



Scalar-induced GWs: Theory and Perspectives

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中国科学院理论物理研究所

第十五届新物理研讨会

2026年9月27日，威海

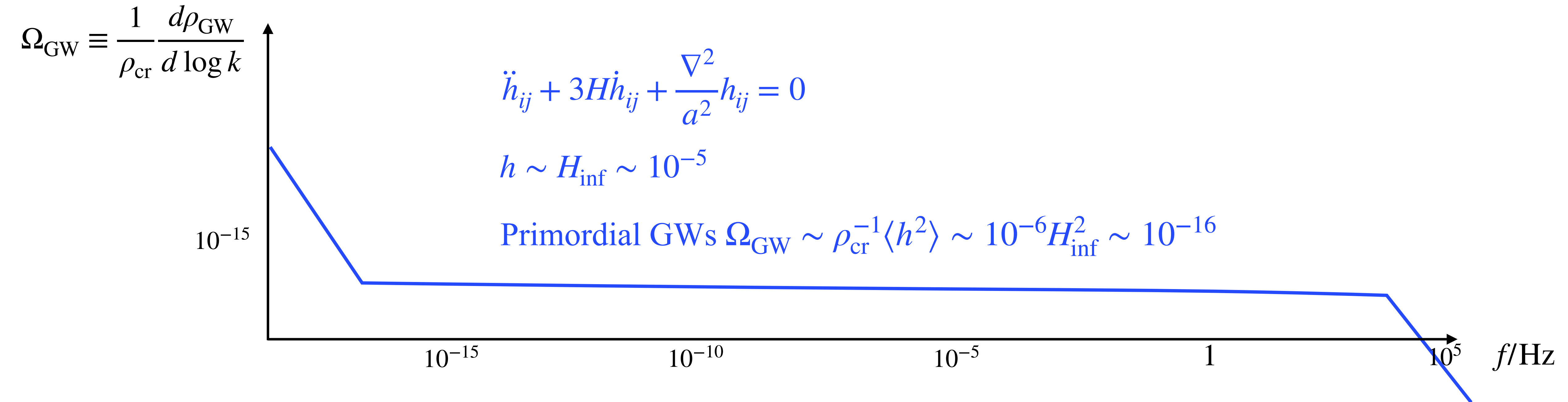
CONTENT

- 理论和观测动机：原初黑洞和诱导引力波
- 诱导引力波的规范问题
- 唯象学和未来的观测
- 非高斯相关的理论问题
- 结语和讨论

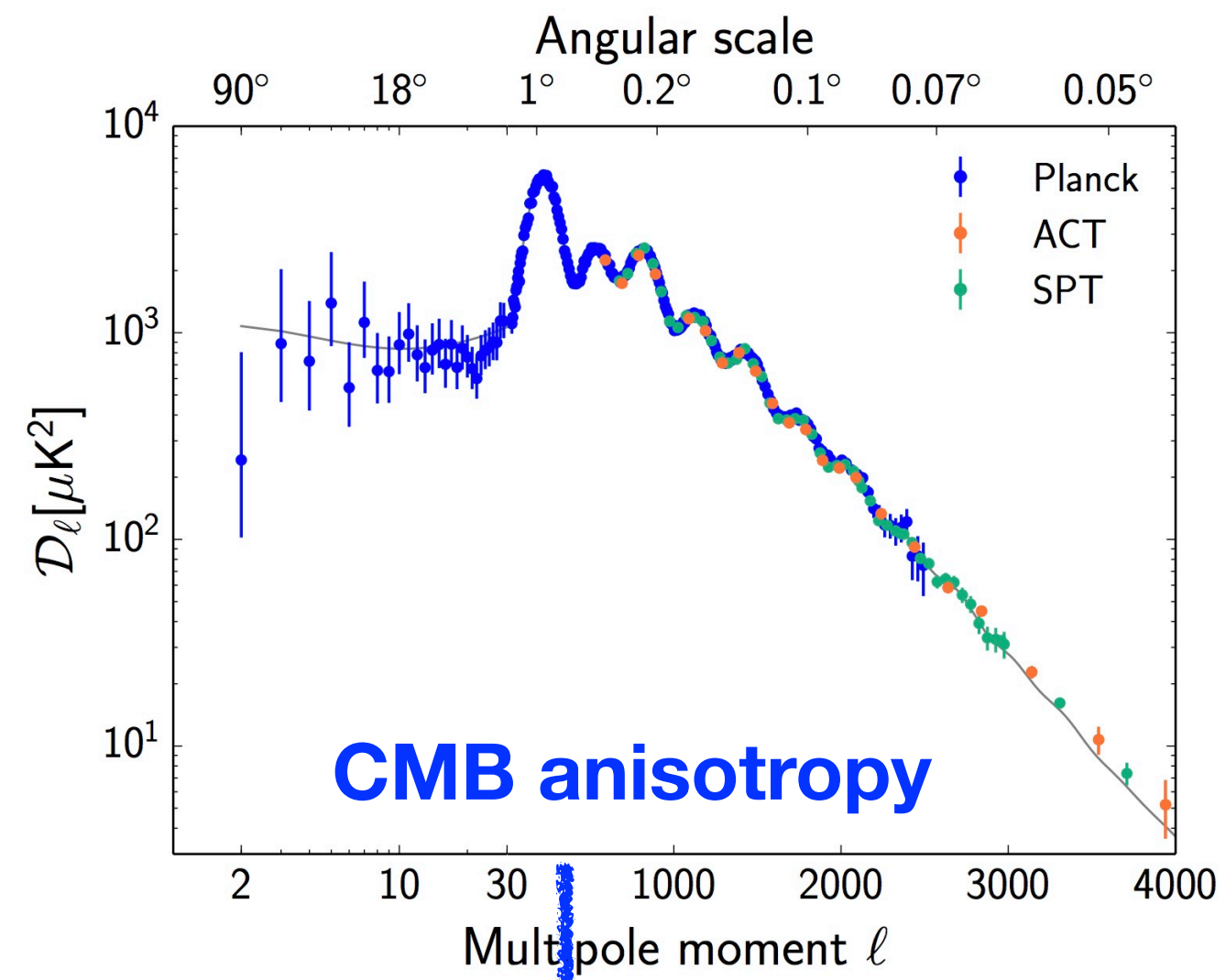
原初引力波

扰动的时空度规：

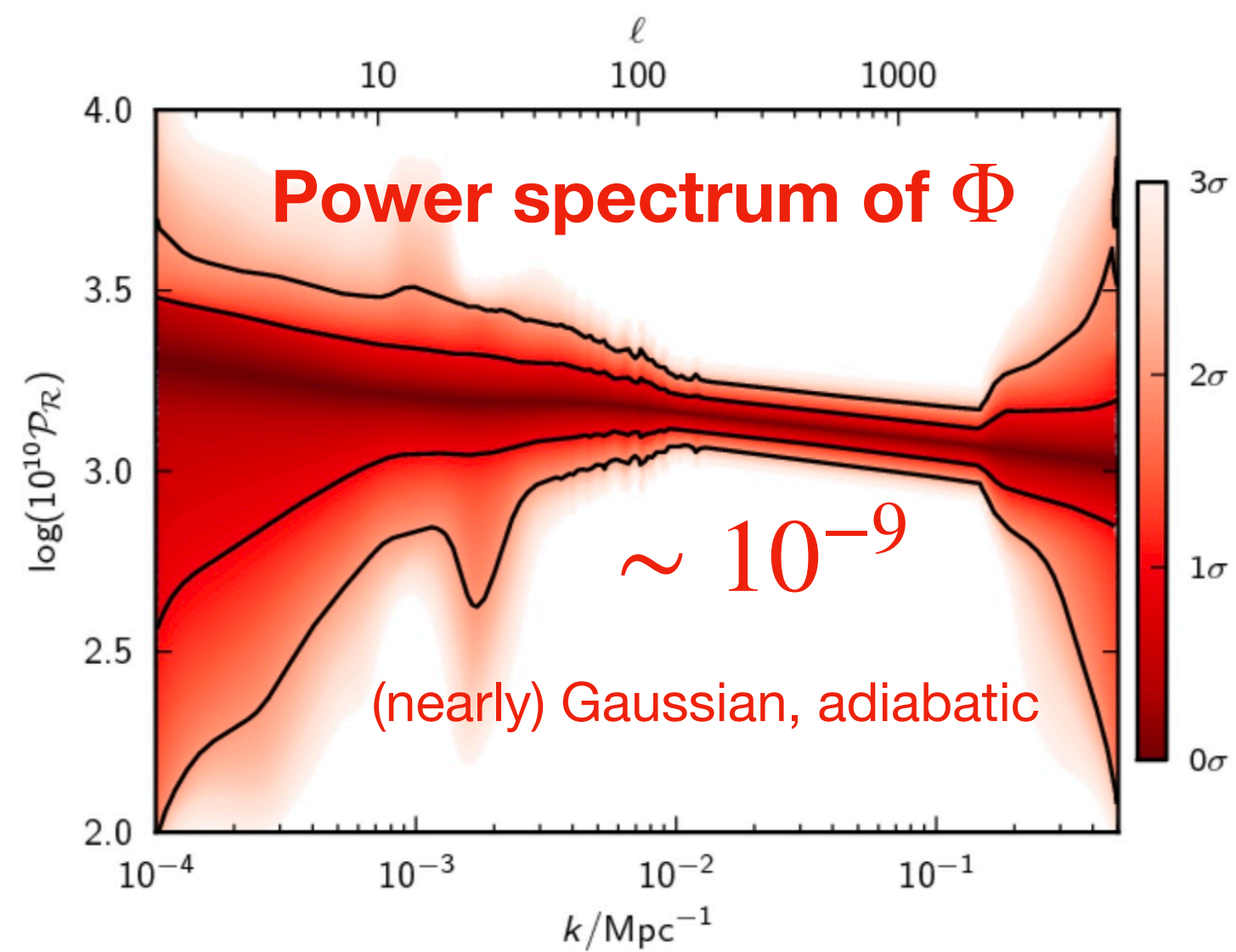
$$ds^2 = a^2(\eta) \left[-(1 + 2\Psi)d\eta^2 + ((1 + 2\Phi)\delta_{ij} + h_{ij})dx^i dx^j \right]$$



Starobinsky 1979



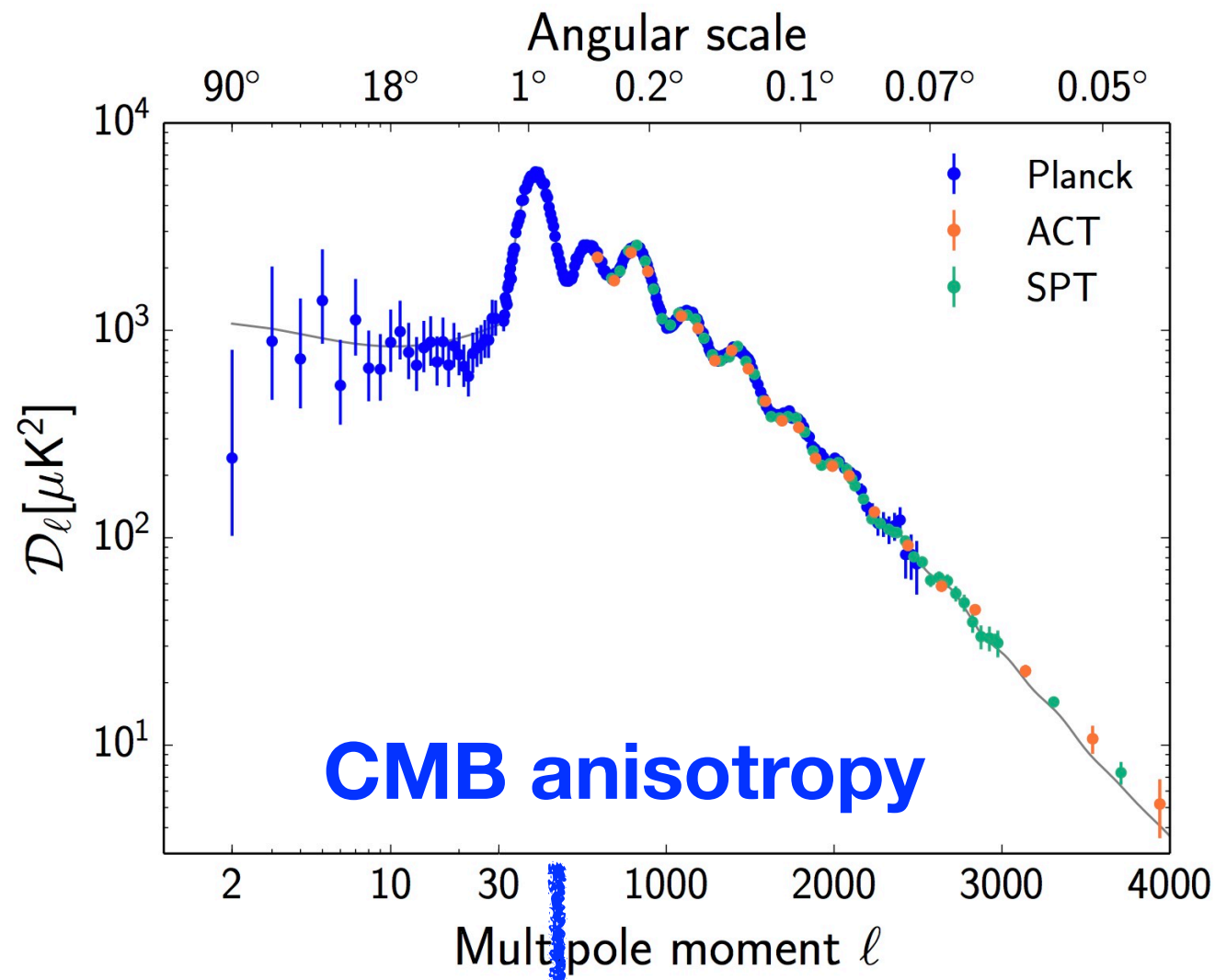
Reconstruction



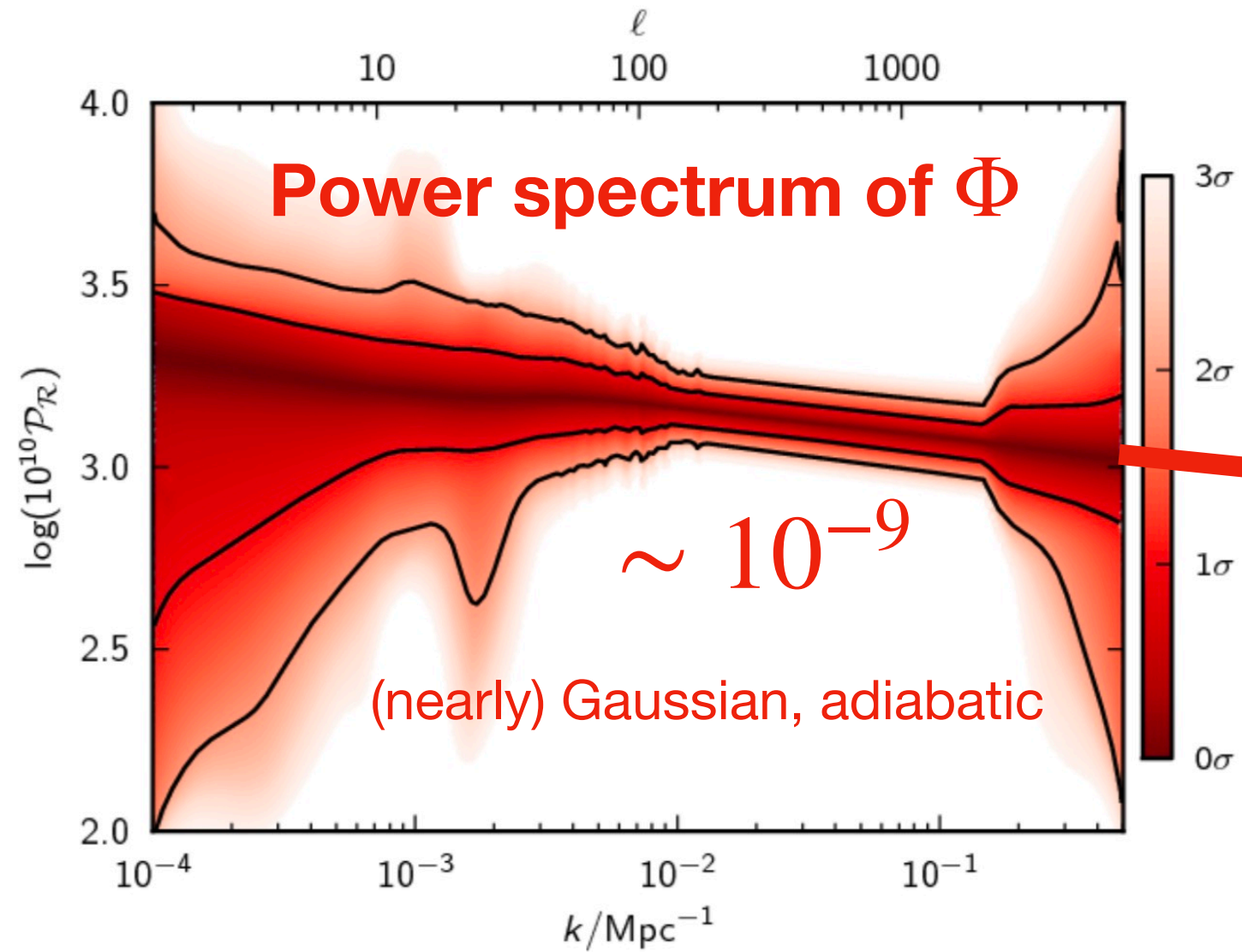
$$\ddot{h}_{ij} + 3H\dot{h}_{ij} + \frac{\nabla^2}{a^2}h_{ij} = \Lambda_{ij}^{lm} \left(2\Phi_{,l}\Phi_{,m} + \frac{\Phi_{,l}\dot{\Phi}_{,m}}{H} + \dots \right)$$

$$h_{ij} \sim \Phi^2 \sim 10^{-9}$$

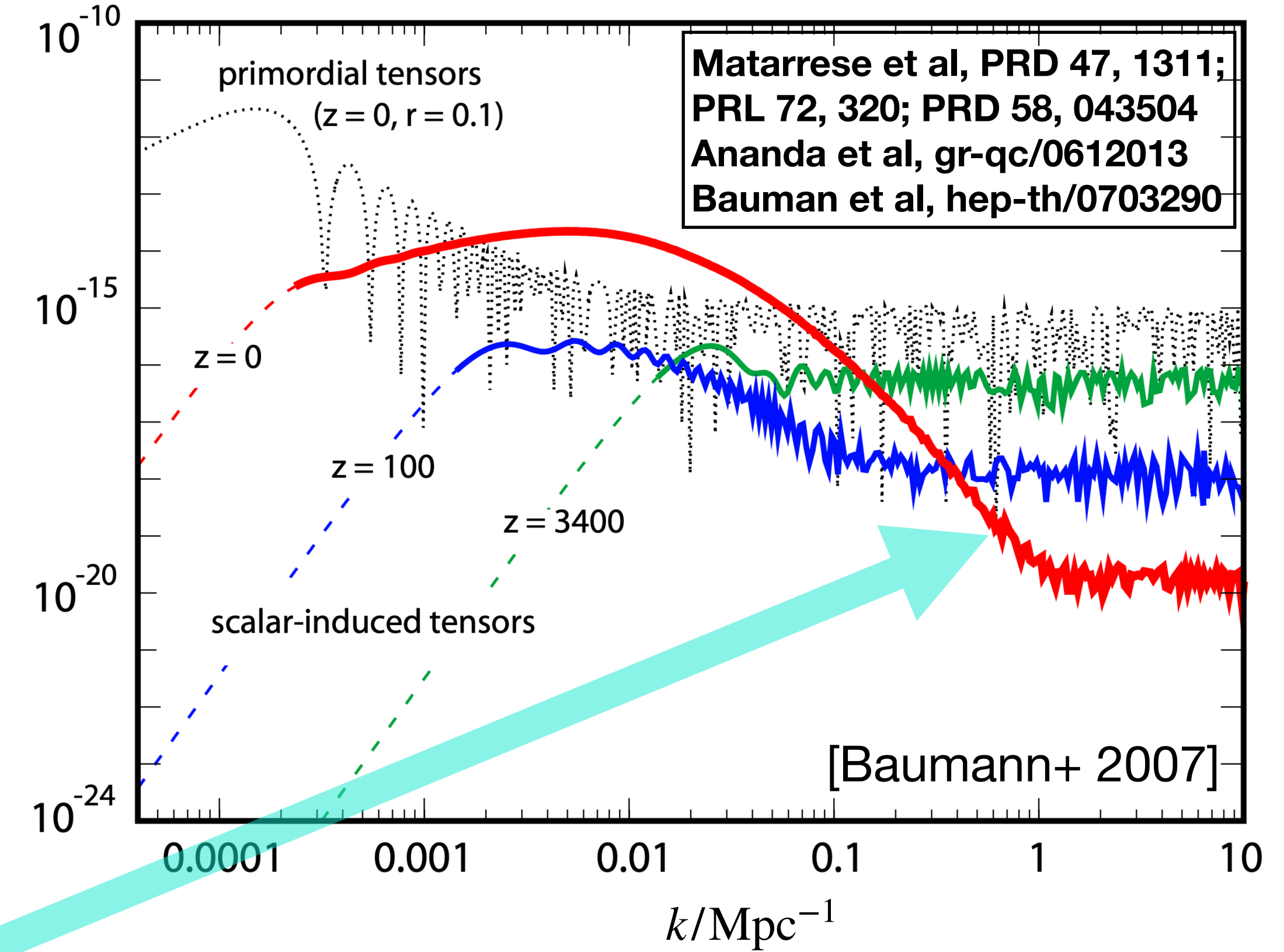
$$\text{Secondary GWs } \langle h_{ij}^2 \rangle \sim 10^{-6} \Phi^4 \sim 10^{-24}$$



Reconstruction



$$\Omega_{\text{GW}} \sim 10^{-6} \mathcal{P}_{\mathcal{R}}^2 \sim 10^{-24}$$

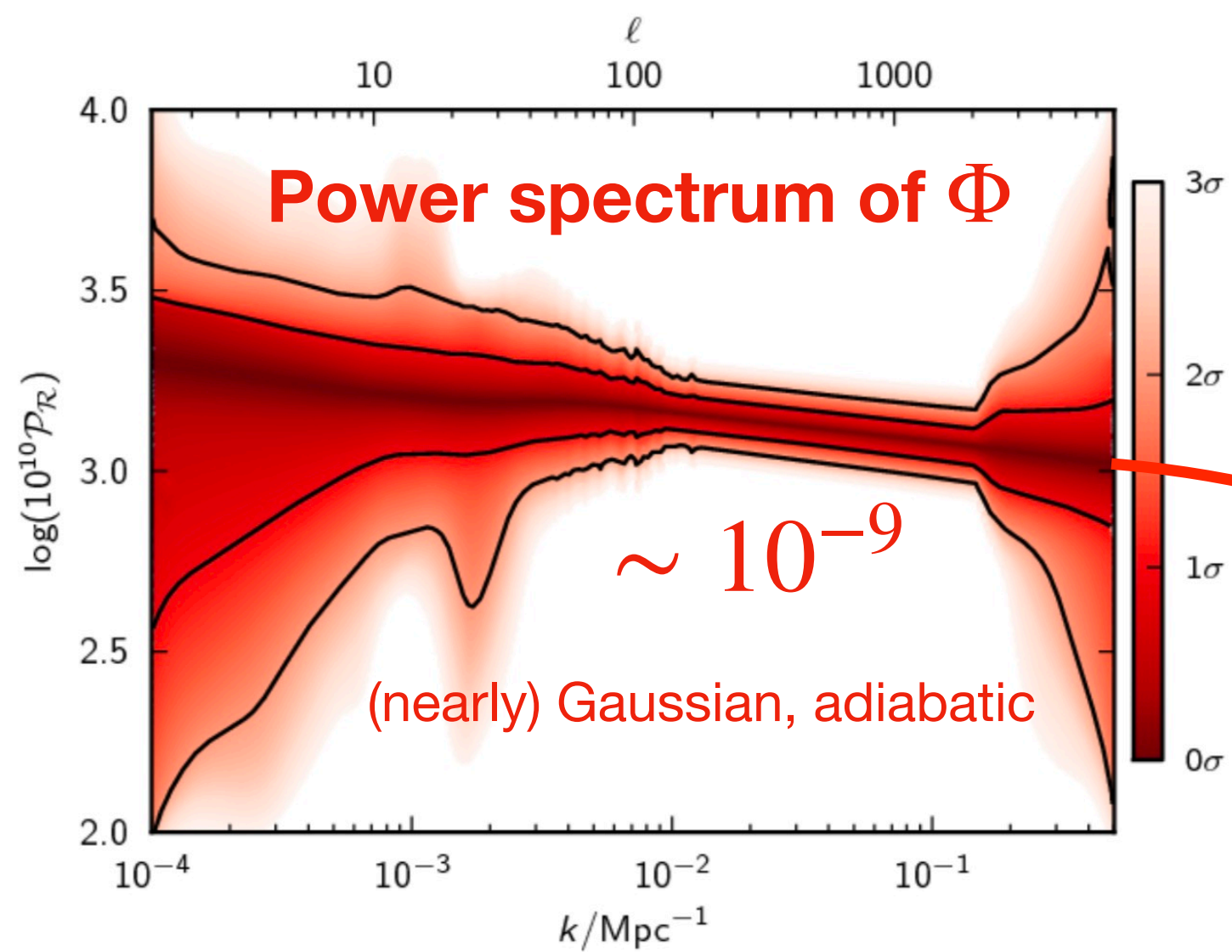
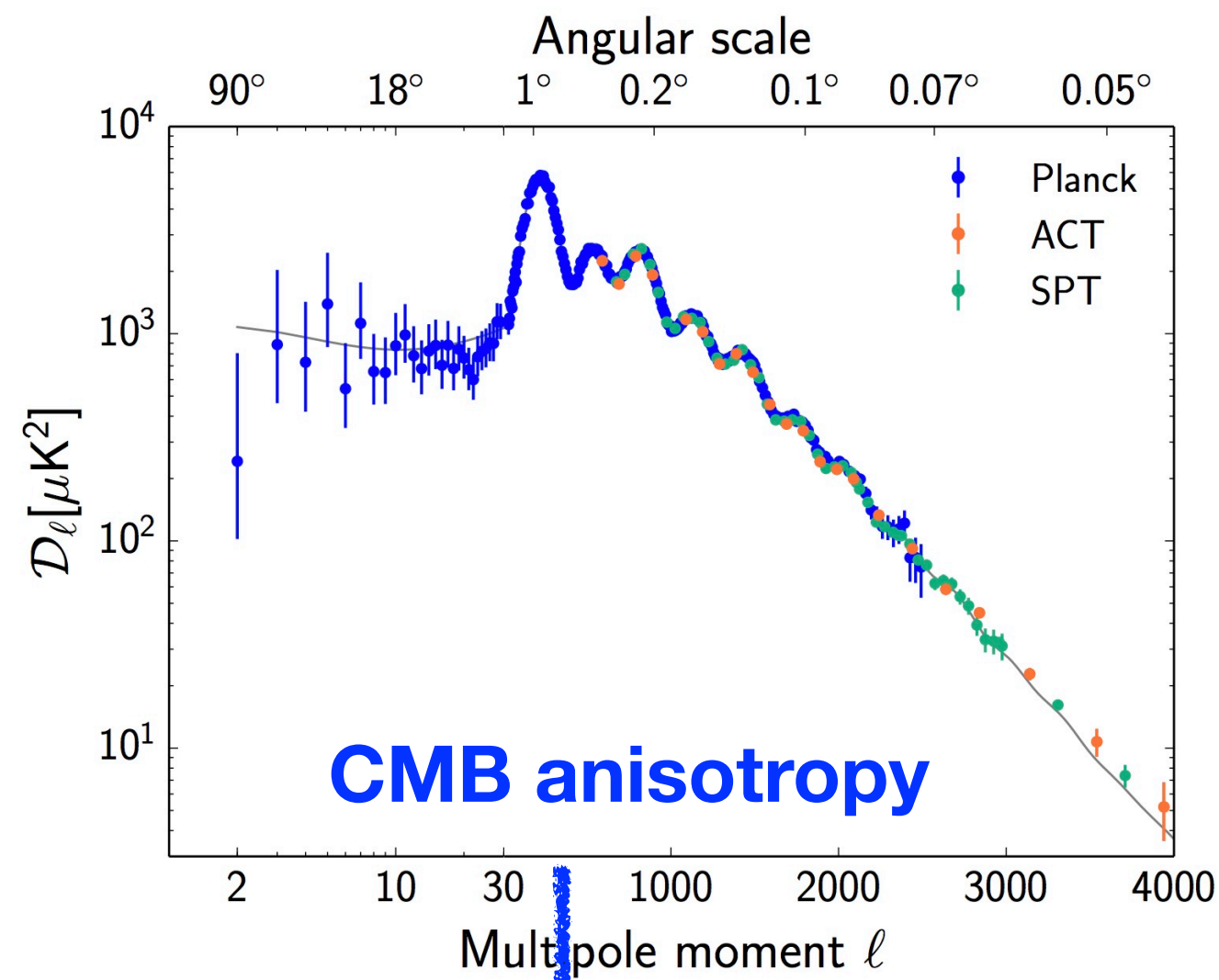


$h\Phi\Phi$ coupling

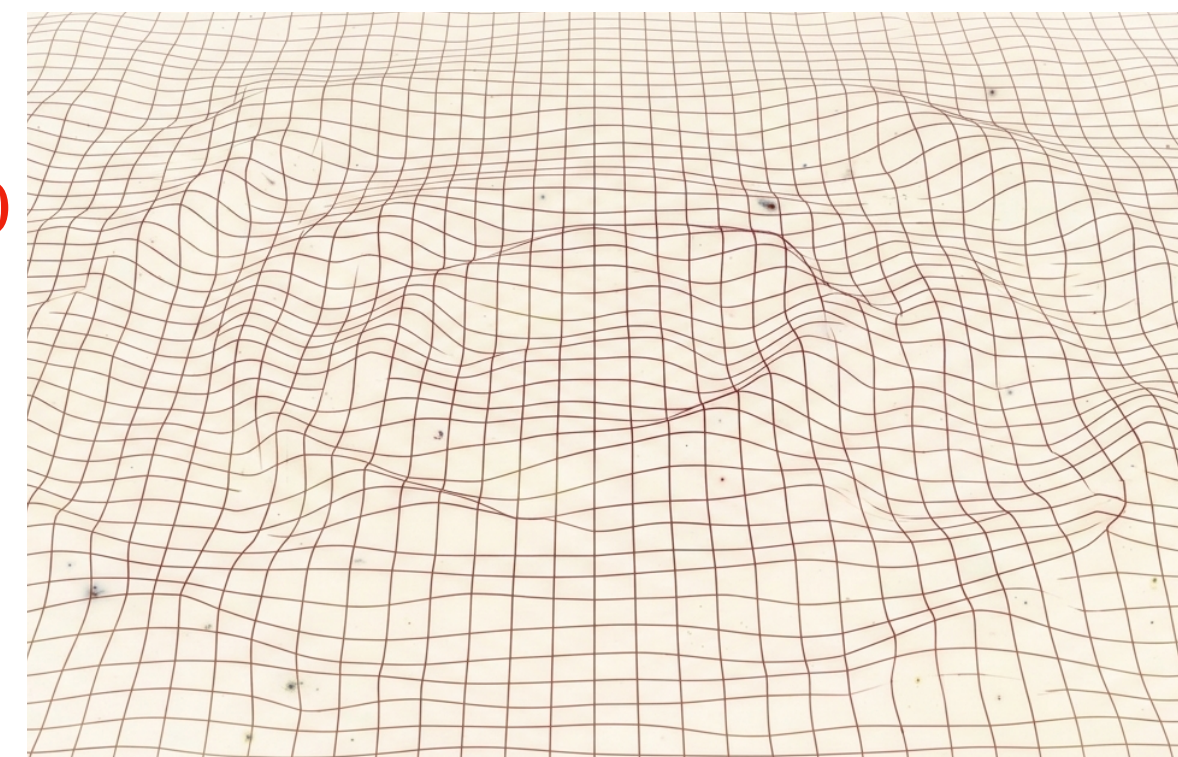
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$$h_{ij} \sim \Phi^2 \sim 10^{-9}$$

$$\text{Secondary GWs } \langle h_{ij}^2 \rangle \sim 10^{-6} \Phi^4 \sim 10^{-24}$$



$$\langle h_{ij}^2 \rangle \sim 10^{-6} \Phi^4 \sim 10^{-10}$$



Matarrese et al, PRD 47, 1311;
PRL 72, 320; PRD 58, 043504
Ananda et al, gr-qc/0612013
Bauman et al, hep-th/0703290

**Scalar Perturbation
Induced GW**

nonlinear
perturbation

$$\sim 10^{-1}$$

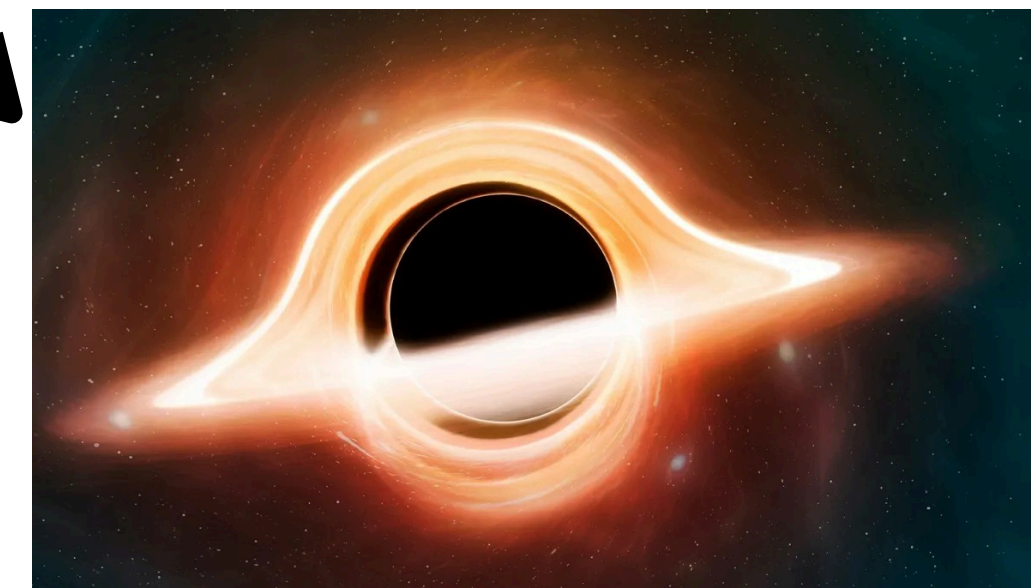
Gaussian?
adiabatic?

gravitational
collapse

crosscheck

Saito, Yokoyama, 0812.4339
Cai, SP, Sasaki, 1810.11000

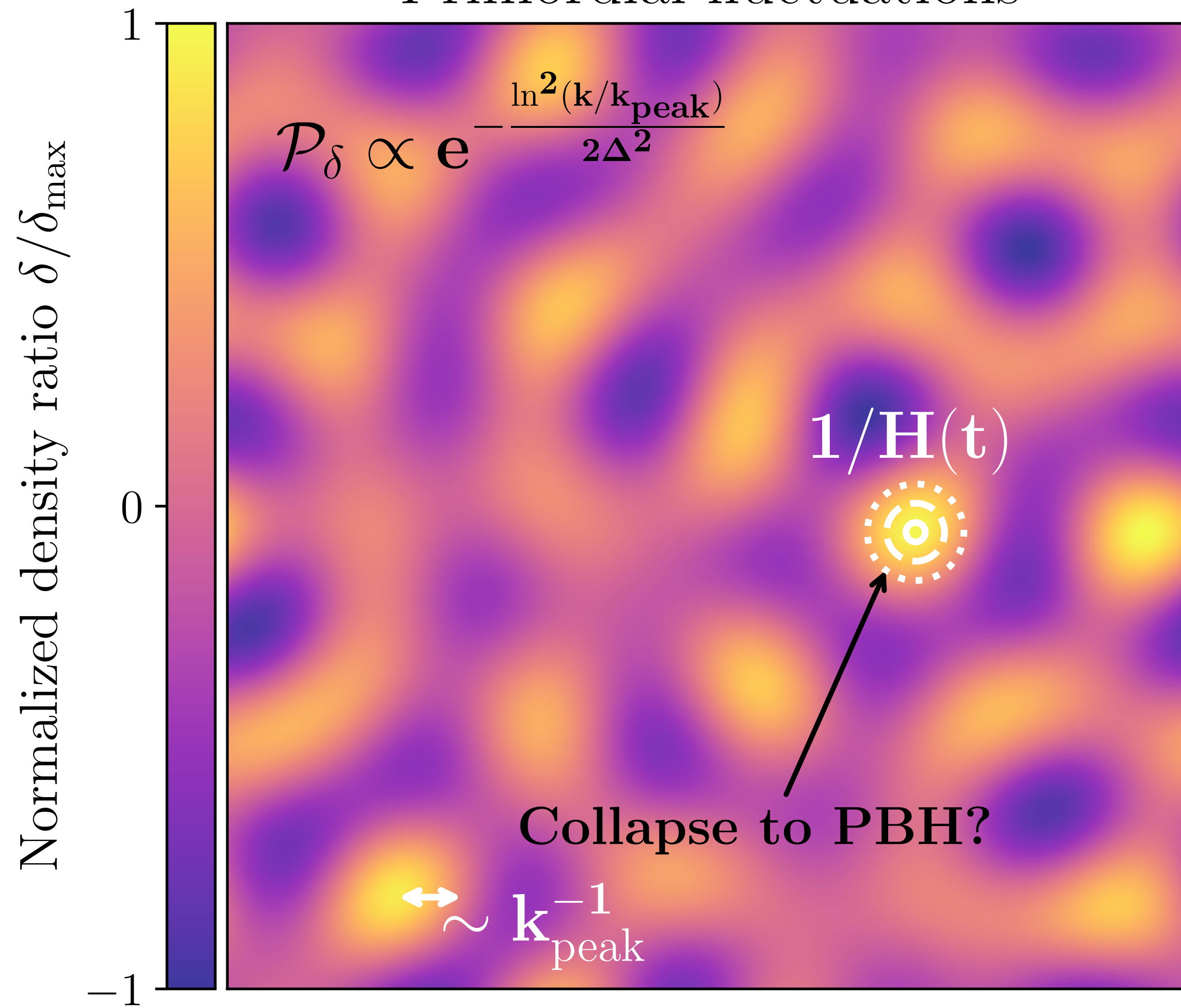
Primordial Black Hole



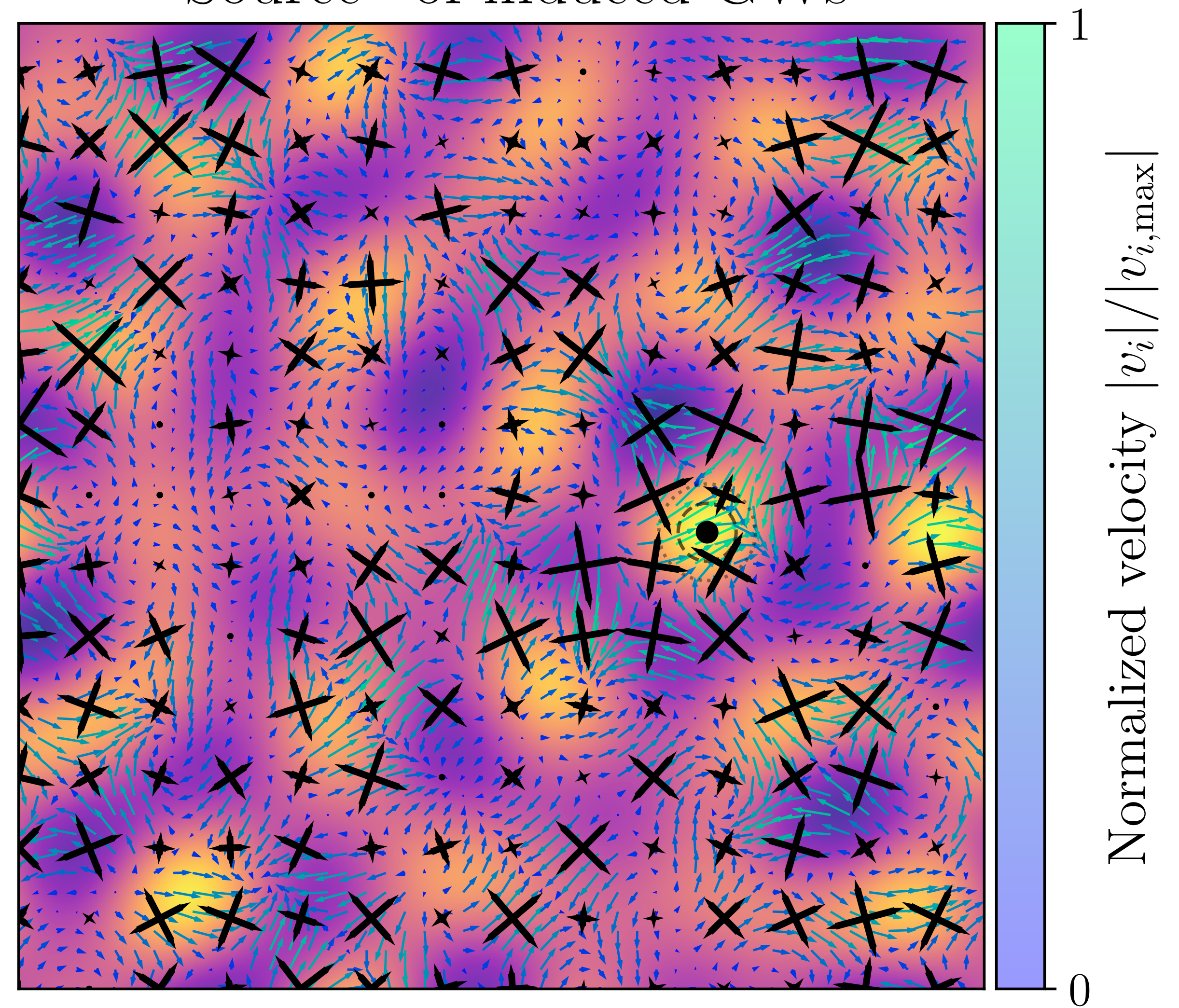
Zeldovich & Novikov 1966
Hawking 1971
Carr & Hawking 1974

$$f_{\text{IGW}} \sim 3\text{Hz} \left(\frac{M_{\text{PBH}}}{10^{16}\text{g}} \right)^{-\frac{1}{2}}$$

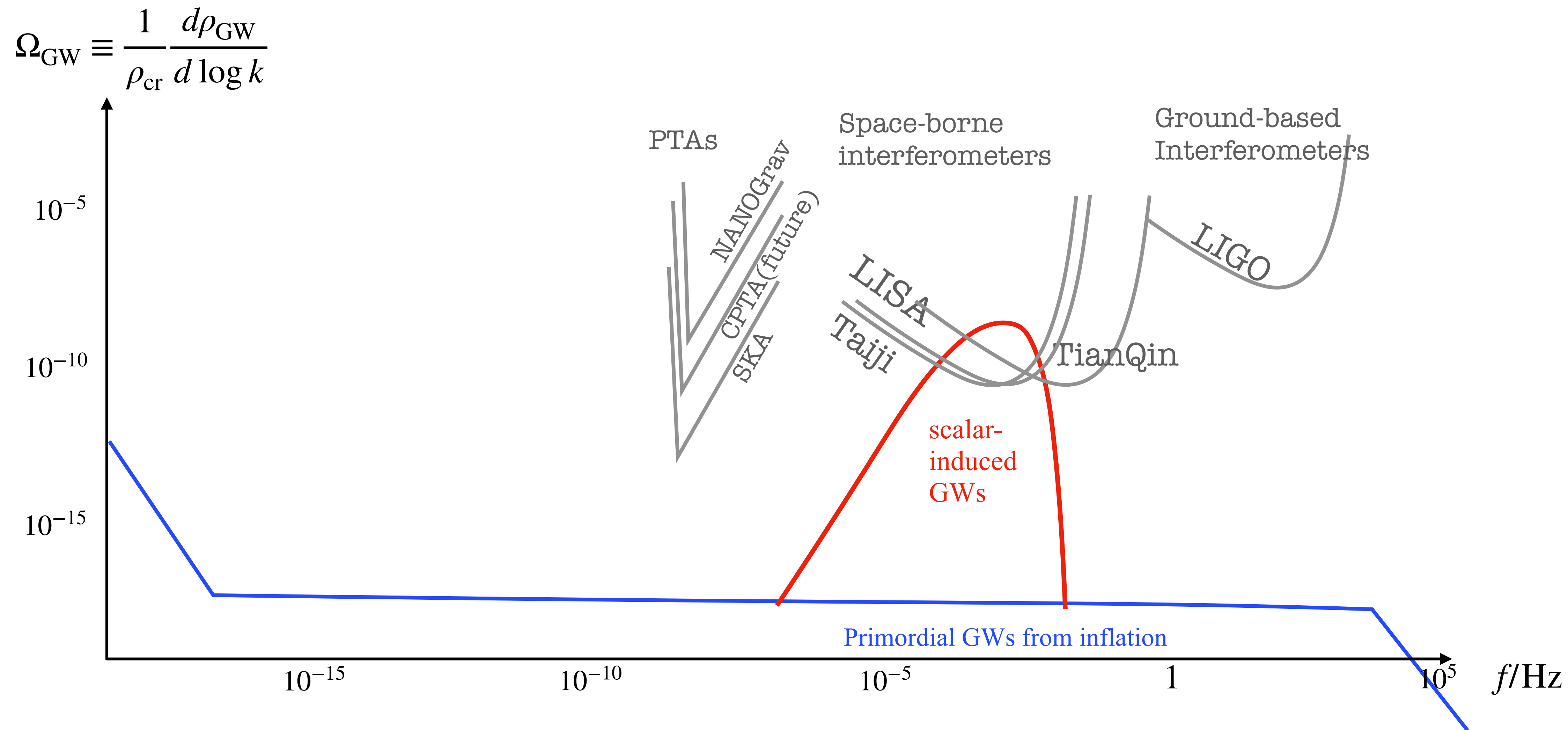
Primordial fluctuations



“Source” of induced GWs



诱导引力波



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诱导引力波规范问题

扰动的时空度规：

$$ds^2 = a^2 \left(- (1 + 2A) d\eta^2 + 2B_{,i} d\eta dx^i + \left((1 + 2(H_L - \frac{1}{3} \partial^2 H_T) \delta_{ij} + 2H_{T,ij} + h_{ij}^{TT}) dx^i dx^j \right) \right)$$

广义相对论的规范变换(坐标变换): $x^\mu \longrightarrow \tilde{x}^\mu = x^\mu + \xi^\mu$ $[\xi^\mu = (T, \partial_i L)]$

$$A \longrightarrow \tilde{A} = A - \frac{1}{a} (aT)'$$
$$H_L \longrightarrow \widetilde{H}_L = H_L - \frac{a'}{a} T - \frac{\partial^2 L}{3}$$

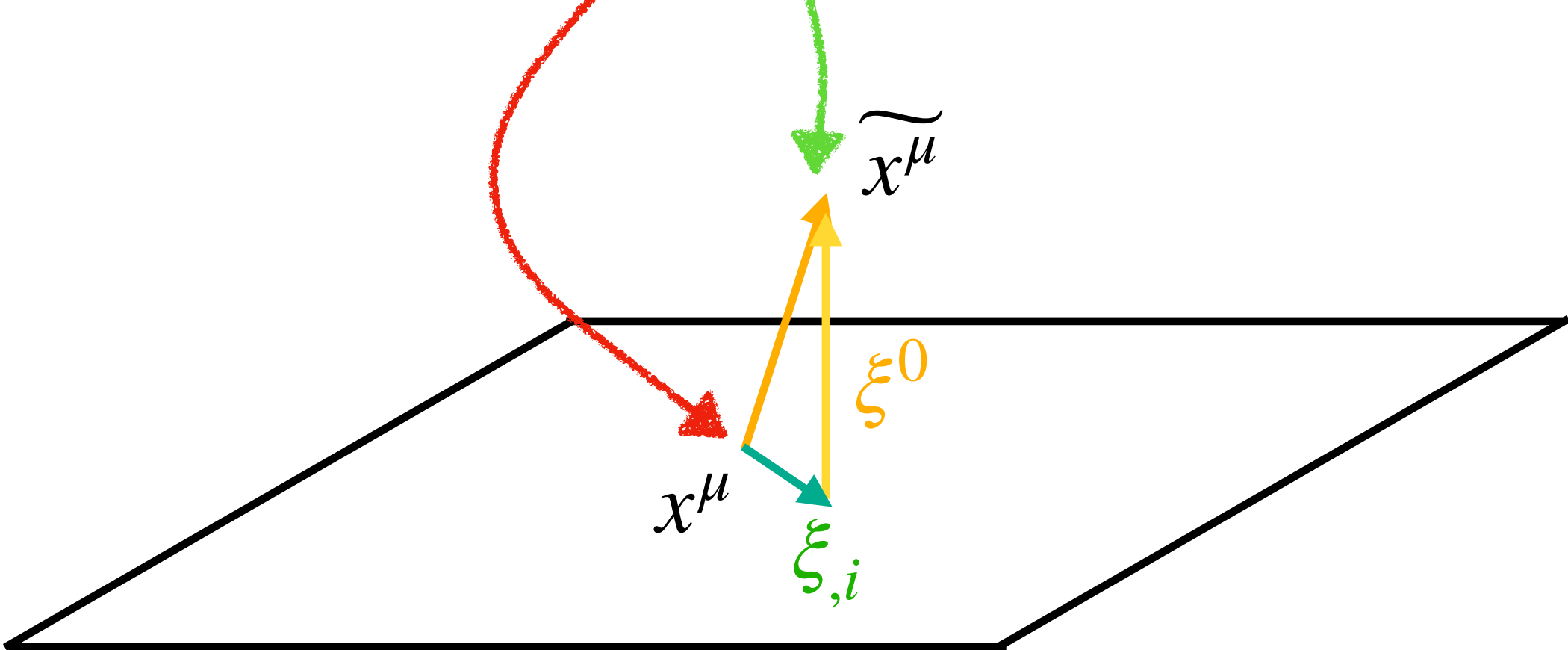
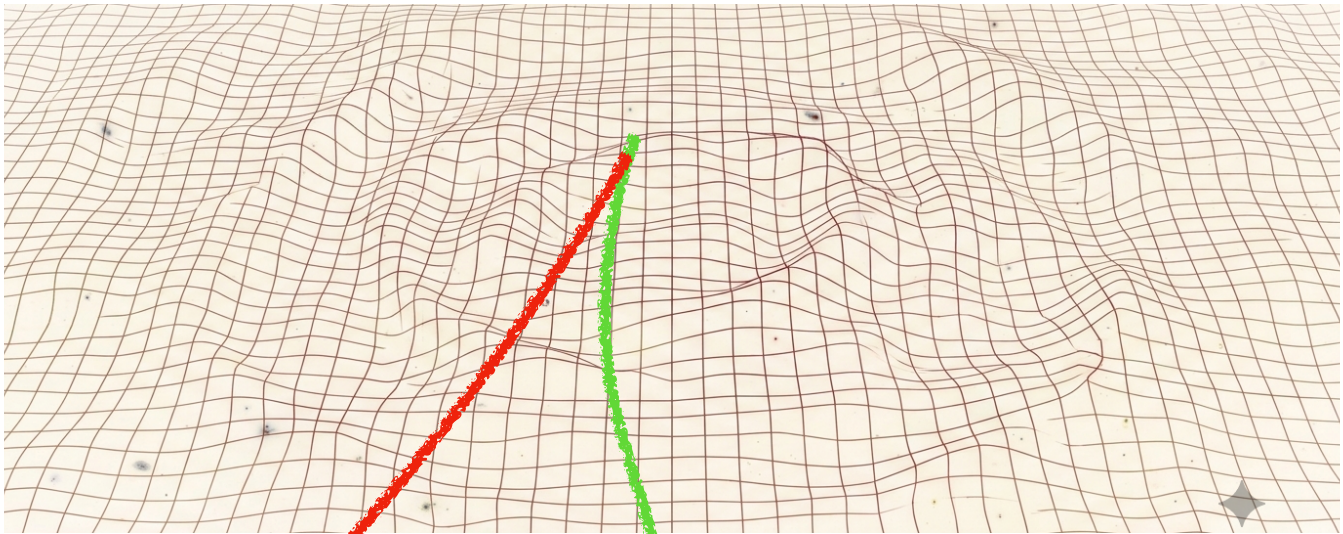
$$H_T \longrightarrow \widetilde{H}_T = H_T - L$$
$$B \longrightarrow \widetilde{B} = B + T - L'$$

$$h_{ij}^{TT} \longrightarrow \widetilde{h}_{ij}^{TT} = h_{ij}^{TT} + \mathcal{O}(\xi^2)$$

(螺旋度分解定理)

可以把坐标变换用一阶扰动写出：

$$T = (H'_T - B) - (\widetilde{H}'_T - \widetilde{B}) \equiv \sigma - \tilde{\sigma}$$
$$L = H_T - \widetilde{H}_T$$



诱导引力波规范问题

- 剩下的两个标量自由度(A 和 H_L)中的 T 和 L 可以用 σ 和 H_T 消掉, 据此得到规范不变量

$$\Psi \equiv A + \frac{1}{a}(a\sigma)' \quad \Phi \equiv H_L - \frac{1}{3}\partial^2 H_T + \frac{a'}{a}\sigma$$

$$\sigma \equiv H'_T - B$$

- 即Bardeen势。通过恰当选取 T , 可使 $\sigma = 0$, 称为无剪切切片(zero-shear slice). 更进一步选取 L 使 $H_T = 0$ 即为纵向规范(也称为牛顿规范)

$$ds^2 = a^2(\eta) \left[-(1 + 2\Psi)d\eta^2 + ((1 + 2\Phi)\delta_{ij} + h_{ij}^{TT})dx^i dx^j \right]$$

- 上面的 h_{ij}^{TT} 在线性阶是规范不变的, 但到了二阶则按照下式变换

$$h_{ij}^{TT} \longrightarrow \tilde{h}_{ij}^{TT} = h_{ij}^{TT} - \Lambda_{ij}{}^{ab} \left((T - 2\sigma)_{,a} T_{,b} + L_{,ak} (L - 2H_T)_{,bk} \right)$$

诱导引力波规范问题

- 仿造我们构造Bardeen势的过程，把 T 和 L 用 σ 和 H_T 消掉，据此得到规范不变量

$$h_{ij}^{TT} \longrightarrow \tilde{h}_{ij}^{TT} = h_{ij}^{TT} + \Lambda_{ij}^{ab} \left((\sigma + \tilde{\sigma})_{,a} (\sigma - \tilde{\sigma})_{,b} + (H_T + \tilde{H}_T)_{,ak} (H_T - \tilde{H}_T)_{,bk} \right)$$

$$\tilde{h}_{ij} + \Lambda_{ij}^{ab} \left(\tilde{\sigma}_{,a} \tilde{\sigma}_{,b} + \tilde{H}_{T,ak} \tilde{H}_{T,bk} \right) = h_{ij} + \Lambda_{ij}^{ab} \left(\sigma_{,a} \sigma_{,b} + H_{T,ak} H_{T,bk} \right) \equiv h_{ij}^{N(2)}$$

- 此即牛顿规范中的TT度规张量，它是规范不变的。在牛顿规范下，度规可以写为

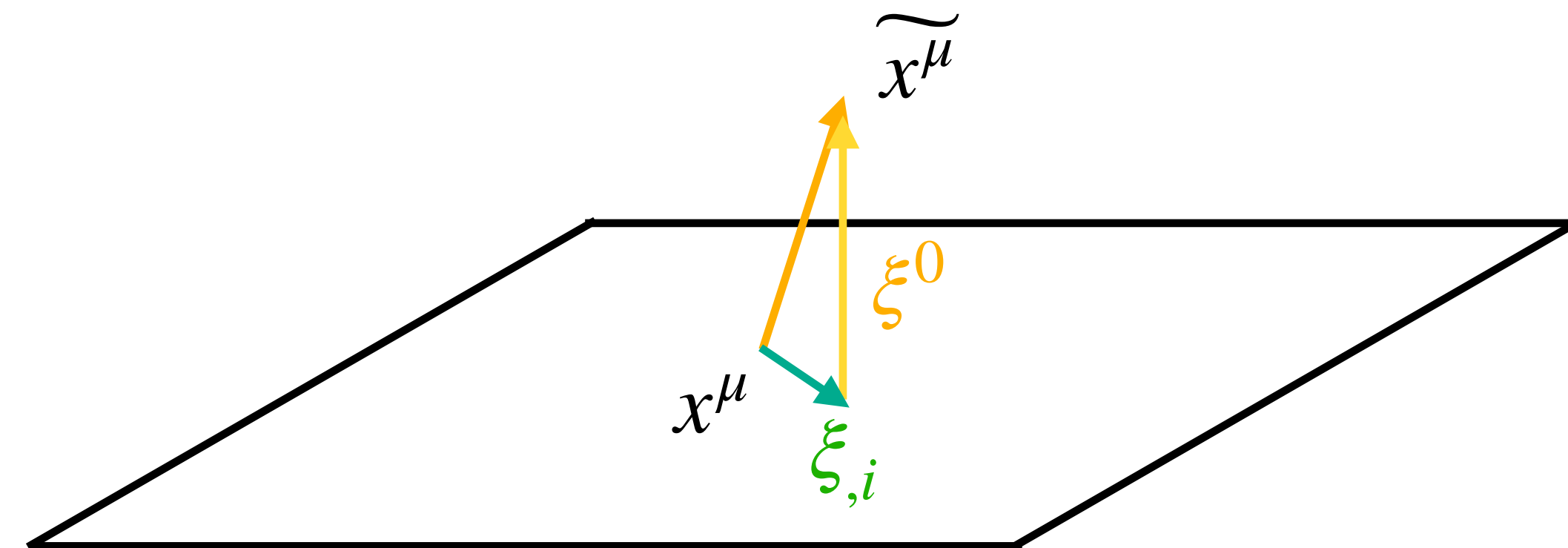
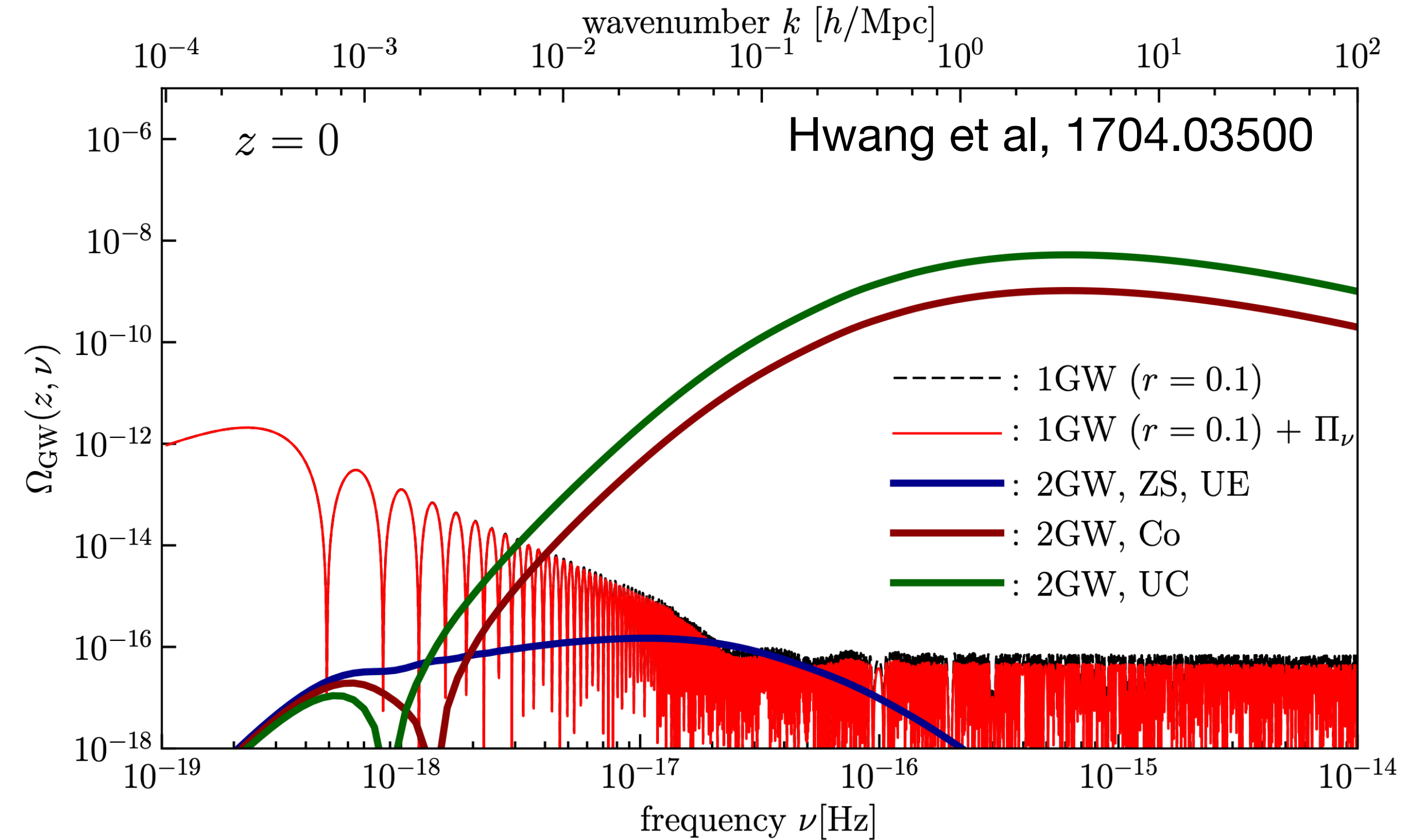
$$ds^2 = a^2(\eta) \left[-(1 + 2\Psi) d\eta^2 + ((1 + 2\Phi)\delta_{ij} + h_{ij}^{N(2)}) dx^i dx^j \right]$$

- 并用此式计算诱导引力波：

$$h_{ij}^{N(2)''} + 2\mathcal{H} h_{ij}^{N(2)'} - \nabla^2 h_{ij}^{N(2)} = 2\Lambda_{ij}^{lm} \left[2\Psi_{,l} \Psi_{,m} + \left(\Psi + \frac{\Psi'}{\mathcal{H}} \right)_{,l} \left(\Psi + \frac{\Psi'}{\mathcal{H}} \right)_{,m} \right]$$

诱导引力波规范问题

- 问题在于类似的规范不变量可以写出无穷多个，为什么牛顿规范是一个优越的参考系？
- 换个角度：哪个规范下的二阶 $h_{ij}^{(2)}$ 对应于观测量？
- 首先提出 $h_{ij}^{(2)}$ 规范问题：Hwang et al 1704.03500
- 二阶微扰论：Domenech, Sasaki, 1709.09804;
Wang, Zhang, 1905.03272
- 规范分类：De Luca et al, 1911.09689; Domenech, Sasaki, 2012.14016; ...



诱导引力波规范问题

Domenech, SP, Ao Wang,
2512.15704, PRL 136, 221402

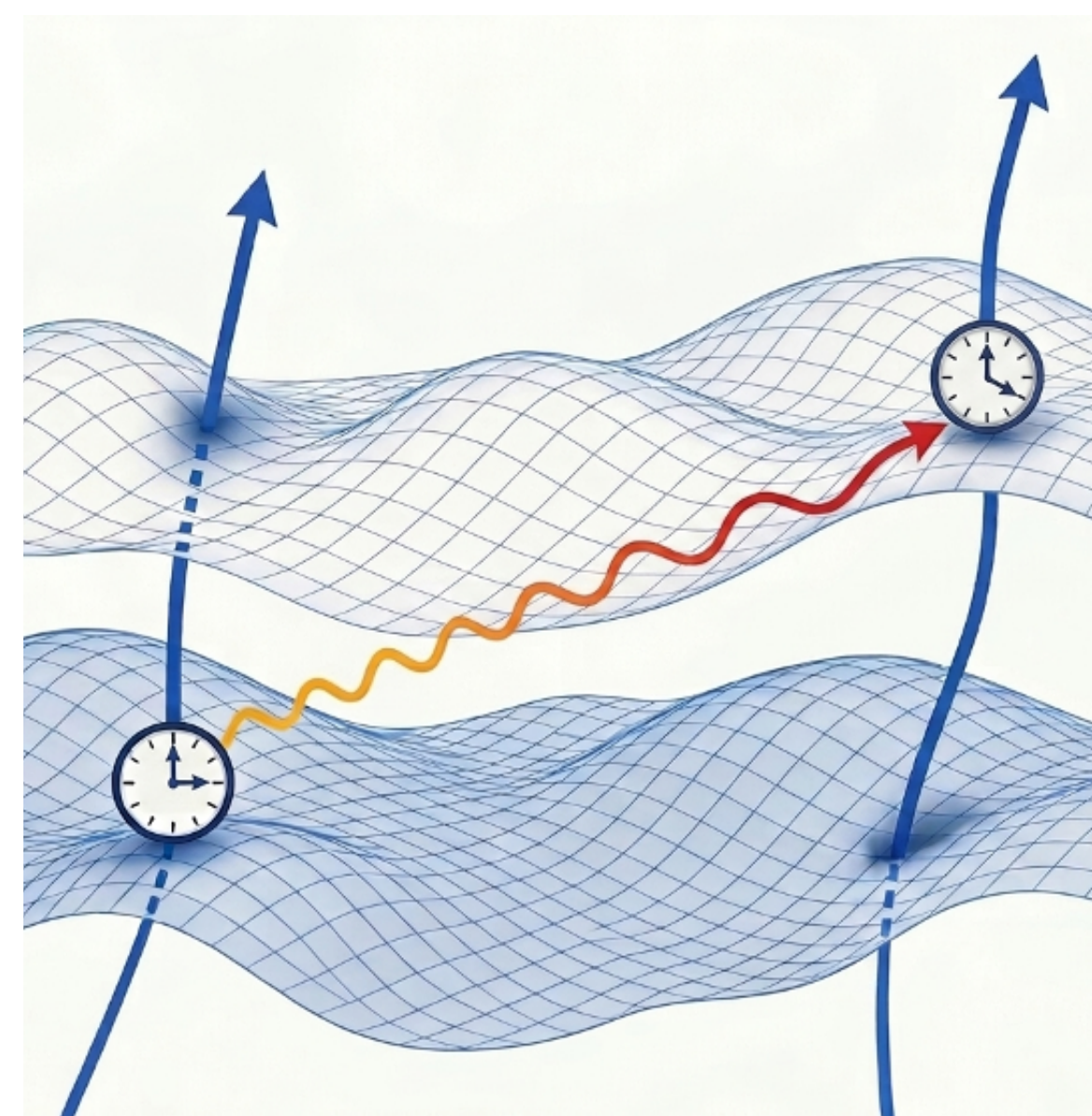
- 换个思路：激光干涉仪/脉冲星测时阵列看到的是什么物理量？
- 答案：沿测地线运动的钟的固有时的变化
- 问题转换为计算自由下落的探测器的固有时

$$\tau^{(1)} = a(\tau) \int_0^L d\lambda \left(\frac{1}{2} n^i n^j h_{ij}^{(1)} - \Phi^{(1)} - \Psi^{(1)} \right) + \dots$$

引力波带来的时间膨胀

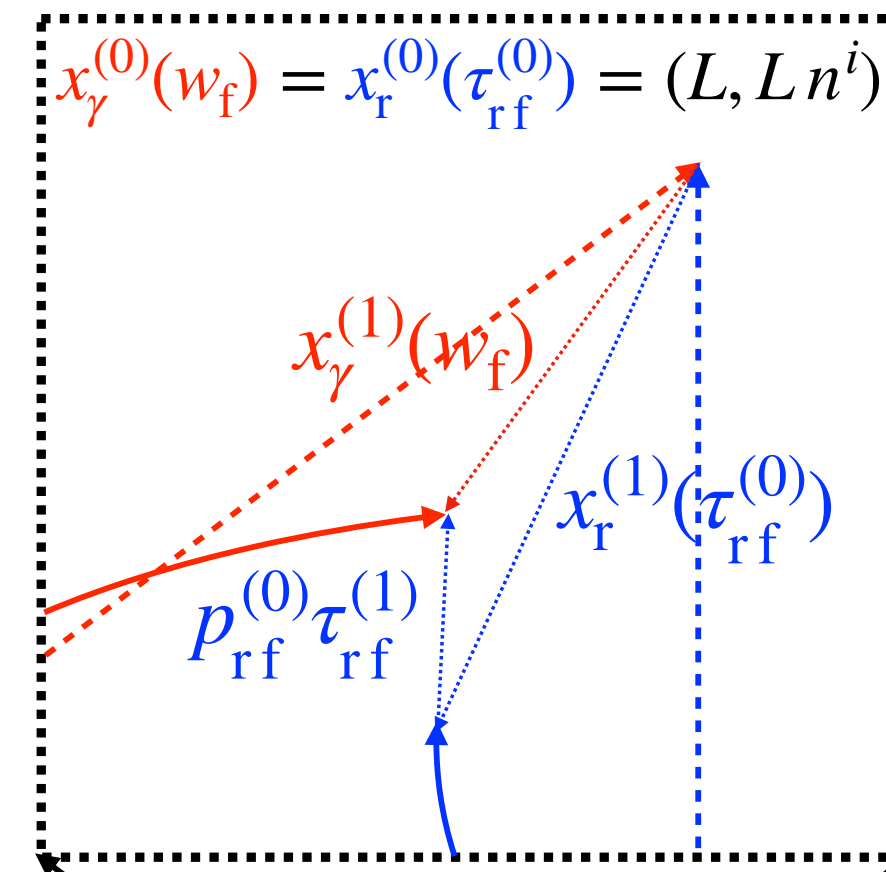
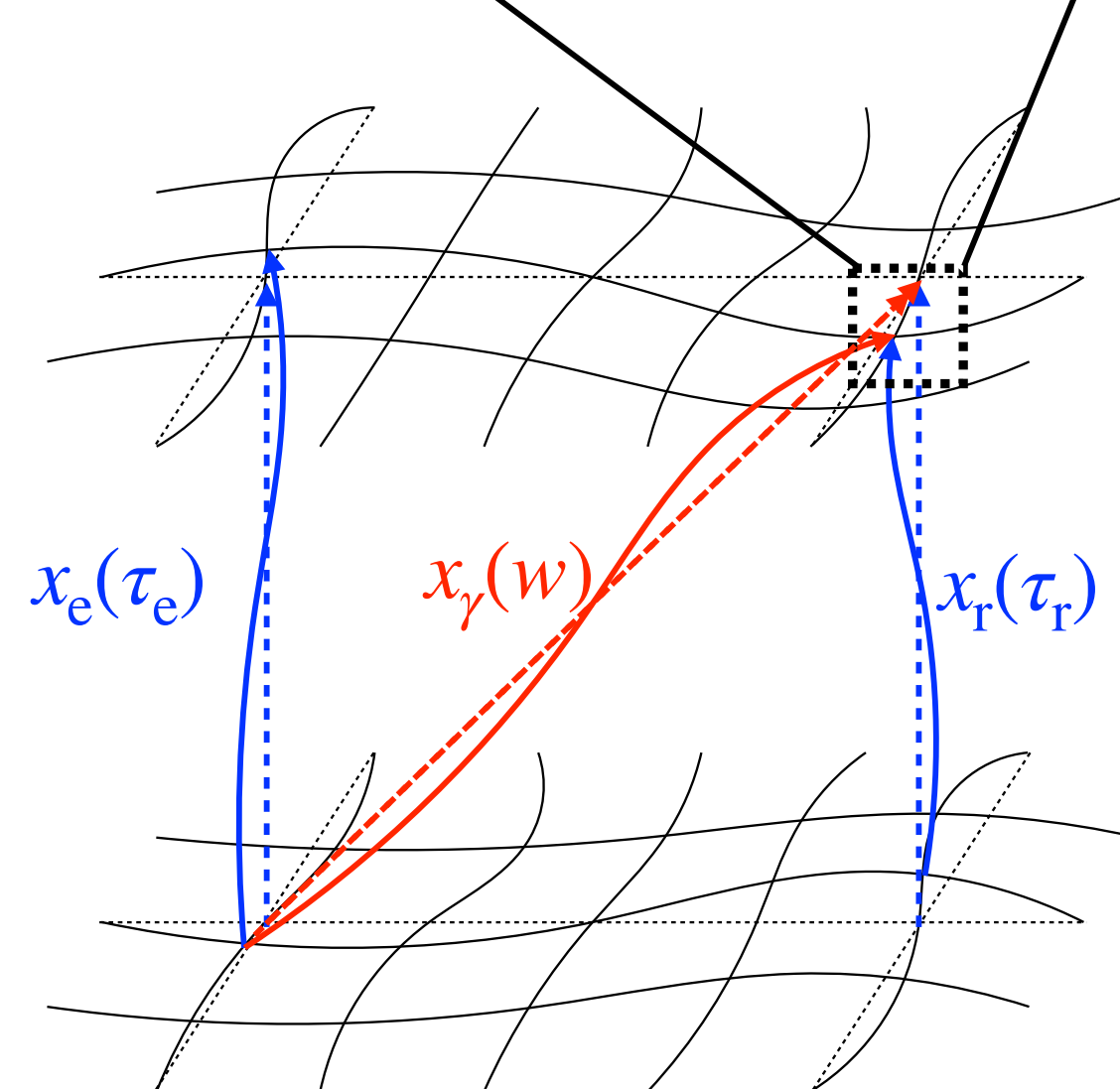
Shapiro delay

探测器运动相关的项



测地钟
(Fermi正规坐标)

测地钟
(Fermi正规坐标)



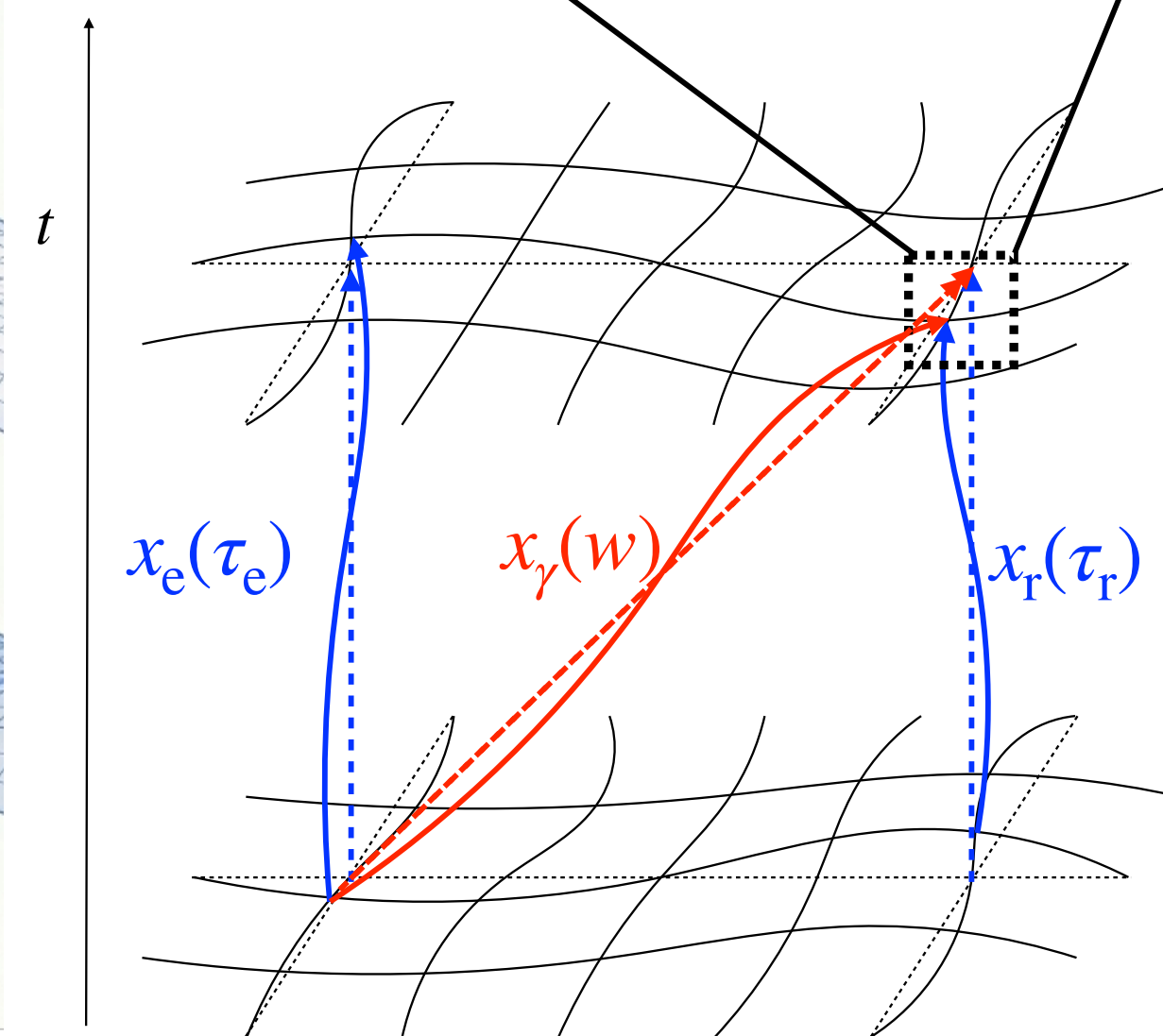
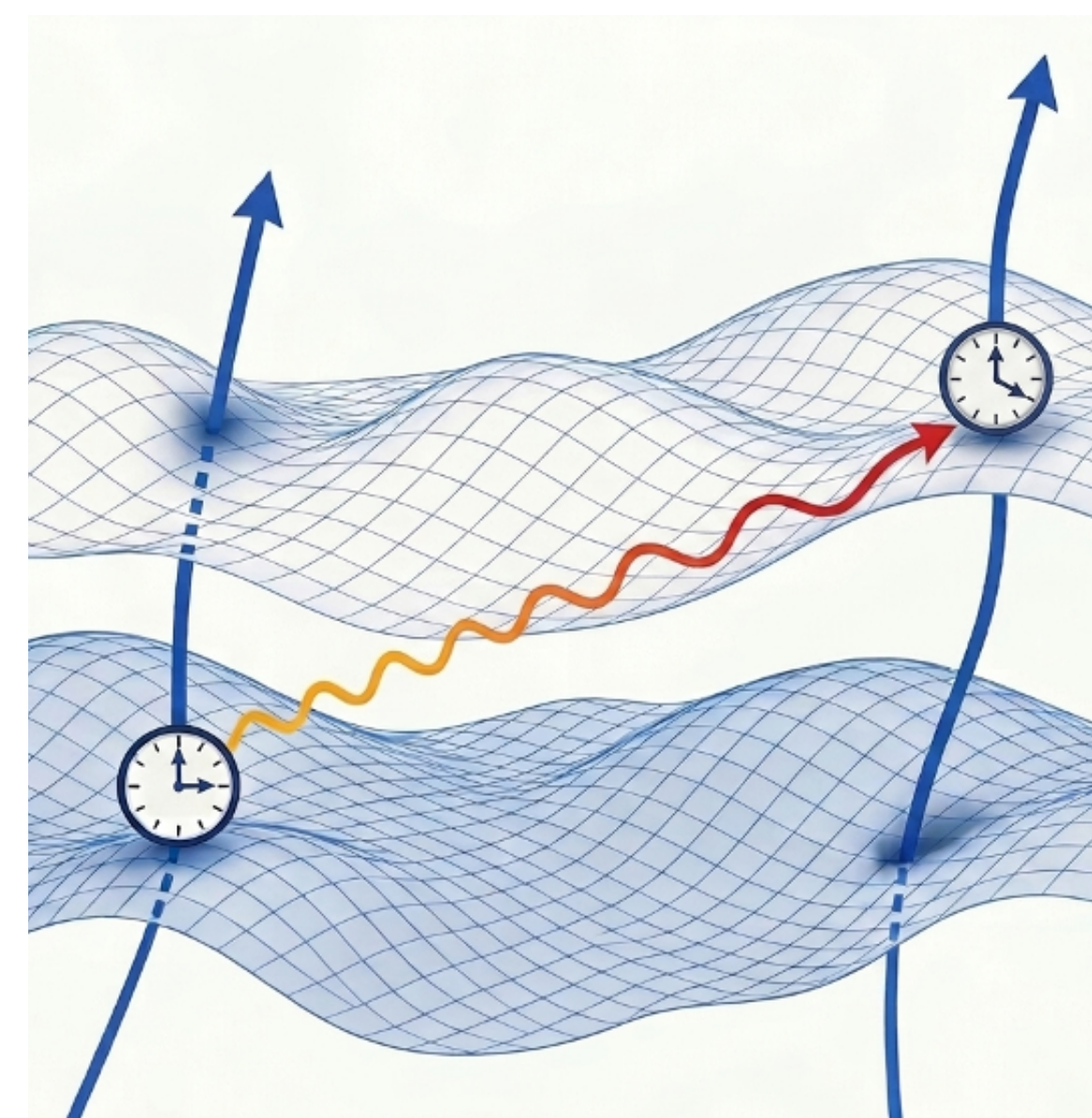
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$$\tau^{(2)} \supset a_{\text{rf}} n^i n^j \left(\frac{1}{2} \int_0^L d\lambda h_{ij}^{N(2)} + V_{\text{rf},i}^{(1)} \int_0^L d\eta V_{r,j} + \dots \right)$$

$$h_{ij}^{N(2)} \equiv h_{ij}^{(2)} + \Lambda_{ij}{}^{lk} \left(\sigma_l^{(1)} \sigma_k^{(1)} + E_{,lm}^{(1)} E_{,mk}^{(1)} \right)$$



EDITORS' SUGGESTION

Observable Gravitational Wave Strain at Second Order

3 JUNE, 2026

The ambiguity in associating gravitational waves with transverse-traceless components of the metric at second order in perturbation theory is resolved by computing the detector response to second order for the first time.

Guillem Domènech, Shi Pi (皮石), and Ao Wang (王奥)

[Phys. Rev. Lett. 136, 221402 \(2026\)](#)

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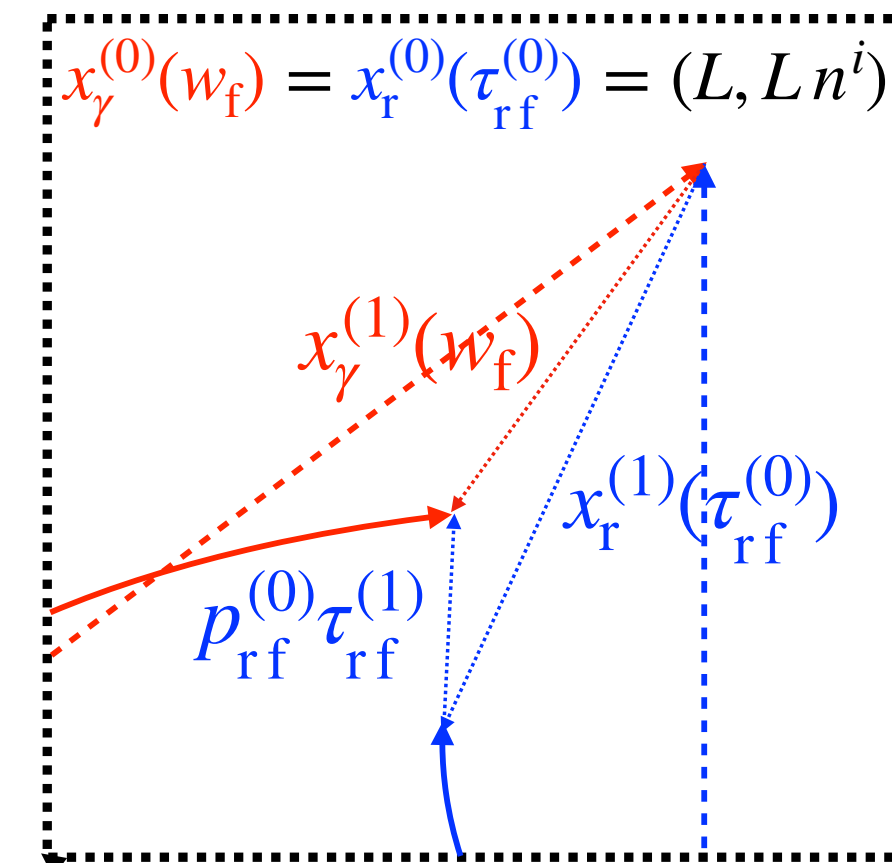
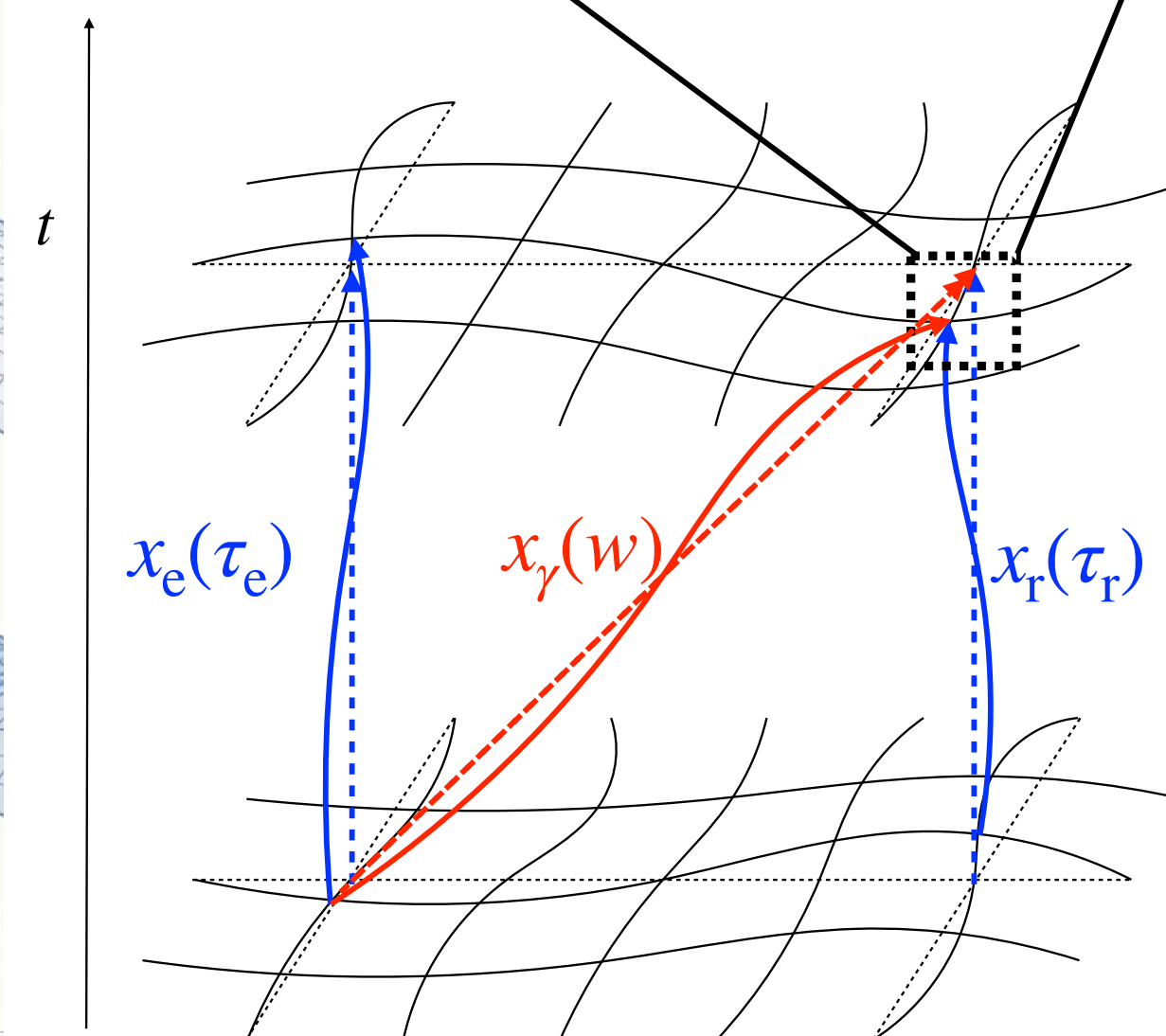
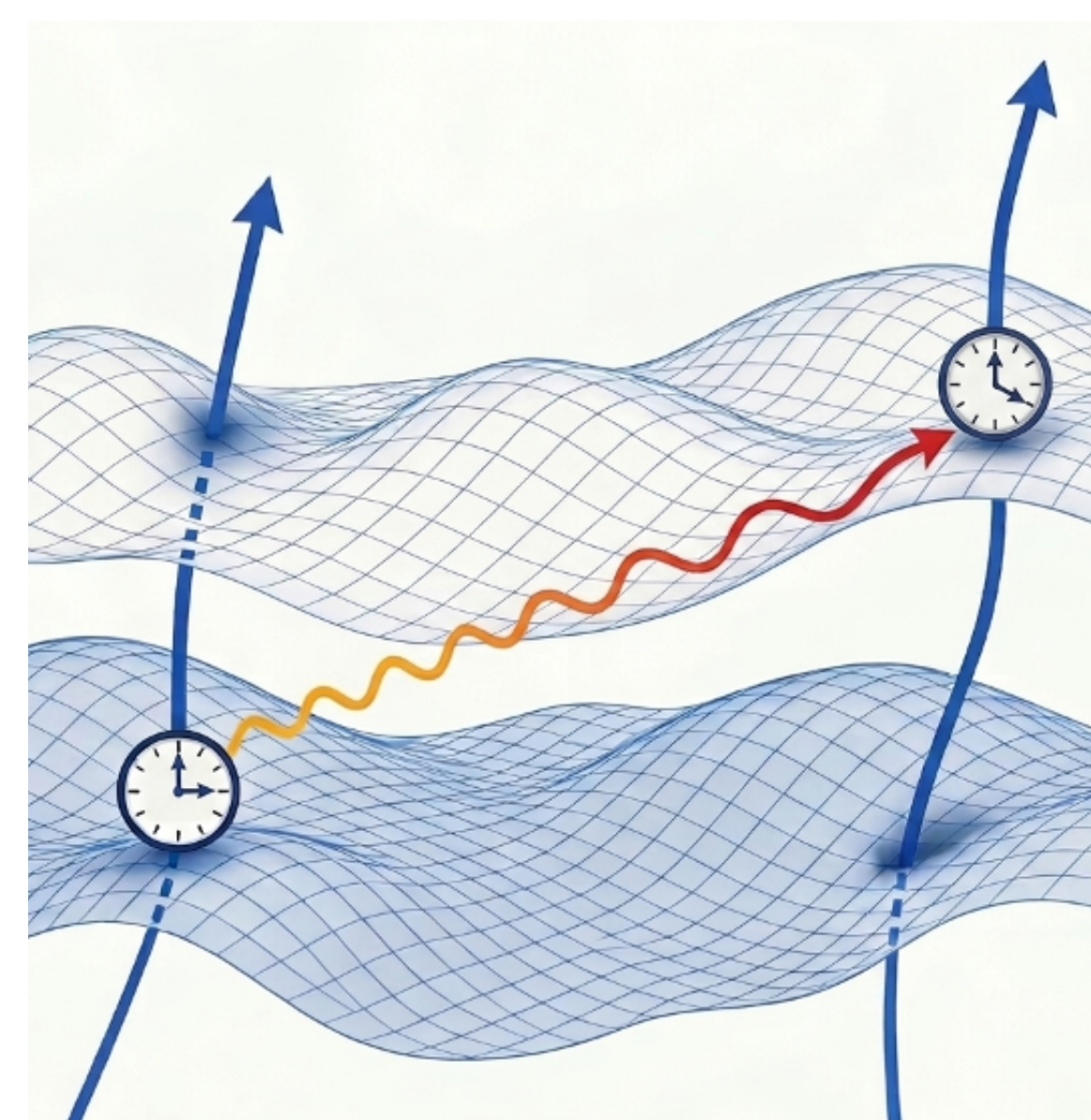
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- PRL编辑推荐语：第一次通过计算探测器响应解决了微扰论中二阶横向无迹分量对应的引力波的不确定性



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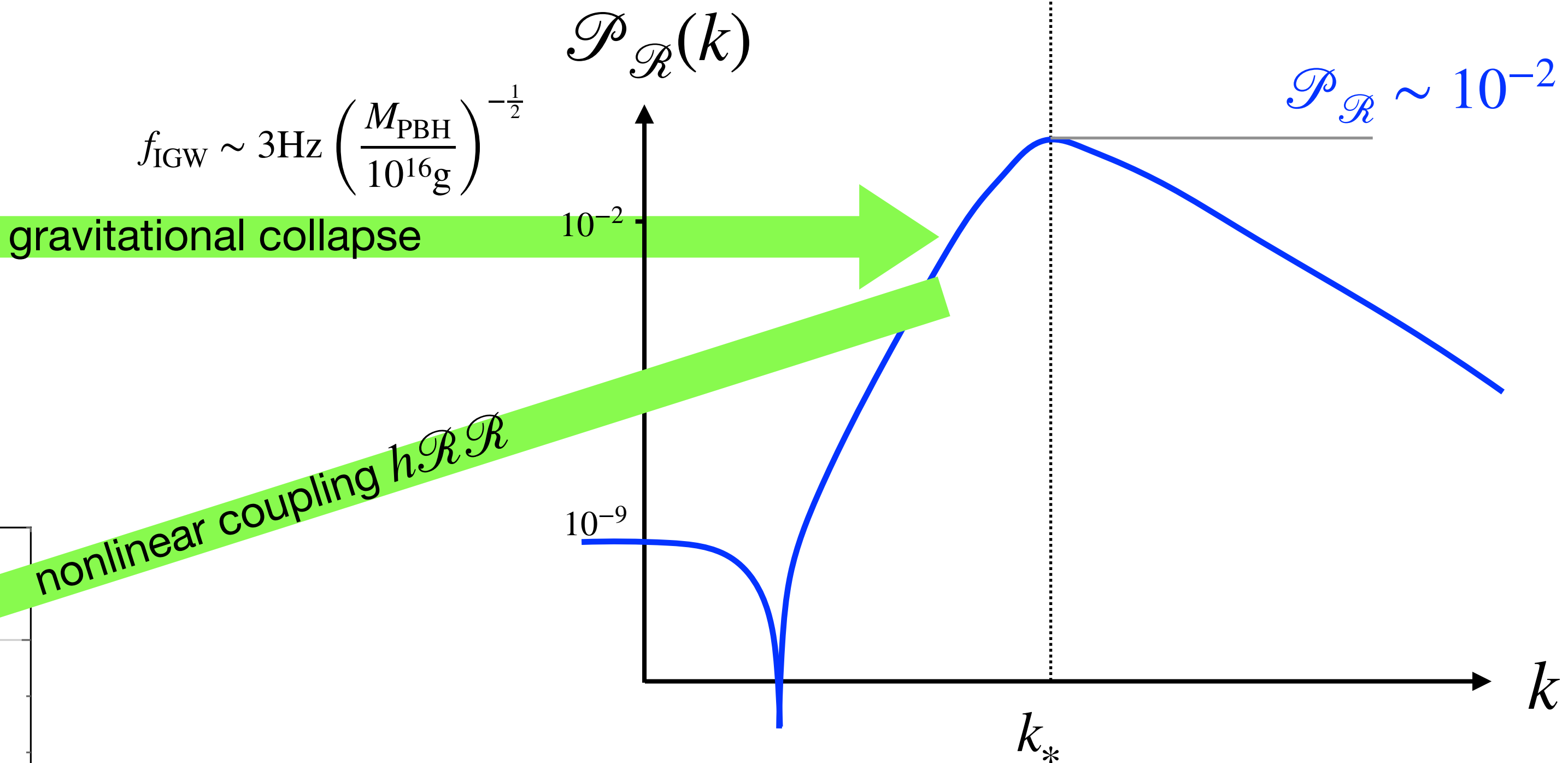
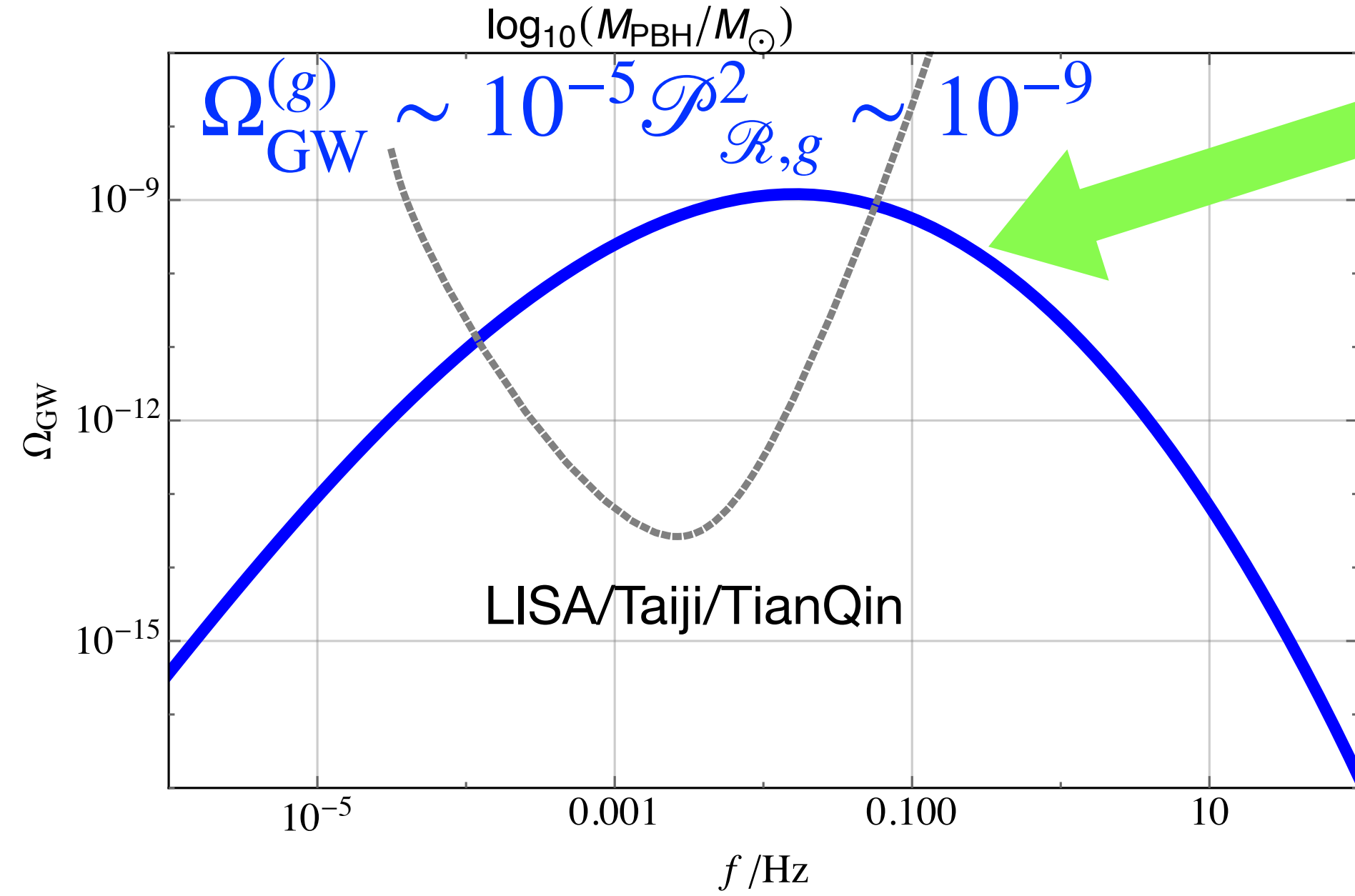
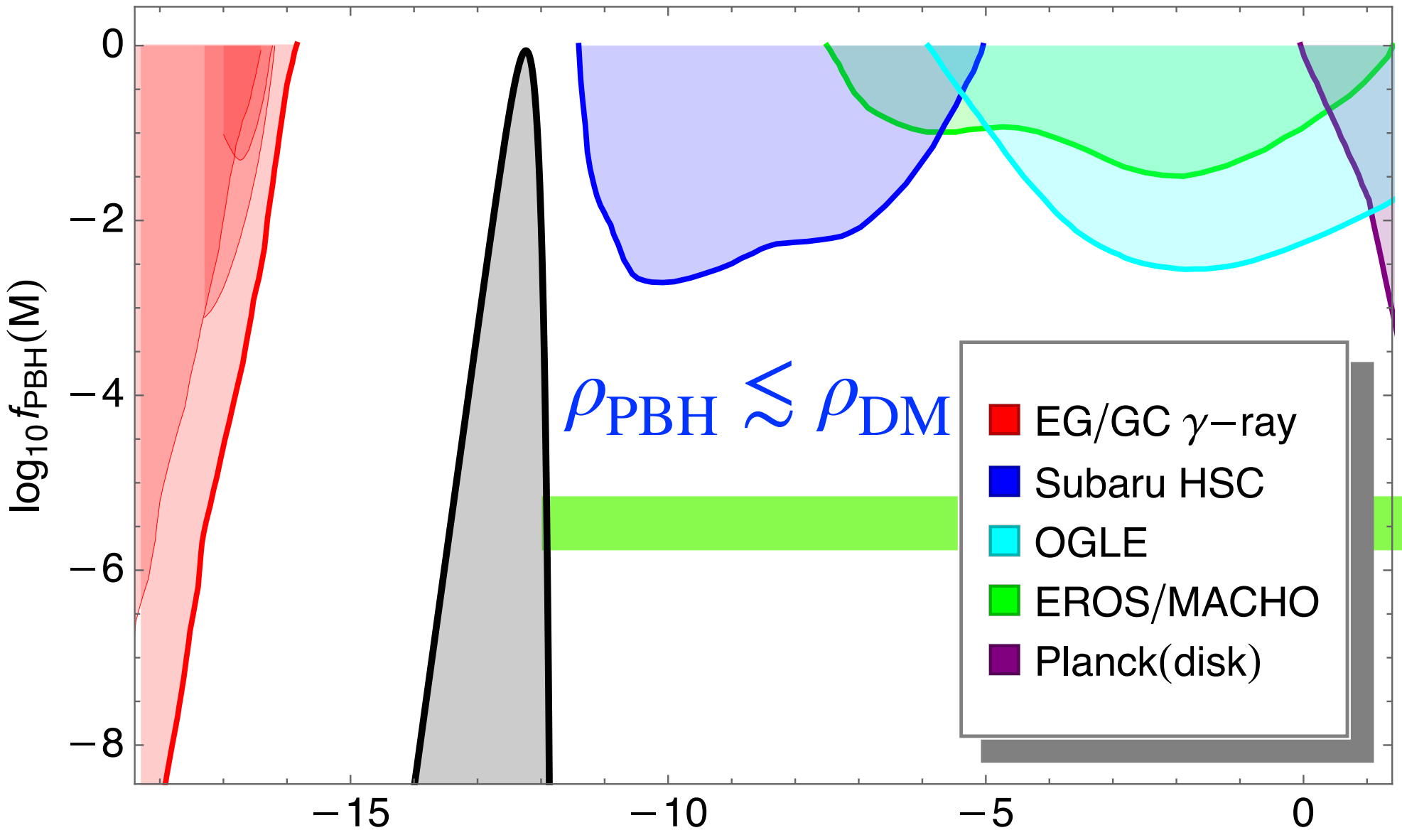
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Phys. Rev. Lett. **136**, 221402 (2026)

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PBH-IGW crosscheck



Cai, SP, Sasaki, PRL 122, 201101

Gravitational Waves Induced by non-Gaussian Scalar Perturbations #1
Rong-gen Cai (CAS, Beijing and Beijing, GUCAS), Shi Pi (Tokyo U., IPMU), Misao Sasaki (Tokyo U., IPMU and CAS, Beijing and Kyoto U., Yukawa Inst., Kyoto and Taiwan, Natl. Taiwan U.) (Oct 25, 2018)
Published in: *Phys.Rev.Lett.* 122 (2019) 20, 201101 • e-Print: [1810.11000](https://arxiv.org/abs/1810.11000) [astro-ph.CO]
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [508 citations](#)

PBH-IGW crosscheck

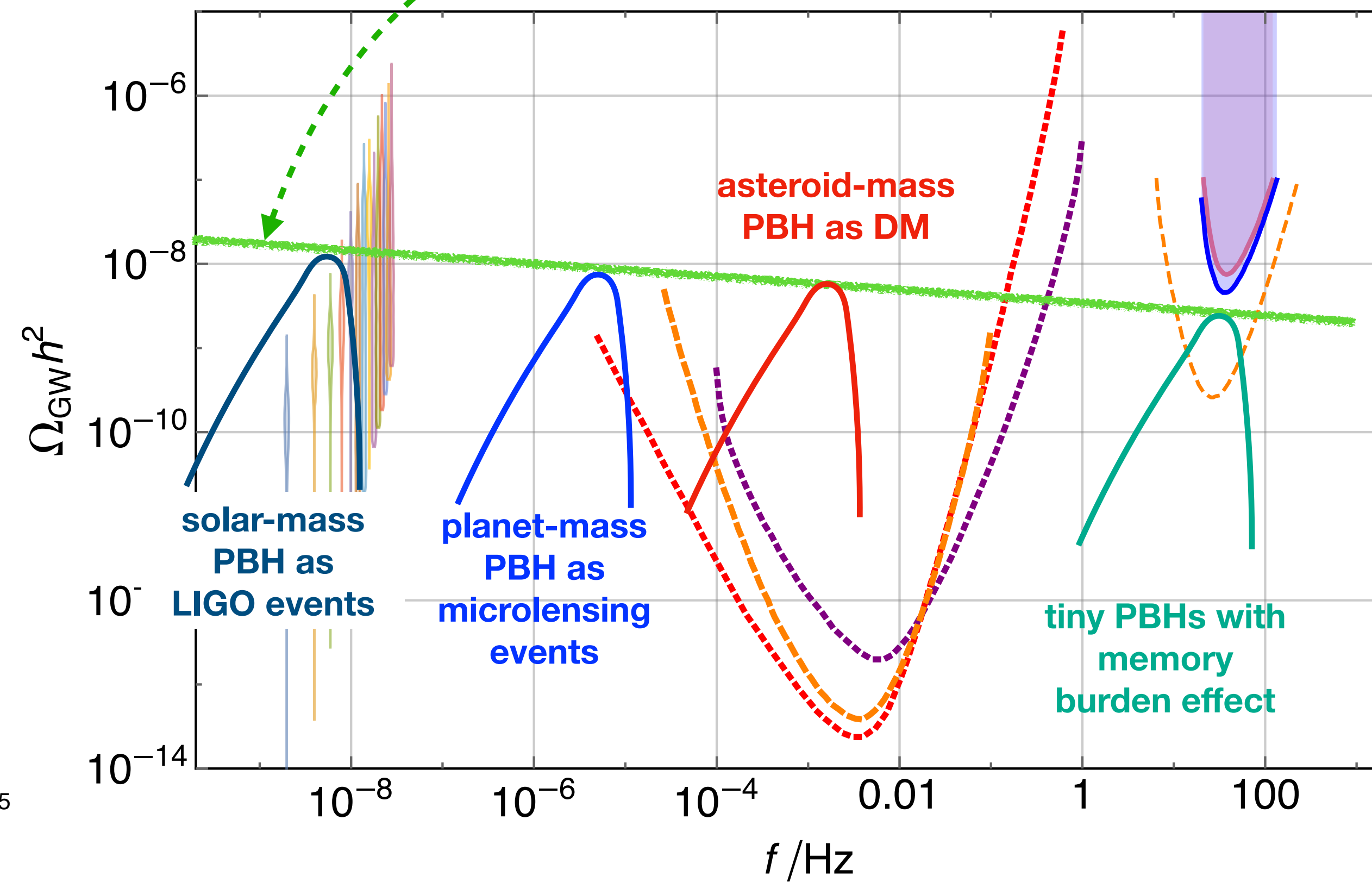
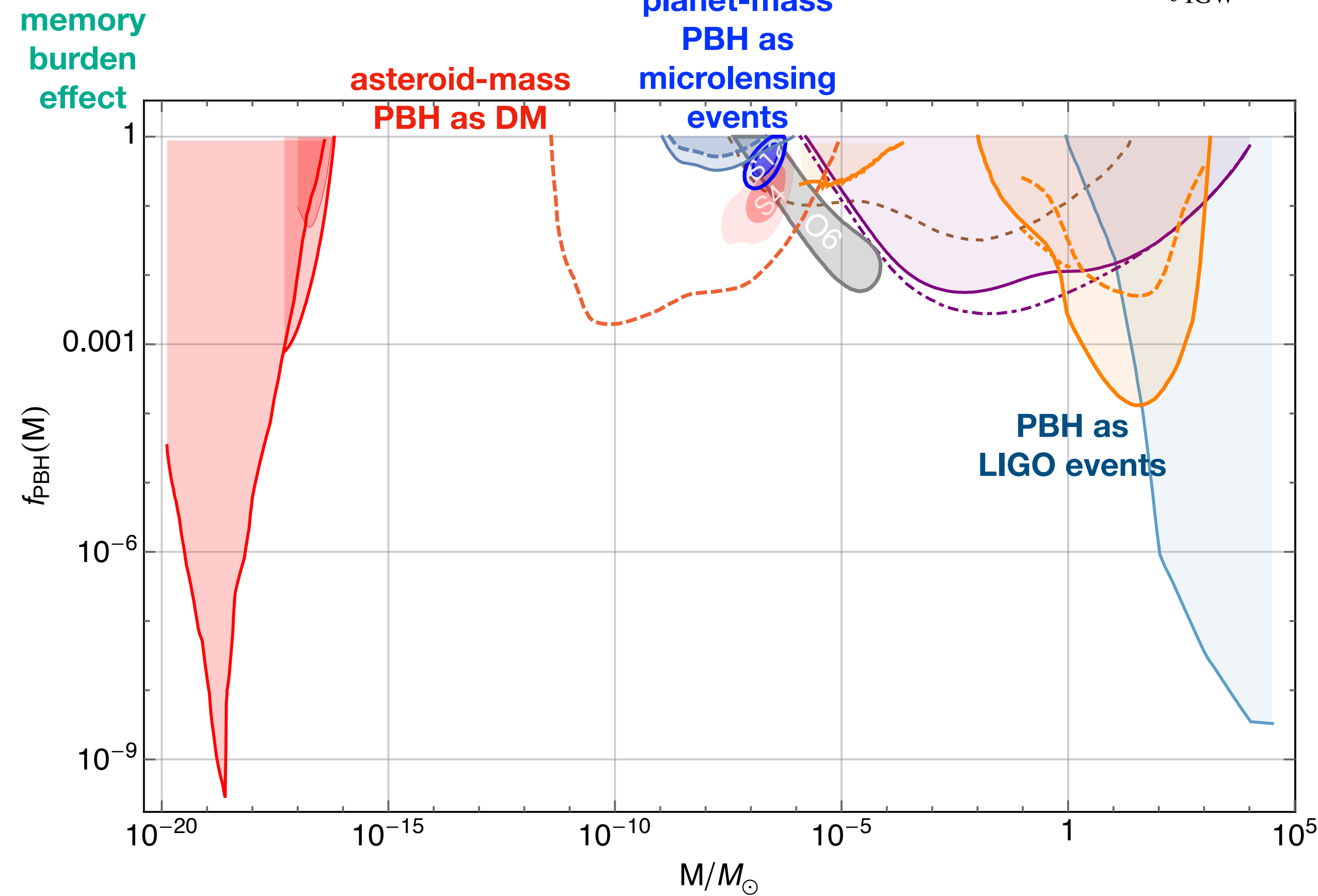
PBH constraints

crosscheck

IGW constraints

$$f_{\text{IGW}} \sim 3\text{Hz} \left(\frac{M_{\text{PBH}}}{10^{16}\text{g}} \right)^{-\frac{1}{2}}$$

Constraint $\text{PBH} \leq \text{DM}$



PBH-IGW crosscheck

PBH constraints

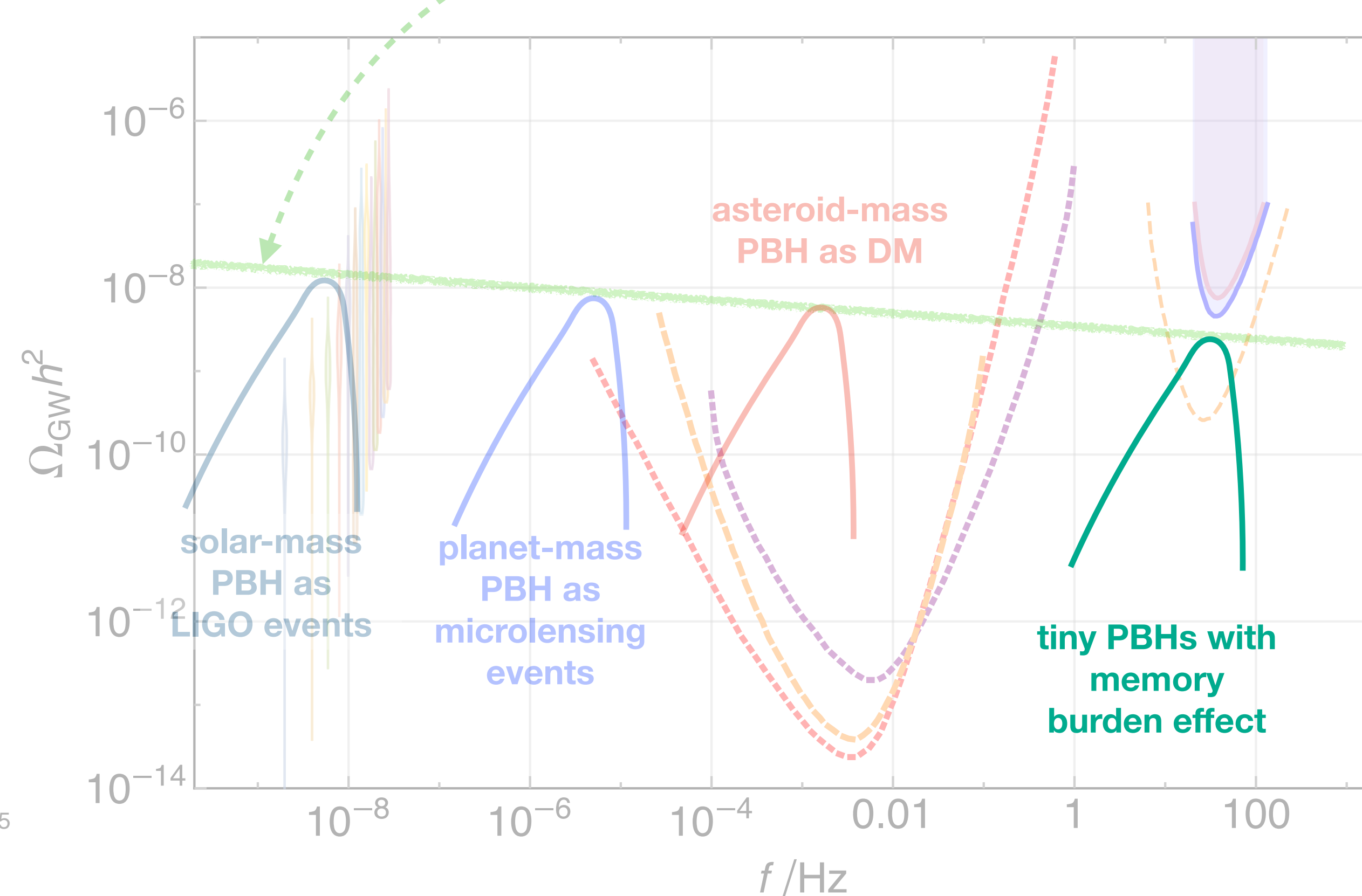
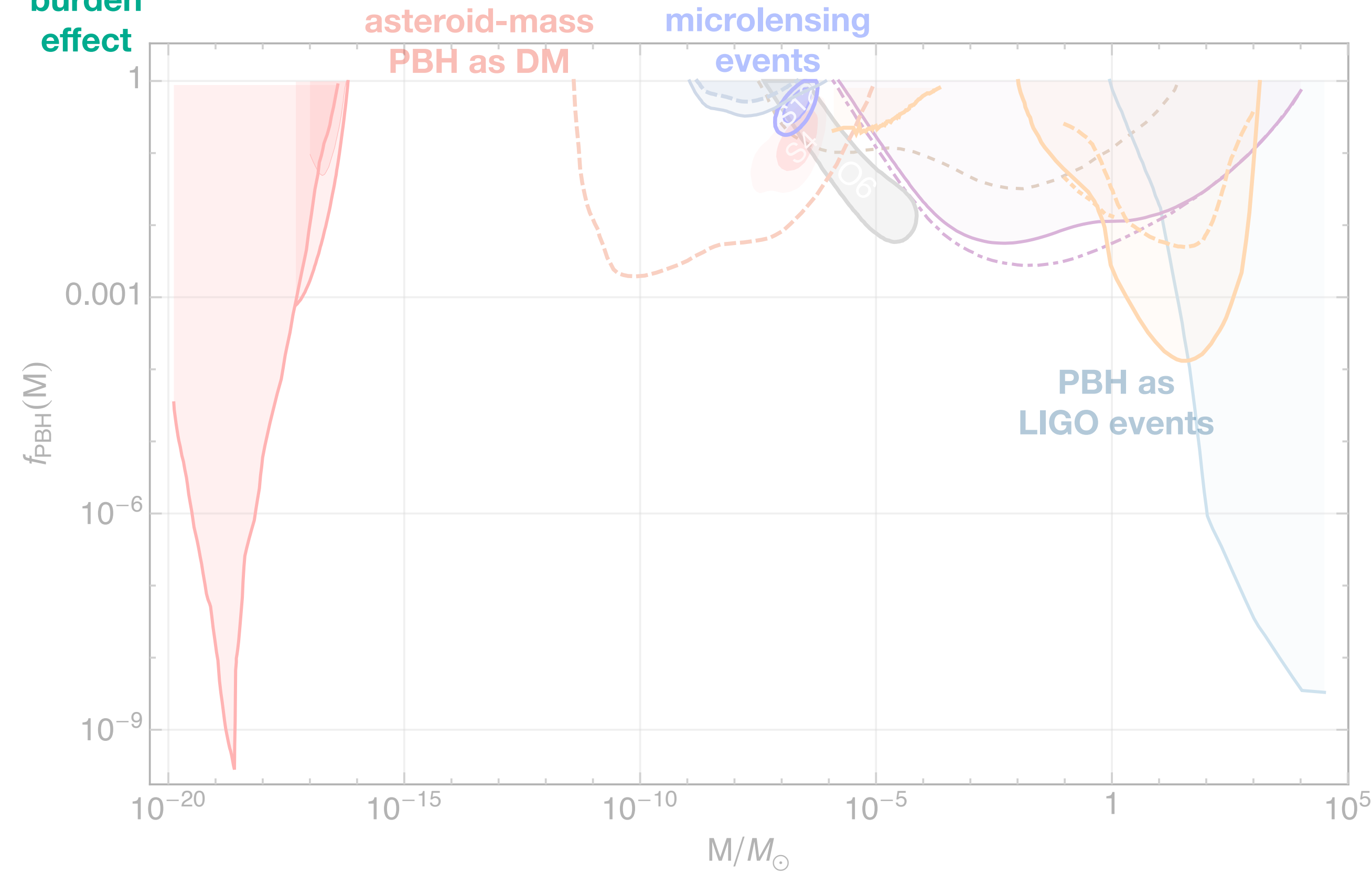
crosscheck

IGW constraints

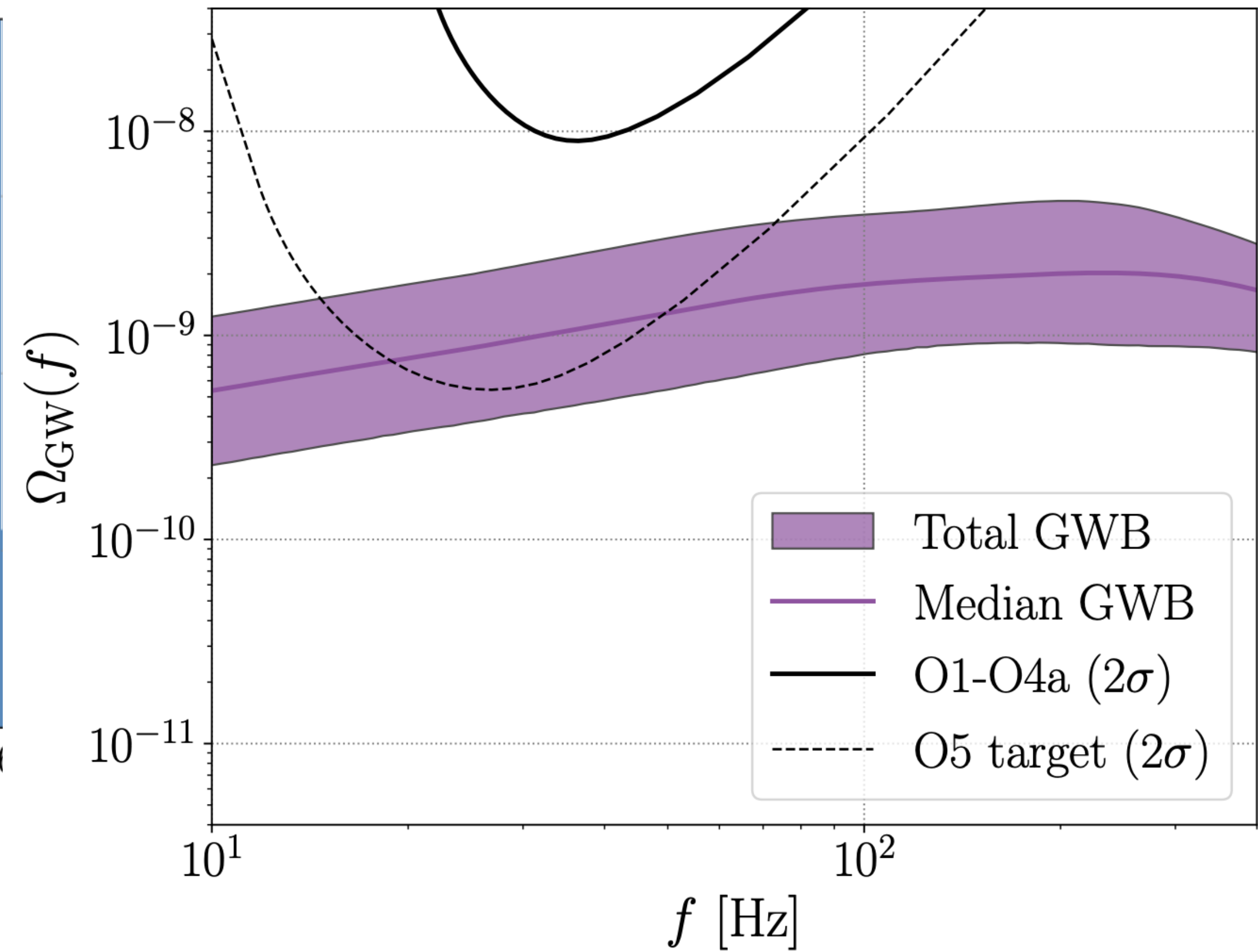
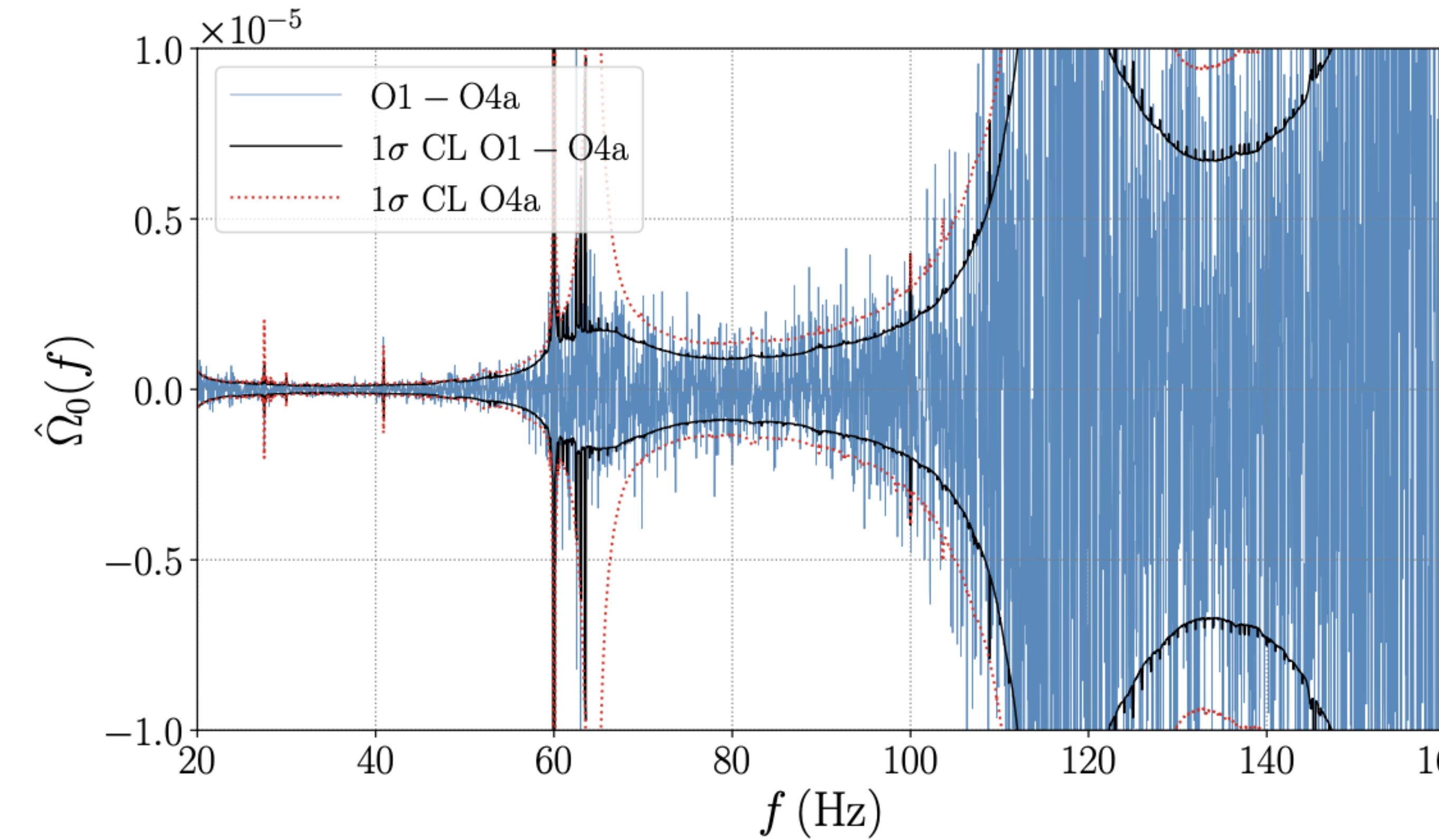
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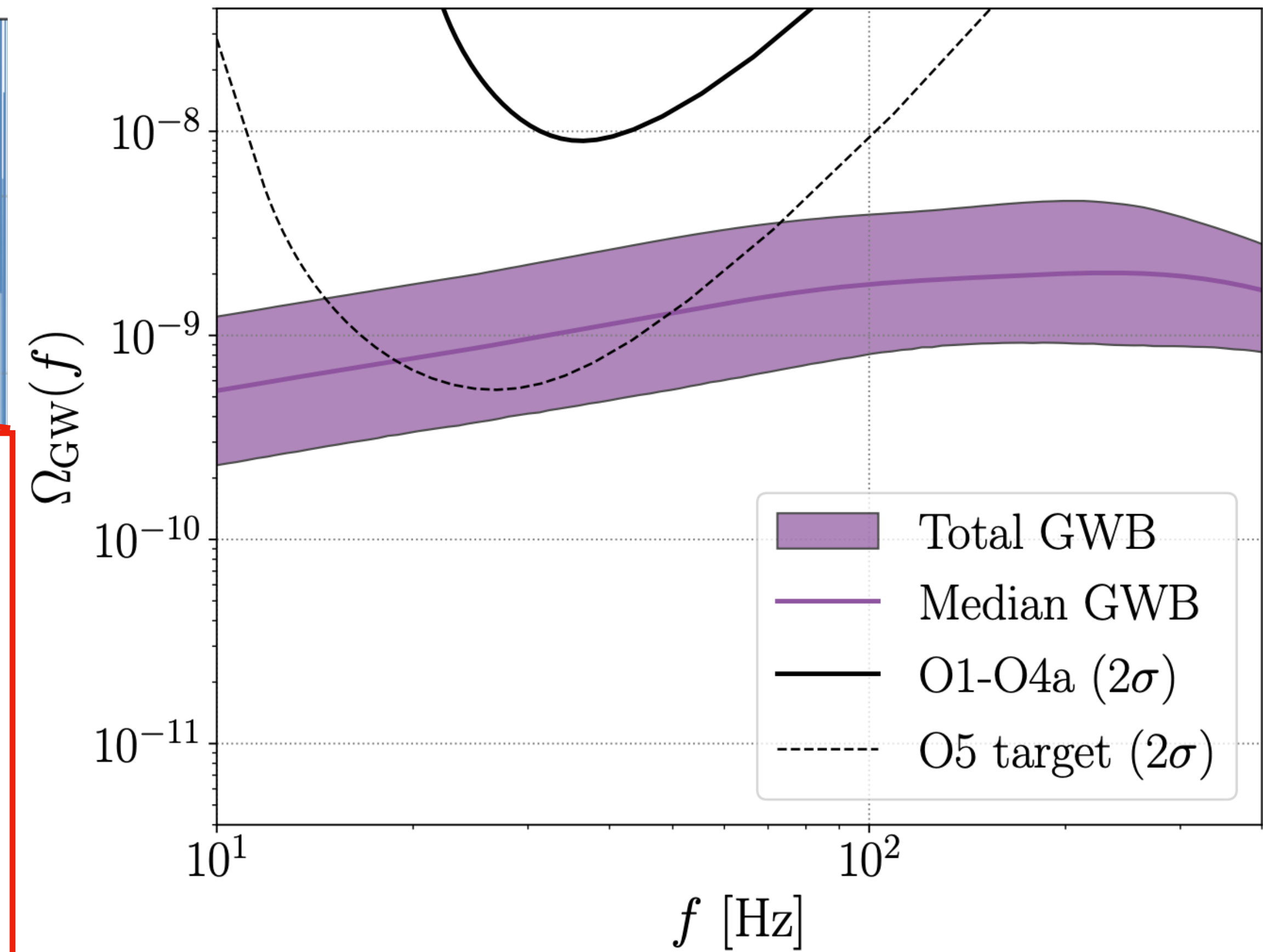
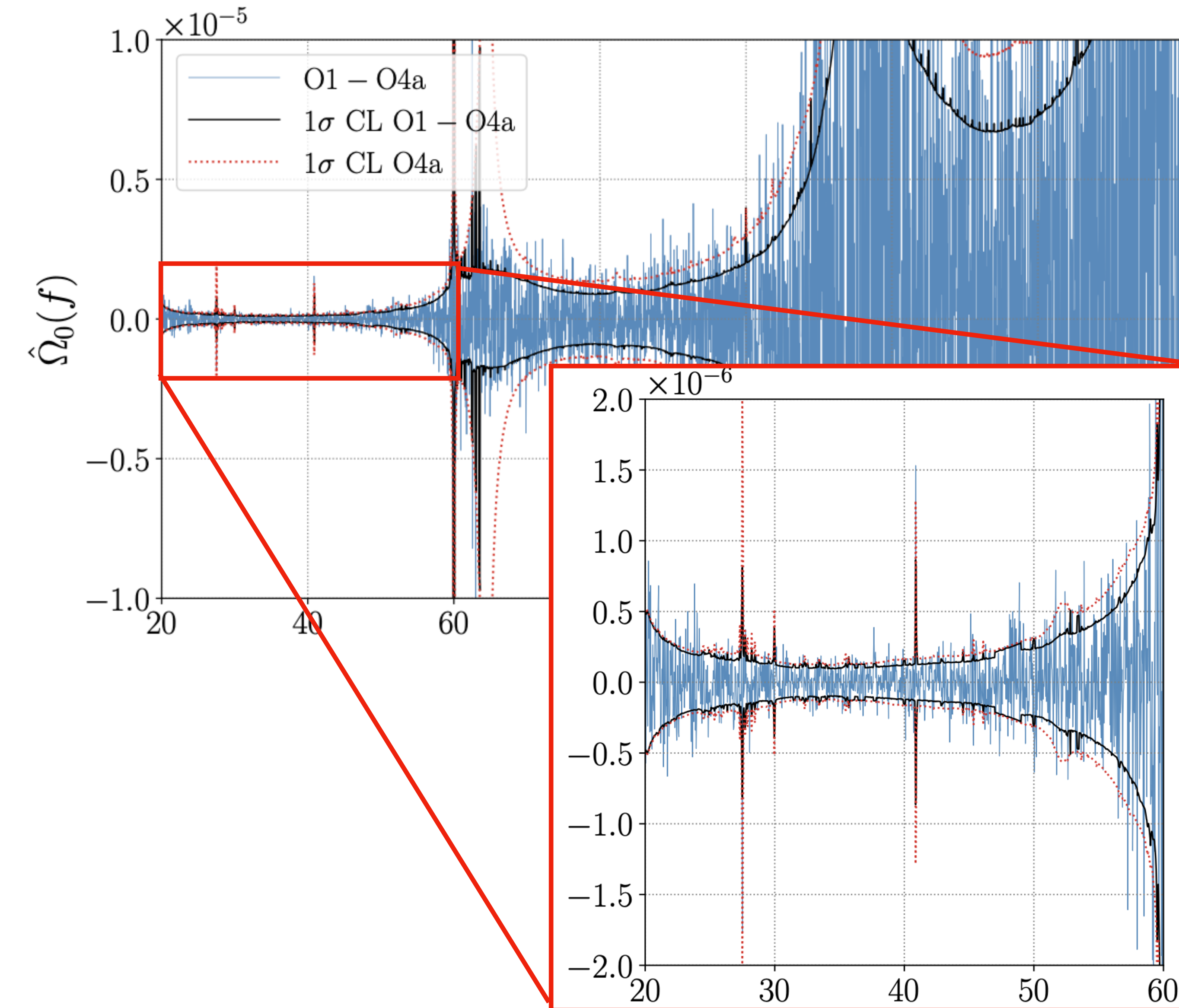
memory
burden
effect



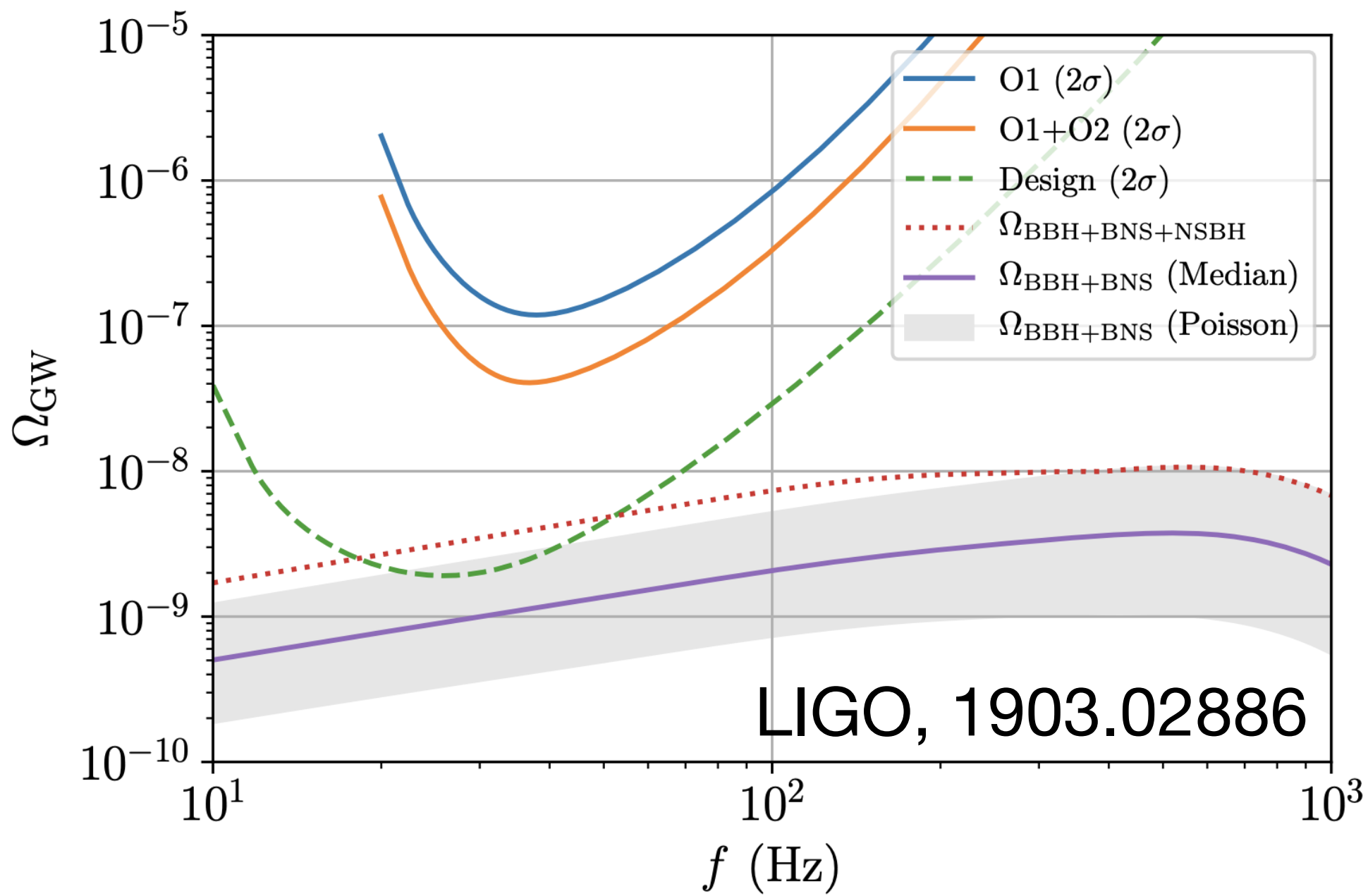
LIGO constraints



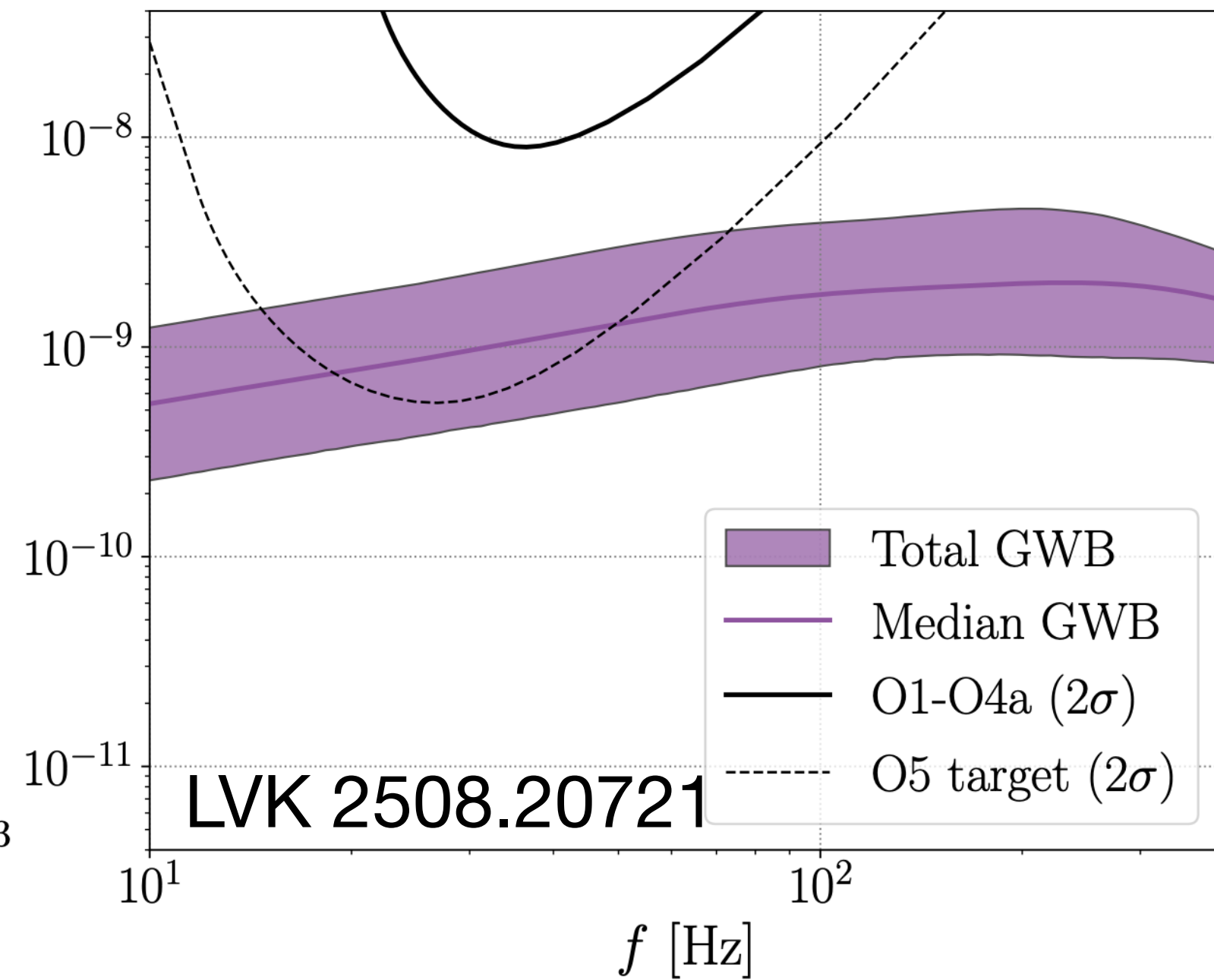
LIGO constraints



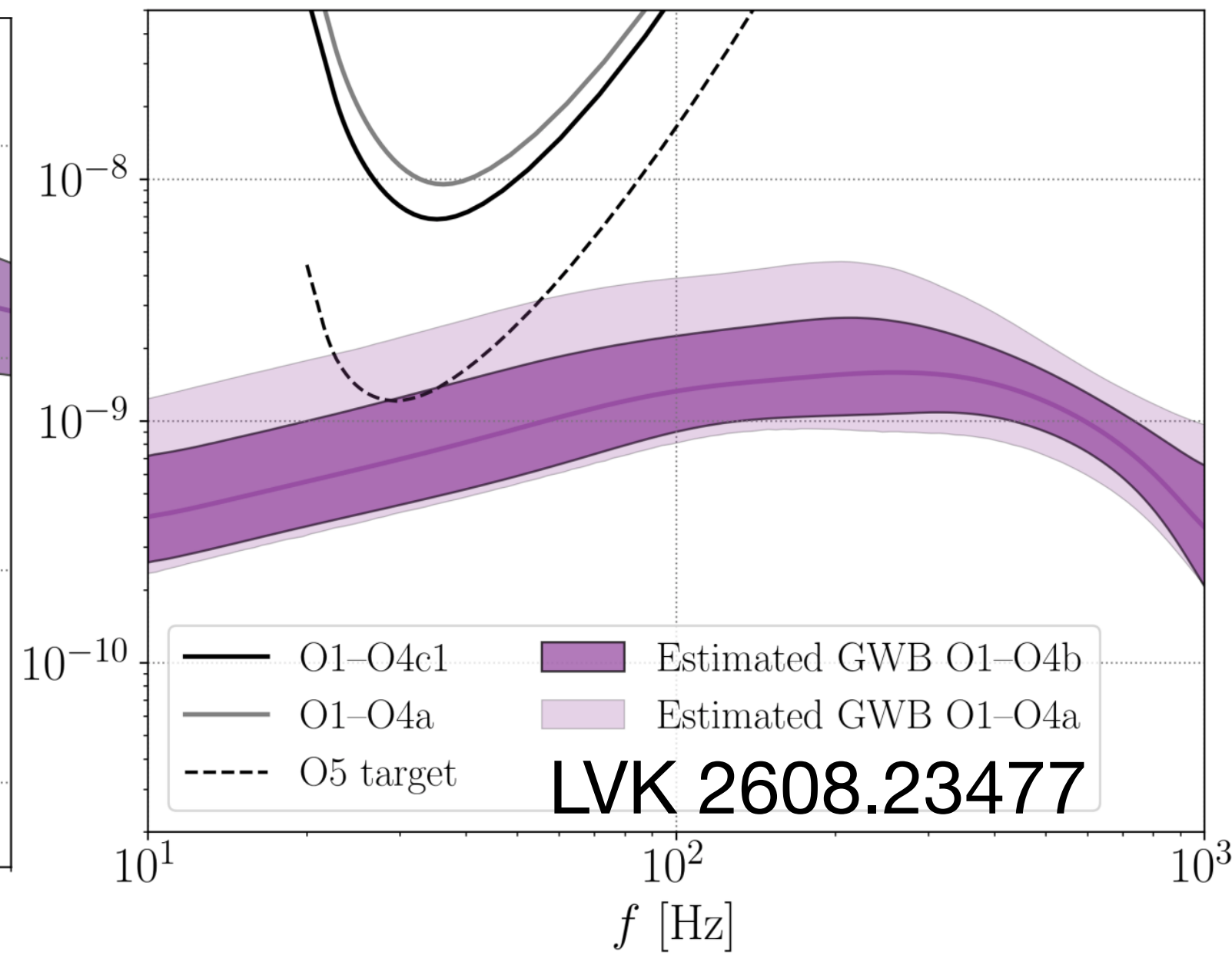
LIGO constraints



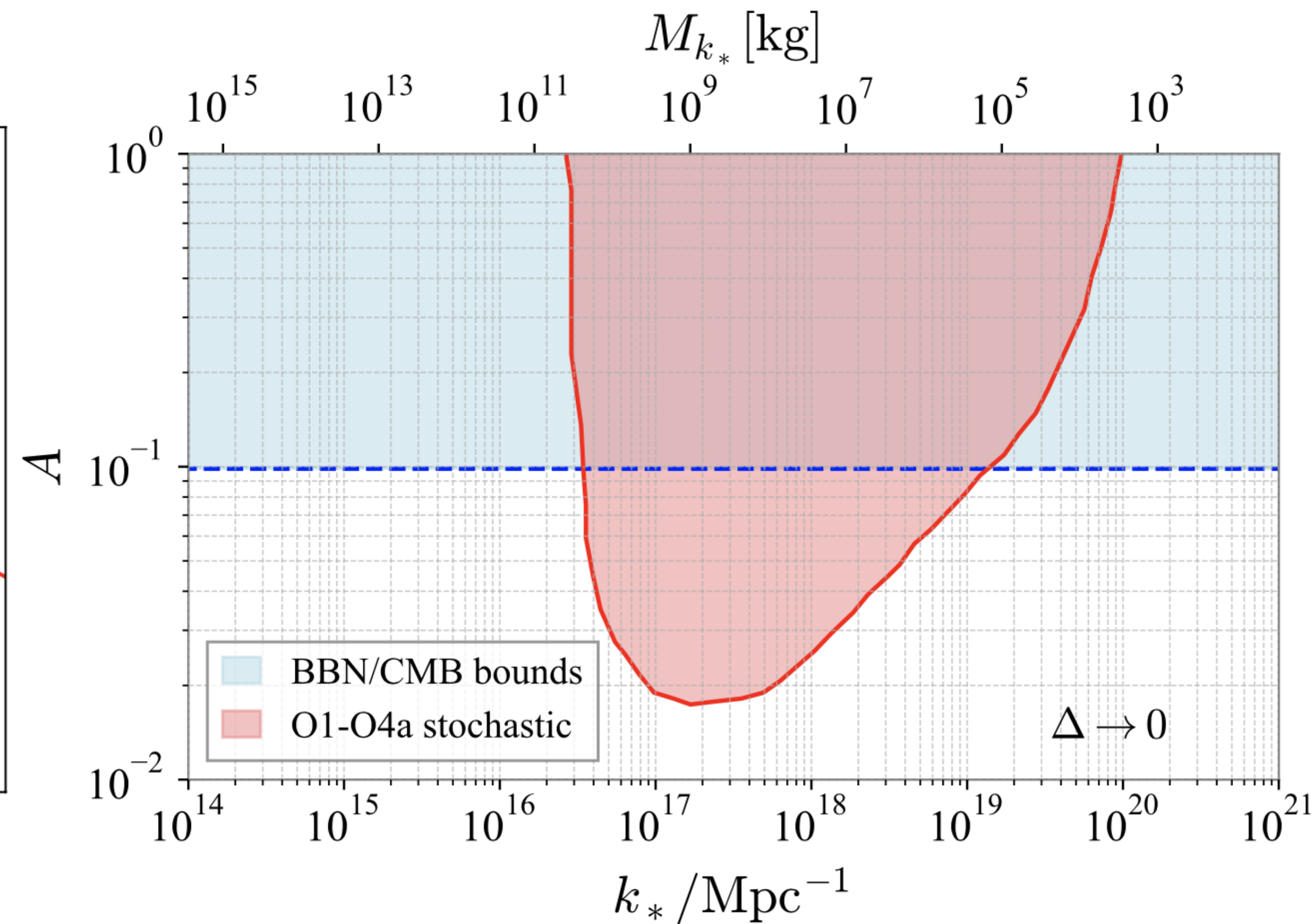
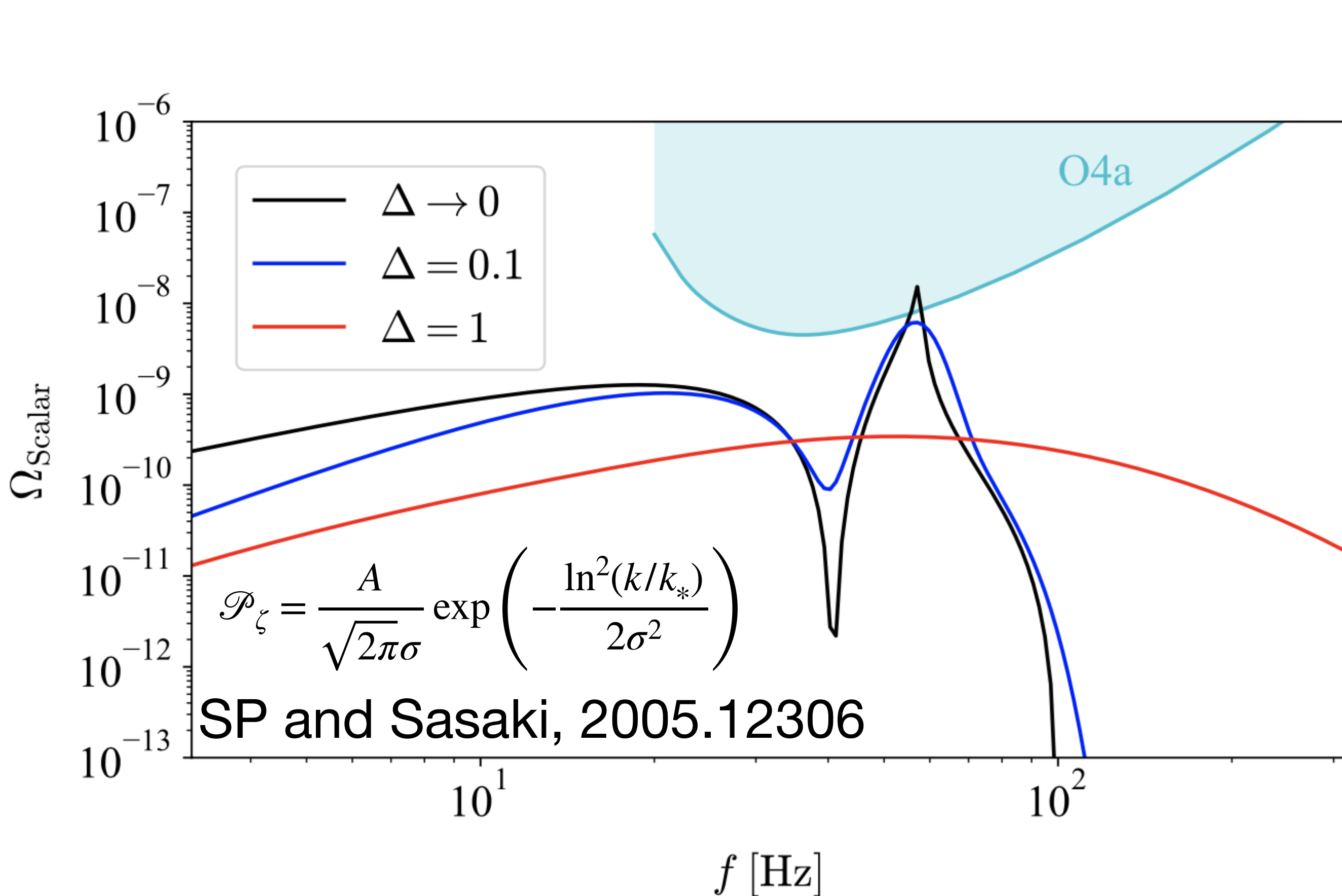
$$\Omega_{\text{GW}}(f) \leq 4.8 \times 10^{-8} @ 25\text{Hz}$$



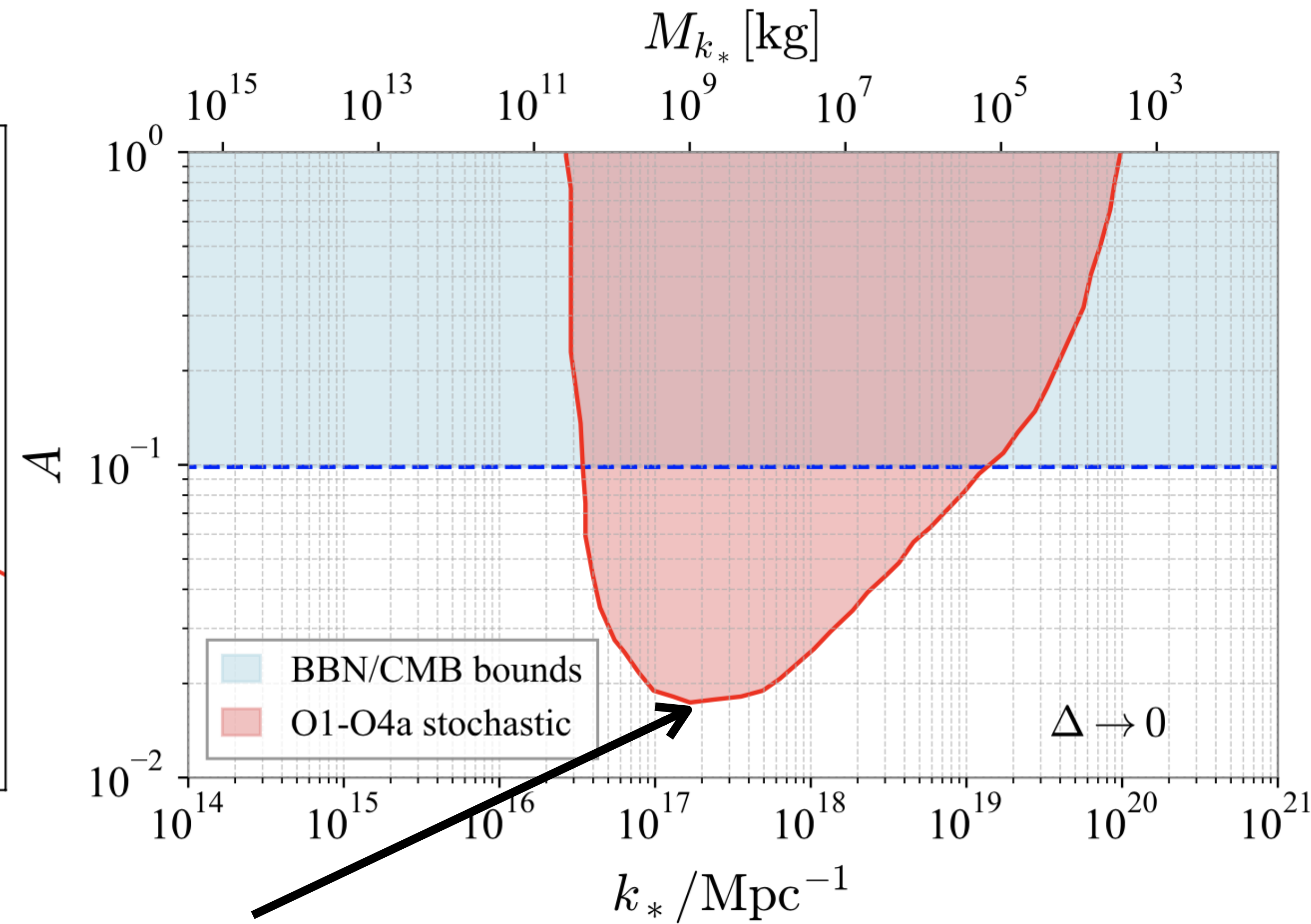
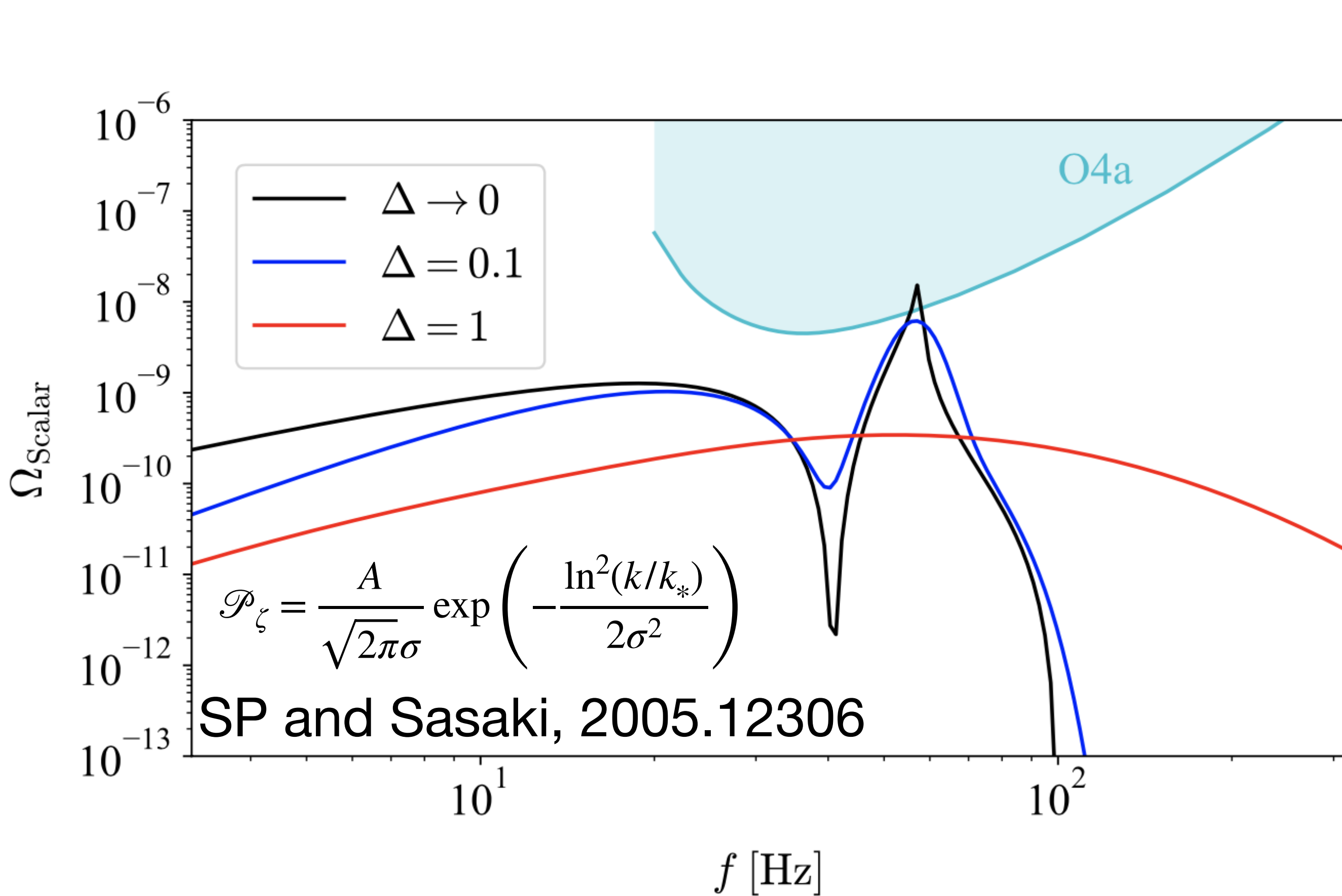
$$\Omega_{\text{GW}}(f) \leq 2.0 \times 10^{-9} @ 25\text{Hz}$$



LIGO constraints



LIGO constraints



$$f_{\text{PBH}} \sim 4 \times 10^8 \left(\frac{M}{M_\odot}\right)^{-1/2} \text{erfc} \frac{\zeta_c}{\sqrt{2A}} \sim 10^9$$

$$f_{\text{PBH}} < 1 \text{ implies } A \lesssim 9 \times 10^{-3}$$

PBH-IGW crosscheck

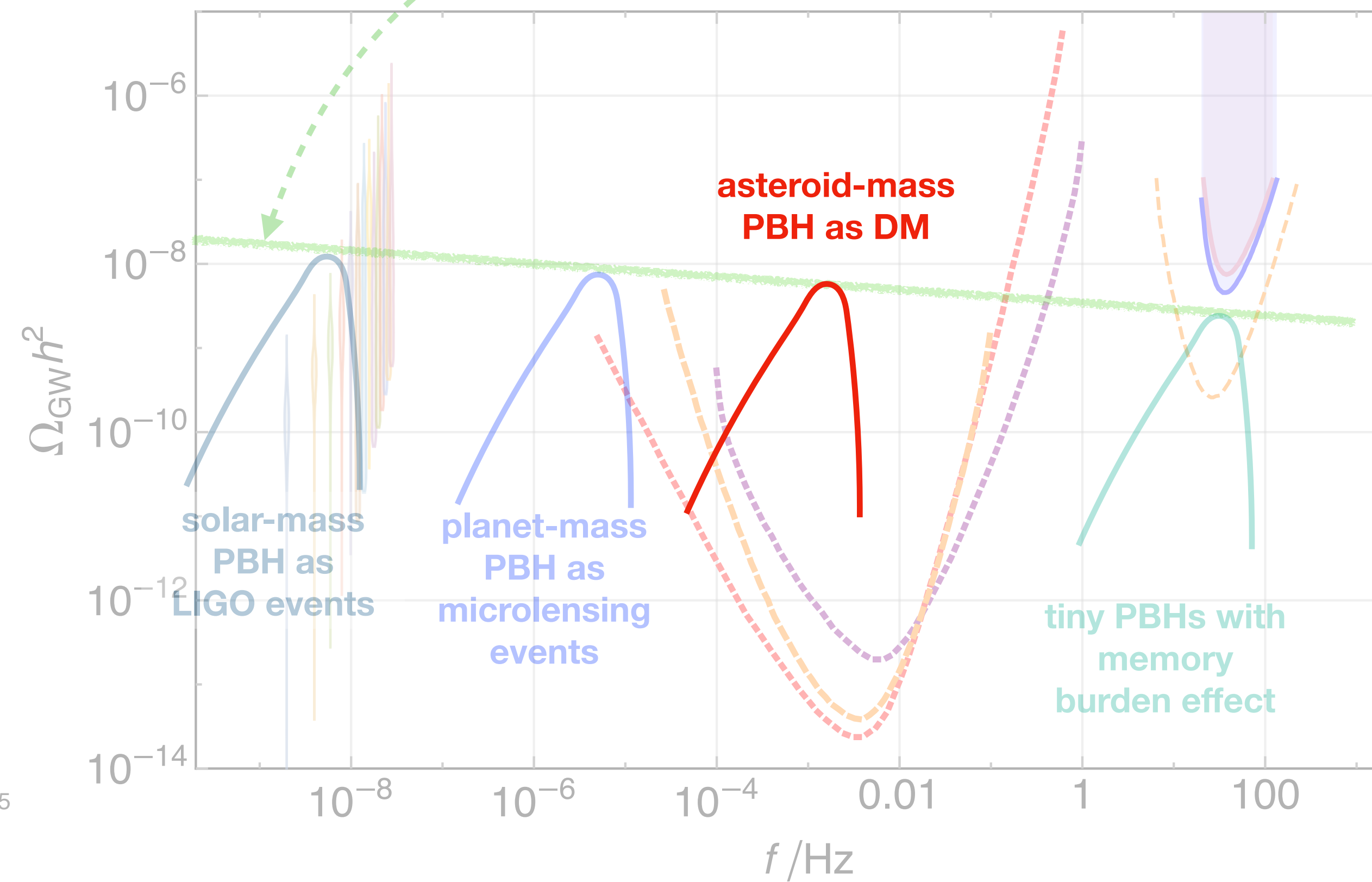
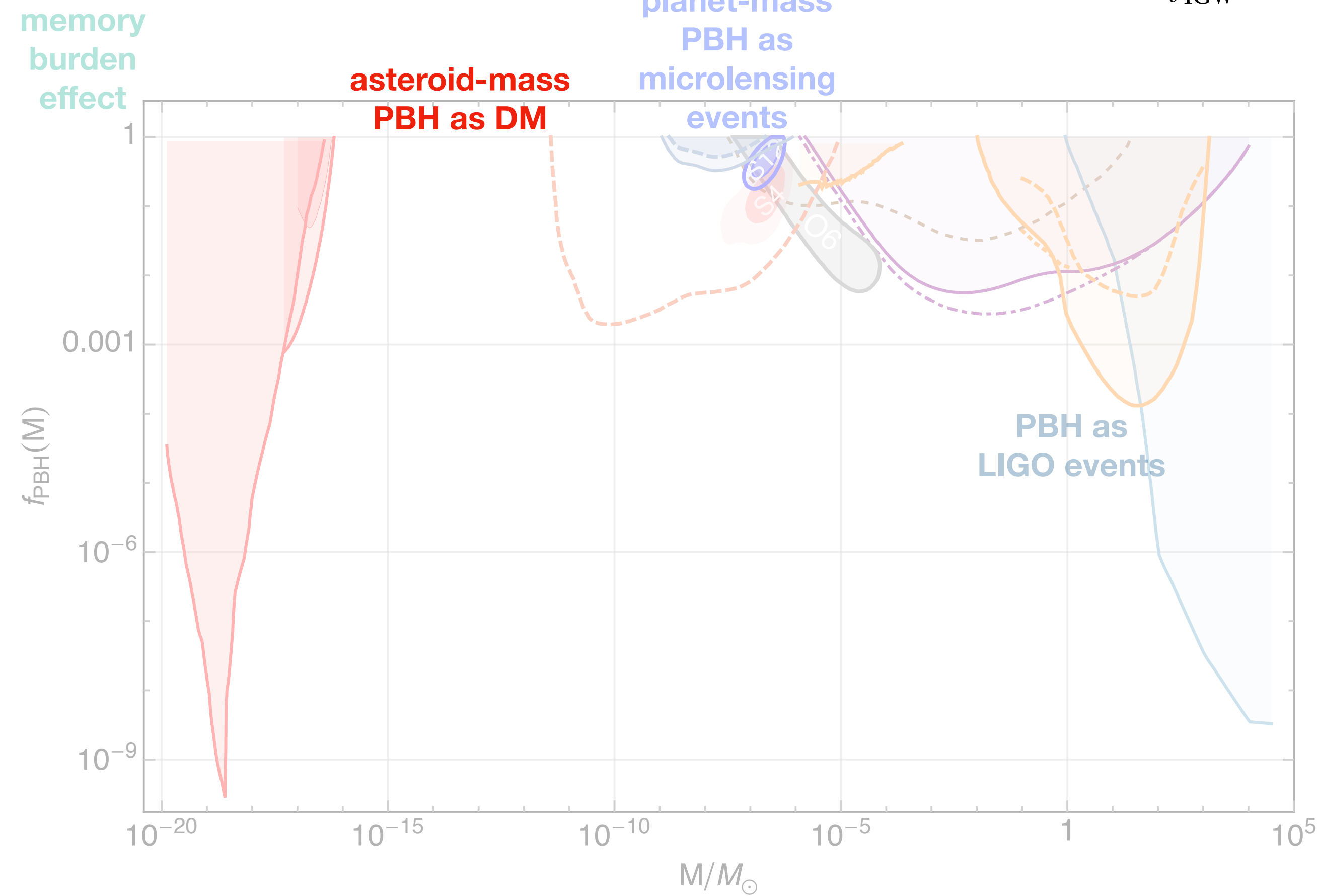
PBH constraints

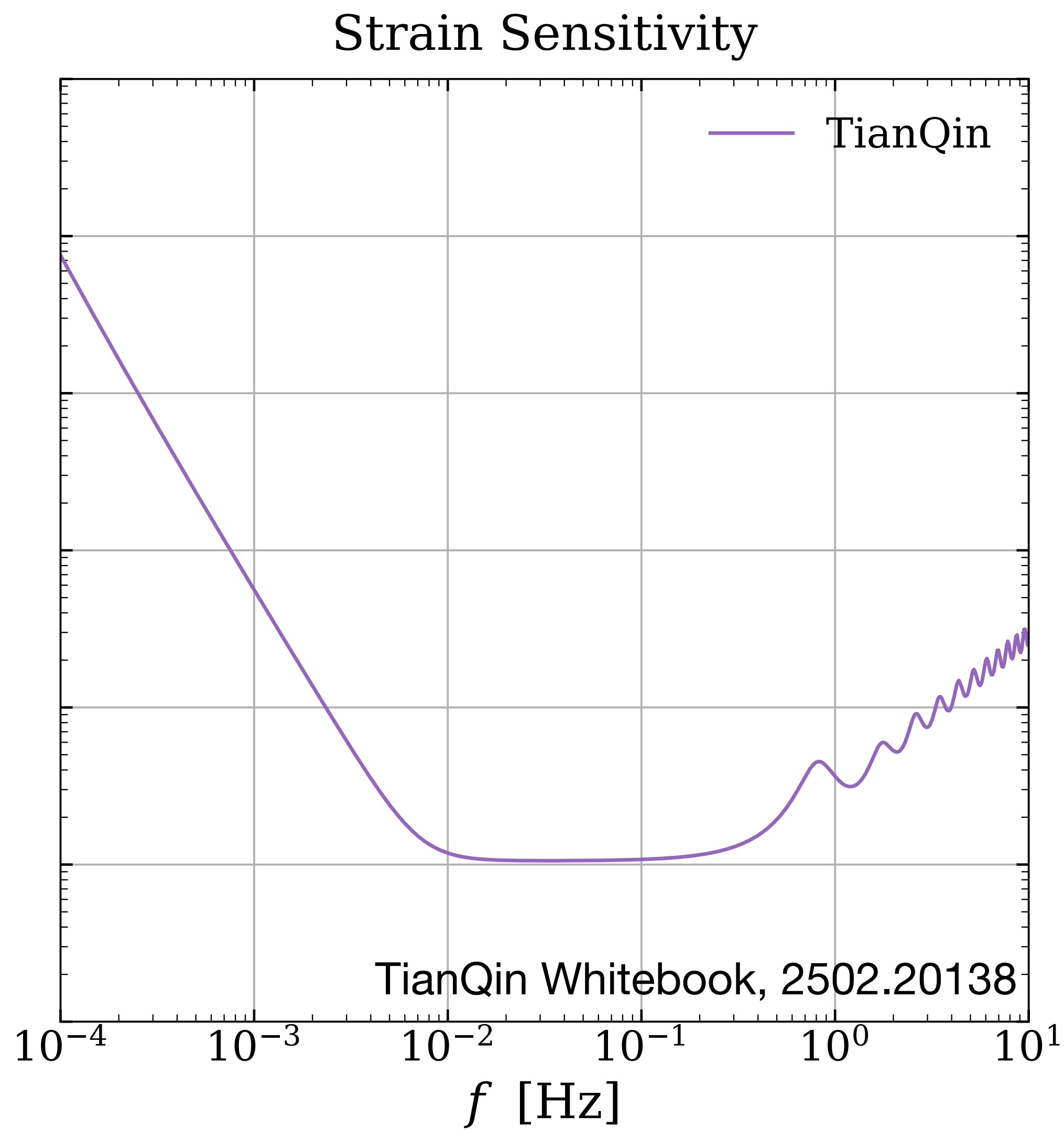
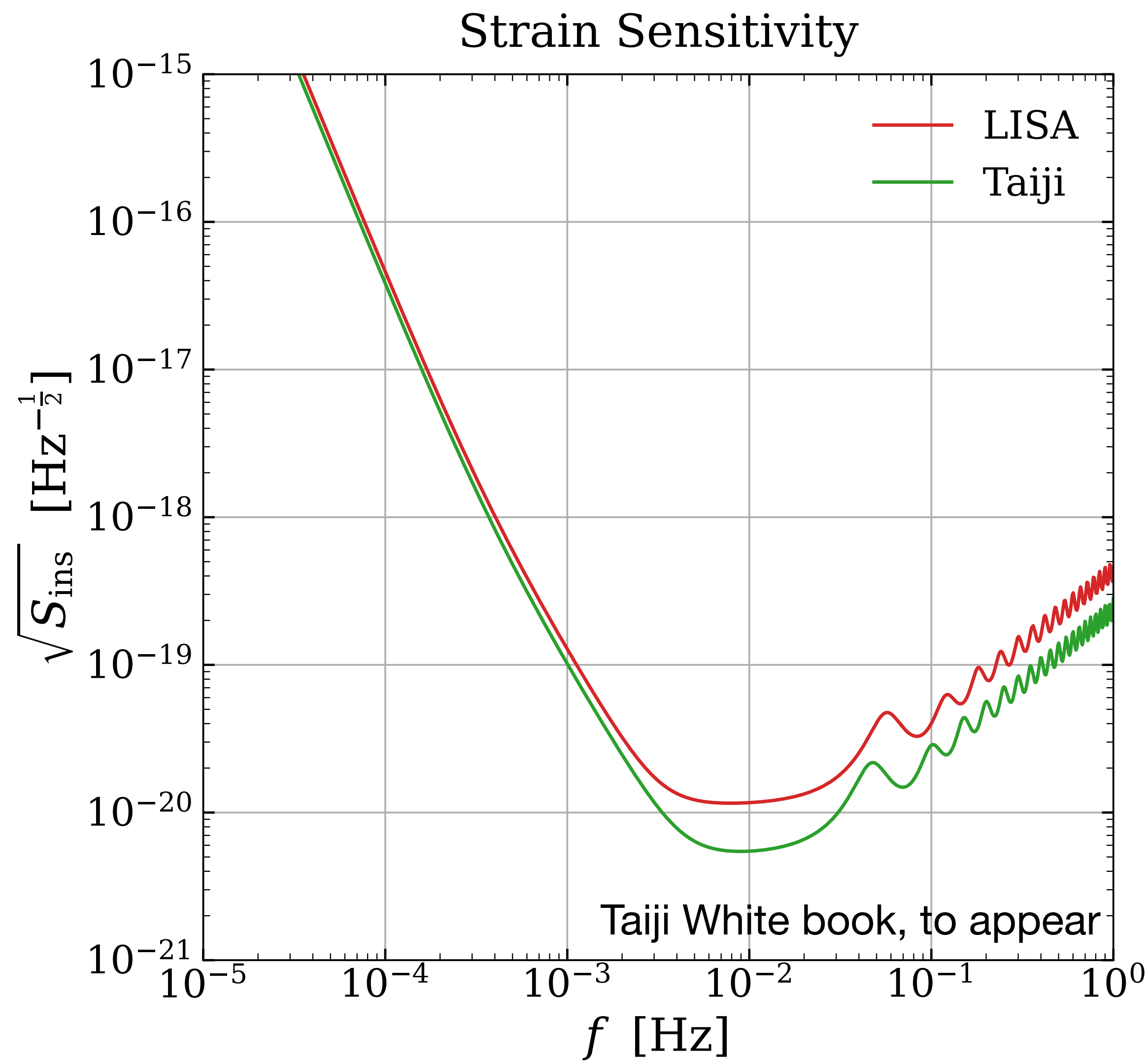
crosscheck

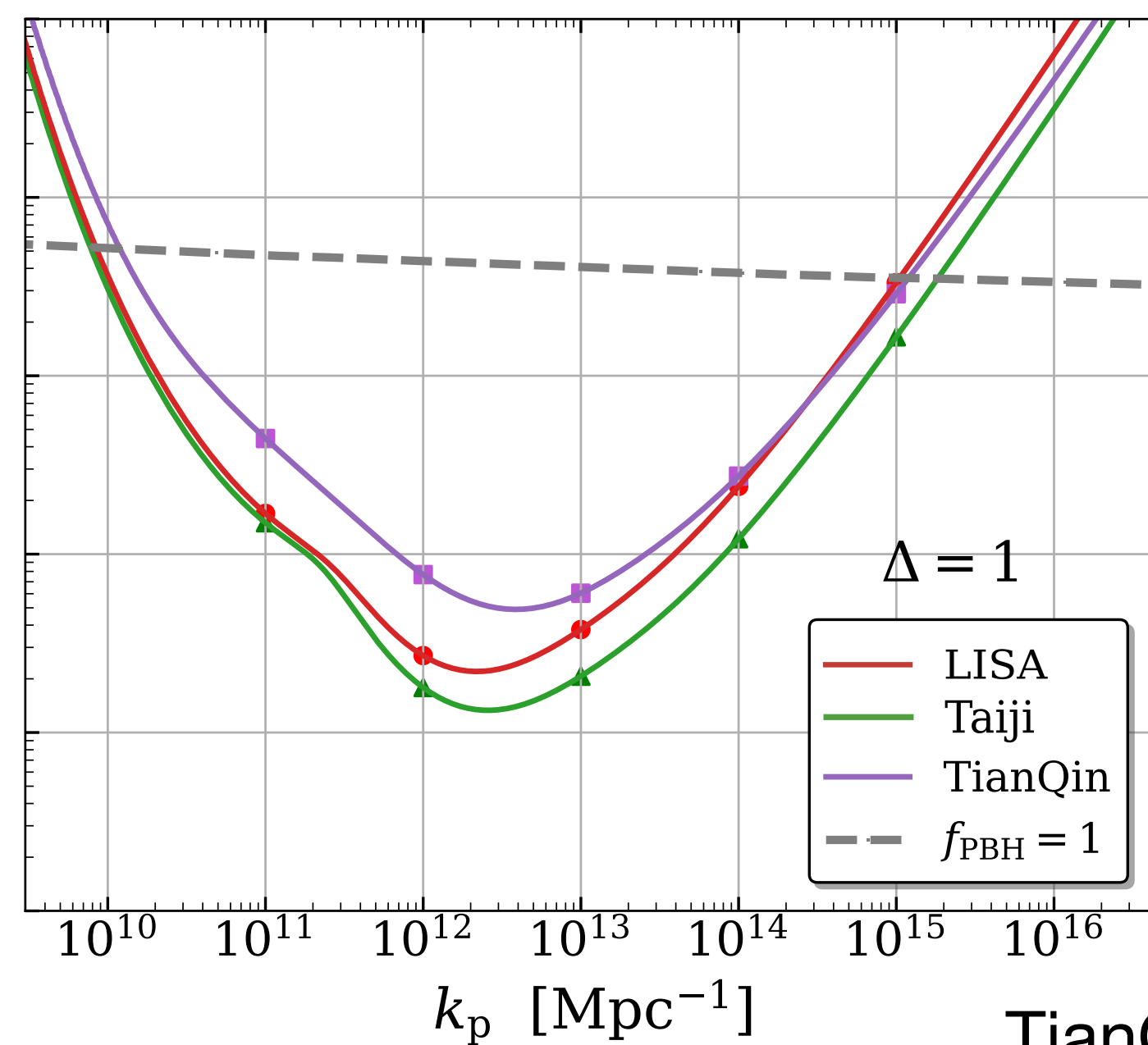
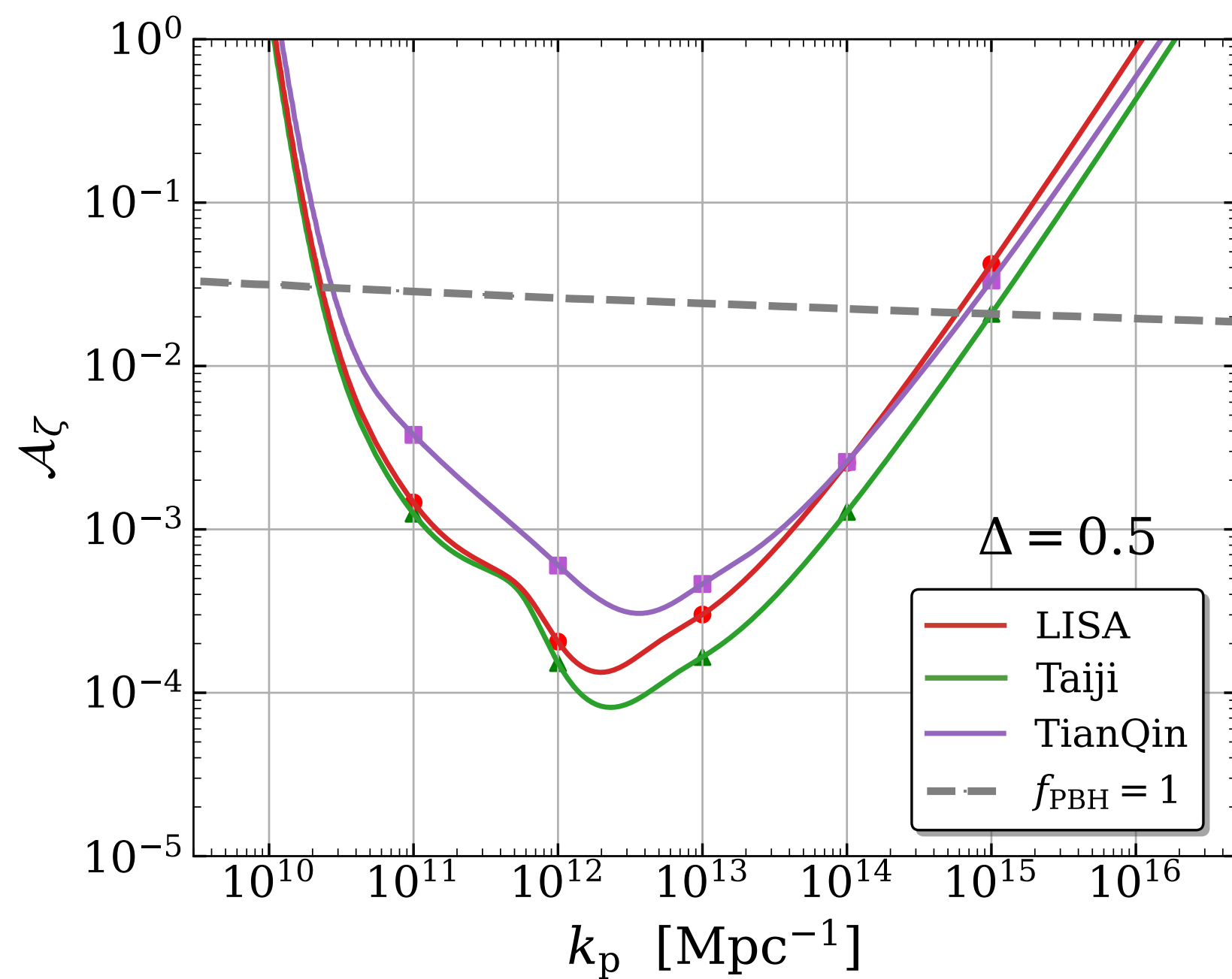
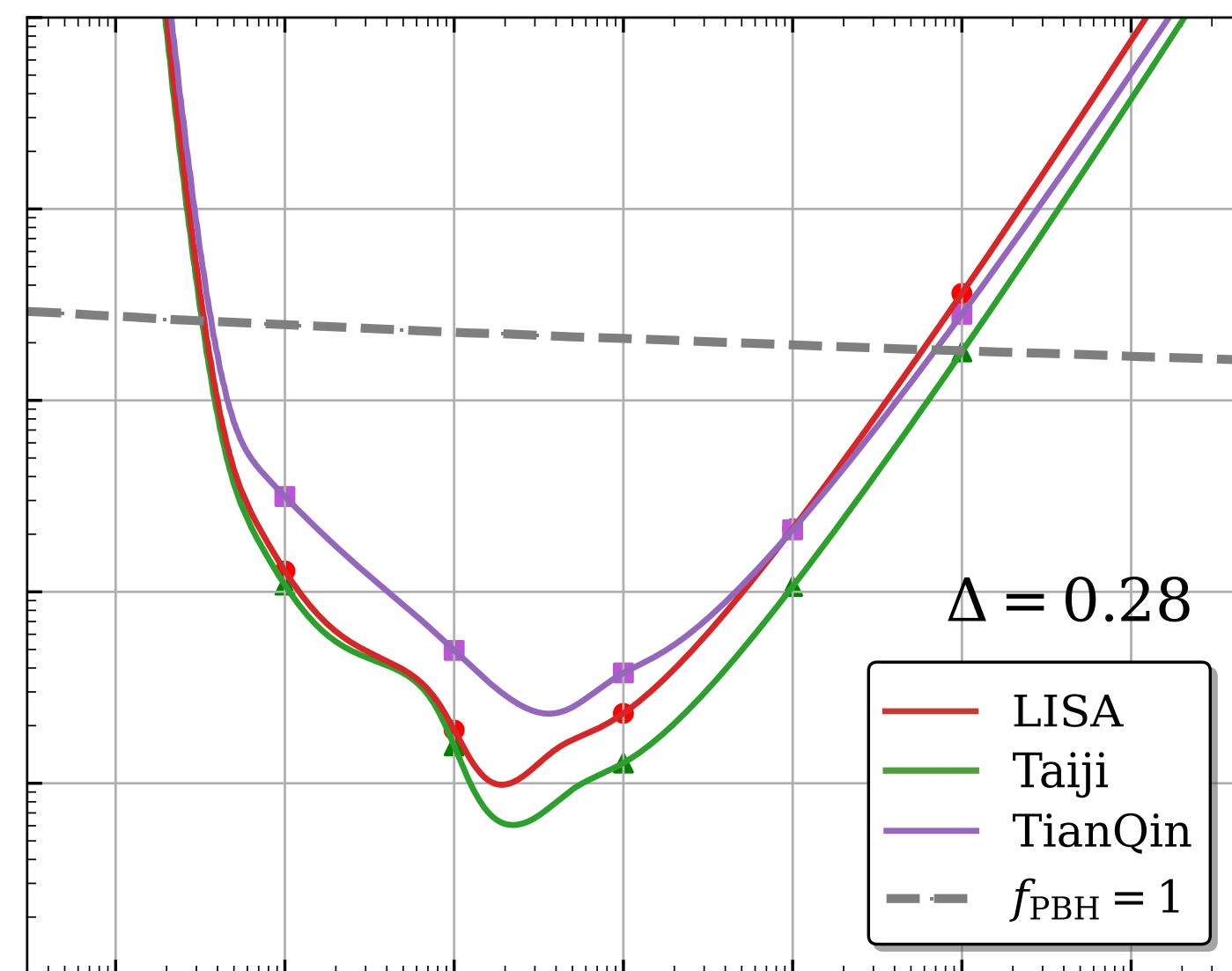
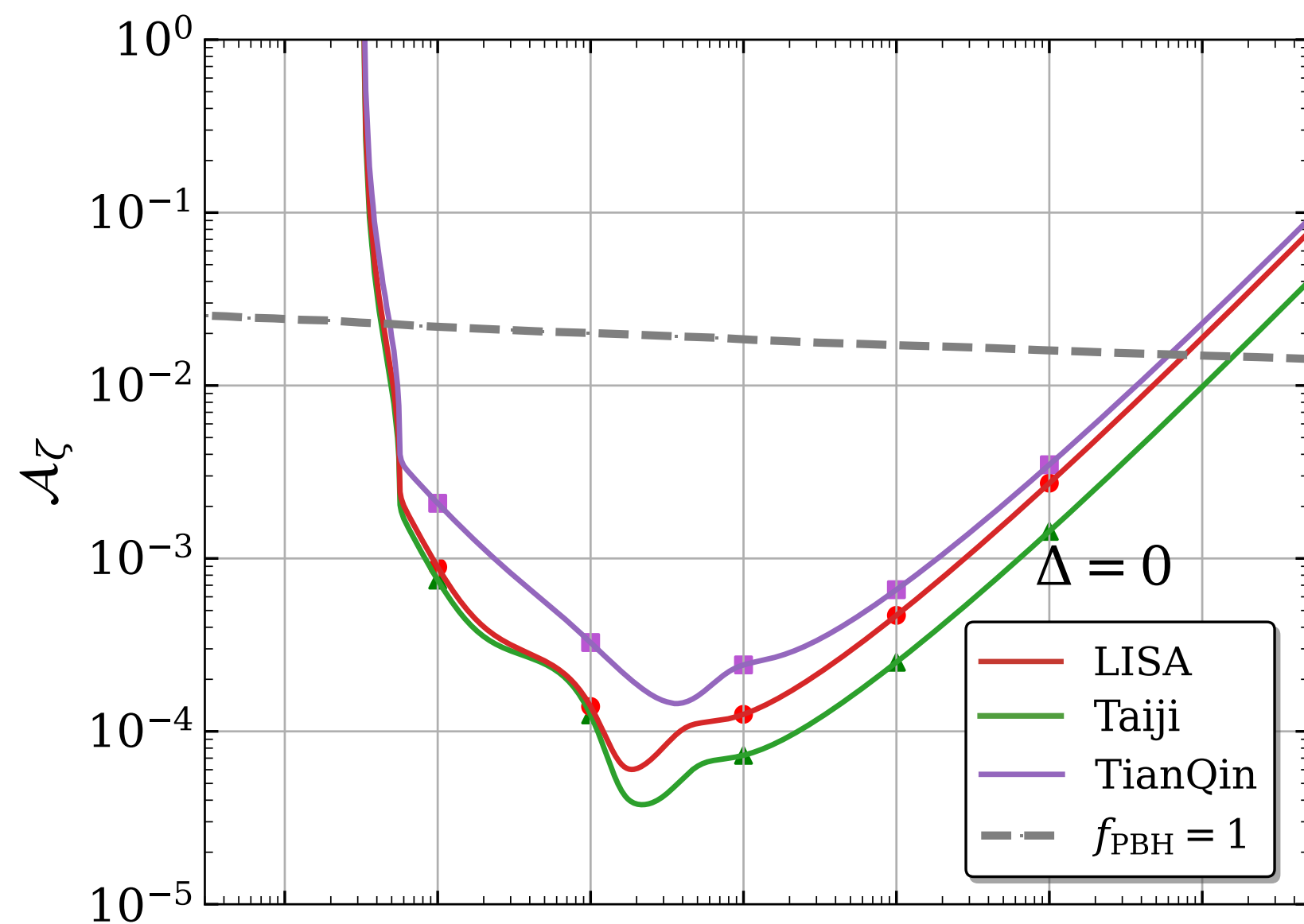
IGW constraints

$$f_{\text{IGW}} \sim 3\text{Hz} \left(\frac{M_{\text{PBH}}}{10^{16}\text{g}} \right)^{-\frac{1}{2}}$$

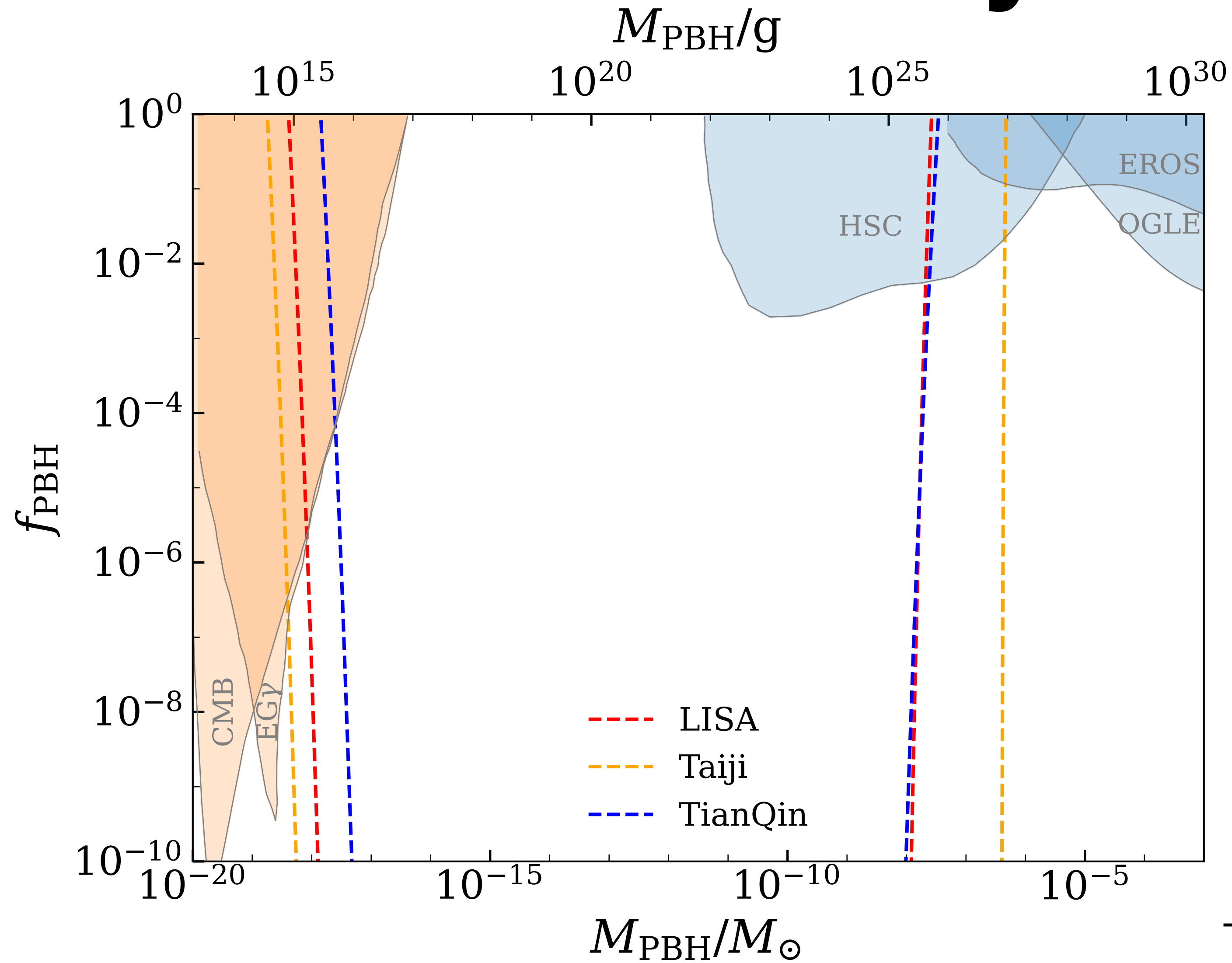
Constraint $\text{PBH} \leq \text{DM}$





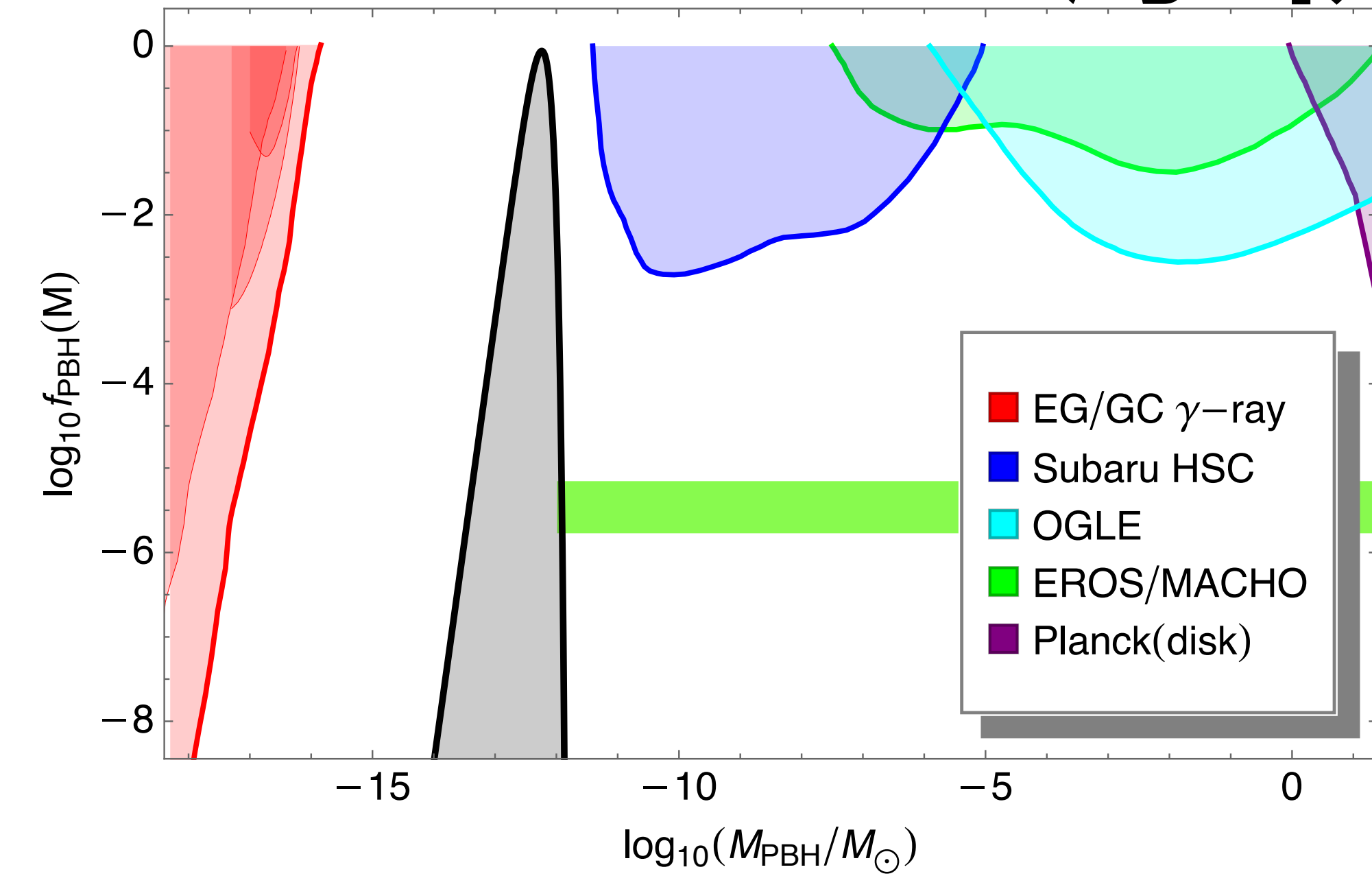


Detectability

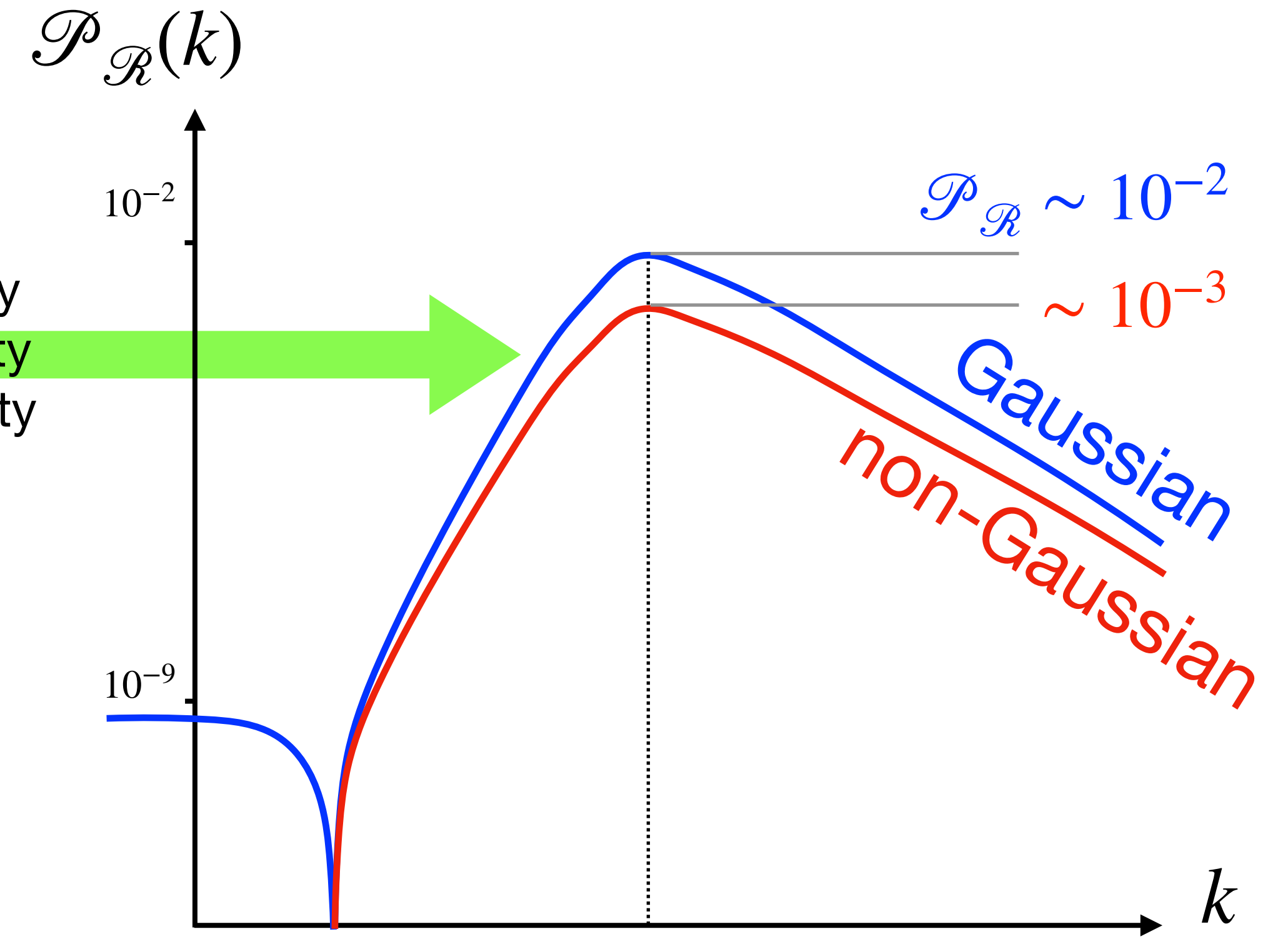


TianQin Whitebook, 2502.20138
 Hong, SP, Wang, Zhang, 2601.05069
 Taiji White book, to appear

原初黑洞暗物质



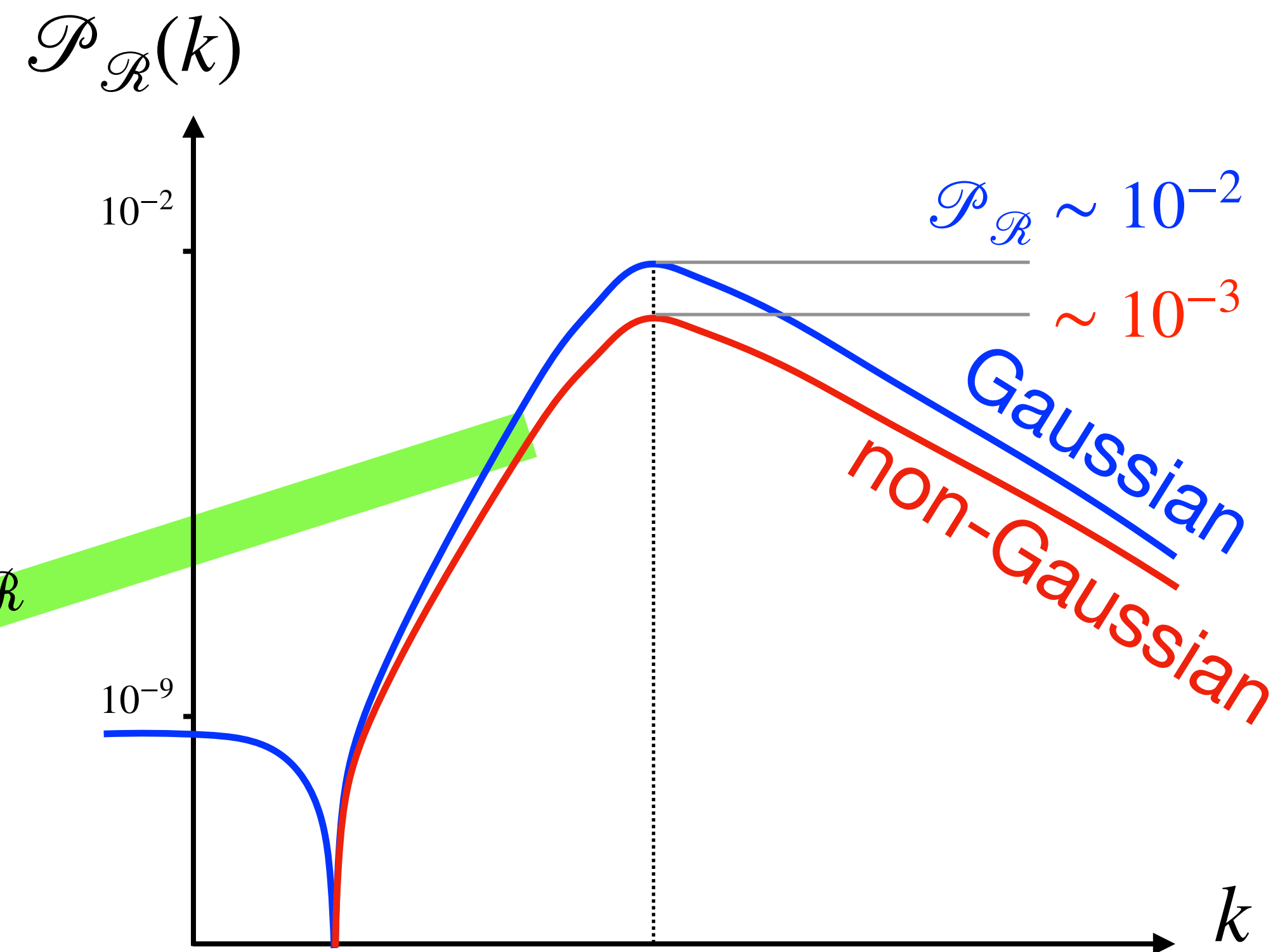
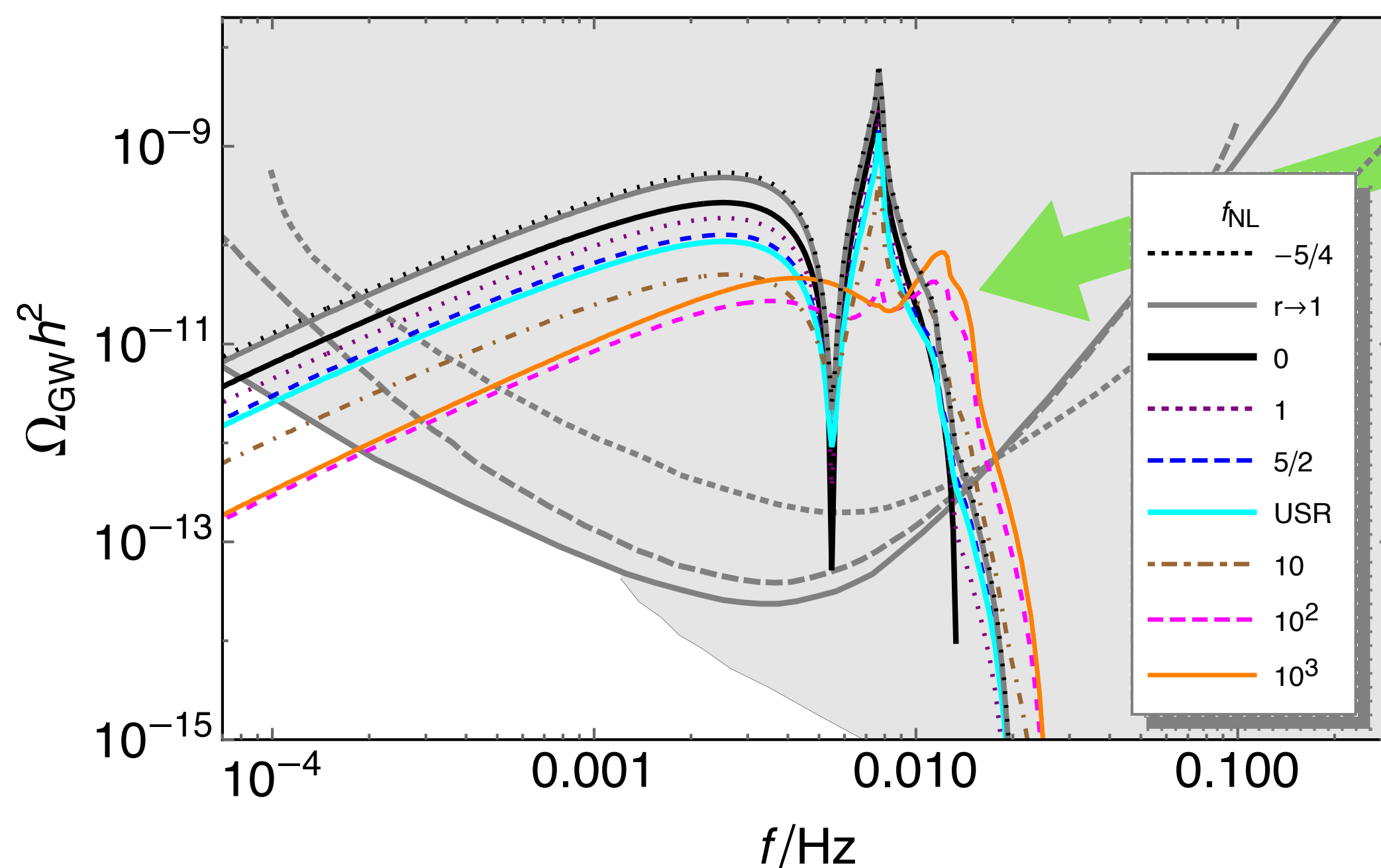
extended PS/peak theory
ineludible non-Gaussianity
primordial non-Gaussianity



- Press-Schechter: Young & Byrnes 1307.4995
- Extended Press-Schechter: Biagetti et al 2105.07810; Gow et al 2211.08348
- Peak theory: Kitajima et al 2109.00791; SP, Sasaki, Takhistov, Wang, 2501.00295
- 计算原初黑洞丰度时必须考虑非高斯性（指数依赖）

原初黑洞暗物质

- Quadratic expansion: Cai, SP, Sasaki 1810.11000 (PRL)
- Higher orders: Adshead+ 2105.01659. Abe+ 2209.13891. Garcia-Saenz+ 2207.14267. Yuan+, 2308.07155; Li+, 2309.07792. Perna+, 2403.06962
- Lattice simulation marks the breakdown of perturbativity: Zeng+ 2508.10812, Caravano+ 2604.03628
- 非高斯性对诱导引力波的影响较弱 (平方依赖)



Cai, SP, Sasaki, PRL 122, 201101

Gravitational Waves Induced by non-Gaussian Scalar Perturbations

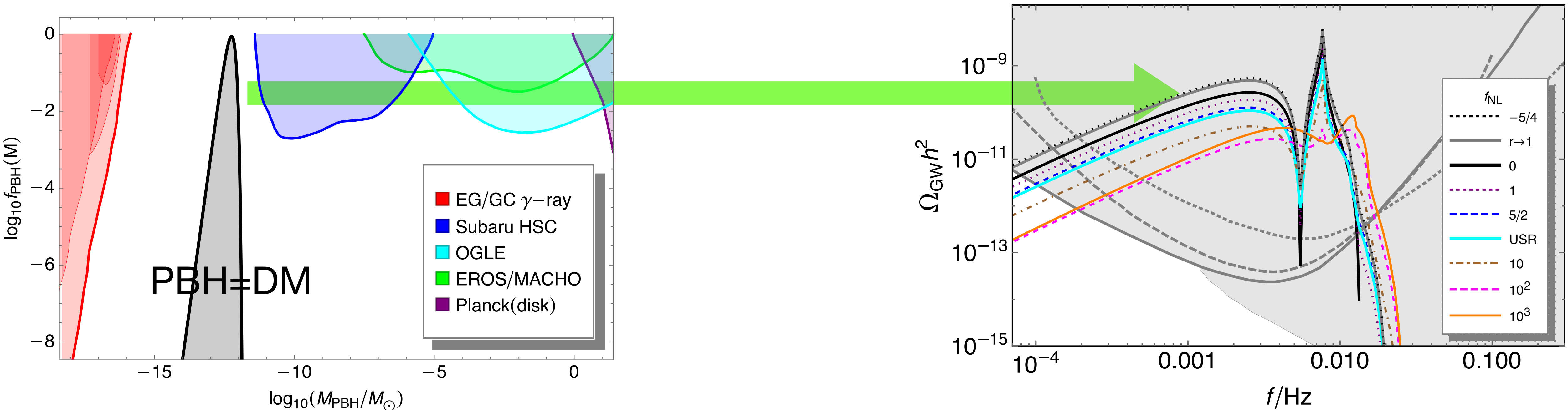
Rong-gen Cai (CAS, Beijing and Beijing, GUCAS), Shi Pi (Tokyo U., IPMU), Misao Sasaki (Tokyo U., IPMU and CAS, Beijing and Kyoto U., Yukawa Inst., Kyoto and Taiwan, Natl. Taiwan U.) (Oct 25, 2018)

Published in: *Phys.Rev.Lett.* 122 (2019) 20, 201101 • e-Print: [1810.11000](https://arxiv.org/abs/1810.11000) [astro-ph.CO]

[pdf](#) [DOI](#) [cite](#) [claim](#)

[reference search](#) [508 citations](#)

原初黑洞暗物质



- 固定原初黑洞丰度时，预言的诱导引力波振幅几乎不变
- 原初黑洞暗物质是太极/天琴/LISA的重要科学目标

Gravitational Waves Induced by non-Gaussian Scalar Perturbations

#1

Rong-gen Cai (CAS, Beijing and Beijing, GUCAS), Shi Pi (Tokyo U., IPMU), Misao Sasaki (Tokyo U., IPMU and CAS, Beijing and Kyoto U., Yukawa Inst., Kyoto and Taiwan, Natl. Taiwan U.) (Oct 25, 2018)

Published in: *Phys.Rev.Lett.* 122 (2019) 20, 201101 • e-Print: [1810.11000](https://arxiv.org/abs/1810.11000) [astro-ph.CO]

[pdf](#) [DOI](#) [cite](#) [claim](#)

[reference search](#) [508 citations](#)

PBH-IGW crosscheck

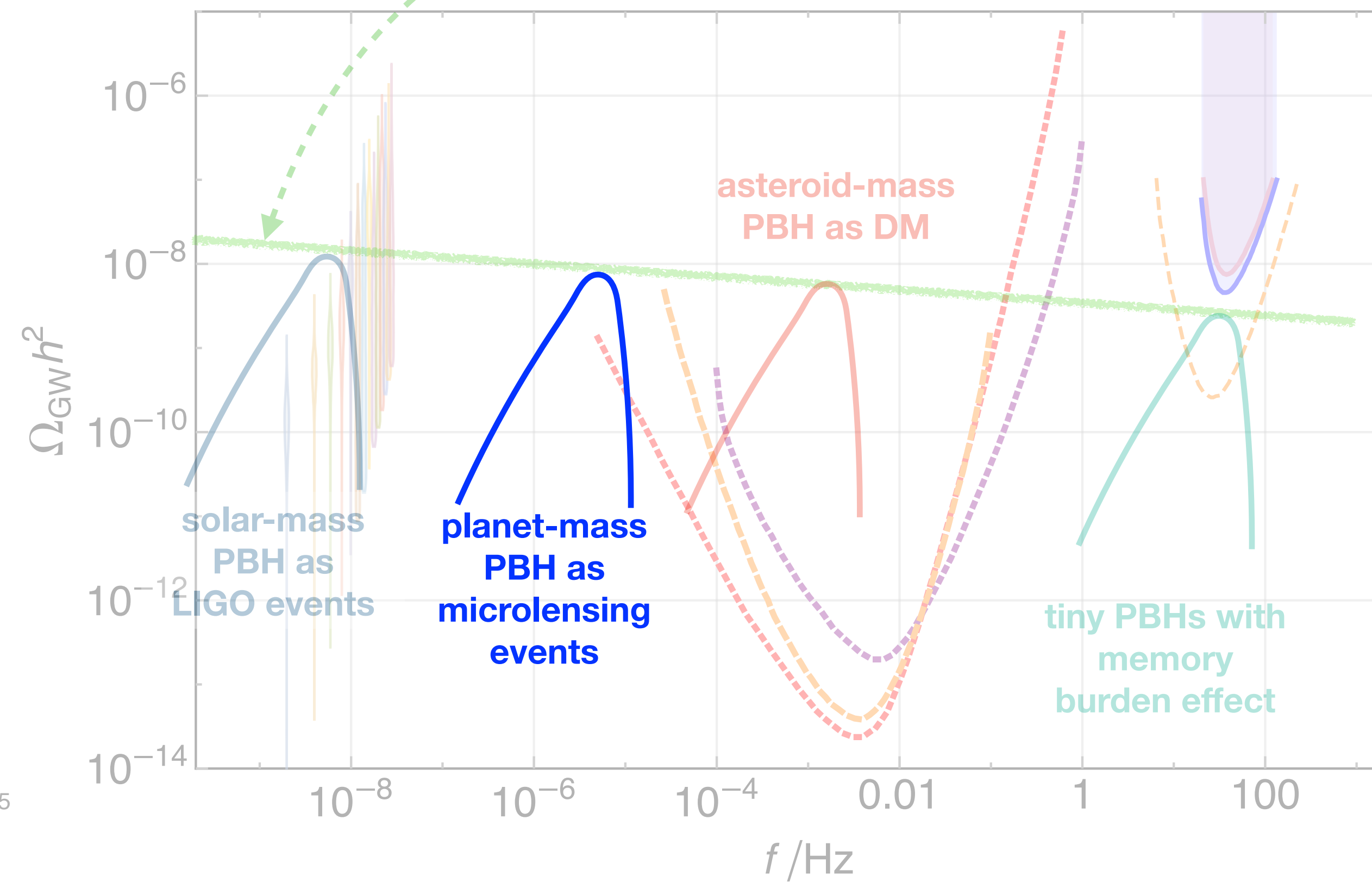
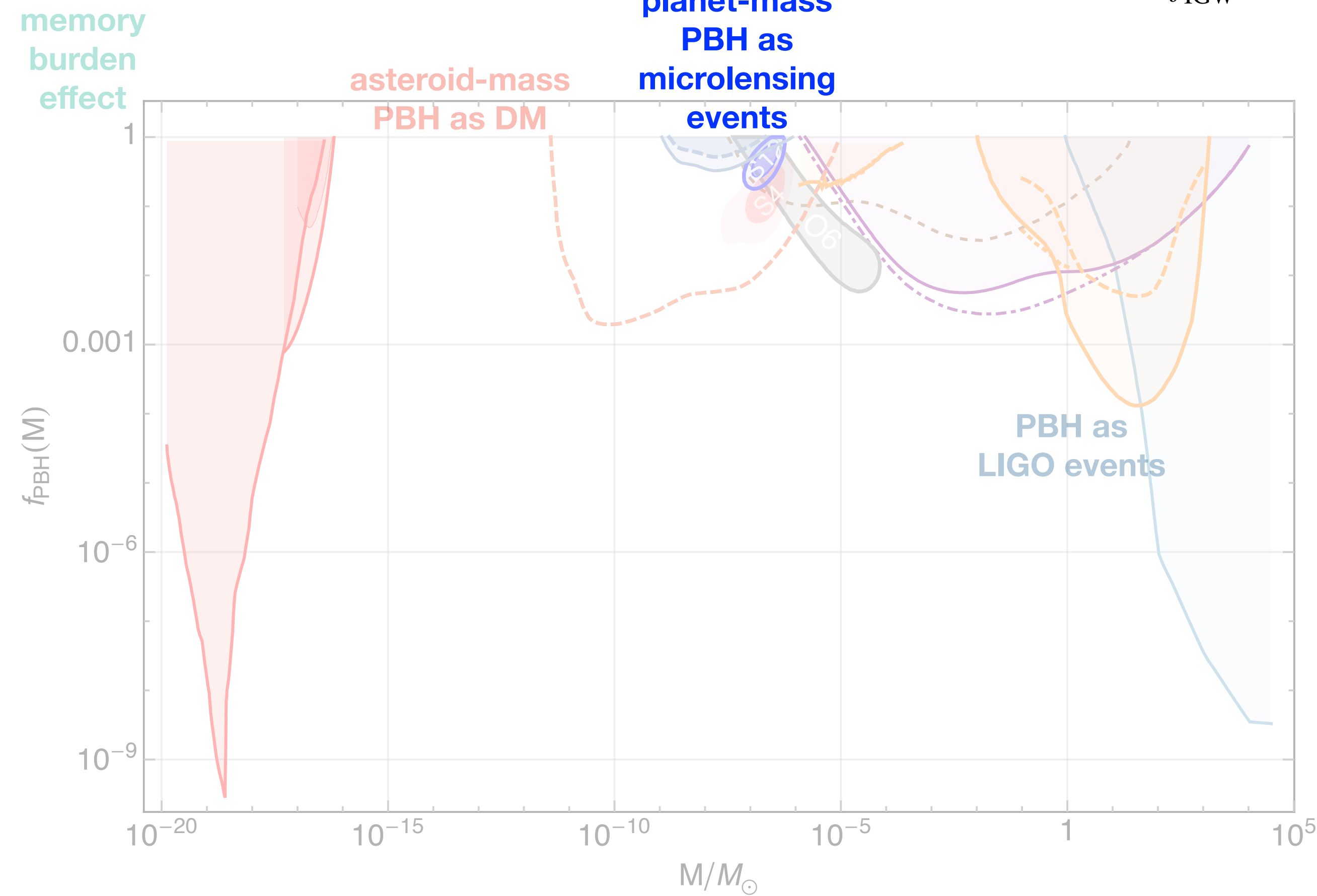
PBH constraints

crosscheck

IGW constraints

$$f_{\text{IGW}} \sim 3\text{Hz} \left(\frac{M_{\text{PBH}}}{10^{16}\text{g}} \right)^{-\frac{1}{2}}$$

Constraint $\text{PBH} \leq \text{DM}$



PBH-IGW crosscheck

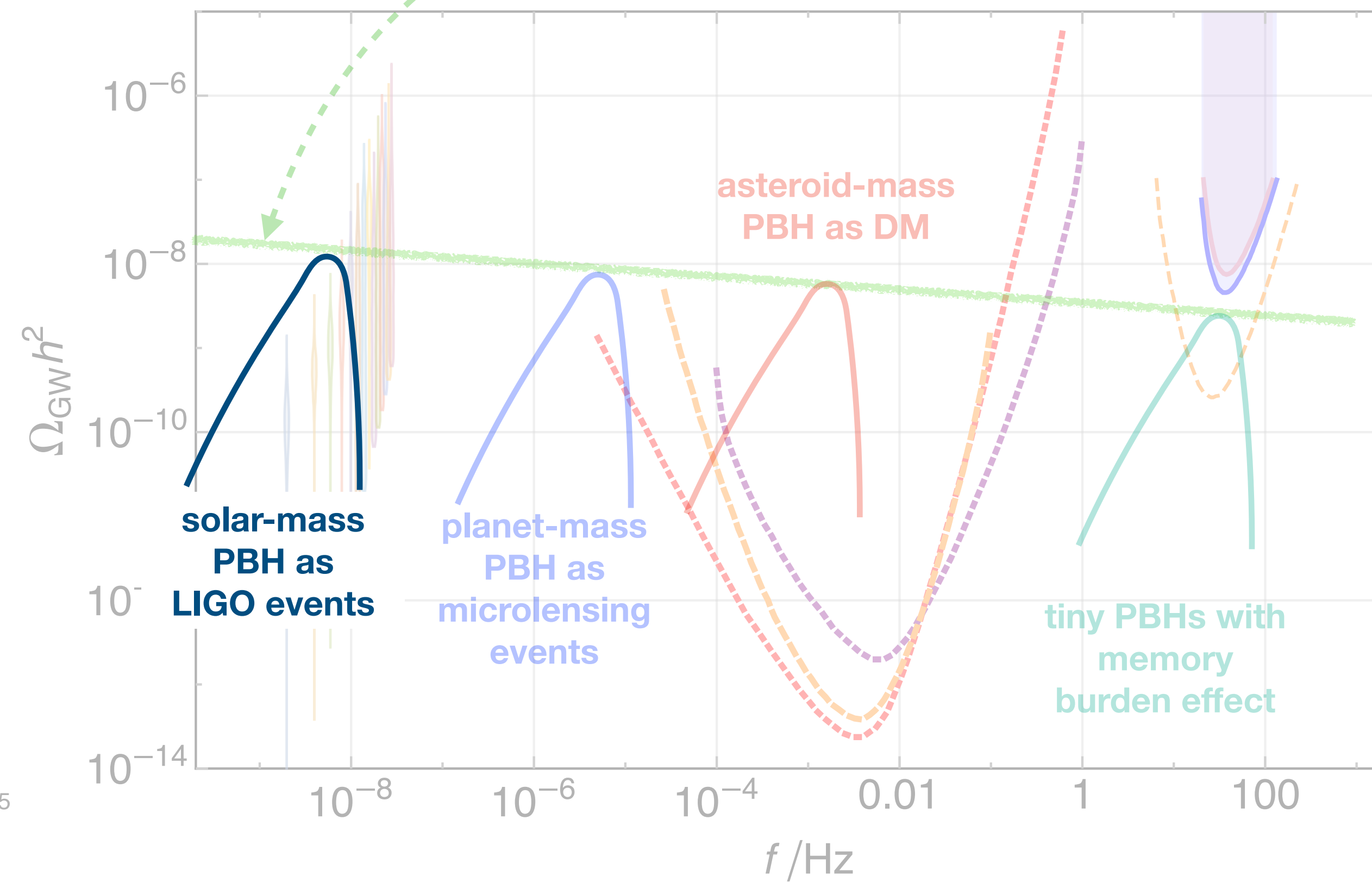
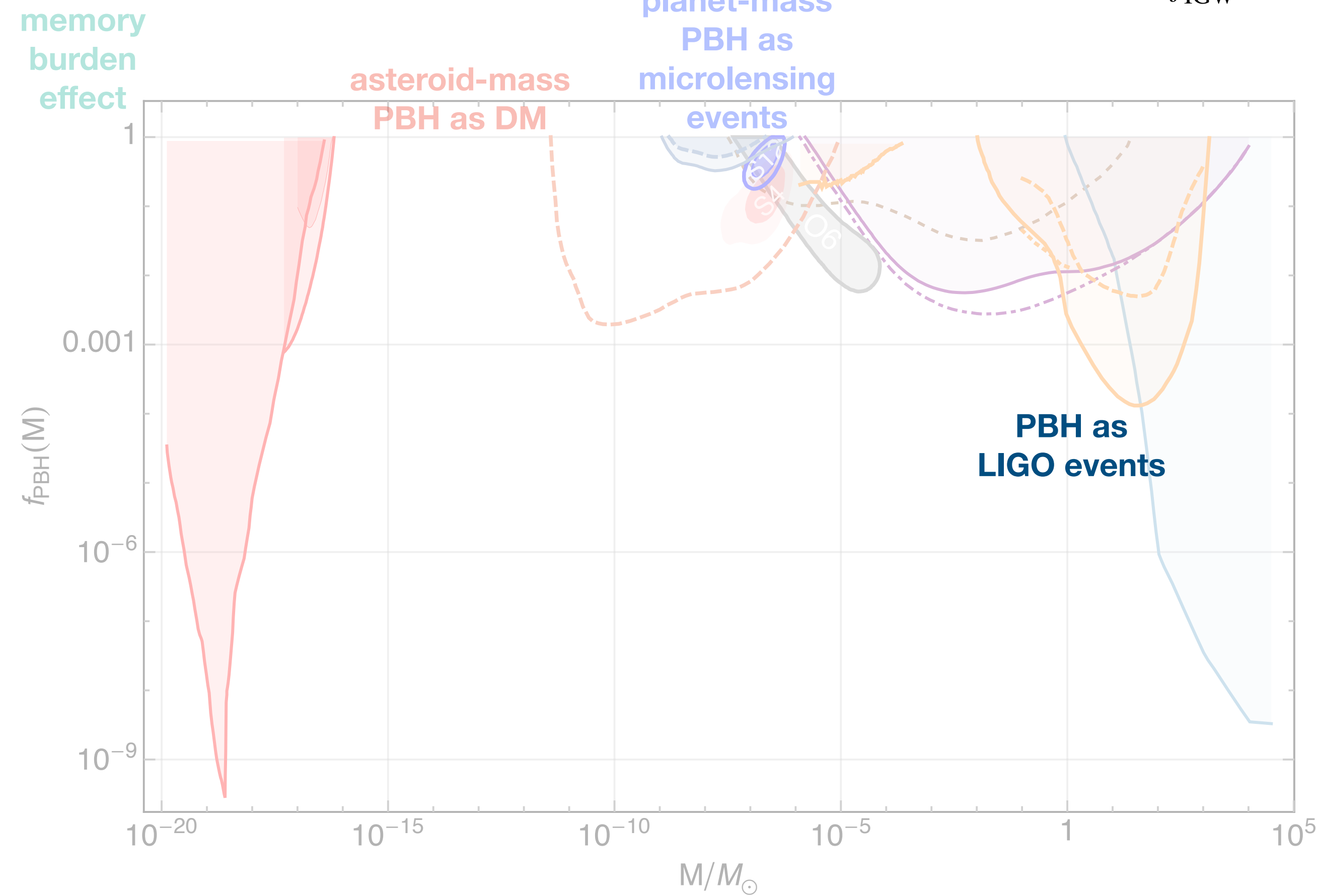
PBH constraints

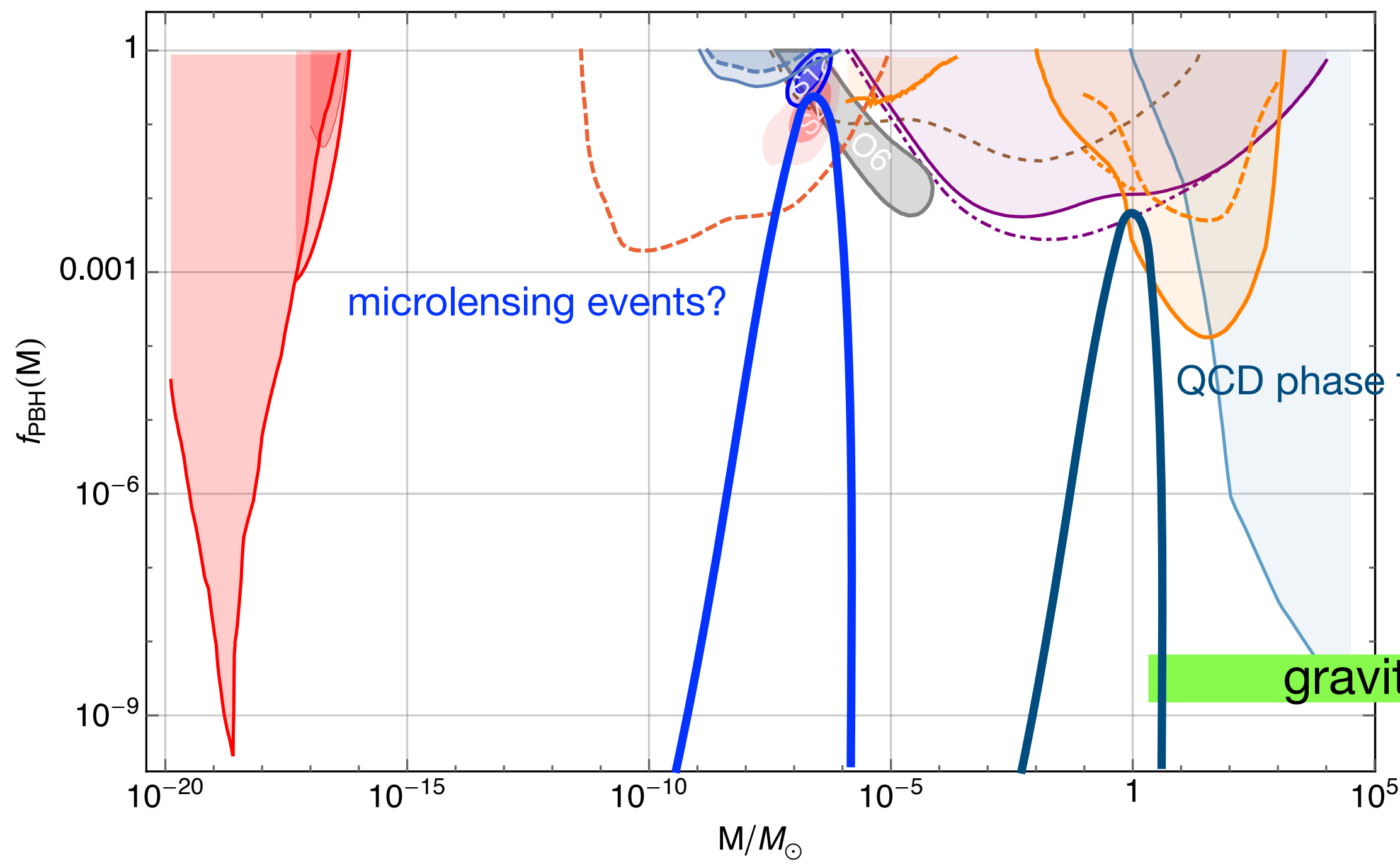
crosscheck

IGW constraints

$$f_{\text{IGW}} \sim 3\text{Hz} \left(\frac{M_{\text{PBH}}}{10^{16}\text{g}} \right)^{-\frac{1}{2}}$$

Constraint $\text{PBH} \leq \text{DM}$





$$f_{\text{IGW}} \sim 3\text{Hz} \left(\frac{M_{\text{PBH}}}{10^{16}\text{g}} \right)^{-\frac{1}{2}}$$

gravitational collapse

$$\mathcal{P}_{\mathcal{R}}(k)$$

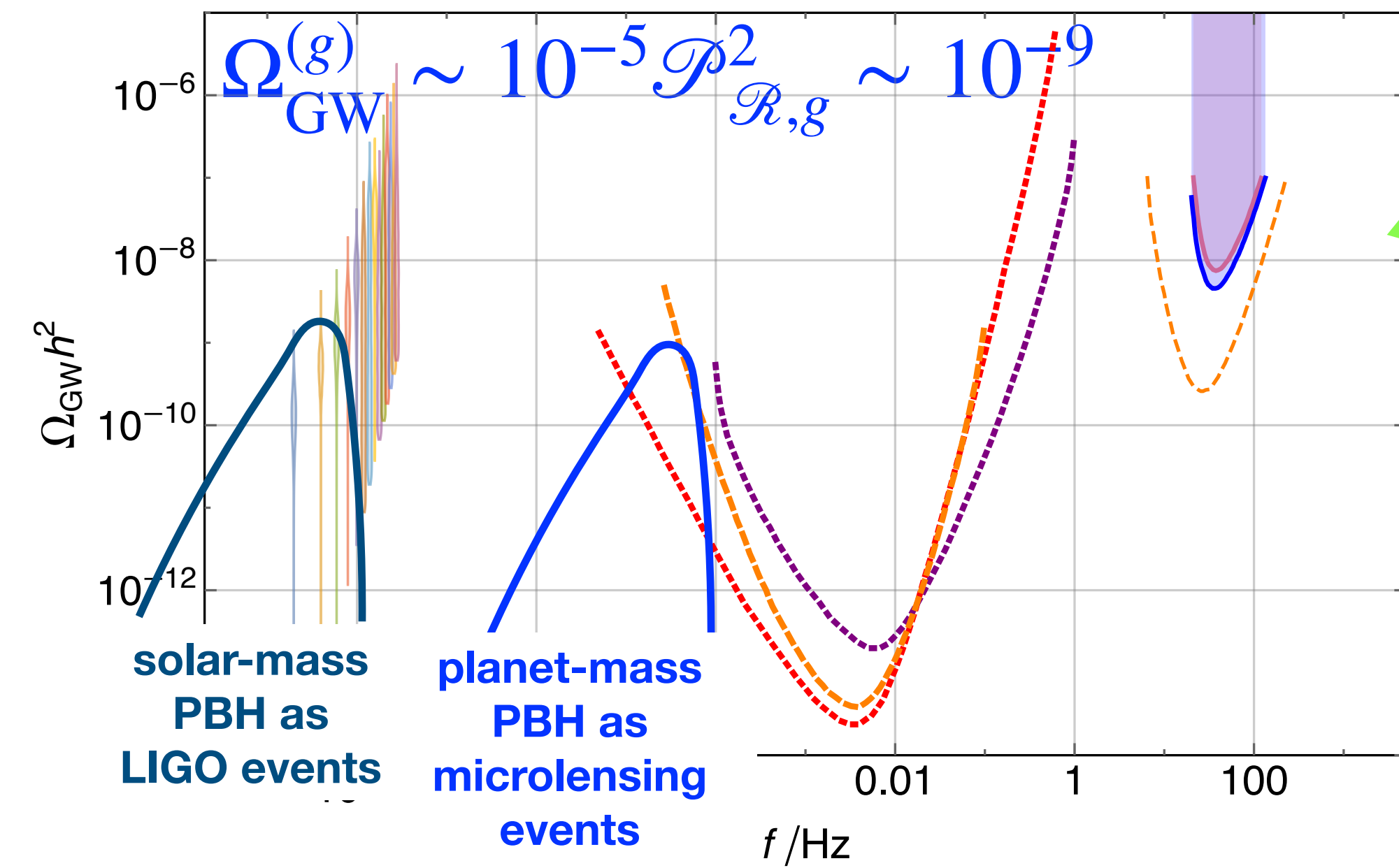
$$\mathcal{P}_{\mathcal{R}} \sim 10^{-2}$$

$$10^{-9}$$

$$10^9 \text{ Mpc}^{-1}$$

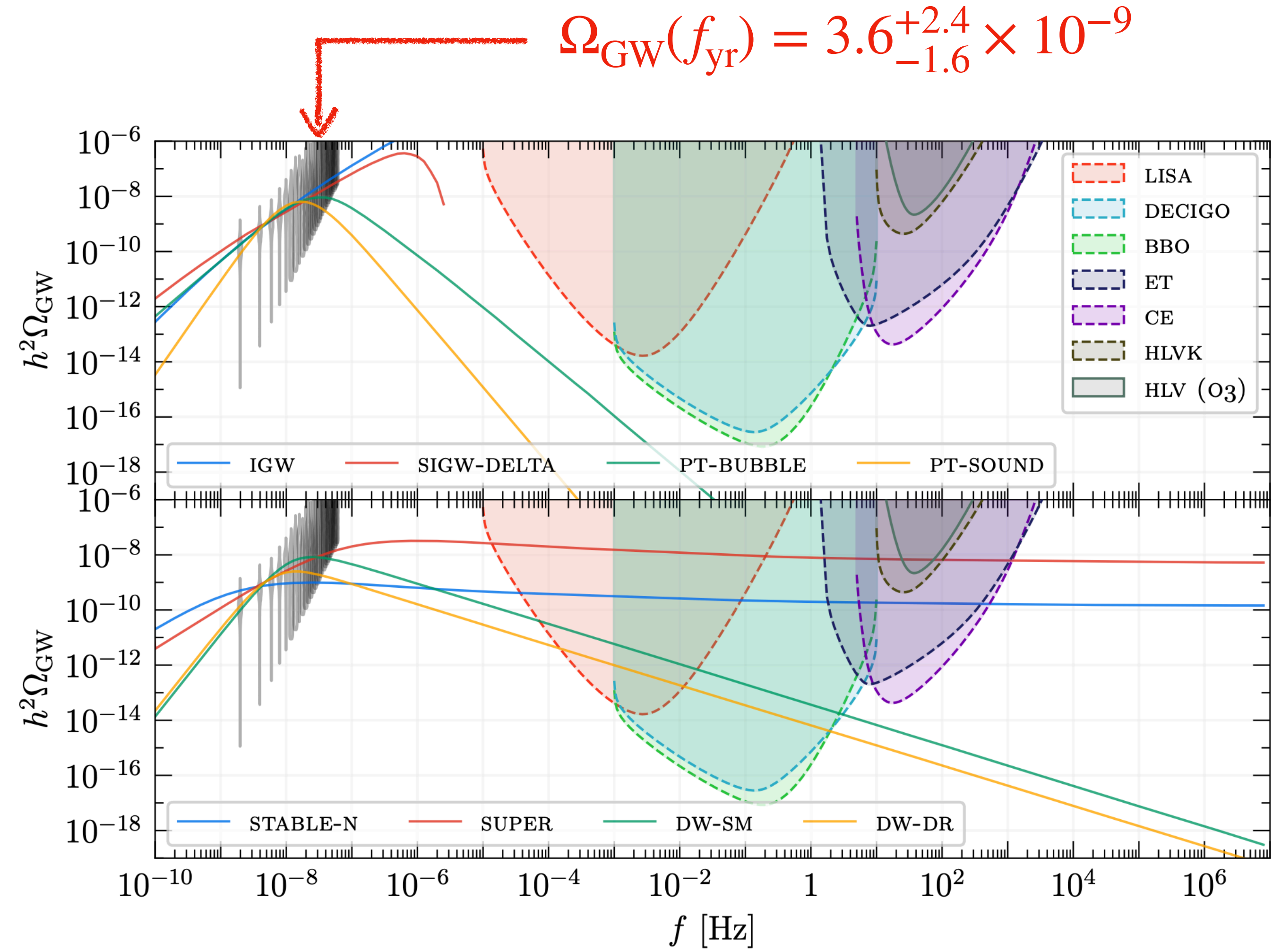
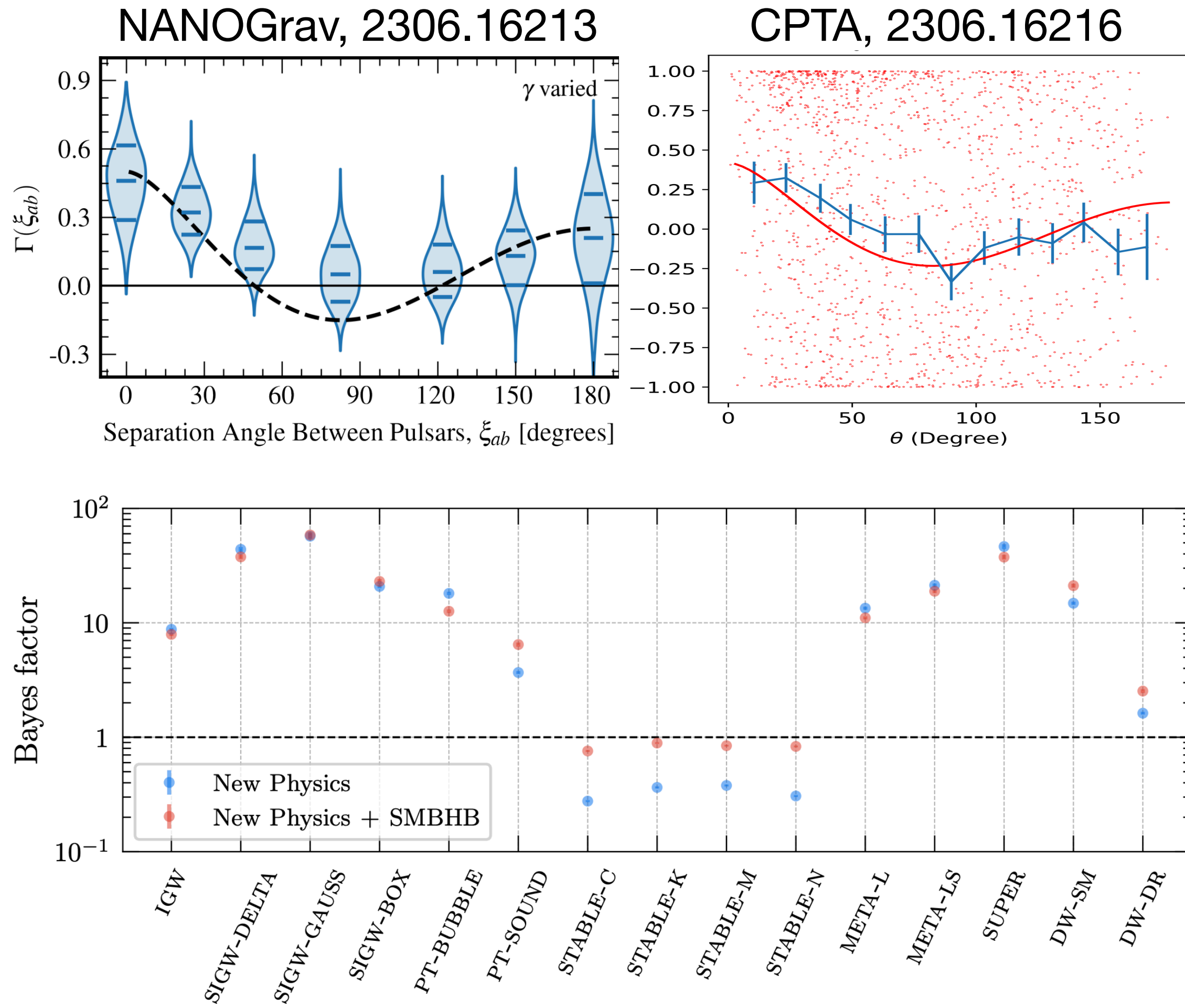
k

nonlinear coupling $h\mathcal{R}\mathcal{R}$

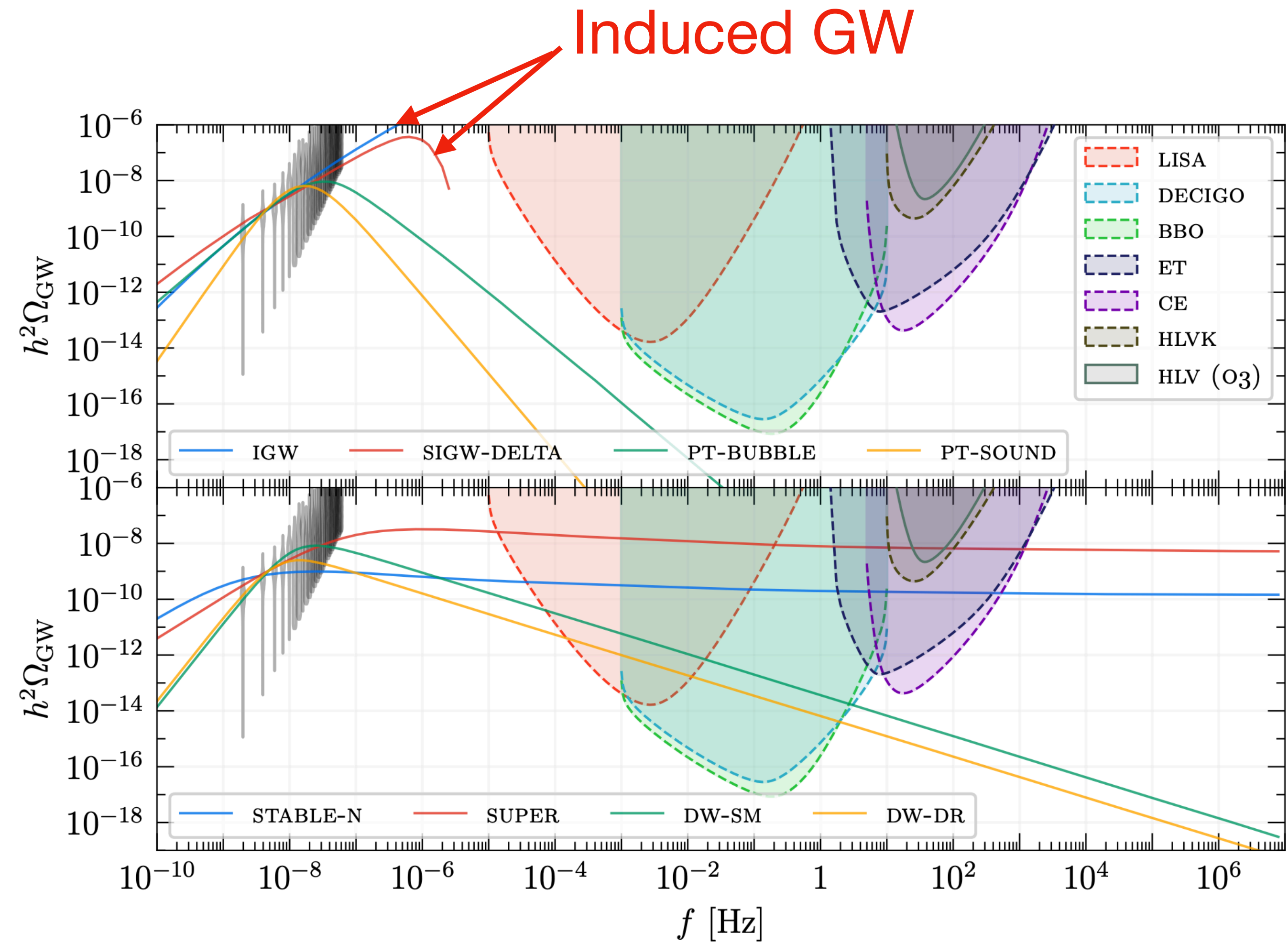
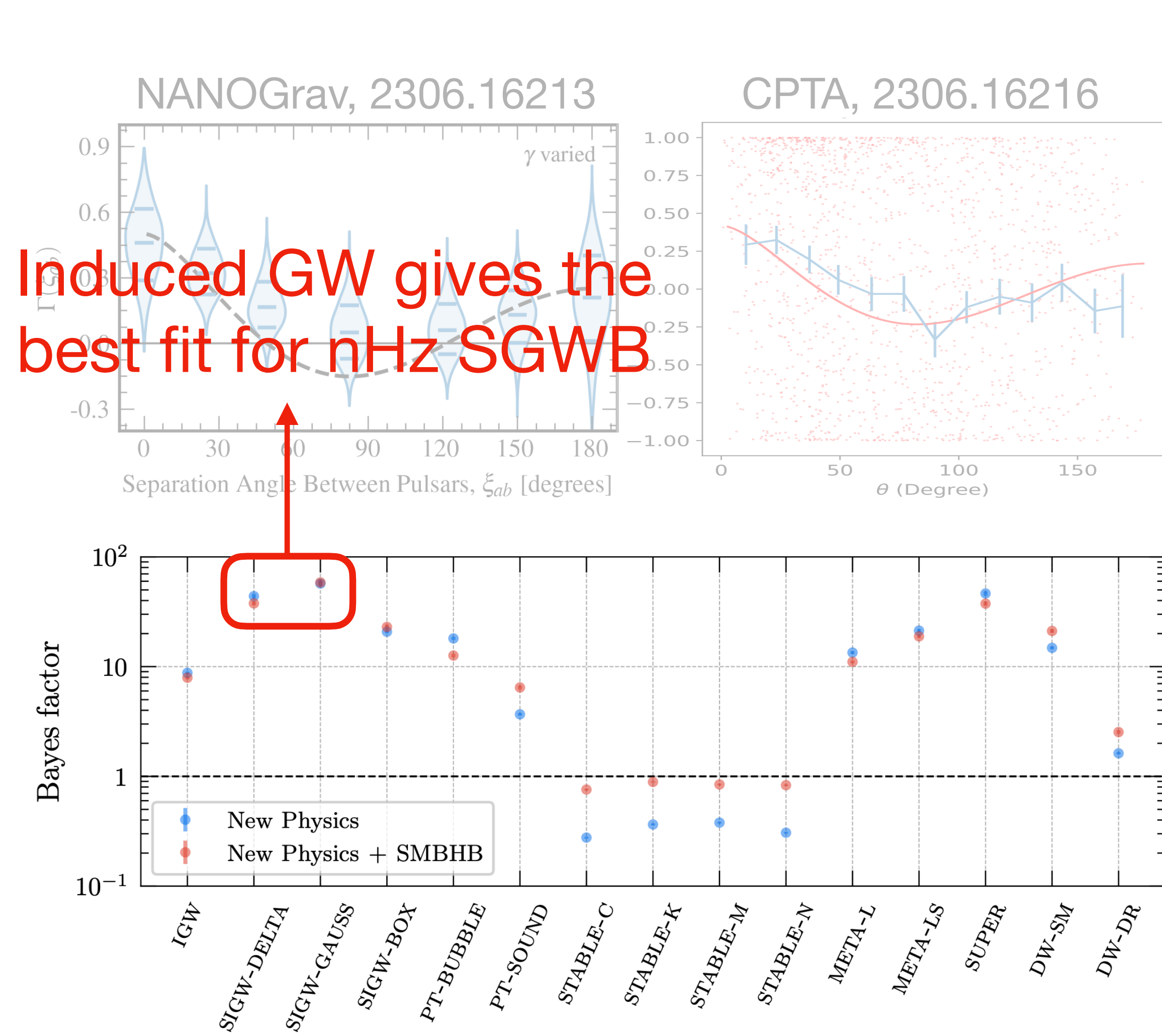


Saito & Yokoyama 0812.4339
Cai, SP, Sasaki, 1810.11000

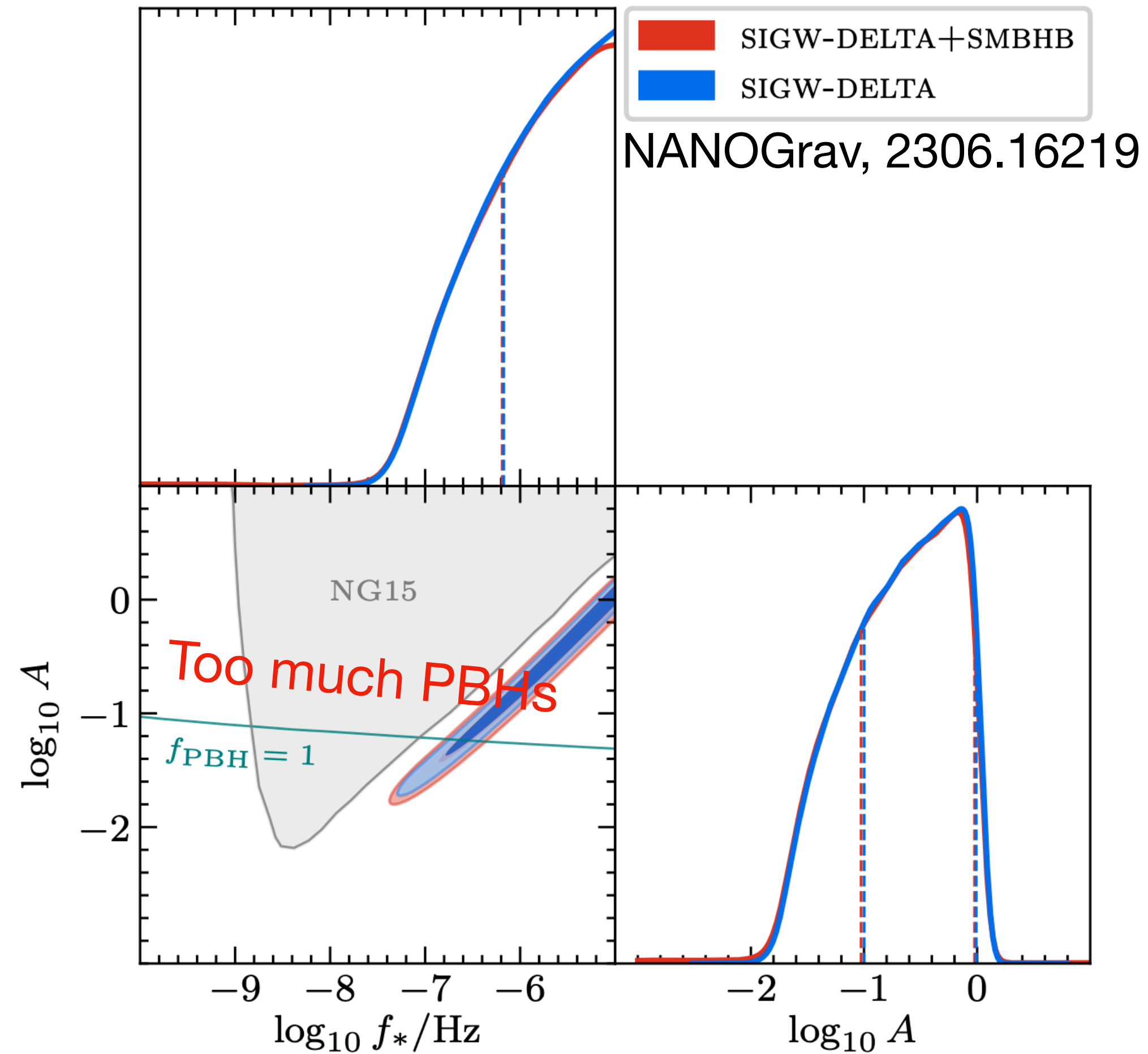
Application: nHz SGWB



Application: nHz SGWB

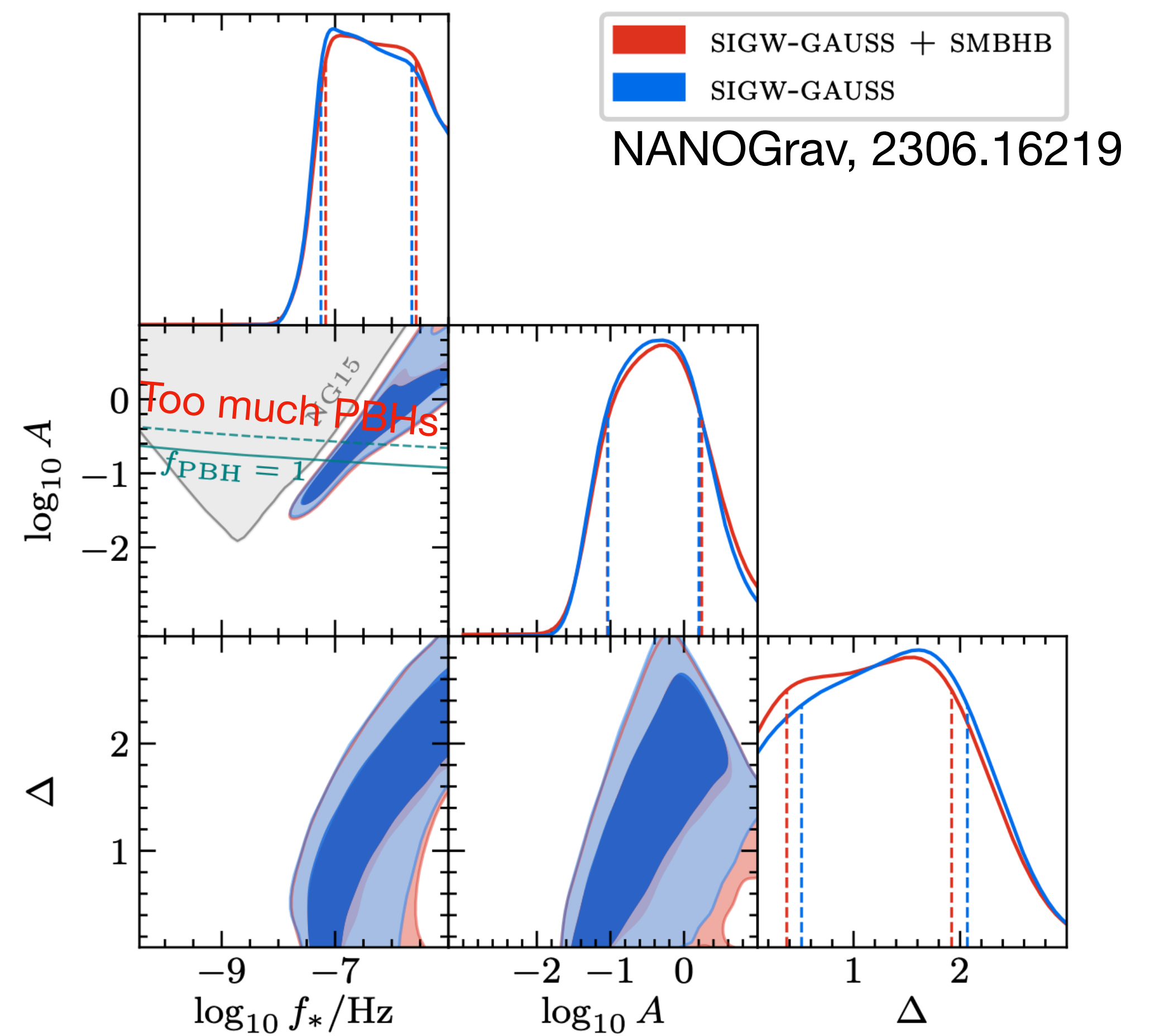


IGW as nHz SGWB



$$\mathcal{P}_{\mathcal{R}} = A \delta(\ln k - \ln k_*)$$

monochromatic



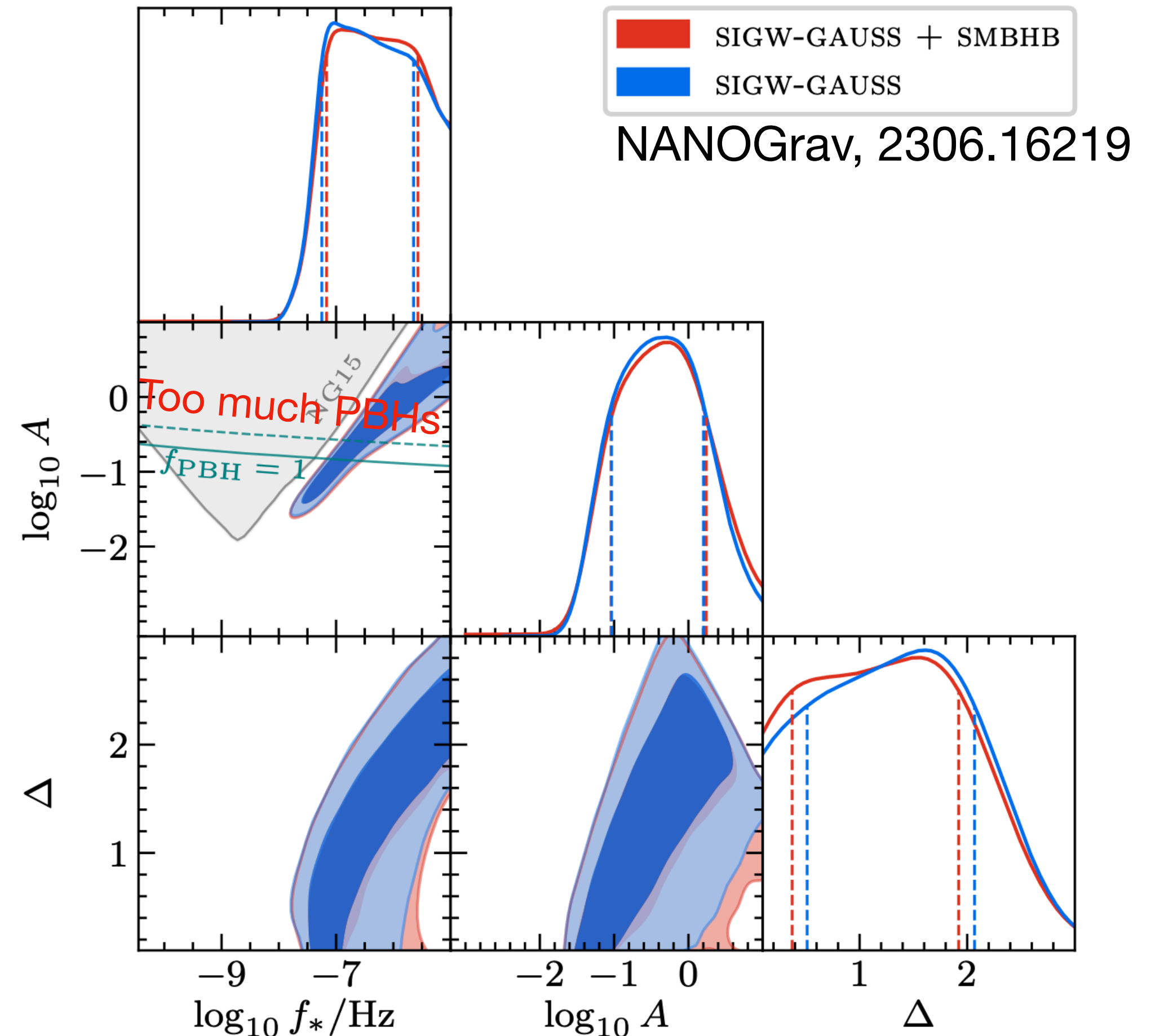
$$\mathcal{P}_{\mathcal{R}} = \frac{A}{\sqrt{2\pi}\Delta} \exp\left(-\frac{(\ln k - \ln k_*)^2}{2\Delta^2}\right)$$

lognormal [SP and Sasaki 2005.12306]

IGW as nHz SGWB

如何解决原初黑洞过量产生问题?

- (1) 利用更保守的方法计算. (Inomata et al 2306.17834; Iovino et al 2406.20089)
- (2) 通过增长物态方程参数 w 来提升临界密度, 从而压低原初黑洞丰度. (Domenech and SP, 2010.03976; Domenech, SP, et al, 2402.18965)
- (3) 通过负非高斯压低原初黑洞丰度, 通常需要利用“对数对偶”形式. (SP and Sasaki 2112.12680; Franciolini et al. 2306.17149)
- (4) 在固定归一化时, 宽谱可自然压低原初黑洞丰度.

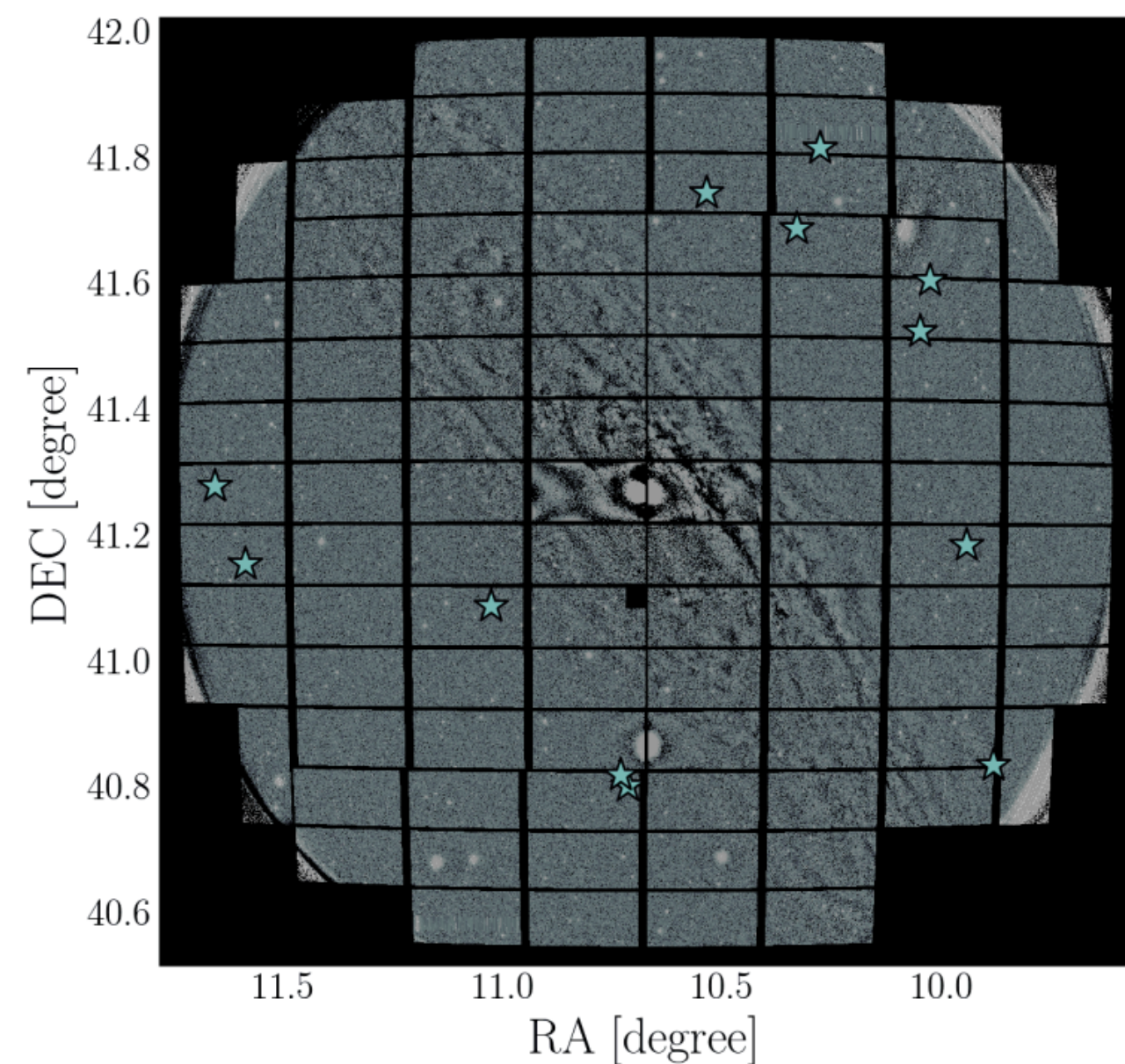


$$\mathcal{P}_{\mathcal{R}} = \frac{A}{\sqrt{2\pi}\Delta} \exp\left(-\frac{(\ln k - \ln k_*)^2}{2\Delta^2}\right)$$

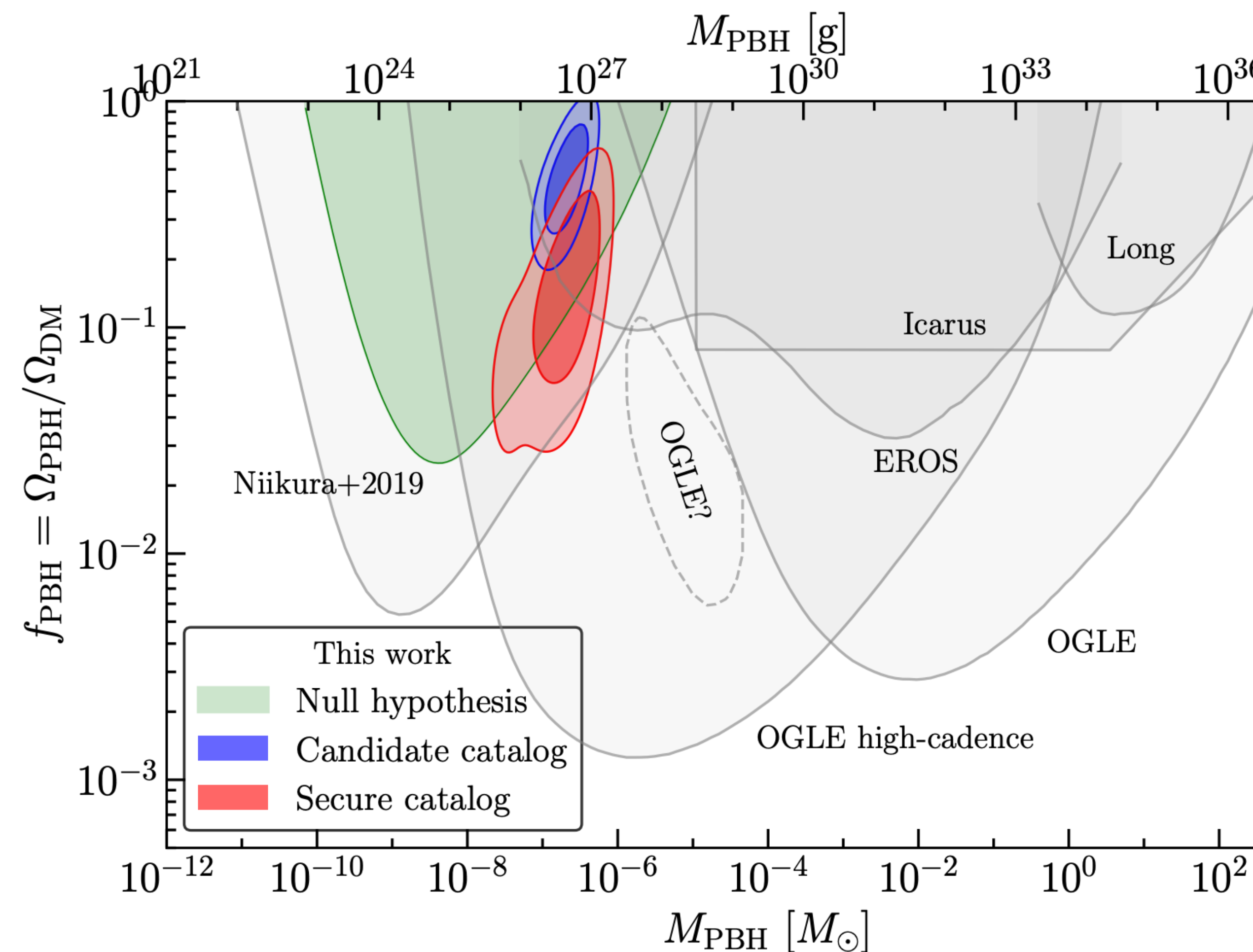
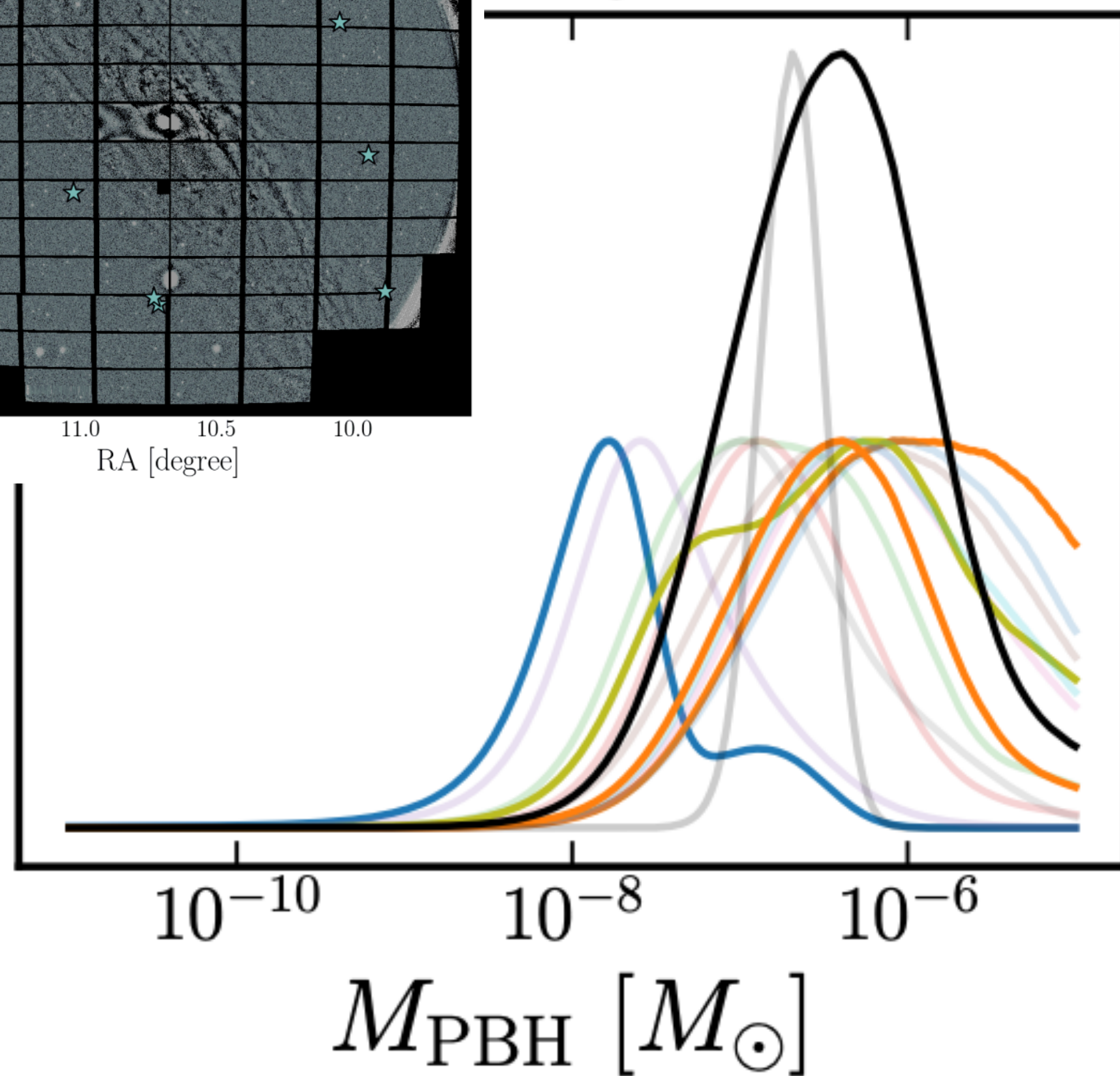
lognormal [SP and Sasaki 2005.12306]

行星质量原初黑洞

Subaru-HSC



$\mathcal{L}_{\text{LC}}$



行星质量原初黑洞

- AMPM is a fast-cadence DECam microlensing survey targeting the LMC to constrain lunar-mass PBHs. With 5 nights operating, it constraints $f_{\text{PBH}} < 0.3$.

AMPM I. — A Targeted Search for Asteroid Mass Primordial Black Hole Microlenses.

Renee Key,^{1,2}★ Edward N. Taylor,¹ Ken C. Freeman,³ Jeremy Mould,^{1,2} Abhijit Saha,⁴ Anais Möller,¹

Timothy M. C. Abbott,⁵ and Alan R. Duffy^{1,2}

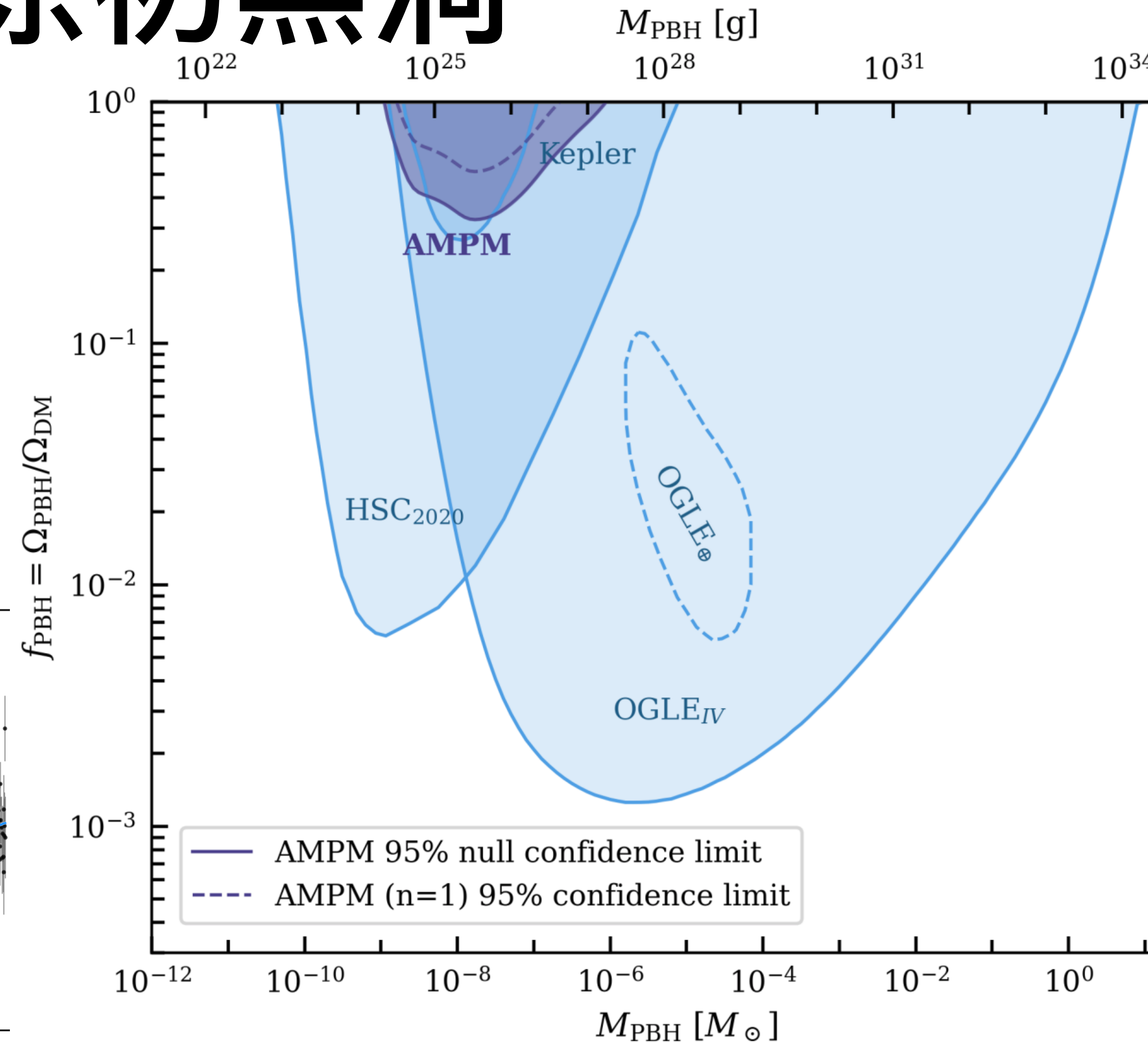
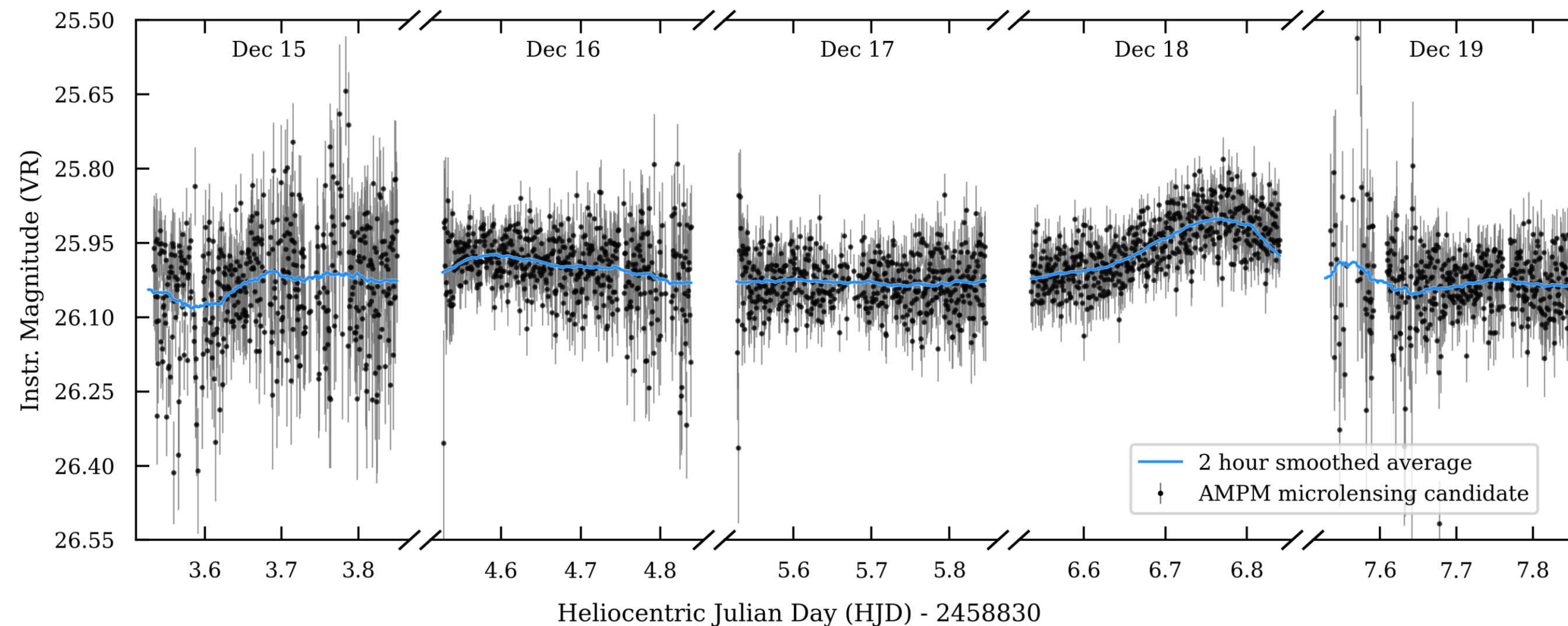
¹Centre for Astrophysics and Supercomputing, Swinburne University of Technology, Melbourne, 3122, VIC, Australia

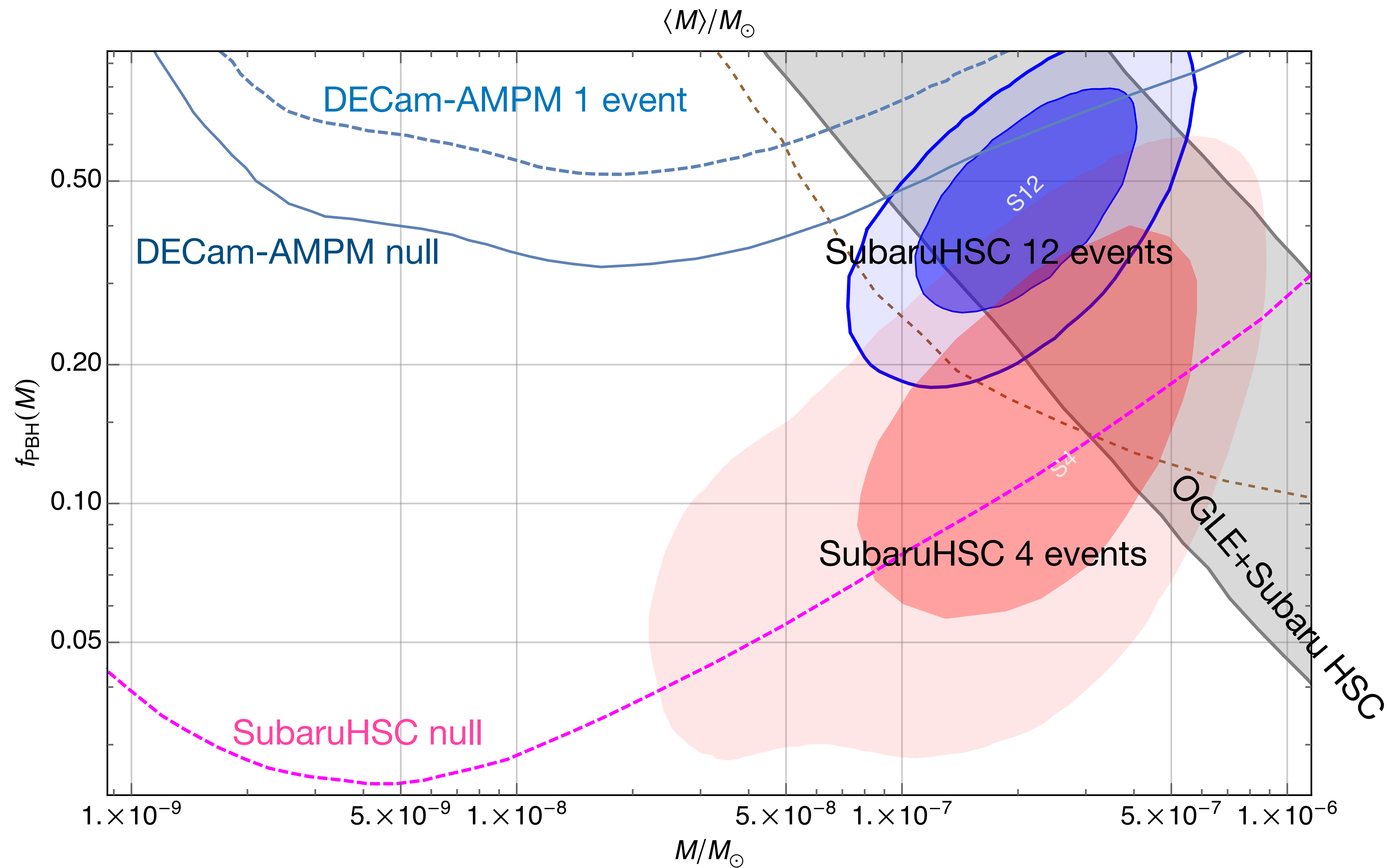
²ARC Centre of Excellence for Dark Matter Particle Physics

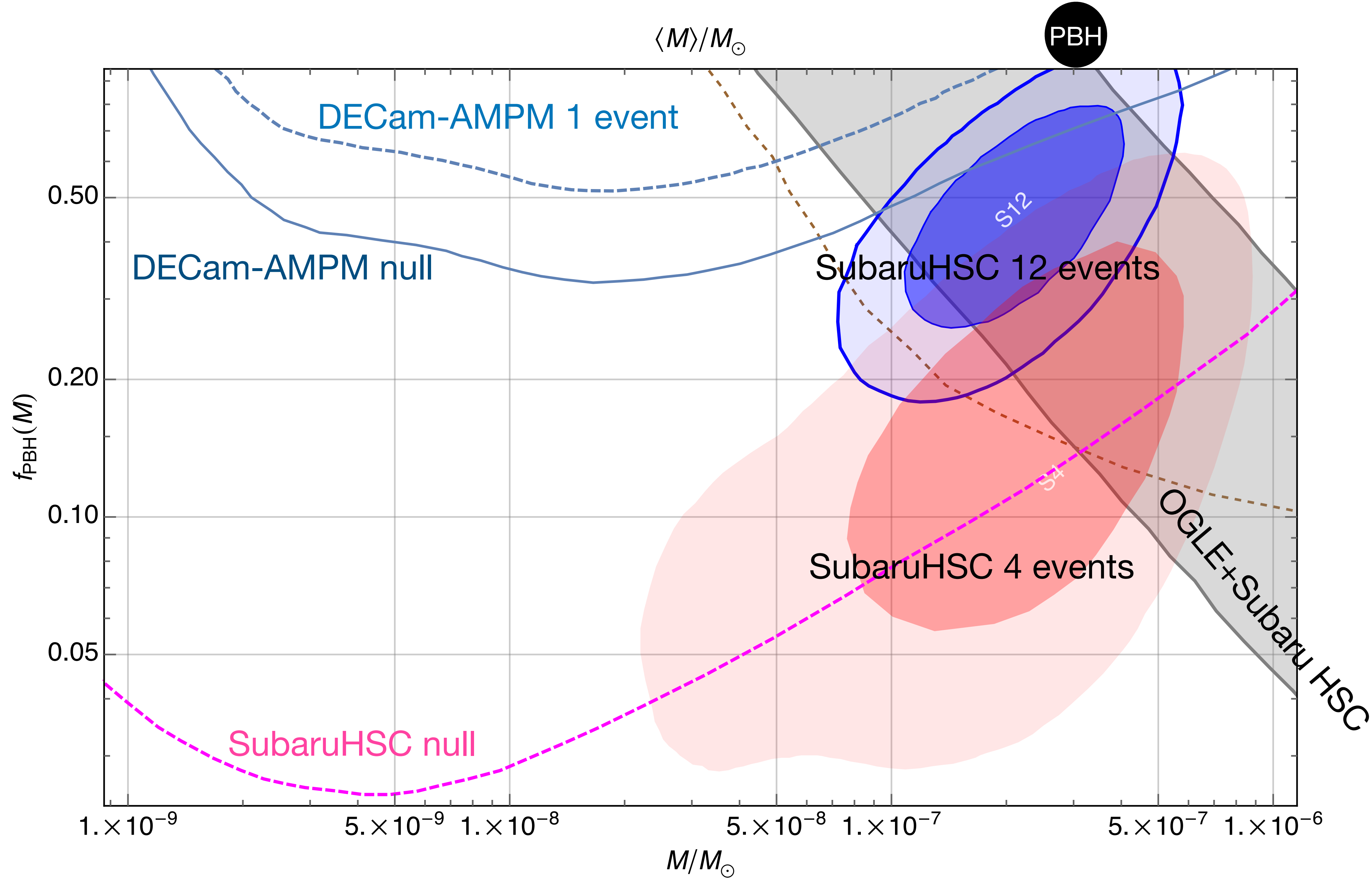
³Research School of Astronomy and Astrophysics Australian National University, Canberra, 2611, ACT, Australia

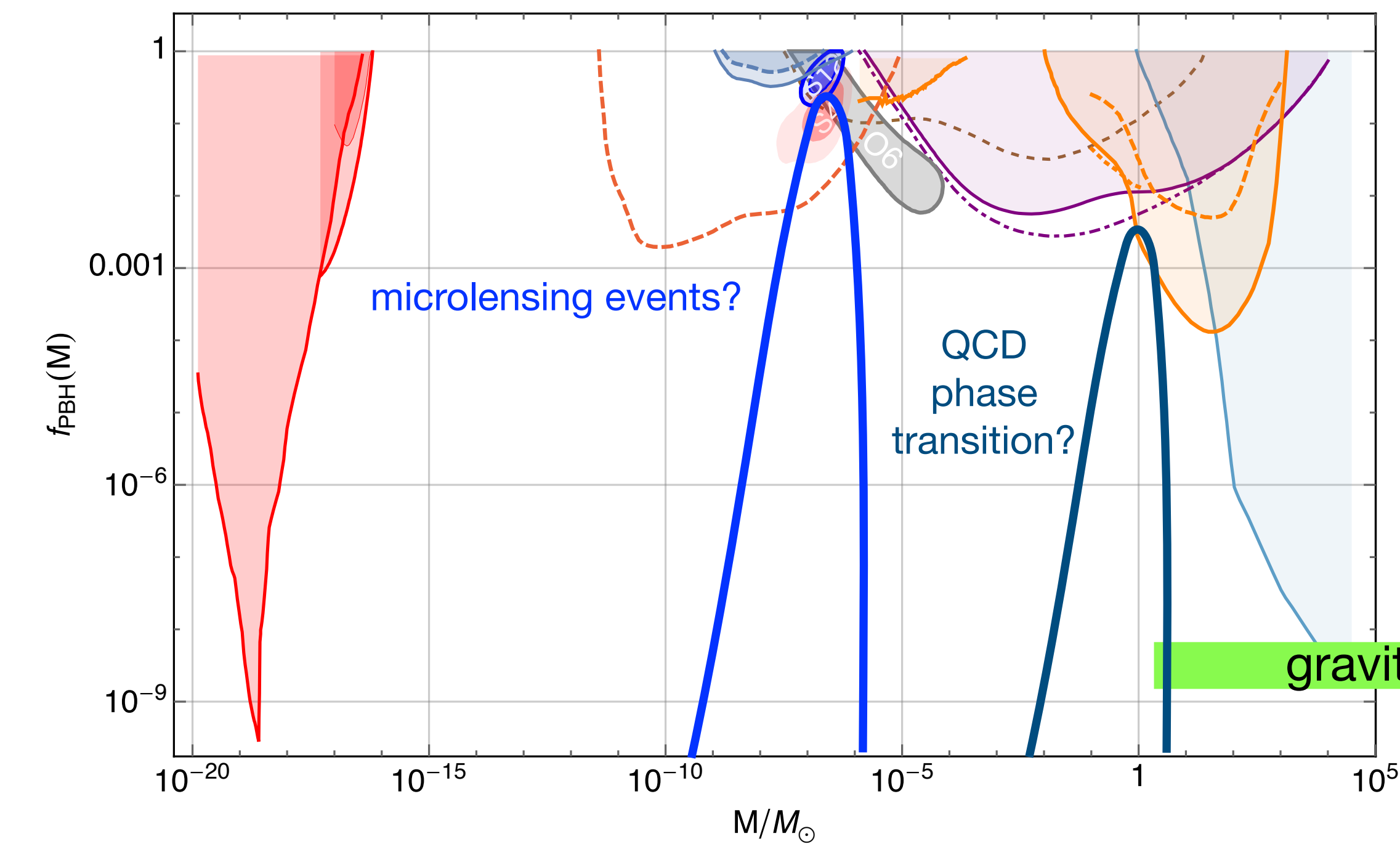
⁴NSF NOIRLab National Optical-Infrared Astronomy Research Laboratory, 950 North Cherry Ave., Tucson, 85719, AZ, USA

⁵Cerro Tololo Inter-American Observatory, Casilla 603, La Serena, Chile



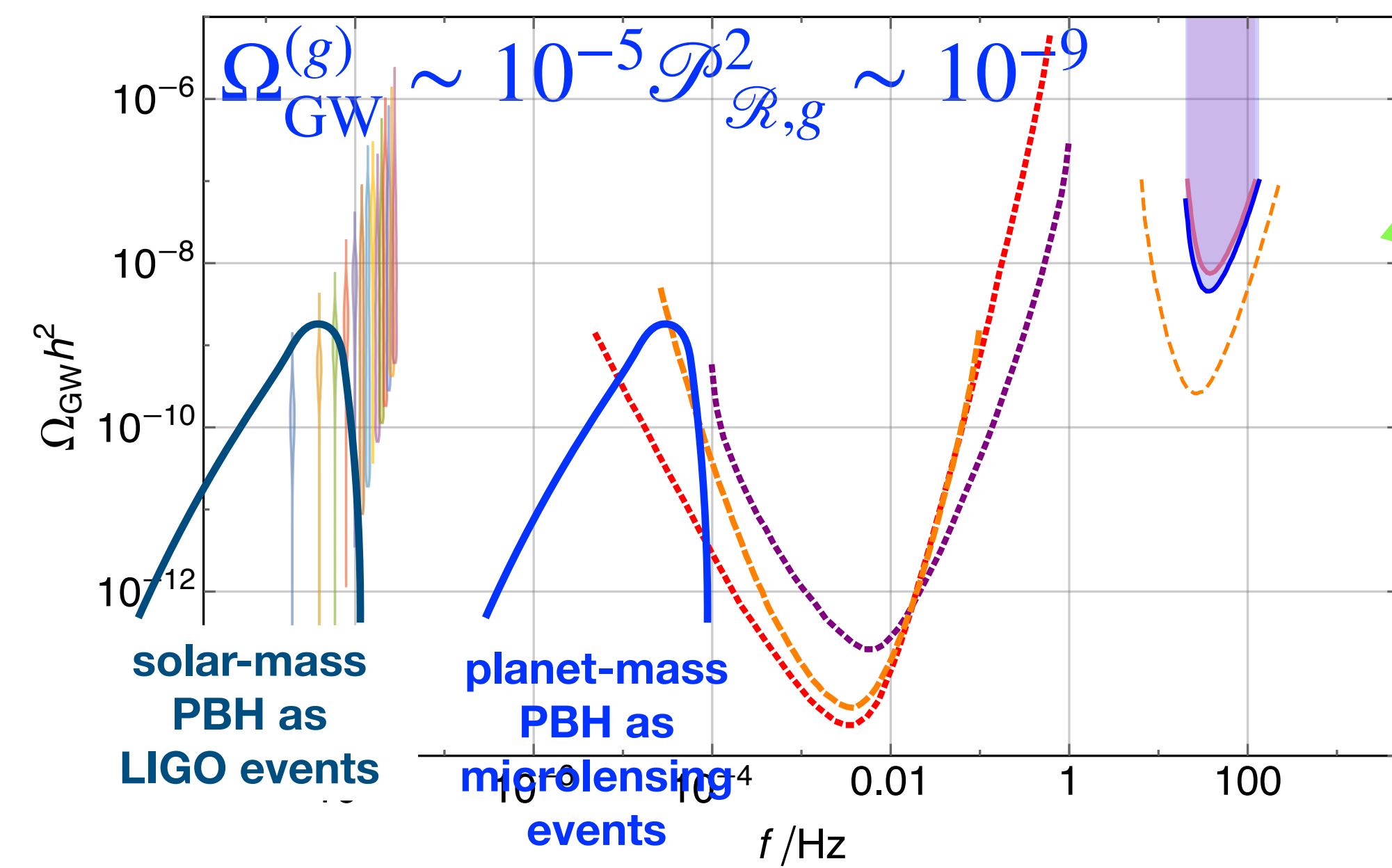




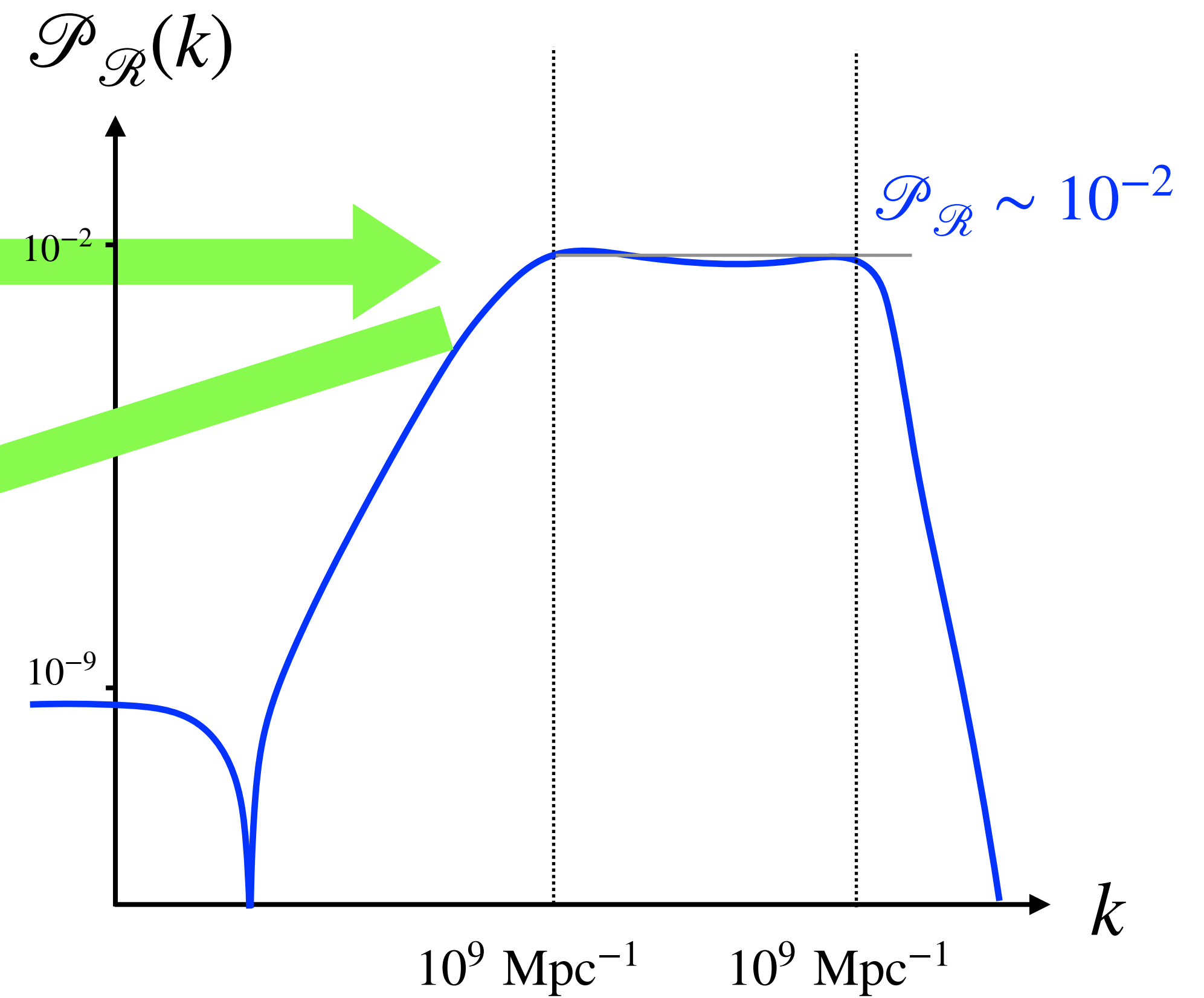


$$f_{\text{IGW}} \sim 3\text{Hz} \left(\frac{M_{\text{PBH}}}{10^{16}\text{g}} \right)^{-\frac{1}{2}}$$

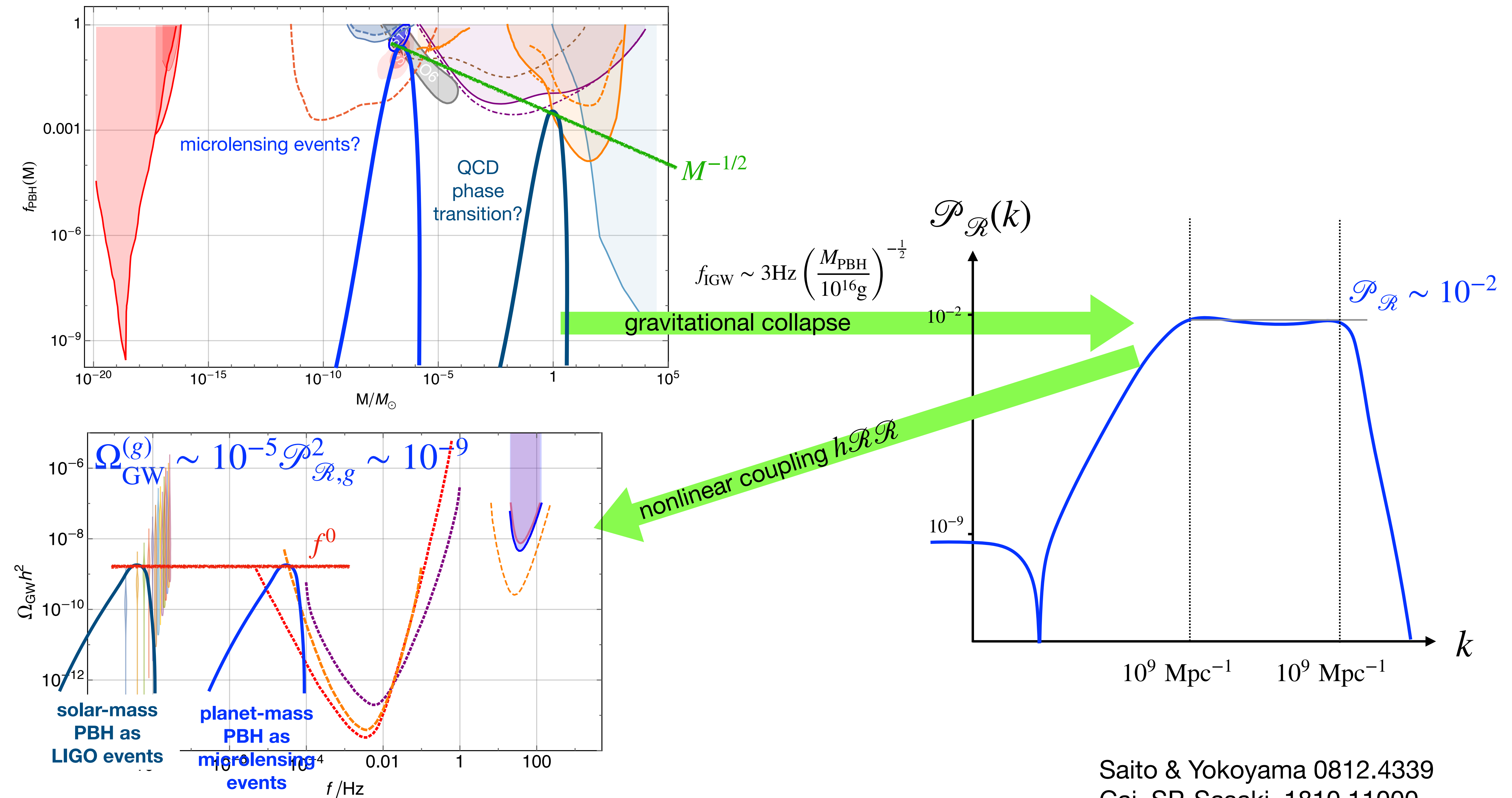
gravitational collapse

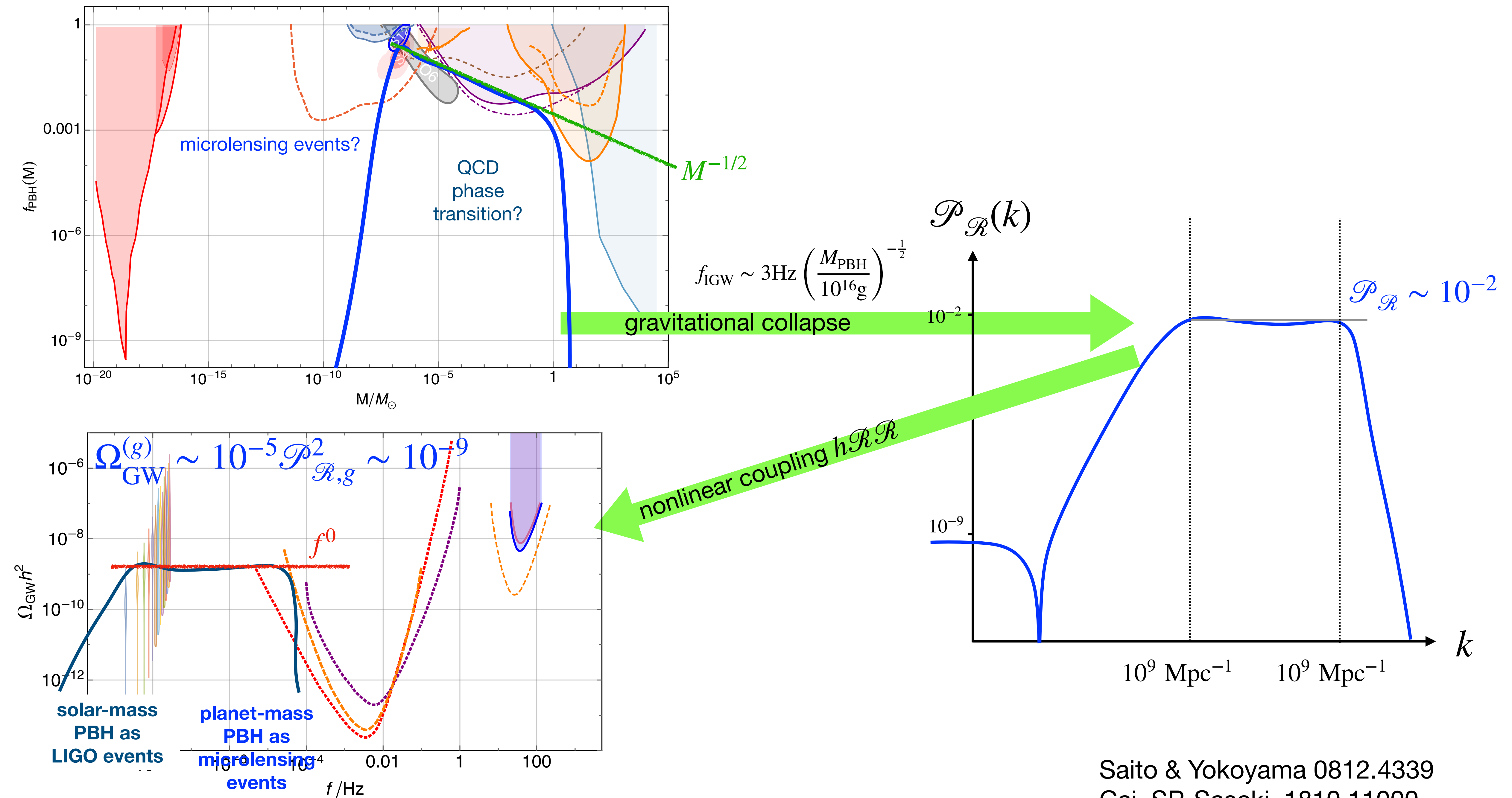


nonlinear coupling $h\mathcal{R}\mathcal{R}$

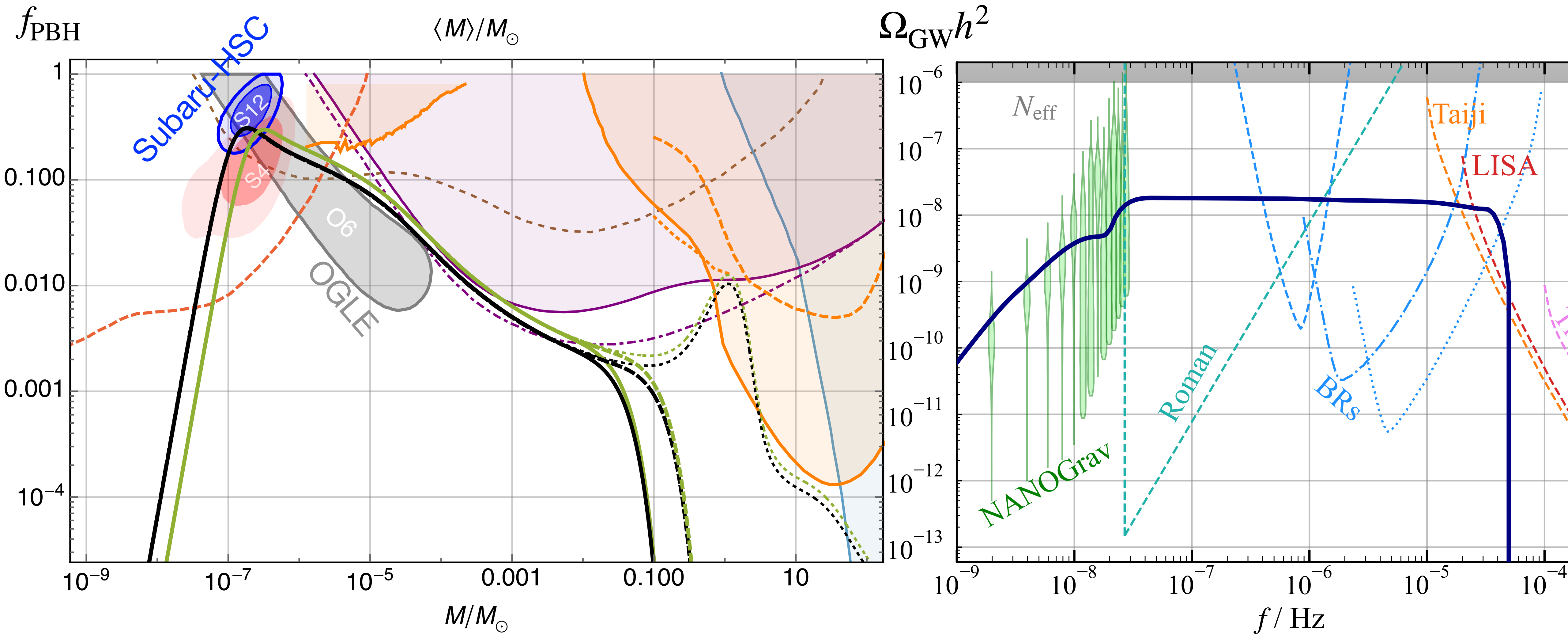


Saito & Yokoyama 0812.4339
Cai, SP, Sasaki, 1810.11000

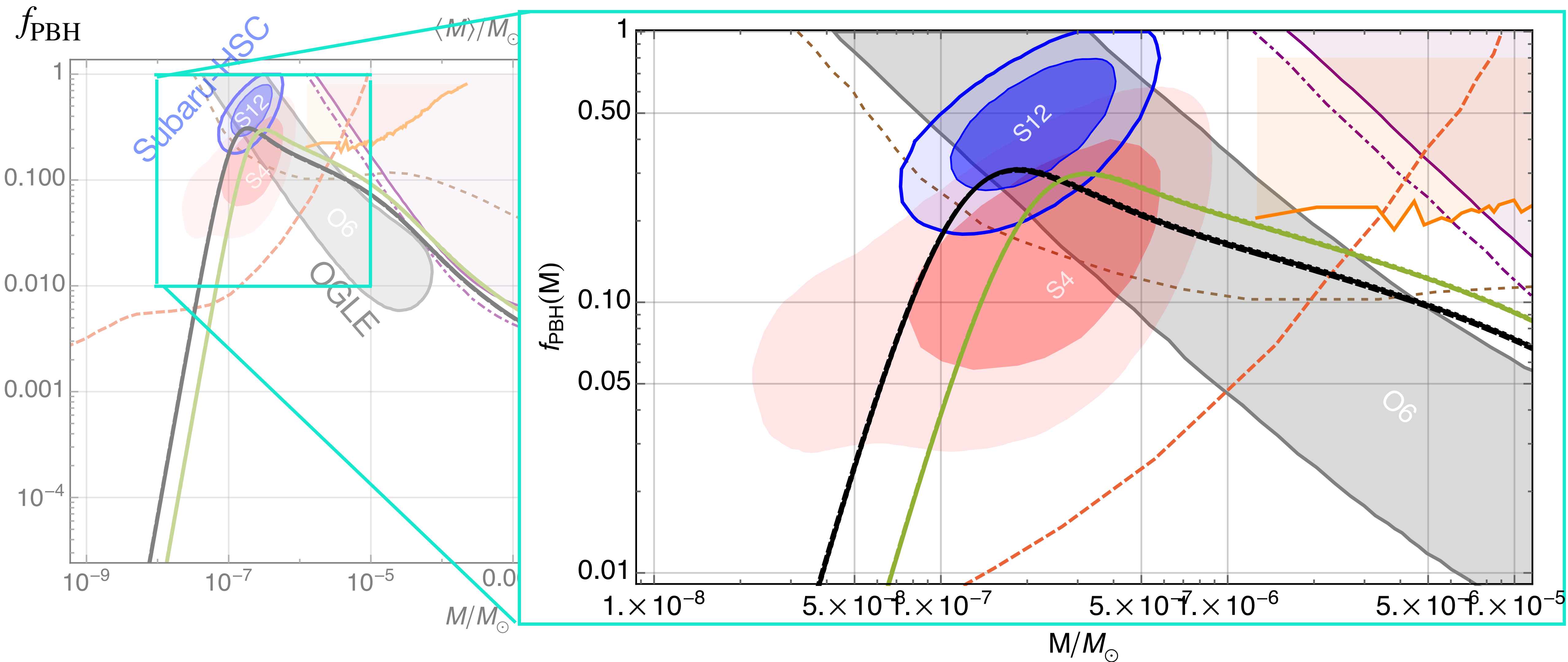




Planet-mass PBH DM



Planet-mass PBH DM



CONTENT

- 理论和观测动机：原初黑洞和诱导引力波
- 诱导引力波的规范问题
- 唯象学和未来的观测
- 非高斯相关的理论问题
 - 对数对偶：非微扰的曲率扰动
 - 局域永恒暴胀和B型原初黑洞
- 结语和讨论

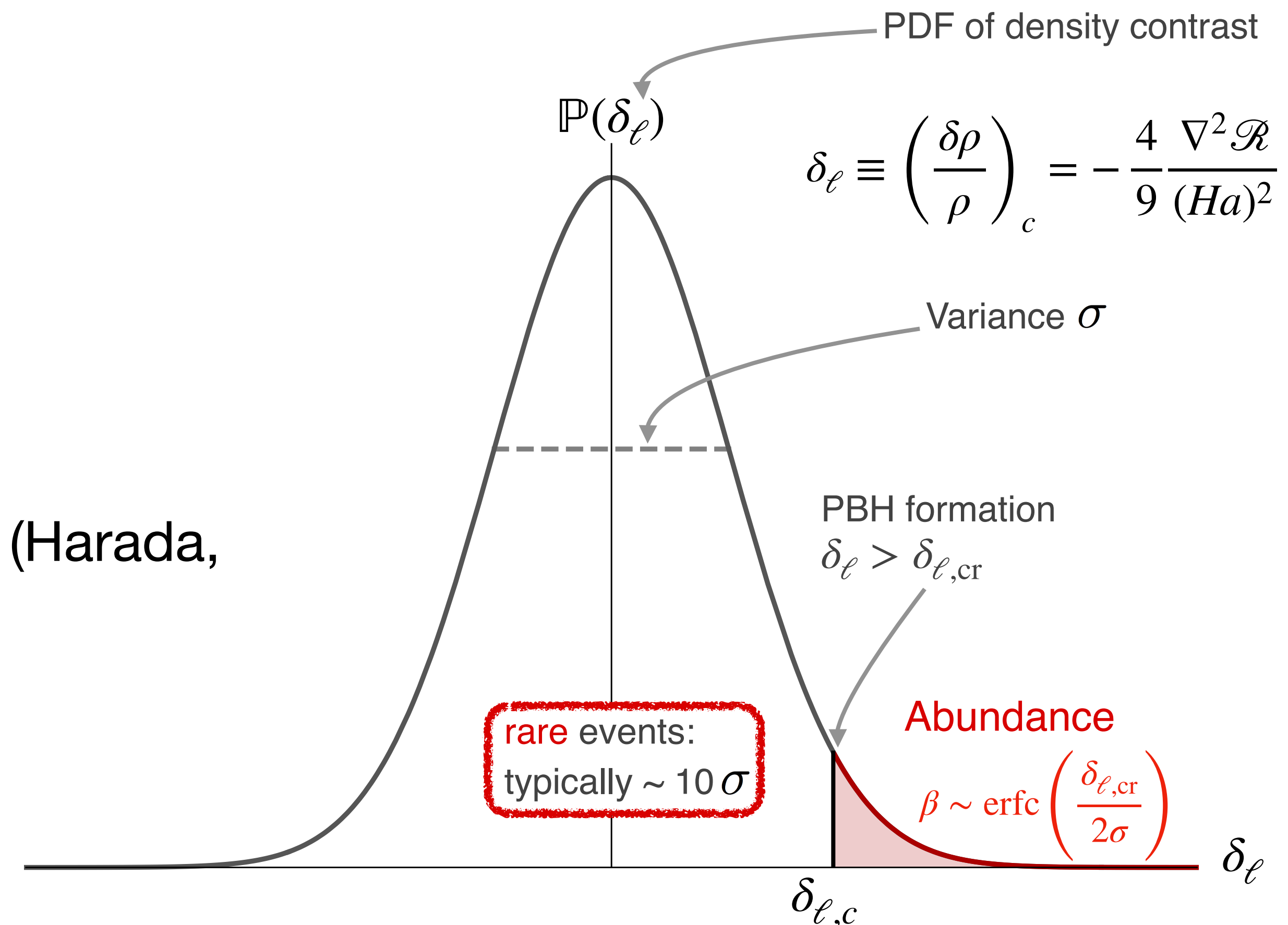
(Simplest) Press-Schechter

$$\left. \begin{array}{l} \mathcal{R} \xrightarrow{(1)} \delta_\ell \\ \mathbb{P}(\mathcal{R}) \xrightarrow{(2)} \mathbb{P}(\delta_\ell) \end{array} \right\} \xrightarrow[\text{(4) Window function}]{\text{(3) given } \delta_{\ell,\text{cr}}} \beta = \int_{\delta_{\ell,\text{cr}}} \mathbb{P}(\delta_\ell) \frac{M(\delta_\ell)}{M_H} d\delta_\ell \sim \text{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma_\delta}\right)$$

Every step is linear/Gaussian:

- (1) Linear Poisson equation.
- (2) Gaussian PDF $\mathbb{P}(\mathcal{R})$ gives Gauss PDF $\mathbb{P}(\delta_\ell)$:

$$\mathbb{P}(\mathcal{R})d\mathcal{R} = \mathbb{P}(\delta_\ell)d\delta_\ell$$
- (3) Critical density contrast $\delta_{\ell,\text{cr}}$ given by HYK limit (Harada, Yoo, Kohri, 1309.4201).
- (4) Window function.



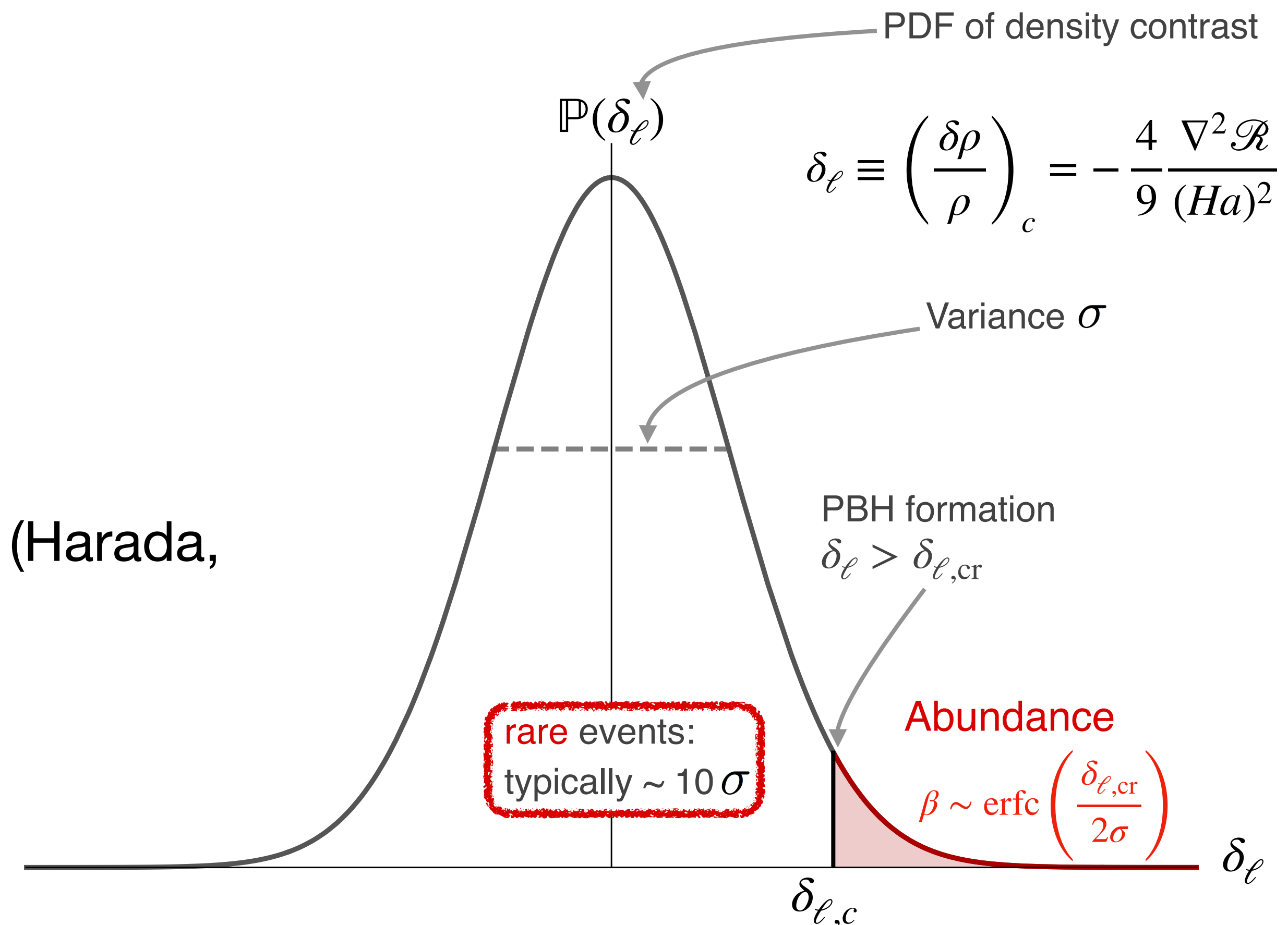
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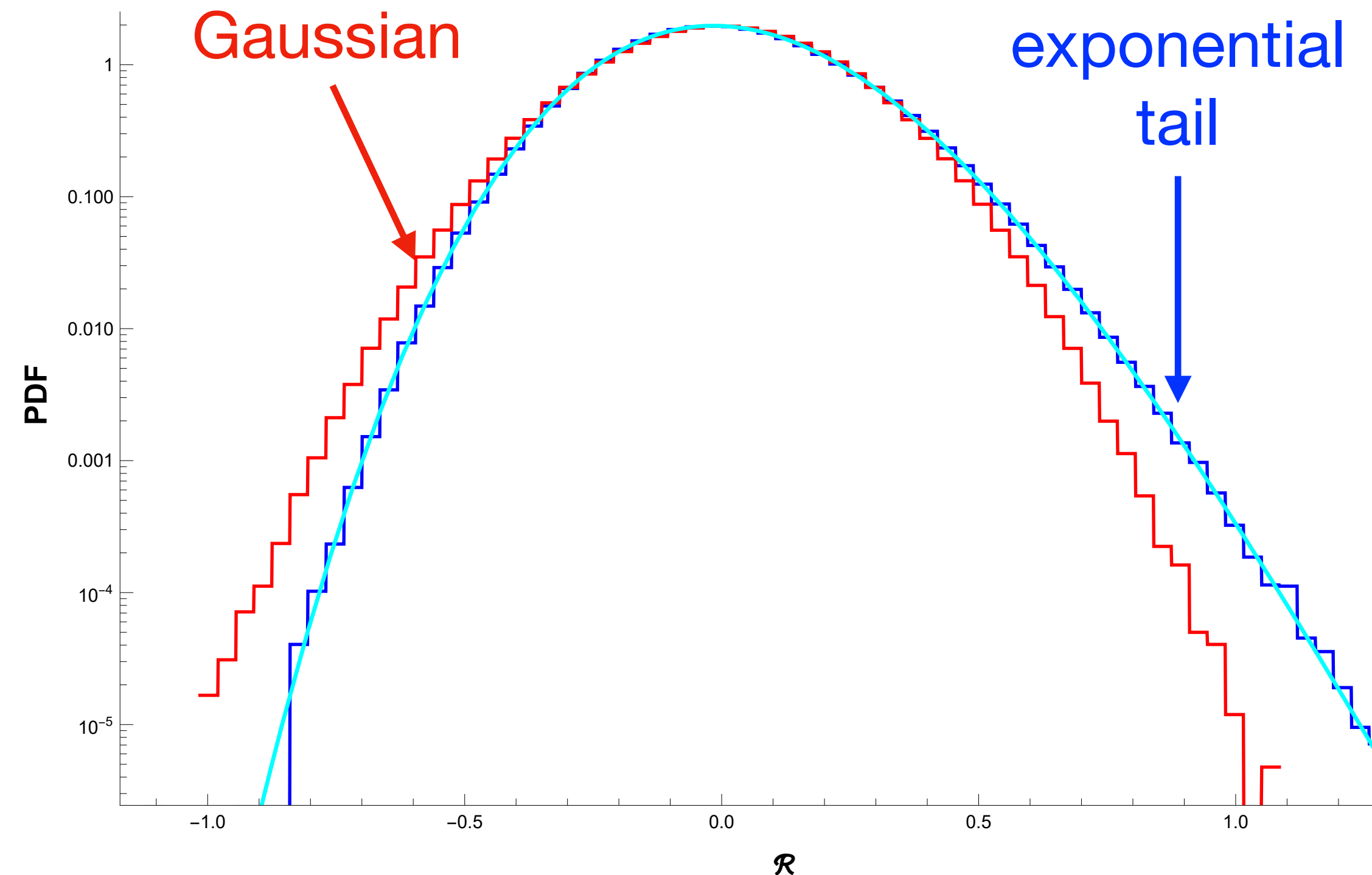


Why non-Gaussianity?

$$\left. \begin{array}{l} \mathcal{R} \xrightarrow{(1)} \mathcal{C}_\ell \\ \mathbb{P}(\mathcal{R}) \xrightarrow{(2)} \mathbb{P}(\mathcal{C}_\ell) \end{array} \right\} \xrightarrow[\text{(4) Window function}]{\text{(3) given } \mathcal{C}_{\text{cr}}} \beta = \int_{\mathcal{C}_{\ell, \text{cr}}}^{4/3} \mathbb{P}(\mathcal{C}_\ell) \frac{M(\mathcal{C}_\ell)}{M_H} d\mathcal{C}_\ell$$

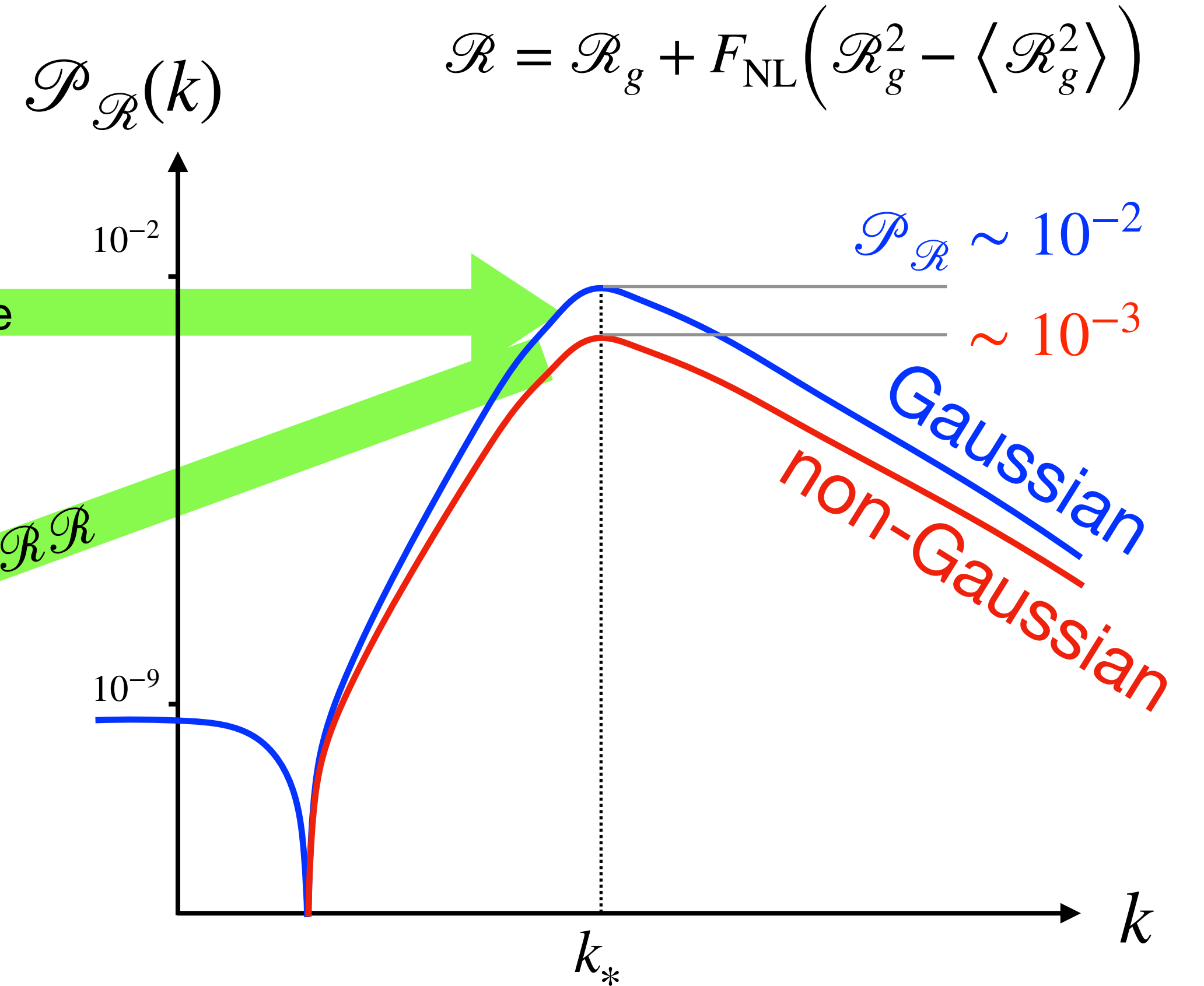
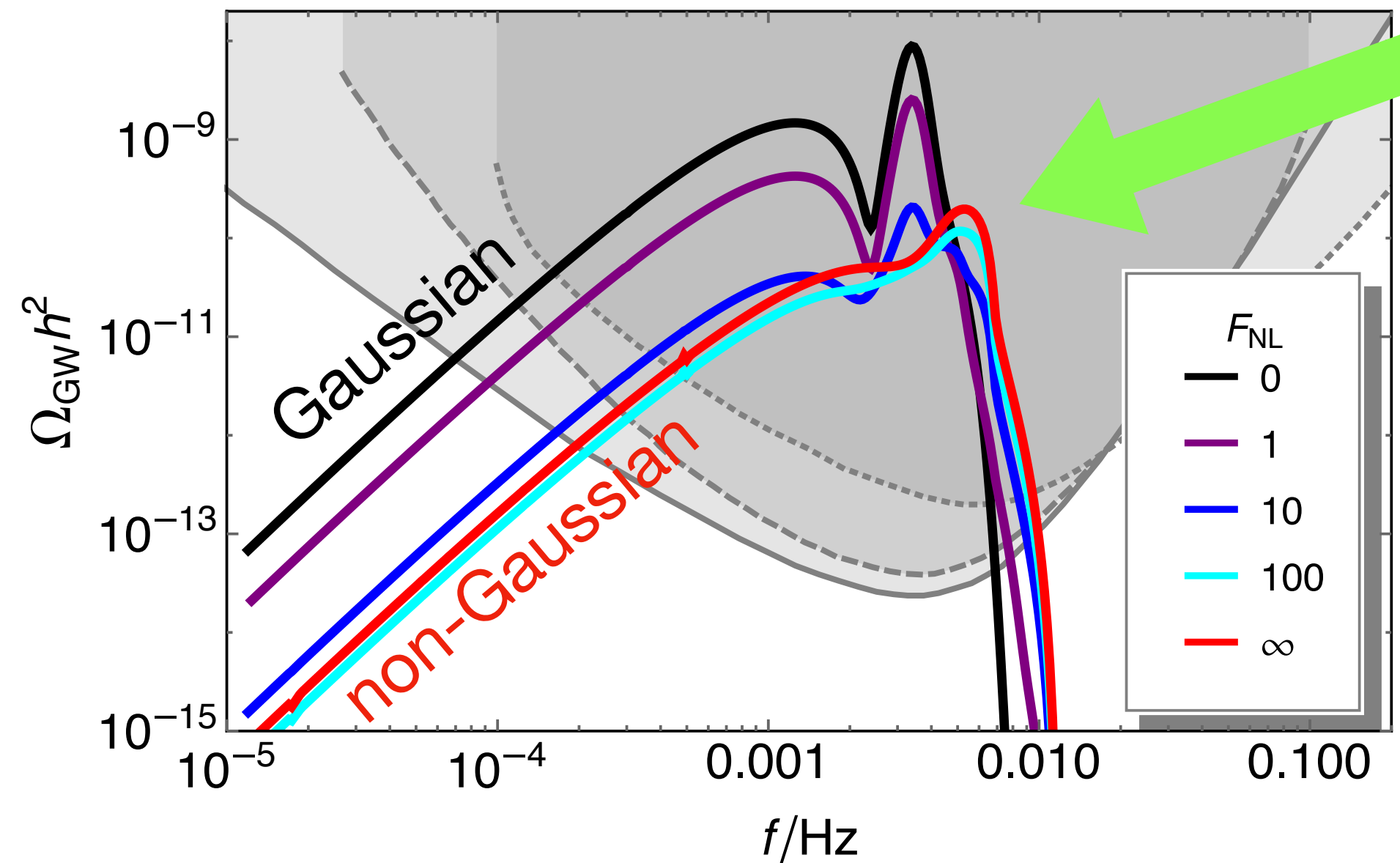
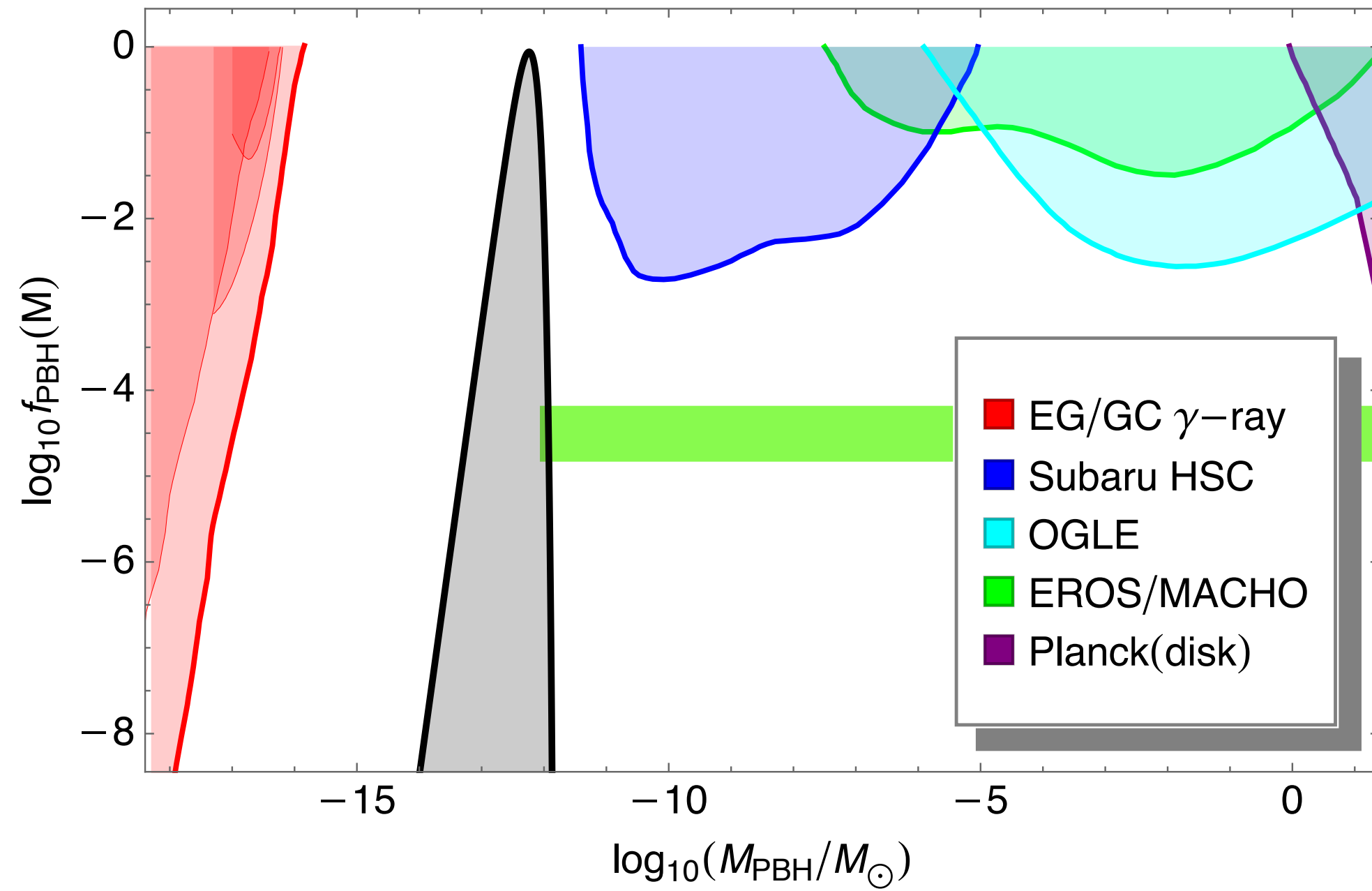
Non-Gaussianity must be taken into account:

- (1) Use compaction function $\mathcal{C}$ which nonlinearly depends on $\mathcal{R}$. (Harada et al 1503.03934; De Luca et al 1904.00970; Gow et al 2211.08348)
- (2) Primordial non-Gaussianity of $\mathcal{R}$ is usually non-perturbative.
- (3) $\mathcal{C}_{\text{cr}}$ depends on profile. (Musco 1809.02127; Escrivà et al 1907.13311)



Including non-Gaussianity

PBH=DM

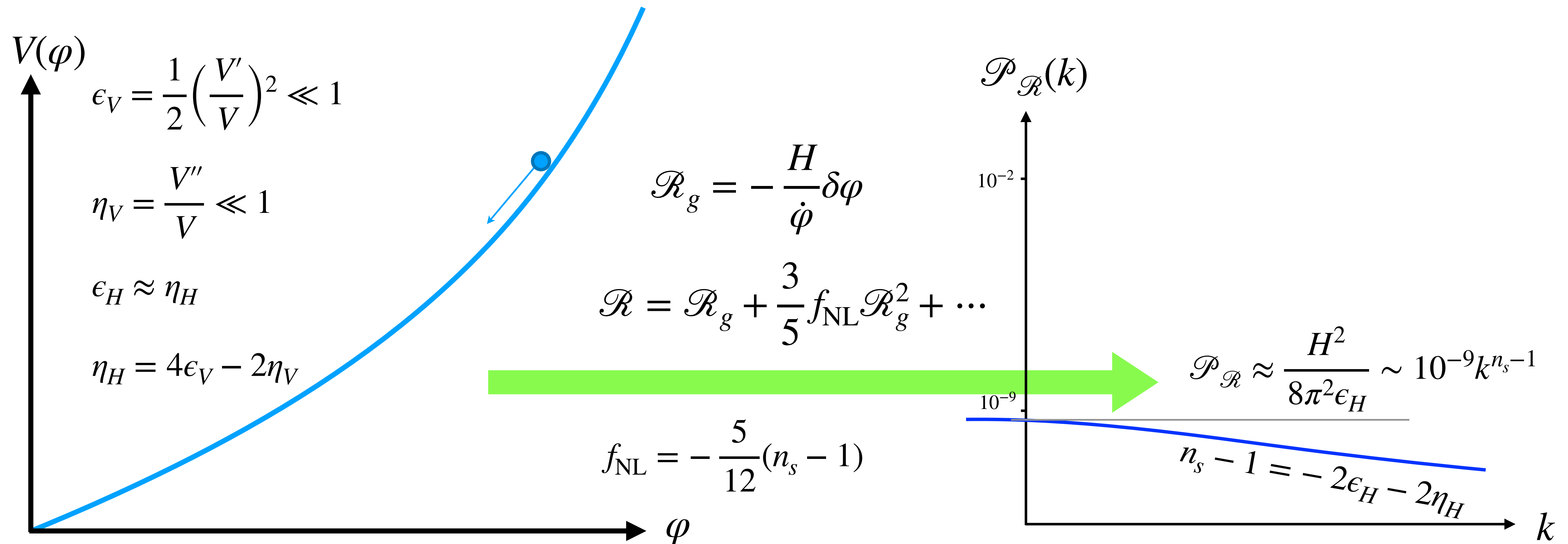


$$F_{\text{NL}} \mathcal{P}_{\mathcal{R}} \sim 10^{-2}$$

$$\Omega_{\text{GW}}^{(\text{NG})} \sim 10^{-5} F_{\text{NL}}^4 \mathcal{P}_{\mathcal{R},g}^4 \sim 10^{-12} > \text{LISA}$$

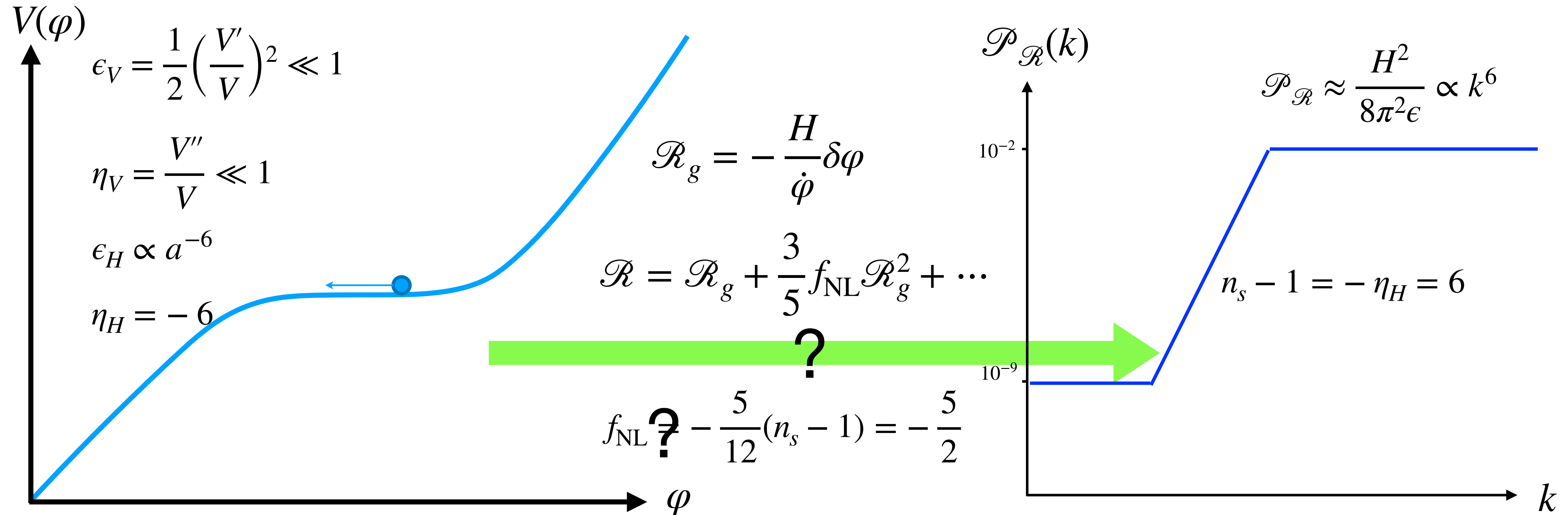
LISA/Taiji/TianQin can probe PBH-DM.

Slow-roll inflation

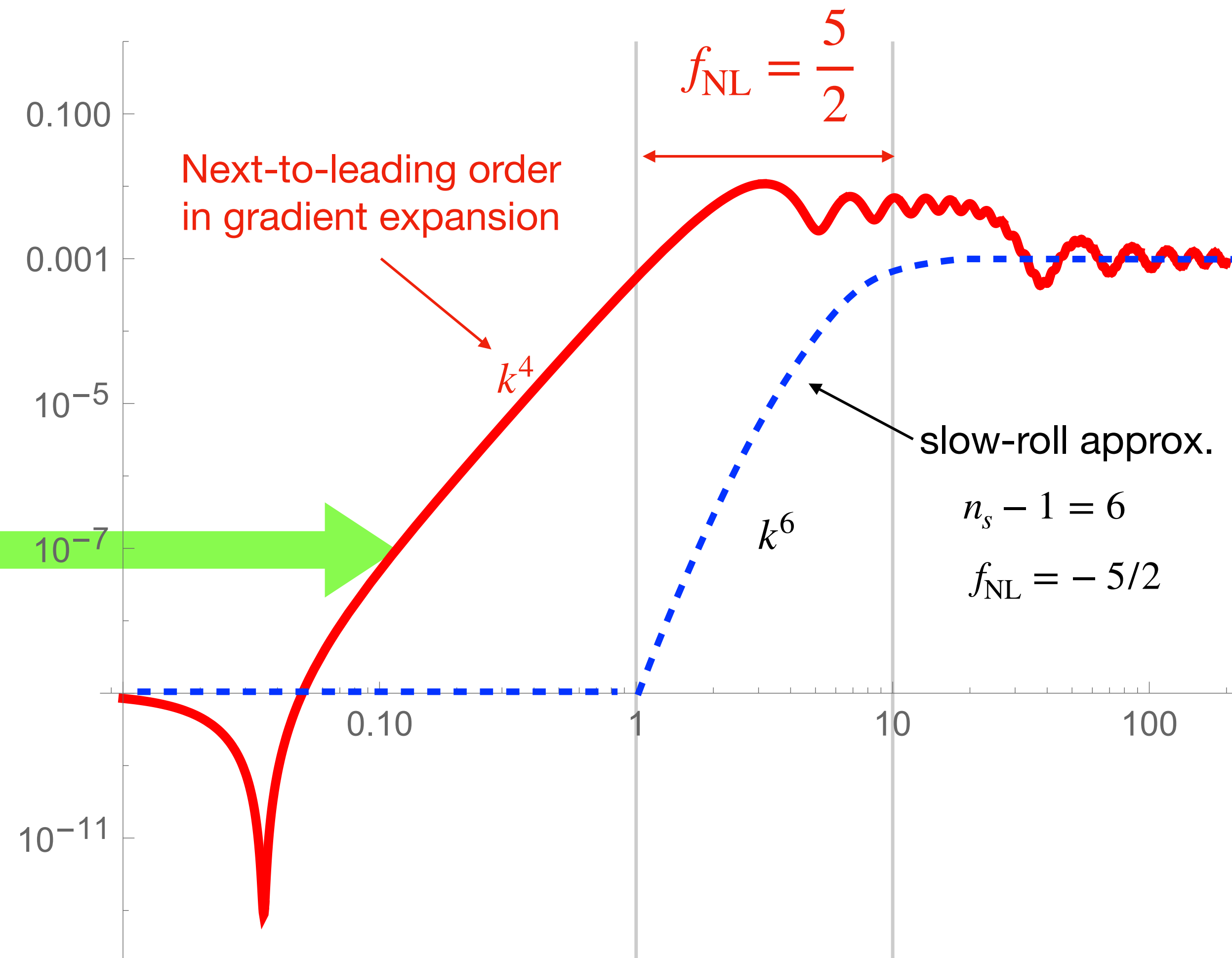
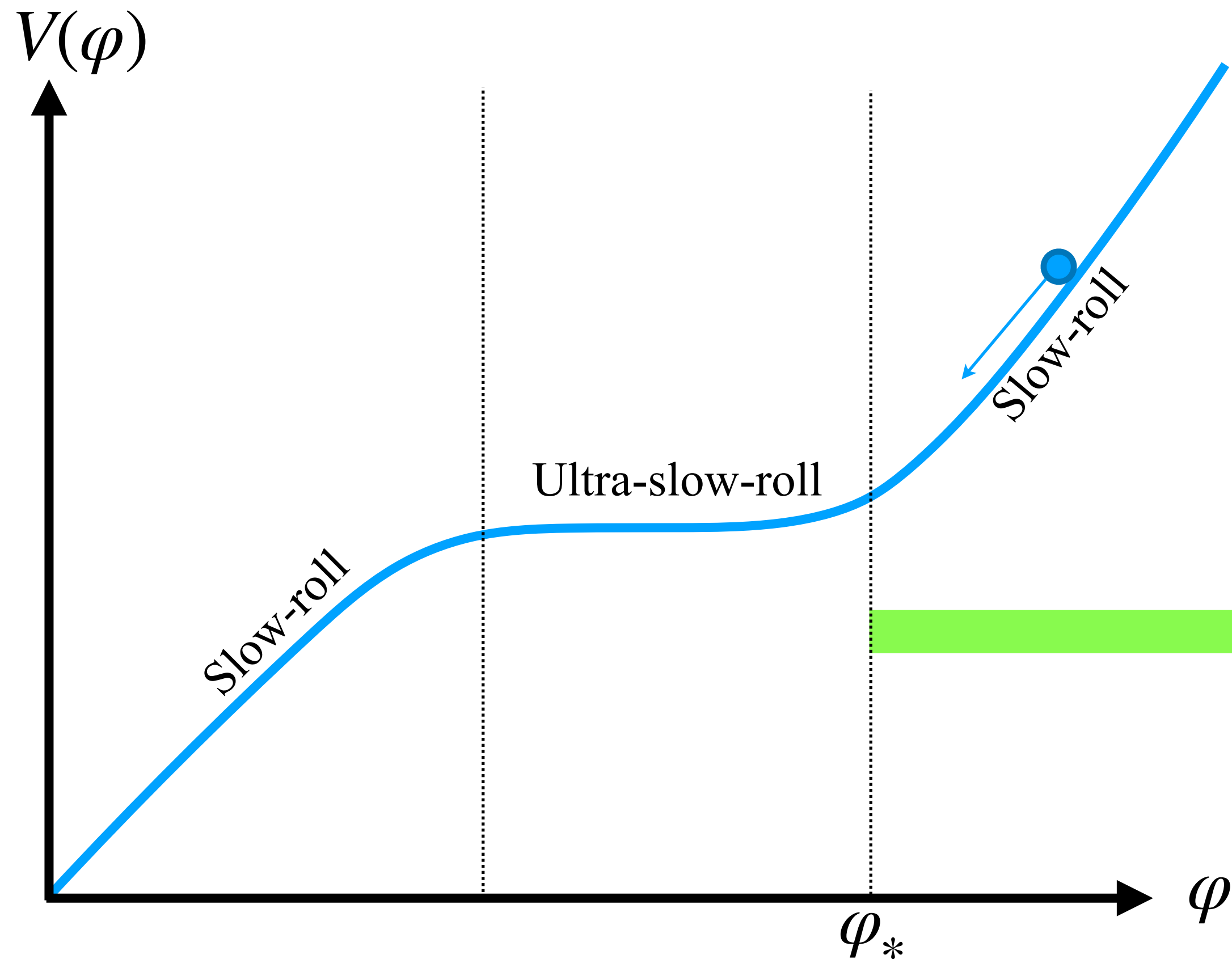


Stewart and Sasaki, astro-ph/9507001
 Lyth and Roquigez, astro-ph/0504045
 Maldadena, astro-ph/0210603

Ultra-slow-roll inflation

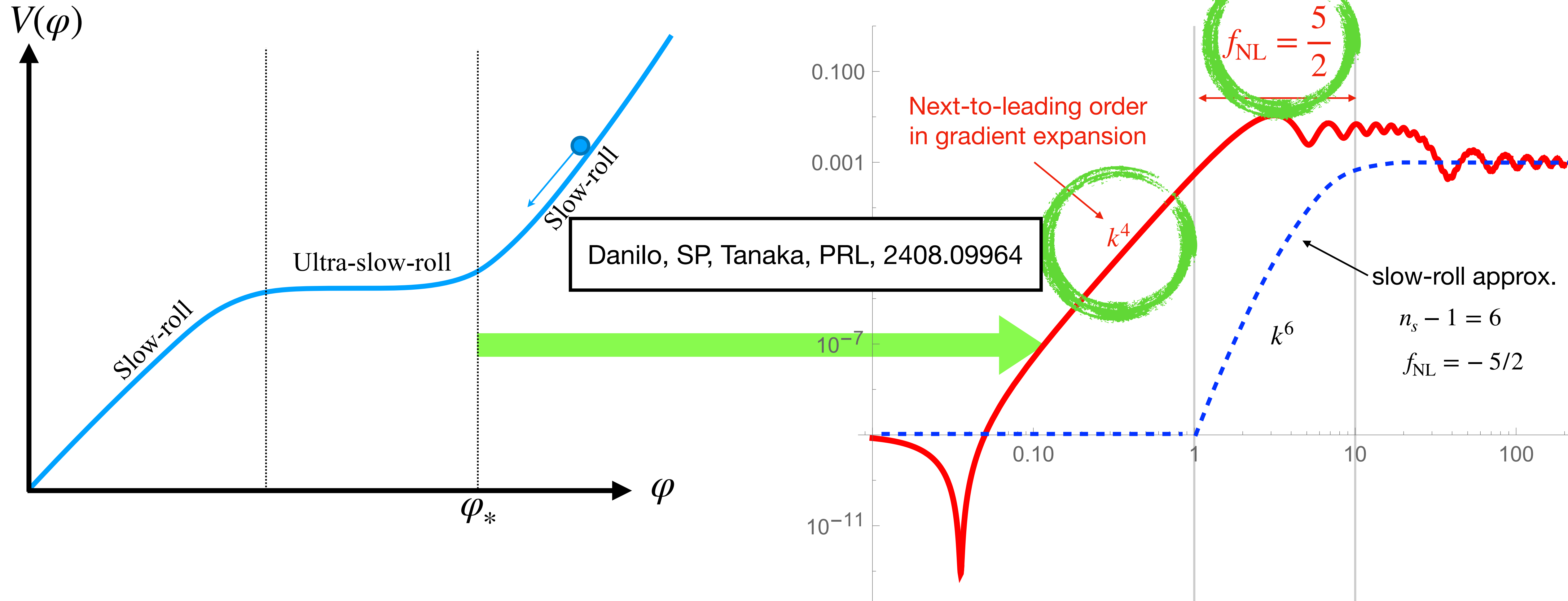


Ultra-slow-roll inflation



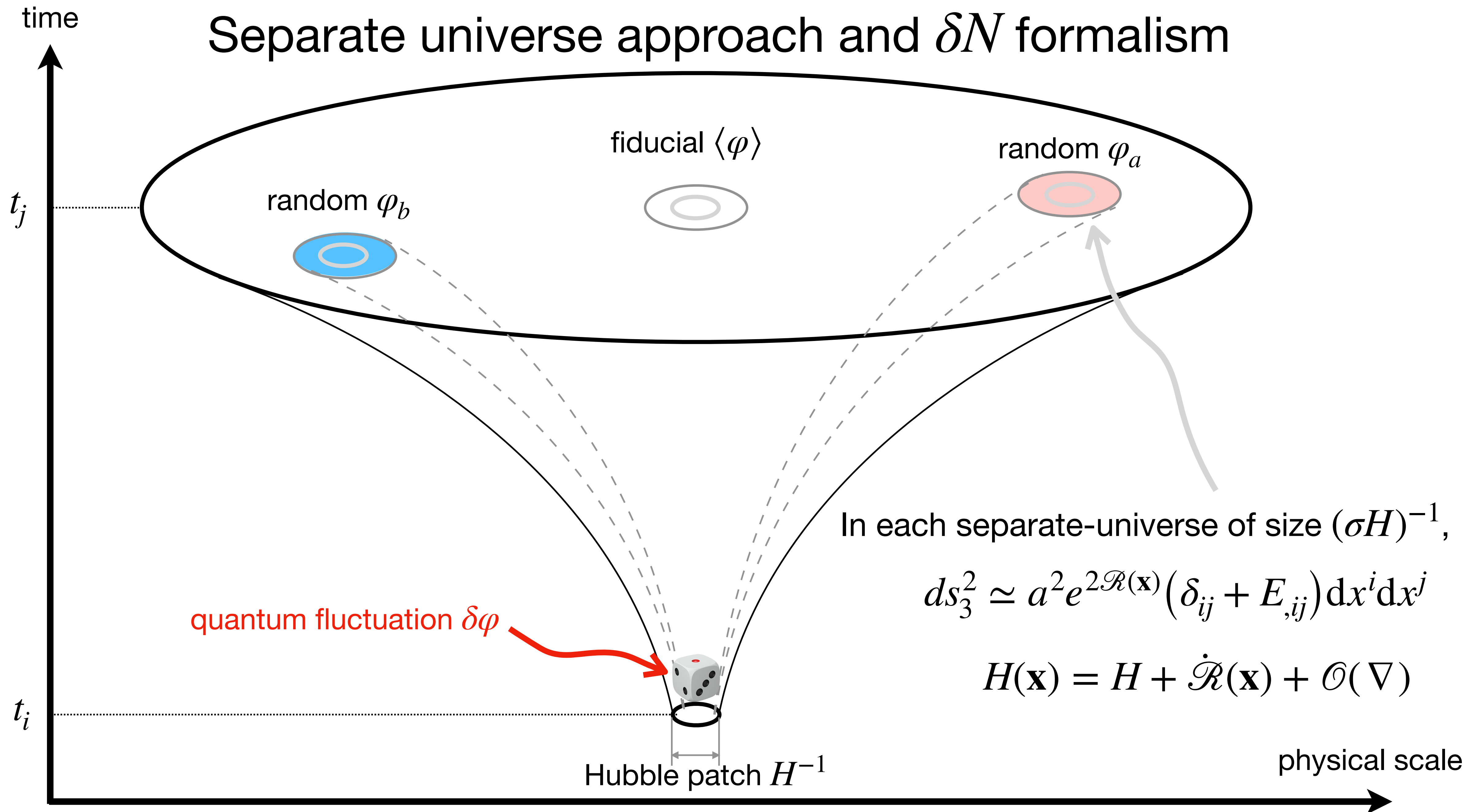
Ultra-slow-roll inflation

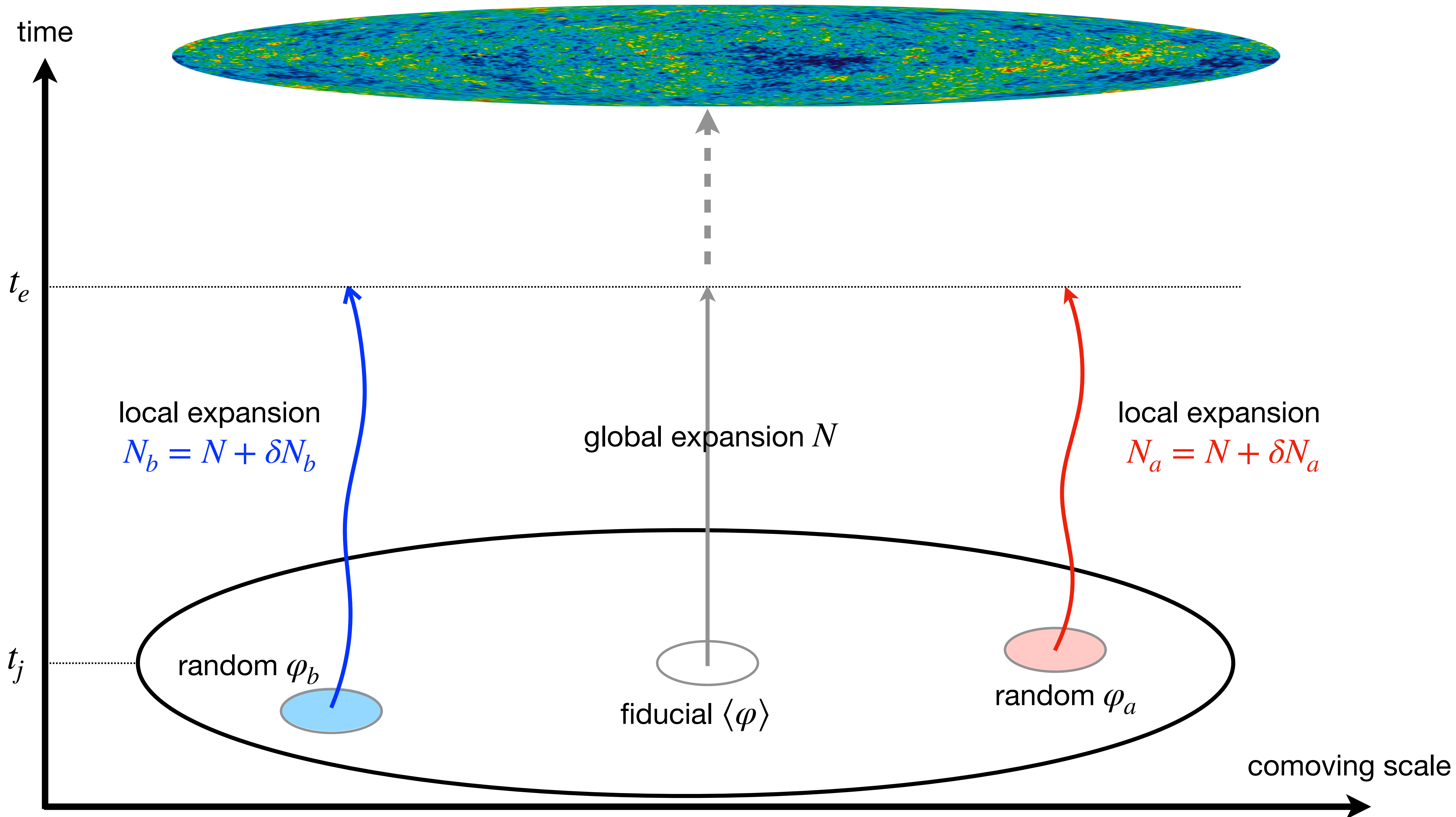
SP, Sasaki, PRL, 2211.13932



Danilo, SP, Tanaka, PRL, 2408.09964

Separate universe approach and δN formalism

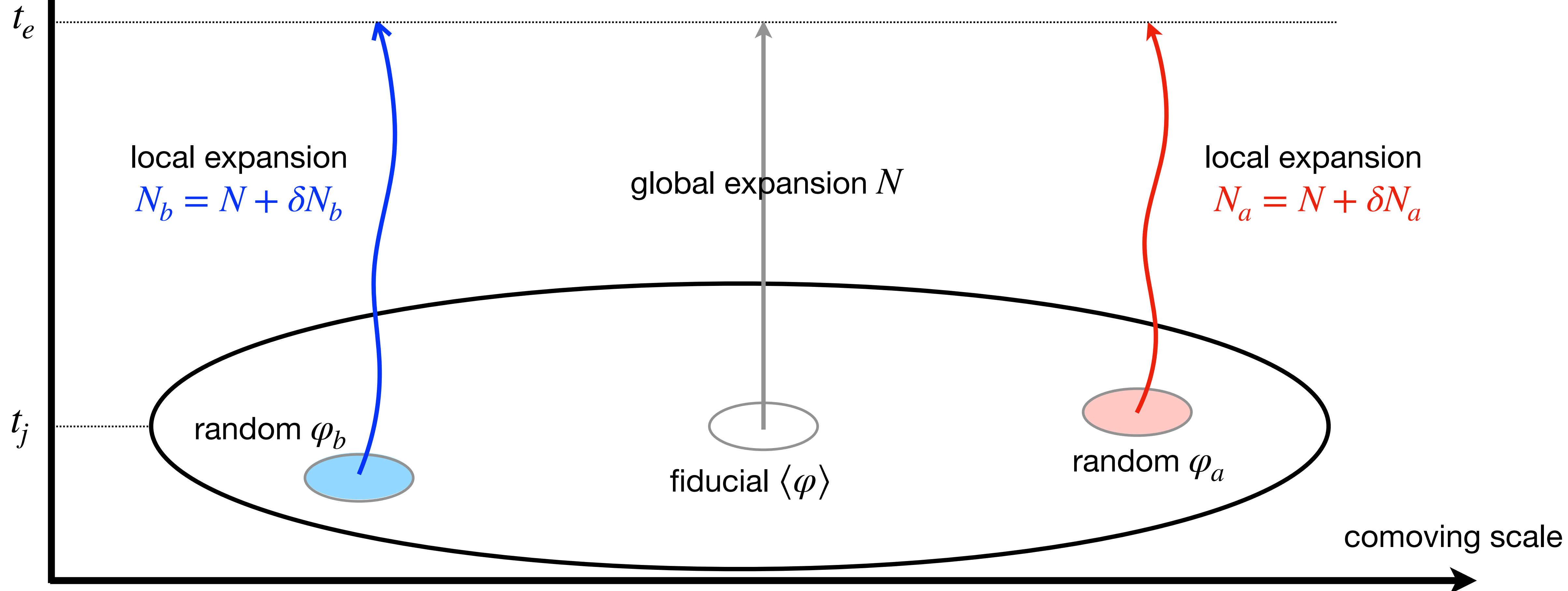




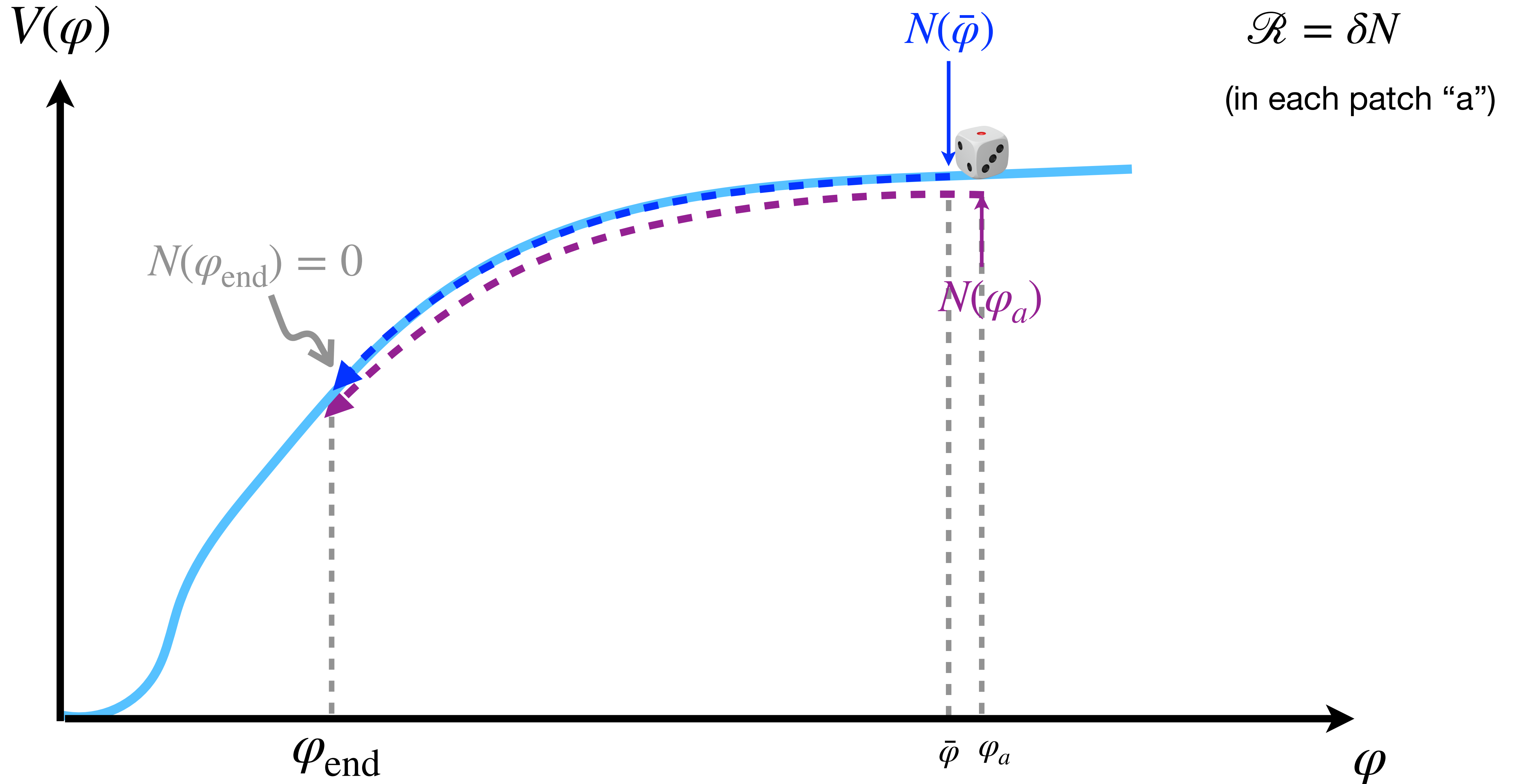
Starobinsky, *JETP Lett.* 42 (1985) 152
 Salopek and Bond, *PRD* 42 (1990) 3936
 Sasaki and Stewart, *astro-ph/9507001*
 Wands, Malik, Lyth, Liddle, *astro-ph/0003278*

$$N_a = \int_{t_j}^{t_e} H_a dt = \int_{t_j}^{t_e} (H + \dot{\mathcal{R}}_a) dt + \frac{1}{3} \int \nabla^2 \sigma dt \approx N + \mathcal{R}(t_e) - \mathcal{R}(t_j) = N + \mathcal{R}_c(t_e) \quad \Rightarrow \quad \mathcal{R}_c = \delta N$$

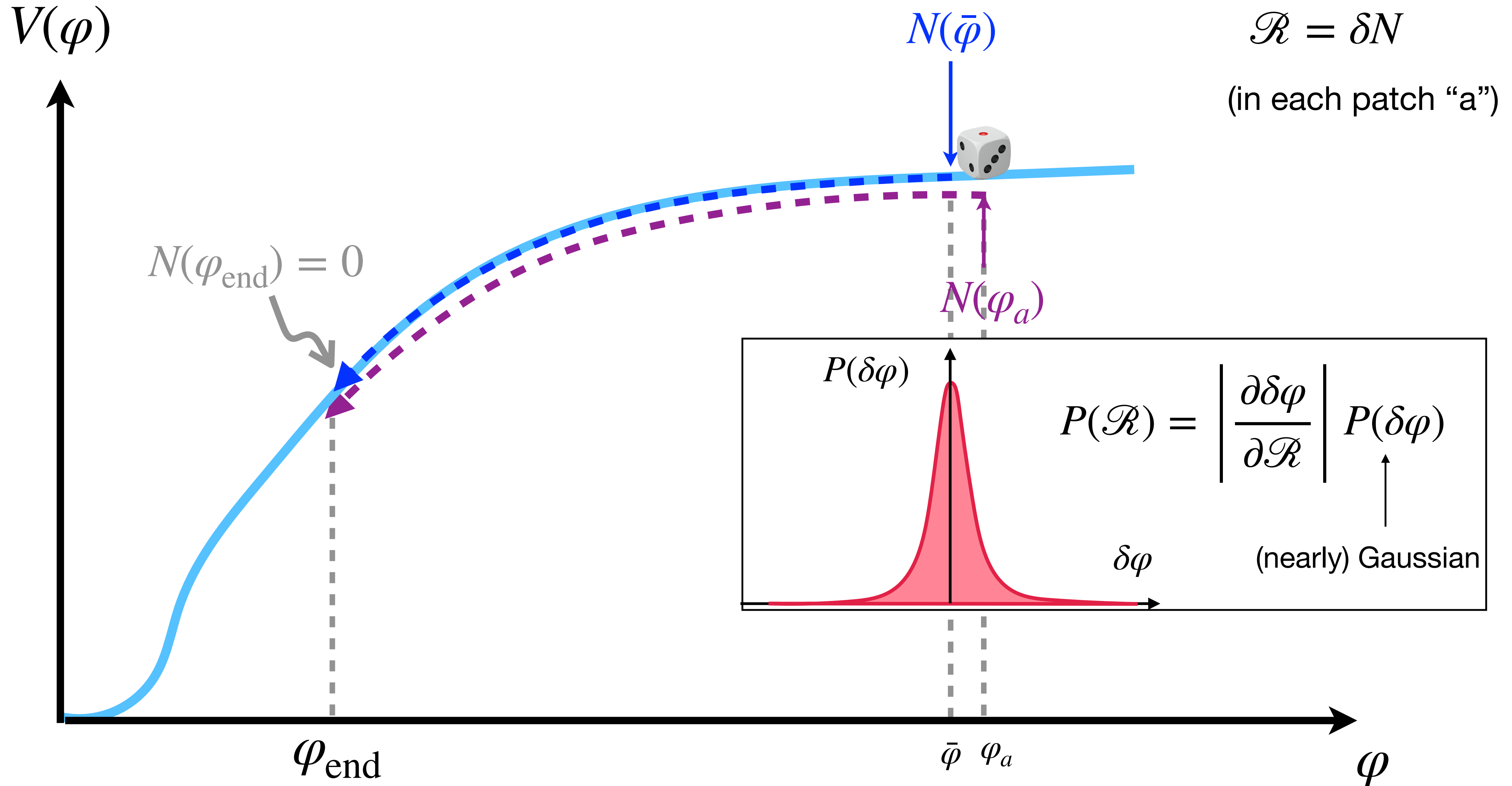
(in each patch “a”)



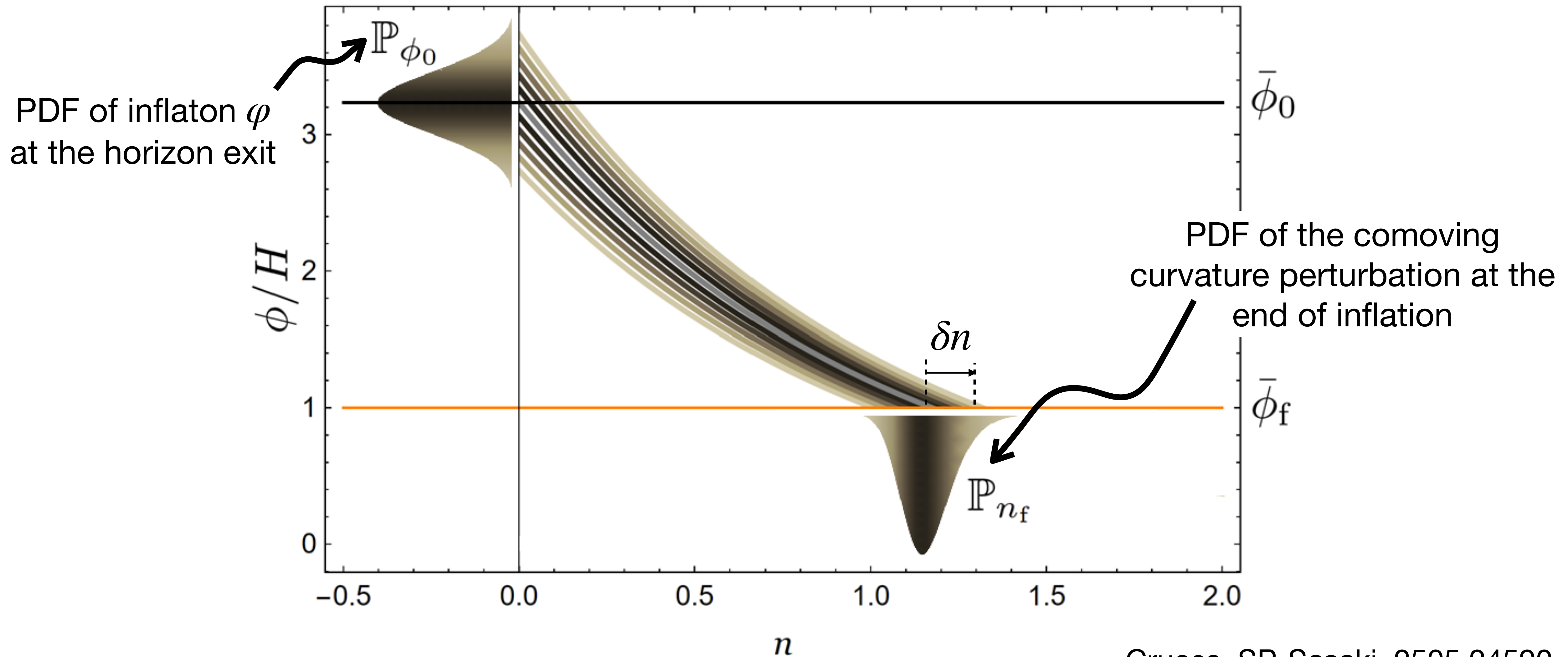
Starobinsky, *JETP Lett.* 42 (1985) 152
Salopek and Bond, *PRD* 42 (1990) 3936
Sasaki and Stewart, *astro-ph/9507001*
Wands, Malik, Lyth, Liddle, *astro-ph/0003278*



Starobinsky, *JETP Lett.* 42 (1985) 152
 Salopek and Bond, *PRD* 42 (1990) 3936
 Sasaki and Stewart, *astro-ph/9507001*
 Wands, Malik, Lyth, Liddle, *astro-ph/0003278*



δN formalism



Ultra-slow-roll inflation

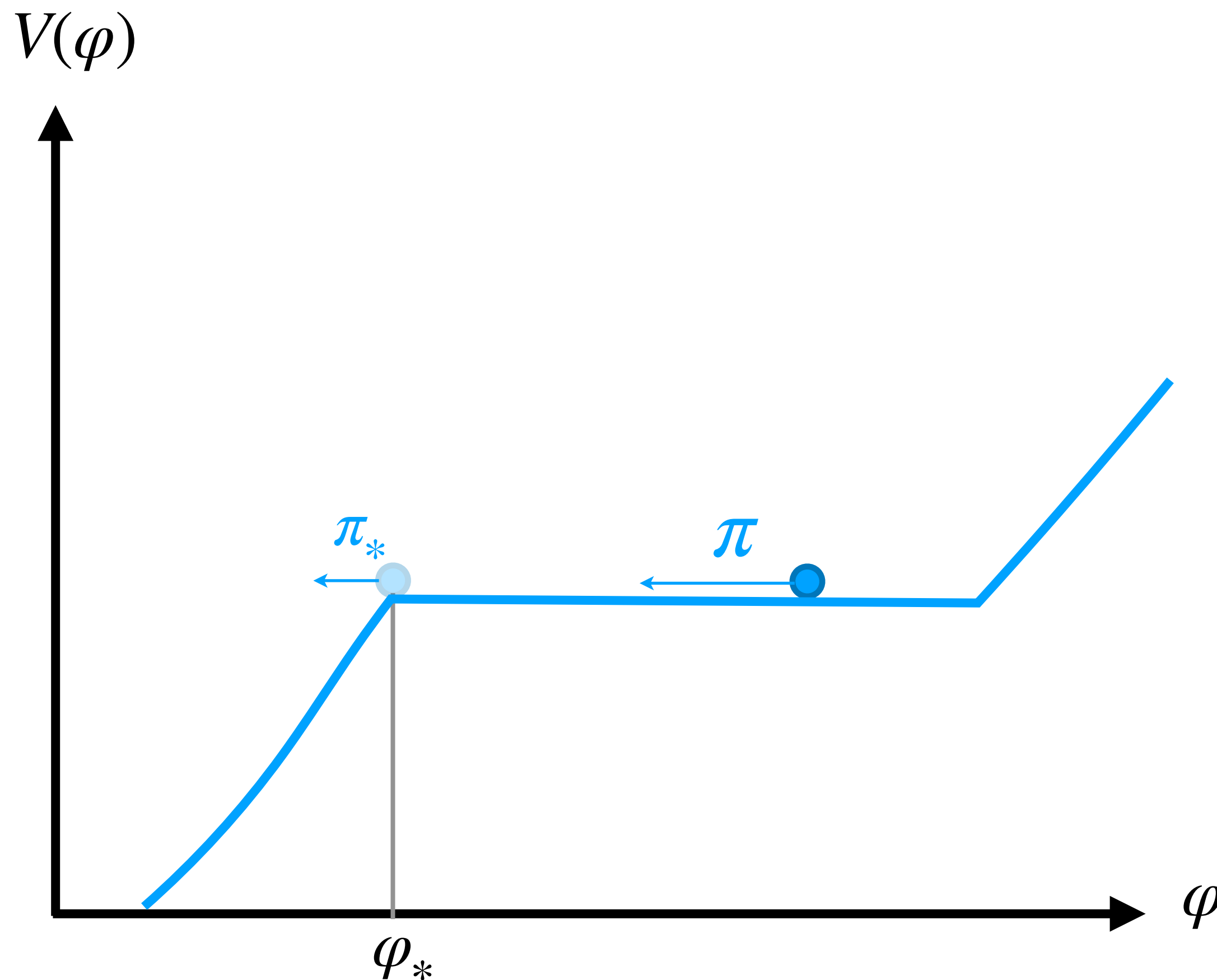
$$dN = H dt$$

$$\varphi'' + 3H\varphi' = 0 \quad (\text{superhorizon, not background})$$

$$\pi' + 3\pi = 0 \quad \left(\pi \equiv \frac{d\varphi}{dN} \right)$$

$$\pi = \pi_* e^{-3(N-N_*)} \quad (\pi = 0 \text{ is the attractor solution})$$

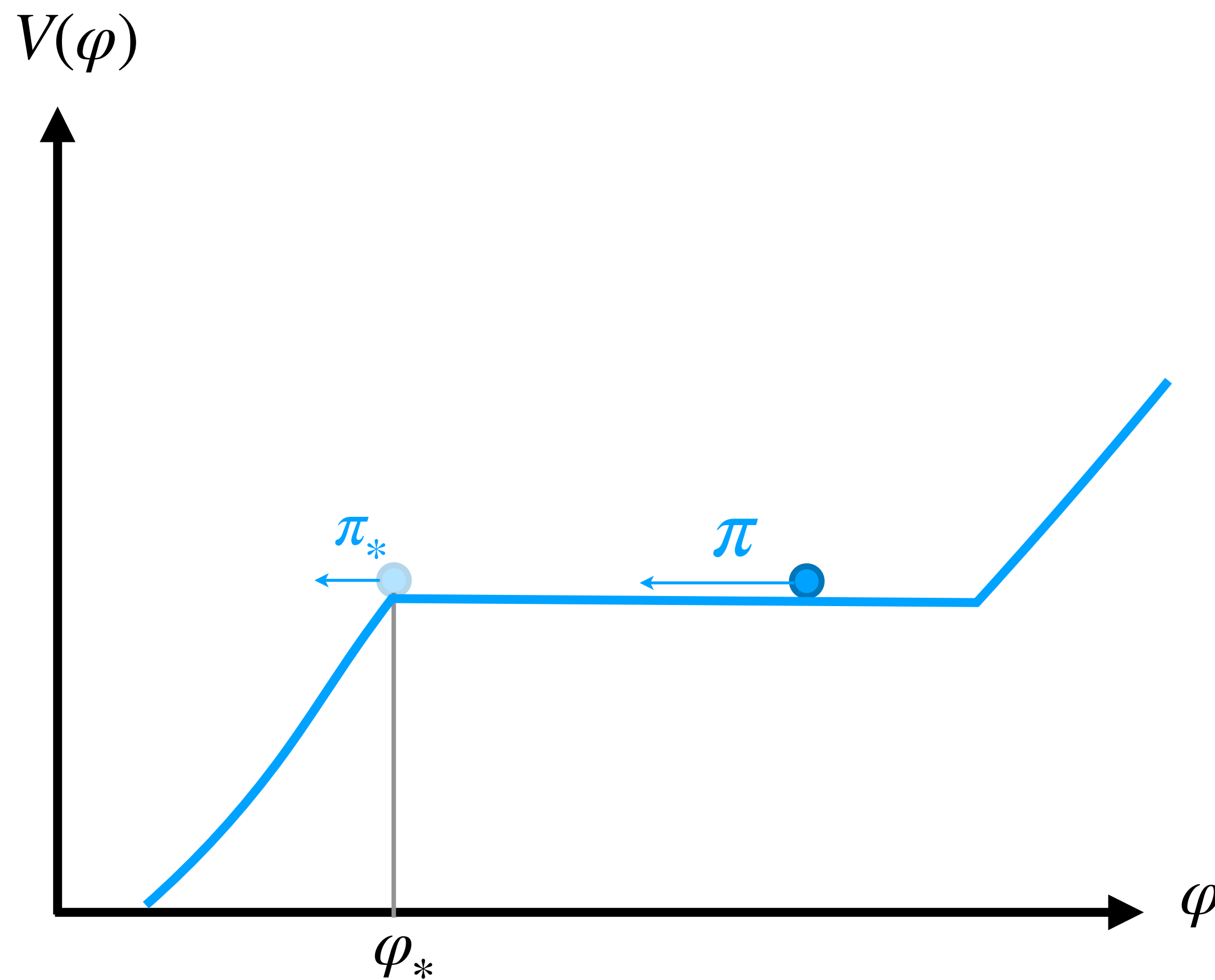
$$N - N_* = -\frac{1}{3} \ln \frac{\pi(N)}{\pi_*}$$



Ultra-slow-roll inflation

In the “fiducial” patch

$$\bar{N} - N_* = -\frac{1}{3} \ln \frac{\pi(\bar{N})}{\pi_*}$$



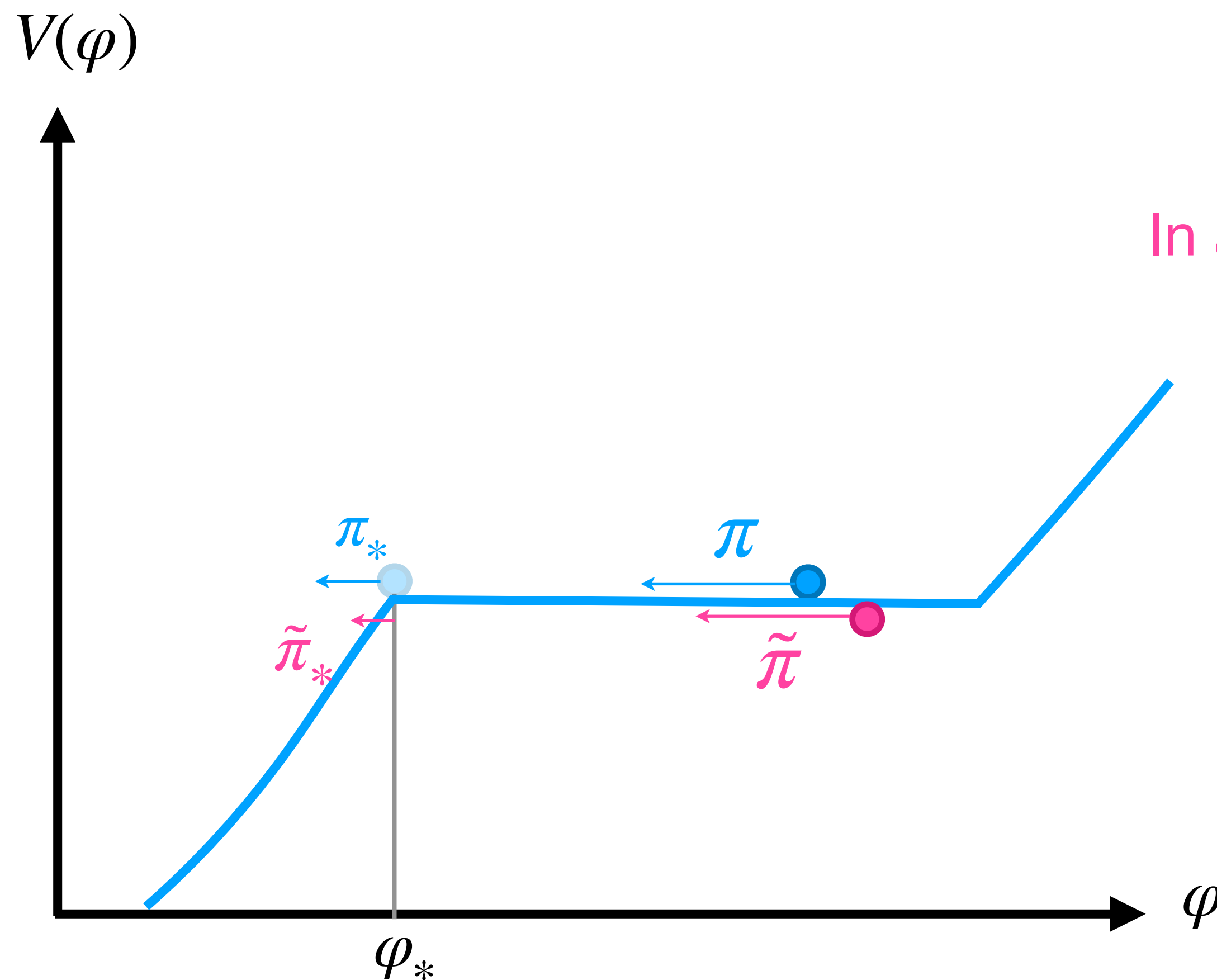
Ultra-slow-roll inflation

In the “fiducial” patch

$$\bar{N} - N_* = -\frac{1}{3} \ln \frac{\pi(\bar{N})}{\pi_*}$$

In a perturbed patch

$$N - N_* = -\frac{1}{3} \ln \frac{\pi(N)}{\pi_*}$$



Ultra-slow-roll inflation

In the “fiducial” patch

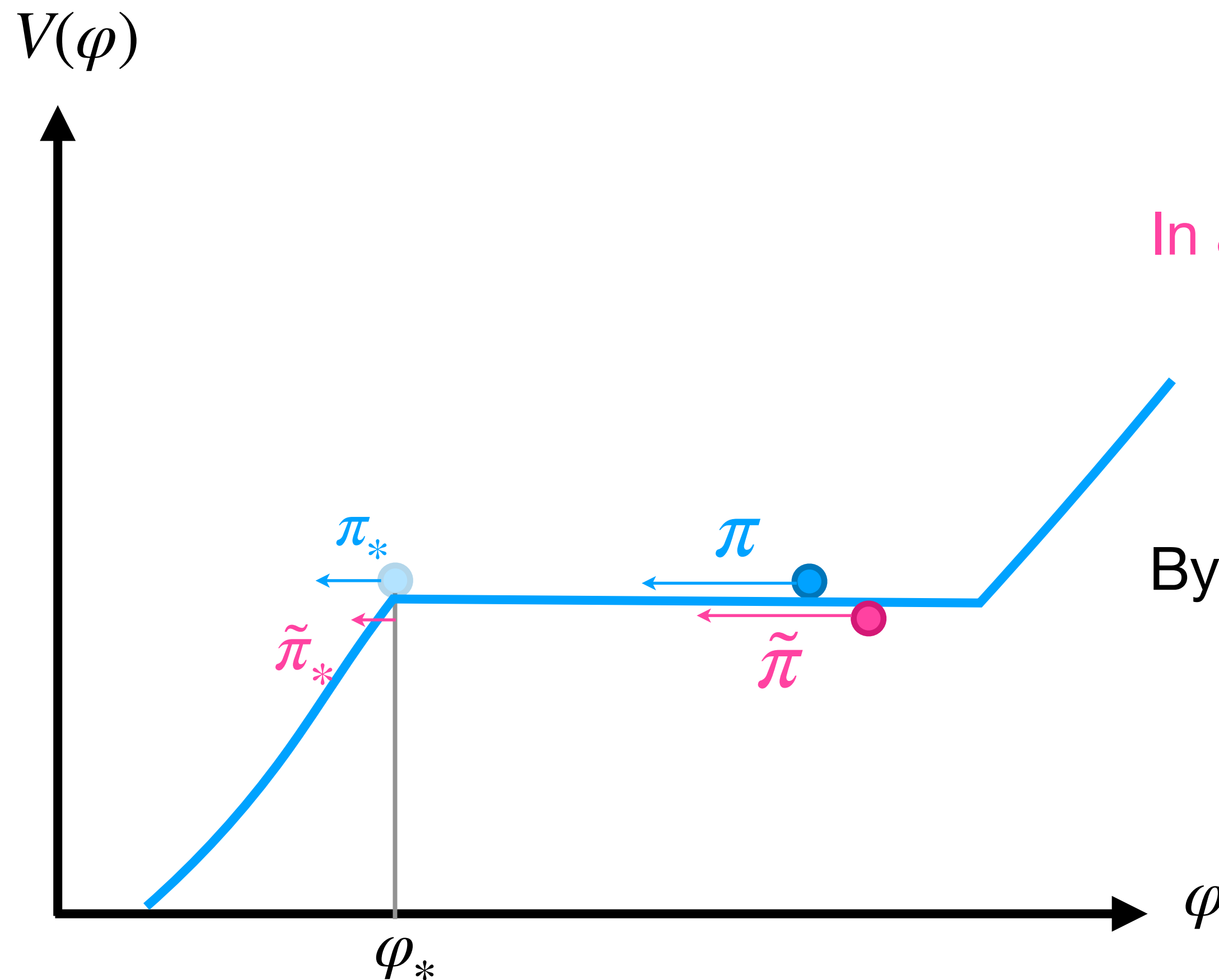
$$\bar{N} - N_* = -\frac{1}{3} \ln \frac{\pi(\bar{N})}{\pi_*}$$

In a perturbed patch

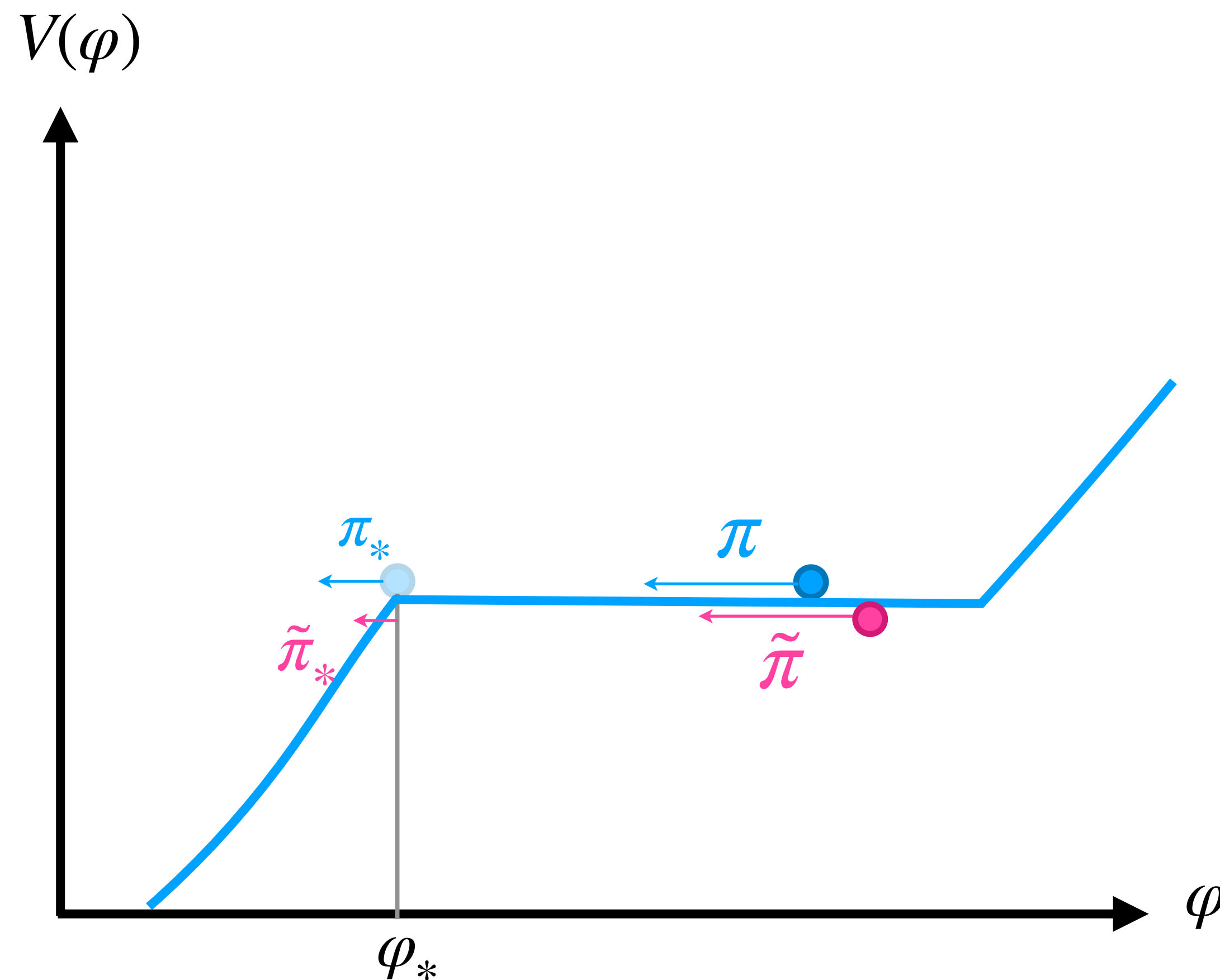
$$N - N_* = -\frac{1}{3} \ln \frac{\pi(N)}{\pi_*}$$

By δN formalism, the curvature perturbation is

$$\begin{aligned} \mathcal{R} = \delta N &= N - \bar{N} = -\frac{1}{3} \ln \frac{\pi(N)}{\pi(\bar{N})} \\ &= -\frac{1}{3} \ln \left(1 + \frac{\delta\pi(N)}{\pi(\bar{N})} \right) \end{aligned}$$



Ultra-slow-roll inflation



$$\mathcal{R} = -\frac{1}{3} \ln \left(1 + \frac{\delta\pi}{\bar{\pi}} \right)$$

$$\left(f_{\text{NL}} = \frac{5}{2}, \quad g_{\text{NL}} = -\frac{25}{3}, \quad \dots \right)$$

Namjoo, Firouzjahi, Sasaki, 1210.3692

Chen, Firouzjahi, Komatsu, Namjoo, Sasaki, 1308.5341

Cai, Chen, Namjoo, Sasaki, Wang, Wang, 1712.09998

Biagetti, Franciolini, Kehagias, Riotto, 1804.07124

Passaglia, Hu, Motohashi, 1812.08243

SP and Sasaki, 2211.13932

SP, 2404.06151, PBH book ch.7

Jackson et al 2410.13683, Cruces et al 2410.17987

Ballestores et al 2412.14106, Caravano et al 2506.11795

Artigas, SP, Tanaka, 2408.09964

Escriva, Garriga, SP, 2512.04986

piecewise quadratic potential

$$V_1(\varphi) = V_0 + \frac{m_1^2}{2}\varphi^2$$

$$\eta_V = \frac{m_1^2}{3H^2}$$

$$\lambda_{\pm} = \frac{3 \pm \sqrt{9 - 12\eta_V}}{2}$$

slow-roll parameter

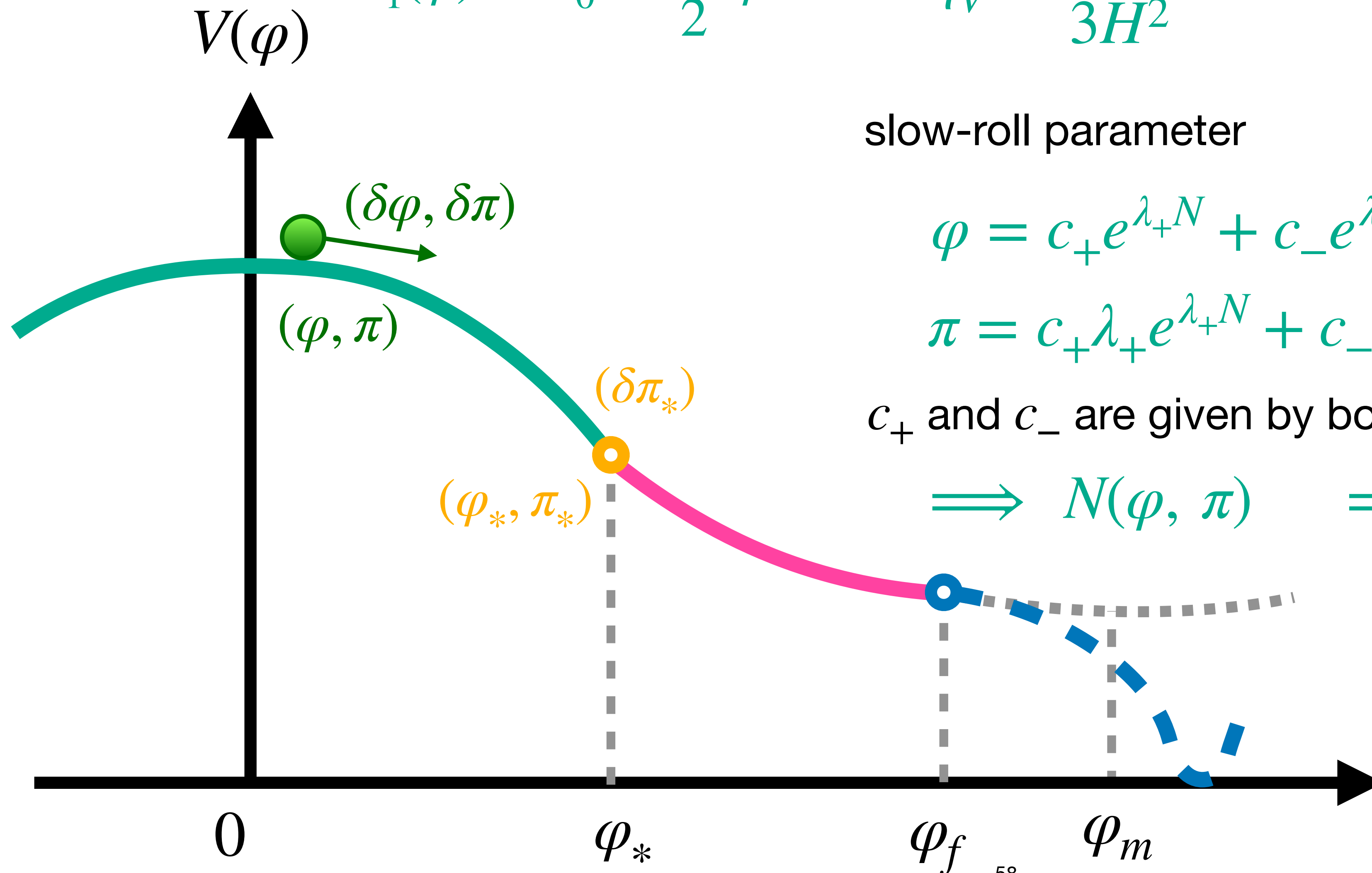
Characteristic roots
(Liyapnov exponent)

$$\varphi = c_+ e^{\lambda_+ N} + c_- e^{\lambda_- N}$$

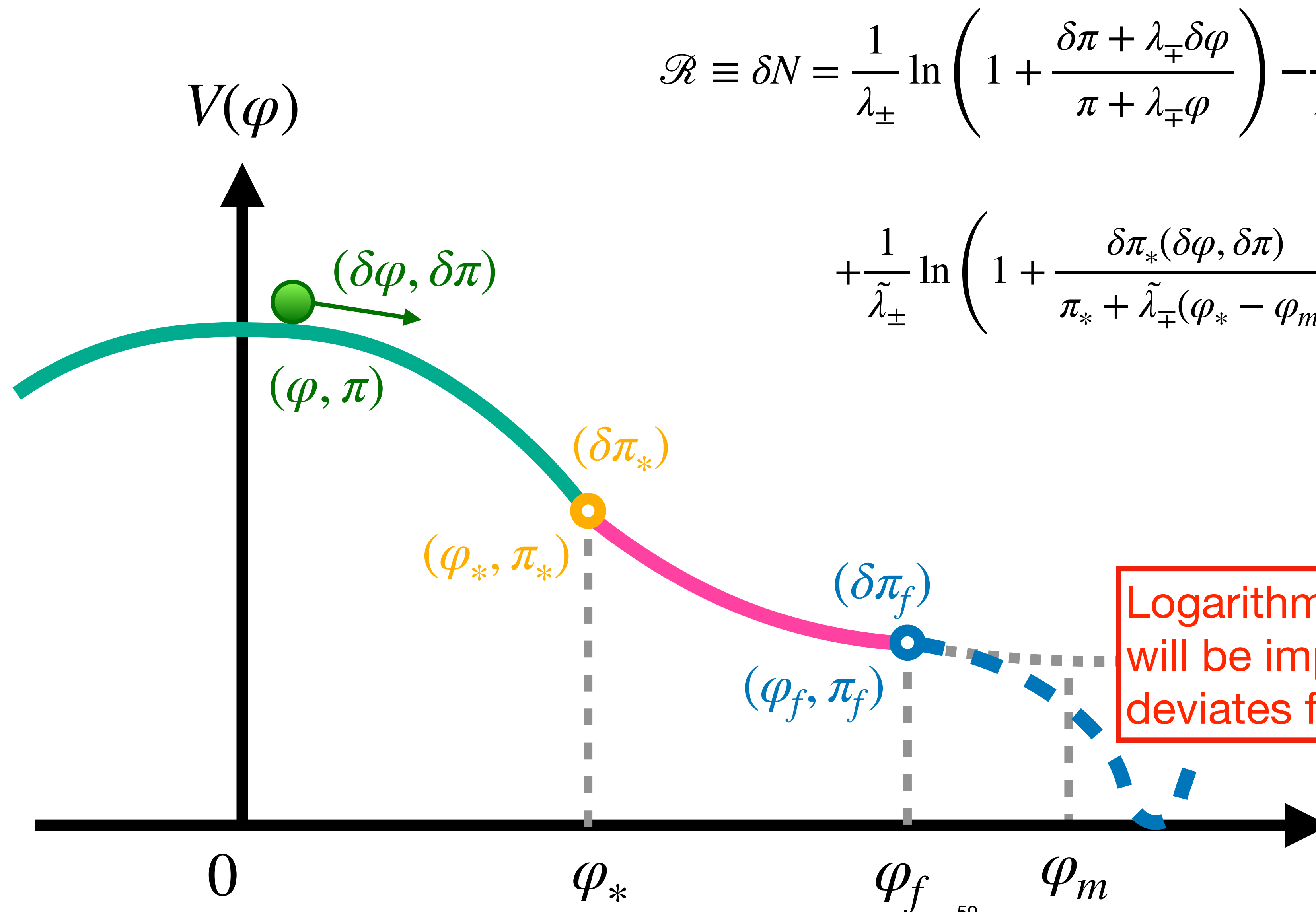
$$\pi = c_+ \lambda_+ e^{\lambda_+ N} + c_- \lambda_- e^{\lambda_- N}$$

c_+ and c_- are given by boundary conditions (φ_*, π_*)

$$\Rightarrow N(\varphi, \pi) \quad \Rightarrow \mathcal{R} = \delta N = \delta N(\delta\varphi, \delta\pi)$$



Logarithmic Duality

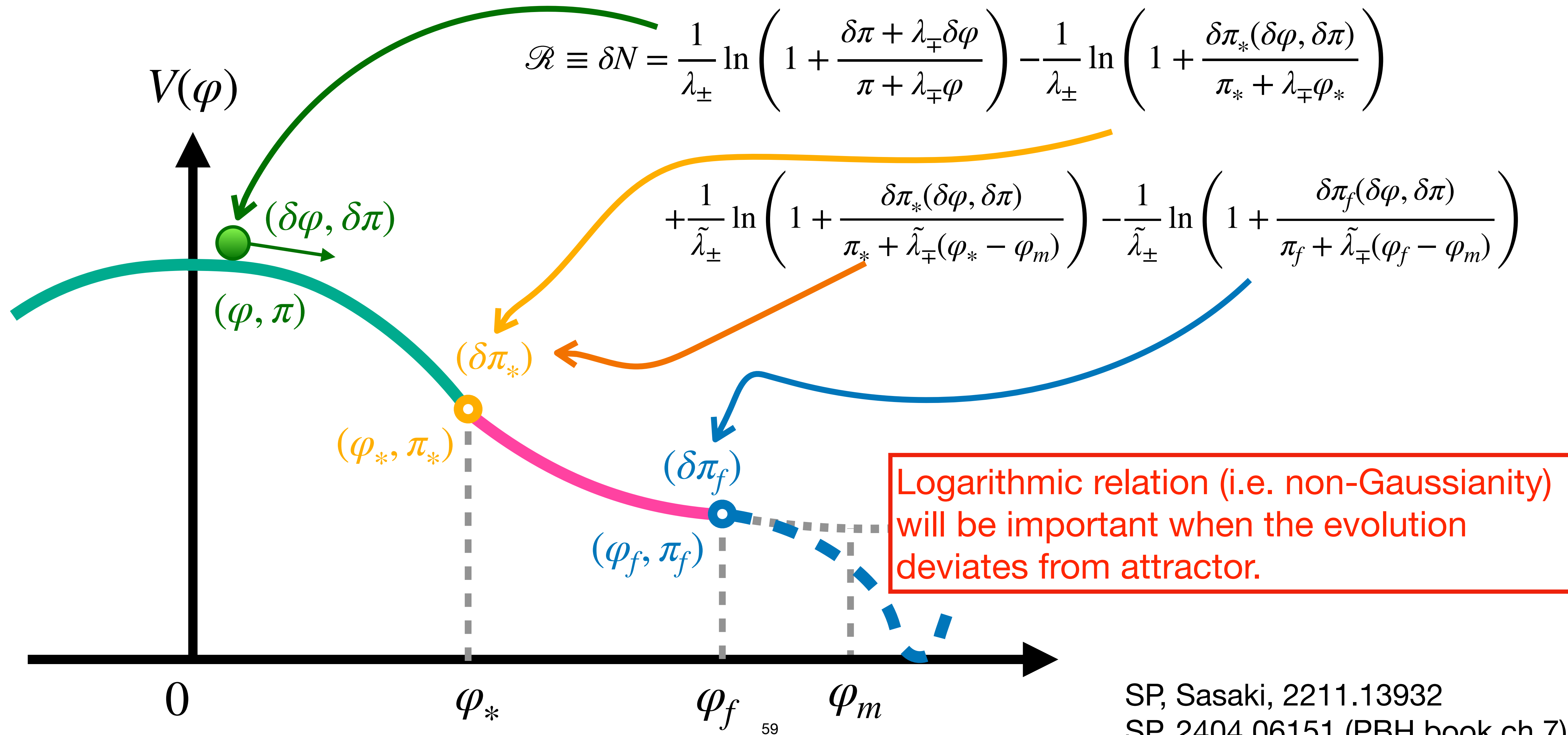


$$\mathcal{R} \equiv \delta N = \frac{1}{\lambda_{\pm}} \ln \left(1 + \frac{\delta\pi + \lambda_{\mp} \delta\varphi}{\pi + \lambda_{\mp} \varphi} \right) - \frac{1}{\lambda_{\pm}} \ln \left(1 + \frac{\delta\pi_*(\delta\varphi, \delta\pi)}{\pi_* + \lambda_{\mp} \varphi_*} \right)$$

$$+ \frac{1}{\tilde{\lambda}_{\pm}} \ln \left(1 + \frac{\delta\pi_*(\delta\varphi, \delta\pi)}{\pi_* + \tilde{\lambda}_{\mp}(\varphi_* - \varphi_m)} \right) - \frac{1}{\tilde{\lambda}_{\pm}} \ln \left(1 + \frac{\delta\pi_f(\delta\varphi, \delta\pi)}{\pi_f + \tilde{\lambda}_{\mp}(\varphi_f - \varphi_m)} \right)$$

Logarithmic relation (i.e. non-Gaussianity) will be important when the evolution deviates from attractor.

Logarithmic Duality

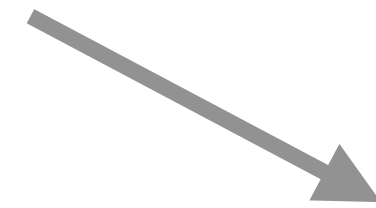


Logarithmic Duality

SP, Sasaki, 2211.13932

SP, 2404.06151 (PBH book ch.7)

$\mathcal{R}(\delta\varphi, \delta\pi)$



$$\mathcal{R} = \frac{1}{\lambda_{\pm}} \ln \left(1 + \frac{\delta\pi + \lambda_{\mp} \delta\varphi}{\pi + \lambda_{\mp} \varphi} \right) - \frac{1}{\lambda_{\pm}} \ln \left(1 + \frac{\delta\pi_{*}}{\pi_{*} + \lambda_{\mp} \varphi_{*}} \right) + \dots$$

Logarithmic Duality

SP, Sasaki, 2211.13932

SP, 2404.06151 (PBH book ch.7)

$$\mathcal{R}(\delta\varphi, \delta\pi)$$

c.f. consistency relation

$$f_{\text{NL}} = \frac{5}{2}\epsilon_V - \frac{5}{6}\eta_V$$

$$\left(f_{\text{NL}} = -\frac{5}{6}\lambda_- \approx -\frac{5}{6}\eta\right)$$

$$\mathcal{R} = -H \frac{\delta\varphi}{\dot{\varphi}} + \frac{3}{5}f_{\text{NL}} \left(-H \frac{\delta\varphi}{\dot{\varphi}}\right)^2$$

$$\lambda_- \ll 1$$

$$\lambda_+ \approx 3$$

$$\mathcal{R} = \frac{1}{\lambda_{\pm}} \ln \left(1 + \frac{\delta\pi + \lambda_{\mp}\delta\varphi}{\pi + \lambda_{\mp}\varphi} \right) - \frac{1}{\lambda_{\pm}} \ln \left(1 + \frac{\delta\pi_*}{\pi_* + \lambda_{\mp}\varphi_*} \right) + \dots$$

Slow-roll inflation
Stewart and Sasaki, 1995
Lyth and Roquigez, 2005

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$$\begin{aligned} \lambda_- &\ll 1 \\ \lambda_+ &\approx 3 \\ \lambda_- &= -1/\mu \\ \lambda_+ &= 3 - 1/\mu \end{aligned}$$

Slow-roll inflation
 Stewart and Sasaki, 1995
 Lyth and Roquigez, 2005

$$\mathcal{R} = -\mu \ln\left(1 - \frac{\mathcal{R}_g}{\mu}\right)$$

Constant-roll

Atal, Garriga, Marcos-Caballero, 1905.13202
 Atal, Cid, Escrivà, Garriga, 1908.11357
 Escrivà, Atal, Garriga, 2306.09990
 Inui, Motohashi, SP, et al, 2409.13500

$$\mathcal{R} = \frac{1}{\lambda_{\pm}} \ln \left(1 + \frac{\delta\pi + \lambda_{\mp}\delta\varphi}{\pi + \lambda_{\mp}\varphi} \right) - \frac{1}{\lambda_{\pm}} \ln \left(1 + \frac{\delta\pi_*}{\pi_* + \lambda_{\mp}\varphi_*} \right) + \dots$$

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$$\lambda_+ \approx 3$$

$$\lambda_- = -1/\mu$$

$$\lambda_+ = 3 - 1/\mu$$

$$\lambda_- = 0$$

$$\lambda_+ = 3$$

$$\mathcal{R} = -\frac{1}{3} \ln \left(1 + \frac{\delta\pi_*}{\pi_*} \right)$$

Ultra-slow-roll

Namjoo, Firouzjahi, Sasaki, 1210.3692

Cai, Chen, et al 1712.09998

Biagetti et al 1804.07124

Passaglia et al 1812.08243

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SP, Sasaki, 2211.13932

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Ultra-slow-roll

Namjoo, Firouzjahi, Sasaki, 1210.3692

Cai, Chen, et al 1712.09998

Biagetti et al 1804.07124

Passaglia et al 1812.08243

$$\lambda_- = -3/2$$

$$\lambda_+ = 9/2$$

$$\mathcal{R} = \frac{2}{3} \ln(1 + \delta)$$

Curvaton scenario,

SP and Sasaki, 2112.12680

Ferrante et al, 2211.01728

Hooper et al. 2308.00756

Logarithmic Duality

SP, Sasaki, 2211.13932

SP, 2404.06151 (PBH book ch.7)

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Lyth and Roquigez, 2005

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Inui, Motohashi, SP, et al, 2409.13500

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Ultra-slow-roll

Namjoo, Firouzjahi, Sasaki, 1210.3692

Cai, Chen, et al 1712.09998

Biagetti et al 1804.07124

Passaglia et al 1812.08243

$$\mathcal{R} = -\frac{1}{\lambda} \ln (f(\mathcal{R}_G))$$

Generations,

Celoria et al, 2103.09244

Kawaguchi et al, 2305.18140

$$\mathcal{R} = \frac{2}{3} \ln (1 + \delta)$$

Curvaton scenario,

SP and Sasaki, 2112.12680

Ferrante et al, 2211.01728

Hooper et al. 2308.00756

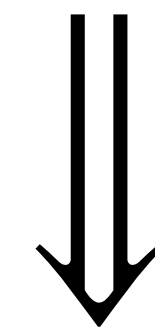
Probability Distribution Function

SP, Sasaki, 2211.13932

SP, 2404.06151 (PBH book ch.7)

For the simplest single-logarithm case:

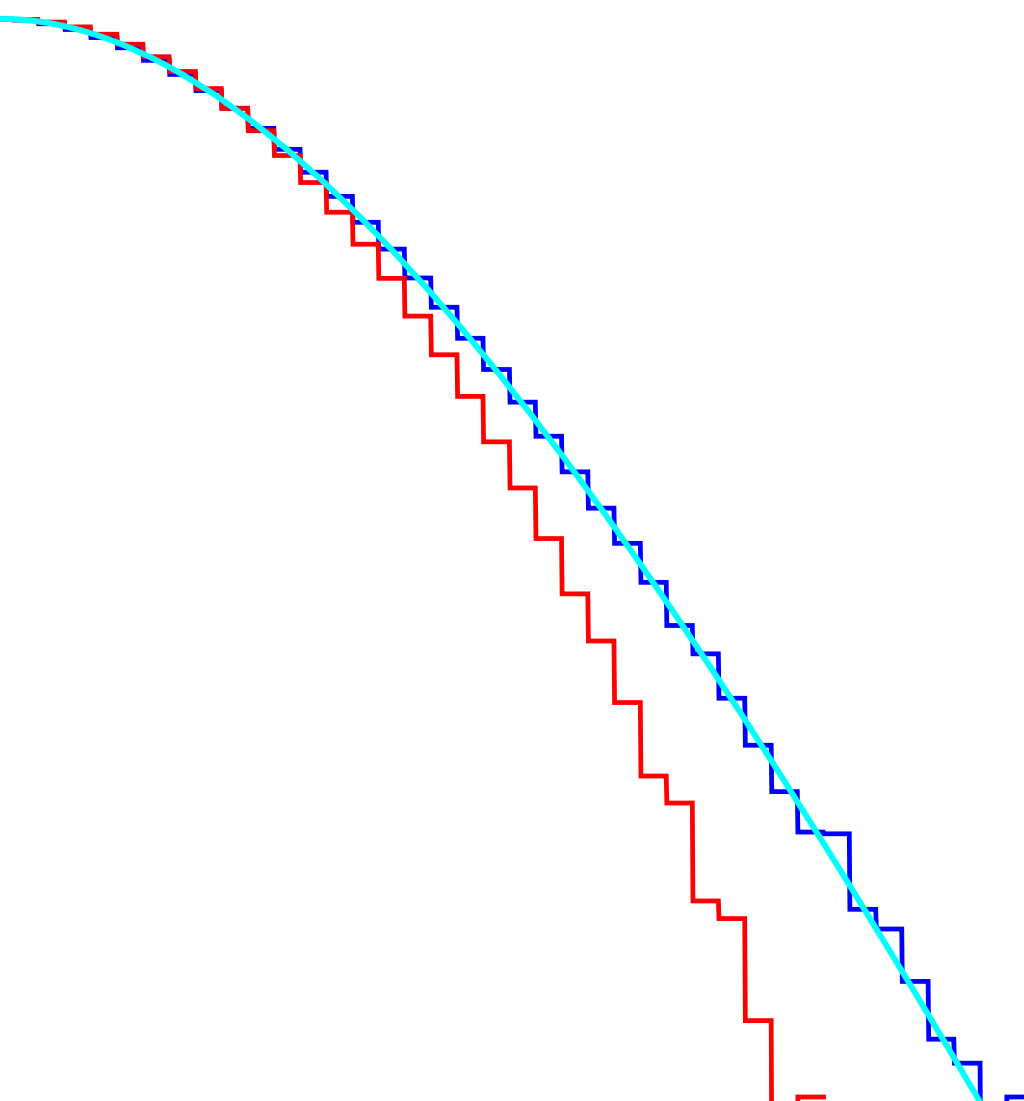
$$\mathcal{R} \equiv \delta N = \frac{1}{\lambda_-} \ln \left(1 + \frac{\delta\pi + \lambda_+ \delta\varphi}{\pi + \lambda_+ \varphi} \right)$$



$$P(\mathcal{R})d\mathcal{R} = P(\delta\varphi)d\delta\varphi$$

Gaussian PDF with variance $\sigma_{\delta\varphi}^2$

$$P(\mathcal{R}) = \frac{e^{\lambda_- \mathcal{R}}}{\sqrt{2\pi}\sigma_{\delta\varphi}} |\lambda_-| \varphi \exp \left[-\frac{\varphi^2}{2\sigma_{\delta\varphi}^2} (e^{\lambda_- \mathcal{R}} - 1)^2 \right]$$



Probability Distribution Function

SP, Sasaki, 2211.13932

SP, 2404.06151 (PBH book ch.7)

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$$\mathcal{R} \equiv \delta N = \frac{1}{\lambda_-} \ln \left(1 + \frac{\delta\pi + \lambda_+ \delta\varphi}{\pi + \lambda_+ \varphi} \right)$$

Gaussian PDF with variance $\sigma_{\delta\varphi}^2$

$$\Downarrow P(\mathcal{R})d\mathcal{R} = \boxed{P(\delta\varphi)}d\delta\varphi$$

$$P(\mathcal{R}) = \frac{e^{\lambda_- \mathcal{R}}}{\sqrt{2\pi}\sigma_{\delta\varphi}} |\lambda_-| \varphi \exp \left[-\frac{\varphi^2}{2\sigma_{\delta\varphi}^2} (e^{\lambda_- \mathcal{R}} - 1)^2 \right]$$

$$\lambda_- < 0$$

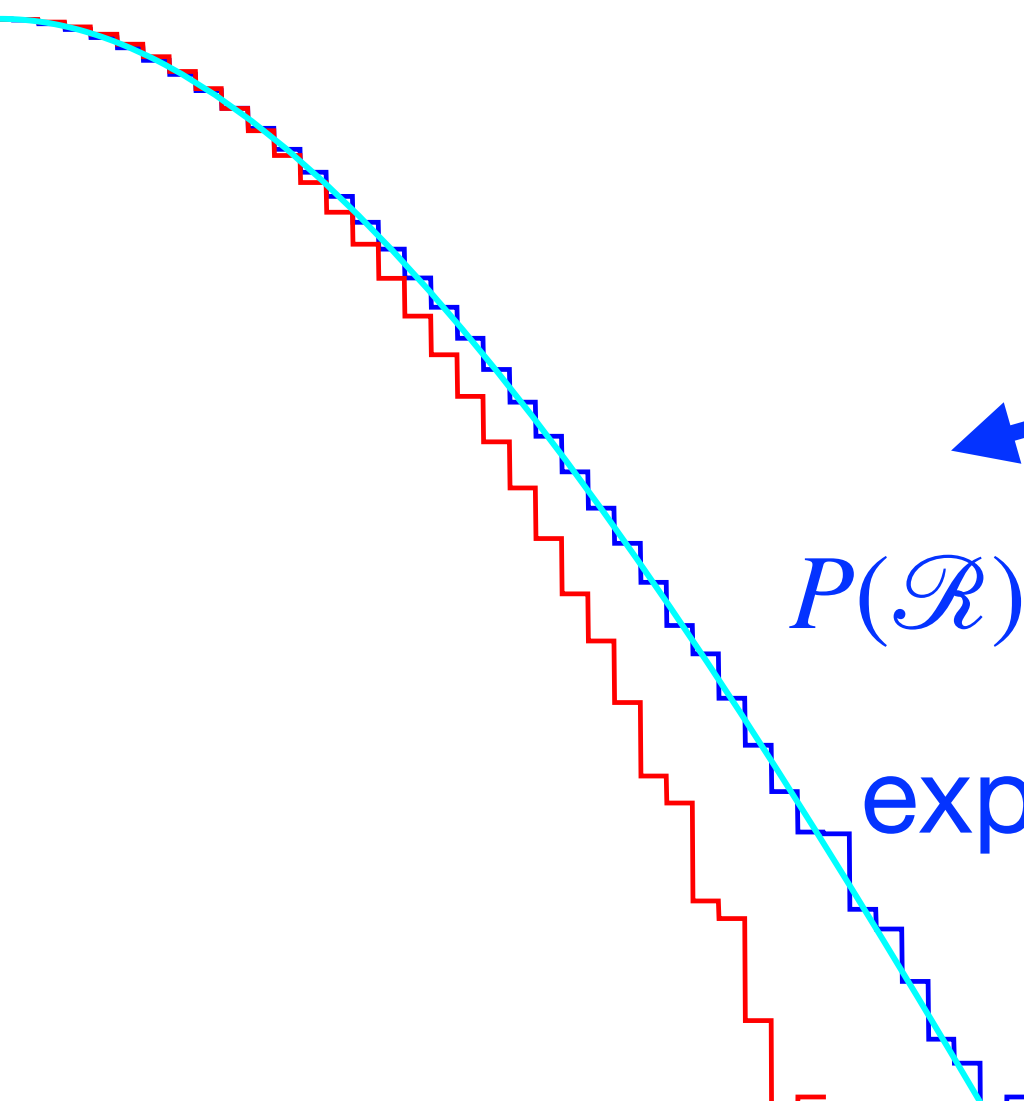
$$P(\mathcal{R}) \sim e^{\lambda_- \mathcal{R}}$$

exponential tail

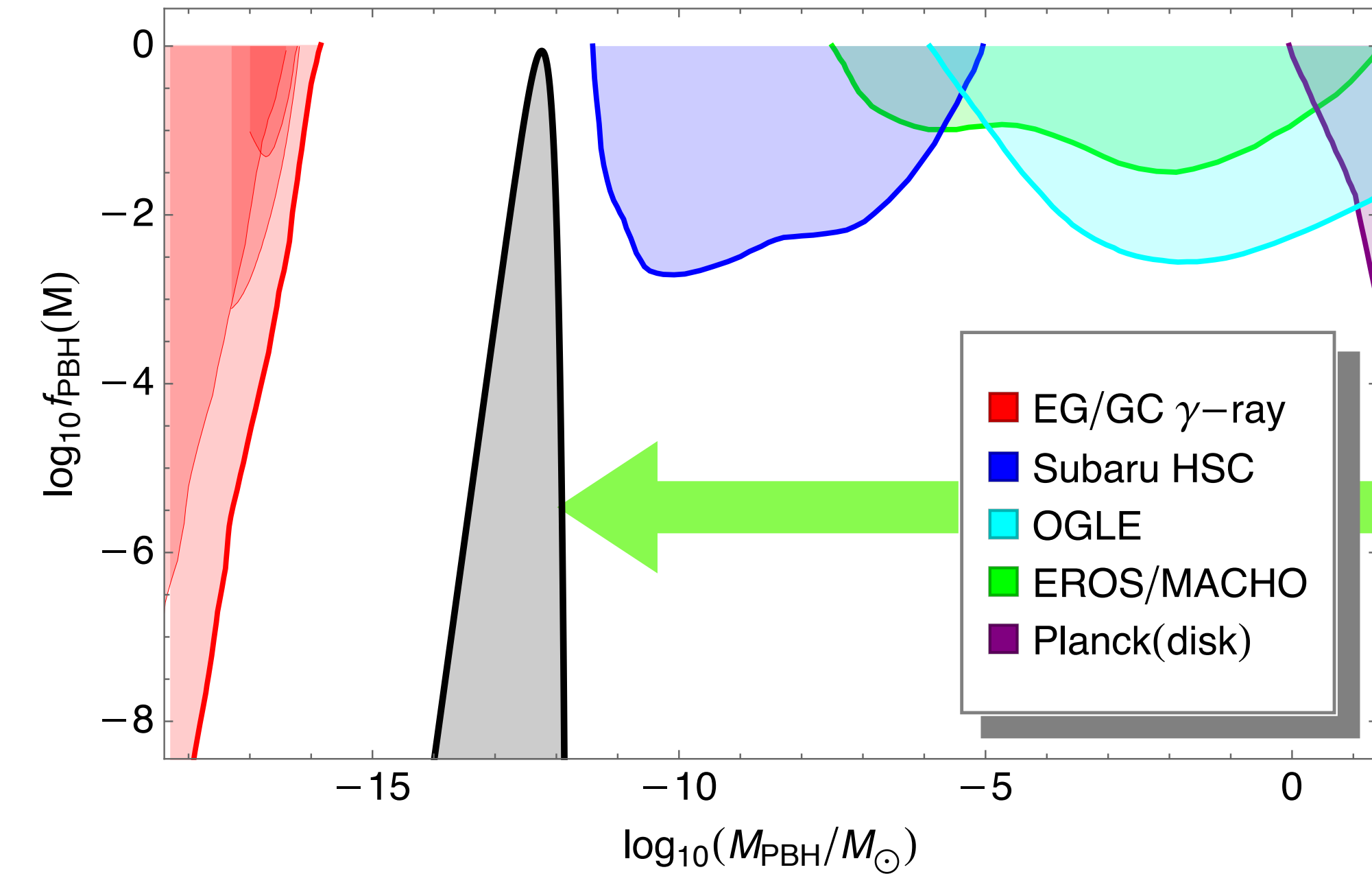
$$\lambda_- > 0$$

$$P(\mathcal{R}) \sim \exp(-c^2 e^{2\lambda_- \mathcal{R}})$$

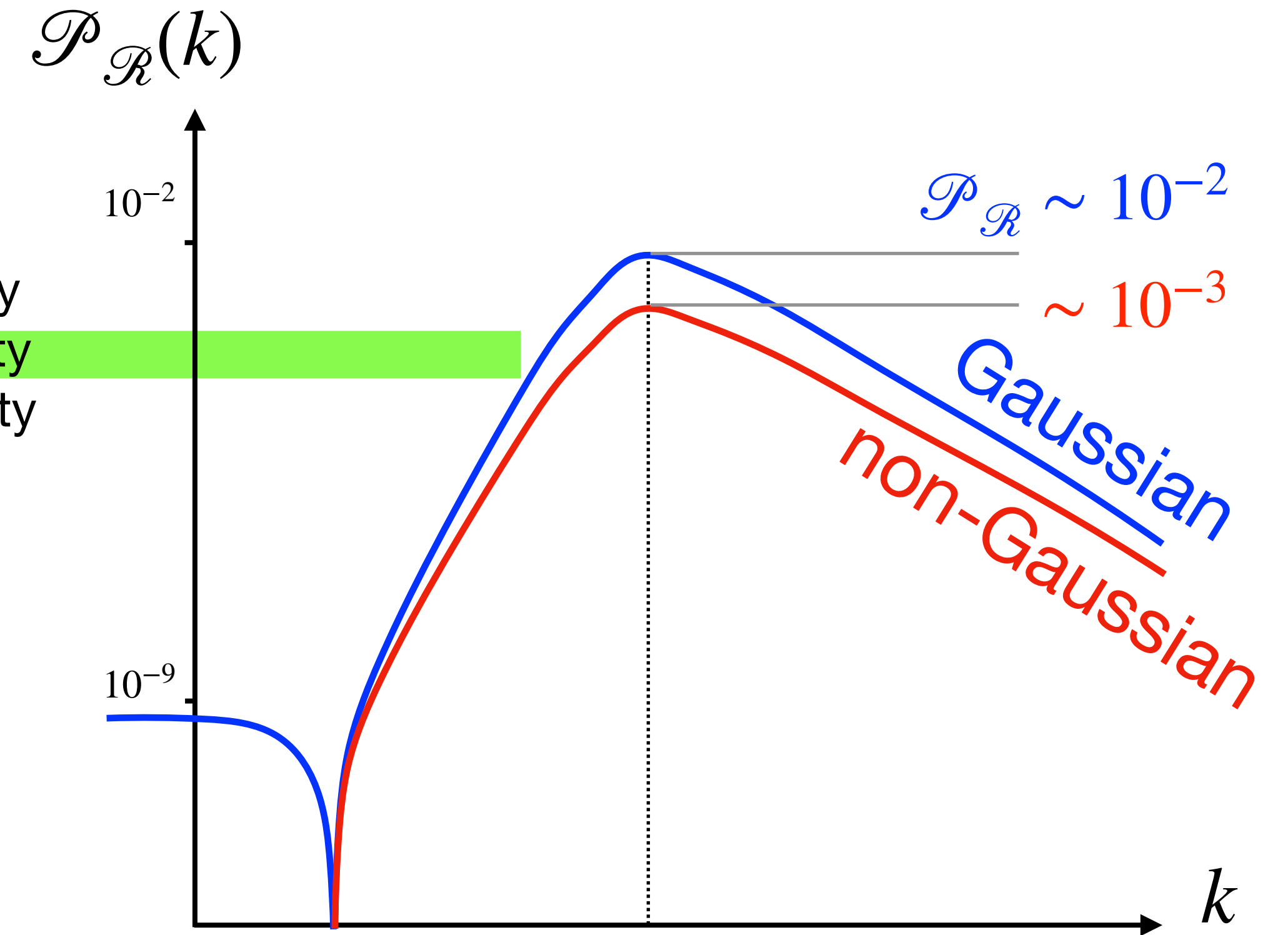
Gumbel-distribution-like tail



PBH Dark Matter



extended PS/peak theory
includible non-Gaussianity
primordial non-Gaussianity



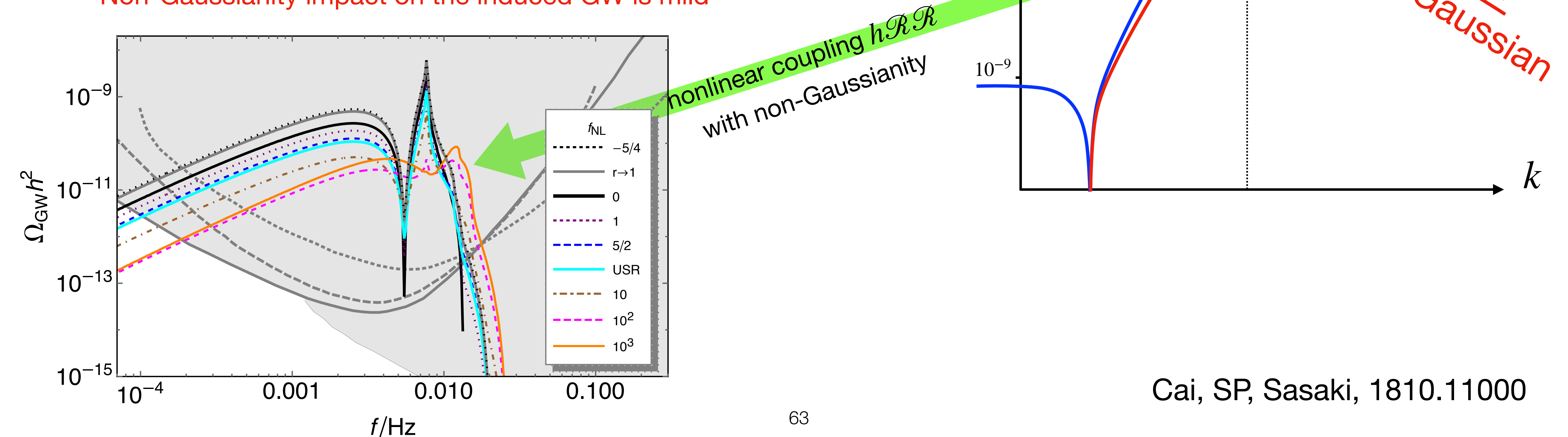
- Press-Schechter: Young & Byrnes 1307.4995
- Extended Press-Schechter: Biagetti et al 2105.07810; Gow et al 2211.08348
- Peak theory: Kitajima et al 2109.00791; SP, Sasaki, Takhistov, Wang, 2501.00295

- Non-Gaussianity must be considered when calculating the PBH abundance

SP, Sasaki, 2211.13932
SP, 2404.06151 (PBH book ch.7)

PBH Dark Matter

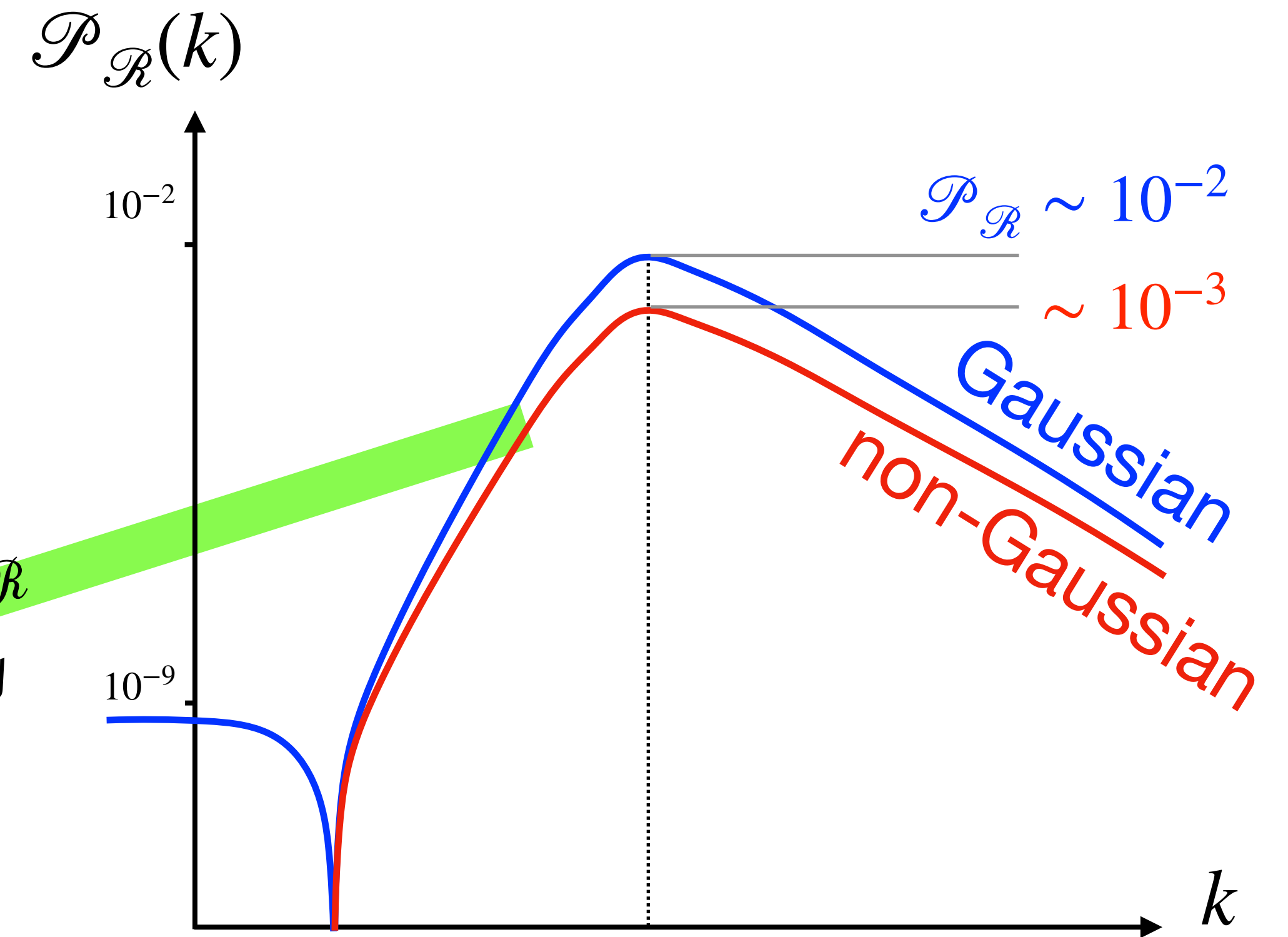
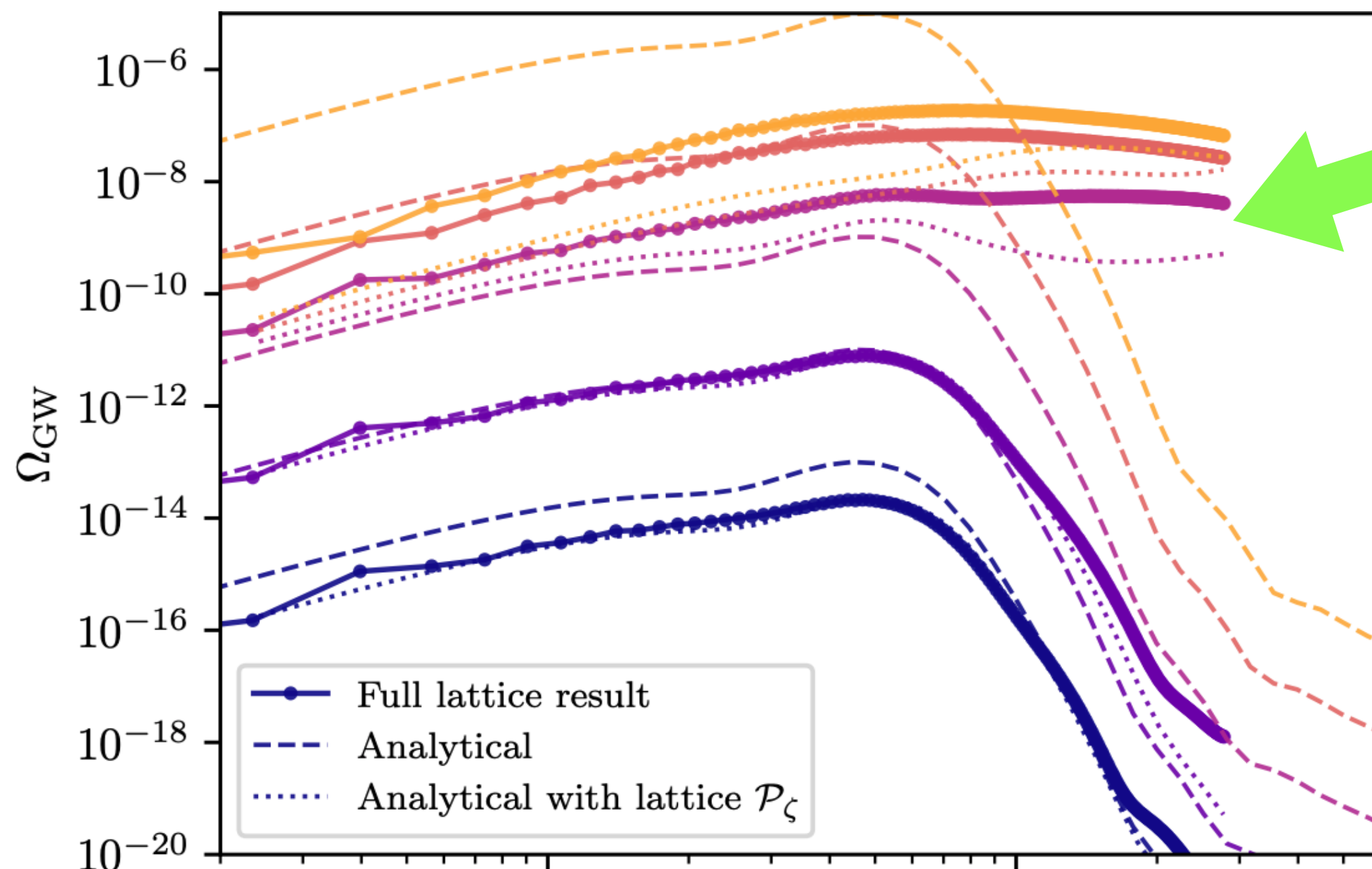
- Quadratic expansion: Cai, SP, Sasaki 1810.11000
- Higher orders: Adshead+ 2105.01659. Abe+ 2209.13891. Garcia-Saenz+ 2207.14267. Yuan+, 2308.07155; Li+, 2309.07792. Perna+, 2403.06962
- Lattice simulation marks the breakdown of perturbativity: Zeng+ 2508.10812, Caravano+ 2604.03628
- Non-Gaussianity impact on the induced GW is mild



Cai, SP, Sasaki, 1810.11000

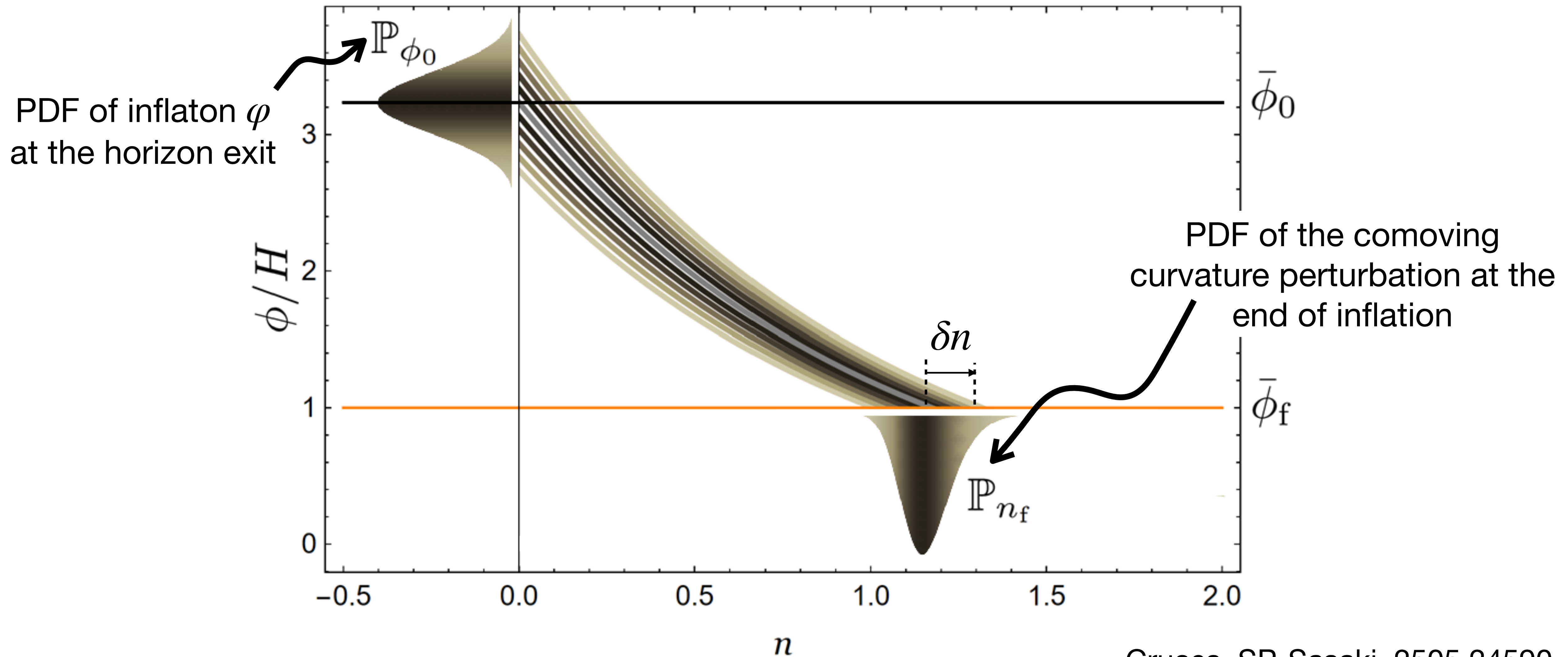
PBH Dark Matter

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- Lattice simulation marks the breakdown of perturbativity: Zeng+ 2508.10812, Caravano+ 2604.03628
- Non-Gaussianity impact on the induced GW is mild (???)

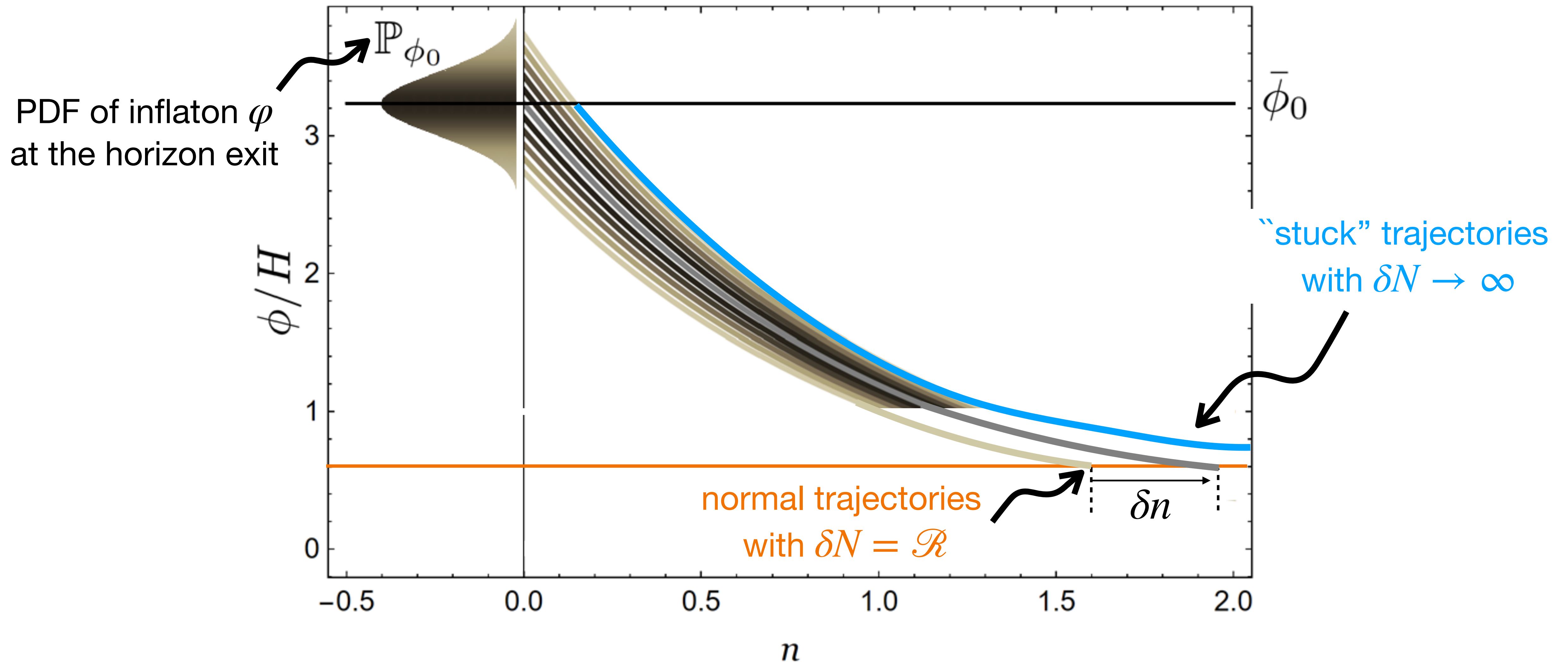


Zeng, Ning, Cai, Wang 2508.10812
Caravano, Franciolini, Renaux-Petel 2604.03628

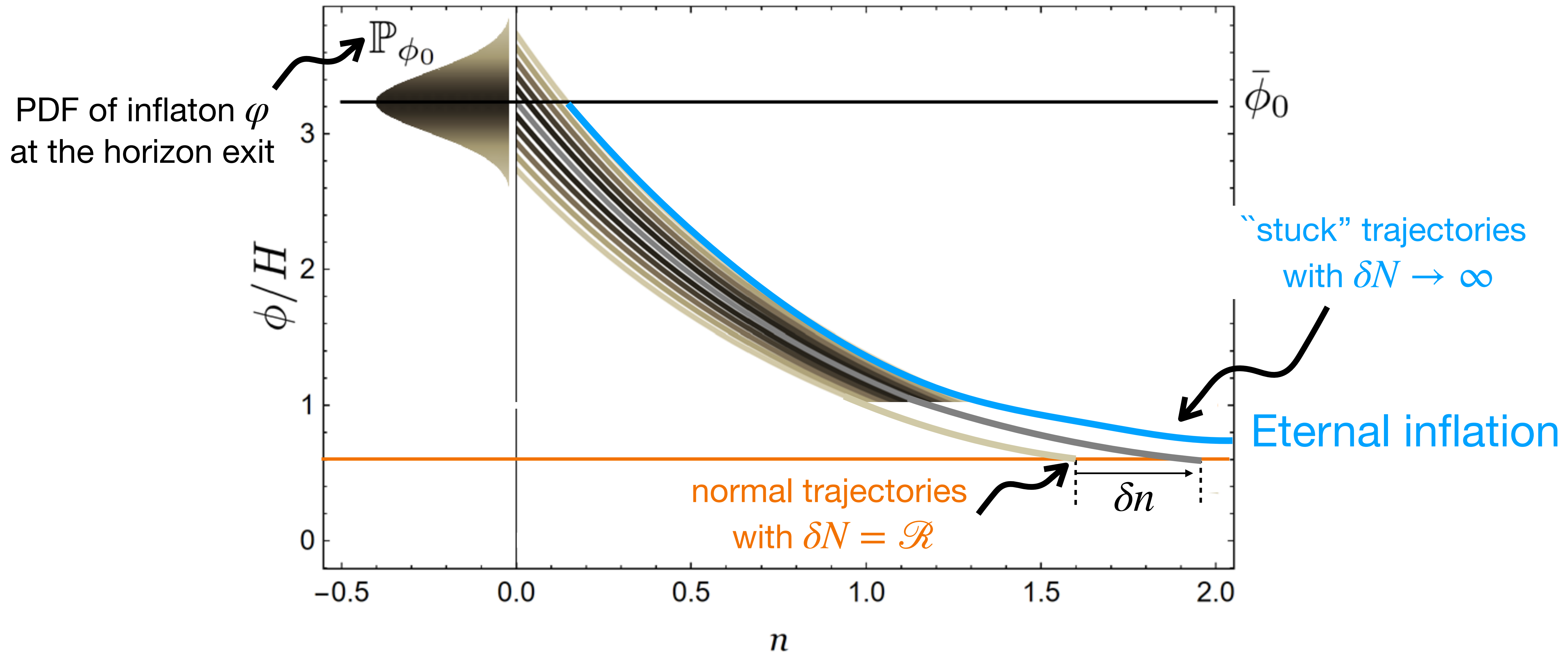
Bubble relics

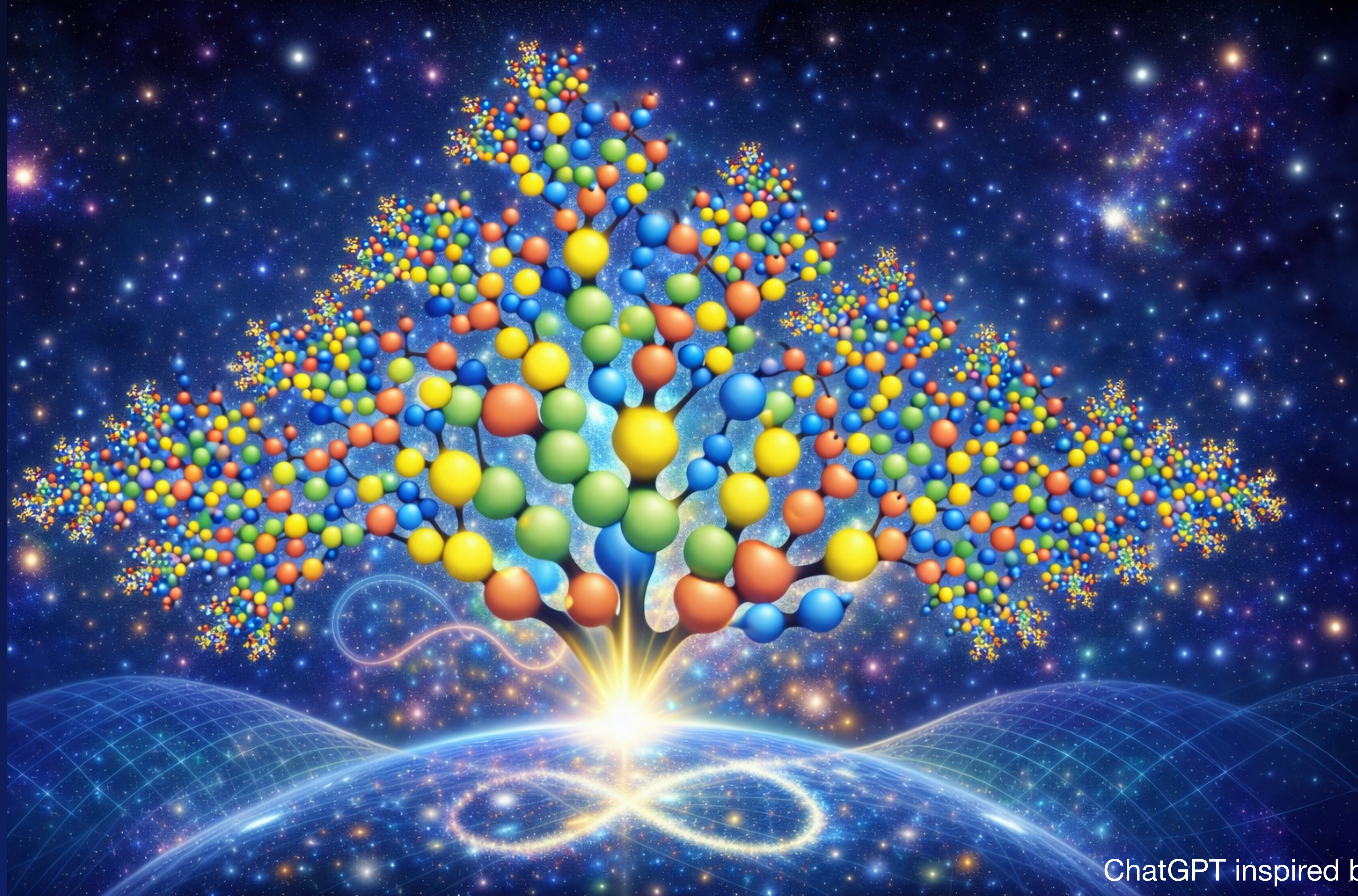


Bubble relics



Bubble relics

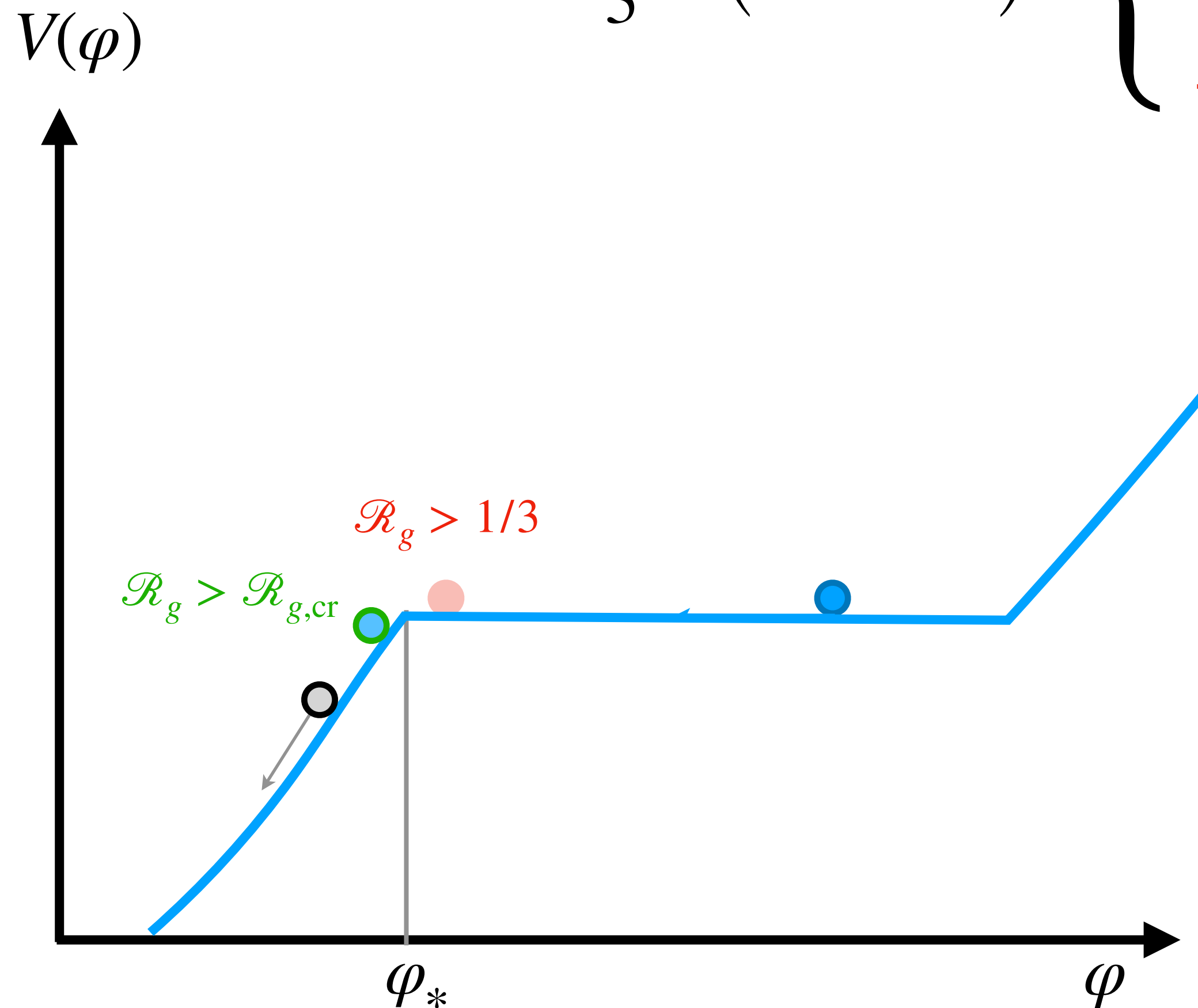




ChatGPT inspired by Linde

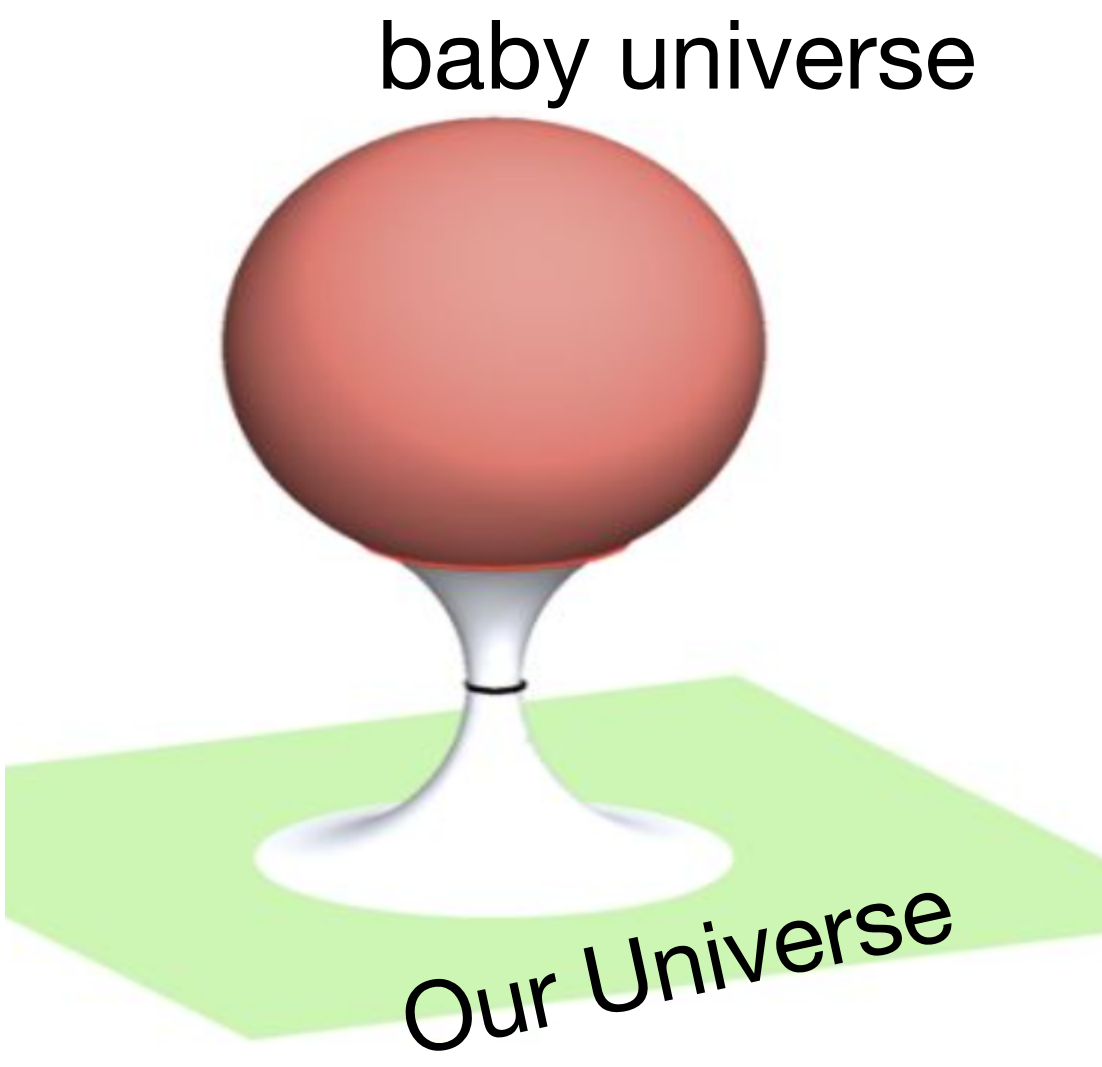
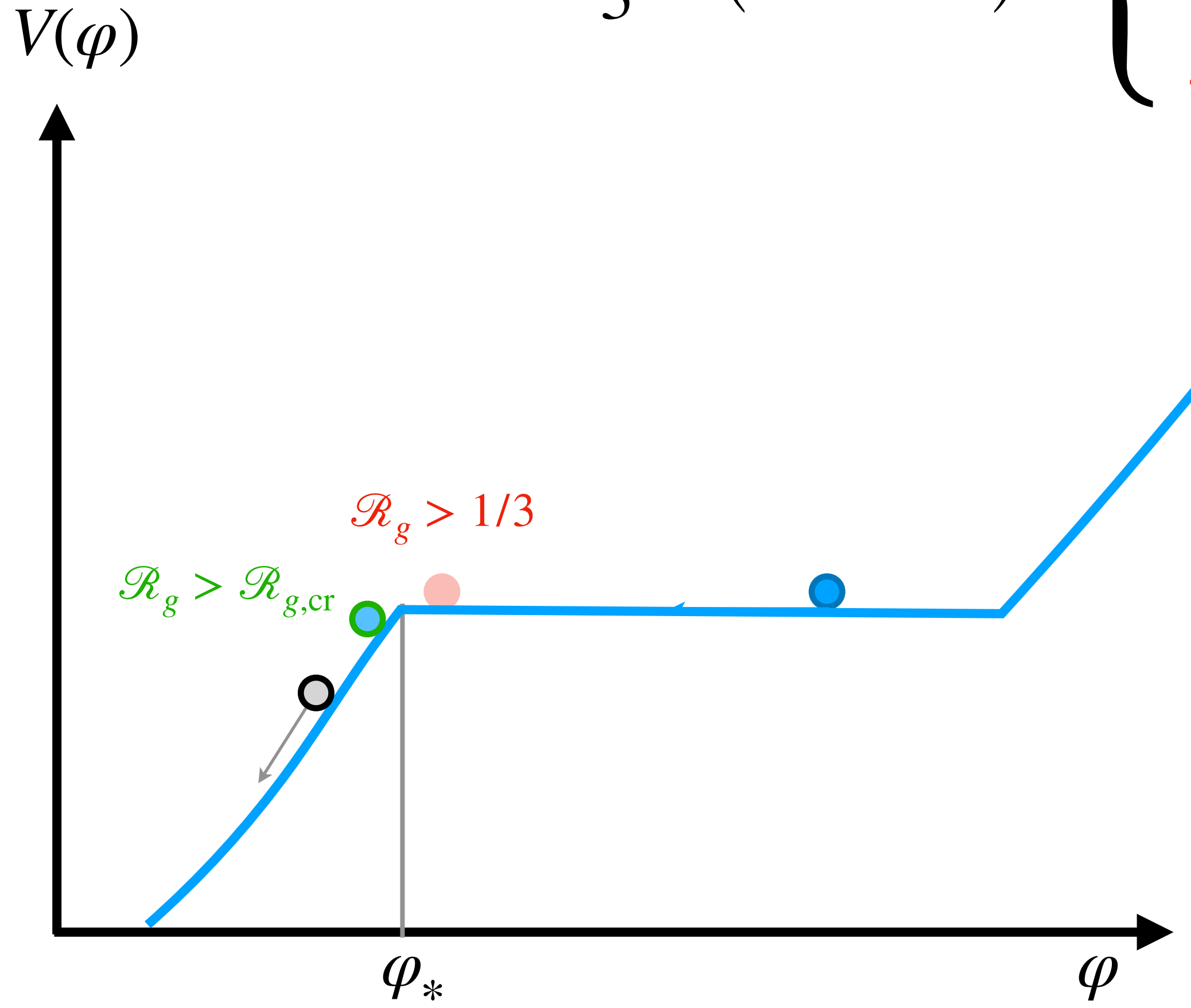
Ultra-slow-roll inflation

$$\delta N = -\frac{1}{3} \ln(1 - 3\mathcal{R}_g) \quad \left\{ \begin{array}{ll} \approx \mathcal{R}_g \ll 1 & \text{with } \mathcal{R}_g \ll 1 \\ = \text{PBH} & \text{with } \mathcal{R}_g > \mathcal{R}_{g,\text{th}} \\ \rightarrow \infty(?) & \text{with } \mathcal{R}_g > 1/3 \end{array} \right. \quad \begin{array}{l} \text{vanilla universe} \\ \text{local closed universe} \\ \text{eternal inflation} \end{array}$$

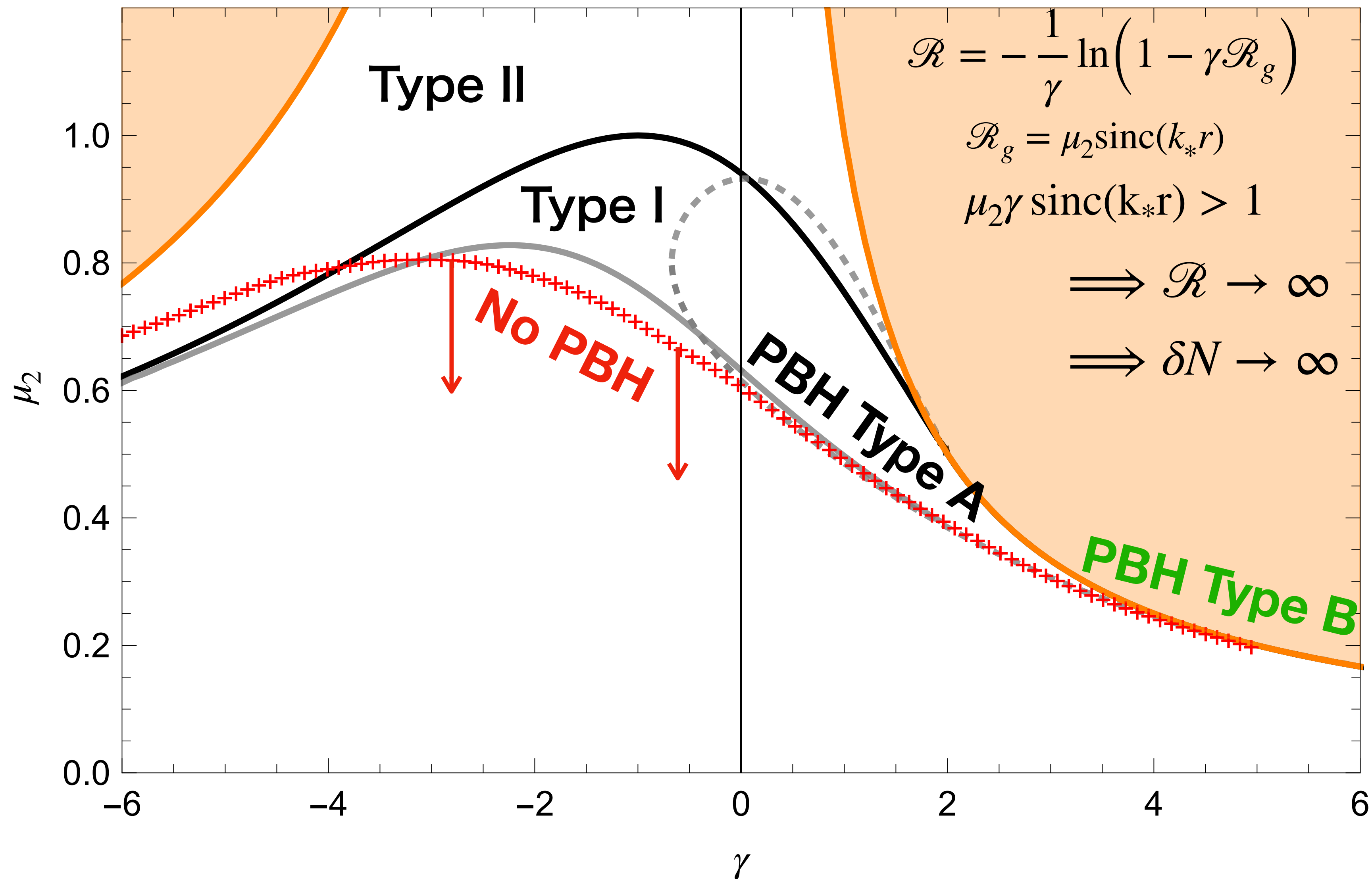


Ultra-slow-roll inflation

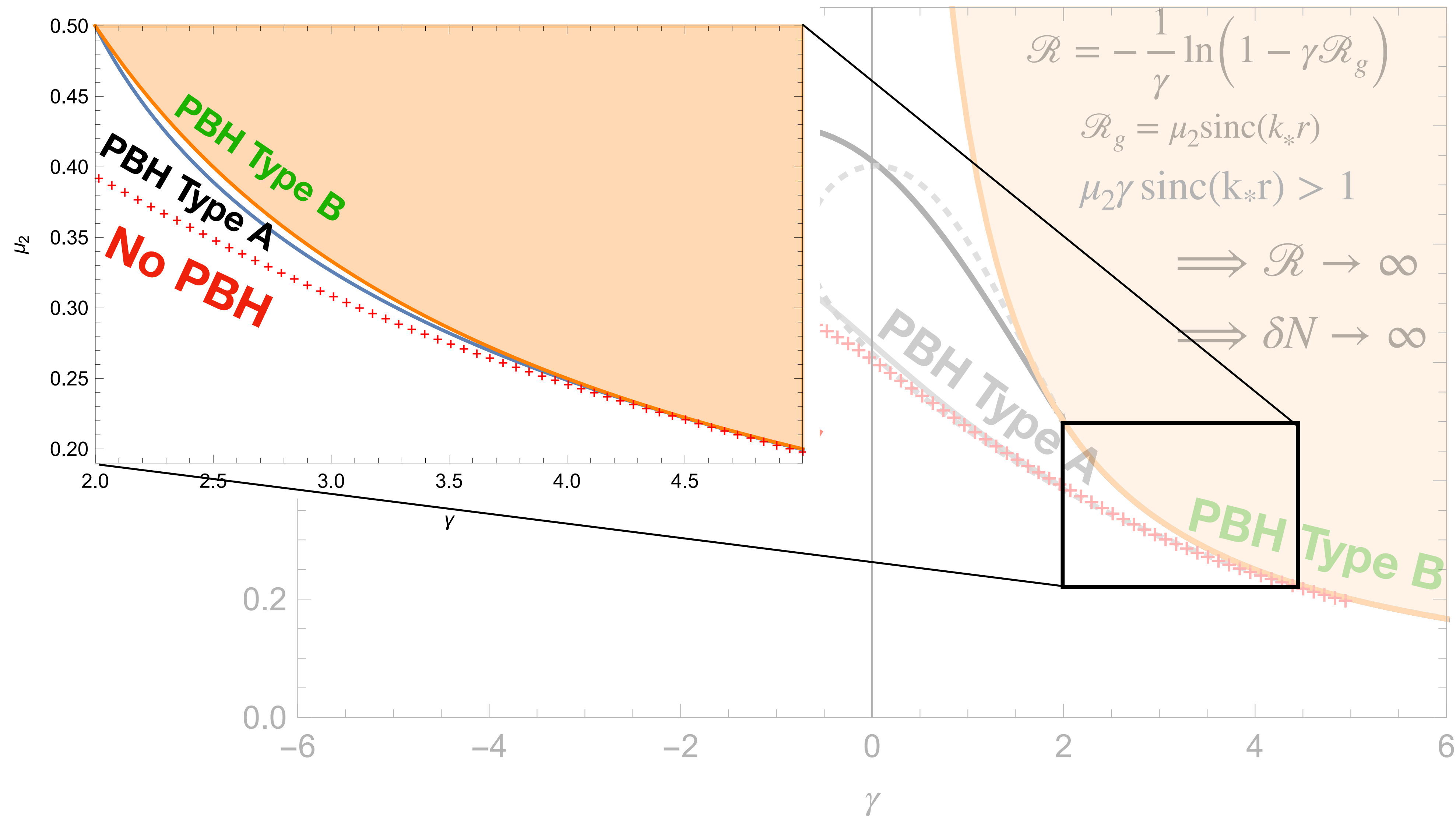
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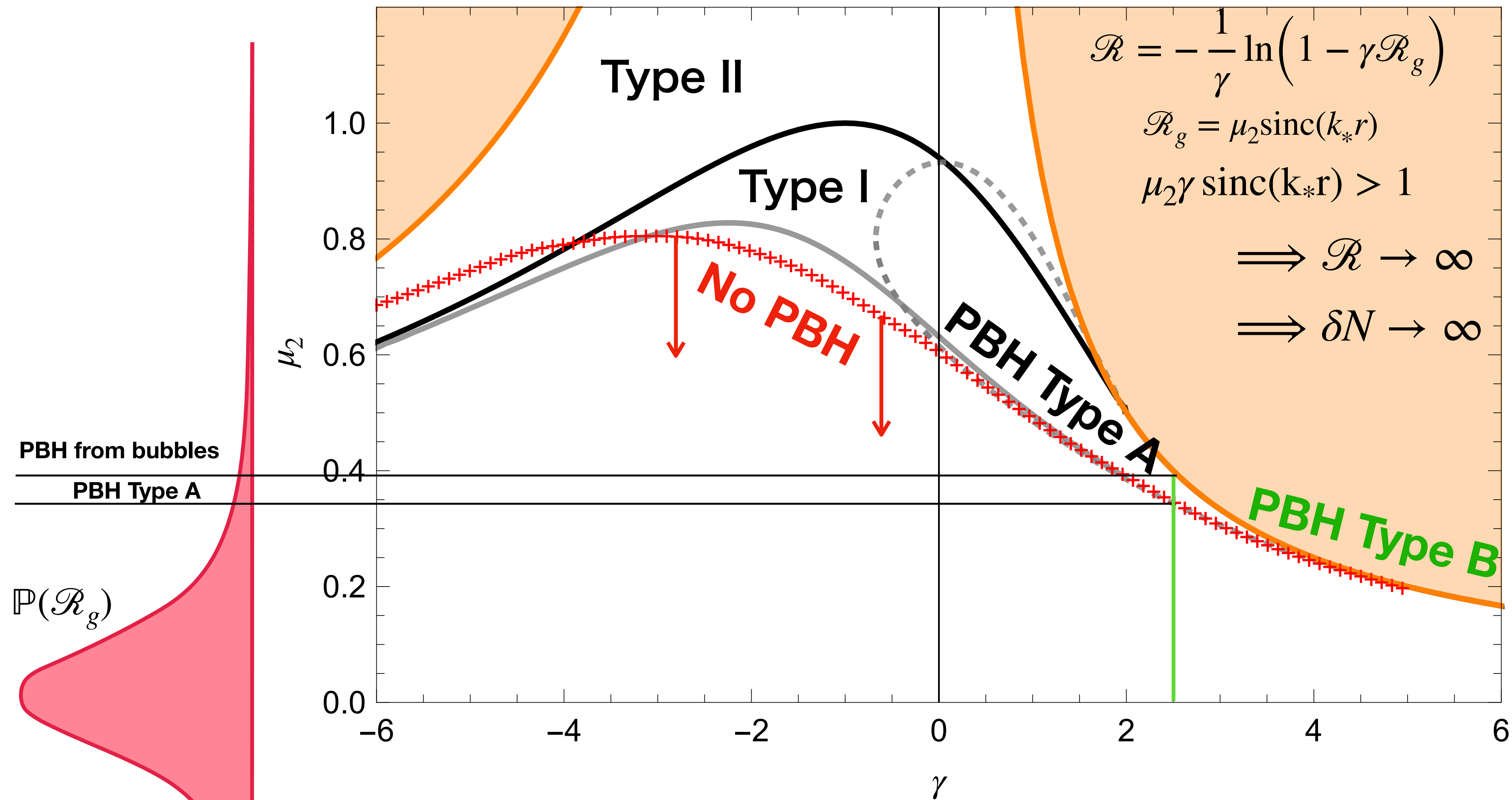
PBH phase diagram



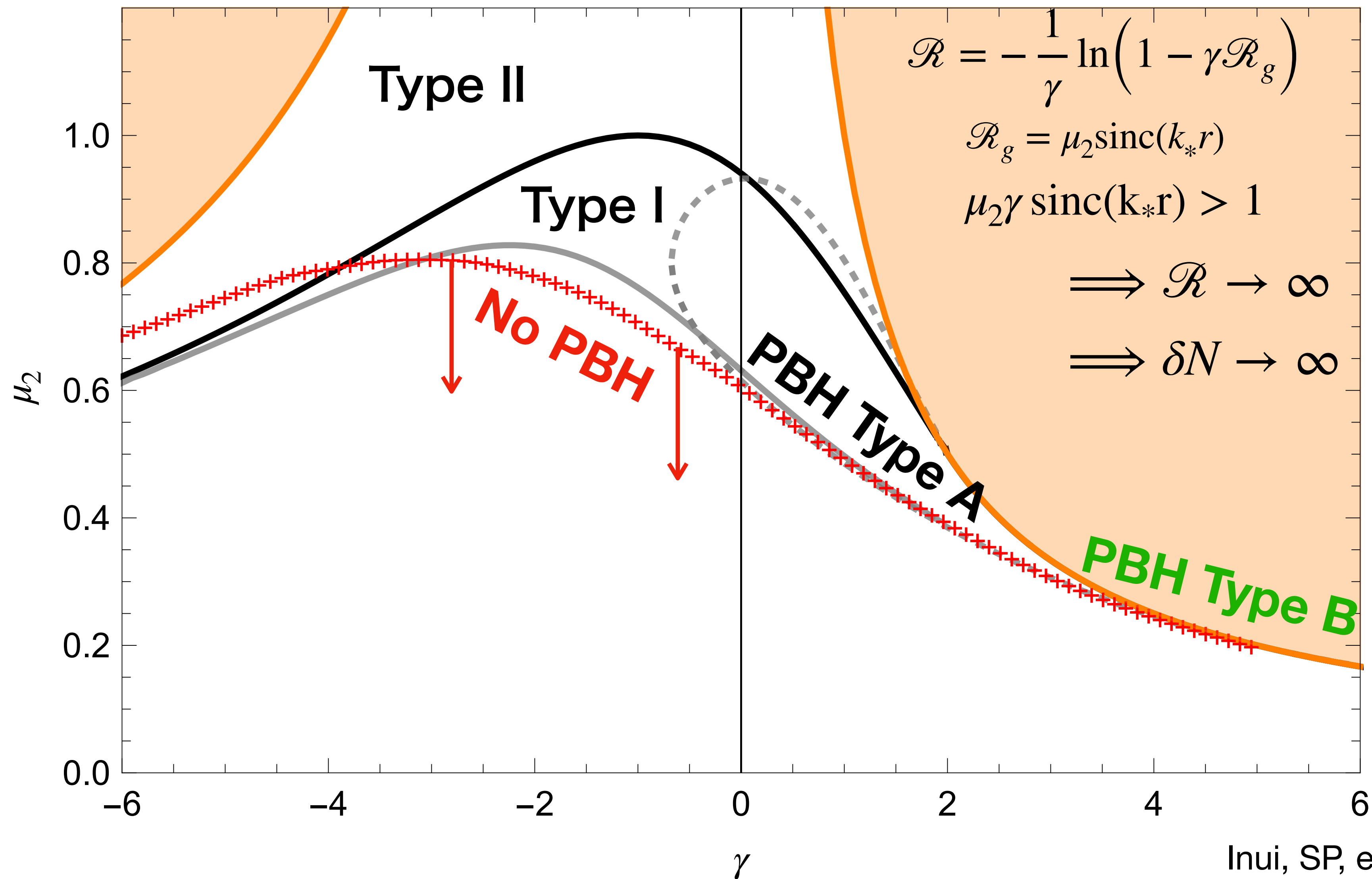
PBH phase diagram



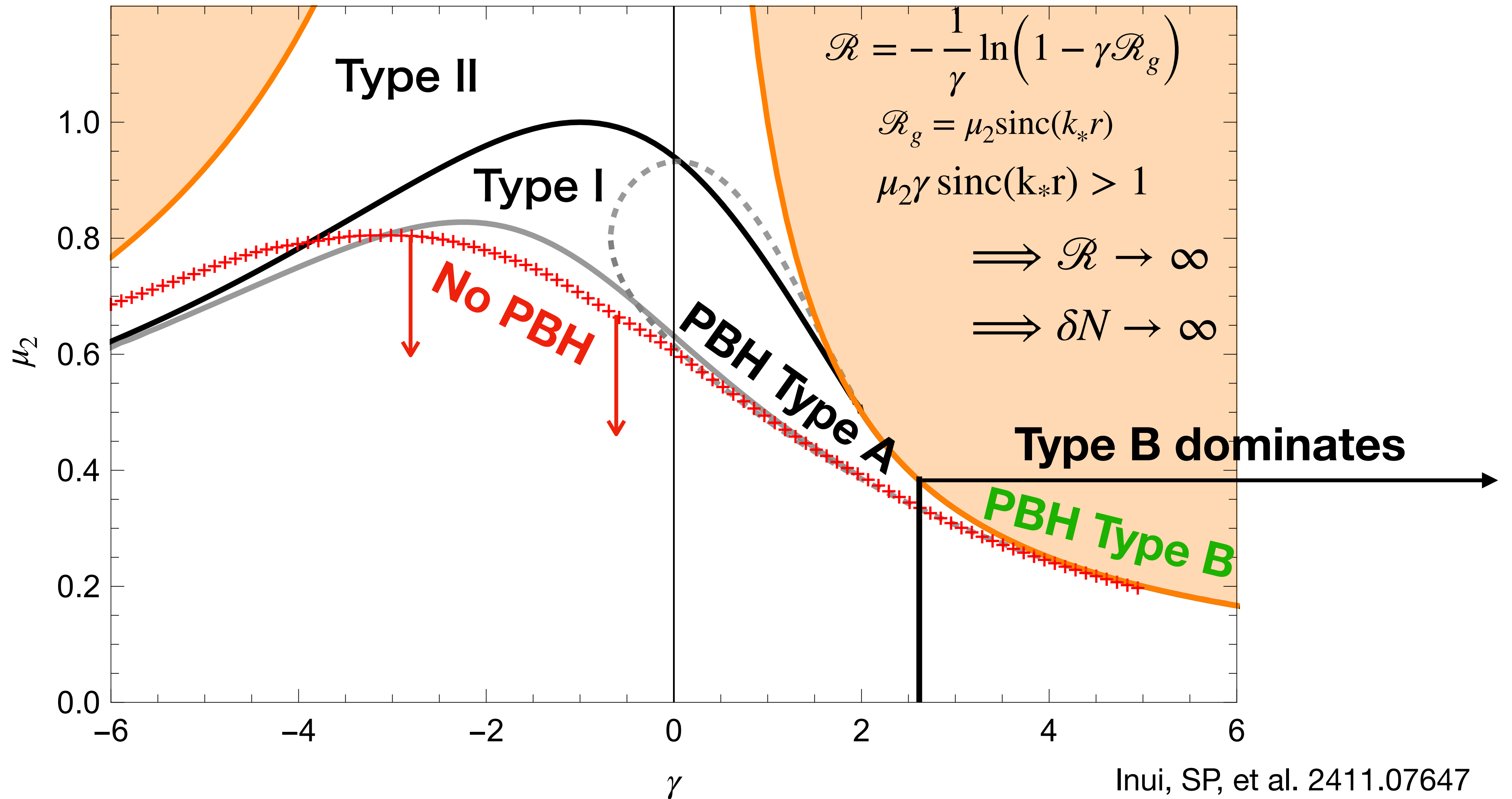
PBH phase diagram



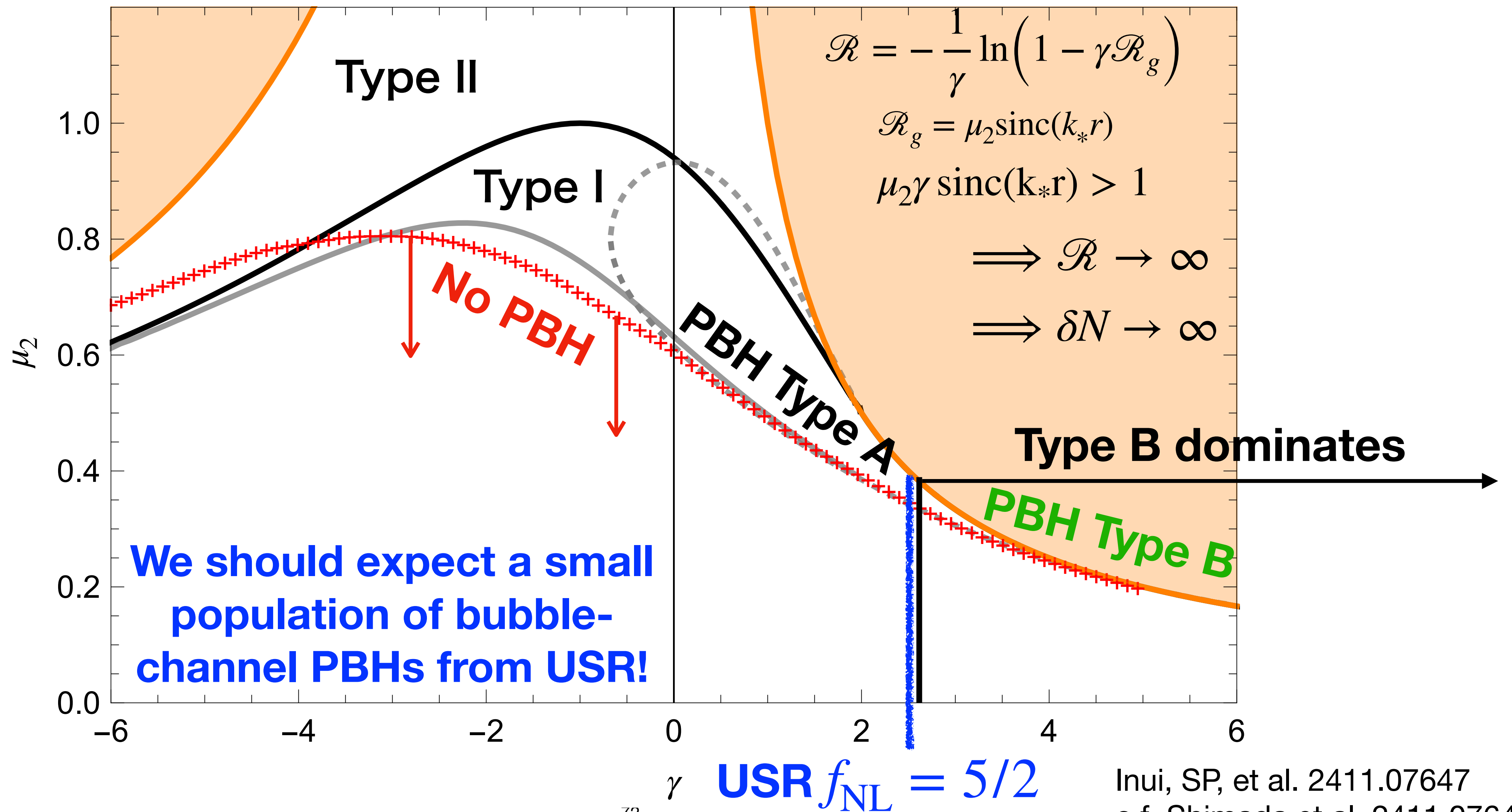
PBH in logarithmic $\mathcal{R}$



PBH in logarithmic $\mathcal{R}$

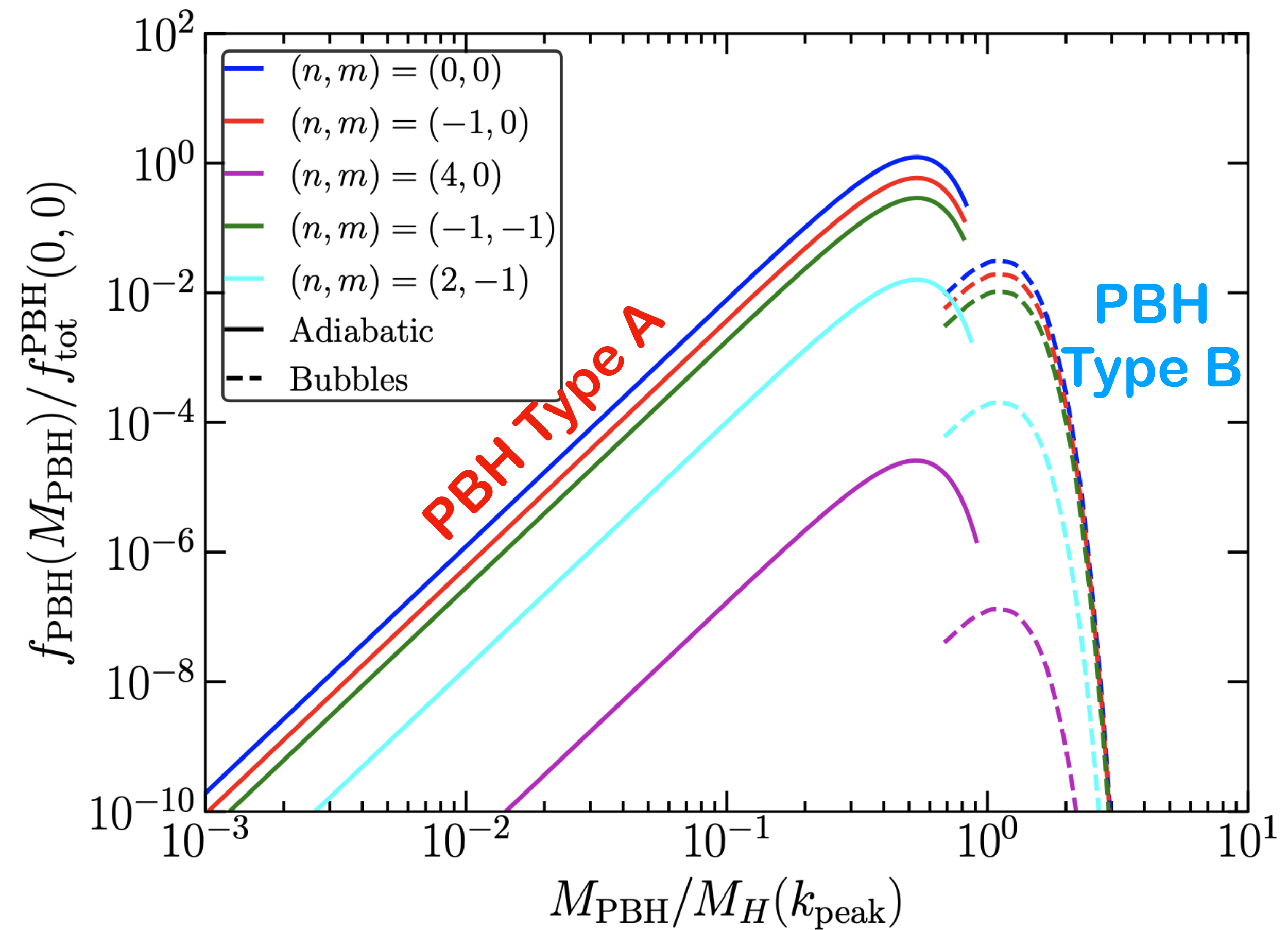
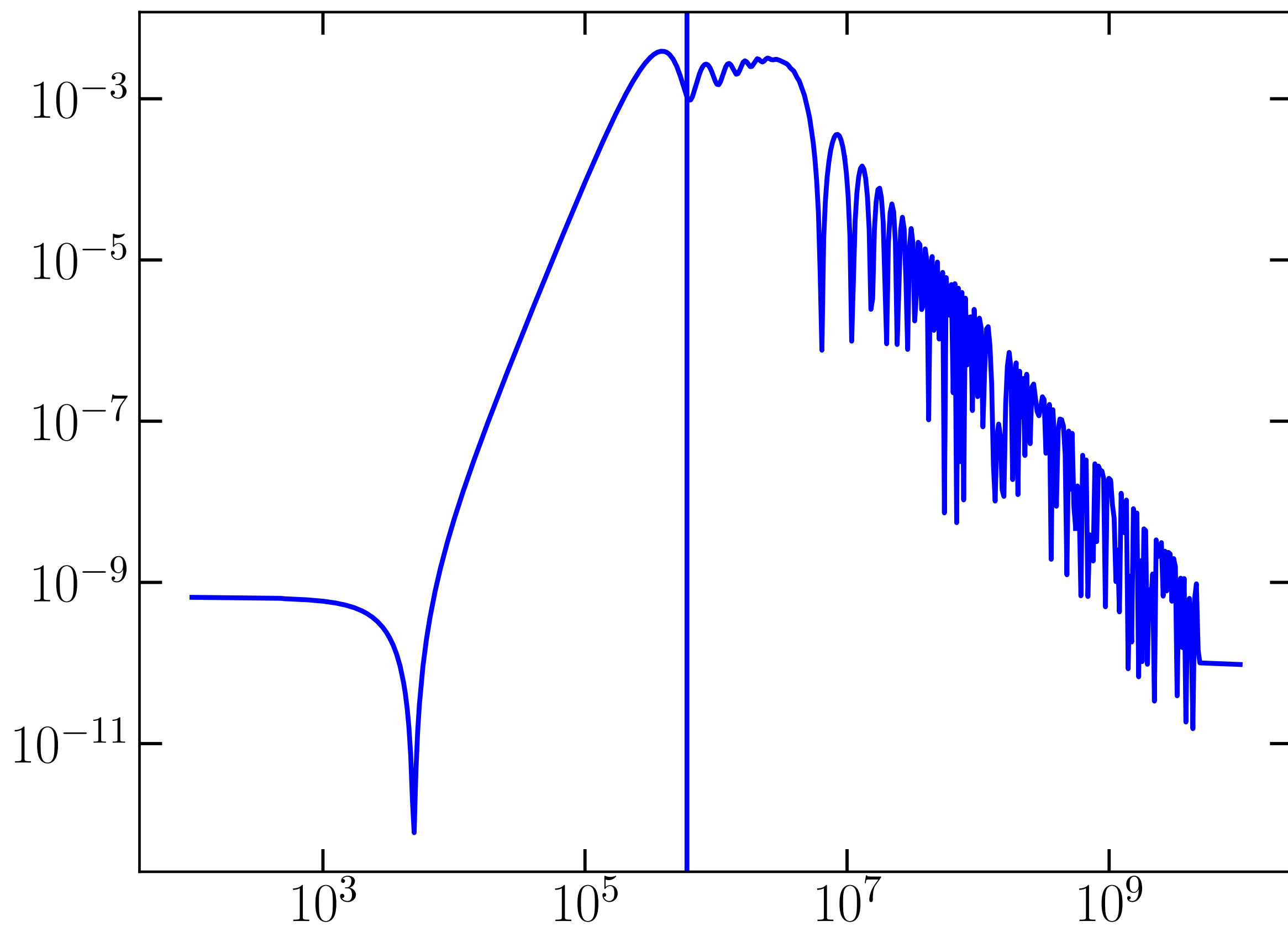


PBH in logarithmic $\mathcal{R}$



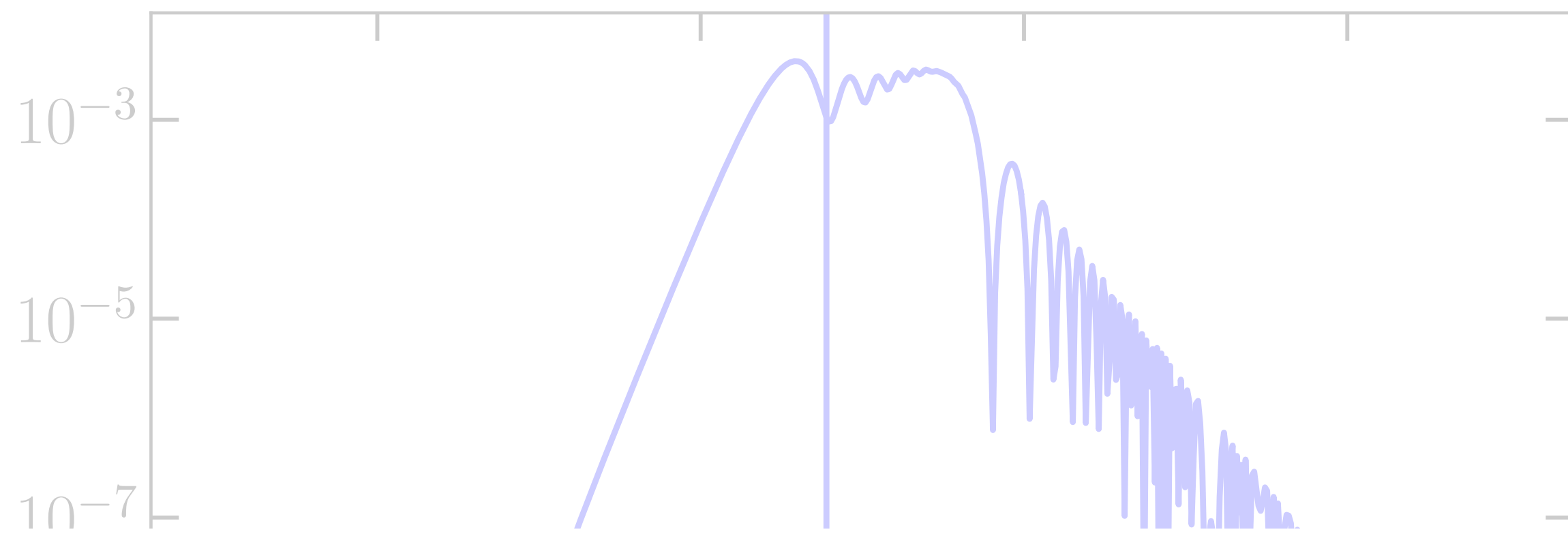
Eternal-inflating PBHs

Escriva, Garriga, SP, 2512.04986

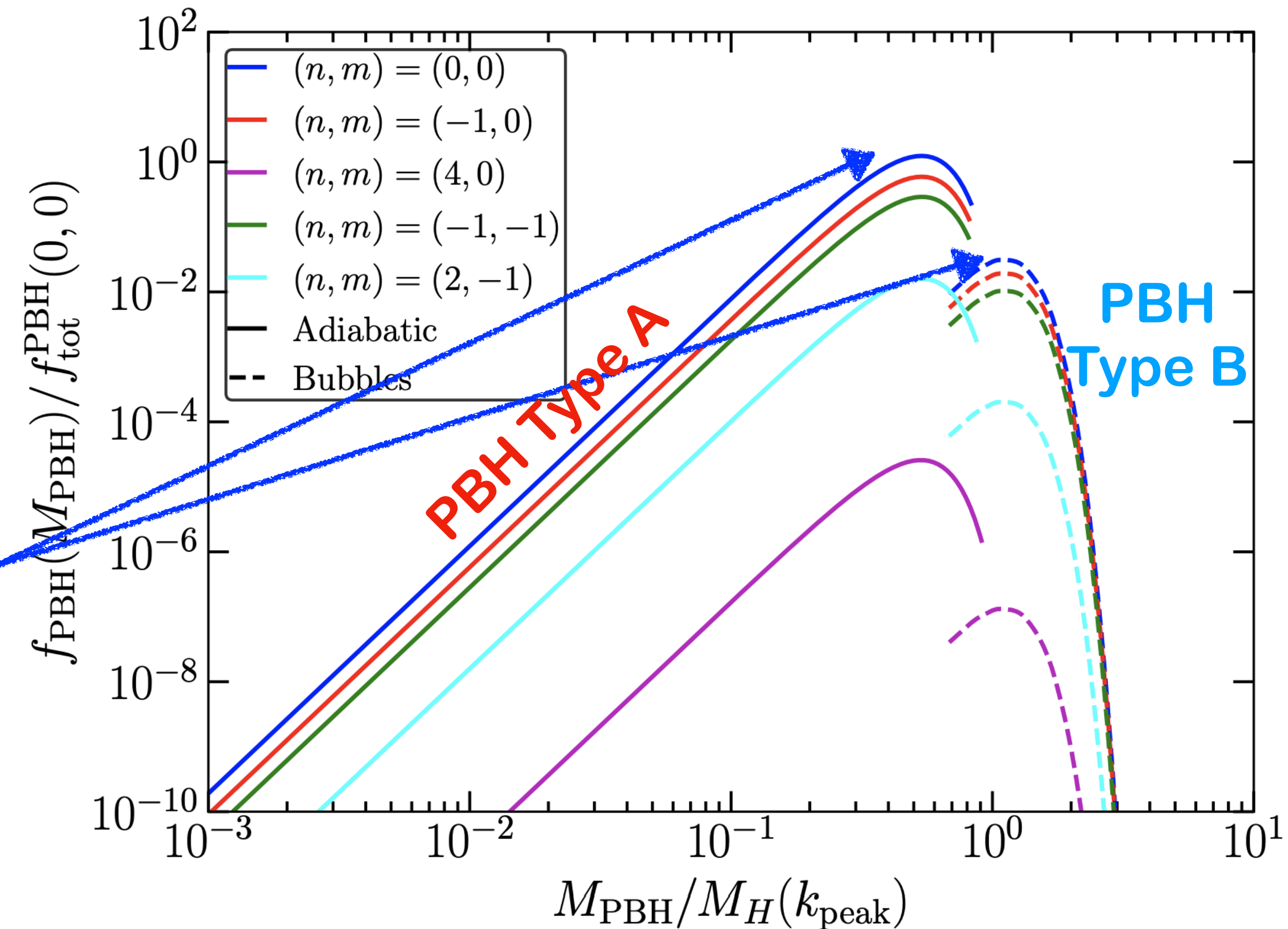


Eternal-inflating PBHs

Escriva, Garriga, SP, 2512.04986



n	m	$f_{\text{PBH}}^{\text{tot}}(n, m) / f_{\text{PBH}}^{\text{tot}}(0, 0)$	$f_{\text{a}}^{\text{tot}}(n, m) / f_{\text{b}}^{\text{tot}}(n, m)$
0.0	0.0	1.0	51.68
-1.0	0.0	4.78×10^{-1}	39.71
-1.0	-1.0	2.35×10^{-1}	36.05
2.0	-1.0	1.29×10^{-2}	104.06
4.0	0.0	2.08×10^{-5}	260.07



结论

- 次级引力波规范问题的核心是找到相应的观测量。诱导引力波经过时，沿测地线运动的观测者看到的固有时变化对应的应力张量符合牛顿规范中的形式。在其他规范中计算则要加上一些附加项。
- 非高斯性是原初黑洞-诱导引力波交叉检验的一个重要因素。我们证明了超慢滚阶段非高斯主要来源于超视界的非线性演化，且推导出非微扰的曲率扰动解析表达式。
- 近期观测到的超短时标微引力透镜体可以用行星质量原初黑洞来解释，这族原初黑洞可能与纳赫兹引力波的起源相同，可用一近似平坦的曲率扰动功率谱来统一描述。