



第十五届“新物理研讨会”

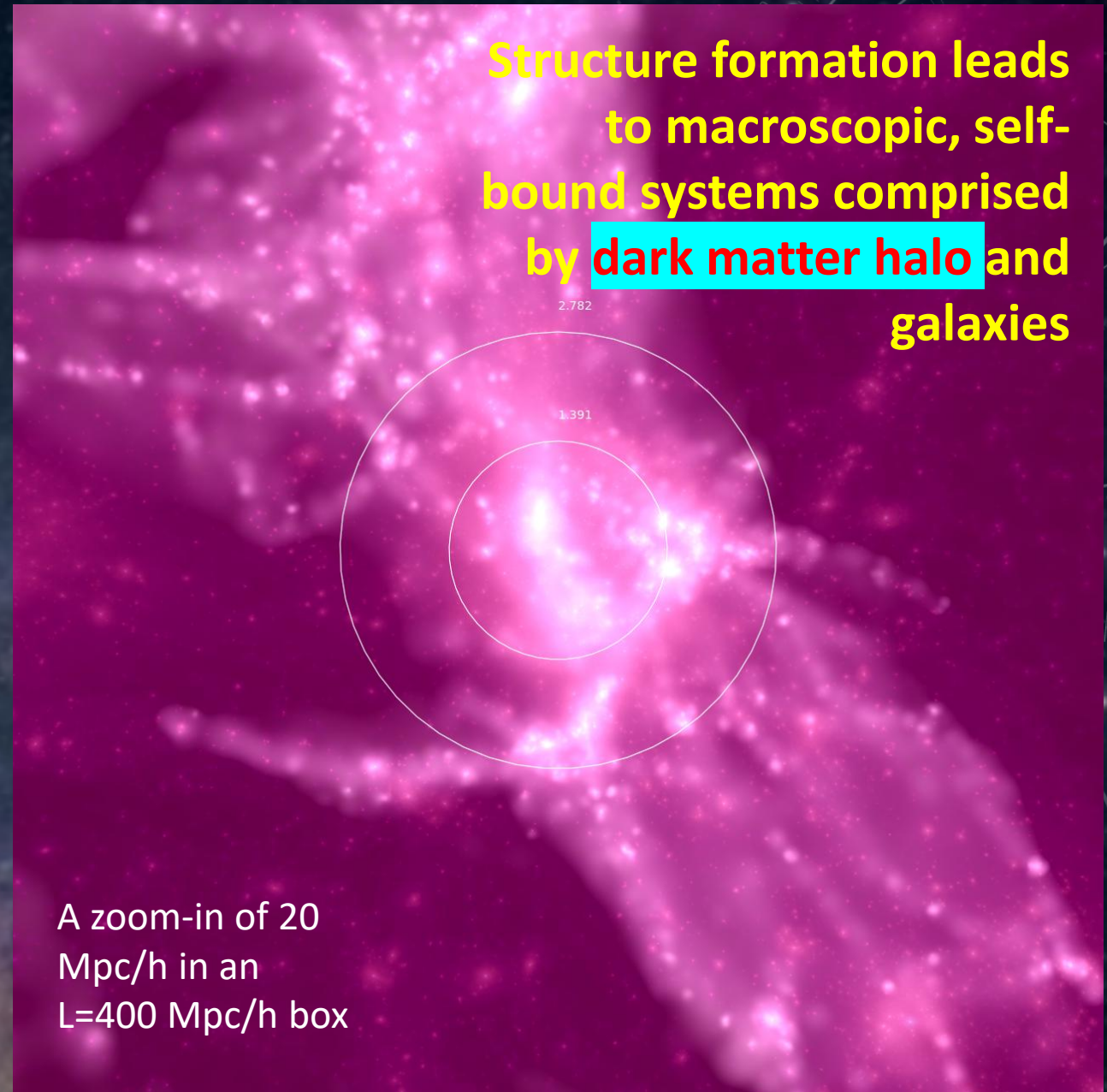
Self-Interacting Dark Matter (SIDM): From Particle Physics to Small-Scale Structures

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August 24, 2026
山东 威海

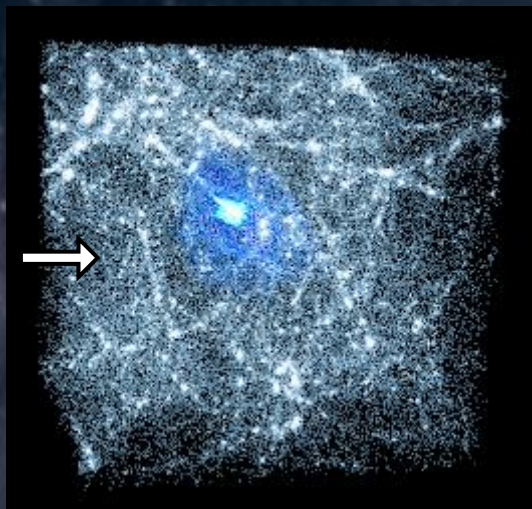
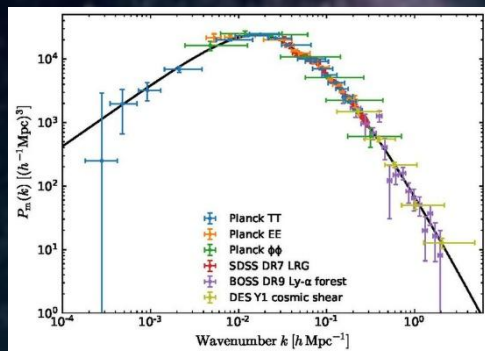
Outline

- ◆ From particle interactions to effective halo physics
- ◆ From N-body dynamics to thermodynamics and kinetics
- ◆ From SIDM evolution to observable predictions
- ◆ Beyond Deterministic Halo Modeling



Small-Scale Structure as a Probe of DM Physics

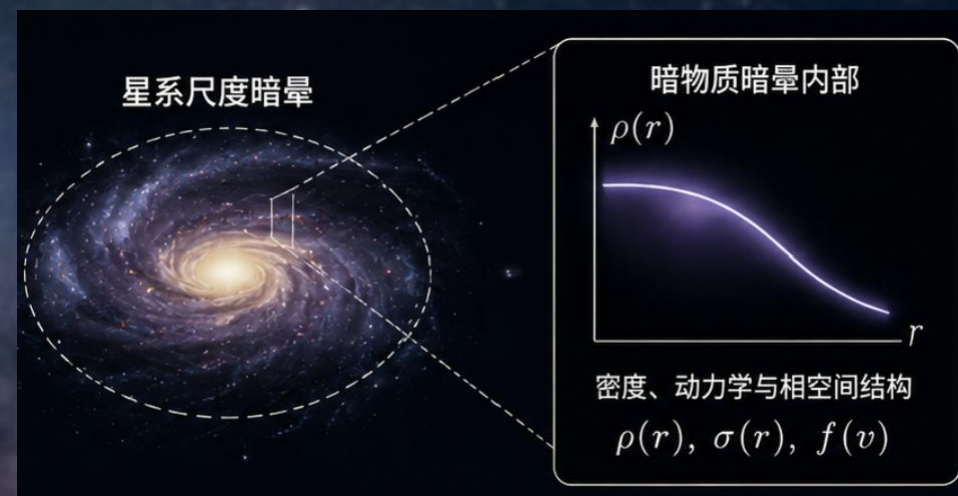
Early Universe Effects
mainly affect the matter power spectrum



**Non-linear
Evolution**

Gravitational collapse
Structure formation
Baryonic processes
Dark matter physics

Small-Scale Processes
sensitive to dark matter micro-physics

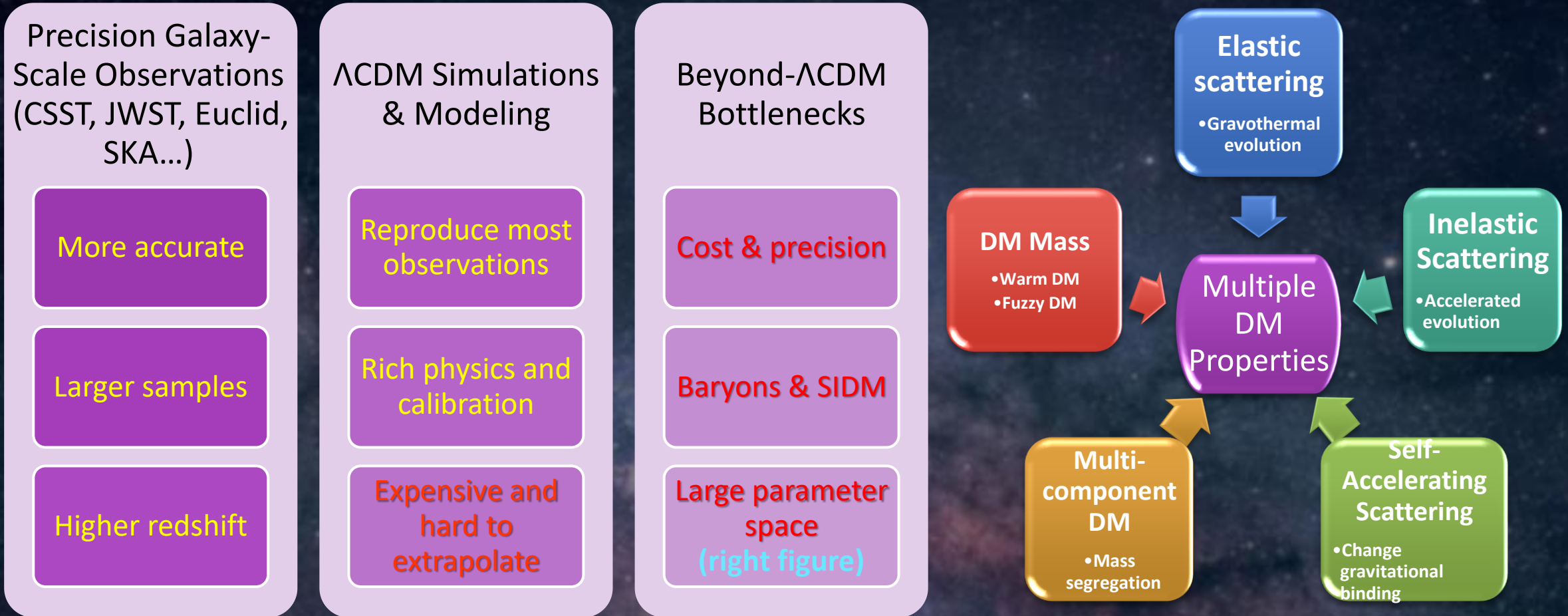


**Complementary observational probes provide a laboratory
for dark matter across a wide range of scales and redshifts**

- Self-Interacting Dark Matter
- Fuzzy Dark Matter
- Warm Dark Matter



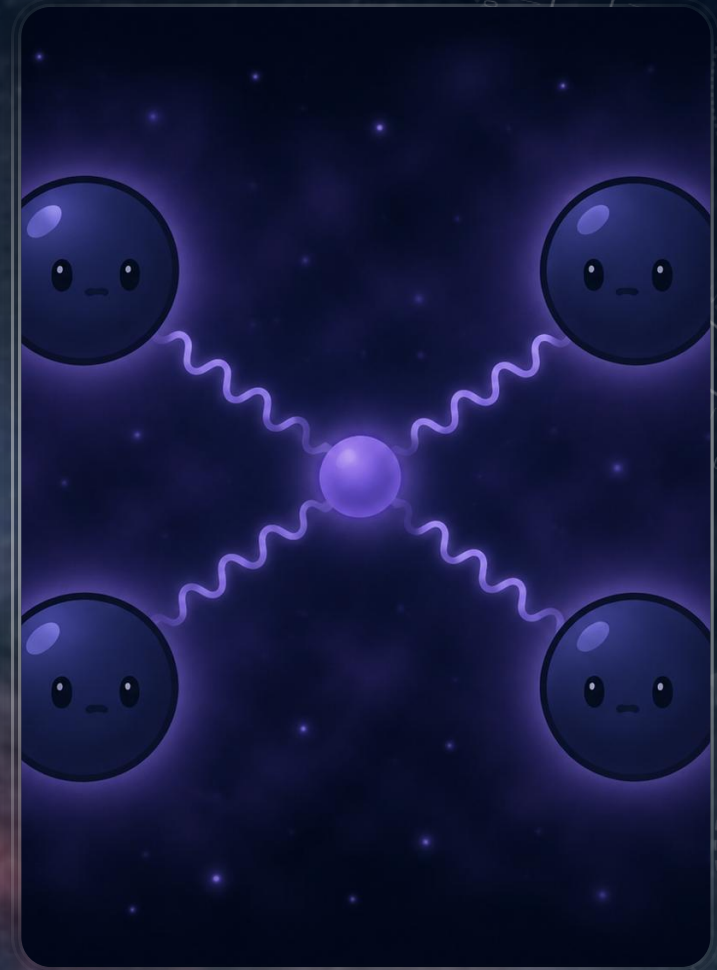
Challenges in Simulation & Modeling



We need an efficient framework connecting
DM Physics->**Galaxy Formation**->**Observables**

From particle interactions to effective halo physics

- Effective constant SIDM cross section
- Multiple species
- Energy injection and dissipation
- Chaotic relaxation after sudden changes



From Microscopic Collisions to Macroscopic Dynamics in SIDM

1. Microscopic world (SIDM)

N particles + collisions



Coarse-graining
Neglect
irrelevant
correlations

2. Single-particle distribution

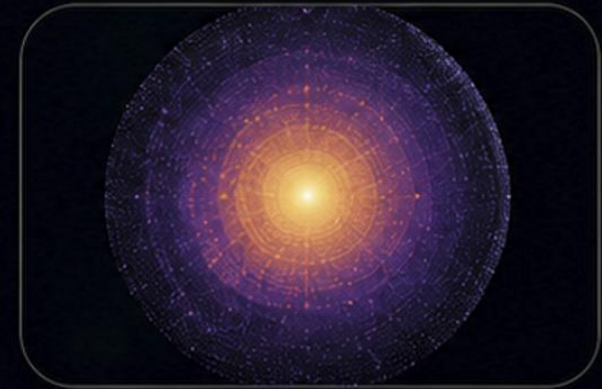
$f(\mathbf{x}, \mathbf{v}, t)$



Take
moments

3. Macroscopic SIDM halo

$\rho(r), T(r), u(r), S(r), \dots$



Correlations generated by collisions become irrelevant on macroscopic scales. The system is fully described by the **single-particle distribution function** (Yu Deng's *Fields medal 2026*)

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla f + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} = \left(\frac{\partial f}{\partial t} \right)_{\text{coll}}$$

In **SIDM**, we ask:
which **reduced variables** or
transport coefficients
retain memory of how
microscopic collisions
reshape a halo?

SIDM extends it to self-gravitating, evolving many body systems

- Long-range gravity
- Non-equilibrium evolution
- Quantum/Microscopic reversible interactions

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \nabla \Phi \frac{\partial f}{\partial \mathbf{v}} = C[f]$$

From SIDM to effective fluid dynamics

SIDM lives here
(short range collisions)

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \nabla \Phi \frac{\partial f}{\partial \mathbf{v}} = C[f]$$

Gravity complicates the story
(Long-range “collisions”)

SIDM thermalizes the inner halo region

$$\rho(r_1) \frac{\sigma}{m} \langle v_{\text{rel}}(r_1) \rangle t_{\text{age}} \sim 1$$

Scatterings
are
typically
rare

*Kaplinghat, Tulin,
Yu, 2016*

Velocity distribution is driven to Maxwellian

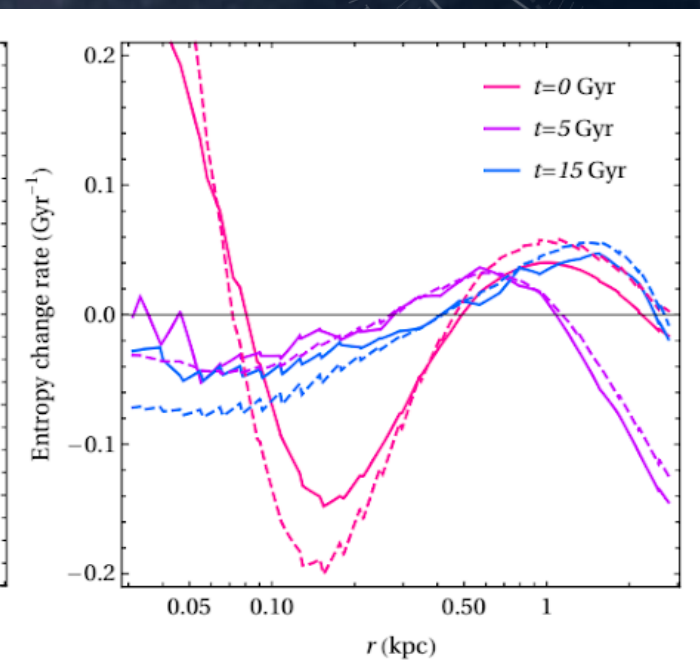
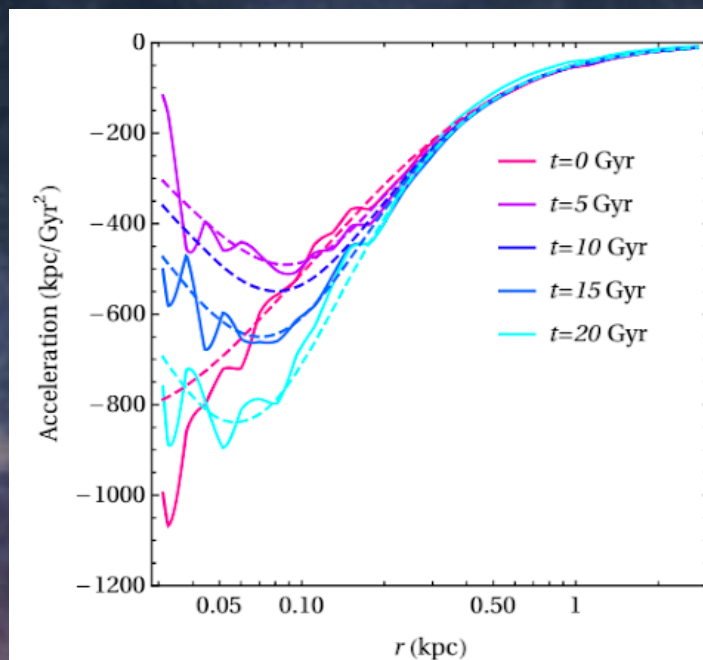
$$f(\mathbf{v}, \mathbf{x}, t) = \rho(\mathbf{x}, t) \left(\frac{m_D}{2\pi T} \right)^{\frac{3}{2}} \exp \left[-\frac{m_D(\mathbf{v} - \mathbf{u})^2}{2T} \right]$$

Pressure equilibrium

$$\nabla(\rho \sigma_{\text{1D}}^2) = -\rho \nabla \Phi$$

Energy transport

$$\frac{1}{4\pi} \frac{\partial L}{\partial r} = -\rho \sigma_{\text{1D}}^2 \frac{Ds}{Dt}$$



Test effective fluid dynamics using
an accurate N-body simulation

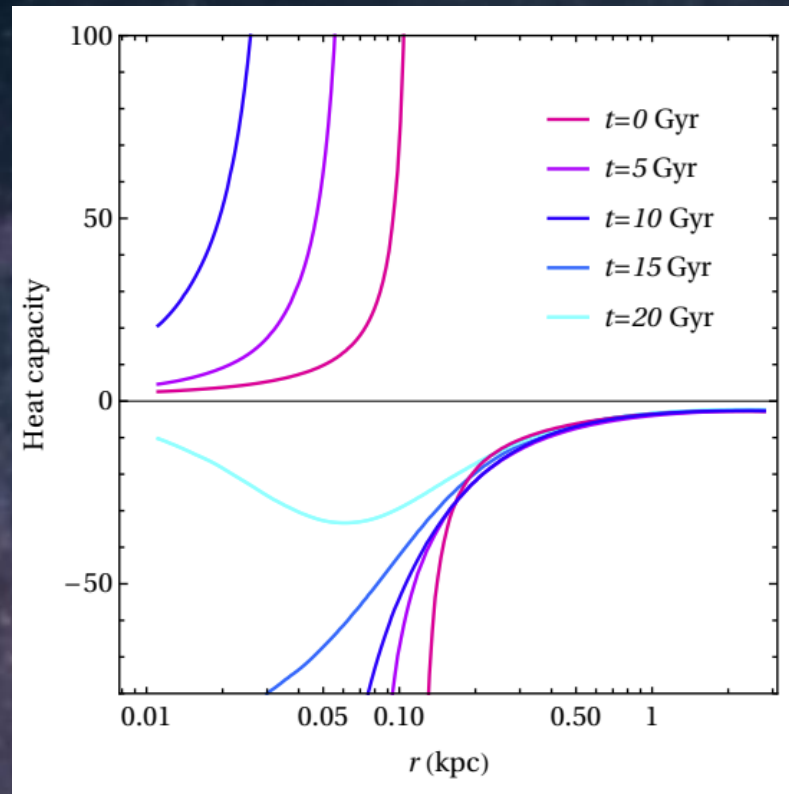
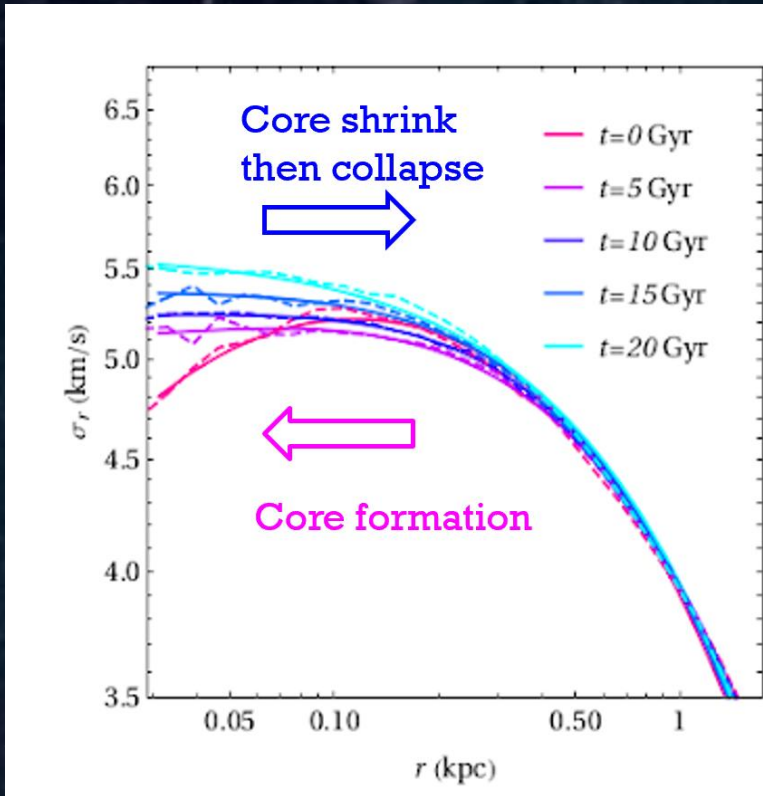
$$\frac{\partial s}{\partial t} \text{ (solid)} \quad \frac{1}{\sigma^2} \frac{\partial \mathcal{E}}{\partial t} \text{ (dashed)}$$

Yang & Yu 2205.03392 JCAP

Gravothermal evolution of SIDM halos

A thermodynamics picture

Spergel & Steinhardt 2000,
Balberg et al. 2002 ApJ,
Koda et al. 2011 MN, etc



Core formation

- heat flux + **capacity** => core formation

Core shrink (heat flux small)

+ heat flux + capacity => quasi-stable core

Core collapse

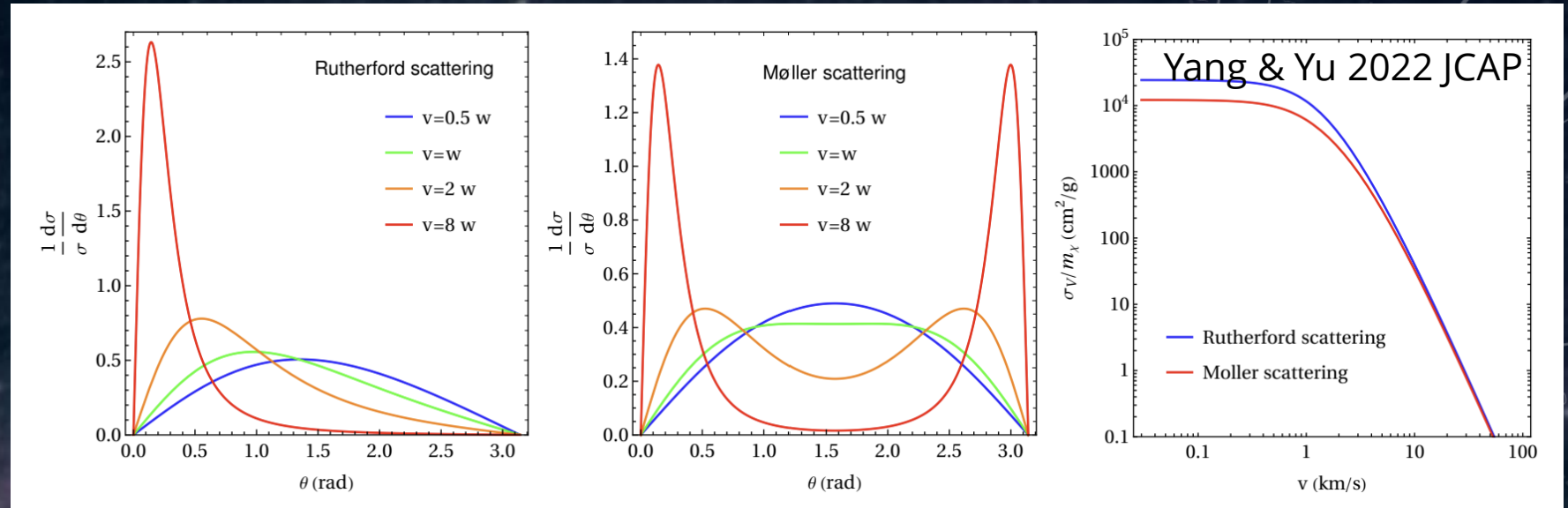
+ heat flux - **capacity** => core collapse

$$C(r) = dE/d\sigma_{1D}^2$$

Yang & Yu 2205.03392 JCAP

From SIDM to effective fluid dynamics

- Particle scatterings can be velocity and angular dependent
- How to calculate its effect in a halo?



$\langle \sigma v \rangle$ computes the scatterings rate, which is the correct quantity for indirect detection

In a halo, heat conductivity κ is the most relevant quantity

$$\frac{\partial}{\partial r} \left(r^2 \kappa m \frac{\partial v^2}{\partial r} \right) = r^2 \rho v^2 \frac{D}{Dt} \ln \frac{v^3}{\rho}$$

Heat conductivity

- **Issue:** SIDM collision is mostly too rare to apply fluid calculations.
- The bulk halo lives in the Long-Mean-Free-Path (LMFP) regime

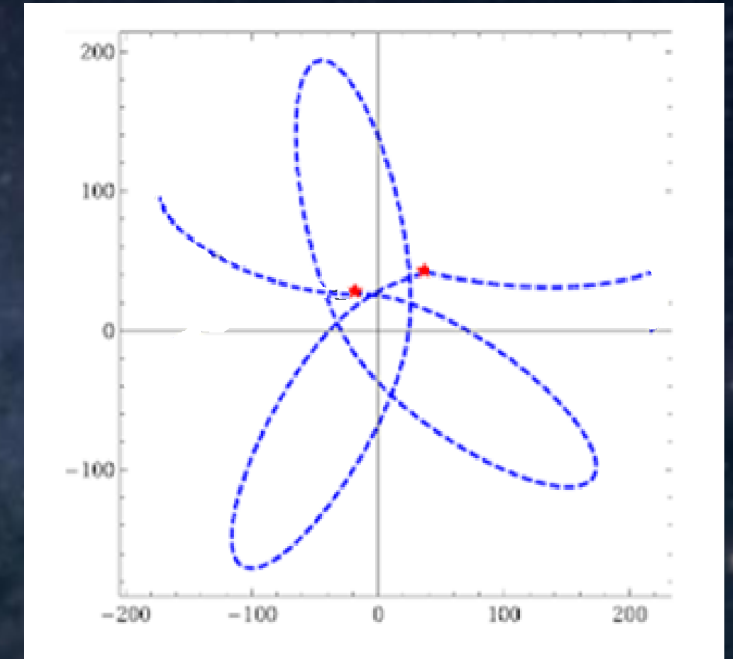
Reducing the differential cross section to an Effective constant cross section

The weighting kernel should align with the kinetic theory calculation of heat conductivity (Lifshitz, *Physical Kinetics*)

$$\sigma_{\text{eff}} = \frac{2 \int v^2 dv d \cos \theta \frac{d\sigma}{d \cos \theta} \sin^2 \theta v^5 \exp \left[-\frac{v^2}{4\nu_{\text{eff}}^2} \right]}{\int v^2 dv d \cos \theta \sin^2 \theta v^5 \exp \left[-\frac{v^2}{4\nu_{\text{eff}}^2} \right]}$$

- Angular dependence is completely integrated out
- *Details of an SIDM model is **hidden** in a single halo*
- Only the **velocity dependence of SIDM** couples to the **halo velocity dispersion**

Two kinds of physics decoupled
Short scale: SIDM
Large scale: Gravity



Yang & Yu 2022 JCAP
See also: S. Yang et al. 2022 ApJ
Tulin, Yu & Zurek 2013

Astrophysical probes of SIDM and its variants

**Multi-
window
observations
break model
degeneracies**

Grav lensing perterber
Galaxy-Galaxy Strong Lensing
Lens flux ratios
DM deficient galaxies
Stellar stream gaps
Dwarf galaxy clustering
Little Red Dots

DM Mass

- Warm DM
- Fuzzy DM

**Elastic
scattering**

- Gravothermal evolution

- Self-Interacting Dark Matter (SIDM)
- MACHO scatterings
- PBH scatterings

**Inelastic
Scattering**

- Accelerated evolution

- Dissipative SIDM
- DM-baryon scatterings
- Indirect signatures

**Multiple
DM
Properties**

- Atomic Dark Matter
- Dark glue-balls
- Boosted Dark Matter
- Multi states/continuum dark matter

**Multi-
component
DM**

- Mass segregation

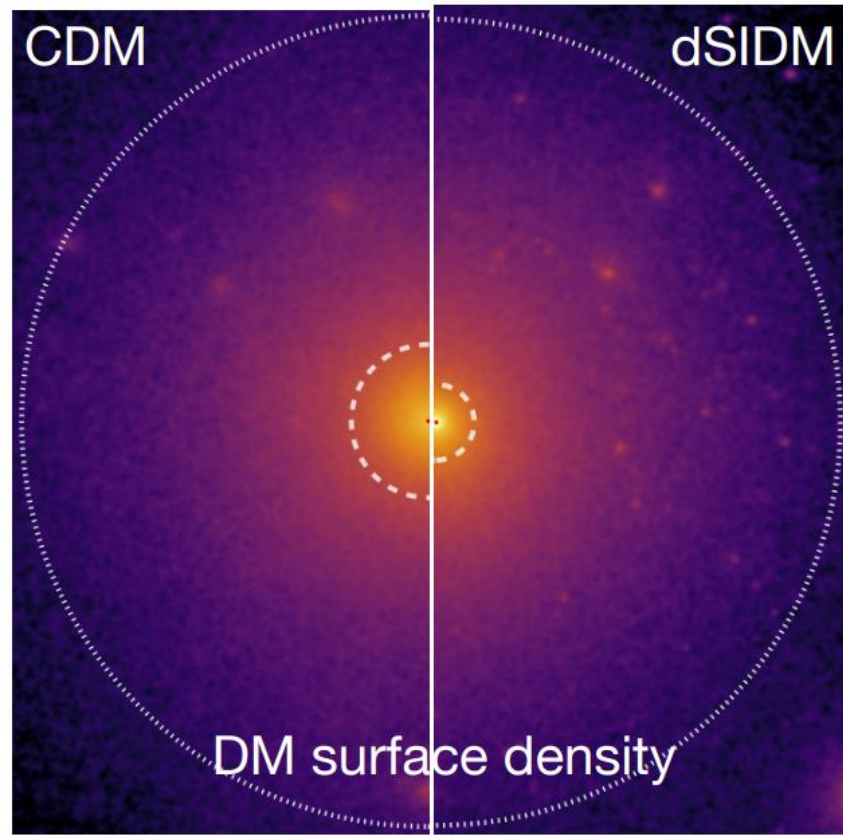
**Self-
Accelerating
Scattering**

- Change gravitational binding

- Exothermic dark matter
- Dynamical dark matter
- Cannibal dark matter
- Bosonovae
- Warm dark matter
- DM induced baryon feedback

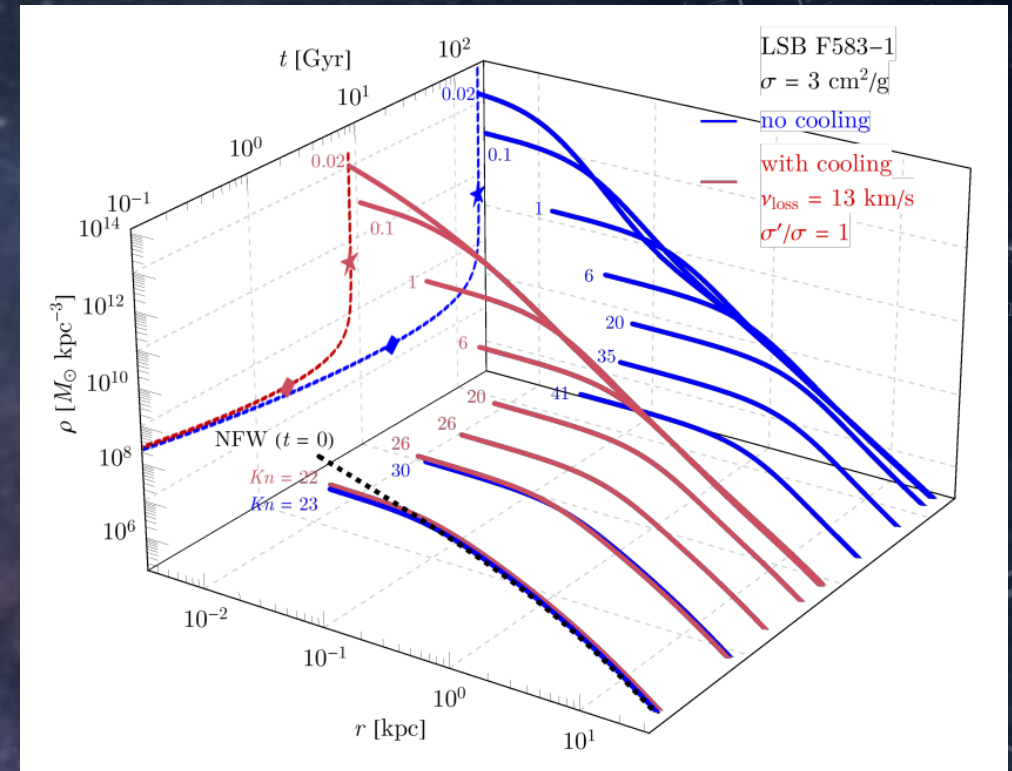
(1) Dissipative SIDM

a classical dwarf of $M_{\text{halo}} \sim 10^{10} M_{\odot}$



More concentrated, spherical halos + promote baryonic disk formation, Xuejian Shen et al. 2021, 2024

- Dissipation tends to make the distribution more compact
- SIDM core formation tends to make the distribution more diffuse



Boosted gravothermal evolution, Essig et al. 2019 & many other studies...

(2) SIDM with multiple species and mass segregation

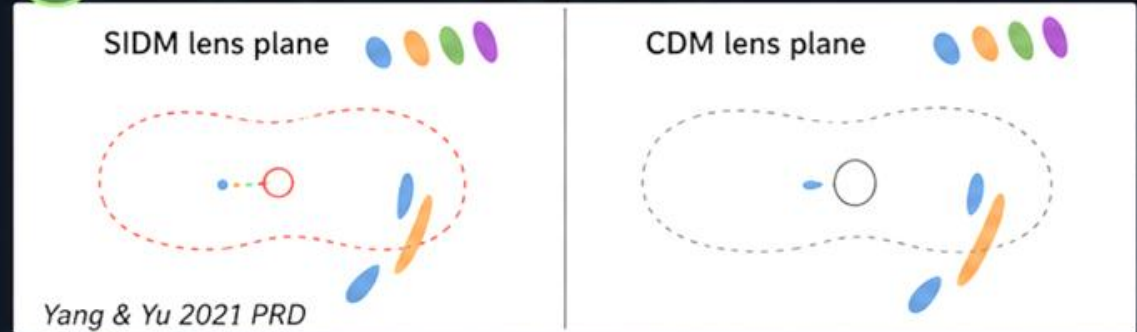
1 Limitation of 1-component SIDM

- Core shrinkage in 1-component SIDM is driven by the **loss of high-velocity** particles, which **reduces the inner halo mass**.



Negative heat capacity:
the system becomes hotter
as it releases heat.

2 Implication for lensing



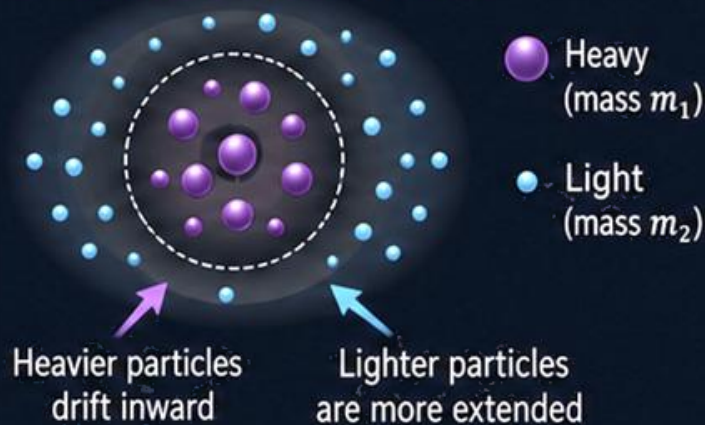
- ✓ Einstein radii remain nearly the same. (Stay tuned)
- ✓ Effects on lensed images are modest, mainly enhancing 2-image events.

3 Mass segregation in 2-component SIDM

Collisions drive energy equilibration:

$$m_1 v_1^2 \sim m_2 v_2^2$$

$$m_1 > m_2 \Rightarrow v_1 < v_2$$



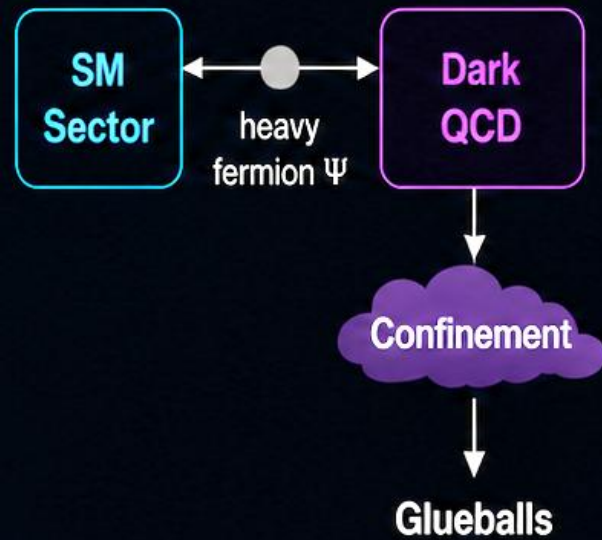
Can we reconcile diffuse and dense observations simultaneously?



**Massive particles
sink inward,
elevating the
central halo mass.**

A Natural Path to **Two-Component SIDM** with **Mass Segregation**

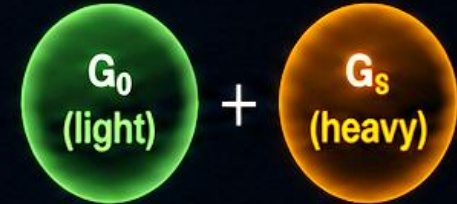
1. Composite ALP (GALP) Framework



2. Two Glueball States

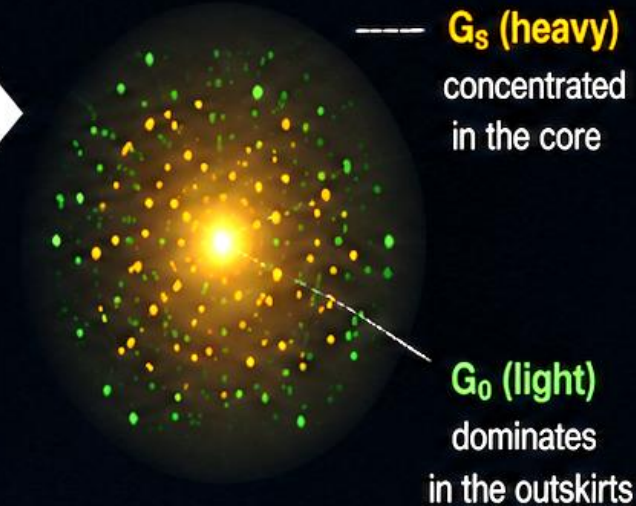


3. Two-Component SIDM



Self-interact via dark force
 $\sigma \sim 0.1 - 10 \text{ cm}^2/\text{g}$

4. Mass Segregation in Halos



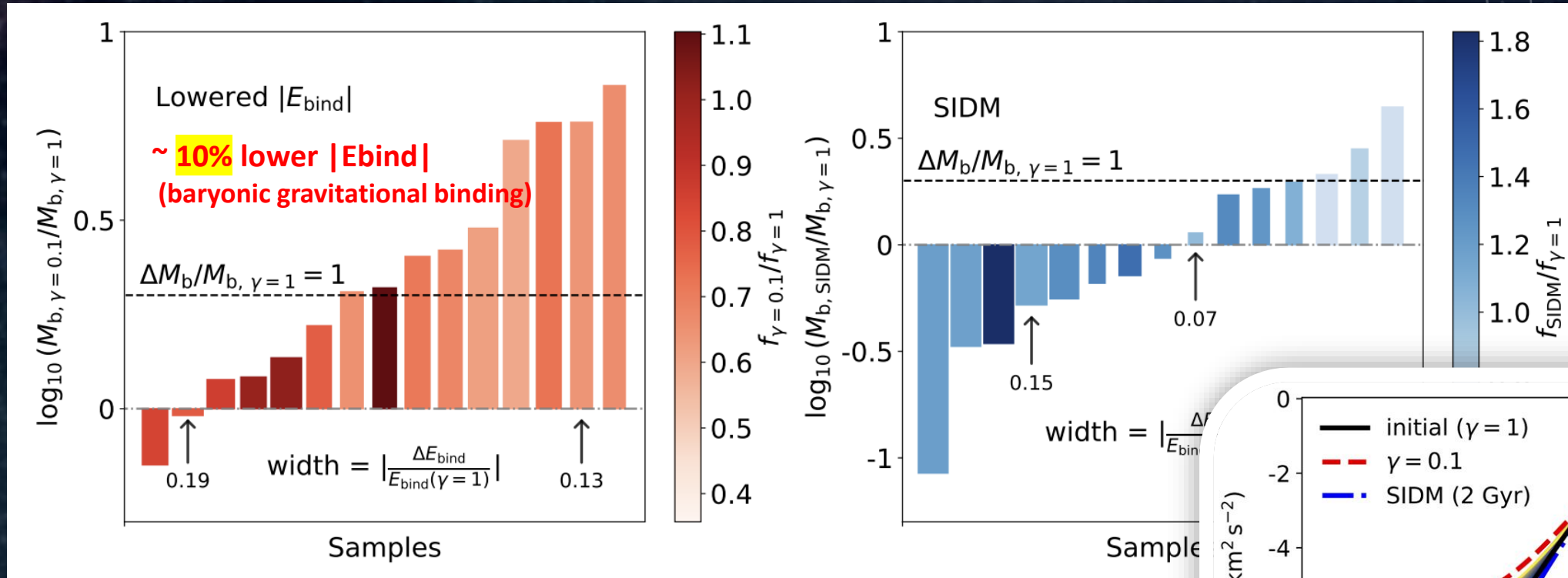
Carenza, Pasechnik, Wang, 2025 PRL



Composite ALP framework naturally produces two stable, self-interacting components with different masses → **mass segregation** in galaxies and clusters.

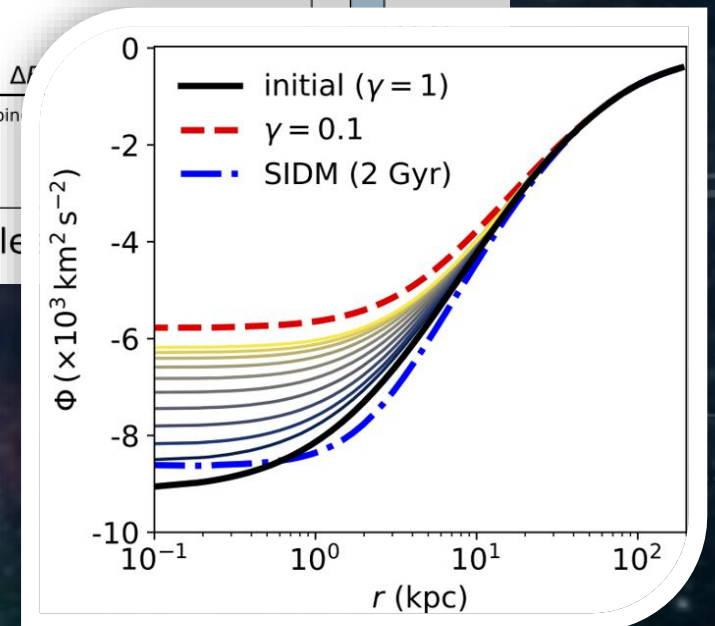
(3) Exothermic dark matter / DM induced baryon feedback

Models: Vogelsberger et al. 2019 MN / Acevedo et al. 2024 PRD



The Collisional formation of DMDG offers a novel probe to test energy injection in dwarf halos

Wang & Yang et al. arXiv:2509.24270, ApJL 997 (2026) 2, L43



(4) Chaotic relaxation after sudden changes

⇒ Out of equilibrium

❖ $\Delta\Phi$: mass changes

- accretion; feedback outflow
- tidal/ram-pressure stripping

❖ ΔK : dynamical heating

- mergers; tidal shocks

⇒ Restore via **violent relaxation**!

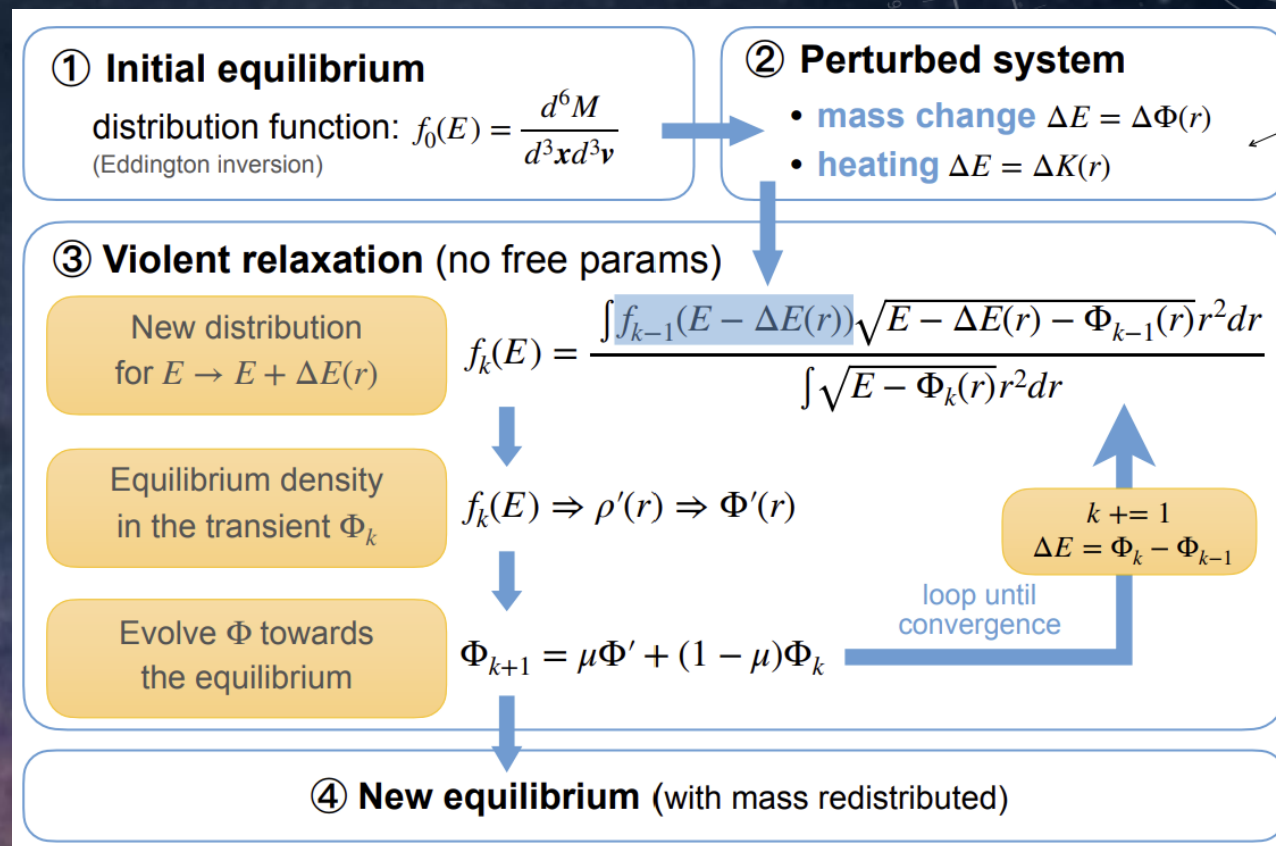
holy grail of since Lynden-Bell 1967

challenge: energy/actions **not** conserved!

CoreCusp2: An accurate model for halo evolution given

- Mass change profiles
- Energy injection profiles

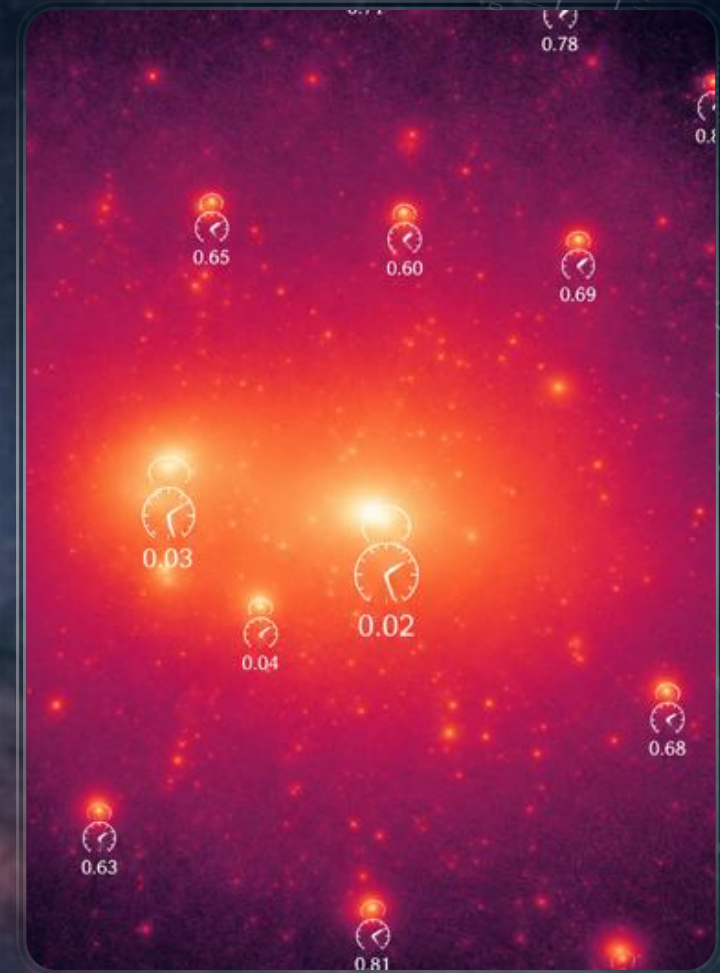
Li, Dekel+ 2023 (arXiv:2206.07069) & Li+ in prep



From Zhaozhou Li (NJU)

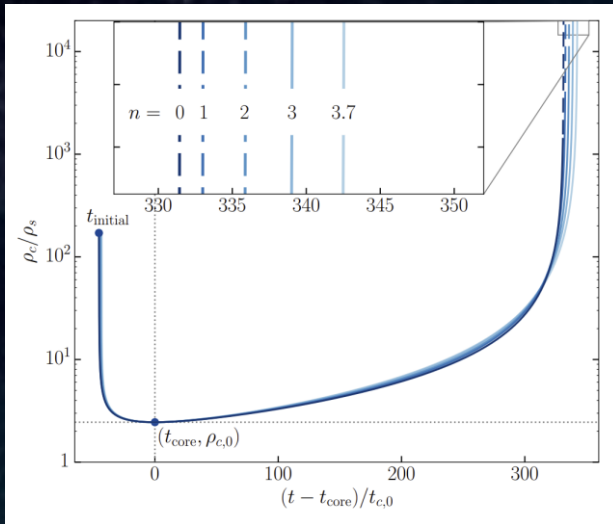
From N-body dynamics to thermodynamics and kinetics

- Cosmological vs controlled simulations
- Example applications
- Effective thermodynamics and the parametric model



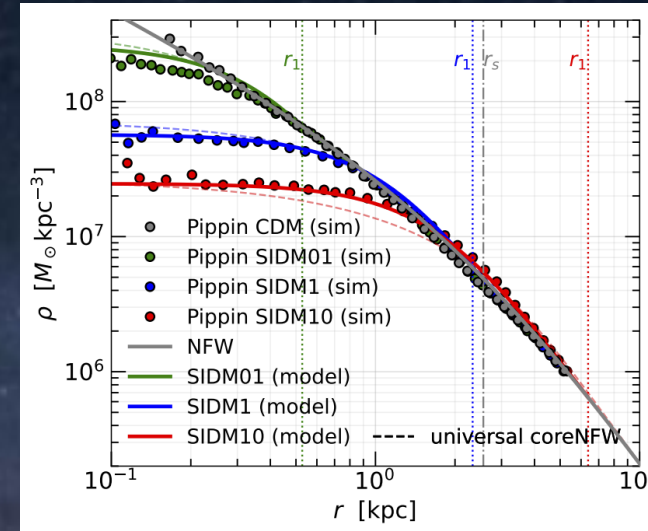
Modeling & Simulating SIDM halos

Evolution in isolation: easy
Evolution in environment: difficult



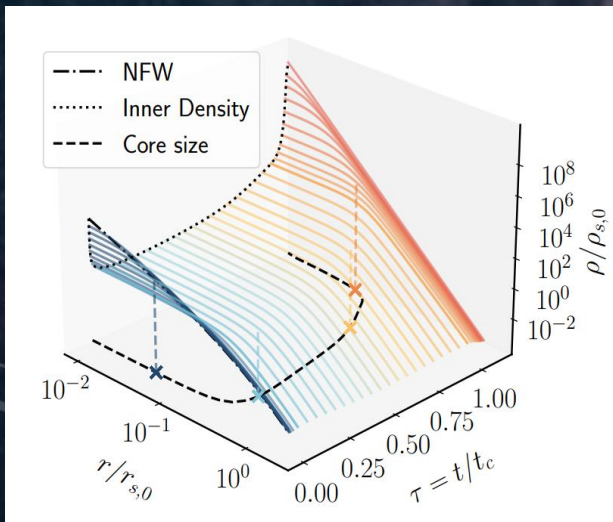
Fluid simulation

- Universality (Outmezguine et al. 2022)
- Seeding SMBH, Feng et al. 2021
- Dissipative SIDM, Essig et al. 2019 PRL
- Balberg et al. 2002



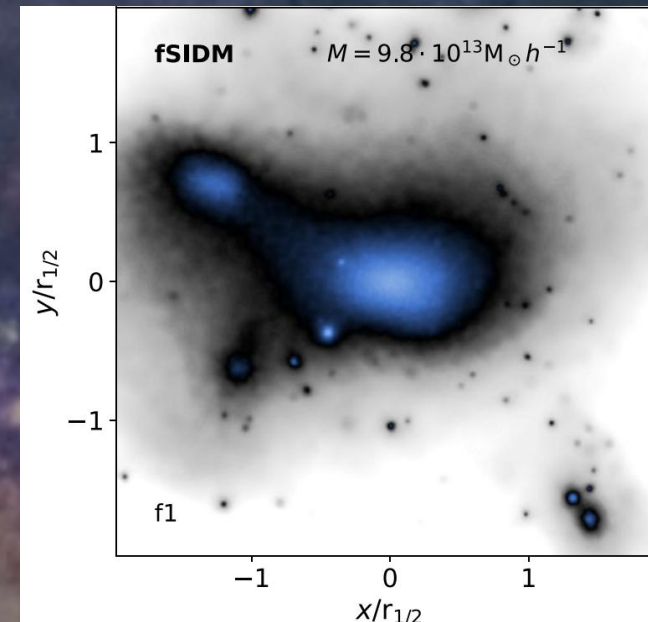
Isothermal-Jeans model

- Jiang et al. 2022
- Kaplinghat et al. 2014



Parametric modeling

- Lensing model. Hou et al. 2025
- Effective constant cross section. Yang et al. 2022
- Integral approach: Yang et al. 2023



N-body simulation Accretion & Environment

- Moritz et al. 2024+
- Palubski et al. 2024
- Robertson et al. 2017
- Rocha et al. 2012
- Kamionkowski et al. 2025
- Gurian & May, 2025

SIDM halos in cosmological simulations

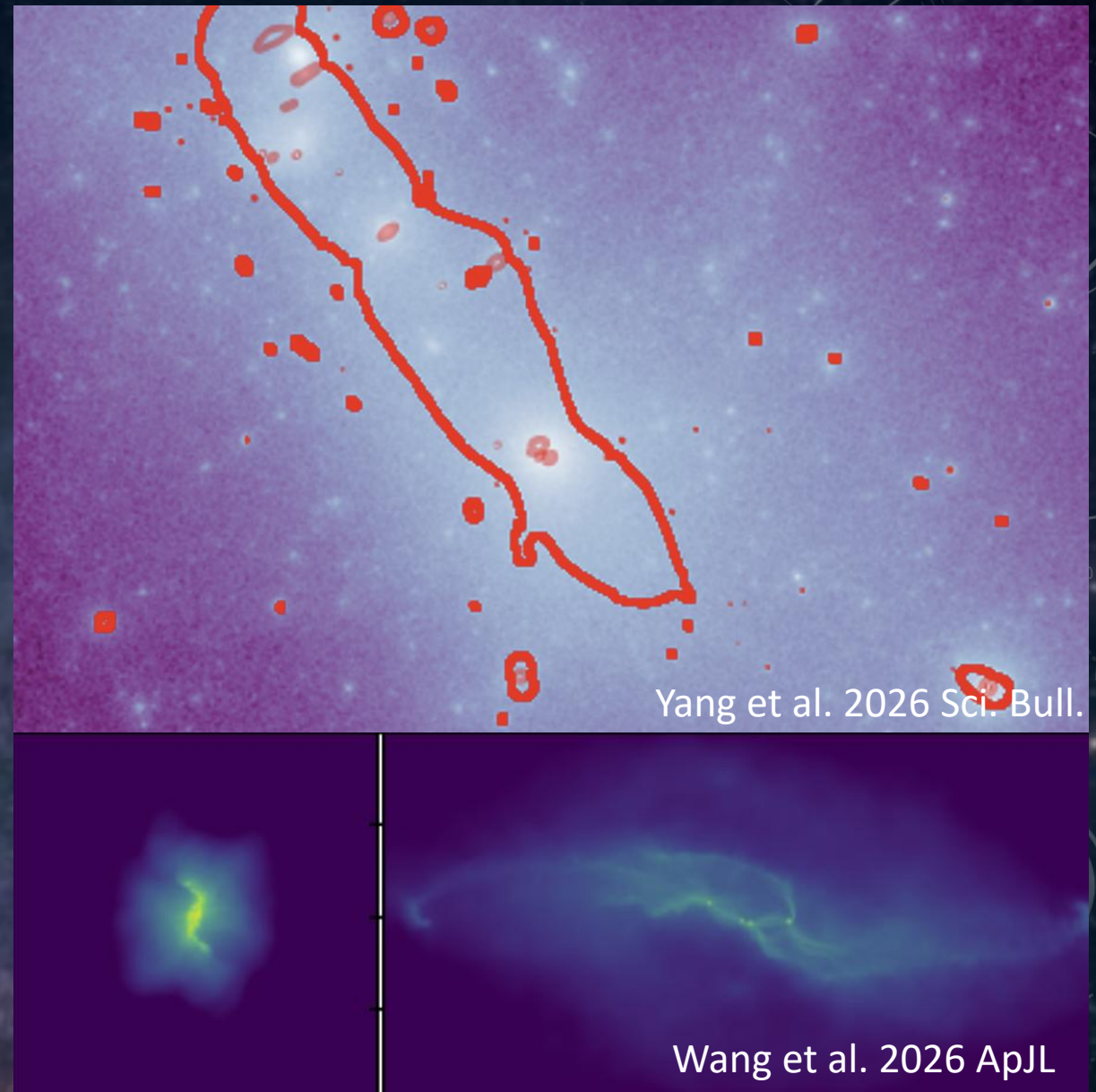
Cosmological simulations

- Structure & halo formation
 - Evolution histories in environment
 - Halos in populations
- (Extremely expensive)

Controlled simulations

- Typically one/few halos in isolation or an external potential
 - Initial condition put by hand & in equilibrium
- (Expensive if go to high resolution)

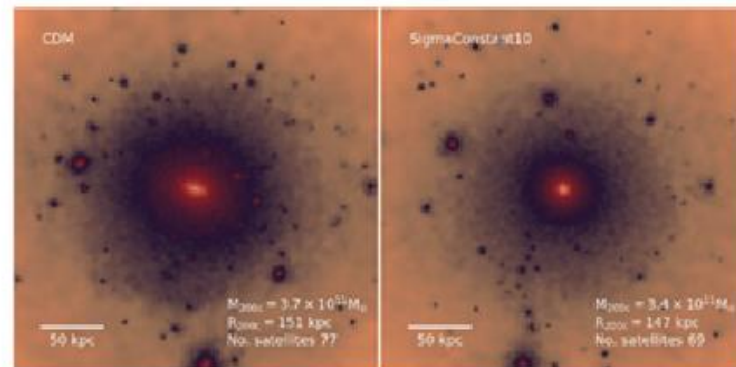
➤ We aim to understand the results and emulate without simulation



Cosmological simulations with v-dependent cross sections

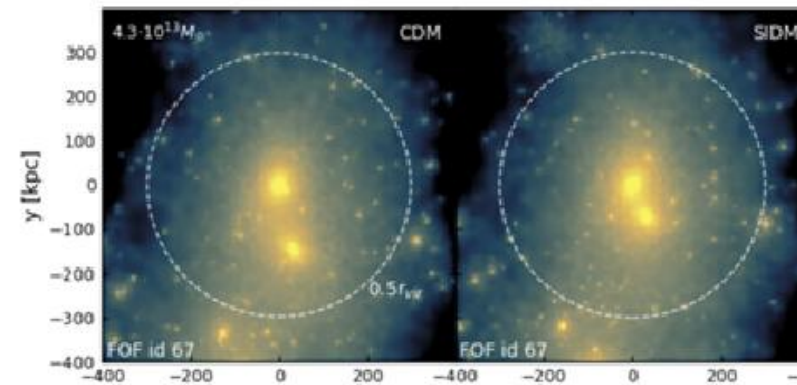
Hydrodynamical
simulations and
Dark matter only
cosmological
zoom-in
simulations

TangoSIDM

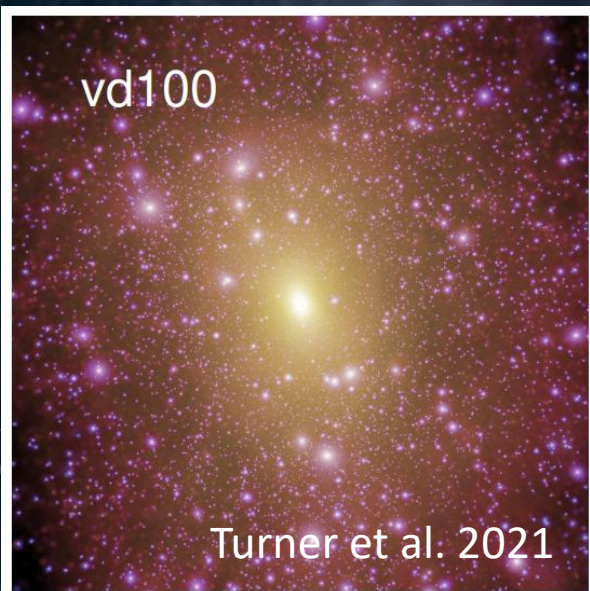


Correa et al. 2022

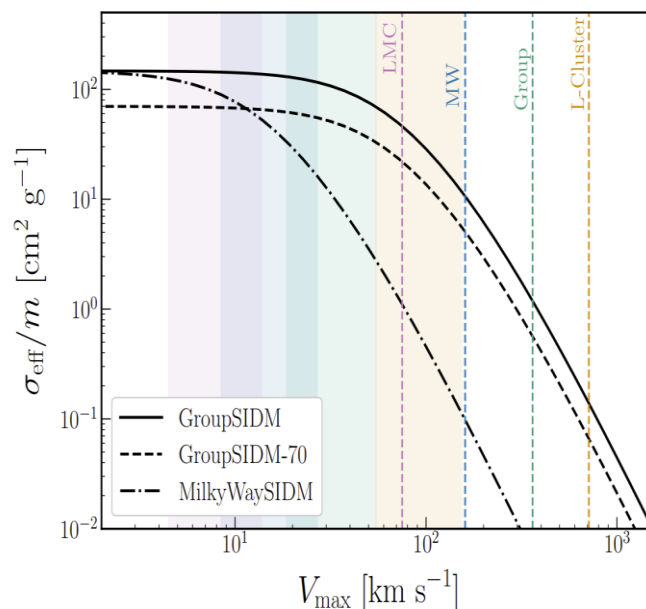
AIDA-TNG



Despali et al. 2025



Turner et al. 2021



Data Published and Ready to Explore

zenodo

Search records...



Communities

My dashboard

Planned intervention: On Wednesday, July 16th 05:00 UTC Zenodo will be unavailable for 15-30 minutes to perform a storage cluster

Published February 26, 2025 | Version v1

Dataset

Open

Data Release: SIDM Concerto: Compilation and Data Release of Self-interacting Dark Matter Zoom-in Simulations

Nadler, Ethan (Contact person)¹ Kong, Demao² Yang, Daneng³ Yu, Hai-Bo²

Hide affiliations

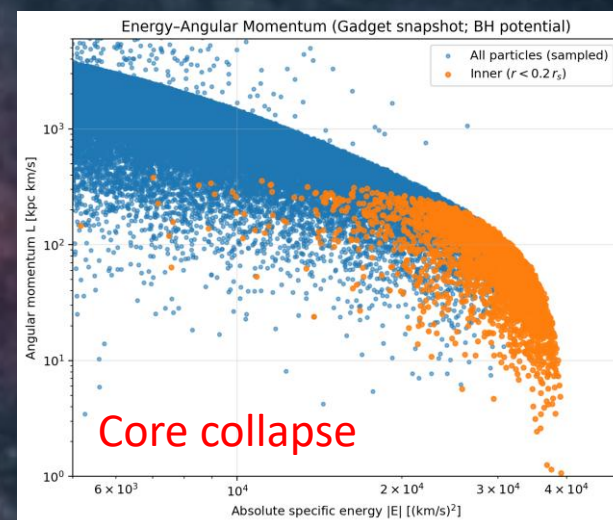
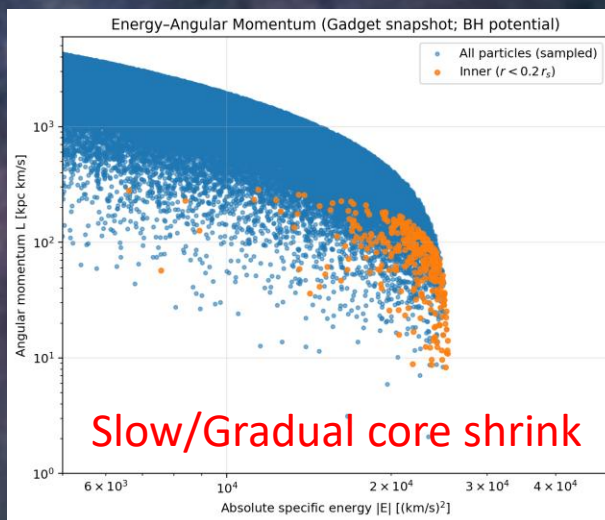
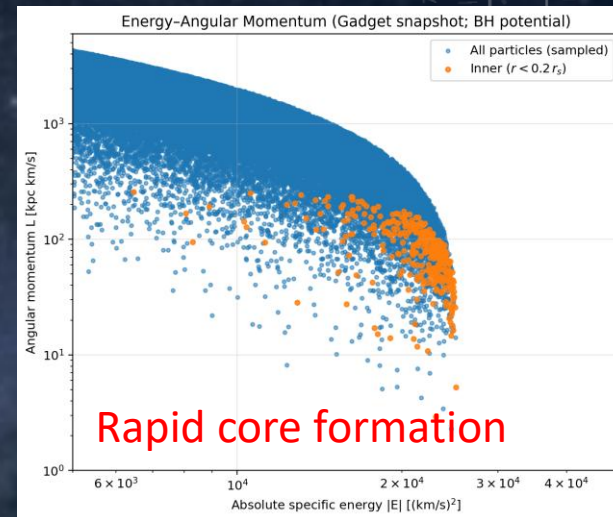
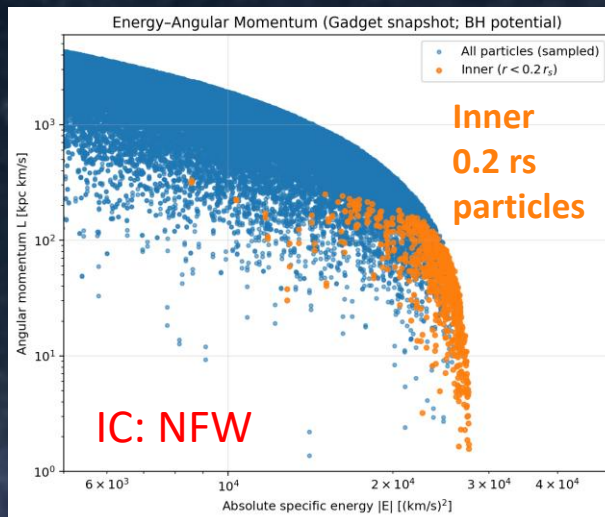
1. University of California, San Diego
2. University of California, Riverside
3. Purple Mountain Observatory

ApJ, 991, 69 (2025)

Example controlled simulation (1): Slow core shrink in SIDM

A kinetic picture of gravothermal evolution under SIDM

- Inner-outer scatterings: Inner low-J particles get kicked to larger-J
- Inner-inner scatterings: inefficient heat transport
- A “lost cone” in spherically symmetric SIDM halos



Angular momentum

Angular momentum

$$|E| = |K + P|$$

$$|E| = |K + P|$$

Example controlled simulation (2): Verify conditioned universality

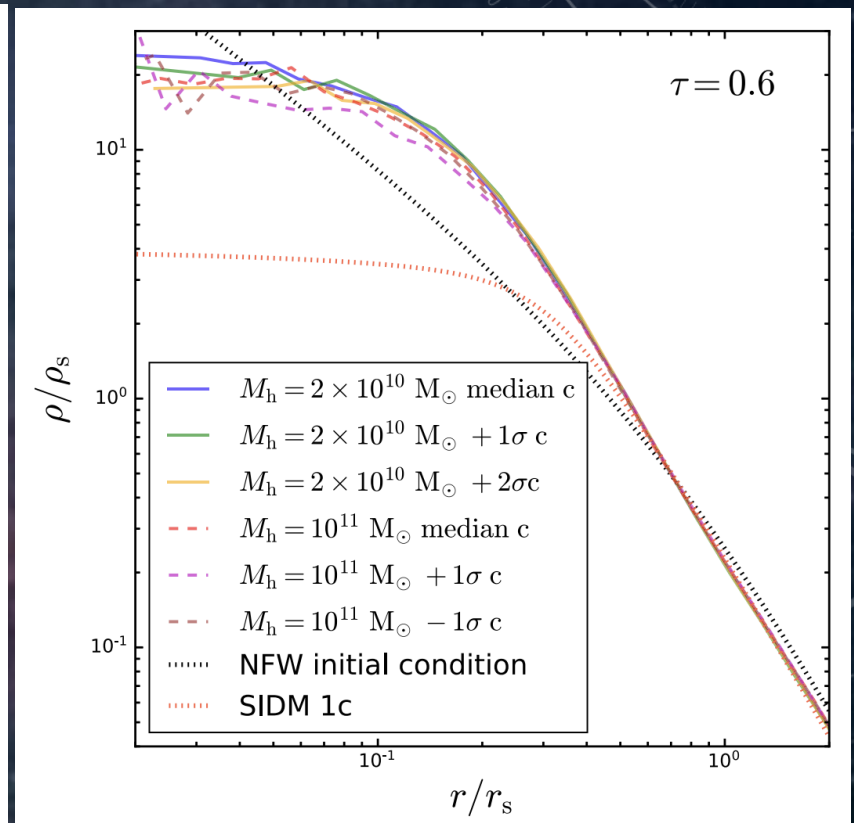
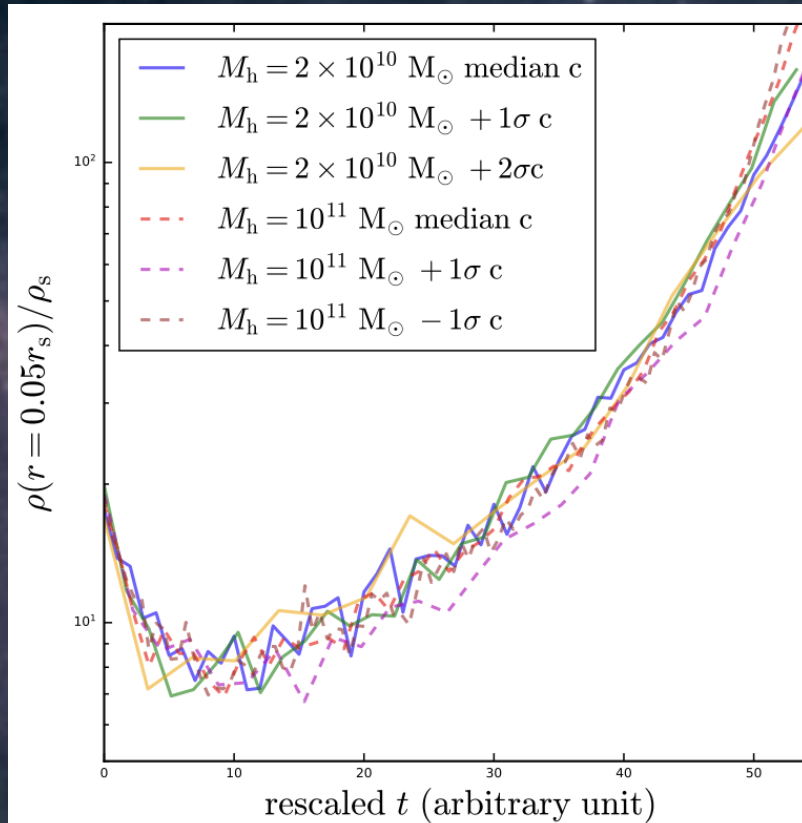
Approximate Universal Evolution when

- All channels in the SIDM 2-comp model specified
- Mass ratio, abundance fixed

Energy transfer for inter- and intra-species interactions are both proportional to the number of scatterings in LMFP

$$\rho_H \nu_H^2 \left(\frac{D}{Dt} \right) \ln \frac{\nu_H^3}{\rho_H} = -\frac{1}{4\pi r^2} \frac{\partial L_H}{\partial r} - R,$$

$$\rho_L \nu_L^2 \left(\frac{D}{Dt} \right) \ln \frac{\nu_L^3}{\rho_L} = -\frac{1}{4\pi r^2} \frac{\partial L_L}{\partial r} + R,$$



Why Does Gravothermal Evolution Appear Universal?

1

CORE PHYSICAL IDEA

🌀 **Heat conduction** breaks time-reversal invariance.

💥 The **arrow of time** is controlled by
SIDM collisions (the collision term).

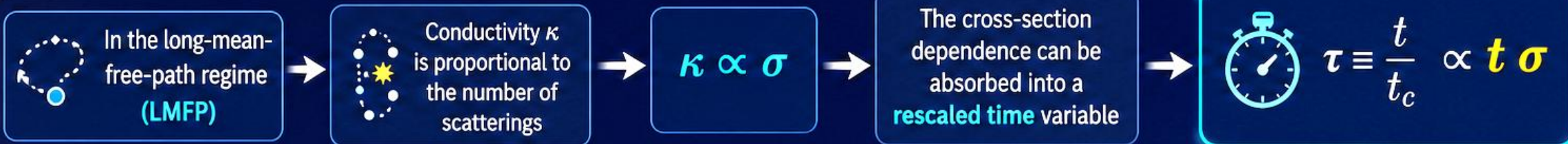
Schematic gravothermal heat-conduction equation

$$\frac{\partial}{\partial r} \left(r^2 \kappa m \frac{\partial v^2}{\partial r} \right) = r^2 \rho v^2 \left[\frac{D}{Dt} \right] \ln \left(\frac{v^3}{\rho} \right)$$

Time-derivative / convective derivative
(sets the arrow of time)

2

WHY UNIVERSALITY EMERGES



Hence gravothermal evolution follows an approximately **universal** track when plotted against τ .



KEY MESSAGE

In the LMFP regime, changing σ mainly **rescales the clock** rather than changing the form of the evolution.

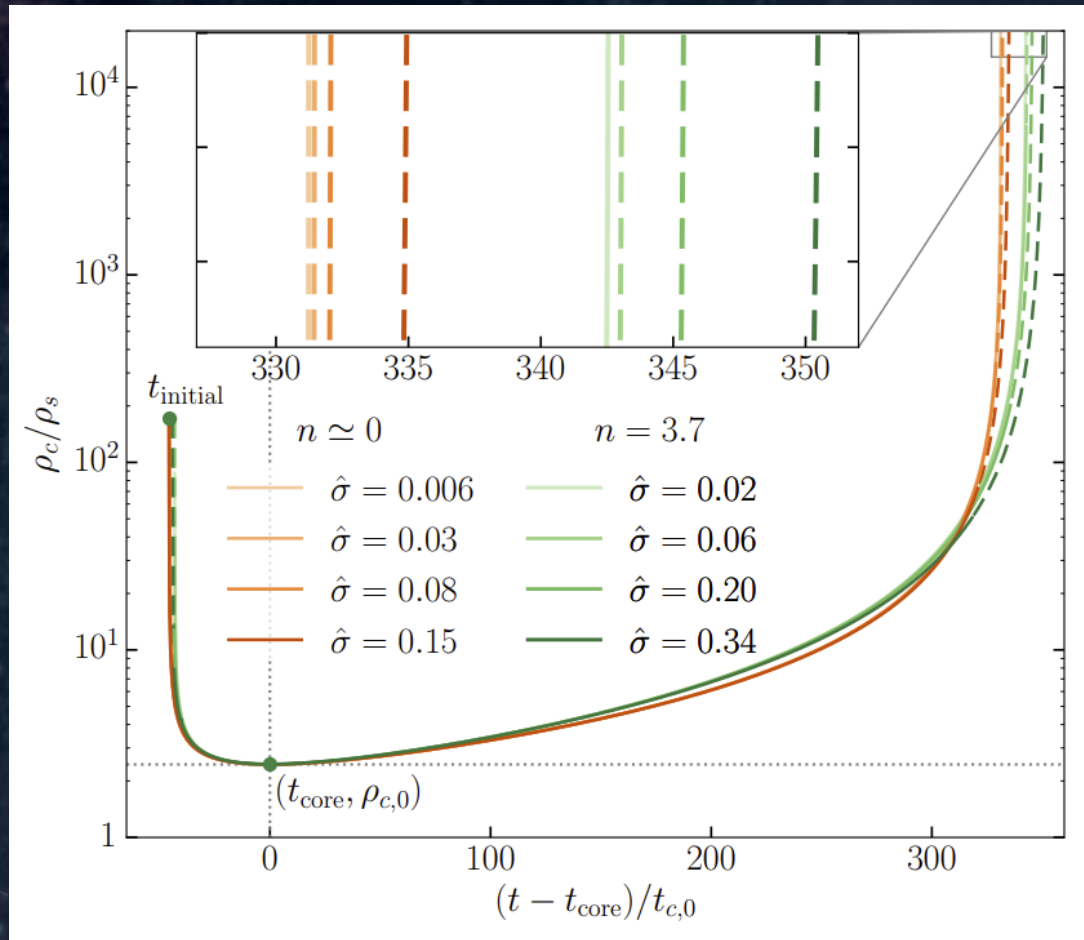


REFERENCES

- Yang+2405.03787 (Simplified discussion here)
- Outmezguine+ 2204.06568 (First paper)
- Yang+ 2305.16176 (Parametric model)
- Zhong, Yang, Yu 2306.08028 (When universality is exact)

Universal gravothermal evolution?

→ The kernel function



The gravothermal evolution is almost universal after normalizing the evolution time by the core collapse time (Outmezguine et al. MNRAS 2022)

- What is its origin?
- How accurate/reliable is it?
- Does it work for v-dependent models?
- How baryons breaks it?
- Does it work for extended SIDM models?

How accurate/reliable is it?

Exact Universality in a fluid PDE

(Zhong, Yang & Yu, MNRAS, 2306.08028)

$$\begin{aligned}\frac{\partial \hat{M}_\chi}{\partial \hat{r}} &= \hat{r}^2 \hat{\rho}_\chi, & \frac{\partial (\hat{\rho}_\chi \hat{v}_\chi^2)}{\partial \hat{r}} &= -\frac{(\hat{M}_\chi + \hat{M}_b) \hat{\rho}_\chi}{\hat{r}^2}, \\ \frac{\partial \hat{\hat{L}}_\chi}{\partial \hat{r}} &= -\hat{\rho}_\chi \hat{r}^2 \hat{v}_\chi^2 D_{\hat{r}} \ln \frac{\hat{v}_\chi^3}{\hat{\rho}_\chi}, & \frac{\hat{\hat{L}}_\chi}{\hat{r}^2} &= -3.4 \gamma \hat{\rho}_\chi \hat{v}_\chi^3 \frac{\partial \hat{v}_\chi^2}{\partial \hat{r}},\end{aligned}\quad (19)$$

- ✓ **Given smooth initial condition and coefficients, this PDE has a unique solution that is smooth in a finite time**
- ✓ **Everything is normalized, independent of NFW rhos, rs, and SIDM physics (sigma/m)**

What breaks it?

- Breakdown of the LMFP regime
- Approximate nature of the conduction fluid model
- Additional physics: baryons, environment

Bottom line

When the heat conductivity in the halo is proportional to the number of scatterings, we expect the “**t** -> **t** **σ**” trick to bring us a reasonable universality

Analytic calculation of SIDM density profiles

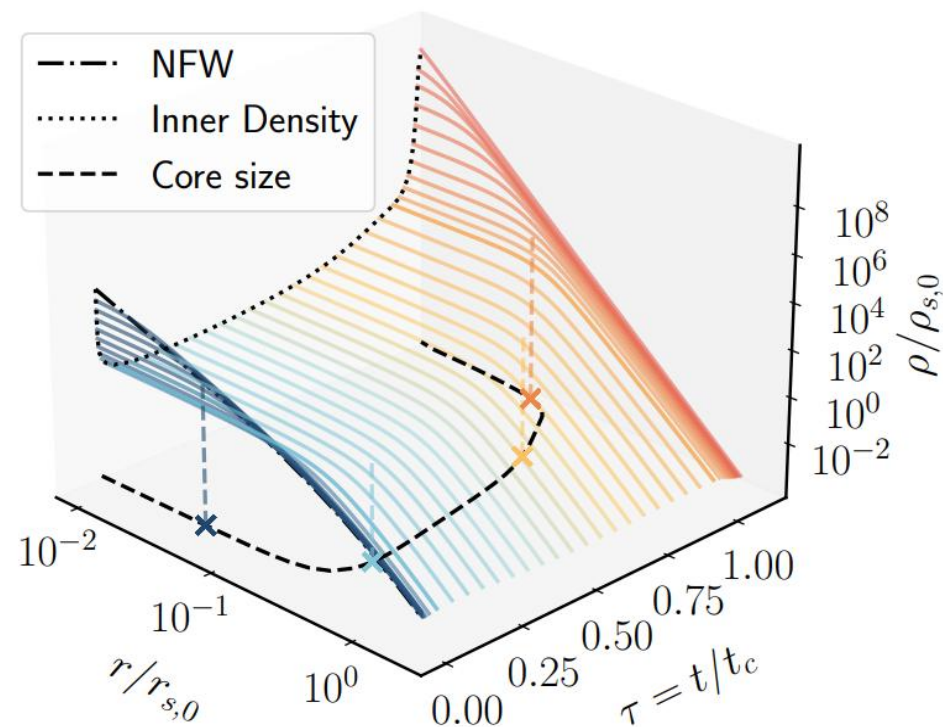
Calibration of the universal solution

$$\rho_{\text{PSIDM}}(r) = \frac{\rho_s}{\left[\left(\frac{r}{r_s} \right)^4 + \left(\frac{r_c}{r_s} \right)^4 \right]^{\frac{\gamma}{4}} \cdot \left[1 + \left(\frac{r}{r_s} \right)^{\beta} \right]^{\frac{3-\gamma}{\beta}}}$$

3 parameter version:
Yang+2305.16176 JCAP

5 parameter version,
improved over the late
core collapse phase:
Hou+2502.14964 JCAP

$$\begin{aligned} \rho_s &= \rho_{s,0} f_{\rho}(\tau) \\ r_s &= r_{s,0} f_s(\tau) \\ r_c &= r_{s,0} f_c(\tau) \\ \beta &= f_{\beta}(\tau) \quad \gamma = f_{\gamma}(\tau) \end{aligned}$$



Hou & Yang et al. 2025 JCAP

Yang+2305.16176 JCAP 2024

<https://github.com/DanengYang/parametricSIDM>

Parametric Model Analysis Tools for Self-Interacting Dark Matter Halos

The parametric model for self-interacting dark matter (SIDM) halos allows for obtaining halo density profiles based on a few calibrated equations under an assumed SIDM model, using parameters from their CDM counterparts, such as NFW scale parameters or halo mass and concentration parameters. The effect of a baryon potential can be incorporated, assuming the density profile conforms to the Hernquist model.

Relevant works

- [1] **Dark matter-only version**: D. Yang, E. O. Nadler, H.-B. Yu, and Y.-M. Zhong, [arXiv:2305.16176](https://arxiv.org/abs/2305.16176), published in [JCAP 02, 032 \(2024\)](https://doi.org/10.1088/1361-6470/acc032)
- [2] **Dark matter plus baryon version**: D. Yang, [arXiv:2405.03787](https://arxiv.org/abs/2405.03787), published in [Phys. Rev. D](https://doi.org/10.1088/1361-6470/acc032)
- [3] **Testing the parametric model using matched halos in cosmological simulations**: D. Yang, E. O. Nadler, and H.-B. Yu, [arXiv:2406.10753](https://arxiv.org/abs/2406.10753), published in [Phys. Dark Universe](https://doi.org/10.1088/1361-6470/acc032)
- [4] For a **lensing specific** parametric model for SIDM halos, see S. Hou, D. Yang, N. Li, and G. Li, [arXiv:2502.14964](https://arxiv.org/abs/2502.14964), with [public codes on GitHub](https://github.com).
- [5] Our method has been implemented in the [SASHIMI program for SIDM subhalos](https://github.com): S. Ando, S. Horigome, E. O. Nadler, D. Yang, and H.-B. Yu, [arXiv:2403.16633](https://arxiv.org/abs/2403.16633)

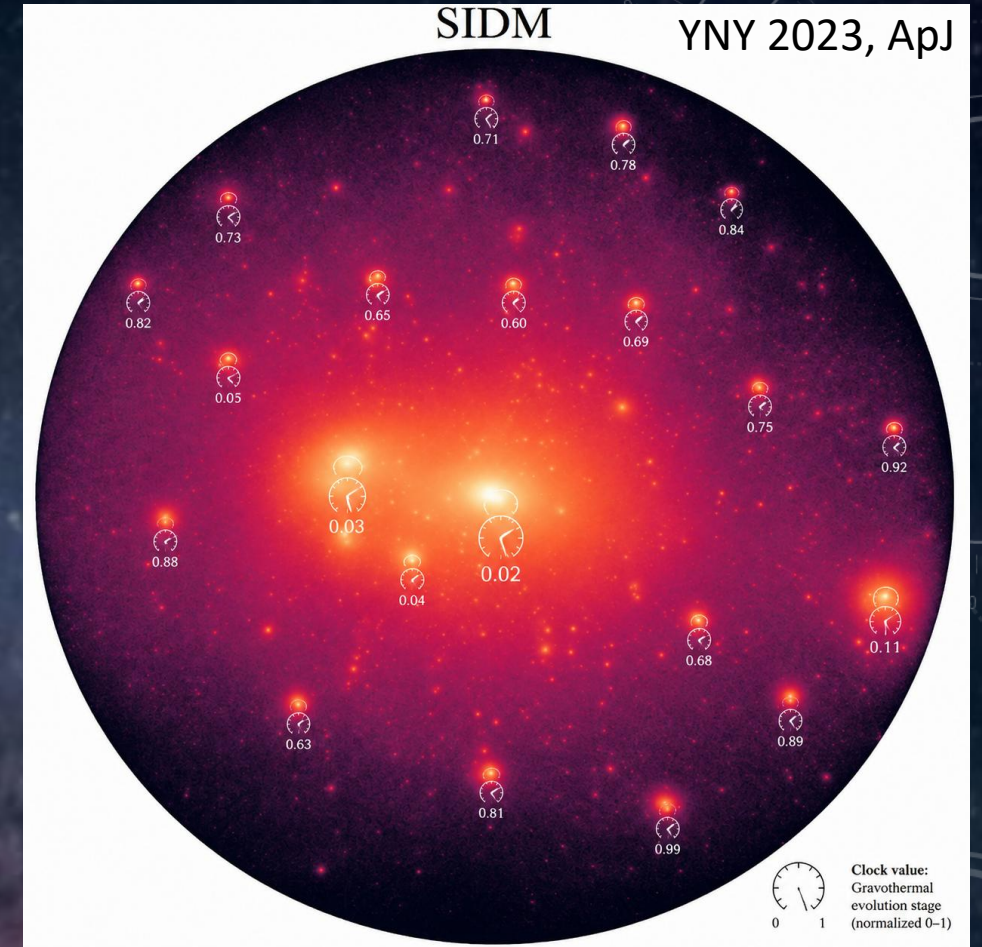
Our codes are free to copy and modify. Please quote the relevant paper(s), if you find them useful. For any questions or comments, please contact [yangdn_at_pmo.ac.cn].

Example applications

For simple stand-alone scripts for SIDM halos

<https://github.com/DanengYang/parametricSIDM>

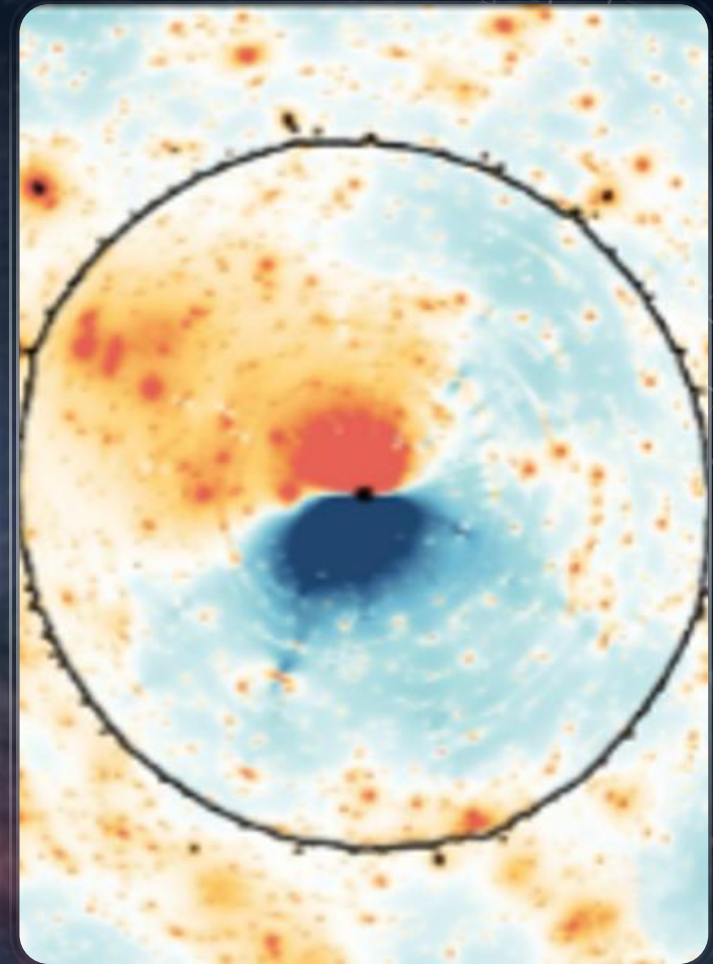
To incorporate accretion histories, we introduce the integral approach



- Without baryons, each halo can be (approximately) characterized by 3 quantities:
2 NFW params + 1 gravothermal phase
- The effect of baryons can (currently) be (only) estimated in closed form equations. (Yang 2024 PRD)

From SIDM evolution to observable predictions

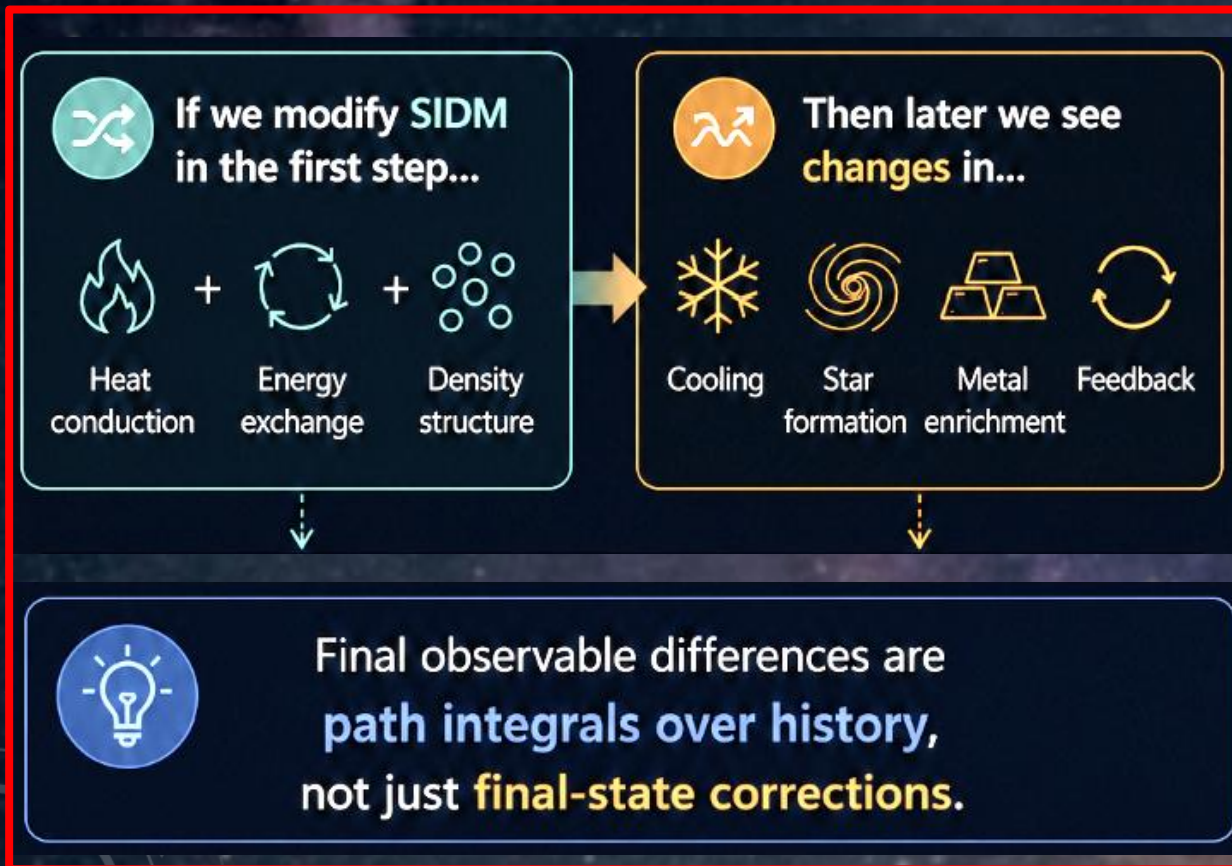
- Integral approach
- Effect of baryons
- Lensing effects
- Observable predictions



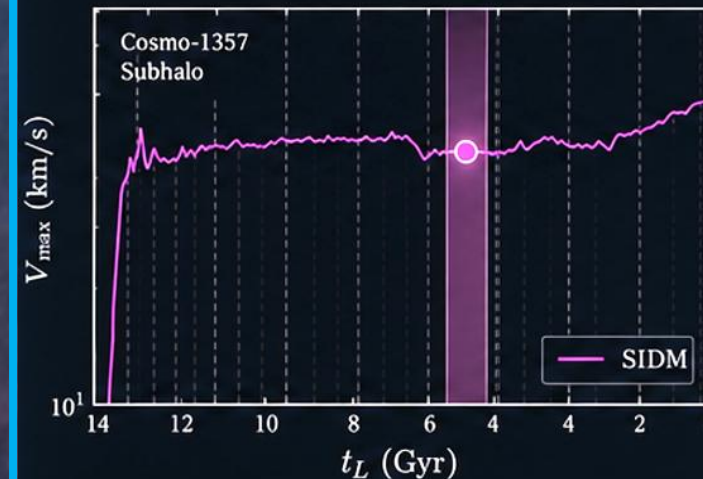
The method: SIDM as controlled deformation on CDM paths



To see how SIDM+mass segregation effects differ in these galaxies, we evolve SIDM & accretion/merger history simultaneously



Integral approach of the parametric model (2305.16176)



Add small increments in gravothermal phase.

$$\Delta\tau = \frac{\Delta t}{t_c}$$

t_c from *instantaneous* CDM halo parameters.

$$V_{\max}(t) = V_{\max,\text{CDM}}(t) + \int_0^{\tau(t)} d\tau' \frac{dV_{\max,\text{Model}}(\tau')}{d\tau'}$$

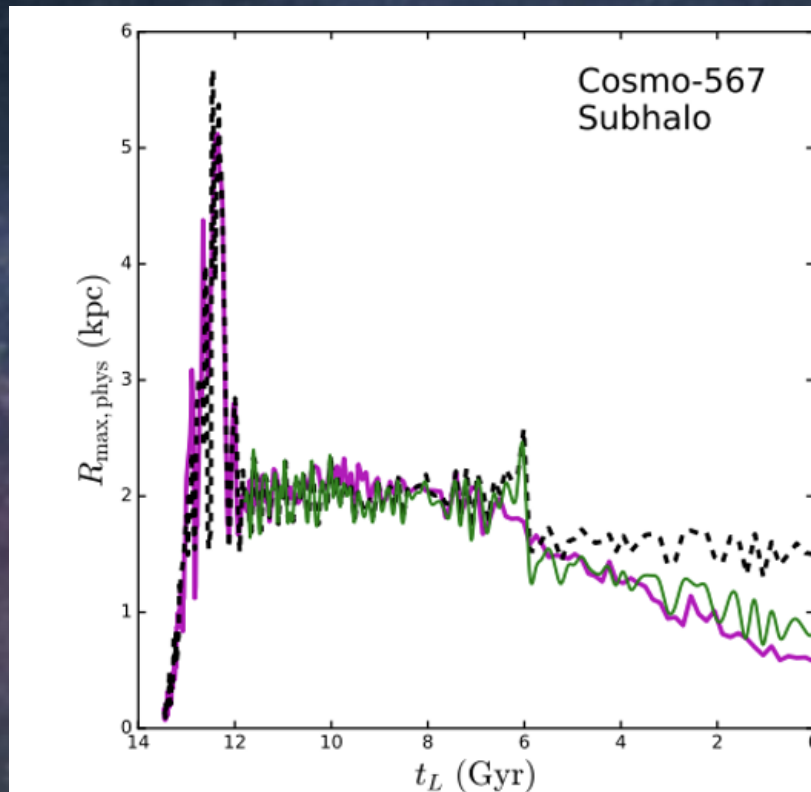
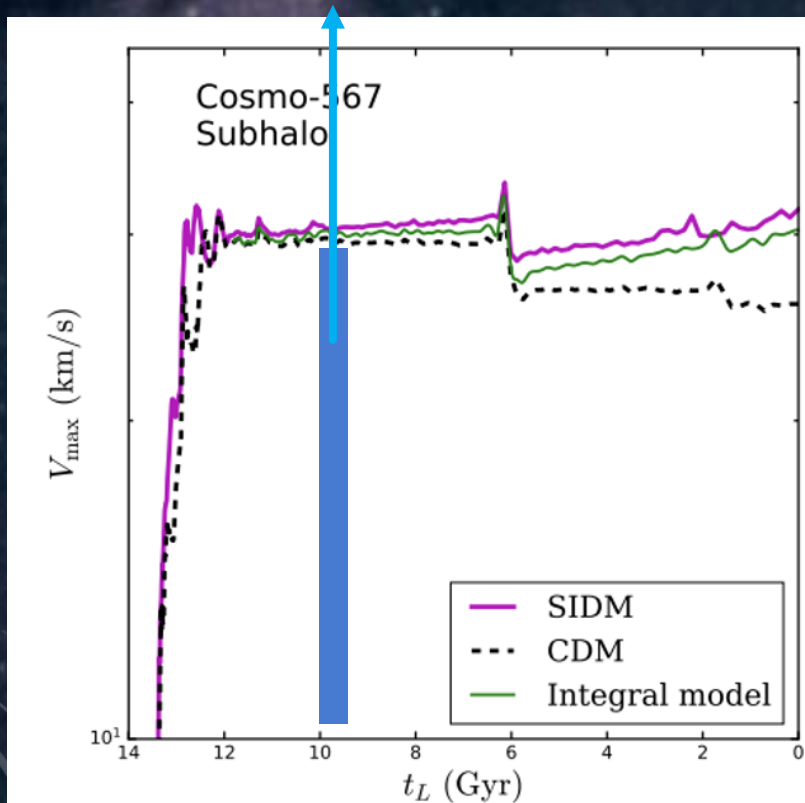
Tested against cosmological simulations

(Yang et al. 2025 PDU, Nadler et al. 2025a,b ApJ)

SIDM as controlled deformation on CDM paths

Each SIDM halo is assumed to have a **fictitious CDM** progenitor which evolves into its current state **in isolation**

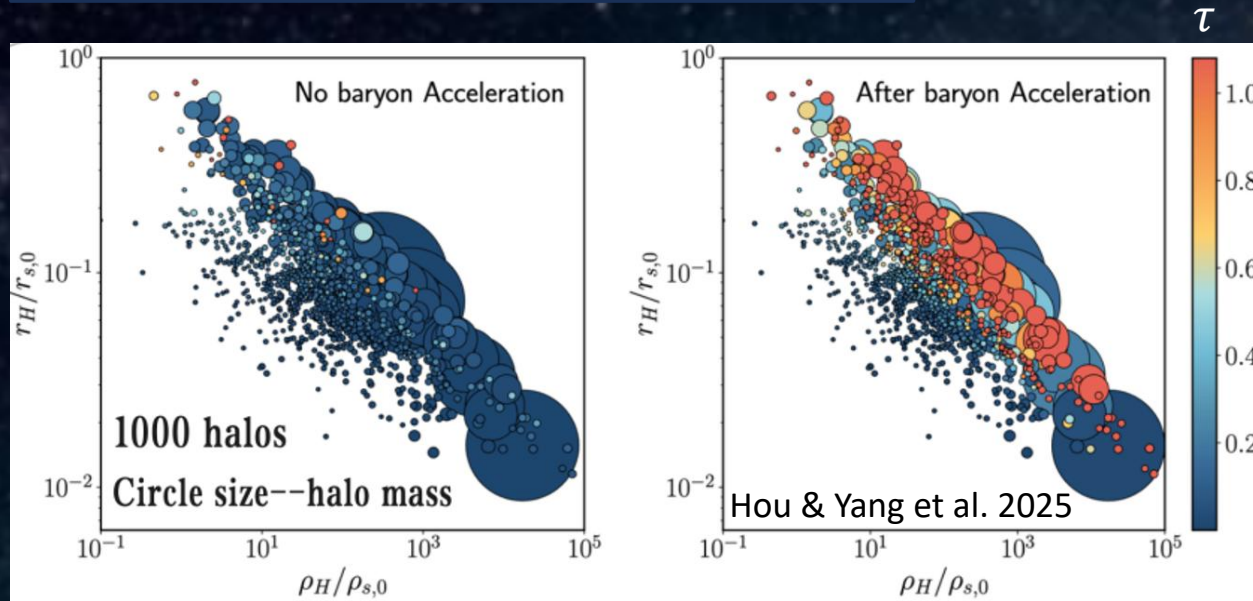
$$V_{\max}^{\text{SIDM}}(t) = V_{\max}^{\text{CDM}}(t) + \int_{t_f}^t \frac{dt'}{t_c(t')} \frac{dV_{\max, \text{Model}}(\tilde{t}')}{d\tilde{t}'},$$
$$r_{\max}^{\text{SIDM}}(t) = r_{\max}^{\text{CDM}}(t) + \int_{t_f}^t \frac{dt'}{t_c(t')} \frac{dr_{\max, \text{Model}}(\tilde{t}')}{d\tilde{t}'},$$



The integral approach in the parametric model offers an **analytic kernel** for computing the **incremental SIDM effect**

Yang+2305.16176 JCAP

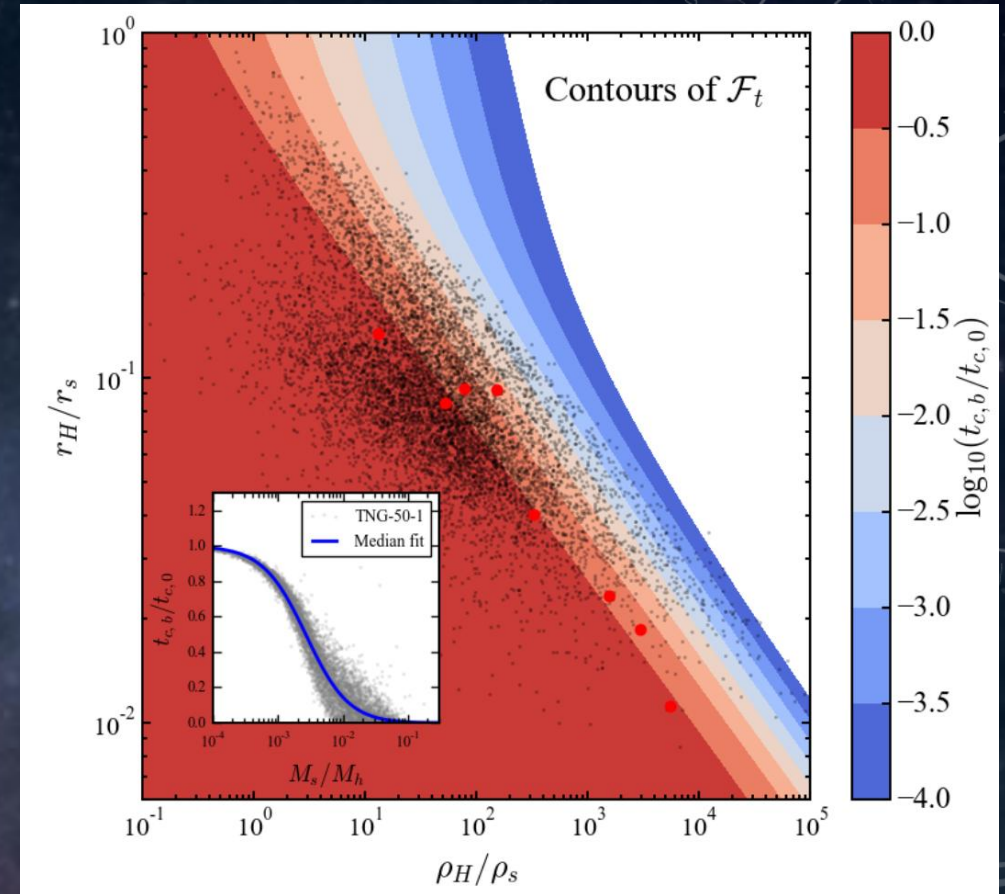
Effect of baryons



- Core collapse is boosted in the presence of baryons (Feng, Yu, Zhong 2021 ApJL)
- Effect can be incorporated in a **closed form equation** (Yang 2405.03787 PRD)
- SIDM core becomes smaller after contraction

$$t_{c,b} = t_{c,0} \mathcal{F}_t(\hat{\rho}_H, \hat{r}_H)$$

$$r_c(\tau) = r_{s,0} f_c(\tau) (\mathcal{F}_t)^2$$



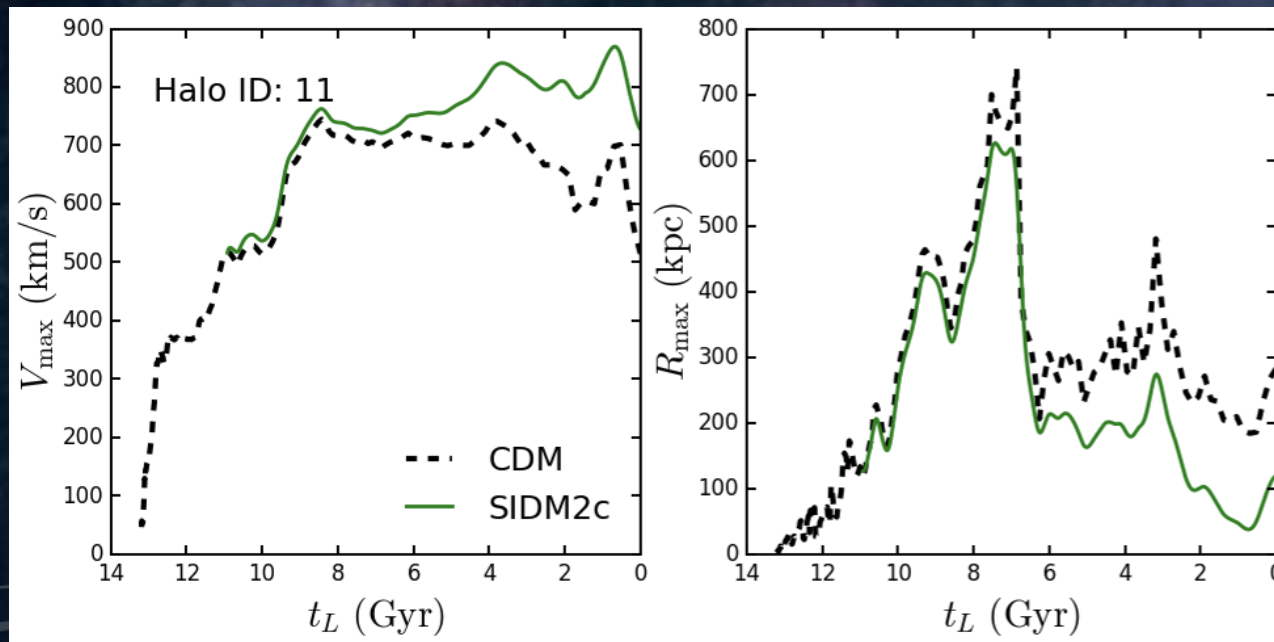
Extend the parametric model and can be implemented as path distortions

$$\mathcal{F}_t(M_s/M_h) \approx \frac{e^{-7.9(M_s/M_h)}}{1 + 2300(M_s/M_h)^{1.3}},$$

applicable for $M_s/M_h \lesssim 0.08$.

Extension & Application: 2-comp SIDM with mass segregation

- Isolated dwarfs are **mostly core forming** ($\tau < 0.2$)
- Dwarfs in clusters are **mostly core collapsing** ($\tau > 0.2$)



Yang, Fan, Hou, Tsai, Sci. Bull. 71 (2026)

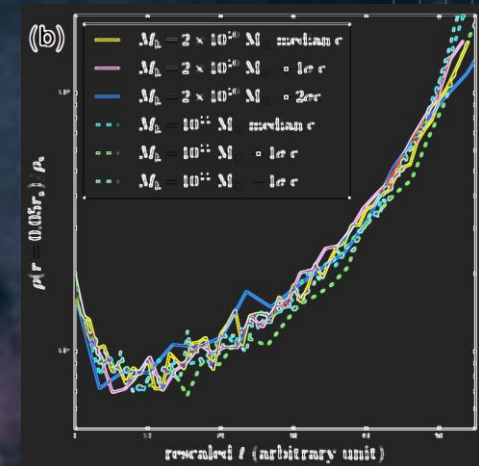
$$X = (V_{\max}, R_{\max}, \tau)$$

$$\frac{dX}{dt} = F_{\text{SIDM}}(X)$$

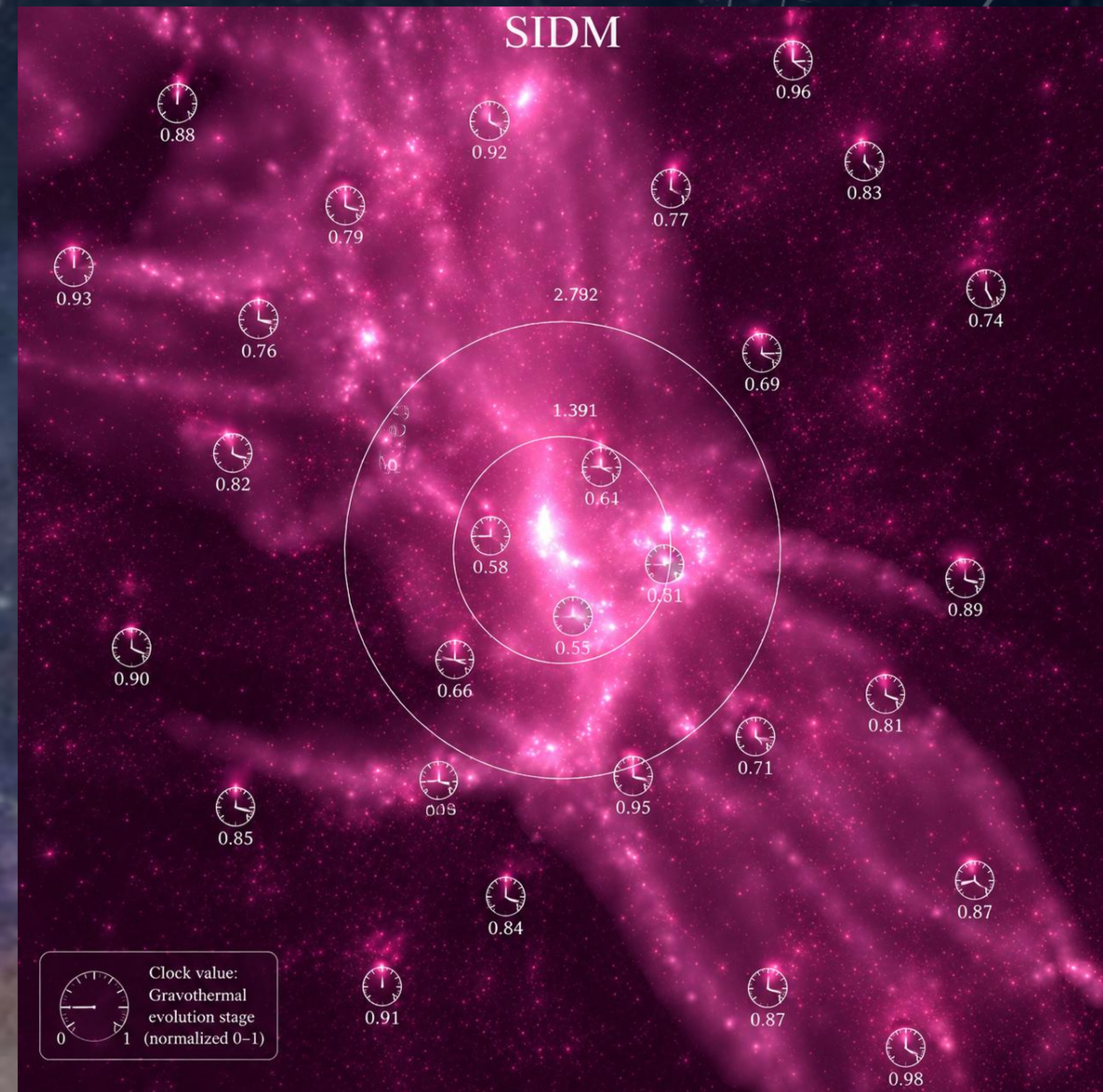
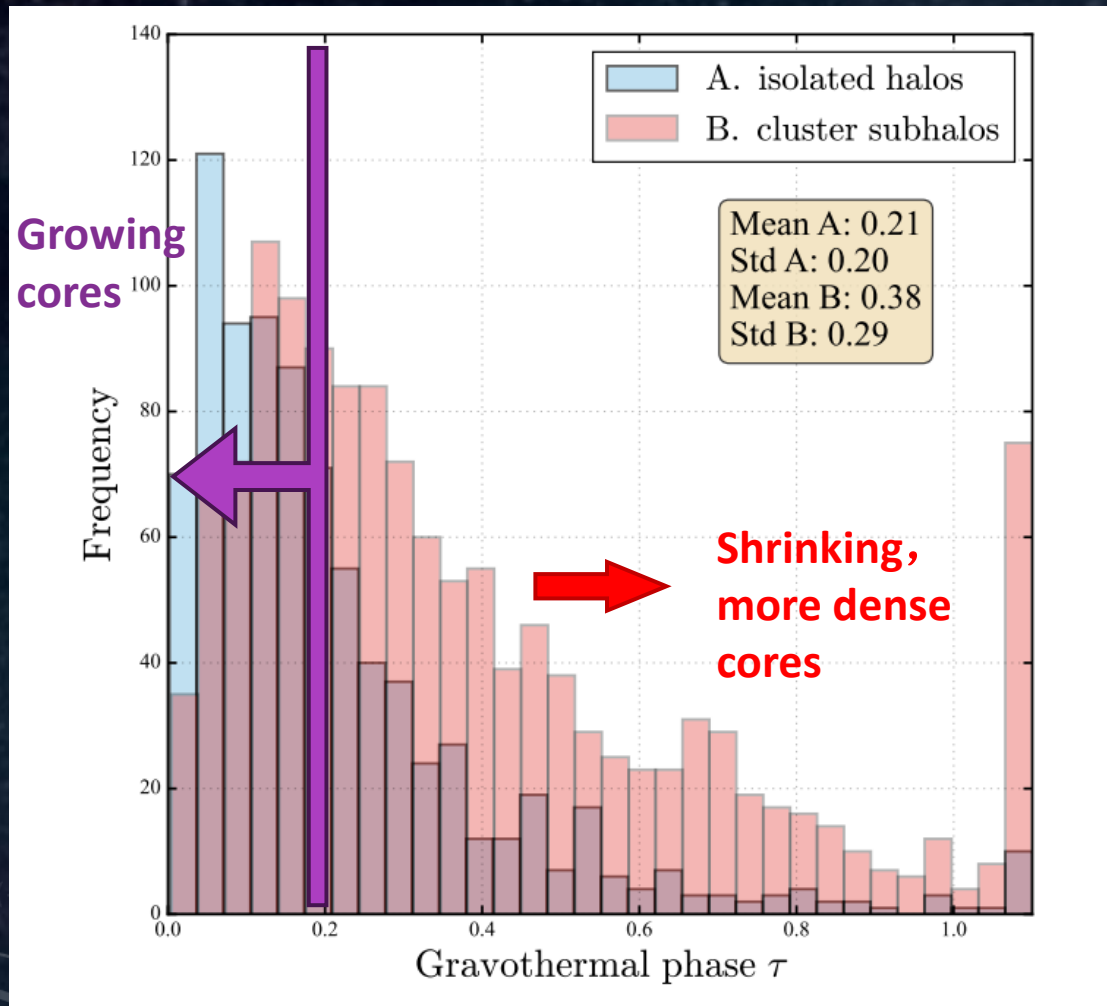
where F_{SIDM} is based on a universal & analytic parametric model

Conditioned universality in 2-comp SIDM

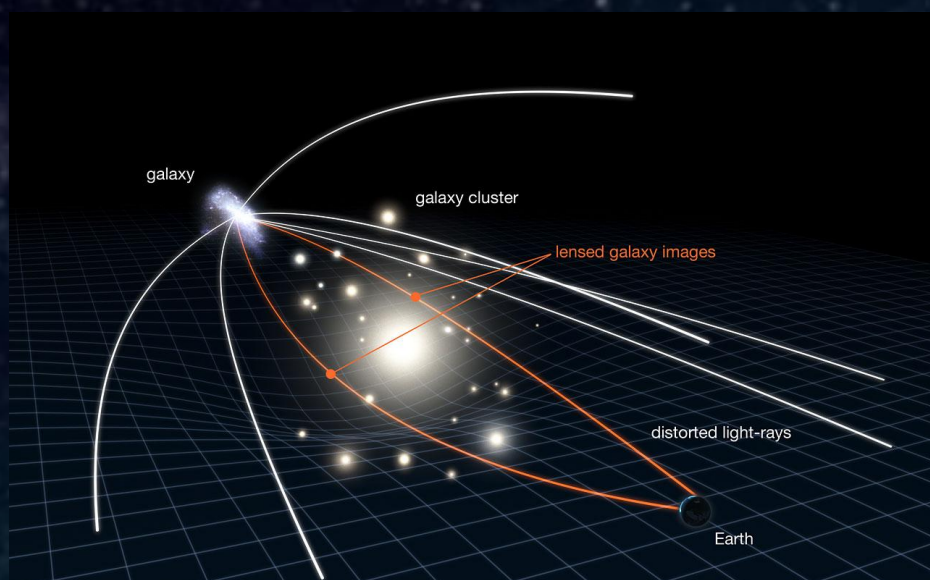
- All channels in the SIDM 2-comp model specified
- Mass ratio, abundance fixed



The method: Parametric SIDM model & CDM cosmological simulation



For lensing observables



$$\hat{\Psi}(\hat{R}) = a \ln \left(1 + b\hat{R} + c\hat{R}^2 \right)^c - \ln \left(p\hat{R} + 1 \right)^s$$

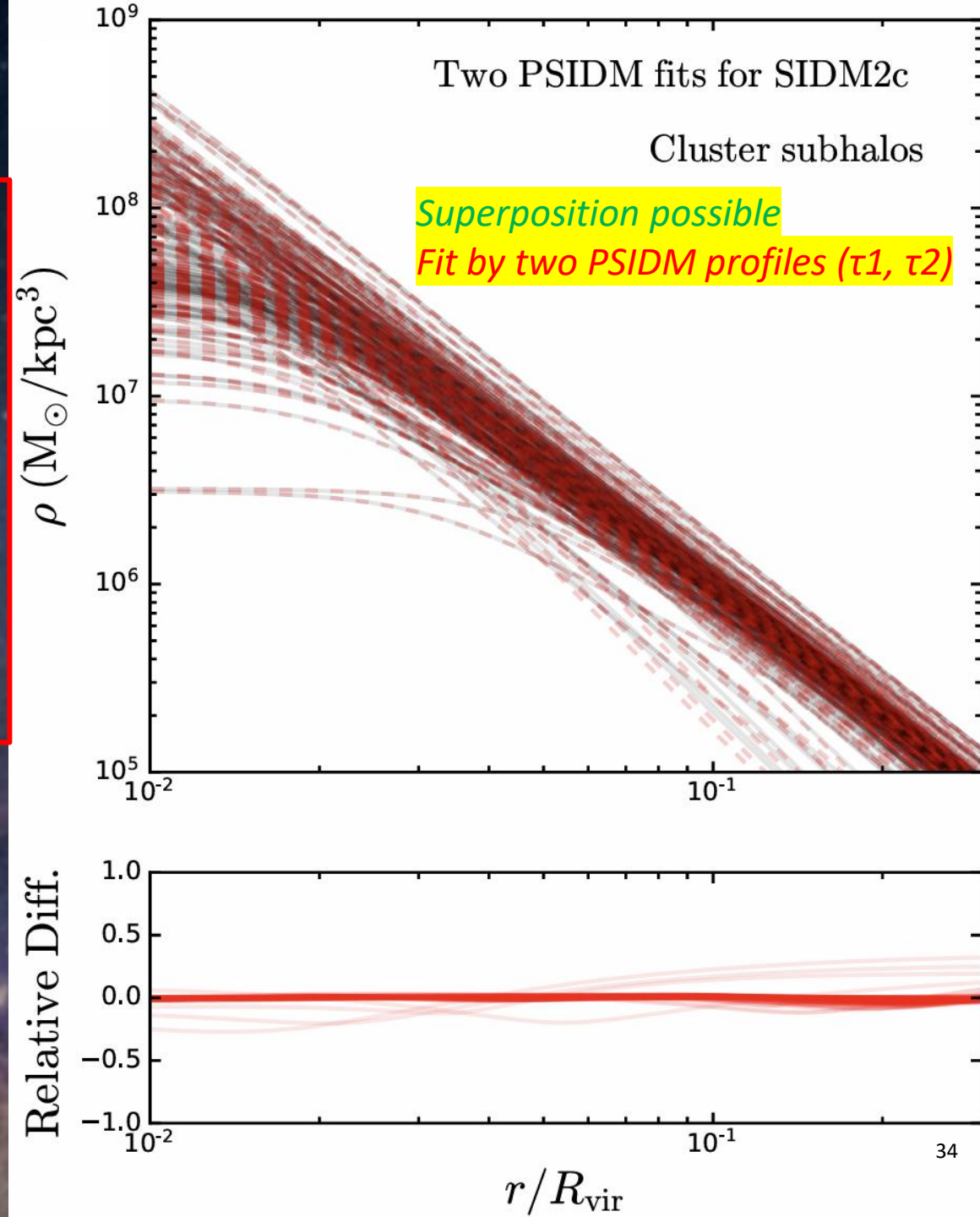
Calibrate the
lensing
potential
instead of the
density profile

Application of
the model is
broader than
SIDM!

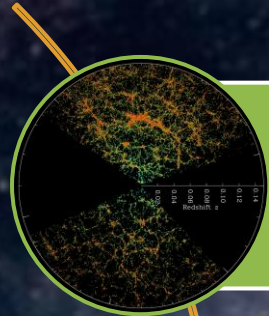
Hou, Yang, Li, Li JCAP 08 (2025) 048

- Constructed based on universality
- A family of central density profiles ranging r^0 to $r^{-2.5}$ with analytic lensing predictions
- Lensing for complex density fields via superposing models (not limited to SIDM)

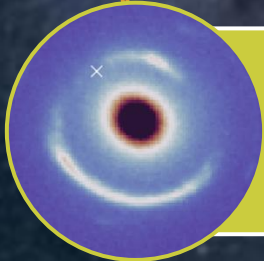
https://github.com/HouSiyan2001/SIDM_Lensing_Model



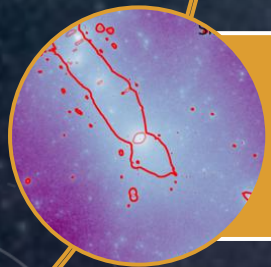
Results: dwarfs can have growing cores and densities simultaneously



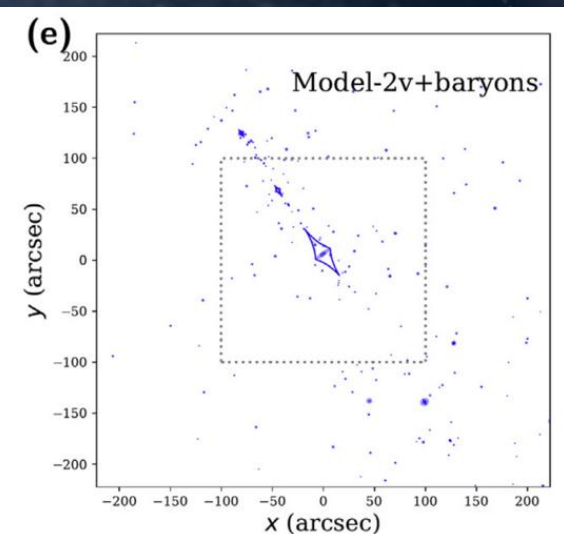
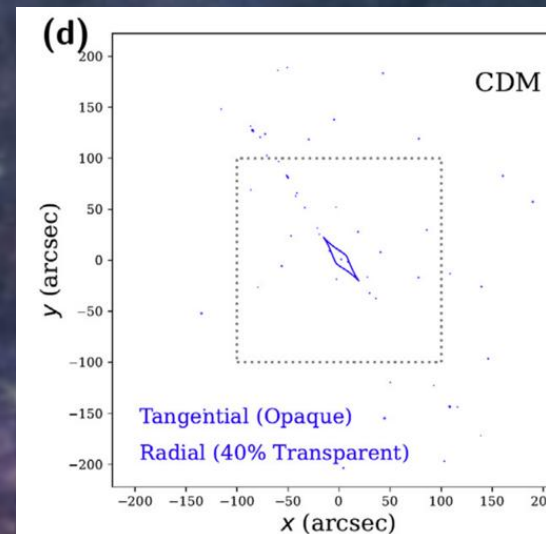
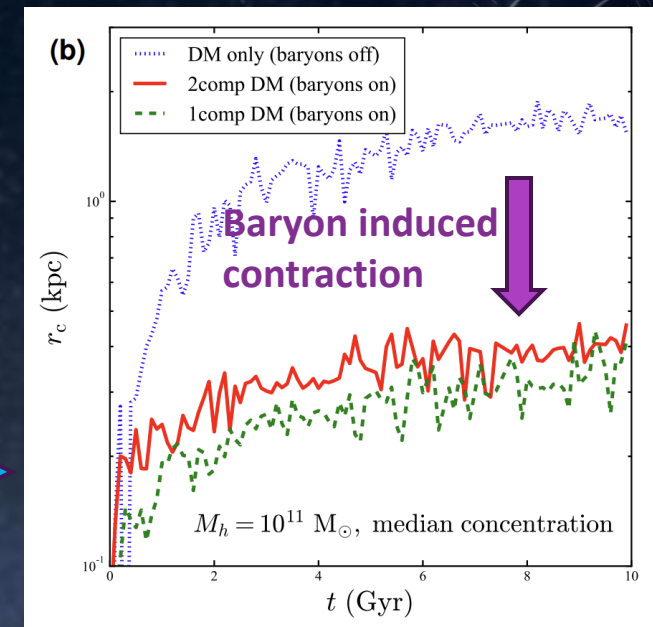
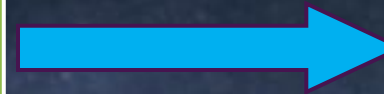
Growing stellar cores in isolated dwarfs



Overall x10 faster gravothermal evolution, explaining strong lensing perturbers

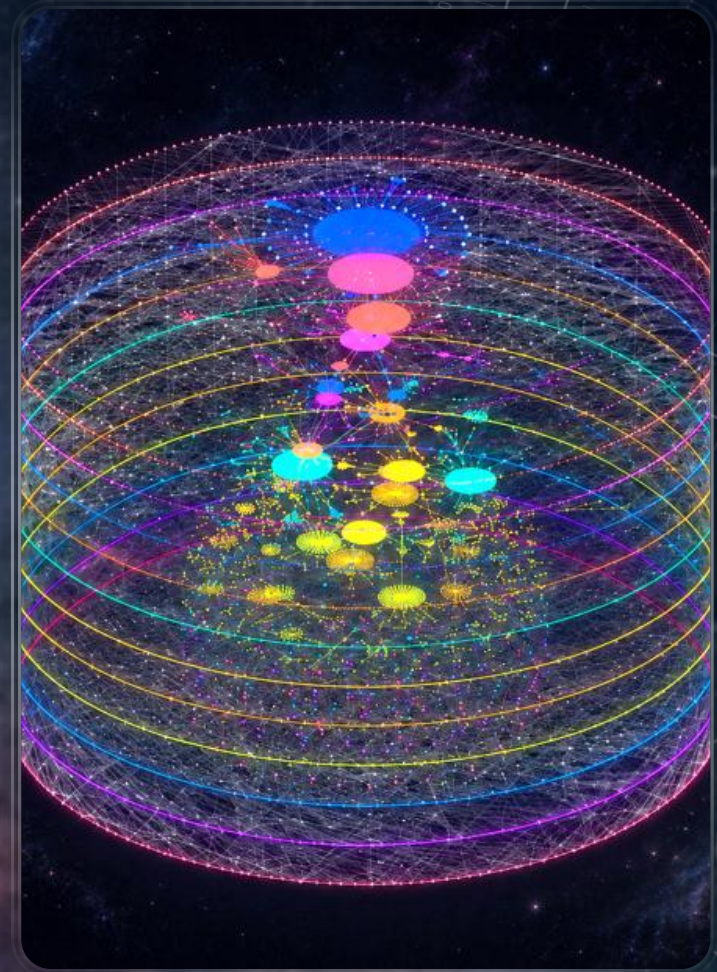


Cluster subhalos evolve faster (gravothermal), explaining GGSL



Beyond Deterministic Halo Modeling

- A path measure model for galaxy formation and SIDM



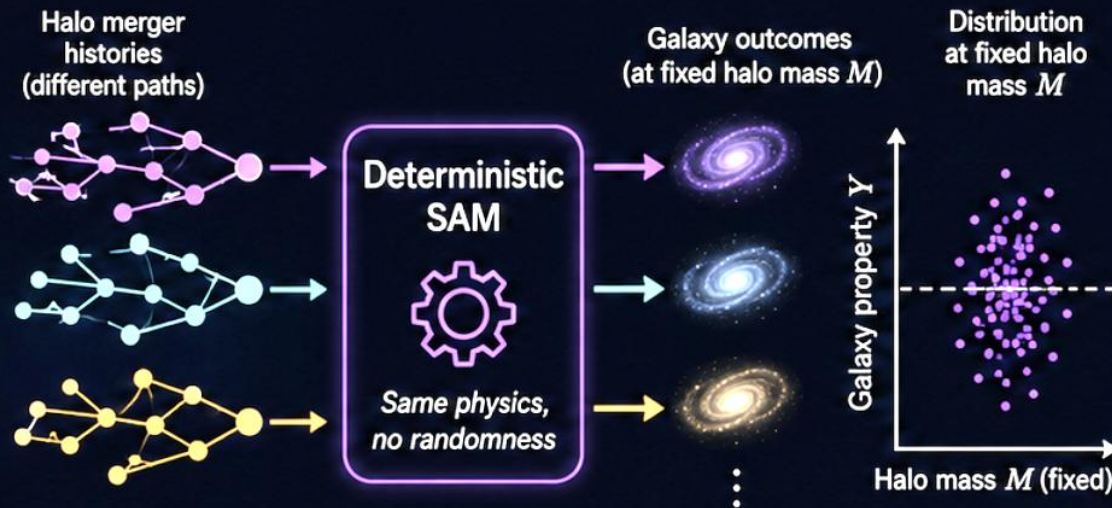
Astrophysical scatter is **not just noise** – it **carries physics**



Coarse-graining transforms **deterministic microscopic dynamics** into an **Effective Stochastic Dynamics**

1. Population (intrinsic) scatter

Can arise even from a deterministic SAM

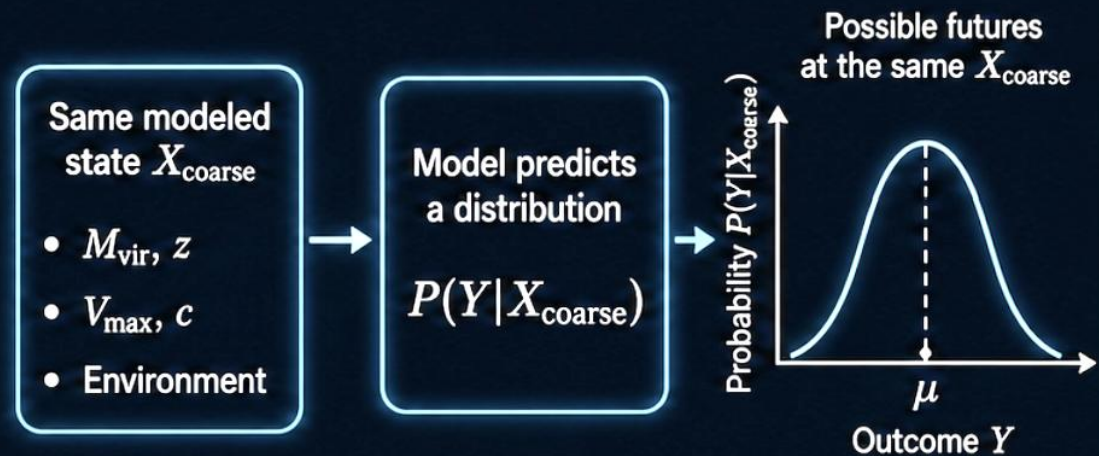


Enhanced
with AI

Different paths → different outcomes

2. Predictive (model) scatter

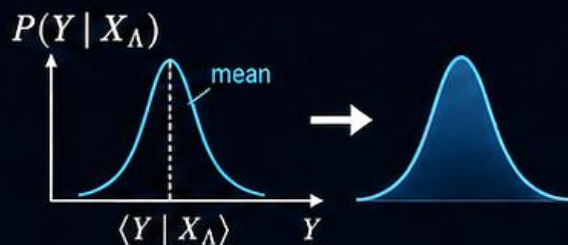
Our focus



Same state → distribution of outcomes

Why predictive scatter matters

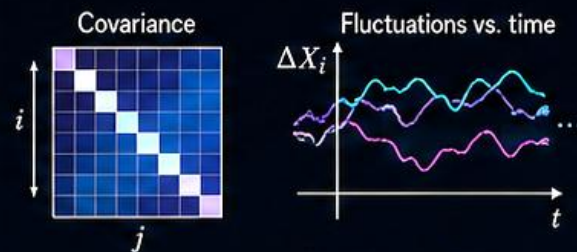
1. Part of the effective prediction



$P(Y | X_\Lambda)$, not just $\langle Y | X_\Lambda \rangle$

At fixed coarse-grained state,
predict a conditional distribution.

2. Encodes unresolved dynamics

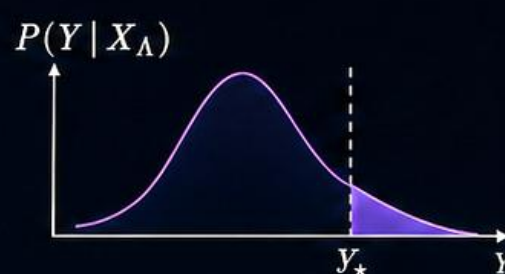


$$D_{\Lambda,ij}(X) = \frac{\langle \Delta X_i \Delta X_j \rangle_c}{\Delta t}$$

Amplitude, covariance, and time dependence
reflect unresolved dynamics.

Physical, but scheme-dependent.

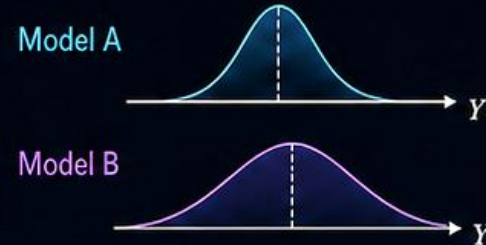
3. Controls rare outcomes



$$P(Y > Y_* | X_\Lambda)$$

Extreme paths and rare galaxies
depend on the fluctuation sector.

4. Can distinguish models



$P_A(Y | X_\Lambda) \neq P_B(Y | X_\Lambda)$
even if $\langle Y | X_\Lambda \rangle_A \approx \langle Y | X_\Lambda \rangle_B$

Compare within the same
coarse-graining scheme.

Enhanced
with AI



**Mean tells
the typical evolution**



**Predictive scatter tells
which evolutions are possible
and how probable they are**



Predictive scatter depends on coarse-graining; its RG-like evolution remains an open question.

A good framework should predict **mean**, **predictive scatter**, and **covariance**.

A Functional View of NP under Effective Dynamical Scatter

Observables can be
viewed as functionals of
paths where distortions
can be introduced
 $O=F[x(t)]$

$$\dot{x}(t) = b(x(t)) + \eta(t)$$

The Langevin equation

The Martin-Siggia-Rose-Janssen-De Dominicis (MSRJD) formalism

$$Z = \int \mathcal{D}\eta \mathcal{D}x \mathcal{D}\hat{x} e^{\int dt \hat{x}^T (\dot{x} - b - \eta)} e^{-\frac{1}{2} \int dt \eta^T D^{-1} \eta}$$

$$S_{\text{tot}} = S_G[G_{0:K}] + S_x[x_{0:K}; G_{0:K}] + S_{\text{bnd}}[x_{0:K}; G_{0:K}]$$

Future extension



S_G :
structure term
(graph evolution)



S_x :
internal term
(formation physics)



S_{bnd} :
boundary term
(environment)

First example: A path measure model decomposition

Hierarchical structure formation

Graph-trajectory measure

$$P(\mathbf{x}, G) = P(\mathbf{x} \mid G) P(G)$$

Graph-conditioned path measure

$$P(\mathbf{x} \mid G) \propto p_{\text{attach}}(\mathbf{x} \mid G) \exp[-S(\mathbf{x}; G)]$$

Evolution in isolation

Evolution in environment

Probabilistic halo graph construction for clustering

Yang & Yu 2023 PR Research:

- Halo graphs can be constructed using **preferential attachment**
- They are power-law graphs

$$\text{Attachment Probability} = \frac{A_i}{\sum_j A_j}$$

Preferential attachment

- Rich becomes richer
- Early attachment advantage

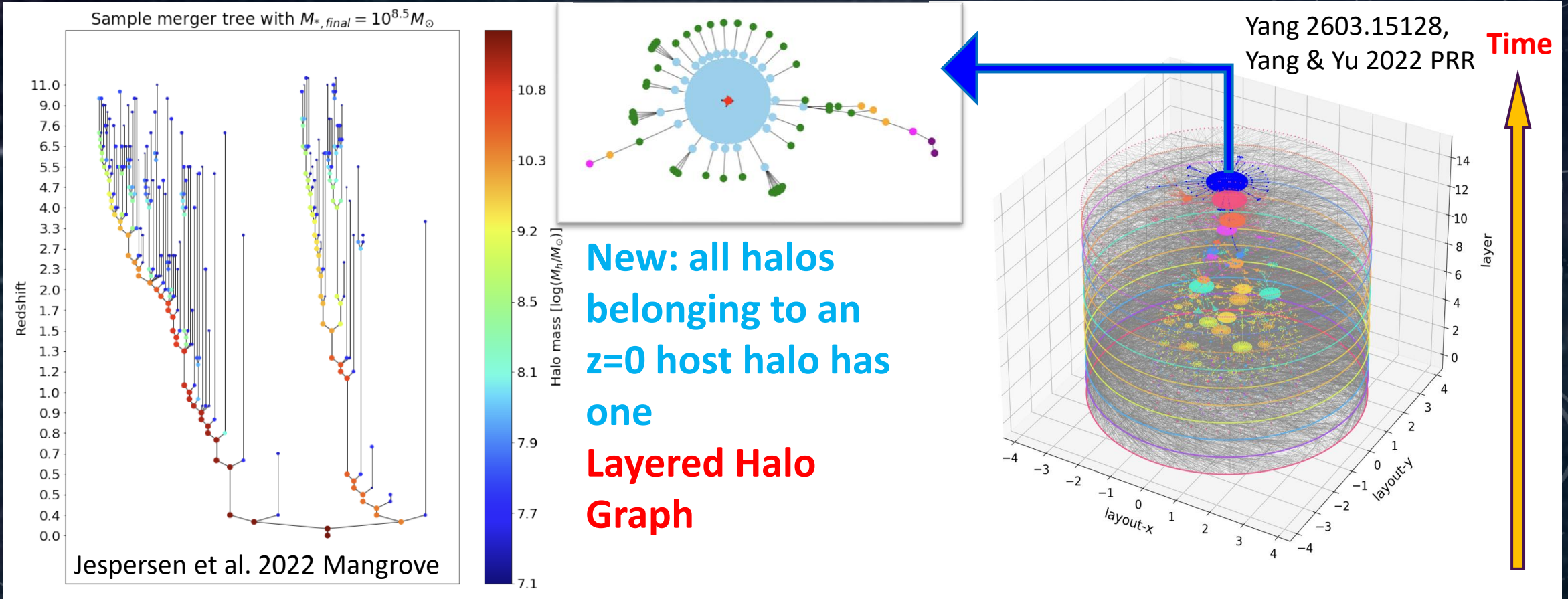
Self-similarity: Hierarchical structure naturally arise



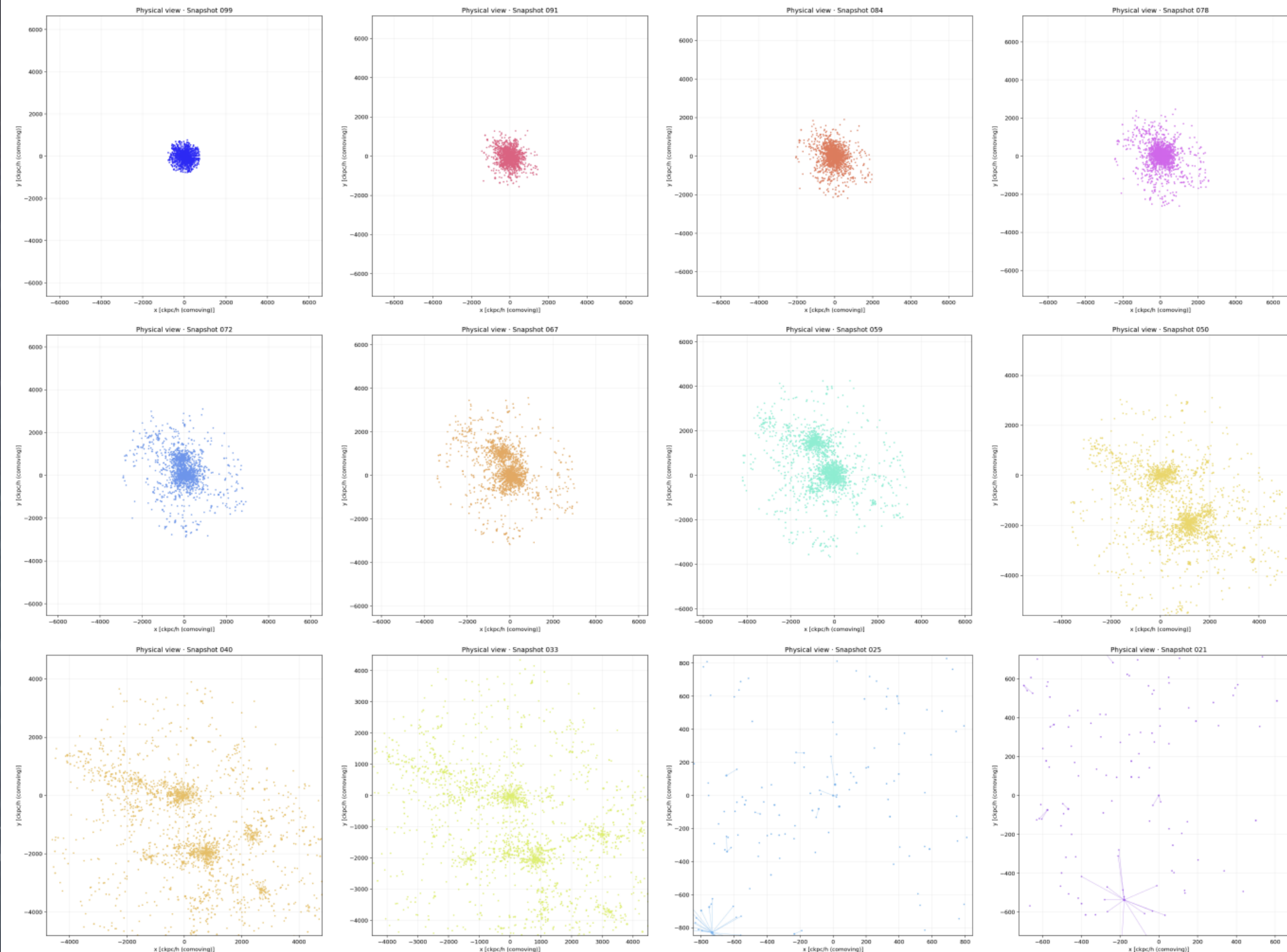
A natural object to encode hierarchical structure formation and environmental features

One halo at $z=0$ has one merger tree

Grouped histories for main and sub halos



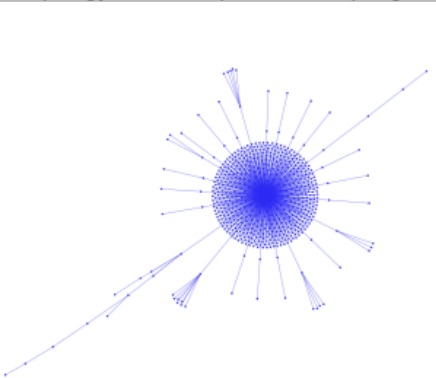
Probability-based construction naturally defines a graph measure $P(G)$



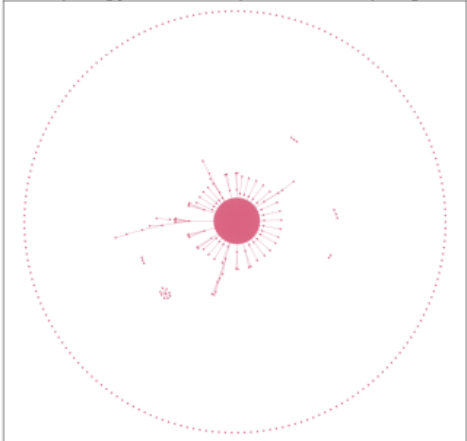
**Physical
(comoving-
coordinate)
view**

**Hierarchical
structures
merge into a
self-bound halo**

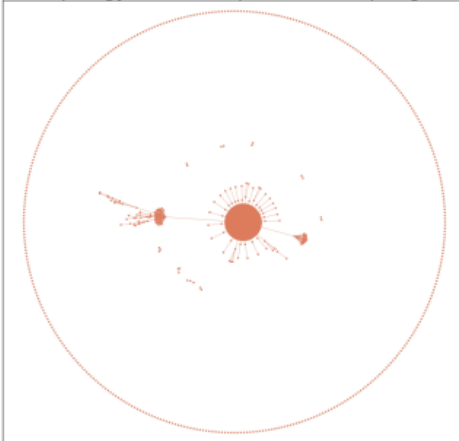
Topology view · Snapshot 099 (spring)



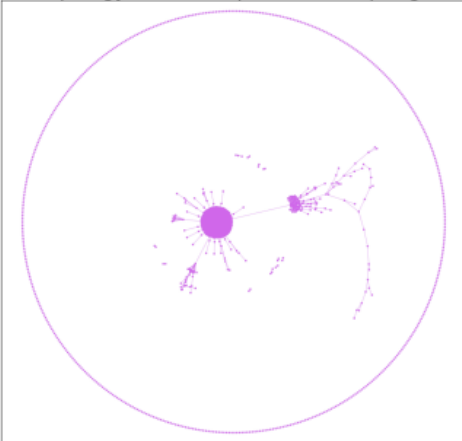
Topology view · Snapshot 091 (spring)



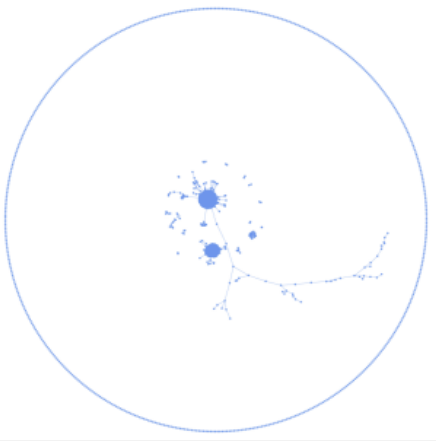
Topology view · Snapshot 084 (spring)



Topology view · Snapshot 078 (spring)



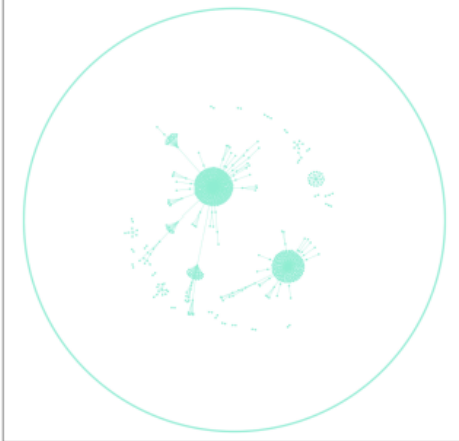
Topology view · Snapshot 072 (spring)



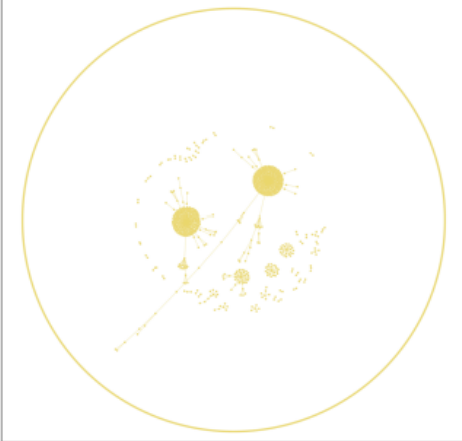
Topology view · Snapshot 067 (spring)



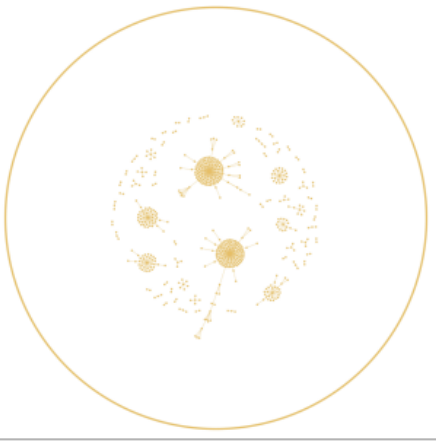
Topology view · Snapshot 059 (spring)



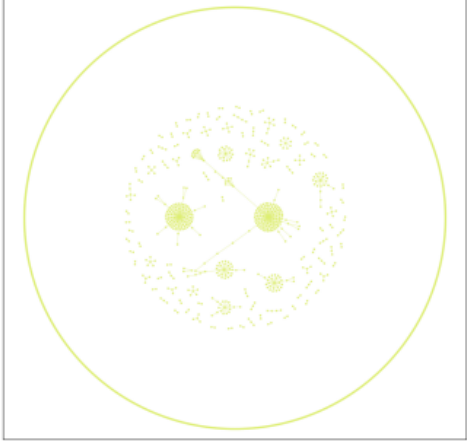
Topology view · Snapshot 050 (spring)



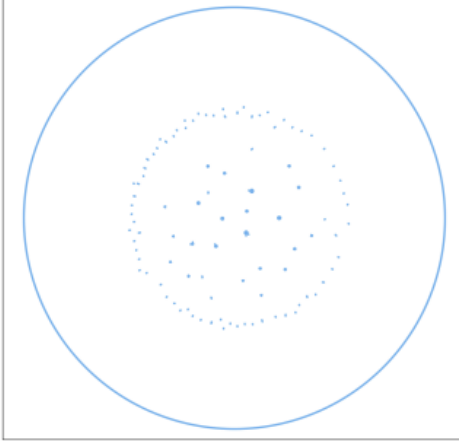
Topology view · Snapshot 040 (spring)



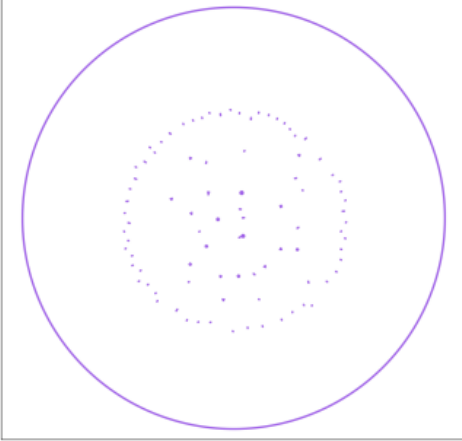
Topology view · Snapshot 033 (spring)



Topology view · Snapshot 025 (spring)



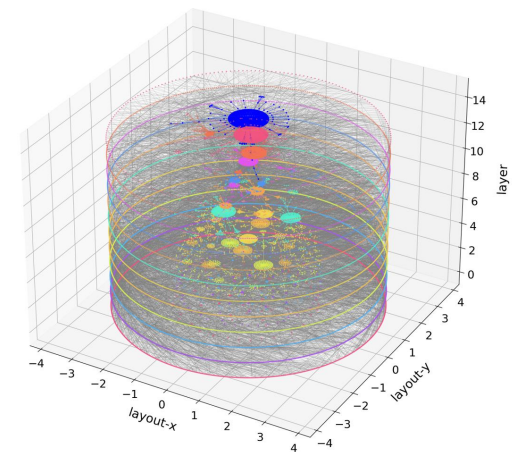
Topology view · Snapshot 021 (spring)



Topological view

- Point has no size
- Edge has no length

Stack the layers



An update: A path measure model from machine learning

Graph encoding,
conditioning



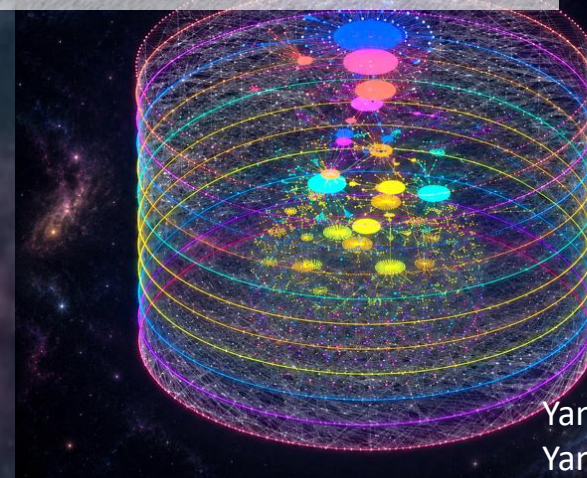
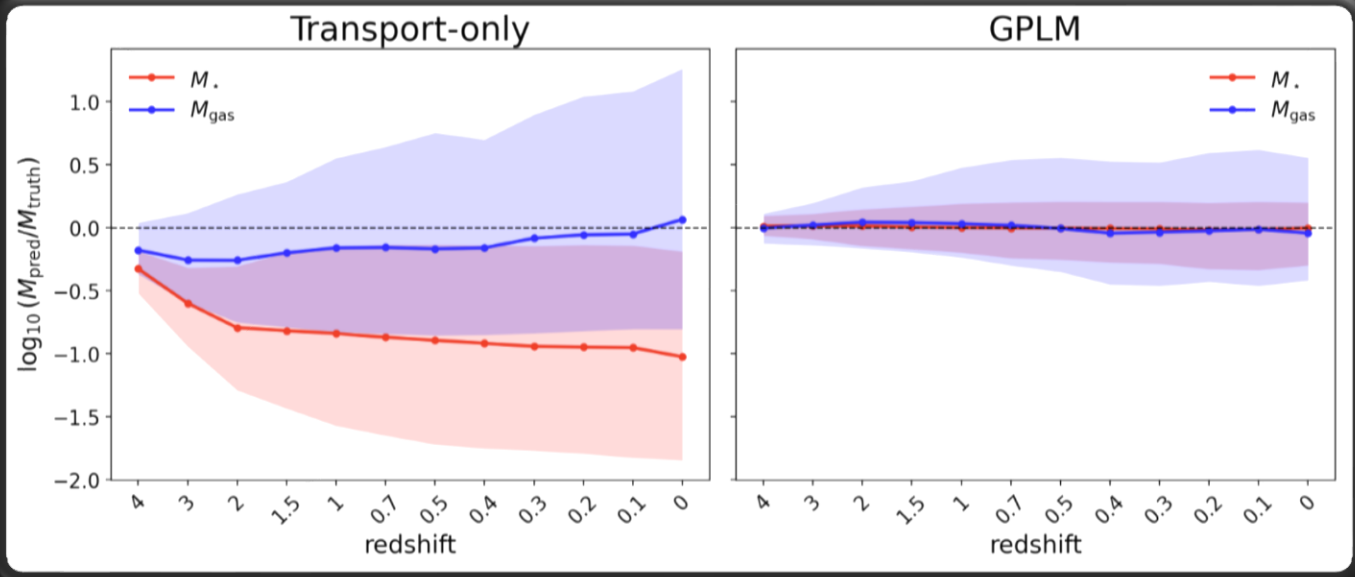
Graph Neural
Network



➤ b: learned residual drift
➤ D: learned diffusion

$$S_{\text{OM}}[\mathbf{x}; G] = \frac{1}{2} \sum_{(i,k) \in \mathcal{V}_{\text{sup}}} \left[(\Delta x_{i,k} - \delta_{i,k})^T \hat{\Sigma}_{i,k}^{-1} (\Delta x_{i,k} - \delta_{i,k}) + \log \det \hat{\Sigma}_{i,k} \right]$$

$$P(\mathbf{x}|G) \propto p_{\text{attach}}(\mathbf{x}|G) e^{-S(\mathbf{x};G)}$$



Yang 2603.15128,
Yang & Yu 2022 PRR

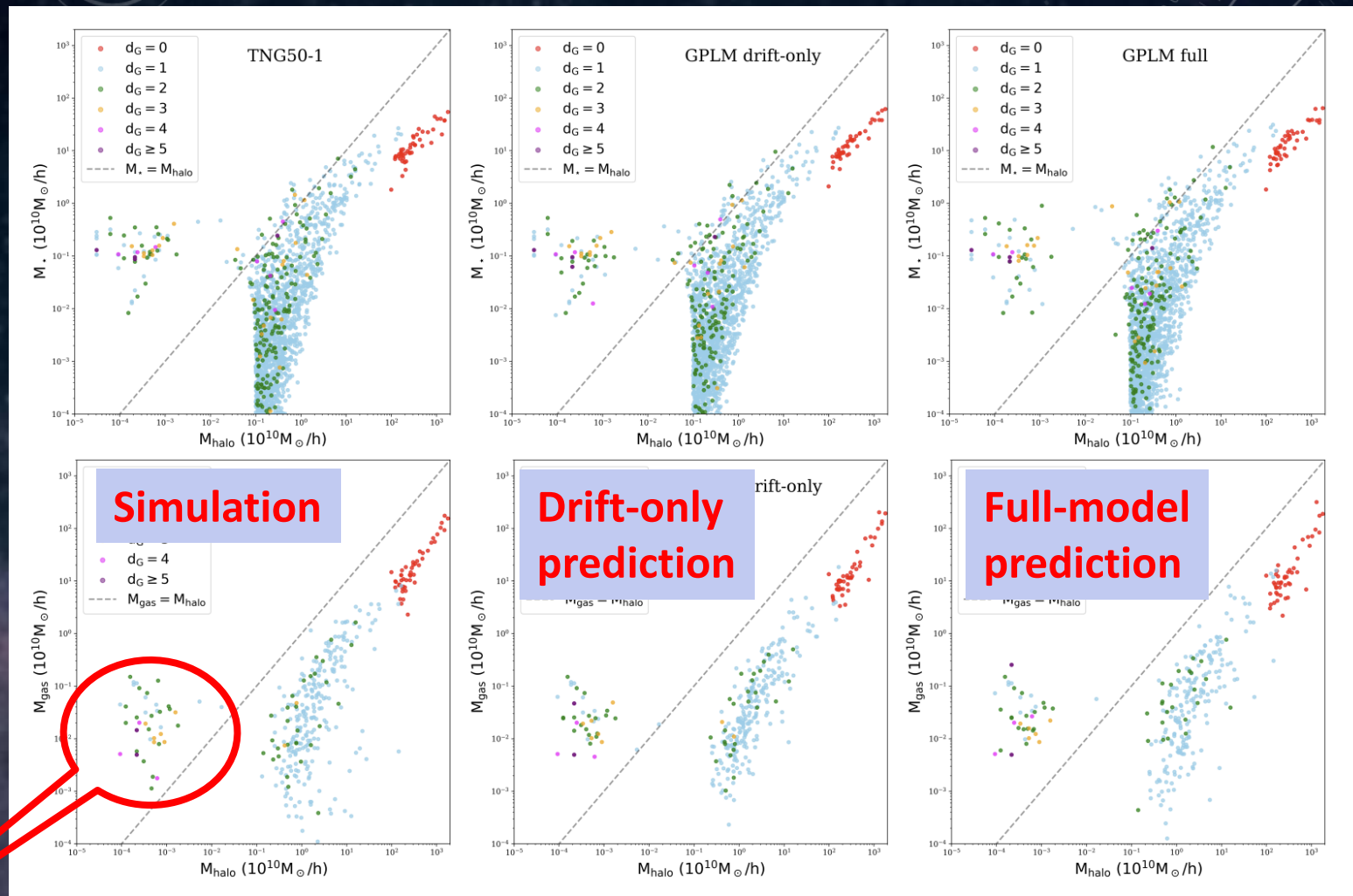
GPLM substantially reduces both the median bias and the spread of the residual distribution.

Training minimizes a discrete OM action build on data across histories: **6-layer (13 total) window**

Model performance

The trained GPLM reliably reproduces the dG-split population structure observed in the simulation

Dark Matter Deficient Galaxies



- Different sampled path batches represent **distinct stochastic realizations** of the same learned effective dynamics.
- The endpoint observables retain **analogous stochastic structure** across these different realizations.

Theoretical Perspectives

Non-Gaussian
scatter

Geometric
interpretation

Optimal Transport

The action can be rewrite as an energy functional of a curve in Riemannian geometry

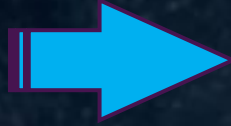
$$S = \frac{1}{2} \int dt v^T g v \quad v \equiv \dot{x} - b(x) \quad \ddot{x}^i + \Gamma_{jk}^i \dot{x}^j \dot{x}^k = F^i(x, \dot{x})$$

Most probable path=geodesic in $g=D^{-1}$ under an effective potential

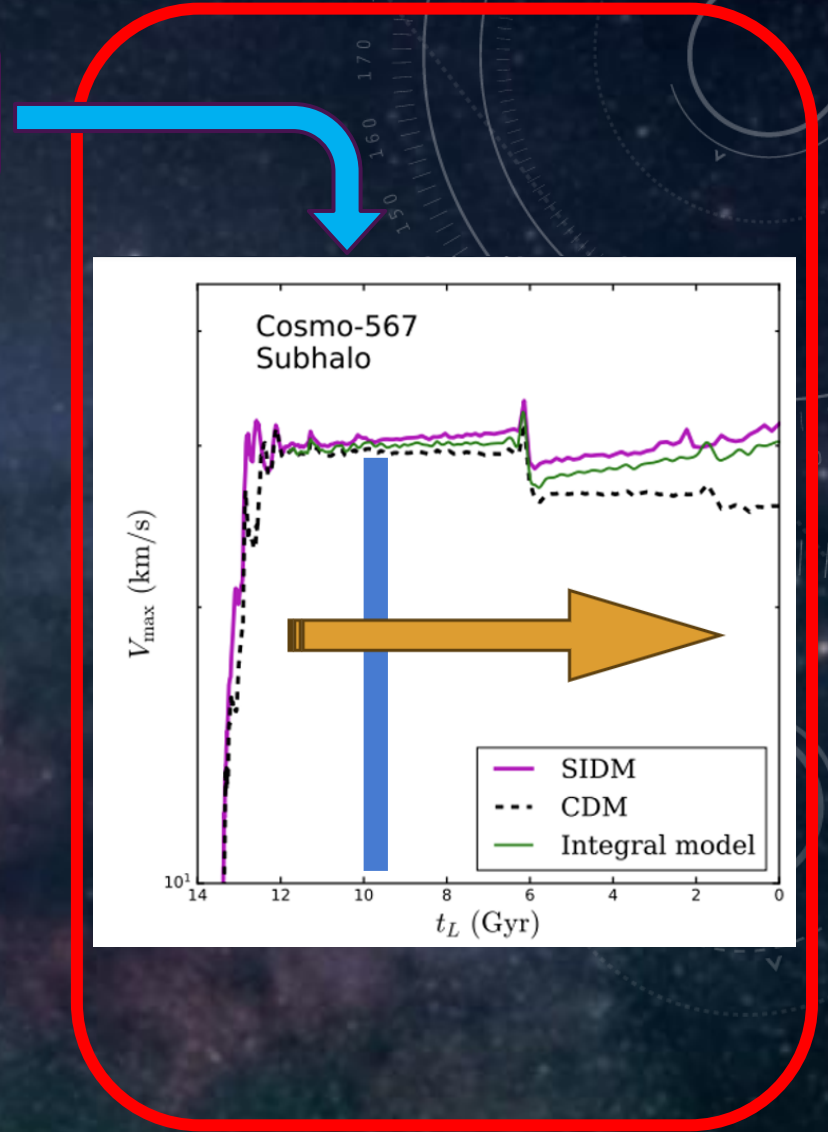
Galaxy evolution path=geodesic in environment-shaped metric+drift forcing

SIDM as controlled deformation on graph paths

Analytic kernel



Neural Network Agent



- **SIDM deforms ($V_{\max}, R_{\max}$) per time step**
- **Environmental Paths change endpoint observables**

$$\mathbf{y}_k \equiv (V_{\max}, R_{\max})_k$$

$$\Delta \mathbf{y}_k = \Delta \mathbf{y}_k^{\text{tr}} + \Delta \tau_k \mathbf{b}_{\text{SIDM}}(s_k; \theta) + \epsilon_k$$

(Non-linear)
Transport

Residual
Drift

Residual
Scatter

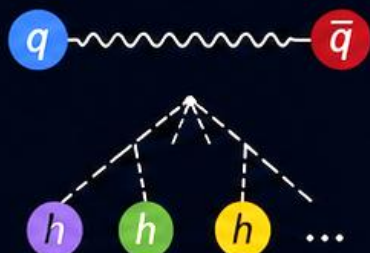
Two effective kernels in larger forward models

Broadly: SIDM, dissipation, heating, mass change...

LUND STRING MODEL (Hadronization Kernel)

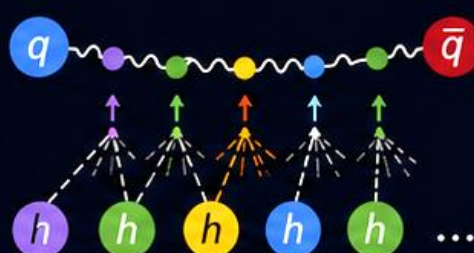
Then (late 1970s)

Simple and not accurate



Now (Today)

Mature, precise, widely used



- ✗ Simple 1D string picture
- ✗ Few parameters
- ✗ Phenomenological
- ✗ Not accurate

- ✓ Extensive extensions (flavors, diquarks, CR, ropes...)
- ✓ Global tuning to collider data
- ✓ Uncertainty estimates
- ✓ Essential kernel inside PYTHIA



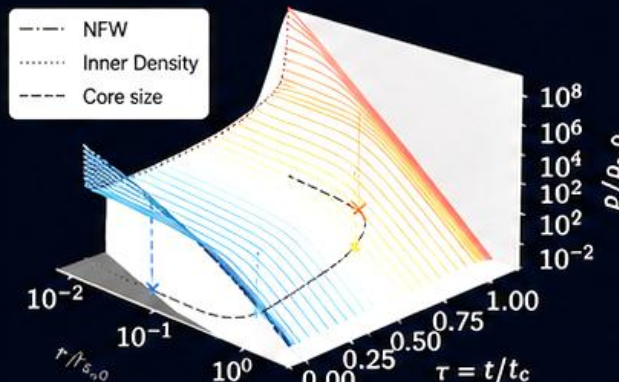
Role: Hadronization kernel inside PYTHIA

Transforms colored partons into hadrons for the rest of the generator.

SIDM PARAMETRIC MODEL (Halo Evolution Kernel)

Today

Very simple but physically motivated



- Parametric evolution ($\tau = t/t_c$)
- Universal inner-profile behavior
- Few calibrated parameters
- Calibrated to controlled simulations
- Preliminary, not yet complete



Role: Halo evolution kernel inside a SAM / galaxy-formation model

Evolves dark-matter halos for the rest of the model.

Future

Toward a next-generation kernel

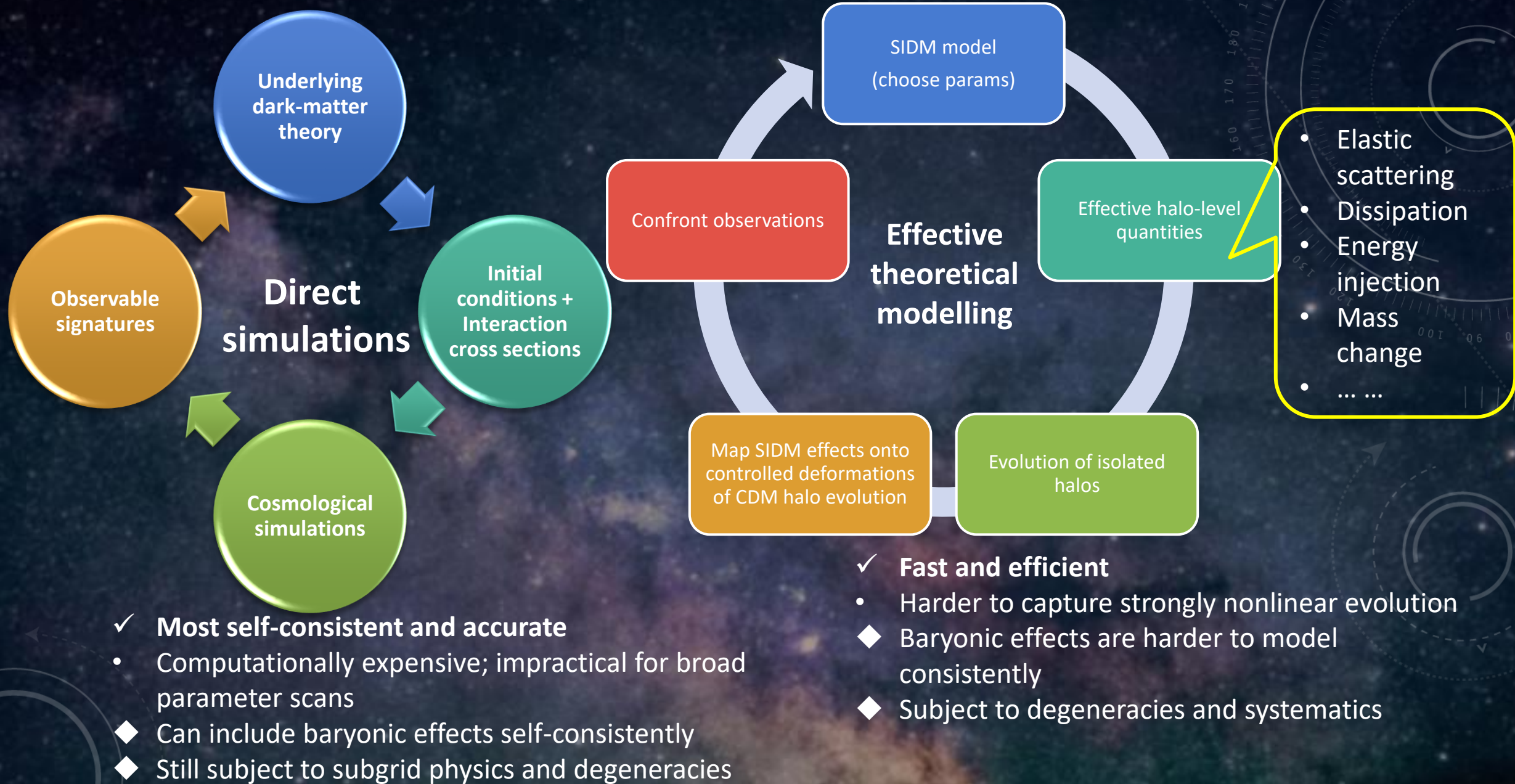


- ✓ Baryons, accretion, tides, stripping
- ✓ Stochastic path ensemble (GPLM)
- ✓ Uncertainty quantification
- ✓ Lensing & multi-probe tests
- ✓ Public tuning & benchmark suite

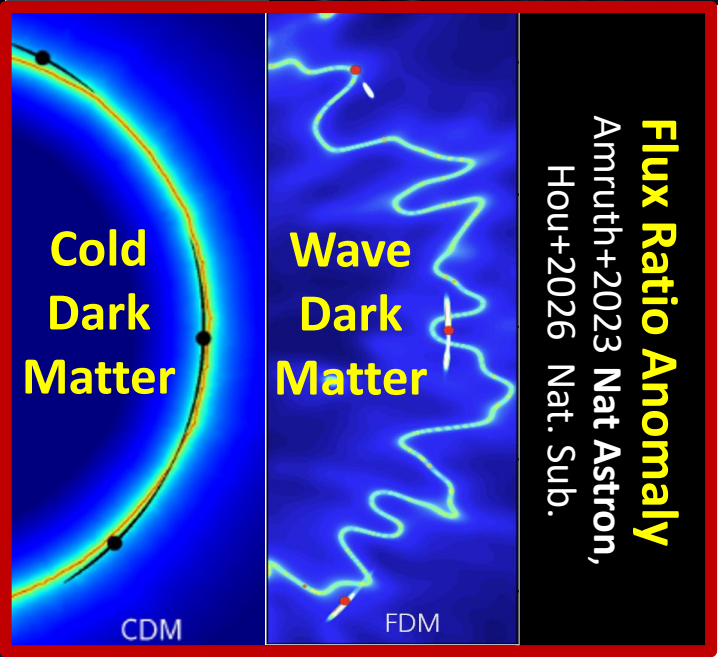


Key Message: Both kernels start simple and are systematically extended, calibrated and validated to become indispensable components of larger predictive frameworks.

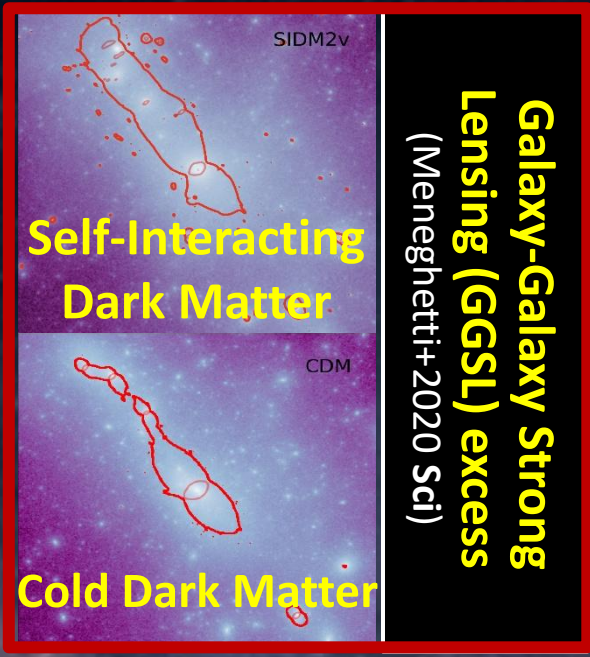
Summary: From Microphysics to Halo Populations



Opportunities in the Era of Next-Generation Observations



Flux Ratio Anomaly
Amruth+2023 Nat Astron,
Hou+2026 Nat. Sub.



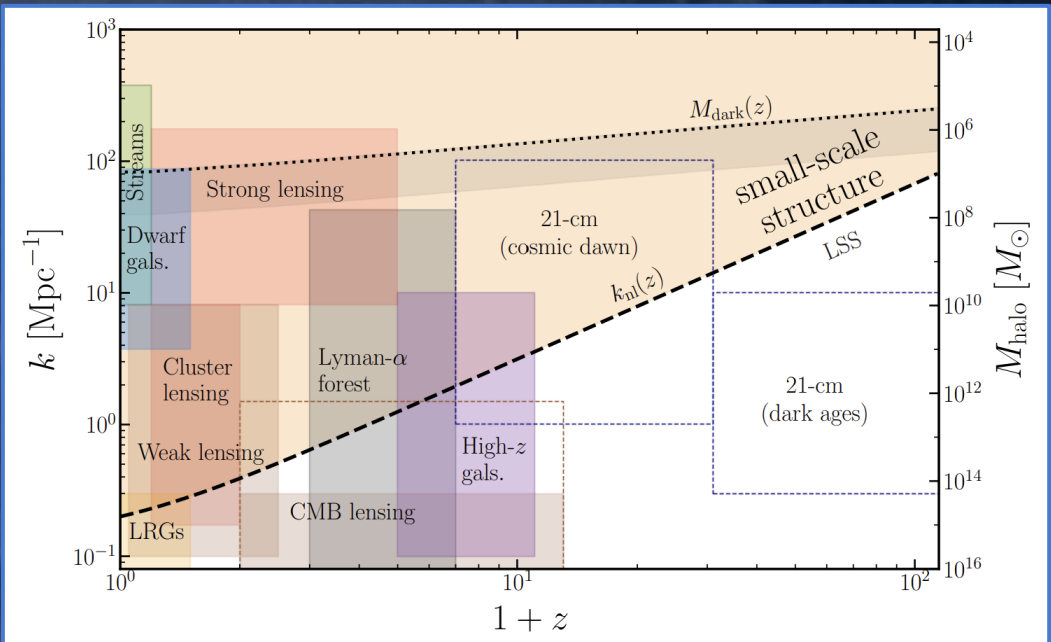
Galaxy-Galaxy Strong Lensing (GGSL) excess
(Meneghetti+2020 Sci)

Different probes create a dark matter laboratory across scales and redshifts

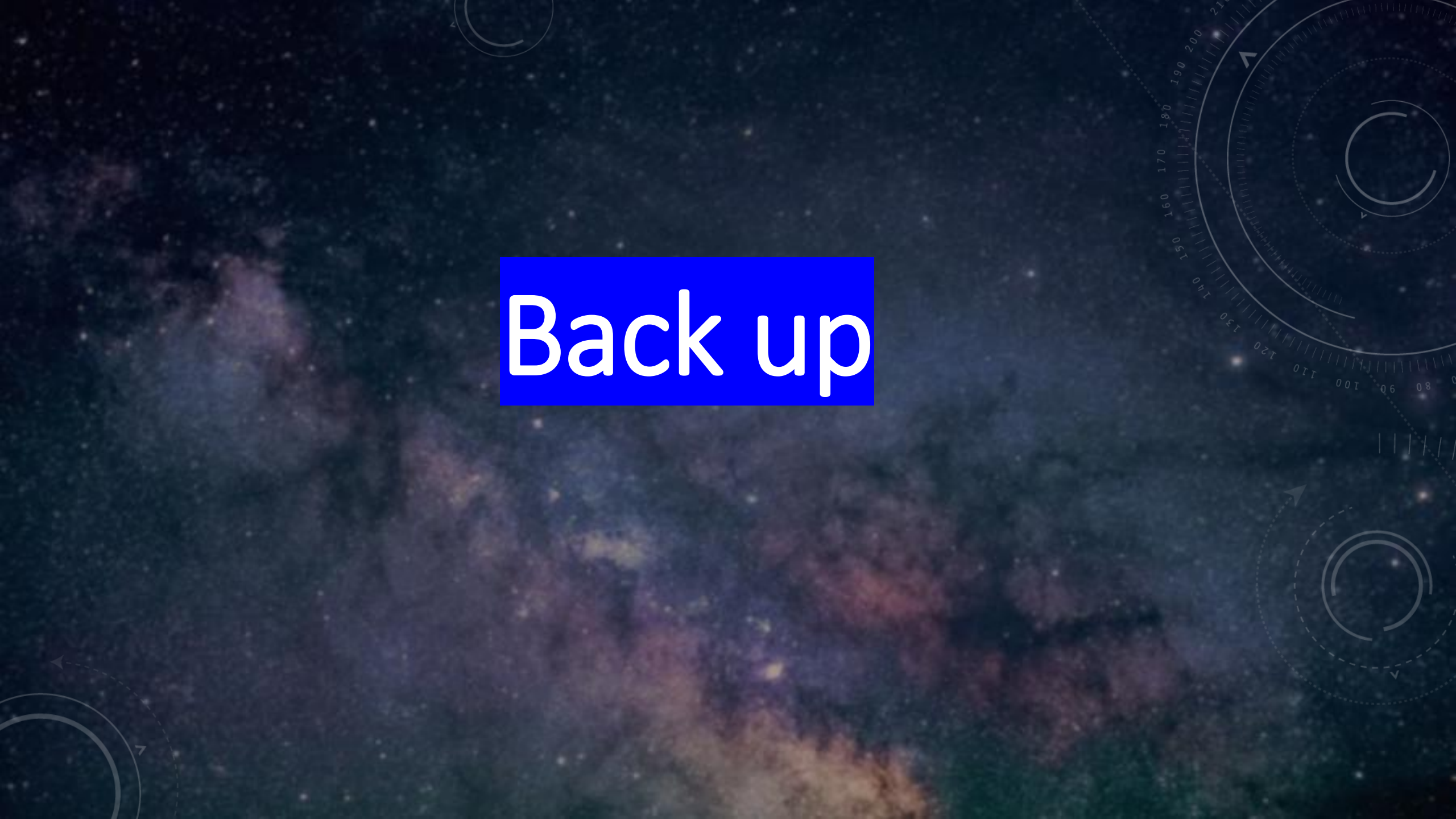
Past: If dark matter exist?
Future: What are the properties of DM?

More precise: Each galaxy–halo system is an independent dark matter probe.
More samples: Large surveys enable population studies and studies of rare systems.

Nadler et al. 2026, Sabti et al. 2022, Boddy et al. 2022



Thanks for your attention!

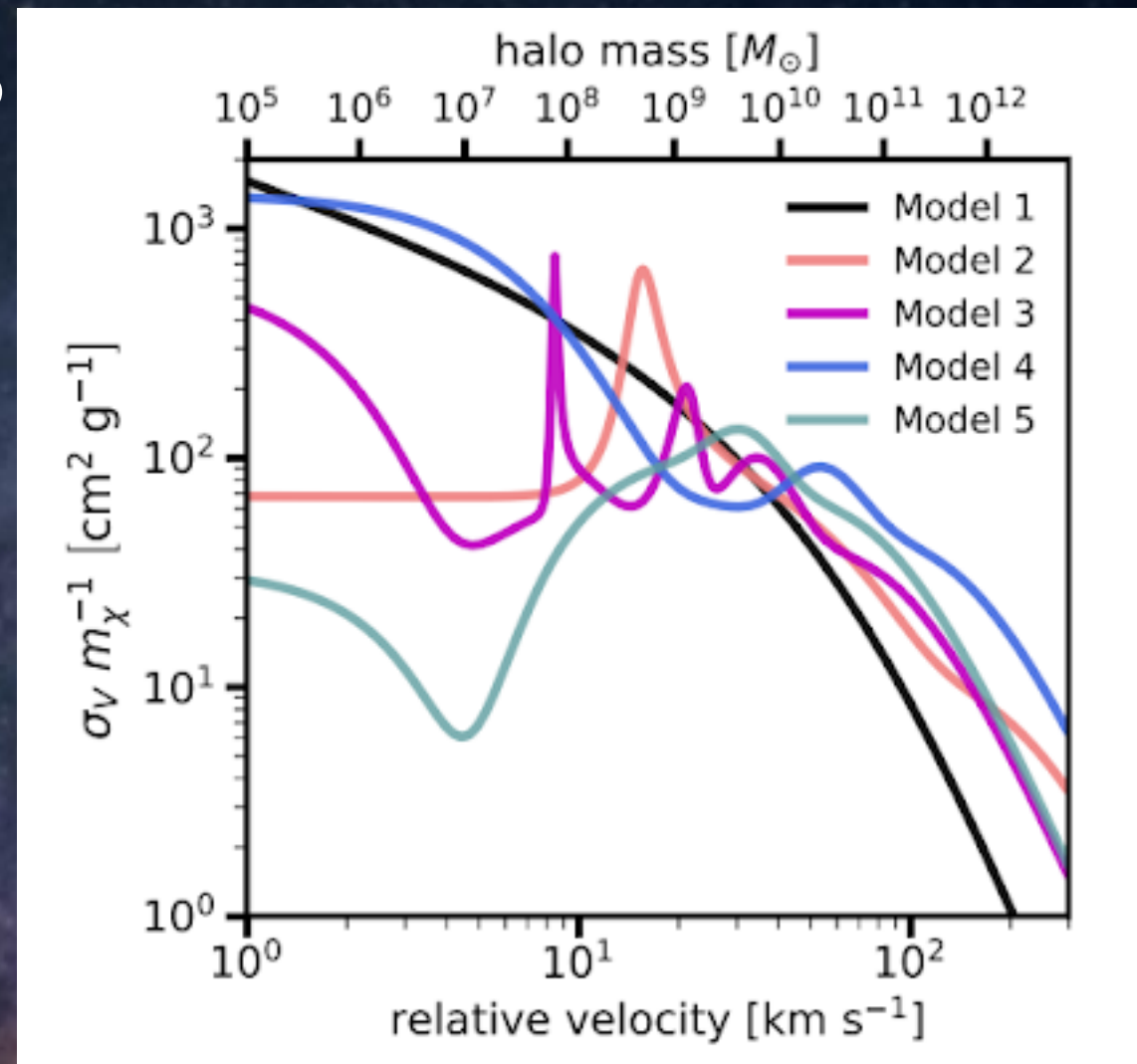
The background is a deep space image featuring a prominent purple and blue nebula. Overlaid on this are several technical graphics: a large circular scale on the right with degree markings from 0 to 210, and several concentric circles with arrows indicating clockwise rotation in the corners.

Back up

SIDM CROSS SECTIONS CAN HAVE NONTRIVIAL VELOCITY DEPENDENCIES

- Yukawa potential/Gravity: v^{-4} at large v
- Massive mediator: flatten the inner dependence
- Quantum resonances

Interface to **new physics** (SIDM) models?



Gilman+2207.1311

INCORPORATING BARYONS

A new equation for the core collapse time

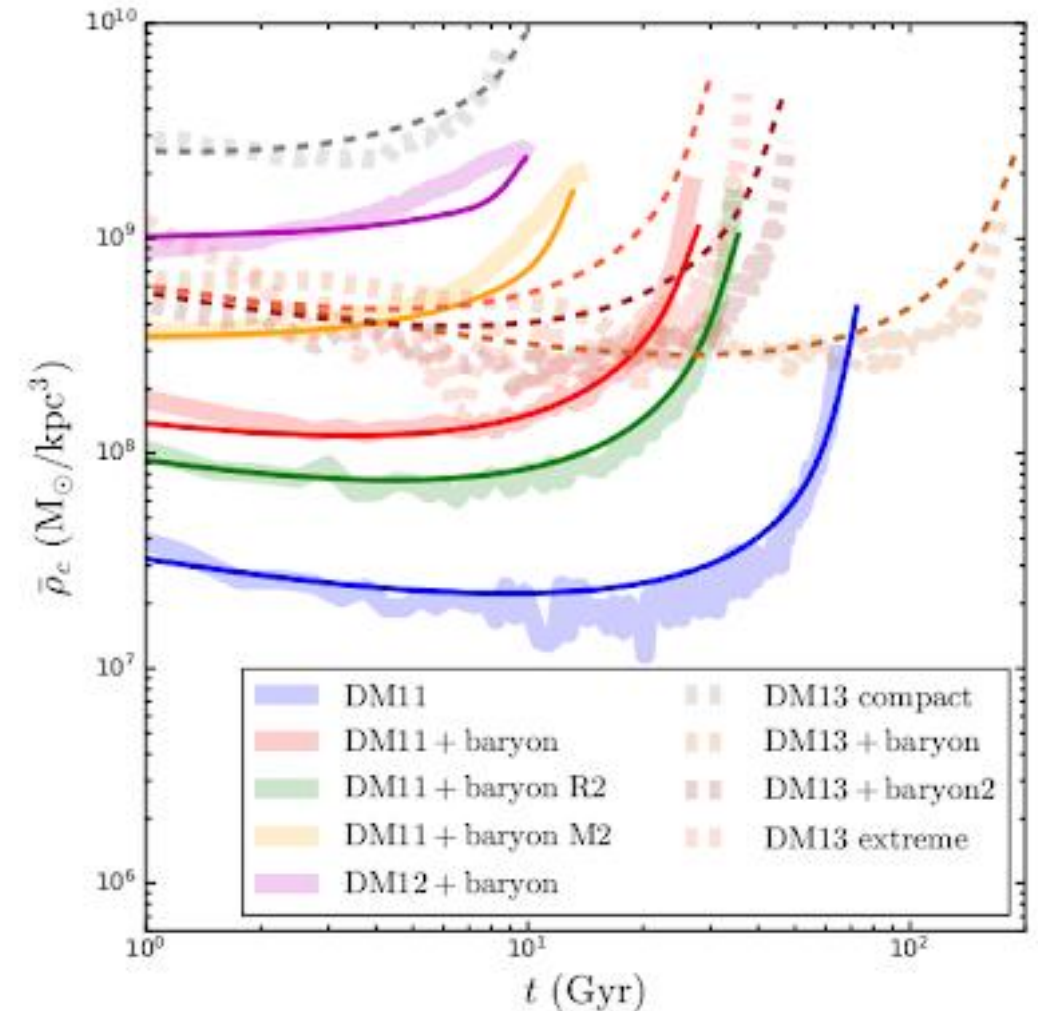
- Based on energy transfer
- Incorporate baryons
- *Collision rate => Energy transport*

$$t_R(r) \propto \frac{M(r)|\Phi(r)|/2}{4\pi r^2 \kappa |\nabla T|}$$

A density profile that allows incorporating adiabatic contraction effect

$$\rho_{\text{CoredDZ}}(r) = \frac{f_{\text{in}}(r)\rho_x f_{\text{out}}(r)}{\frac{(r^k + r_c^k)^{1/k}}{r_x} \left(1 + \left(\frac{r}{r_x}\right)^{1/2}\right)^{2(3.5-a)}}$$

N-body simulation: bands
Parametric model: lines



PUTTING BARYON EFFECTS INTO A FORM FACTOR

$$t_{c,b} = t_{c,0} \mathcal{F}_t(\hat{\rho}_H, \hat{r}_H)$$

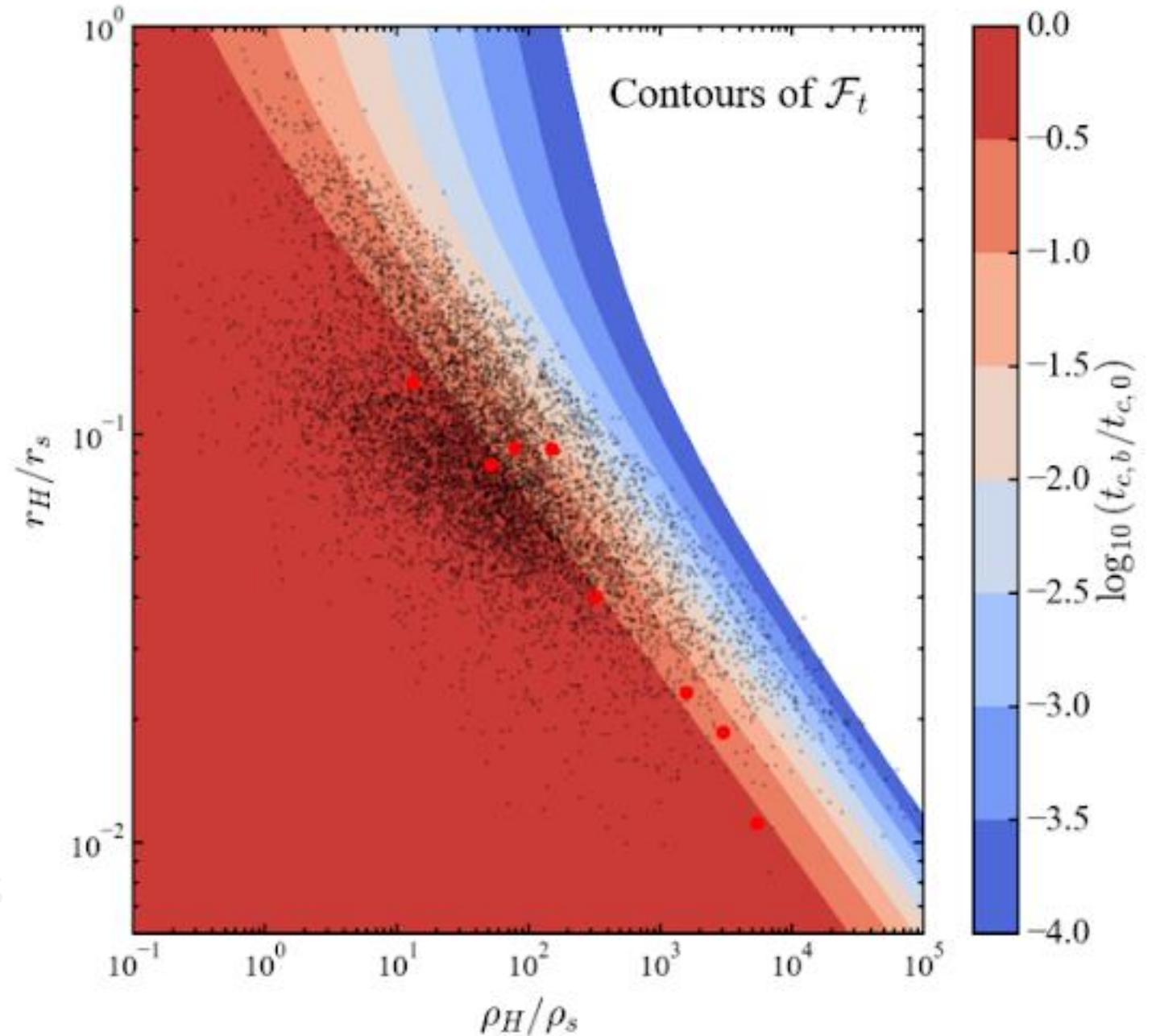
The ratio of core collapse times w&o baryons

$$\mathcal{F}_t = \left(\frac{1}{\hat{r}_{\text{eff}}} + \frac{\gamma \hat{\rho}_H \hat{r}_H^3}{\hat{r}_{\text{eff}} (\hat{r}_{\text{eff}} + \hat{r}_H)^2} \right)^{-1} \left(1 + \alpha \frac{\hat{\rho}_H \hat{r}_H^2}{2} \right)^{-\frac{1}{2}}$$

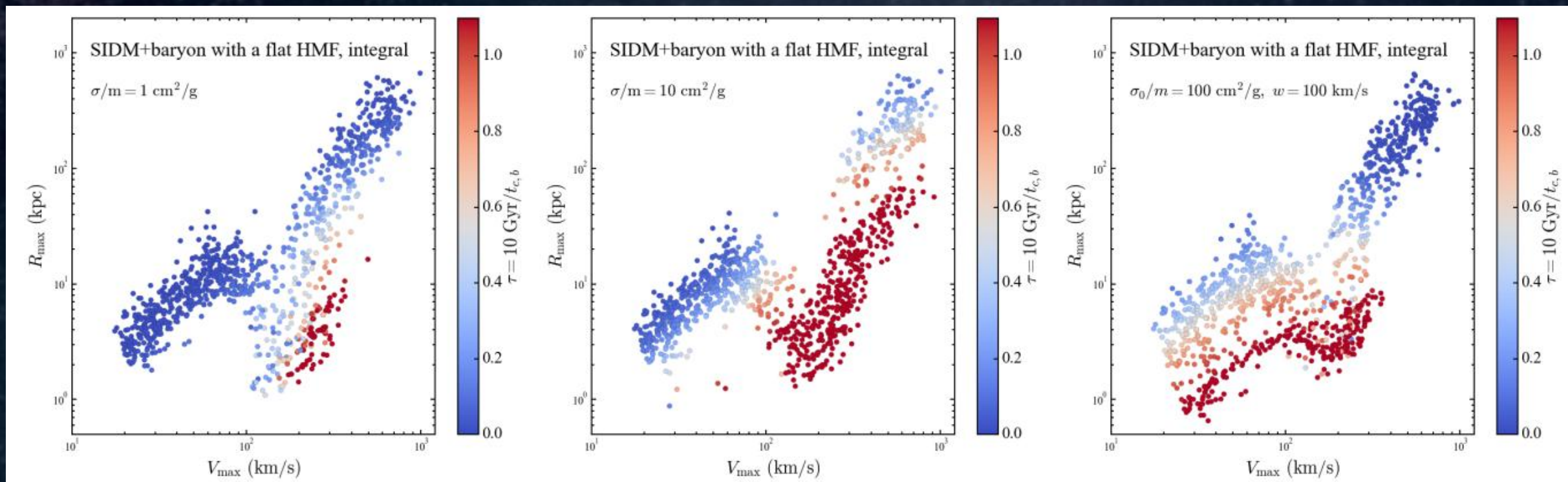
Lower-left: marginal effect

Upper-right: can be orders of
magnitude large

ρ_H , r_H : Hernquist scale radius
and density of the stellar
component



APPLICATION 1: MC GENERATOR FOR SIDM HALOS



- Three SIDM models
- Baryon contents based on known cosmological relations

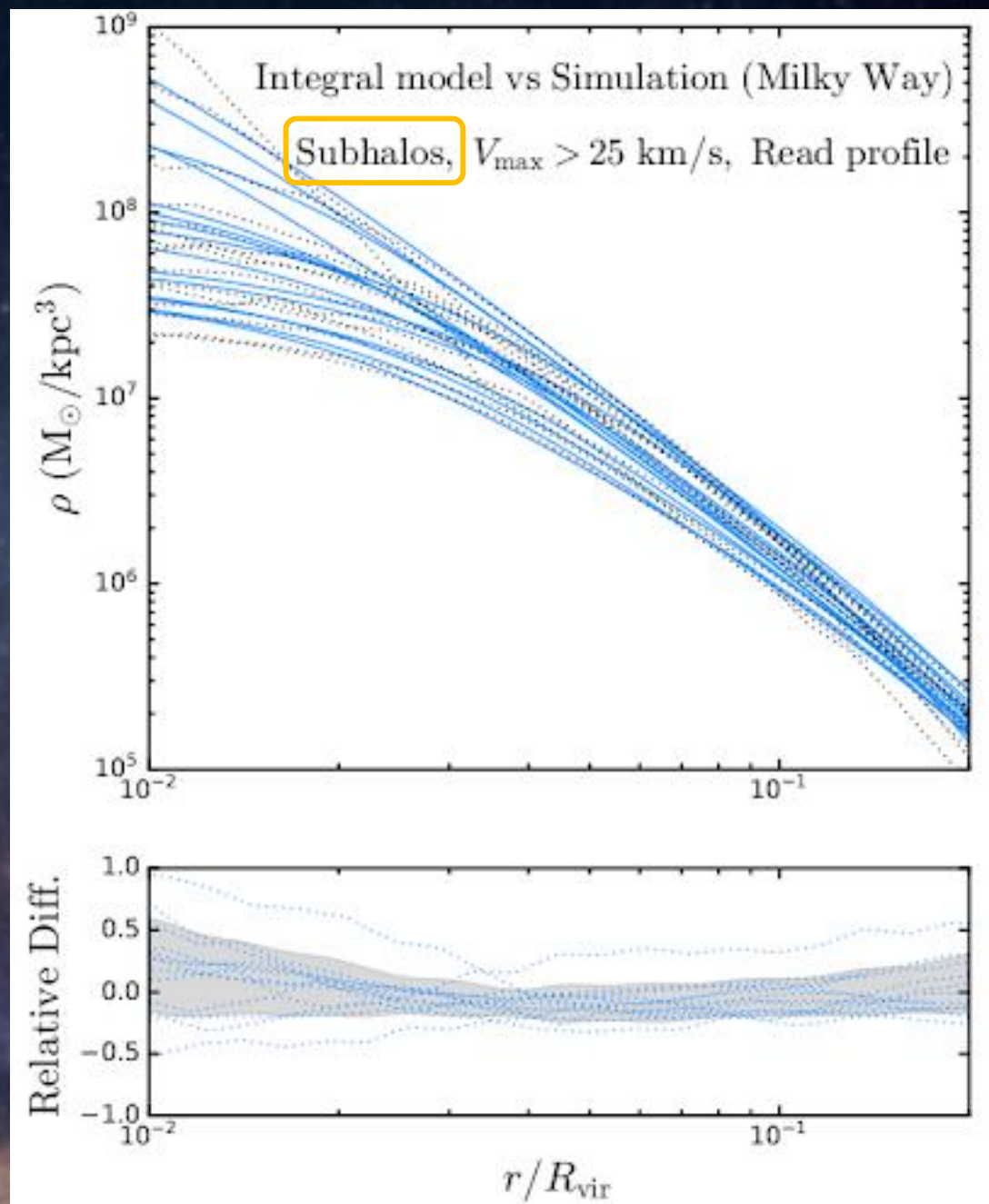
PARAMETRIC ANALYSIS TOOLS FOR SIDM HALOS

An efficient tool for obtaining SIDM predictions

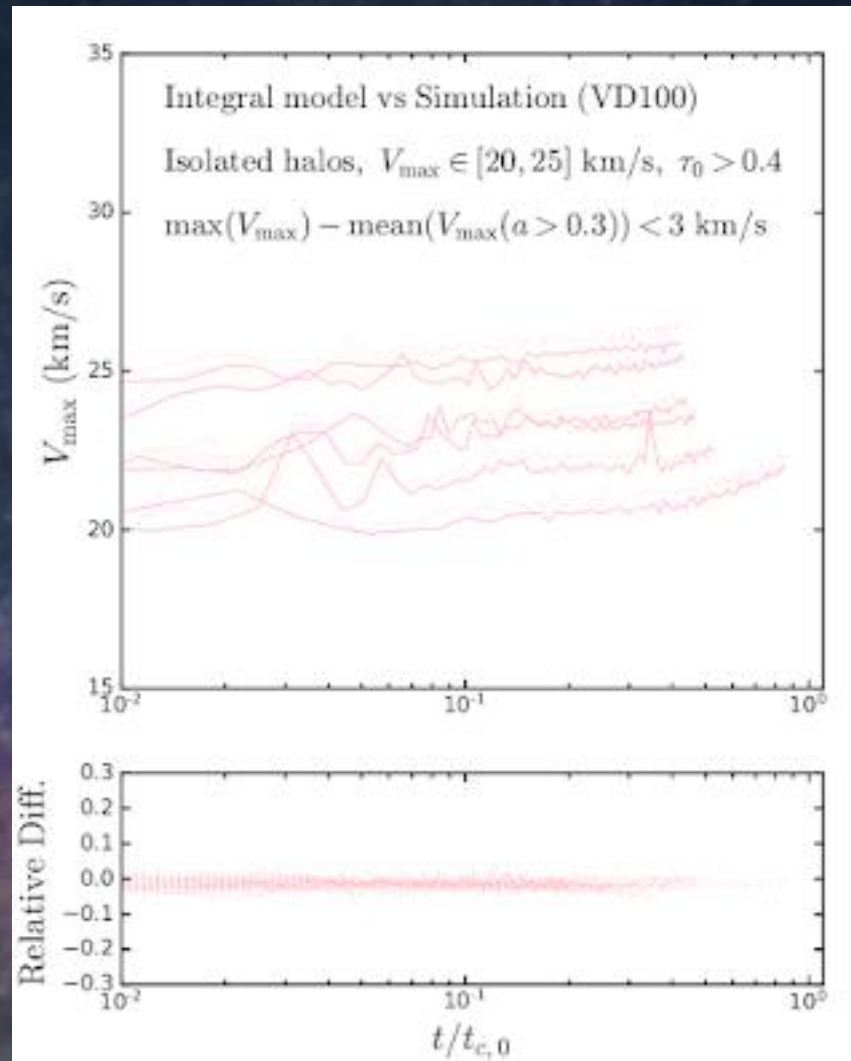
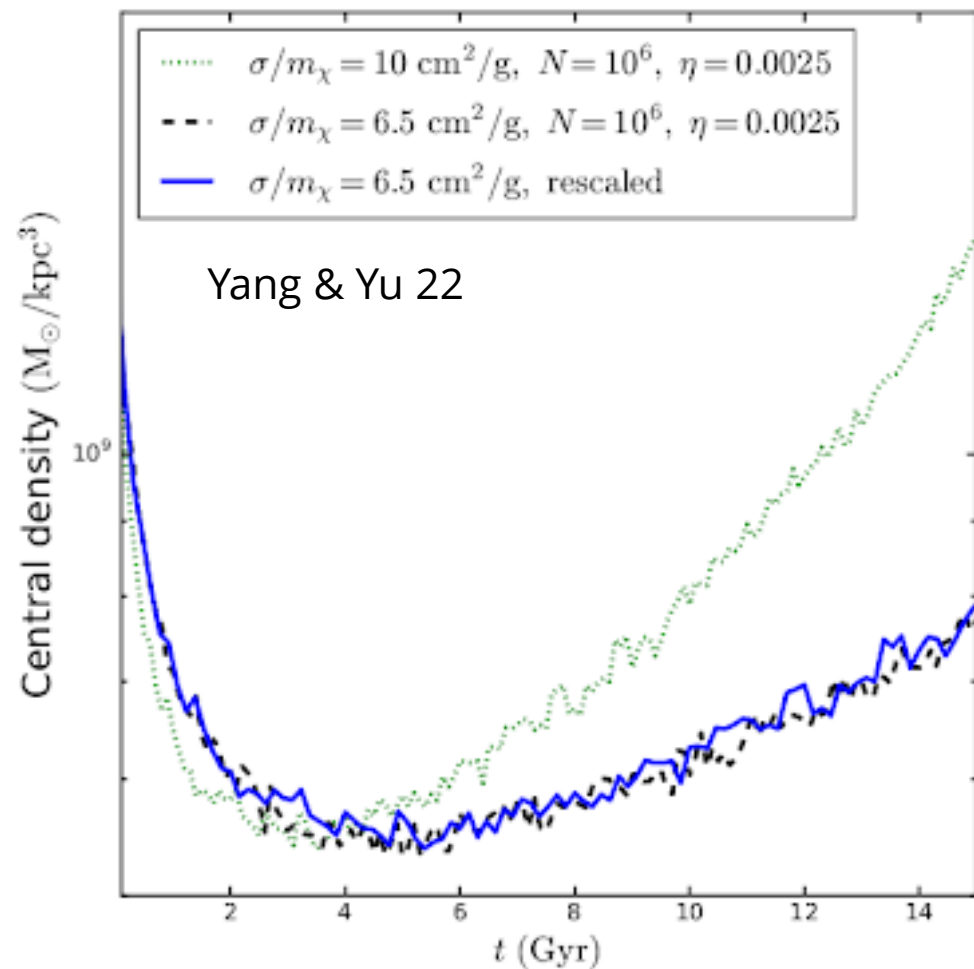
- Based on a few analytic functions/trajectories of the *gravothermal phase*
- Grounded in theory principles: not just an empirical model
- Tested against a large number of halos in cosmological simulations
- Has been extended to incorporate *mass changes* and *baryon potentials* [arXiv:2405.03787](https://arxiv.org/abs/2405.03787)

<https://github.com/DanengYang/parametricSIDM>

With Ethan O. Nadler, Hai-Bo Yu, Yi-Ming Zhong, S. Ando, S. Horigome



HOW WELL IS TC BEING PROPORTIONAL TO SIGMA/M?



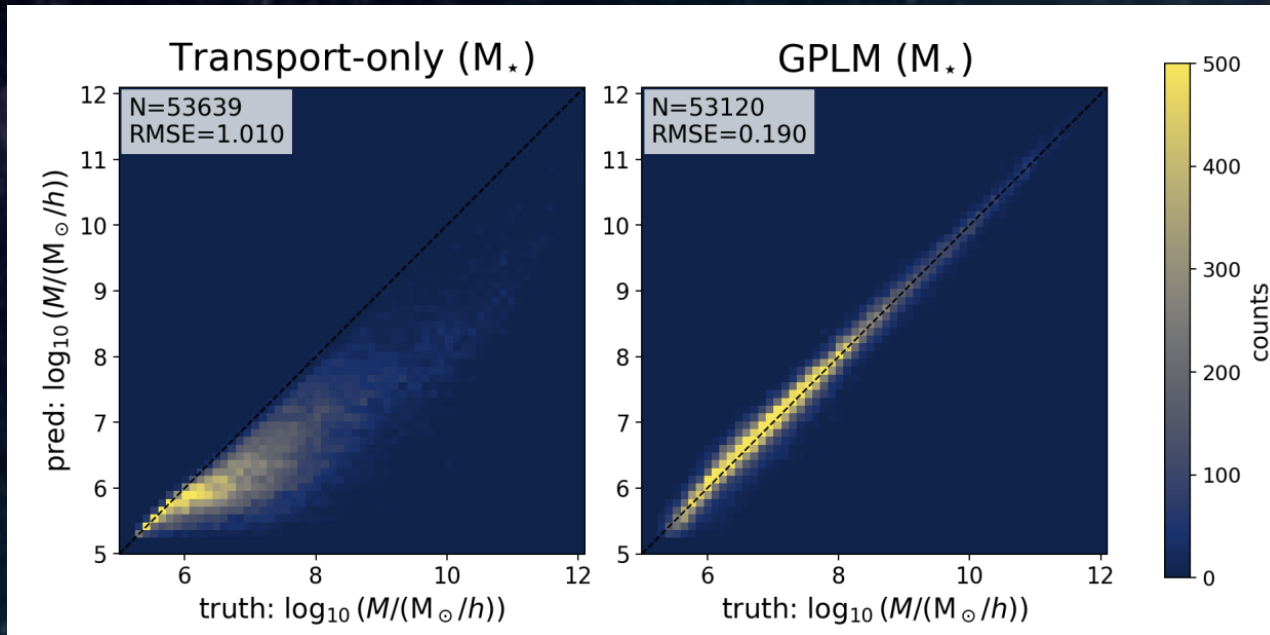
“Isolated” halos
in cosmological
simulation (Yang,
Nadler, Yu 23)
with

- $t/t_c > 0.4$
- Exclude cases with major mergers

Model vs
simulation

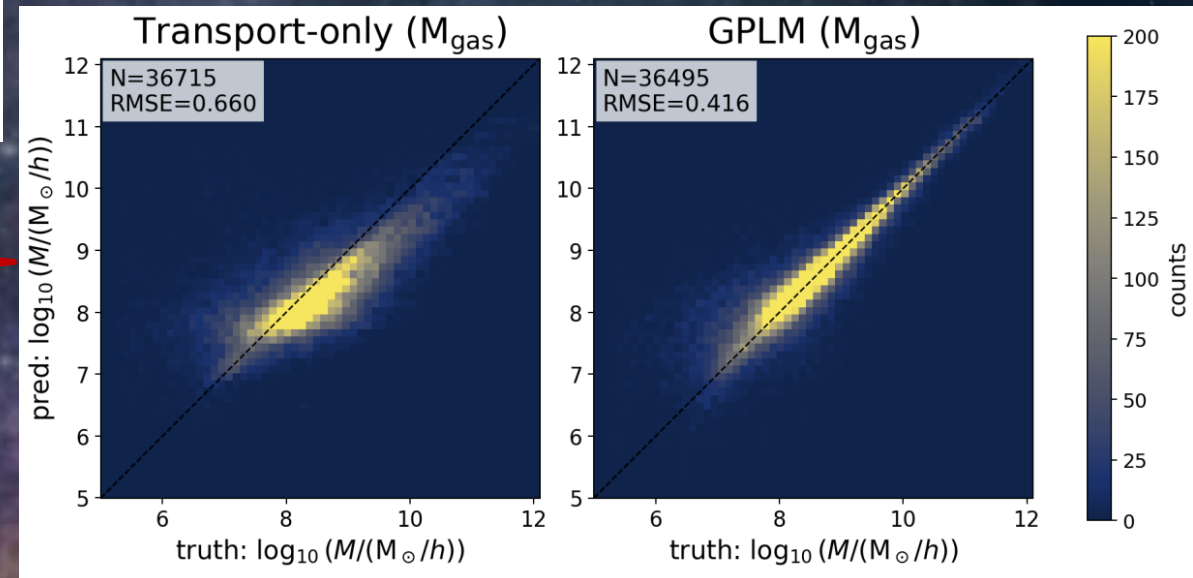
More in
YNY2406.10753

Model performance

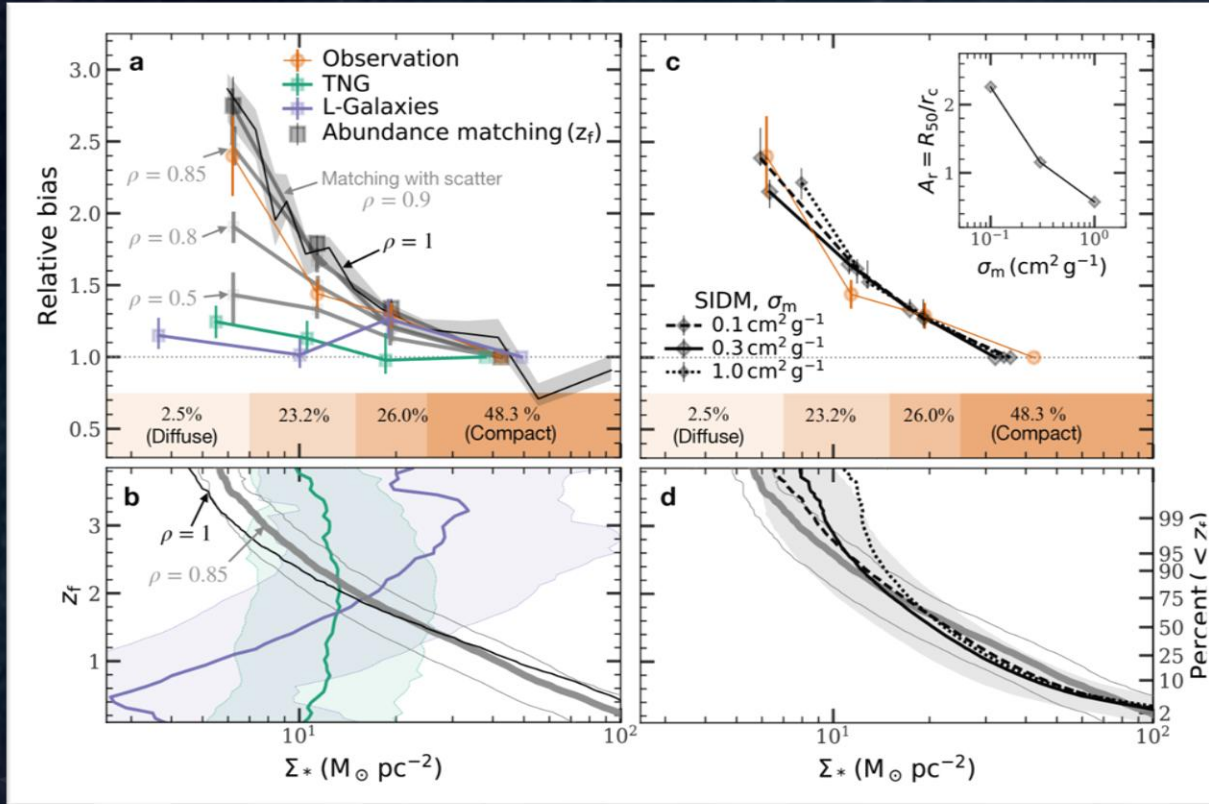


Prediction vs Truth in Stellar Mass

Prediction vs Truth in Gas Mass



Example (where path matters) 1: time-dependent cores in SIDM



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Article | Published: 21 May 2025

Unexpected clustering pattern in dwarf galaxies challenges formation models

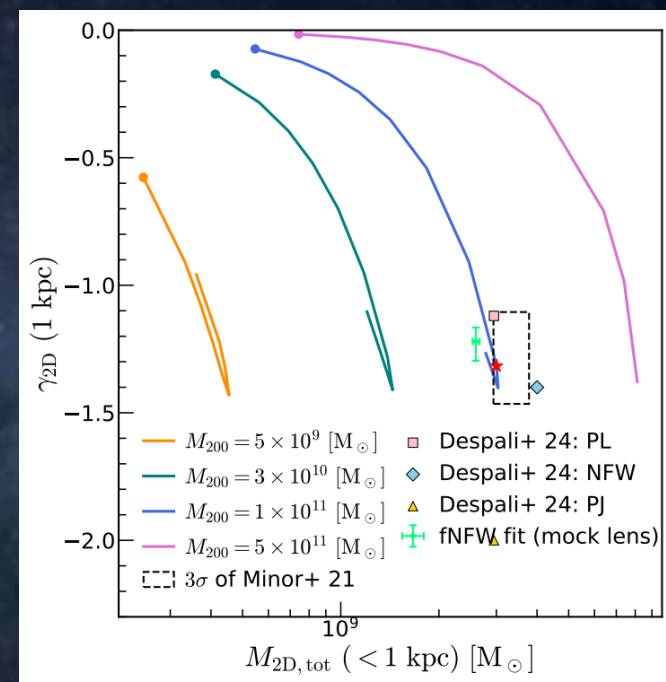
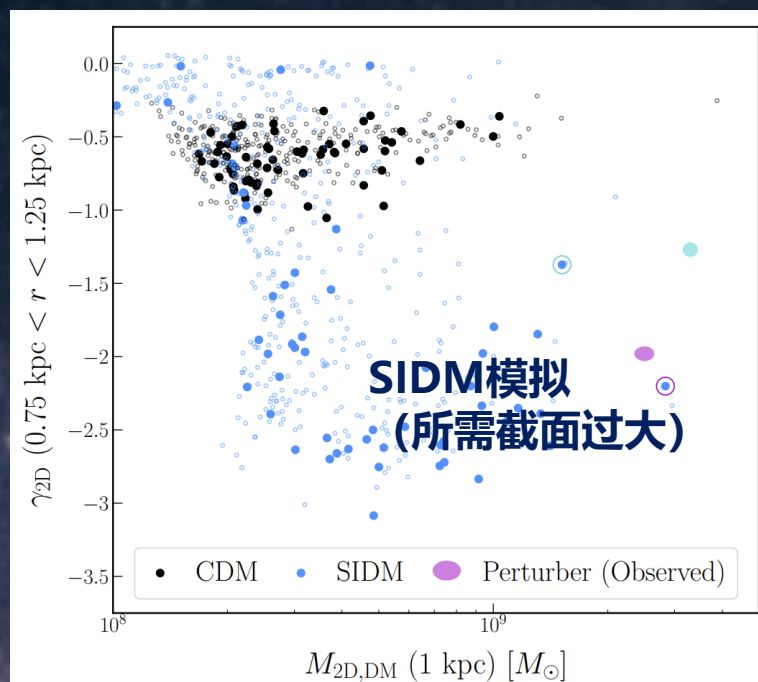
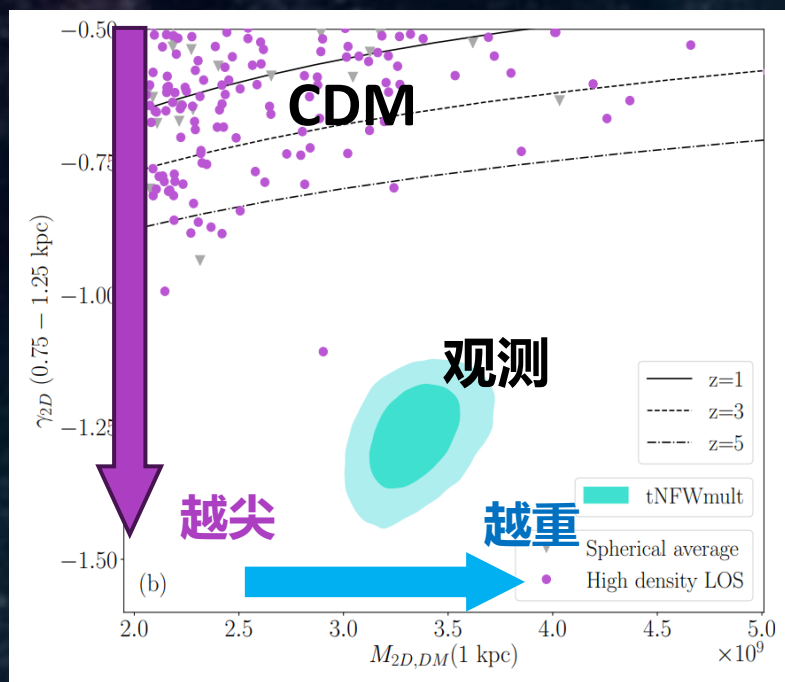
[Ziwen Zhang](#), [Yangyao Chen](#), [Yu Rong](#), [Huiyuan Wang](#) , [Houjun Mo](#), [Xiong Luo](#) & [Hao Li](#)

[Nature](#) **642**, 47–52 (2025) | [Cite this article](#)

1274 Accesses | 84 Altmetric | [Metrics](#)

- ◆ Decreasing surface density
- ◆ Hydro baryonic simulation gives inverted feature
- ◆ Self-Interacting Dark Matter (SIDM), with $\sigma/m=0.3 \text{ cm}^2/\text{g}$ (SIDM03) can explain the observation

(2) Strong lensing perturbers could be core-collapsed SIDM subhalos



Minor et al. 2011.10627
The dark substructure has
unexpected high concentration

Nadler, Yang, Yu, ApJL 2023
Strong SIDM with
 $\sigma/m > 100 \text{ cm}^2/\text{g}$ in group
substructures

S. Li & R. Li+2504.11800
Little dark dots
 $\sigma/m > 138 \text{ cm}^2/\text{g}$
Why M11 does not host a
visible galaxy?

(1) Cored dwarf hosts

Dashed lines: 1 component SIDM03

Following Zhang et al. *Nature* (2025)

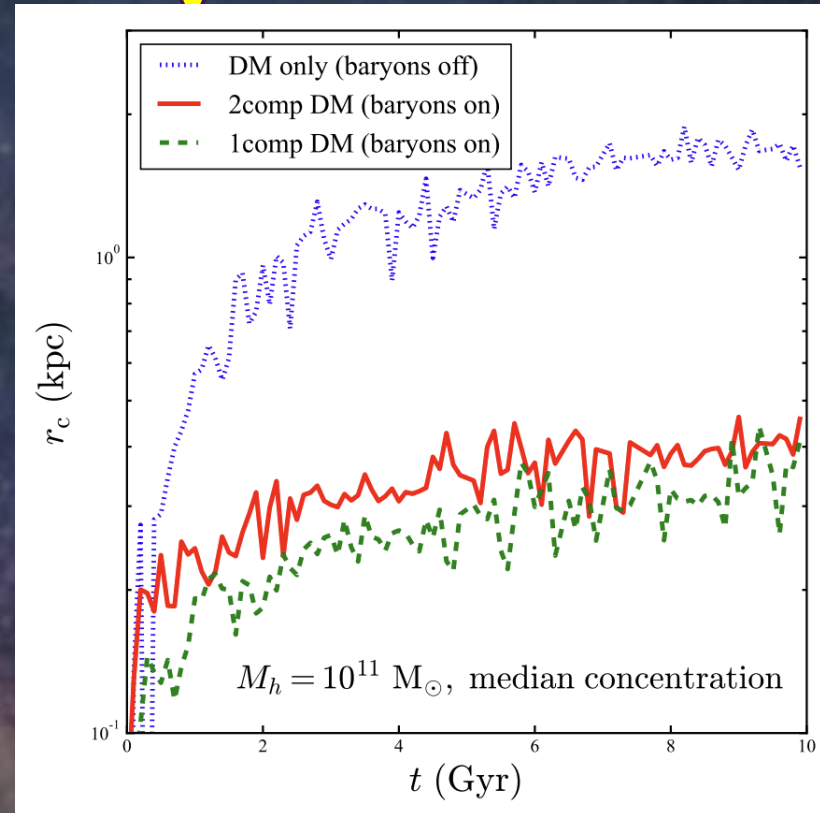
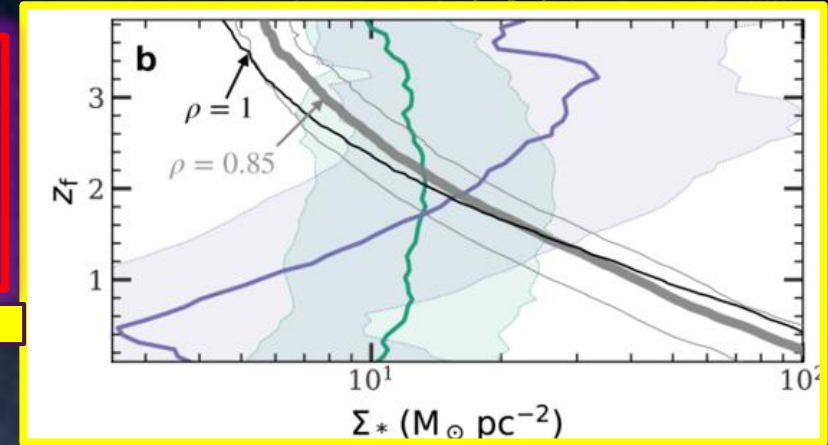
$$\sigma/m \sim 0.3 \text{ cm}^2/\text{g}$$

Test against Massive dwarfs of
Median,
+1 sigma,
-1 sigma concentration

Solid lines: **SIDM2c** with two component

Produce roughly identical results

We need a time
dependent
decreasing surface
density in dwarfs



Flat halos
evolve similar
to 1
component
SIDM halos

Hint: dense
halos become
denser

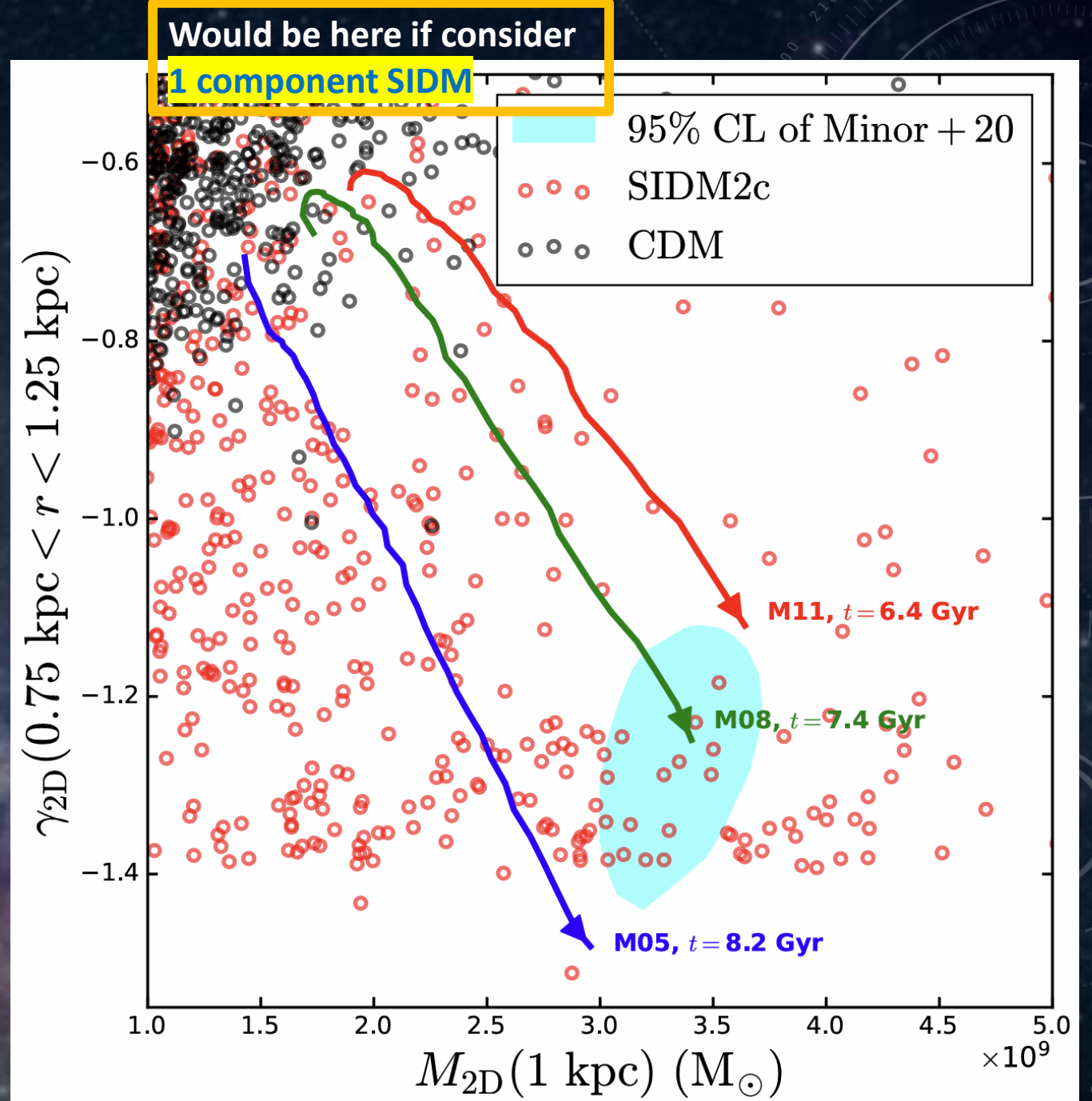
(2) SIDM + mass segregation

Boosted gravothermal evolution such that a small SIDM strength can work!

SIDM strength $\lesssim 1 \text{ cm}^2/\text{g}$

No candidate in CDM

Trajectories from controlled simulations; dots for cosmological



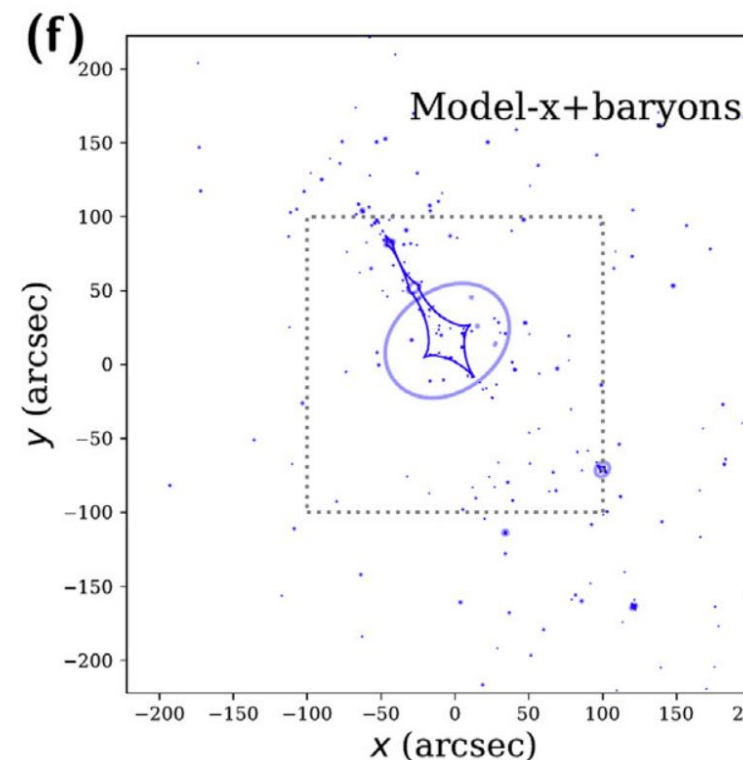
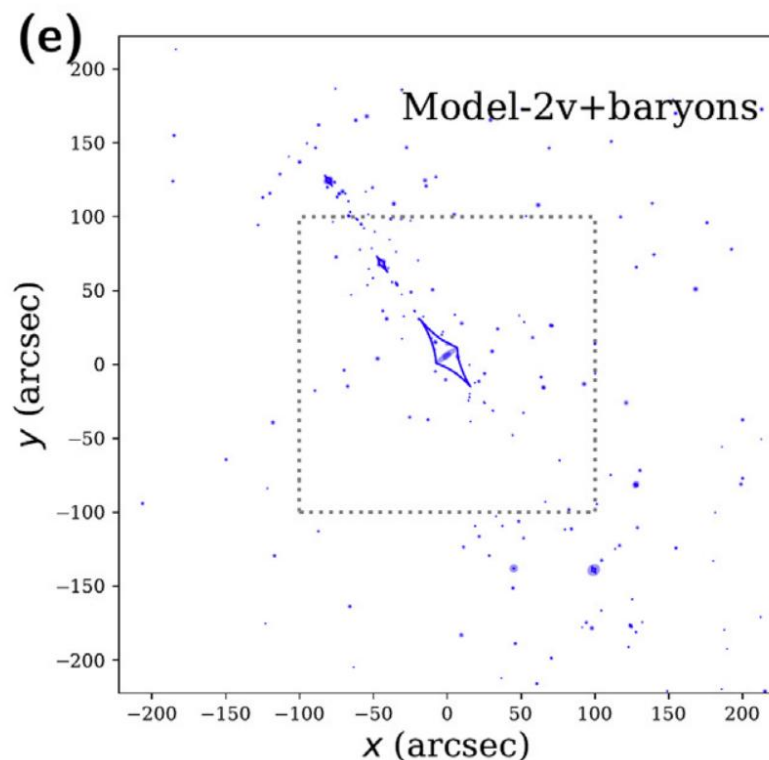
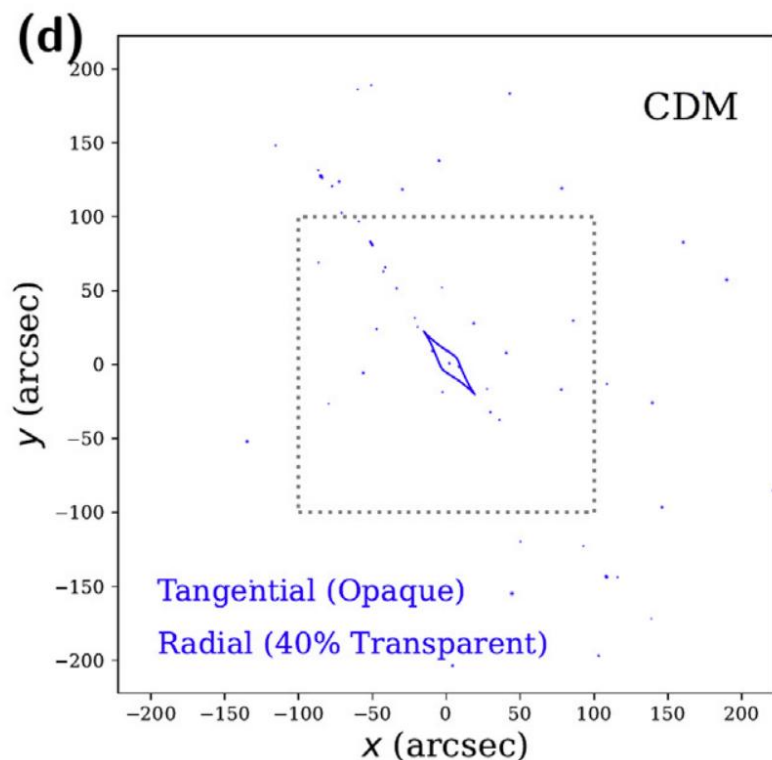
(3) Gravitational lensing by cluster substructures

GGSL cross section, averaged over 11 angles

0.30 (CDM)

1.36 (SIDM2v, fiducial)

4.15 (SIDMx)



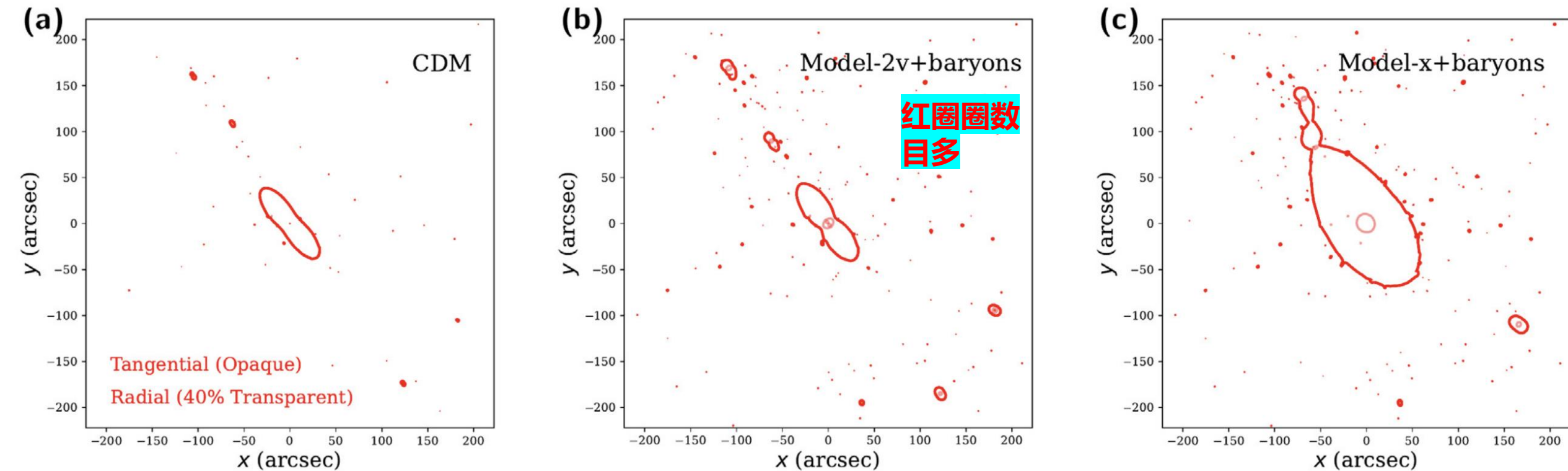
(3) Gravitational lensing by cluster substructures

Number of small scale lenses, effective radii $>0.5''$, average over 11 angles

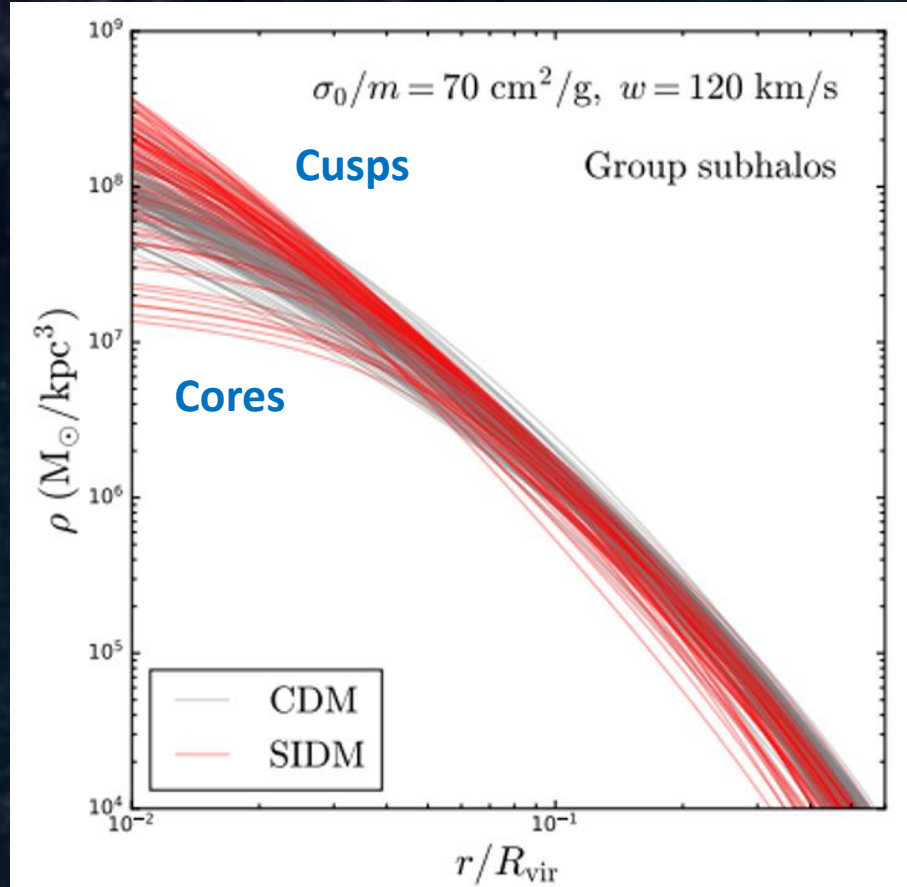
3.7 (CDM)

17.4 (SIDM2v, fiducial)

29.6 (SIDMx)



Diversifying “endpoint” halo profiles by SIDM

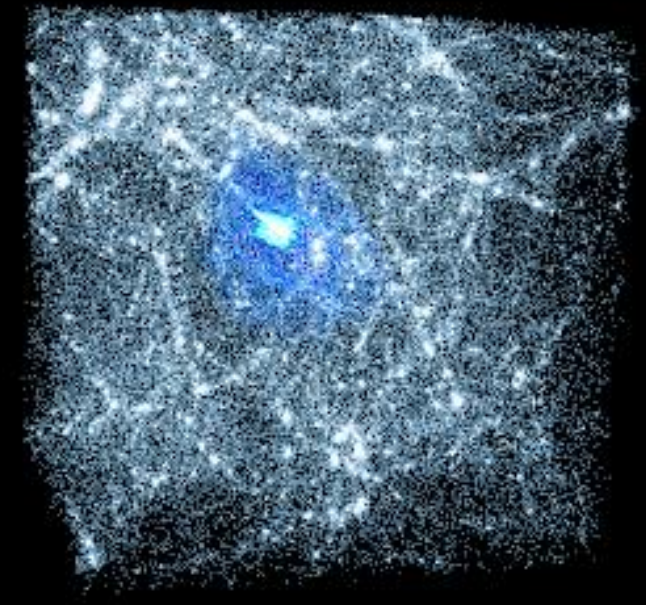


We observe only a snapshot/light-cone of the universe.

Observables correspond to **endpoints** of histories.

However, physics work along the evolution **path**.

First-principle calculations/simulations challenging and expensive



Smearing out unresolved dynamics introduce **intrinsic scatters**

Indirect detection of SIDM

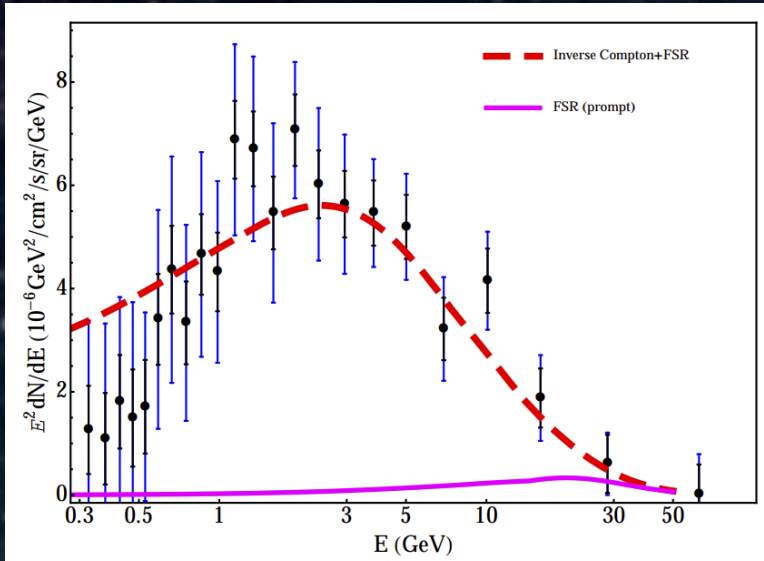


FIG. 1. γ -ray spectrum from Inverse Compton emission and final state radiation produced by annihilation of a 50 GeV dark matter particle through a light mediator into e^+e^- final state. The spectrum is compared to the Galactic Center excess [10].

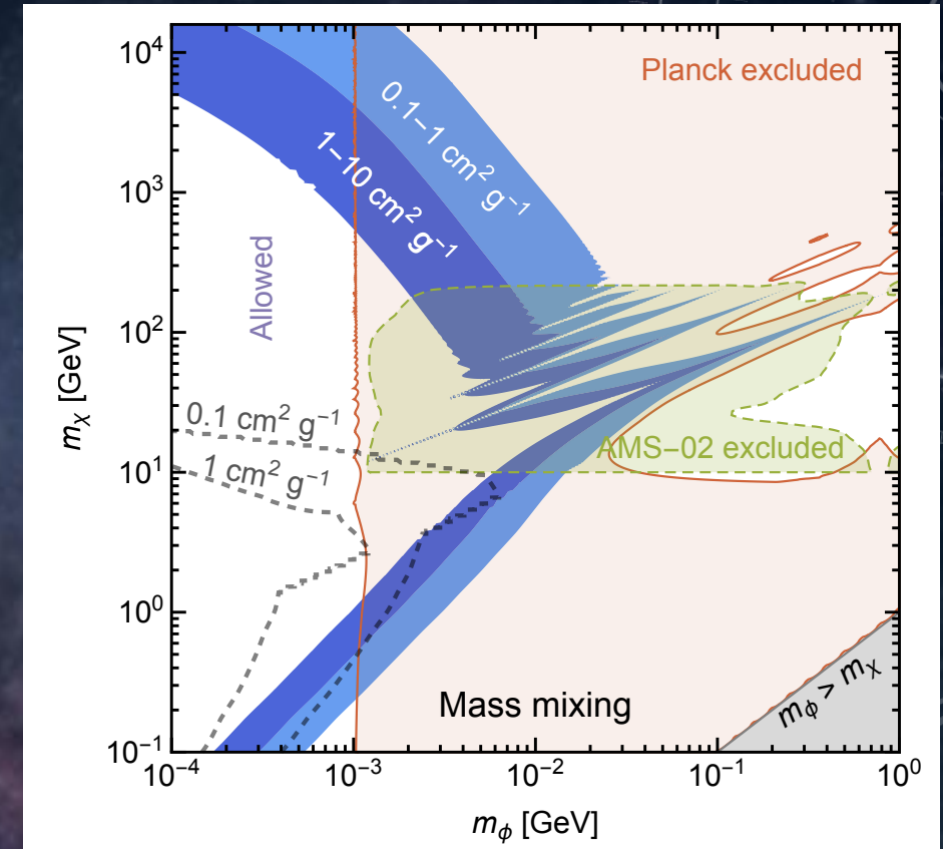
Kaplinghat, Linden, and Yu,
1501.03507 PRL
Galactic Center Excess in Gamma
Rays from Annihilation of Self-
Interacting Dark Matter

Golden window:
SIDM + WIMP
miracle + Galactic
Center Excess
simultaneously

Parameter window
closed by later
studies

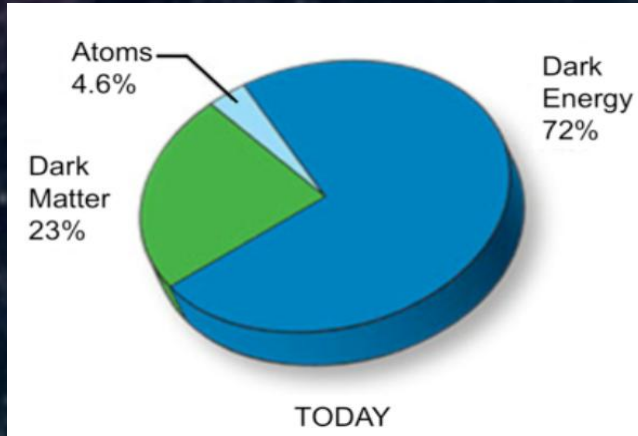
Viable SIDM
parameter region
with $m_\phi < \text{MeV}$

Only weak Gamma
signatures



Bringmann et al. 1612.00845 PRL
Strong constraints on self-interacting dark matter
with light mediators

Indirect detection safe SIDM models



Credit: Swinburne

Asymmetry: The amount of **electrons** in the universe is more than that of the anti-electrons

The net excess equals the excess in **protons**

The 5% baryonic matter is asymmetric

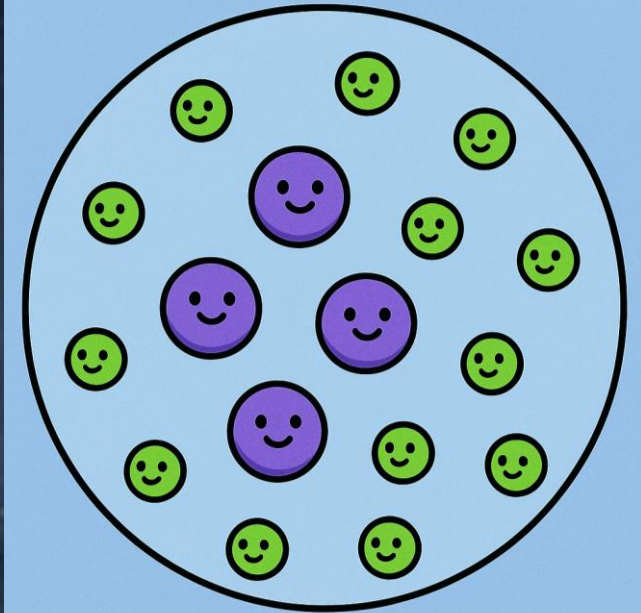
The 25% of dark matter can also be asymmetric

Asymmetric dark matter bypassed the relic abundance problem by introducing a dark “baryon number” into the theory

Self-interacting dark matter implied by nano-Hertz gravitational waves, Han et al. 2306.16966

arXiv:2504.02303 (PRD) & 2506.14898 with Yue-Lin Sming Tsai, Siyuan Hou, Yi-Zhong Fan (PMO)

HALO WITH MASS SEGREGATION OF TWO-COMPONENT SELF-INTERACTING DARK MATTER



Dark electrons and dark protons can exist simultaneously in the dark sector: **at least two components**

功率谱可以用来产生密度分布

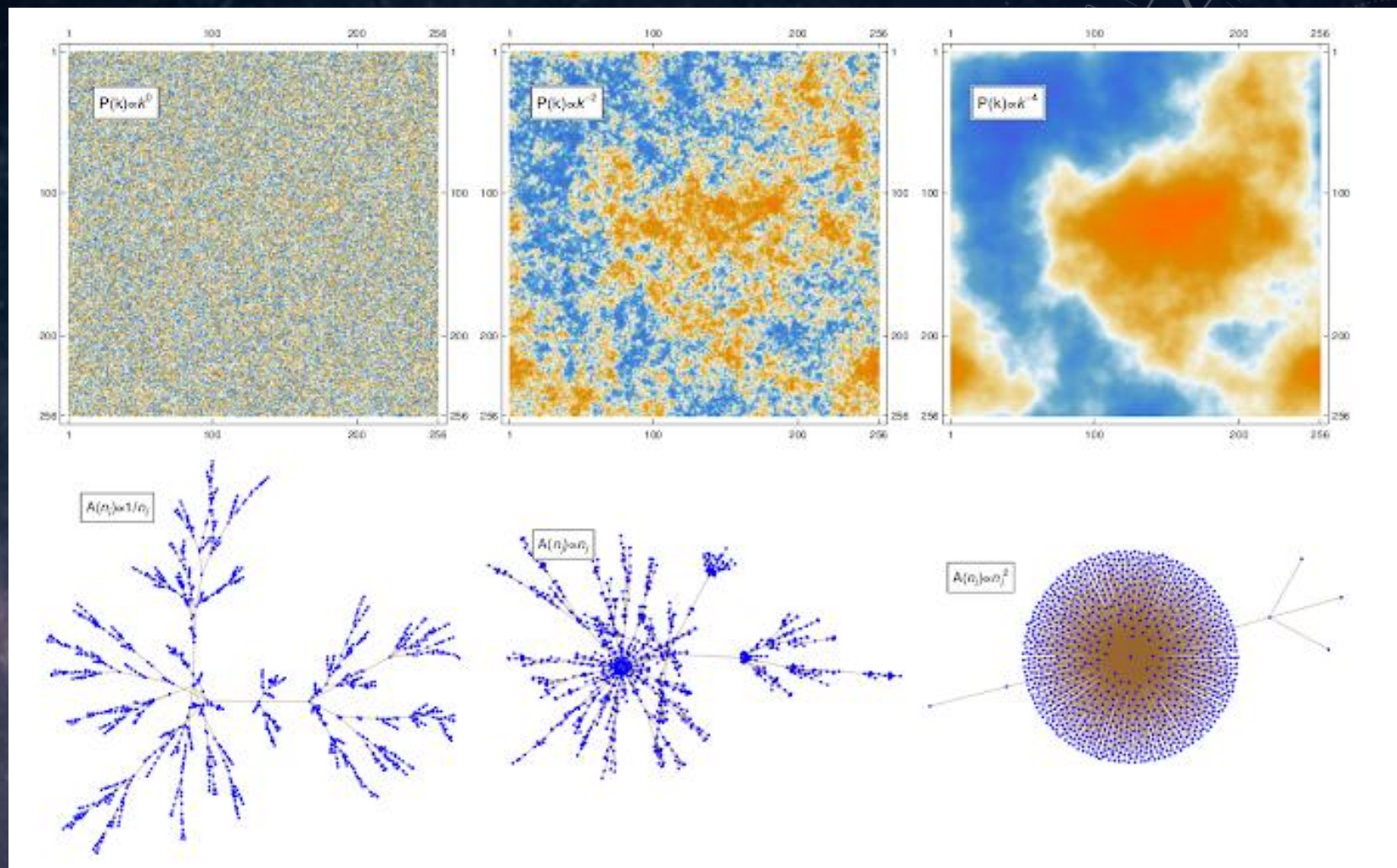
一个二维的例子

尺度不变的物质功率谱
意味着结构的多样性

(Yang & Yu 2022 PRR)

k: node degree,
i.e., number of
links connected
to a node

$$\text{Attachment Probability} = \frac{A_i}{\sum_j A_j}$$



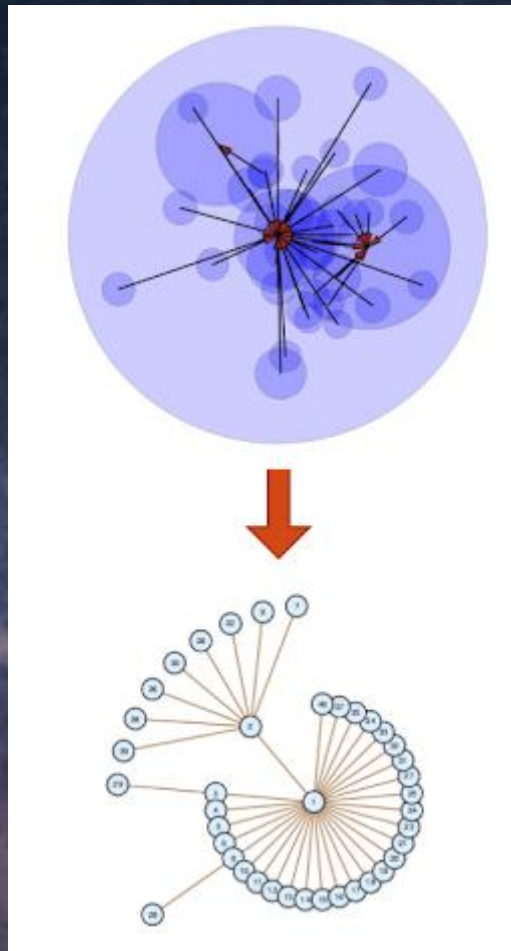
White noise

Scale free

Winner takes all

如何从暗晕结构理解星系的多样性？

- ◆ 物理规律同等地影响每一个星系，然而，星系地形态和结构展现出巨大的多样性
- ◆ 暗晕间结构的关联组成一个复杂系统，可以用网络图来描述
- ◆ 优先依附为复杂系统中多样性的来源提供了一种理解

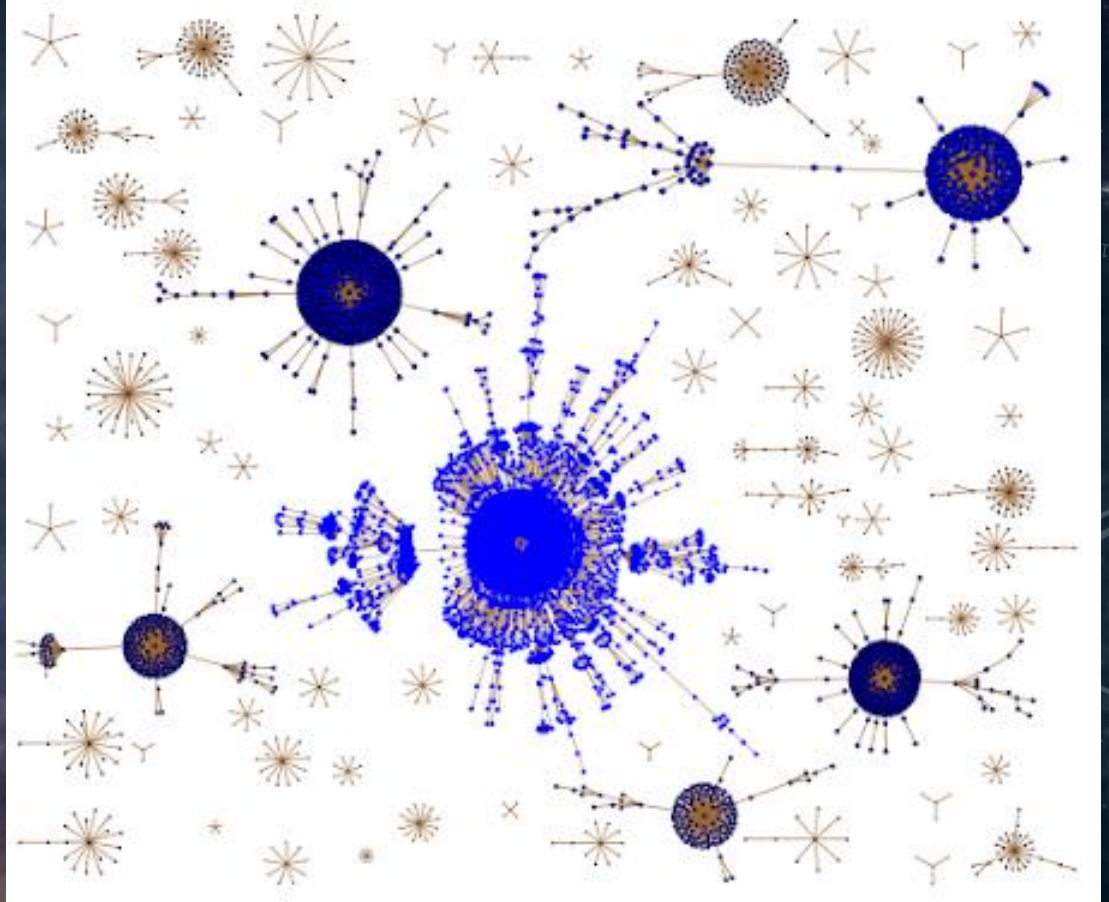
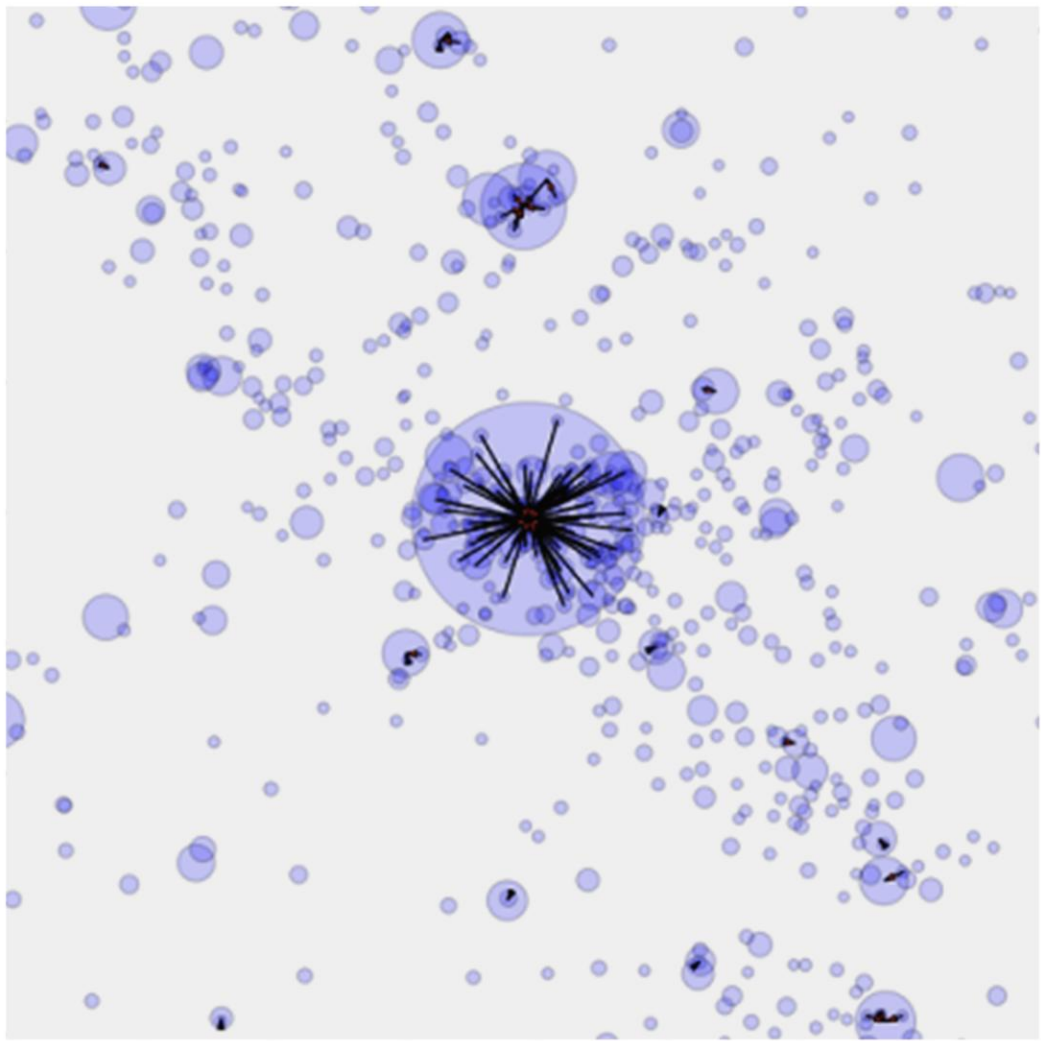


Yang & Yu, 2206.05578 [astro-ph.CO]
Phys. Rev. Research 5, 043187 (2023)

- 自相似性是尺度不变性的一种体现
- 原初功率谱是近似无尺度的
- 层级结构形成伴随着自相似演化
- 引力系统的演化方程是无尺度的

Cosmological halo formation can be encoded by graphs

Yang & Yu, 2206.05578 [astro-ph.CO]
Phys. Rev. Research 5, 043187 (2023)



MSRJD in GPLM: Discrete Realization of Stochastic Evolution

$$\Delta x_{i,k} \mid s_{i,k}, G \sim \mathcal{N}(\delta_{i,k}, \hat{\Sigma}_{i,k})$$

$$\delta_{i,k} \equiv b_{\theta,i,k} \Delta t_k \quad \hat{\Sigma}_{i,k} \equiv D_{\theta,i,k} \Delta t_k$$

$$\Delta x_{i,k} = b_{\theta,i,k}(s_{i,k}, G_k) \Delta t_k + \xi_{i,k}$$

$$\langle \xi_{i,k} \xi_{j,k'}^T \rangle = D_{\theta,i,k} \Delta t_k \delta_{ij} \delta_{kk'}$$

$$S_{\text{MSRJD}}[x, \hat{x}; G] = \int dt \left[\hat{x}^T (\dot{x}_{\text{res}} - b_{\theta}(x^{\text{tr}}, G)) - \frac{1}{2} \hat{x}^T D_{\theta}(x^{\text{tr}}, G) \hat{x} \right]$$

enforces the stochastic dynamics

Gaussian Onsager-Machlup (OM) action

$$S_{\text{OM}}[\mathbf{x}; G] = \frac{1}{2} \sum_{(i,k) \in \mathcal{V}_{\text{sup}}} \left[(\Delta x_{i,k} - \delta_{i,k})^T \hat{\Sigma}_{i,k}^{-1} (\Delta x_{i,k} - \delta_{i,k}) + \log \det \hat{\Sigma}_{i,k} \right]$$

non-Gaussianity could be handled by higher-order terms

Path Space Tools

Operator average

Controlled likelihood
deformation

Likelihood Ratio;
forward-backward asymmetry

MSRJD FROM NOISE MEASURE (1)

$$\dot{x}(t) = b(x(t)) + \eta(t)$$

$$Z = \int \mathcal{D}\eta \, P[\eta]$$

$$P[\eta] \propto \exp \left[-\frac{1}{2} \int dt \, \eta^T D^{-1} \eta \right]$$

$$1 = \int \mathcal{D}x \, \delta[\dot{x} - b(x) - \eta]$$

$$\delta[\dot{x} - b - \eta] = \int \mathcal{D}\hat{x} \, \exp \left[\int dt \, \hat{x}^T (\dot{x} - b - \eta) \right]$$

$$Z = \int \mathcal{D}\eta \, \mathcal{D}x \, \mathcal{D}\hat{x} \, e^{\int dt \, \hat{x}^T (\dot{x} - b - \eta)} \, e^{-\frac{1}{2} \int dt \, \eta^T D^{-1} \eta}$$

MSRJD FROM NOISE MEASURE (2)

$$\int \mathcal{D}\eta \, e^{-\frac{1}{2}\eta^T D^{-1}\eta - \hat{x}^T \eta} \propto \exp\left[-\frac{1}{2} \int dt \, \hat{x}^T D \hat{x}\right]$$

$$Z = \int \mathcal{D}x \, \mathcal{D}\hat{x} \, e^{-S_{\text{MSRJD}}[x, \hat{x}]}$$

$$S_{\text{MSRJD}} = \int dt \left[\hat{x}^T (\dot{x} - b(x)) - \frac{1}{2} \hat{x}^T D \hat{x} \right]$$

$$A \equiv (\dot{x}_{\text{res}} - b_{\theta})$$

$$-\frac{1}{2} \hat{x}^T D \hat{x} + \hat{x}^T A = -\frac{1}{2} (\hat{x} - D^{-1} A)^T D (\hat{x} - D^{-1} A) + \frac{1}{2} A^T D^{-1} A$$

$$\int \mathcal{D}\hat{x} \, \exp\left[-\frac{1}{2} (\hat{x} - \mu)^T D (\hat{x} - \mu)\right] \propto (\det D)^{-1/2}$$

$$Z \propto \int \mathcal{D}x \, \exp\left[-\frac{1}{2} \int dt \, A^T D^{-1} A\right] \times \exp\left[-\frac{1}{2} \int dt \, \log \det D_{\theta}\right]$$

Dark Matter Deficient Galaxy (DMDG) Probabilities

DMDG operator:

$$O_{\text{DMDG}}^{(i)}(\mathbf{x}_i) = \Theta\left(\frac{M_{\star}^{(i)}(z=0) + M_{\text{gas}}^{(i)}(z=0)}{M_{\text{halo}}^{(i)}(z=0)} - 1\right)$$

Operator average

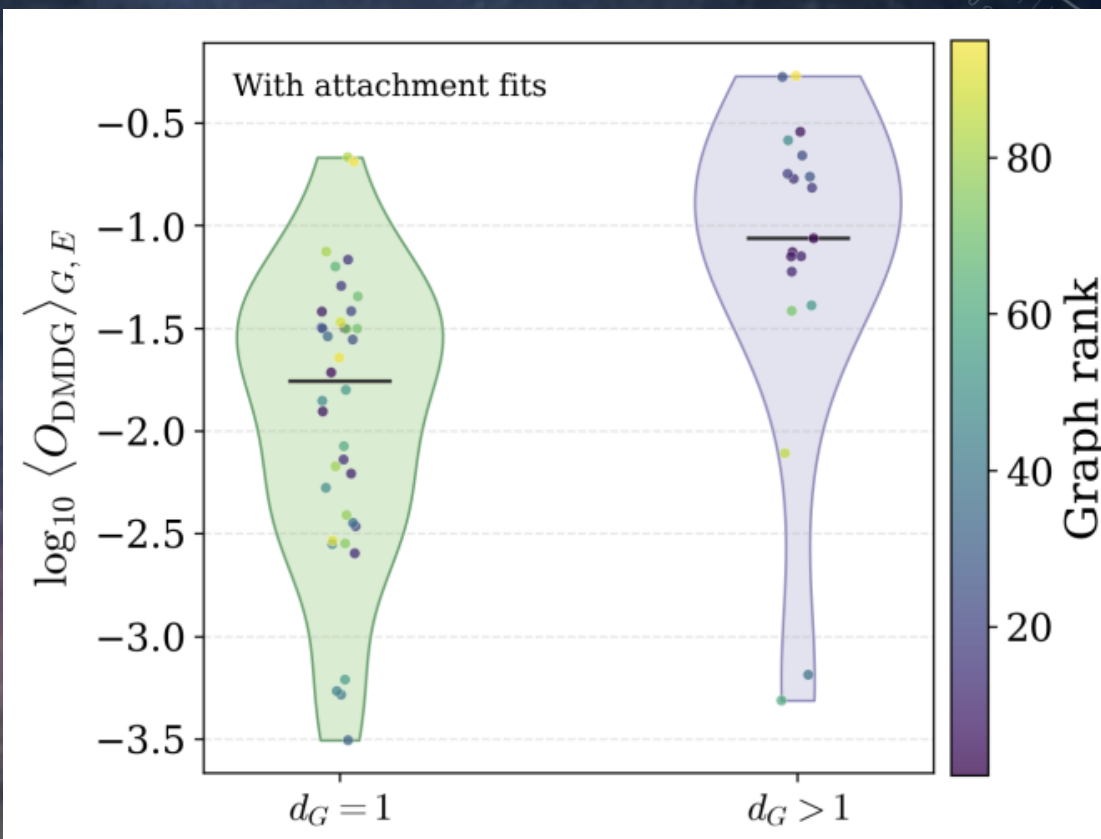
$$\langle O_{\text{DMDG}} \rangle_{G,E} \equiv \int \mathcal{D}x \left[\frac{\sum_{i \in E} O_{\text{DMDG}}^{(i)}[\mathbf{x}]}{N_E} \right] P_{\text{tot}}(\mathbf{x} \mid G, x_0)$$

dG=1: satellites

dG>1: higher order satellites

Higher tidal stripping in dG>1 satellites leads to more DMDGs but with greater scatter

➤ **Most probable path for DMDG formation?**



Gas-Rich response as a controlled likelihood deformation

Gas-Rich operator:

$$O_{\text{GR}}(i) \equiv \Theta[M_{\text{gas},i}(z=0) - M_{\star,i}(z=0)]$$

Controlled deformation:

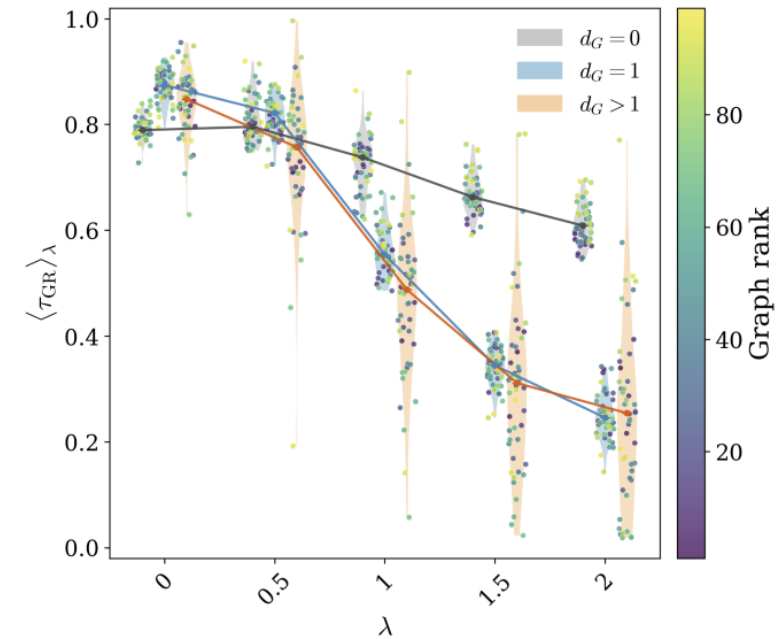
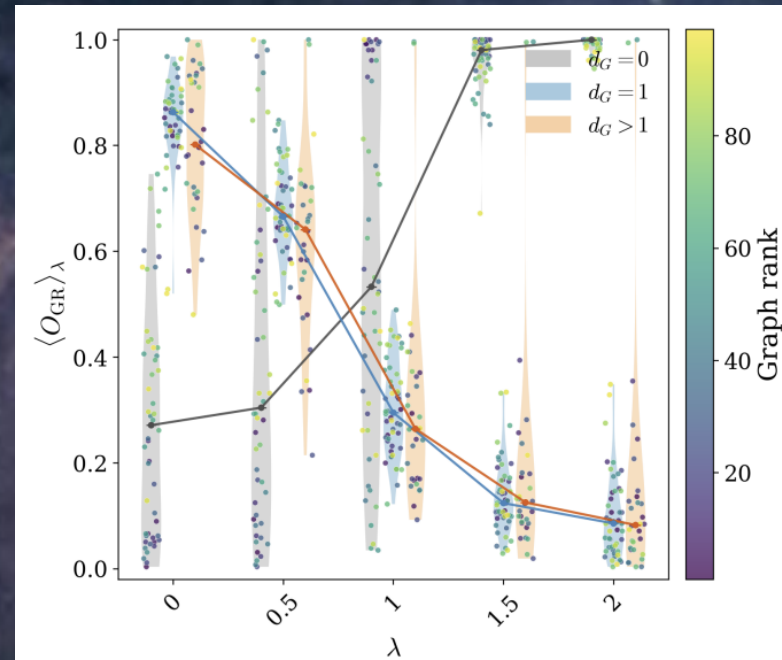
$$b_{\text{gas},\lambda}(x_k, G) = b_{\text{gas,off}}(x_k, G) + \lambda[b_{\text{gas,full}}(x_k, G) - b_{\text{gas,off}}(x_k, G)]$$

Ram-pressure stripping causes gas of satellites to feed into the host

$d_G=0$: host

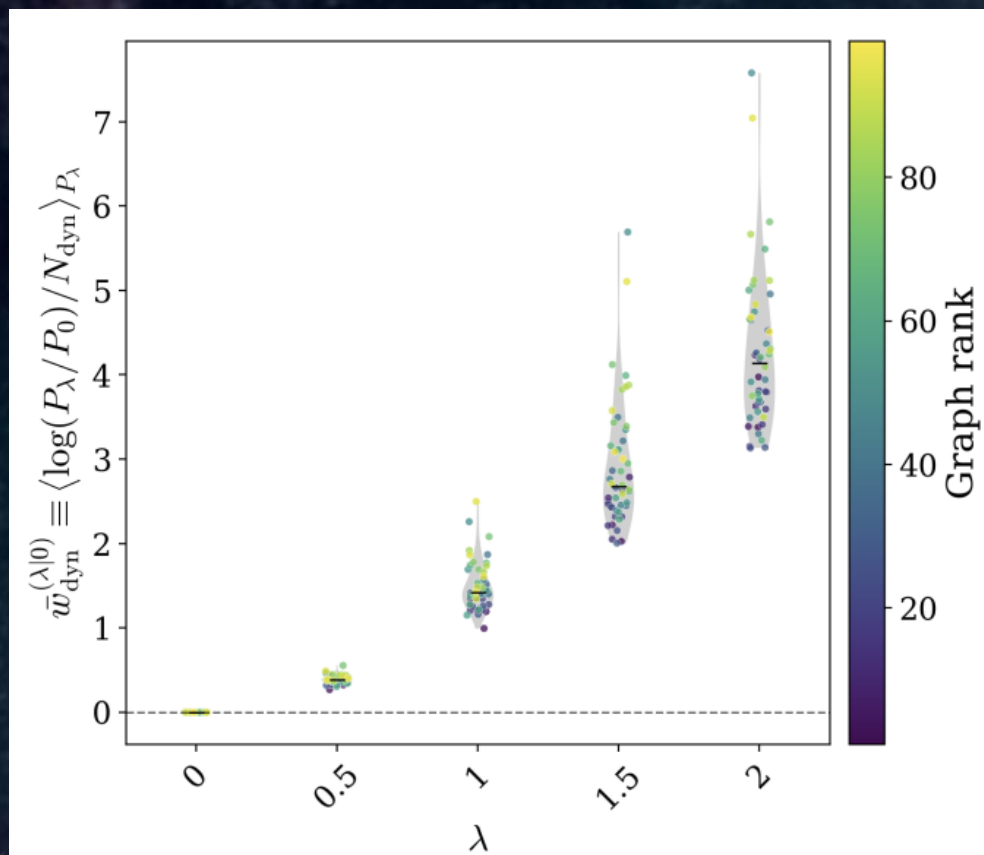
$d_G=1$: satellites

$d_G>1$: higher order satellites



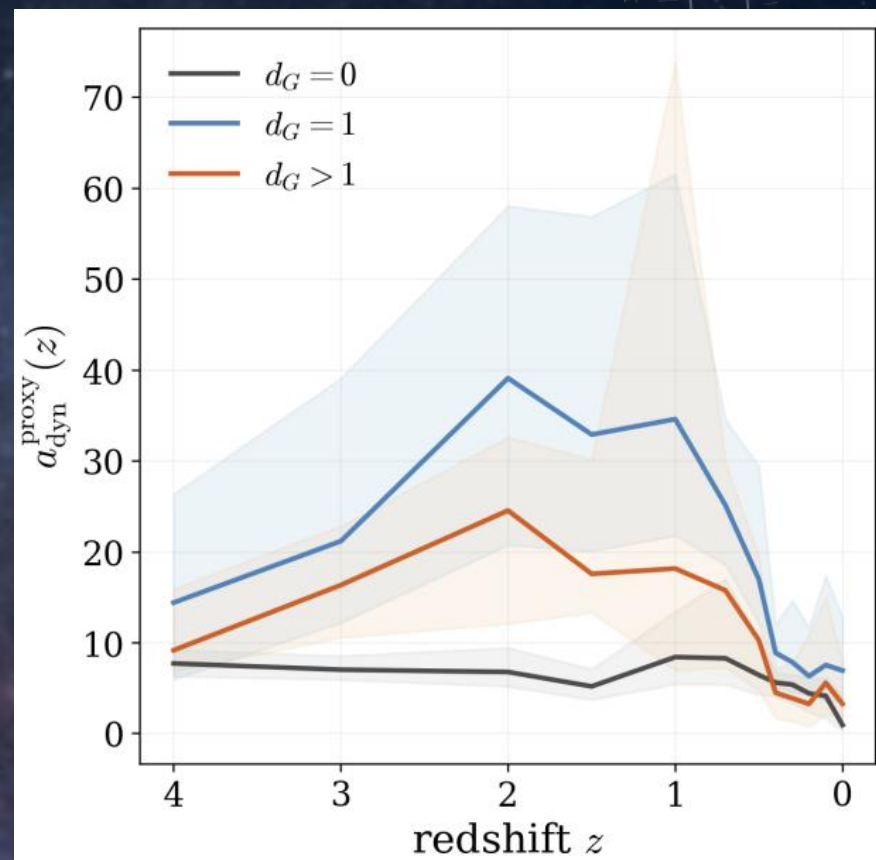
Path-Space likelihood diagnostics

A path-level KL-like distance



Larger λ drives a larger departure from $\lambda=0$ and broadens the graph-to-graph scatter

A local forward-reverse asymmetry

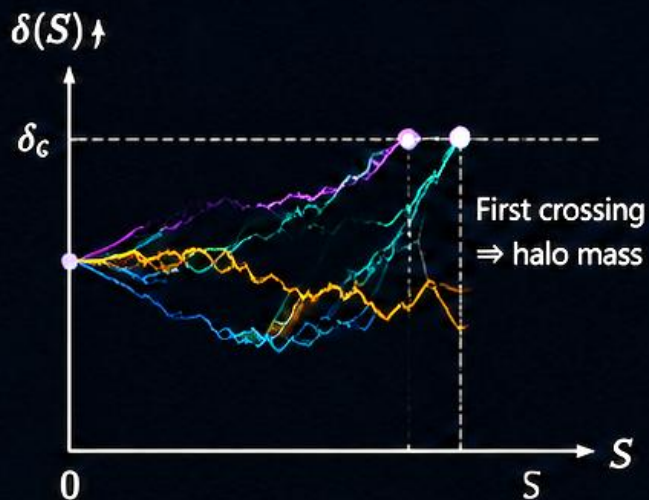


Peaks around cosmic noon, indicating stronger history-dependent evolution there

From Stochastic Halo Growth to Stochastic Layered Graphs

Excursion-set theory: halo growth is a **stochastic trajectory** of density fluctuations across smoothing scales. Bond et al. 1991

1. Excursion-set: random walks



Stochasticity comes from random initial fluctuations $\Rightarrow$ **random walks**.

2. Stochastic growth (Langevin equation)

$$\frac{d\delta}{dS} = \mu(S) + \sqrt{D(S)}\eta(S)$$

- $\mu(S)$: drift (e.g., gravity)
- $D(S)$: diffusion (power spectrum)
- $\eta(S)$: Gaussian white noise $\langle \eta(S)\eta(S') \rangle = \delta_D(S - S')$



Drift + random forcing $\Rightarrow$ **stochastic trajectories**

3. From trajectories to layered halo graphs

Time / scale

$k = 0$
(early)

$k = 1$

...

$k = K - 1$

$k = K$
(late)



- Halos (nodes)
- Mergers (within layer)
- Progenitor links (across layers)

Joint probability

$$P(G_{0:K}) = P(G_0) \prod_{k=0}^{K-1} P(G_{k+1} | G_k)$$

4. Joint model: halo graph + galaxy fields

Halo graph action

$$S_G[G_{0:K}]$$

+

Galaxy/field action

$$S_x[\mathbf{x}_{0:K}; G_{0:K}]$$

$\Rightarrow$

Total action

$$S_{\text{tot}} = S_G + S_x$$

+

5. Outlook



Model halo graphs and galaxy fields within one **joint stochastic description** of their coupled formation histories.

Which graph is most consistent with observations?

$$P(\mathcal{G} \mid d_{\text{end}}) \propto P(\mathcal{G}) \int \mathcal{D}\mathbf{x} P(d_{\text{end}} \mid \mathbf{x}, \mathcal{G}) P(\mathbf{x} \mid \mathcal{G})$$

SIMULATION

- Full history
- Model dependent
- Fewer

Can We Learn
from
Observational
Data?

OBSERVATION

- z-dependent snapshots
- Real data with uncertainty
- A lot more

The vast amount of data z-dependent could, in principle, be used to fine-tune GPLM

$$P(d_{\text{all}} \mid \vartheta) \sim \prod_z \int \mathcal{D}G \mathcal{D}\mathbf{x} P(d_z \mid \mathbf{x}_z, G) P_{\vartheta}(\mathbf{x}, G)$$

One cluster,
one layered halo graph

