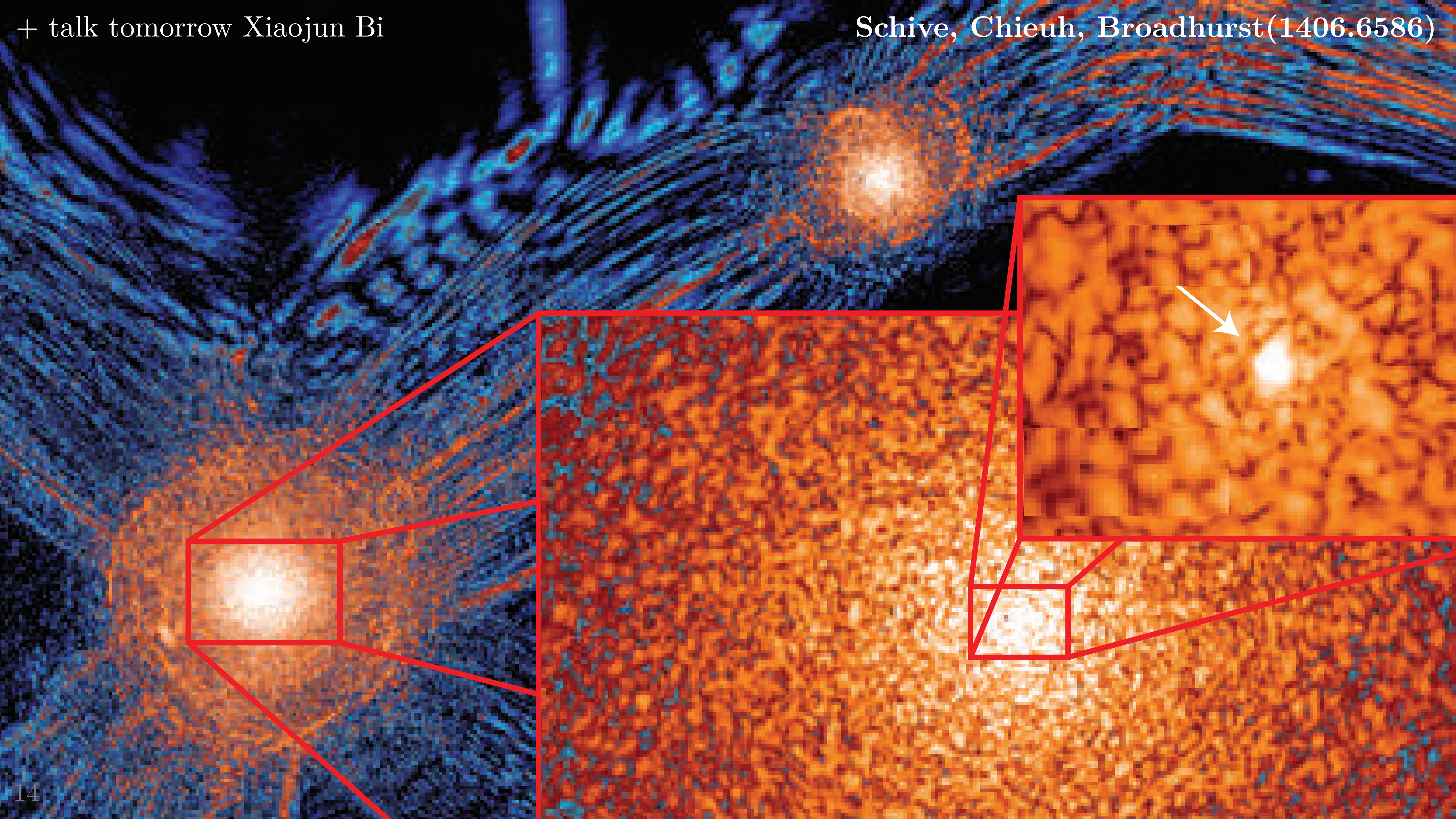
$$\varphi(t, \mathbf{x}) \equiv \frac{1}{\sqrt{2m}} \Re [\Psi(t, \mathbf{x}) e^{-imt}]$$



split in “fast” and “slow” parts

Schrodinger-Poisson system

$$i\hbar \frac{\partial}{\partial t} \Psi = -\frac{\hbar^2}{2m} \nabla^2 \Psi + m\Phi \Psi, \quad \nabla^2 \Phi = 4\pi G m \Psi^* \Psi$$



+ talk tomorrow Xiaojun Bi

Schive, Chieuh, Broadhurst(1406.6586)

# vector = spin-1 wave dark matter

\* assuming classical fields

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{4} X^{\mu\nu} X_{\mu\nu} + \frac{1}{2} m^2 X^\mu X_\mu + \frac{1}{8\pi G} R + \dots \right] + \text{non-grav, interactions}$$

$$X_{\mu\nu} = \nabla_\mu X_\nu - \nabla_\nu X_\mu$$



# vector = spin-1 wave dark matter

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{4} X^{\mu\nu} X_{\mu\nu} + \frac{1}{2} m^2 X^\mu X_\mu + \frac{1}{8\pi G} R + \dots \right] + \text{non-grav, interactions}$$

$$X_{\mu\nu} = \nabla_\mu X_\nu - \nabla_\nu X_\mu$$

non-relativistic limit

$$\mathbf{X}(t, \mathbf{x}) = \frac{1}{\sqrt{2m}} \Re [\boldsymbol{\Psi}(t, \mathbf{x}) e^{-imt}]$$



split in “fast” and “slow” parts

Schrodinger-Poisson system

$$i\partial_t \boldsymbol{\Psi} = -\frac{\nabla^2 \boldsymbol{\Psi}}{2m} + m\Phi \boldsymbol{\Psi}, \quad \nabla^2 \Phi = 4\pi G m \boldsymbol{\Psi}^\dagger \boldsymbol{\Psi}$$

Spin - 1      Adshead & Lozanov (2101.07265)

Spin -  $2s+1$       Jain & MA (2109.04892)



# non-relativistic limit = multicomponent Schrödinger-Poisson

$$i\partial_t \boldsymbol{\Psi} = -\frac{\nabla^2 \boldsymbol{\Psi}}{2m} + m\Phi \boldsymbol{\Psi}, \quad \nabla^2 \Phi = 4\pi G m \boldsymbol{\Psi}^\dagger \boldsymbol{\Psi}$$

$[\boldsymbol{\Psi}]_i = \psi_i$  with  $i = 1$       scalar case

$[\boldsymbol{\Psi}]_i = \psi_i$  with  $i = 1, 2, 3$       vector case

at this level this is just  $2s+1$  equal-mass scalar fields  
but not when non-gravitational interactions are included

Zhang, Jain & MA (2111.08700)

# non-relativistic limit = conserved spin angular momentum

$$i\partial_t \boldsymbol{\Psi} = -\frac{\nabla^2 \boldsymbol{\Psi}}{2m} + m\Phi \boldsymbol{\Psi}, \quad \nabla^2 \Phi = 4\pi G m \boldsymbol{\Psi}^\dagger \boldsymbol{\Psi}$$

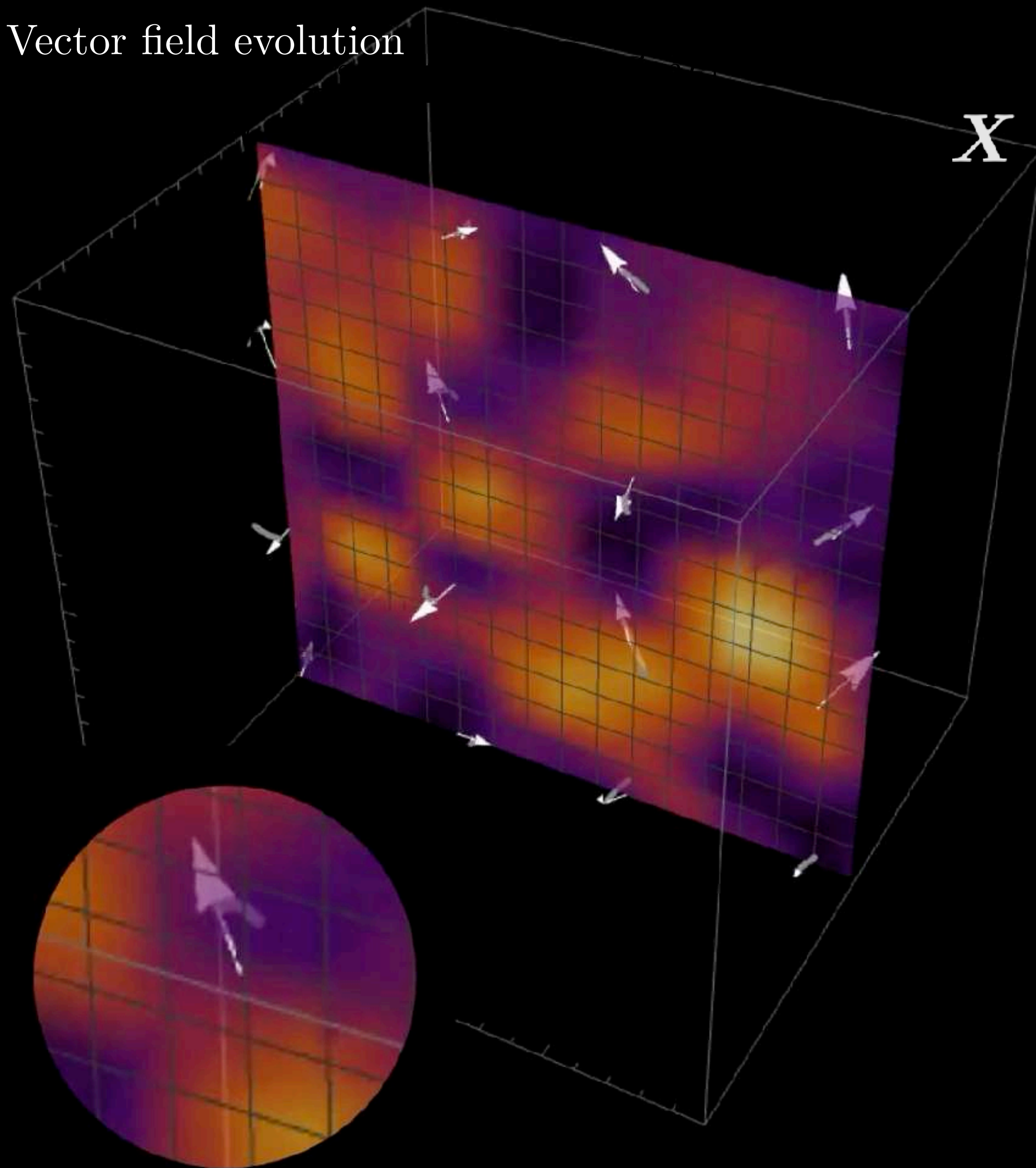
$[\boldsymbol{\Psi}]_i = \psi_i$  with  $i = 1$       scalar case

$[\boldsymbol{\Psi}]_i = \psi_i$  with  $i = 1, 2, 3$       vector case

Spin angular momentum  $S = \hbar \int d^3x i \boldsymbol{\Psi} \times \boldsymbol{\Psi}^\dagger$

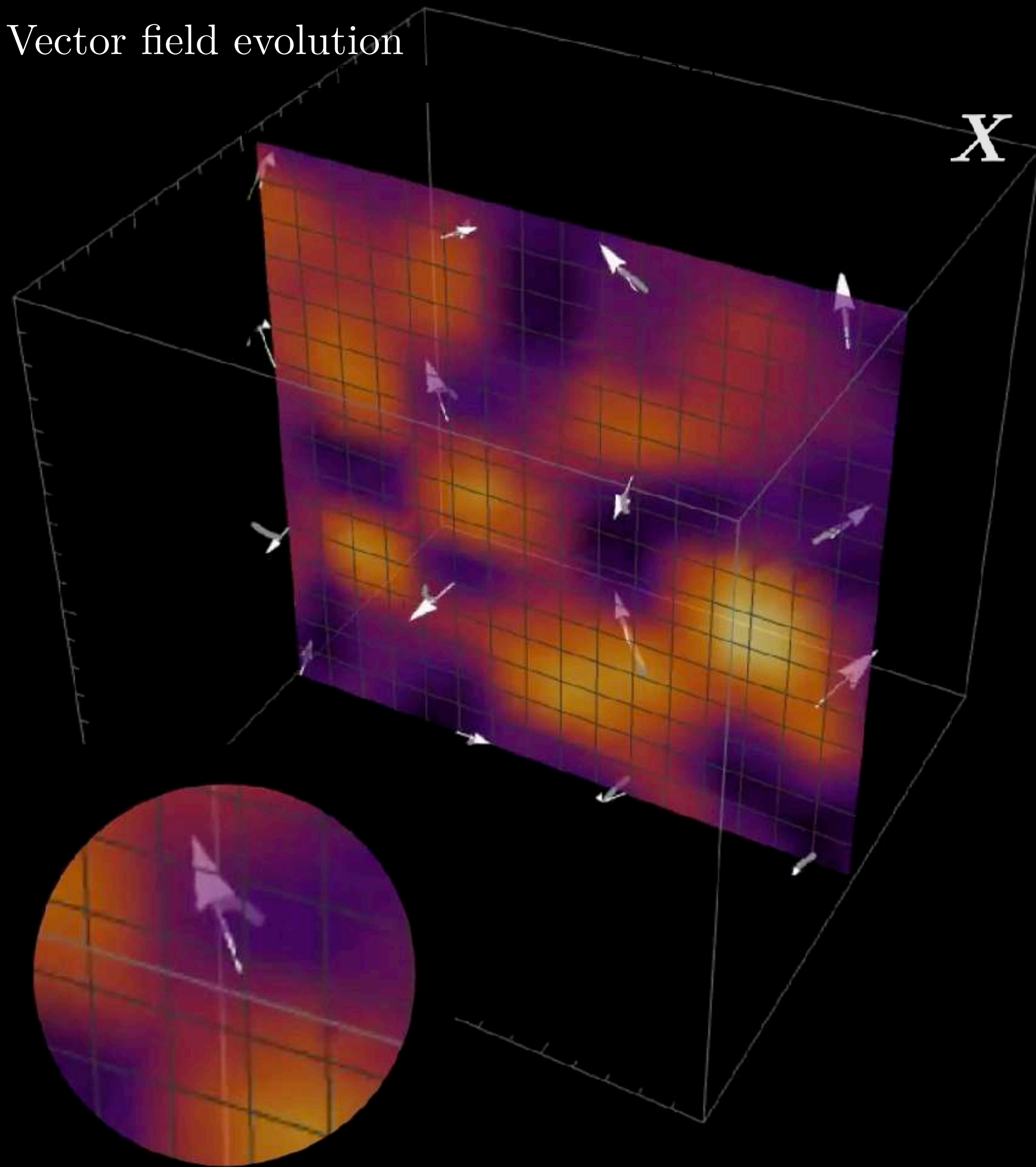
Jain & MA (2109.04892)

# Vector field evolution

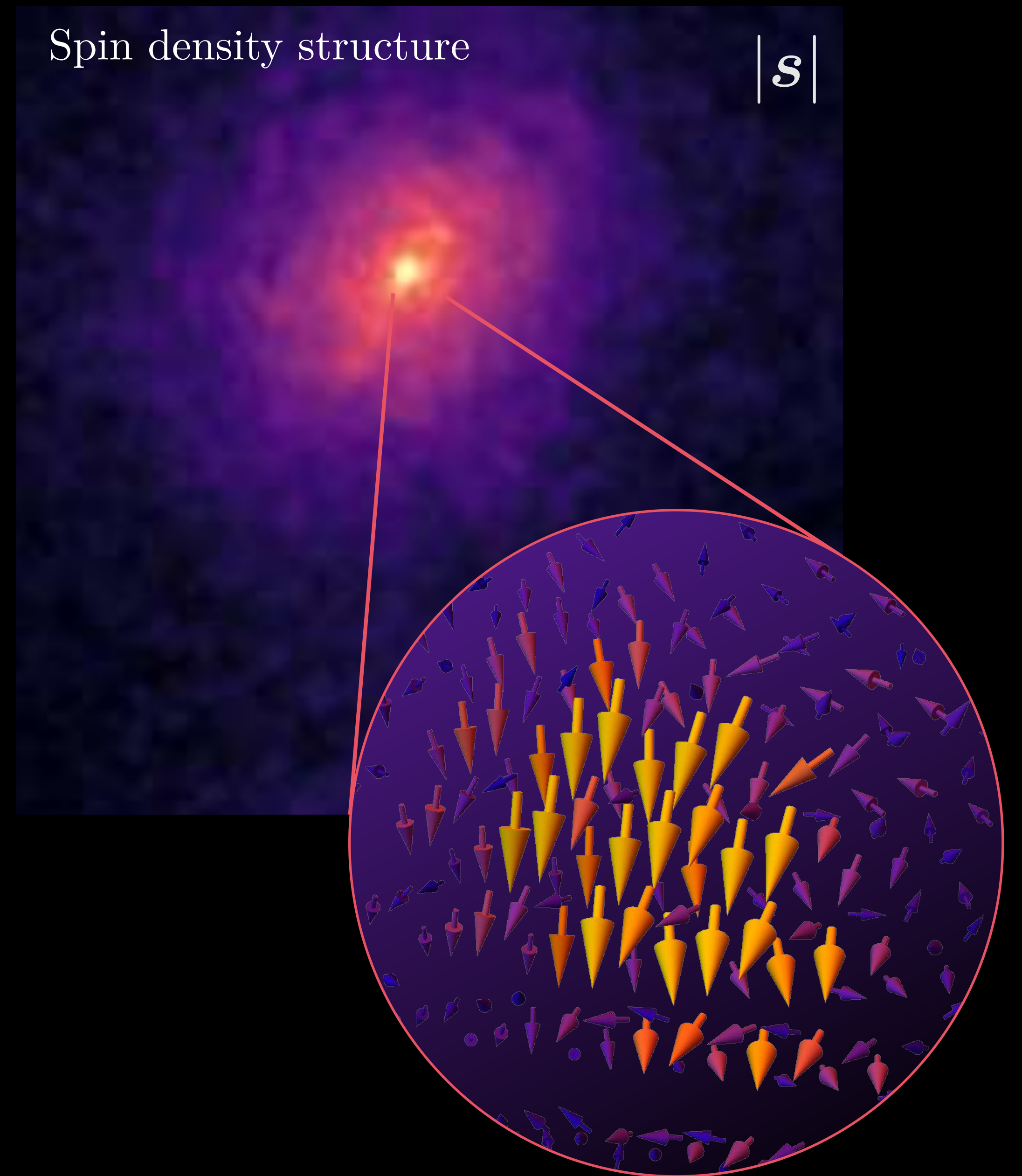




Vector field evolution



Spin density structure





# how spin can effect small scale structure in wave DM

wave interference

solitons

Bose effects

interactions

initial conditions+growth of structure

# what effect is relevant?

wave interference

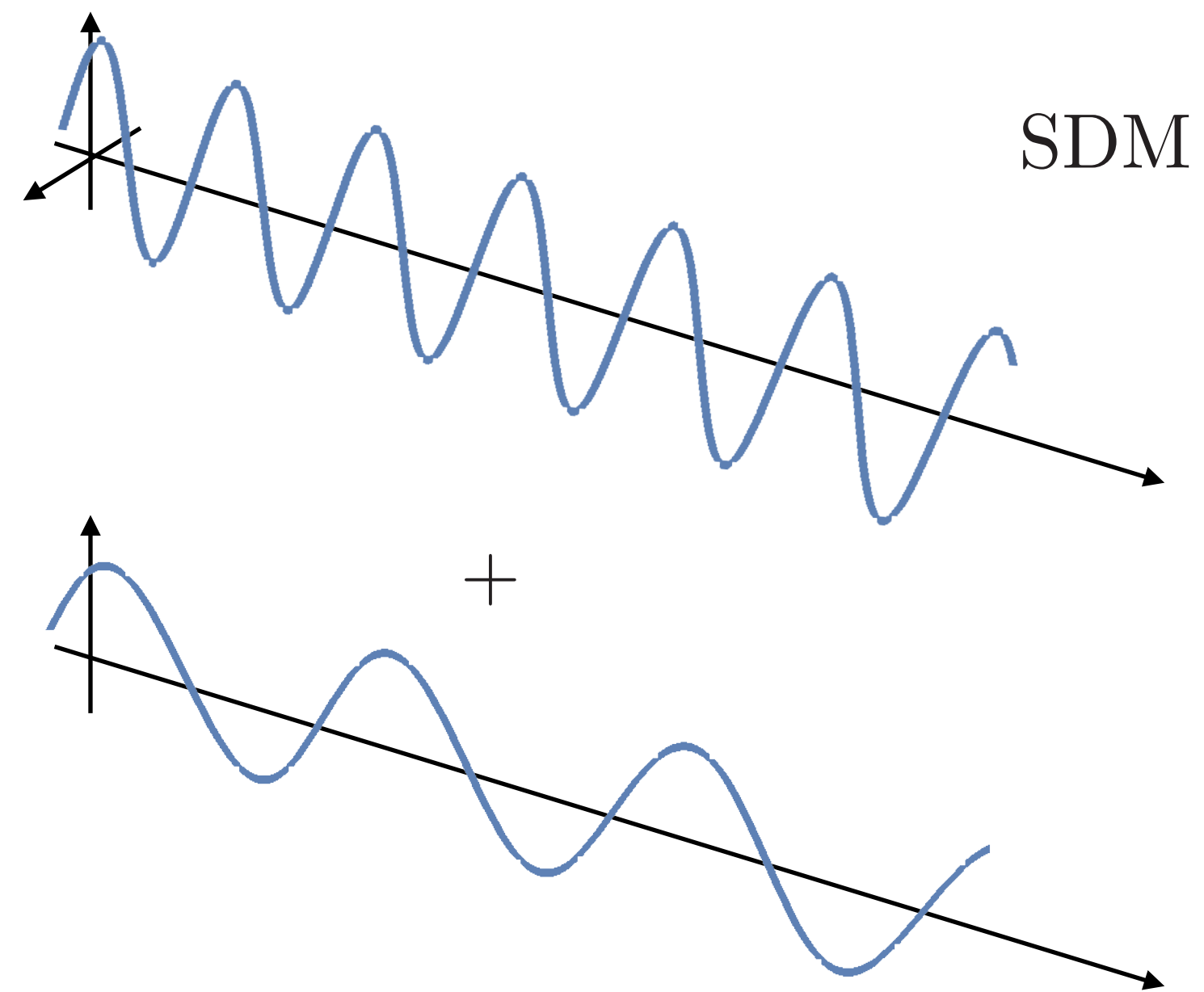
solitons

Bose effects

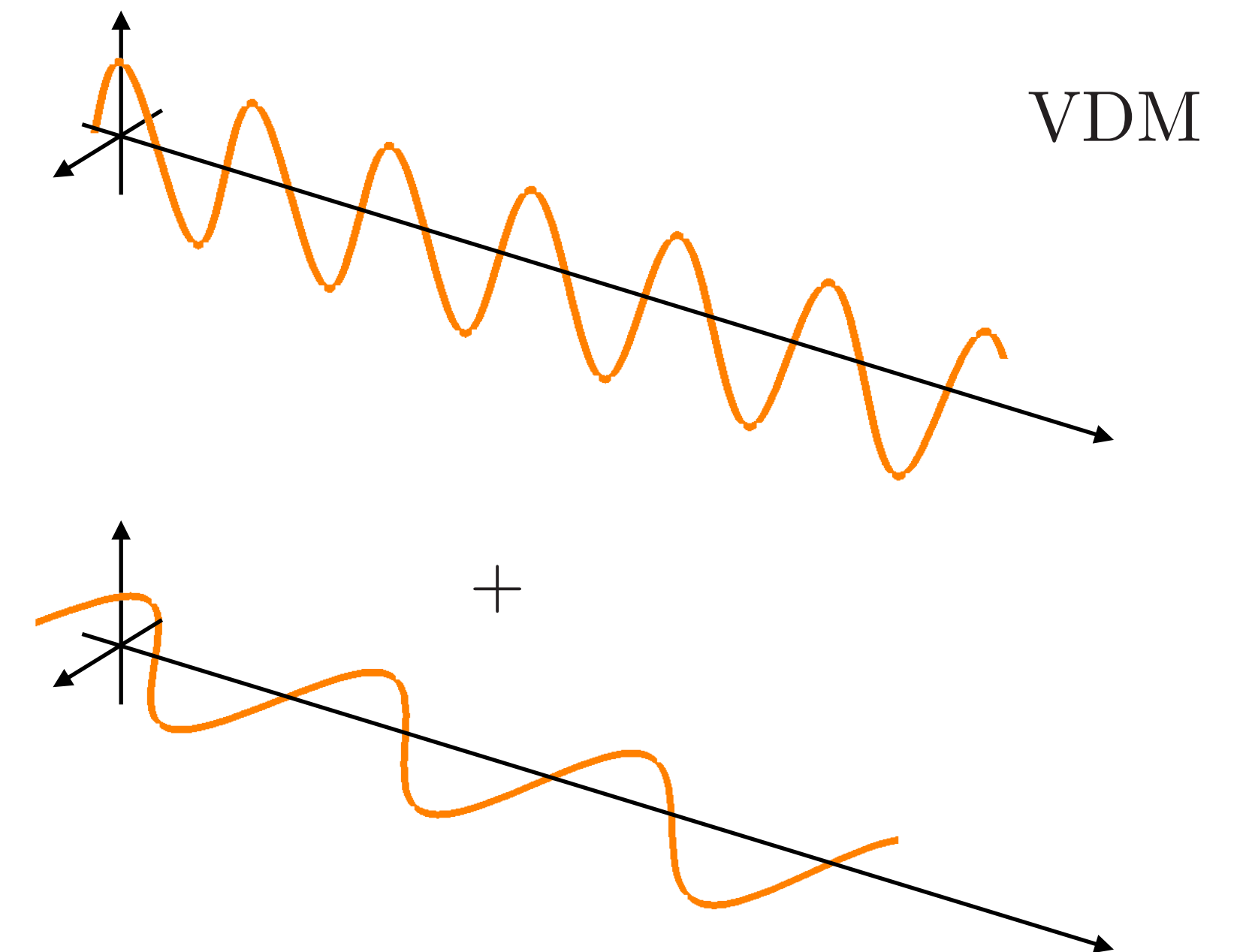
interactions

initial conditions+growth of structure

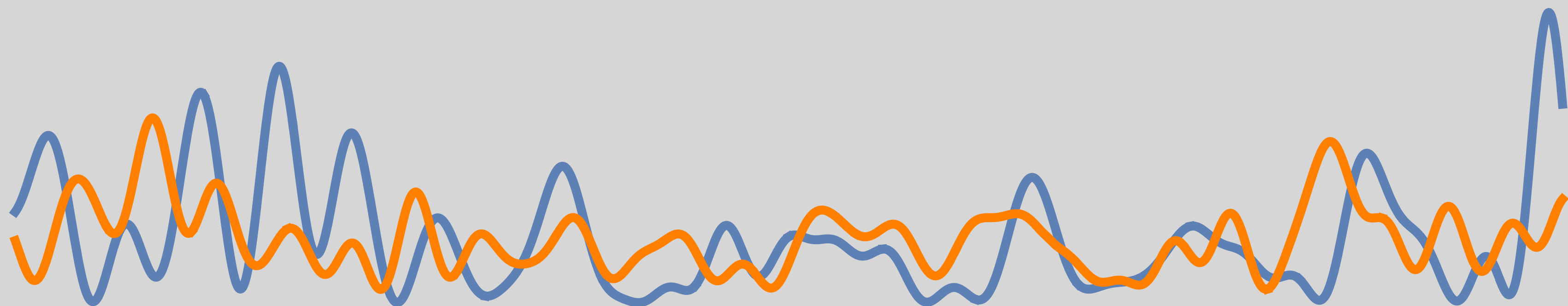
# wave interference



$$|\Psi_a(\mathbf{x}) + \Psi_b(\mathbf{x})|^2 \neq |\Psi_a(\mathbf{x})|^2 + |\Psi_b(\mathbf{x})|^2$$



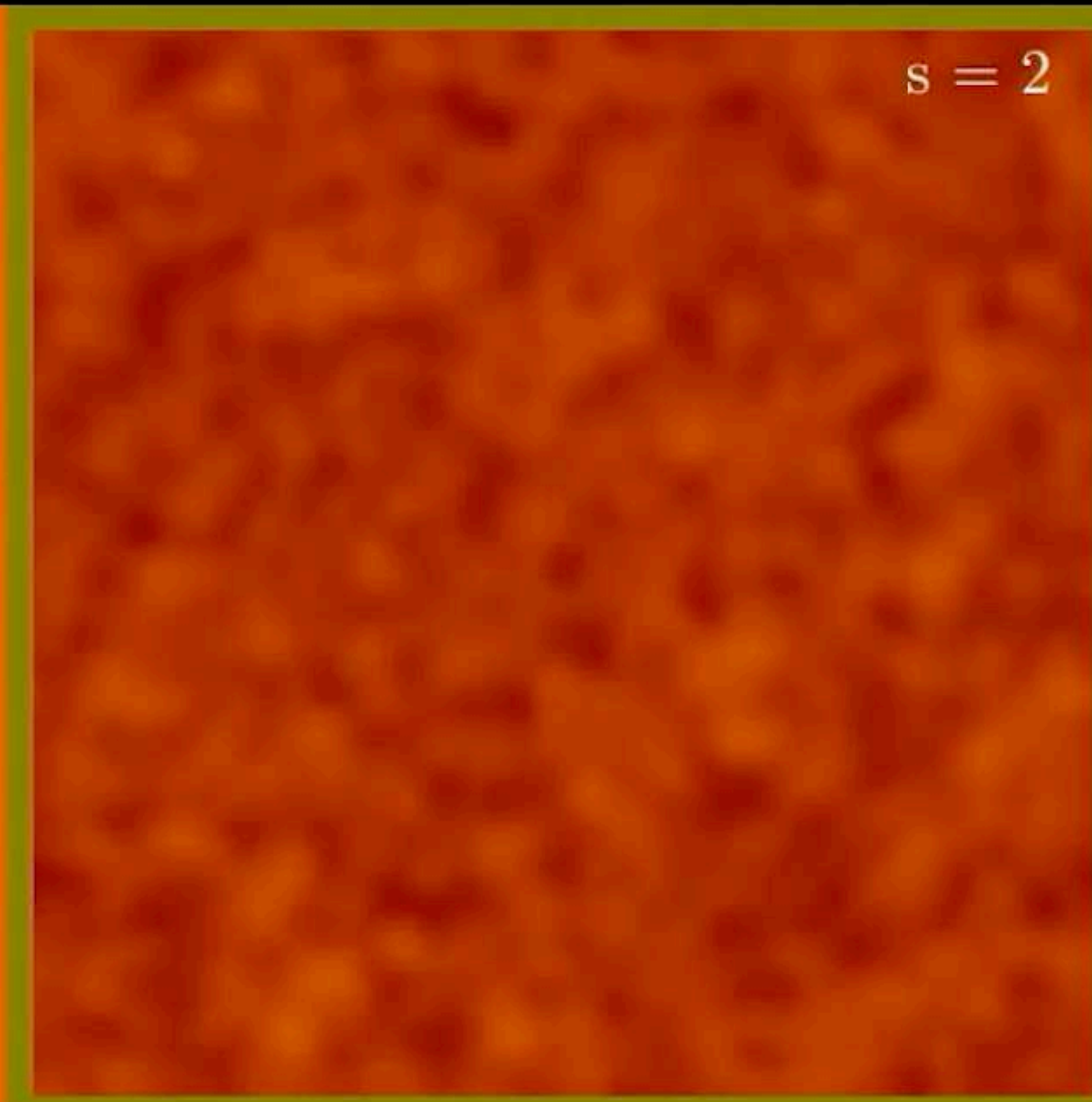
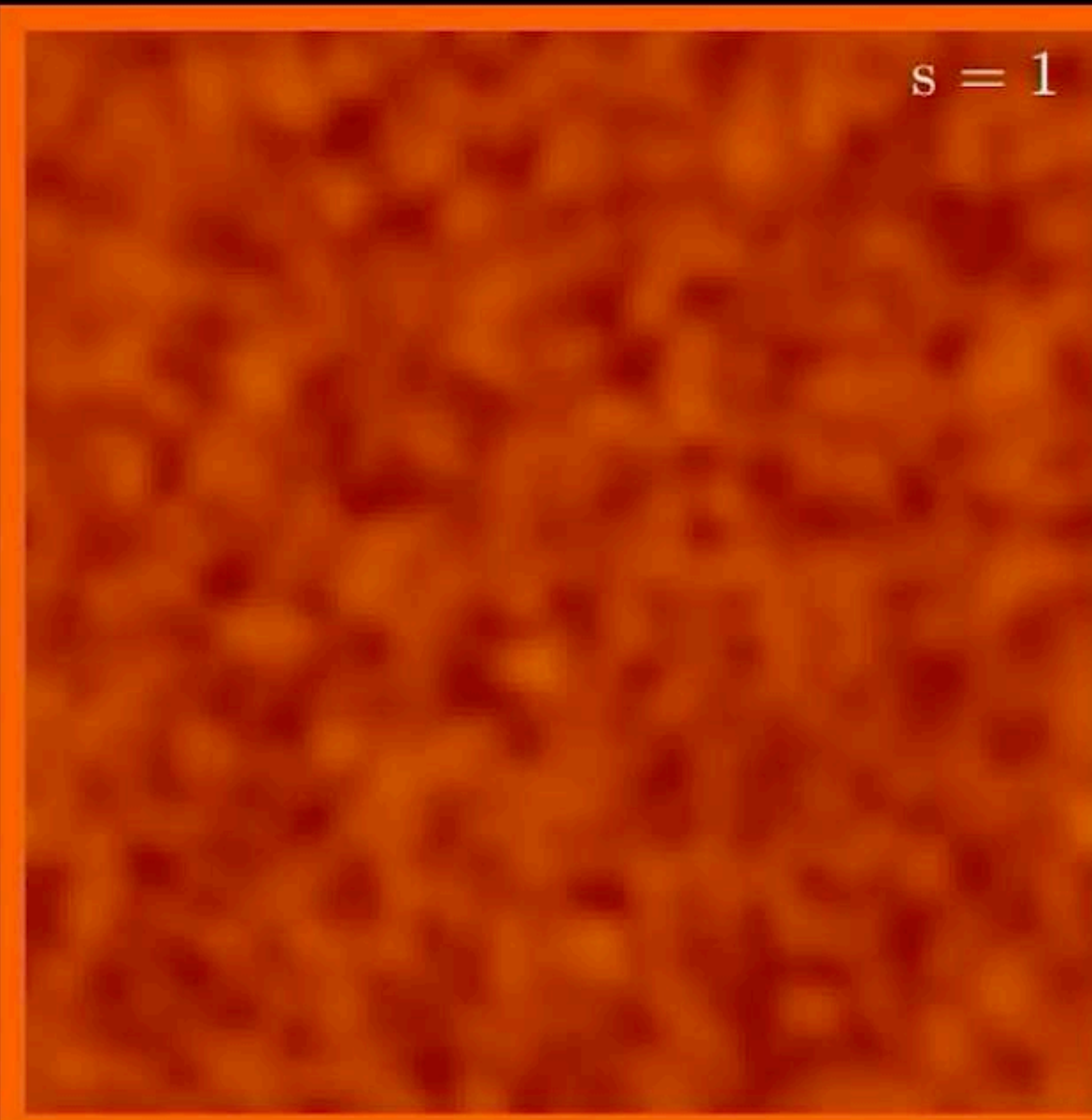
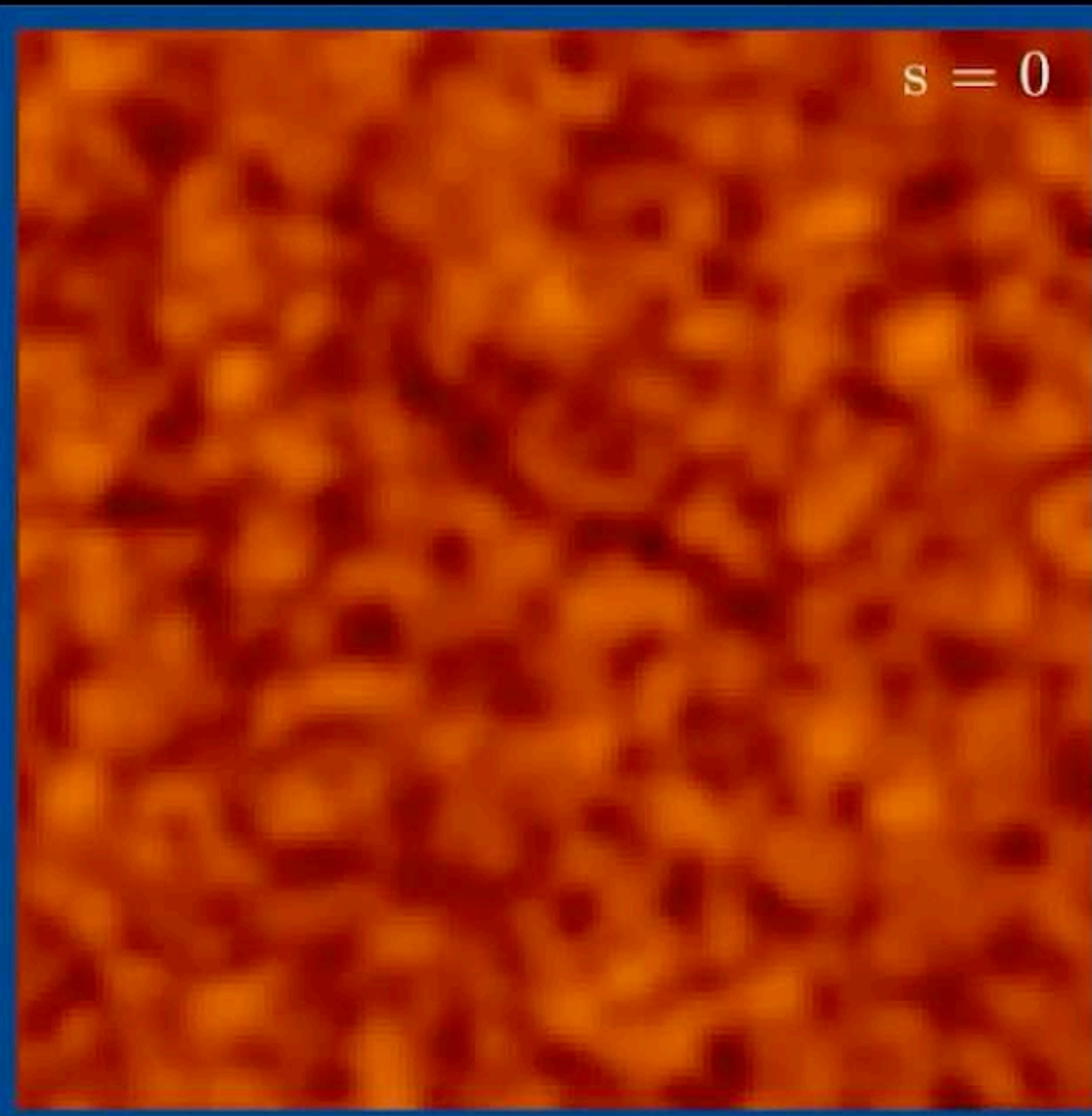
$$|\Psi_a(\mathbf{x}) + \Psi_b(\mathbf{x})|^2 = |\Psi_a(\mathbf{x})|^2 + |\Psi_b(\mathbf{x})|^2$$





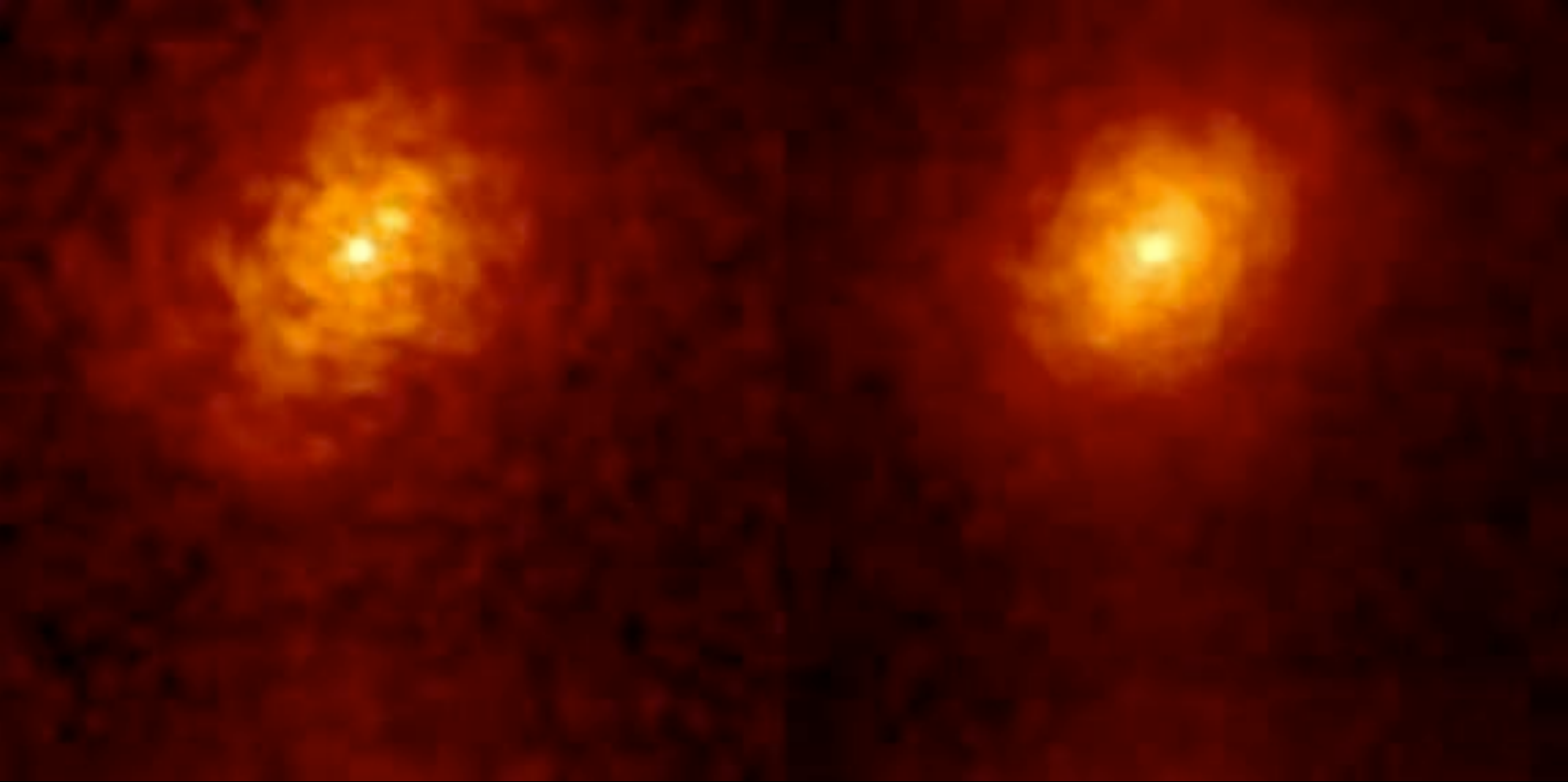
# reduced interference

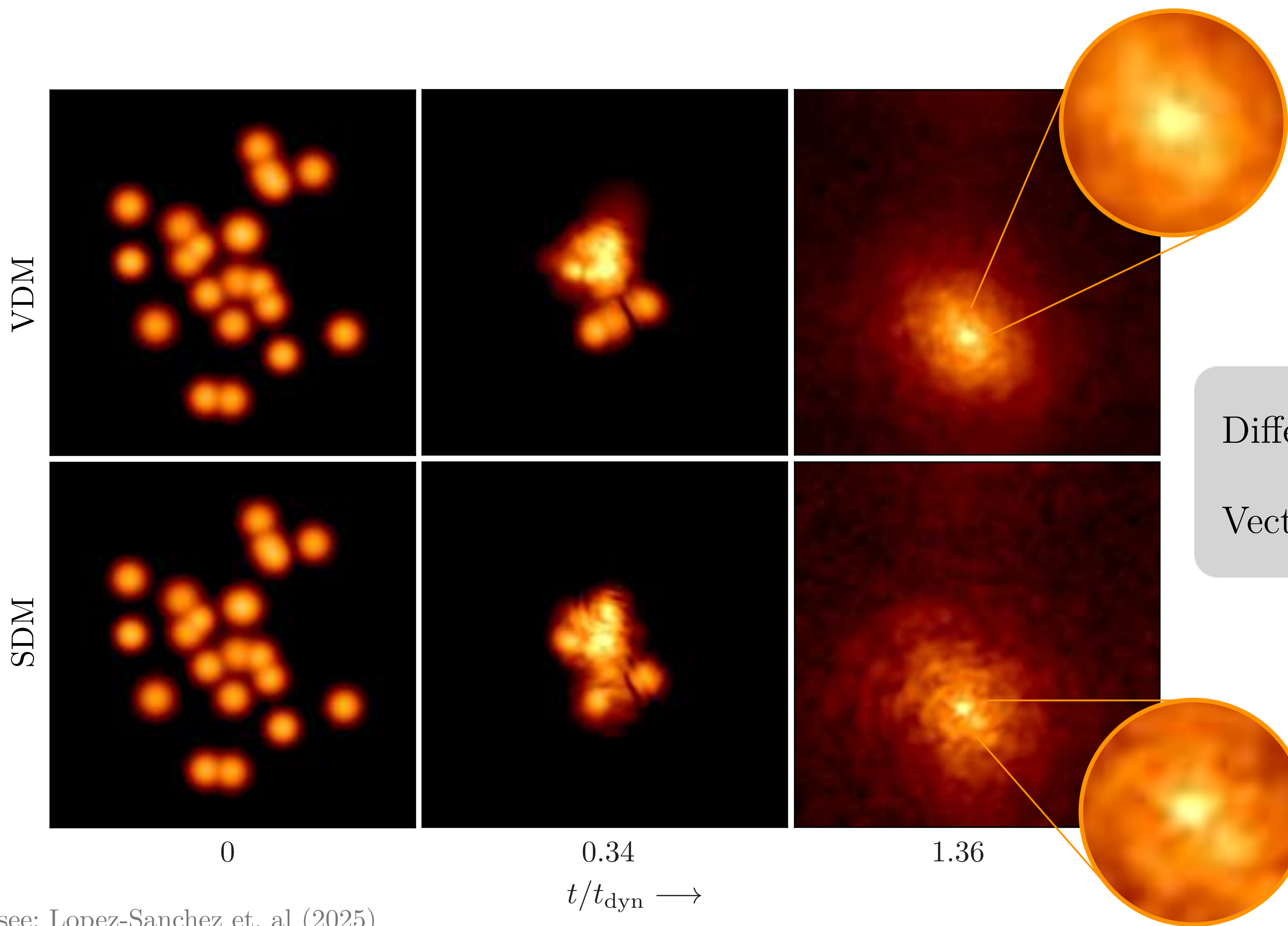
$$\frac{\delta\rho}{\rho} \propto \frac{1}{\sqrt{2s+1}}$$











Difference between

Vector & Scalar Dark Matter

also see: Lopez-Sanchez et. al (2025)  
Chen, Yao & Marsh (2026)

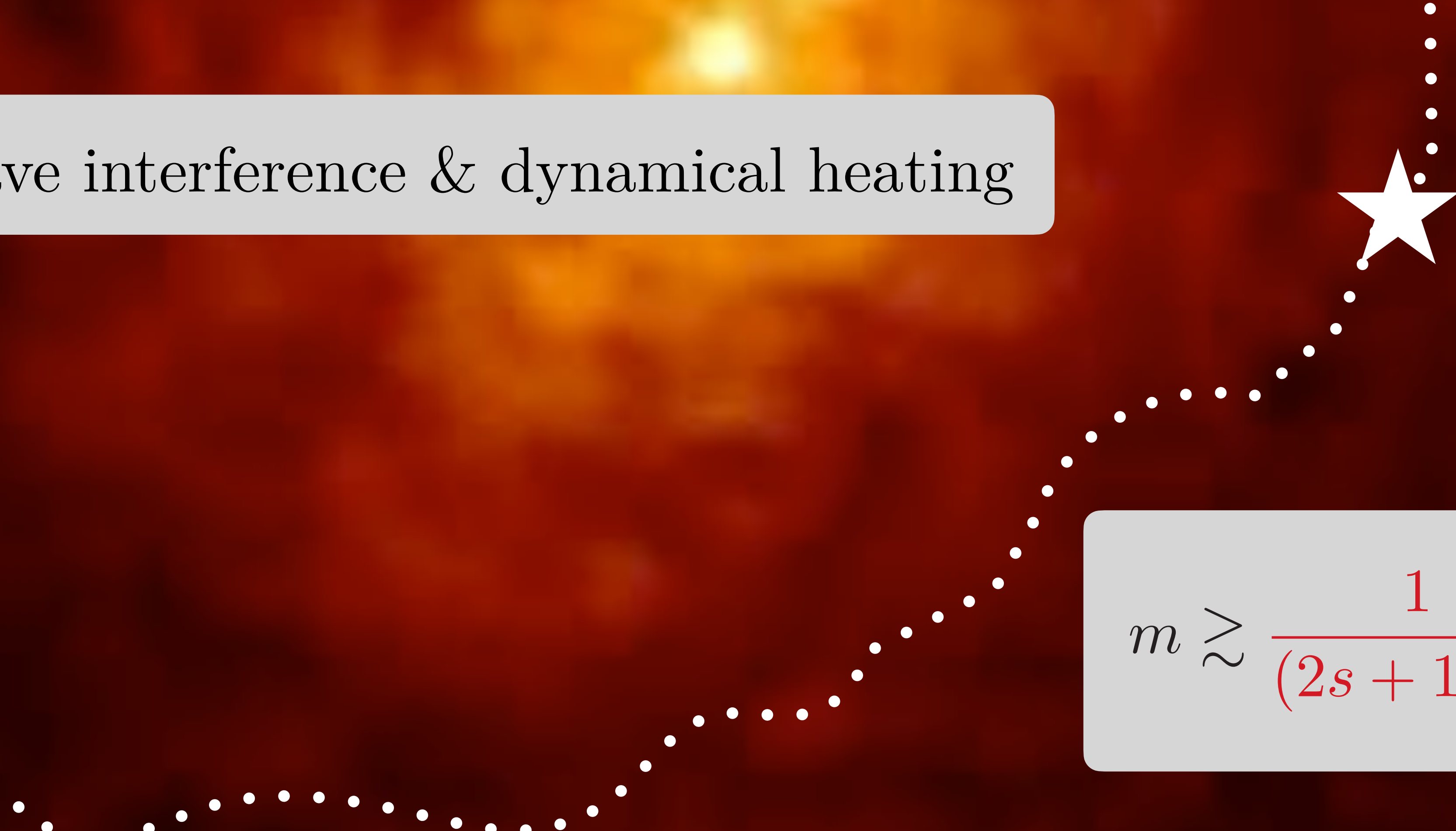
MA, Karur, Jain & Mocz (2203.11935)



wave interference



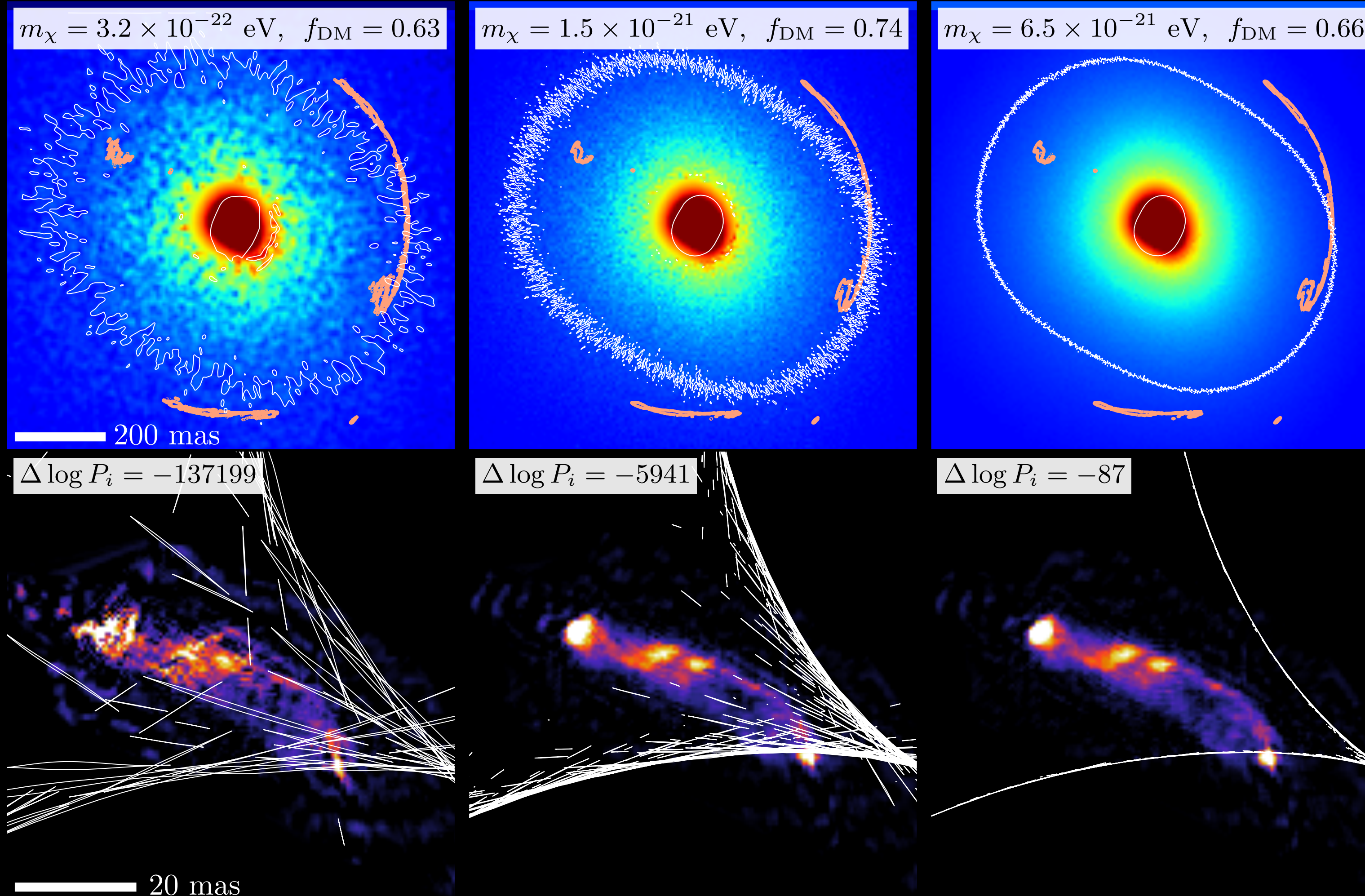
wave interference & dynamical heating


$$m \gtrsim \frac{1}{(2s+1)^{1/3}} \left[ 3 \times 10^{-19} \text{eV} \right]$$

Dalal & Kratsov (2203.05750)

MA, Jain, Karur & Mocz (2203.11935)

# wave interference & lensing

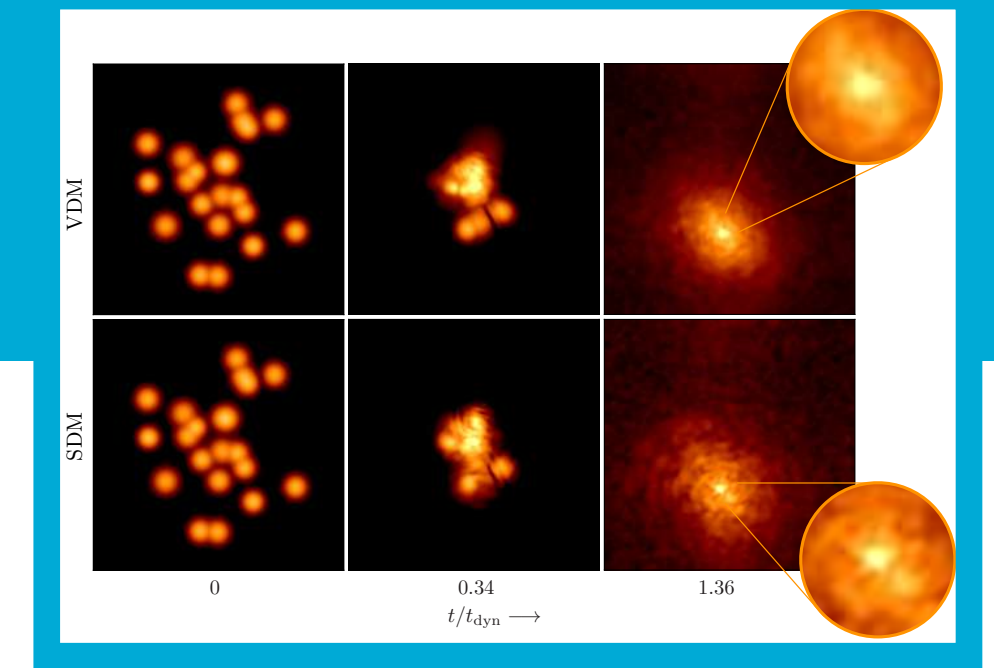


$$m \gtrsim \frac{1}{(2s+1)} \left[ 4.4 \times 10^{-21} \text{ eV} \right]$$

Powell et. al (2023)

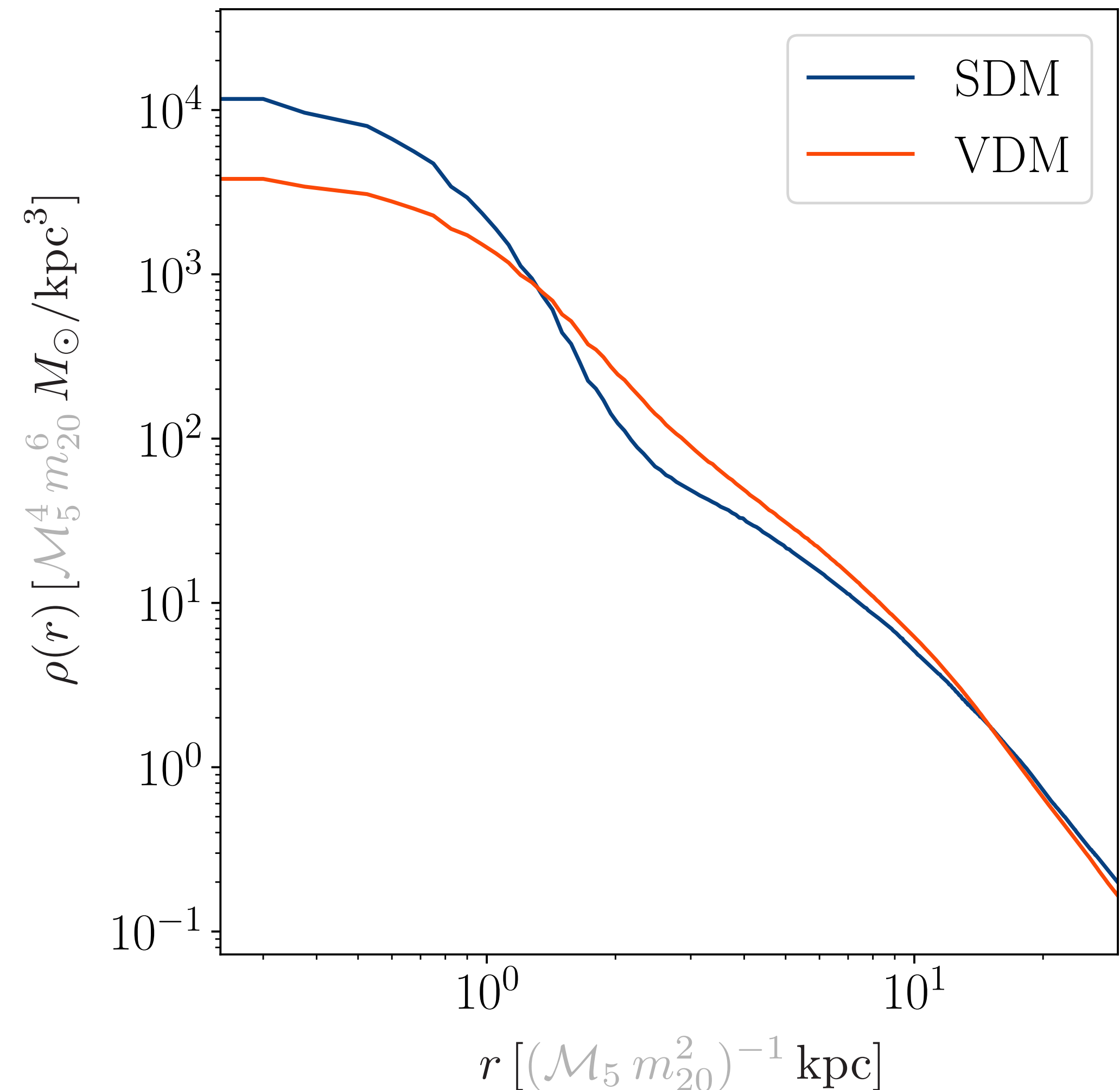
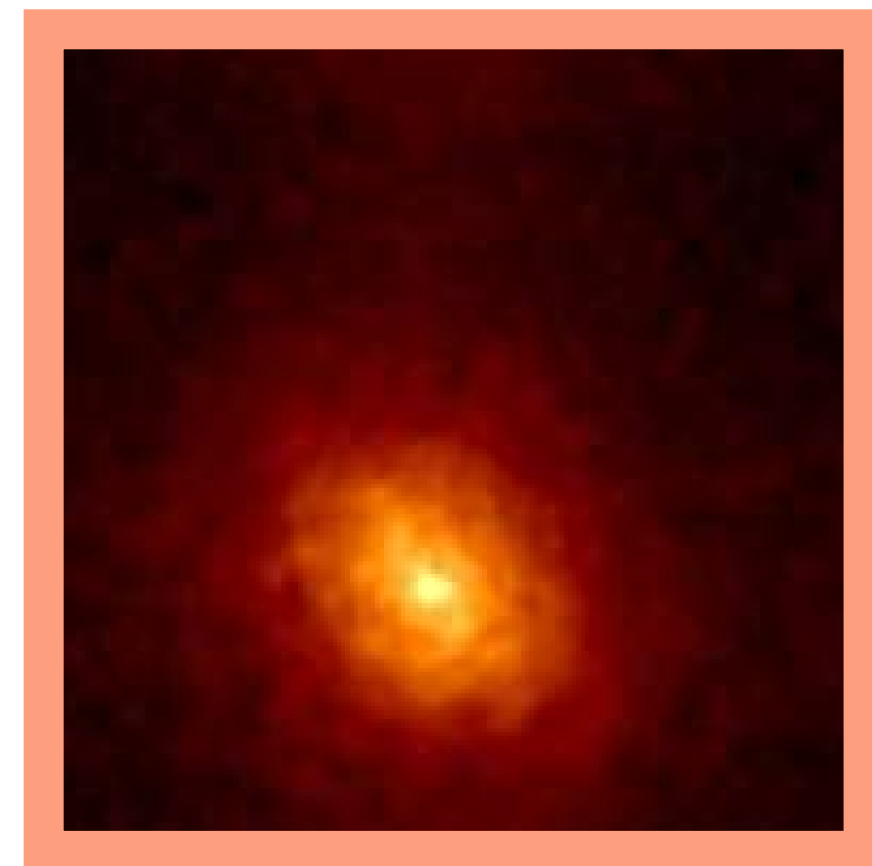
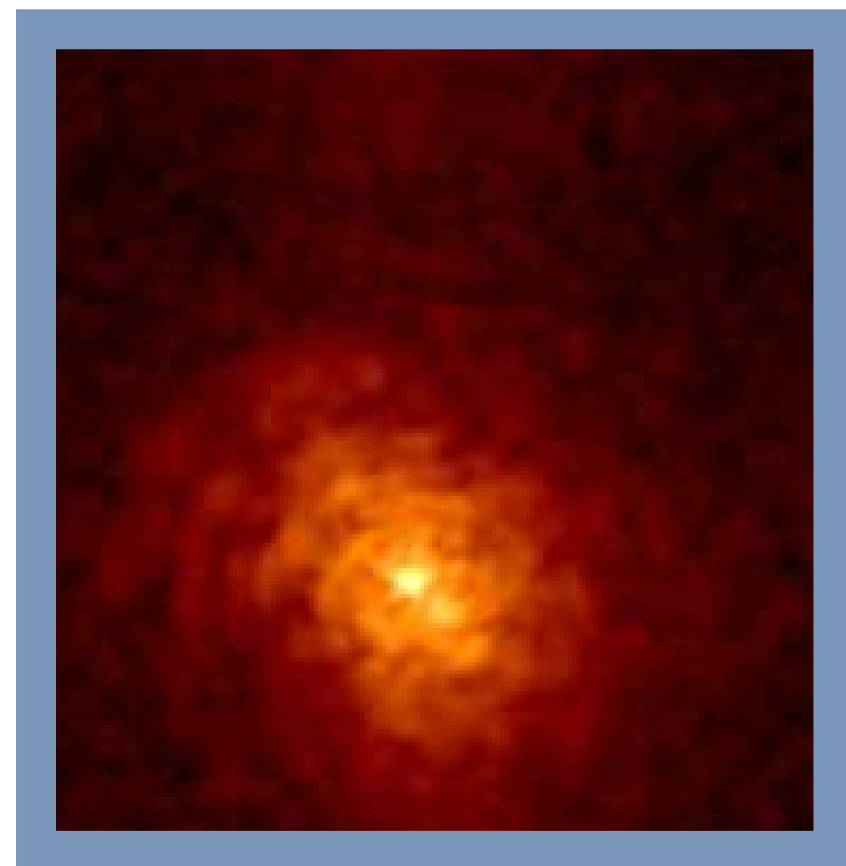


# wave interference & radial density profiles



scalar vs. **vector** dark matter

- less dense & broader core
- smoother transition to  $r^{-(2-3)}$  tail



# how spin can effect small scale structure in wave DM

wave interference

solitons

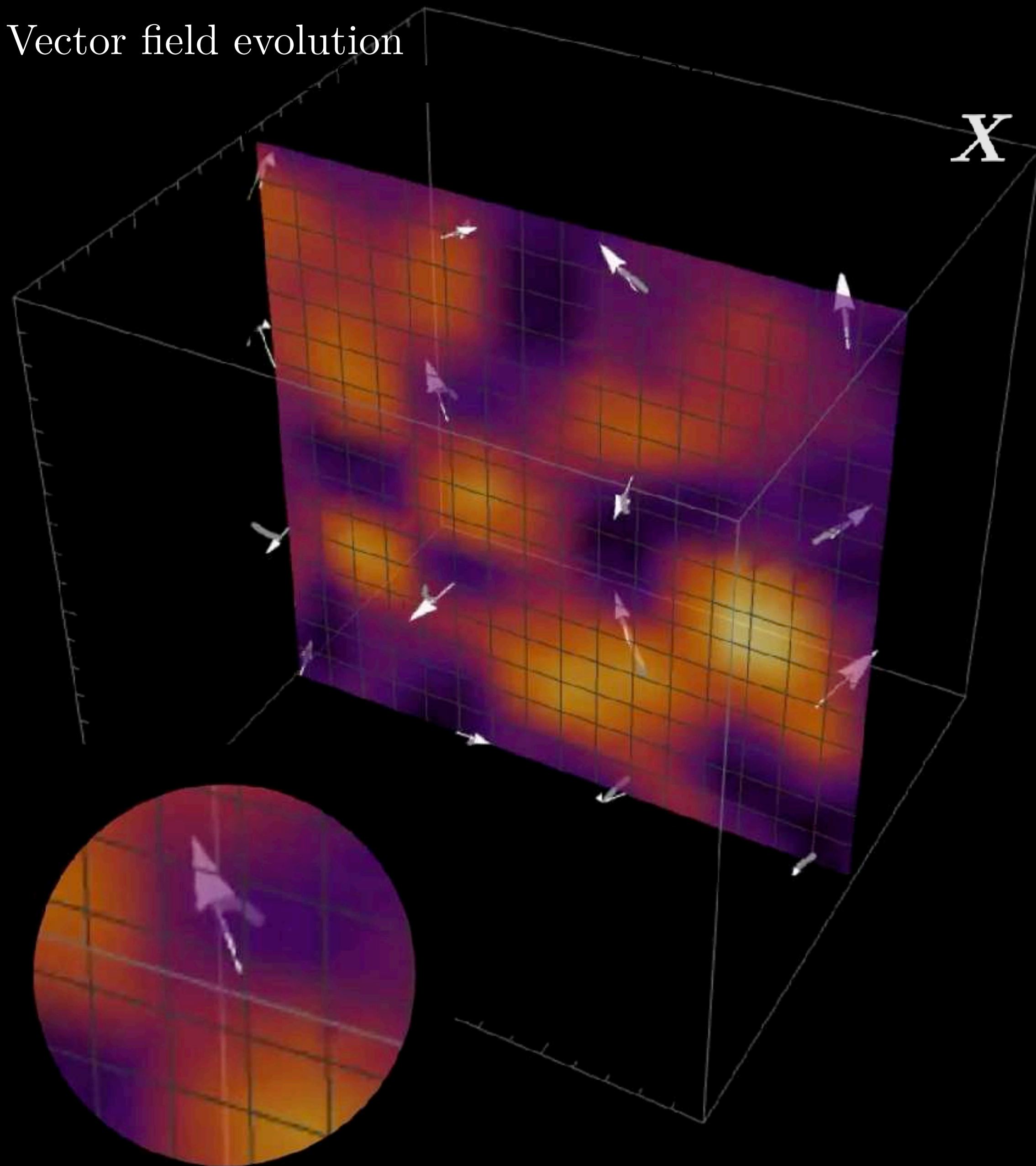
Bose effects

interactions

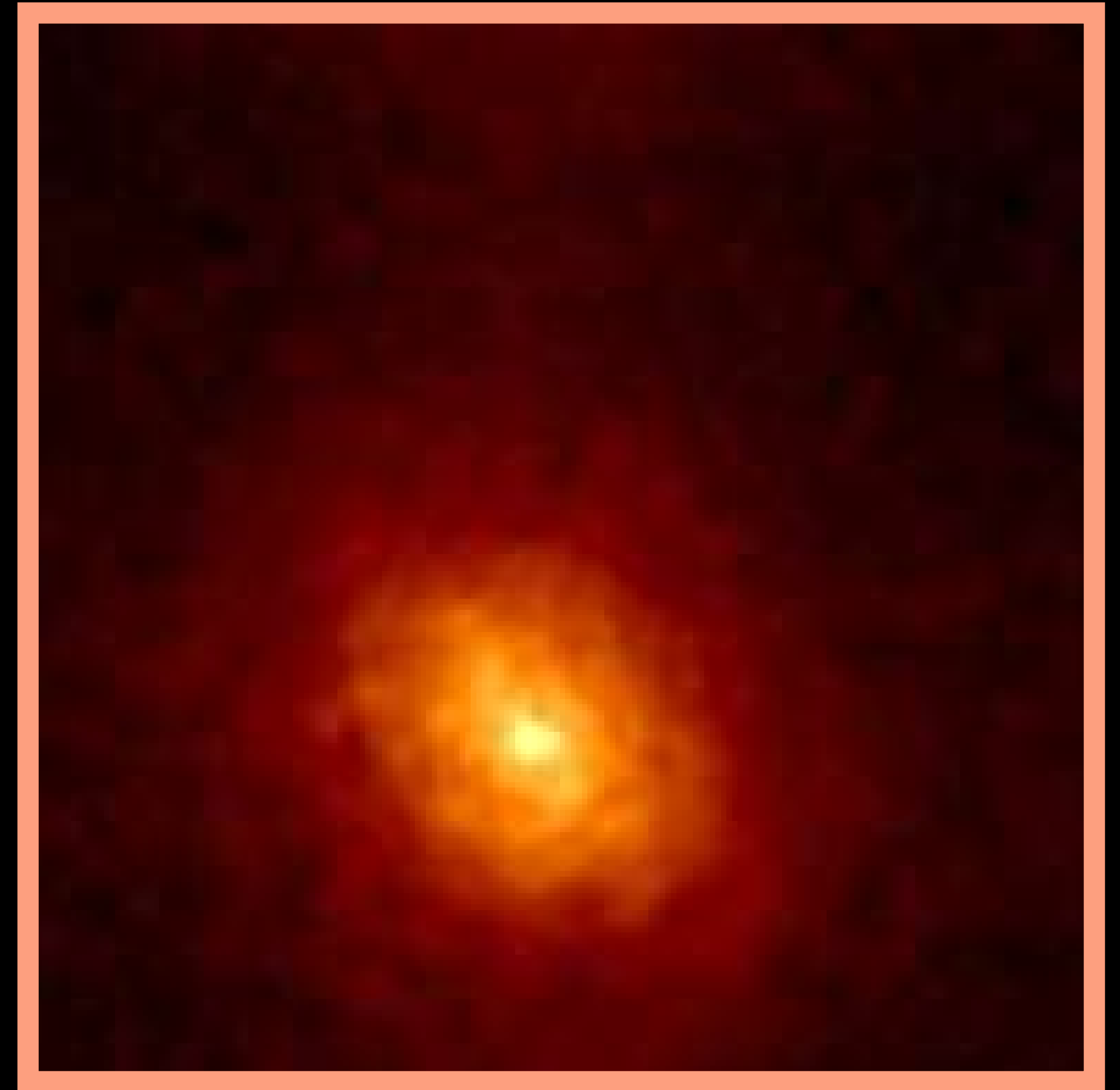
initial conditions+growth of structure



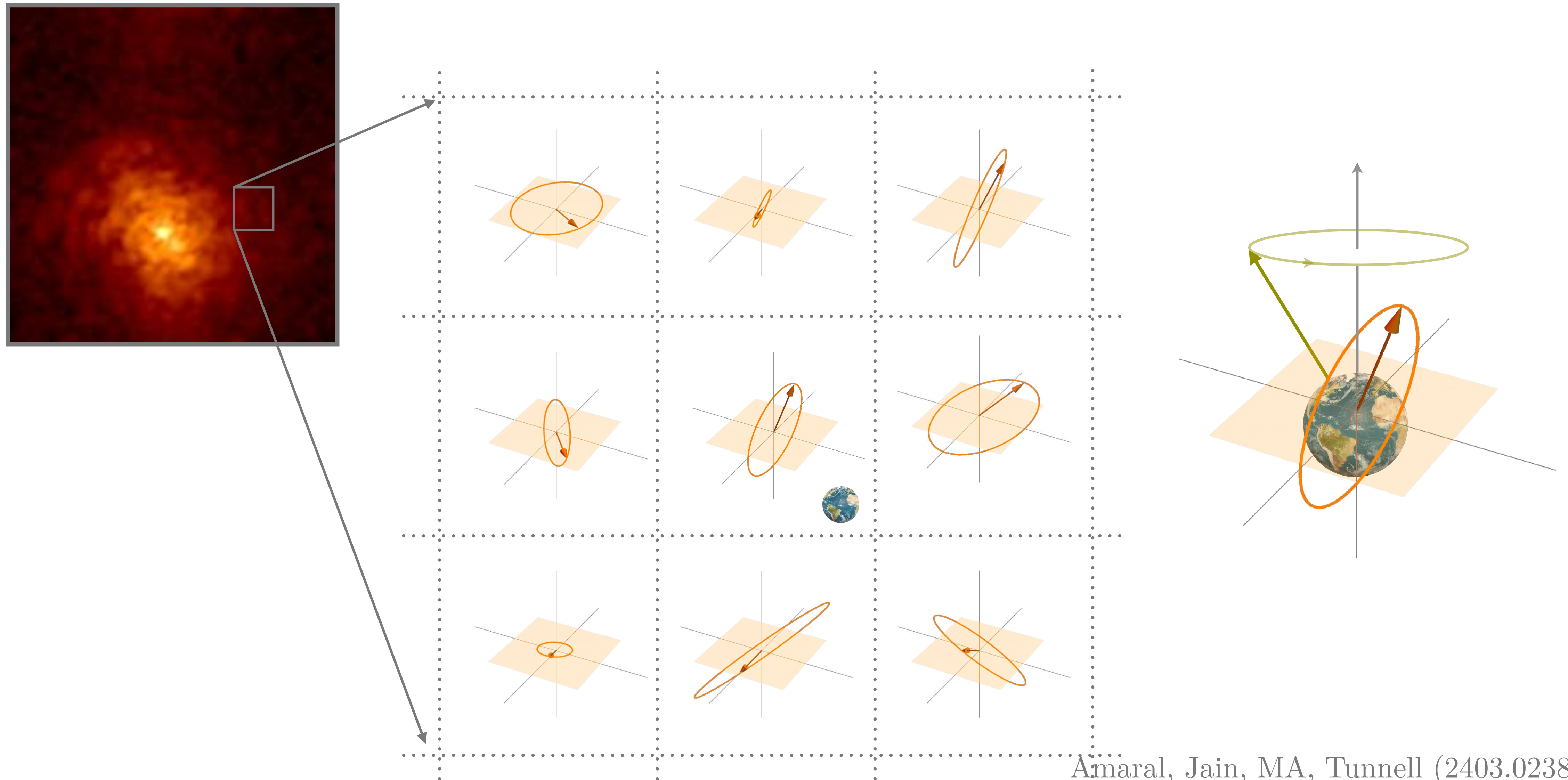
Vector field evolution



Amaral et. al (2024)

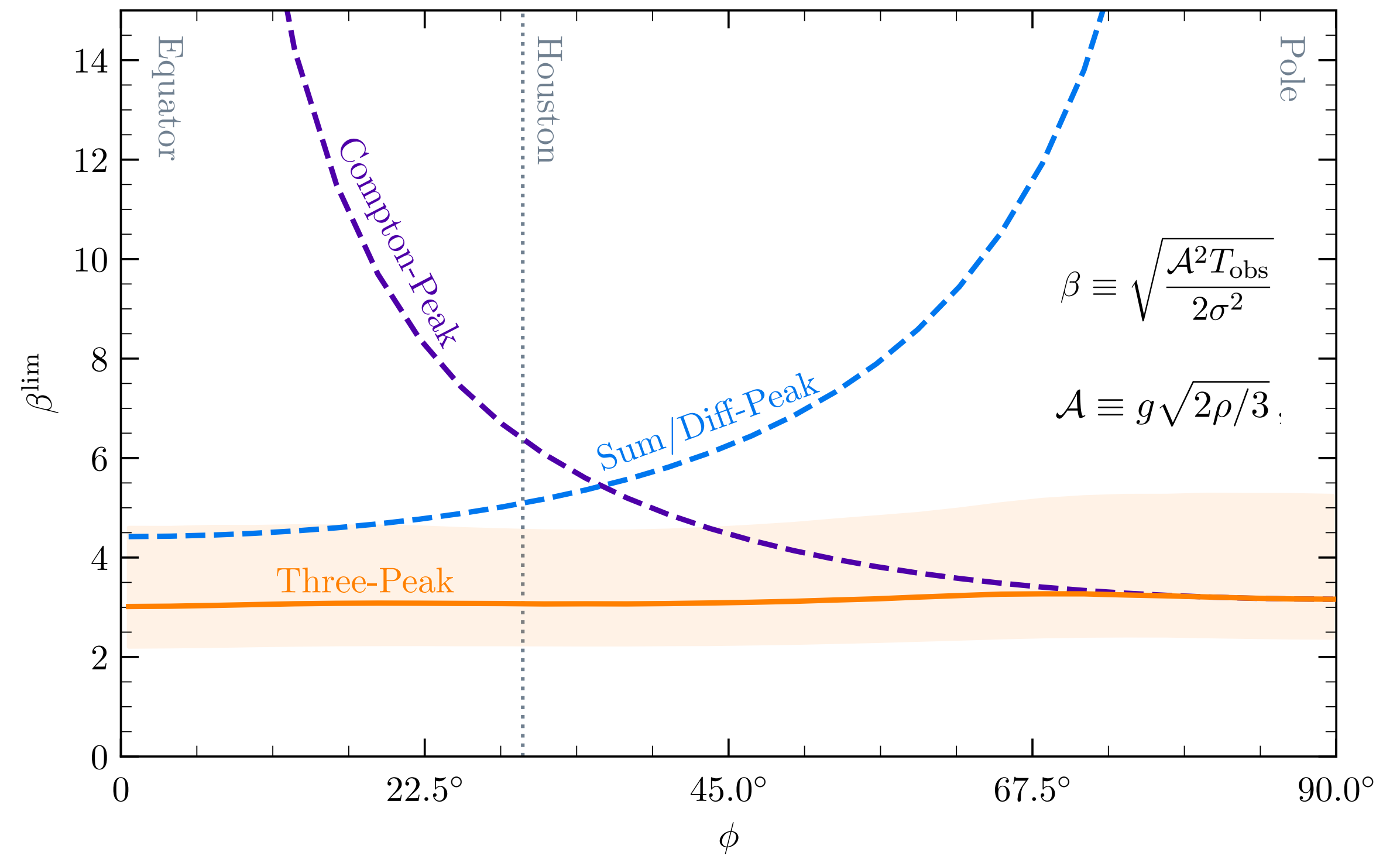
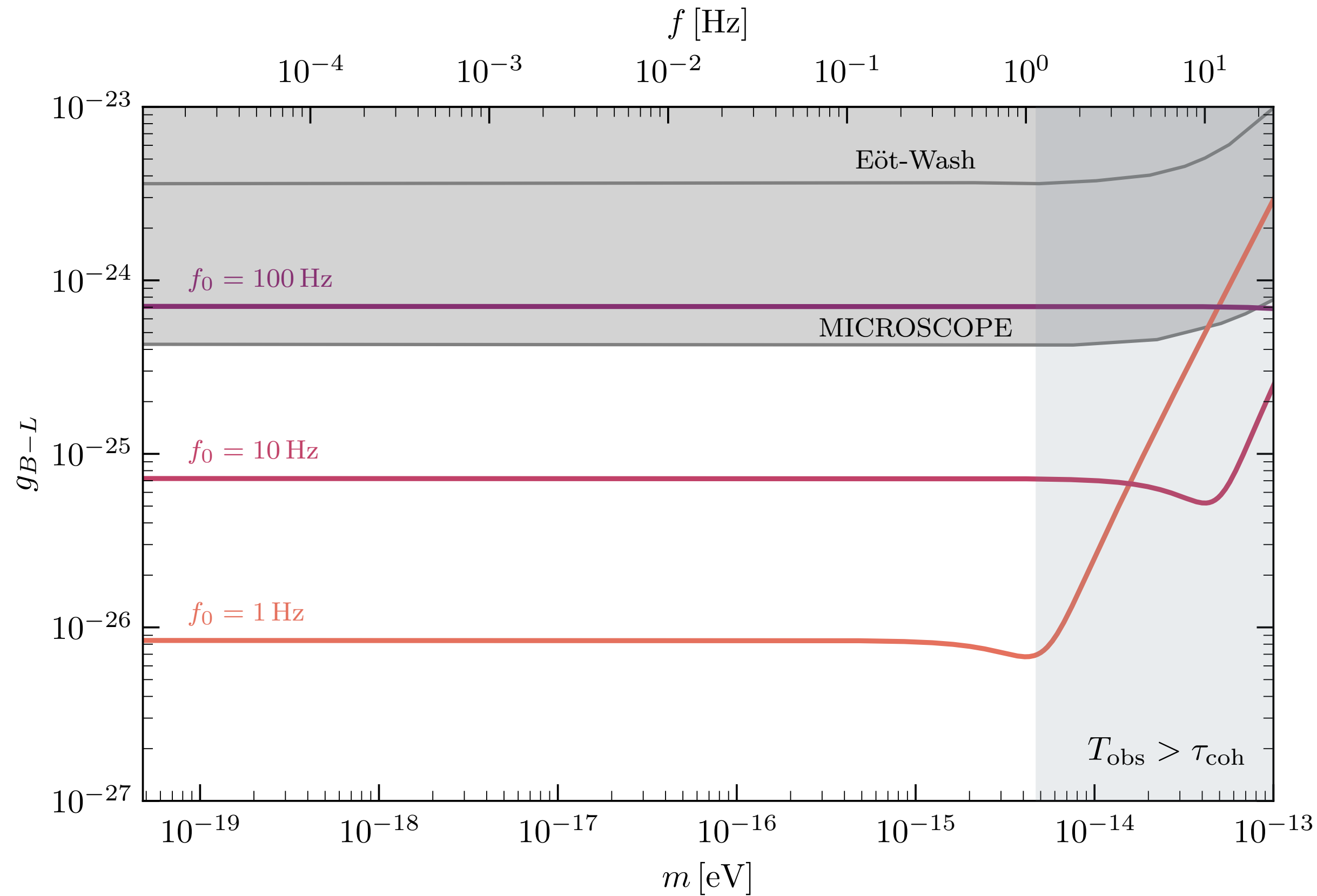


# interference effects and direct detection



# interference/polarization effects and direct detection

$$\mathcal{L} \supset -\frac{1}{4}A'^{\mu\nu}A'_{\mu\nu} - \frac{1}{2}m^2A'^{\mu}A'_{\mu} - g_{B-L}j_{B-L}^{\mu}A'_{\mu}$$





# what effects are relevant

wave interference

solitons

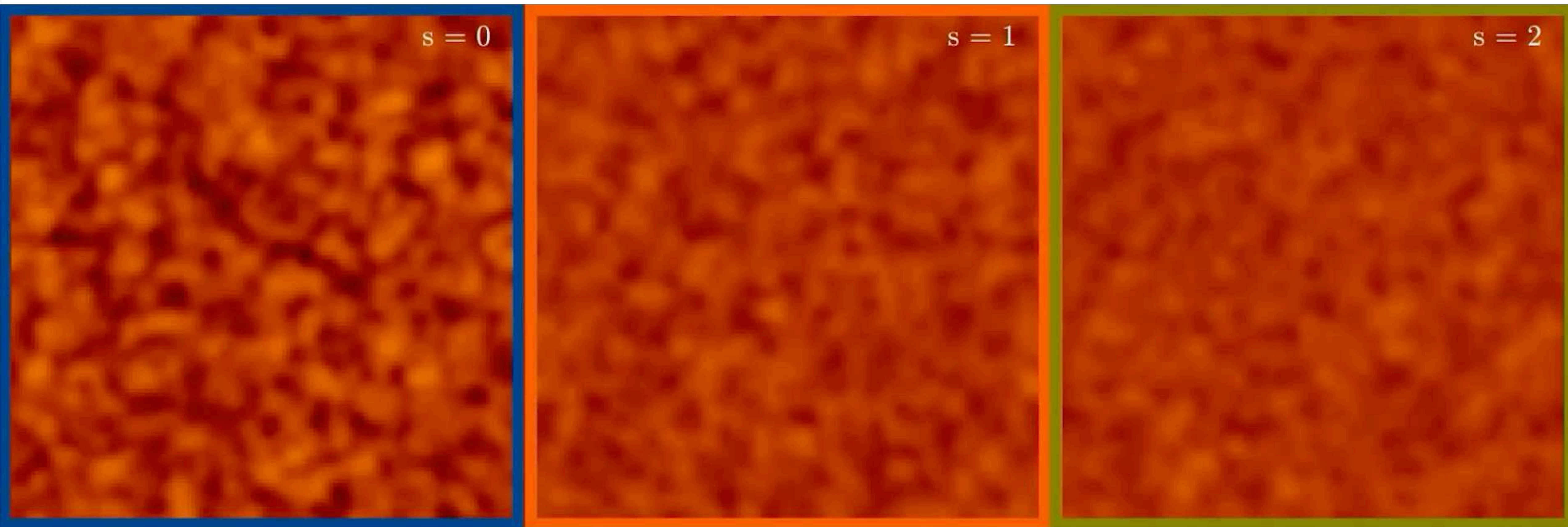
Bose effects

interactions

initial conditions+growth of structure

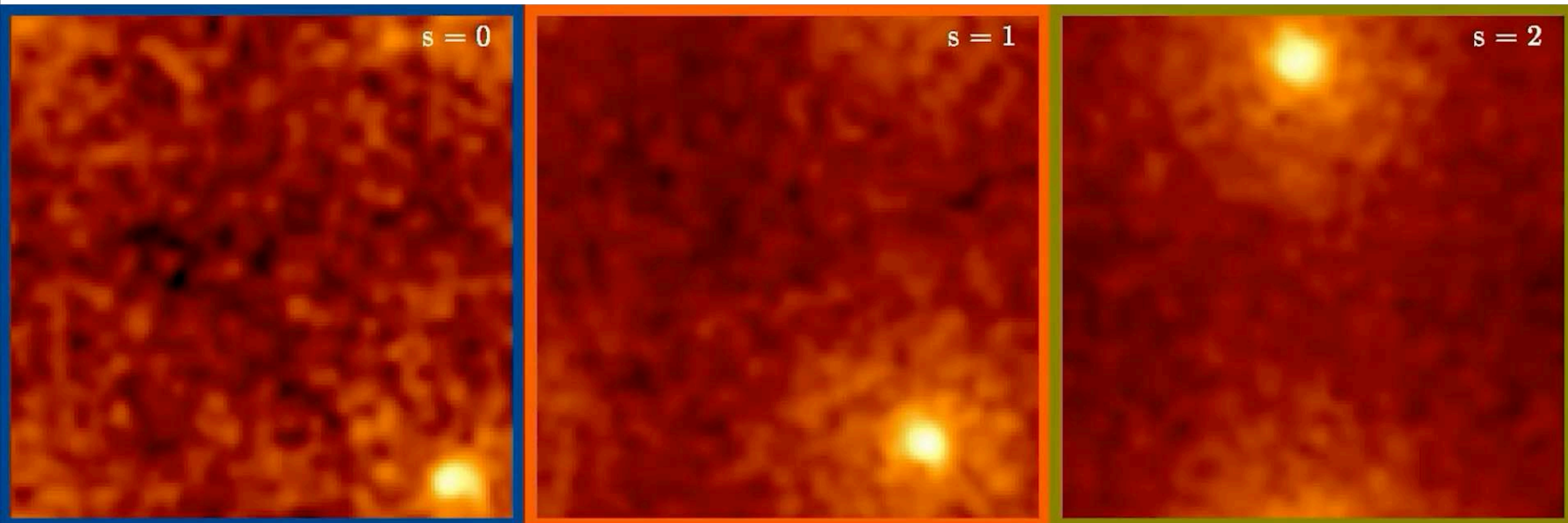


# interference & Bose: condensation in the kinetic regime





# what are these blobs?



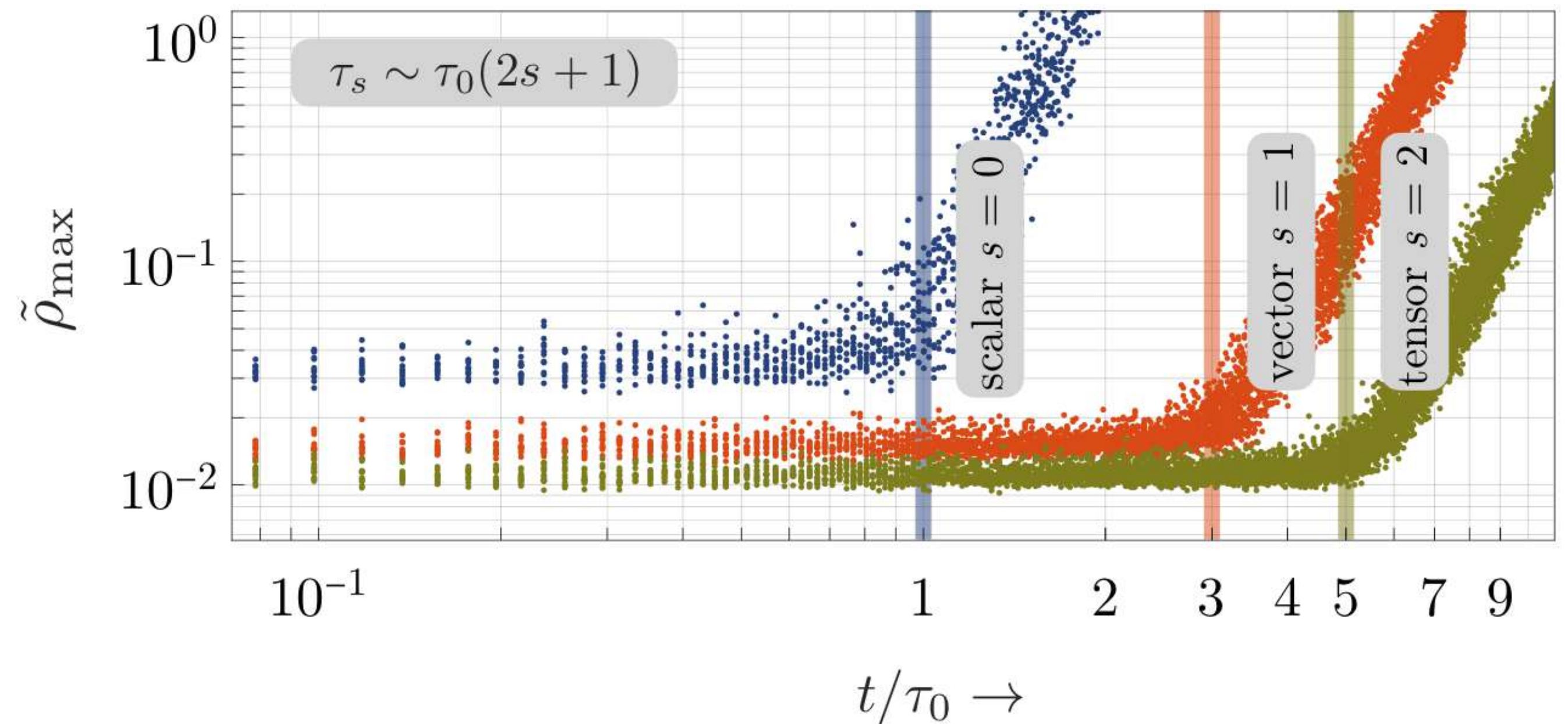
# condensation in the kinetic regime

- nucleation time scale

$$\tau_s \sim (2s + 1) \tau_{s=0}$$

$$\tau_{s=0} = [n \sigma_{\text{gr}} v \mathcal{N}]^{-1}$$

$$\sigma_{\text{gr}} \sim (Gm/v^2)^2, \quad \mathcal{N} \sim n \lambda_{\text{dB}}^3$$

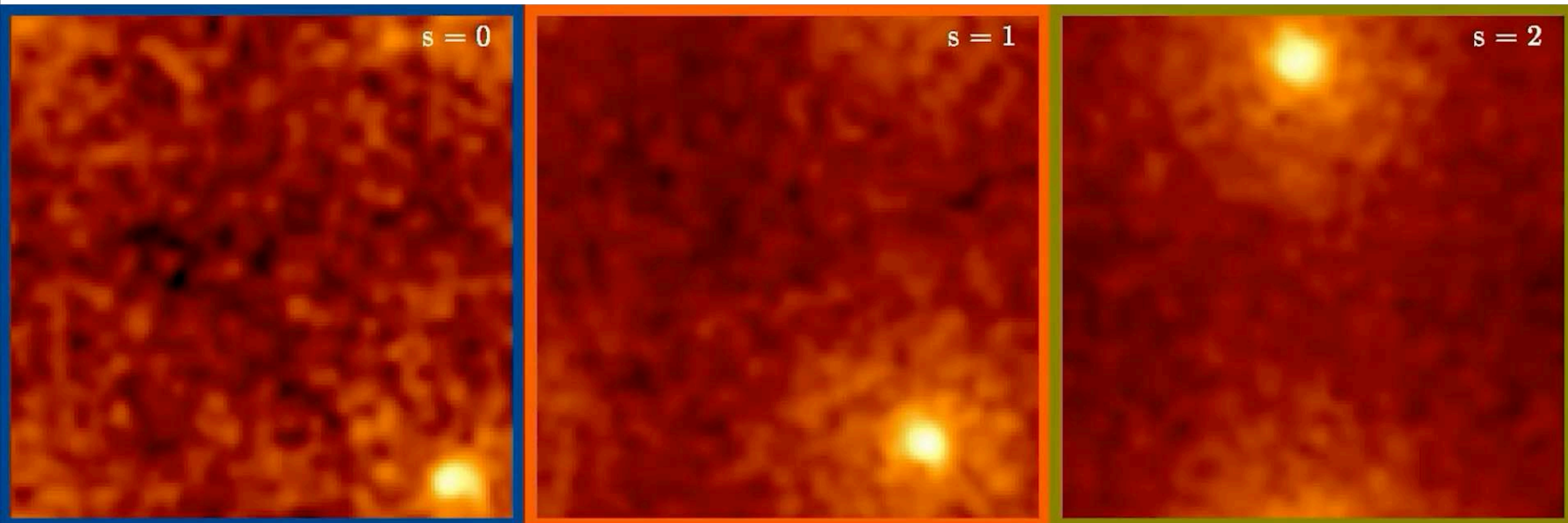


MA, Jain, Thomas, Wanichwecharungruang (2304.01985)

see Levkov, Panin & Tkachev (2018) for scalar case

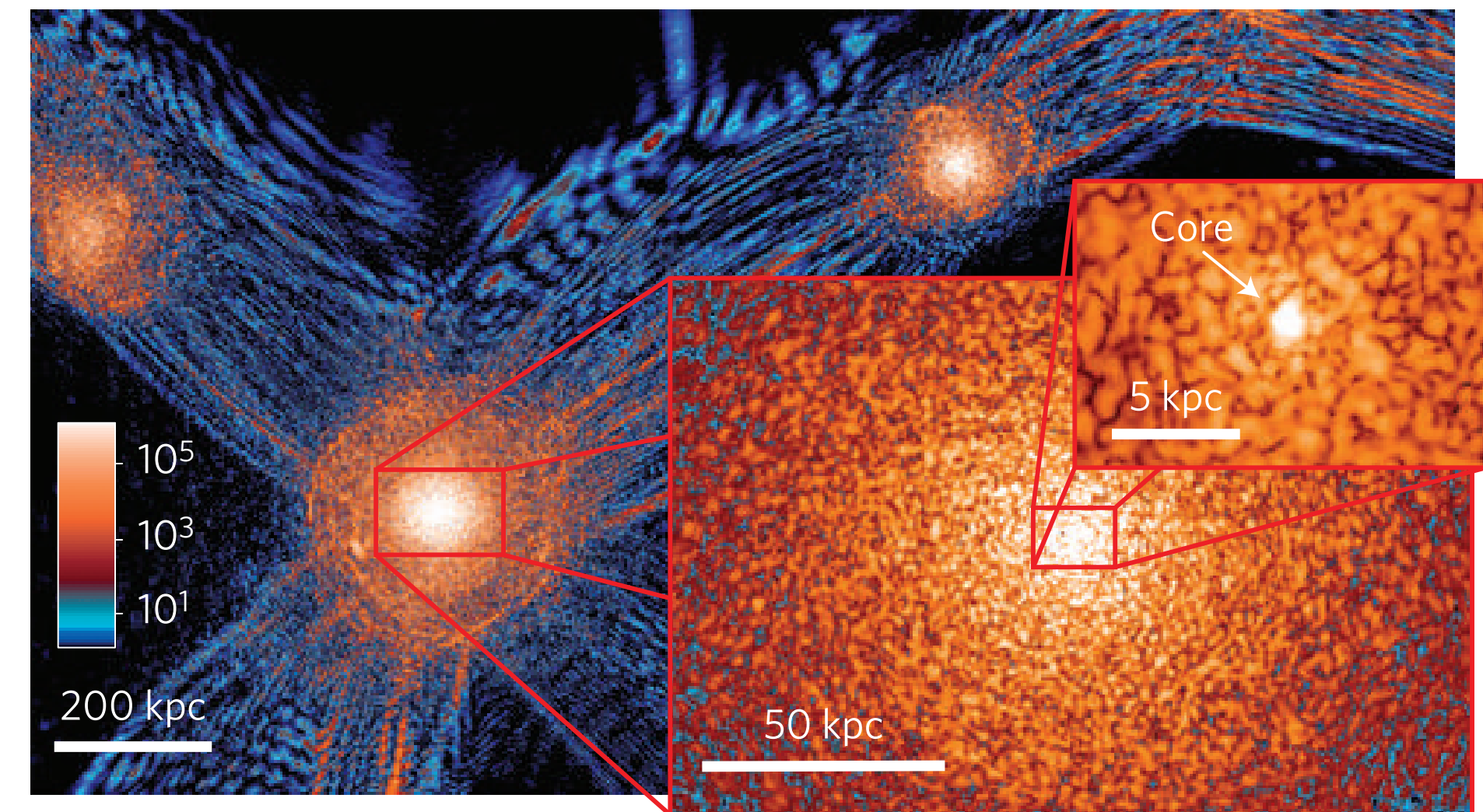
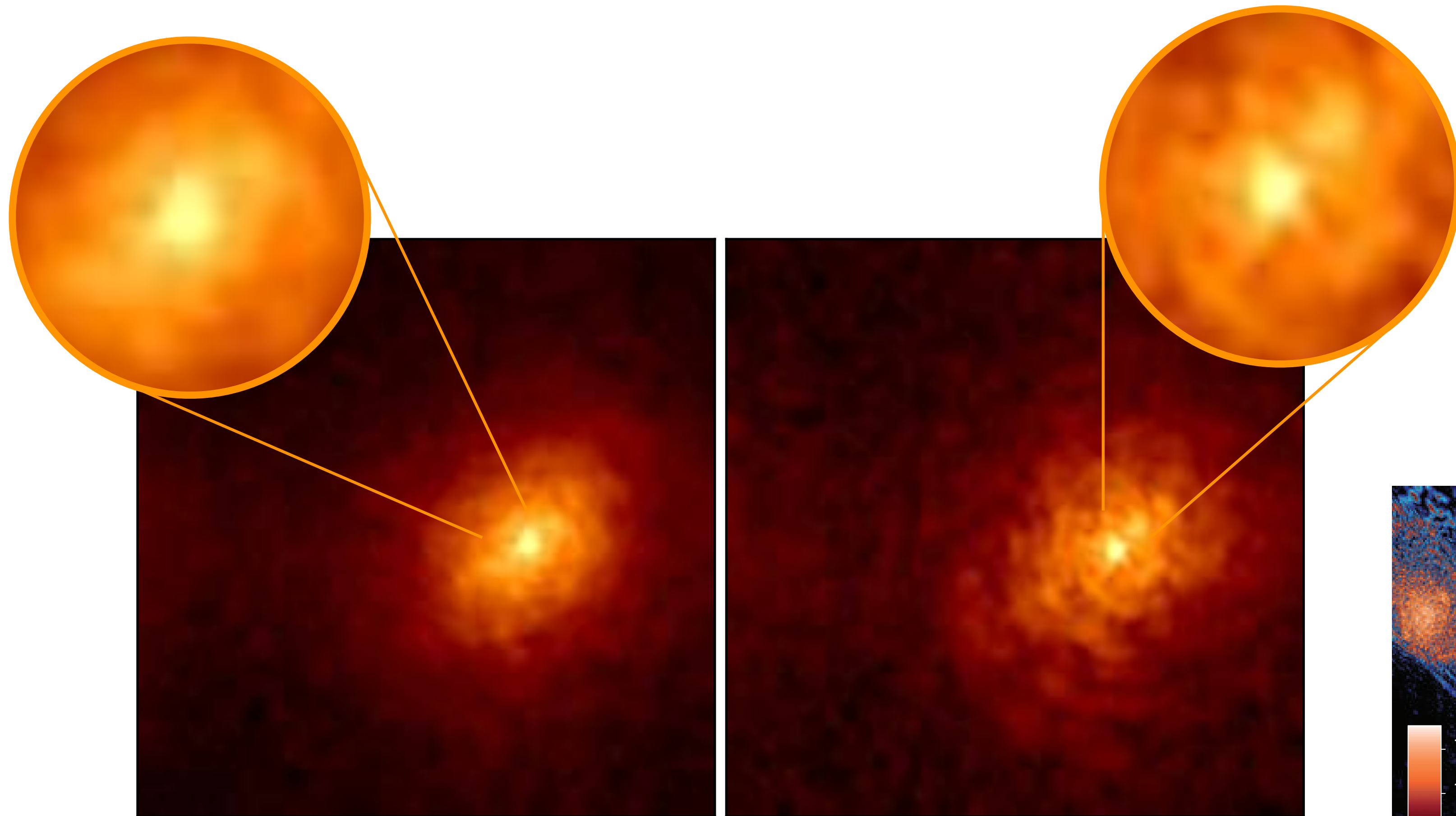


# what are these blobs?



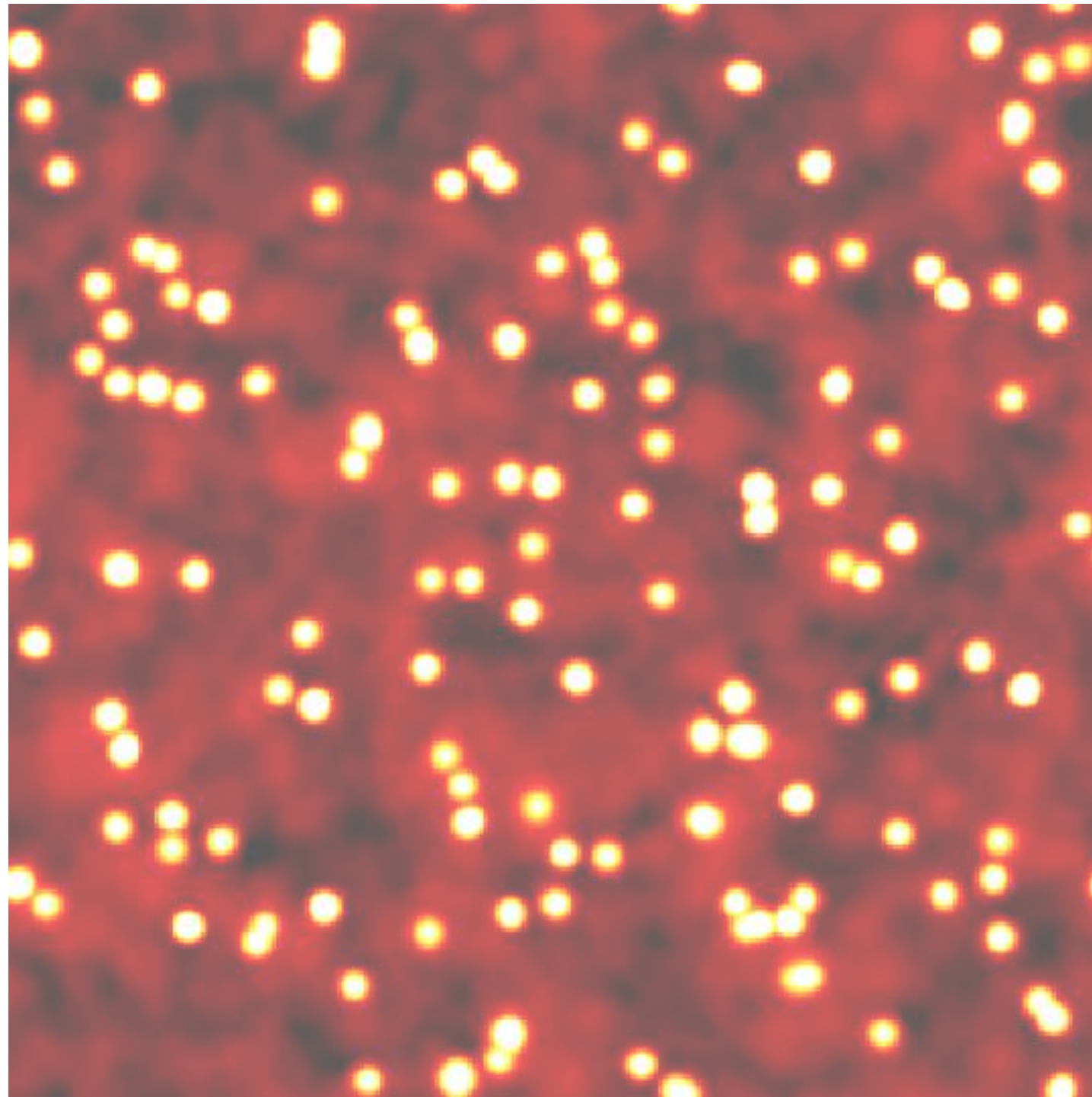


# what are these blobs?

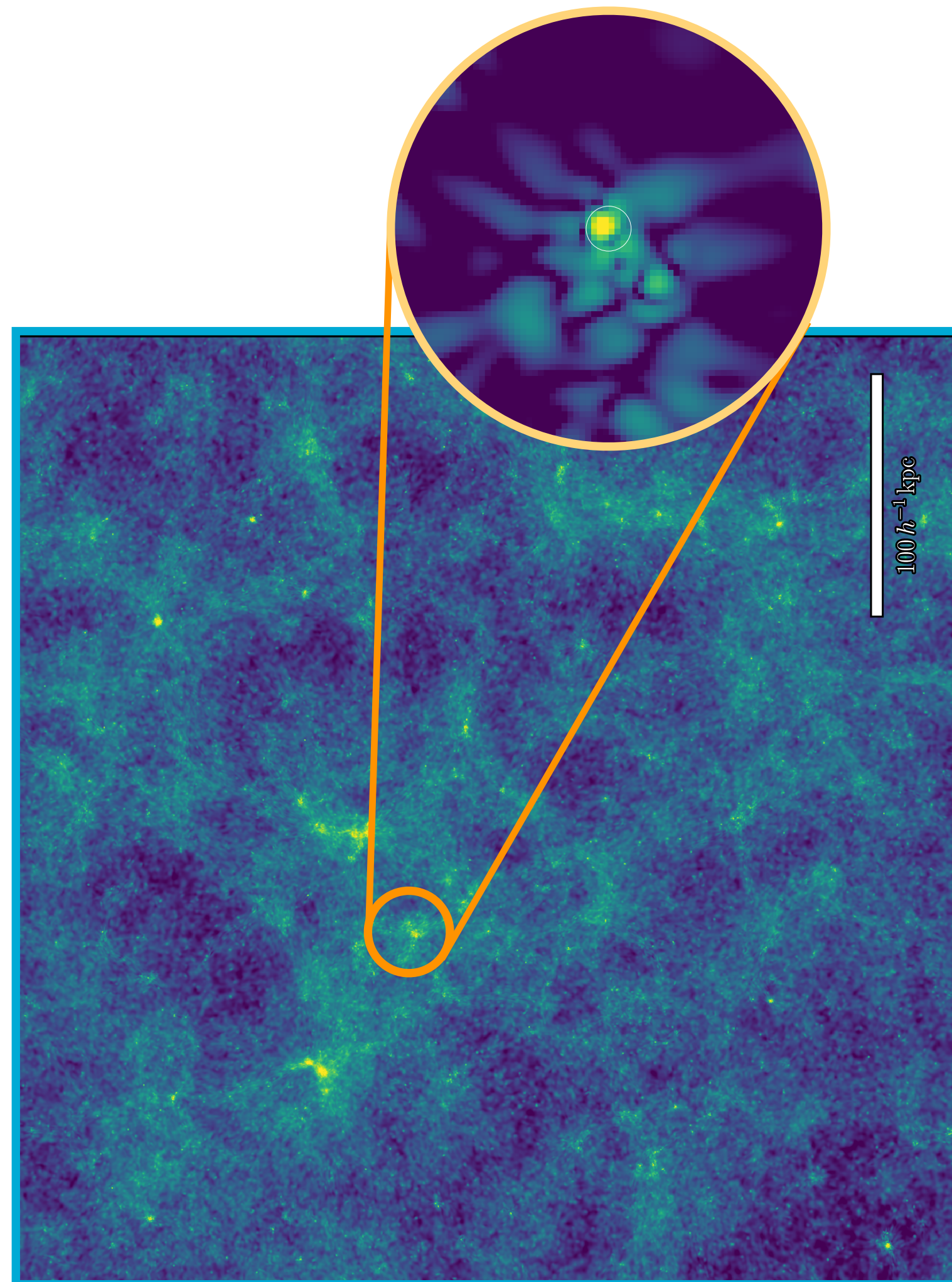




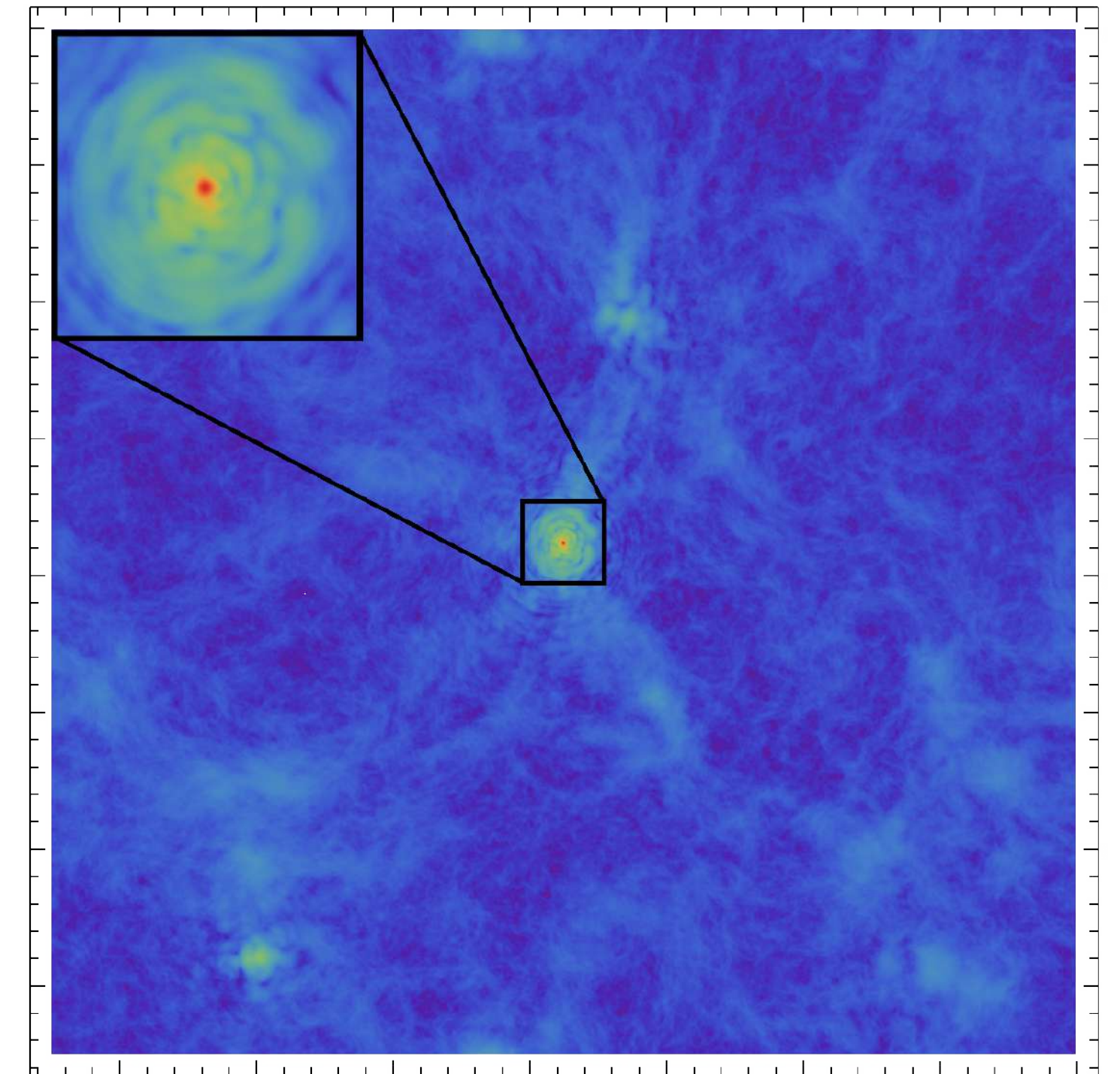
# what are these blobs?



MA & Mocz (2019)



MA, May & Mirbabayi (2025)



Gorghetto, Hardy, March-Russell,  
Song & West (2022)



# ③ ② Solitons in Warm, Noisy and Spinning Wave Dark Matter ①

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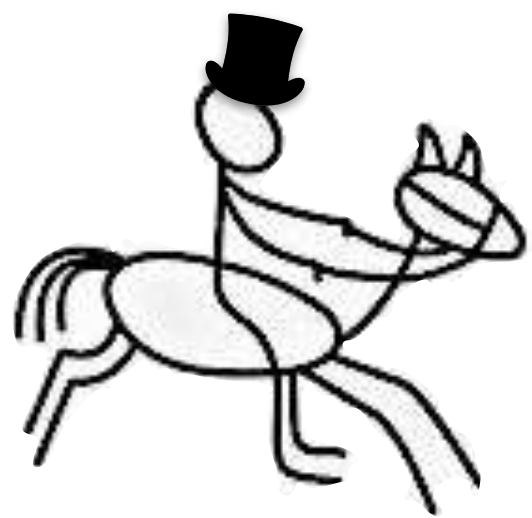


# solitons

very-long lived, dynamical excitation in a field, with nonlinearities balancing dispersion



Image Credit: Heriot-Watt University



- discovered in nonlinear waves in water in canals (John Scott Russell, 1834)
- **optics, hydrodynamics, BECs**, high energy physics, and cosmology

# soliton in a scalar field (non-topological)

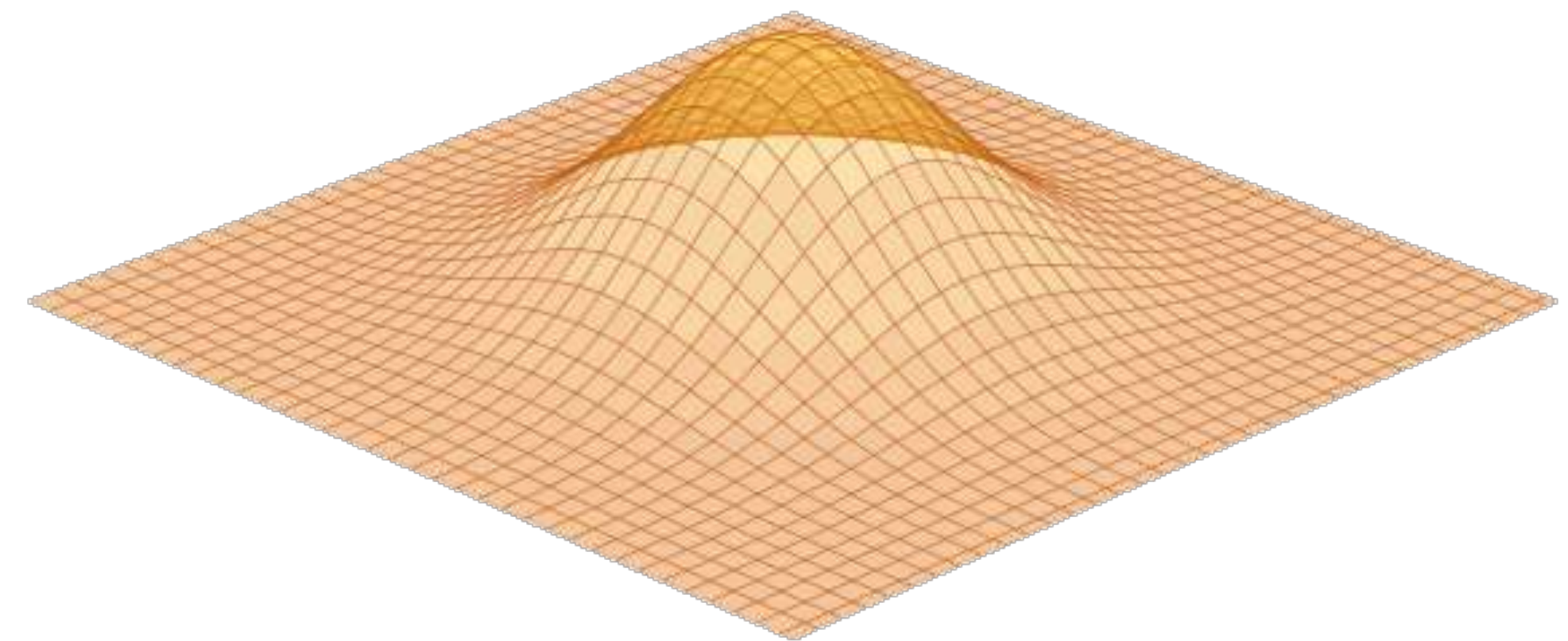
spatially localized

coherently oscillating (components)

exceptionally long-lived



$$\rho_{\text{sol}}(t, \boldsymbol{x}) \approx \rho(r)$$



$$\varphi_{\text{sol}}(t, \boldsymbol{x}) \approx f(r) \cos(\omega t)$$

$$\omega \approx m$$

# soliton in a vector field (non-topological)

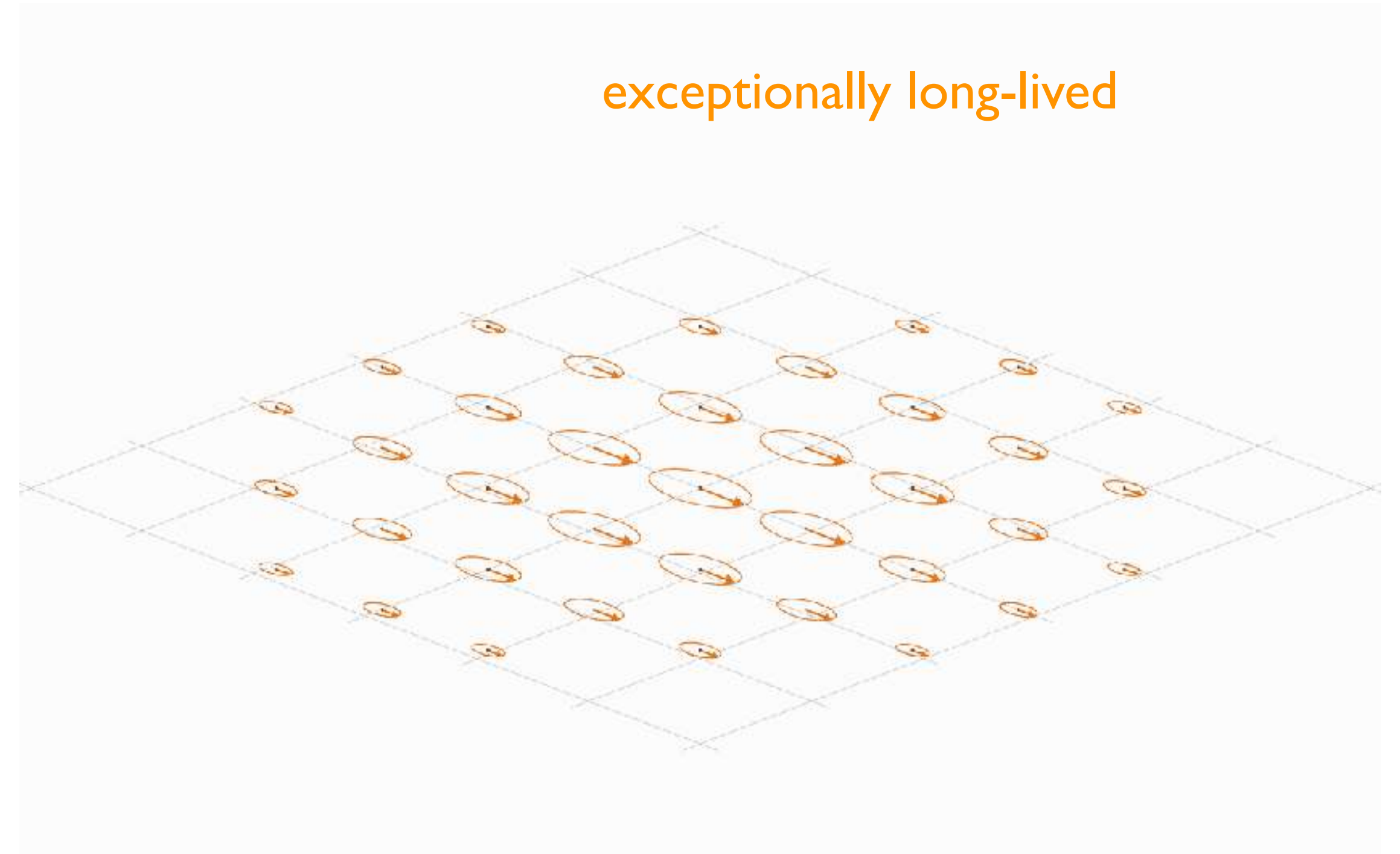


$$\rho_{\text{sol}}(t, \mathbf{x}) \approx \rho(r)$$

spatially localized

coherently oscillating (components)

exceptionally long-lived



$$\mathbf{X}_{\text{sol}}(t, \mathbf{x}) \approx f_x(r) \cos(\omega t + \theta_x) \hat{x} + f_y(r) \cos(\omega t + \theta_y) \hat{y} + f_z(r) \cos(\omega t) \hat{z}$$

$$\omega \approx m$$



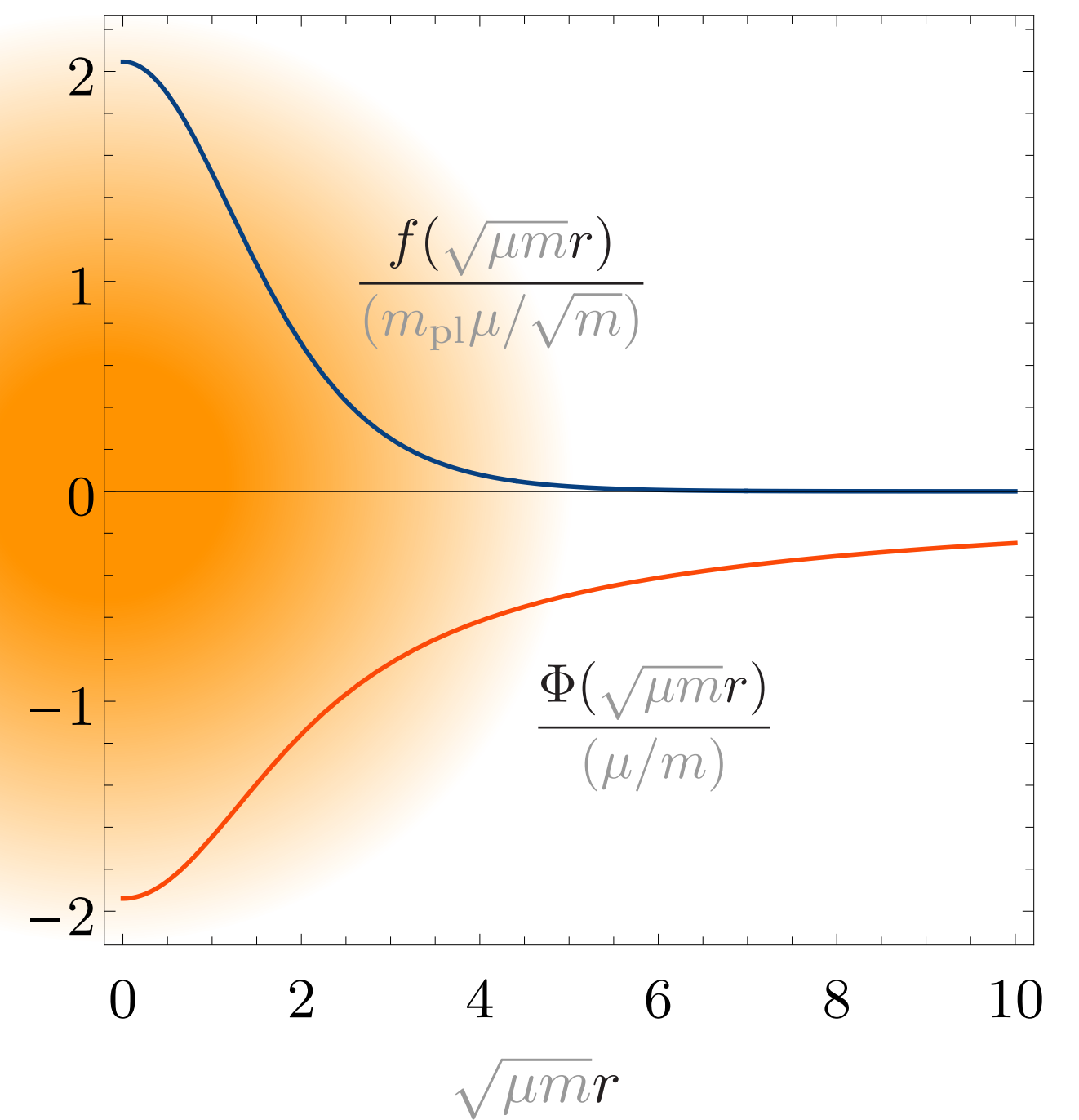
# non-relativistic solitons - scalar

$$\varphi(t, \mathbf{x}) = \frac{1}{\sqrt{2m}} \Re [\Psi(t, \mathbf{x}) e^{-imt}]$$

$$i\hbar \frac{\partial}{\partial t} \Psi = -\frac{\hbar^2}{2m} \nabla^2 \Psi + m\Phi \Psi, \quad \nabla^2 \Phi = 4\pi G m \Psi^* \Psi$$

$$\Psi_{\text{sol}}(t, r) = f(r) e^{i\mu t} \epsilon \quad \epsilon^* \epsilon = 1 \quad \omega = m - \mu$$

$$-\mu f = -\frac{1}{2m} \nabla^2 f + m\Phi f \quad \nabla^2 \Phi = 4\pi G m f^2$$



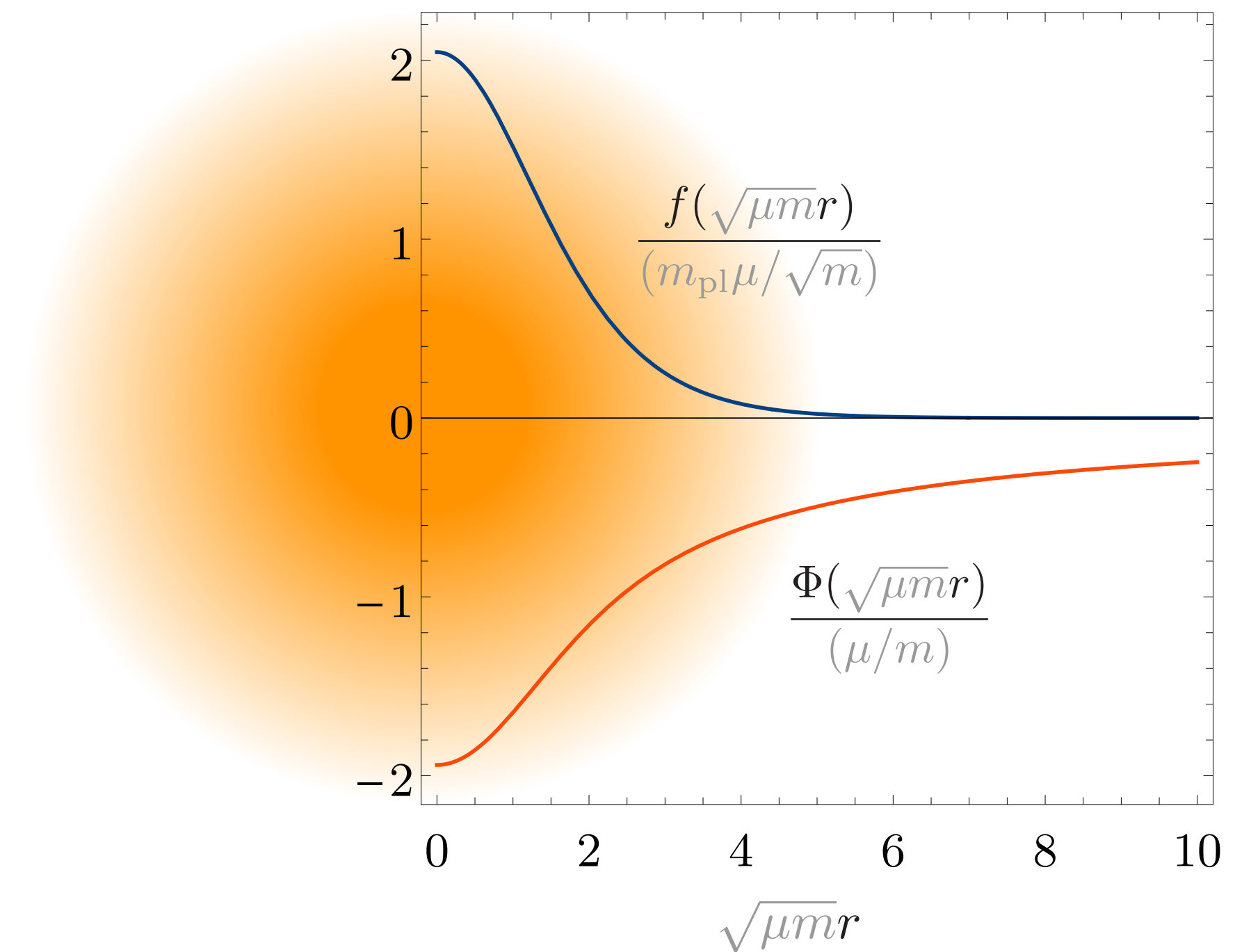
# non-relativistic solitons - vector

$$\mathbf{X}(t, \mathbf{x}) = \frac{1}{\sqrt{2m}} \Re [\boldsymbol{\Psi}(t, \mathbf{x}) e^{-imt}]$$

$$i\hbar \frac{\partial}{\partial t} \boldsymbol{\Psi} = -\frac{\hbar^2}{2m} \nabla^2 \boldsymbol{\Psi} + m\Phi \boldsymbol{\Psi}, \quad \nabla^2 \Phi = 4\pi Gm \boldsymbol{\Psi}^\dagger \boldsymbol{\Psi}$$

$$\boldsymbol{\Psi}_{\text{sol}}(t, r) = f(r) e^{i\mu t} \boldsymbol{\epsilon} \quad \text{with} \quad \boldsymbol{\epsilon}^\dagger \boldsymbol{\epsilon} = 1 \quad \omega = m - \mu$$

$$-\mu f = -\frac{1}{2m} \nabla^2 f + m\Phi f \quad \nabla^2 \Phi = 4\pi Gm f^2$$

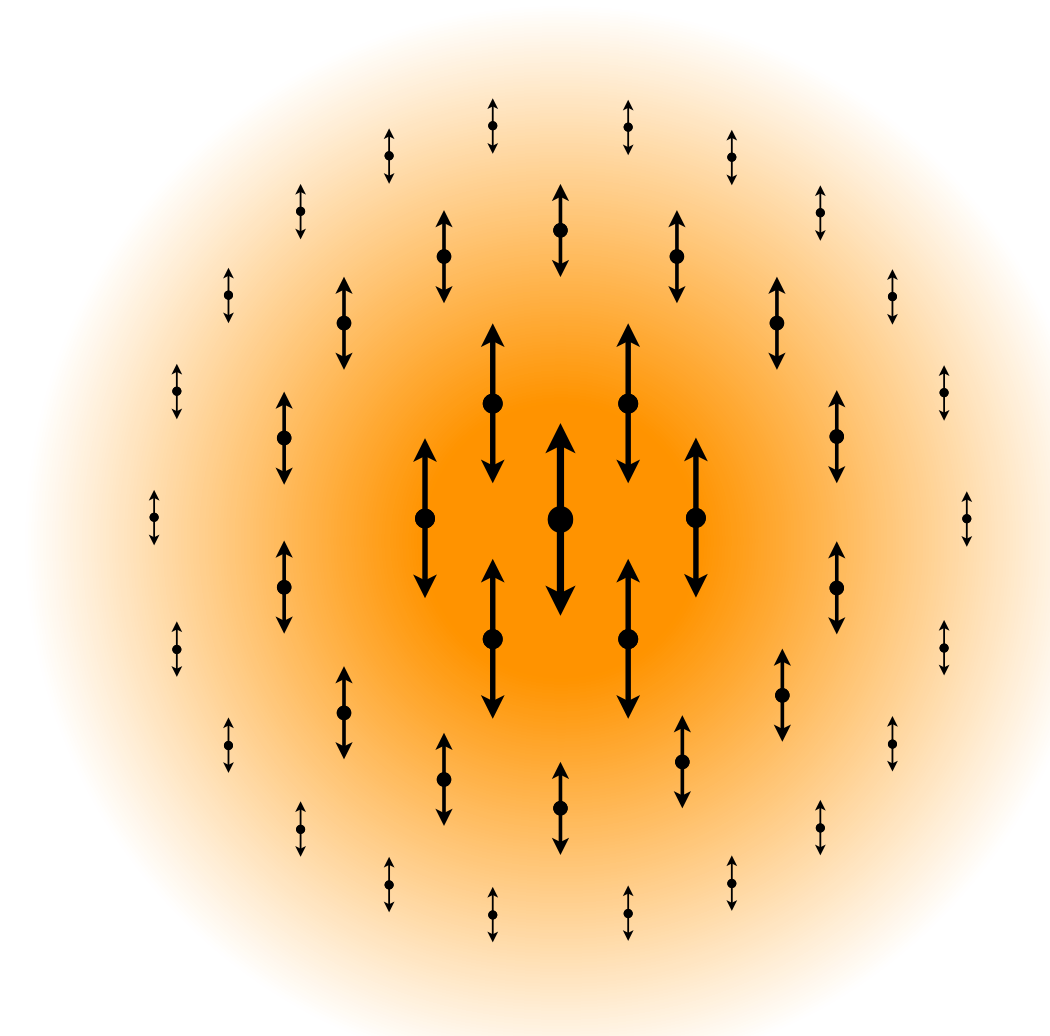


# “polarized” vector solitons

Jain & MA (2109.04892)

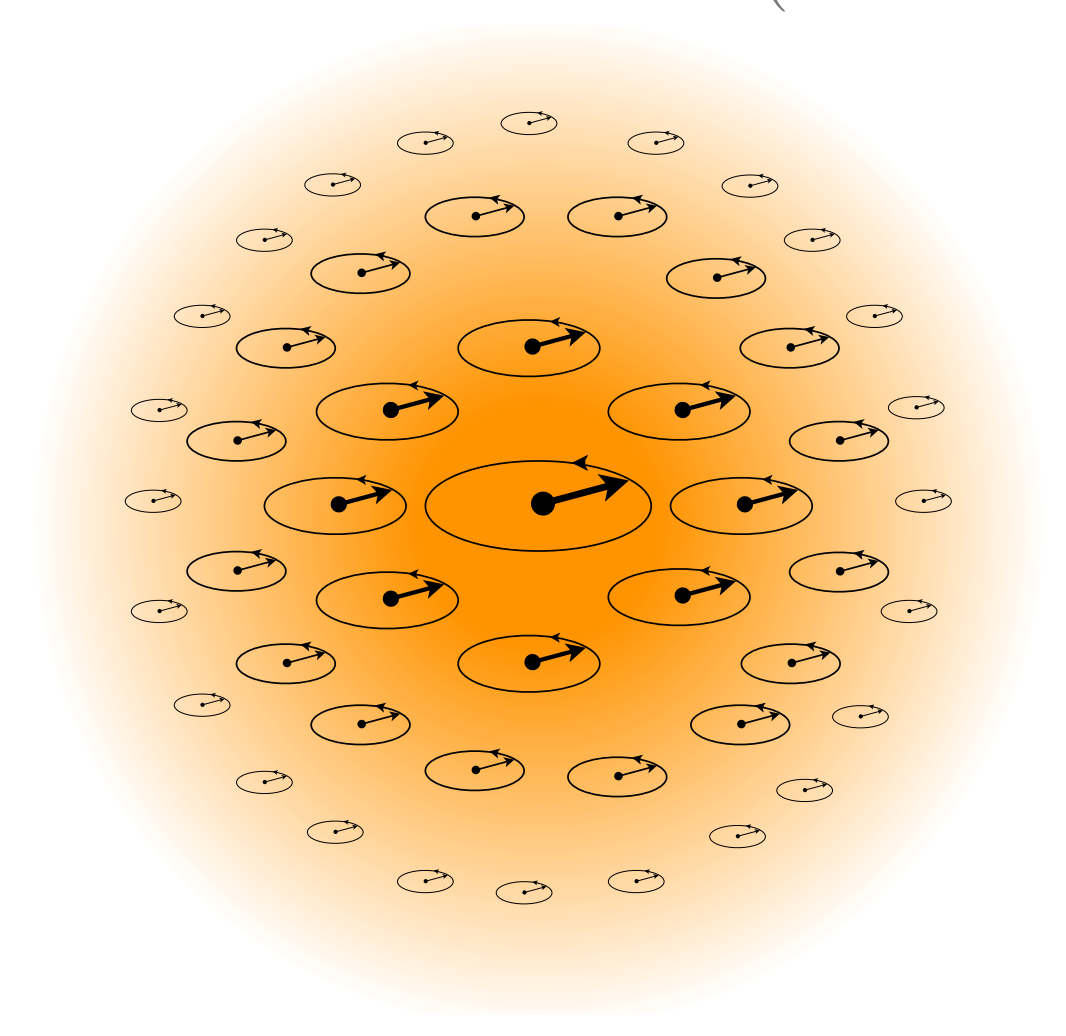
$$\mathbf{X}(t, \mathbf{x}) = \frac{1}{\sqrt{2m}} \Re [\boldsymbol{\Psi}(t, \mathbf{x}) e^{-imt}]$$

$$\boldsymbol{\Psi}_{\text{sol}}(t, r) = f(r) e^{i\mu t} \boldsymbol{\epsilon} \quad \text{with} \quad \boldsymbol{\epsilon}^\dagger \boldsymbol{\epsilon} = 1$$



$$\boldsymbol{\epsilon}_0 = \hat{\mathbf{z}}$$

$$f(r) \cos[(m - \mu)t] \hat{\mathbf{z}}$$



$$\boldsymbol{\epsilon}_\pm = \hat{\mathbf{x}} \pm i\hat{\mathbf{y}}/\sqrt{2}$$

$$f(r) [\hat{\mathbf{x}} \cos[(m - \mu)t] + \hat{\mathbf{y}} \sin[(m - \mu)t]]$$

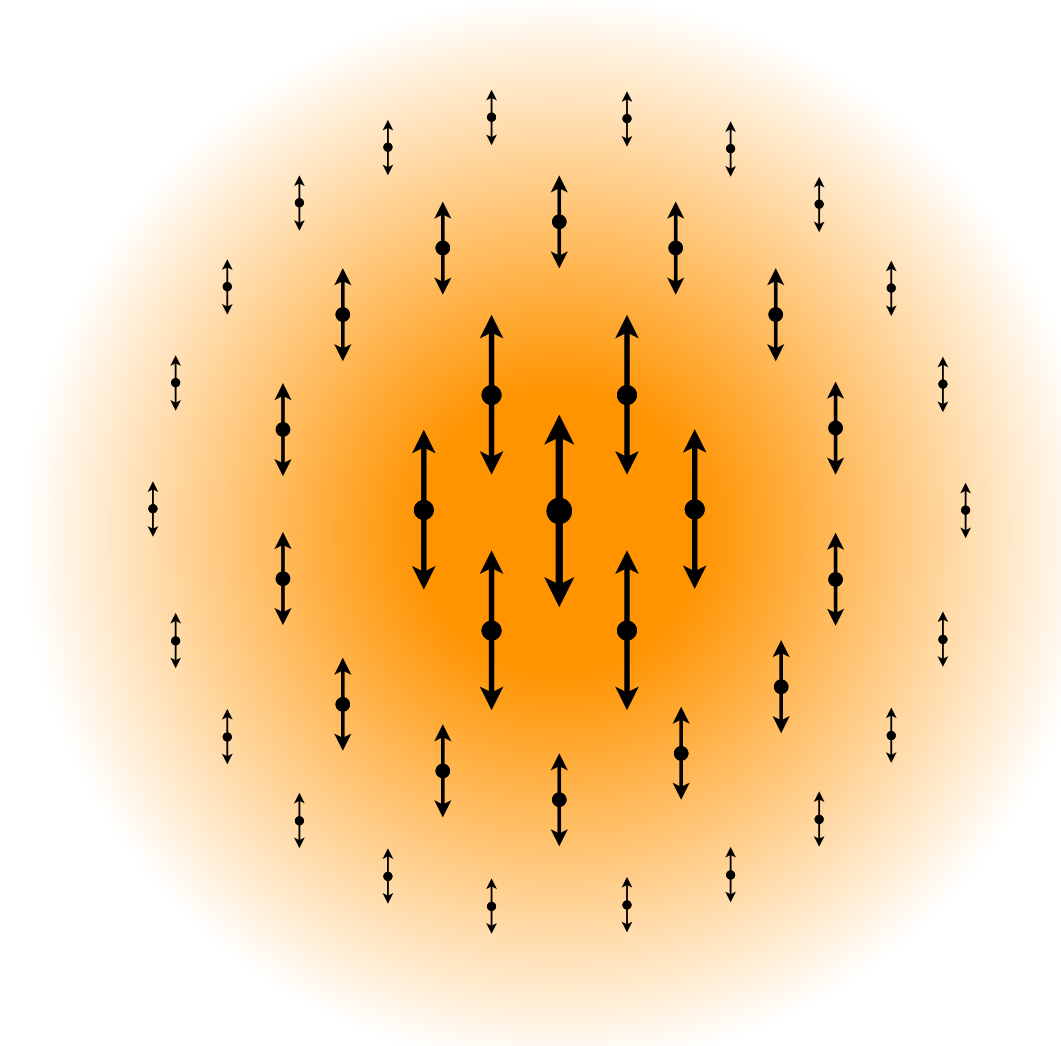


# “polarized” vector solitons (with macroscopic spin)

$$\mathbf{S} = \hbar \int d^3x i \boldsymbol{\Psi} \times \boldsymbol{\Psi}^\dagger$$

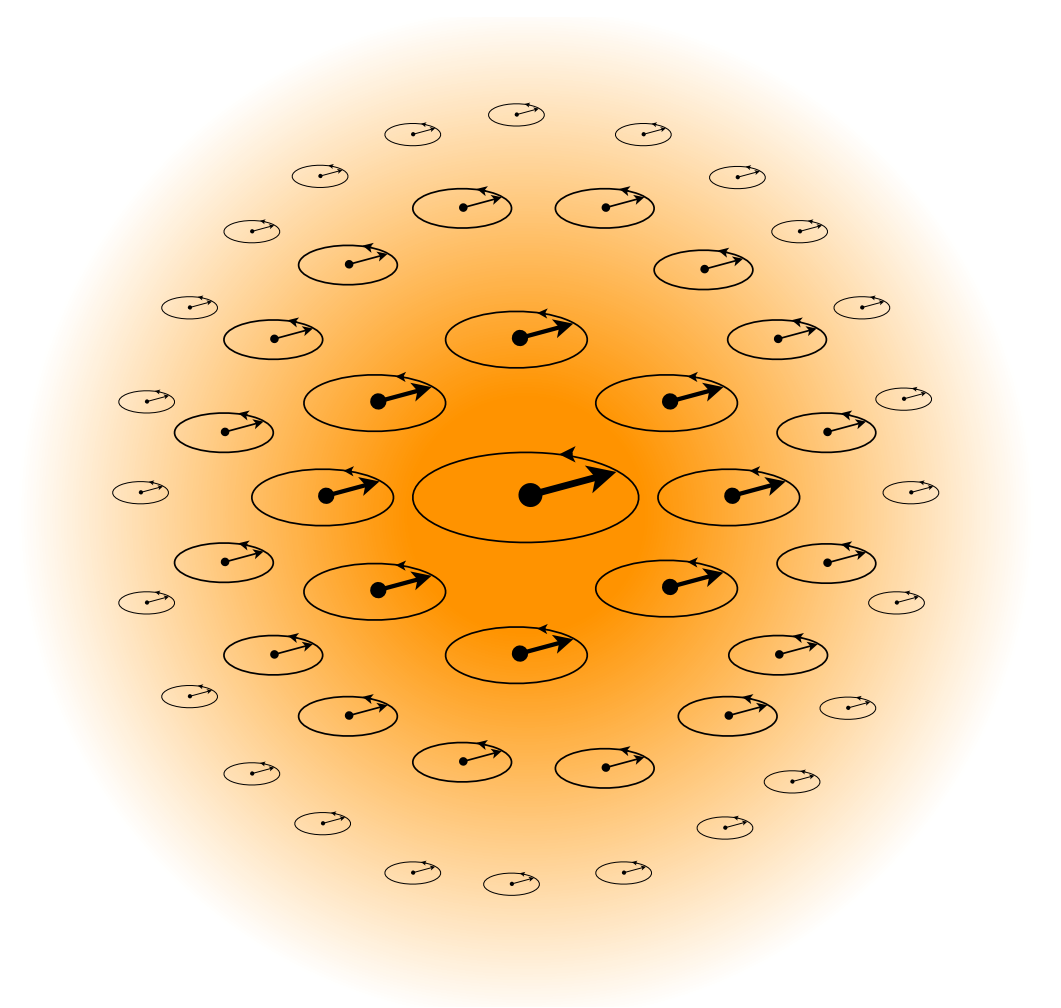
macroscopic intrinsic spin!

$$\mathbf{S}_{\text{tot}} = i(\boldsymbol{\epsilon} \times \boldsymbol{\epsilon}^\dagger) \frac{M_{\text{sol}}}{m} \hbar$$



$$\boldsymbol{\epsilon}_0 = \hat{z}$$

$$\mathbf{S}_{\text{tot}} = 0$$



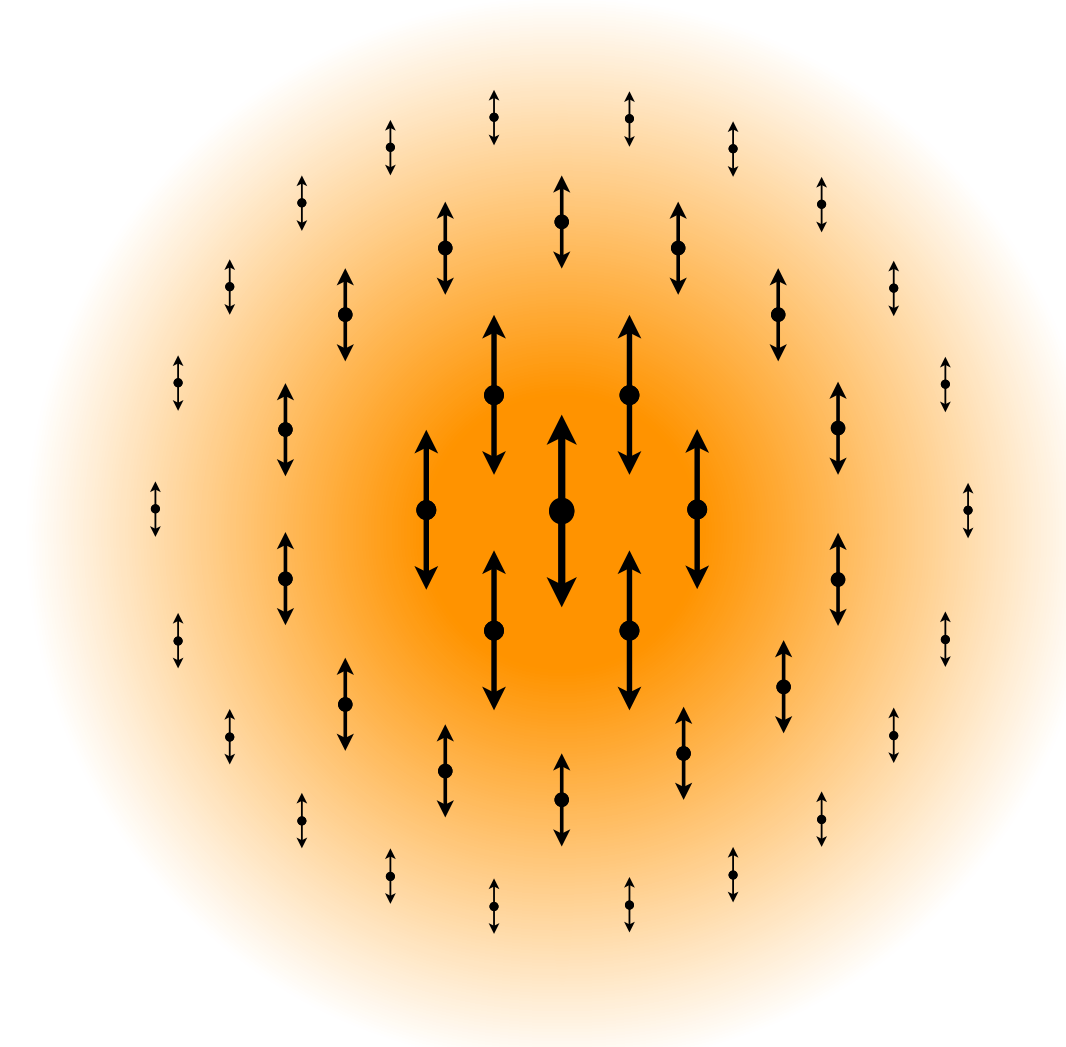
$$\boldsymbol{\epsilon}_\pm = \hat{x} \pm i\hat{y}/\sqrt{2}$$

$$\mathbf{S}_{\text{tot}} = \hbar \frac{M_{\text{sol}}}{m} \hat{z}$$

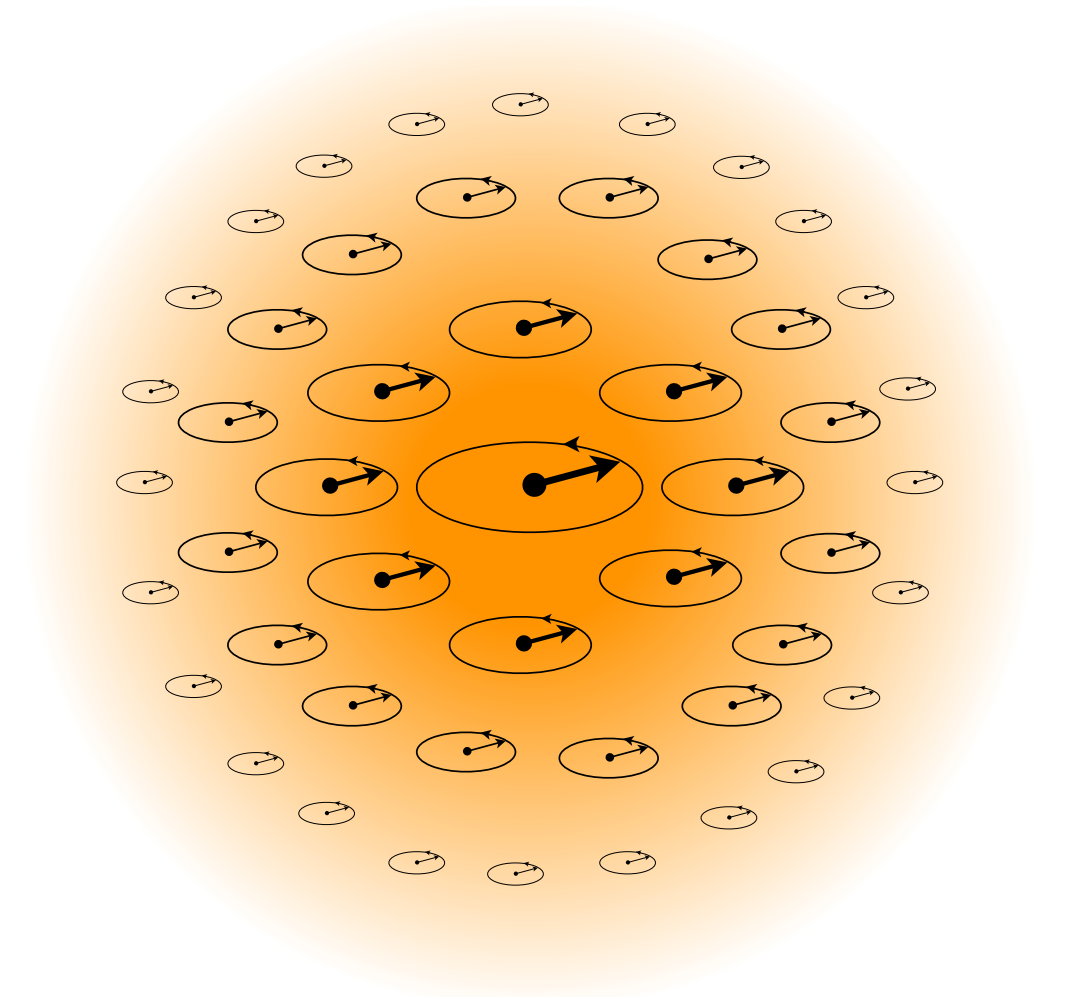
# “polarized” vector solitons (with macroscopic spin)

$$\mathbf{S}_{\text{tot}} = i(\boldsymbol{\epsilon} \times \boldsymbol{\epsilon}^\dagger) \frac{M_{\text{sol}}}{m} \hbar$$

- all lowest energy for fixed  $M$
- bases for partially-polarized solitons

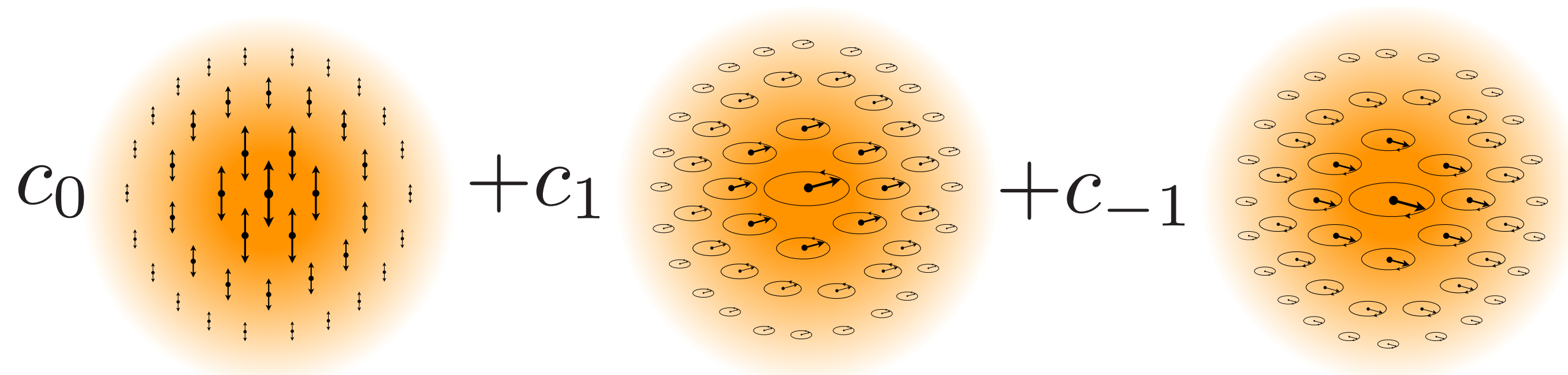


$$\mathbf{S}_{\text{tot}} = 0$$



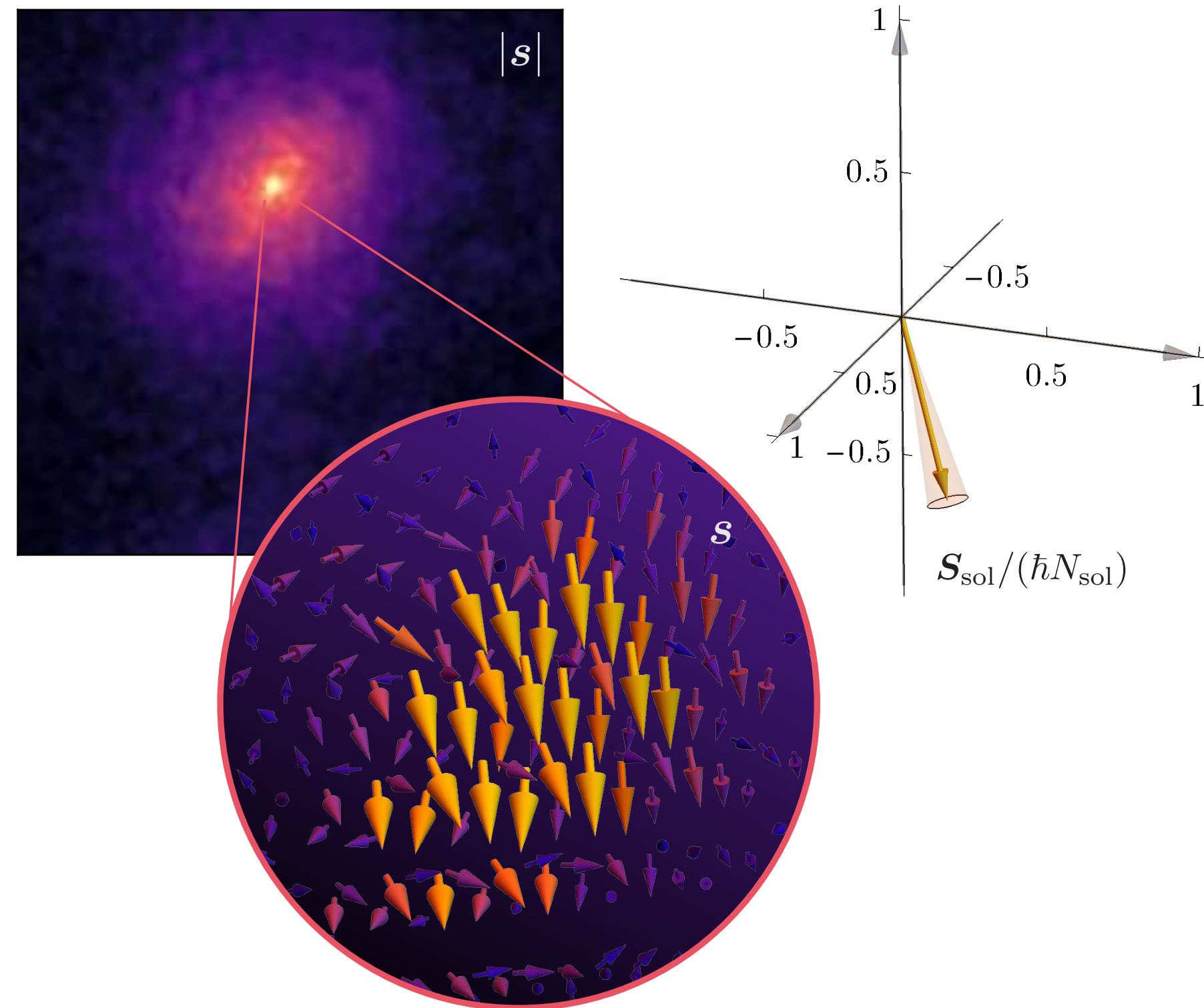
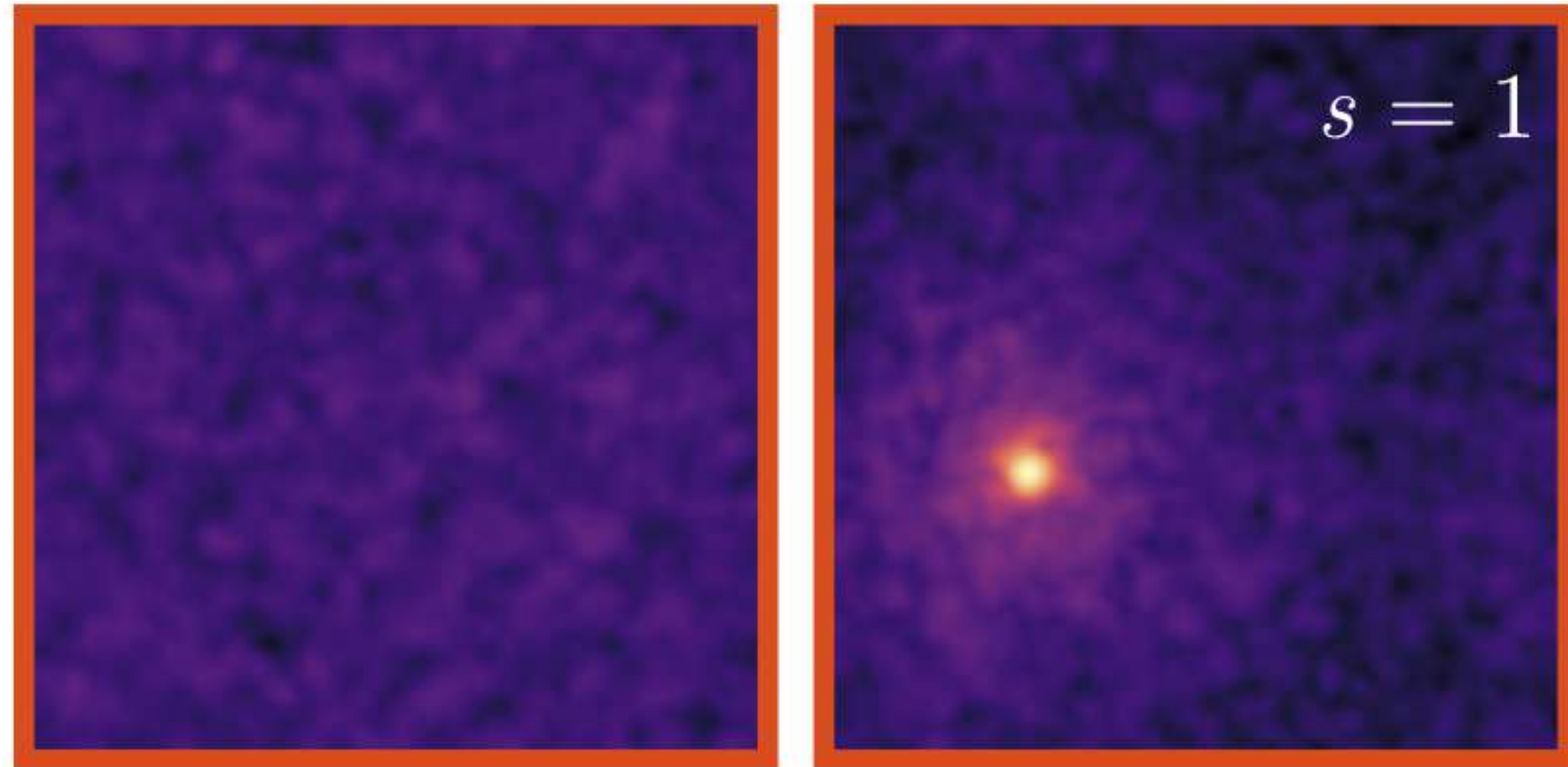
$$\mathbf{S}_{\text{tot}} = \hbar \frac{M_{\text{sol}}}{m} \hat{z}$$

$$0 \leq |\mathbf{S}_{\text{tot}}| \leq \frac{M_{\text{sol}}}{m} \hbar$$



# born to spin

spin density



$$S_{\text{core}} \sim \hbar \frac{M_{\text{core}}}{m}$$

even when initial total spin is negligible

MA, Jain, Karur & Mocz(2022)

Gorghetto et. al (2022)



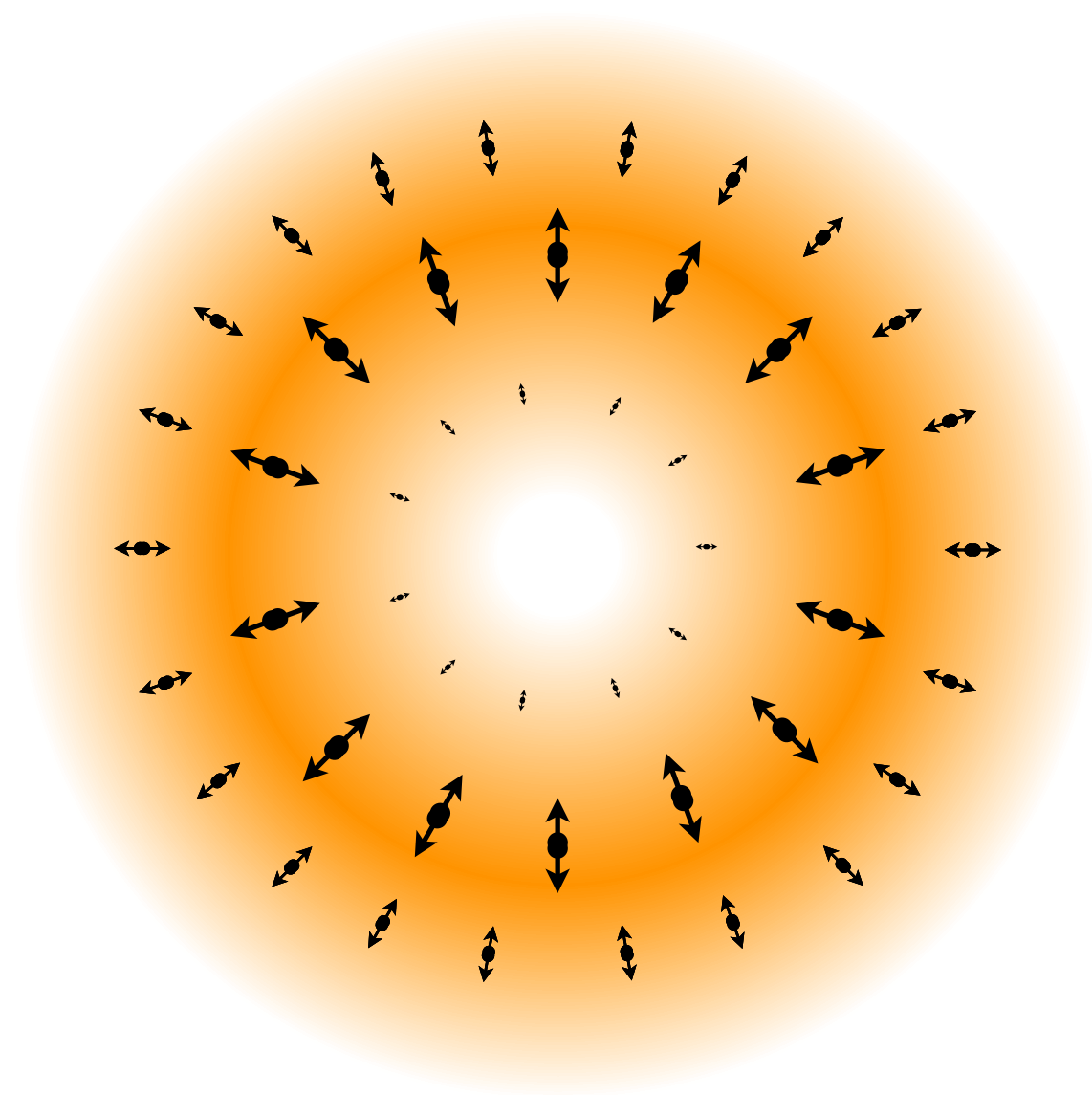
# a different higher energy soliton: the “hedgehogs”

earlier literature

$$W_j(\mathbf{x}, t) = f(r) \frac{x^j}{r} \cos \omega t ,$$

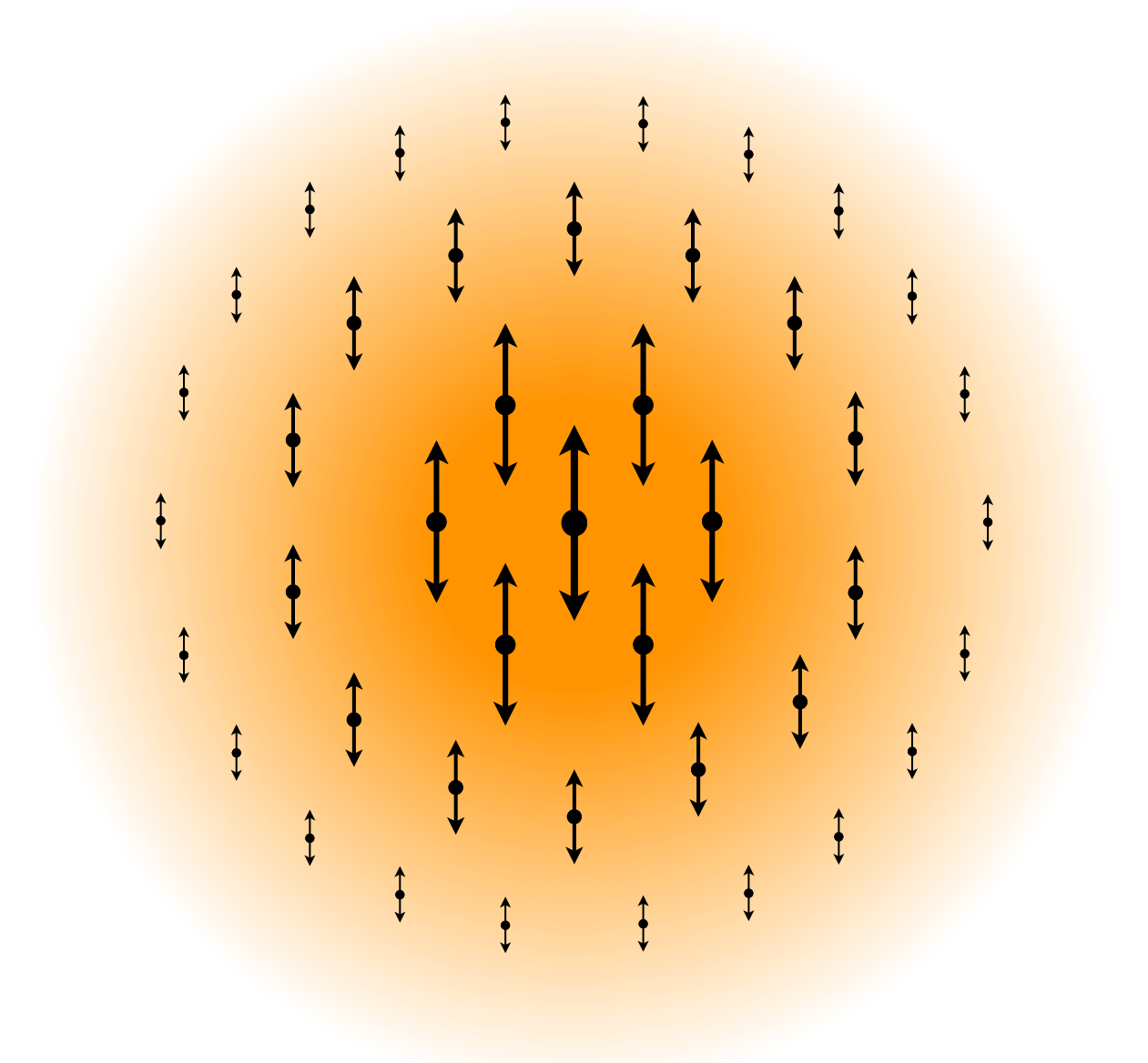
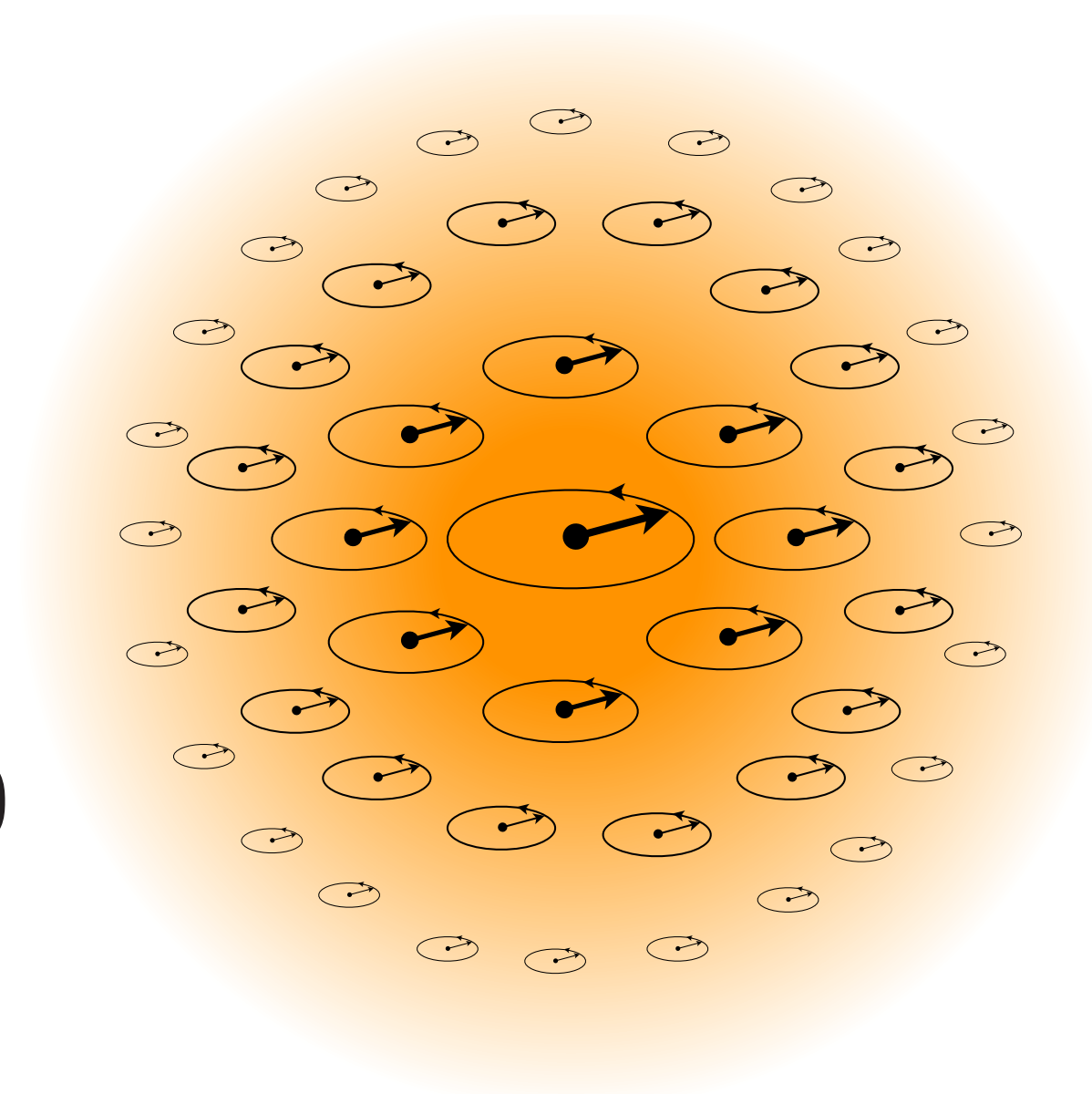
hedgehogs  
not ground states

at least when non-relativistic  
Lozanov & Adshead (2021)



$$E_{\text{hh}}^s > E$$

$$E_{\text{hh}}^{s=1} \approx 0.33E < 0$$

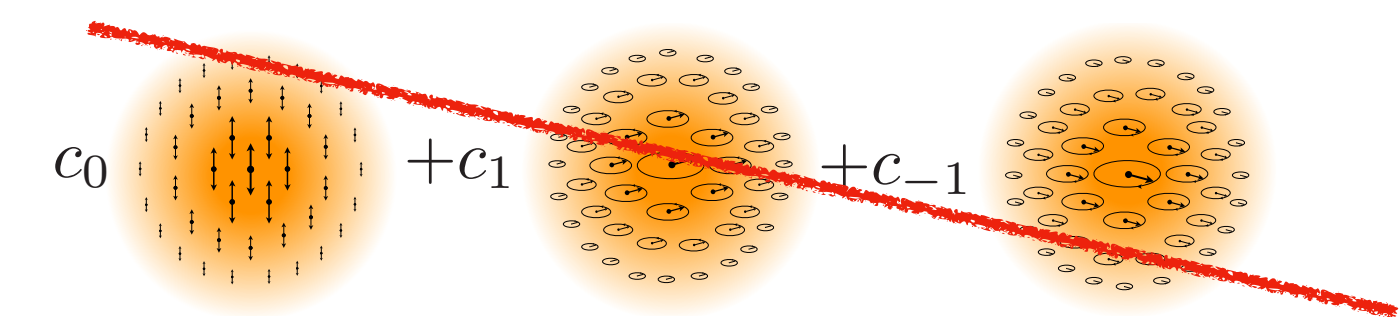
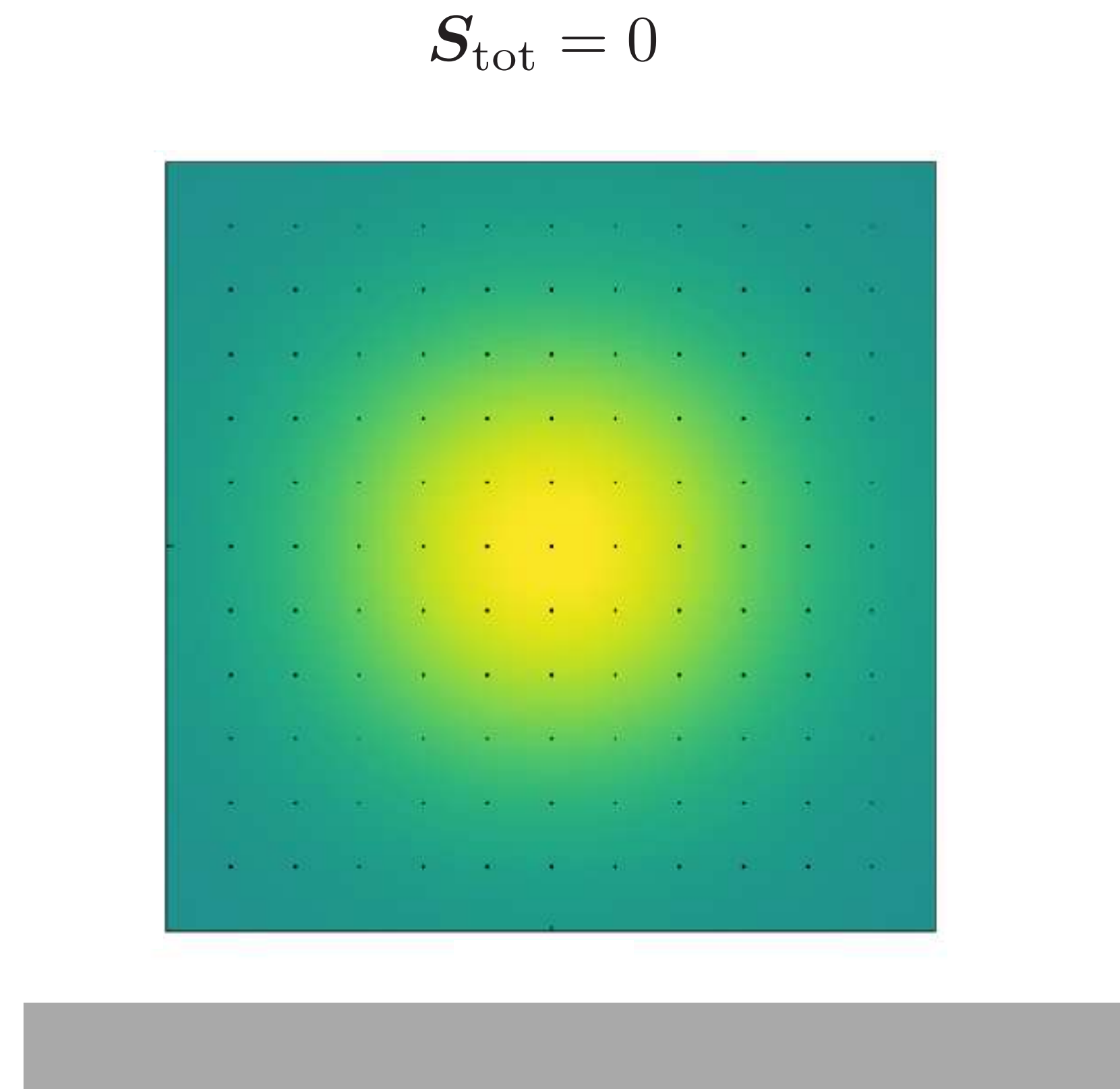
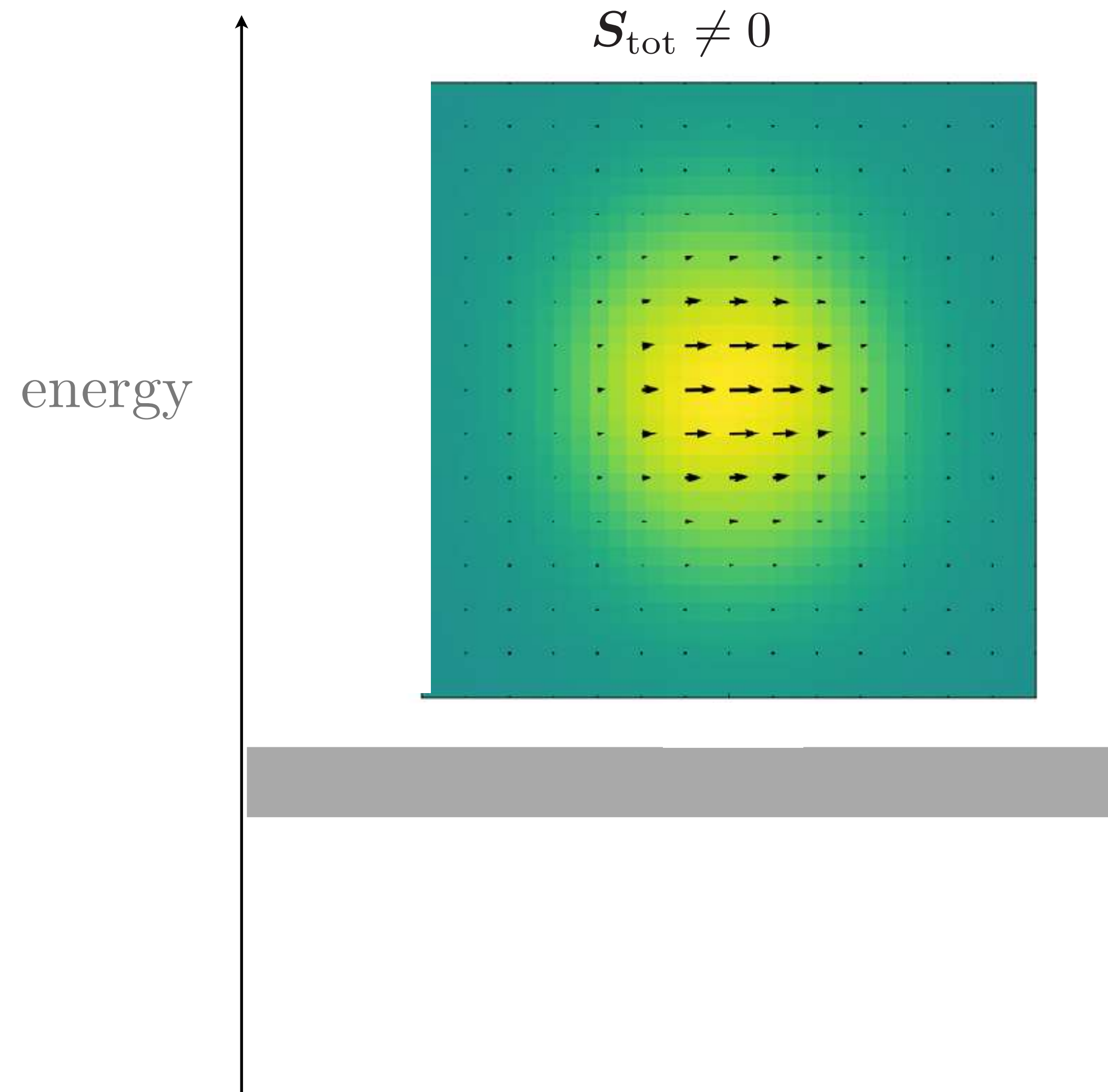


$$c_0 \text{ (vertical arrows)} + c_1 \text{ (radial arrows)} + c_{-1} \text{ (rotational arrows)}$$

# non-gravitational self-interactions

Zhang, Jain & MA (2022)

Also see Jain (2021), Zhang & Ling (2023)





# i-SPin: An integrator for multicomponent Schrodinger-Poisson systems with self-interactions

arXiv: 2211.08433

Mudit Jain & Mustafa Amin

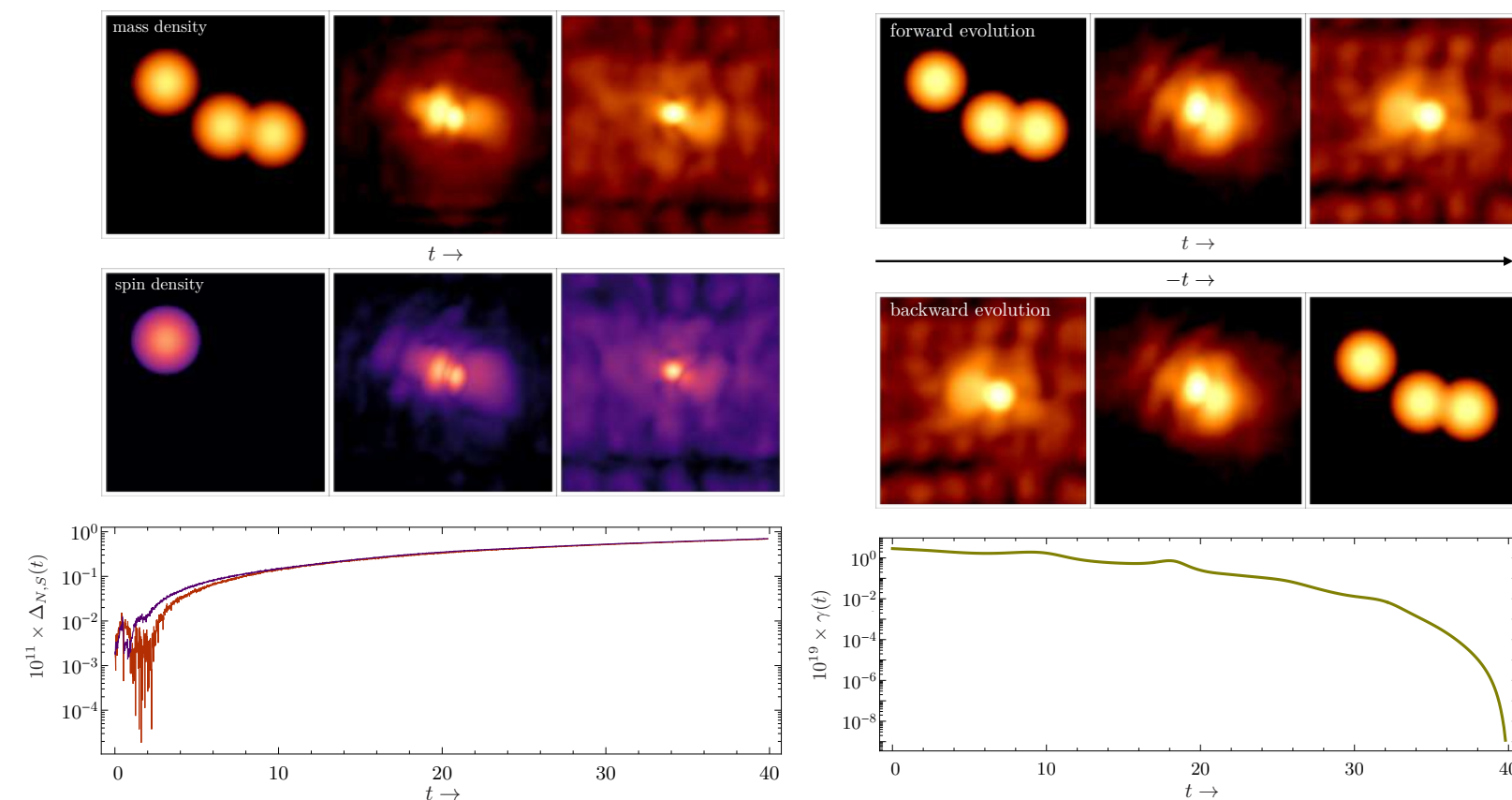
**i-SPin:** An algorithm (and publicly available code) to numerically evolve multicomponent Schrodinger-Poisson (SP) systems, including attractive/repulsive self-interactions + gravity

**problem:** If SP system represents the non-relativistic limit of a massive vector field, non-gravitational self-interactions (in particular, *spin-spin* type interactions) introduce new challenges related to mass and spin conservation which are not present in purely gravitational systems.

**solution:** Above challenges addressed with a novel analytical solution for the non-trivial ‘kick’ step in the algorithm (sec 4.3.2)

**features:** (i) second order accurate evolution (ii) spin and mass conserved to machine precision (iii) reversible

**generalizations:**  $n$ -component fields with  $SO(n)$  symmetry, an expanding universe relevant for cosmology, and the inclusion of external potentials relevant for laboratory settings



$$i\hbar \partial_t \Psi = -\frac{\hbar^2}{2m} \nabla^2 \Psi + m \Phi \Psi - \frac{\lambda(\hbar c)^3}{4(mc^2)^2} [(\Psi \cdot \Psi) \Psi^\dagger + 2(\Psi^\dagger \cdot \Psi) \Psi]$$

$$V_{\text{rel}}(\rho, \mathcal{S}) = -\frac{\lambda(\hbar c)^3}{8(mc^2)^2} \left[ 3\rho^2 - \frac{(\mathcal{S} \cdot \mathcal{S})}{\hbar^2} \right]$$

$$\text{number density } \rho = \Psi^\dagger \Psi$$

$$\text{spin density } \mathcal{S} = i\hbar \Psi \times \Psi^\dagger$$

## i-SPin 2: An integrator for general spin-s Gross-Pitaevskii systems

arXiv: 2305.01675

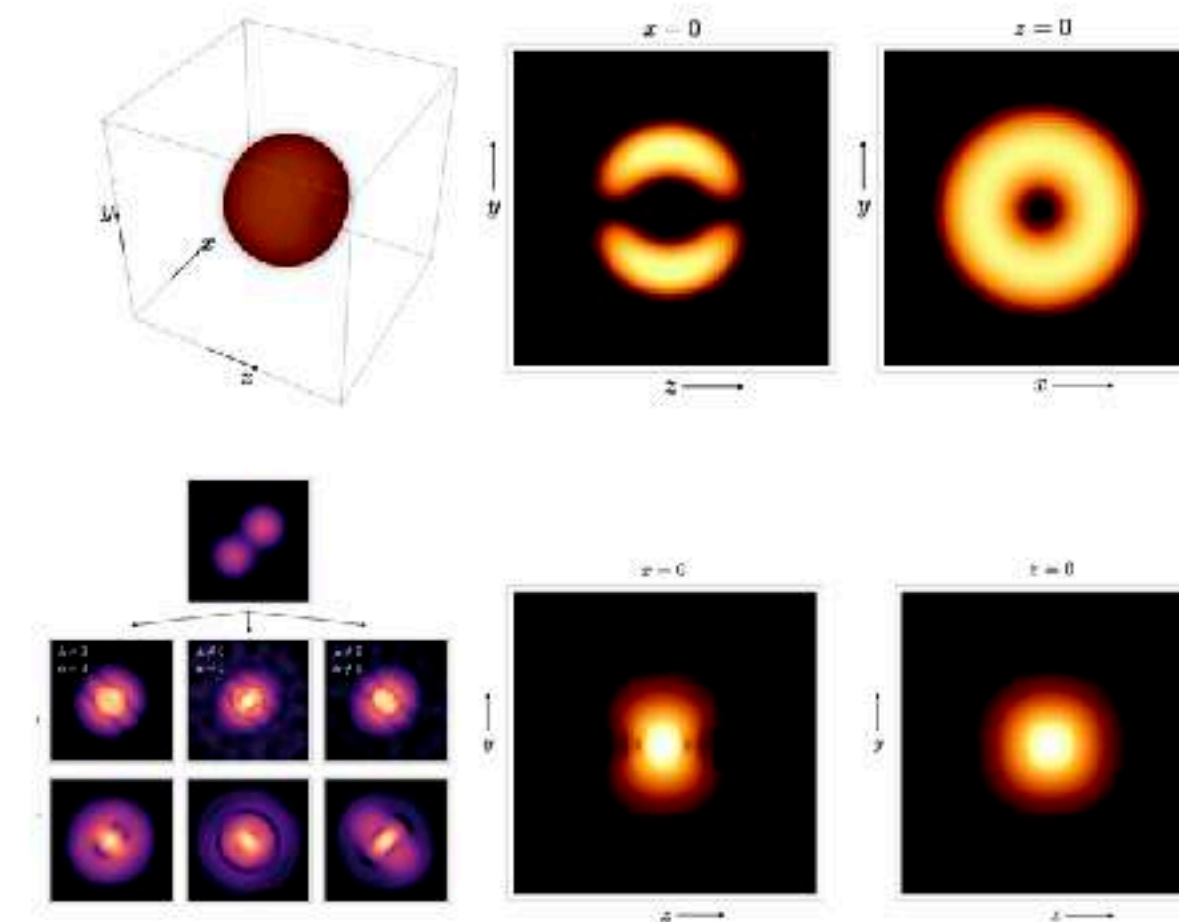
Mudit Jain, Mustafa Amin & H. Pu

**i-SPin 2:** An algorithm for evolving general spin-s Gross-Pitaevskii / non-linear Schrodinger systems carrying a variety of interactions, where the  $2s+1$  components of the ‘spinor’ field represent the different spin-multiplicity states.

**Allowed interactions:** Nonrelativistic interactions up to quartic order in the Schrodinger field (both short and long-range, and spin-dependent and spin-independent interactions), including explicit spin-orbit couplings. The algorithm allows for spatially varying external and/or self-generated vector potentials that couple to the spin density of the field.

**Applications:** (a) Laboratory systems such as spinor Bose-Einstein condensates (BECs). (b) Cosmological/astrophysical systems such as self-interacting bosonic dark matter.

**Numerical features:** Our symplectic algorithm is second-order accurate in time, and is extensible to the known higher-order accurate methods.



$$\mathcal{S}_{\text{nr}} = \int dt d^3x \left[ \frac{i}{2} \psi_n^\dagger \dot{\psi}_n + \text{c.c.} - \frac{1}{2\mu} \nabla \psi_n^\dagger \cdot \nabla \psi_n - \mu \rho V(\mathbf{x}) - \gamma \mathcal{S} \cdot \bar{\mathbf{B}}(\mathbf{x}, t) - V_{\text{rel}}(\rho, \mathcal{S}) - \frac{\xi}{2} \frac{1}{(2s+1)} |\psi_n \hat{A}_{nn'} \psi_{n'}|^2 + i g_{ij} \psi_n^\dagger [\hat{S}_i]_{nn'} \nabla_j \psi_n \right],$$

with  $\bar{\mathbf{B}}(\mathbf{x}, t) = f(t) \mathbf{B}(\mathbf{x})$ , and

$$V_{\text{rel}}(\rho, \mathcal{S}) = -\frac{1}{2\mu^2} [\lambda \rho^2 + \alpha (\mathcal{S} \cdot \mathcal{S})].$$

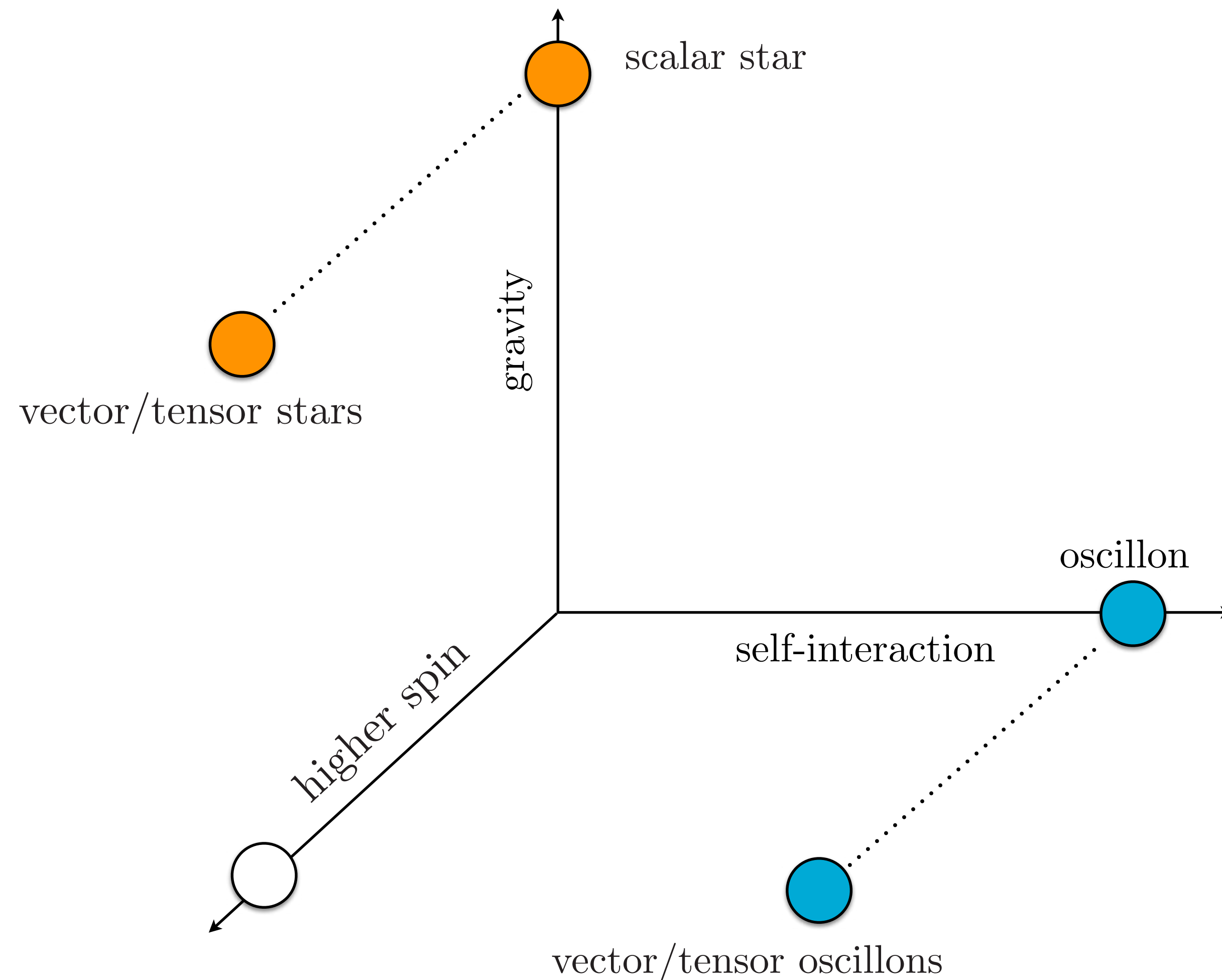
$$\text{number density } \rho = \psi_n^\dagger \psi_n$$

$$\text{spin density } \mathcal{S} = \psi_n^* \hat{S}_{nn'} \psi_{n'}$$



# solitons in $s \geq 0$ fields

spatially localized, coherently oscillating, long-lived



## Long History!

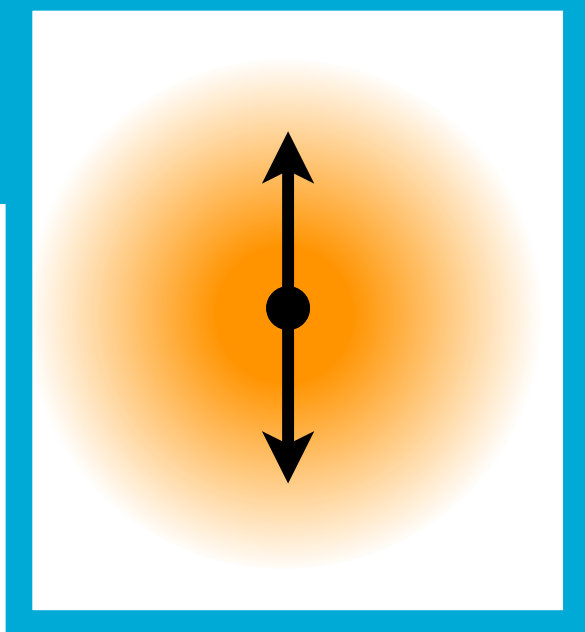
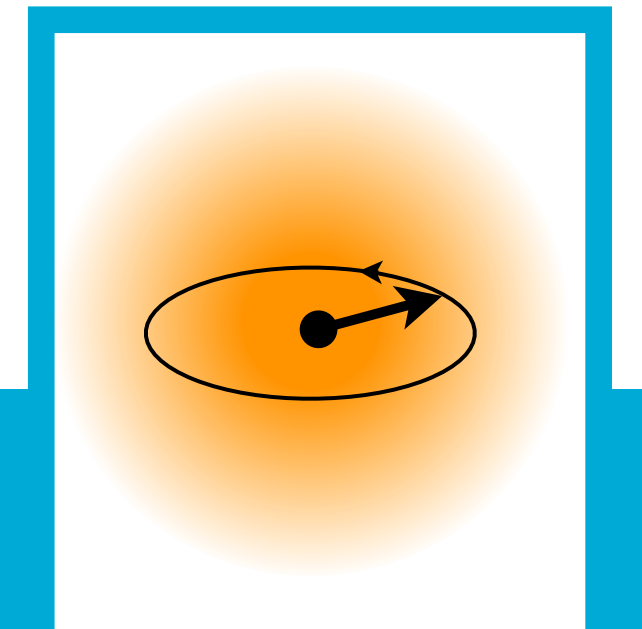
Geons  
Oscillons/I-balls  
Q-balls  
Boson Stars  
non-rel. Solitons

\* existence, stability, radiation — rich physics

see Review: Zhou [2411.16604]

\*these are non-topological solitons, for topological solitons such as strings, domain walls etc. see talk by Ligong Bian

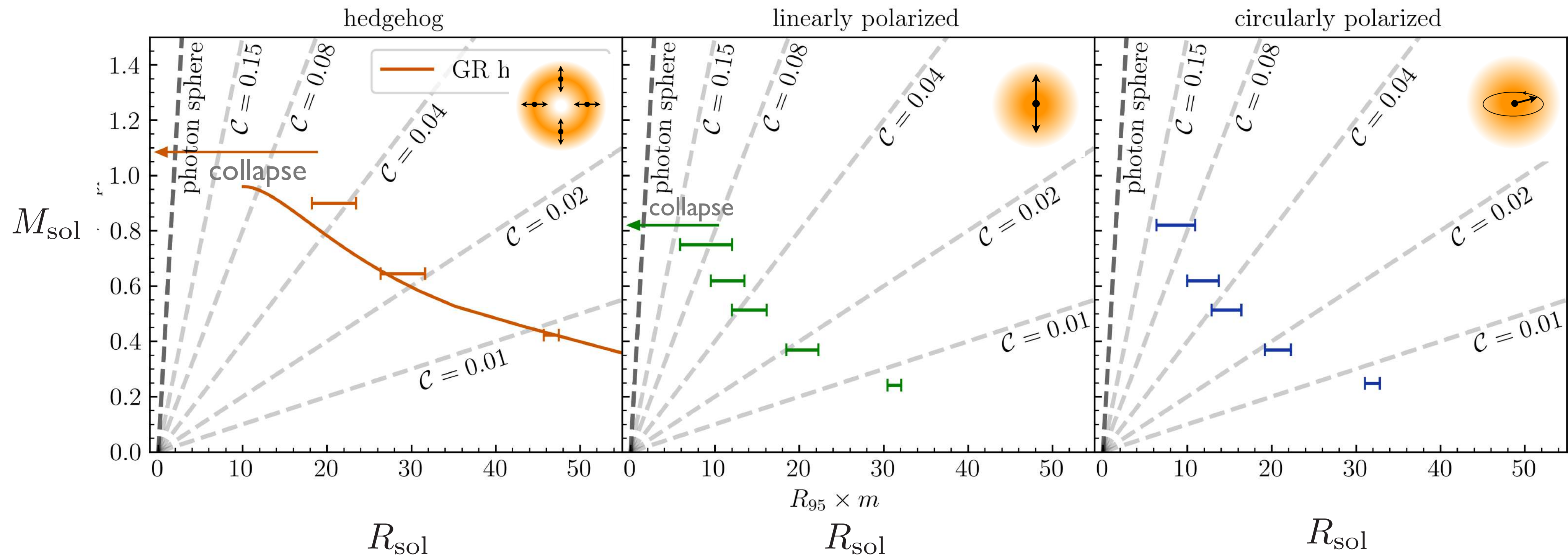
# probing intrinsic spin of solitons



# compactness of polarized solitons

more resistance to collapse to BH for circularly polarized stars

$$\mathcal{C}_{\text{hedgehog}} < \mathcal{C}_{\text{linearly polarized}} < \mathcal{C}_{\text{circularly polarized}}$$

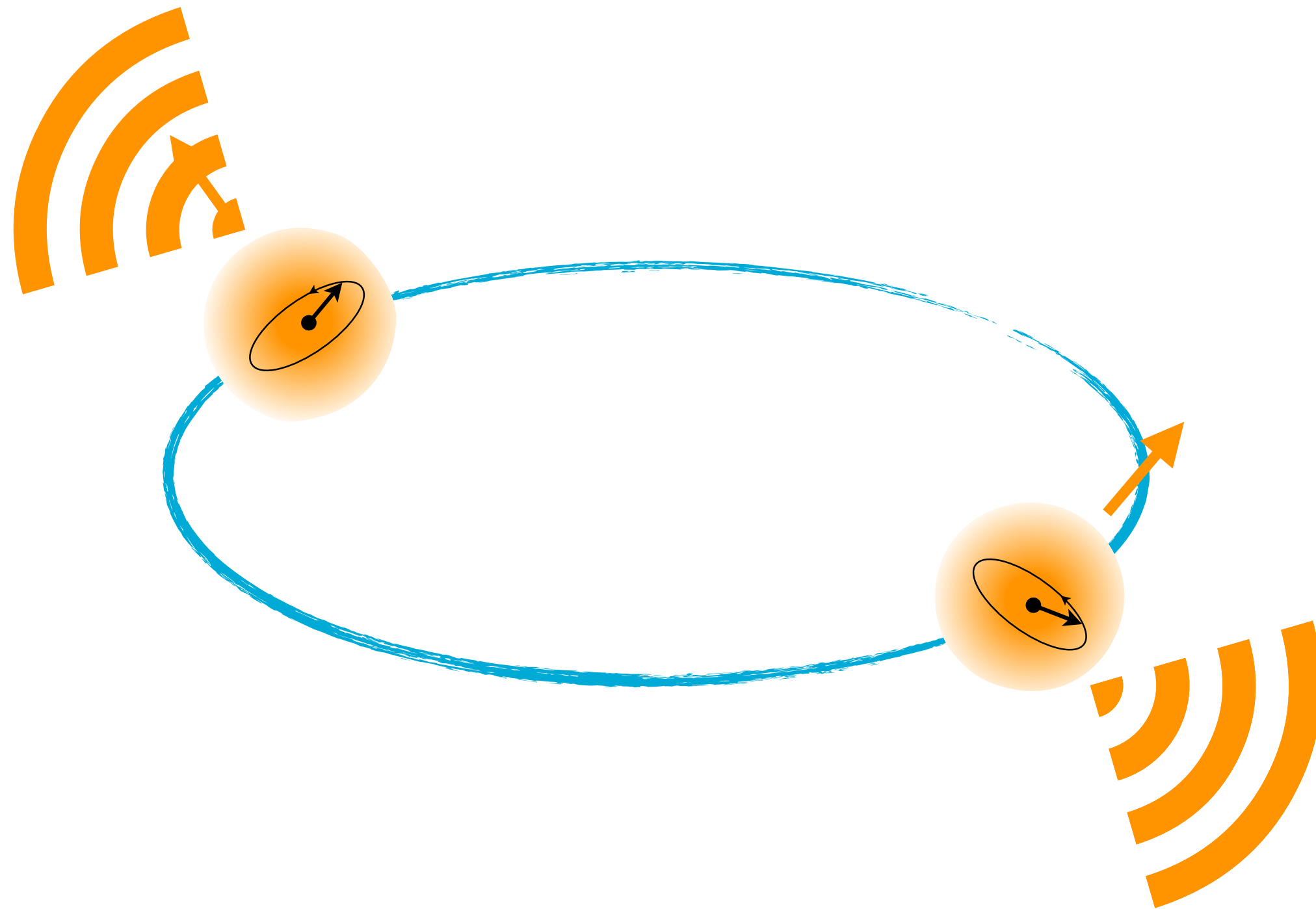


$$\mathcal{C} = GM/Rc^2$$



# gravitational waves and spin

$$V = -\frac{GM_1M_2}{r} \left[ 1 + \mathcal{O}(v^2/c^2) - \frac{2}{rc} [\hat{\mathbf{r}} \times (\mathbf{v}_1 - \mathbf{v}_2)] \cdot \sum_{a=1}^2 \frac{\mathbf{S}_a}{M_a} \right. \\ \left. + \frac{1}{r^2 c^2} \left\{ \frac{\mathbf{S}_1}{M_1} \cdot \frac{\mathbf{S}_2}{M_2} - 3 \left( \frac{\mathbf{S}_1}{M_1} \cdot \hat{\mathbf{r}} \right) \left( \frac{\mathbf{S}_2}{M_2} \cdot \hat{\mathbf{r}} \right) + \sum_{a=1}^2 \frac{C_{ES^2}^{(a)}}{2M_1M_2} [S_a^2 - 3(\mathbf{S}_a \cdot \hat{\mathbf{r}})^2] \right\} + \dots \right]$$



# what effects are relevant?

wave interference

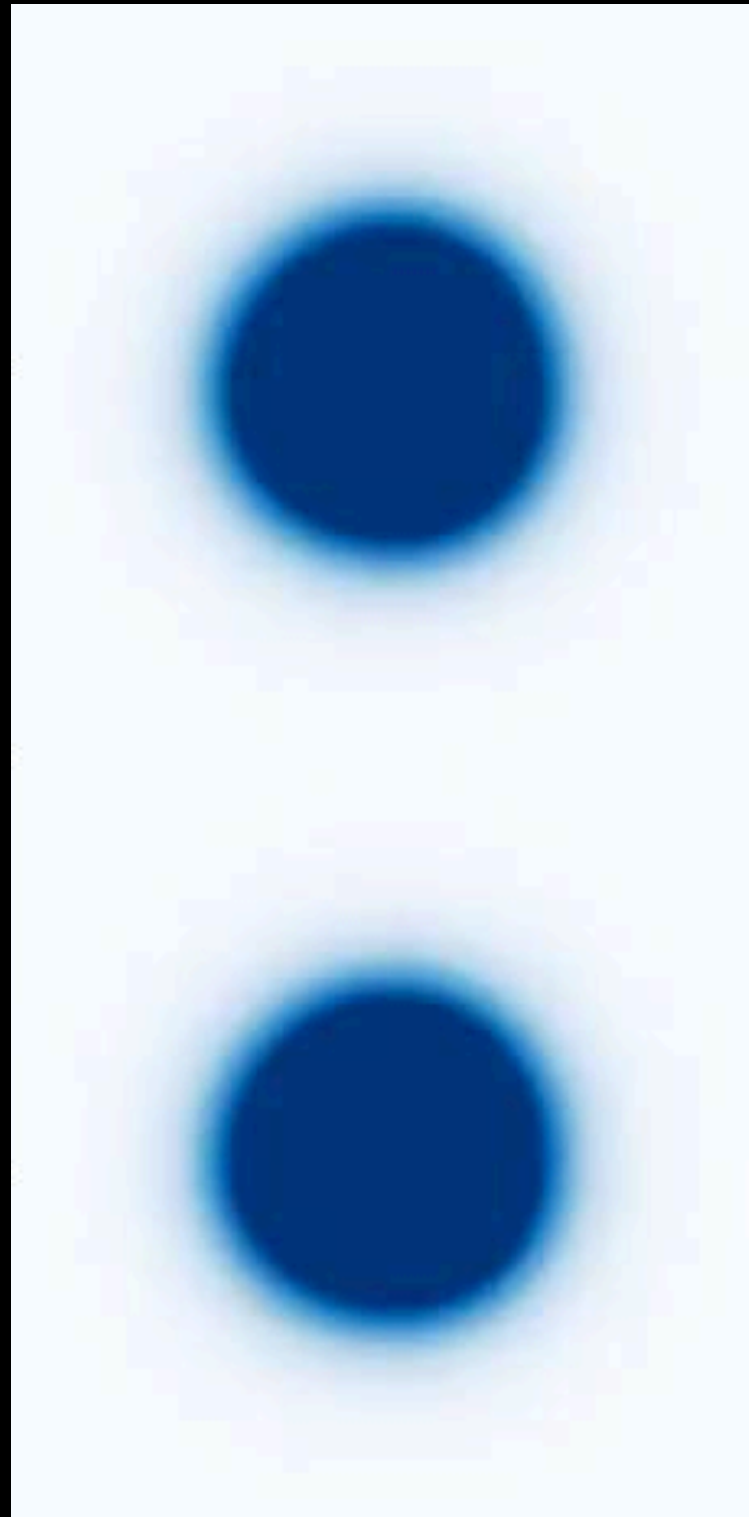
solitons

Bose effects

interactions

initial conditions+growth of structure

# Photons from Dark Photon Solitons via Parametric Resonance



$$\mathcal{L}_{\text{int}} \sim g\chi FF$$

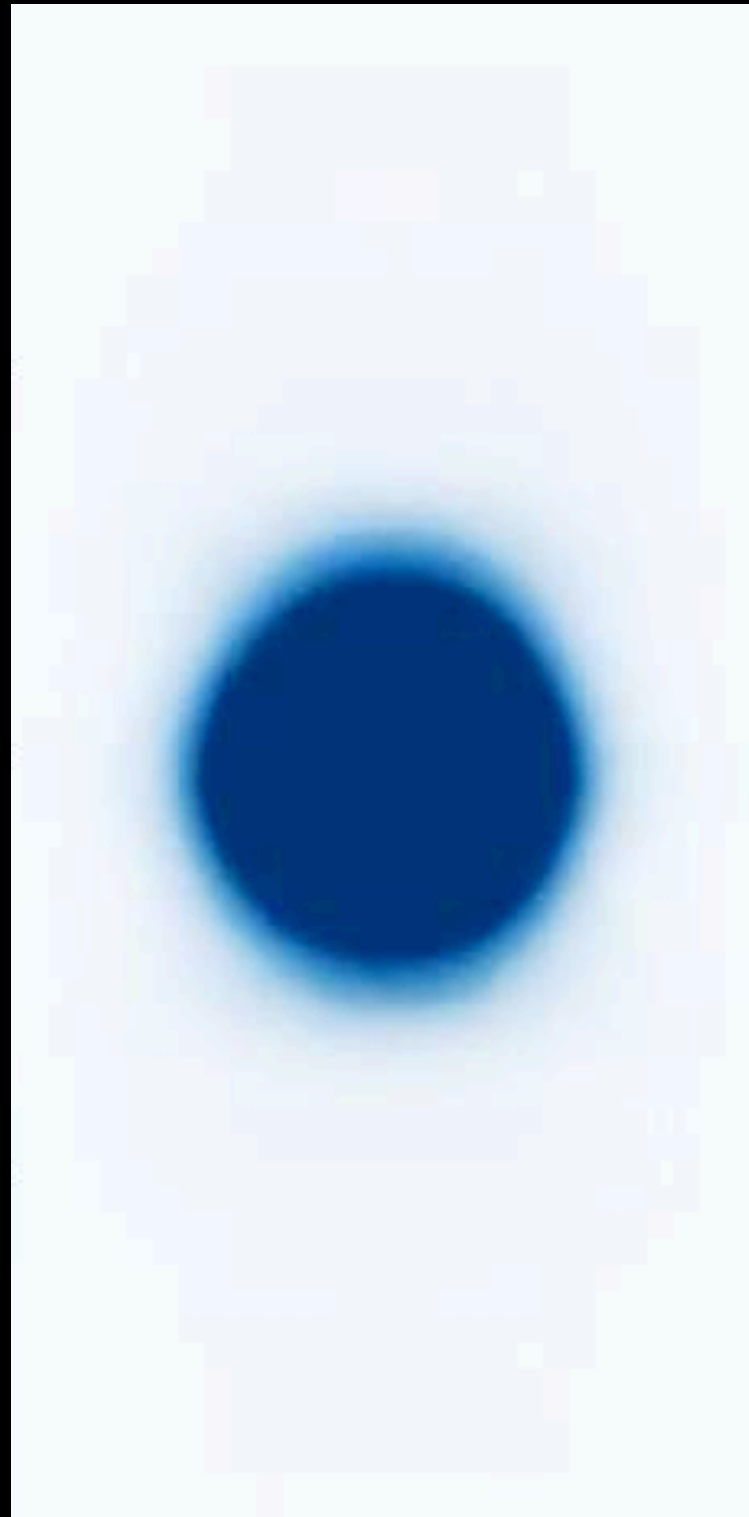
$$\mathcal{L}_{\text{int}} \sim g^2 XXFF$$

MA, Mou, Schiappacasse & Long (2019, 2022)

Amaral, Schiappacasse & Zhang (2025)



# Photons from Dark Photon Solitons via Parametric Resonance



$$\mathcal{L}_{\text{int}} \sim g\chi FF$$

$$\mathcal{L}_{\text{int}} \sim g^2 XXFF$$

MA, Mou, Schiappacasse & Long (2019, 2022)

Amaral, Schiappacasse & Zhang (2025)

# spin of soliton & polarization of photons

$$\mathcal{O}_1 = -\frac{1}{2}F_{\mu\nu}\tilde{F}^{\mu\nu}(X \cdot X)$$

$$\mathcal{O}_2 = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu}(X \cdot X)$$

$$\mathcal{O}_3 = F_{\mu\rho}F^{\nu\rho}X^\mu X_\nu$$

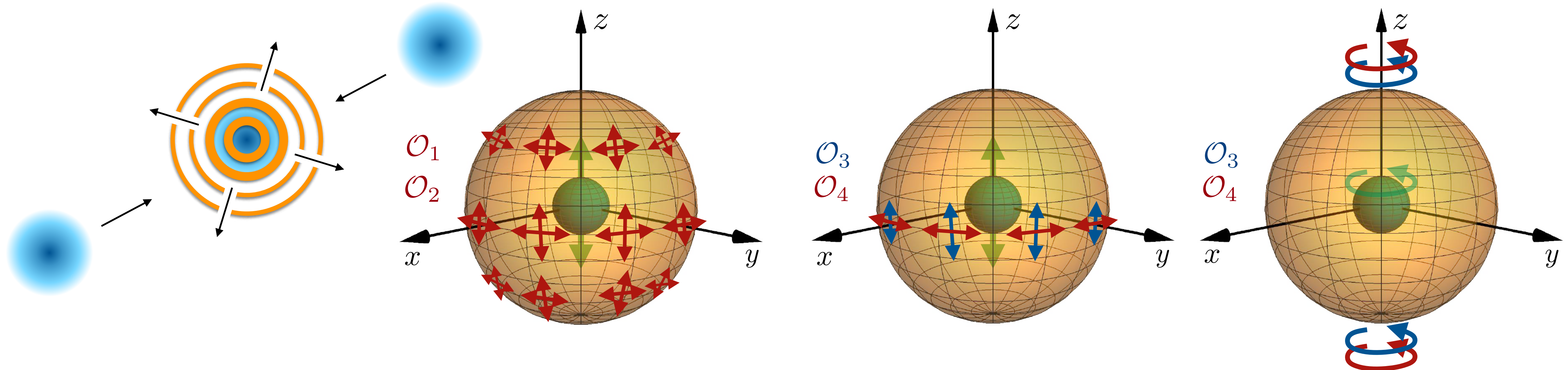
$$\mathcal{O}_4 = \tilde{F}_{\mu\rho}\tilde{F}^{\nu\rho}X^\mu X_\nu$$

$$\mathcal{O}_5 = F_{\mu\rho}F^{\nu\rho}\partial^\mu X_\nu$$

explosive photon production (under certain conditions)

$$\mu R \gtrsim 1, \quad \mu \sim g^2 X^2 m$$

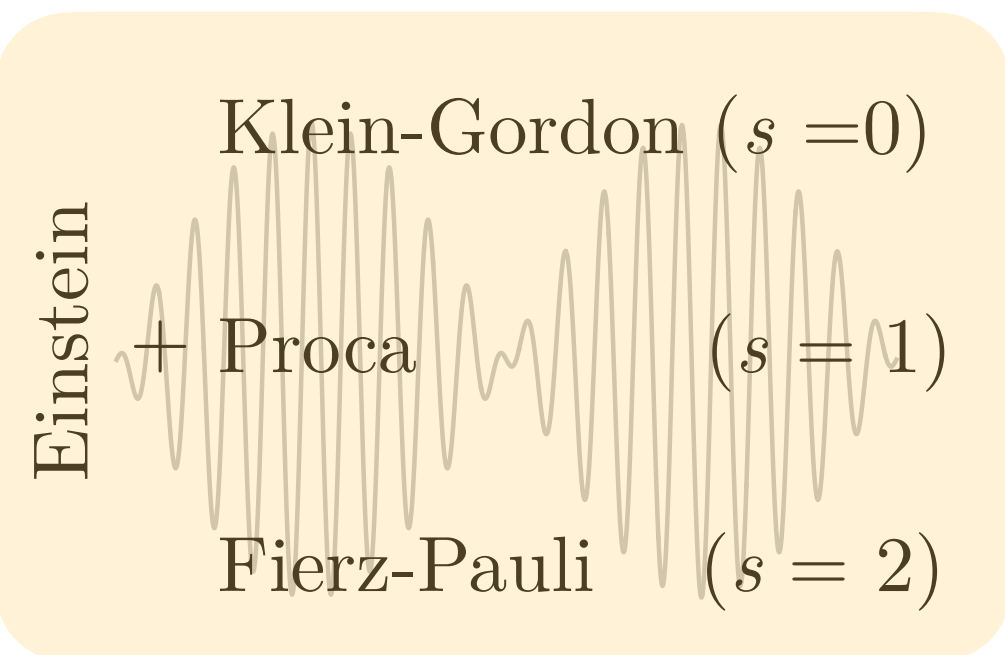
$R$  = soliton radius,  $\mu$  = Floquet exponent



generalization to arbitrary spin

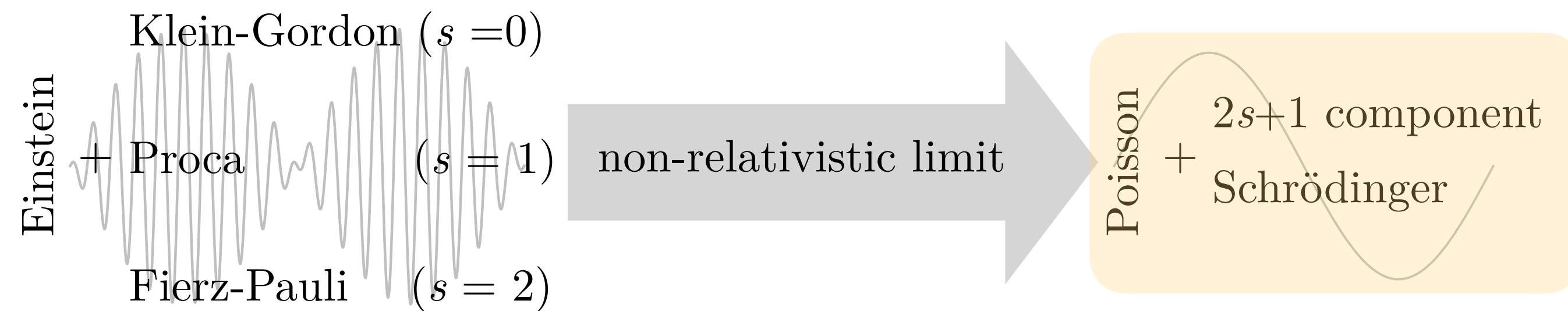


# spin- $s$ field as dark matter



# non-relativistic limit = multicomponent Schrödinger-Poisson

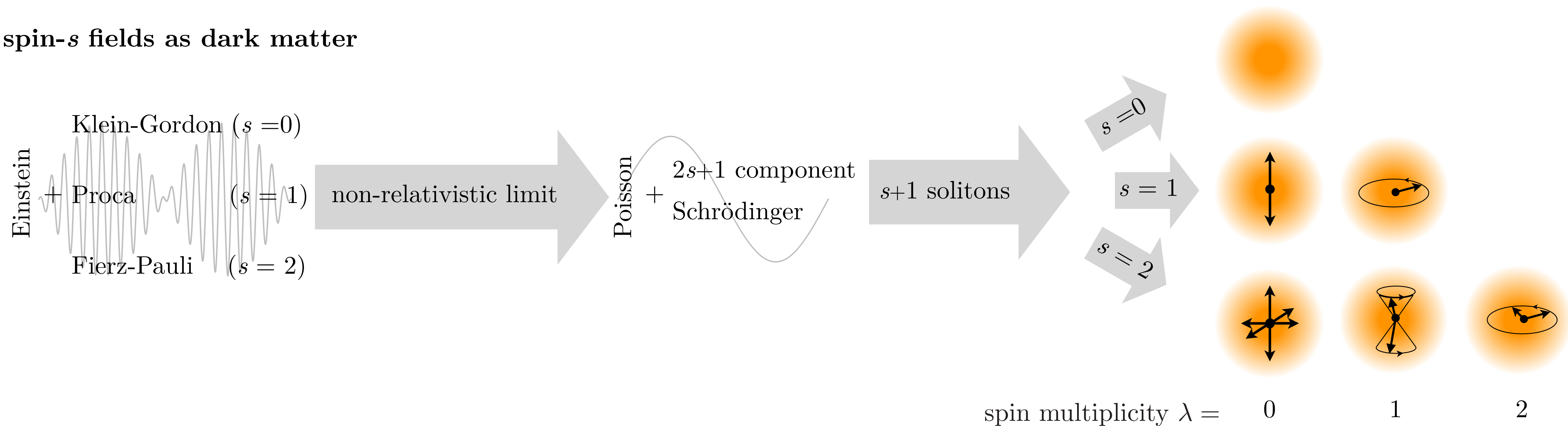
spin- $s$  fields as light dark matter



- phenomenology/numerical simulations
- interference  $\sim 1/(2s+1)$

# extremally polarized solitons

spin- $s$  fields as dark matter



**macroscopic spin**  $\mathbf{S}_{\text{tot}}/\hbar = \lambda N \hat{z}$

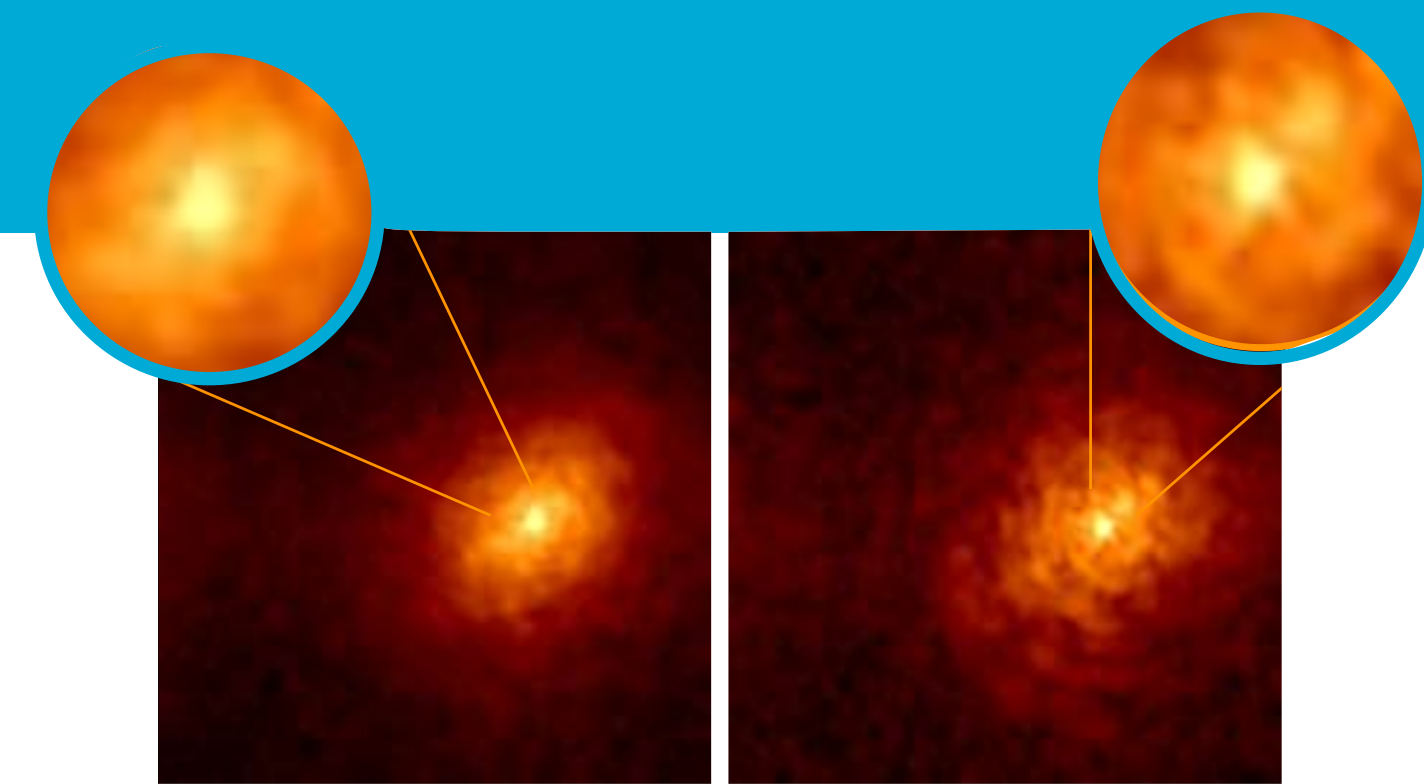
$N =$  # of particles in soliton



# summary so far

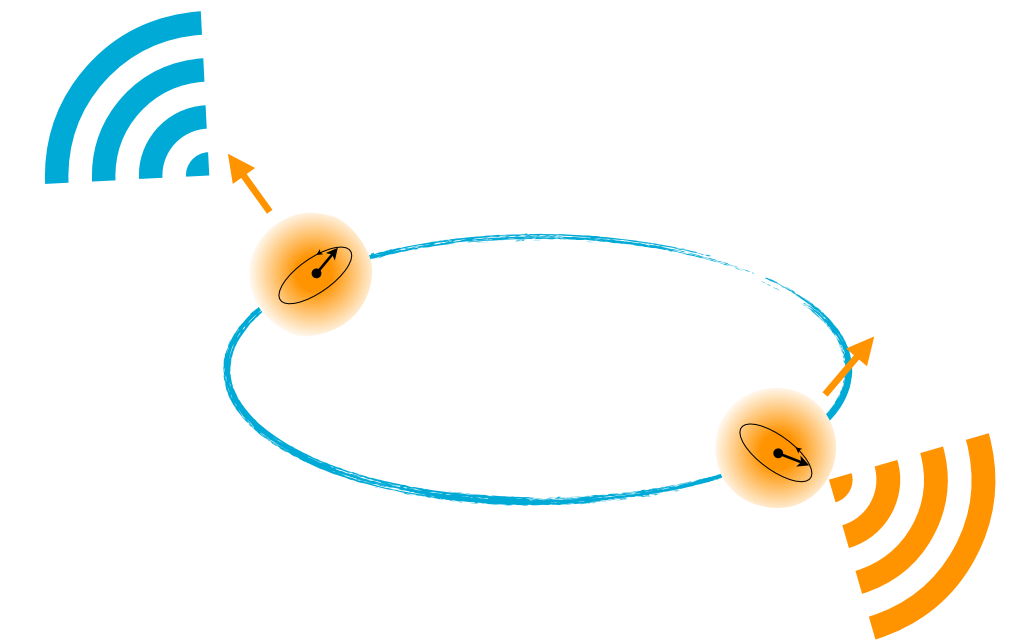
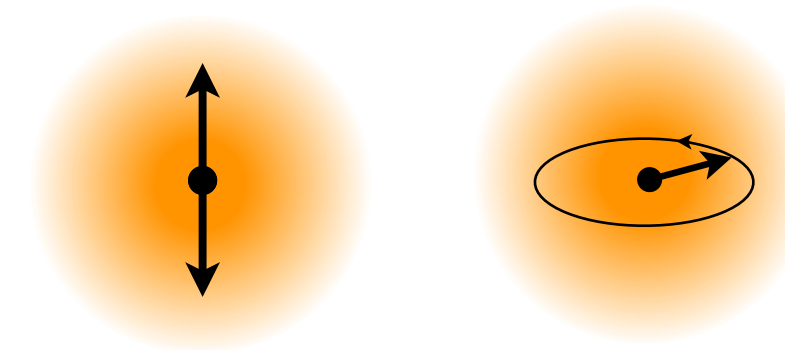
## reduced wave interference

- dynamical heating, lensing, nucleation times, core-halo relation ...



## solitons & spin

- polarized emission, compactness ...



# summary so far

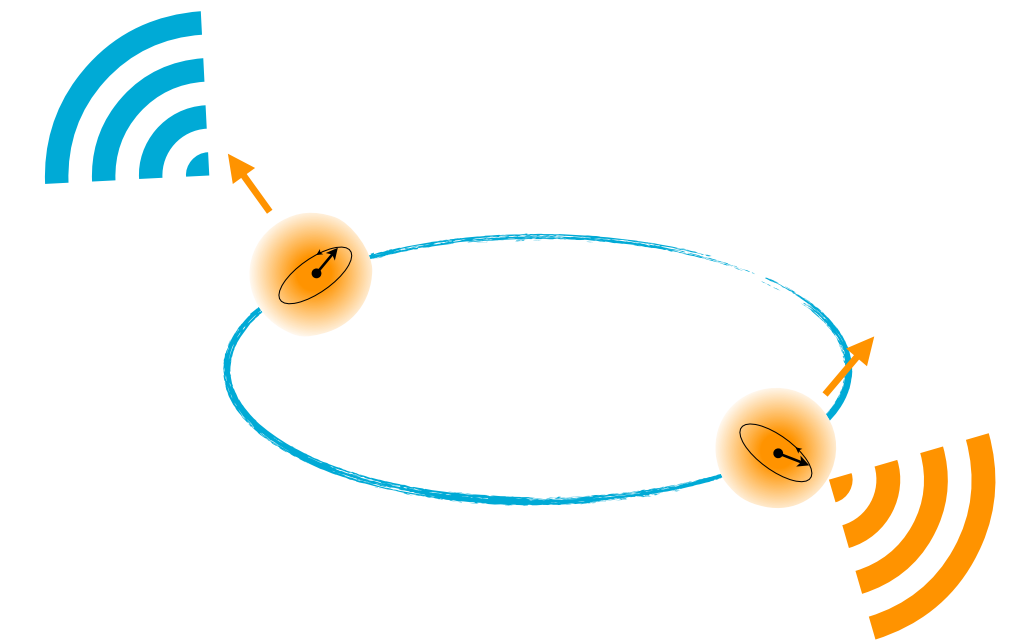
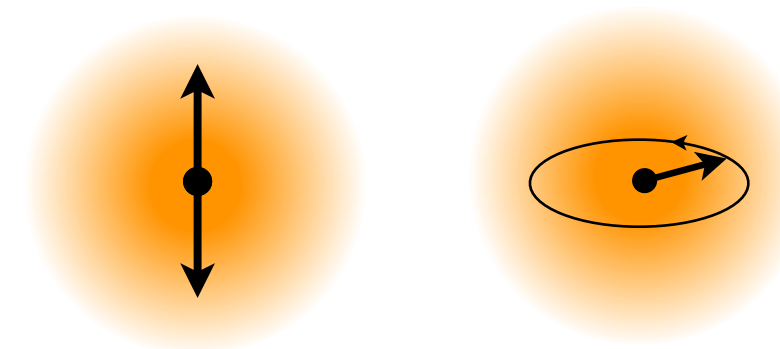
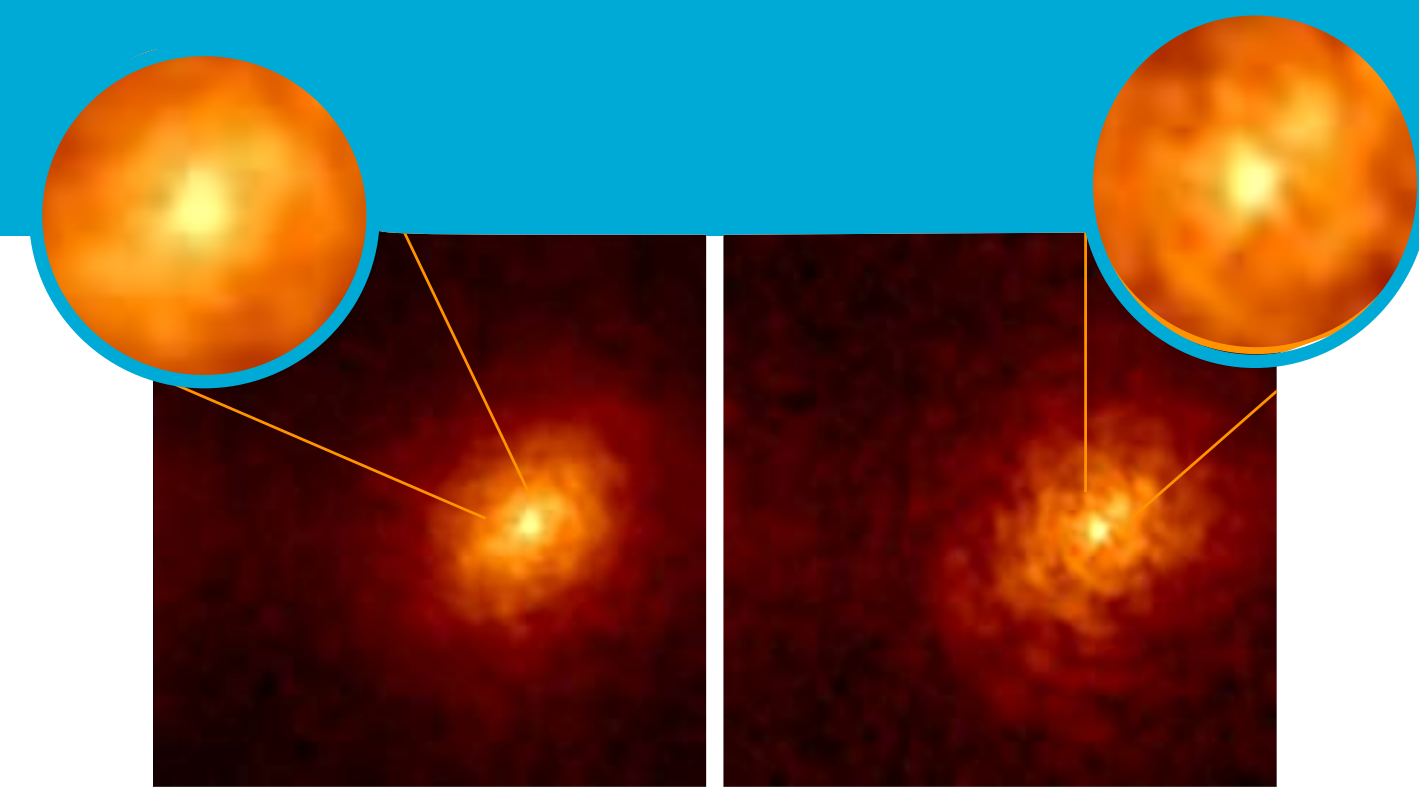
reduced wave interference

- dynamical heating, lensing, nucleation times, core-halo relation ...

solitons & spin

- polarized emission, compactness ...

initial conditions+growth of structure



**production mechanisms and growth of structure**



# ③ ② Solitons in Warm, Noisy and Spinning Dark Matter Fields ①

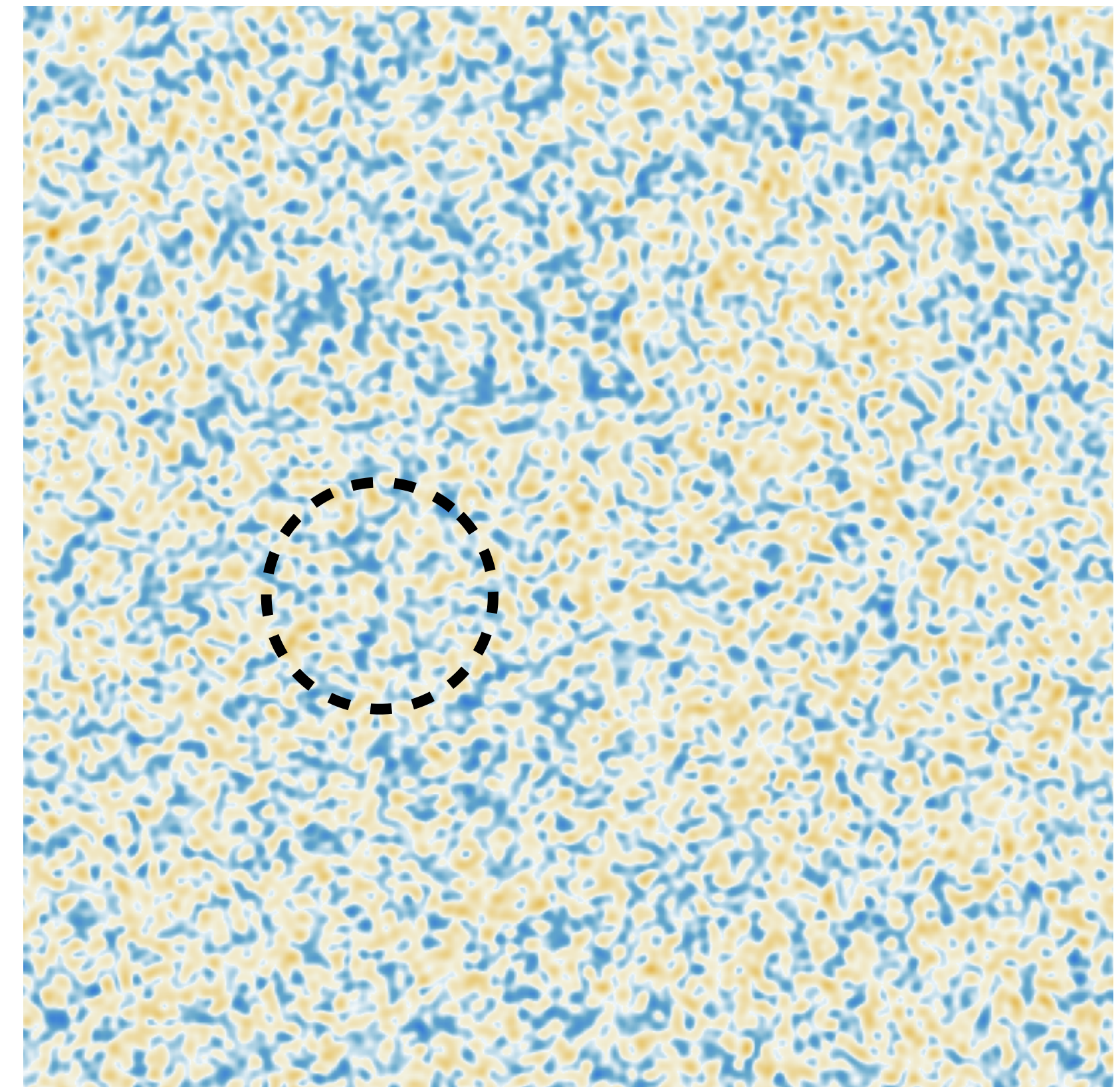
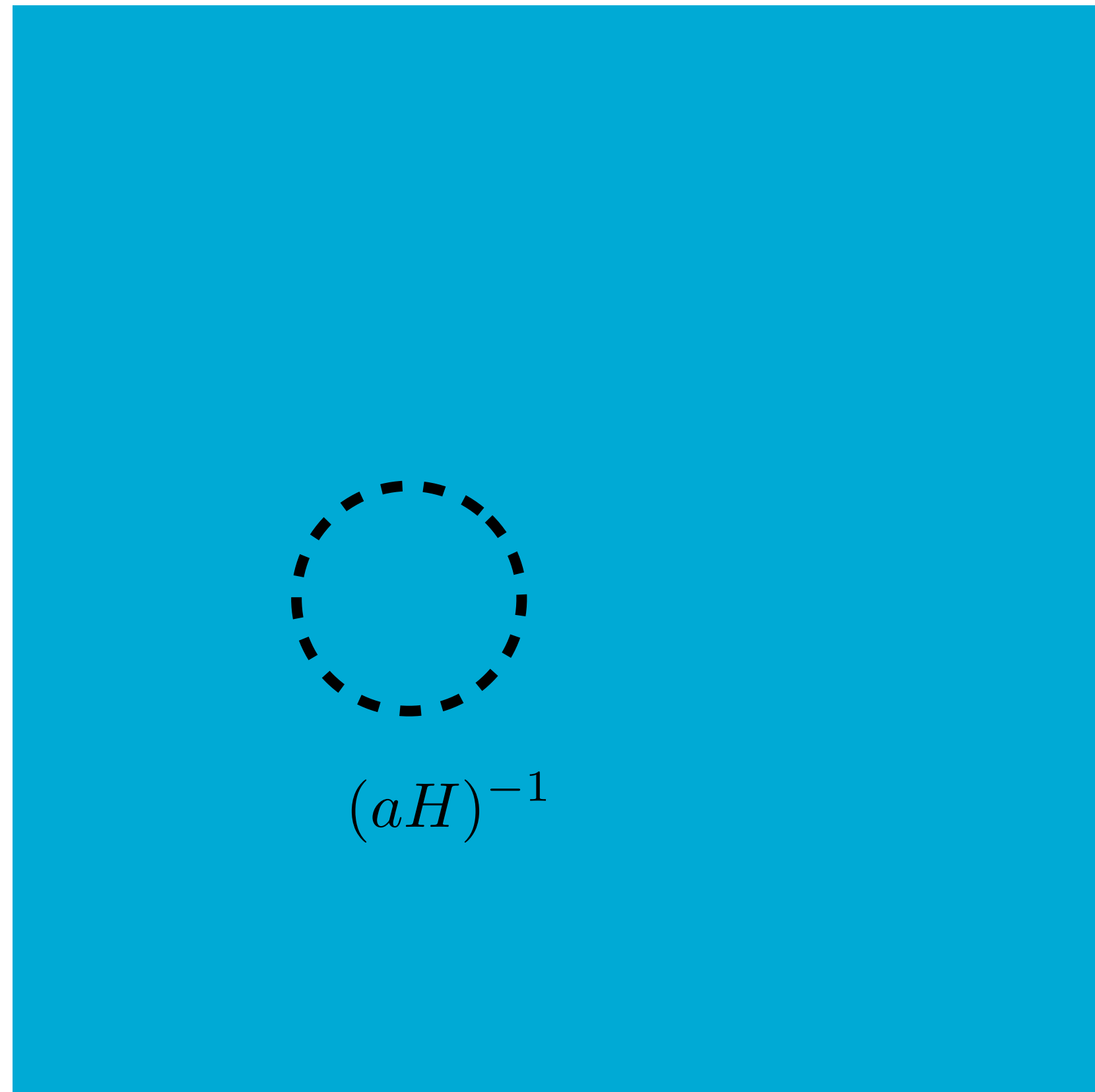
Mustafa A. Amin





# production mechanisms

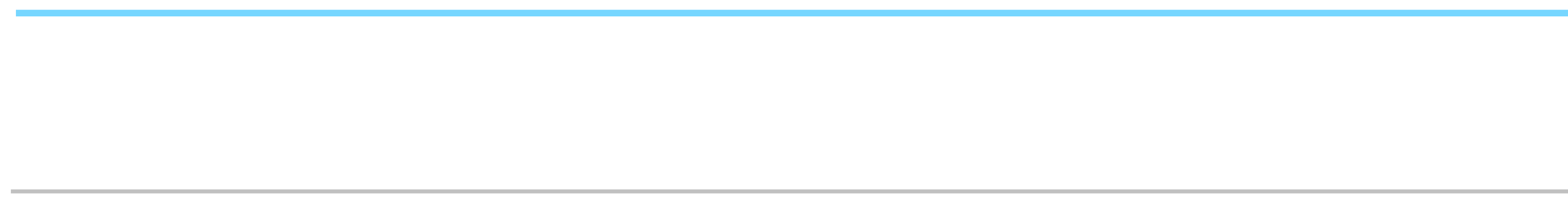
\* for simplicity focus on spin-0 first



inflationary vs. post-inflationary

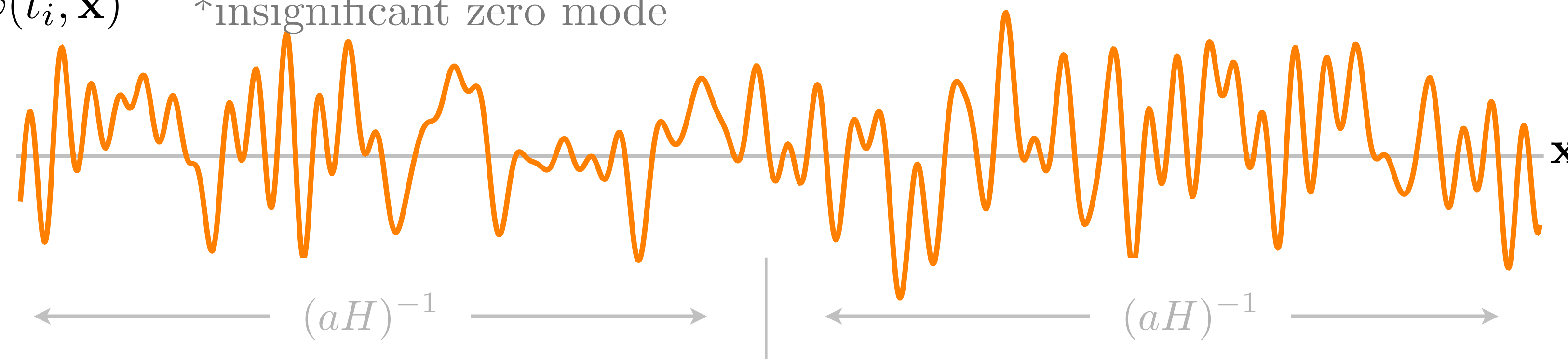
# Quiet vs. Noisy Fields

$\varphi(t_i, \mathbf{x})$  \* significant zero mode



eg. axions with  $\mathbb{PQ}$  before inflation  
inflaton just at end of inf.

$\varphi(t_i, \mathbf{x})$  \* insignificant zero mode

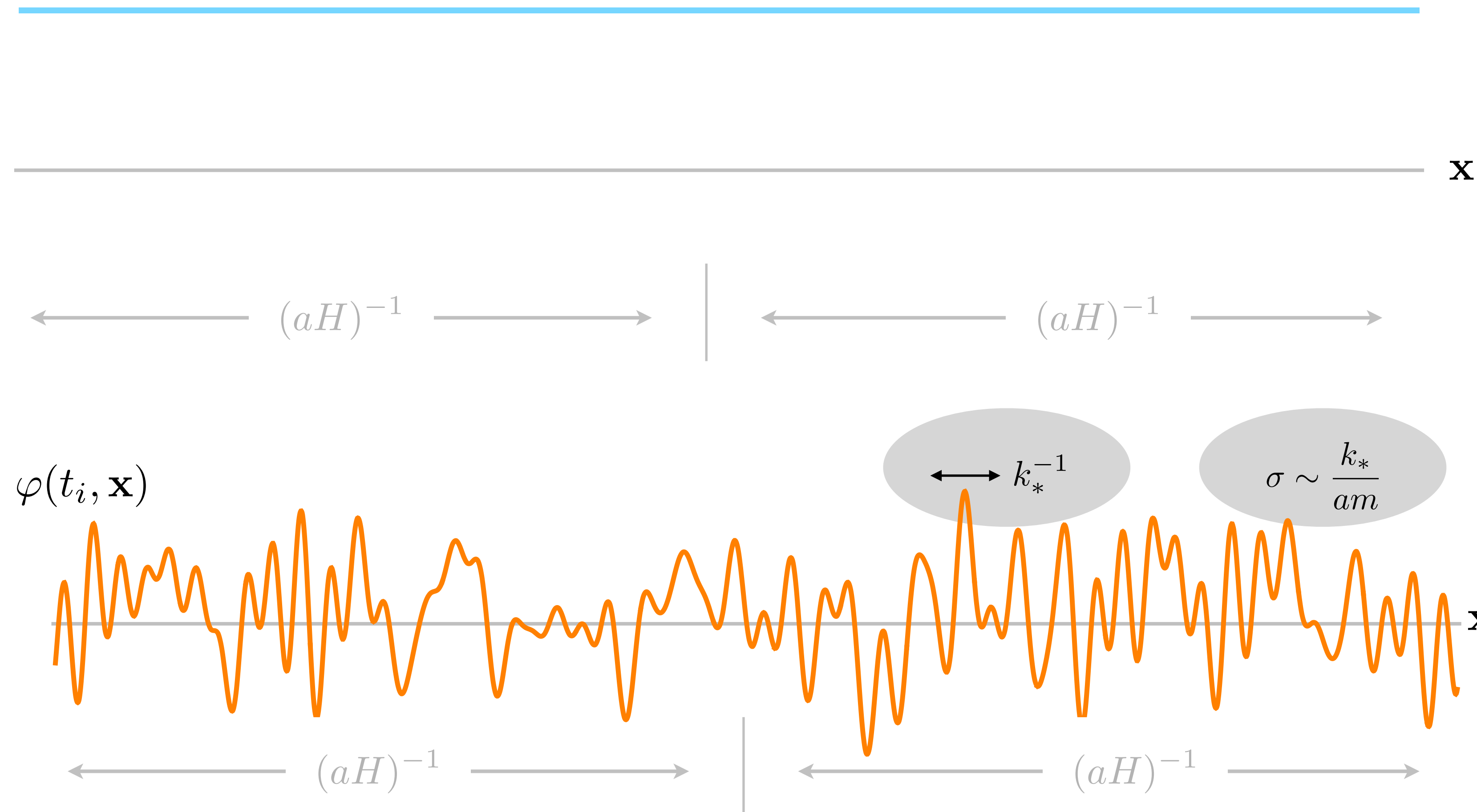


eg. axions with  $\mathbb{PQ}$  after inflation  
field after inflation fragmentation

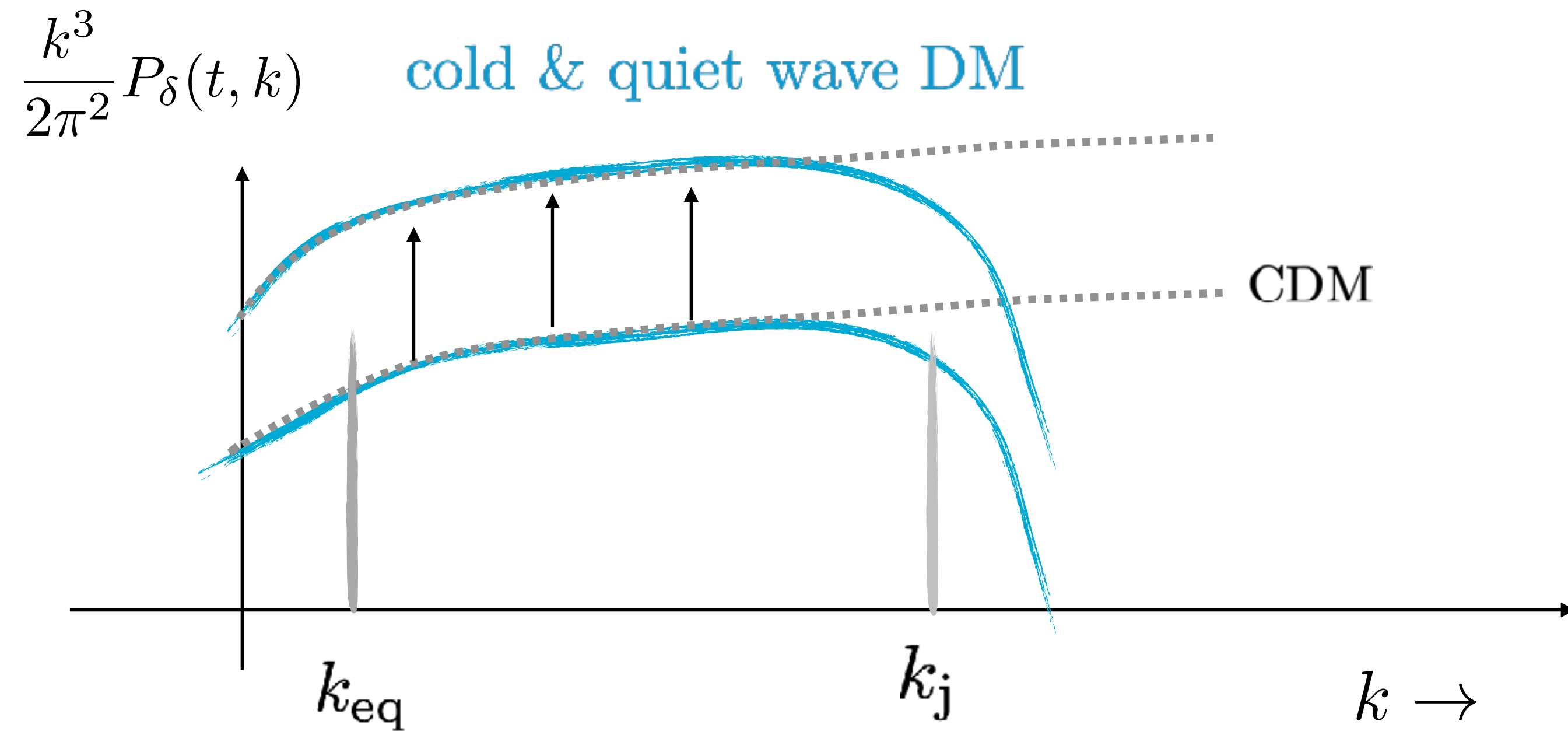


# Cold & Quiet vs. Warm & Noisy Fields

$\varphi(t_i, \mathbf{x})$



# cold & quiet wave dark matter



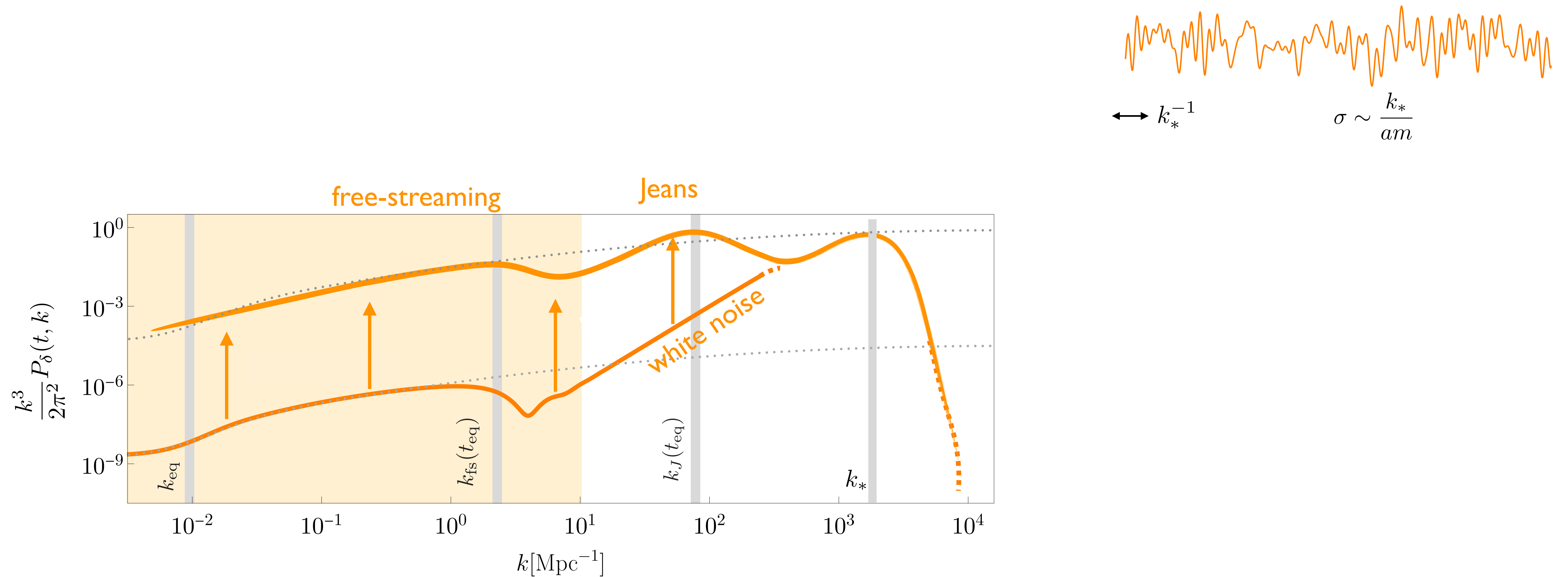
$$\varphi(t_i, \mathbf{x})$$

$\mathbf{x}$

Wave-dynamical Jeans — “quantum”

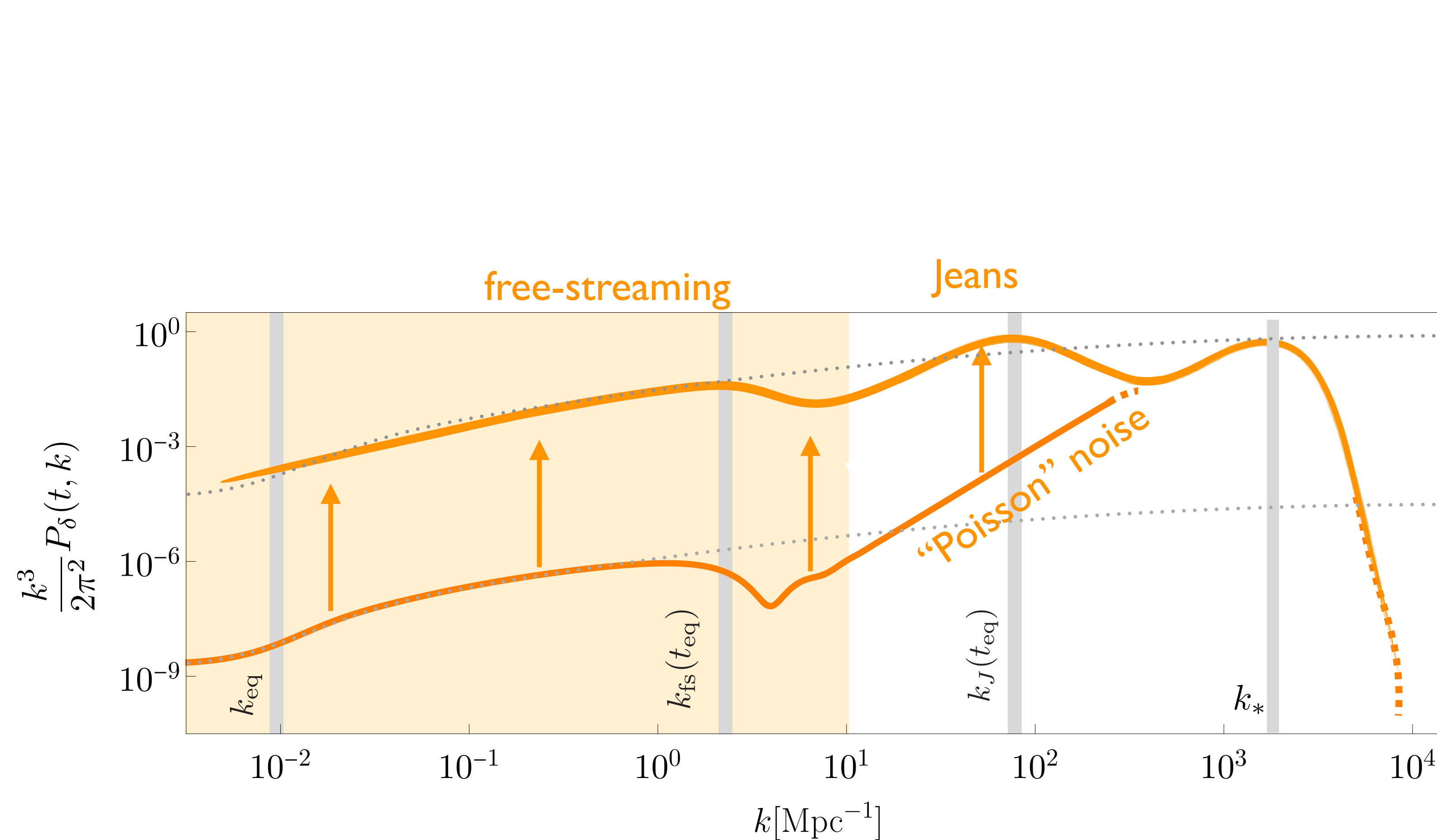
$$k_j \sim 200 \text{ Mpc}^{-1} \sqrt{\frac{m}{10^{-19} \text{ eV}}}$$

# warm & noisy wave dark matter





# warm & noisy wave dark matter



$$\longleftrightarrow k_*^{-1} \quad \sigma \sim \frac{k_*}{am}$$

free-streaming

$$k_{\text{fs}}(a_{\text{eq}}) \sim 10 \text{ Mpc}^{-1} \left( \frac{m}{10^{-19} \text{ eV}} \right) \left( \frac{10^3 \text{ Mpc}^{-1}}{k_*} \right)$$

Jeans

$$k_J(a_{\text{eq}}) \sim 50 \text{ Mpc}^{-1} \left( \frac{m}{10^{-19} \text{ eV}} \right) \left( \frac{10^3 \text{ Mpc}^{-1}}{k_*} \right)$$

“Poisson” noise

$$(k/k_*)^3, \quad k_* \gtrsim 10^3 \text{ Mpc}^{-1}$$

# field power spectrum of noisy cosmological fields

## non-gravitational production *after* inflation (“causal”)

phase transitions

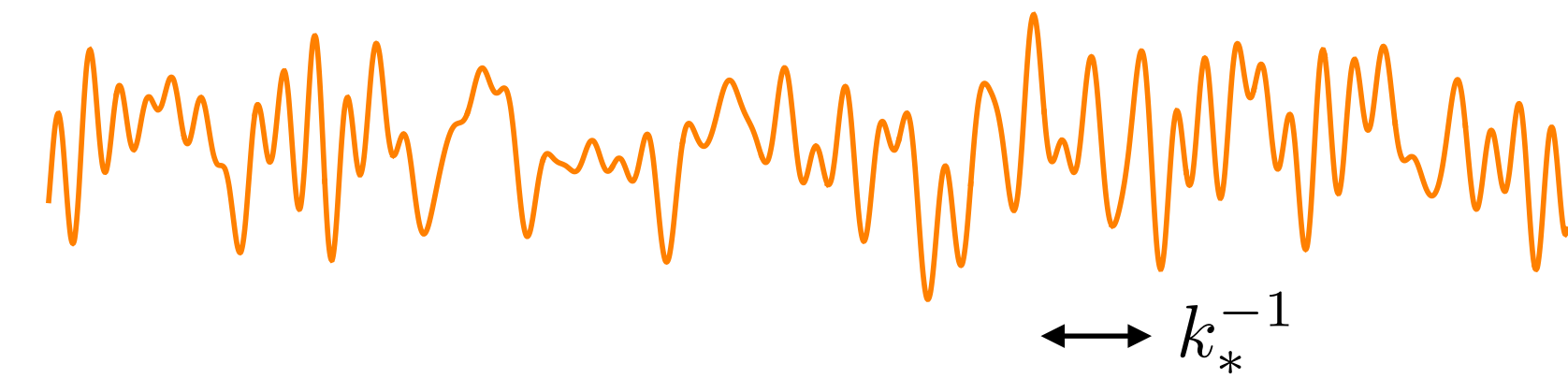
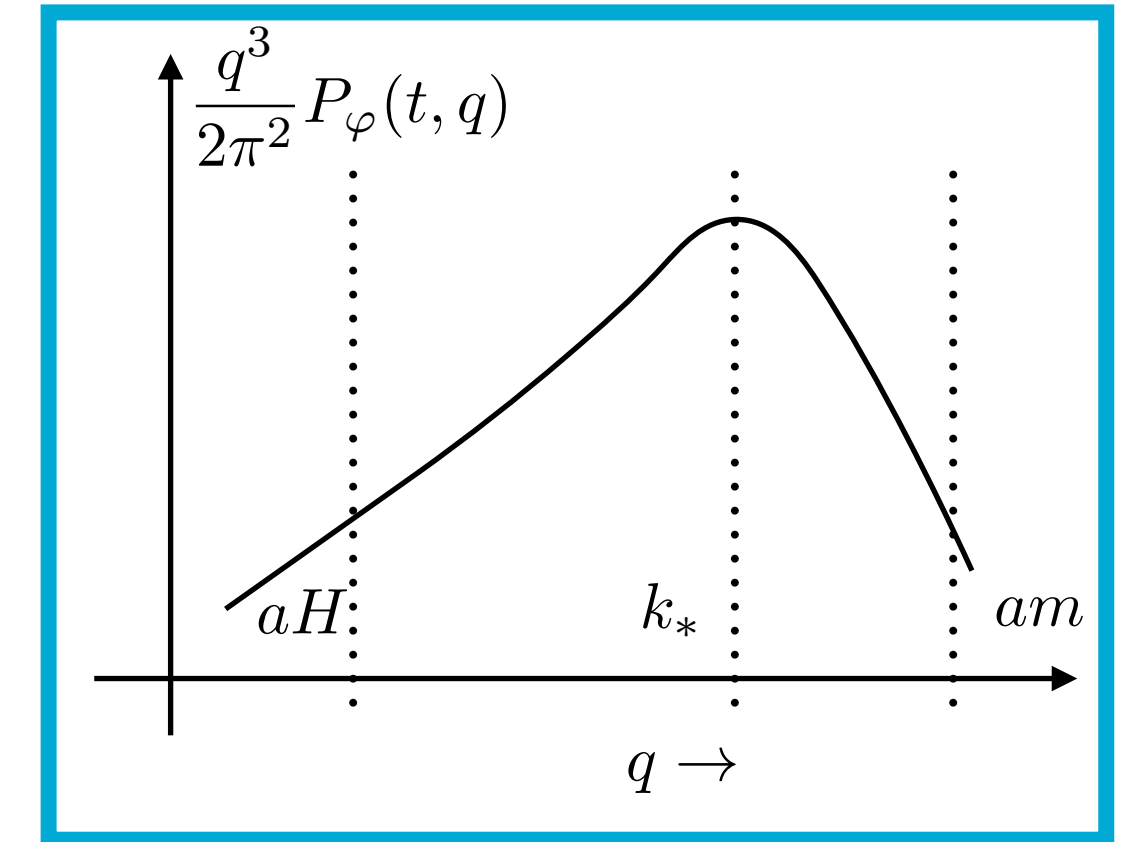
- *axion-like fields (including QCD)*

resonant/tachyonic energy transfer from fields, defects eg. strings

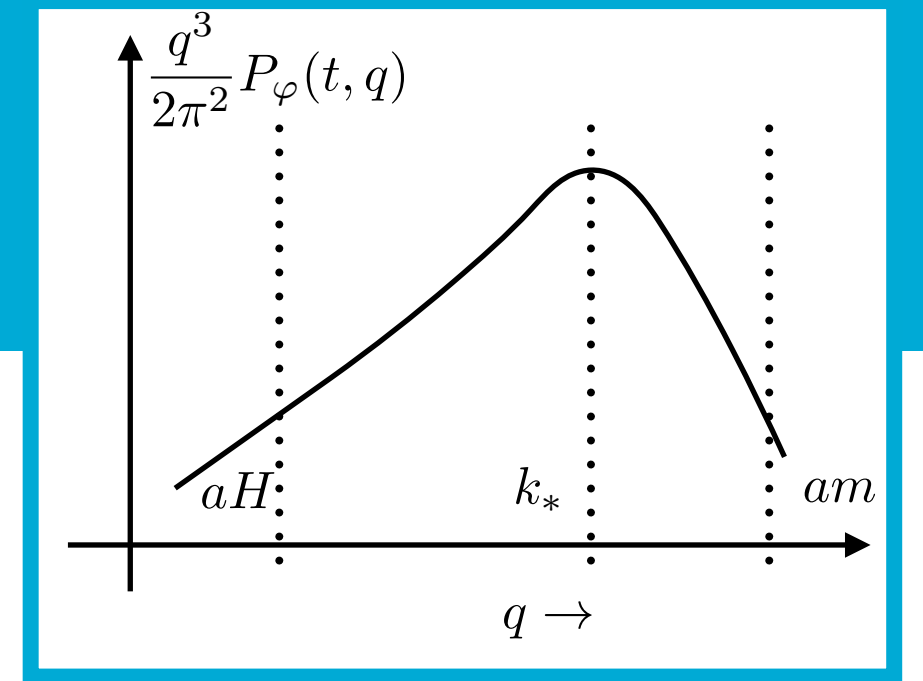
also works for thermal production, but nothing new there

## inflationary gravitational particle production

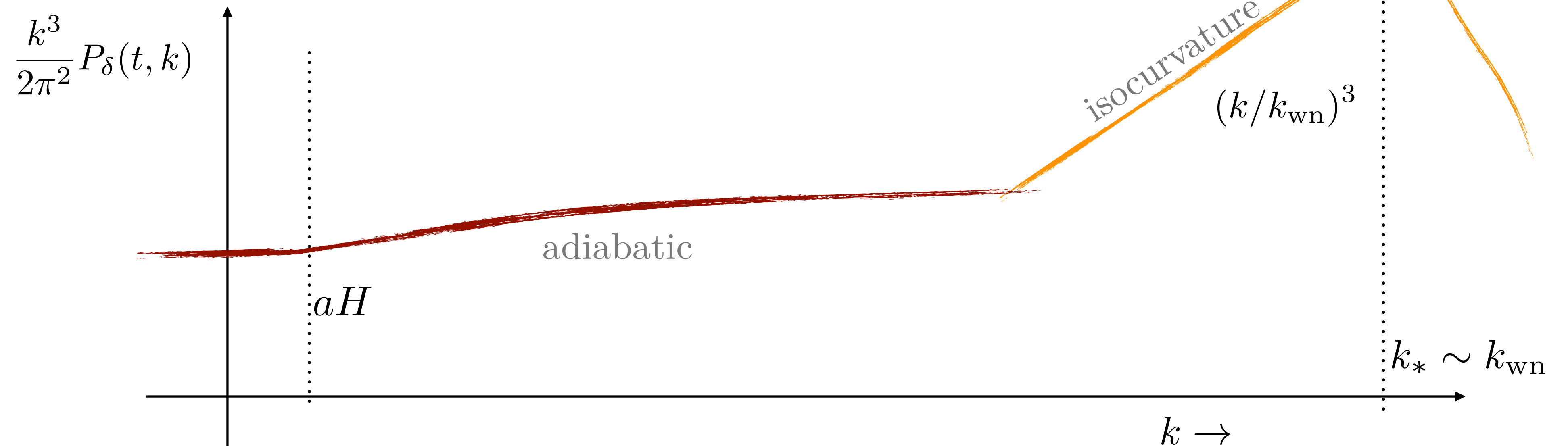
- eg. dark photon dark matter



# initial conditions

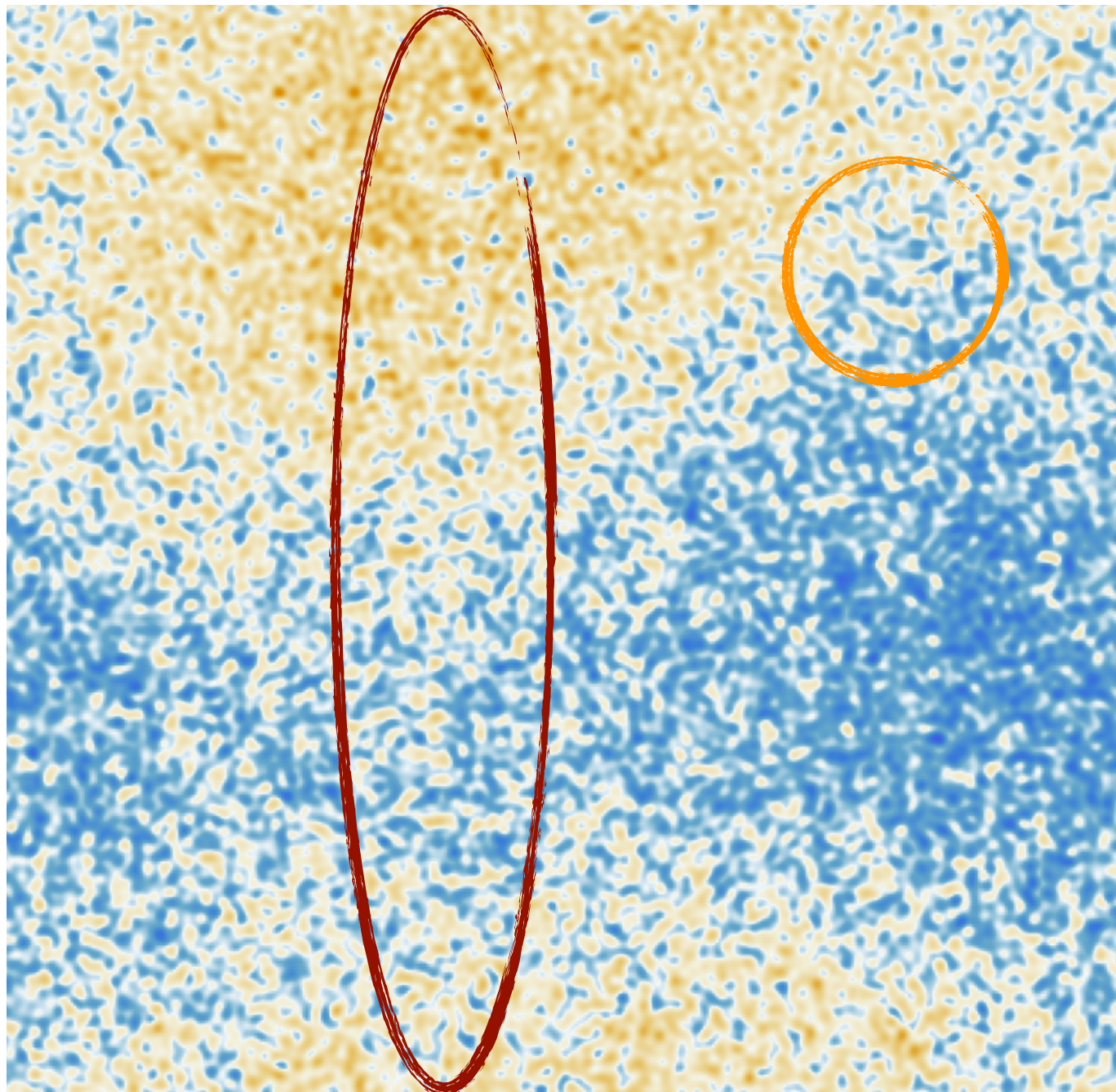


1. small length scale “isocurvature” component
2. larger scale **adiabatic** component



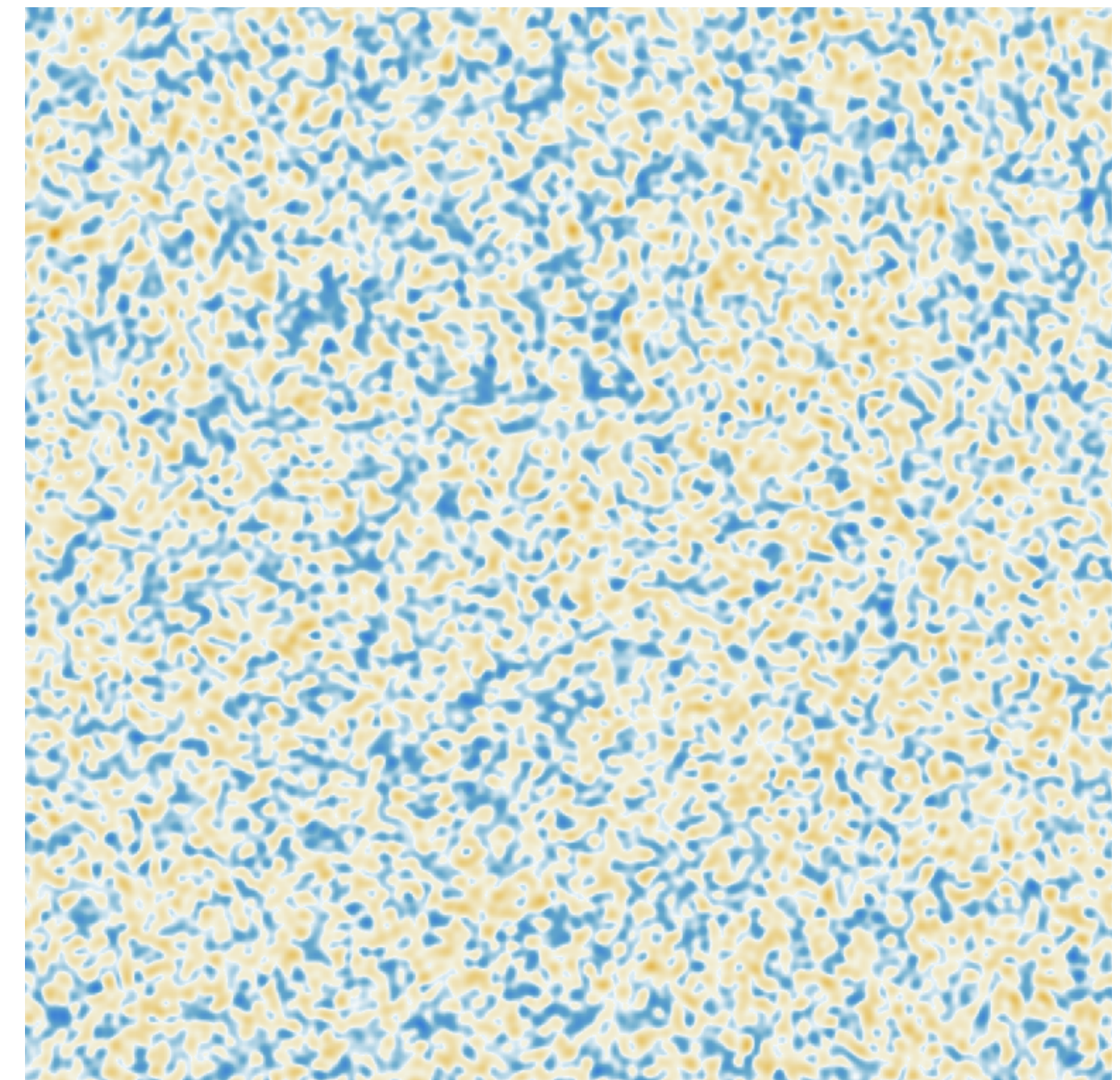
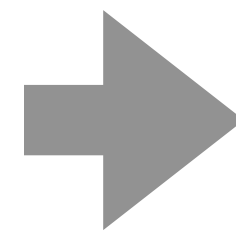
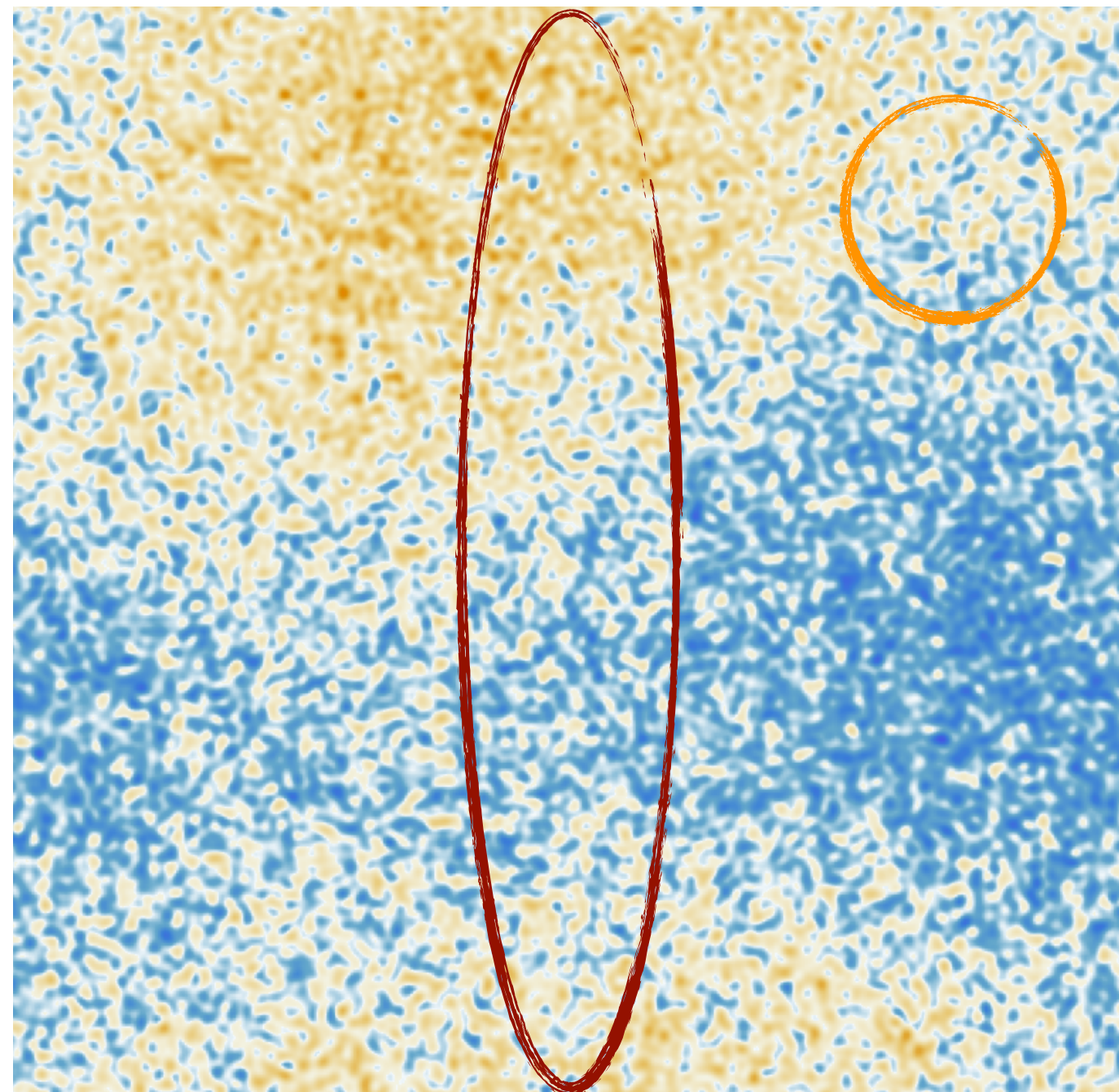
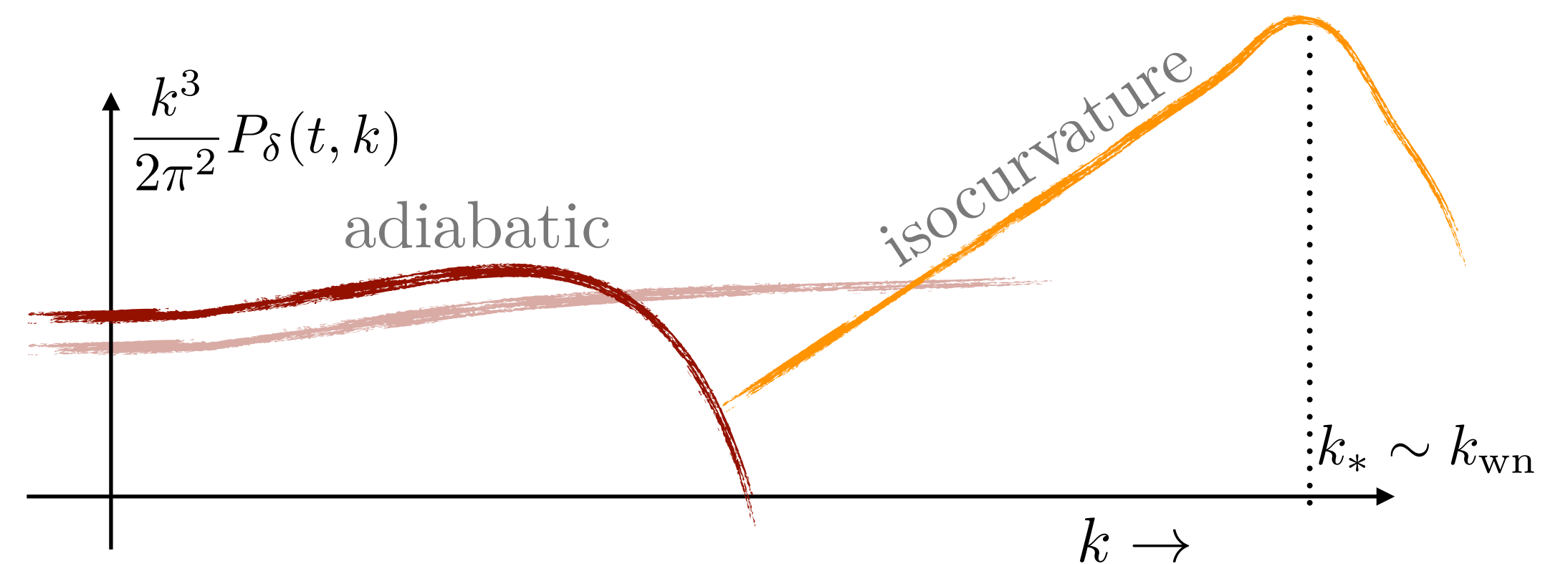
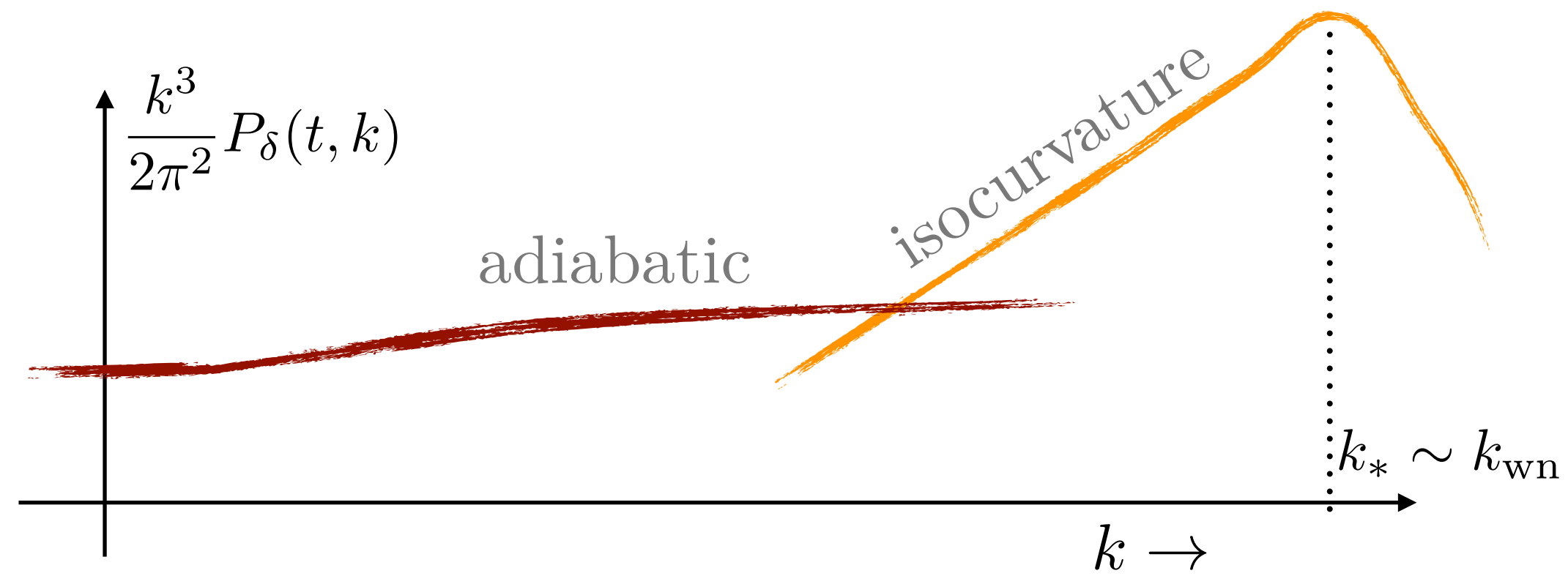


# density evolution during radiation domination



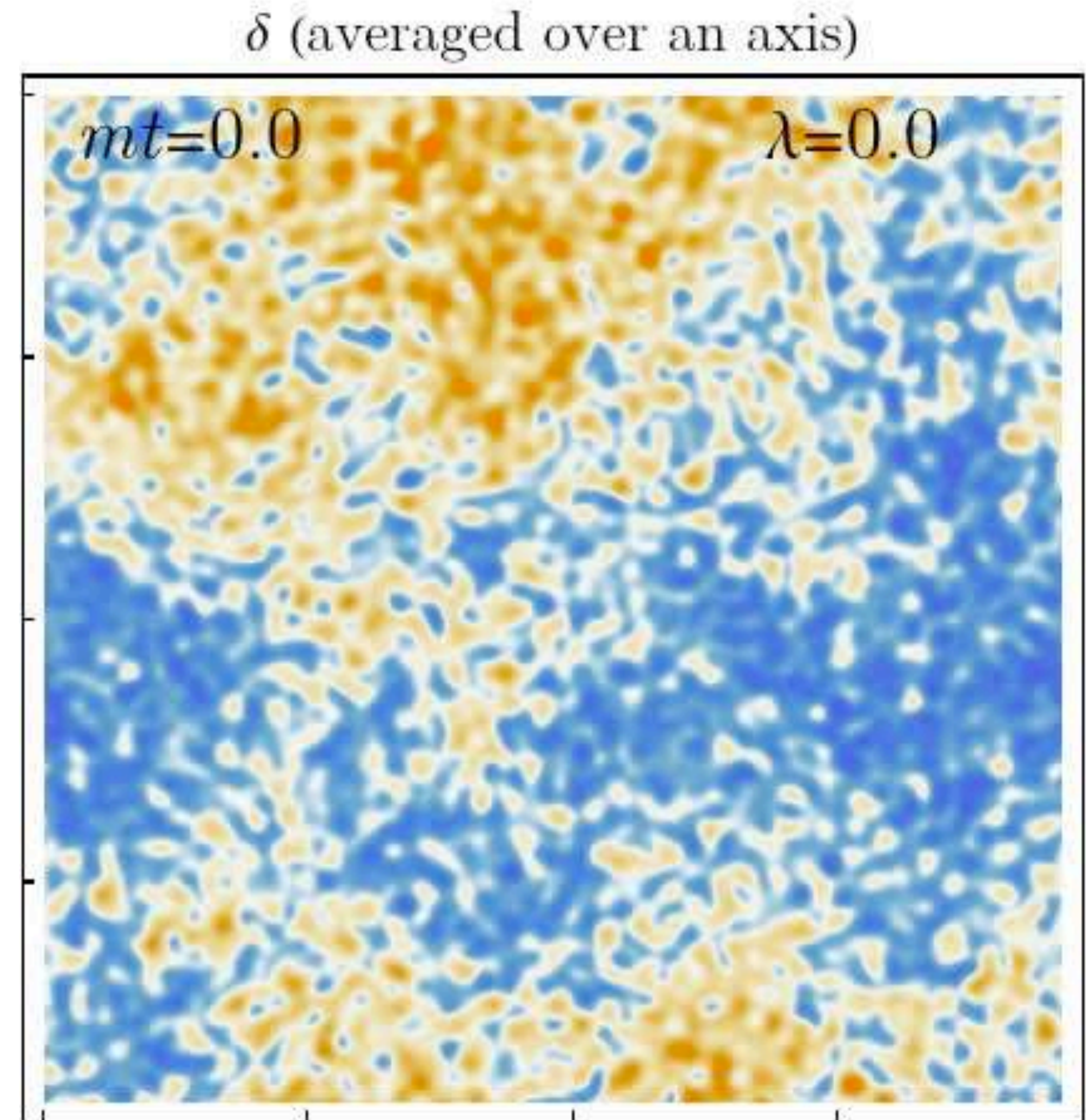
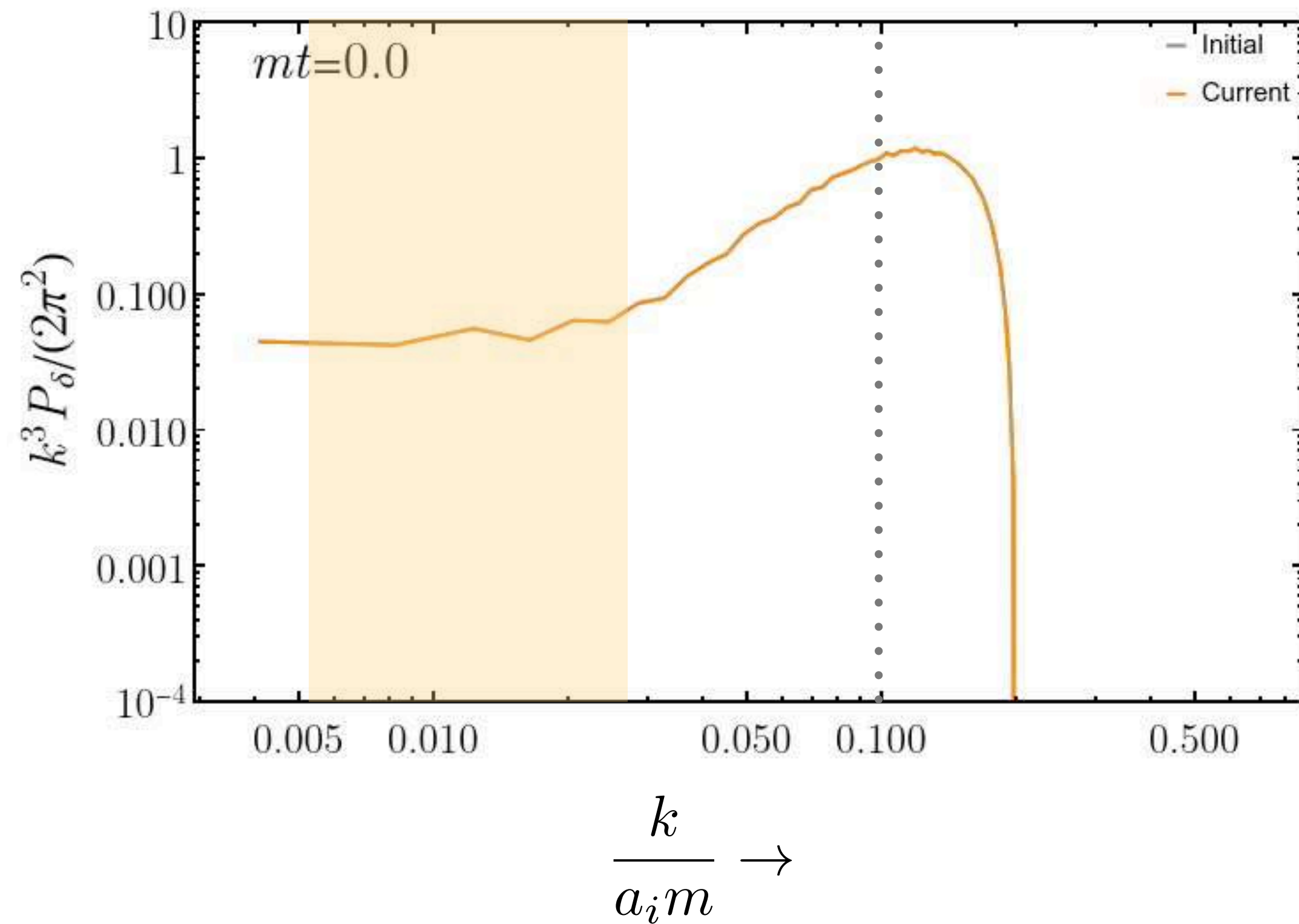


# density evolution during radiation domination



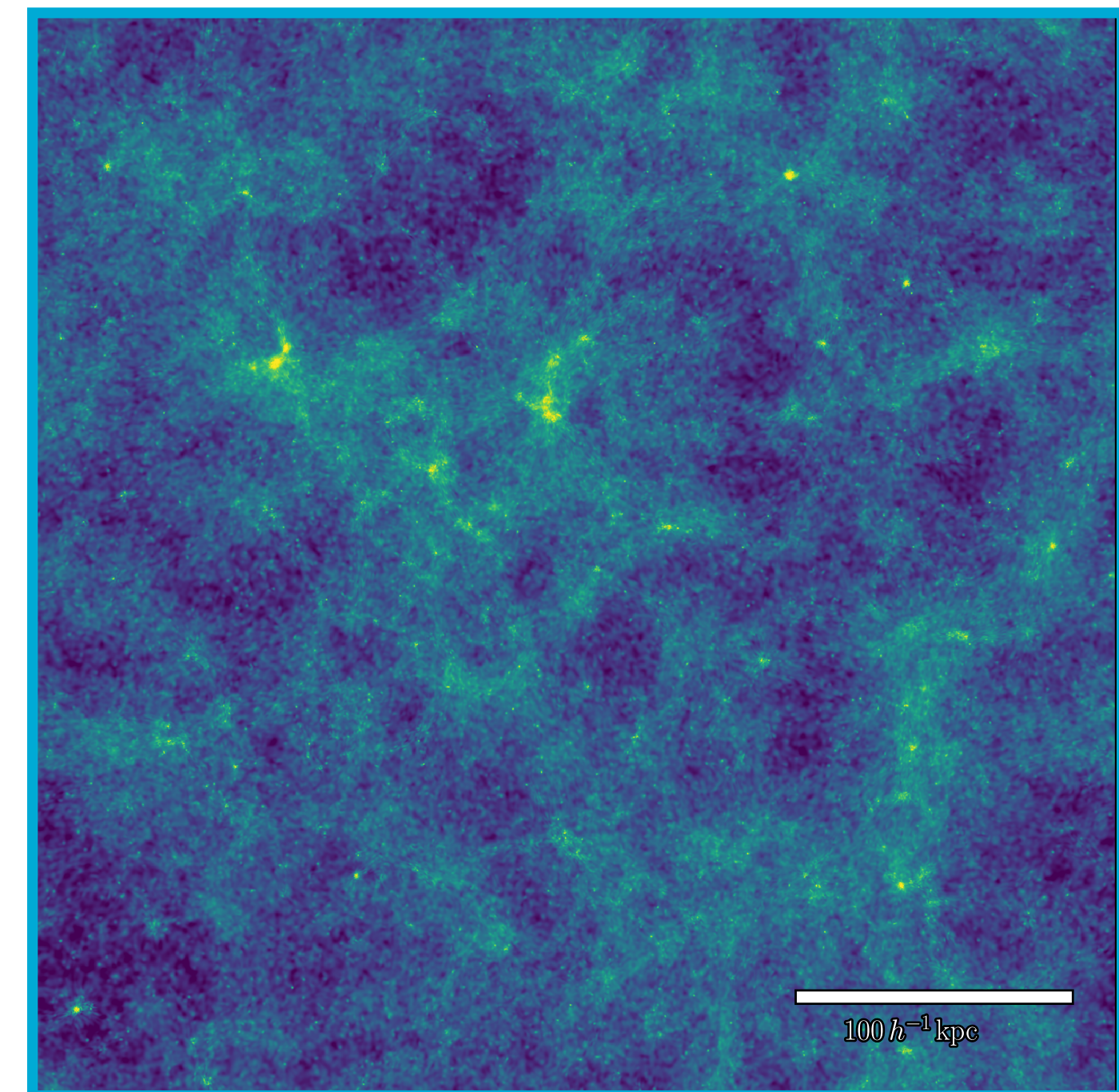
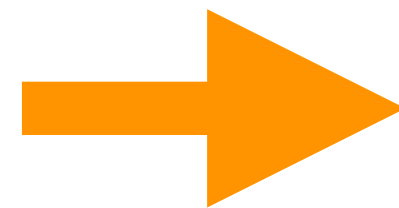
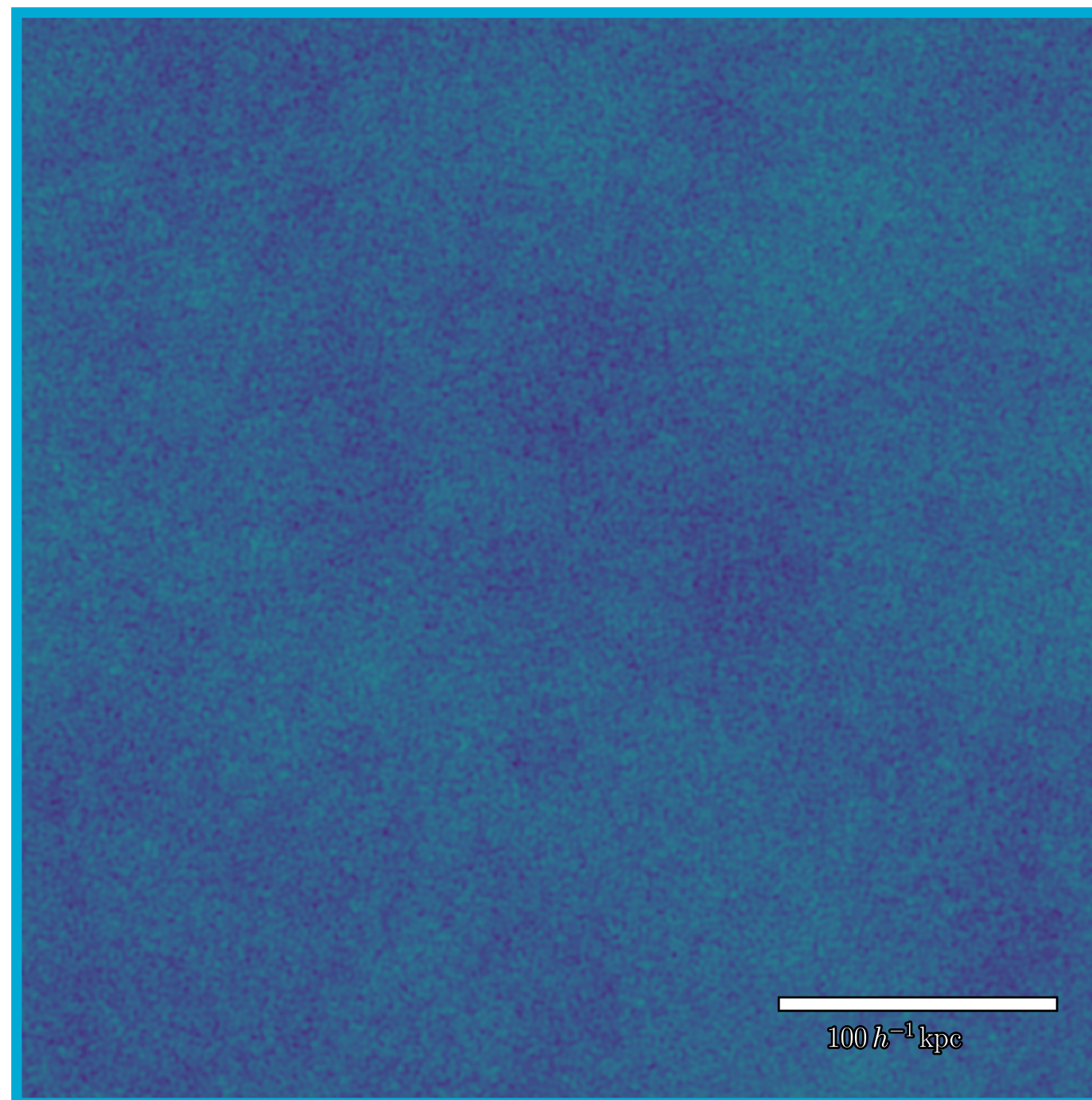
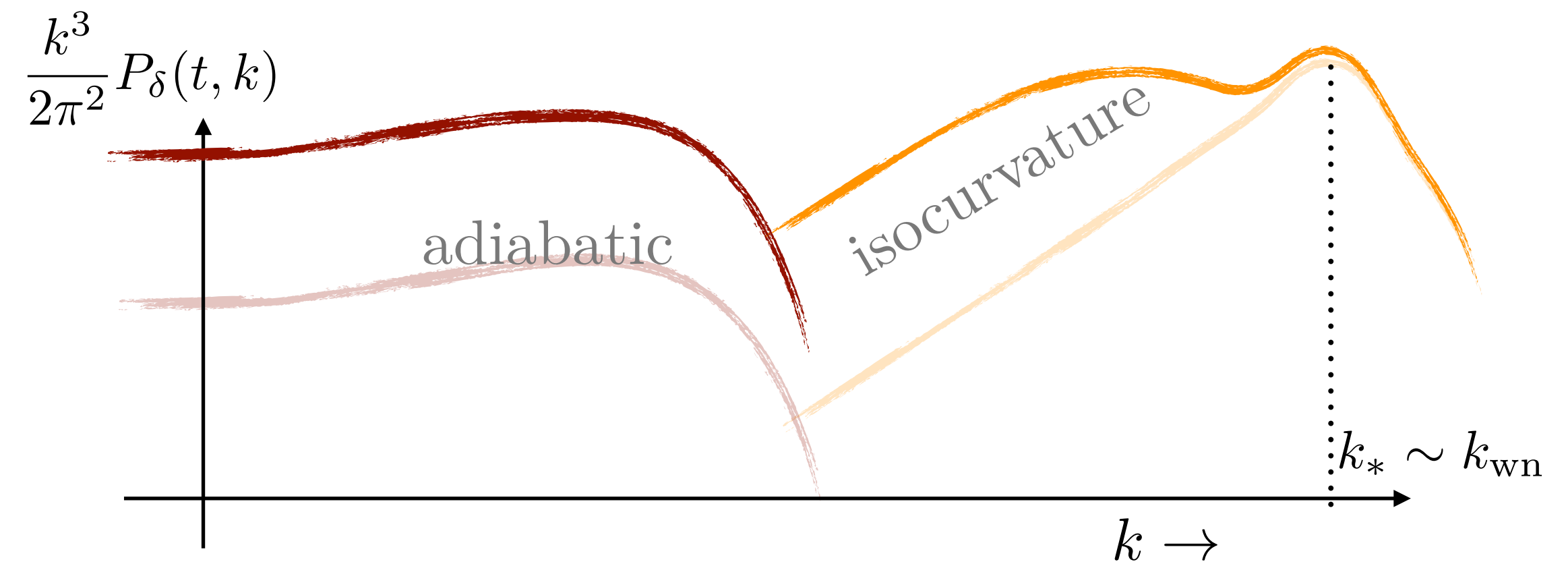
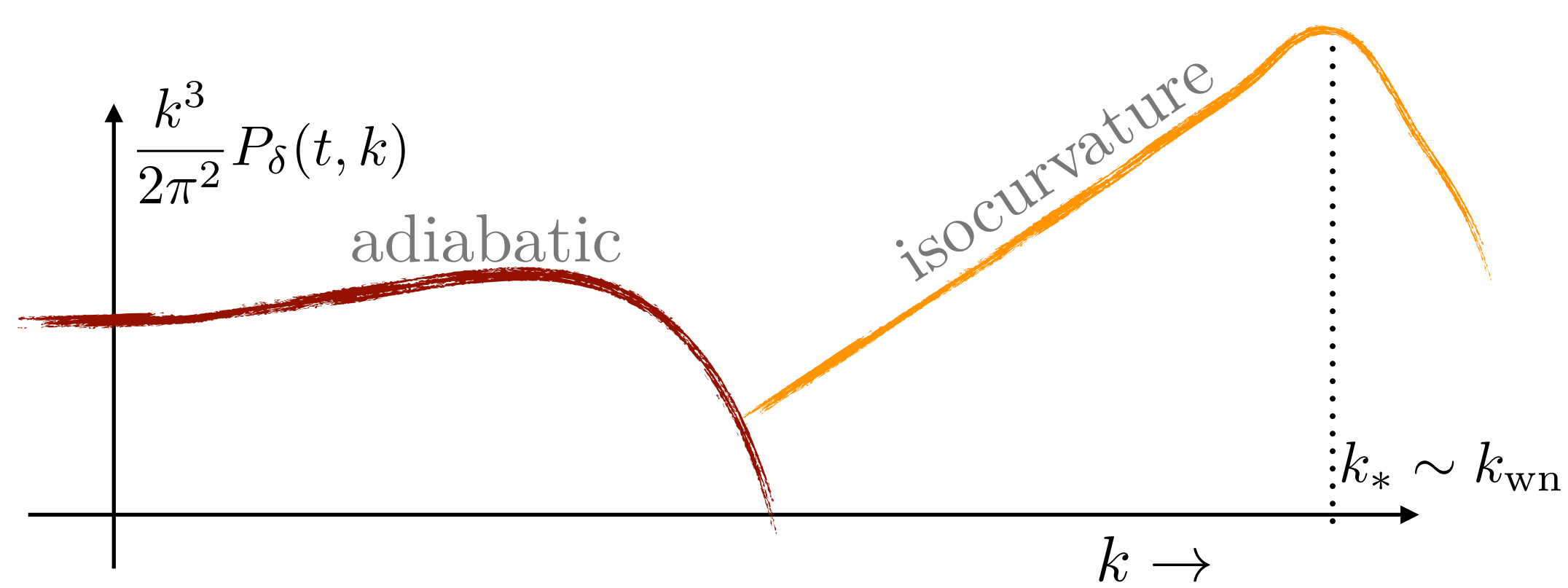


# free streaming — numerical



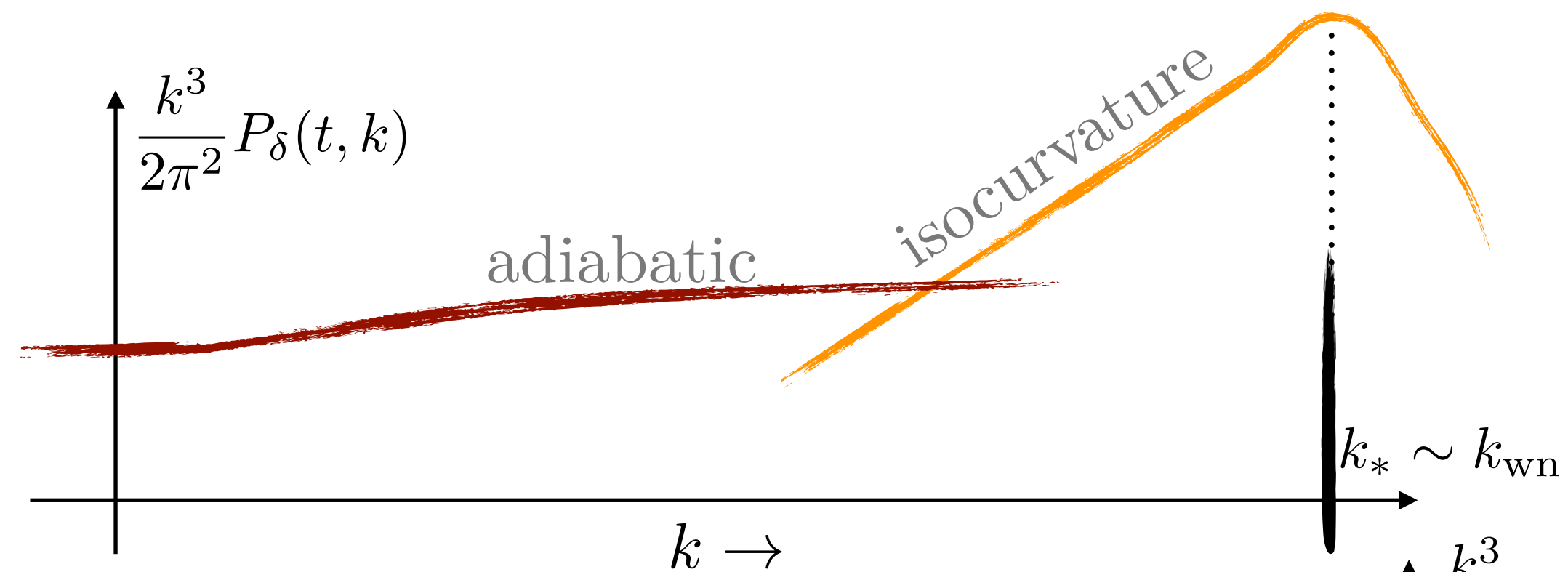
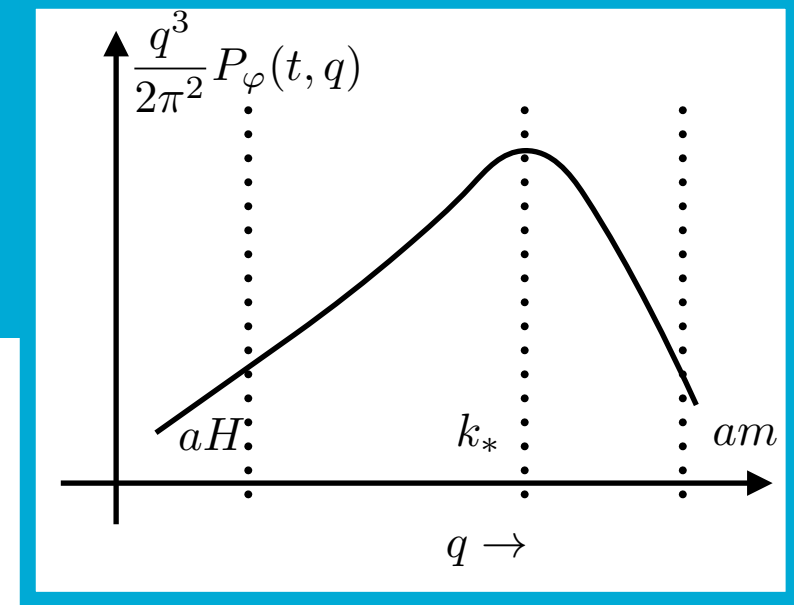


# density evolution during matter domination

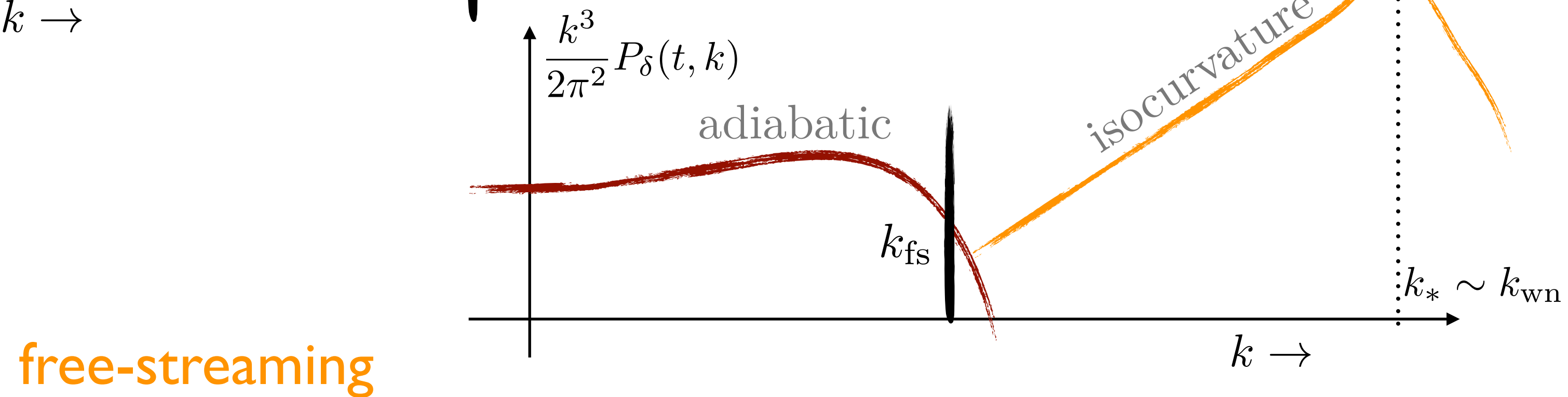




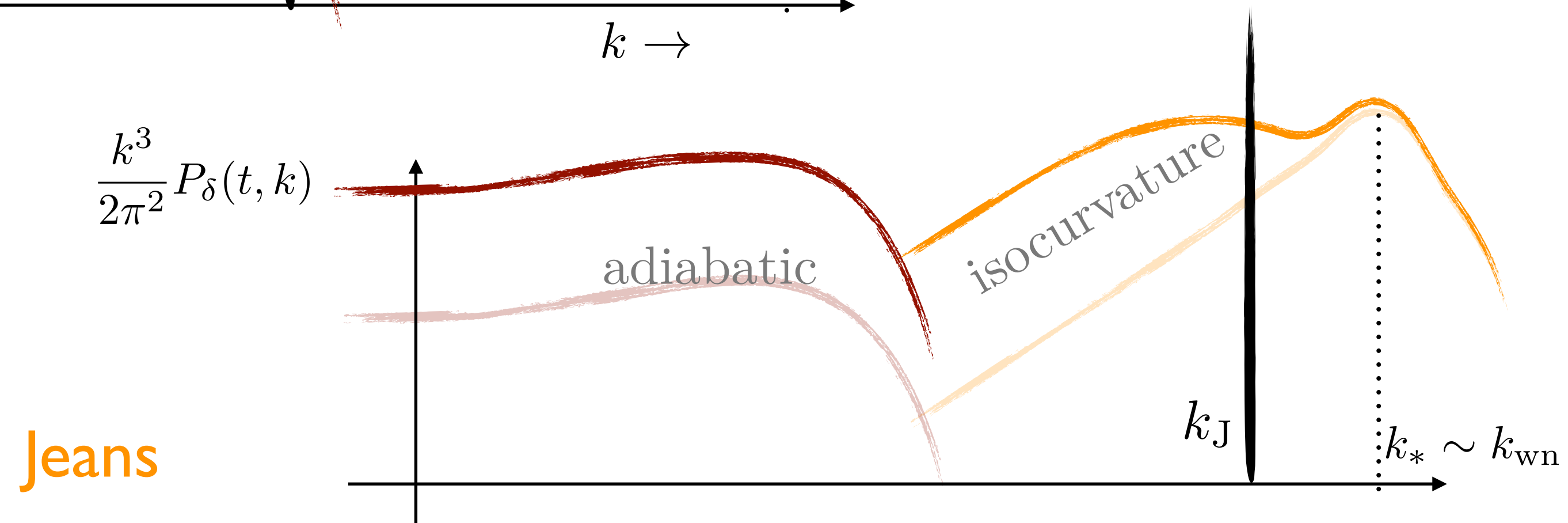
# evolution of the matter power spectrum



white noise



free-streaming



Jeans

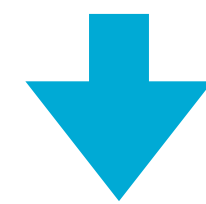
# the lower bound on wave DM mass



Then in density power spectrum:

1. white-noise excess
2. free-streaming suppression

1. and 2. not seen for  $k < k_{\text{obs}} \sim 10 \text{ Mpc}^{-1}$



$$m \gtrsim 10^{-19} \text{ eV}$$

MA & Mirbabayi (2022/24)  
2211.09775



# comparison with literature

$$m \gtrsim 1 \times 10^{-21} \text{ eV}$$

Banik et. al (2019) — Streams

$$m \gtrsim 3 \times 10^{-21} \text{ eV}$$

Nadler et. al (2021) — MW satellites

$$m \gtrsim 2 \times 10^{-20}$$

Rogers and Peiris (2021) —  $\text{Ly}\alpha$

$$m \gtrsim 4 \times 10^{-21} \text{ eV}$$

Powell et. al (2023) — lensing

$$m \gtrsim 3 \times 10^{-19} \text{ eV}$$

Dalal & Kravtsov (2022) — dynamical heating  
+May, Dalal & Kravtsov (2025)

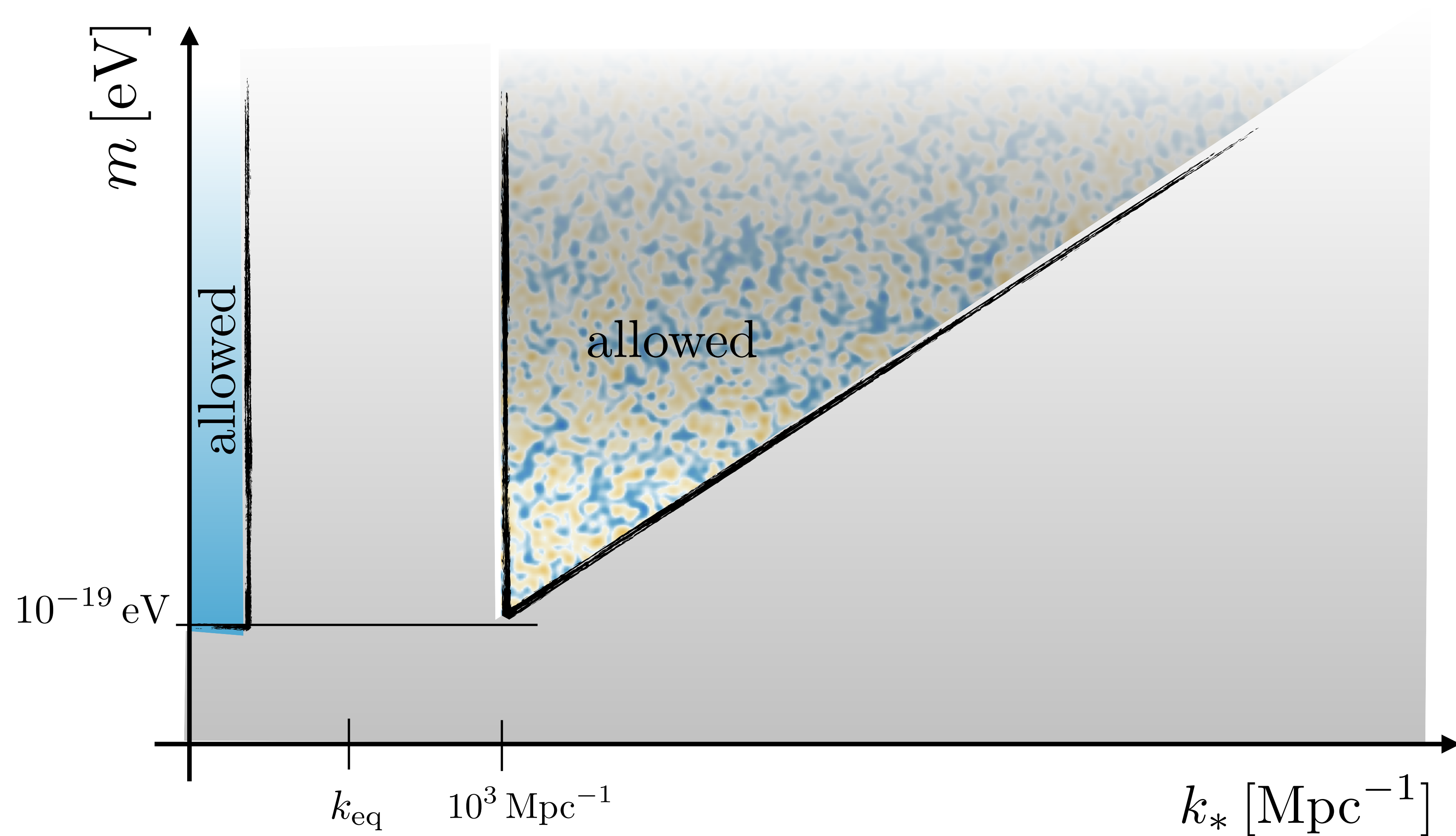
$$m \gtrsim 10^{-19} \text{ eV}$$

MA & Mirbabayi (2022/24)

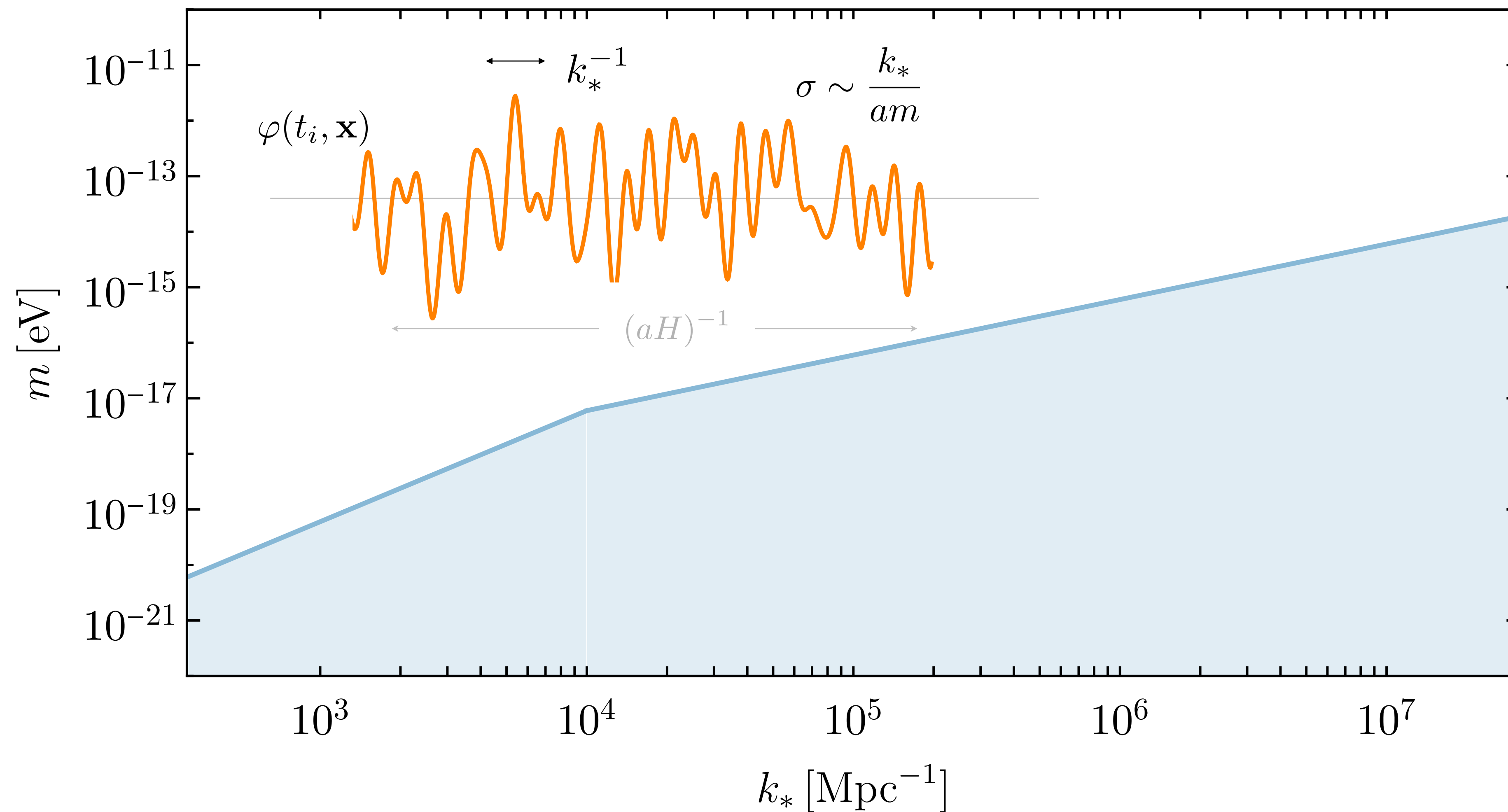
\* constraints come from different observations and assumptions about production mechanisms, compare with care.

\* For thermal production, bound is  $\sim$  few keV — why the difference ? — Ask me.

# the lower bound on wave DM mass

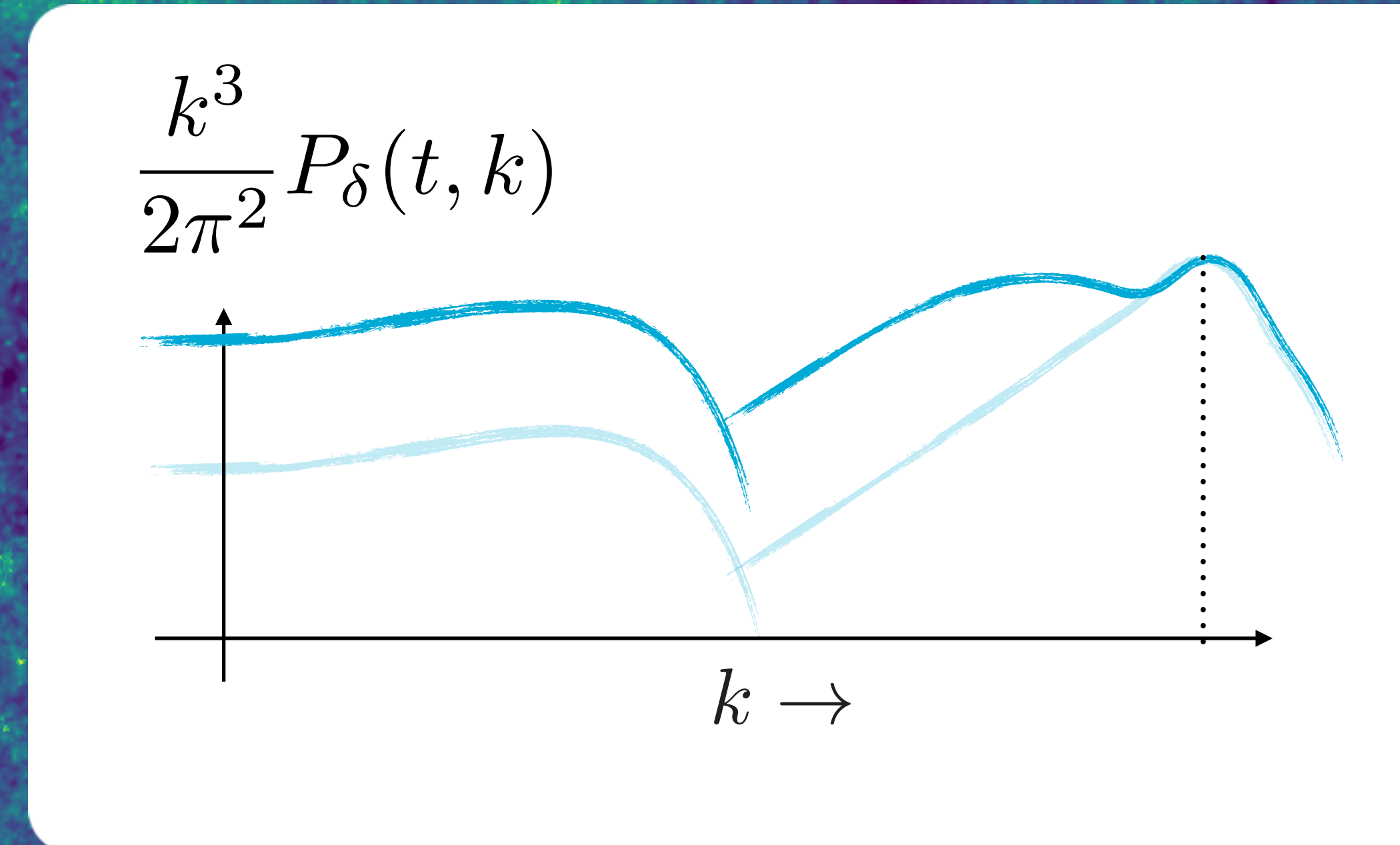
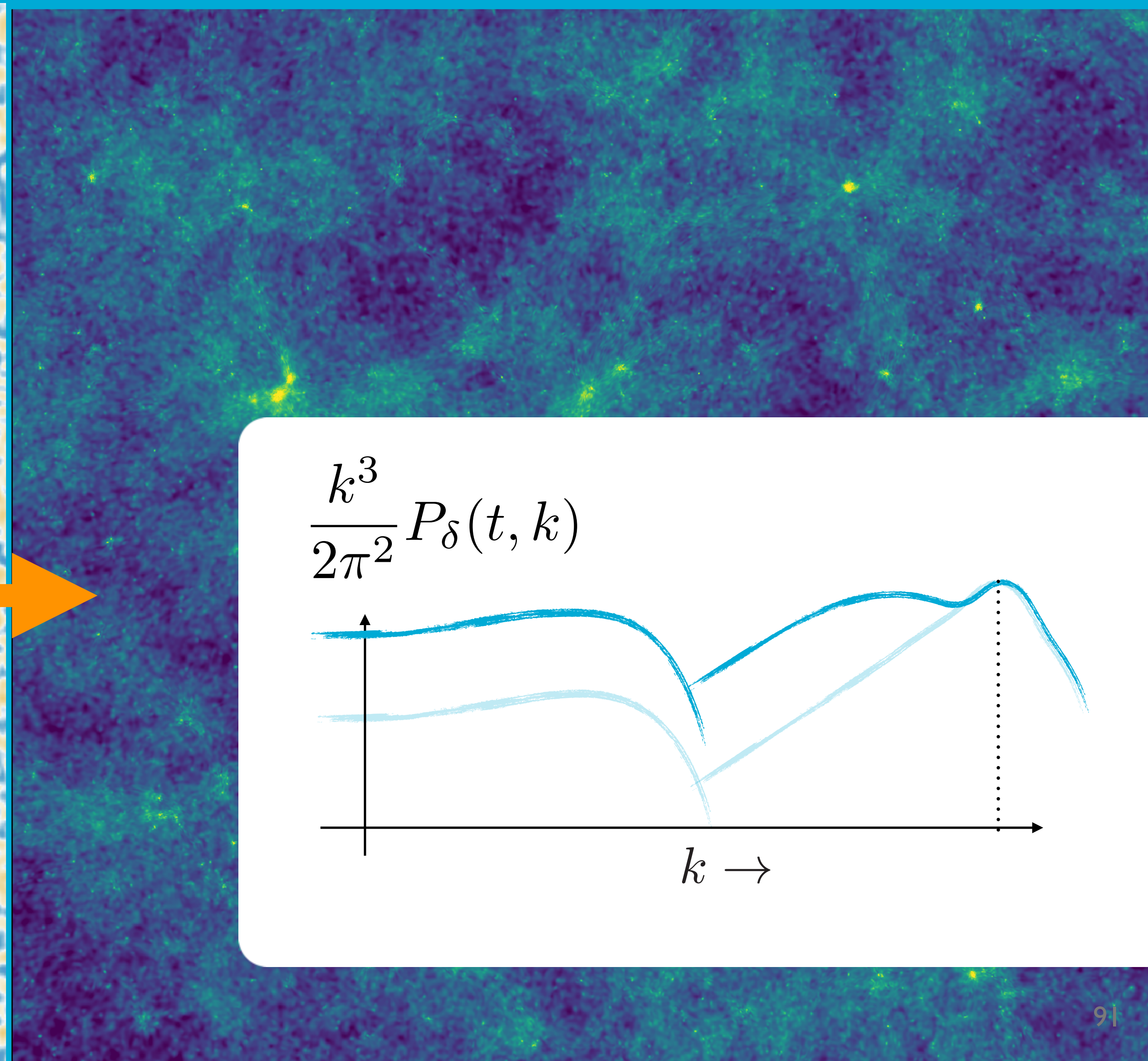
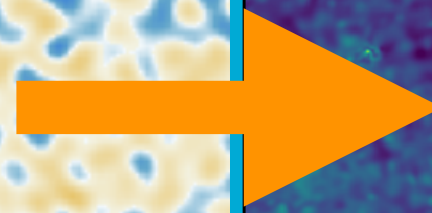
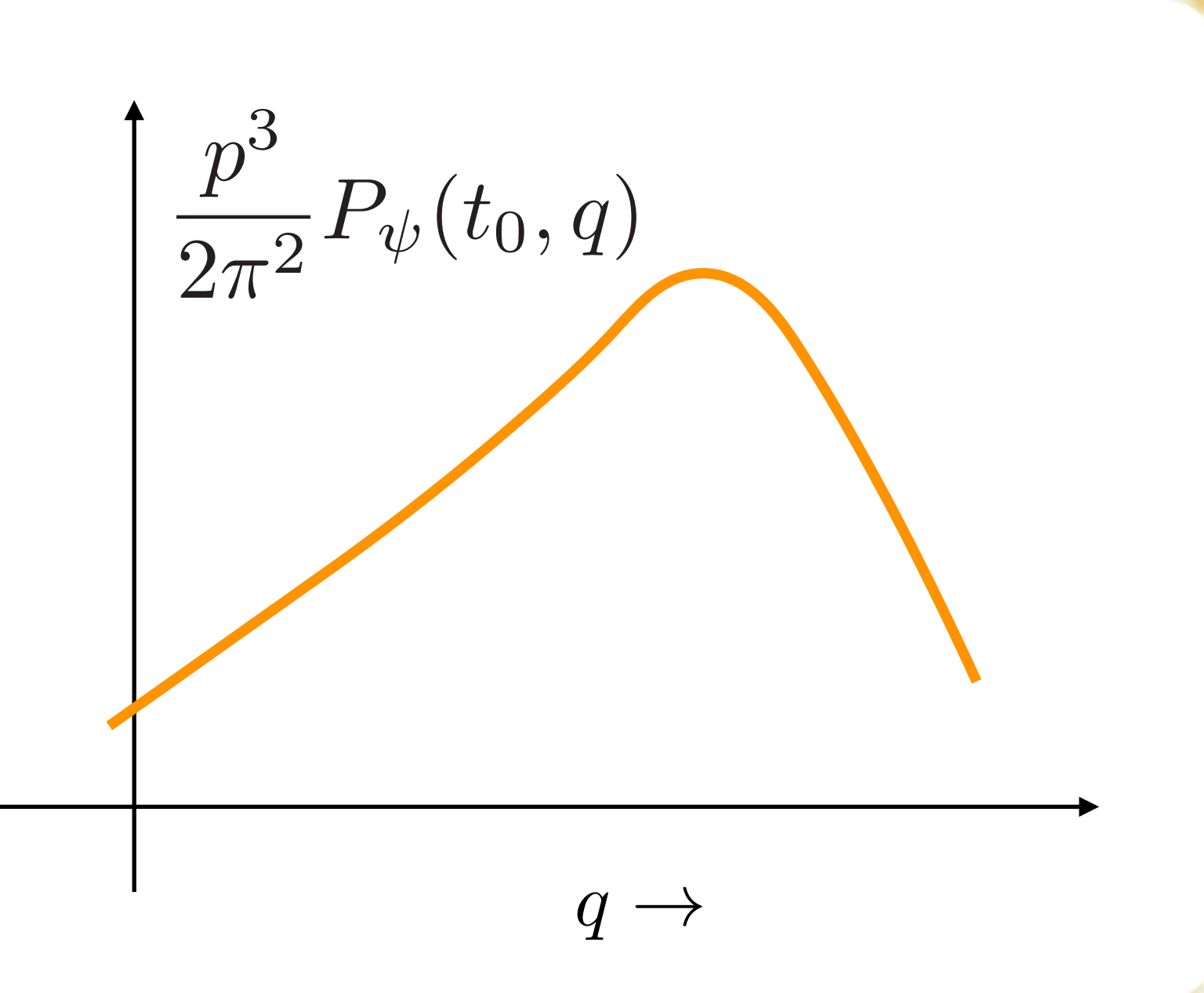
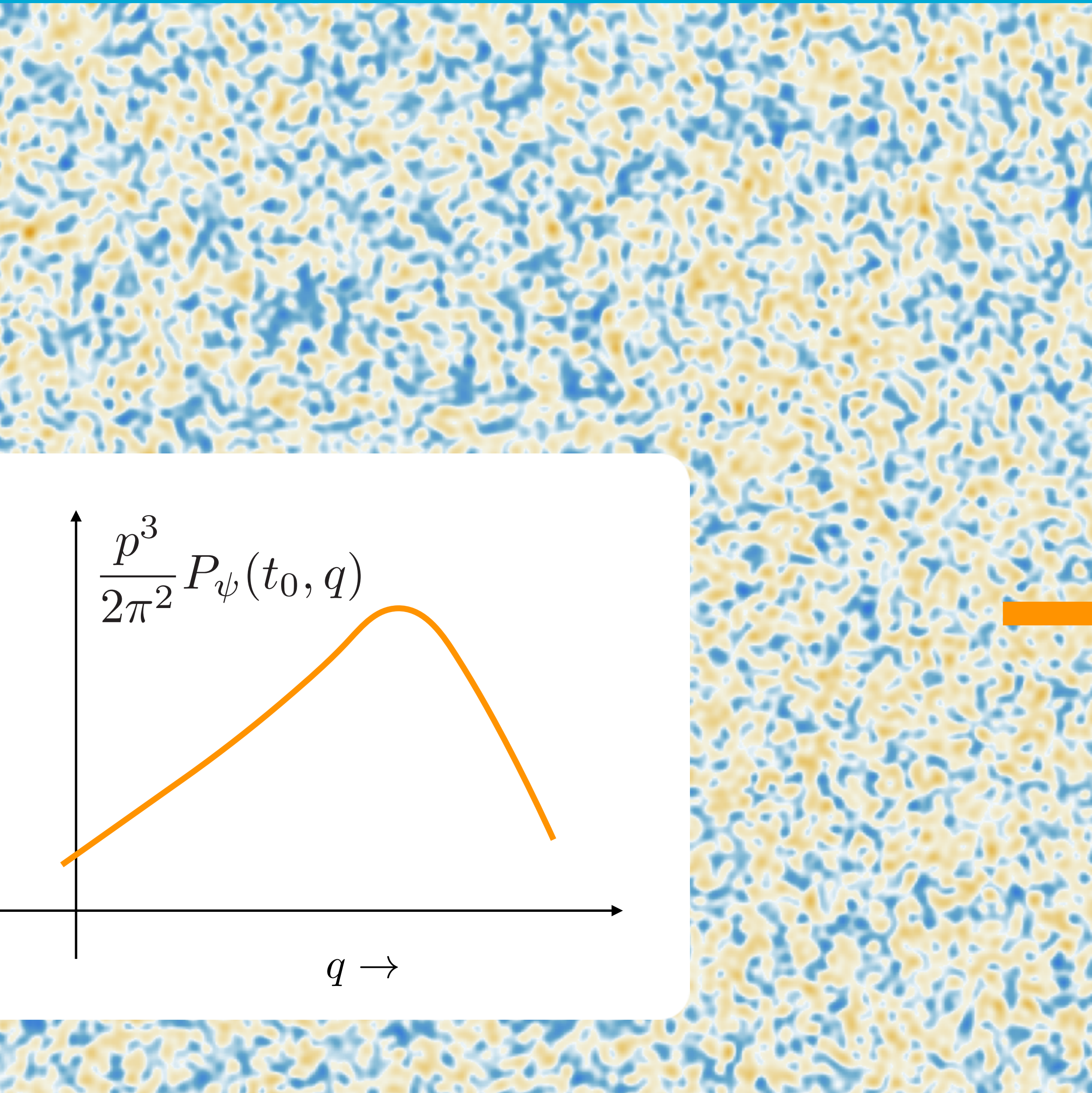


# from MW satelites



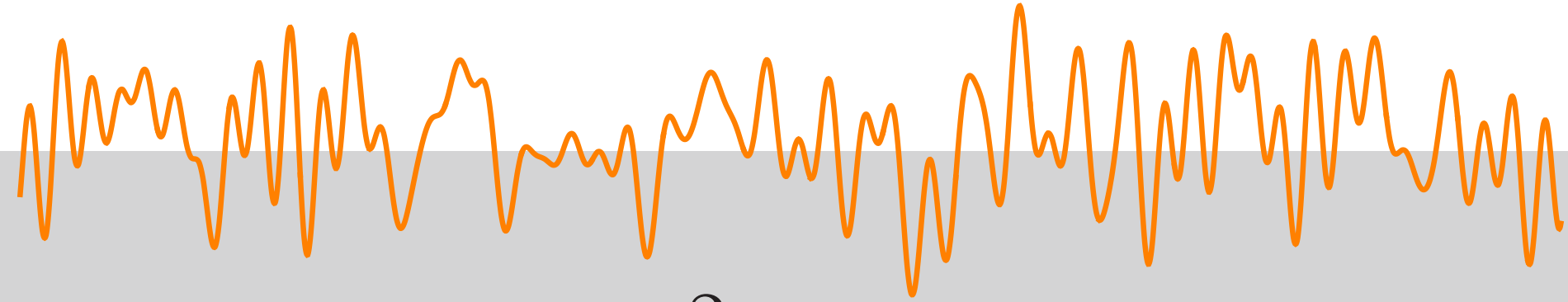


# key input and output





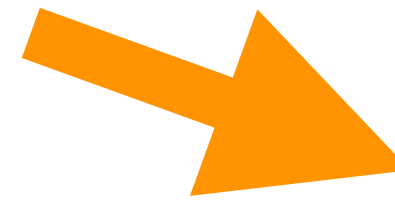
# how is the calculation done?



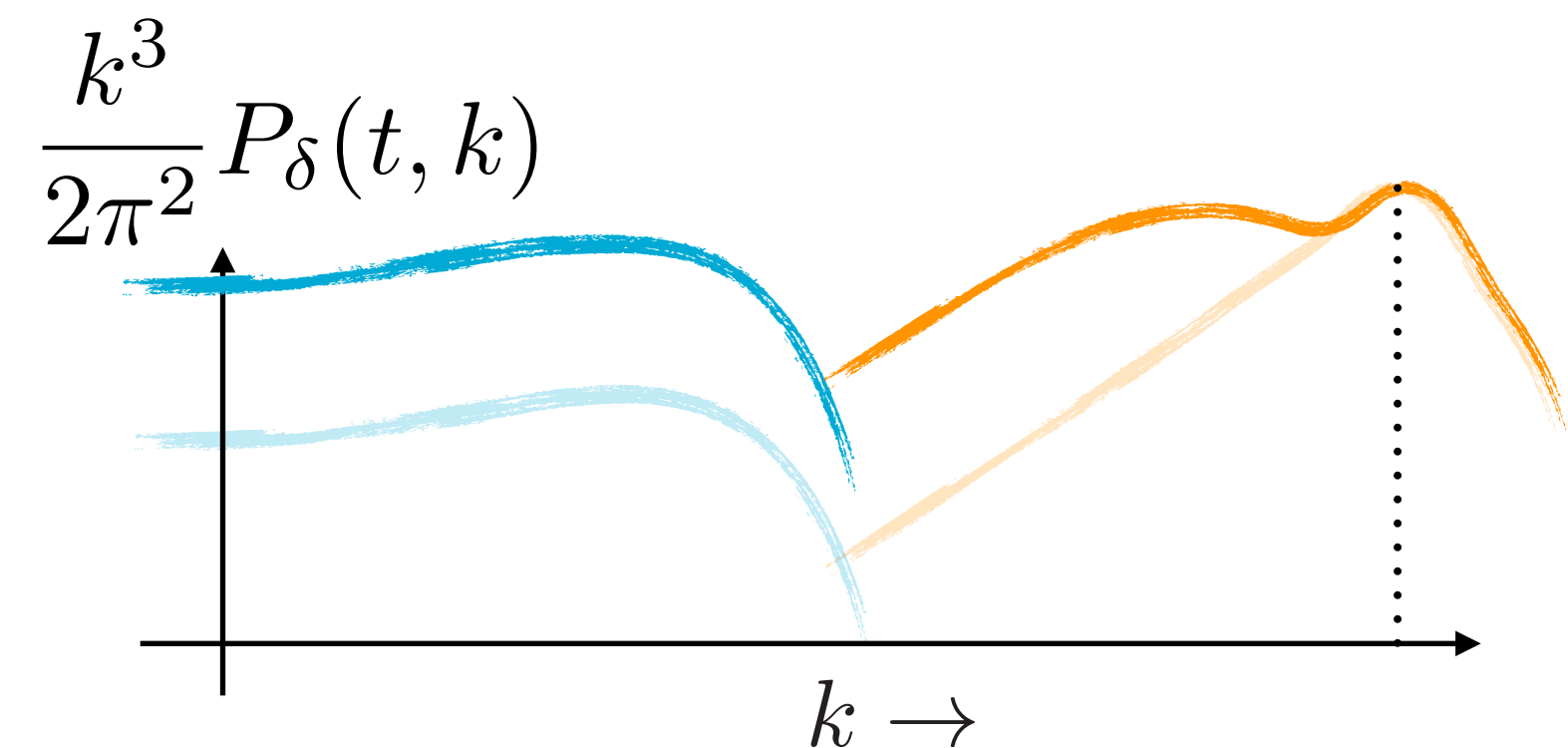
$$i\partial_\eta\psi = -\frac{\nabla^2}{2m}\psi + ma^2\Phi\psi$$

$$\nabla^2\Phi = \frac{4\pi G}{a}(\rho - \bar{\rho}), \quad \rho = m|\psi|^2$$

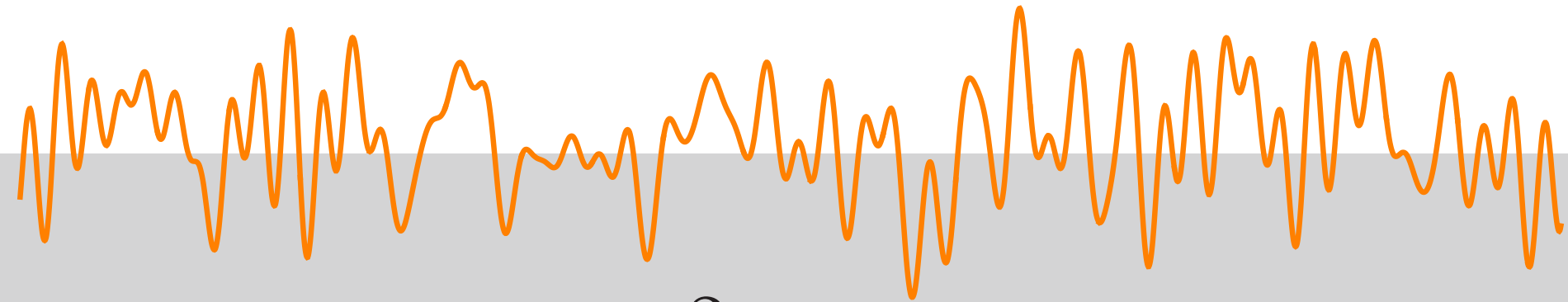
$$d\eta = a^{-2}dt$$



$$\langle \delta_{\mathbf{k}} \delta_{\mathbf{k}'} \rangle \propto P_\delta(k)$$



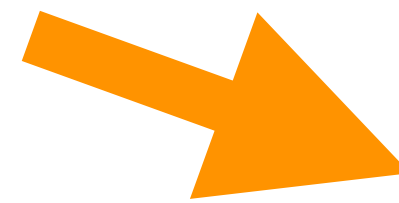
# 2-pt density correlation function = 4 pt field corr. func.



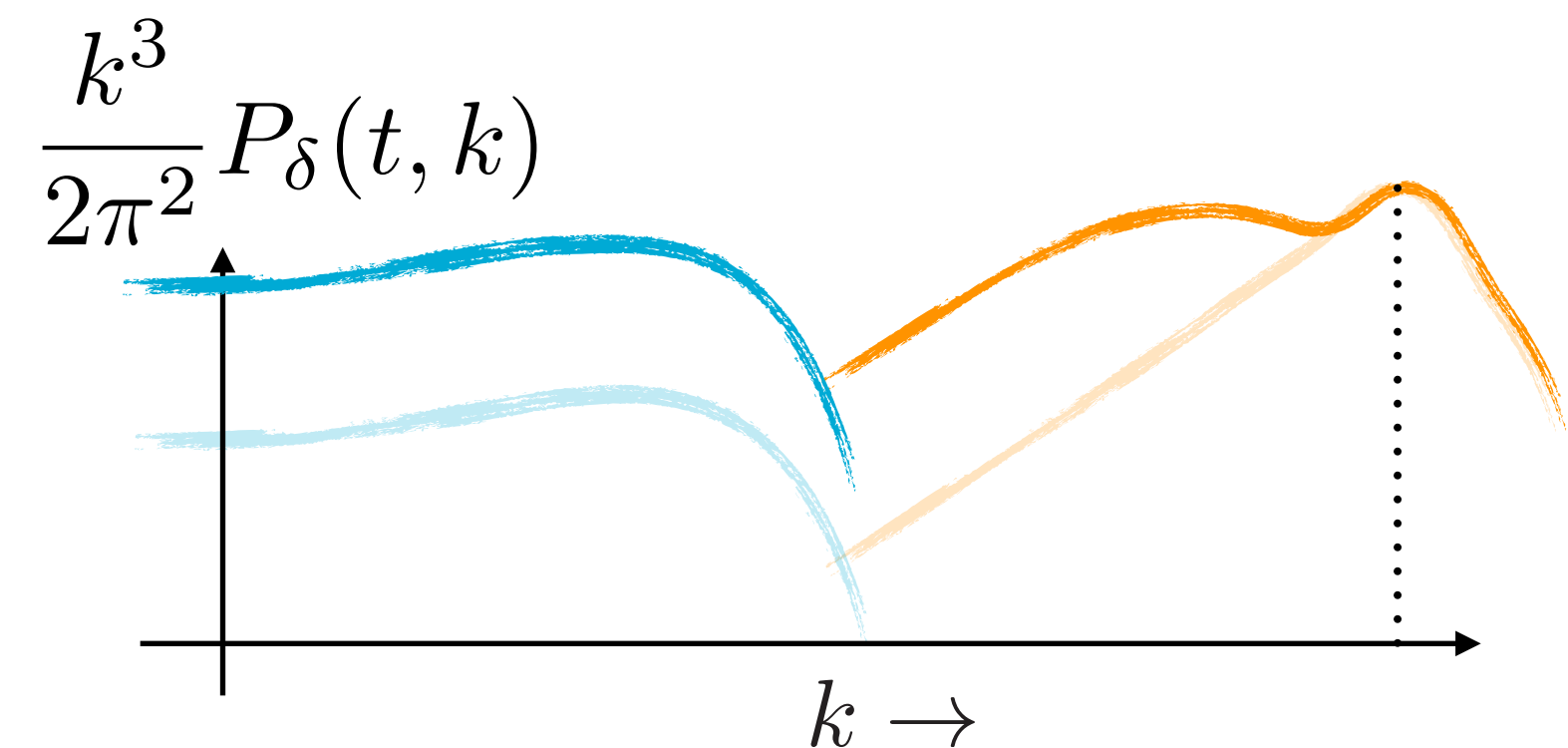
$$i\partial_\eta\psi = -\frac{\nabla^2}{2m}\psi + ma^2\Phi\psi$$

$$\nabla^2\Phi = \frac{4\pi G}{a}(\rho - \bar{\rho}), \quad \rho = m|\psi|^2$$

$$d\eta = a^{-2}dt$$



$$\langle\psi\psi\psi\psi\rangle \sim \langle\delta_{\mathbf{k}}\delta_{\mathbf{k}'}\rangle \propto P_\delta(k)$$





# power spectrum evolution

$$P_\delta(\eta, k) = P_\delta(\eta_0, k) + 3\bar{H}_0^2 \int_{\eta_0}^{\eta} d\eta' a(\eta') T_k^{(b)}(\eta, \eta') T_k^{(c)}(\eta, \eta')$$

$$T_k^{(i)}(\eta, \eta_0) = T_k^{\text{fs}(i)}(\eta, \eta_0) + \frac{3\bar{H}_0^2}{2} \int_{\eta_0}^{\eta} d\eta' a(\eta') T_k^{\text{fs}(b)}(\eta, \eta') T_k^{(i)}(\eta', \eta_0) \quad i = \text{b, c}$$

pure free-streaming      gravitational clustering (with free-streaming)

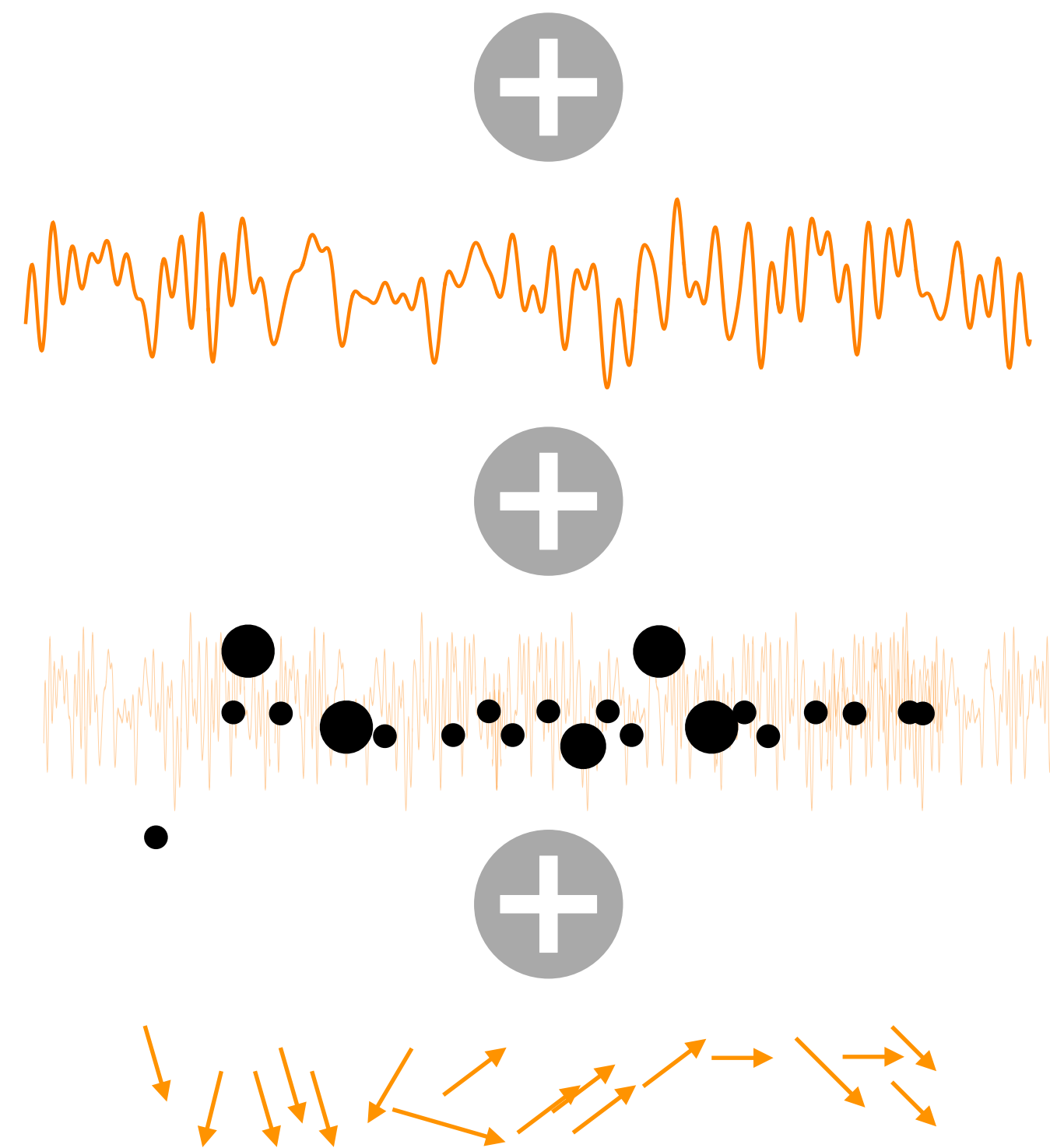
$$T_k^{\text{fs}(b)}(\eta, \eta') = \frac{2m}{k^2} \sin \left[ \frac{k^2}{2m} (\eta - \eta') \right] \int_{\mathbf{p}} f(p) e^{-i \frac{\mathbf{p} \cdot \mathbf{k}}{m} (\eta - \eta')},$$

$$T_k^{\text{fs}(c)}(\eta, \eta') = \int_{\mathbf{p}} f(|\mathbf{p} + \mathbf{k}/2|) f(|\mathbf{p} - \mathbf{k}/2|) e^{-i \frac{\mathbf{p} \cdot \mathbf{k}}{m} (\eta - \eta')}$$

$$\bar{H}_0^2 = (8\pi G/3)\bar{\rho} \quad \Delta_{\mathbf{k}} f(p) \equiv (-im/k^2) [f(|\mathbf{p} + \mathbf{k}/2|) - f(|\mathbf{p} - \mathbf{k}/2|)] \quad 94$$

# generality of the framework

<https://github.com/mustafaaamin/warm-and-random>

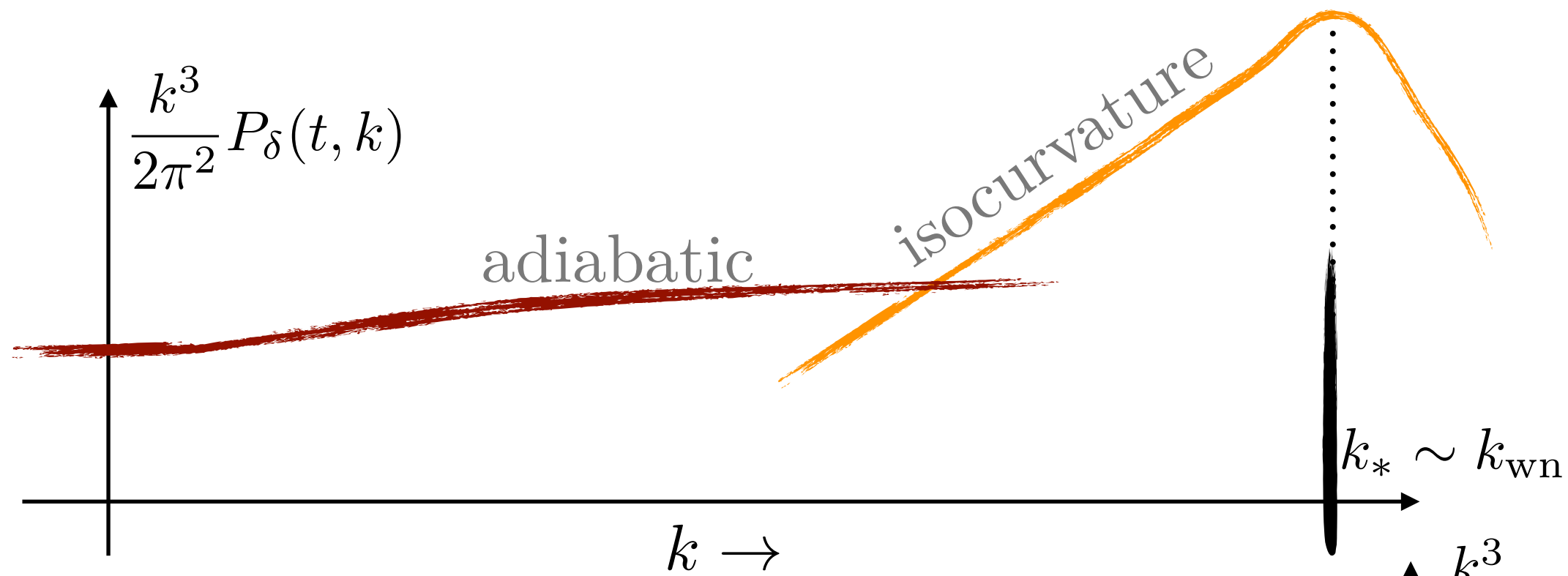
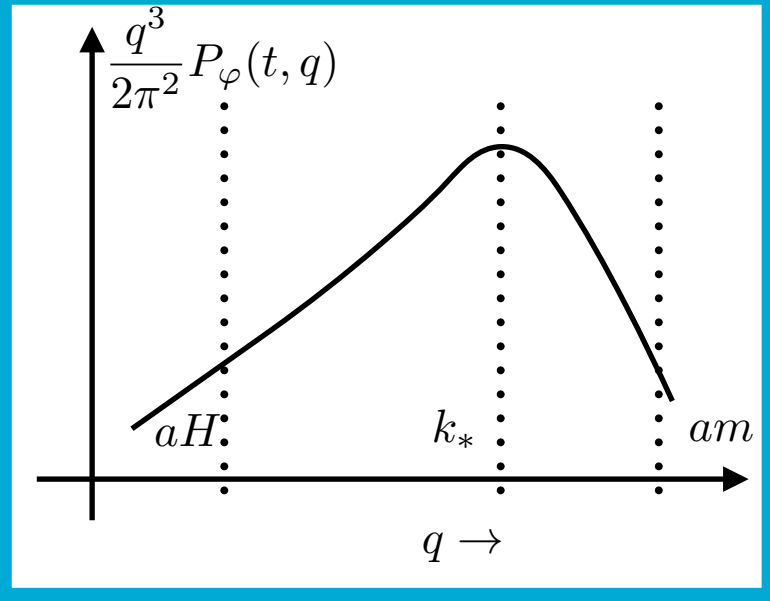


- works for point particles, solitons and fields with **arbitrary initial spectra**
- includes effects of **noise and warmth** (not captured by CLASS, AxiCAMB etc.)\*
- works for **arbitrary # of components**
- can include non-gravitational interactions\*

2211.09775, 2408.05591, 2503.20881, 2506.12131, 2510.15046, 2510.17977

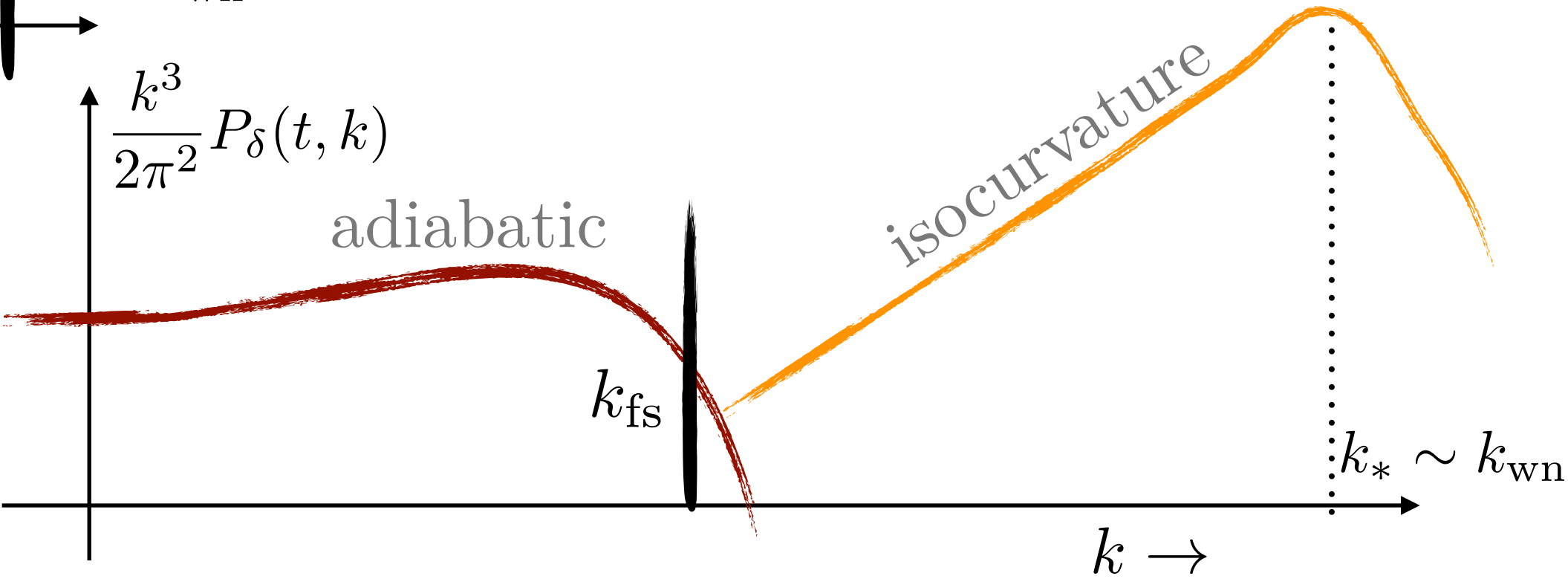
with Delos, Ling, May, Mirbabayi & Yang

# nonlinear structures

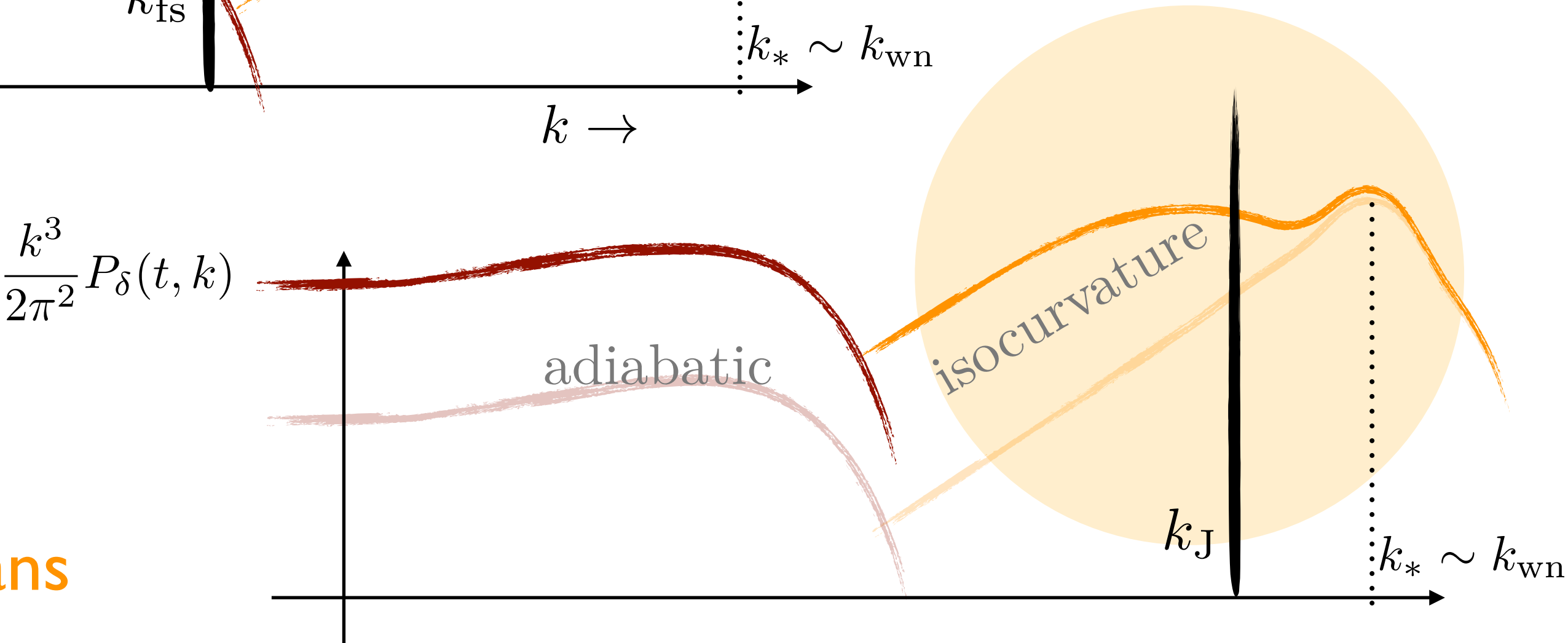


white noise

free-streaming

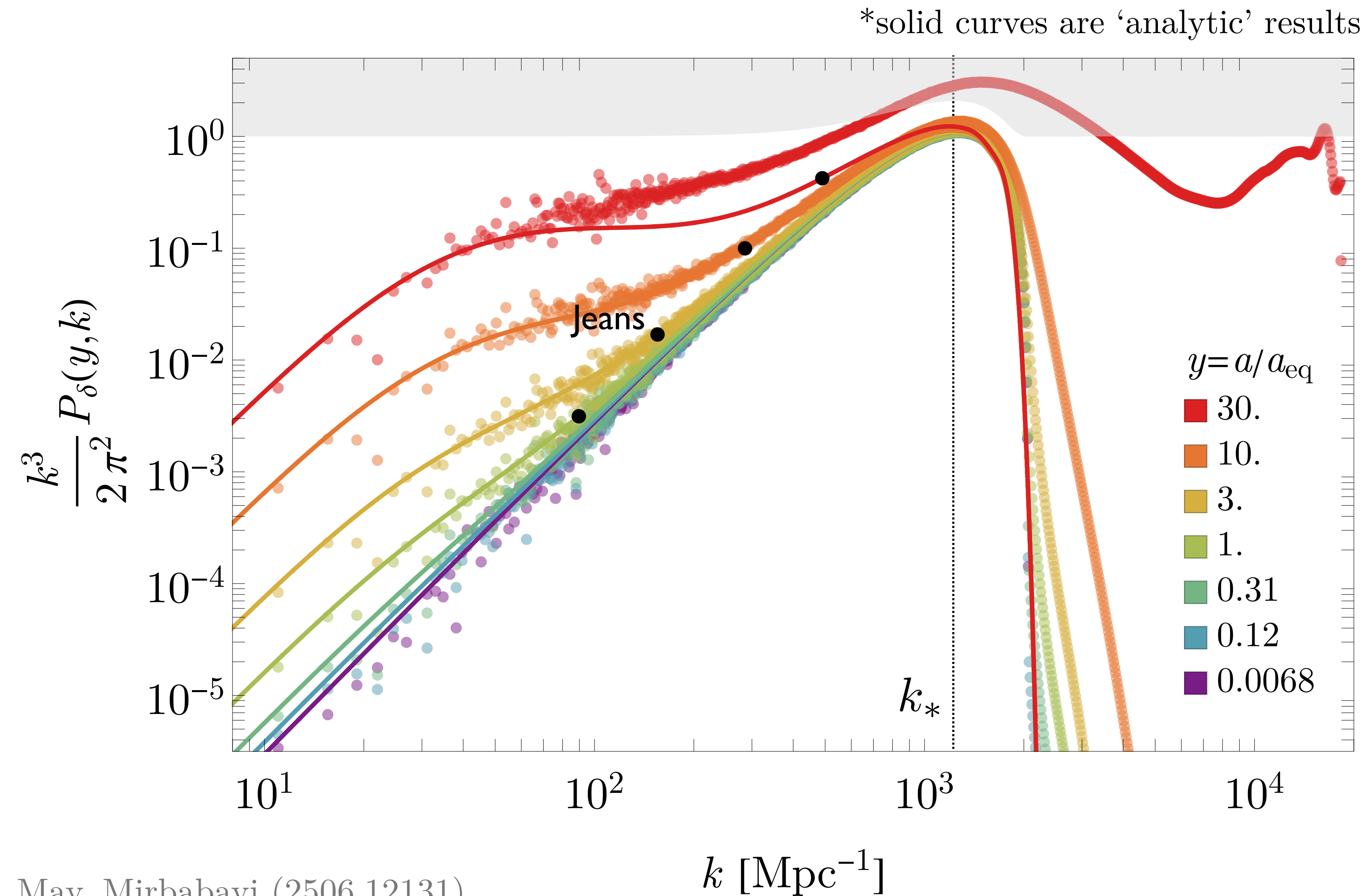


Jeans





# comparison with simulations



$a/a_{\text{eq}} = 0.85$

$100 h^{-1} \text{ kpc}$

$a/a_{\text{eq}} = 5.6$

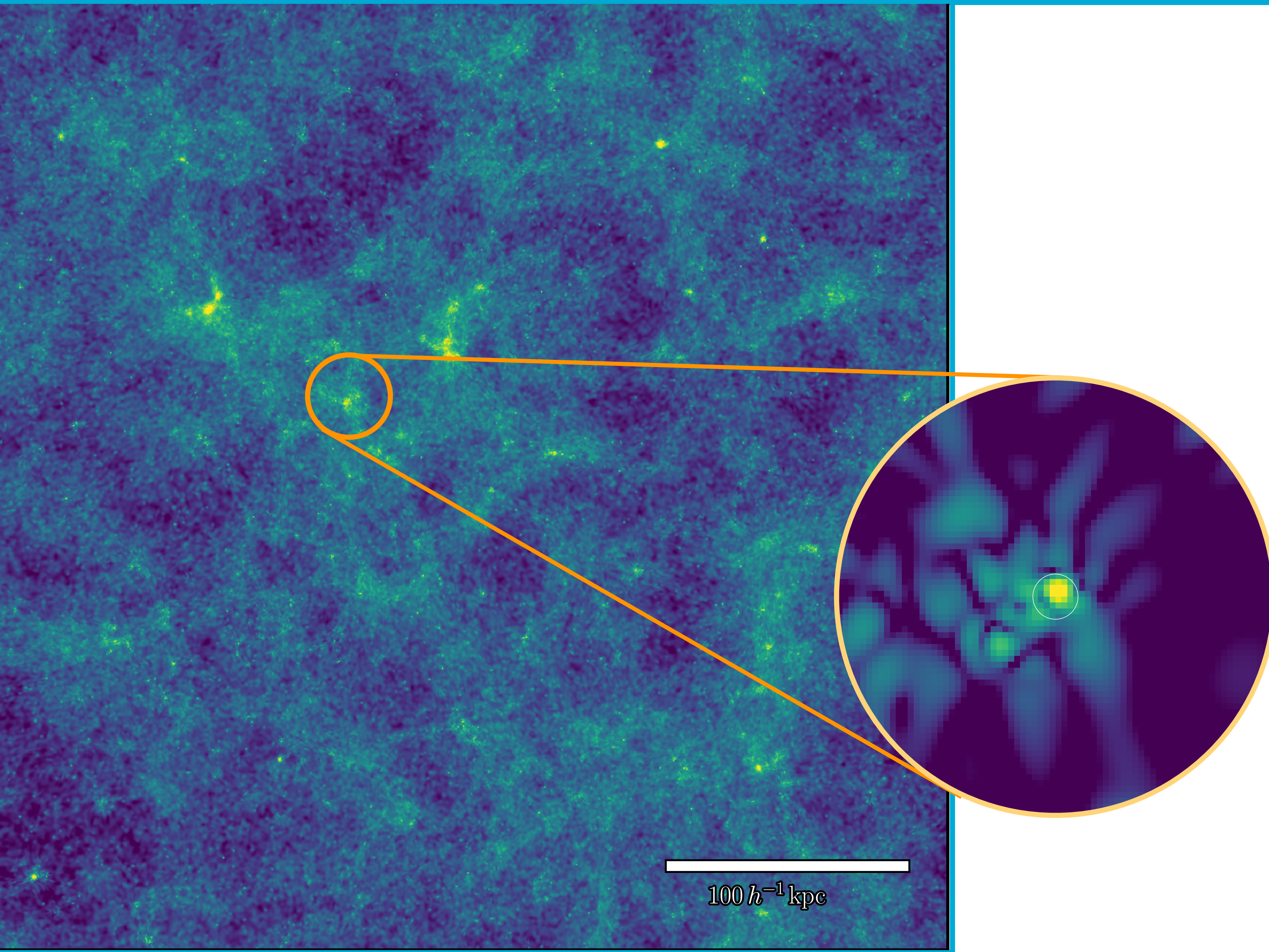
$100 h^{-1} \text{ kpc}$

$a/a_{\text{eq}} = 33.8$

$100 h^{-1} \text{ kpc}$



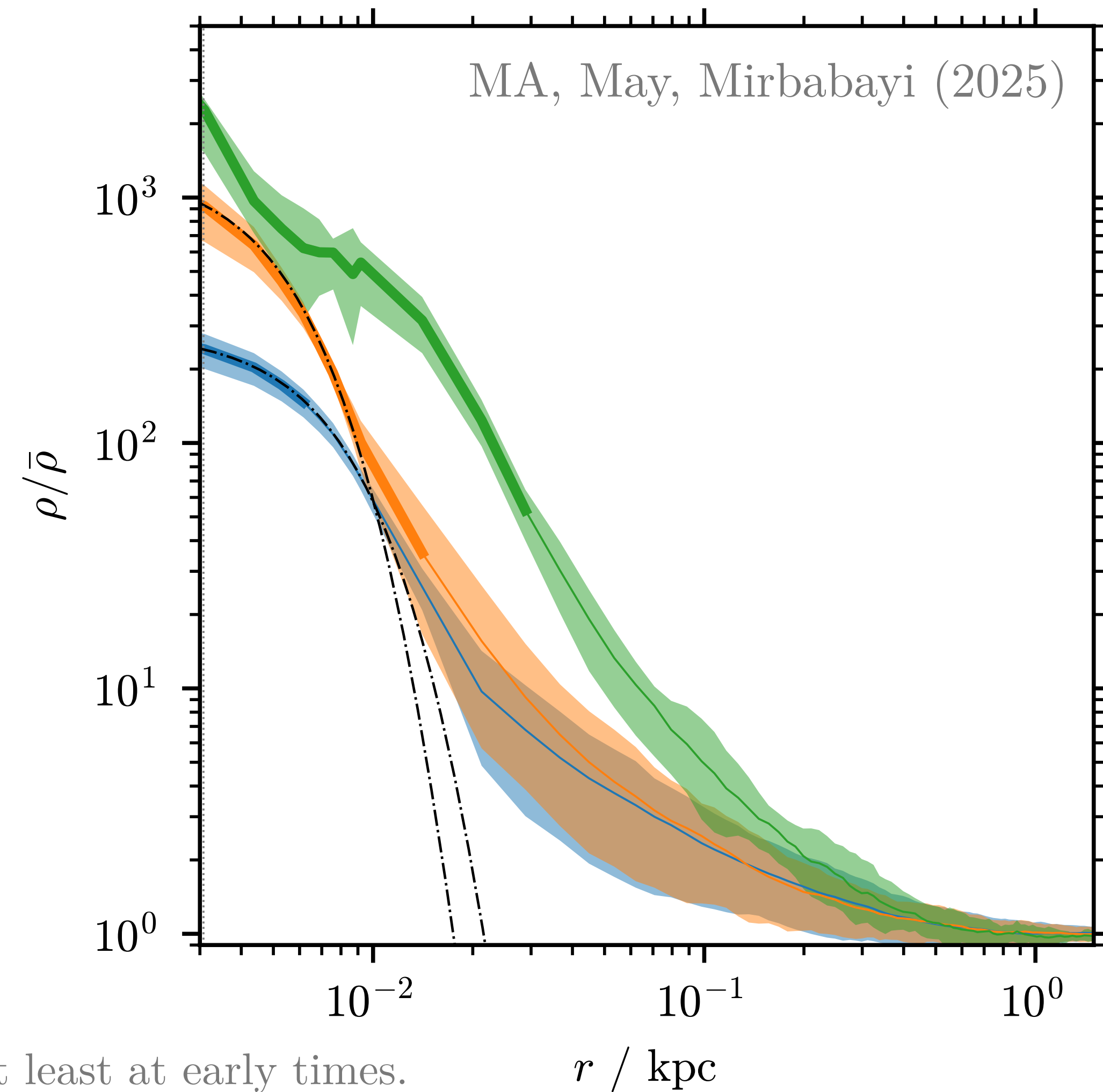
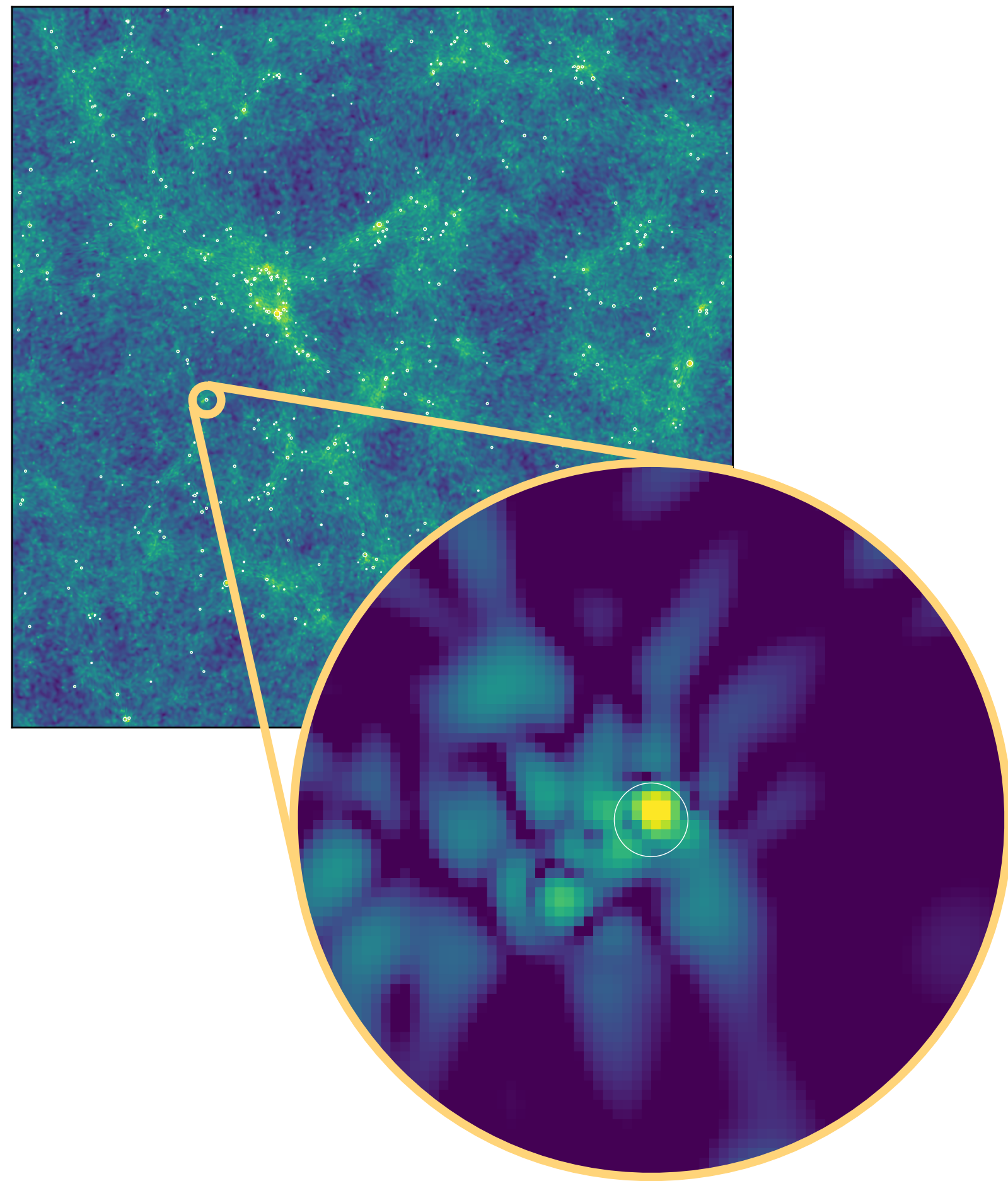
# early nonlinear structure formation



halos+solitons at  $z > 100$



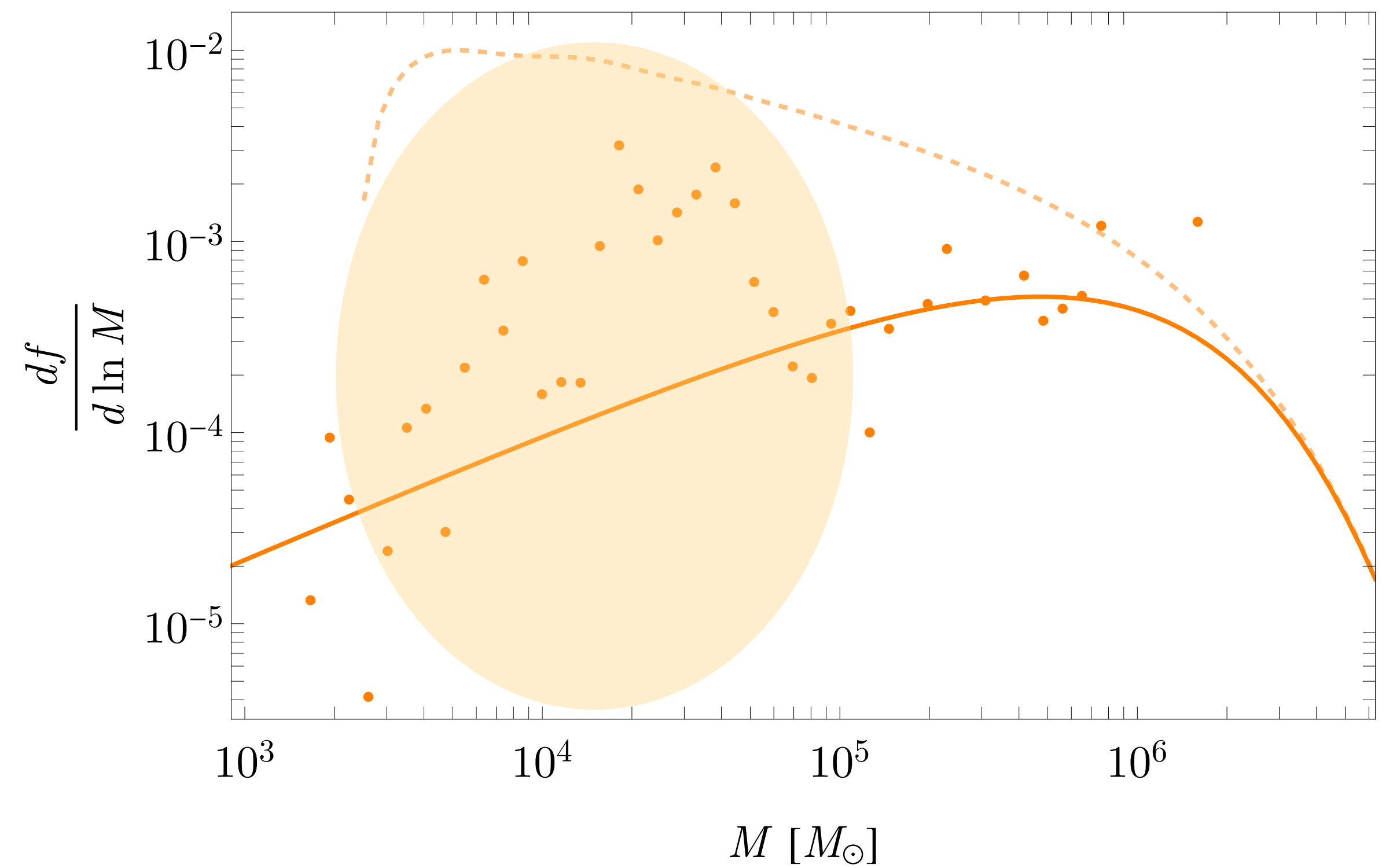
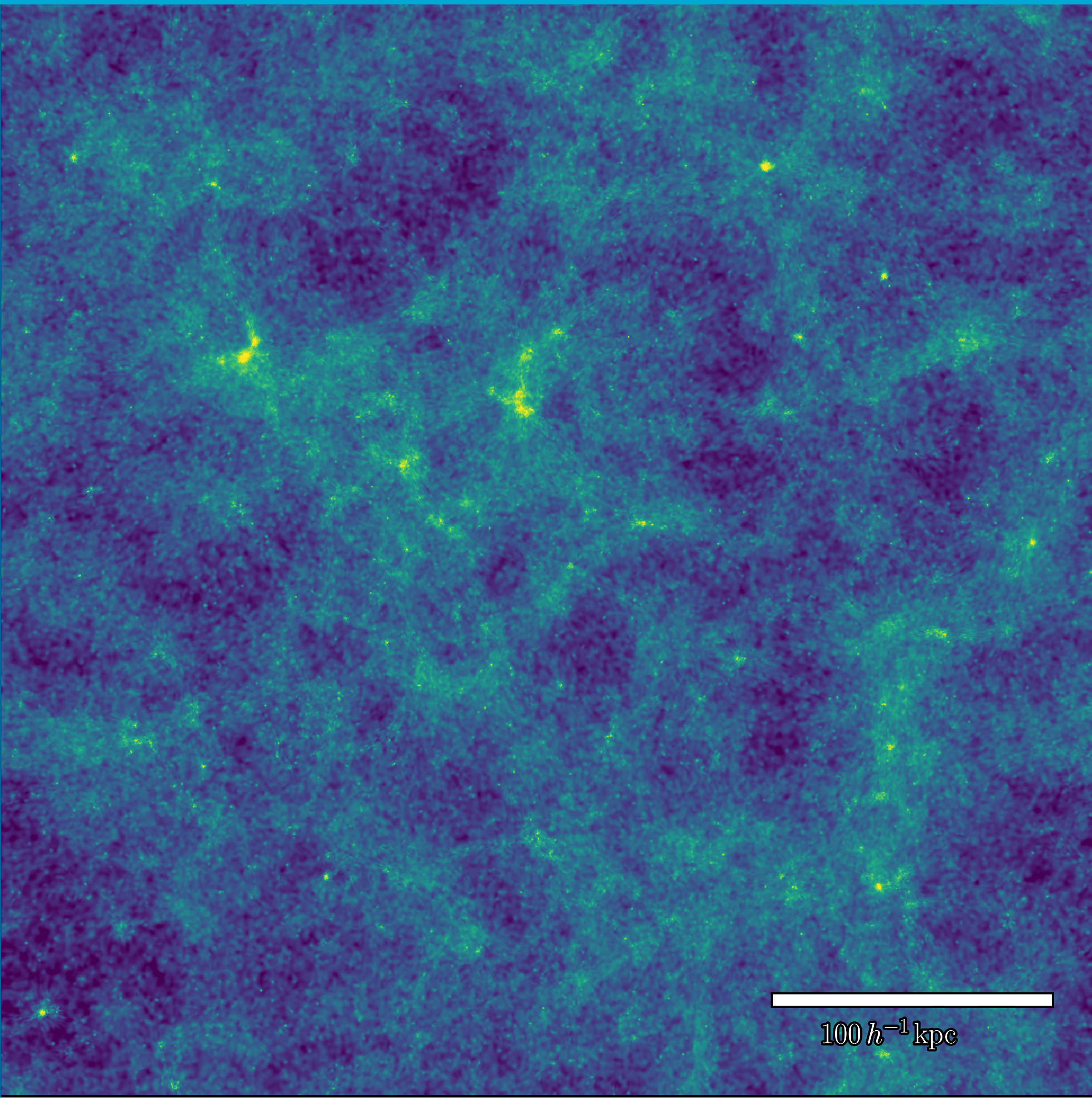
# solitons — when do they form, in what fraction ?



\* can be order unity fraction of mass in collapsed objects, at least at early times.

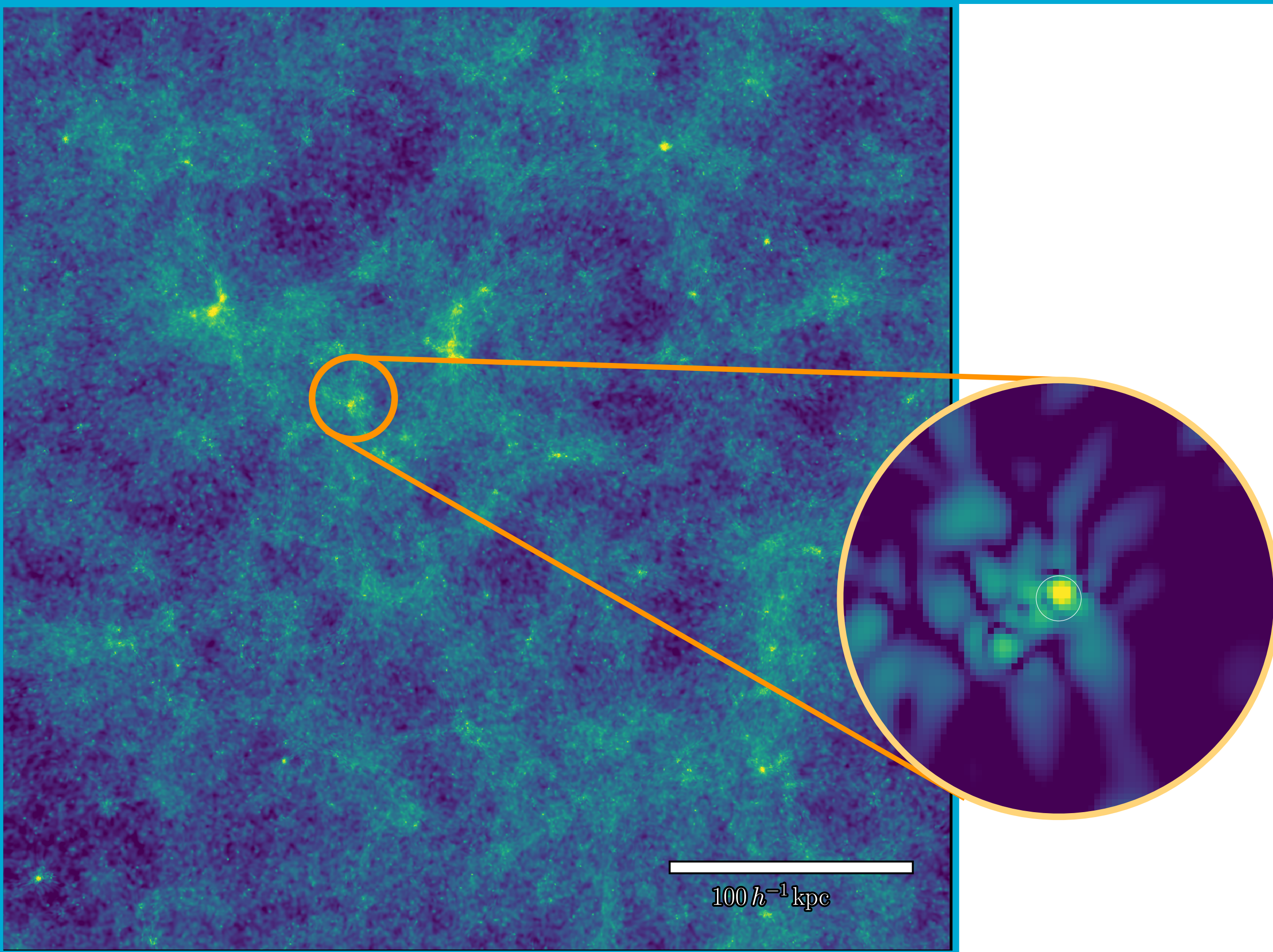


we can “predict” the halo mass function



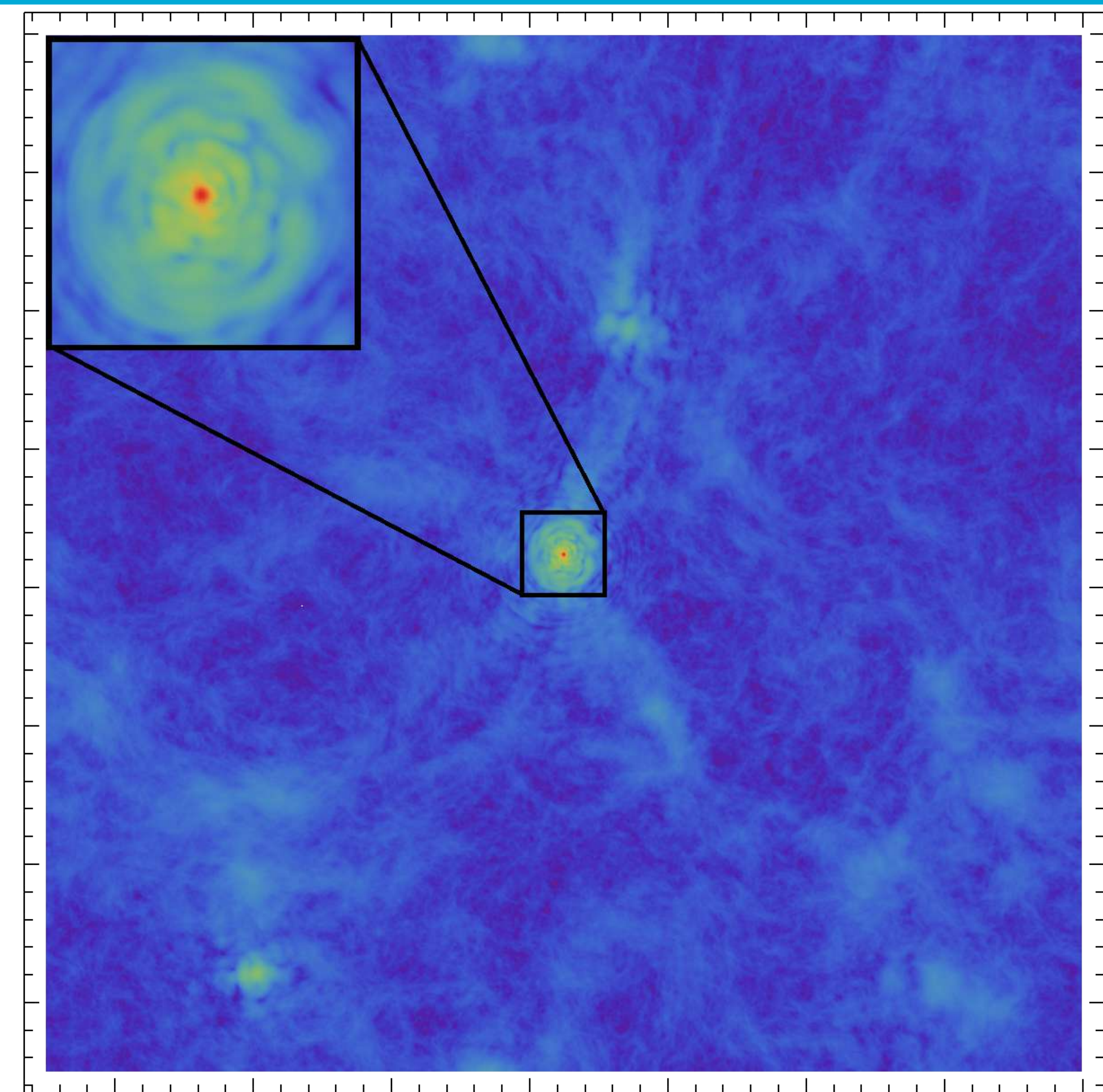


# scalars and vectors — “cosmological” simulations



MA, May & Mirbabayi (2025)

scalars

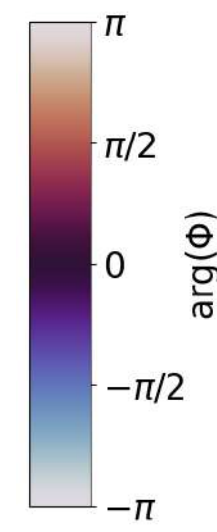
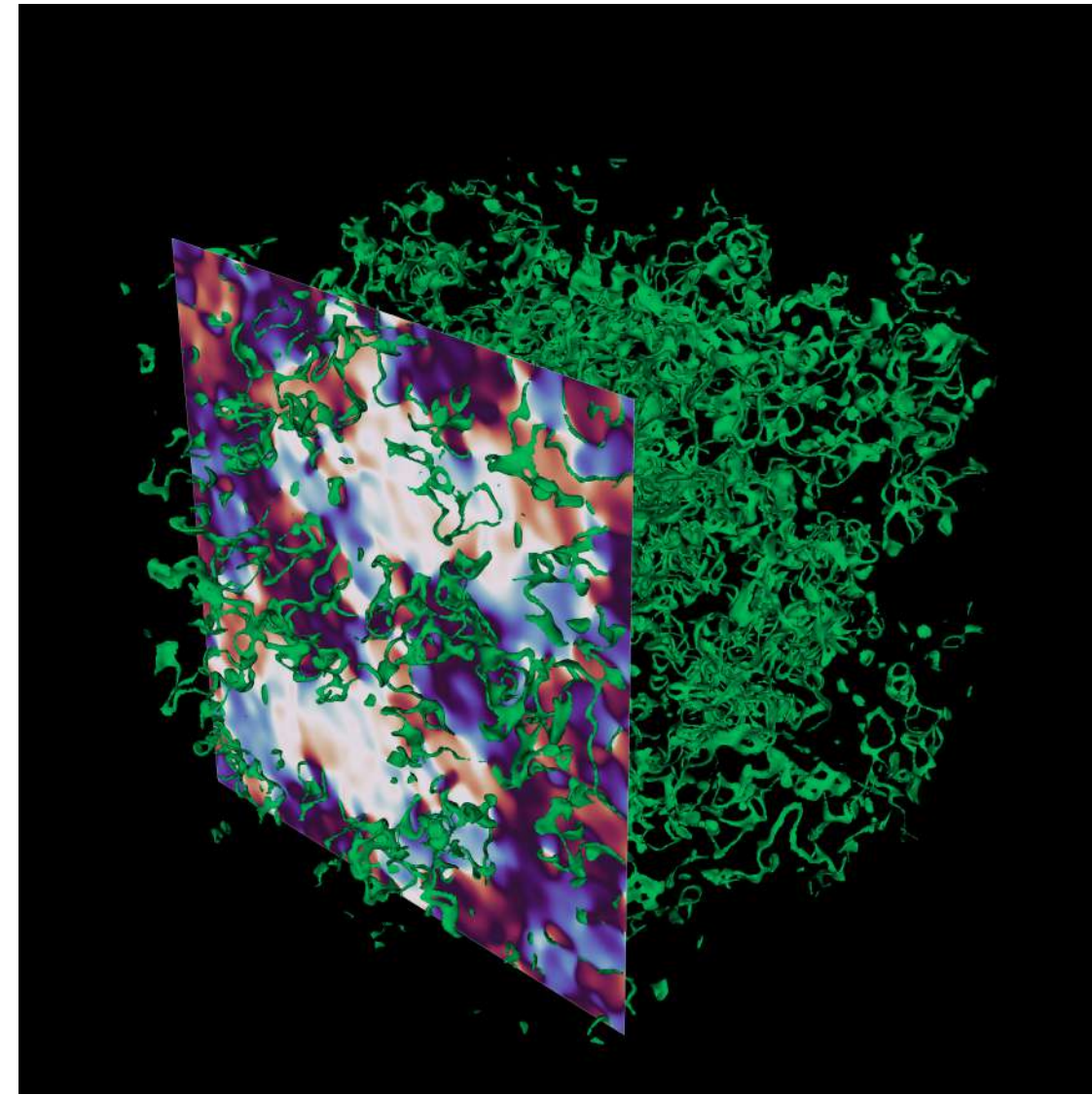
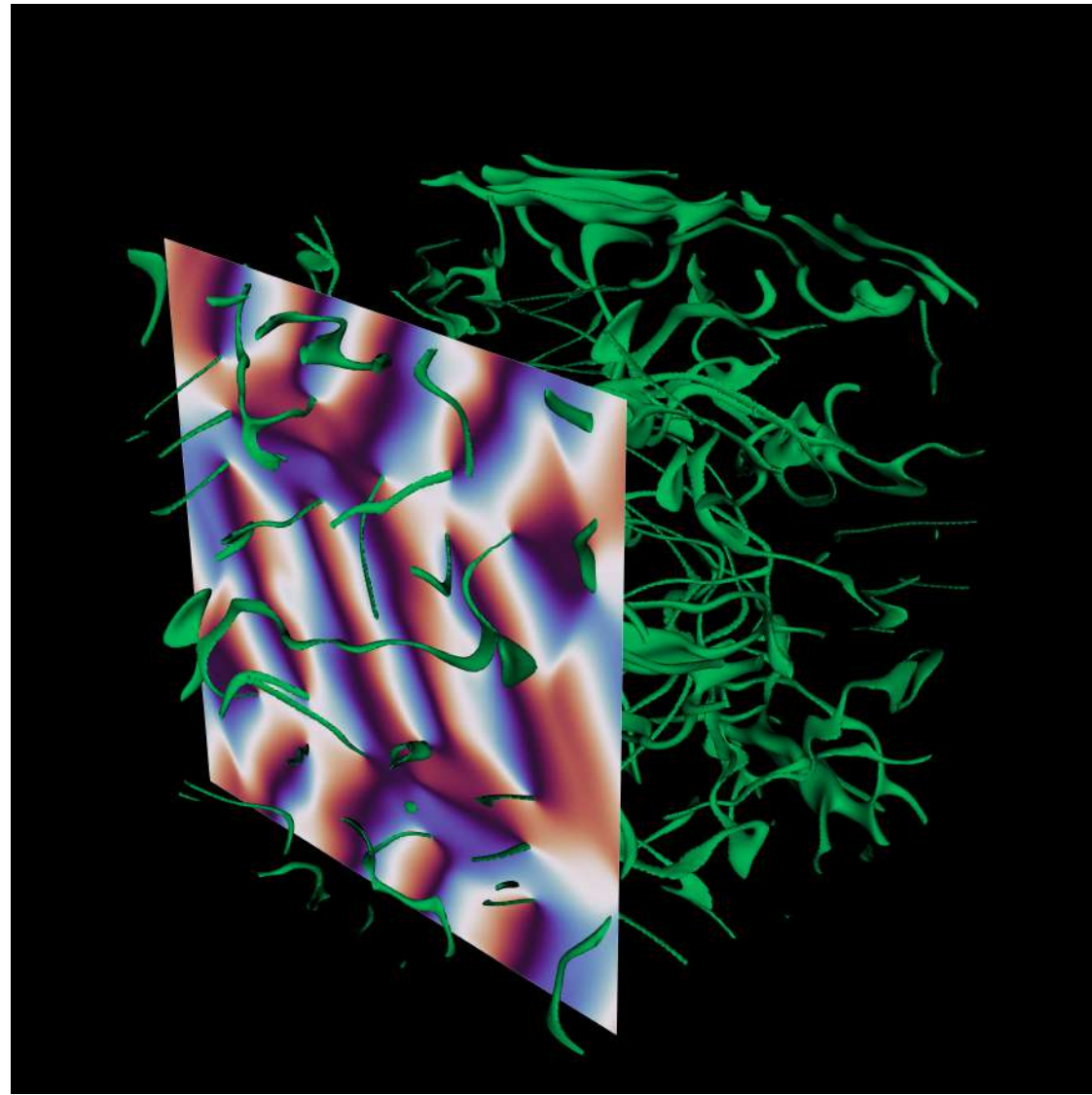


Gorghetto, Hardy, March-Russell, Song & West (2022)

vectors

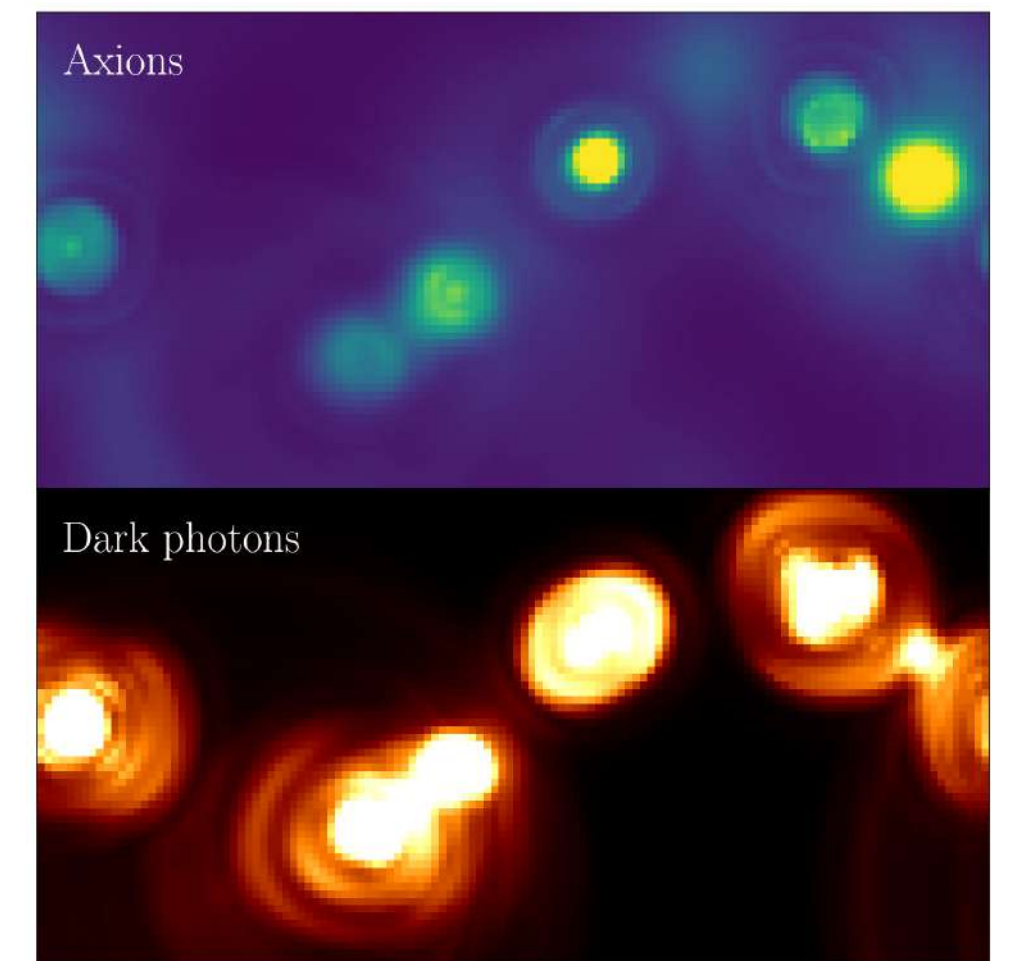
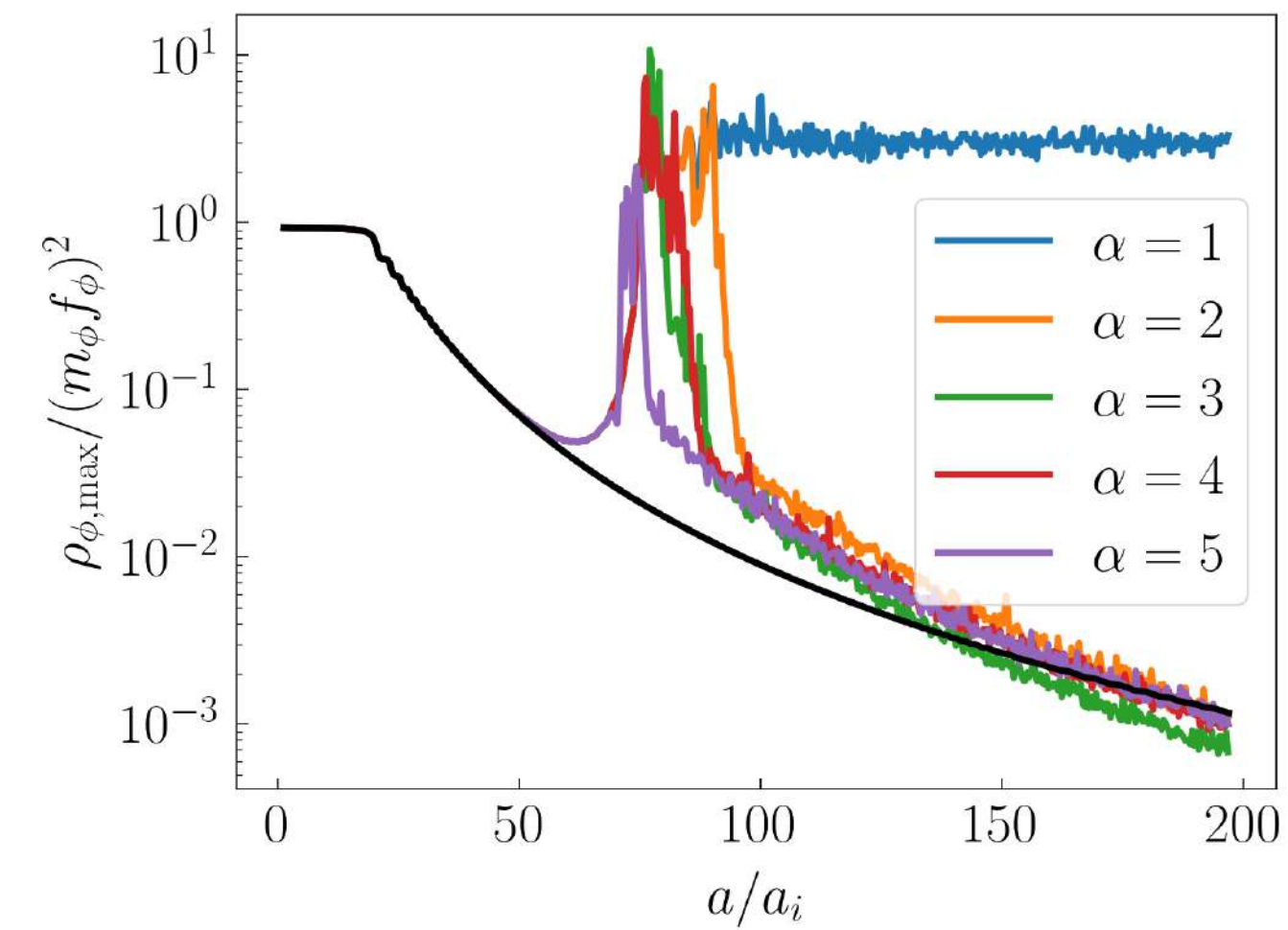


# production mechanism ultralight vectors



solitons, DM abundance ?

East & Huang (2022)



Zhang et. al (2025)



# production mechanism + cosmology questions

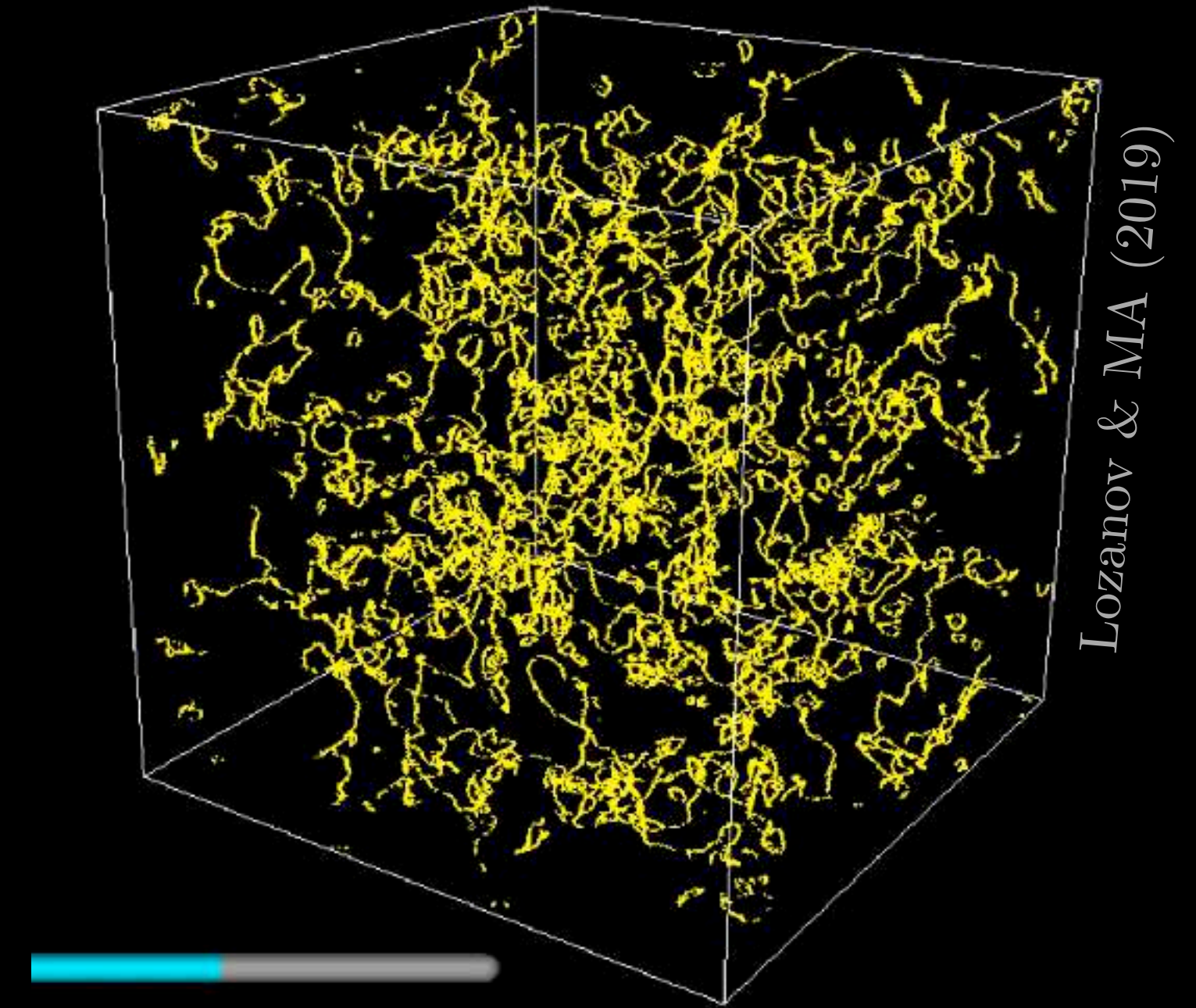
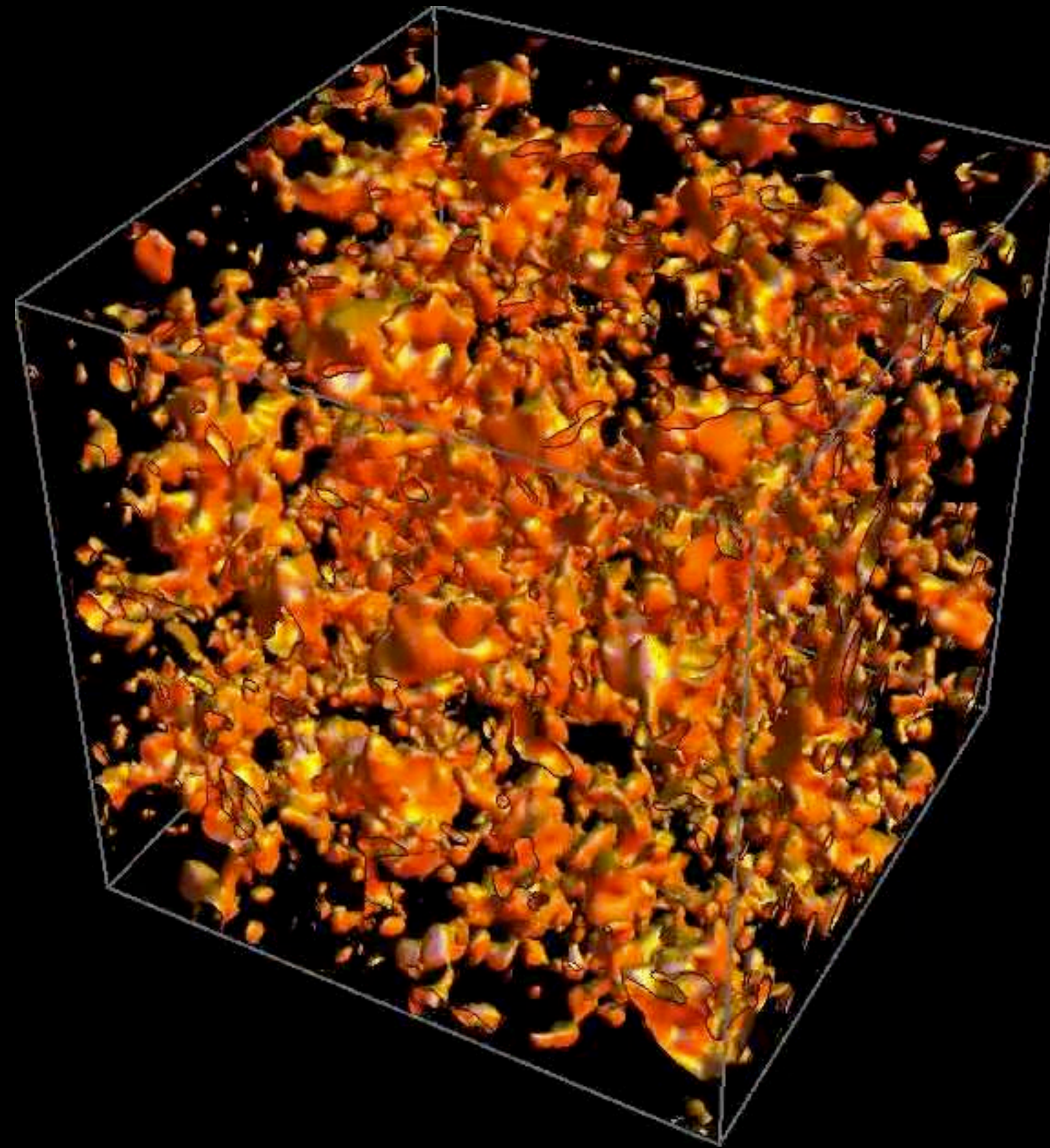
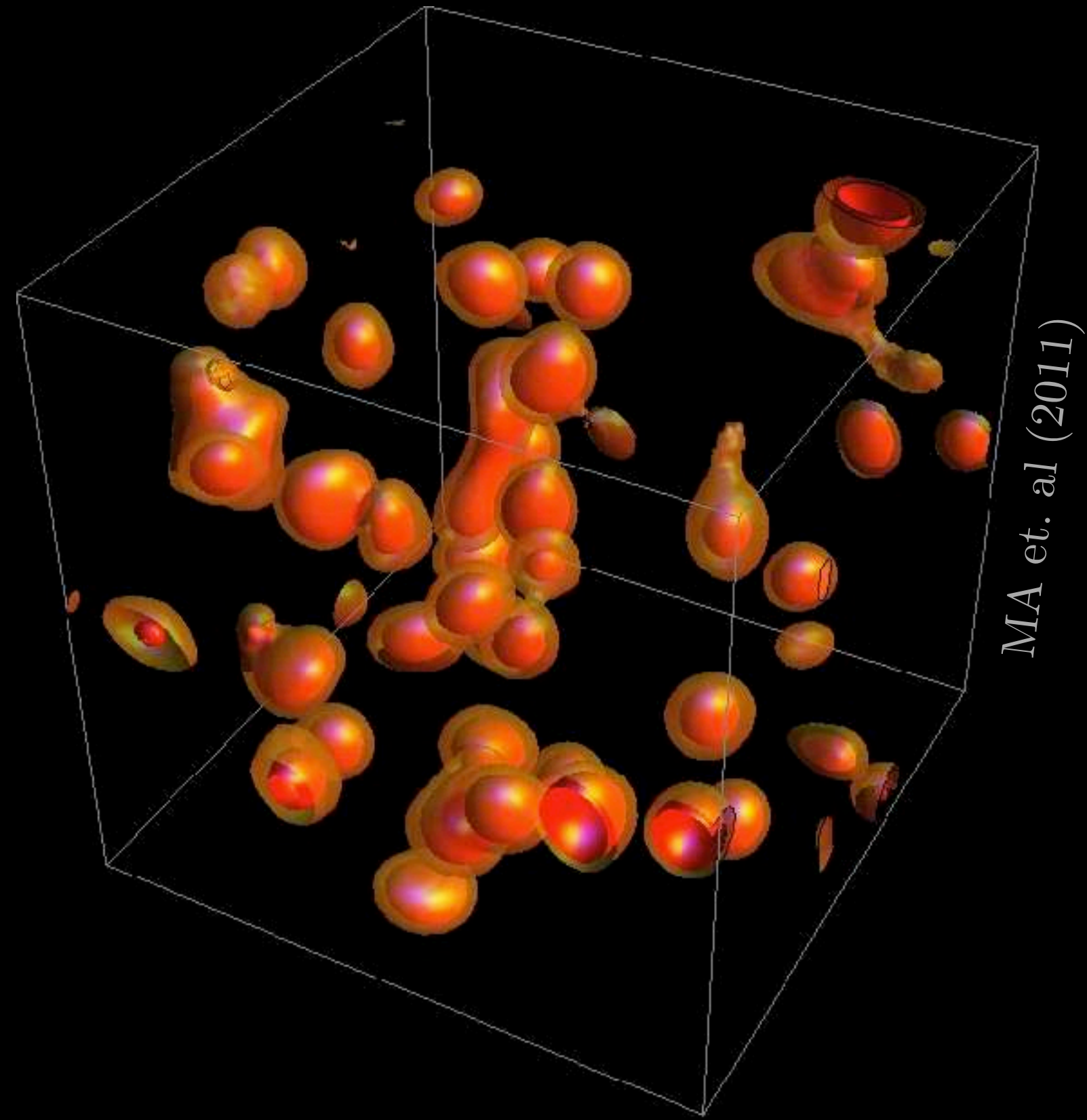
production mechanisms for ultralight vectors post-inflation

mixed scalar and vectors

mass functions of halos, survival probability of solitons

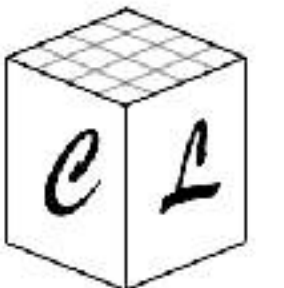
baryons, connections to observables





- more early universe cases

many more with  
Figueroa et. al





# many things I did not discuss

pulsar timing arrays, gravitational waves

more on non-gravitational self-interactions ( $\sim$  Hai-Bo Yu's talk)

conversions around compact objects

the classical vs. quantum nature of wave DM

# summary

## reduced wave interference

- dynamical heating, lensing, nucleation times, core-halo relation ...

## solitons & spin

- polarized emission, compactness ...

## initial conditions+growth of structure

- free-streaming suppression+enhancement, mass bound, very early growth of structure

