第十五届“新物理研讨会”

Self-Interacting Dark Matter (SIDM):
From Particle Physics to Small-Scale Structures

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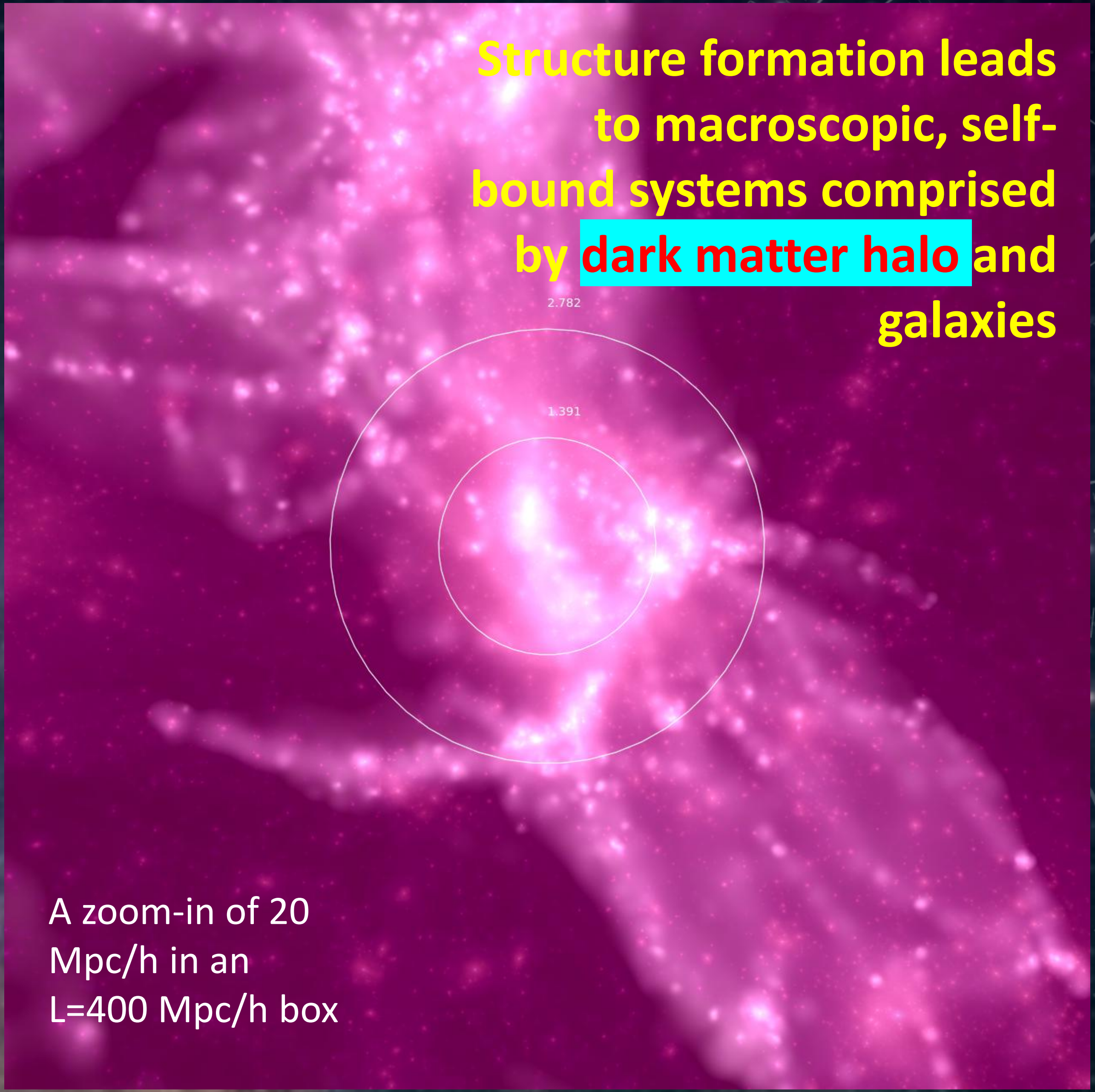
山东 威海

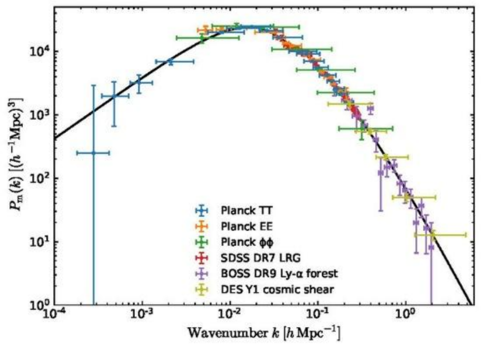
Outline

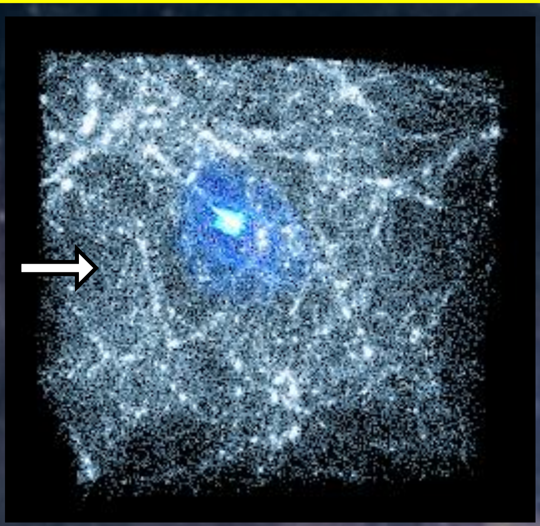
- From particle interactions to effective halo physics
- From N-body dynamics to thermodynamics and kinetics
- From SIDM evolution to observable predictions
- Beyond Deterministic Halo Modeling



Small-Scale Structure as a Probe of DM Physics

Early Universe Effects
mainly affect the matter power spectrum

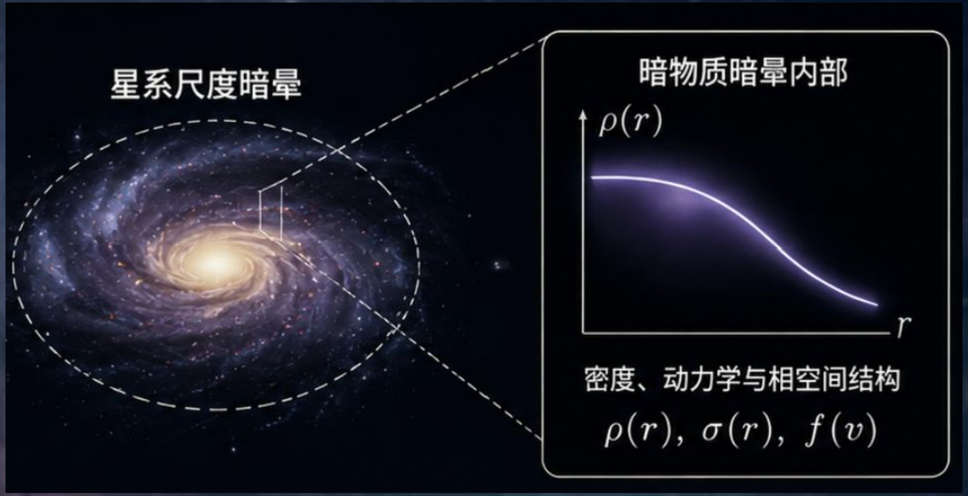




Non-linear Evolution

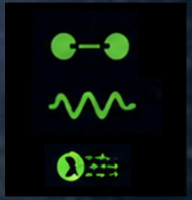
Gravitational collapse
Structure formation
Baryonic processes
Dark matter physics

Small-Scale Processes
sensitive to dark matter micro-physics



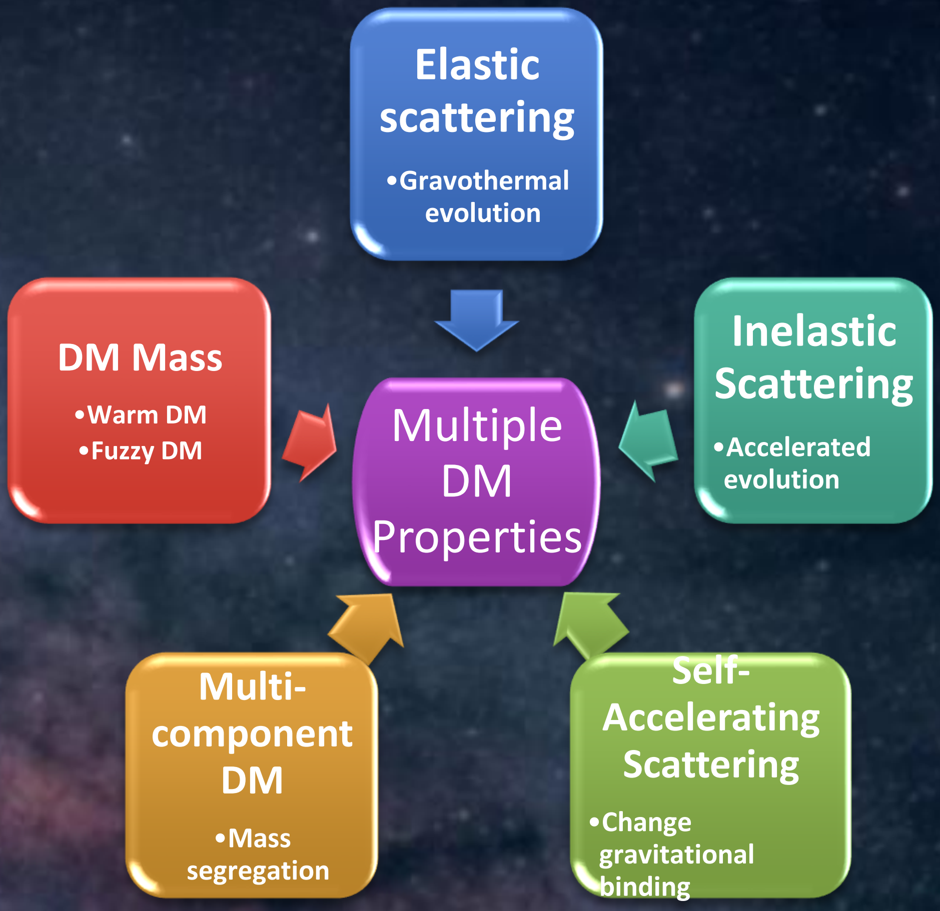
Complementary observational probes provide a laboratory for dark matter across a wide range of scales and redshifts

- Self-Interacting Dark Matter
- Fuzzy Dark Matter
- Warm Dark Matter



Challenges in Simulation & Modeling

Precision Galaxy-Scale Observations (CSST, JWST, Euclid, SKA…)	ΛCDM Simulations & Modeling	Beyond-ΛCDM Bottlenecks
More accurate	Reproduce most observations	Cost & precision
Larger samples	Rich physics and calibration	Baryons & SIDM
Higher redshift	Expensive and hard to extrapolate	Large parameter space (right figure)

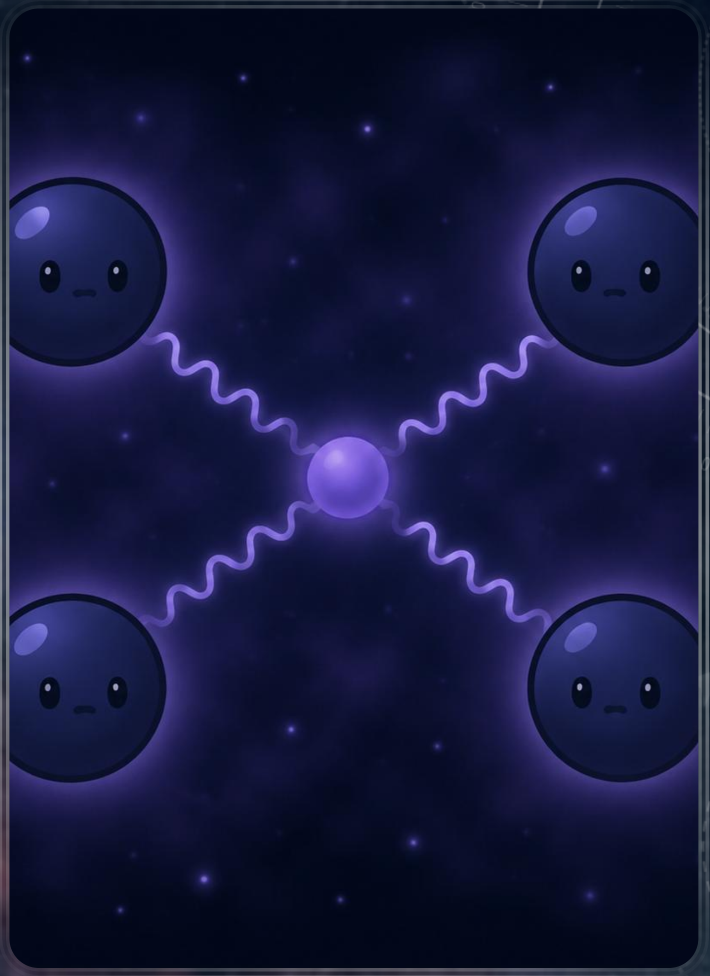


We need an efficient framework connecting

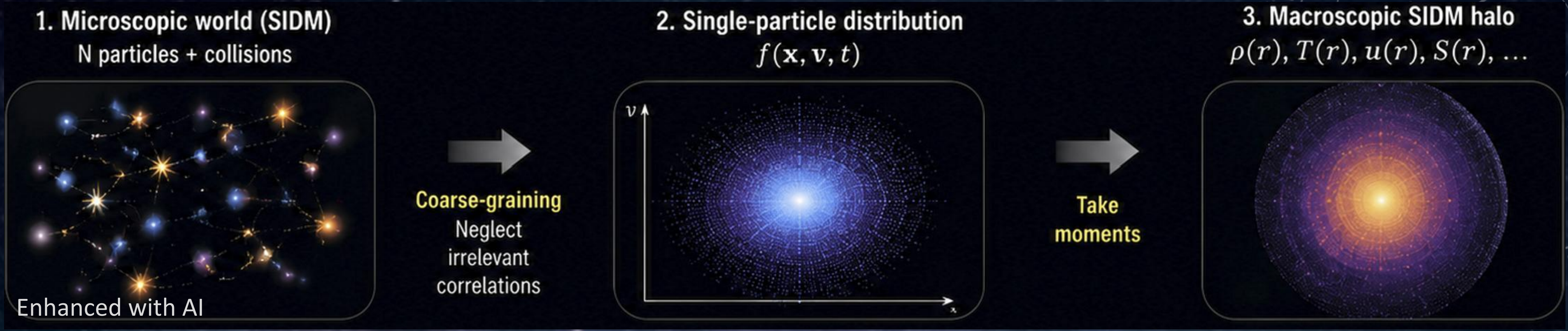
DM Physics->Galaxy Formation->Observables

From particle interactions to effective halo physics

- Effective constant SIDM cross section
- Multiple species
- Energy injection and dissipation
- Chaotic relaxation after sudden changes



From Microscopic Collisions to Macroscopic Dynamics in SIDM



Correlations generated by collisions become irrelevant on macroscopic scales. The system is fully described by the single-particle distribution function (Yu Deng's *Fields medal 2026*)

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m}\cdot\nabla f + \mathbf{F}\cdot\frac{\partial f}{\partial \mathbf{p}} = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}}$$

In SIDM, we ask: which reduced variables or transport coefficients retain memory of how microscopic collisions reshape a halo?

SIDM extends it to self-gravitating, evolving many body systems

- Long-range gravity
- Non-equilibrium evolution
- Quantum/Microscopic reversible interactions

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f - \nabla\Phi\frac{\partial f}{\partial \mathbf{v}} = C[f]$$

From SIDM to effective fluid dynamics

SIDM lives here
(short range collisions)

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f - \nabla\Phi\frac{\partial f}{\partial \mathbf{v}} = C[f]$$

Gravity complicates the story
(Long-range “collisions”)

SIDM thermalizes the inner halo region

$$\rho(r_1)\frac{\sigma}{m}\langle v_{\rm rel}(r_1)\rangle t_{\rm age} \sim 1$$

Velocity distribution is driven to Maxwellian

$$f(\mathbf{v},\mathbf{x},t) = \rho(\mathbf{x},t)\left(\frac{m_D}{2\pi T}\right)^{\frac{3}{2}}\exp\left[-\frac{m_D(\mathbf{v}-\mathbf{u})^2}{2T}\right]$$

Scatterings are typically rare

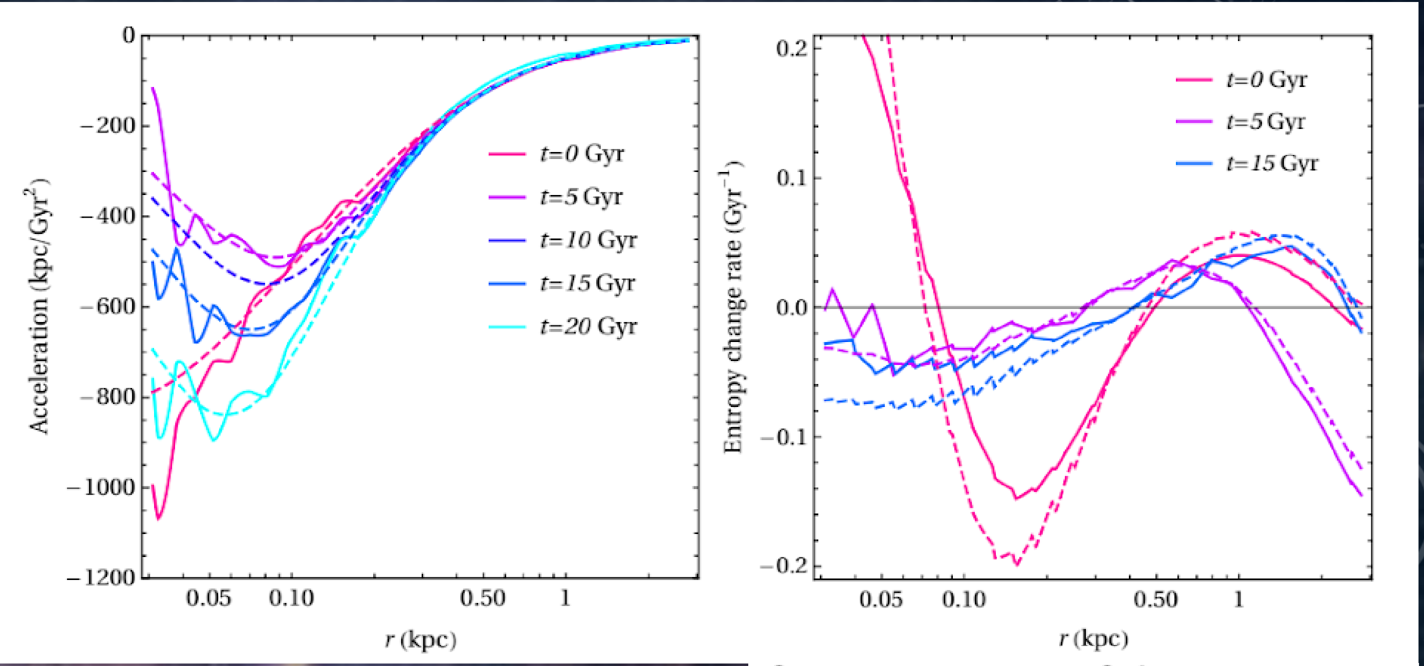
Kaplinghat, Tulin, Yu, 2016

Pressure equilibrium

$$\nabla(\rho\sigma_{\rm 1D}^2) = -\rho\nabla\Phi$$

Energy transport

$$\frac{1}{4\pi}\frac{\partial L}{\partial r} = -\rho\sigma_{\rm 1D}^2\frac{Ds}{Dt}$$



$$\frac{\partial s}{\partial t}\ \text{(solid)}\ \frac{1}{\sigma^2}\frac{\partial \mathcal{E}}{\partial t}\ \text{(dashed)}$$

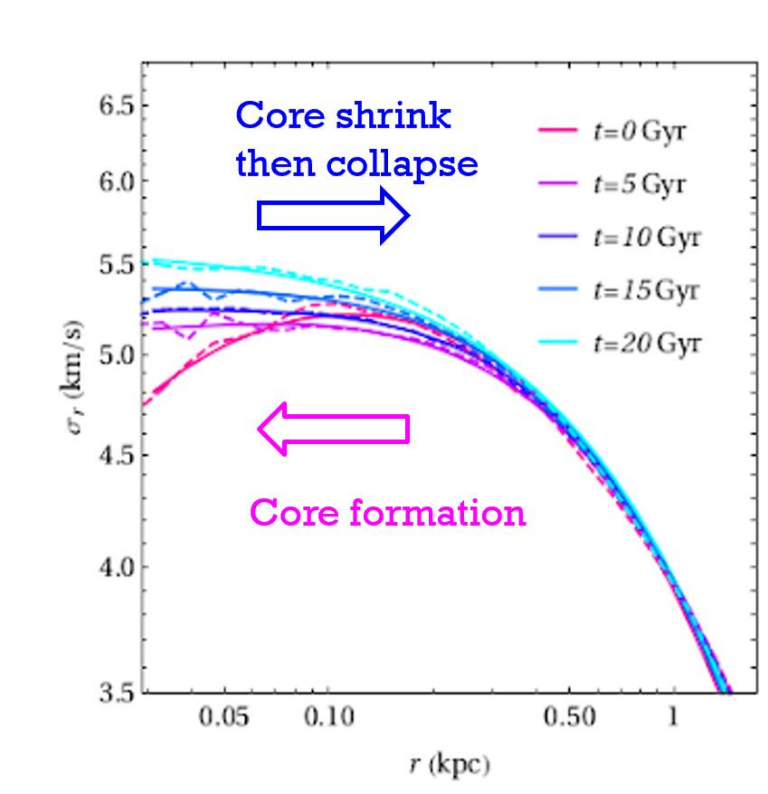
Test effective fluid dynamics using an accurate N-body simulation

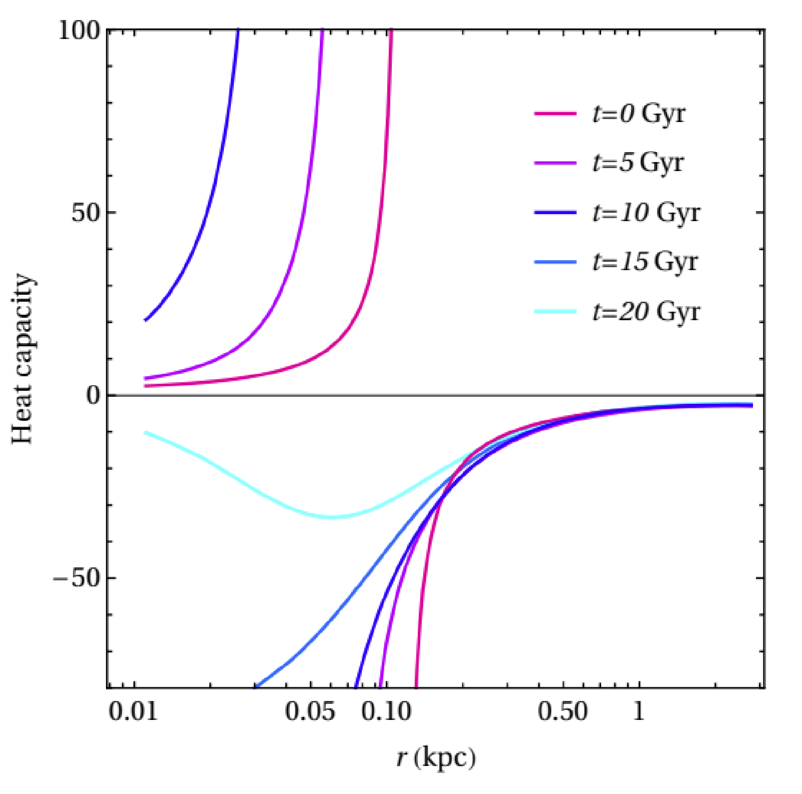
Yang & Yu 2205.03392 JCAP

Gravothermal evolution of SIDM halos

A thermodynamics picture

Spergel & Steinhardt 2000,
Balberg et al. 2002 ApJ,
Koda et al. 2011 MN, etc





$$C(r) = dE/d\sigma_{1D}^2$$

Core formation

- heat flux + **capacity** => core formation

Core shrink (heat flux small)

+ heat flux + capacity => quasi-stable core

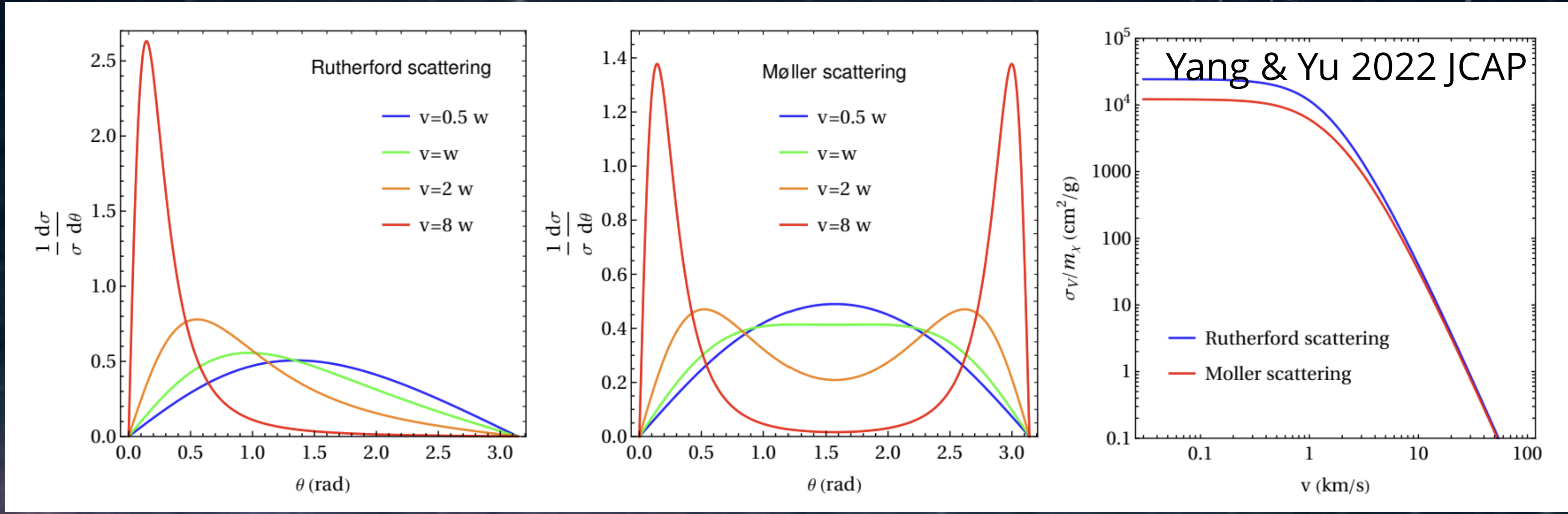
Core collapse

+ heat flux - **capacity** => **core collapse**

Yang & Yu 2205.03392 JCAP

From SIDM to effective fluid dynamics

- Particle scatterings can be velocity and angular dependent
- How to calculate its effect in a halo?



<σv> computes the scatterings rate, which is the correct quantity for indirect detection

In a halo, heat conductivity ***κ*** is the most relevant quantity

$$\frac{\partial}{\partial r}\left(r^2 \kappa m \frac{\partial \nu^2}{\partial r}\right) = r^2 \rho \nu^2 \frac{D}{Dt} \ln \frac{\nu^3}{\rho}$$

Heat conductivity

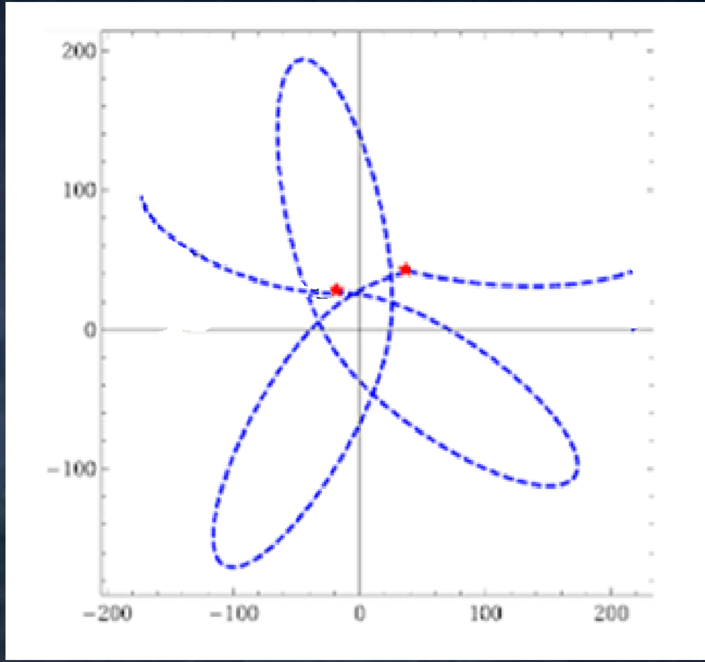
- **Issue:** SIDM collision is mostly too rare to apply fluid calculations.
- The bulk halo lives in the Long-Mean-Free-Path (LMFP) regime

Reducing the differential cross section to an Effective constant cross section

Two kinds of physics decoupled
Short scale: SIDM
Large scale: Gravity

The weighting kernel should align with the kinetic theory calculation of heat conductivity (Lifshitz, *Physical Kinetics*)

$$\sigma_{\rm eff} = \frac{2\int v^2 dv d\cos\theta \boxed{\frac{d\sigma}{d\cos\theta}} \boxed{\sin^2\theta v^5} \boxed{\exp\left[-\frac{v^2}{4\nu_{\rm eff}^2}\right]}}{\int v^2 dv d\cos\theta \boxed{\sin^2\theta v^5} \boxed{\exp\left[-\frac{v^2}{4\nu_{\rm eff}^2}\right]}}$$



Yang & Yu 2022 JCAP
See also: S. Yang et al. 2022 ApJ
Tulin, Yu & Zurek 2013

- Angular dependence is completely integrated out
- *Details of an SIDM model is **hidden** in a single halo*
- Only the ***velocity dependence of SIDM*** couples to the ***halo velocity dispersion***

Astrophysical probes of SIDM and its variants

Multi-window observations break model degeneracies	
	Grav lensing perterber
	Galaxy-Galaxy Strong Lensing
	Lens flux ratios
	DM deficient galaxies
	Stellar stream gaps
	Dwarf galaxy clustering
	Little Red Dots

Multiple DM Properties

Elastic scattering
- Gravothermal evolution

- Self-Interacting Dark Matter (SIDM)
- MACHO scatterings
- PBH scatterings

DM Mass
- Warm DM
- Fuzzy DM

Inelastic Scattering
- Accelerated evolution

- Dissipative SIDM
- DM-baryon scatterings
- Indirect signatures

Multi-component DM
- Mass segregation

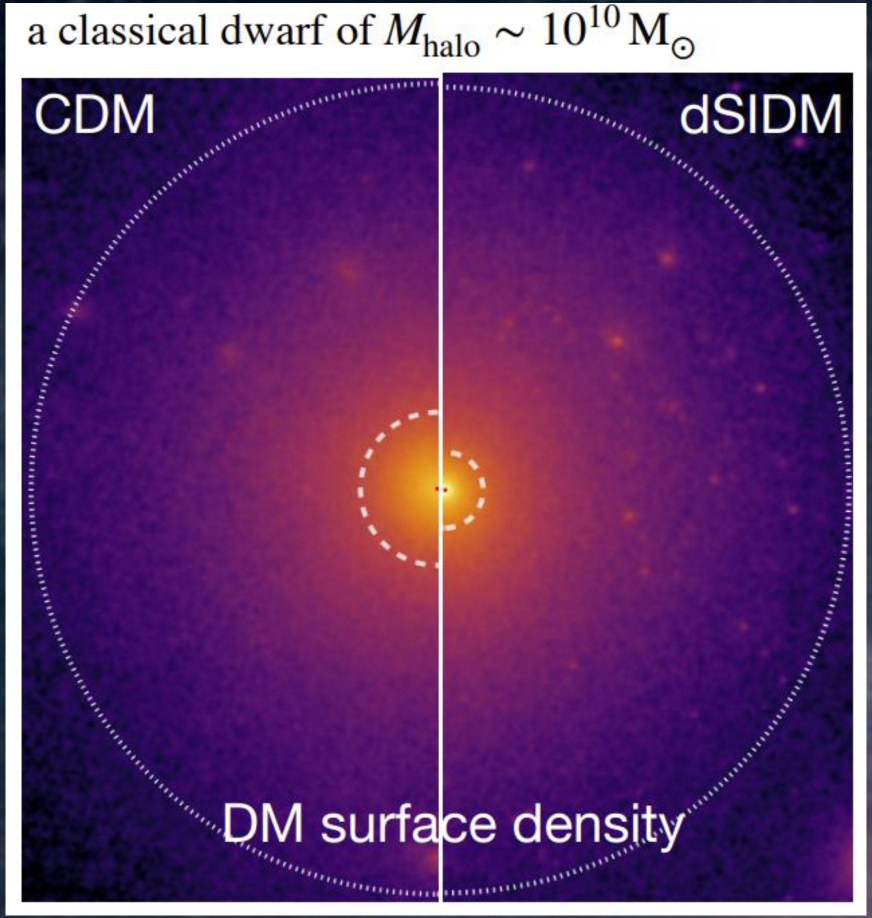
- Atomic Dark Matter
- Dark glue-balls
- Boosted Dark Matter
- Multi states/continuum dark matter

Self-Accelerating Scattering
- Change gravitational binding

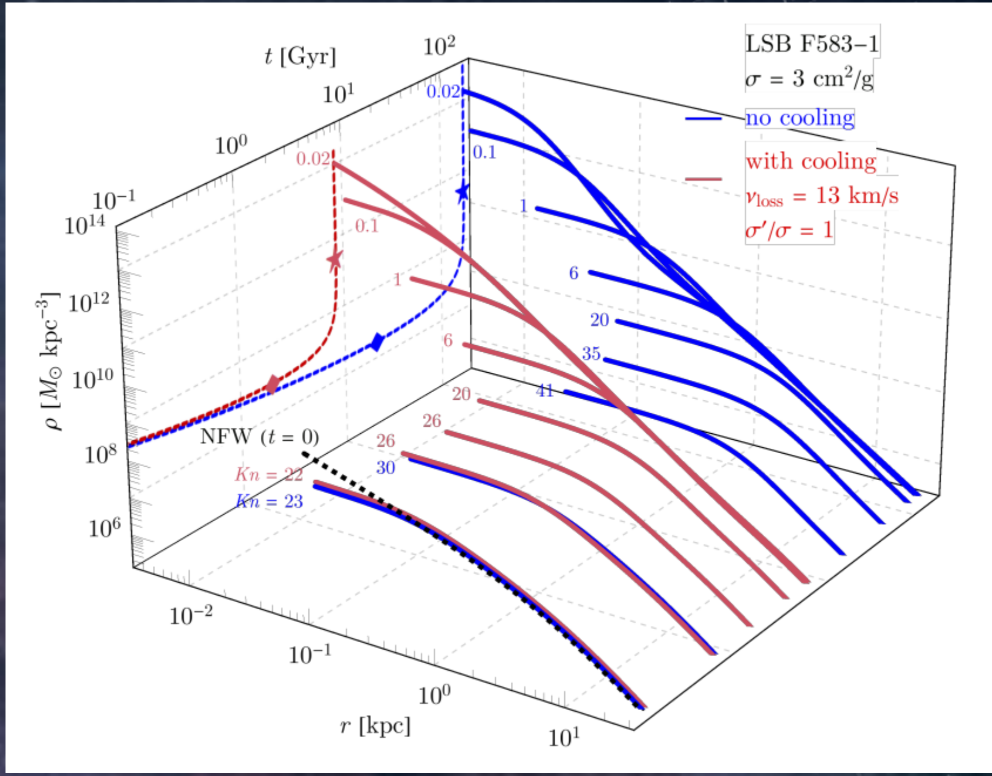
- Exothermic dark matter
- Dynamical dark matter
- Cannibal dark matter
- Bosenovae
- Warm dark matter
- DM induced baryon feedback

(1) Dissipative SIDM

- Dissipation tends to make the distribution more compact
- SIDM core formation tends to make the distribution more diffuse



More concentrated, spherical halos + promote baryonic disk formation, Xuejian Shen et al. 2021, 2024



Boosted gravothermal evolution, Essig et al. 2019 & many other studies…

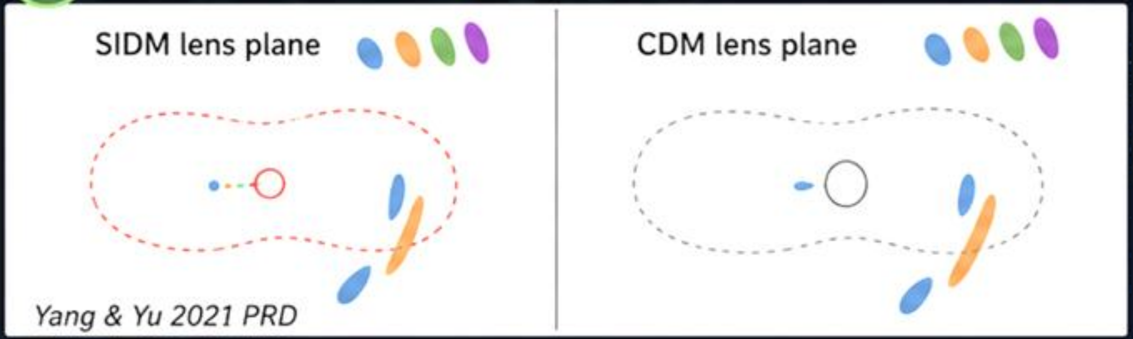
(2) SIDM with multiple species and mass segregation

1 Limitation of 1-component SIDM

- Core shrinkage in **1-component SIDM** is driven by the **loss of high-velocity** particles, which **reduces the inner halo mass**.

Negative heat capacity: the system becomes hotter as it releases heat.

2 Implication for lensing



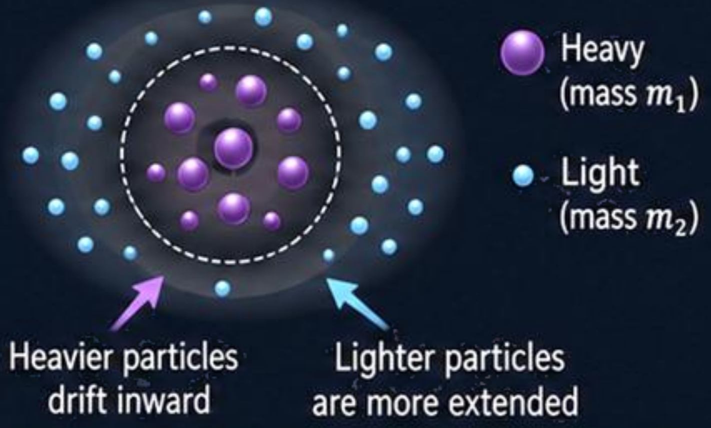
- ✓ Einstein radii remain nearly the same. (Stay tuned)
- ✓ Effects on lensed images are modest, mainly enhancing 2-image events.

3 Mass segregation in 2-component SIDM

Collisions drive energy equilibration:

$$m_1 v_1^2 \sim m_2 v_2^2$$

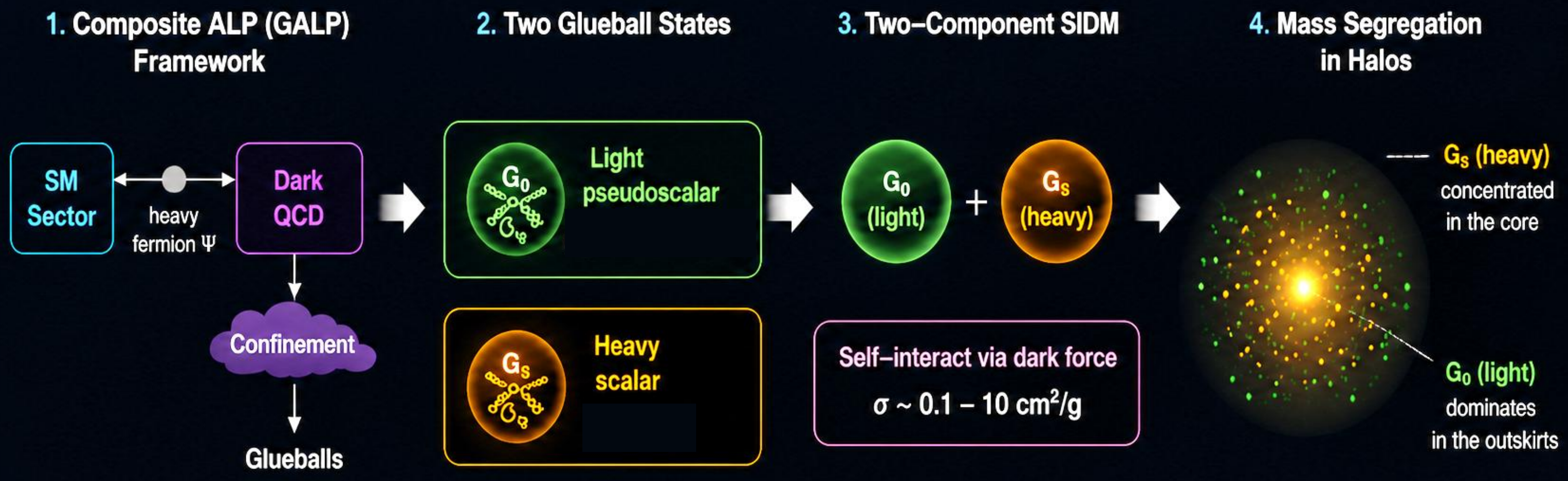
$$m_1 > m_2 \Rightarrow v_1 < v_2$$



Can we reconcile diffuse and dense observations simultaneously?

Massive particles sink inward, elevating the central halo mass.

A Natural Path to Two–Component SIDM with Mass Segregation



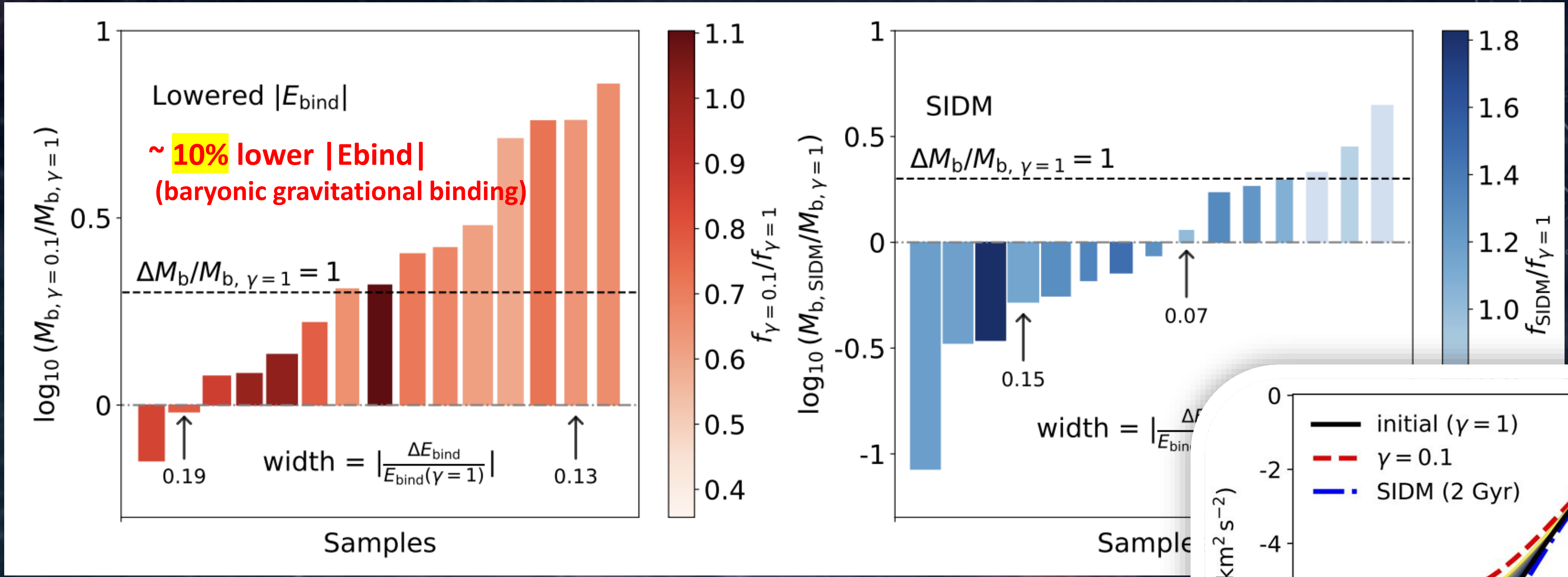
Carenza, Pasechnik, Wang, 2025 PRL

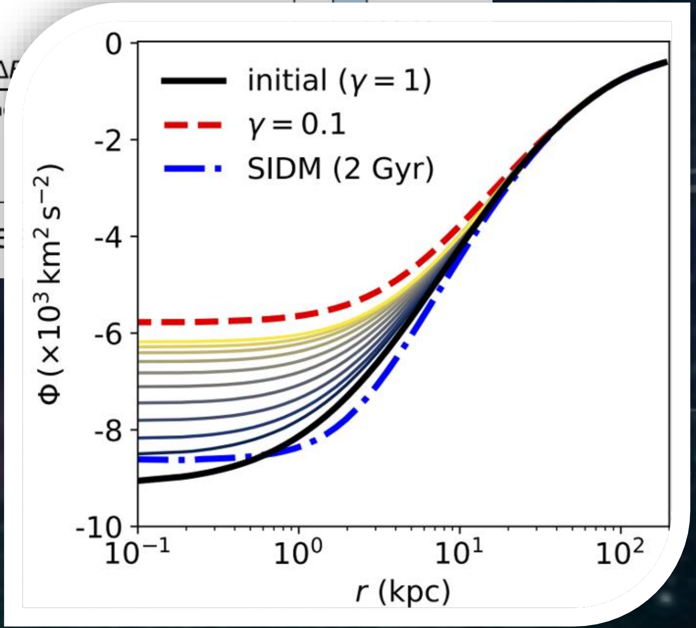
Composite ALP framework naturally produces two stable, self–interacting components with different masses → mass segregation in galaxies and clusters.

Enhanced with AI

(3) Exothermic dark matter / DM induced baryon feedback

Models: Vogelsberger et al. 2019 MN / Acevedo et al. 2024 PRD





The Collisional formation of DMDG offers a novel probe to test energy injection in dwarf halos

Wang & Yang et al. arXiv:2509.24270, ApJL 997 (2026) 2, L43

(4) Chaotic relaxation after sudden changes

⇨ Out of equilibrium

❖ **$\Delta\Phi$:** mass changes

- accretion; feedback outflow
- tidal/ram-pressure stripping

❖ **ΔK:** dynamical heating

- mergers; tidal shocks

⇨ Restore via **violent relaxation!**

holy grail of since Lynden-Bell 1967

challenge: energy/actions **not** conserved!

CoreCusp2: An accurate model for halo evolution given

- Mass change profiles
- Energy injection profiles

Li, Dekel+ 2023 (arXiv:2206.07069) & Li+ in prep

① **Initial equilibrium**

distribution function: $f_0(E) = \frac{d^6M}{d^3\mathbf{x}d^3\mathbf{v}}$

(Eddington inversion)

② **Perturbed system**

- **mass change** $\Delta E = \Delta\Phi(r)$
- **heating** $\Delta E = \Delta K(r)$

③ **Violent relaxation** (no free params)

New distribution for $E \rightarrow E + \Delta E(r)$

$$f_k(E) = \frac{\int f_{k-1}(E - \Delta E(r))\sqrt{E - \Delta E(r) - \Phi_{k-1}(r)}\,r^2dr}{\int \sqrt{E - \Phi_k(r)}\,r^2dr}$$

Equilibrium density in the transient Φ_k

$$f_k(E) \Rightarrow \rho'(r) \Rightarrow \Phi'(r)$$

Evolve Φ towards the equilibrium

$$\Phi_{k+1} = \mu\Phi' + (1-\mu)\Phi_k$$

loop until convergence

$k \mathrel{+}= 1$

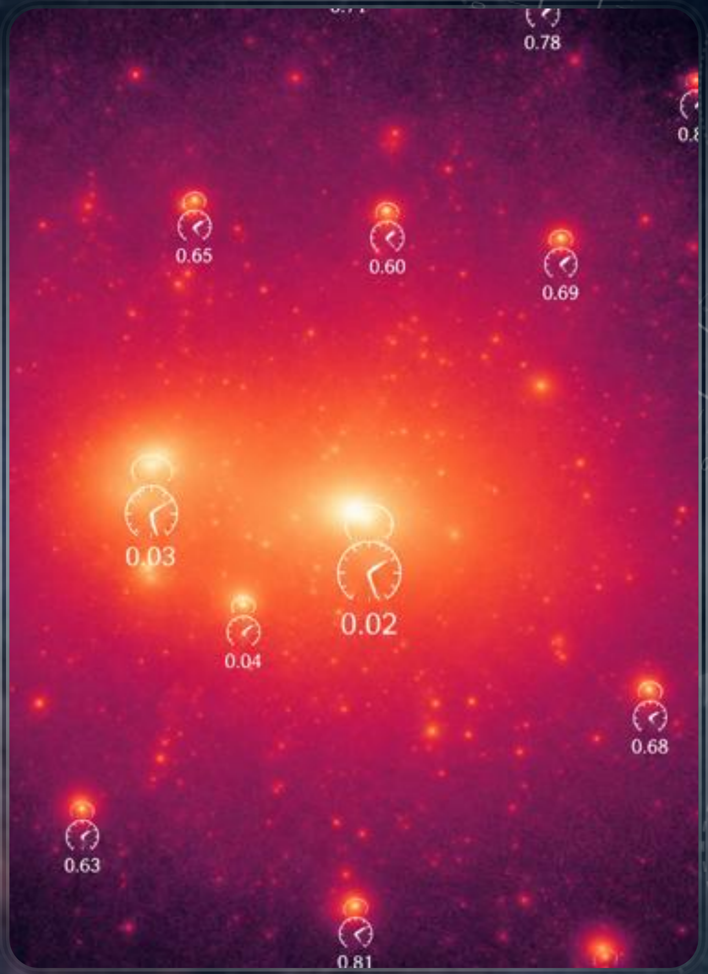
$\Delta E = \Phi_k - \Phi_{k-1}$

④ **New equilibrium** (with mass redistributed)

From Zhaozhou Li (NJU)

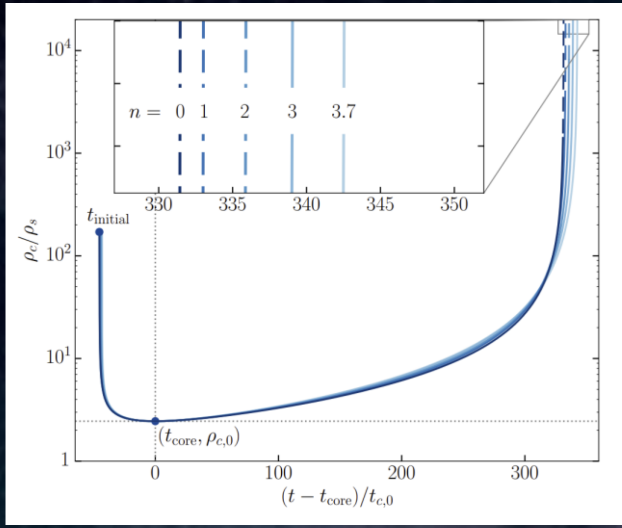
From N-body dynamics to thermodynamics and kinetics

- Cosmological vs controlled simulations
- Example applications
- Effective thermodynamics and the parametric model



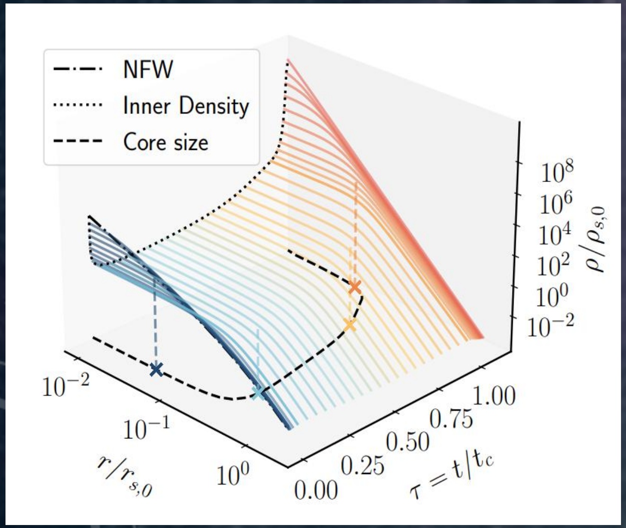
Modeling & Simulating SIDM halos

Evolution in isolation: easy
Evolution in environment: difficult



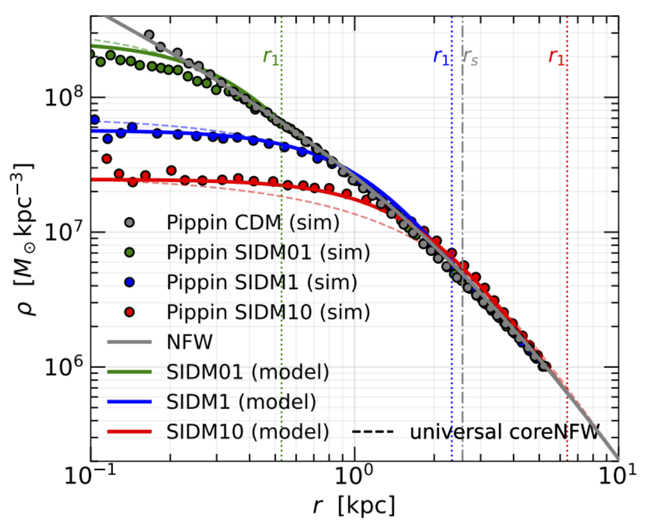
Fluid simulation

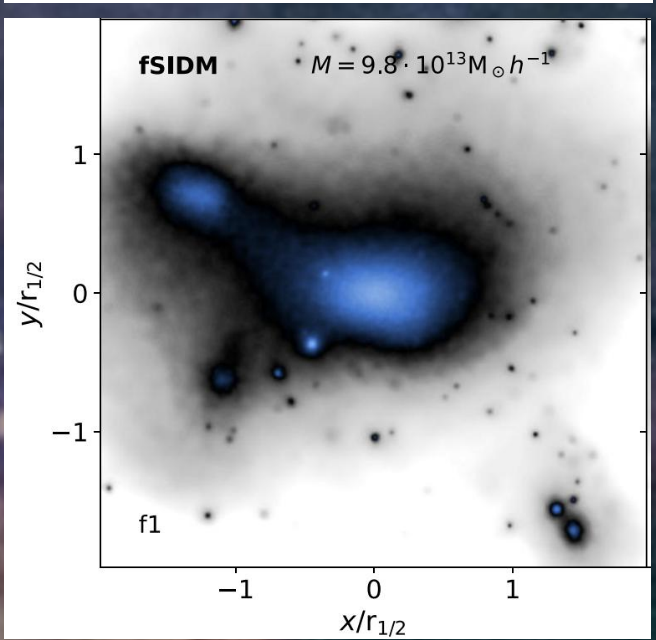
- Universality (Outmezguine et al. 2022)
- Seeding SMBH, Feng et al. 2021
- Dissipative SIDM, Essig et al. 2019 PRL
- Balberg et al. 2002



Parametric modeling

- Lensing model. Hou et al. 2025
- Effective constant cross section. Yang et al. 2022
- Integral approach: Yang et al. 2023





Isothermal-Jeans model

- Jiang et al. 2022
- Kaplinghat et al. 2014

N-body simulation

Accretion & Environment

Moritz et al. 2024+
Palubski et al. 2024
Robertson et al. 2017
Rocha et al. 2012
Kamionkowski et al. 2025
Gurian & May, 2025

SIDM halos in cosmological simulations

Cosmological simulations

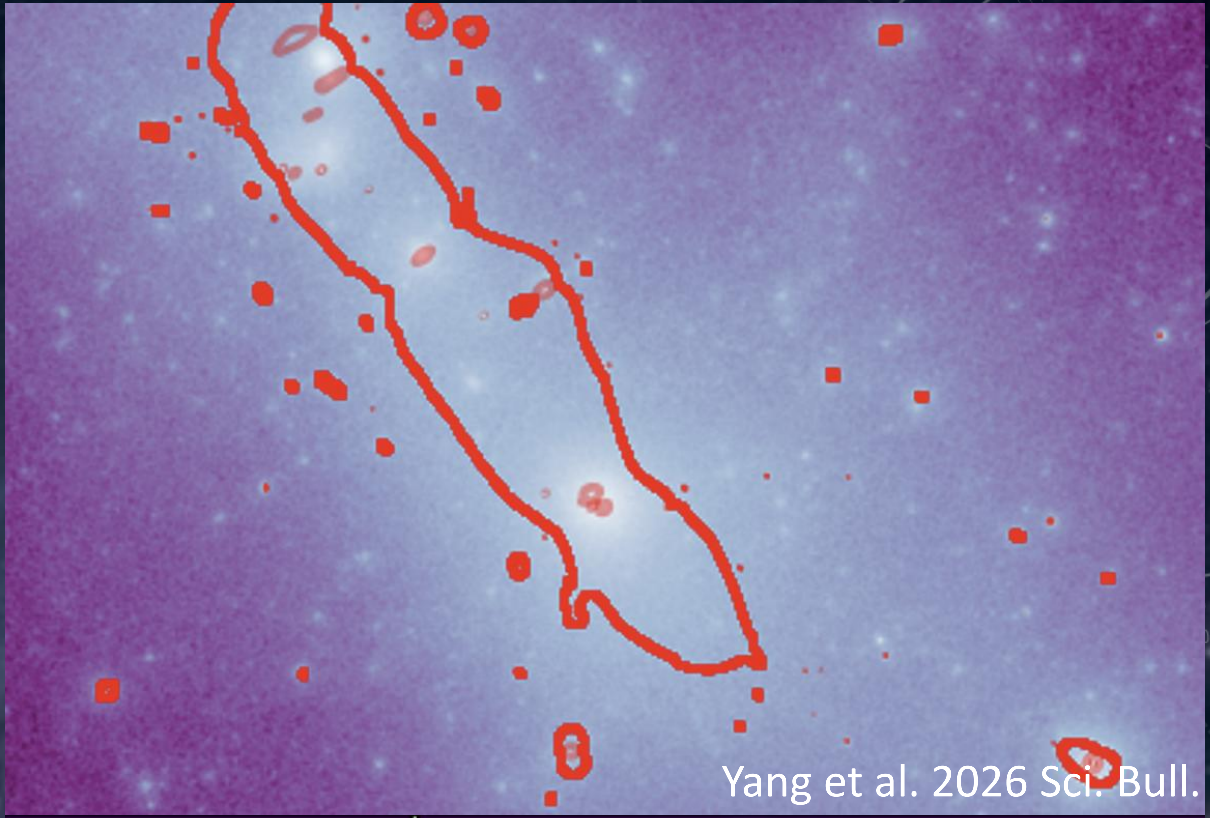
- Structure & halo formation
- Evolution histories in environment
- Halos in populations

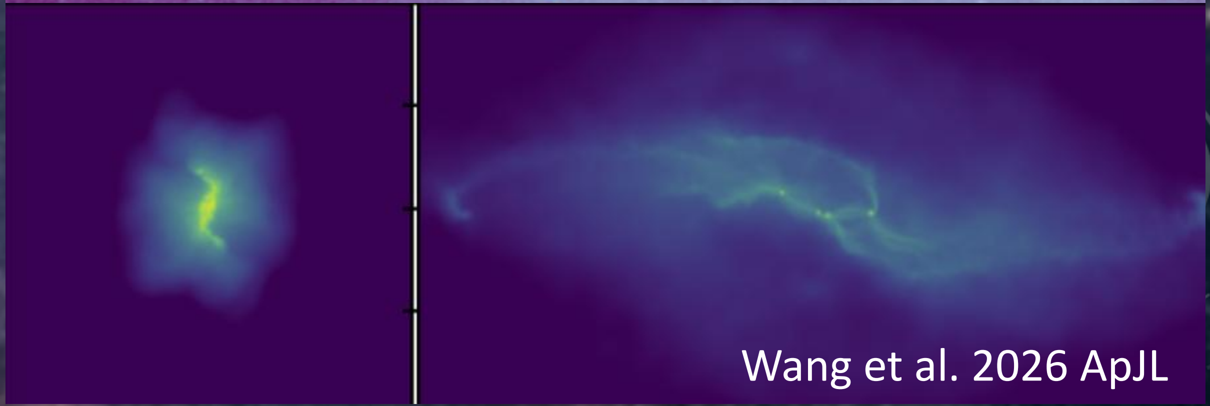
(Extremely expensive)

Controlled simulations

- Typically one/few halos in isolation or an external potential
- Initial condition put by hand & in equilibrium

(Expensive if go to high resolution)

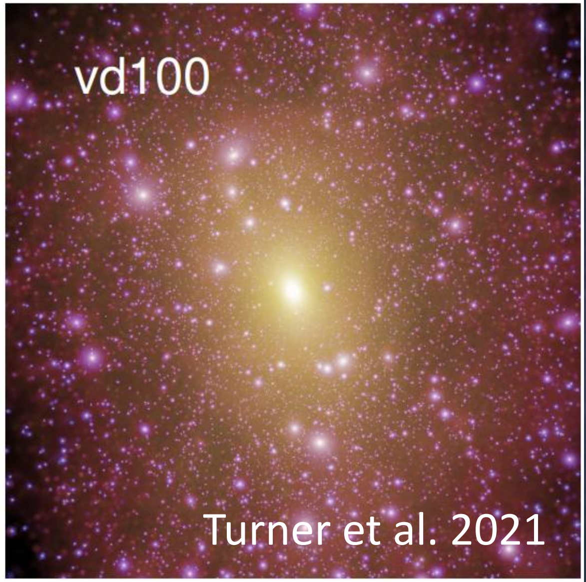




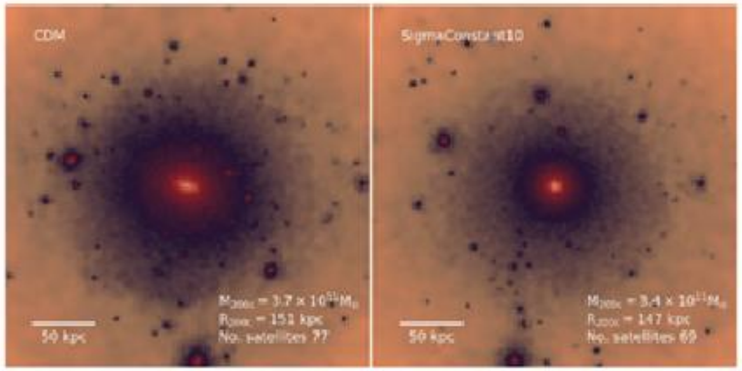
➢ **We aim to understand the results and emulate without simulation**

Cosmological simulations with v-dependent cross sections

Hydrodynamical simulations and Dark matter only cosmological zoom-in simulations

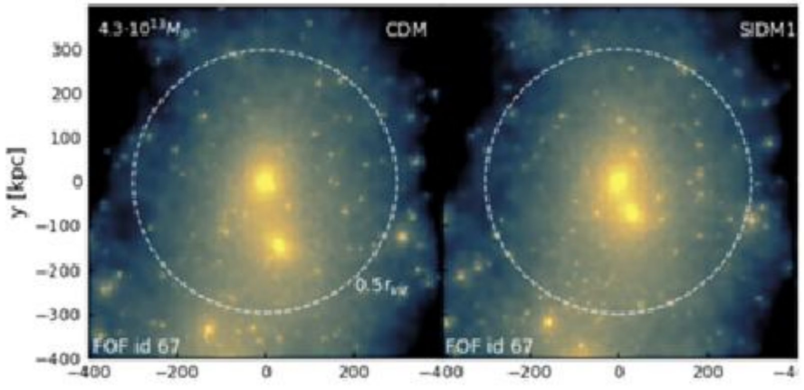


TangoSIDM

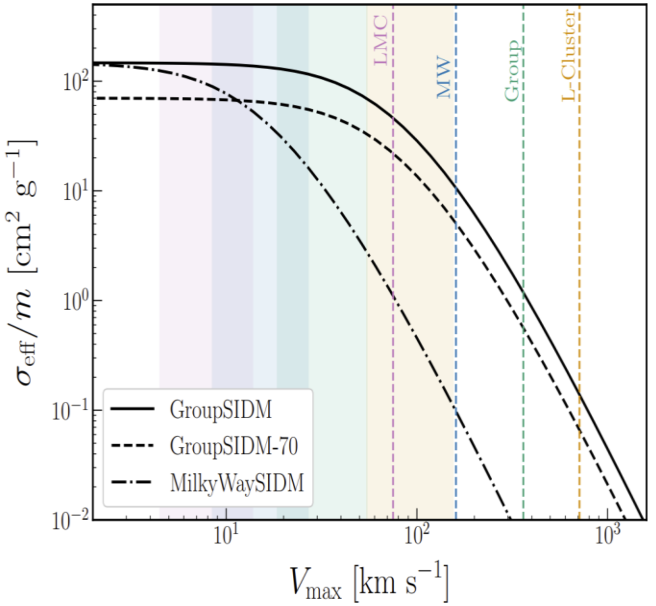


Correa et al. 2022

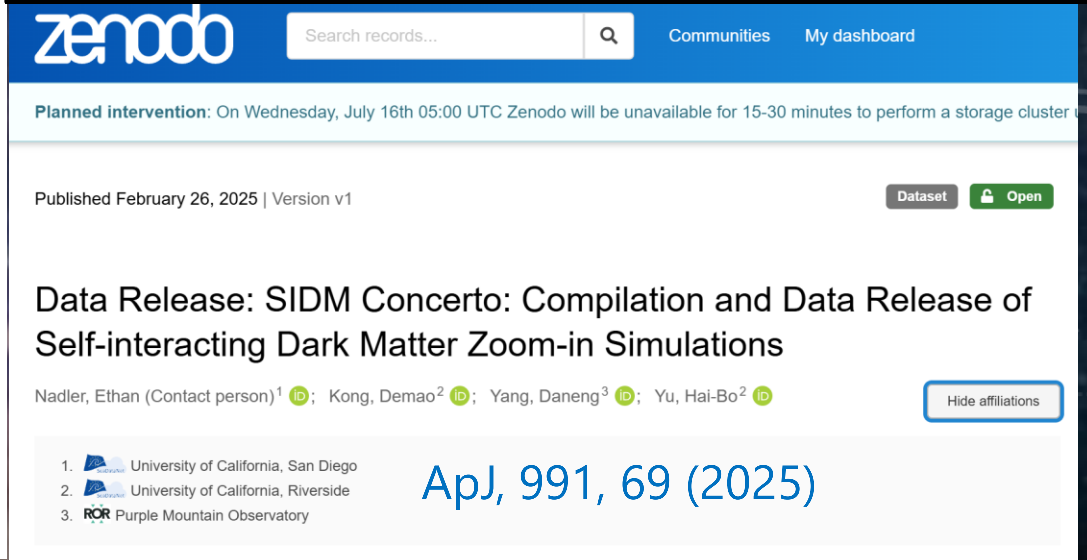
AIDA-TNG



Despali et al. 2025



Data Published and Ready to Explore

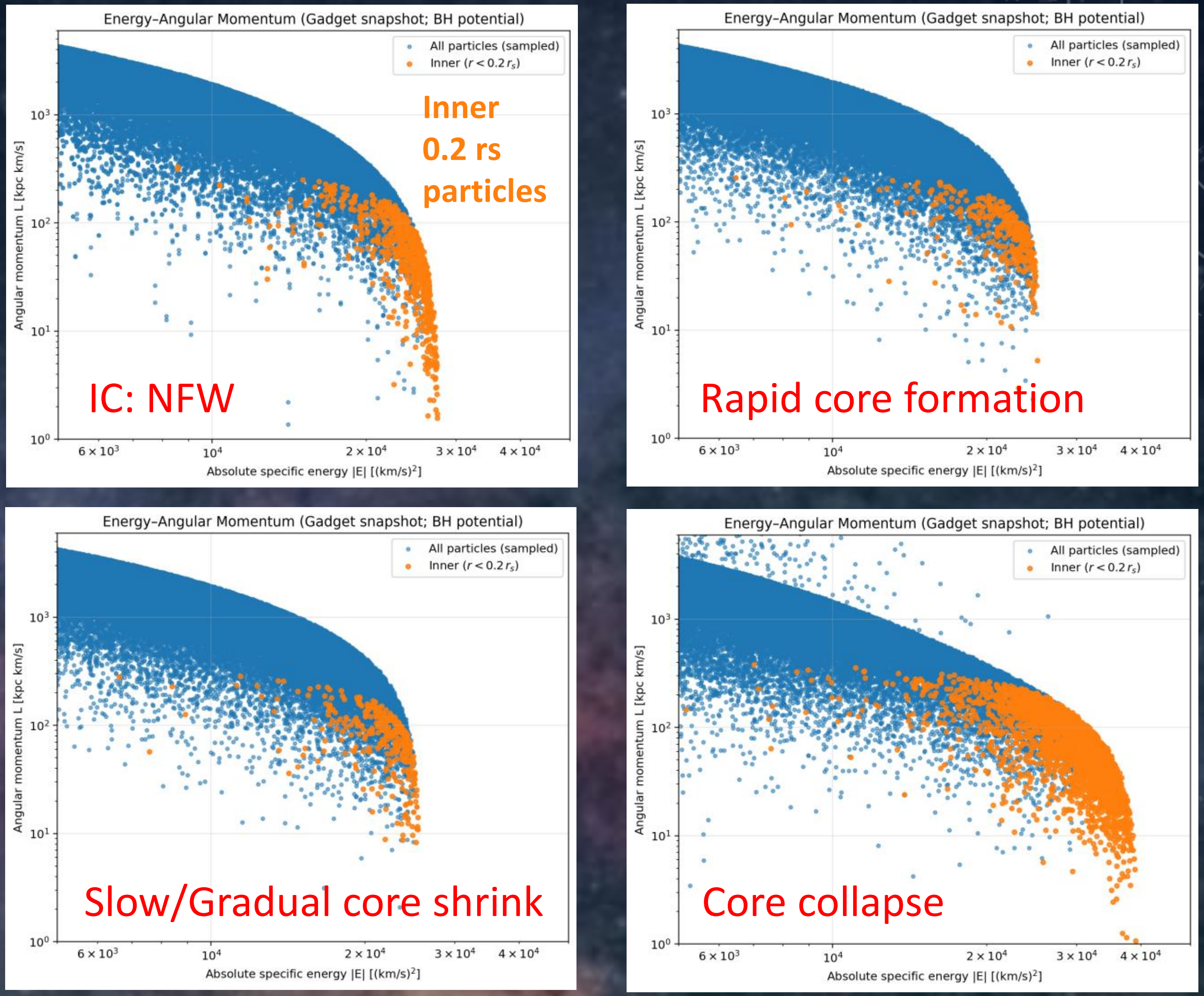


ApJ, 991, 69 (2025)

Example controlled simulation (1): Slow core shrink in SIDM

- Inner-outer scatterings: Inner low-J particles get kicked to larger-J
- Inner-inner scatterings: inefficient heat transport
- A "lost cone" in spherically symmetric SIDM halos

A kinetic picture of gravothermal evolution under SIDM



Angular momentum

Angular momentum

|E| = |K + P|

|E| = |K + P|

Example controlled simulation (2): Verify conditioned universaliry

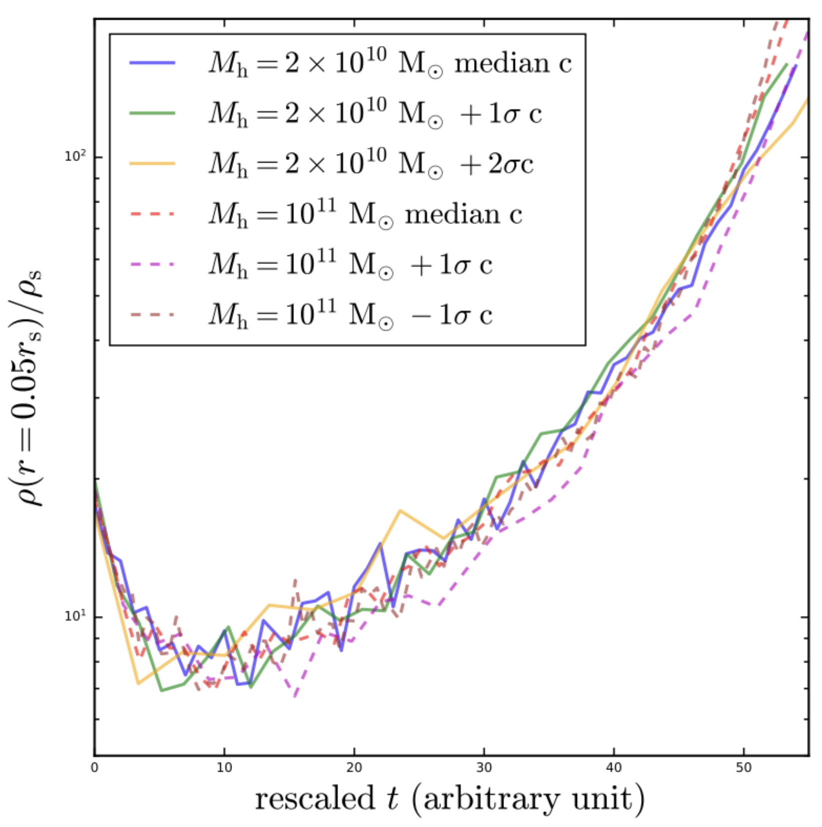
Approximate Universal Evolution when

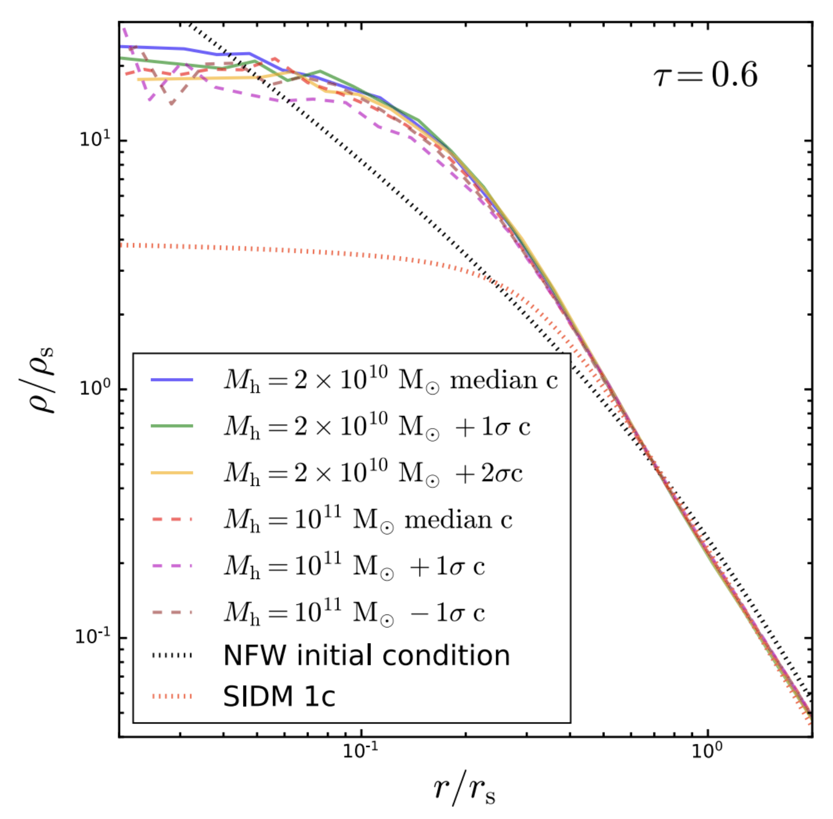
- All channels in the SIDM 2-comp model specified
- Mass ratio, abundance fixed

Energy transfer for inter- and intra-species interactions are both proportional to the number of scatterings in LMFP

$$\rho_{\mathrm{H}}\nu_{\mathrm{H}}^{2}\left(\frac{D}{Dt}\right)\ln\frac{\nu_{\mathrm{H}}^{3}}{\rho_{\mathrm{H}}} = -\frac{1}{4\pi r^{2}}\frac{\partial L_{\mathrm{H}}}{\partial r} - R,$$

$$\rho_{\mathrm{L}}\nu_{\mathrm{L}}^{2}\left(\frac{D}{Dt}\right)\ln\frac{\nu_{\mathrm{L}}^{3}}{\rho_{\mathrm{L}}} = -\frac{1}{4\pi r^{2}}\frac{\partial L_{\mathrm{L}}}{\partial r} + R,$$





Yang, Fan, Hou, Tsai, Sci. Bull. 71 (2026)

Why Does Gravothermal Evolution Appear Universal?

1 CORE PHYSICAL IDEA

Heat conduction breaks time-reversal invariance.

The **arrow of time** is controlled by SIDM collisions (the collision term).

Schematic gravothermal heat-conduction equation

$$\frac{\partial}{\partial r}\left(r^2 \kappa m \frac{\partial v^2}{\partial r}\right) = r^2 \rho \nu^2 \boxed{\frac{D}{Dt}} \ln\left(\frac{v^3}{\rho}\right)$$

Time-derivative / convective derivative
(sets the arrow of time)

2 WHY UNIVERSALITY EMERGES

In the long-mean-free-path regime (LMFP) → Conductivity κ is proportional to the number of scatterings → $\kappa \propto \sigma$ → The cross-section dependence can be absorbed into a **rescaled time** variable → $\tau \equiv \frac{t}{t_c} \propto t\,\sigma$

Hence gravothermal evolution follows an approximately **universal** track when plotted against τ.

KEY MESSAGE

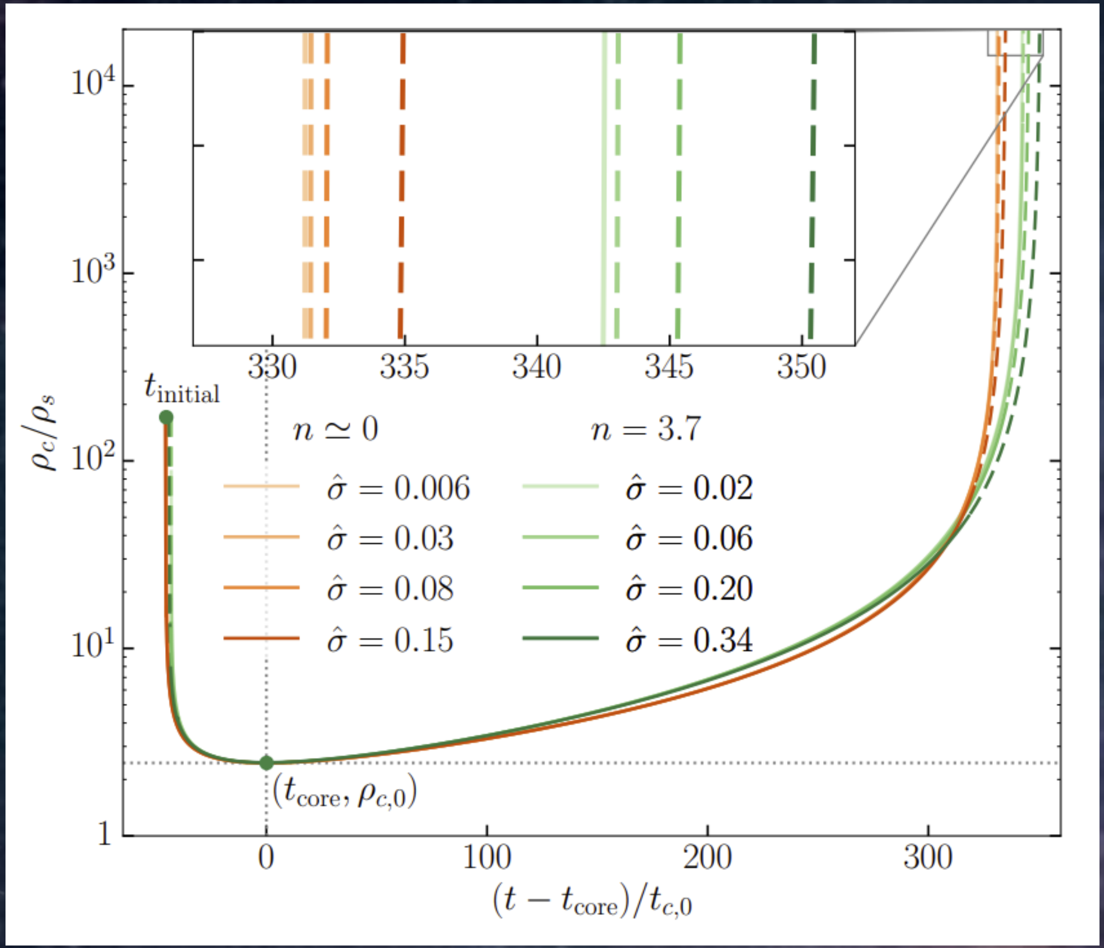
In the LMFP regime, changing σ mainly **rescales the clock** rather than changing the form of the evolution.

REFERENCES

- Yang+ 2405.03787 (Simplified discussion here)
- Outmezguine+ 2204.06568 (First paper)
- Yang+ 2305.16176 (Parametric model)
- Zhong, Yang, Yu 2306.08028 (When universality is exact)

Universal gravothermal evolution?

→ **The kernel function**



The gravothermal evolution is almost universal after normalizing the evolution time by the core collapse time (Outmezguine et al. MNRAS 2022)

- What is its origin?
- How accurate/reliable is it?
- Does it work for v-dependent models?
- How baryons breaks it?
- Does it work for extended SIDM models?

How accurate/reliable is it?

Exact Universality in a fluid PDE
(Zhong, Yang & Yu, MNRAS, 2306.08028)

$$\frac{\partial \hat{M}_\chi}{\partial \hat{r}} = \hat{r}^2 \hat{\rho}_\chi, \qquad \frac{\partial(\hat{\rho}_\chi \hat{v}_\chi^2)}{\partial \hat{r}} = -\frac{(\hat{M}_\chi + \hat{M}_b)\hat{\rho}_\chi}{\hat{r}^2},$$
$$\frac{\partial \hat{\hat{L}}_\chi}{\partial \hat{r}} = -\hat{\rho}_\chi \hat{r}^2 \hat{v}_\chi^2 D_{\hat{\hat{t}}} \ln \frac{\hat{v}_\chi^3}{\hat{\rho}_\chi}, \qquad \frac{\hat{\hat{L}}_\chi}{\hat{r}^2} = -3.4\gamma \hat{\rho}_\chi \hat{v}_\chi^3 \frac{\partial \hat{v}_\chi^2}{\partial \hat{r}}, \qquad (19)$$

- ✓ **Given smooth initial condition and coefficients, this PDE has a unique solution that is smooth in a finite time**
- ✓ **Everything is normalized, independent of NFW rhos, rs, and SIDM physics (sigma/m)**

What breaks it?

- Breakdown of the LMFP regime
- Approximate nature of the conduction fluid model
- Additional physics: baryons, environment

Bottom line

When the heat conductivity in the halo is proportional to the number of scatterings, we expect the "**t -> t σ** " trick to bring us a reasonable universality

Analytic calculation of SIDM density profiles

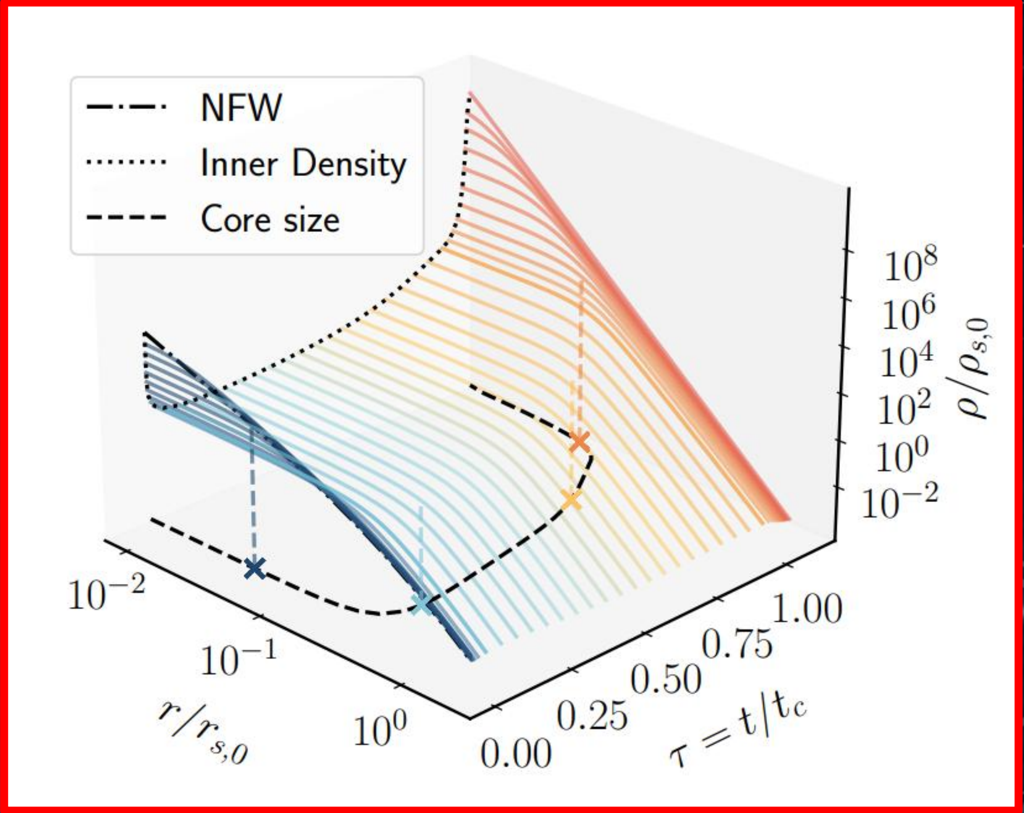
Calibration of the universal solution

$$\rho_{\mathrm{PSIDM}}(r) = \frac{\boxed{\rho_s}}{\left[\left(\frac{r}{\boxed{r_s}}\right)^4 + \left(\frac{\boxed{r_c}}{r_s}\right)^4\right]^{\frac{\gamma}{4}} \cdot \left[1 + \left(\frac{r}{r_s}\right)^{\boxed{\beta}}\right]^{\frac{3-\boxed{\gamma}}{\beta}}}$$

3 parameter version:
Yang+2305.16176 JCAP

5 parameter version,
improved over the late
core collapse phase:
Hou+2502.14964 JCAP

$$\rho_s = \rho_{s,0}\, f_\rho(\boxed{\tau})$$
$$r_s = r_{s,0}\, f_s(\tau)$$
$$r_c = r_{s,0} f_c(\tau)$$
$$\beta = f_\beta(\tau) \qquad \gamma = f_\gamma(\tau)$$



Hou & Yang et al. 2025 JCAP

Yang+2305.16176 JCAP 2024
https://github.com/DanengYang/parametricSIDM

README   GPL-3.0 license

Parametric Model Analysis Tools for Self-Interacting Dark Matter Halos

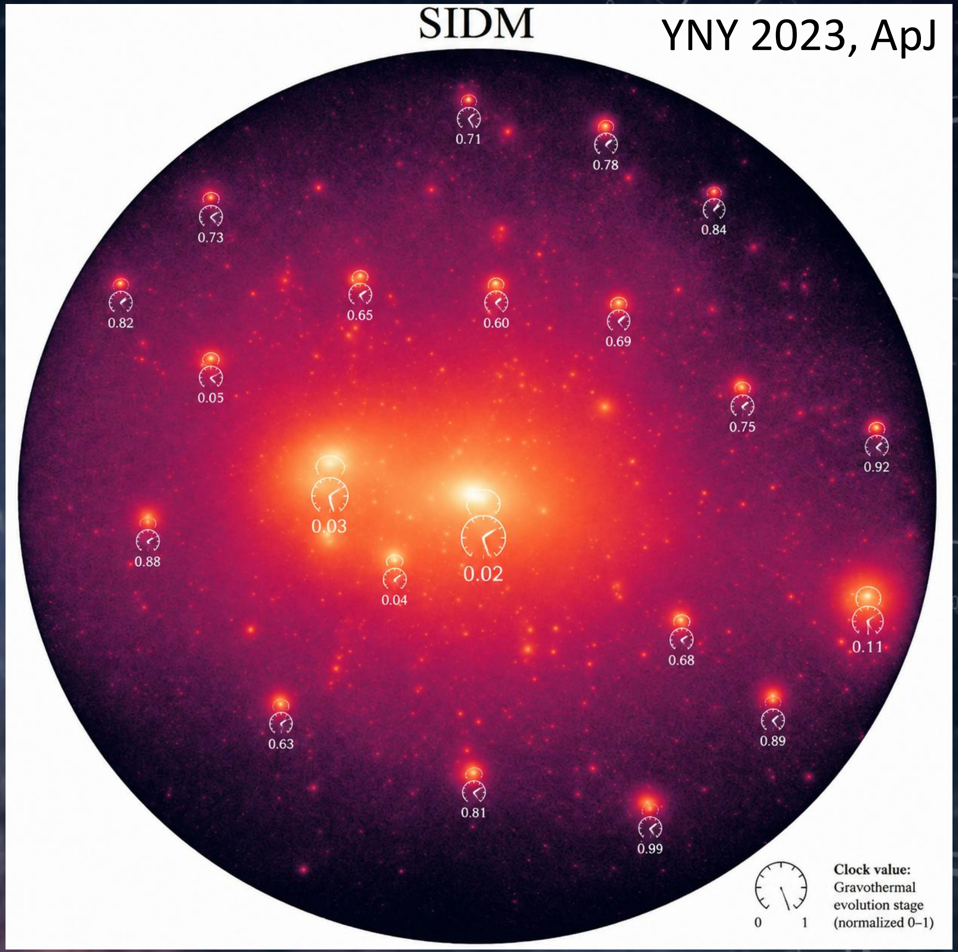
The parametric model for self-interacting dark matter (SIDM) halos allows for obtaining halo density profiles based on a few calibrated equations under an assumed SIDM model, using parameters from their CDM counterparts, such as NFW scale parameters or halo mass and concentration parameters. The effect of a baryon potential can be incorporated, assuming the density profile conforms to the Hernquist model.

Relevant works

- [1] **Dark matter-only version**: D. Yang, E. O. Nadler, H.-B. Yu, and Y.-M. Zhong, arXiv:2305.16176, published in JCAP 02, 032 (2024)
- [2] **Dark matter plus baryon version**: D. Yang, arXiv:2405.03787, published in Phys. Rev. D
- [3] **Testing** the parametric model using matched halos in **cosmological simulations**: D. Yang, E. O. Nadler, and H.-B. Yu, arXiv:2406.10753, published in Phys. Dark Universe
- [4] For a **lensing specific** parametric model for SIDM halos, see S. Hou, D. Yang, N. Li, and G. Li, arXiv:2502.14964, with public codes on GitHub.
- [5] Our method has been implemented in the **SASHIMI program** for SIDM subhalos: S. Ando, S. Horigome, E. O. Nadler, D. Yang, and H.-B. Yu, arXiv:2403.16633

Our codes are free to copy and modify. Please quote the relevant paper(s), if you find them useful. For any questions or comments, please contact [yangdn_at_pmo.ac.cn].

Example applications

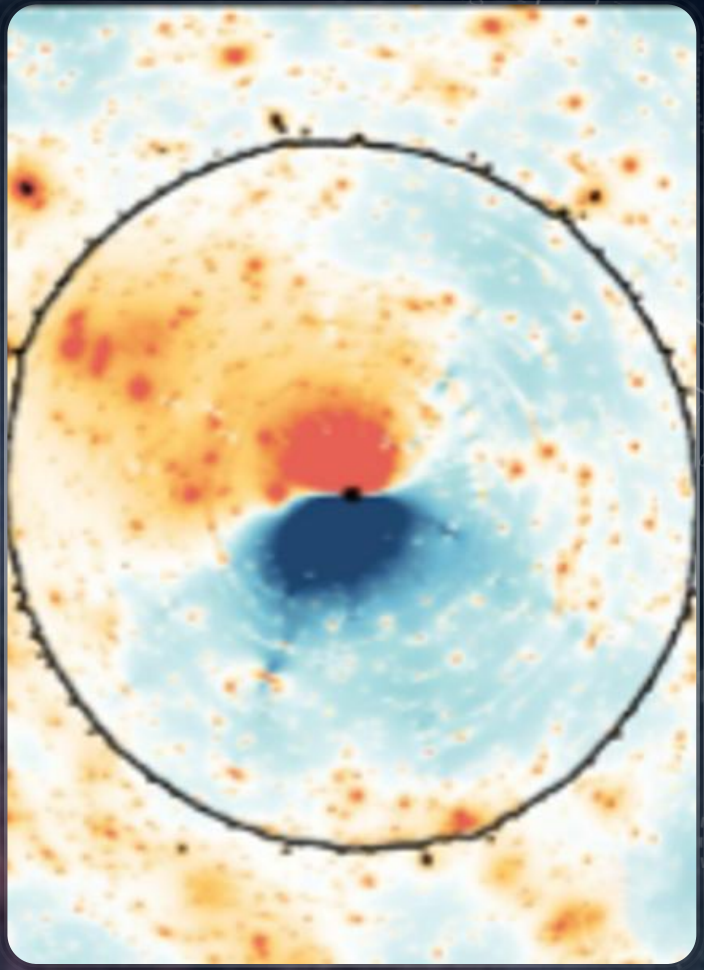


For simple stand-alone scripts for SIDM halos
https://github.com/DanengYang/parametricSIDM
To incorporate accretion histories, we introduce the integral approach

- Without baryons, each halo can be (approximately) characterized by 3 quantities:
  2 NFW params + 1 gravothermal phase
- The effect of baryons can (currently) be (only) estimated in closed form equations. (Yang 2024 PRD)

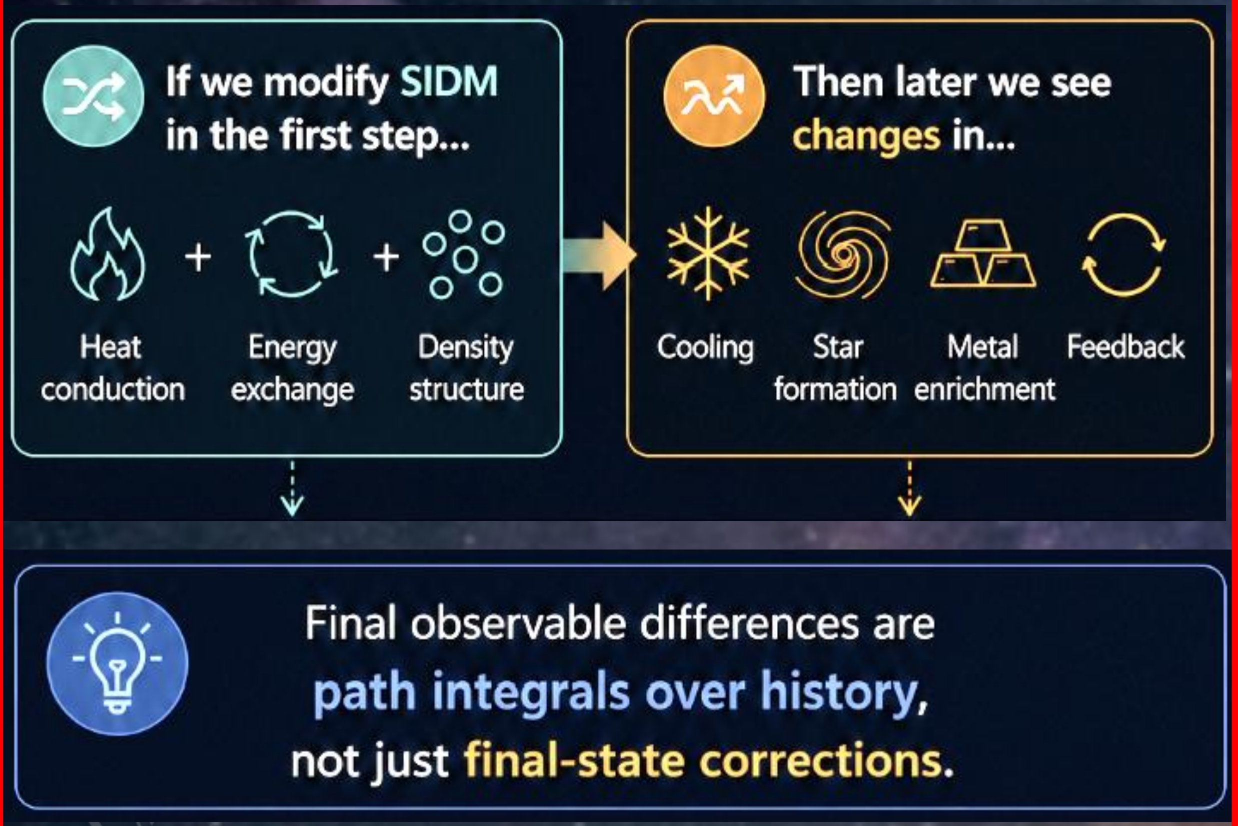
From SIDM evolution to observable predictions

- Integral approach
- Effect of baryons
- Lensing effects
- Observable predictions

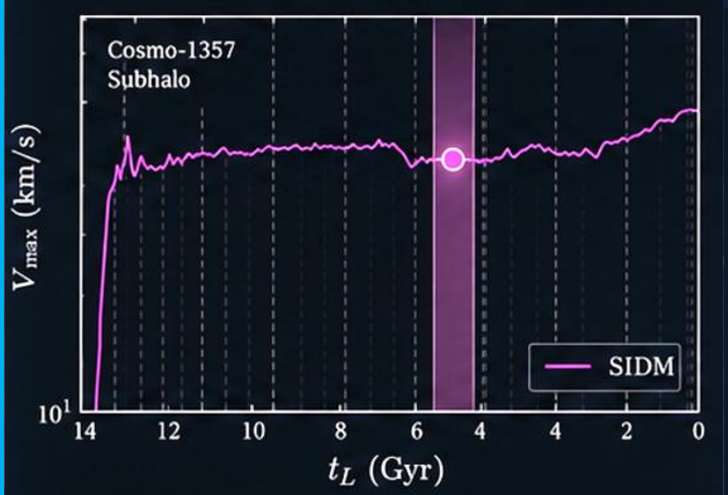


The method: SIDM as controlled deformation on CDM paths

To see how SIDM+mass segregation effects differ in these galaxies, we evolve SIDM & accretion/merger history simultaneously



Integral approach of the parametric model (2305.16176)



Add small increments in **gravothermal phase.**

$$\Delta\tau = \frac{\Delta t}{t_c}$$

t_c from *instantaneous* CDM halo parameters.

$$V_{\rm max}(t) \;=\; V_{\rm max,CDM}(t) + \int_0^{\tau(t)} d\tau' \frac{dV_{\rm max,Model}(\tau')}{d\tau'}$$

Tested against cosmological simulations

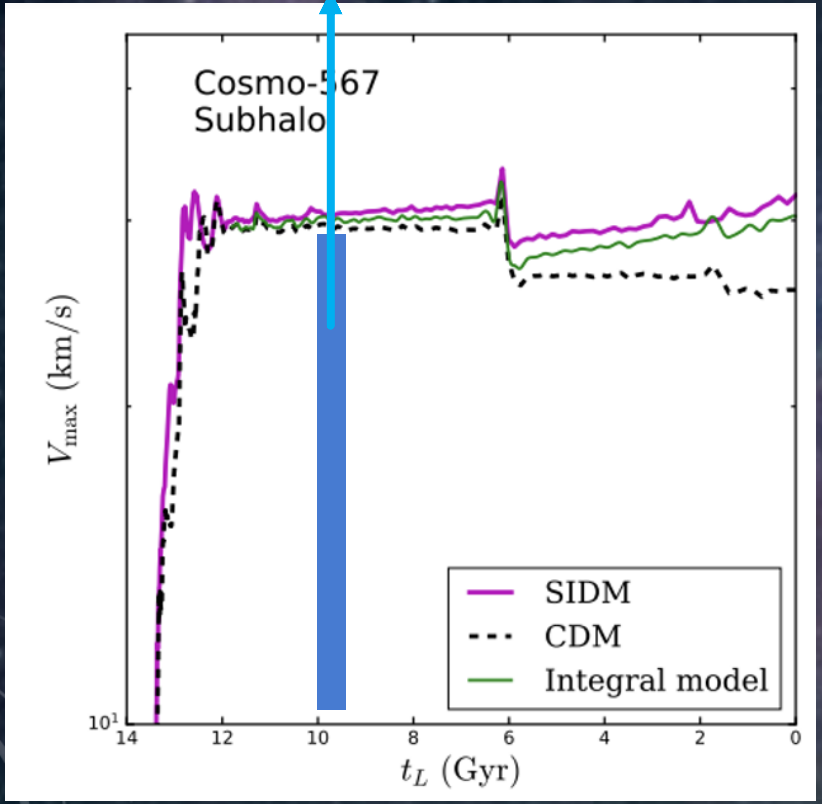
(Yang et al. 2025 PDU, Nadler et al. 2025a,b ApJ)

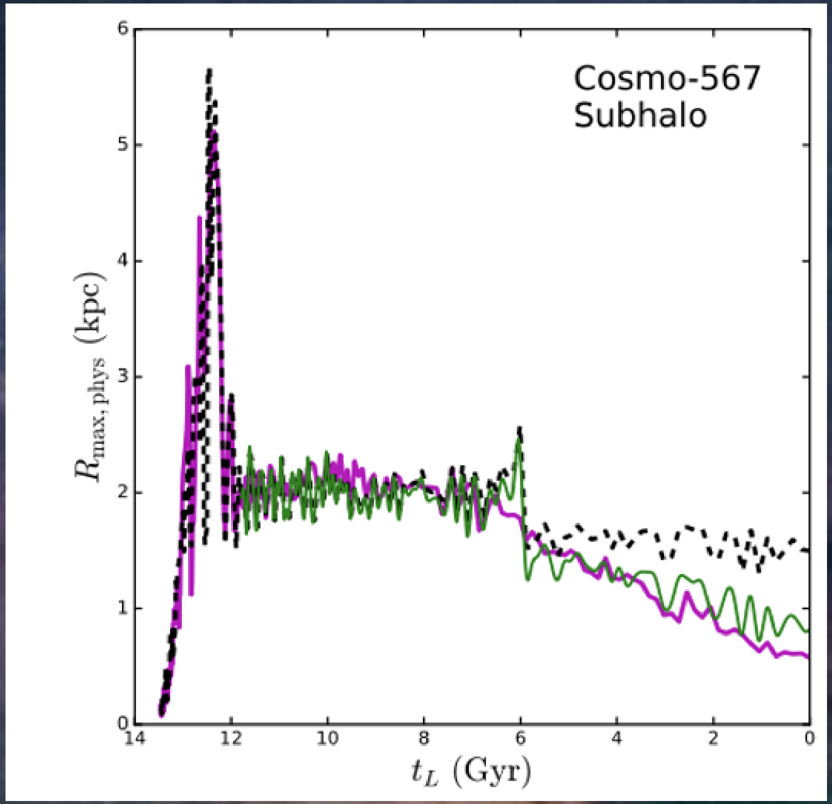
SIDM as controlled deformation on CDM paths

Each SIDM halo is assumed to have a **fictitious CDM** progenitor which evolves into its current state **in isolation**

$$V_{\max}^{\mathrm{SIDM}}(t) = V_{\max}^{\mathrm{CDM}}(t) + \int_{t_f}^{t} \frac{dt'}{t_c(t')} \boxed{\frac{dV_{\max,\mathrm{Model}}(\tilde{t}')}{d\tilde{t}'}},$$

$$r_{\max}^{\mathrm{SIDM}}(t) = r_{\max}^{\mathrm{CDM}}(t) + \int_{t_f}^{t} \frac{dt'}{t_c(t')} \boxed{\frac{dr_{\max,\mathrm{Model}}(\tilde{t}')}{d\tilde{t}'}},$$

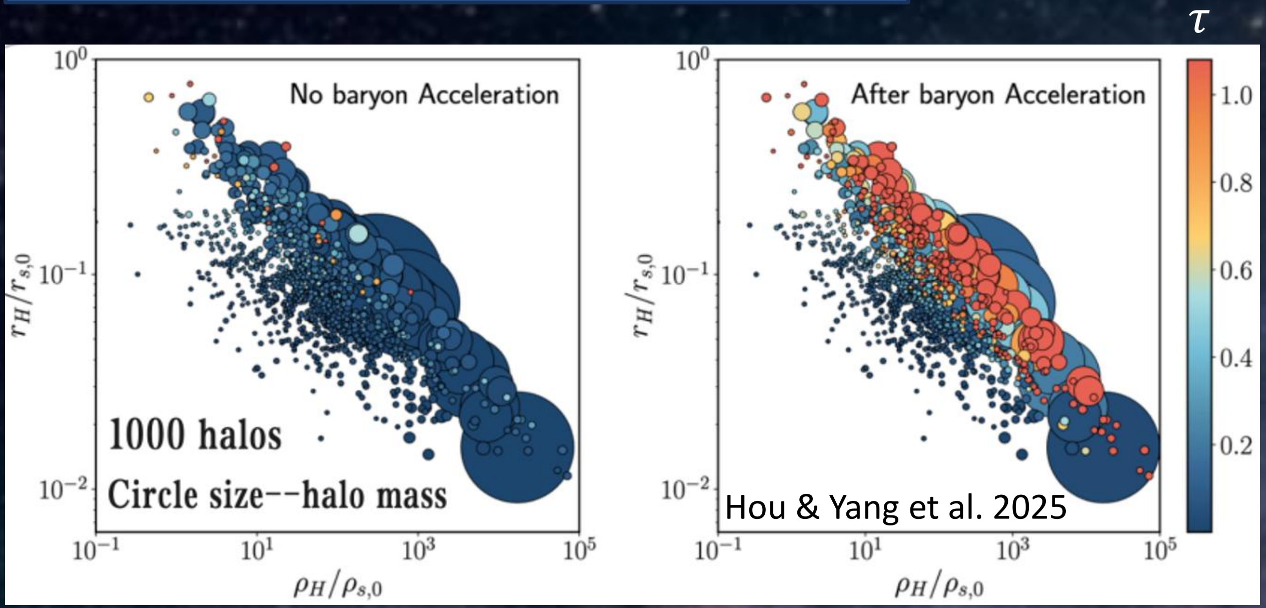




The integral approach in the parametric model offers an analytic kernel for computing the incremental SIDM effect

Yang+2305.16176 JCAP

Effect of baryons

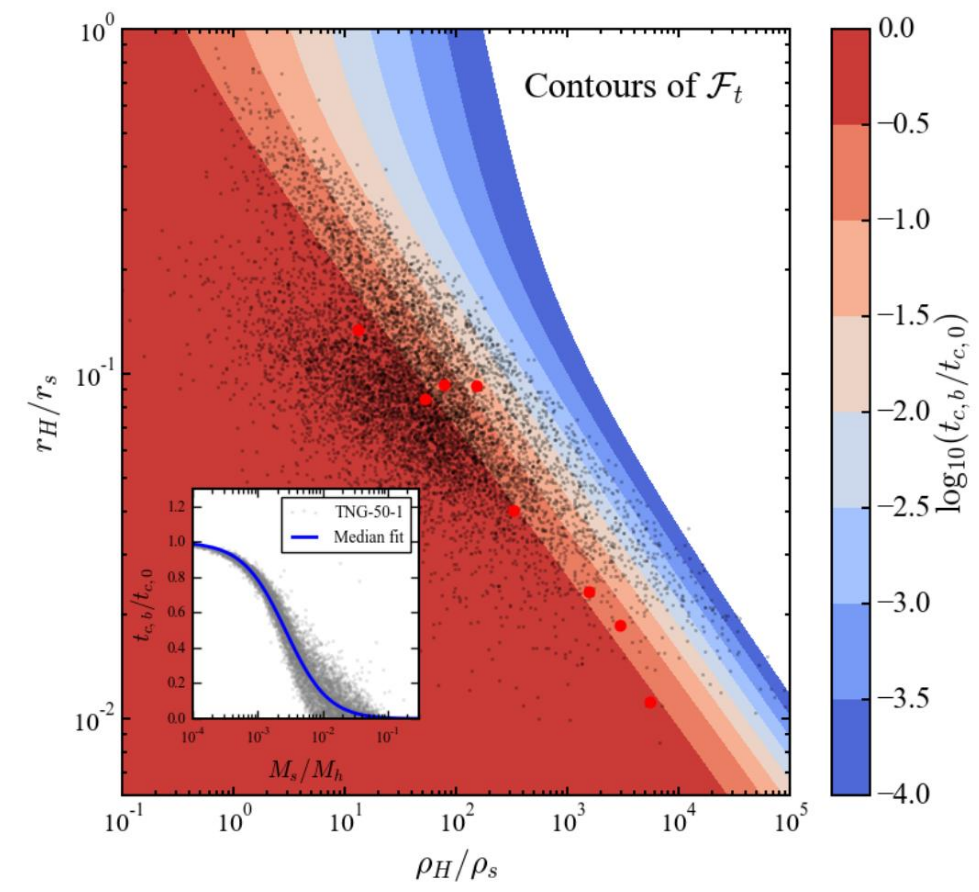


- Core collapse is boosted in the presence of baryons (Feng, Yu, Zhong 2021 ApJL)
- Effect can be incorporated in a **closed form equation** (Yang 2405.03787 PRD)

$$t_{c,b} = t_{c,0}\mathcal{F}_t(\hat{\rho}_H, \hat{r}_H)$$

- SIDM core becomes smaller after contraction

$$r_c(\tau) = r_{s,0} f_c(\tau)(\mathcal{F}_t)^2$$



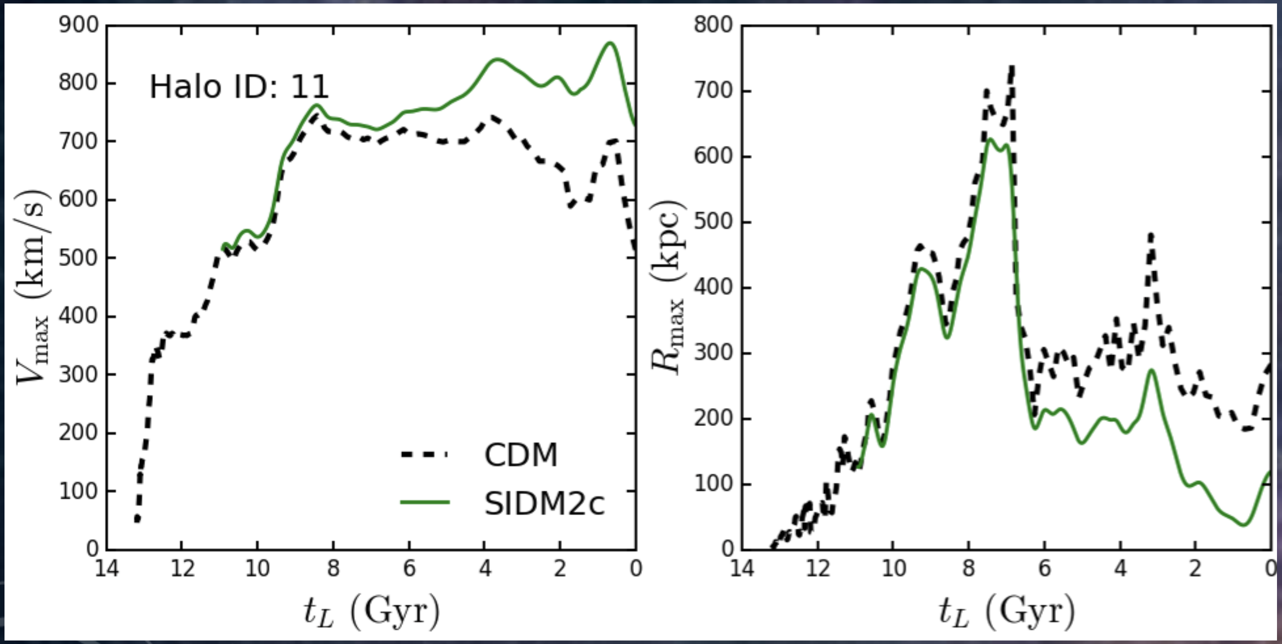
Extend the parametric model and can be implemented as path distortions

$$\mathcal{F}_t(M_s/M_h) \approx \frac{e^{-7.9(M_s/M_h)}}{1 + 2300(M_s/M_h)^{1.3}},$$

applicable for $M_s/M_h \lesssim 0.08$.

Extension & Application: 2-comp SIDM with mass segregation

- Isolated dwarfs are mostly core forming (tau<0.2)
- Dwarfs in clusters are mostly core collapsing (tau>0.2)



Yang, Fan, Hou, Tsai, Sci. Bull. 71 (2026)

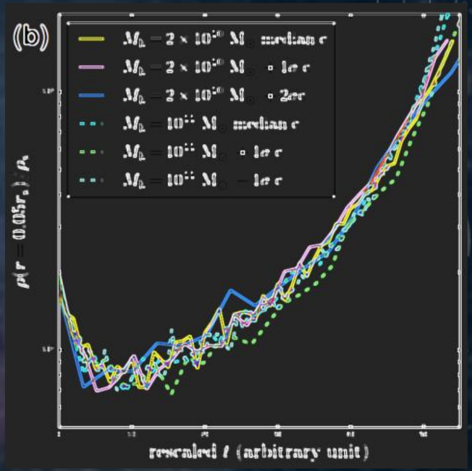
$$X = (V_{\max}, R_{\max}, \tau)$$

$$\frac{dX}{dt} = F_{\mathrm{SIDM}}(X)$$

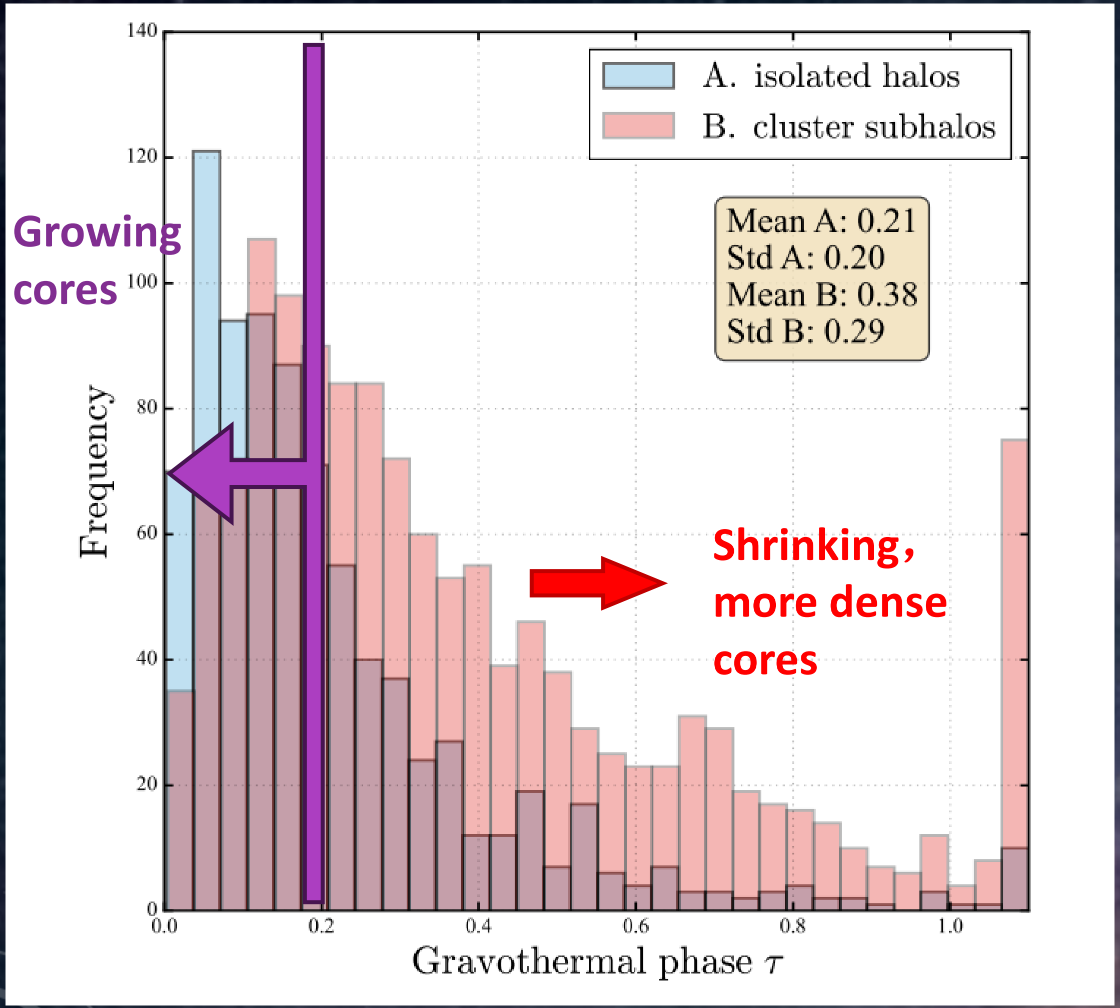
where F_SIDM is based on a universal & analytic parametric model

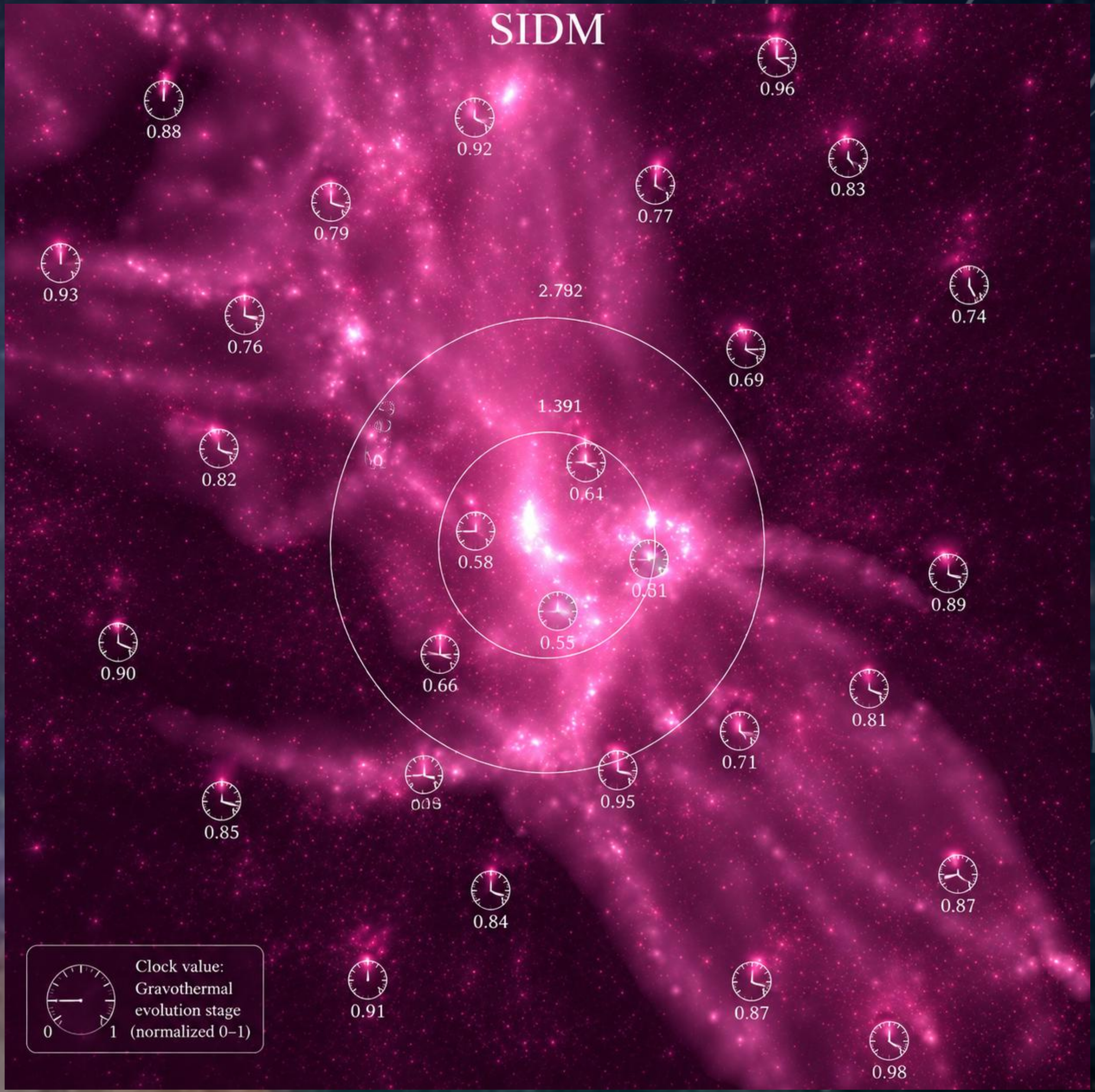
Conditioned universality in 2-comp SIDM

- All channels in the SIDM 2-comp model specified
- Mass ratio, abundance fixed



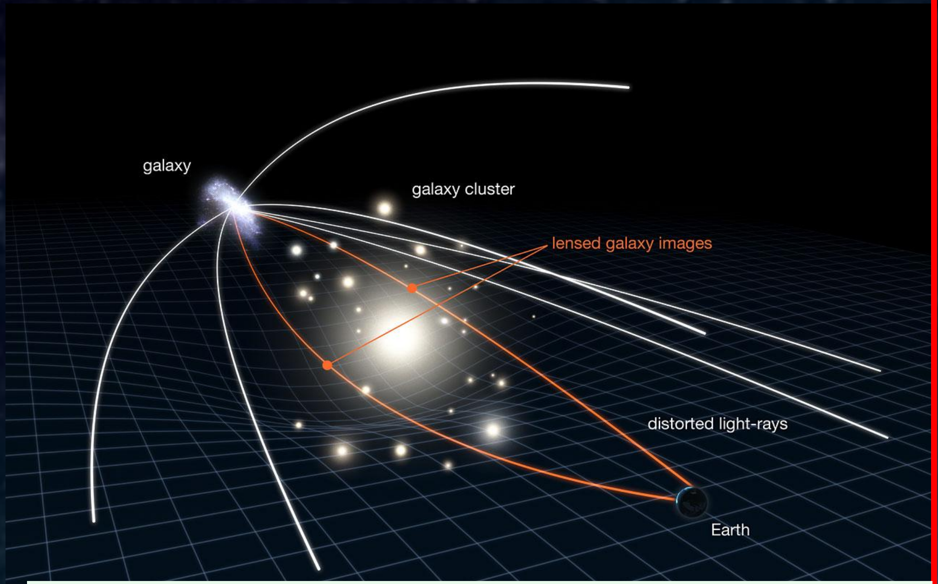
The method: Parametric SIDM model & CDM cosmological simulation





Yang, Fan, Hou, Tsai, Sci. Bull. 71 (2026)

For lensing observables



$$\hat{\Psi}(\hat{R}) = a \ln\left(1 + b\hat{R} + c\hat{R}^2\right)^c - \ln\left(p\hat{R} + 1\right)^s$$

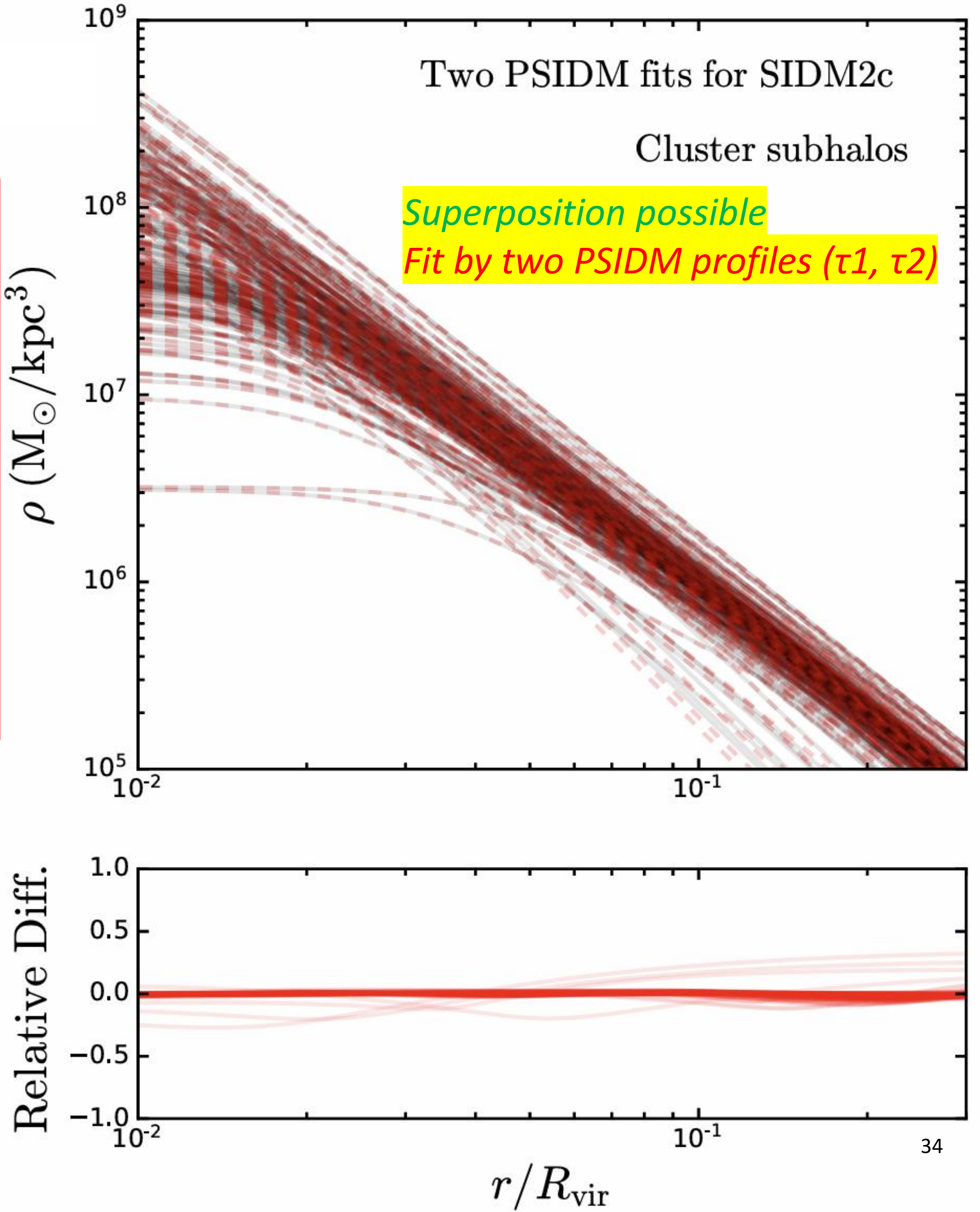
Calibrate the lensing potential instead of the density profile

Application of the model is broader than SIDM!

Hou, Yang, Li, Li **JCAP 08 (2025) 048**

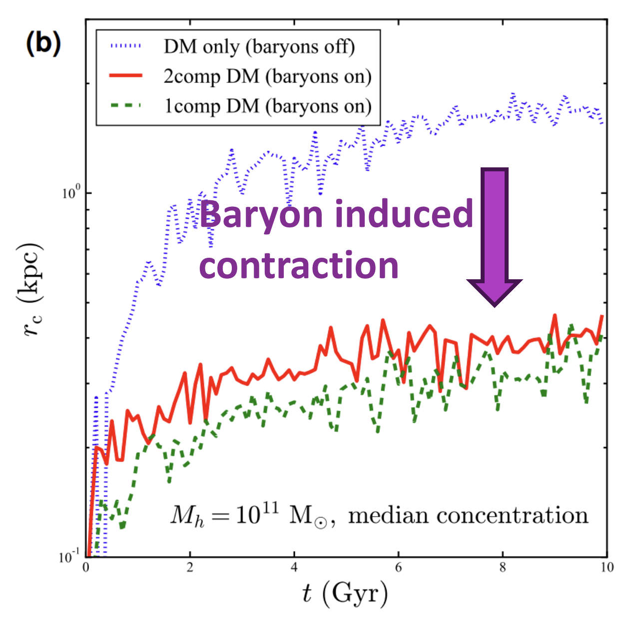
- **Constructed based on universality**
- **A family of central density profiles ranging r^0 to $r^{-2.5}$ with analytic lensing predictions**
- **Lensing for complex density fields via superposing models (not limited to SIDM)**

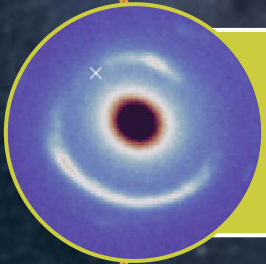
https://github.com/HouSiyuan2001/SIDM_Lensing_Model



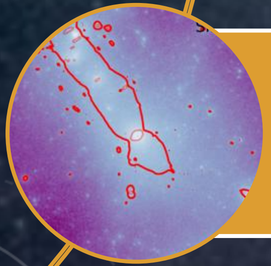
Results: dwarfs can have growing cores and densities simultaneously

Growing stellar cores in isolated dwarfs

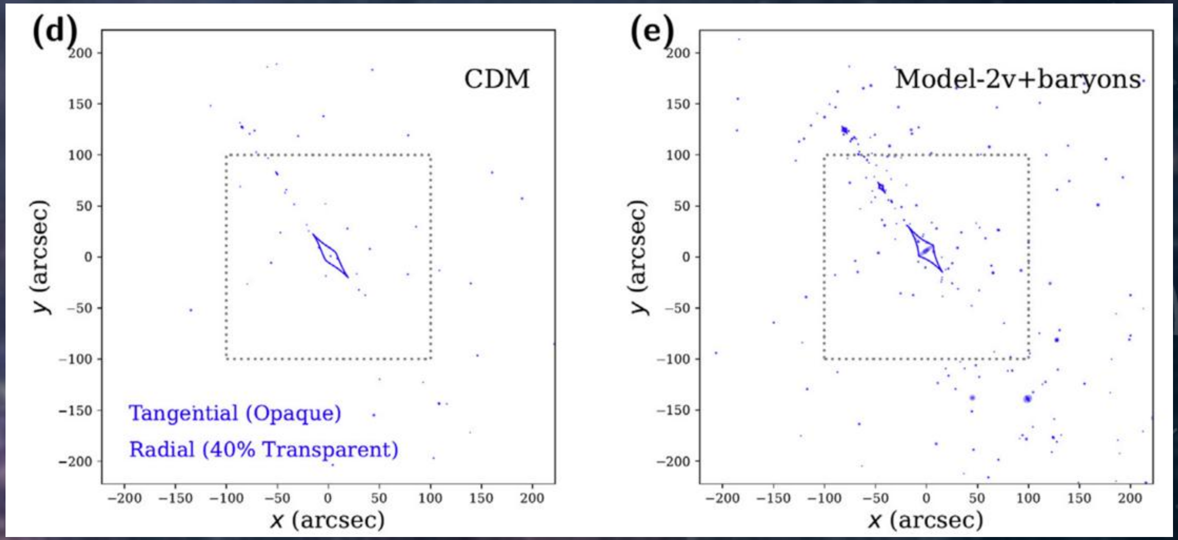




Overall x10 faster gravothermal evolution, explaining strong lensing perturbers



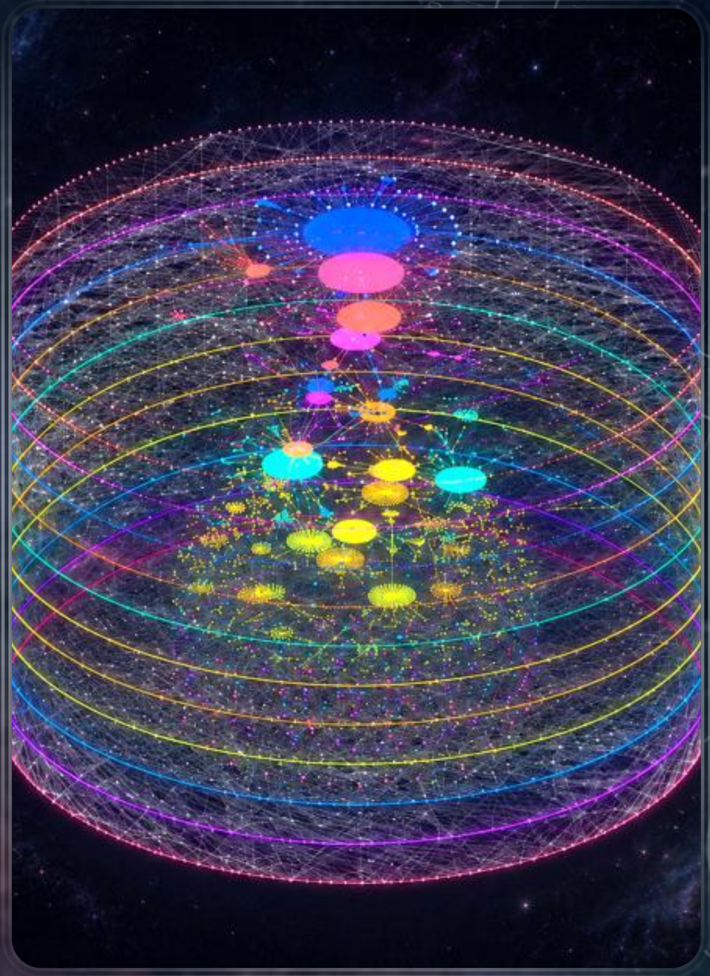
Cluster subhalos evolve faster (gravothermal), explaining GGSL



Yang, Fan, Hou, Tsai, Sci. Bull. 71 (2026)

Beyond Deterministic Halo Modeling

- A path measure model for galaxy formation and SIDM



Why does a probability measure emerge?

Full simulation data → **Snapshots** → **Merger trees & catalogs**

Coarse-graining transforms deterministic microscopic dynamics into an Effective Stochastic Dynamics

Microscopic (fully resolved)

Deterministic dynamics

$$\dot{z} = F(z)$$

$$z = (x, h)$$

x : resolved variables

h : unresolved variables

Coarse-graining

Integrate out unresolved degrees of freedom h

$$\Lambda \frac{\partial P_\Lambda[x]}{\partial \Lambda} = \mathcal{R}_\Lambda[P_\Lambda]$$

The full probability measure flows with resolution scale Λ

Effective (coarse-grained)

Stochastic dynamics

$$dx = b_\Lambda(x)\, dt + \sqrt{2D_\Lambda(x)}\, dW_t$$

$$\Lambda \frac{db_\Lambda}{d\Lambda} = \beta_b, \qquad \Lambda \frac{dD_\Lambda}{d\Lambda} = \beta_D$$

$$\Lambda \downarrow \quad \Rightarrow \quad D_\Lambda \uparrow$$

Predict a probability measure over histories

$$P_\Lambda[x(t)]$$

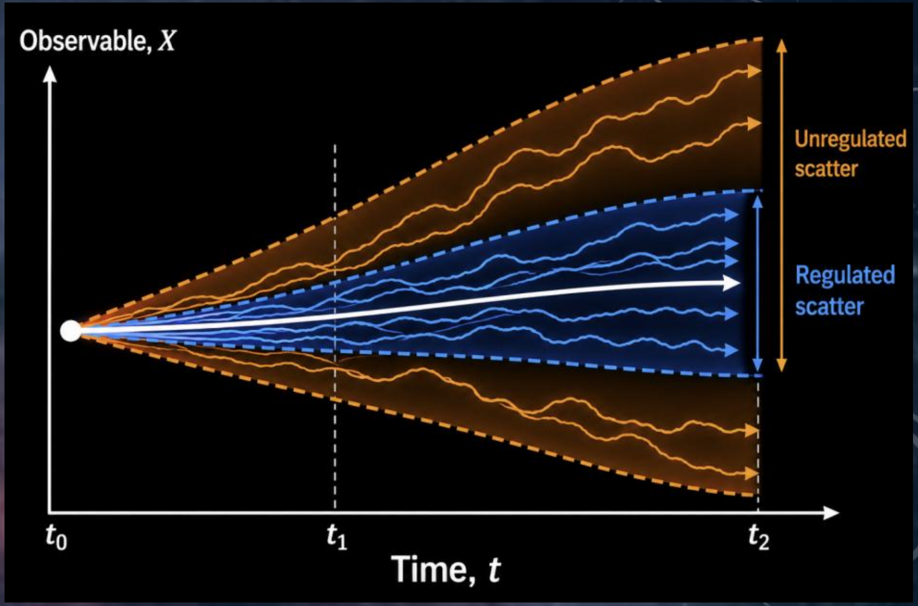
$\Lambda \to \infty$

more physics resolved
smaller scatter

$\Lambda = l^{-1}$

$\Lambda \downarrow$

more coarse-graining
larger scatter



Unresolved differences grow dynamically over time

Why path measure models matter?

Deterministic predictions based on merger histories G are incomplete

For an endpoint variable Y, the missing piece is the unresolved, correlated predictive scatter.

$$\mathrm{Var}(Y) = \underbrace{\mathrm{Var}_G\left[\mathbb{E}(Y \mid G)\right]}_{\text{different resolved histories } G \text{ (SAM captures this)}} + \underbrace{\mathbb{E}_G\left[\mathrm{Var}(Y \mid G)\right]}_{\text{same resolved history } G + \text{ unresolved correlated fluctuations (path measure adds this)}}$$

Law of Total Variance

Residuals are **not i.i.d.** noise

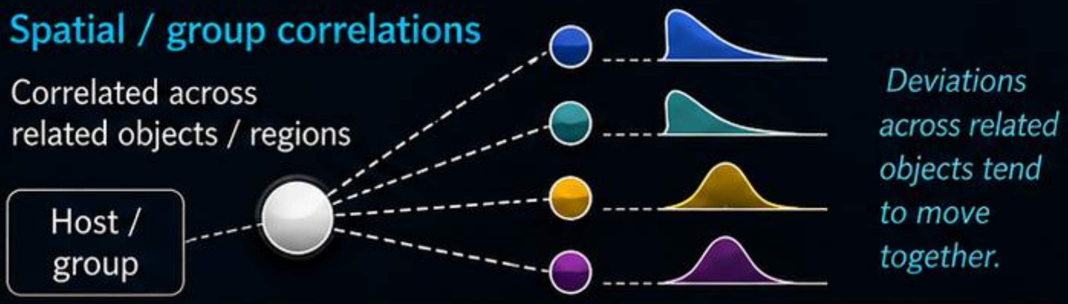
1 Temporal correlations

$$\mathrm{Cov}(\delta x_t, \delta x_{t+k} \mid G) \neq 0$$

Residuals at different times along a path are correlated.

2 Spatial / group correlations

Correlated across related objects / regions



1. Systematic treatment of correlated scatter
2. Improved predictions for rare objects and sub-populations
3. Scatter itself is informative: D_λ, skewness, and tails can distinguish physical models
4. Nonlinear baryonic physics makes unresolved scatter dynamically important

Physical gain: a path measure naturally and systematically incorporates correlated scatter, improving predictions and revealing structure inside the scatter itself.

A Functional View of NP under Effective Dynamical Scatter

Observables can be viewed as functionals of paths where distortions can be introduced O=F[x(t)]

$$\dot{x}(t) = b(x(t)) + \eta(t)$$

The Langevin equation

The Martin-Siggia-Rose-Janssen-De Dominicis (MSRJD) formalism

$$Z = \int \mathcal{D}\eta\, \mathcal{D}x\, \mathcal{D}\hat{x}\; e^{\int dt\, \hat{x}^T(\dot{x}-b-\eta)}\; e^{-\frac{1}{2}\int dt\, \eta^T D^{-1}\eta}$$

Future extension

$$S_{\mathrm{tot}} = S_G[G_{0:K}] + S_x[\mathrm{x}_{0:K}; G_{0:K}] + S_{\mathrm{bnd}}[\mathrm{x}_{0:K}; G_{0:K}]$$

S_G: structure term (graph evolution)

S_x: internal term (formation physics)

S_{bnd}: boundary term (environment)

First example: A path measure model decomposition

Hierarchical structure formation

Graph-trajectory measure

$$P(\mathbf{x}, G) = \boxed{P(\mathbf{x} \mid G)} P(G)$$

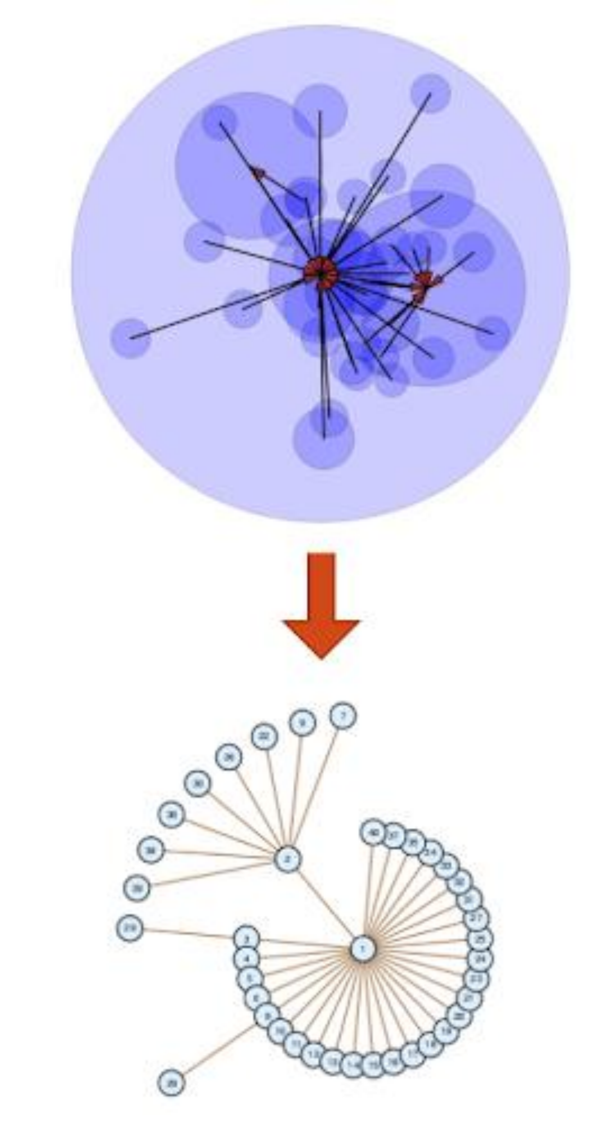
Graph-conditioned path measure

$$\boxed{P(\mathbf{x} \mid G)} \propto p_{\mathrm{attach}}(\mathbf{x} \mid G) \exp\left[-S(\mathbf{x}; G)\right]$$

Evolution in isolation

Evolution in environment

Probabilistic halo graph construction for clustering



Yang & Yu 2023 PR Research:

- ➢ Halo graphs can be constructed using **preferential attachment**
- ➢ They are power-law graphs

$$\text{Attachment Probability} = \frac{A_i}{\sum_j A_j}$$

Preferential attachment

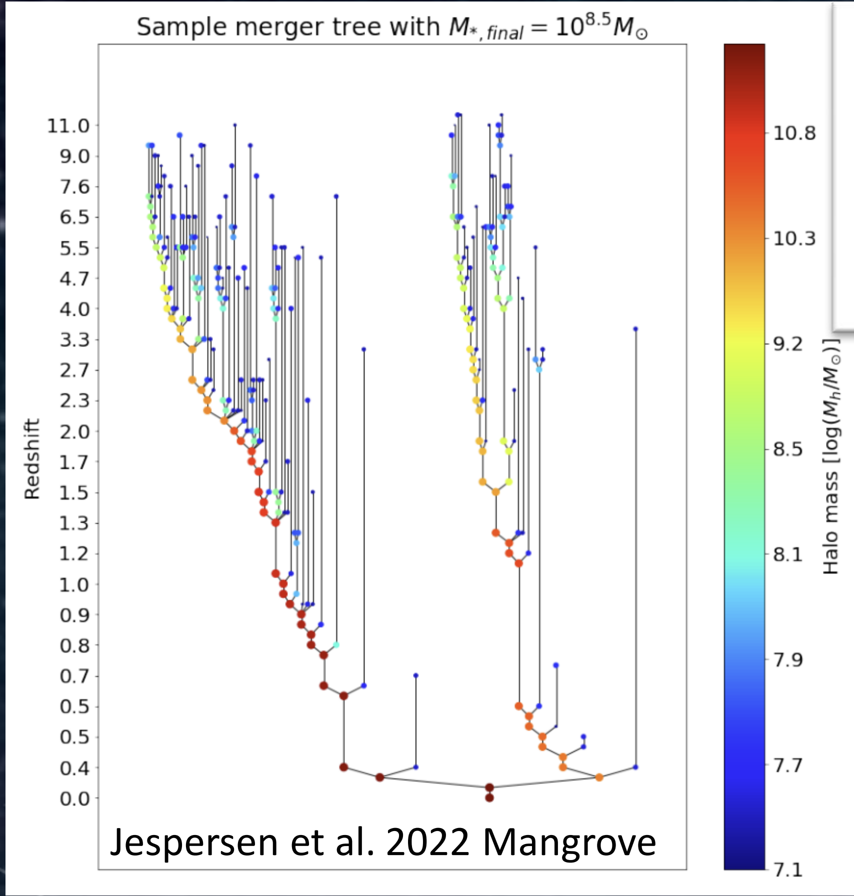
- Rich becomes richer
- Early attachment advantage

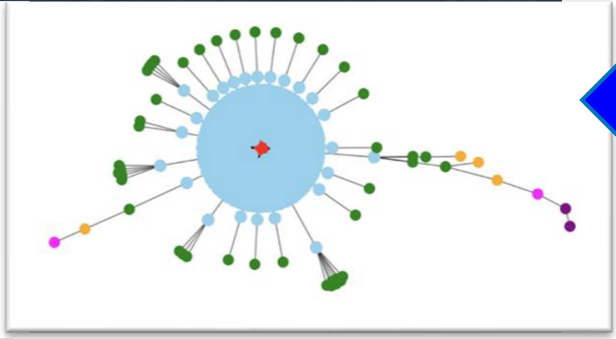
Self-similarity: Hierarchical structure naturally arise

A natural object to encode hierarchical structure formation and environmental features

One halo at z=0 has one merger tree

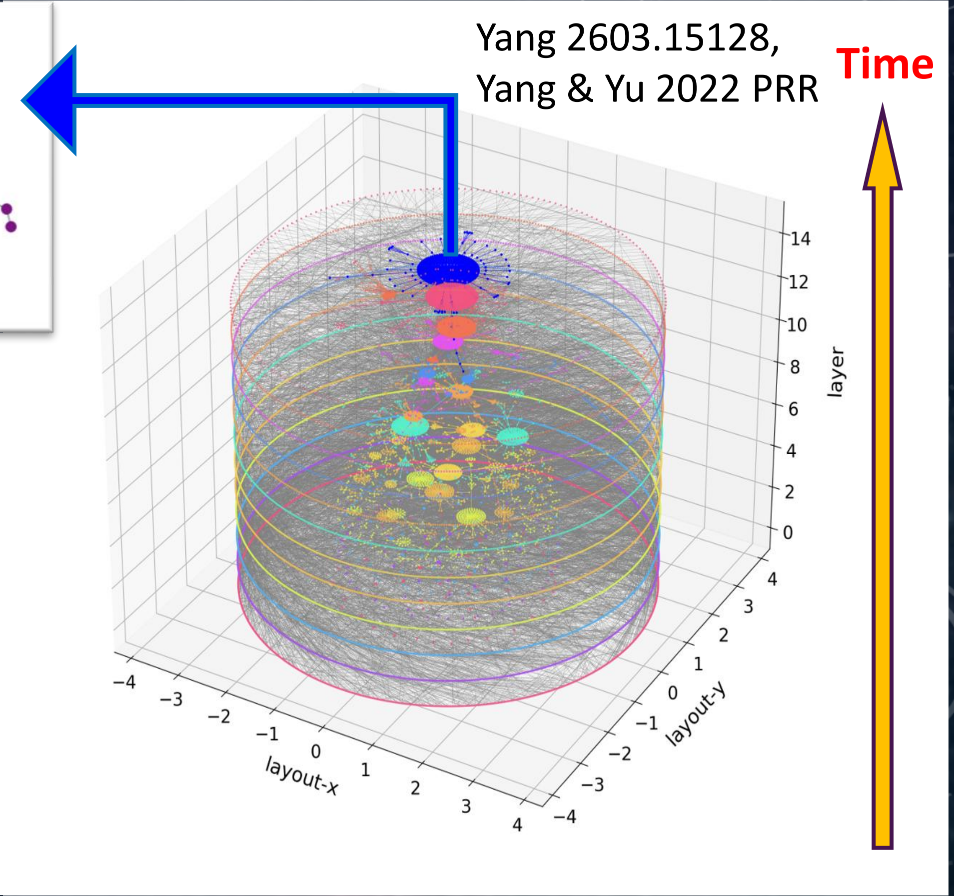
Grouped histories for main and sub halos



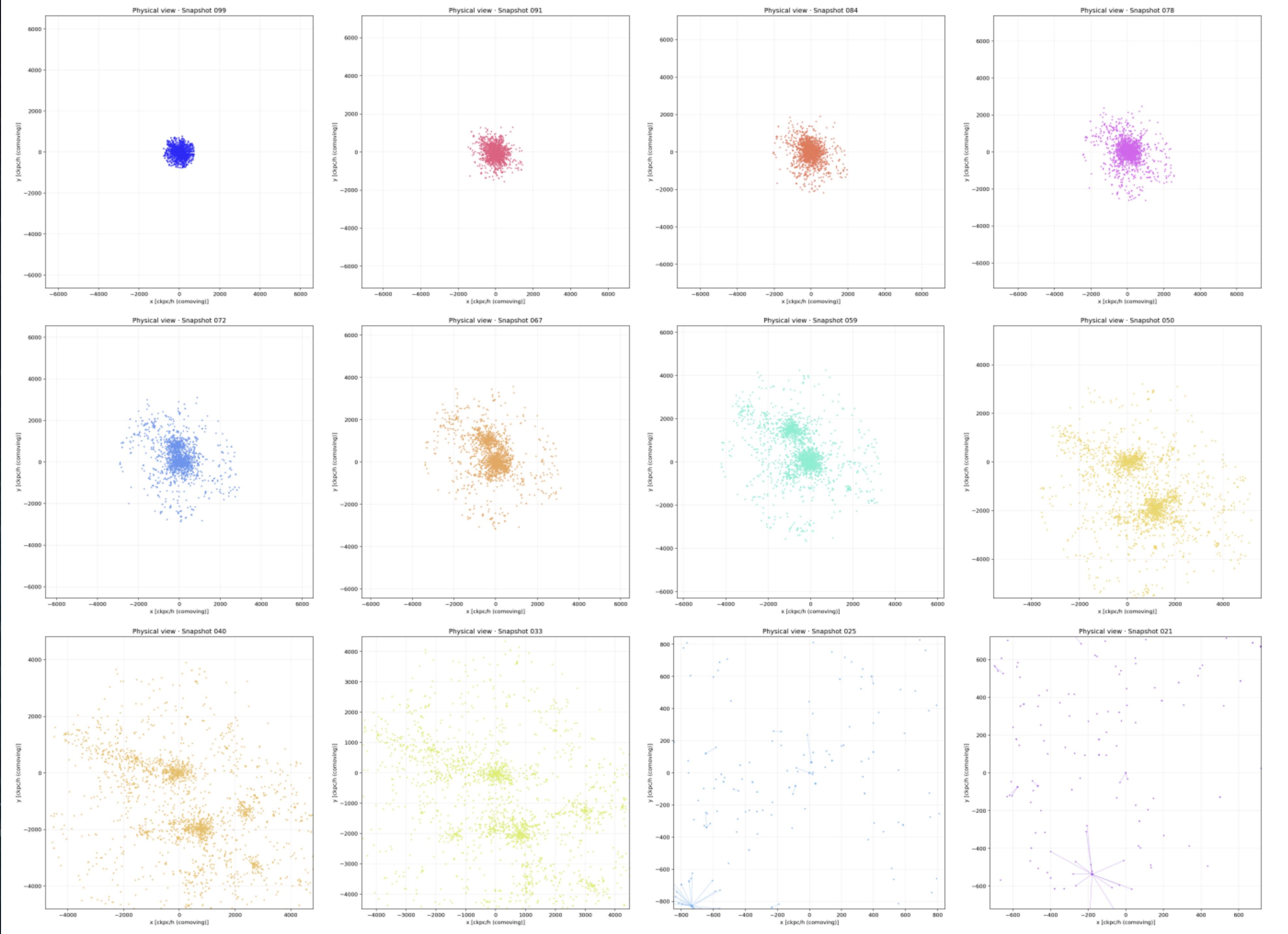


New: all halos belonging to an z=0 host halo has one

Layered Halo Graph

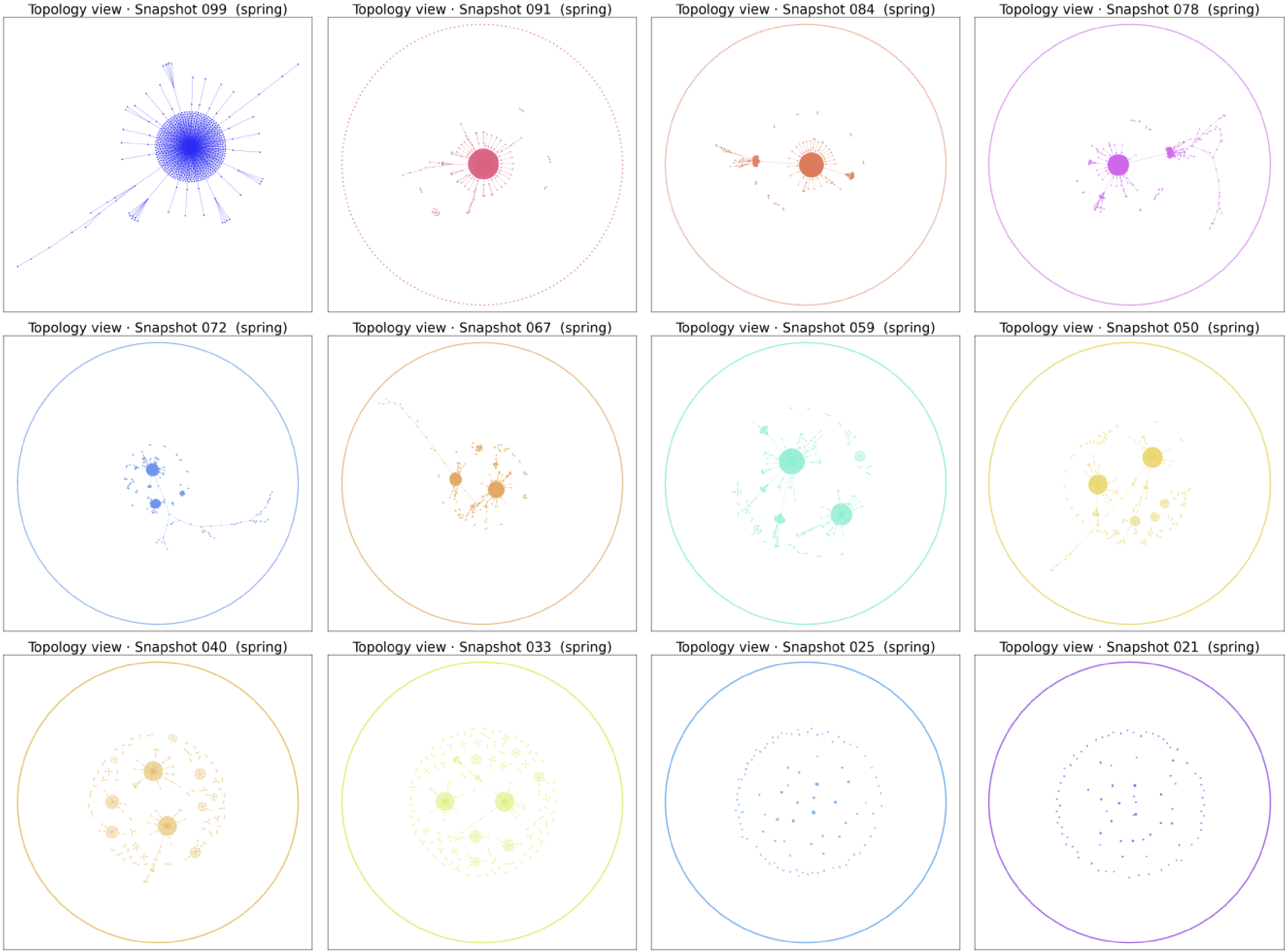


Probability-based construction naturally defines a graph measure P(G)



Physical (comoving-coordinate) view

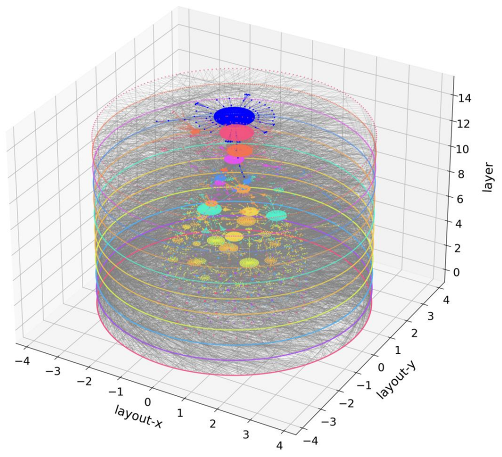
Hierarchical structures merge into a self-bound halo



Topological view

- Point has no size
- Edge has no length

Stack the layers

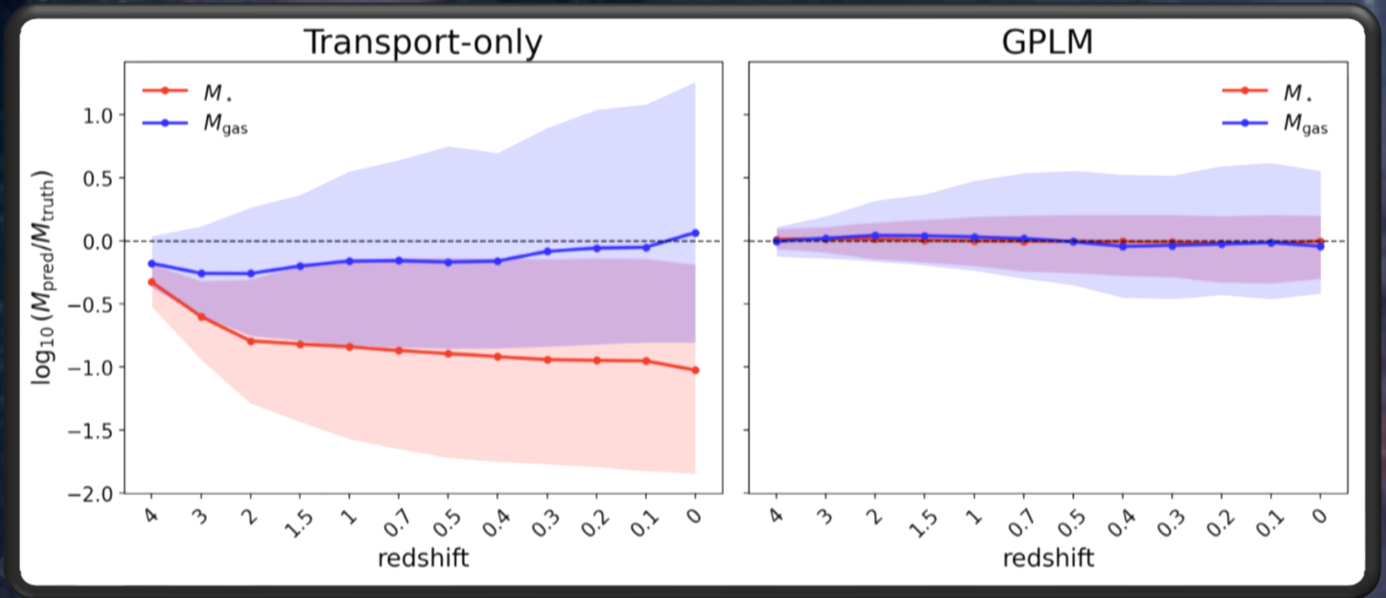


An update: A path measure model from machine learning

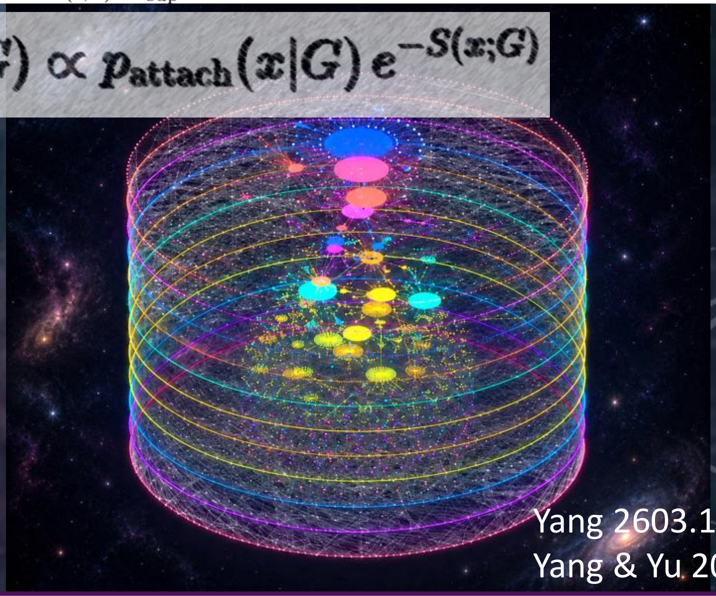
Graph encoding, conditioning → Graph Neural Network →
- b: learned residual drift
- D: learned diffusion

$$S_{\mathrm{OM}}[\mathbf{x};G]=\frac{1}{2}\sum_{(i,k)\in\mathcal{V}_{\mathrm{sup}}}\left[(\Delta x_{i,k}-\delta_{i,k})^{\top}\widehat{\Sigma}_{i,k}^{-1}(\Delta x_{i,k}-\delta_{i,k})+\log\det\widehat{\Sigma}_{i,k}\right]$$

$$P(x|G)\propto p_{\mathrm{attach}}(x|G)\,e^{-S(x;G)}$$



GPLM substantially reduces both the median bias and the spread of the residual distribution.

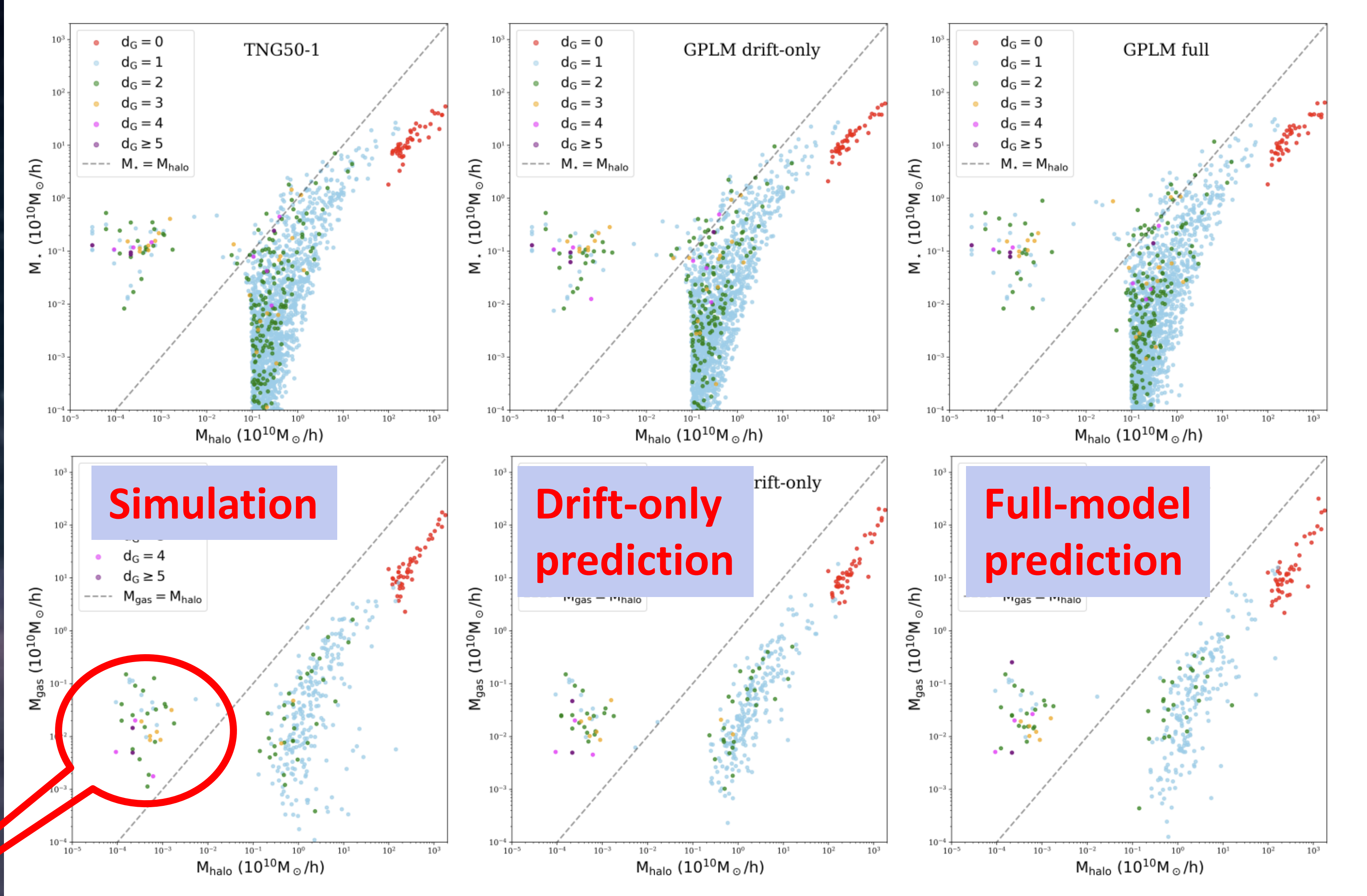


Yang 2603.15128,
Yang & Yu 2022 PRR

Training minimizes a discrete OM action build on data across histories： 6-layer (13 total) window

Model performance

The trained GPLM reliably reproduces the dG-split population structure observed in the simulation



Dark Matter Deficient Galaxies

- *Different sampled path batches represent **distinct stochastic realizations** of the same learned effective dynamics.*
- *The endpoint observables retain **analogous stochastic structure** across these different realizations.*

Theoretical Perspectives

Non-Gaussian scatter

Geometric interpretation

Optimal Transport

The action can be rewrite as an energy functional of a curve in Riemannian geometry

$$S = \frac{1}{2} \int dt\, v^T g v$$

$$v \equiv \dot{x} - b(x)$$

$$\ddot{x}^i + \Gamma^i_{jk} \dot{x}^j \dot{x}^k = F^i(x, \dot{x})$$

Most probable path=geodesic in g=D^{-1} under an effective potential

Galaxy evolution path=geodesic in environment-shaped metric+drift forcing

SIDM as controlled deformation on graph paths

Analytic kernel → **Neural Network Agent**

- **SIDM deforms (Vmax,Rmax) per time step**
- **Environmental Paths change endpoint observables**

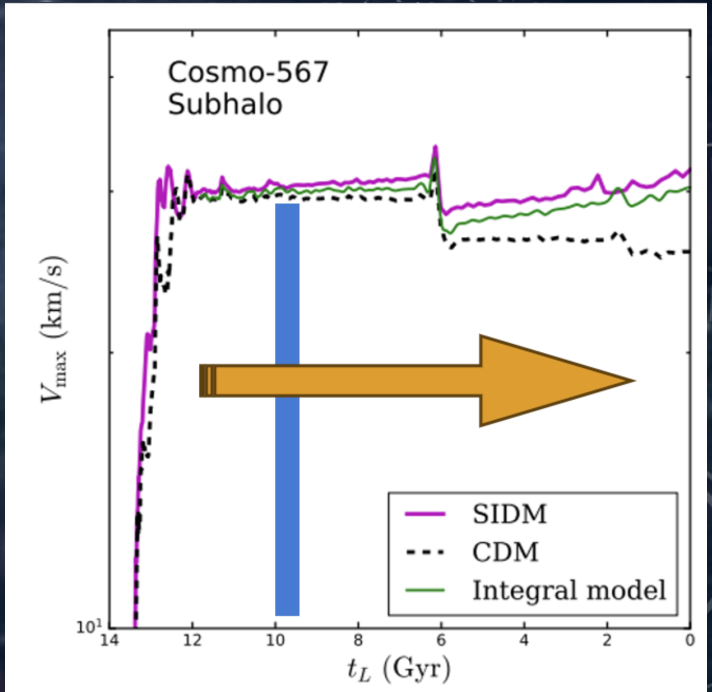
$$\mathbf{y}_k \equiv (V_{\max}, R_{\max})_k$$

$$\Delta\mathbf{y}_k = \Delta\mathbf{y}_k^{\mathrm{tr}} + \Delta\tau_k \mathbf{b}_{\mathrm{SIDM}}(s_k;\theta) + \boldsymbol{\epsilon}_k$$

(Non-linear) Transport

Residual Drift

Residual Scatter



Enhanced with AI

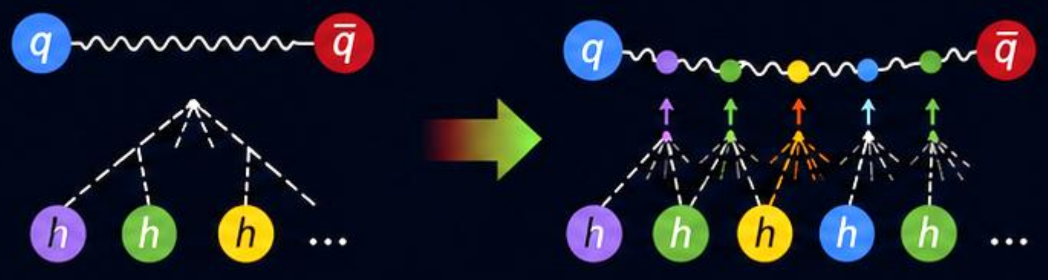
Two effective kernels in larger forward models

Broadly: SIDM, dissipation, heating, mass change…

LUND STRING MODEL (Hadronization Kernel)

Then (late 1970s)
Simple and not accurate

Now (Today)
Mature, precise, widely used



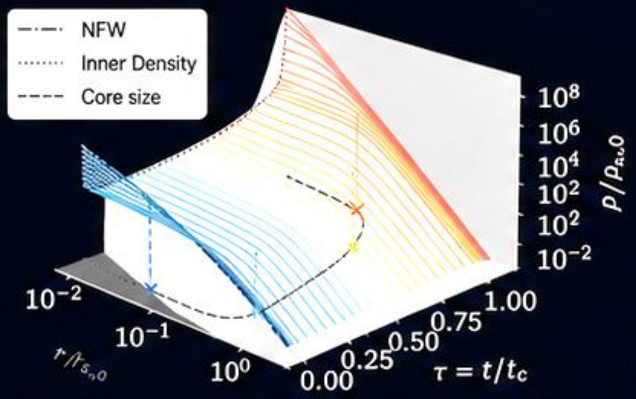
- ✗ Simple 1D string picture
- ✗ Few parameters
- ✗ Phenomenological
- ✗ Not accurate

- ✓ Extensive extensions (flavors, diquarks, CR, ropes…)
- ✓ Global tuning to collider data
- ✓ Uncertainty estimates
- ✓ Essential kernel inside PYTHIA

Role: Hadronization kernel inside PYTHIA
Transforms colored partons into hadrons for the rest of the generator.

SIDM PARAMETRIC MODEL (Halo Evolution Kernel)

Today
Very simple but physically motivated



- Parametric evolution ($\tau = t/t_c$)
- Universal inner-profile behavior
- Few calibrated parameters [redacted]
- Calibrated to controlled simulations
- Preliminary, not yet complete

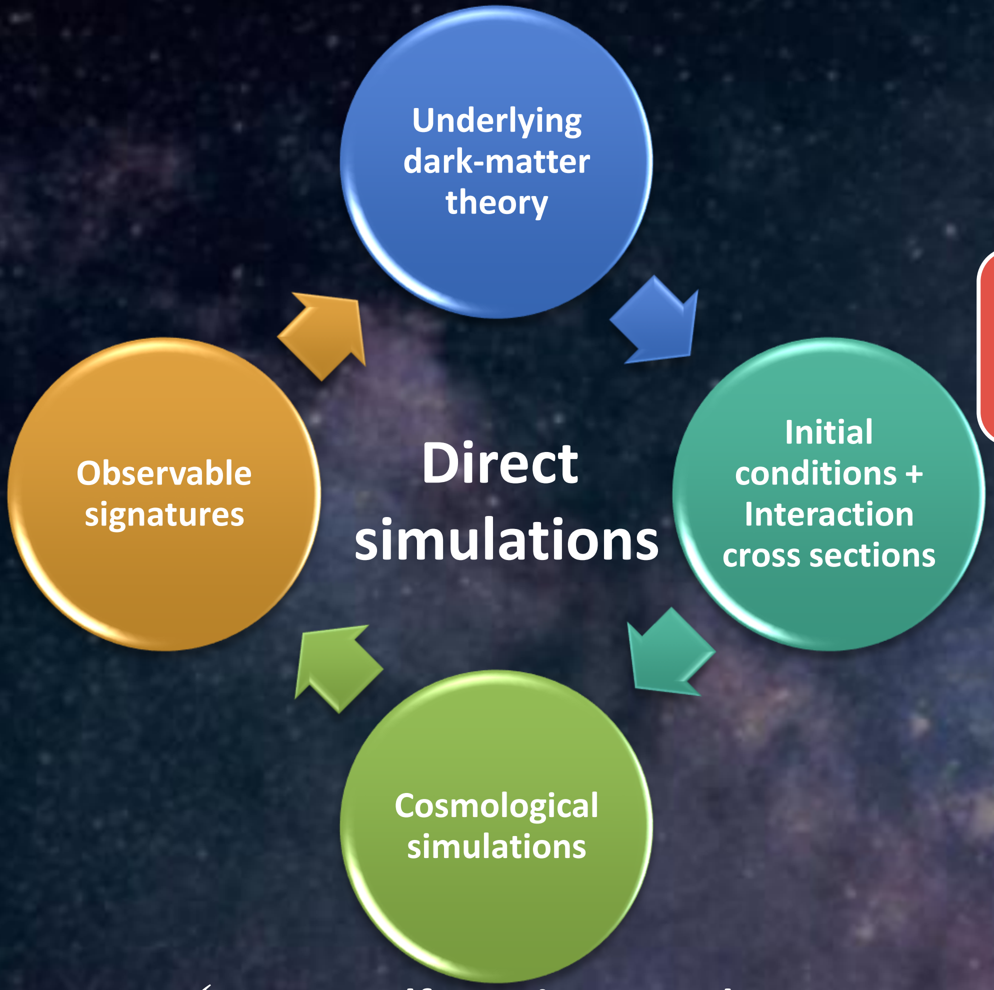
Future
Toward a next-generation kernel

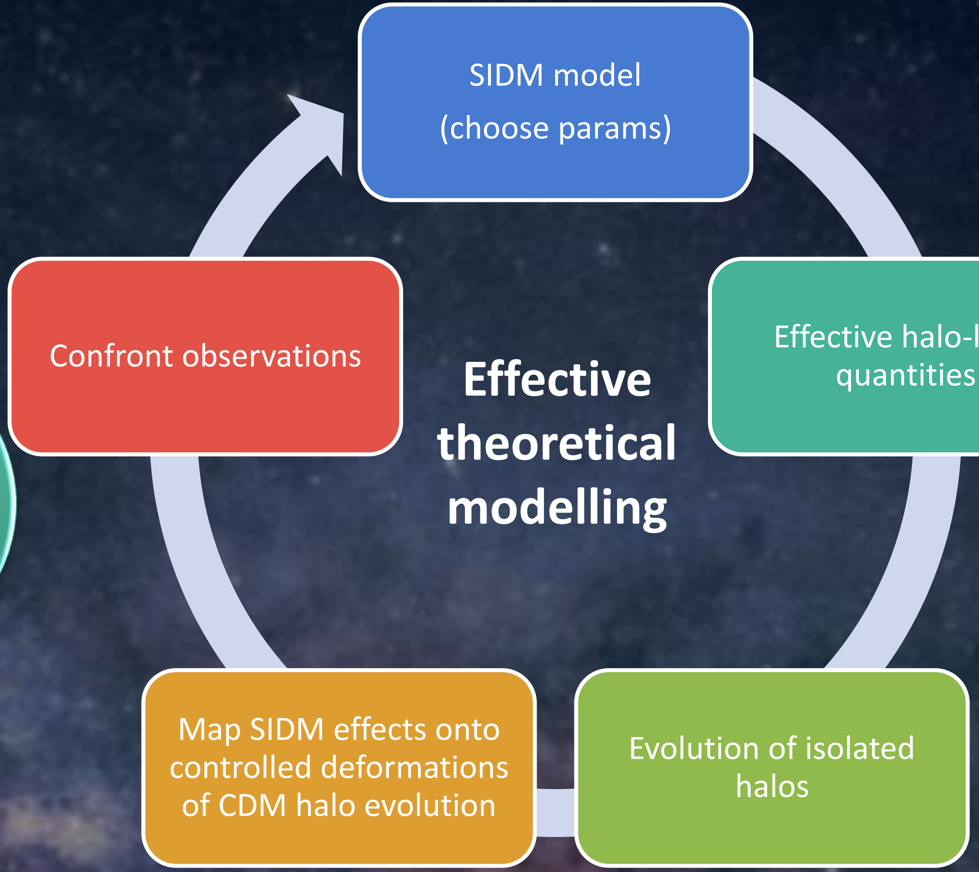
- ✓ Baryons, accretion, tides, stripping
- ✓ Stochastic path ensemble (GPLM)
- ✓ Uncertainty quantification
- ✓ Lensing & multi-probe tests
- ✓ Public tuning & benchmark suite

Role: Halo evolution kernel inside a SAM / galaxy-formation model
Evolves dark-matter halos for the rest of the model.

Key Message: Both kernels start simple and are systematically extended, calibrated and validated to become indispensable components of larger predictive frameworks.

Summary: From Microphysics to Halo Populations



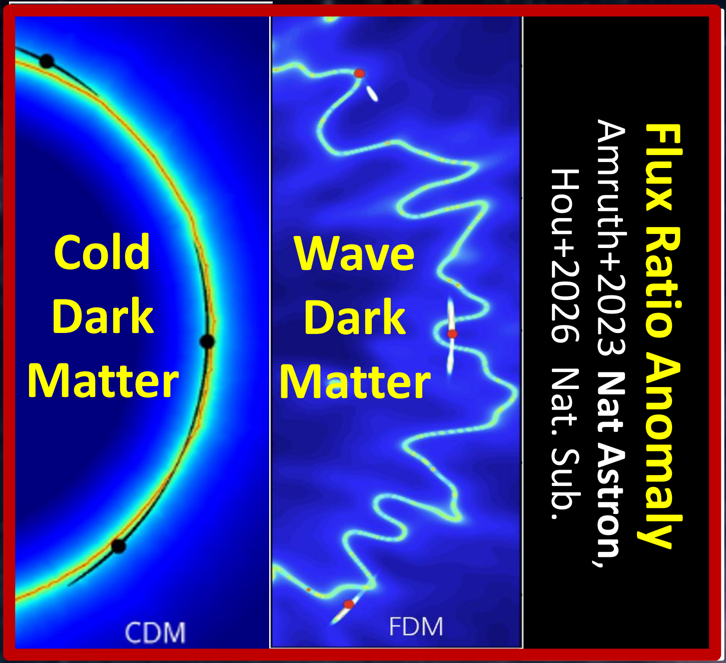


- Elastic scattering
- Dissipation
- Energy injection
- Mass change
- … …

✓ **Most self-consistent and accurate**
- Computationally expensive; impractical for broad parameter scans
- ◆ Can include baryonic effects self-consistently
- ◆ Still subject to subgrid physics and degeneracies

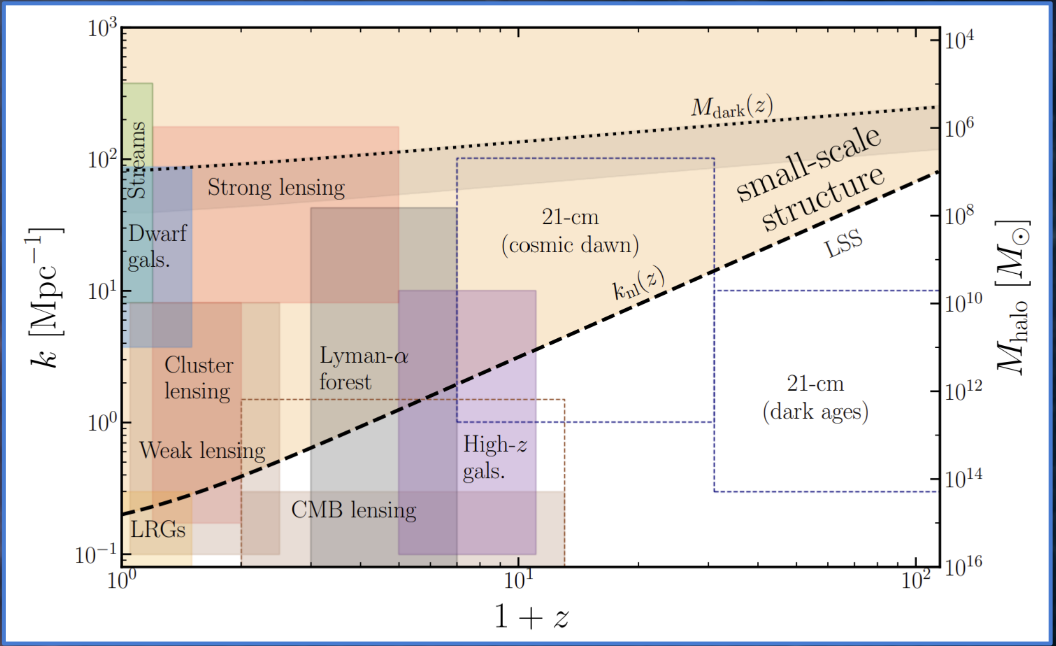
✓ **Fast and efficient**
- Harder to capture strongly nonlinear evolution
- ◆ Baryonic effects are harder to model consistently
- ◆ Subject to degeneracies and systematics

Opportunities in the Era of Next-Generation Observations



More precise: Each galaxy–halo system is an independent dark matter probe.
More samples: Large surveys enable population studies and studies of rare systems.

Nadler et al. 2026, Sabti et al. 2022, Boddy et al. 2022



Different probes create a dark matter laboratory across scales and redshifts

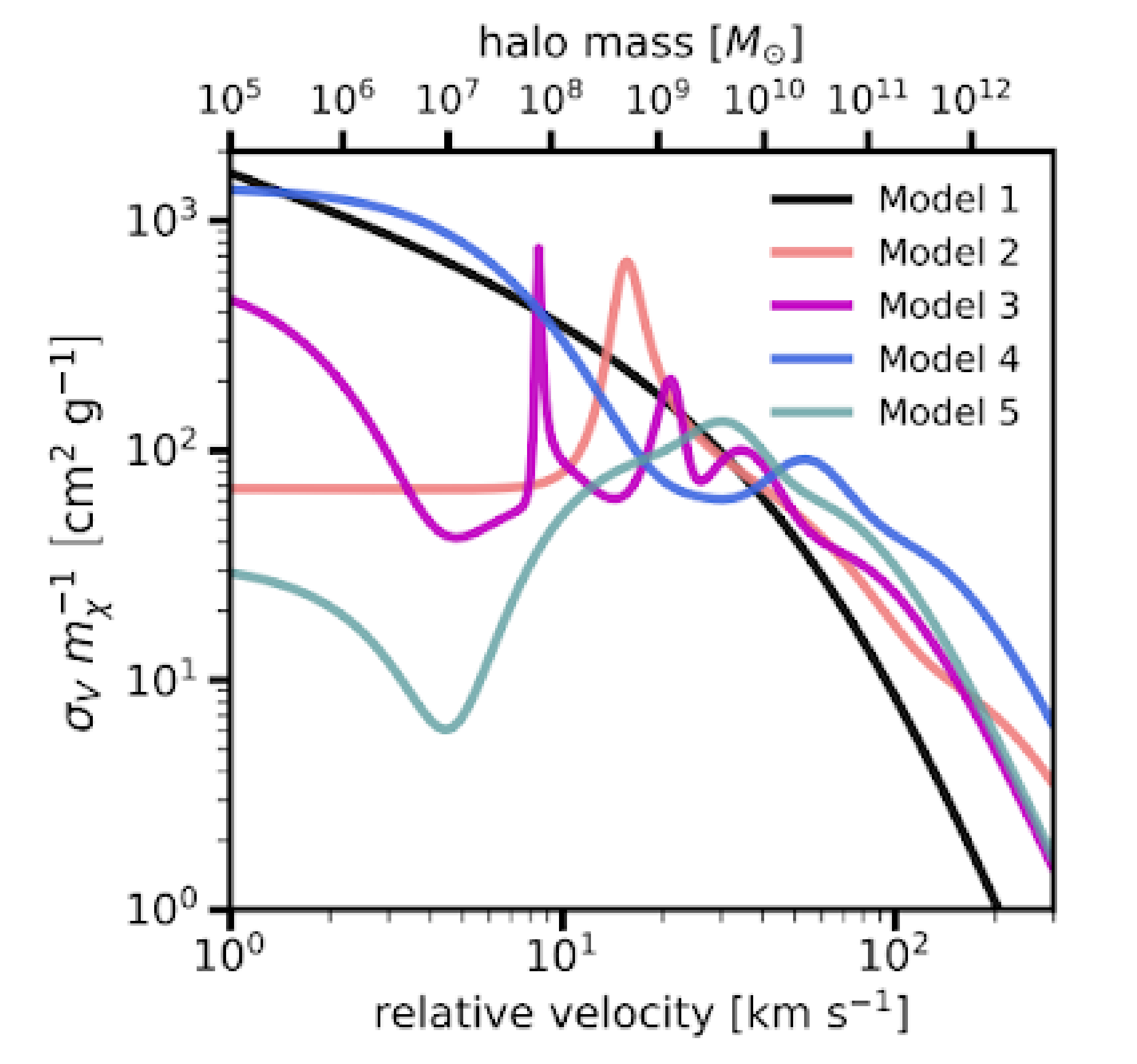
Past: If dark matter exist?
Future: What are the properties of DM?

Thanks for your attention!

Back up

SIDM CROSS SECTIONS CAN HAVE NONTRIVIAL VELOCITY DEPENDENCIES

- Yukawa potential/Gravity: v^-4 at large v
- Massive mediator: flatten the inner dependence
- Quantum resonances



Interface to **new physics** (SIDM) models?

Gilman+2207.13111

INCORPORATING BARYONS

N-body simulation: bands
Parametric model: lines

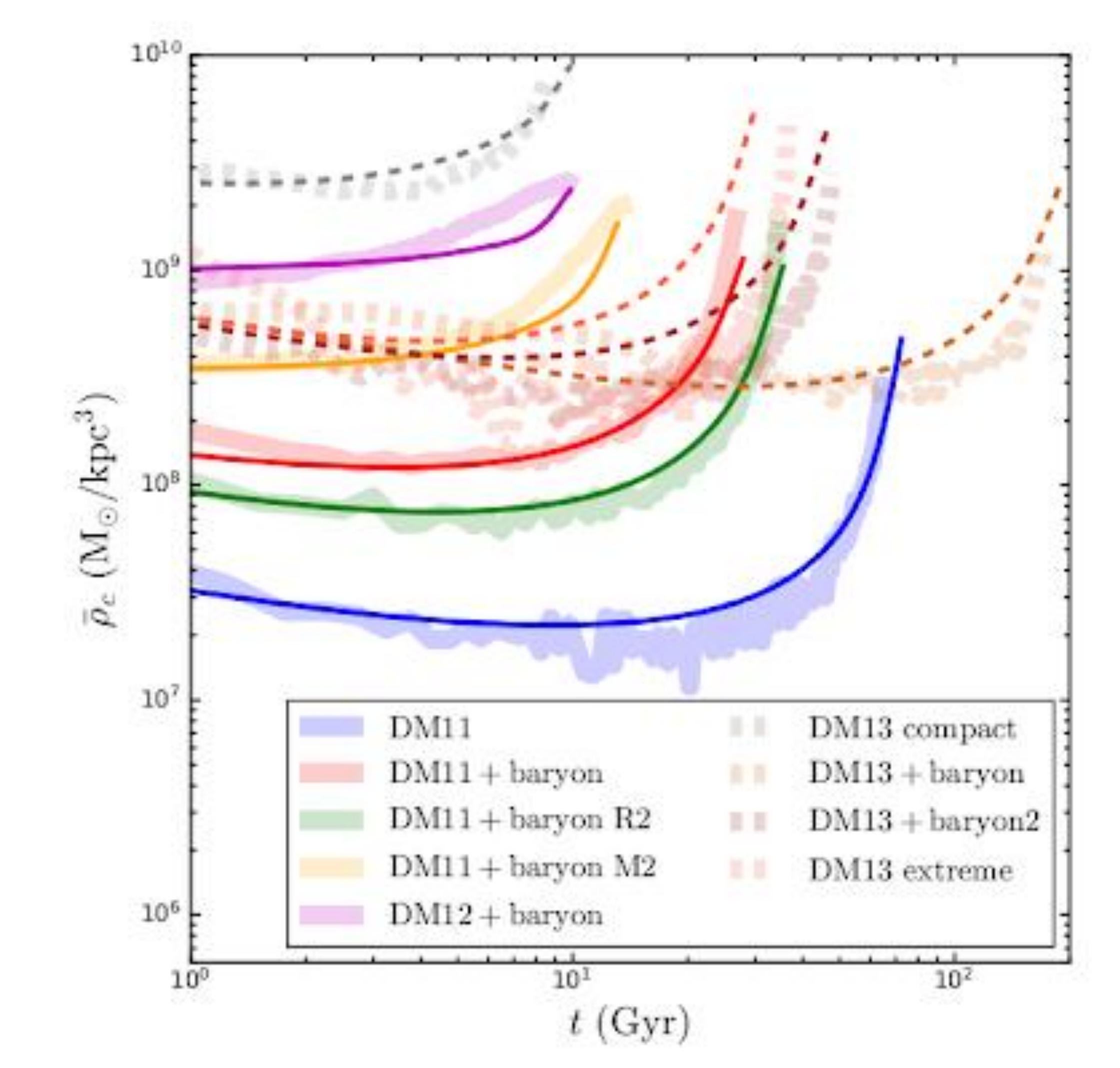
A new equation for the core collapse time

- Based on energy transfer
- Incorporate baryons
- *Collision rate => Energy transport*

$$t_R(r) \propto \frac{M(r)|\Phi(r)|/2}{4\pi r^2 \kappa |\nabla T|}$$

A density profile that allows incorporating adiabatic contraction effect

$$\rho_{\rm CoredDZ}(r) = \frac{f_{\rm in}(r)\rho_x f_{\rm out}(r)}{\frac{(r^k+r_c^k)^{1/k}}{r_x}\left(1+\left(\frac{r}{r_x}\right)^{1/2}\right)^{2(3.5-a)}}$$



arXiv:2405.03787

PUTING BARYON EFFECTS INTO A FORM FACTOR

$$t_{c,b} = t_{c,0}\mathcal{F}_t(\hat{\rho}_H, \hat{r}_H)$$

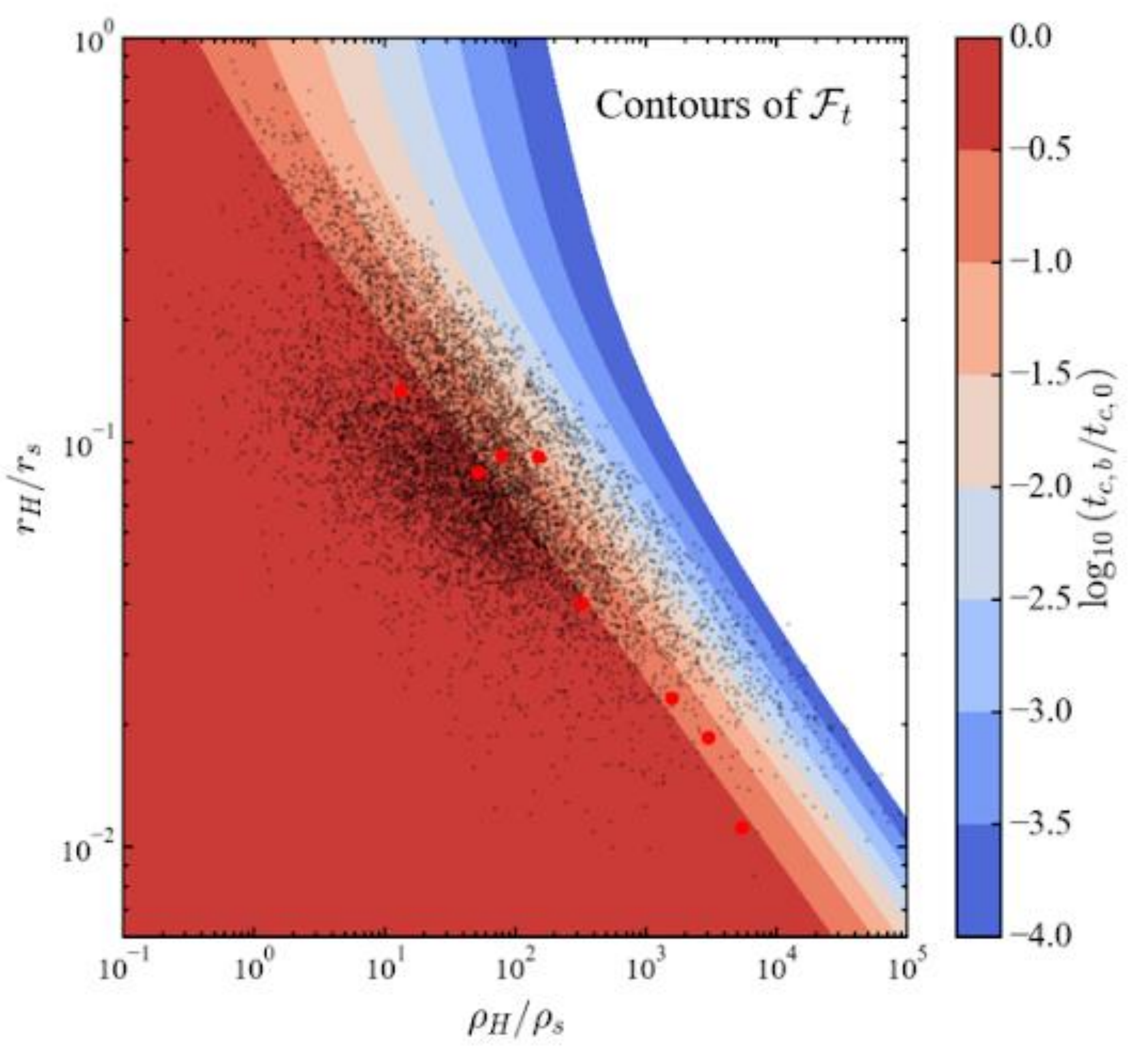
The ratio of core collapse times w&o baryons

$$\mathcal{F}_t = \left(\frac{1}{\hat{r}_{\rm eff}} + \frac{\gamma\hat{\rho}_H\hat{r}_H^3}{\hat{r}_{\rm eff}(\hat{r}_{\rm eff}+\hat{r}_H)^2}\right)^{-1}\left(1+\alpha\frac{\hat{\rho}_H\hat{r}_H^2}{2}\right)^{-\frac{1}{2}}$$

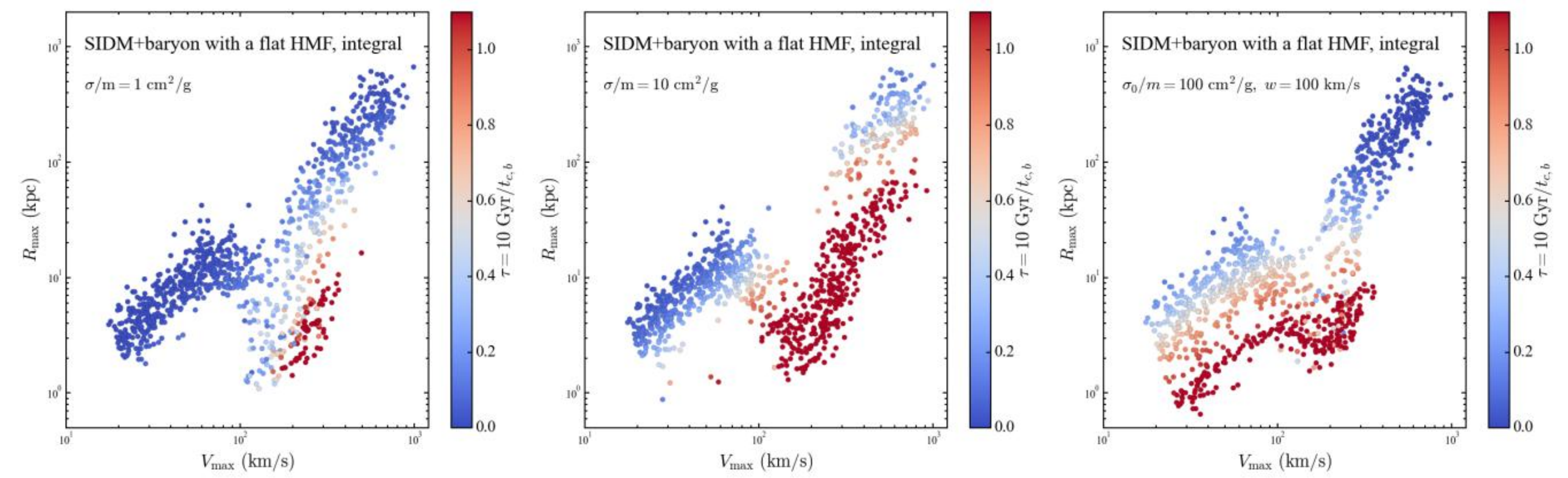
Lower-left: marginal effect

Upper-right: can be orders of magnitude large

ρH, rH: Hernquist scale radius and density of the stellar component



APPLICATION 1: MC GENERATOR FOR SIDM HALOS



- Three SIDM models
- Baryon contents based on known cosmological relations

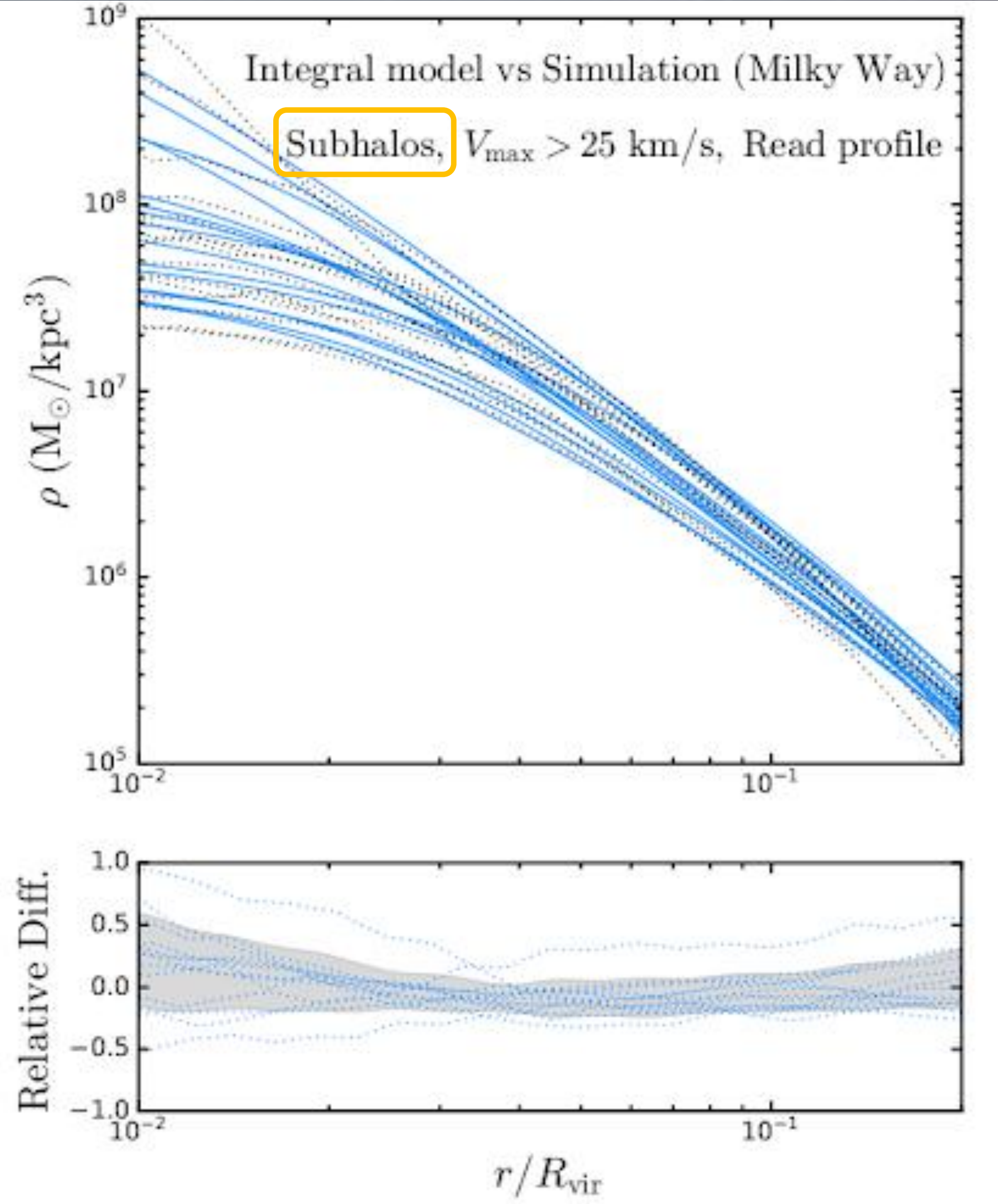
PARAMETRIC ANALYSIS TOOLS FOR SIDM HALOS

An efficient tool for obtaining SIDM predictions

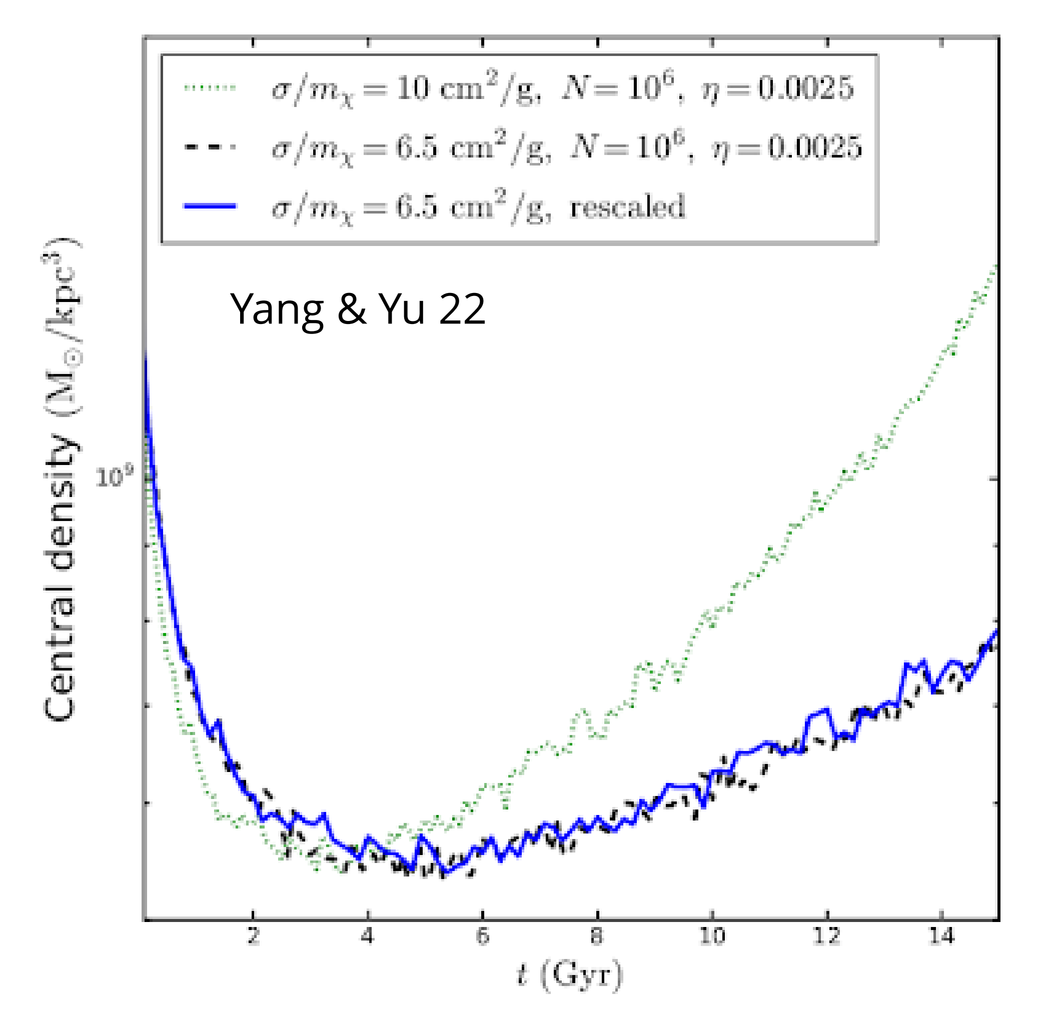
- Based on a few analytic functions/trajectories of the *gravothermal phase*
- Grounded in theory principles: not just an empirical model
- Tested against a large number of halos in cosmological simulations
- Has been extended to incorporate *mass changes* and *baryon potentials* arXiv:2405.03787

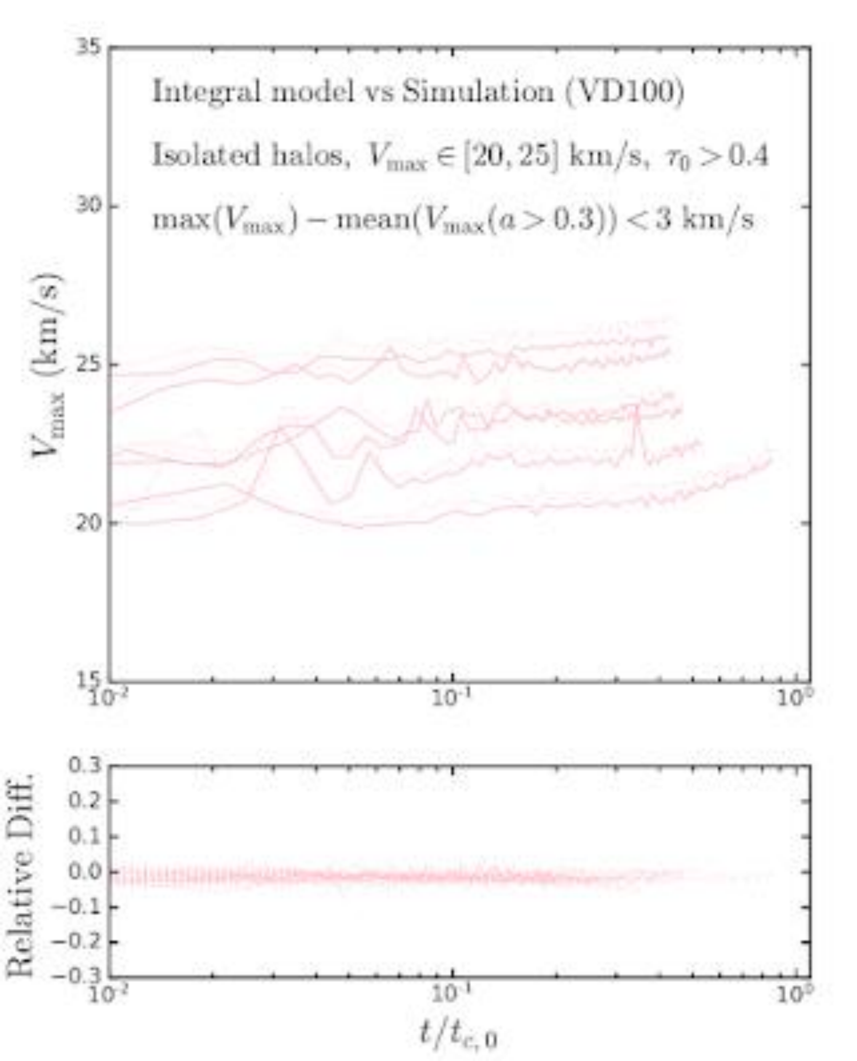
https://github.com/DanengYang/parametricSIDM

With Ethan O. Nadler, Hai-Bo Yu, Yi-Ming Zhong, S. Ando, S. Horigome



HOW WELL IS TC BEING PROPORTIONAL TO SIGMA/M?





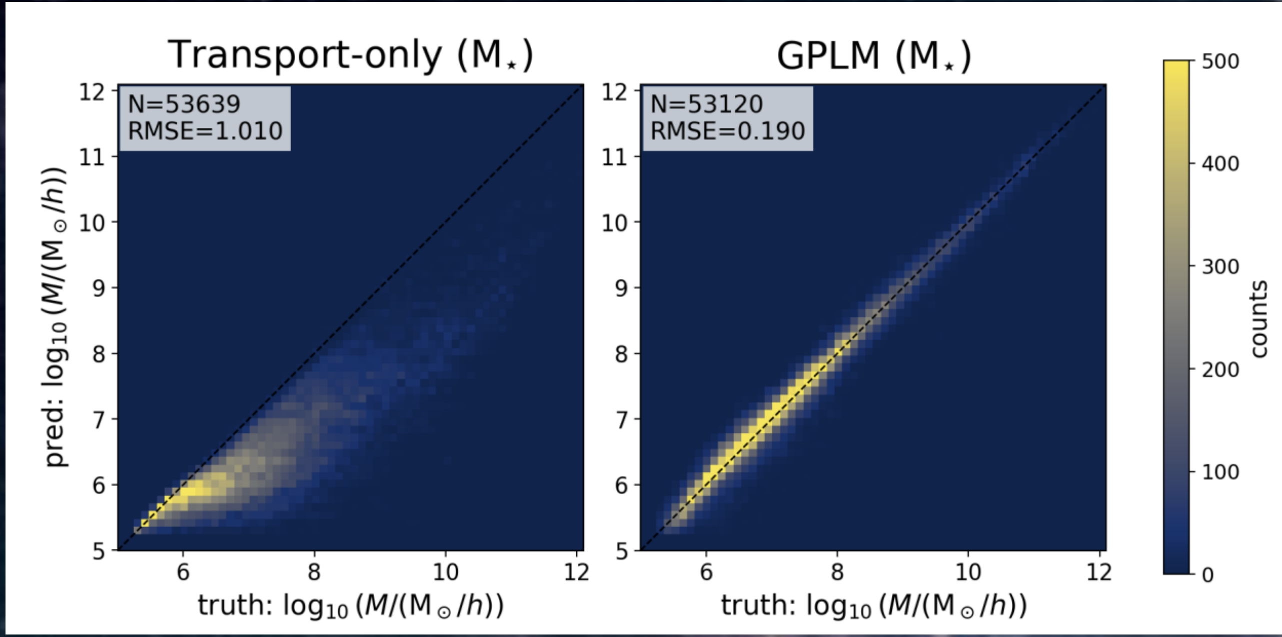
"Isolated" halos in cosmological simulation (Yang, Nadler, Yu 23) with

- t/tc>0.4
- Exclude cases with major mergers

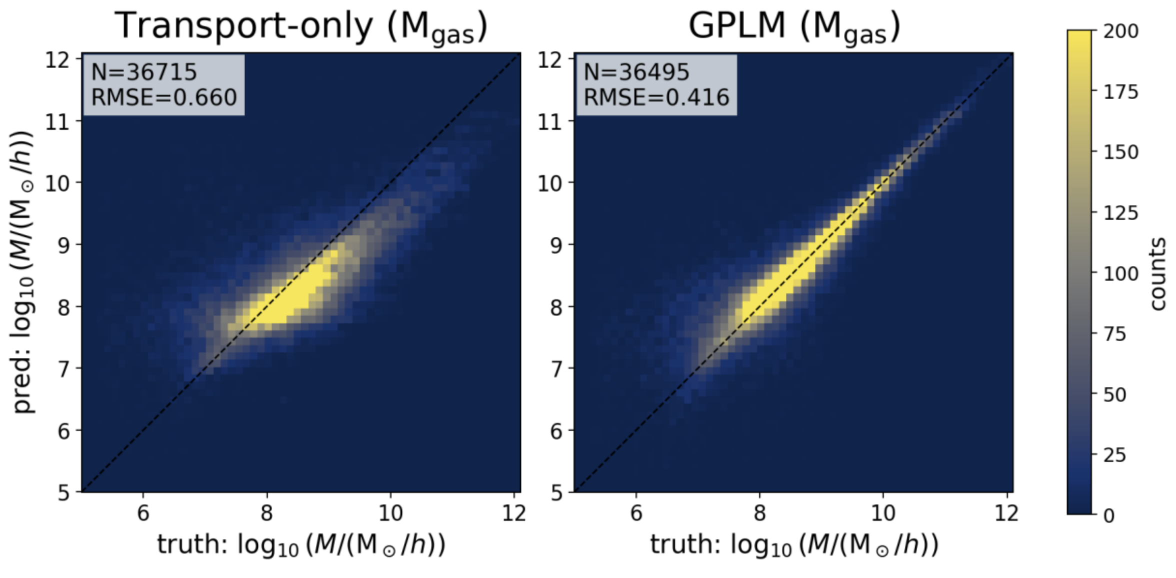
Model vs simulation

More in YNY2406.10753

Model performance

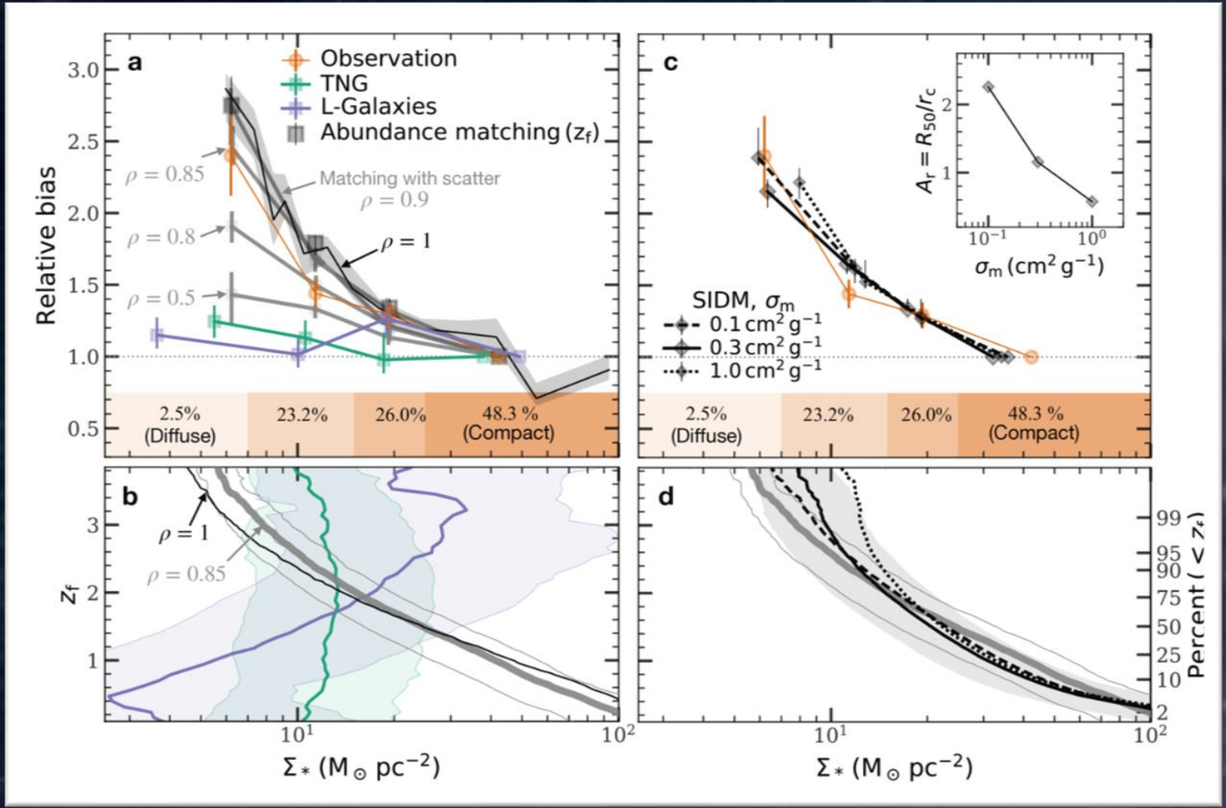


Prediction vs Truth in Stellar Mass



Prediction vs Truth in Gas Mass

Example (where path matters) 1: time-dependent cores in SIDM



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Article | Published: 21 May 2025

Unexpected clustering pattern in dwarf galaxies challenges formation models

Ziwen Zhang, Yangyao Chen, Yu Rong, Huiyuan Wang, Houjun Mo, Xiong Luo & Hao Li

Nature **642**, 47–52 (2025) | Cite this article

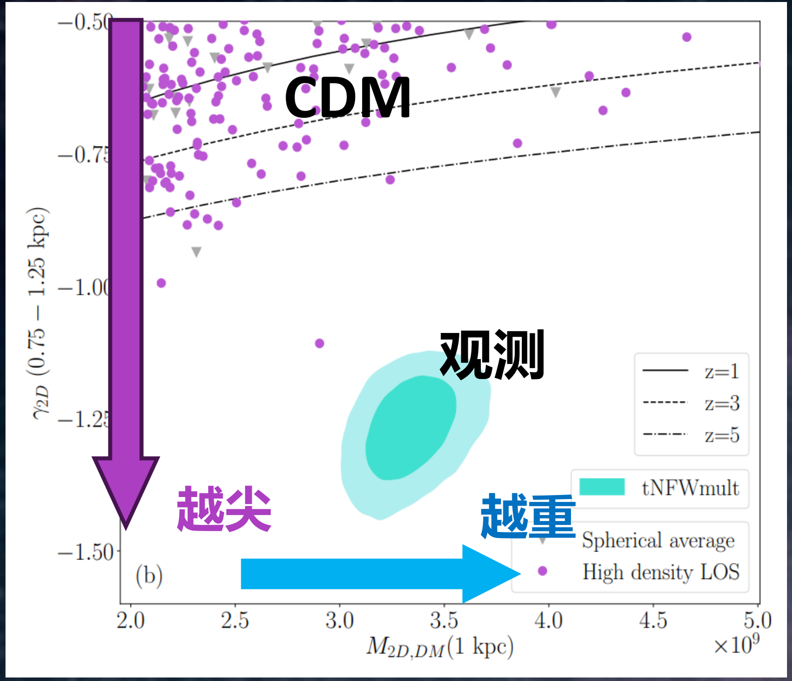
1274 Accesses | **84** Altmetric | Metrics

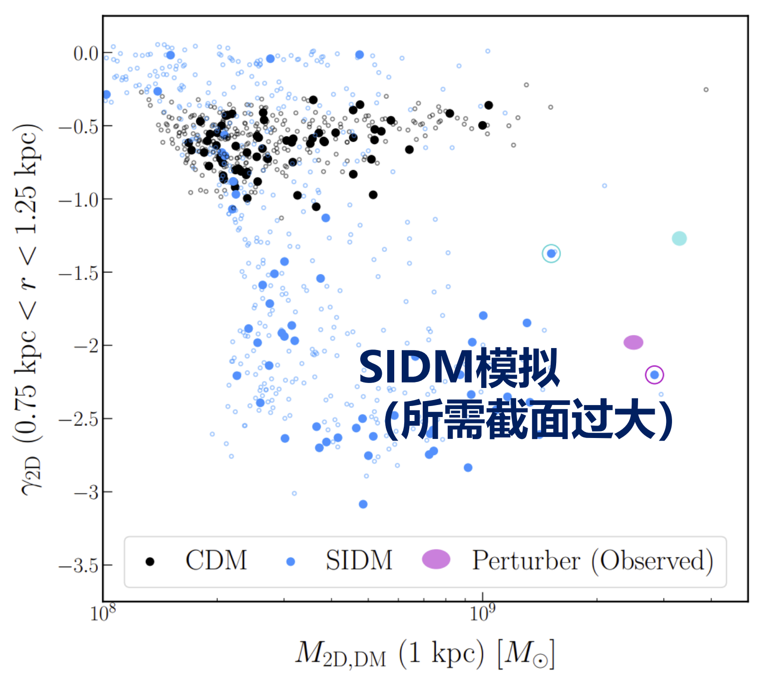
- Decreasing surface density
- Hydro baryonic simulation gives inverted feature
- Self-Interacting Dark Matter (SIDM), with

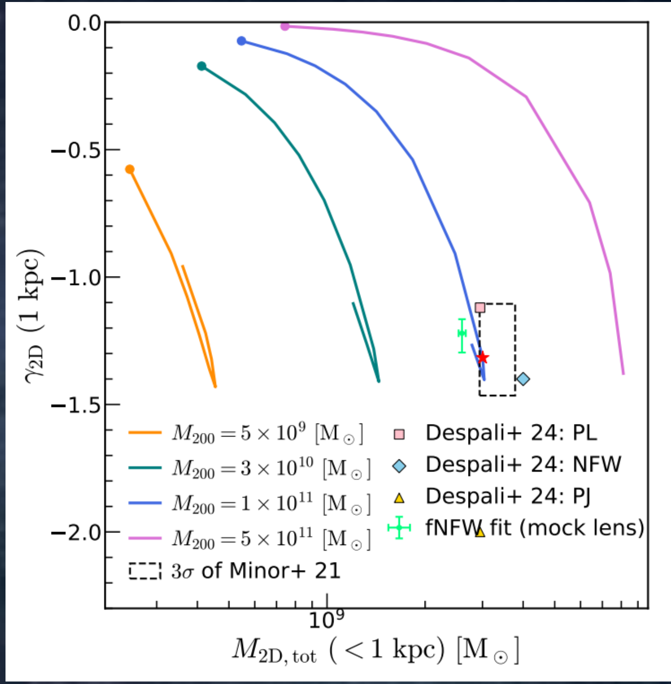
  σ/m=0.3 cm^2/g (SIDM03)

  can explain the observation

(2) Strong lensing perturbers could be core-collapsed SIDM subhalos







Minor et al. 2011.10627
The dark substructure has
unexpected high concentration

Nadler, Yang, Yu, ApJL 2023
Strong SIDM with
$\sigma/m > 100\ cm^2/g$ in group
substructures

S. Li & R. Li+2504.11800
Little dark dots
$\sigma/m > 138\ cm^2/g$
Why M11 does not host a
visible galaxy?

(1) Cored dwarf hosts

Dashed lines: 1 component SIDM03

Following Zhang et al. *Nature* (2025)

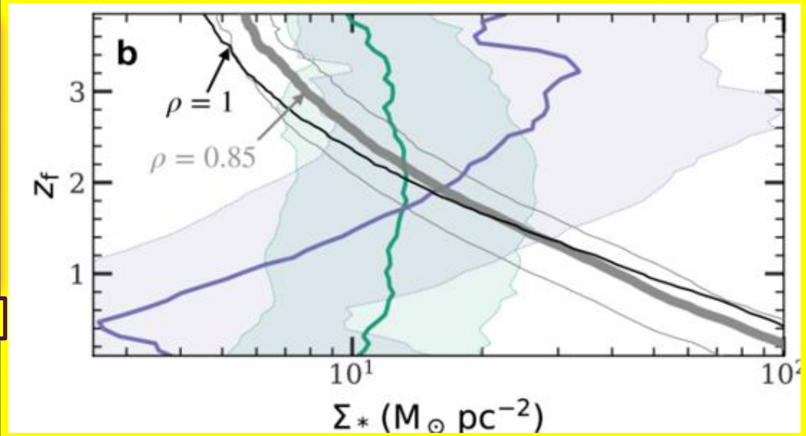
$\sigma/m \sim 0.3\ \mathrm{cm}^2/\mathrm{g}$

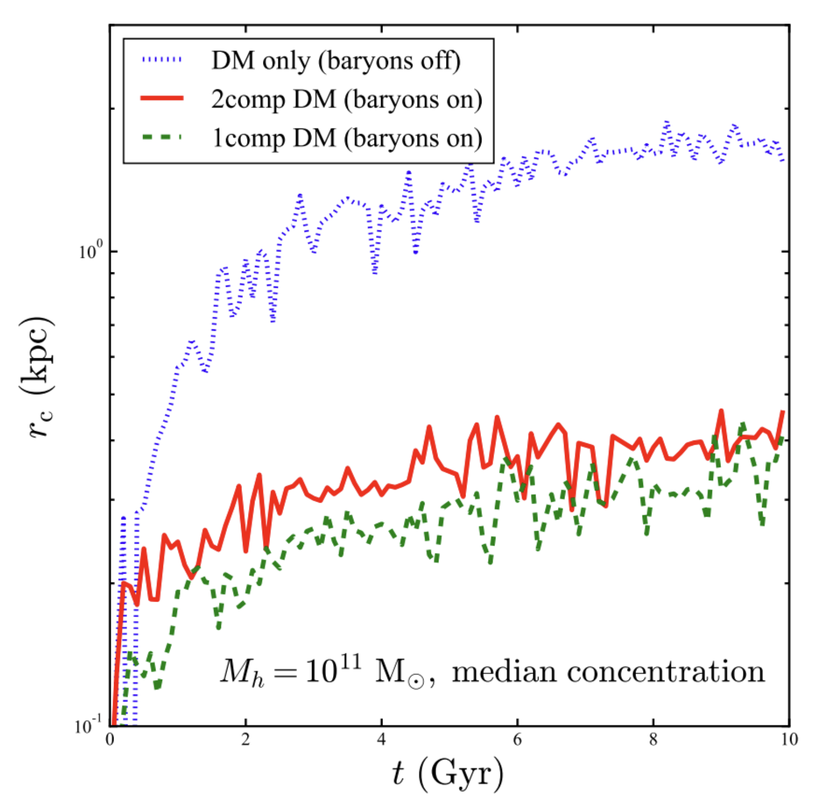
Test against Massive dwarfs of
Median,
+1 sigma,
-1 sigma concentration

Solid lines: **SIDM2c** with two component

Produce roughly identical results

We need a time dependent decreasing surface density in dwarfs





Flat halos evolve similar to 1 component SIDM halos

Hint: dense halos become denser

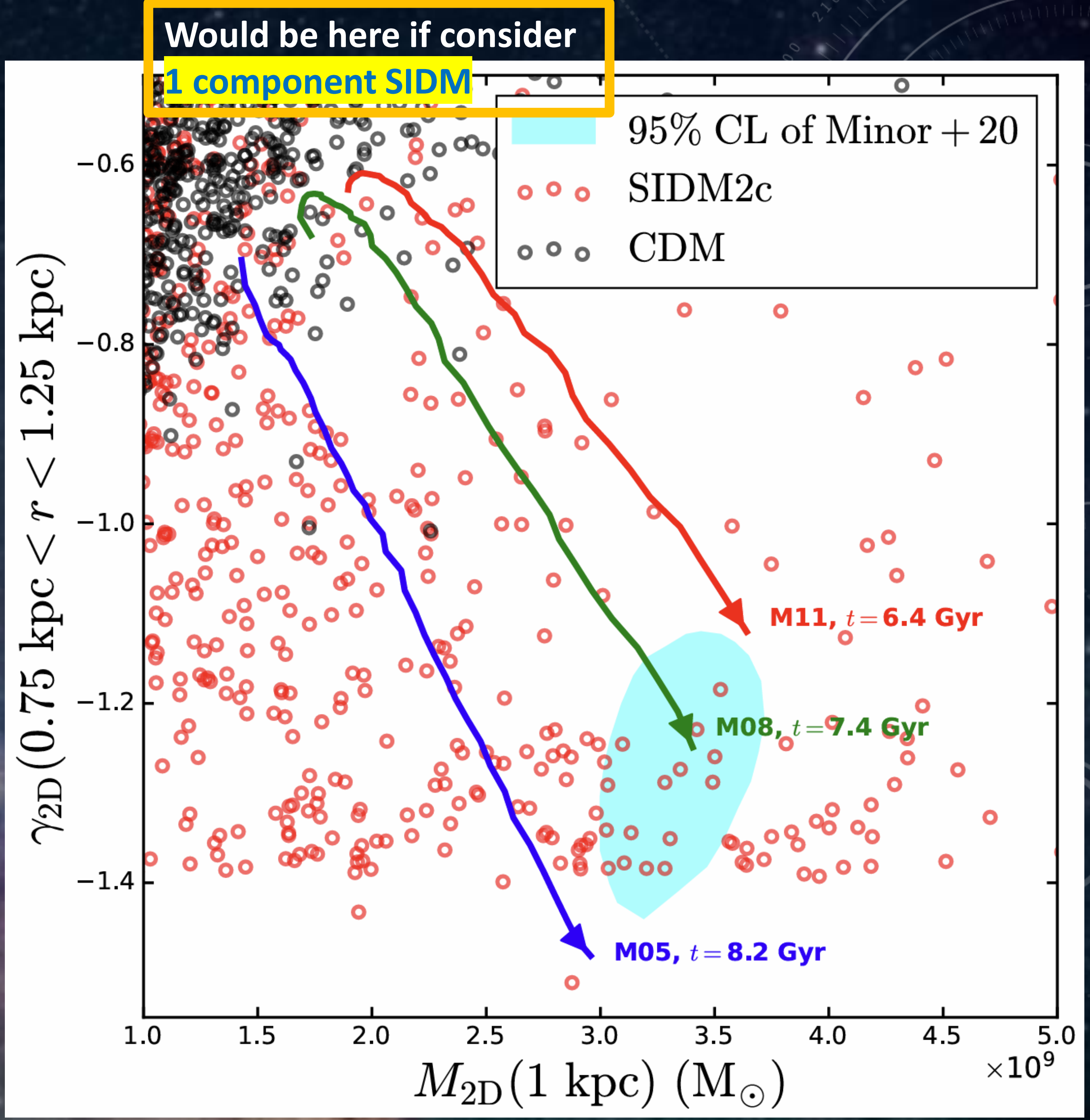
(2) SIDM + mass segregation

Boosted gravothermal evolution such that a small SIDM strength can work!

$\boxed{\textbf{SIDM strength} \lesssim 1\ \mathrm{cm^2/g}}$

No candidate in CDM

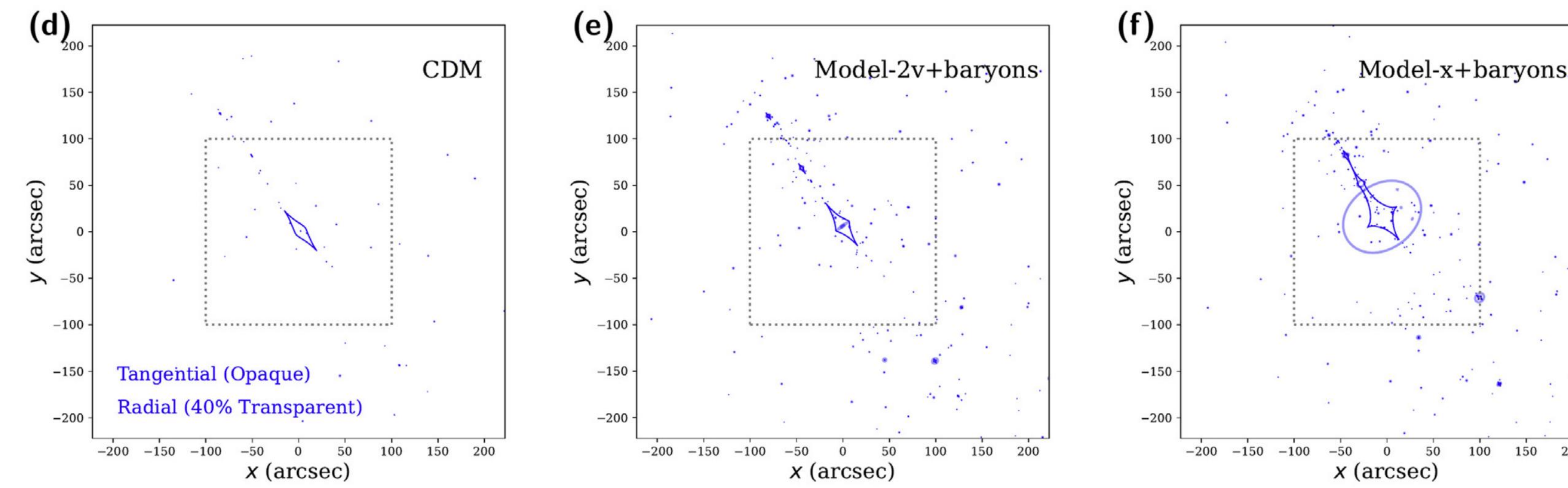
Trajectories from controlled simulations; dots for cosmological



(3) Gravitational lensing by cluster substructures

GGSL cross section, averaged over 11 angles

0.30 (CDM) 1.36 (SIDM2v,fiducial) 4.15 (SIDMx)

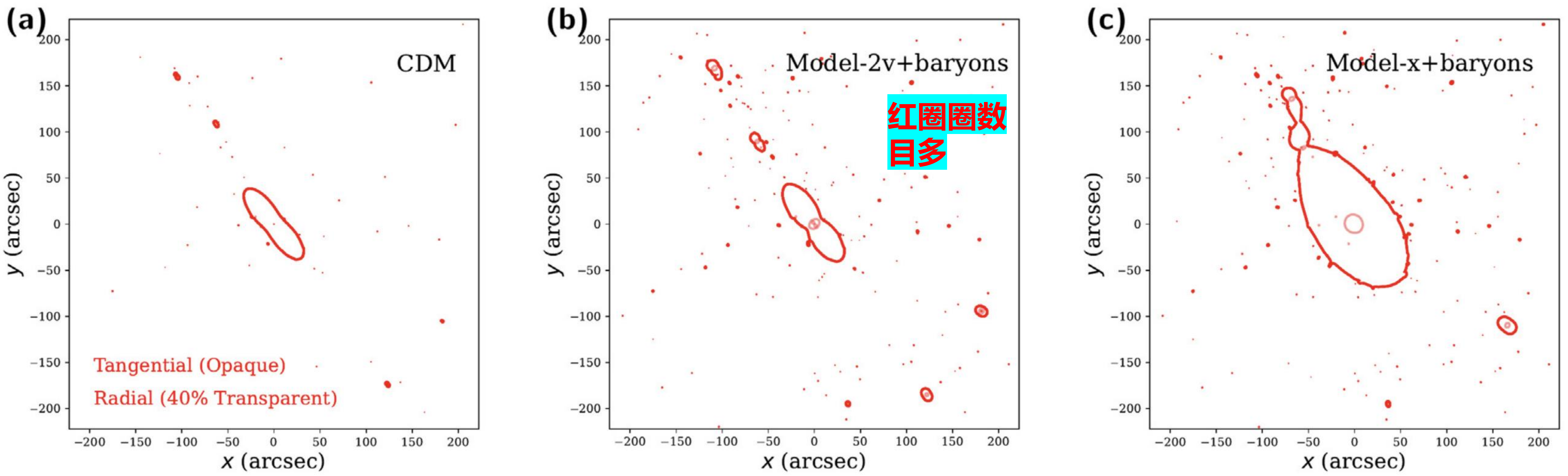


小圈圈面积之和大

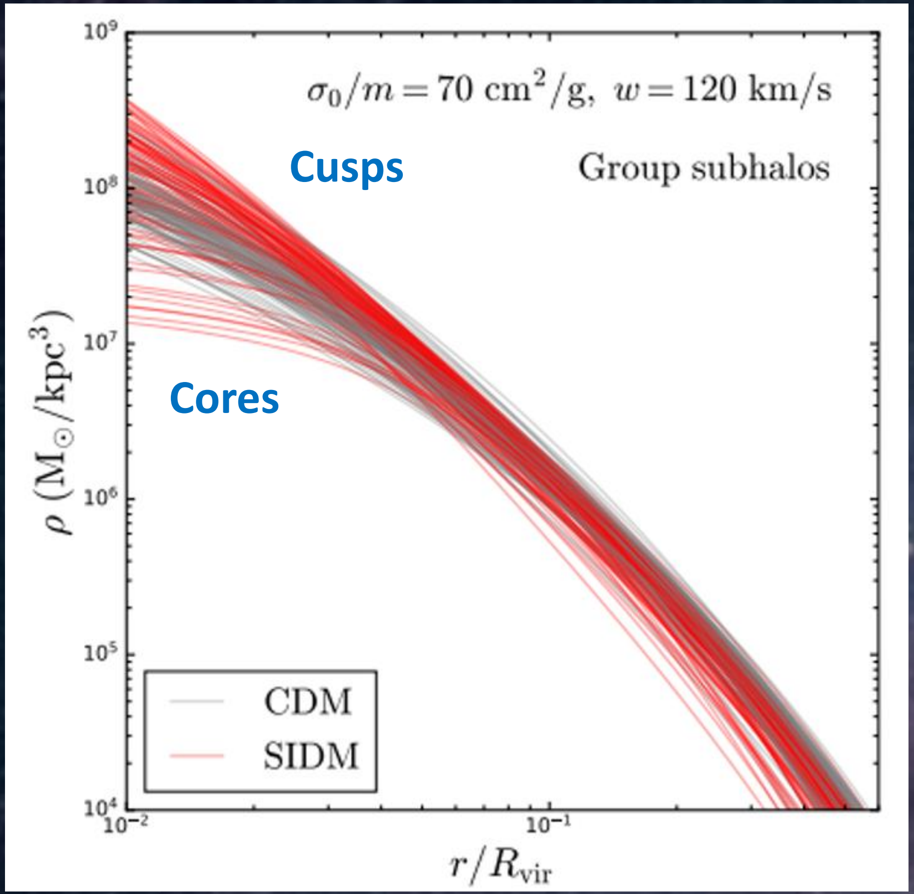
(3) Gravitational lensing by cluster substructures

Number of small scale lenses, effective radii>0.5'', average over 11 angles

3.7 (CDM) **17.4 (SIDM2v,fiducial)** **29.6 (SIDMx)**



Diversifying "endpoint" halo profiles by SIDM

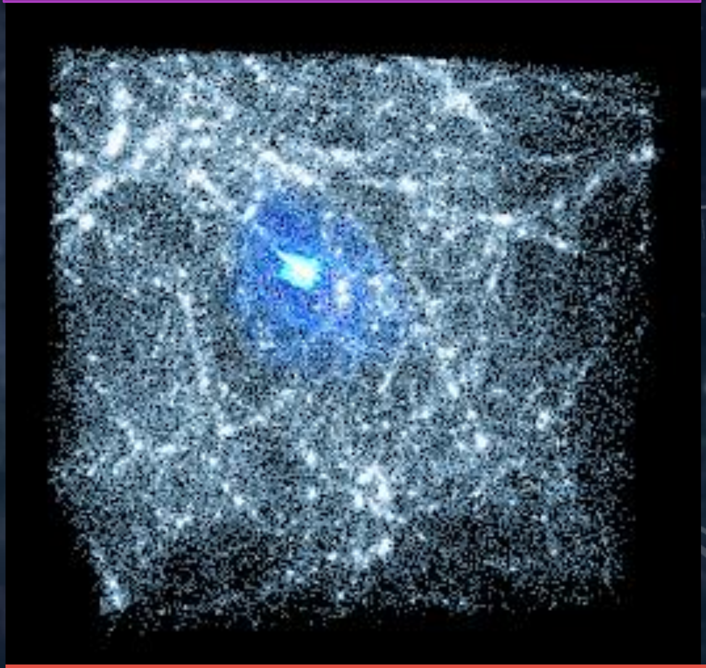


We observe only a snapshot/light-cone of the universe.

Observables correspond to **endpoints** of histories.

However, physics work along the evolution **path**.

First-principle calculations/simulations challenging and expensive



Smearing out unresolved dynamics introduce **intrinsic scatters**

Indirect detection of SIDM

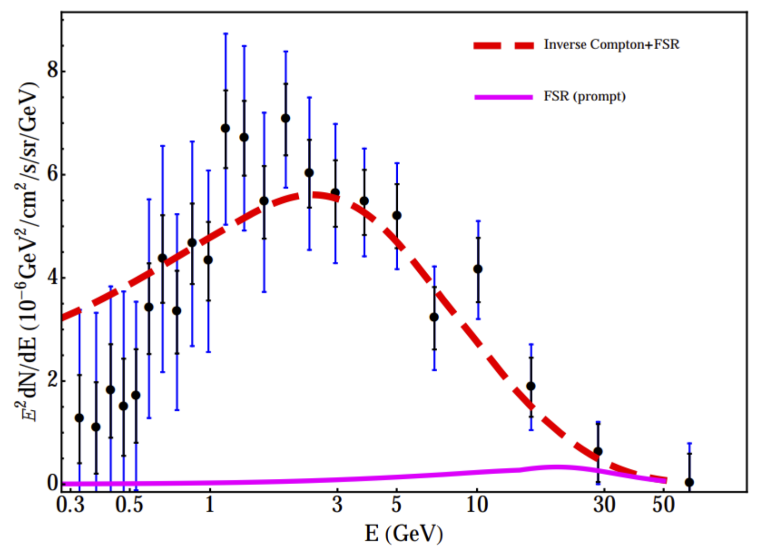


FIG. 1. γ-ray spectrum from Inverse Compton emission and final state radiation produced by annihilation of a 50 GeV dark matter particle through a light mediator into e^+e^- final state. The spectrum is compared to the Galactic Center excess [10].

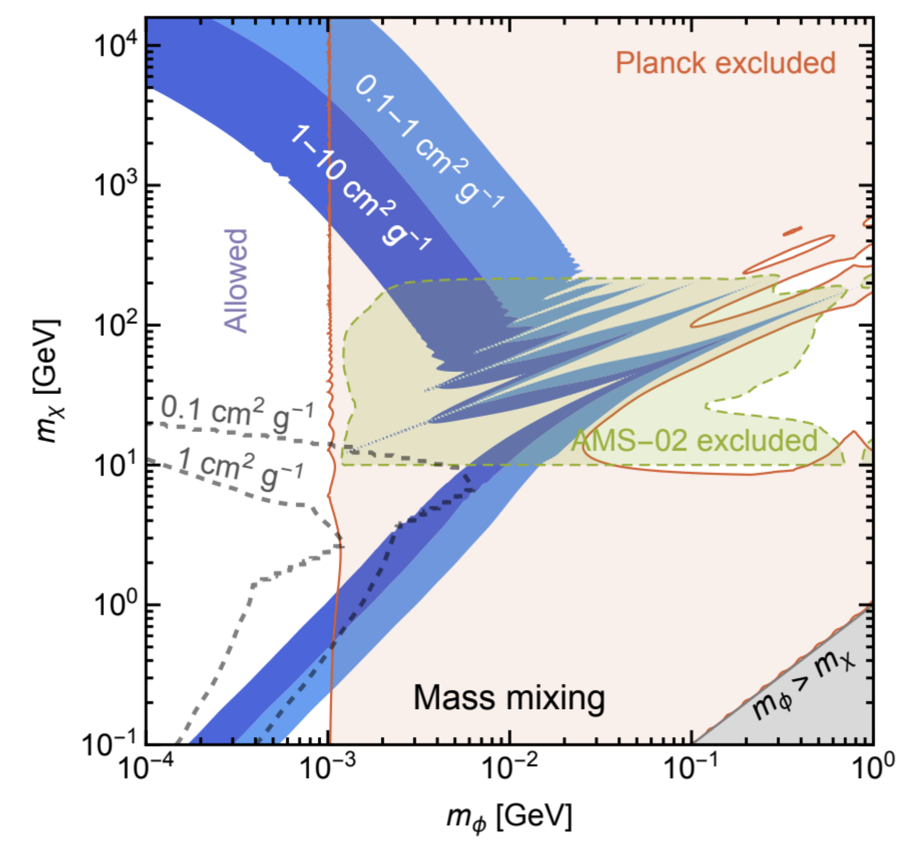
Kaplinghat, Linden, and Yu, 1501.03507 PRL
Galactic Center Excess in Gamma Rays from Annihilation of Self-Interacting Dark Matter

Golden window: SIDM + WIMP miracle + Galactic Center Excess simultaneously

Parameter window closed by later studies

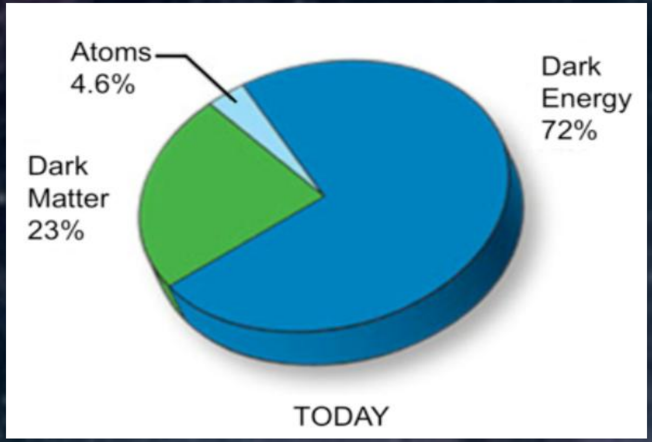
Viable SIDM parameter region with mphi<MeV

Only weak Gamma signatures



Bringmann et al. 1612.00845 PRL
Strong constraints on self-interacting dark matter with light mediators

Indirect detection safe SIDM models



Credit: Swinburne

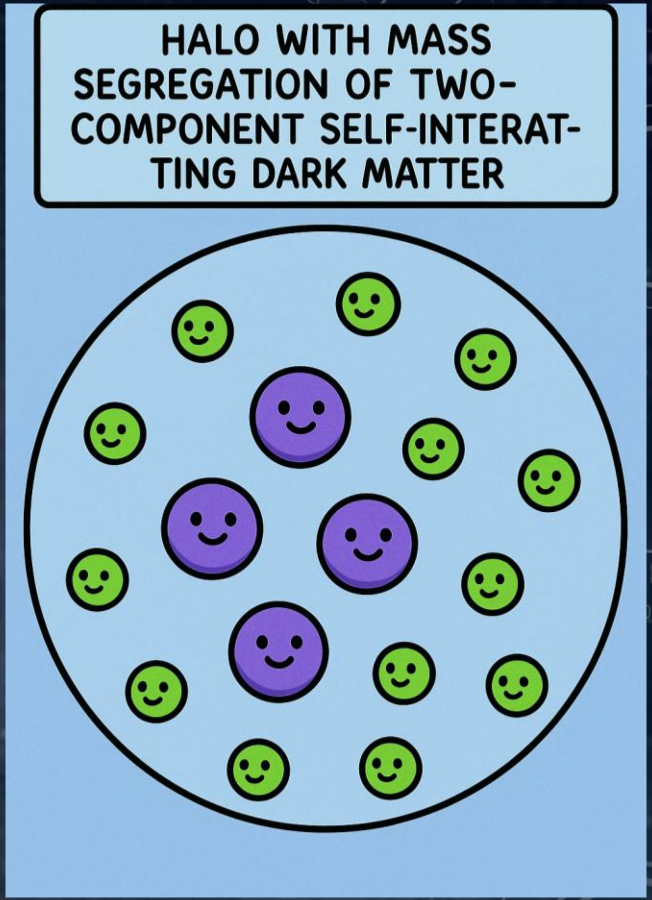
Asymmetry: The amount of **electrons** in the universe is more than that of the anti-electrons

The net excess equals the excess in protons

The 5% baryonic matter is asymmetric

The 25% of dark matter can also be asymmetric

Asymmetric dark matter bypassed the relic abundance problem by introducing a dark "baryon number" into the theory



Dark electrons and dark protons can exist simultaneously in the dark sector: at least two components

Self-interacting dark matter implied by nano-Hertz gravitational waves, Han et al. 2306.16966

arXiv:2504.02303 (PRD) & 2506.14898 with Yue-Lin Sming Tsai, Siyuan Hou, Yi-Zhong Fan (PMO)

功率谱可以用来产生密度分布

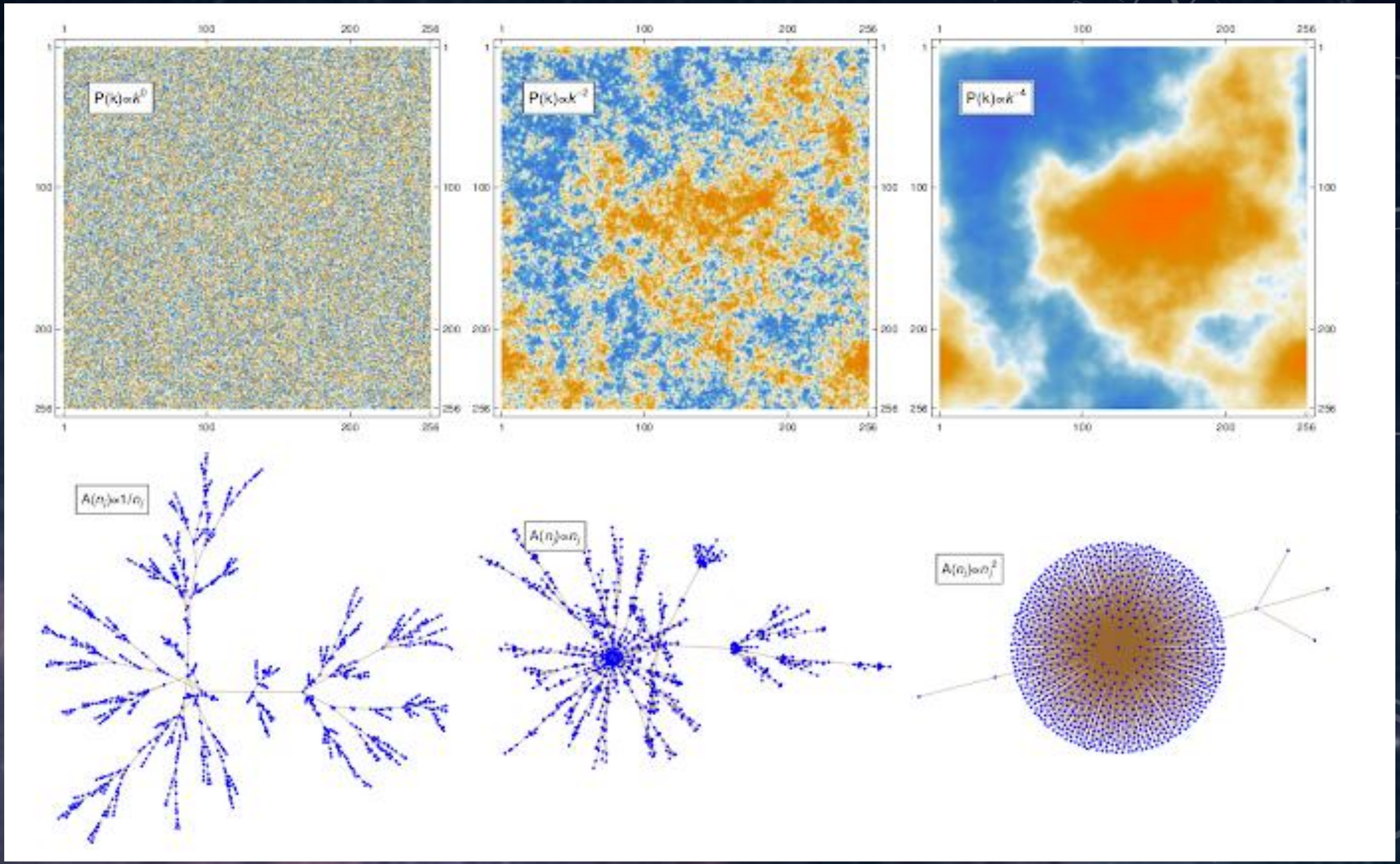
一个二维的例子

尺度不变的物质功率谱意味着结构的多样性

（Yang & Yu 2022 PRR）

k: node degree, i.e., number of links connected to a node

$$\text{Attachment Probability} = \frac{A_i}{\sum_j A_j}$$

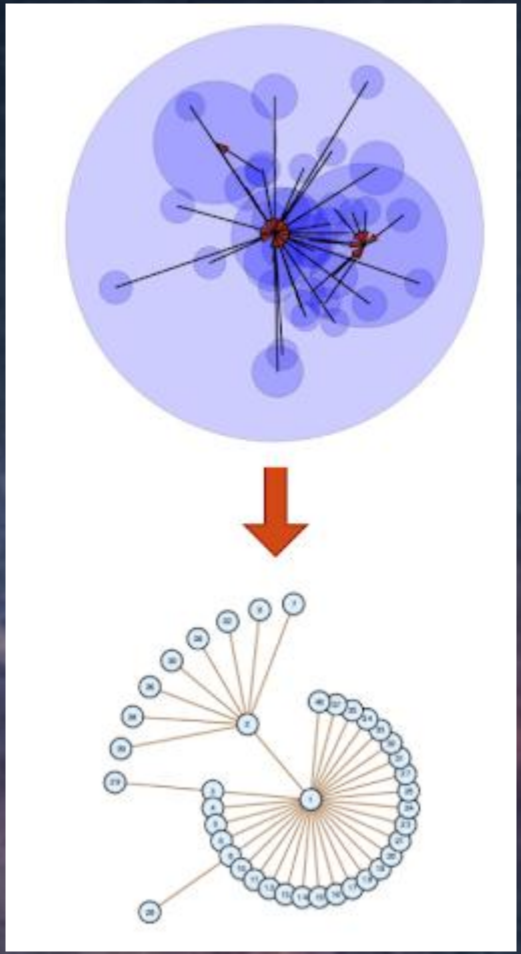


White noise　　Scale free　　Winner takes all

如何从暗晕结构理解星系的多样性？

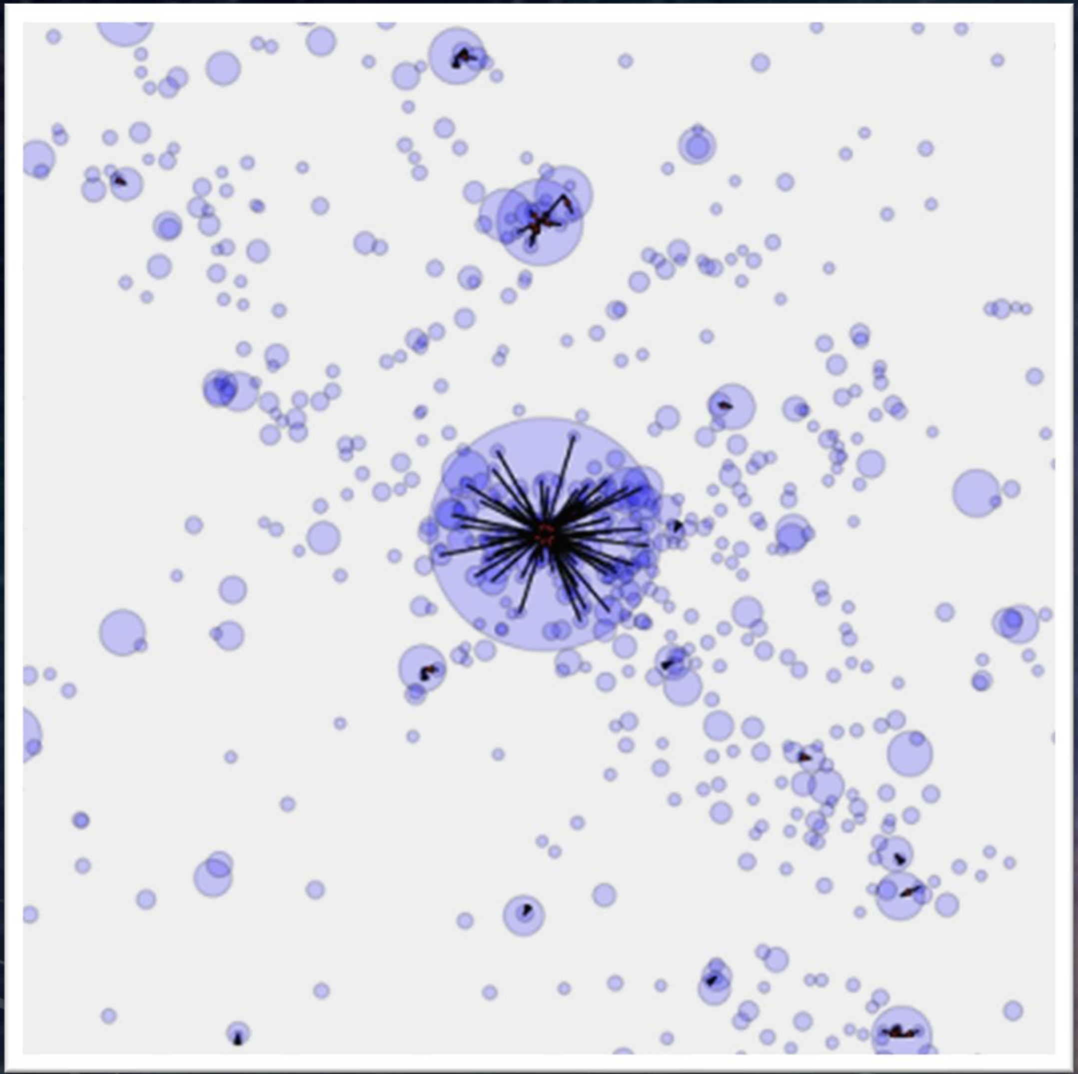
- 物理规律同等地影响每一个星系，然而，星系地形态和结构展现出巨大的多样性
- 暗晕间结构的关联组成一个复杂系统，可以用网络图来描述
- 优先依附为复杂系统中多样性的来源提供了一种理解

Yang & Yu, 2206.05578 [astro-ph.CO]
Phys. Rev. Research 5, 043187 (2023)

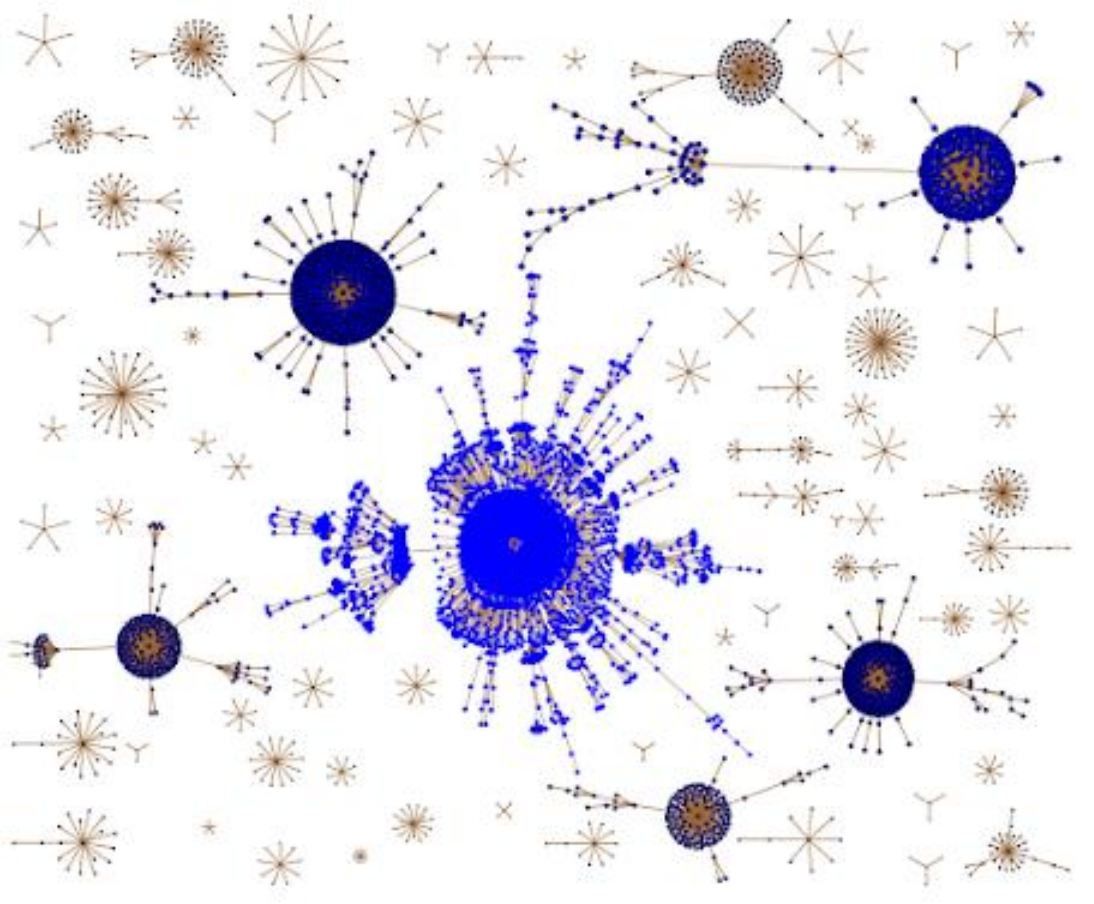


- 自相似性是尺度不变性的一种体现
- 原初功率谱是近似无尺度的
- 层级结构形成伴随着**自相似演化**
- 引力系统的演化方程**是无尺度的**

Cosmological halo formation can be encoded by graphs



Yang & Yu, 2206.05578 [astro-ph.CO]
Phys. Rev. Research 5, 043187 (2023)



MSRJD in GPLM: Discrete Realization of Stochastic Evolution

$$\Delta x_{i,k} \mid s_{i,k}, G \sim \mathcal{N}\left(\delta_{i,k}, \widehat{\Sigma}_{i,k}\right)$$

$$\delta_{i,k} \equiv b_{\theta,i,k}\Delta t_k \qquad \widehat{\Sigma}_{i,k} \equiv D_{\theta,i,k}\Delta t_k$$

$$\left\{\begin{array}{l} \Delta x_{i,k} = b_{\theta,i,k}(s_{i,k}, G_k)\,\Delta t_k + \xi_{i,k} \\ \langle \xi_{i,k}\,\xi_{j,k'}^T \rangle = D_{\theta,i,k}\,\Delta t_k\,\delta_{ij}\,\delta_{kk'} \end{array}\right.$$

$$S_{\mathrm{MSRJD}}[x,\hat{x};G] = \int dt\left[\hat{x}^T\left(\dot{x}_{\mathrm{res}} - b_\theta(x^{\mathrm{tr}}, G)\right) - \frac{1}{2}\hat{x}^T D_\theta(x^{\mathrm{tr}}, G)\hat{x}\right]$$ enforces the stochastic dynamics

Gaussian Onsager-Machlup (OM) action $$S_{\mathrm{OM}}[\mathbf{x};G] = \frac{1}{2}\sum_{(i,k)\in\mathcal{V}_{\mathrm{sup}}}\left[(\Delta x_{i,k} - \delta_{i,k})^{\mathsf{T}}\widehat{\Sigma}_{i,k}^{-1}(\Delta x_{i,k} - \delta_{i,k}) + \log\det\widehat{\Sigma}_{i,k}\right]$$

non-Gaussianity could be handled by higher-order terms

Path Space Tools

Operator average	Controlled likelihood deformation	Likelihood Ratio; forward-backward asymmetry

MSRJD FROM NOISE MEASURE (1)

$$\dot{x}(t) = b(x(t)) + \eta(t)$$

$$Z = \int \mathcal{D}\eta \; P[\eta]$$

$$P[\eta] \propto \exp\left[-\frac{1}{2}\int dt\, \eta^T D^{-1} \eta\right]$$

$$1 = \int \mathcal{D}x \; \delta[\dot{x} - b(x) - \eta]$$

$$\delta[\dot{x} - b - \eta] = \int \mathcal{D}\hat{x} \; \exp\left[\int dt\, \hat{x}^T (\dot{x} - b - \eta)\right]$$

$$Z = \int \mathcal{D}\eta\, \mathcal{D}x\, \mathcal{D}\hat{x}\; e^{\int dt\, \hat{x}^T (\dot{x} - b - \eta)} \; e^{-\frac{1}{2}\int dt\, \eta^T D^{-1} \eta}$$

MSRJD FROM NOISE MEASURE (2)

$$\int \mathcal{D}\eta\, e^{-\frac{1}{2}\eta^T D^{-1}\eta - \hat{x}^T\eta} \;\propto\; \exp\left[-\frac{1}{2}\int dt\, \hat{x}^T D\hat{x}\right]$$

$$Z = \int \mathcal{D}x\,\mathcal{D}\hat{x}\; e^{-S_{\mathrm{MSRJD}}[x,\hat{x}]}$$

$$S_{\mathrm{MSRJD}} = \int dt \left[\hat{x}^T(\dot{x} - b(x)) - \frac{1}{2}\hat{x}^T D\hat{x}\right]$$

$$A \equiv (\dot{x}_{\mathrm{res}} - b_\theta)$$

$$-\frac{1}{2}\hat{x}^T D\hat{x} + \hat{x}^T A = -\frac{1}{2}(\hat{x} - D^{-1}A)^T D(\hat{x} - D^{-1}A) + \frac{1}{2}A^T D^{-1}A$$

$$\int \mathcal{D}\hat{x}\;\exp\left[-\frac{1}{2}(\hat{x}-\mu)^T D(\hat{x}-\mu)\right] \propto (\det D)^{-1/2}$$

$$Z \propto \int \mathcal{D}x\;\exp\left[-\frac{1}{2}\int dt\, A^T D^{-1} A\right] \times \exp\left[-\frac{1}{2}\int dt\, \log\det D_\theta\right]$$

Dark Matter Deficient Galaxy (DMDG) Probabilities

DMDG operator:

$$O^{(i)}_{\mathrm{DMDG}}(\mathbf{x}_i) = \Theta\left(\frac{M^{(i)}_\star(z=0) + M^{(i)}_{\mathrm{gas}}(z=0)}{M^{(i)}_{\mathrm{halo}}(z=0)} - 1\right)$$

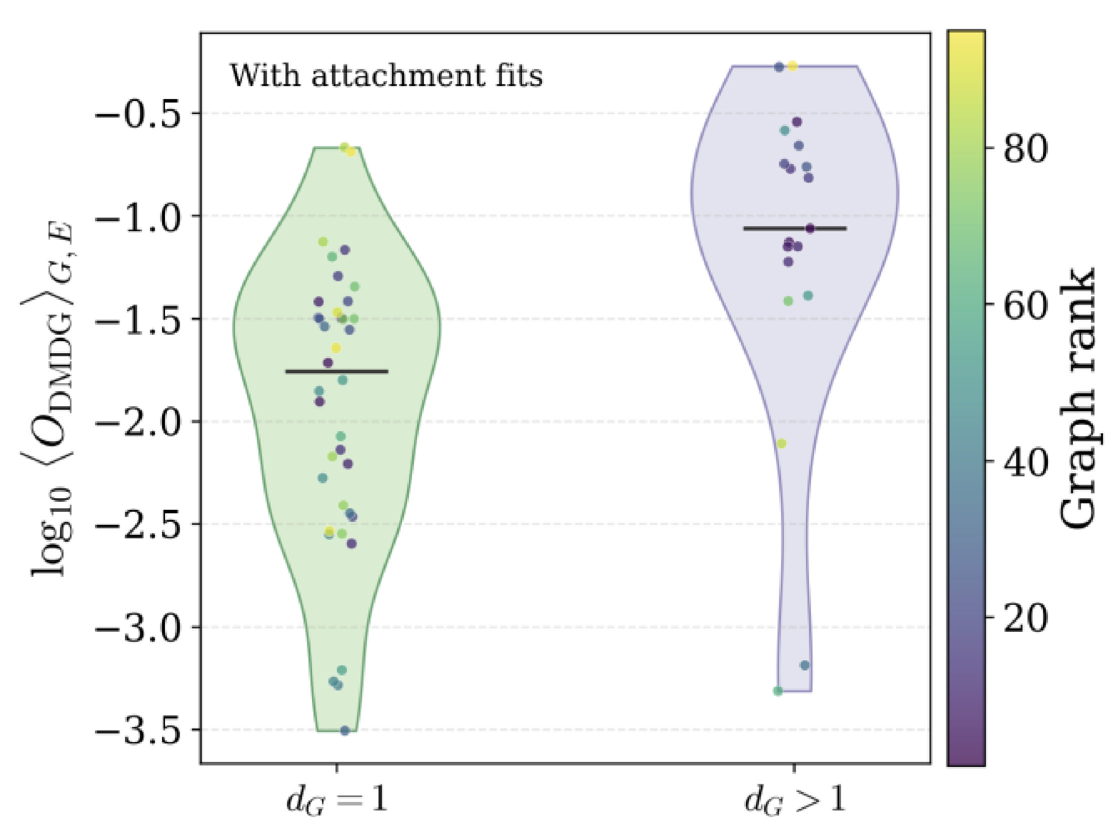
dG=1: satellites
dG>1: higher order satellites

Higher tidal stripping in dG>1 satellites leads to more DMDGs but with greater scatter

➢ **Most probable path for DMDG formation?**

Operator average

$$\langle O_{\mathrm{DMDG}}\rangle_{G,E} \equiv \int \mathcal{D}x \left[\frac{\sum_{i\in E} O^{(i)}_{\mathrm{DMDG}}[\mathbf{x}]}{N_E}\right] P_{\mathrm{tot}}(\mathbf{x} \mid G, x_0)$$



Gas-Rich response as a controlled likelihood deformation

Gas-Rich operator: $O_{\mathrm{GR}}(i) \equiv \Theta[M_{\mathrm{gas},i}(z{=}0) - M_{\star,i}(z{=}0)]$

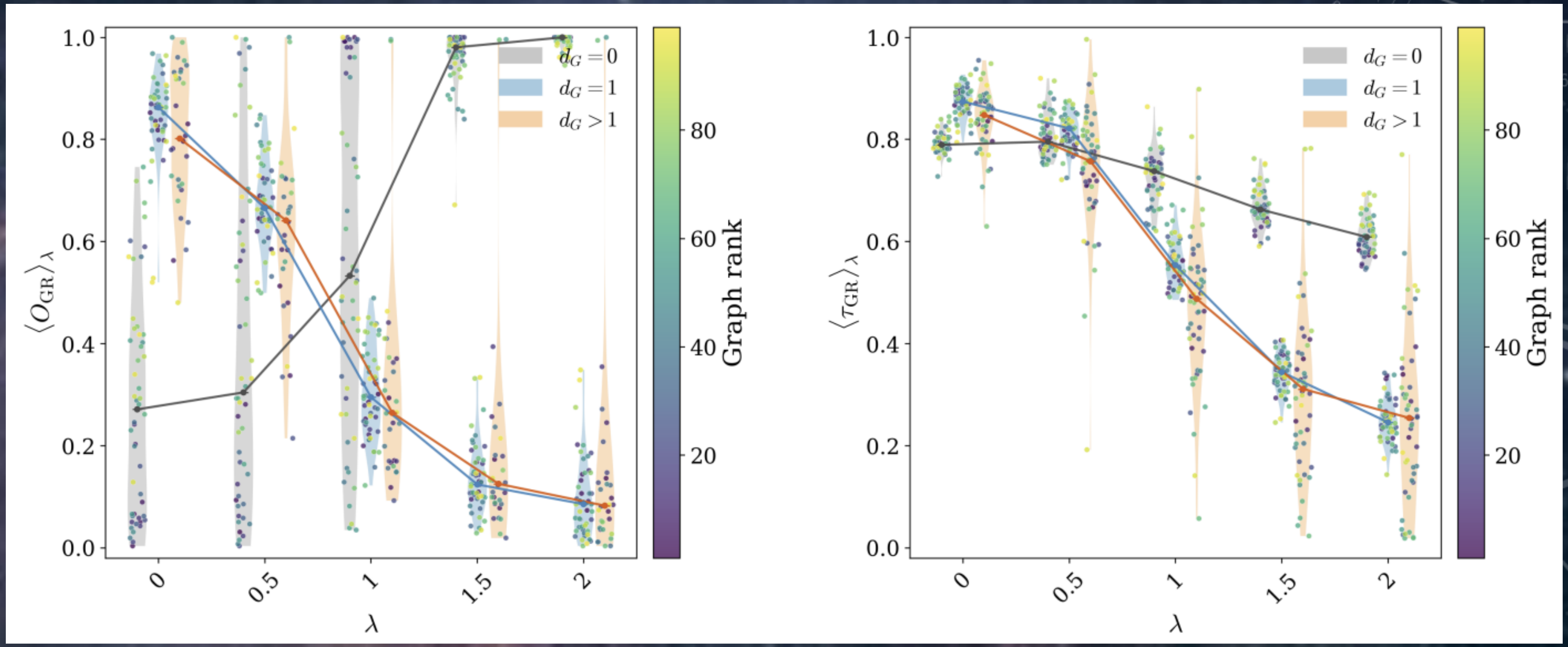
Controlled deformation: $b_{\mathrm{gas},\lambda}(x_k, G) = b_{\mathrm{gas,off}}(x_k, G) + \lambda[b_{\mathrm{gas,full}}(x_k, G) - b_{\mathrm{gas,off}}(x_k, G)]$

Ram-pressure stripping causes gas of satellites to feed into the host

dG=0: host

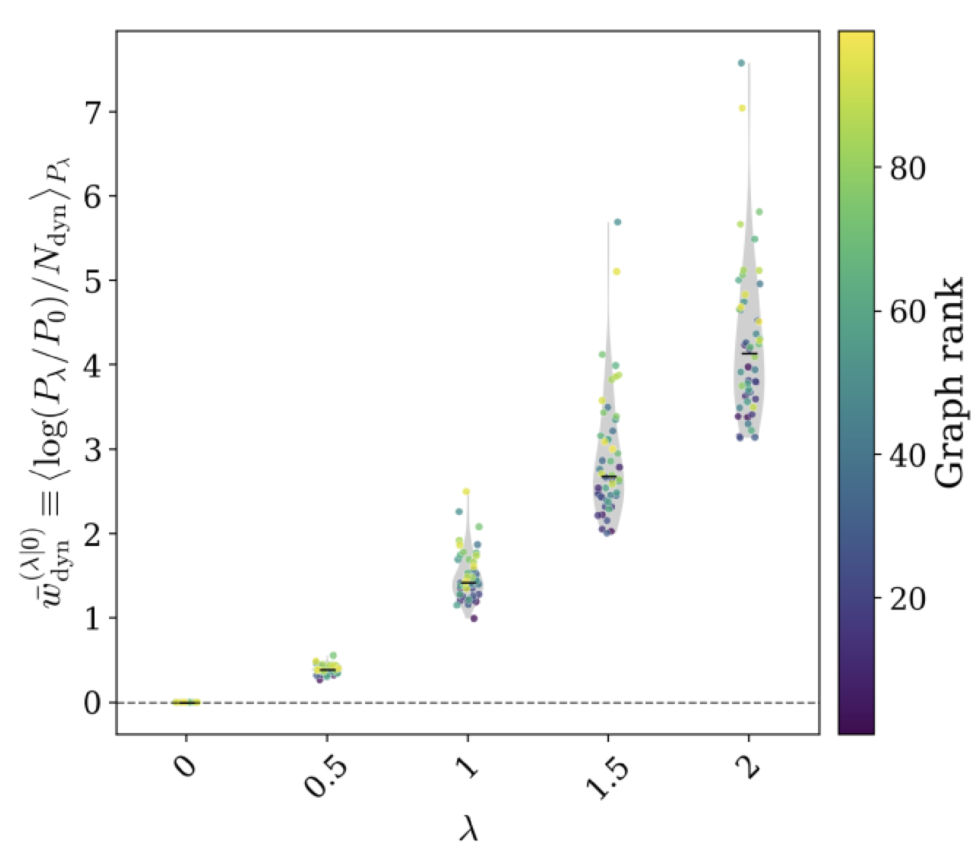
dG=1: satellites

dG>1: higher order satellites



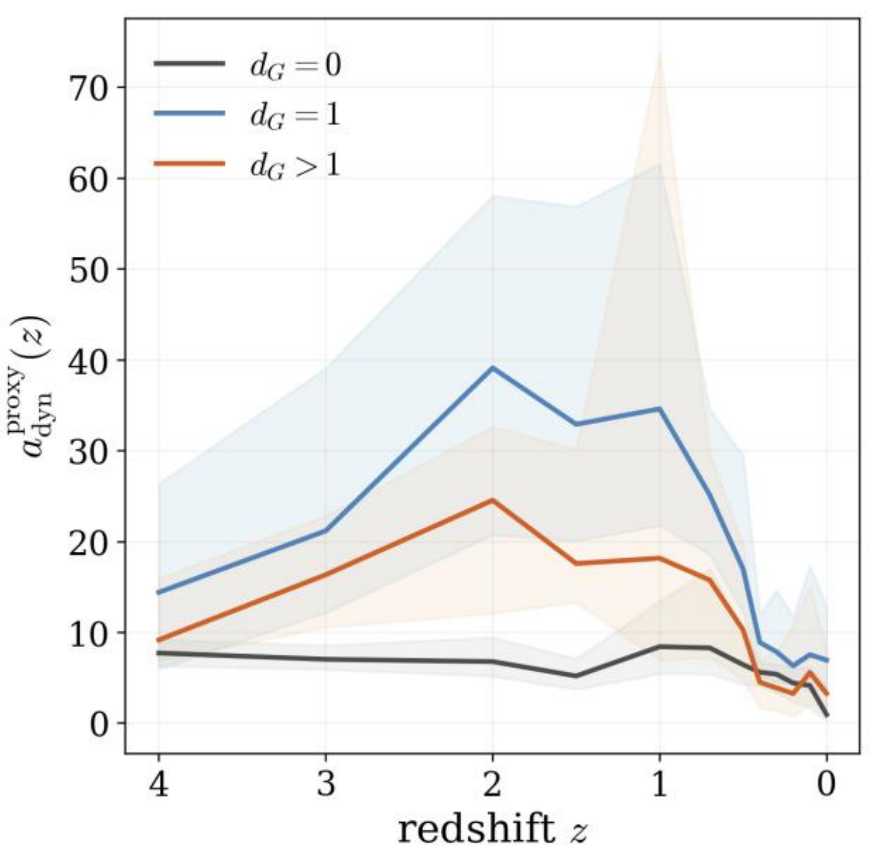
Path-Space likelihood diagnostics

A path-level KL-like distance



Larger λ drives a larger departure from λ=0 and broadens the graph-to-graph scatter

A local forward-reverse asymmetry

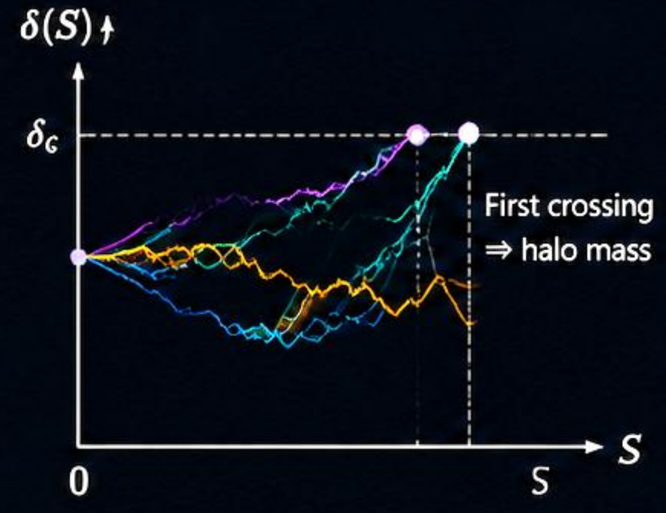


Peaks around cosmic noon, indicating stronger history-dependent evolution there

From Stochastic Halo Growth to Stochastic Layered Graphs

Excursion-set theory: halo growth is a stochastic trajectory of density fluctuations across smoothing scales. Bond et al. 1991

1. Excursion-set: random walks



Stochasticity comes from random initial fluctuations $\Rightarrow$ random walks.

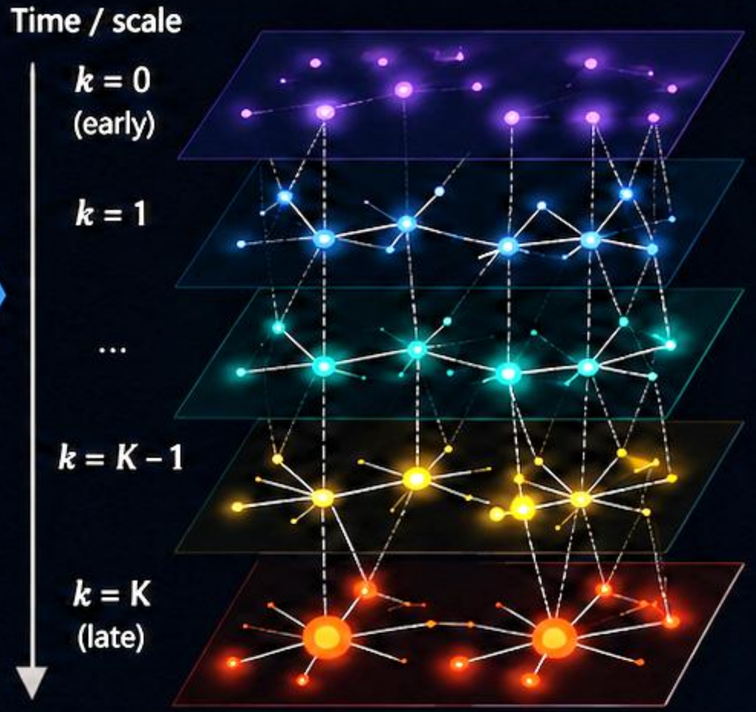
2. Stochastic growth (Langevin equation)

$$\frac{d\delta}{dS} = \mu(S) + \sqrt{D(S)}\,\eta(S)$$

- $\mu(S)$: drift (e.g., gravity)
- $D(S)$: diffusion (power spectrum)
- $\eta(S)$: Gaussian white noise $\langle \eta(S)\eta(S') \rangle = \delta_D(S - S')$

Drift + random forcing $\Rightarrow$ stochastic trajectories

3. From trajectories to layered halo graphs



- Halos (nodes)
- Mergers (within layer)
- Progenitor links (across layers)

Joint probability

$$P(G_{0:K}) = P(G_0) \prod_{k=0}^{K-1} P(G_{k+1} \mid G_k)$$

4. Joint model: halo graph + galaxy fields

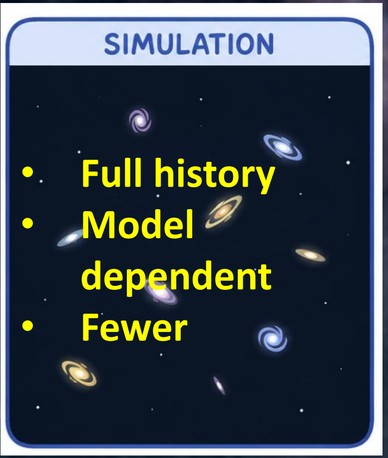
Halo graph action		Galaxy/field action		Total action
$S_G[G_{0:K}]$	+	$S_x[\mathbf{x}_{0:K}; G_{0:K}]$	$\Rightarrow$	$S_{\mathrm{tot}} = S_G + S_x$

5. Outlook

Model halo graphs and galaxy fields within one joint stochastic description of their coupled formation histories.

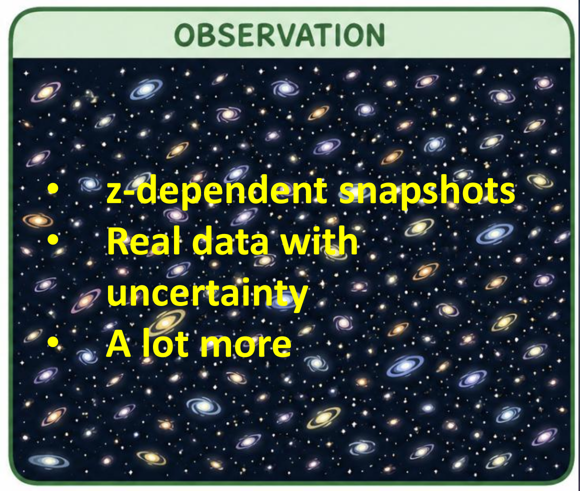
Which graph is most consistent with observations?

$$P(\mathcal{G} \mid d_{\text{end}}) \propto P(\mathcal{G}) \int \mathcal{D}\mathbf{x}\, \boxed{P(d_{\text{end}} \mid \mathbf{x}, \mathcal{G})}\, \boxed{P(\mathbf{x} \mid \mathcal{G})}$$



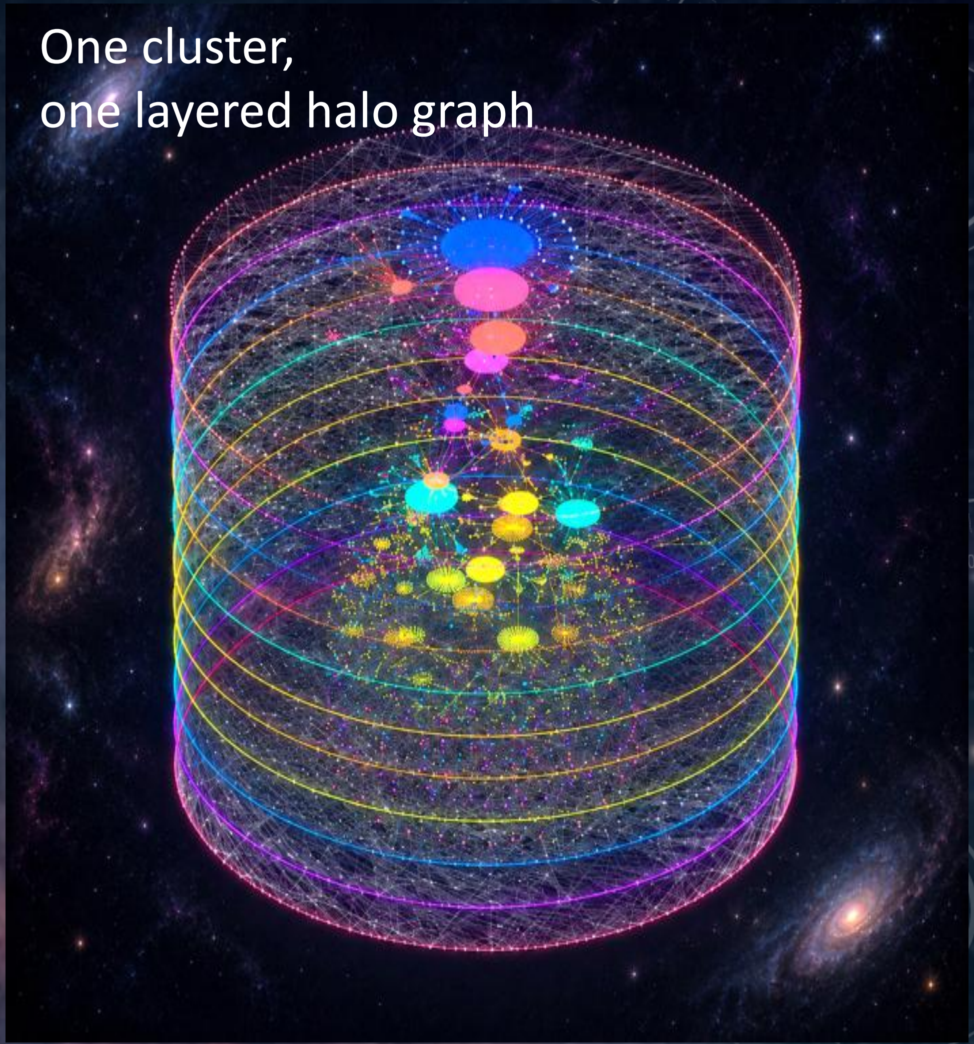
- Full history
- Model dependent
- Fewer

Can We Learn from Observational Data?



- z-dependent snapshots
- Real data with uncertainty
- A lot more

One cluster,
one layered halo graph



The vast amount of data z-dependent could, in principle, be used to fine-tune GPLM

$$P(d_{\text{all}} \mid \vartheta) \sim \prod_{z} \int \mathcal{D}G\, \mathcal{D}\mathbf{x}\, P(d_z \mid \mathbf{x}_z, G)\, \boxed{P_{\vartheta}(\mathbf{x}, G)}$$