

轴子暗物质产生的数值模拟

边立功

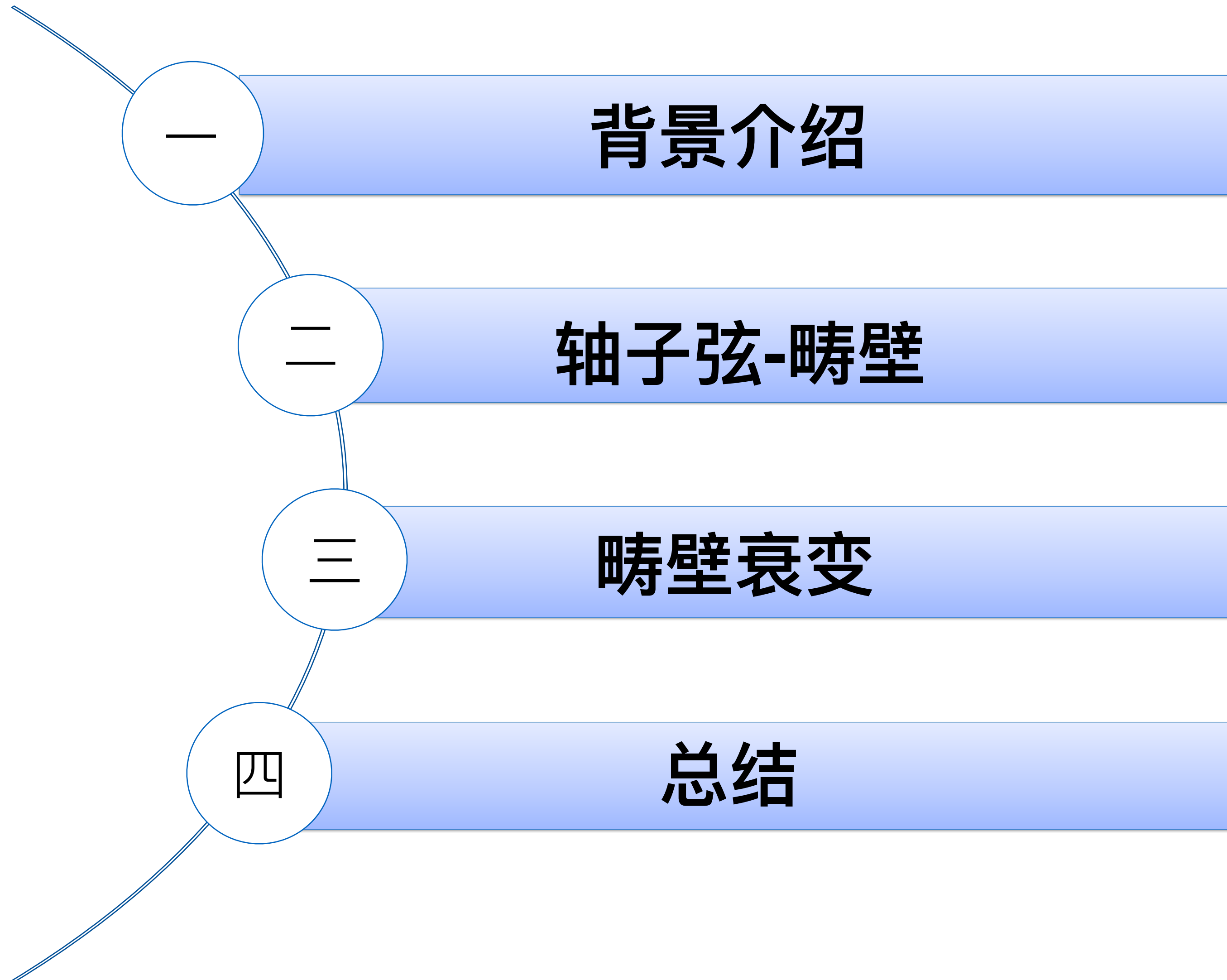
重庆大学

lgbycl@cqu.edu.cn

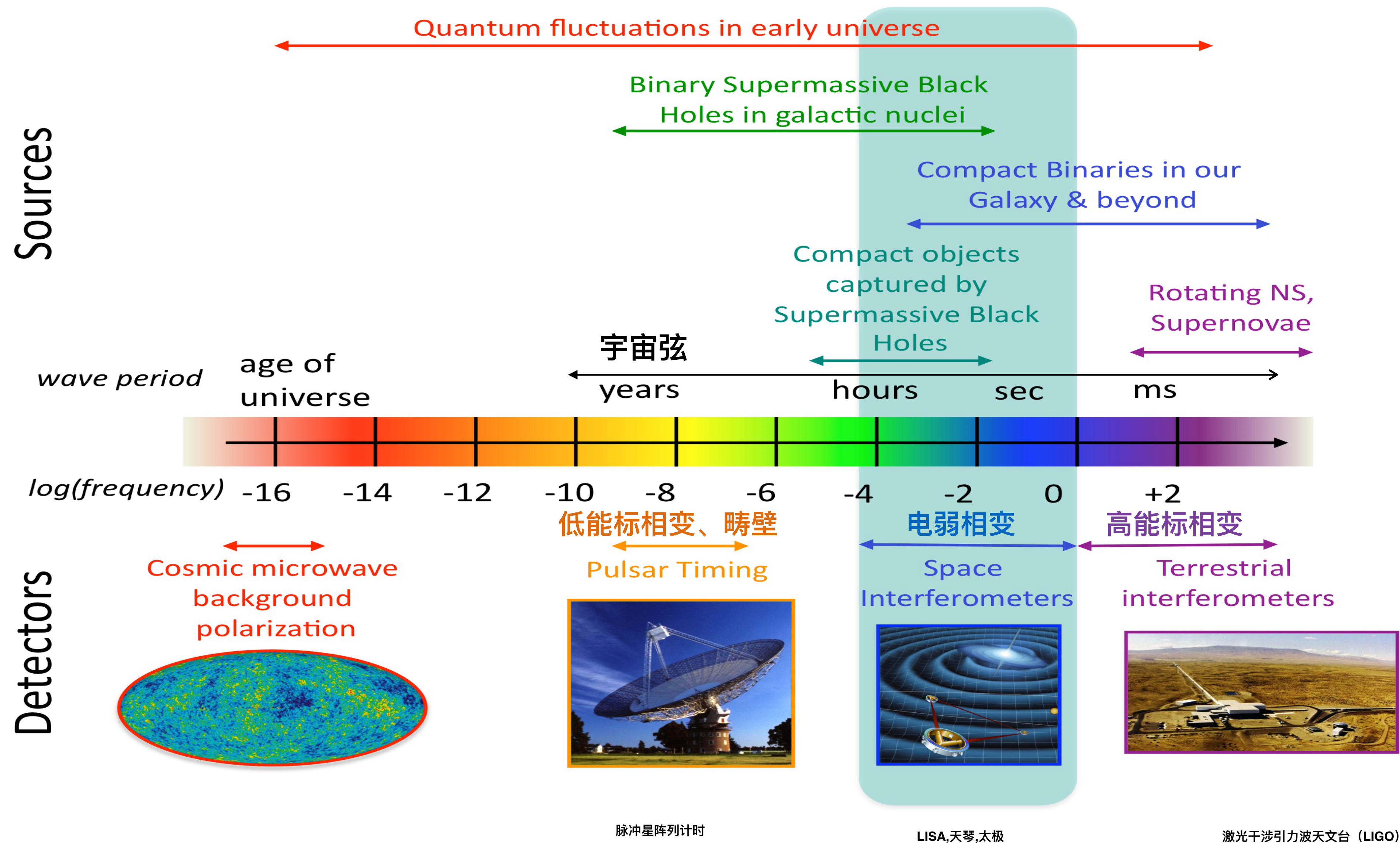
2026新物理研讨会

2026年8月23日-29日@威海

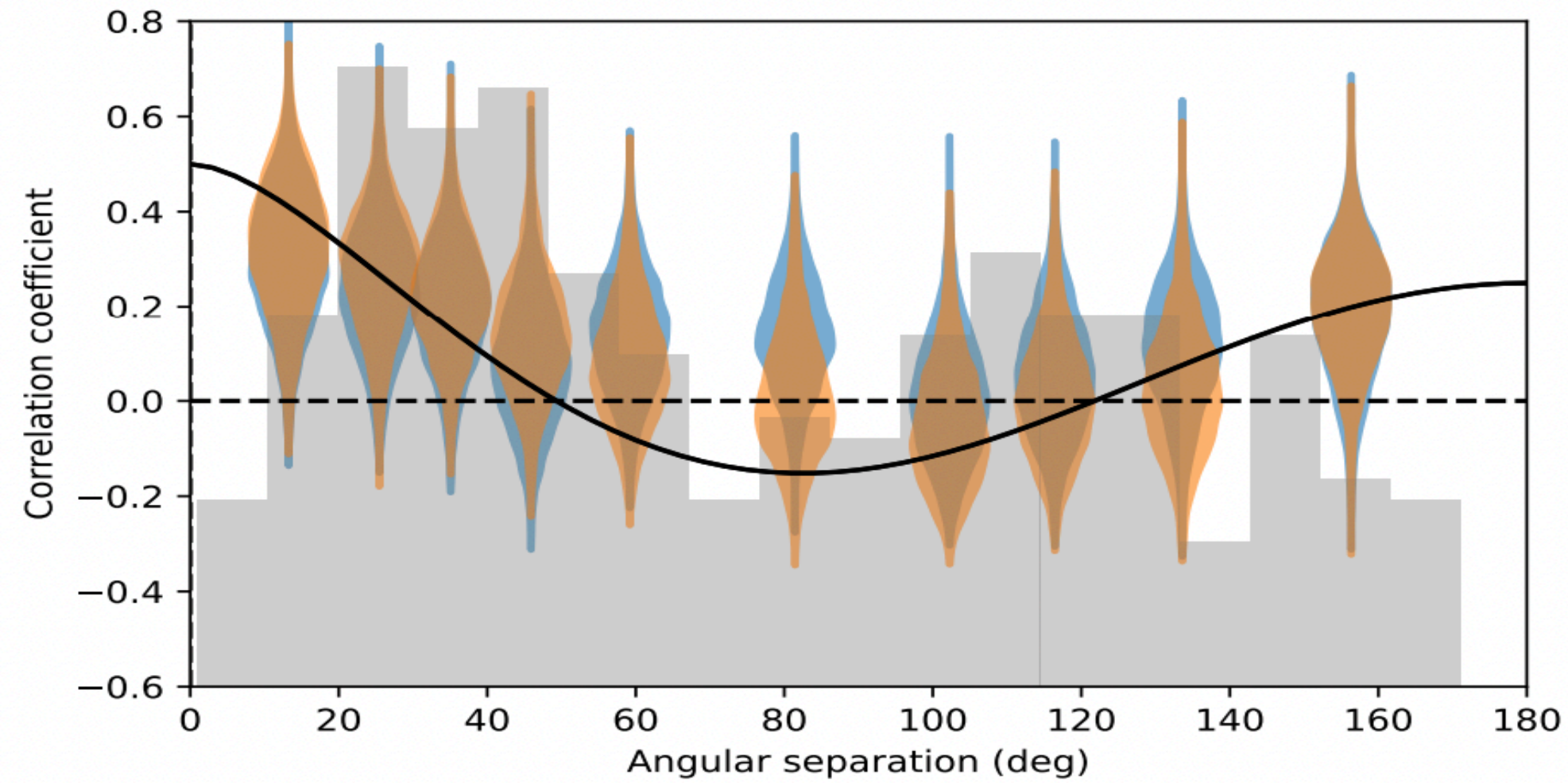
报告提纲



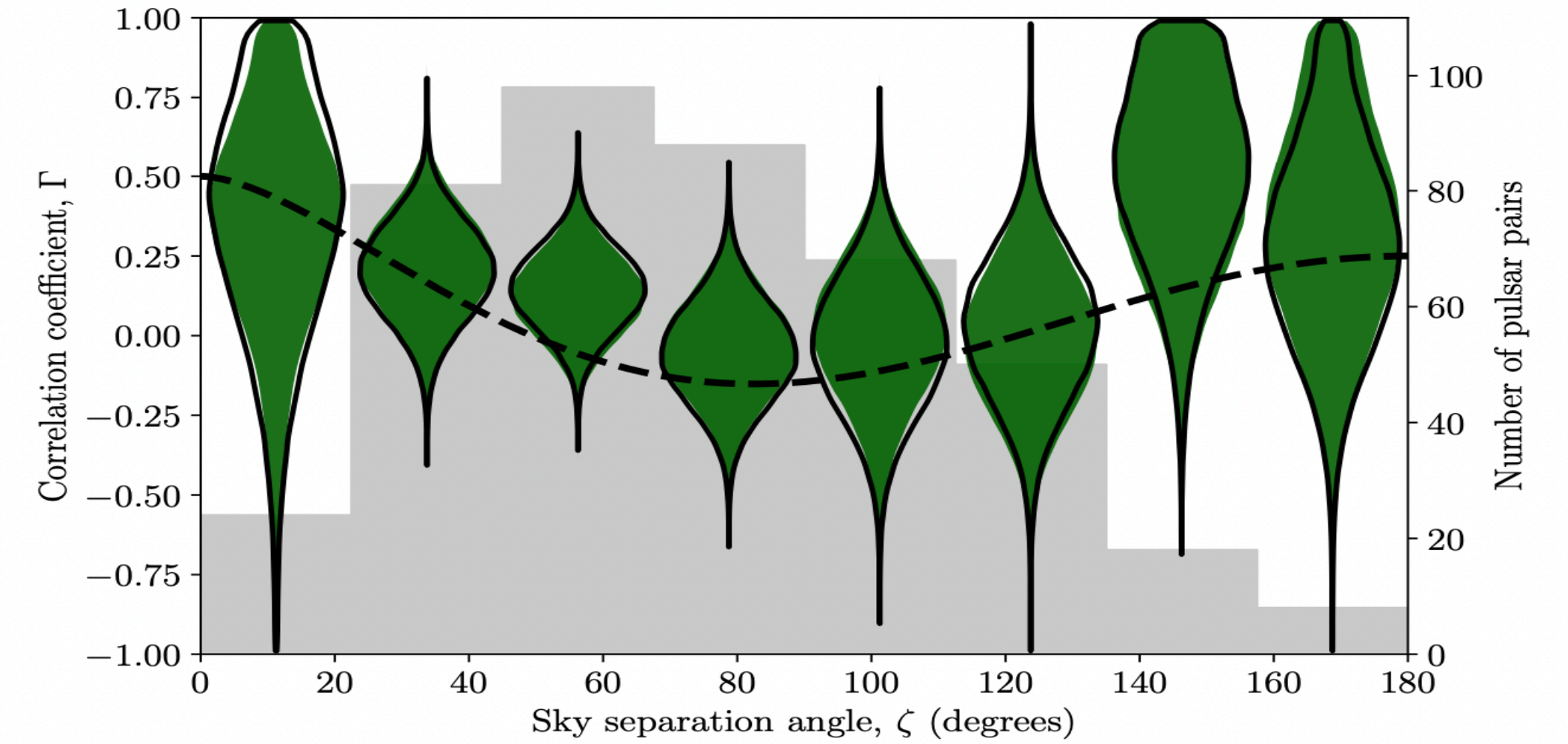
The Gravitational Wave Spectrum



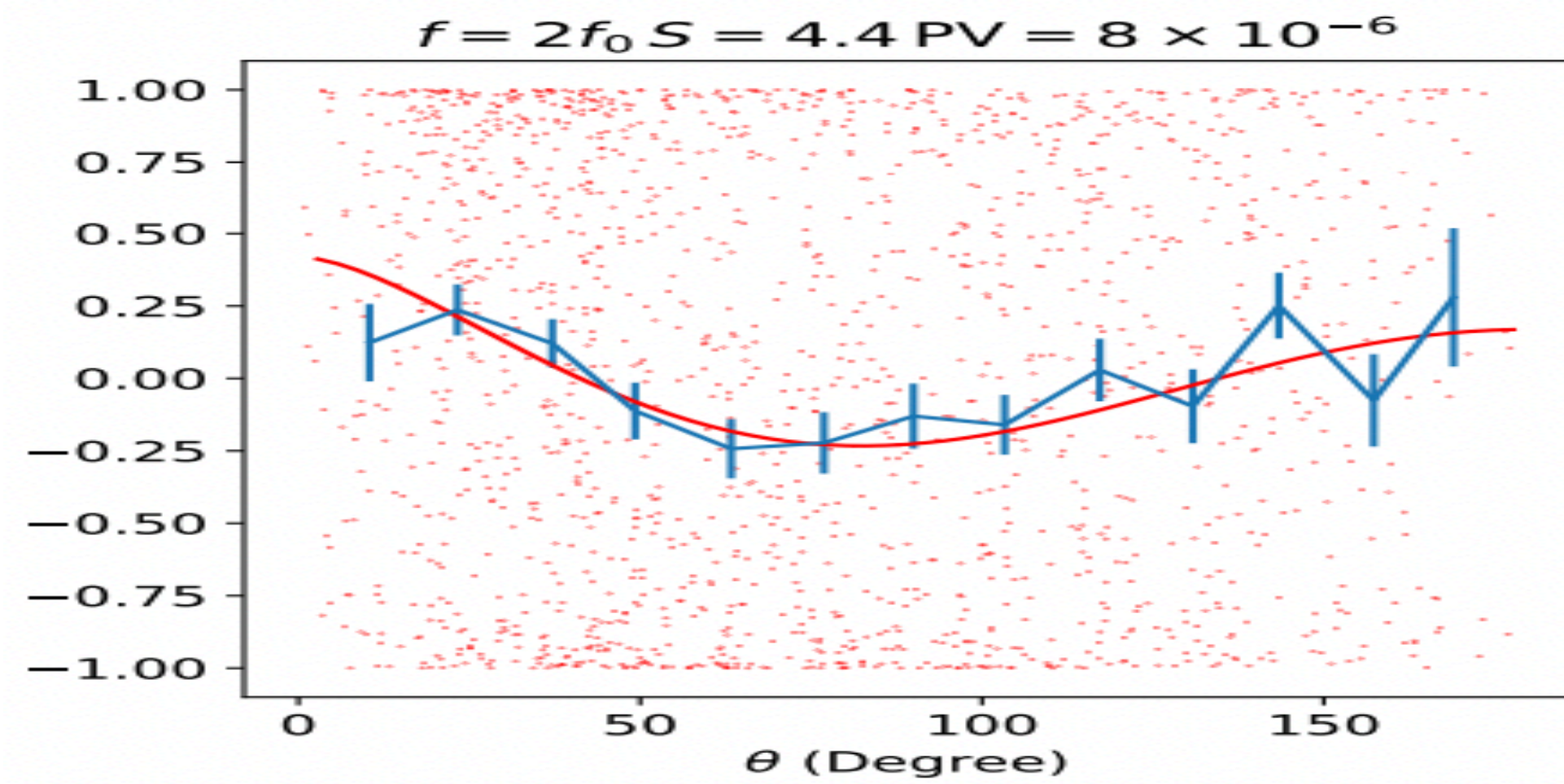
New dataset from PTAs



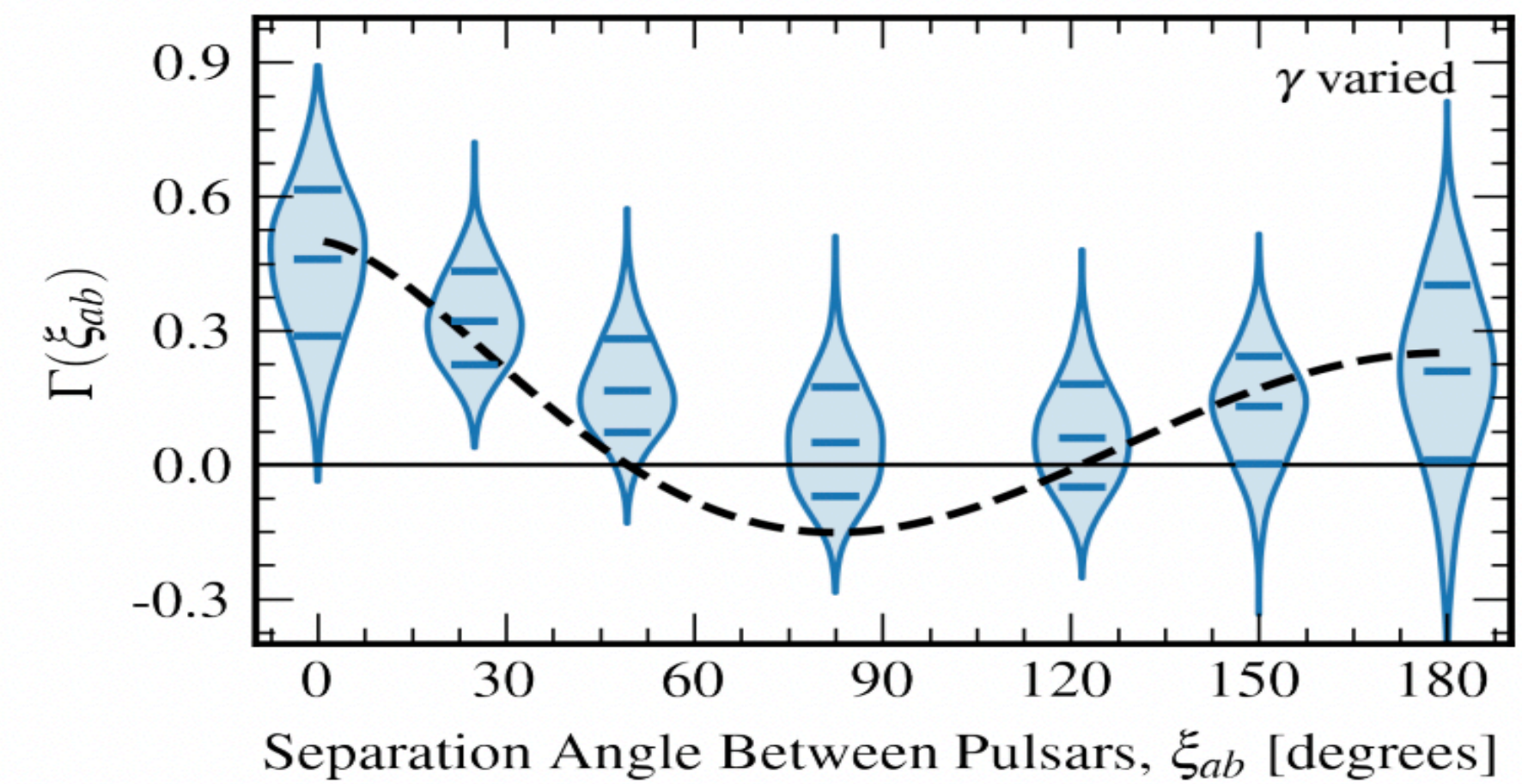
EPTA,2306.16214



PPTA,2306.16215

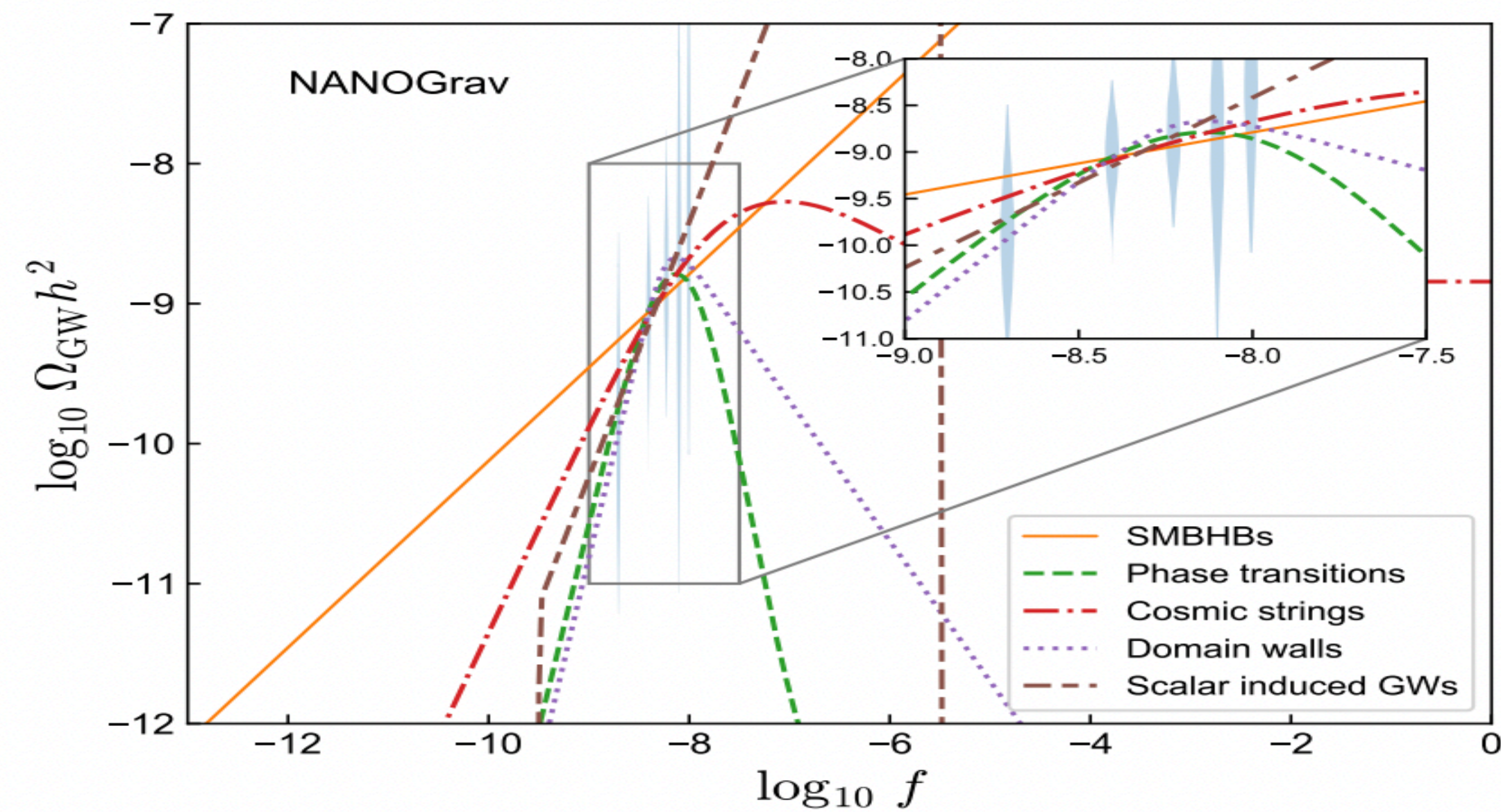
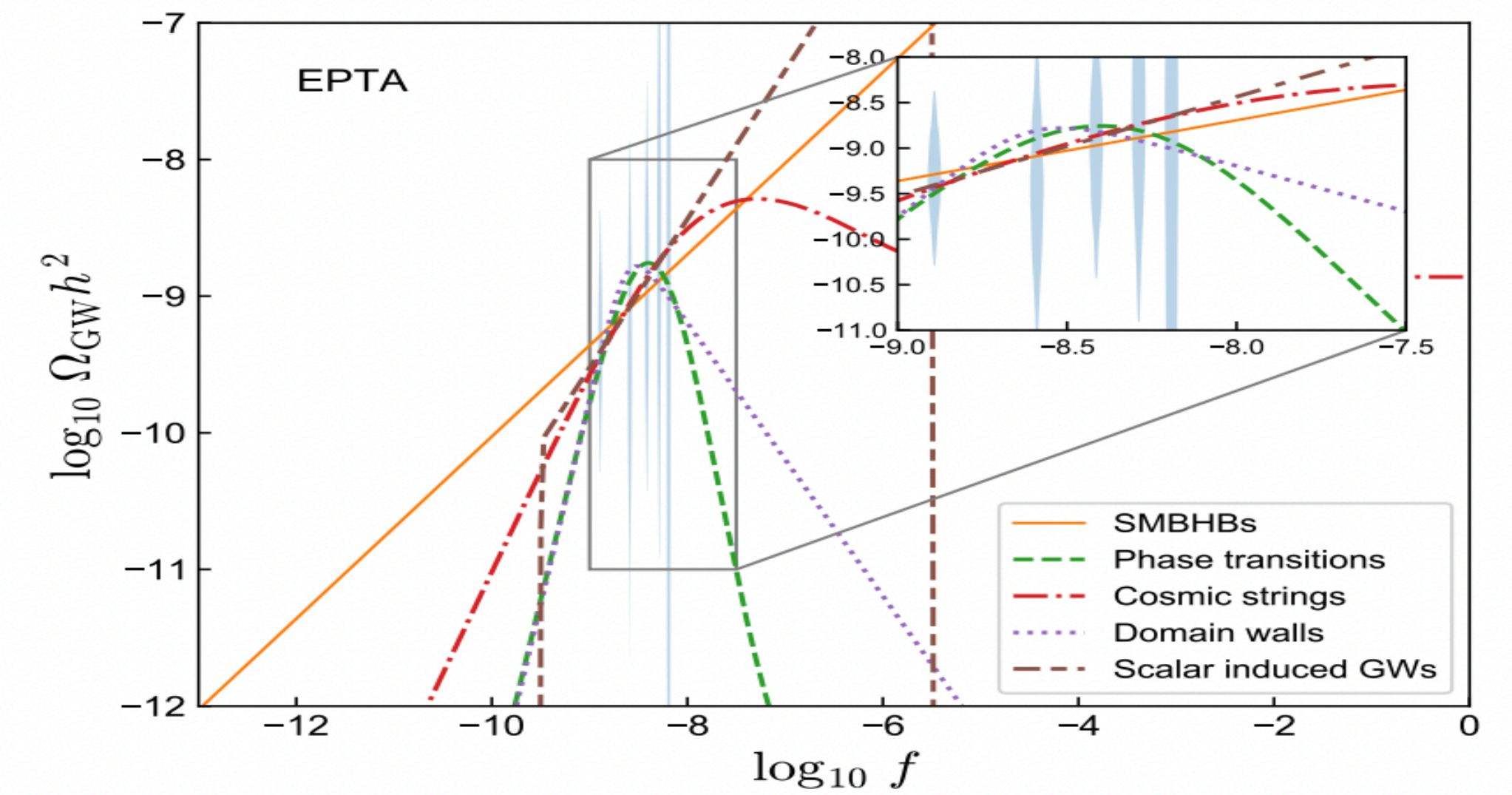
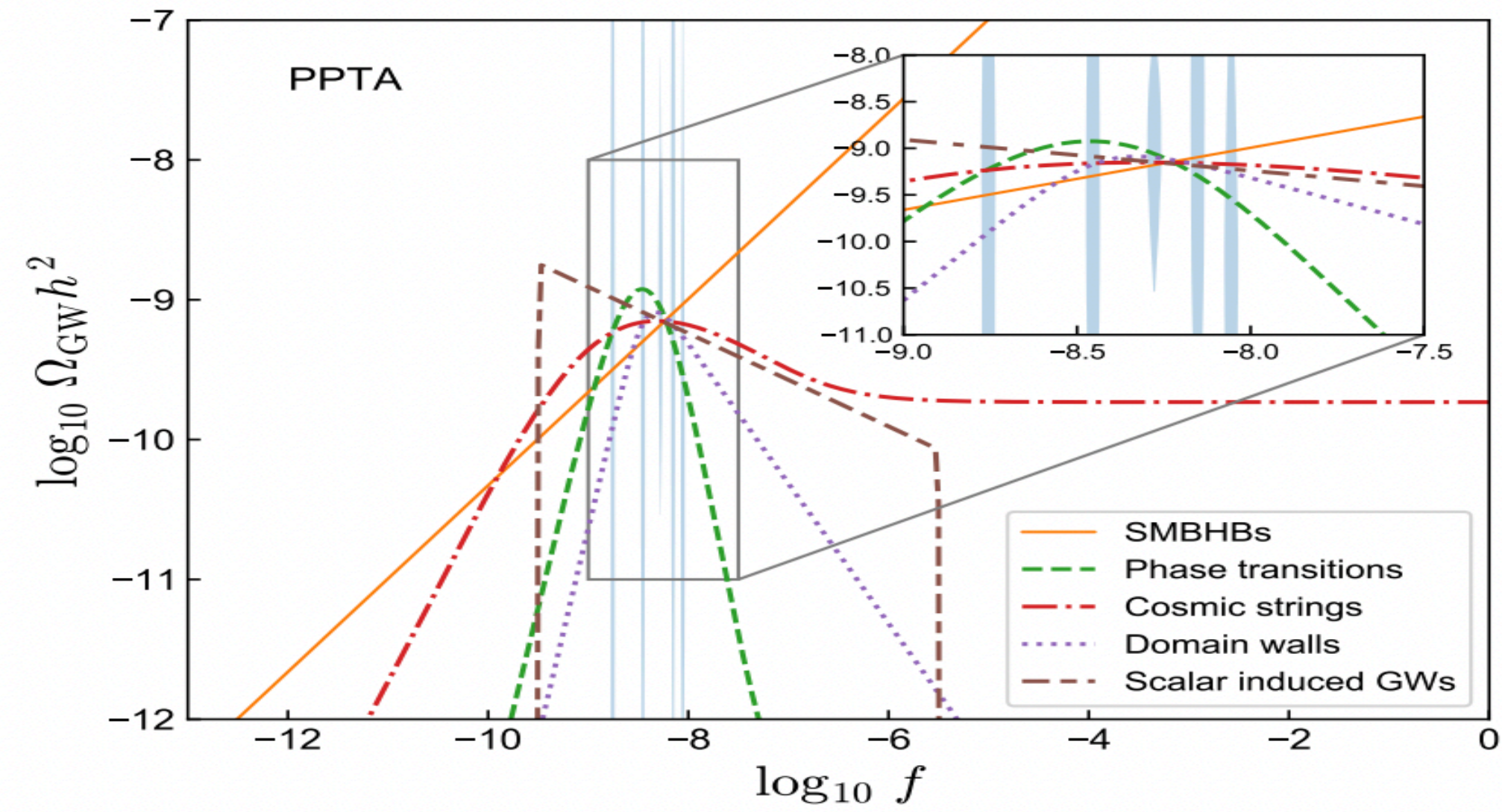


CPTA ,2306.16216

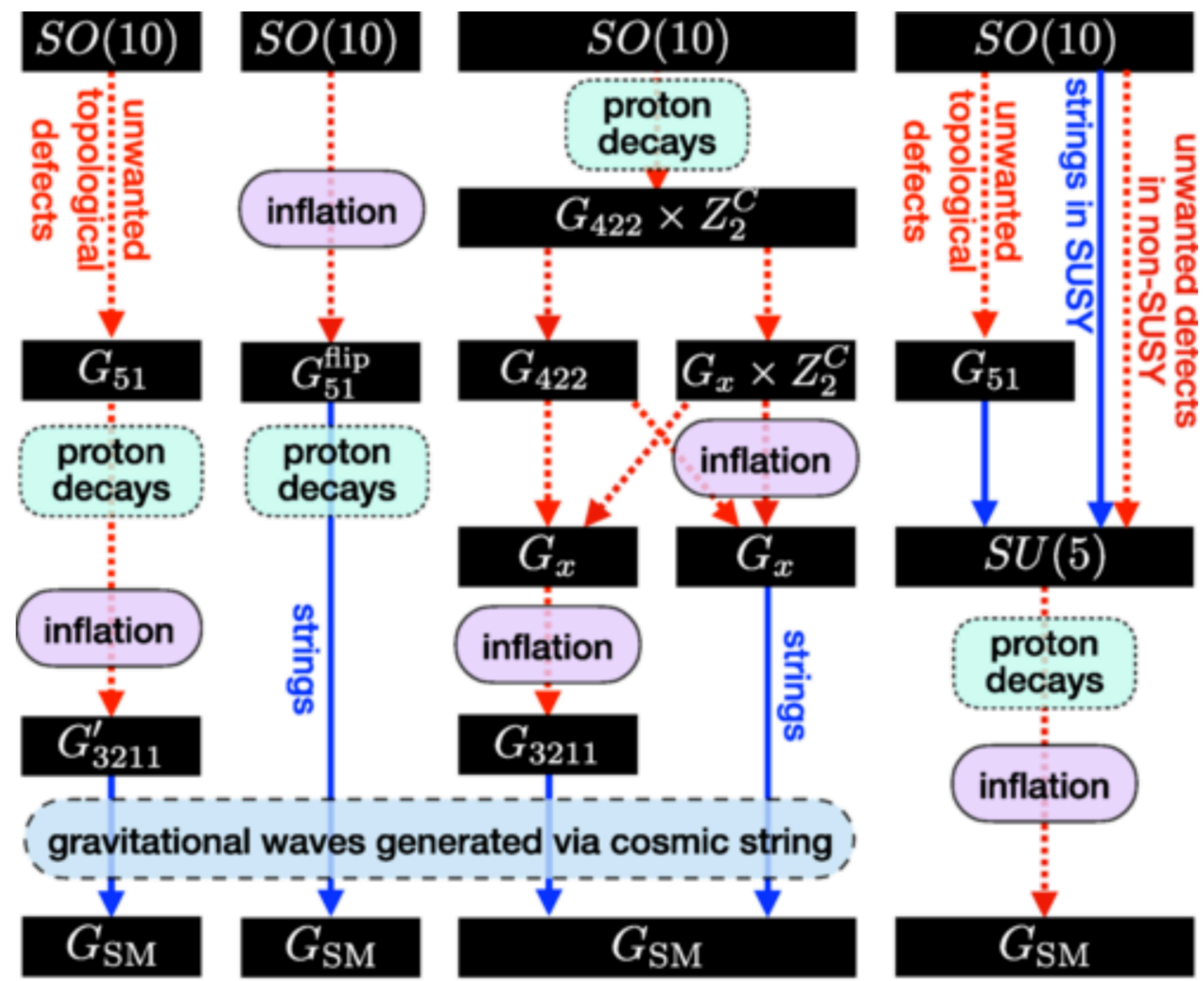


NANOGrav,2306.16213

Gravitational wave sources for Pulsar Timing Arrays

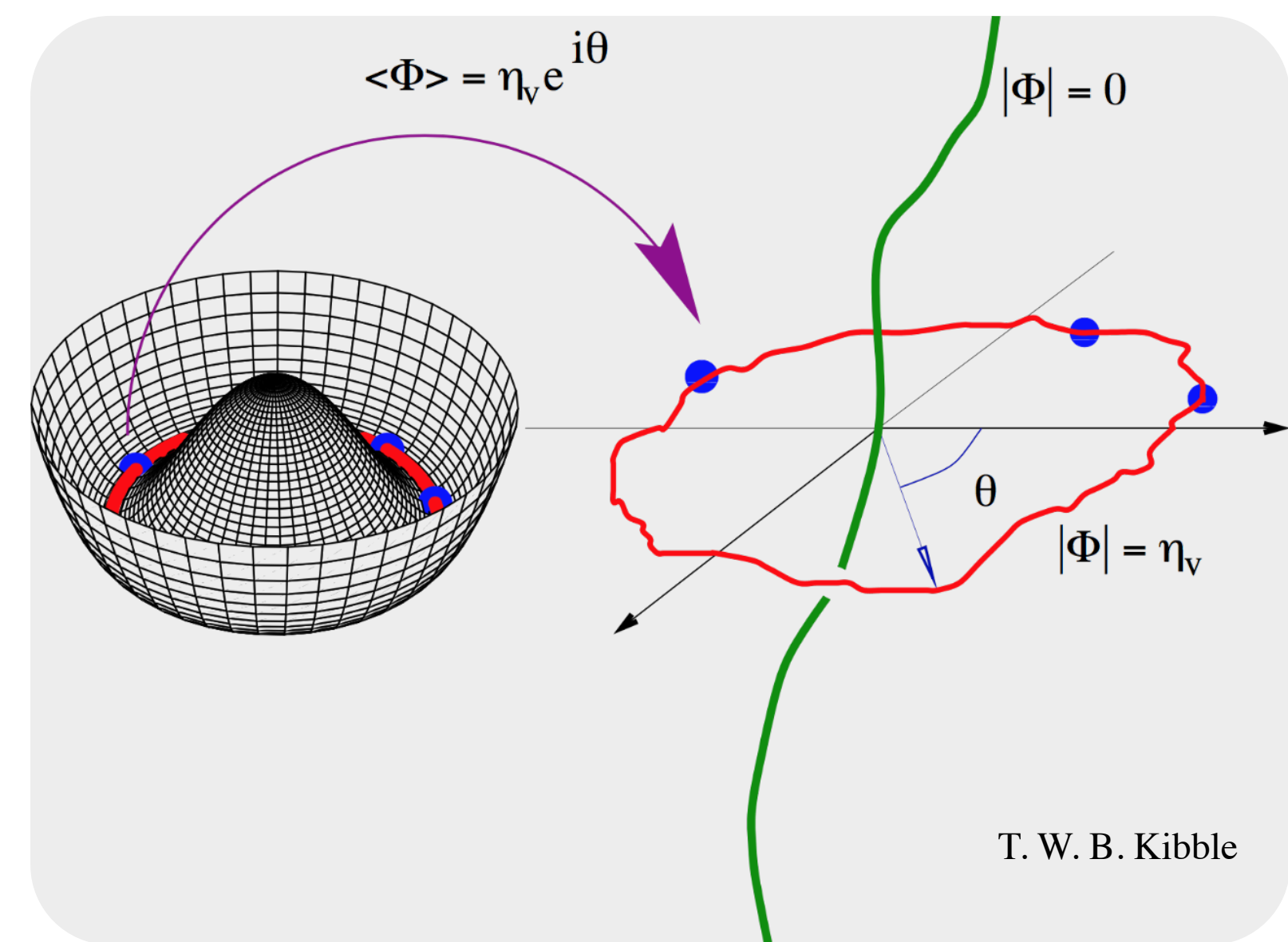


$\pi_1(\mathcal{M}) \neq \mathcal{I}$ non-contractible loops in $\mathcal{M}$

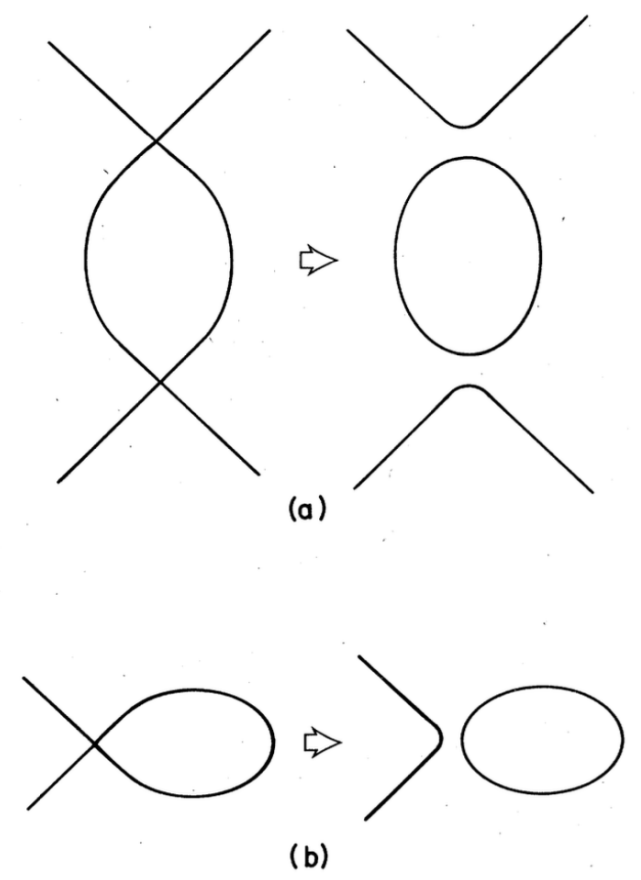


Stephen F. King, Silvia Pascoli, Jessica Turner, Ye-Ling Zhou, Phys. Rev. Lett. **126**, 021802

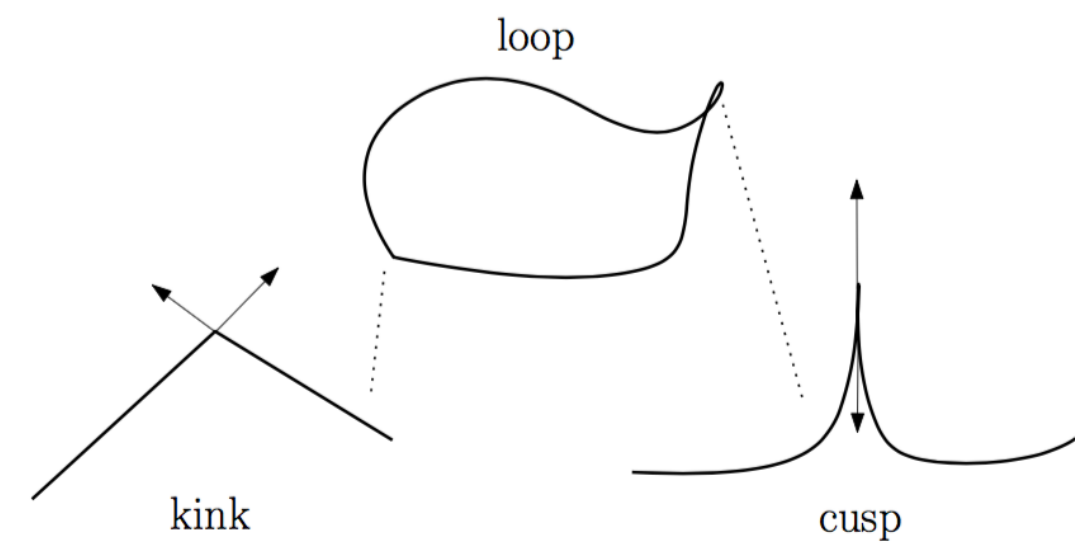
相变后U(1) 对称性自发破缺 宇宙弦



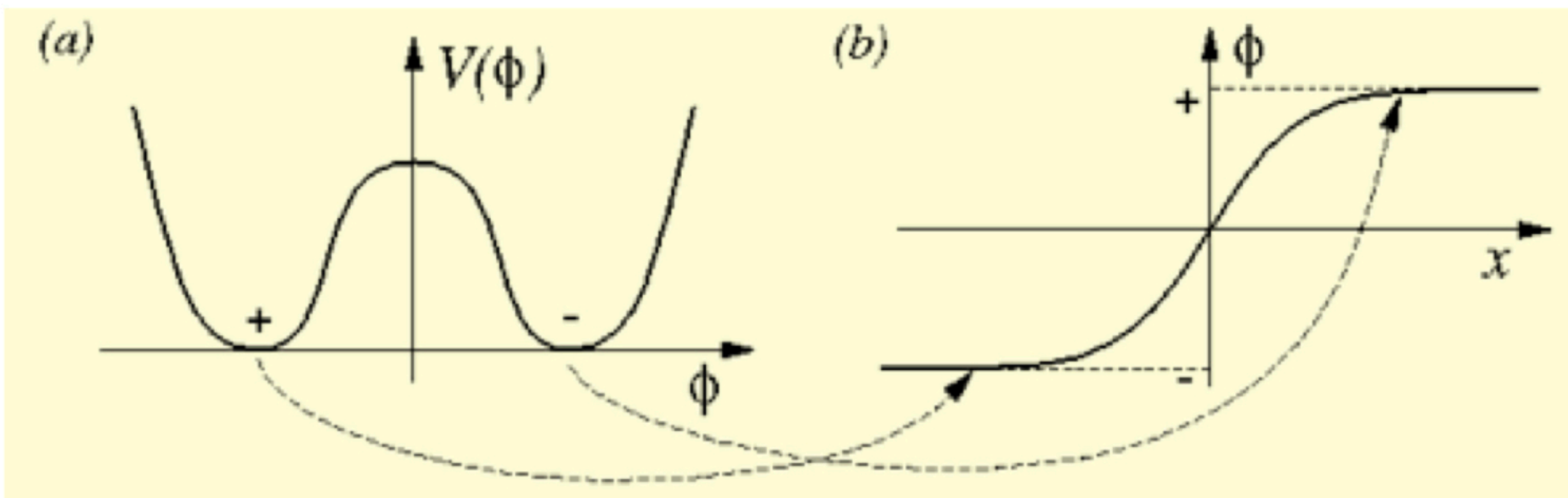
宇宙弦成圈



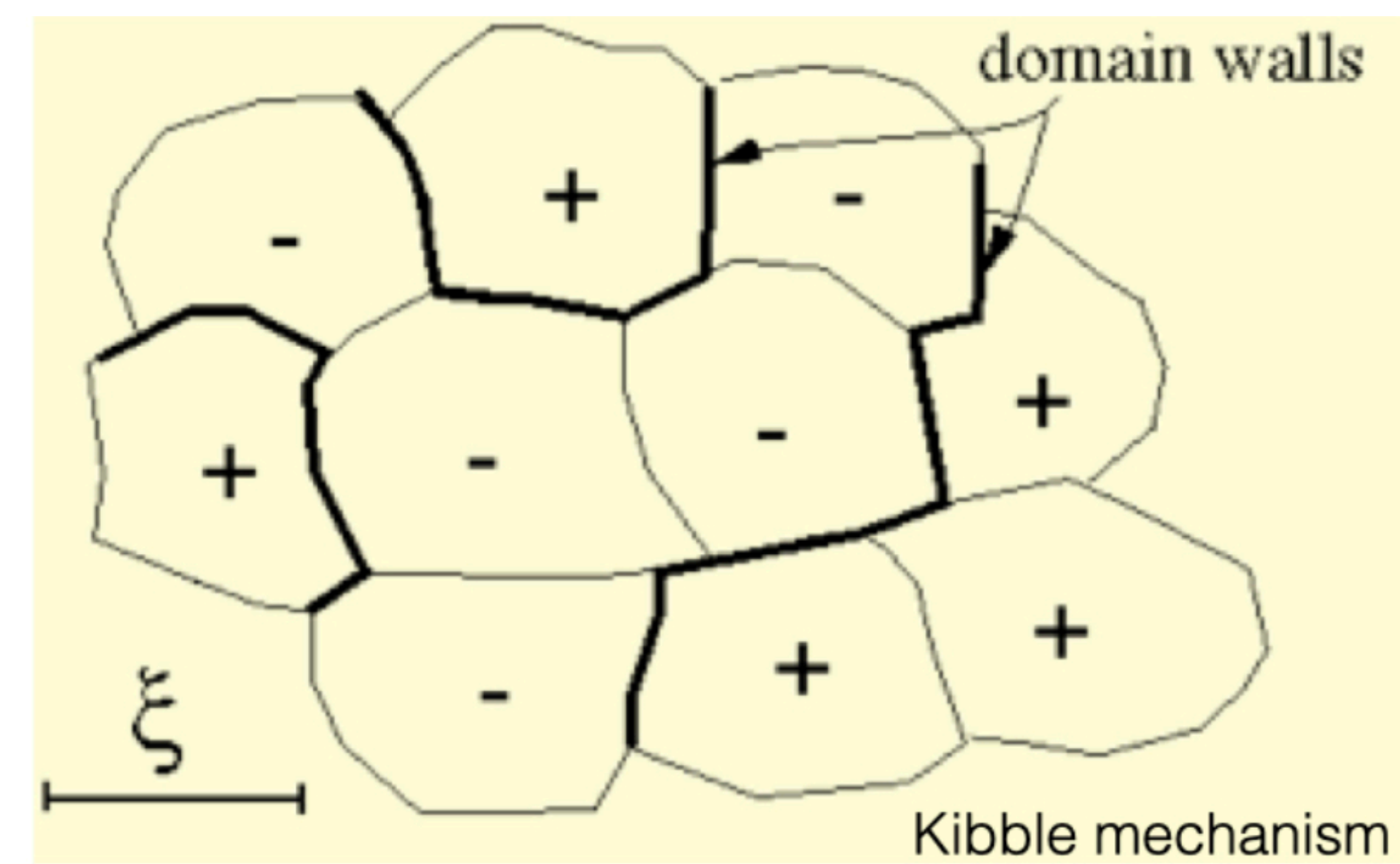
引力波辐射



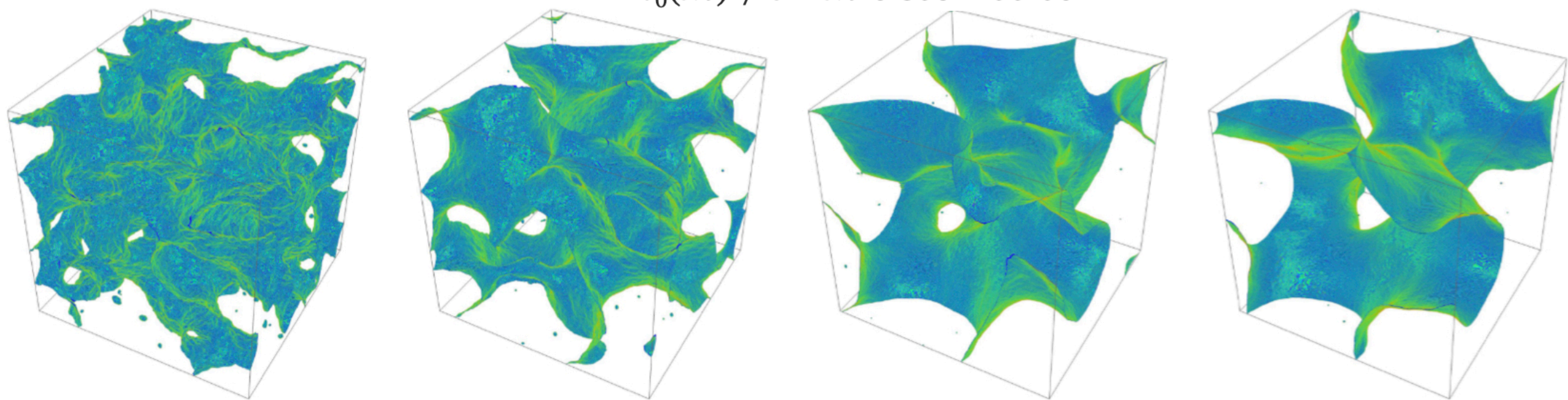
Yann Gouttenoire et al JCAP07(2020)032



势能与场的空间分布



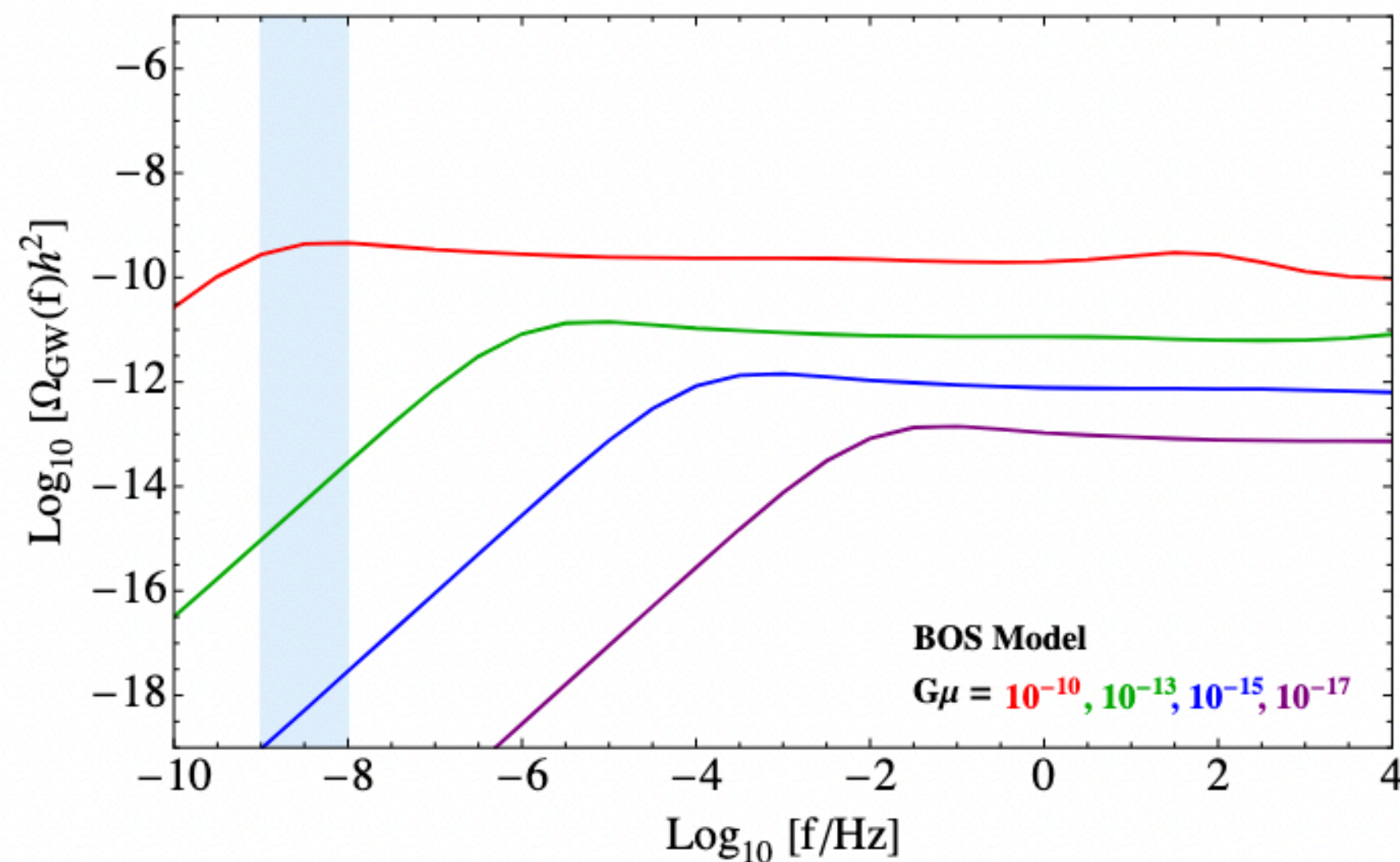
$\pi_0(\mathcal{M}) \neq \mathcal{I}$ $\mathcal{M}$ disconnected



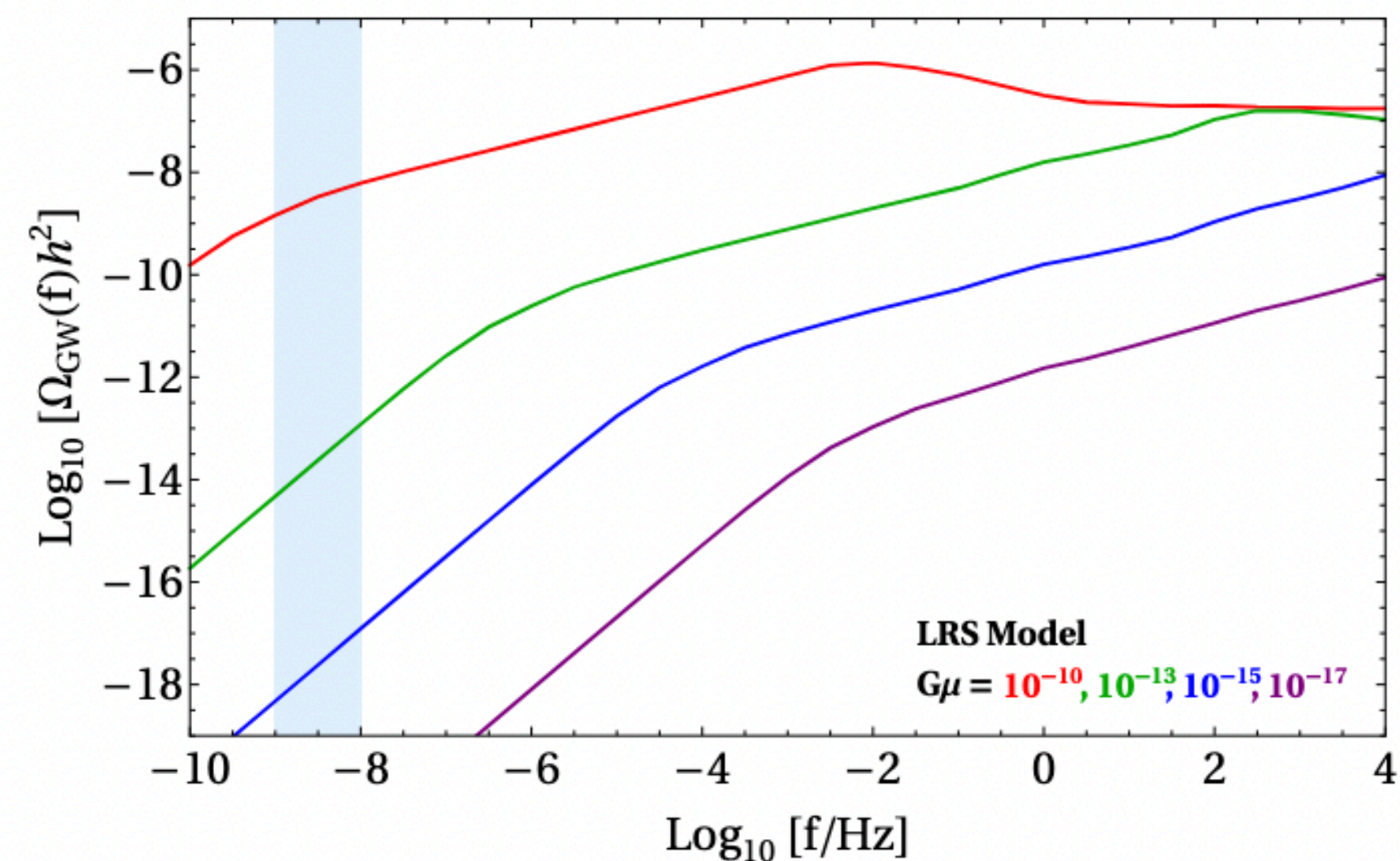
3D snapshots of the pure DW networks

► Nambu-Goto宇宙弦引力波

Nambu-Goto (NG) action is the leading-order approximation when the curvature scale of the strings is much larger than their width



loop production function is numerically obtained



loop distribution of the scaling loops is directly extracted from numerical simulation

► Nambu-Goto vs Lattice simulation 引力波

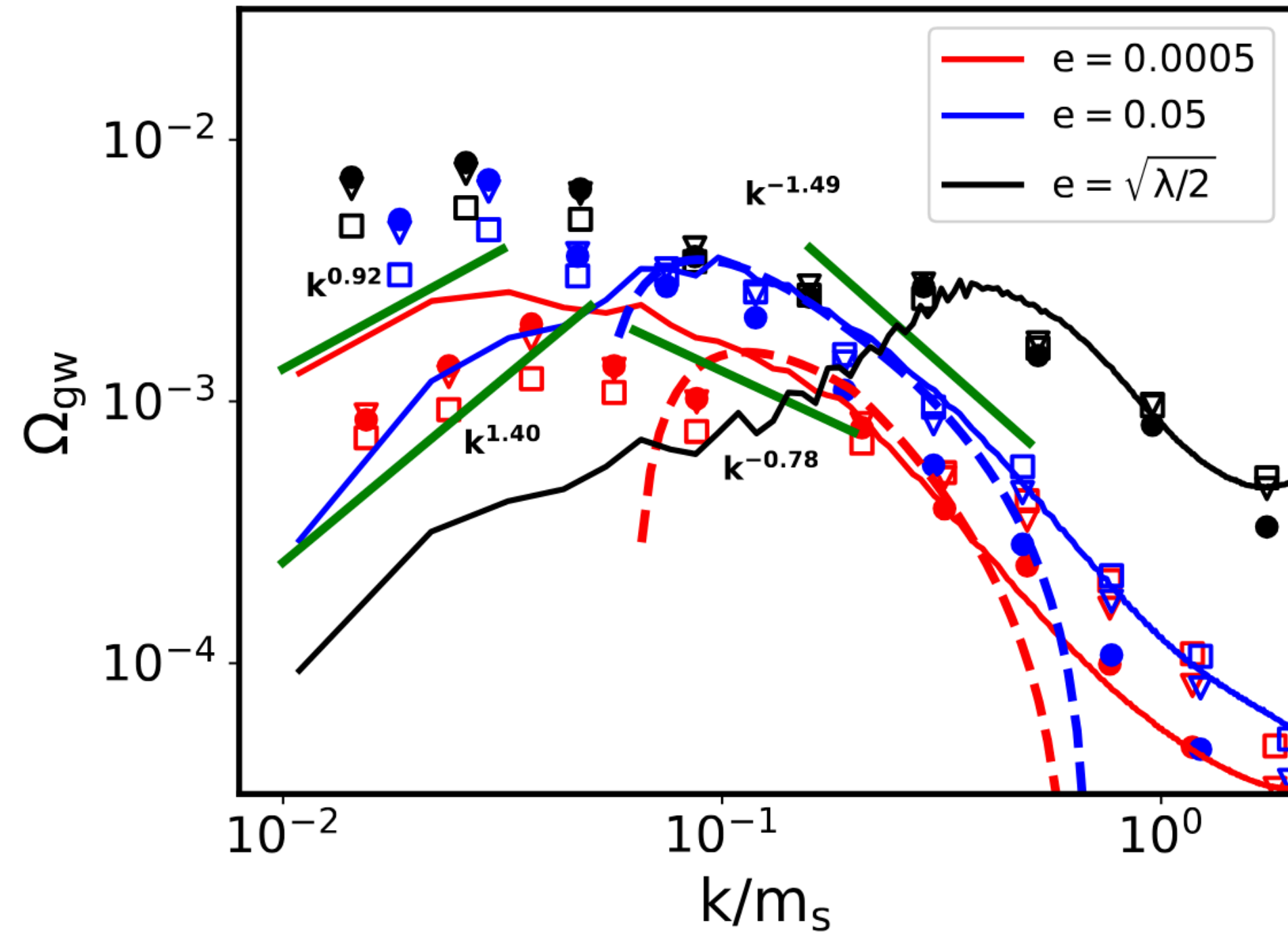


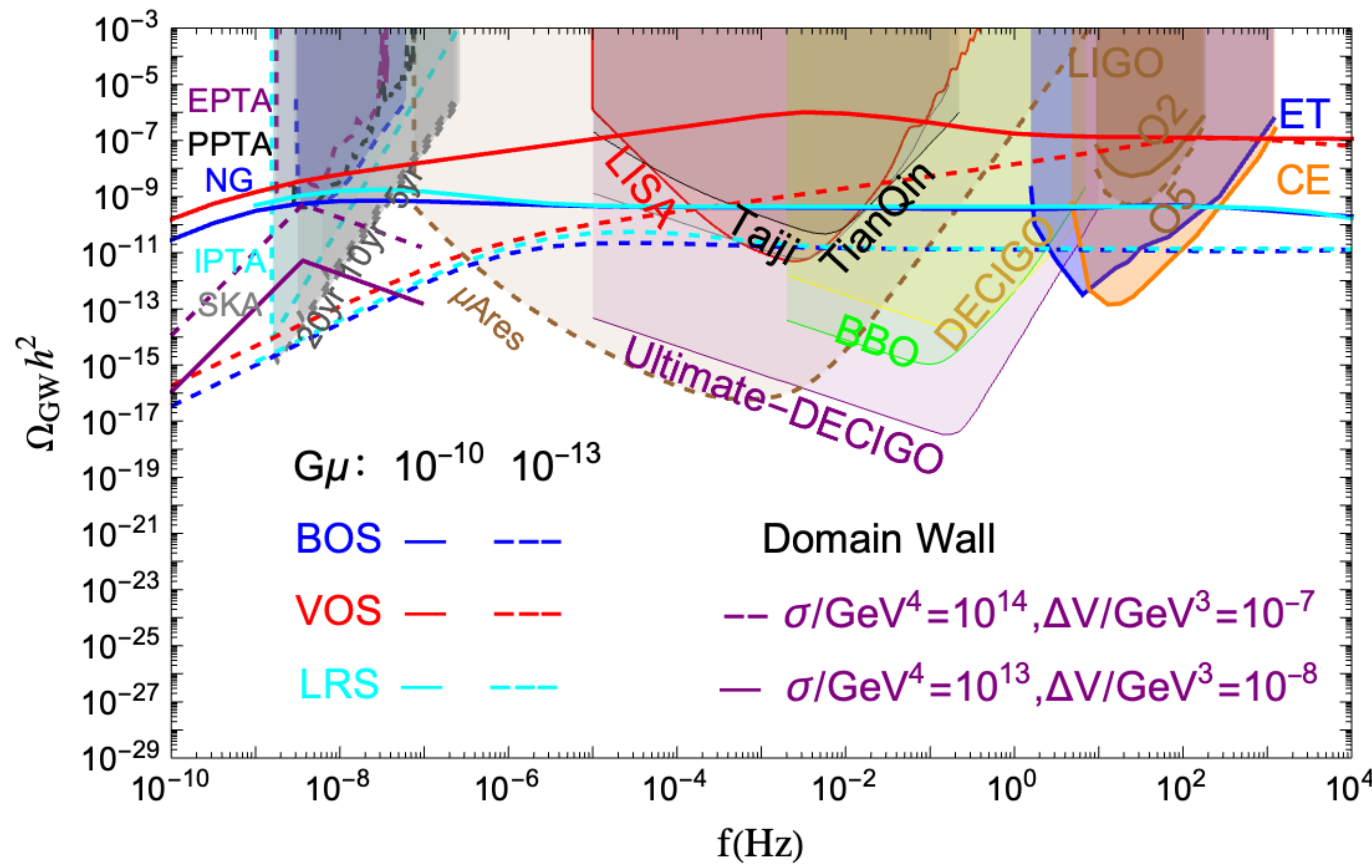
Figure 2: GW spectra for different gauge couplings at $\ln(m_s/H) = 5.5$. Solid lines show the spectra obtained from lattice simulations, while dashed lines show the spectra reconstructed using the transfer function $F(x, y)$. Circles, inverted triangles, and squares denote the NG spectra assuming GW emission dominated by kink-kink collisions, kinks, and cusps, respectively.

$$f_{\text{diff}} = \sqrt{\frac{1}{N_k} \sum_k f_{\text{diff}}(k)}, \quad f_{\text{diff}}(k) = \frac{(x_l(k) - x_{NG}(k))^2}{x_l(k)^2}$$

$x = \log_{10} \Omega_{gw}$ and x_l, x_{NG} denote the results obtained from the lattice simulation and the NG method

Table I: Average differences f_{diff} between lattice results and NG method results

f_{diff}	$e = 0.0005$	$e = 0.05$	$e = \sqrt{\lambda/2}$
cusp	0.092	0.035	0.158
kink	0.068	0.048	0.154
kink-kink collision	0.043	0.065	0.147



The action in expanding universe with spatially flat FLRW metric:

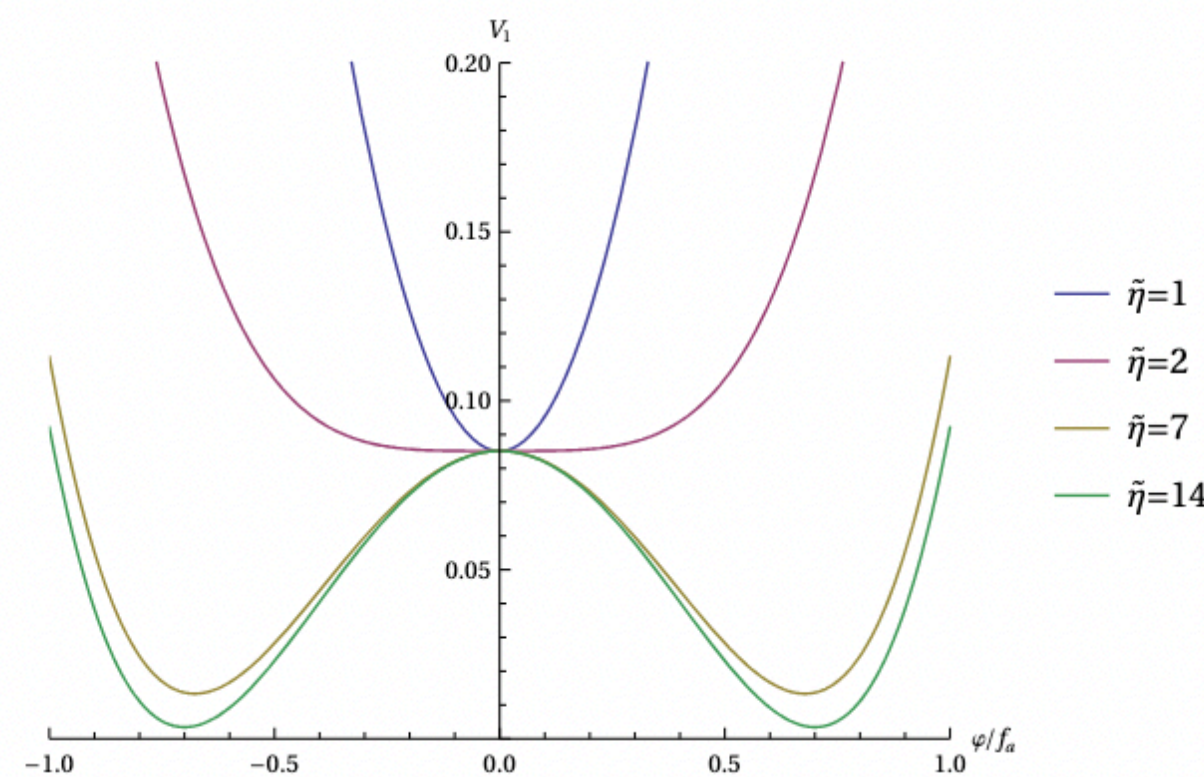
$$S = - \int dx^4 \sqrt{-g} \left(\partial_\mu \varphi^* \partial^\mu \varphi + \frac{1}{2} \partial_\mu h \partial^\mu h + V(\varphi, h, T) \right)$$

Thermal effective potential

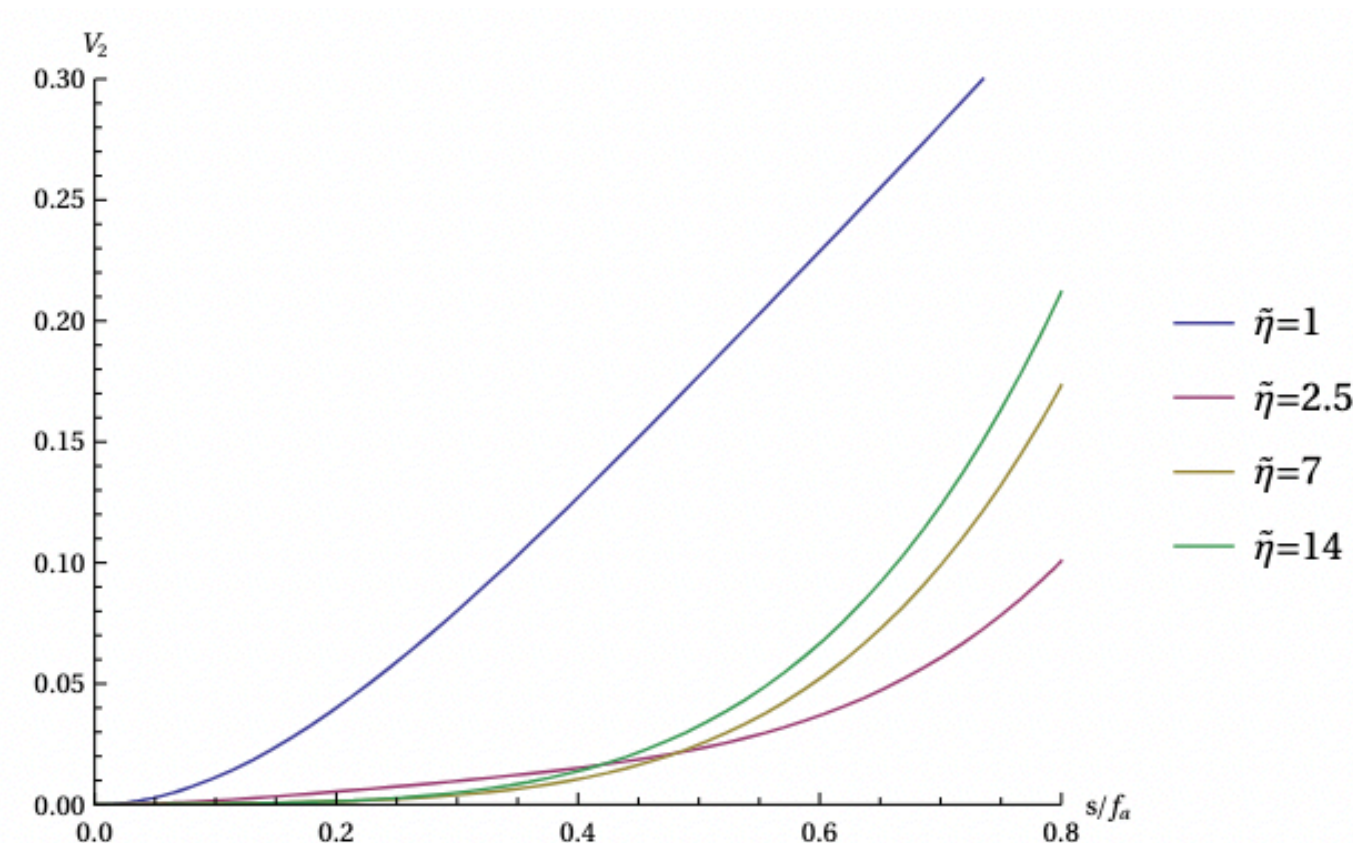
$$V(\varphi, h, T) = V_1(\varphi, T) + V_2(\varphi, h, T)$$

$$V_1(\varphi, T) = \lambda_\varphi \left(|\varphi|^2 - \frac{f_a^2}{2} \right)^2 + \left(\frac{\lambda_\varphi}{3} + \frac{\lambda_{\varphi h}}{6} \right) T^2 |\varphi|^2 \quad V_2(\varphi, s, T) = \frac{1}{2} \gamma (T^2 - T_0^2) h^2 + \frac{1}{3} A T h^3 + \frac{1}{4} \lambda_h h^4 + \frac{1}{2} \lambda_{\varphi h} |\varphi|^2 h^2$$

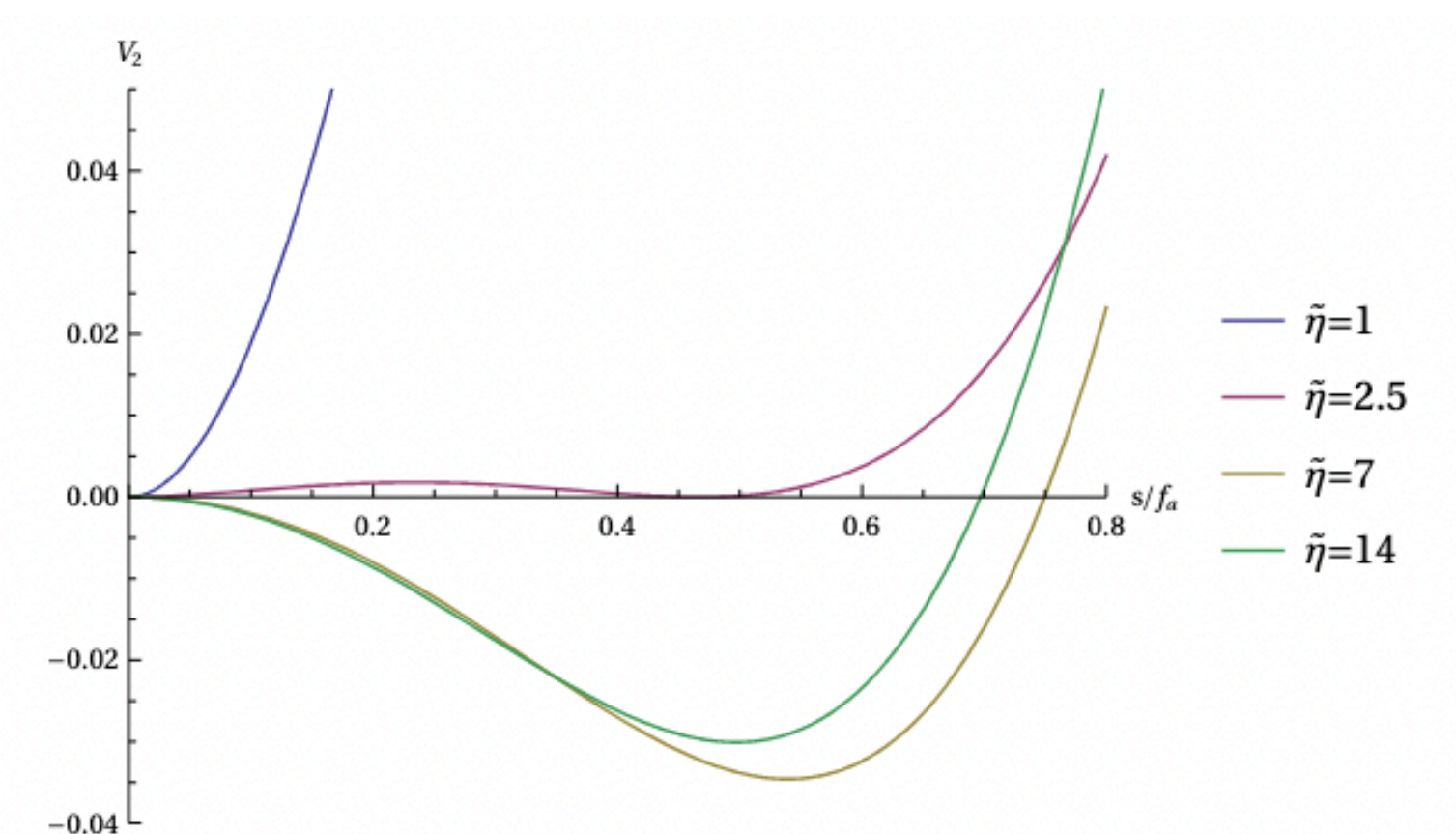
Potential Shape



$h = 0$



$\phi = \phi_{\text{vac}}$



$\phi = 0$

Equations of motion

Only PQ era

$$\left\{ \begin{array}{l} \tilde{\varphi}'' - \tilde{\nabla}^2 \tilde{\varphi} + 2 \frac{a'}{a} \tilde{\varphi}' = -a^2 \frac{\partial \tilde{V}}{\partial |\tilde{\varphi}|} \frac{\tilde{\varphi}}{2|\tilde{\varphi}|}, \\ \tilde{h}'' - \tilde{\nabla}^2 \tilde{h} + 2 \frac{a'}{a} \tilde{h}' = -a^2 \frac{\partial \tilde{V}}{\partial \tilde{h}}, \end{array} \right. \quad \begin{array}{l} \tilde{\varphi} = \frac{\varphi}{f_*} \quad \tilde{h} = \frac{h}{f_*} \quad f_* = f_a \\ \tilde{V} = \frac{V(f_* \tilde{\varphi}, f_* \tilde{h}, T)}{f_*^2 \omega_*^2} \quad \omega_* = a_i H_i \end{array}$$

Initial condition

thermal spectrum

$$\mathcal{P}_{\phi_i}(k) = \frac{1}{\omega_k (e^{\omega_k/T} - 1)} \quad \mathcal{P}_{\pi_{\phi_i}}(k) = \frac{\omega_k}{e^{\omega_k/T} - 1}$$

$$\omega_k = \sqrt{k^2/R^2 + m_{\text{eff}}^2} \quad m_\phi^2 = (\lambda_\phi/3 + \lambda_{\phi h}/6)T^2 - \lambda_\phi v_\phi^2$$

two-point correlation functions

$$\begin{aligned} \langle \phi_i(\mathbf{k}) \phi_j(\mathbf{k}') \rangle &= (2\pi)^3 \mathcal{P}_\phi(k) \delta(\mathbf{k} - \mathbf{k}') \delta_{ij}, \\ \langle \dot{\phi}_i(\mathbf{k}) \dot{\phi}_j(\mathbf{k}') \rangle &= (2\pi)^3 \mathcal{P}_{\dot{\phi}}(k) \delta(\mathbf{k} - \mathbf{k}') \delta_{ij}, \\ \langle \phi_i(\mathbf{k}) \dot{\phi}_j(\mathbf{k}') \rangle &= 0. \end{aligned}$$

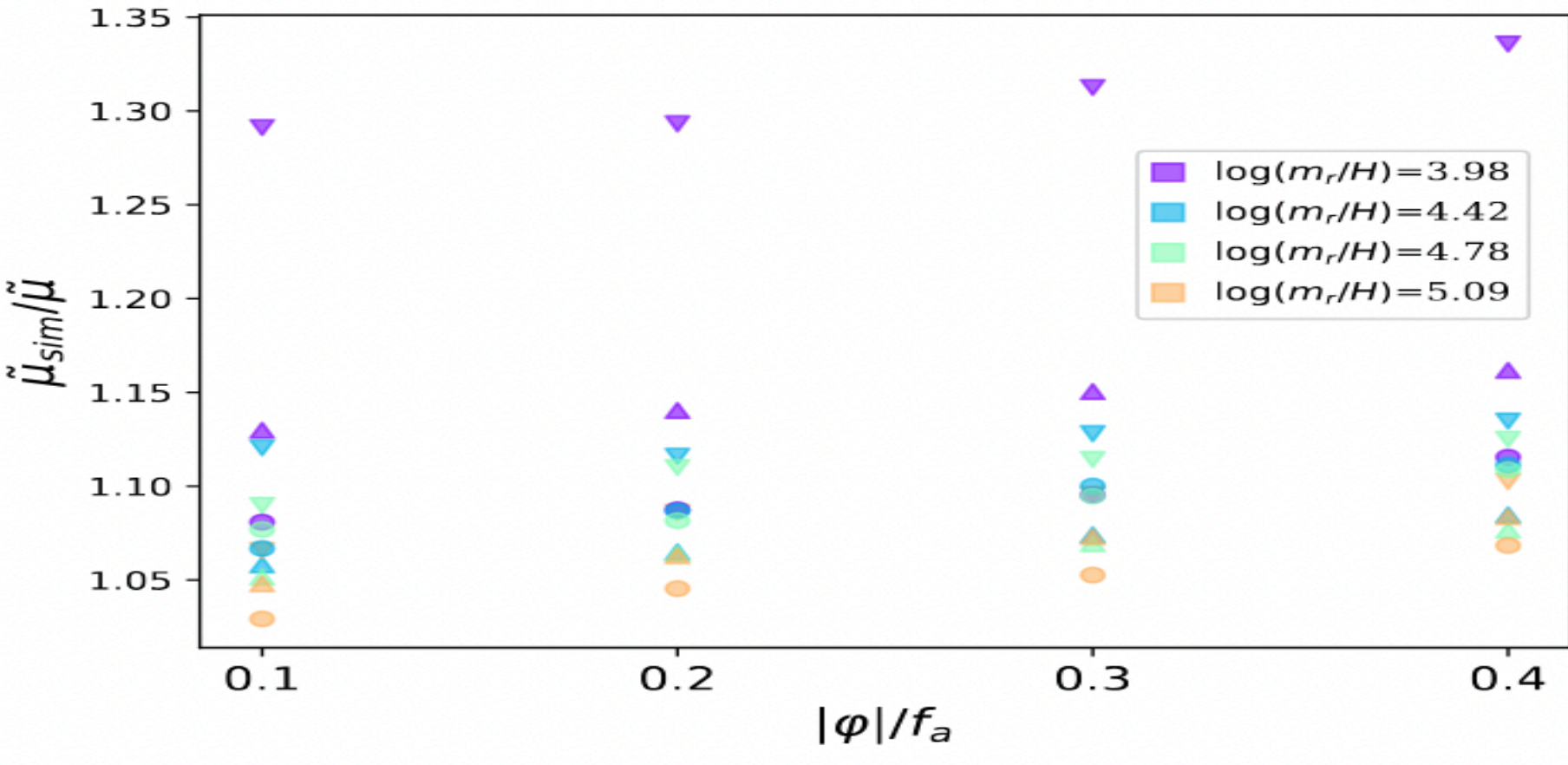
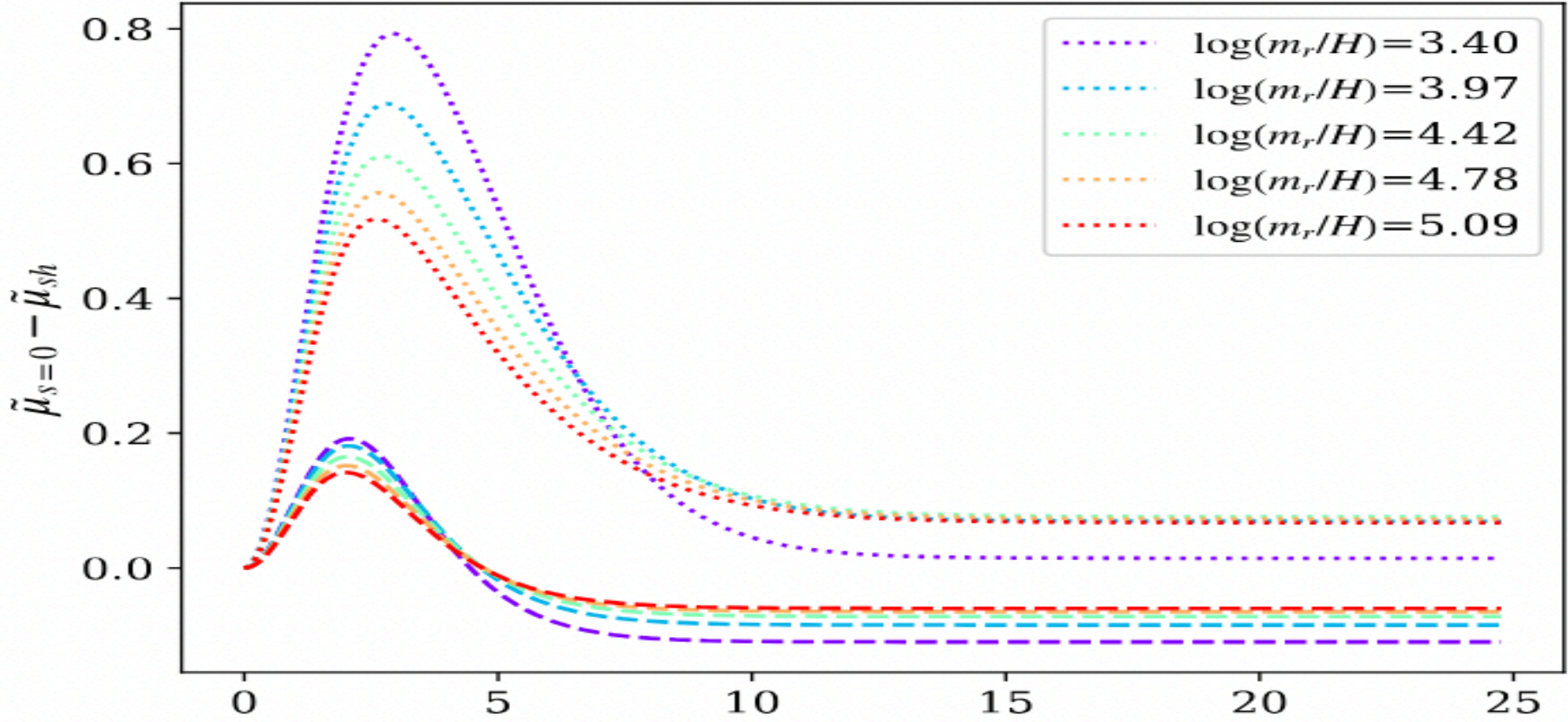
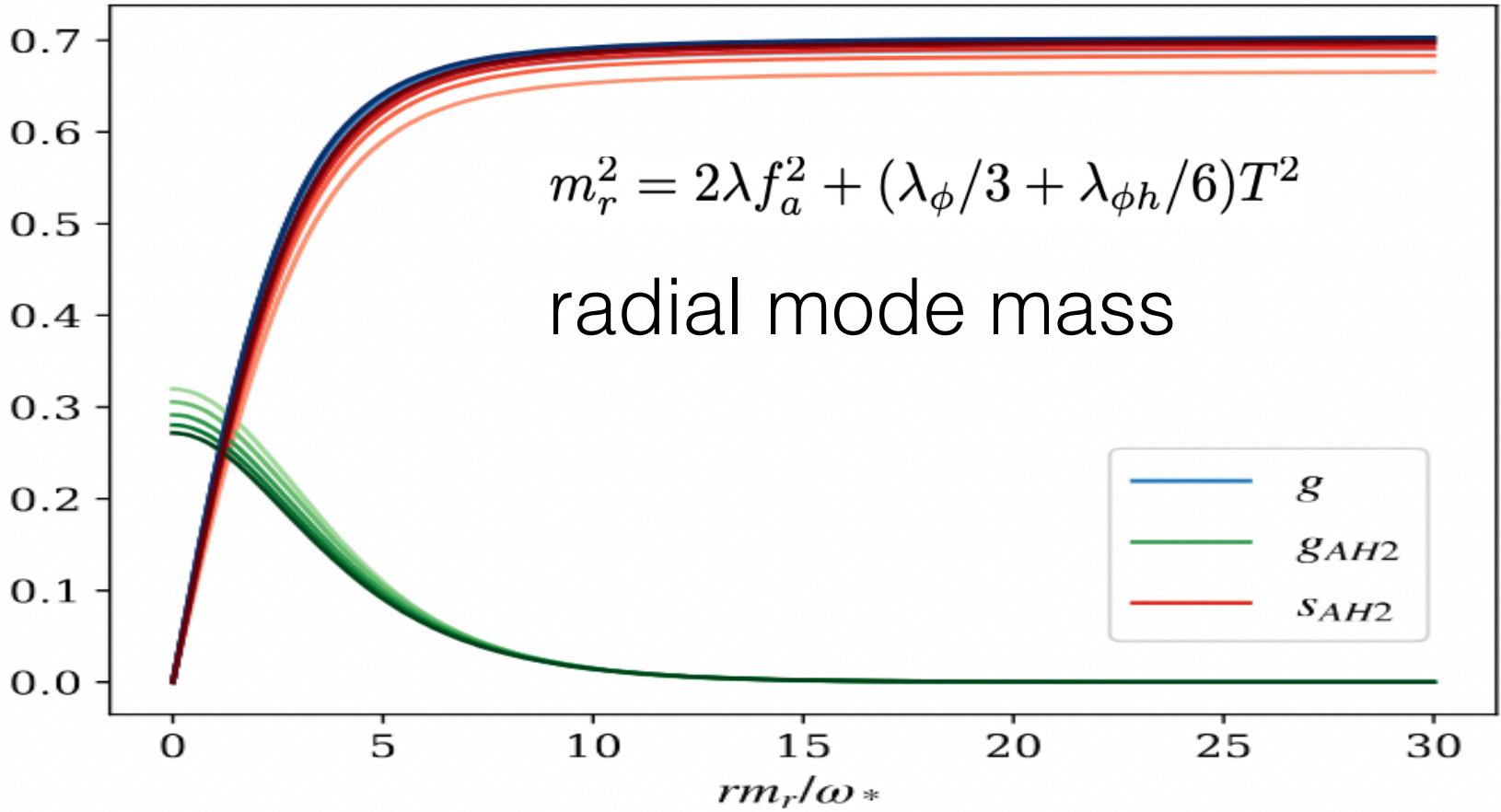
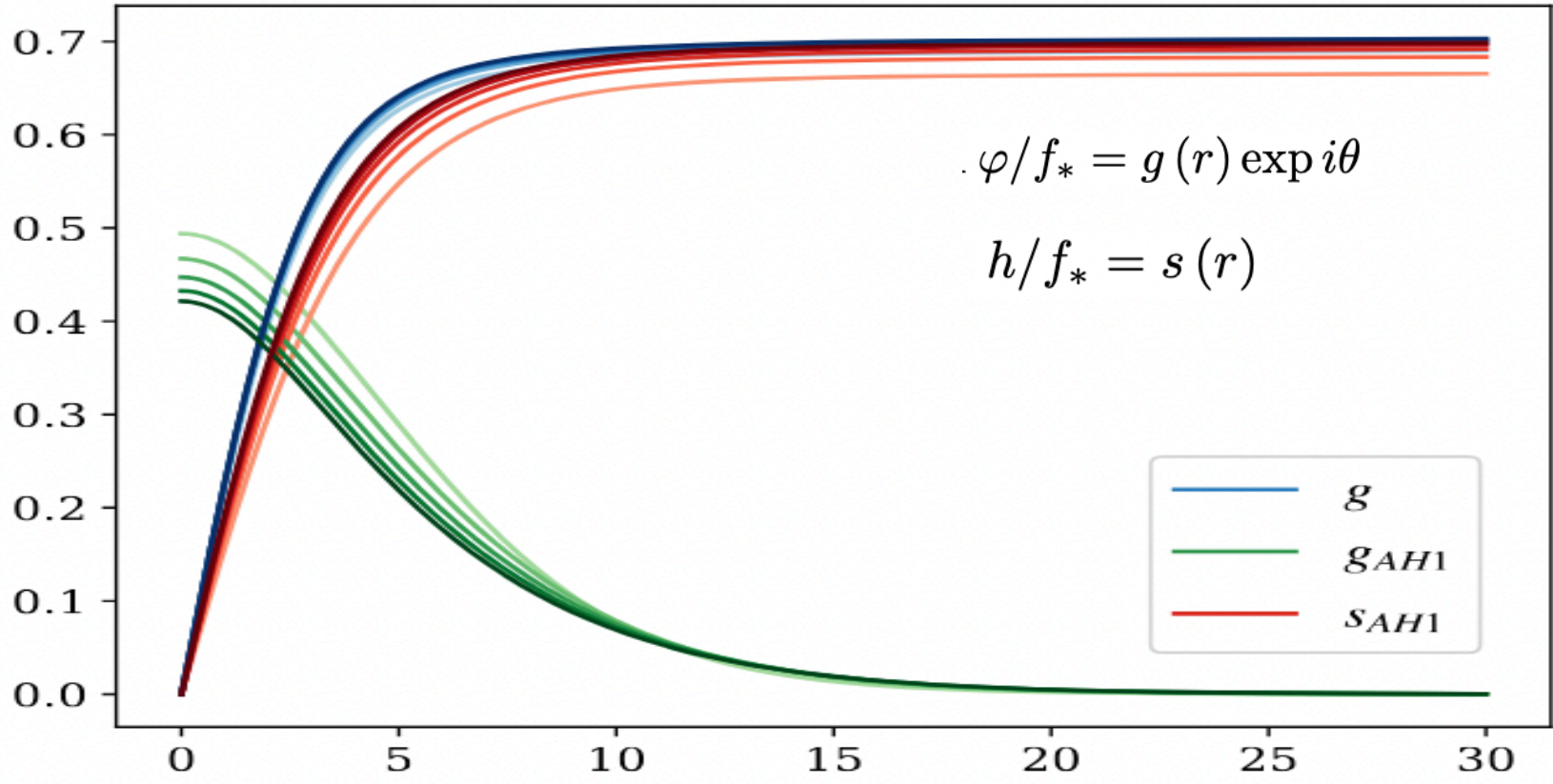
$$\begin{aligned} \langle |\phi_i(\mathbf{k})|^2 \rangle &= \left(\frac{N}{\delta x_{\text{phy}}} \right)^3 \mathcal{P}_{\phi_i}(k), & \langle \phi_i(\mathbf{k}) \rangle &= 0, \\ \langle |\dot{\phi}_i(\mathbf{k})|^2 \rangle &= \left(\frac{N}{\delta x_{\text{phy}}} \right)^3 \mathcal{P}_{\dot{\phi}_i}(k), & \langle \dot{\phi}_i(\mathbf{k}) \rangle &= 0, \end{aligned}$$

► 场的空间构型

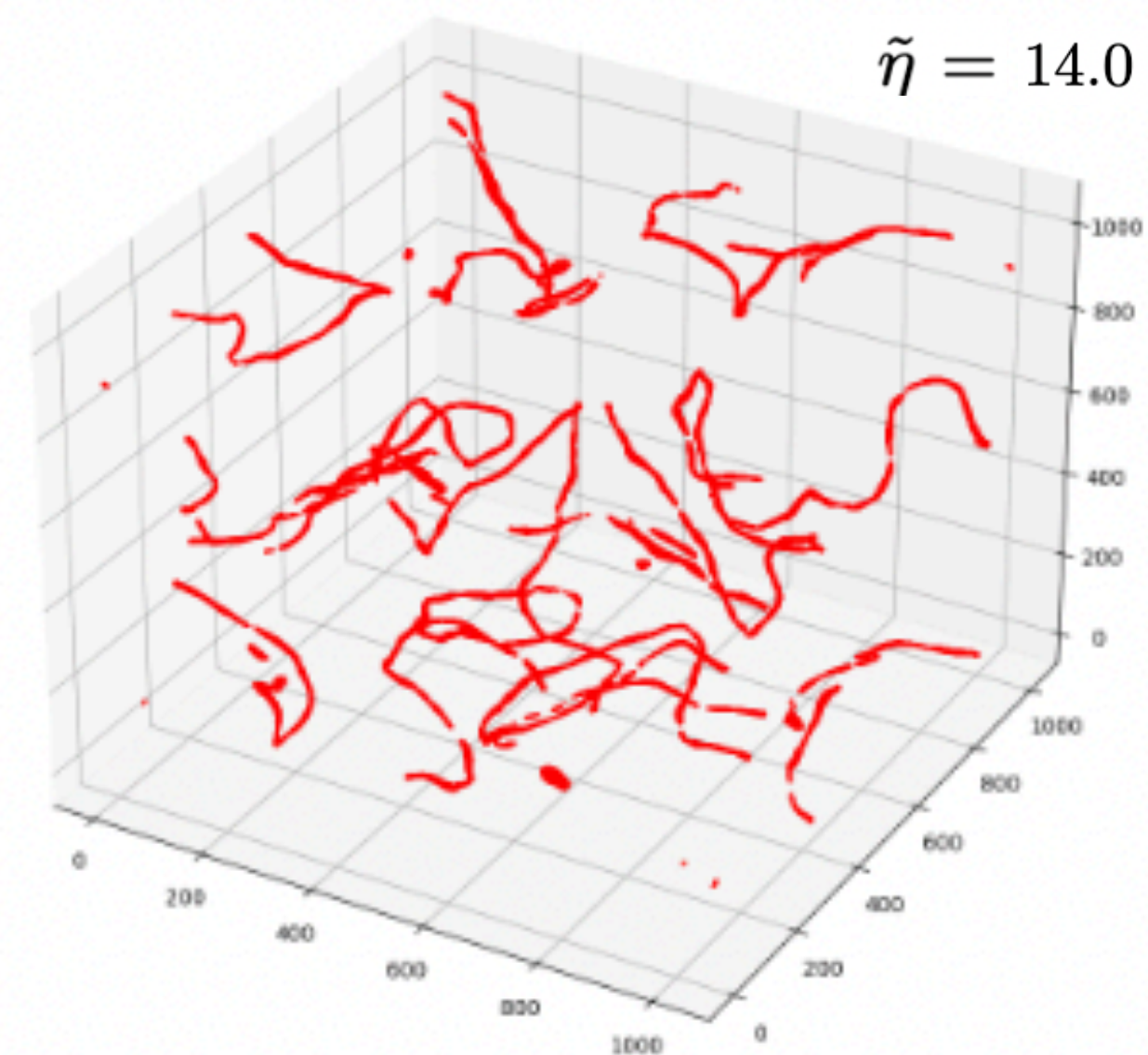
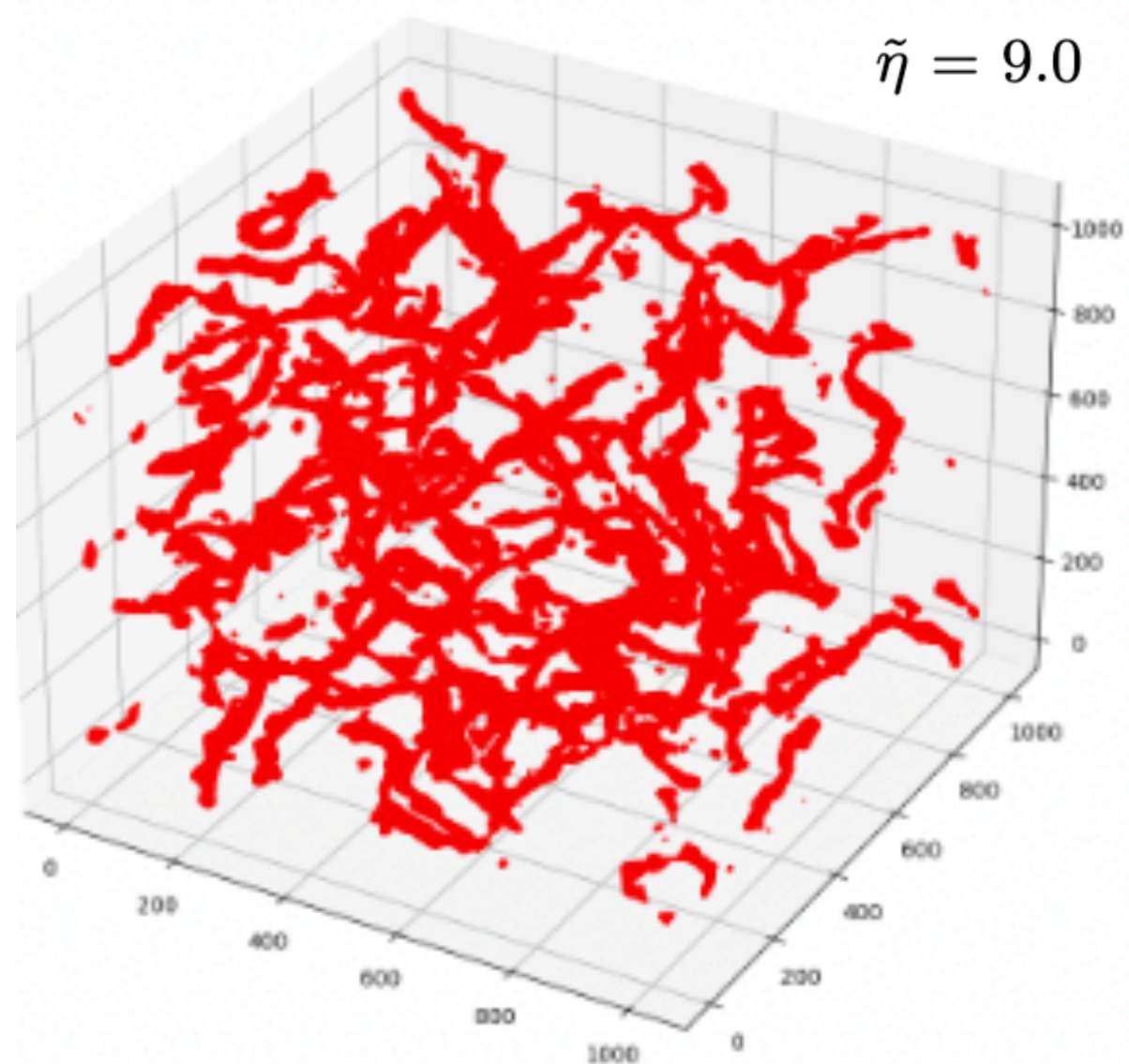
$$g''(r) + \frac{1}{r}g'(r) - \frac{1}{r^2}g(r) = \lambda_\varphi \frac{f_*^2}{\omega_*^2} g(g^2 - \frac{1}{2}) + (\frac{\lambda_\varphi}{3} + \frac{\lambda_{\varphi h}}{6}) \frac{T^2}{\omega_*^2} g + \frac{\lambda_{\phi h}}{2} \frac{f_*^2}{\omega_*^2} g s^2,$$
$$s''(r) + \frac{1}{r}s'(r) = \lambda_{\phi h} \frac{f_*^2}{\omega_*^2} g^2 s + \gamma \frac{T^2 - T_0^2}{\omega_*^2} s + AT \frac{f_*}{\omega_*^2} s^2 + \lambda_h \frac{f_*^2}{\omega_*^2} s^3.$$

Surface tension

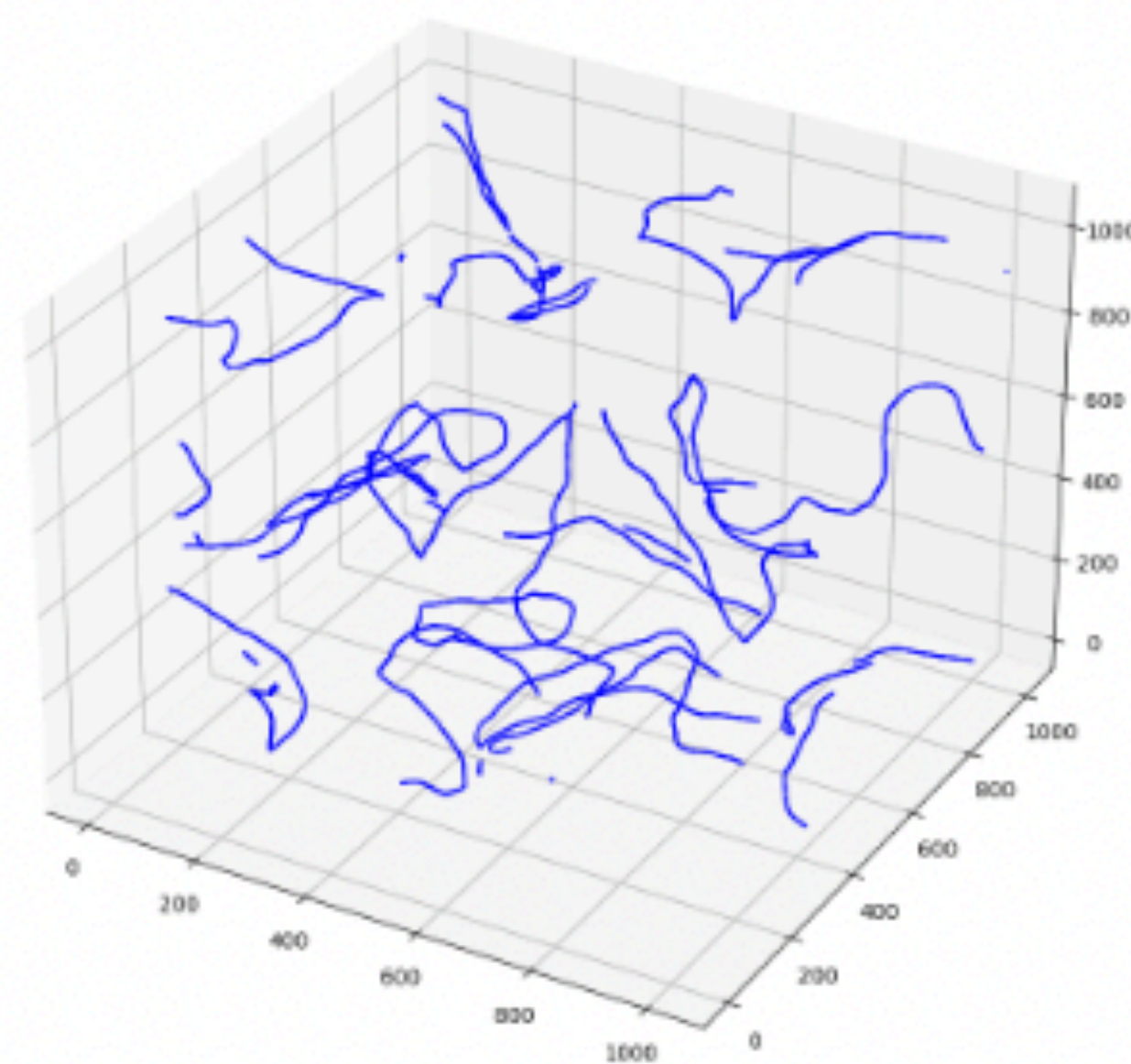
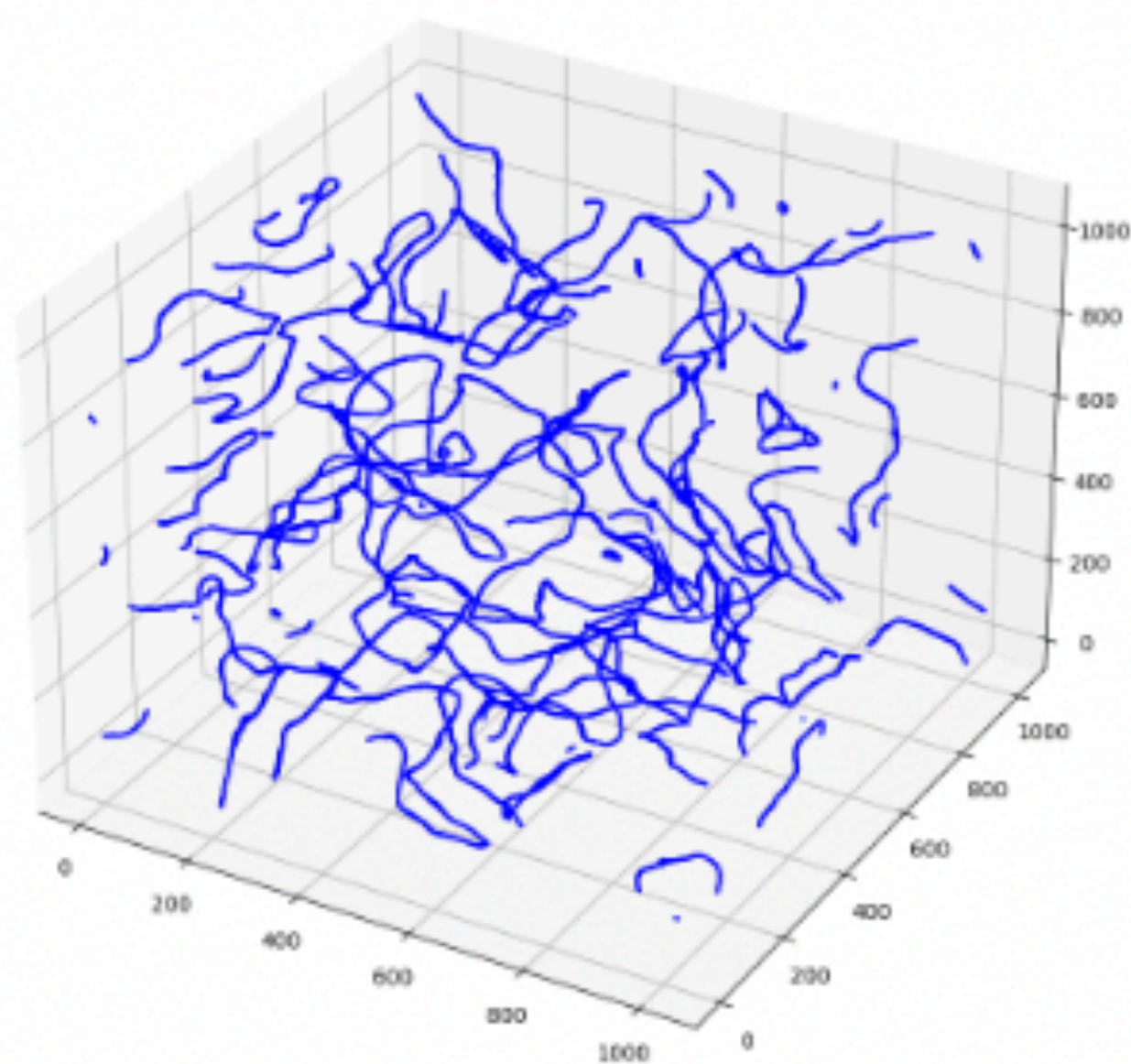
$$\tilde{\mu} = \mu/f_*^2 = \int \tilde{\rho} r dr d\theta$$
$$\tilde{\mu} = 2\pi \int \left((g'^2 + \frac{g^2}{r^2} + (s'^2 + \tilde{V}(g,s)) \right) r dr$$



► 轴子-希格斯弦演化



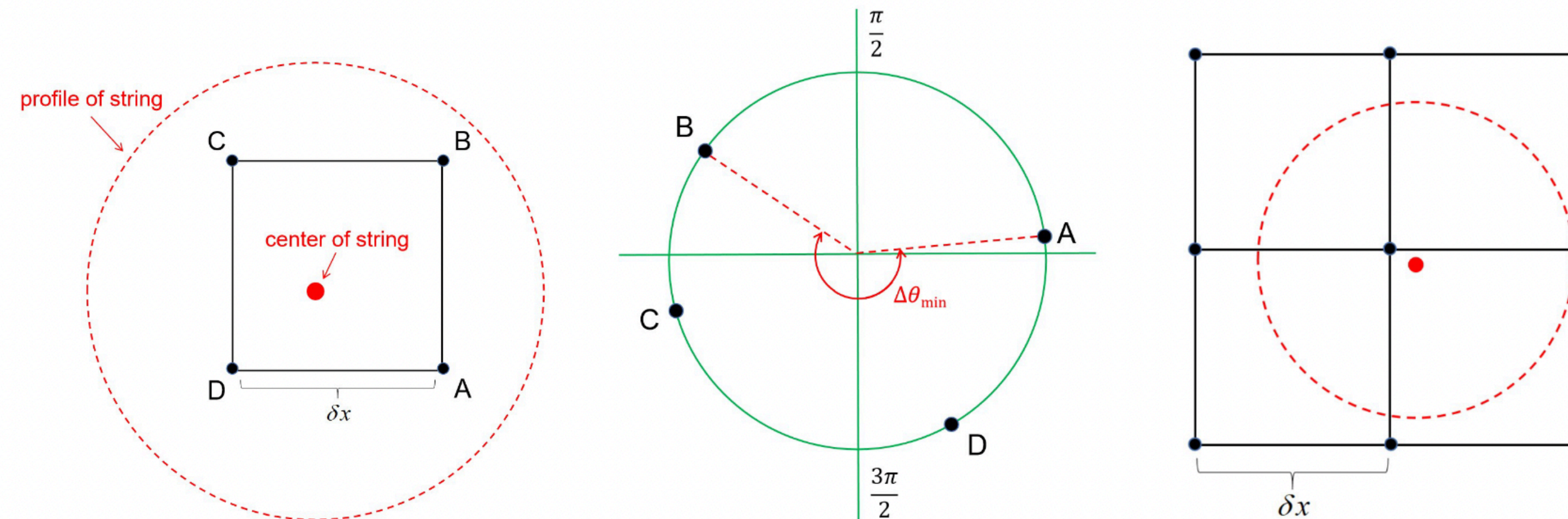
$$s > 0.35 f_a$$



$$|\varphi| < 0.1 f_a$$

► Identification of Cosmic string

String penetrates the square loop if the minimum phase range which contains the four points is greater than π and the phase changes continuously

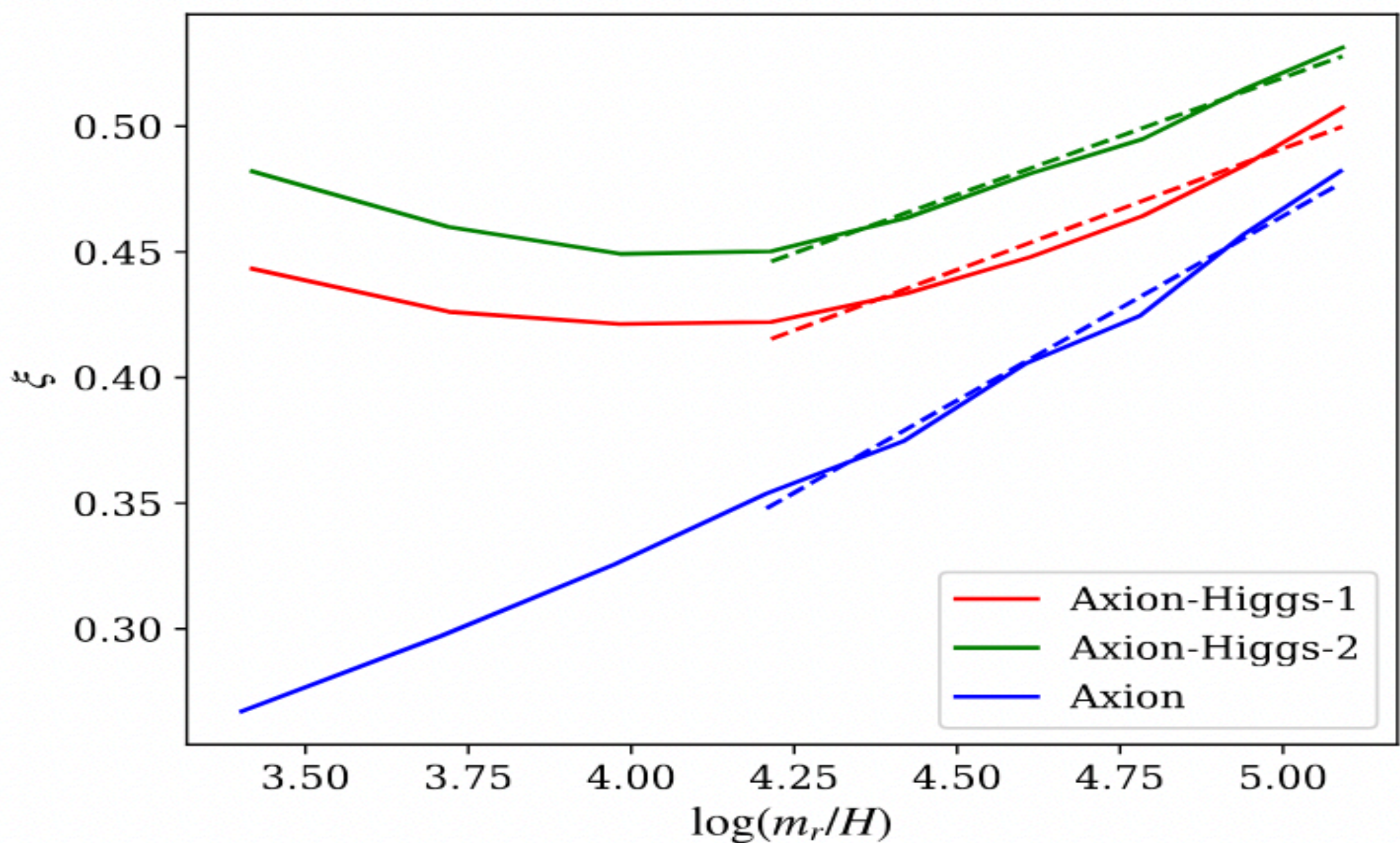


For a specific square loop, assuming that the minimum phase at four points is $\theta_{\min}$

- (1) $\theta_{\min} < \pi$.
- (2) There exists at least one phase at another point minus $\theta_{\min}$ is greater than π .
- (3) There exists at least one phase at another point minus $\theta_{\min}$ is smaller than π .
- (4) Denote the phase closest to π in all phases greater than π as θ_a , and denote the phase closest to π in all phases smaller than π as θ_b , it is required to meet $\theta_a - \theta_b < \pi$.
- (5) Calculate the difference between the phases at each of two adjacent points in a counterclockwise direction, the multiplication of the four differences is required to be negative.

► 宇宙弦的标度参数

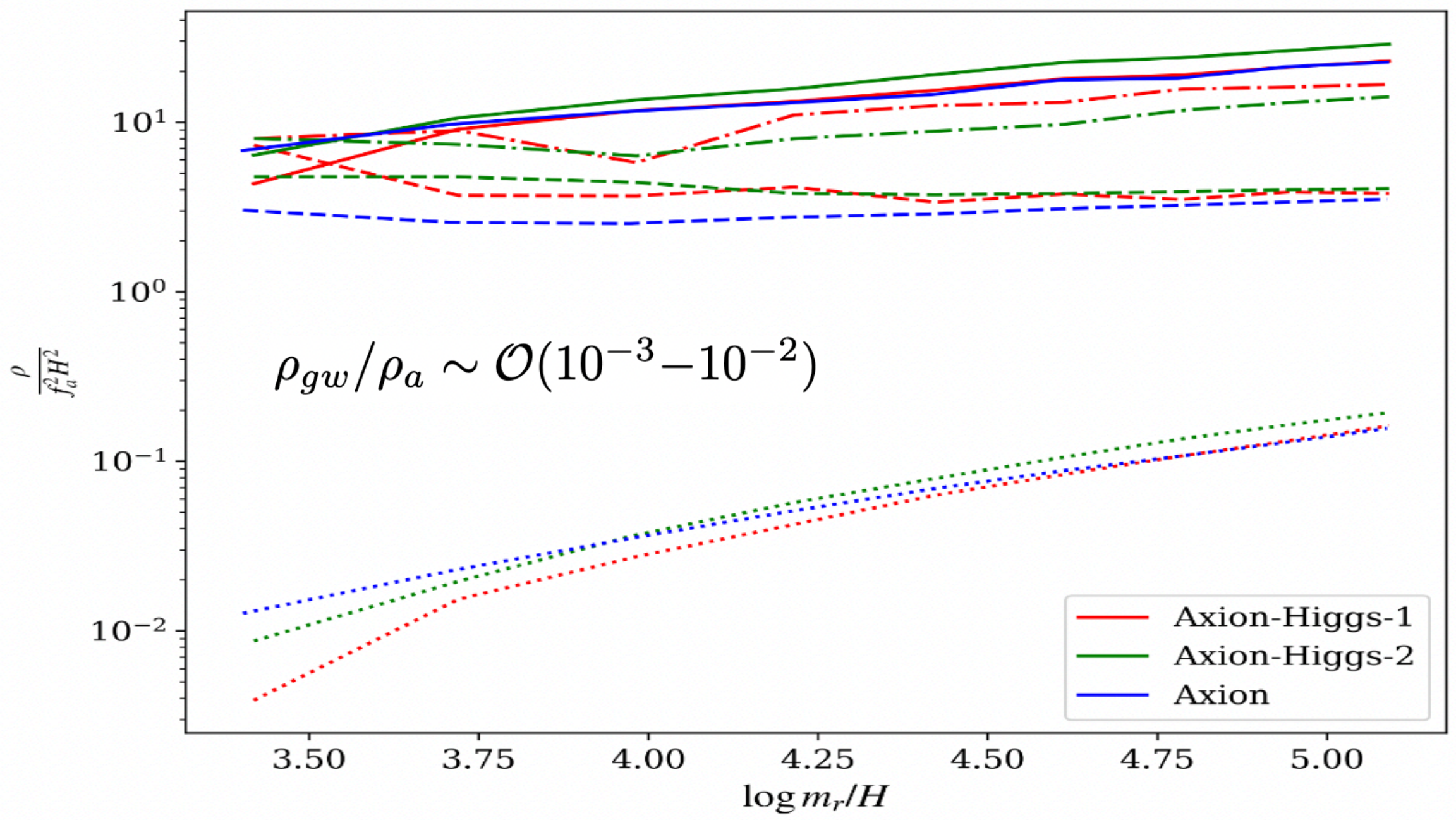
$\xi(t) = \frac{l_s t^2}{a(t)^2 V}$ Scaling parameters



$$\xi = \alpha \log(m_r/H) + \beta$$
$$\alpha = 0.096, 0.093, 0.15$$
$$\beta = -0.009, 0.054, -0.27$$

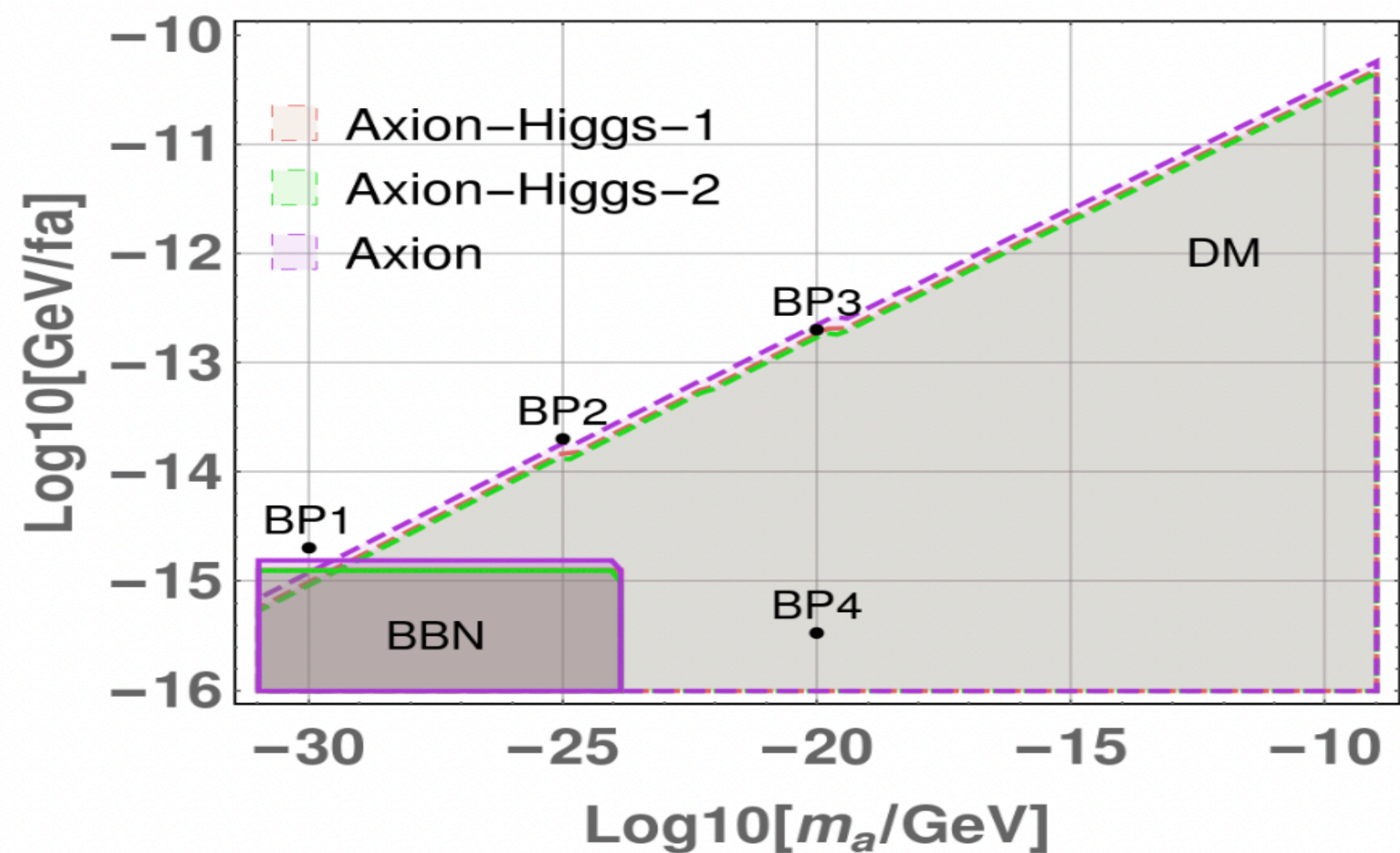
linear growth of $\log(m_r/H)$, same as arXiv:2007.04990, 1806.04677

Energy density



$$\frac{d\rho_{str}(t)}{dt} = -2H(t)\rho_{str}(t) - \left[\frac{d\rho_{str}(t)}{dt} \right]_{emi},$$
$$\frac{d\rho_a(t)}{dt} = -4H(t)\rho_a(t) + \left[\frac{d\rho_{str}(t)}{dt} \right]_{emi1},$$
$$\frac{d\rho_g(t)}{dt} = -4H(t)\rho_g(t) + \left[\frac{d\rho_{str}(t)}{dt} \right]_{emi2}.$$

► 宇宙弦辐射暗物质

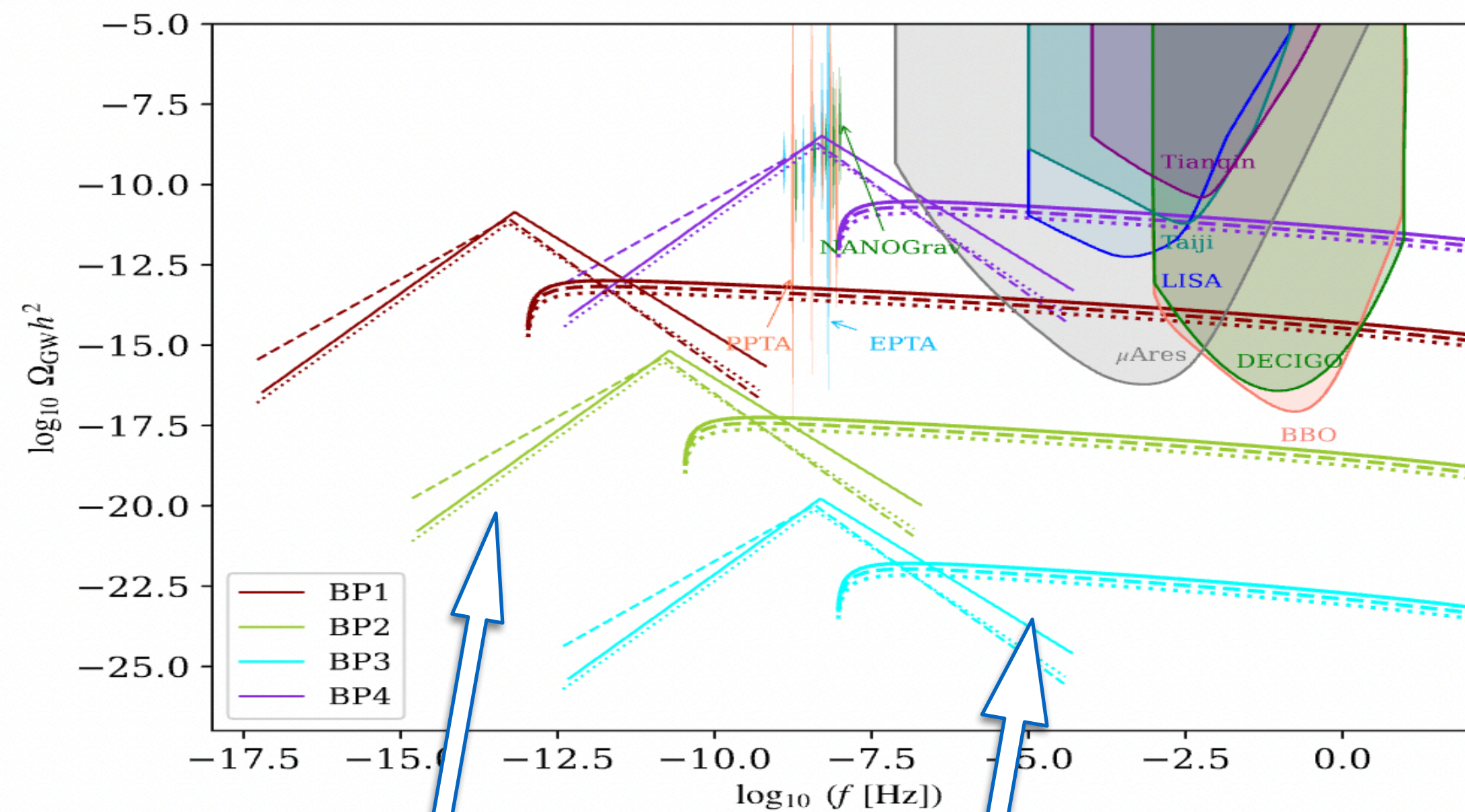


Dark Matter relic abundance

$$\Omega_{DM} h^2 \approx 1.6 \times 10^9 f_a^2 m_a^2 \xi (-2.25 + \log \frac{f_a}{m_a \xi^{1/2}}) \times \epsilon^{-1} (m_a^2 m_{pl}^2 g_*^{1/3})^{-3/4}.$$

Dark Radiation

$$\Delta N_{eff} = \xi \left(\frac{f_a}{10^{15} \text{GeV}} \right)^2 \left(-1.21 \times 10^{-3} + 1.67 \times 10^{-5} \log^2 \left(\frac{1.12 \times 10^{39} \frac{f_a}{10^{15} \text{GeV}}}{\xi^{1/2}} \right) \right)$$



$$(d(a^4 \rho_g)/dt)/(d(a^4 \rho_a)/dt) = \gamma G \mu^2 / f_a^2$$

Method 1

$$F_g(x, y) = \frac{H}{\Gamma_g} \frac{1}{a^3} \frac{\partial}{\partial t} \left(a^3 \frac{\rho_g}{\partial k} \right)$$

$$\Gamma_g \simeq 8\pi \xi r G f_a^4 H^3 \log^3 m_r / H$$

Method 2

ξ_{str} of the string network is essentially the typical bubble diameter for SFOPT???

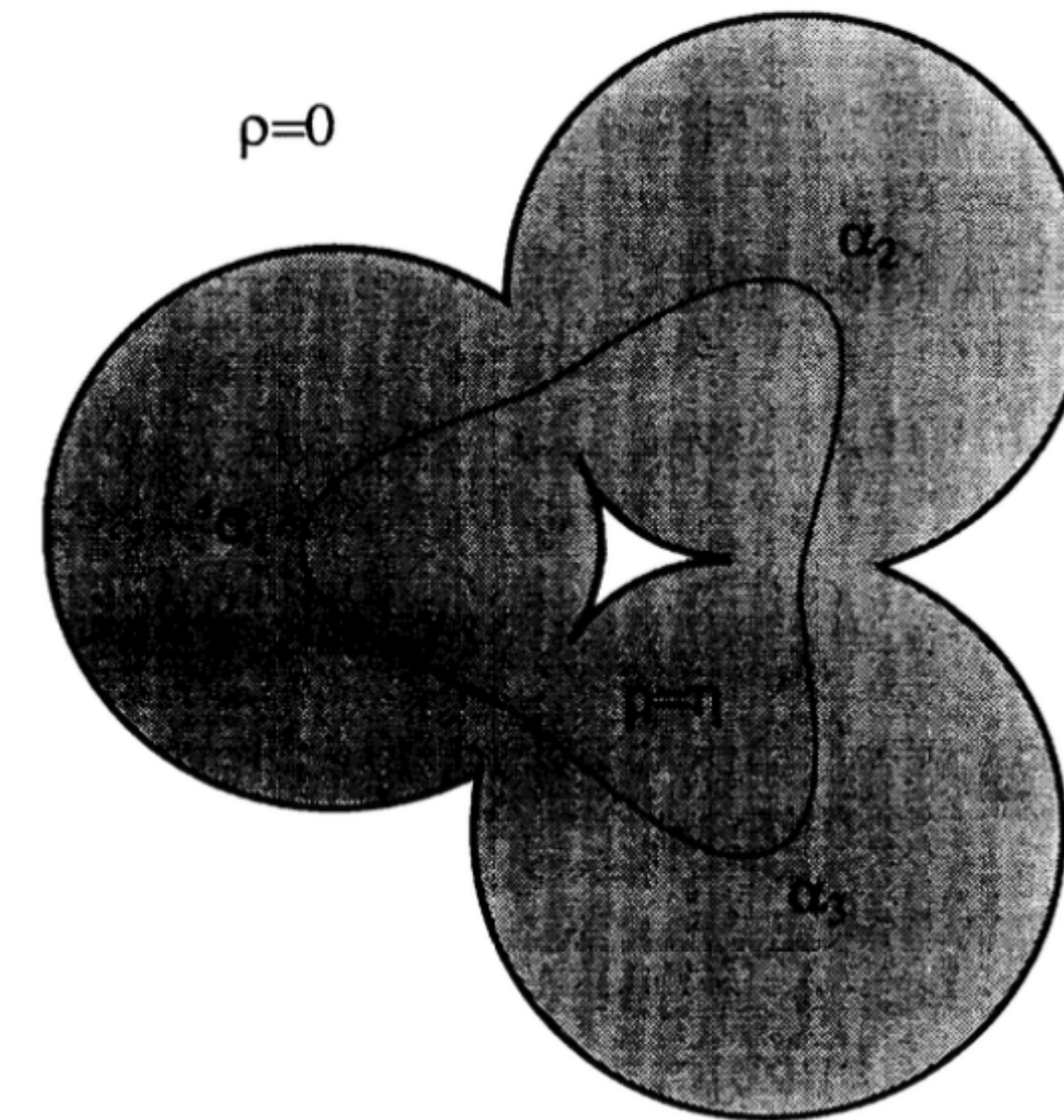
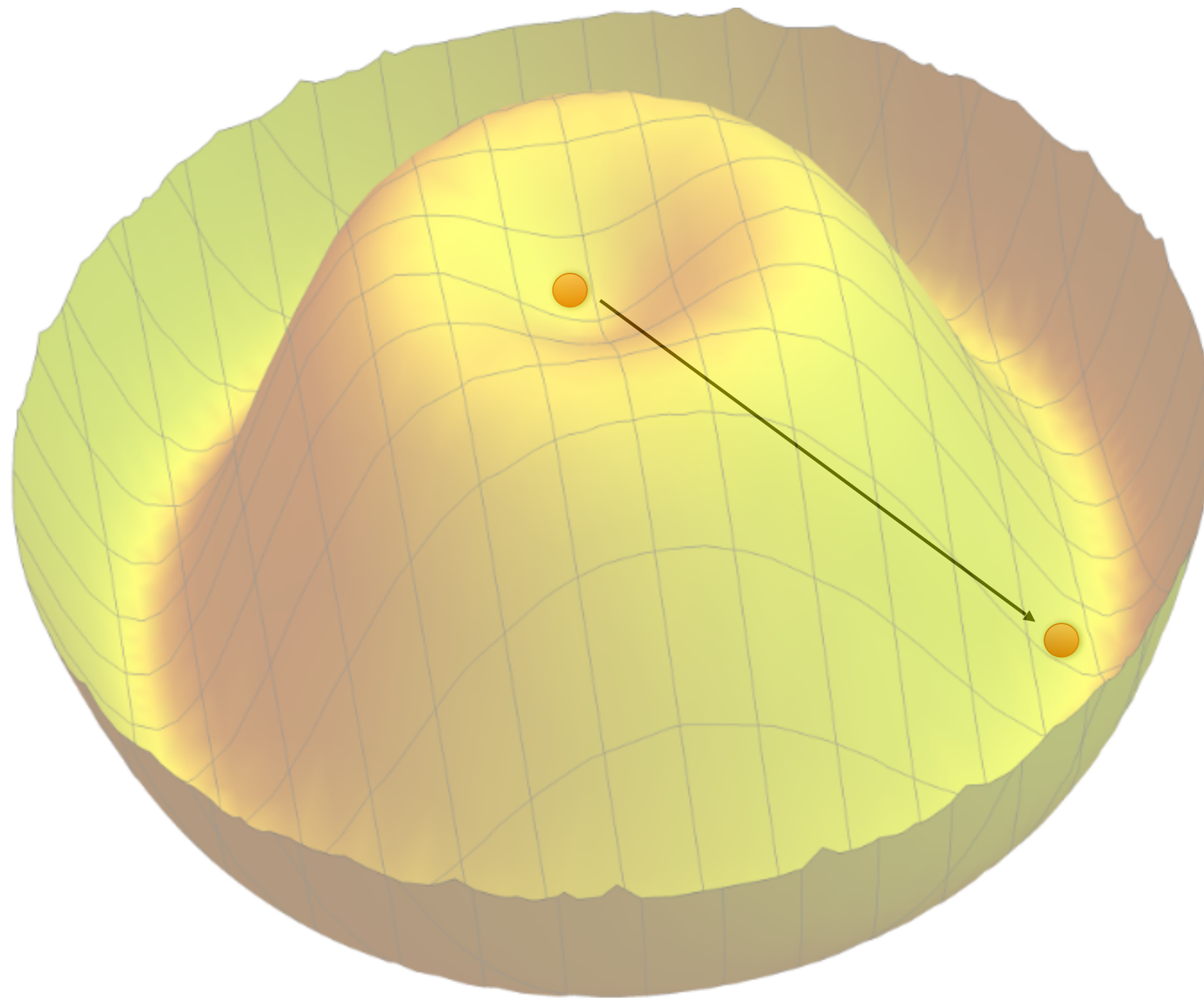
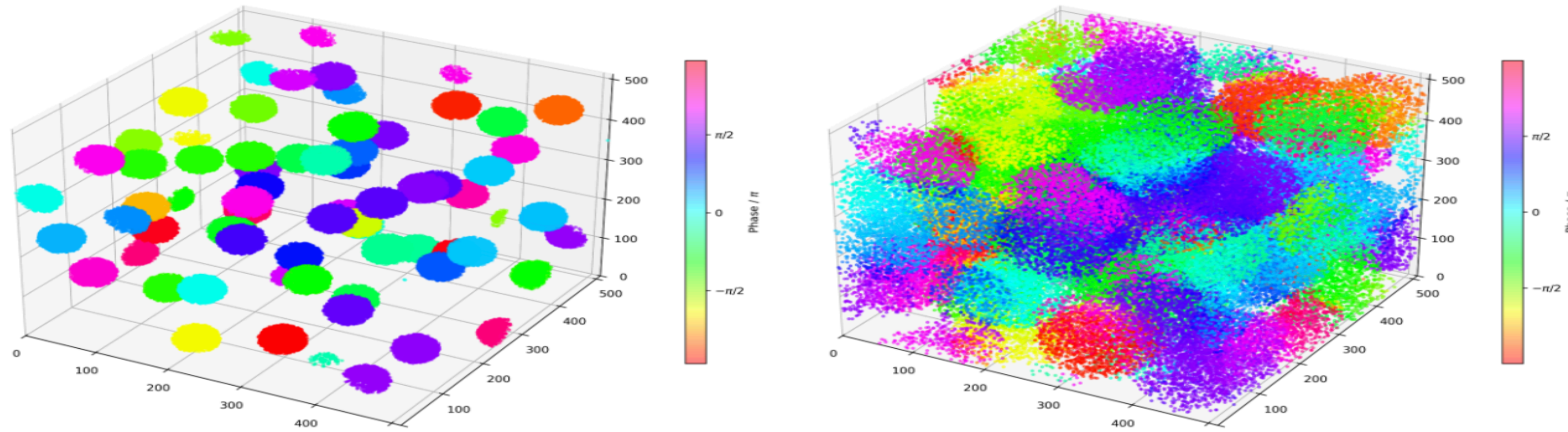
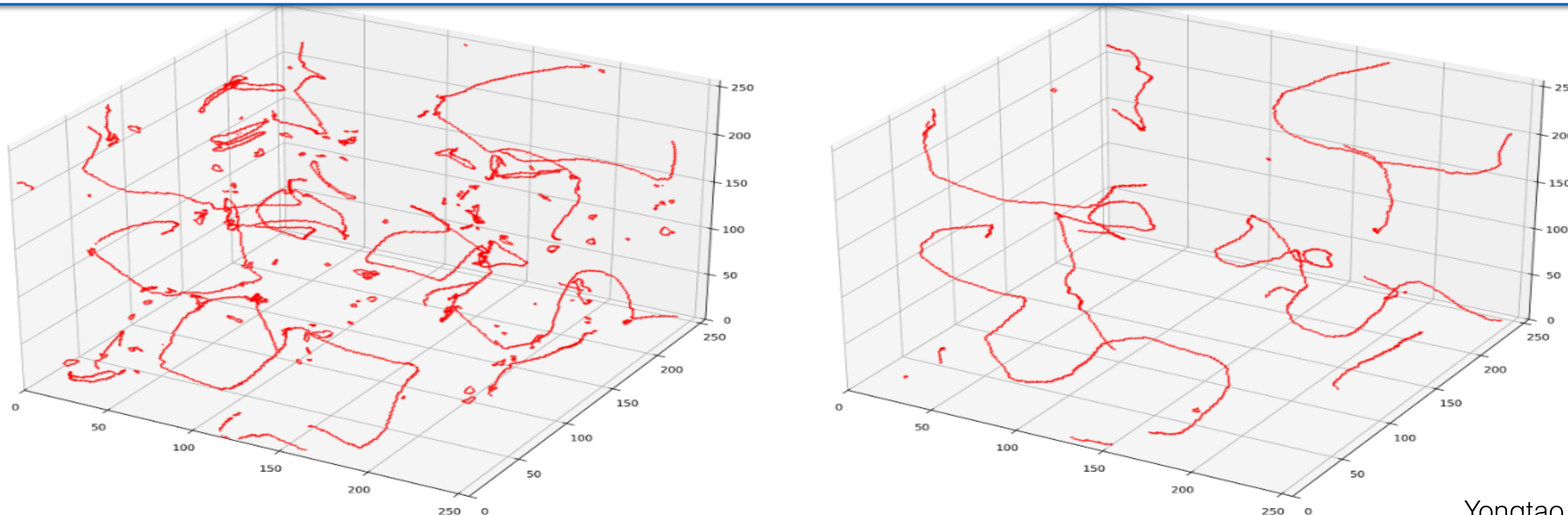


FIG. 1. Three bubbles of the broken symmetry phase ($\rho=\eta$) colliding. If the phase change of the scalar field around the loop γ is $\pm 2\pi$, a string (or antistring) is formed. If the phases α_i are ordered, then the requirement for a string is $\alpha_1 + \pi < \alpha_3 < \alpha_2 + \pi$.

snapshots of the phase field at $\tilde{\tau} = 1.0$ and 5.0, masked by $|\varphi| > 0.5\varphi_v$



Snapshots of the string network at $\tilde{\tau} = 7.0$ and 9.0



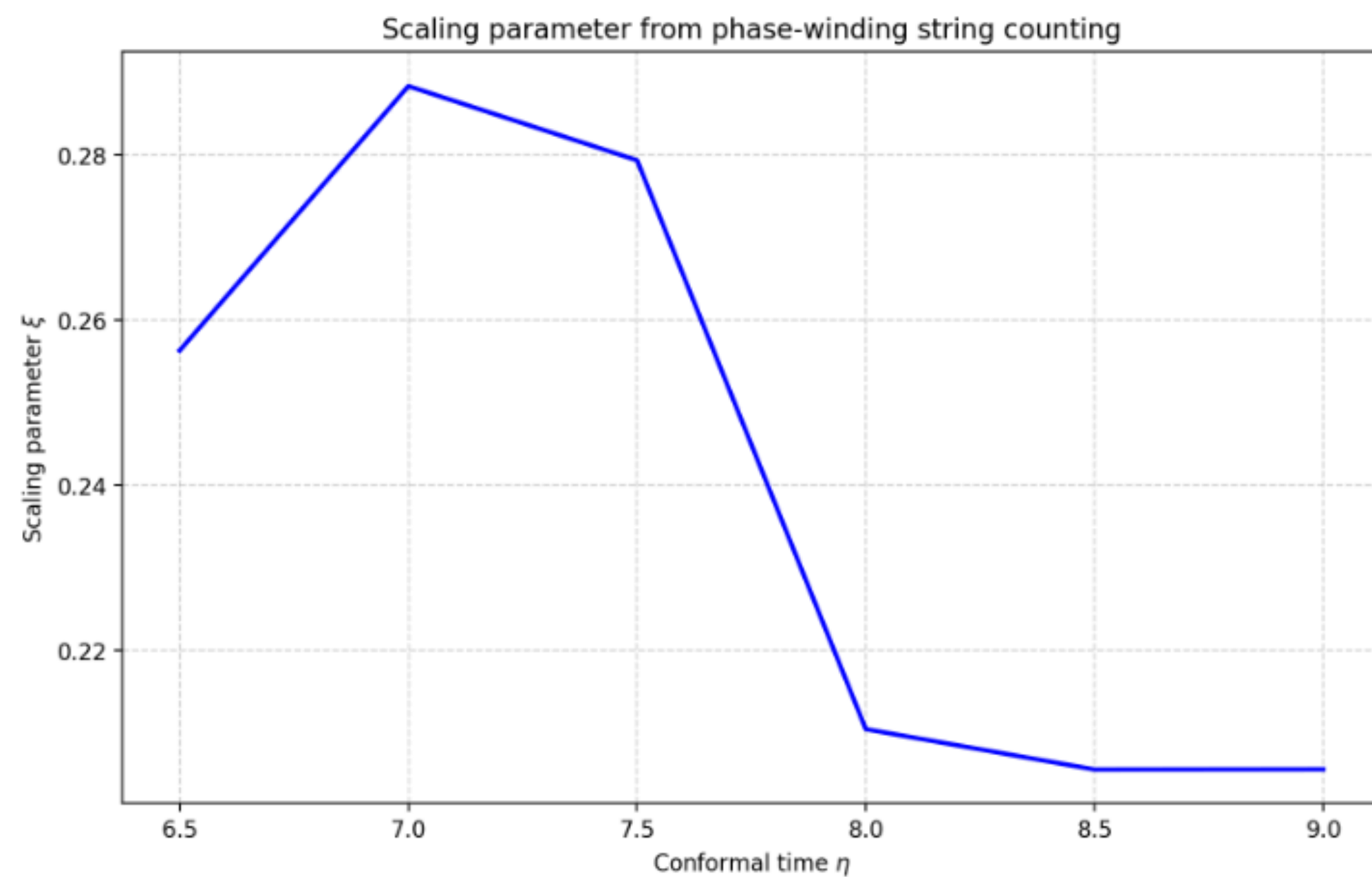
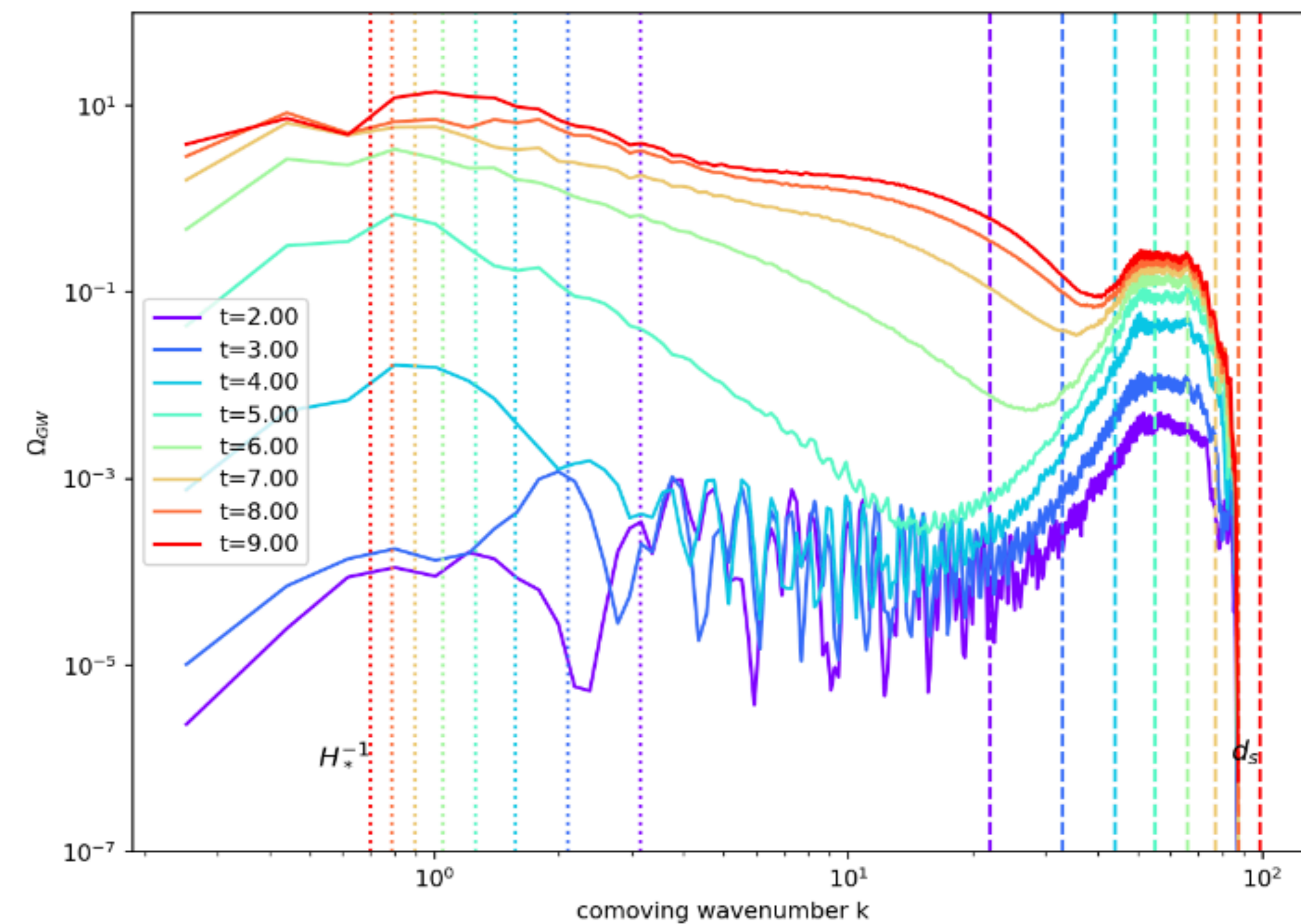


Fig. 2: Evolution of the scaling parameter in the simulation.



The action in expanding universe with spatially flat FLRW metric:

$$S = -\int d^4x \sqrt{-g} \left(g^{\mu\nu} \frac{1}{2} \partial_\mu \varphi^* \partial_\nu \varphi + V(\varphi) \right)$$

PQ complex scalar: $\varphi = \phi_1 + i\phi_2$

$$V(\varphi) = \underbrace{\frac{1}{4} \lambda (|\varphi|^2 - v^2)^2 + \frac{\lambda}{6} T^2 |\varphi|^2}_{\text{PQ era}} + \underbrace{\frac{m^2(T)v^2}{N_{DW}^2} (1 - \cos(N_{DW}\theta)) - \Xi v^3 (\varphi e^{-i\delta} + \text{h.c.})}_{\text{QCD era}}$$

$\min \left[\frac{\alpha_a \Lambda^4}{f_a^2 (T / \Lambda)^{6.68}}, m_a^2 \right]$
axion field
bias term

PQ era, PQ symmetry broken, second order phase transition, $T_c \sim 10^9\text{-}10^{11}\text{GeV}$

QCD era, axion acquires a non-zero mass due to QCD non-perturbative effect, $T \sim 100\text{MeV}$

Axion(global) strings form and enters the scaling regime

String-domain wall hybrid networks form and eventually decay

Gravitational waves and axion radiated by topological defects of two eras

→ Detection of axion dark matter

Equations of motion

PQ era-the first stage

$$\begin{cases} \phi_1'' + 2\frac{a'}{a}\phi_1' - \nabla^2\phi_1 = -a^2[\lambda\phi_1(\phi_1^2 + \phi_2^2 - v^2 + \frac{1}{3}T^2) - \frac{m^2(T)v^2}{N_{\text{DW}}^2}(\cos\theta \cos N_{\text{DW}}\theta + N_{\text{DW}} \sin\theta \sin N_{\text{DW}}\theta) - 2\Xi v^3 \cos\delta] \\ \phi_2'' + 2\frac{a'}{a}\phi_2' - \nabla^2\phi_2 = -a^2[\lambda\phi_2(\phi_1^2 + \phi_2^2 - v^2 + \frac{1}{3}T^2) - \frac{m^2(T)v^2}{N_{\text{DW}}^2}(\sin\theta \cos N_{\text{DW}}\theta - N_{\text{DW}} \cos\theta \sin N_{\text{DW}}\theta) - 2\Xi v^3 \sin\delta] \end{cases}$$

Initial condition

thermal spectrum

$$\mathcal{P}_{\phi_1}(k) = \mathcal{P}_{\phi_2}(k) = \frac{n_k}{w_k} = \frac{1}{w_k} \frac{1}{e^{w_k/T} - 1}, \quad \mathcal{P}_{\dot{\phi}_1}(k) = \mathcal{P}_{\dot{\phi}_2}(k) = n_k w_k = \frac{w_k}{e^{w_k/T} - 1}$$

$$w_k = \sqrt{k^2/R^2 + m_{\text{eff}}^2} \quad m_{\text{eff}}^2 = \lambda(T^2/3 - v^2)$$

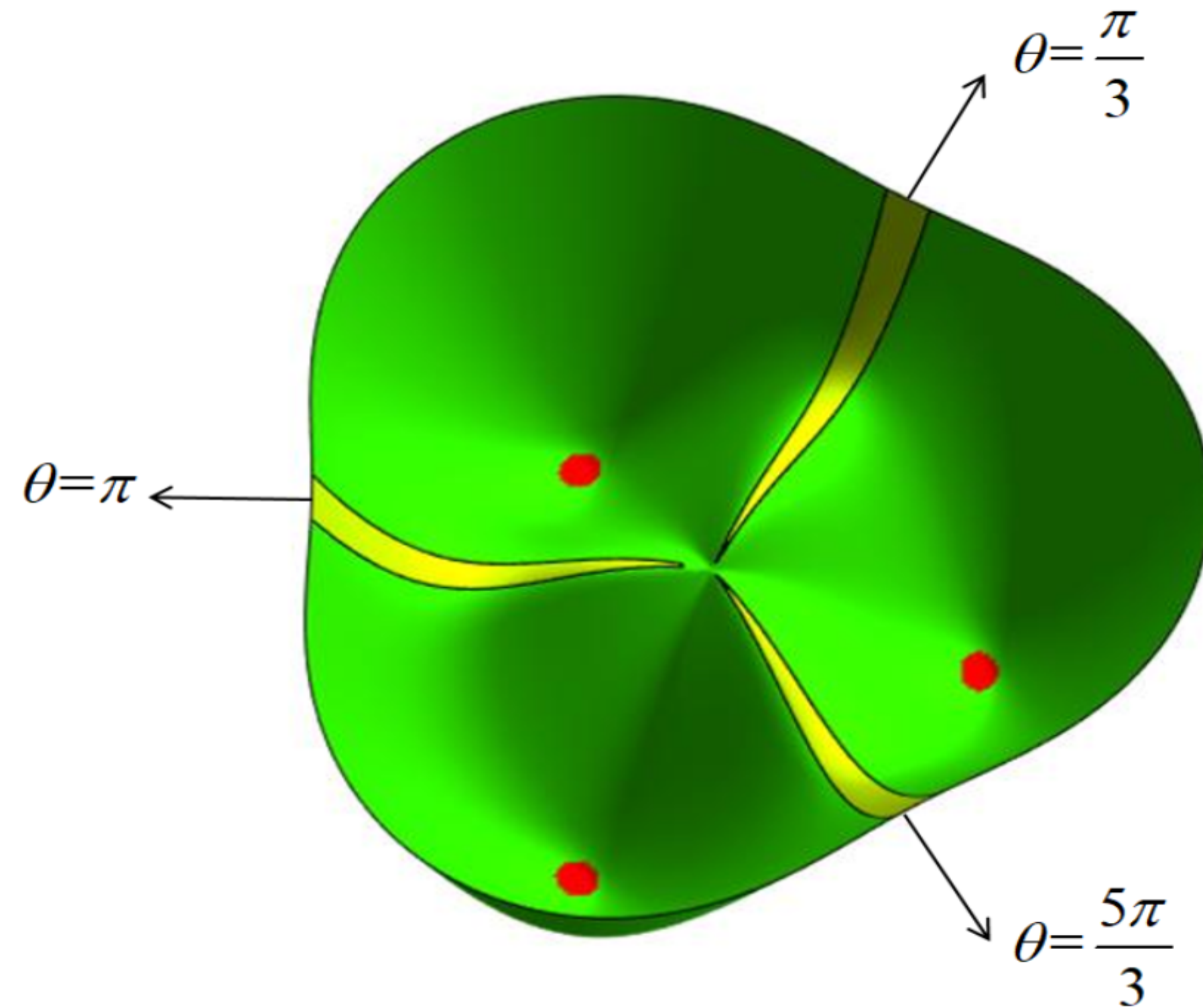
two-point correlation functions

$$\begin{aligned} \langle \phi_i(\mathbf{k})\phi_j(\mathbf{k}') \rangle &= (2\pi)^3 \mathcal{P}_\phi(k) \delta(\mathbf{k} - \mathbf{k}') \delta_{ij}, \\ \langle \dot{\phi}_i(\mathbf{k})\dot{\phi}_j(\mathbf{k}') \rangle &= (2\pi)^3 \mathcal{P}_{\dot{\phi}}(k) \delta(\mathbf{k} - \mathbf{k}') \delta_{ij}, \\ \langle \phi_i(\mathbf{k})\dot{\phi}_j(\mathbf{k}') \rangle &= 0. \end{aligned}$$

$$\begin{aligned} \langle |\phi_i(\mathbf{k})|^2 \rangle &= \left(\frac{N}{\delta x_{\text{phy}}} \right)^3 \mathcal{P}_{\phi_i}(k), & \langle \phi_i(\mathbf{k}) \rangle &= 0, \\ \langle |\dot{\phi}_i(\mathbf{k})|^2 \rangle &= \left(\frac{N}{\delta x_{\text{phy}}} \right)^3 \mathcal{P}_{\dot{\phi}_i}(k), & \langle \dot{\phi}_i(\mathbf{k}) \rangle &= 0, \end{aligned}$$

► Domain wall's identification

Top view of the shape of potential energy



Comoving area density

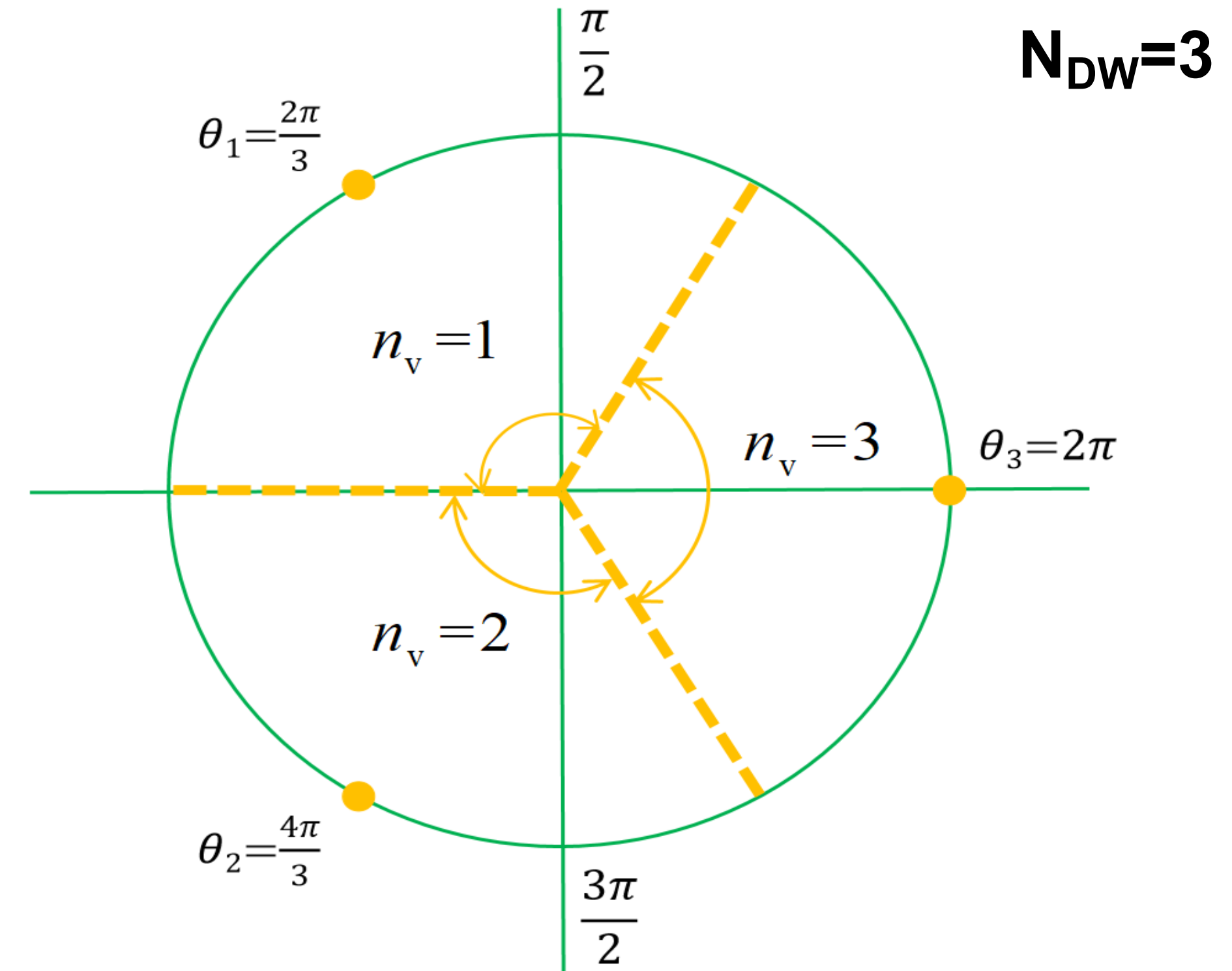
$$A/V = C \sum_{\text{links}} \delta \frac{|\nabla \theta|}{|\theta_{,x}| + |\theta_{,y}| + |\theta_{,z}|}$$

$\theta_{,i}$ ($i = x, y, z$) : spatial derivatives of the dimensionless axion field $\theta(x)$

Area parameter A of DW (scaling parameter of DW)

$$A = \Delta A \sum_{\text{links}} \delta \frac{|\nabla \theta|}{|\theta_{,x}| + |\theta_{,y}| + |\theta_{,z}|}$$

The distribution of fields in phase space



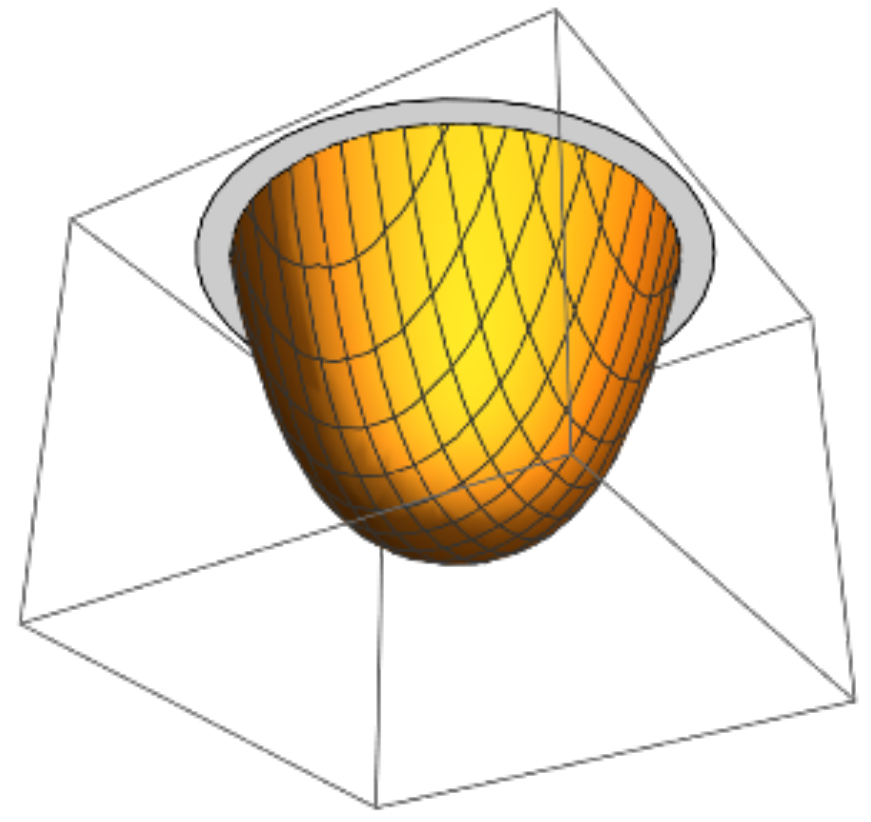
$$\xi_{\text{dw}} \equiv \mathcal{A} = \frac{\rho_{\text{wall}}}{\sigma_{\text{wall}}} t, \quad \text{with } \rho_{\text{wall}} = \frac{\sigma_{\text{wall}} A}{R(t)V}$$

$\Delta A = (\delta x)^2$ is the comoving area of one grid surface

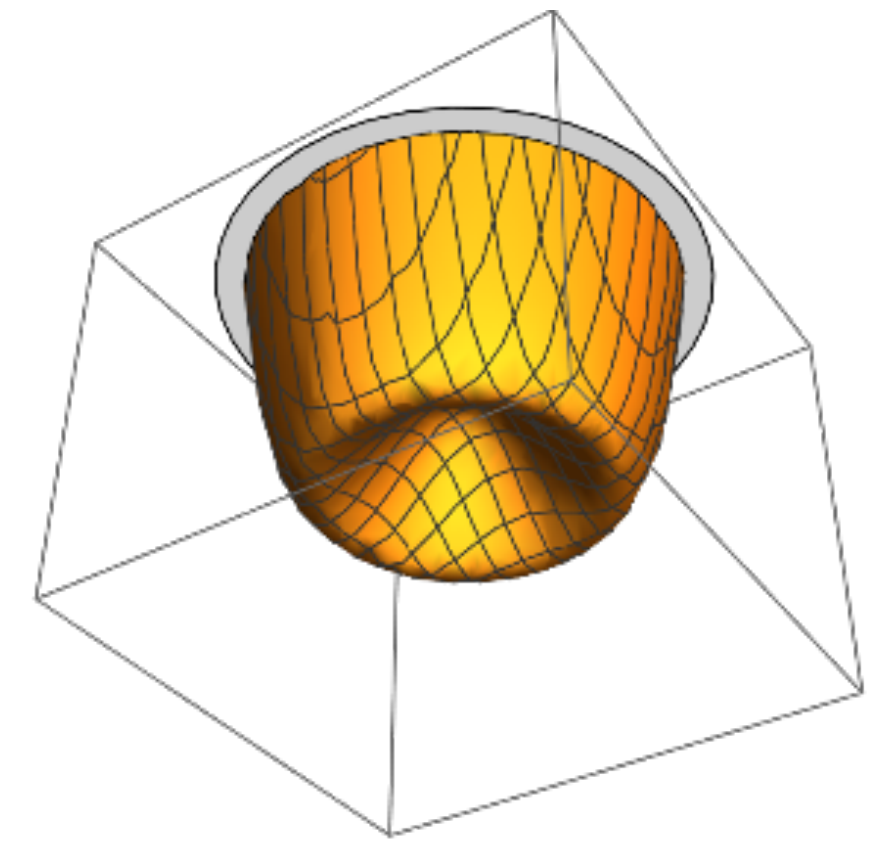
► 势能形状

PQ era:

before PQ transition

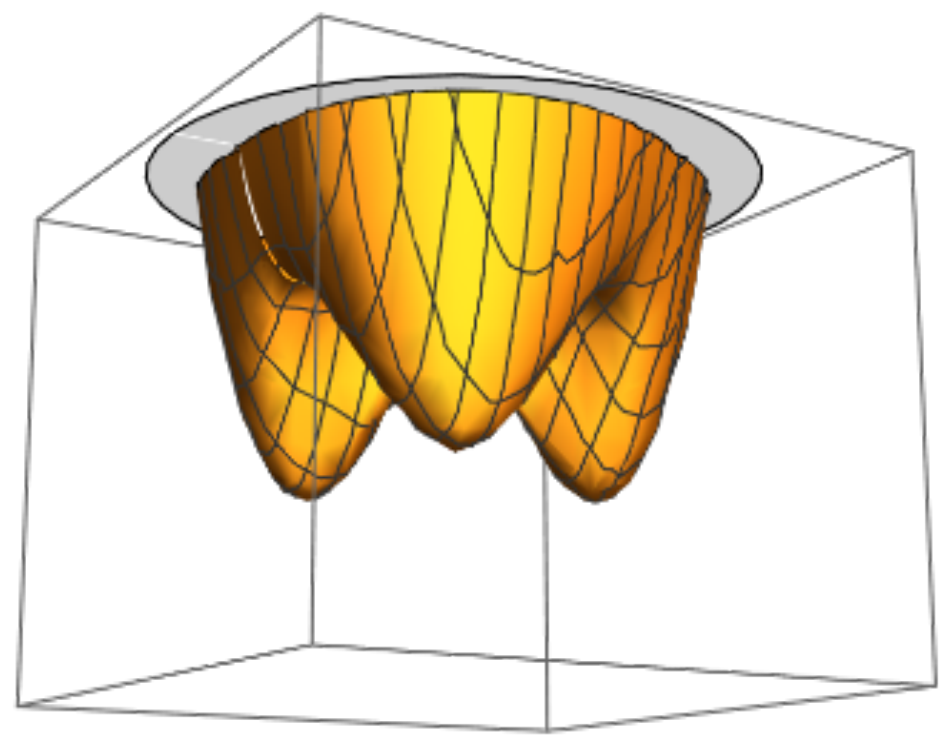


after PQ transition

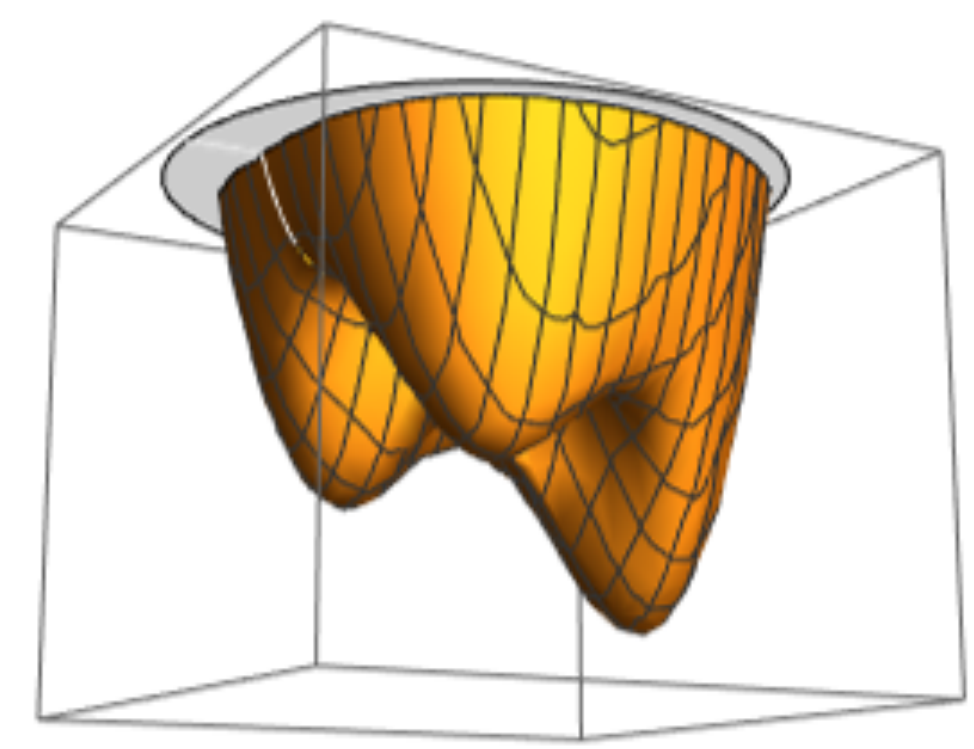


QCD era:

with nonzero axion mass
without bias term



with bias term

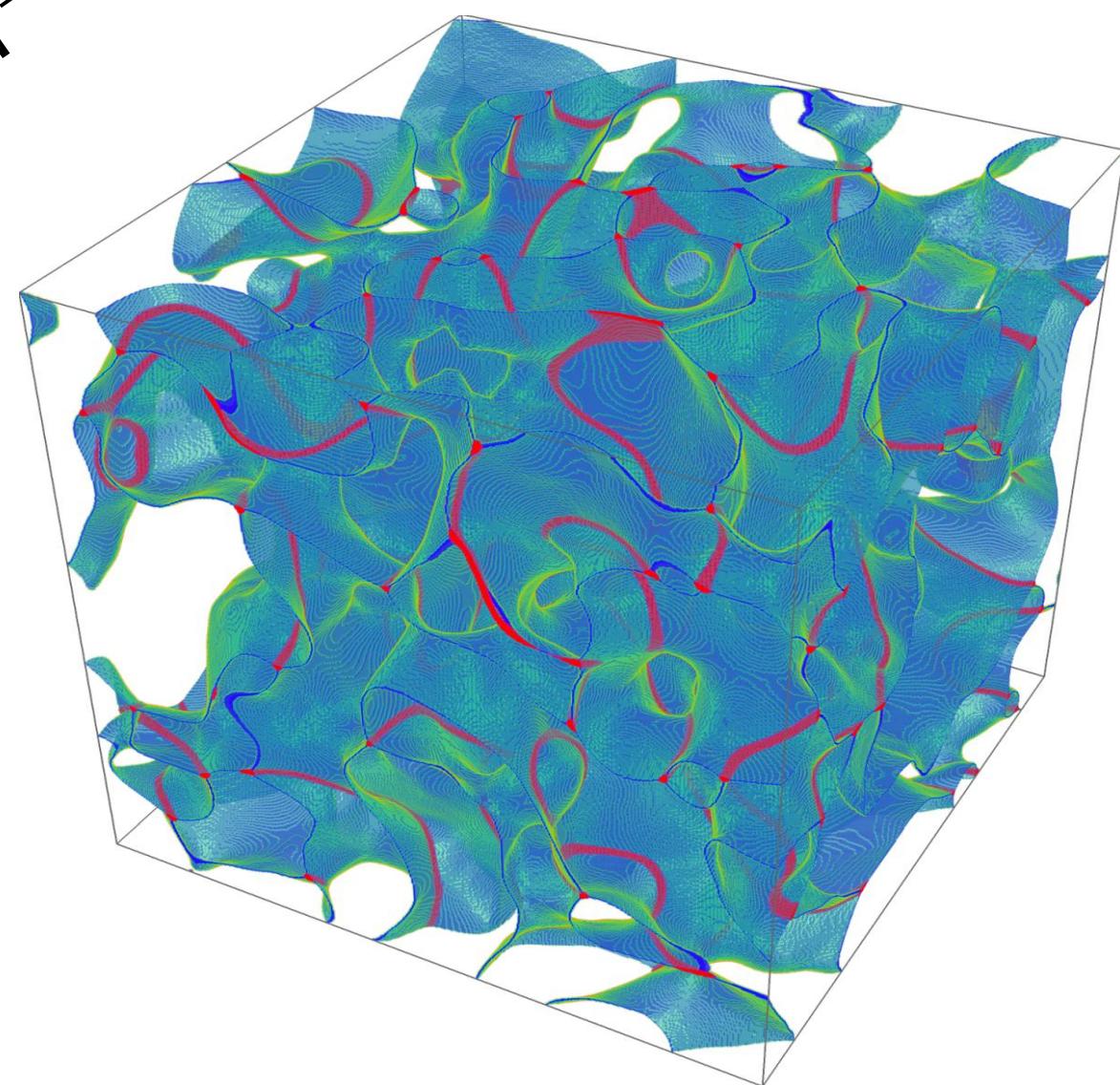


$N_{DW}=3$

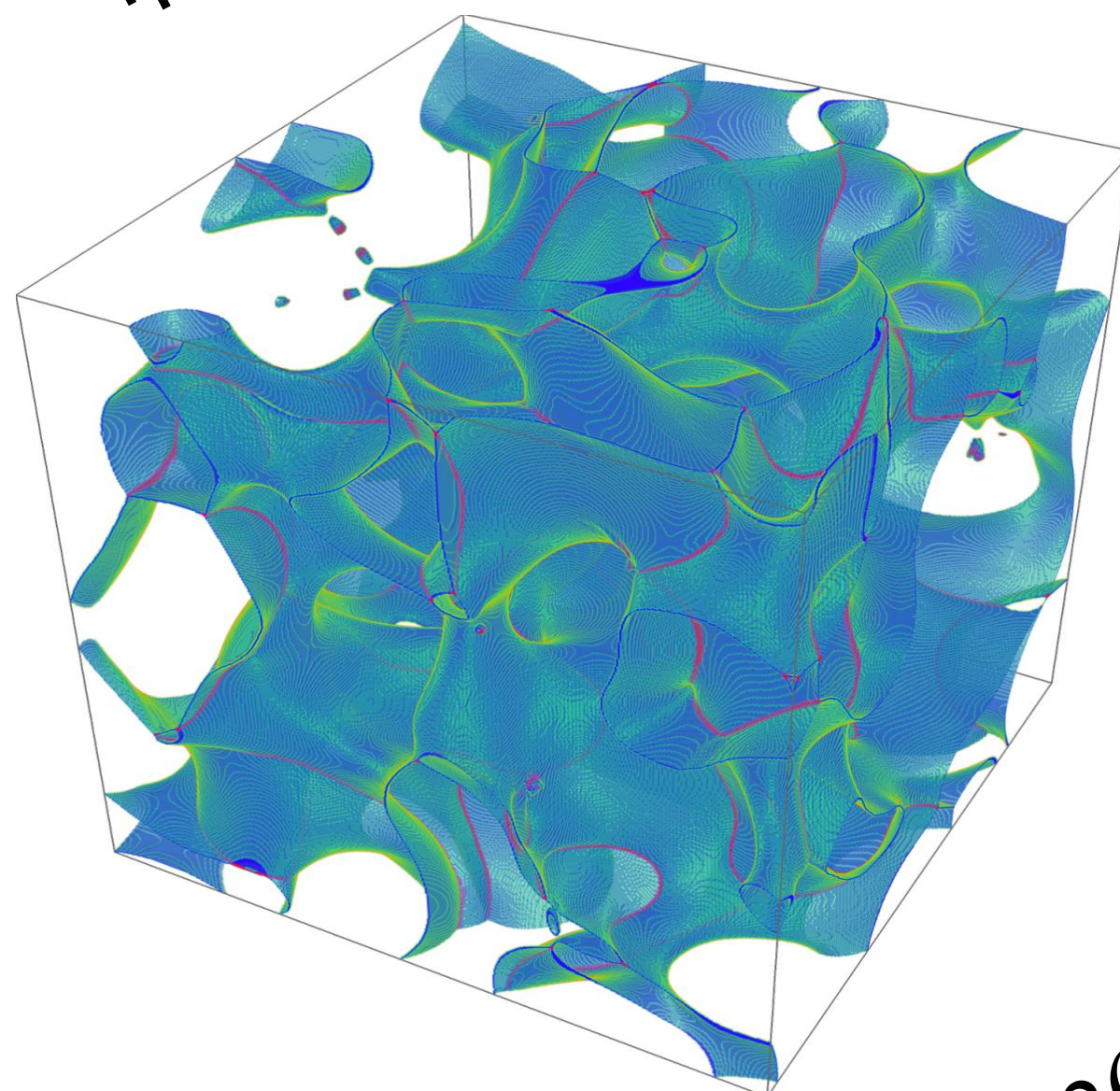
弦-畴壁网络演化

Axion string-domain wall hybrid network destruction with gravitational waves and axions emission

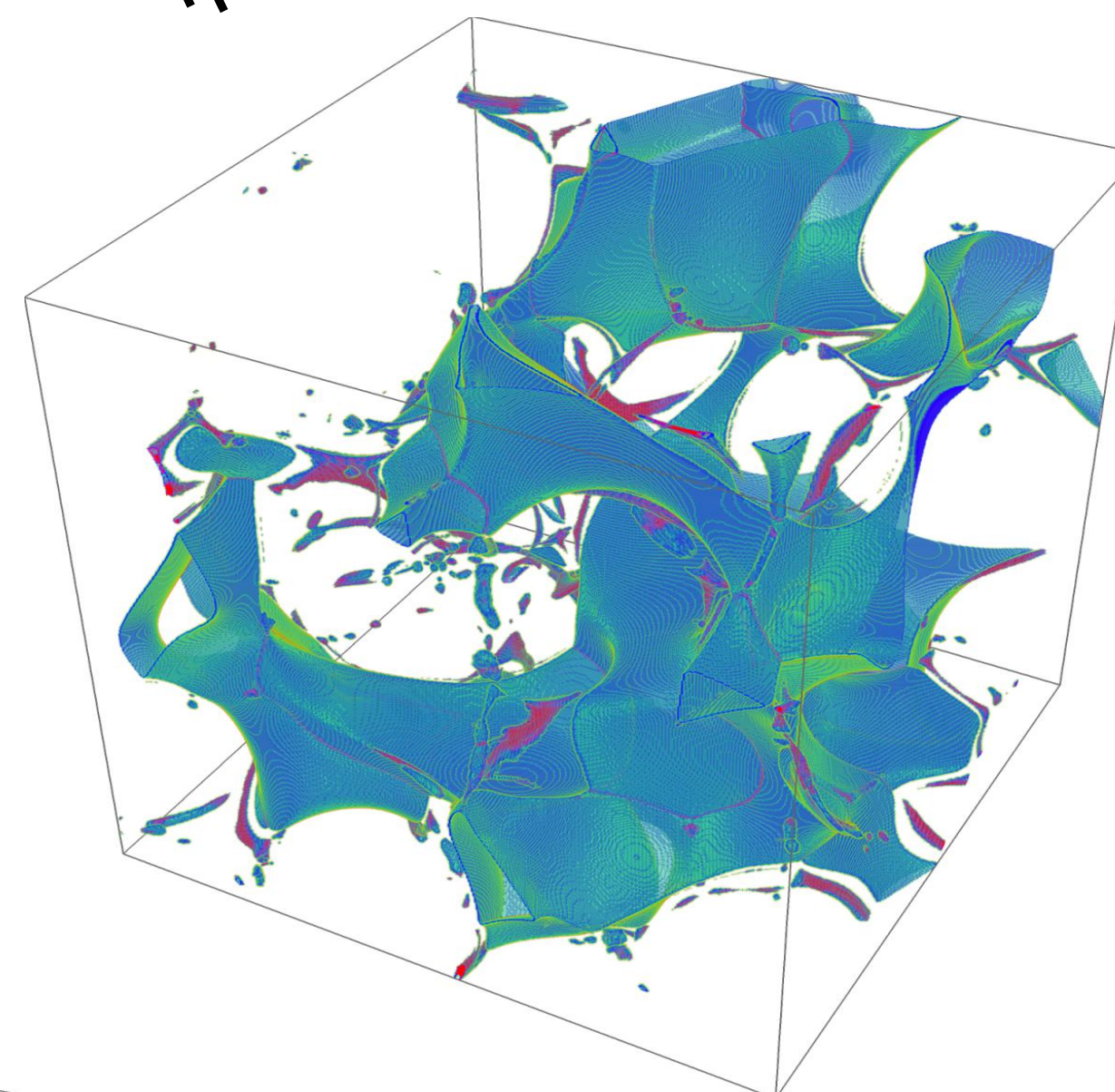
$\eta=4.6$



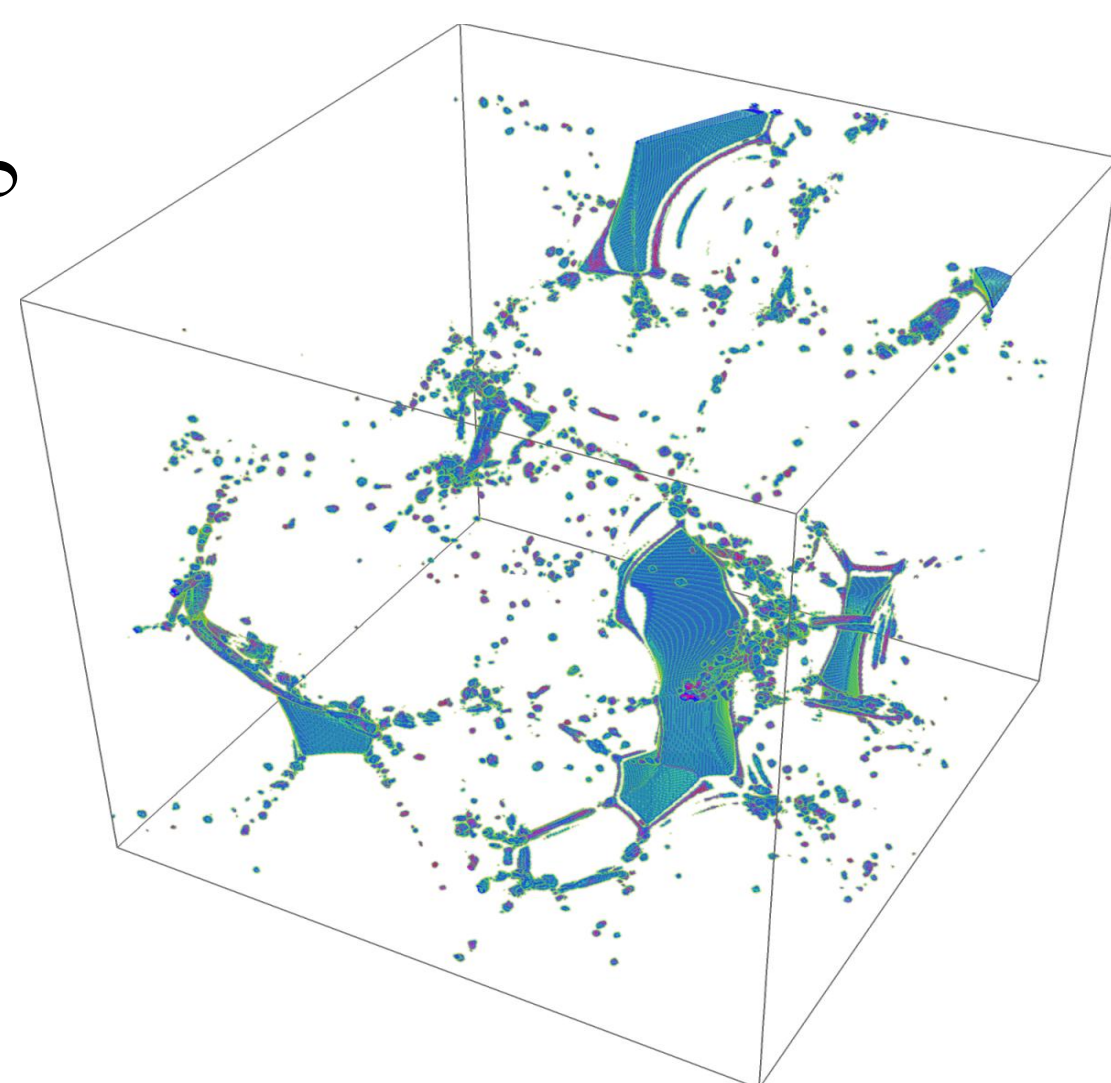
$\eta=10$



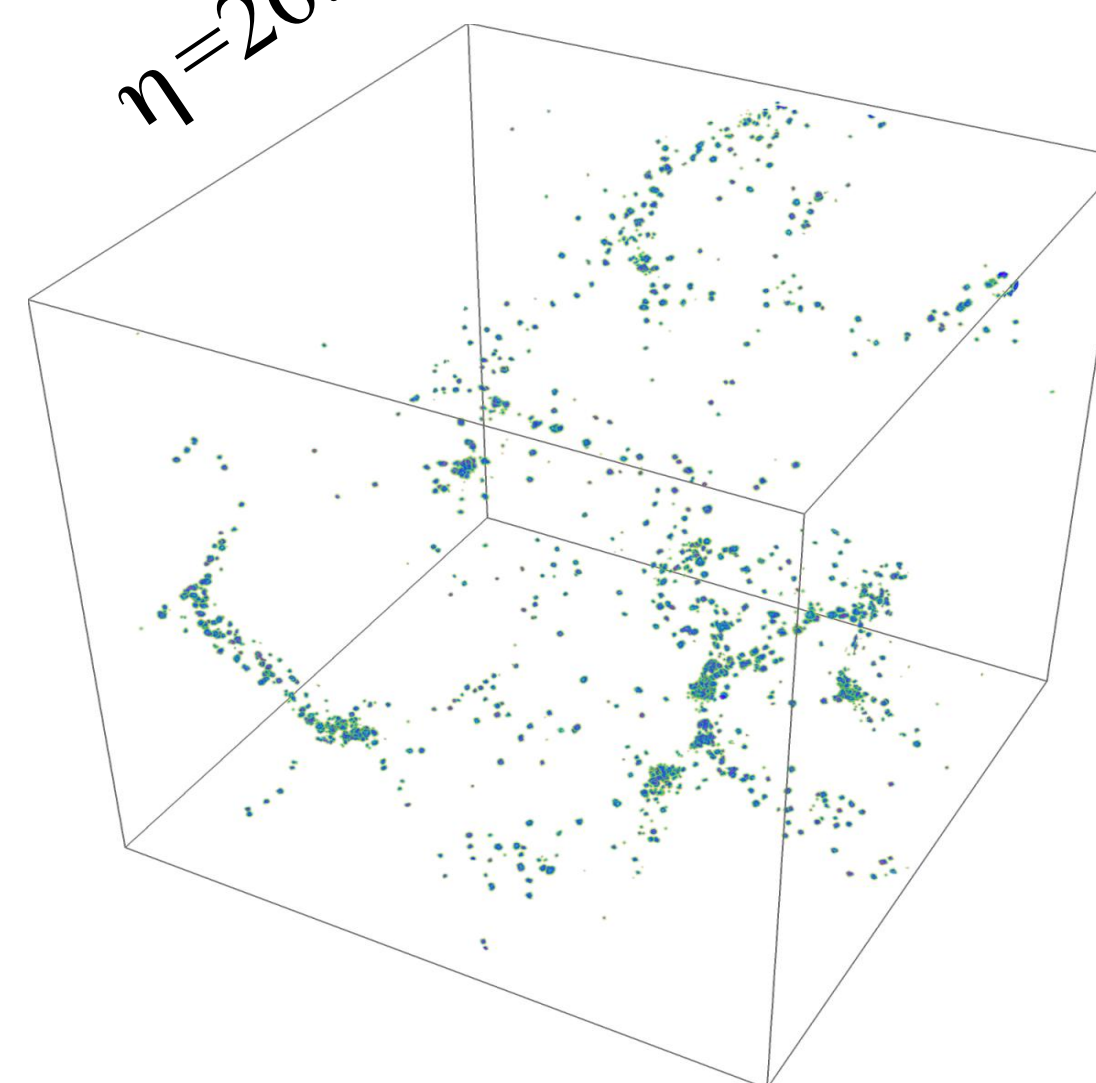
$\eta=15.4$



$\eta=20.8$

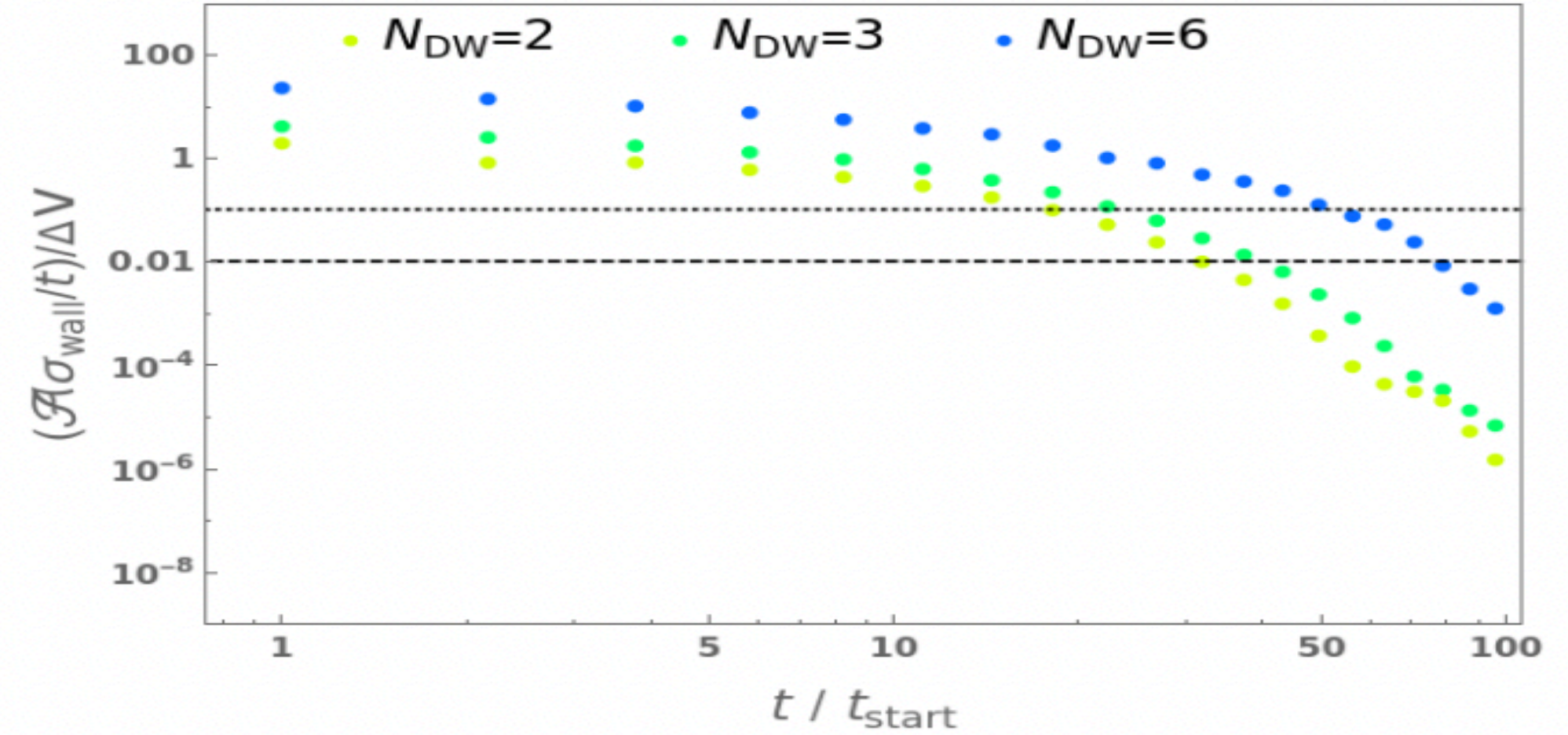
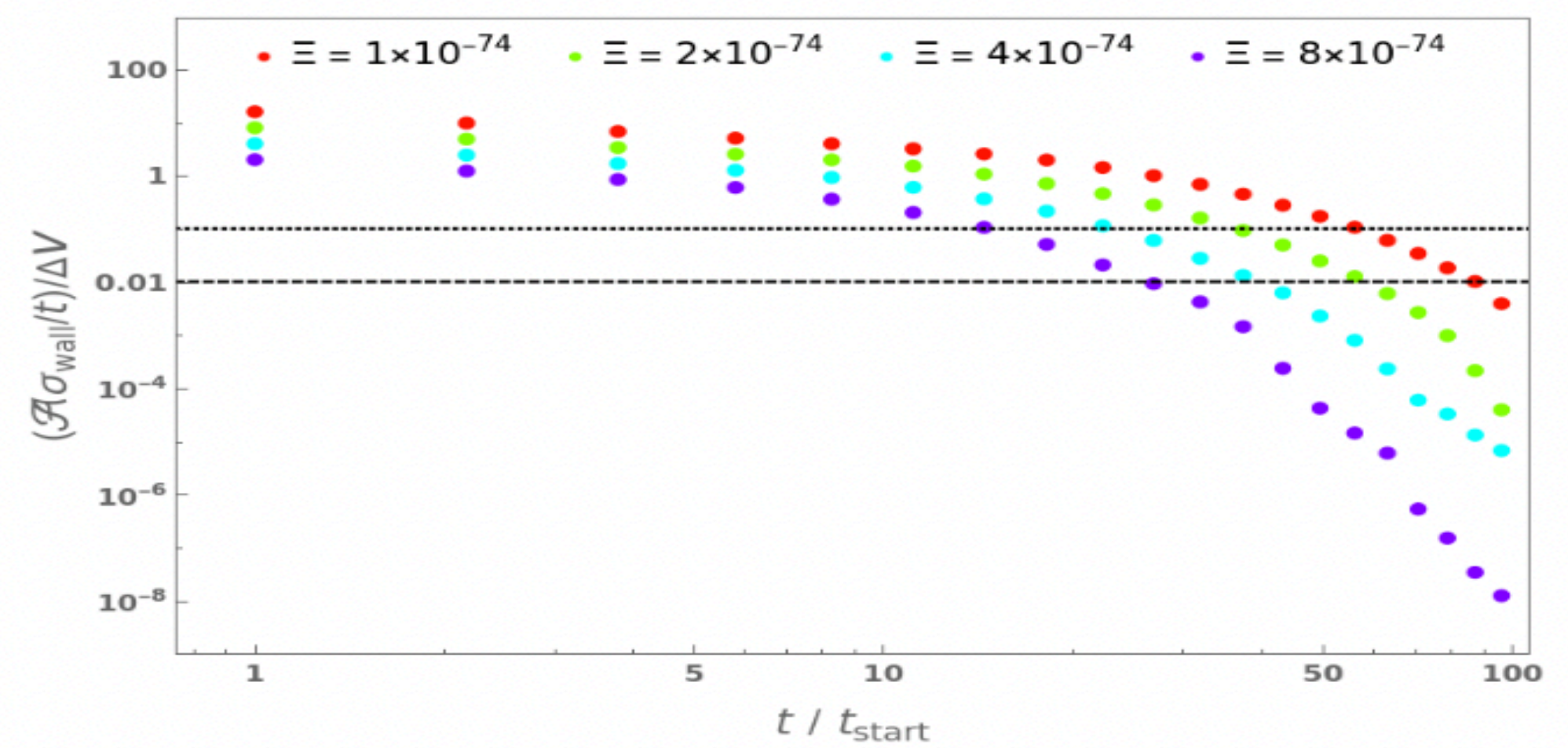
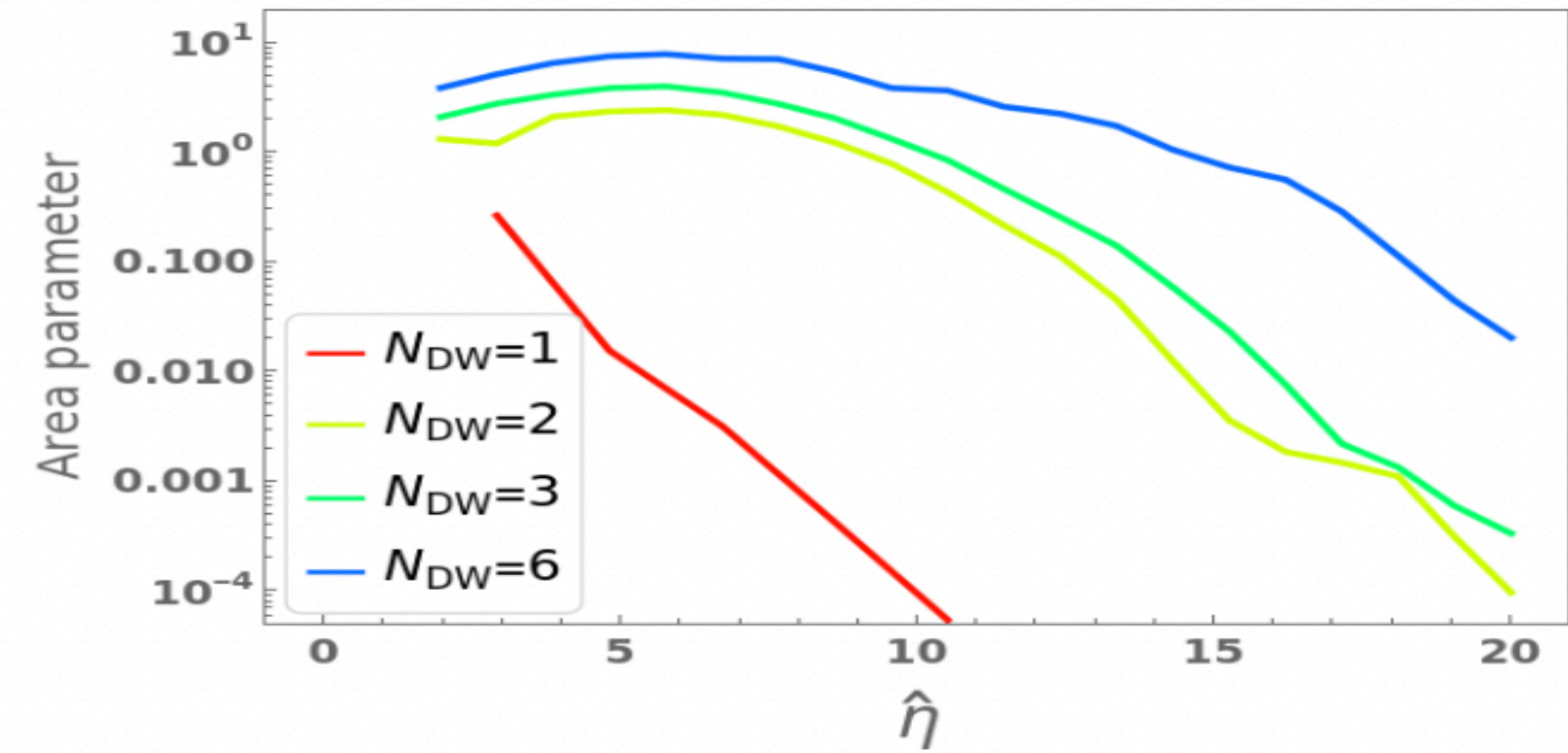
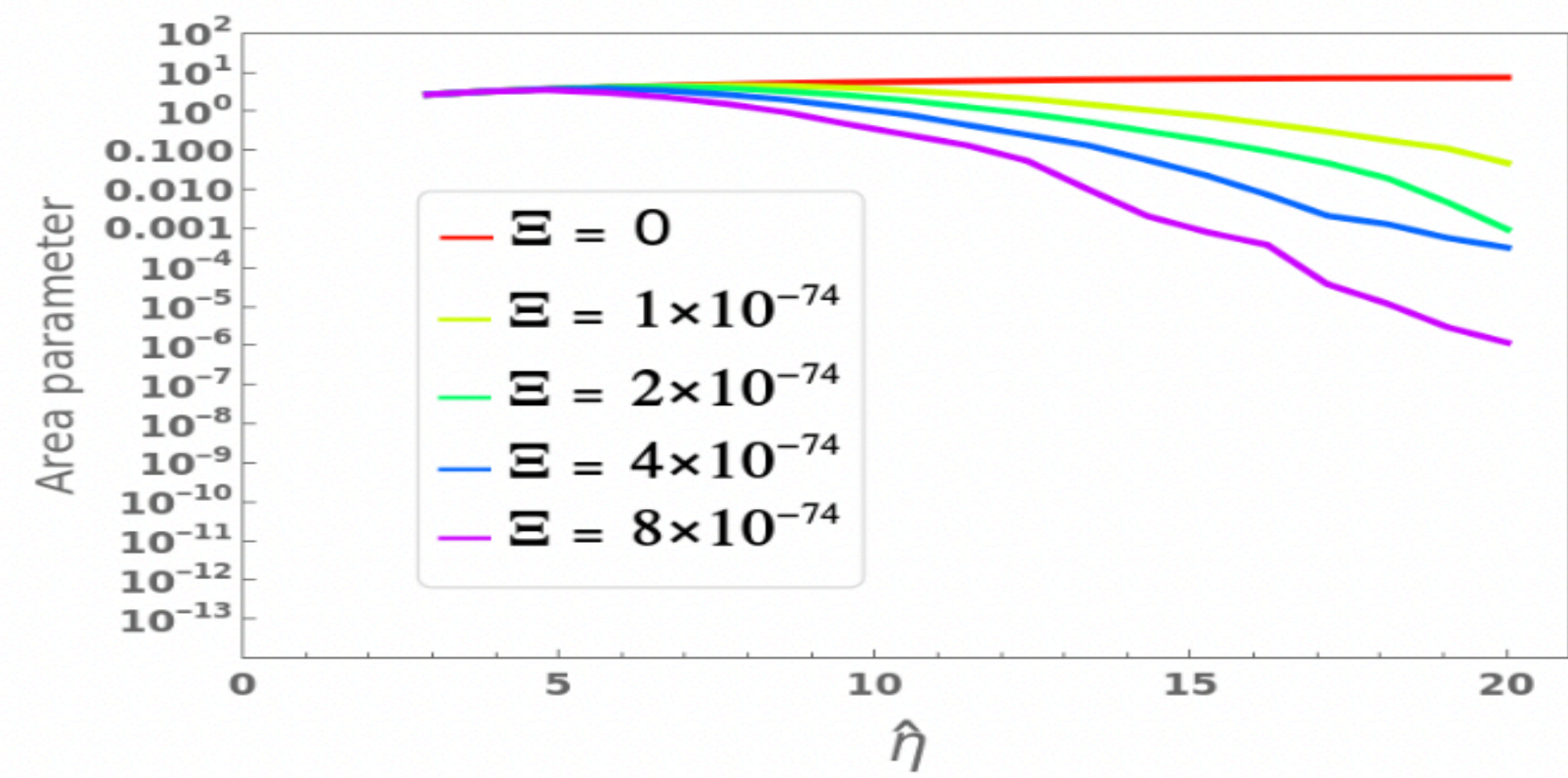


$\eta=26.2$



$$N_{\text{DW}} = 3$$

► 畴壁标度参数



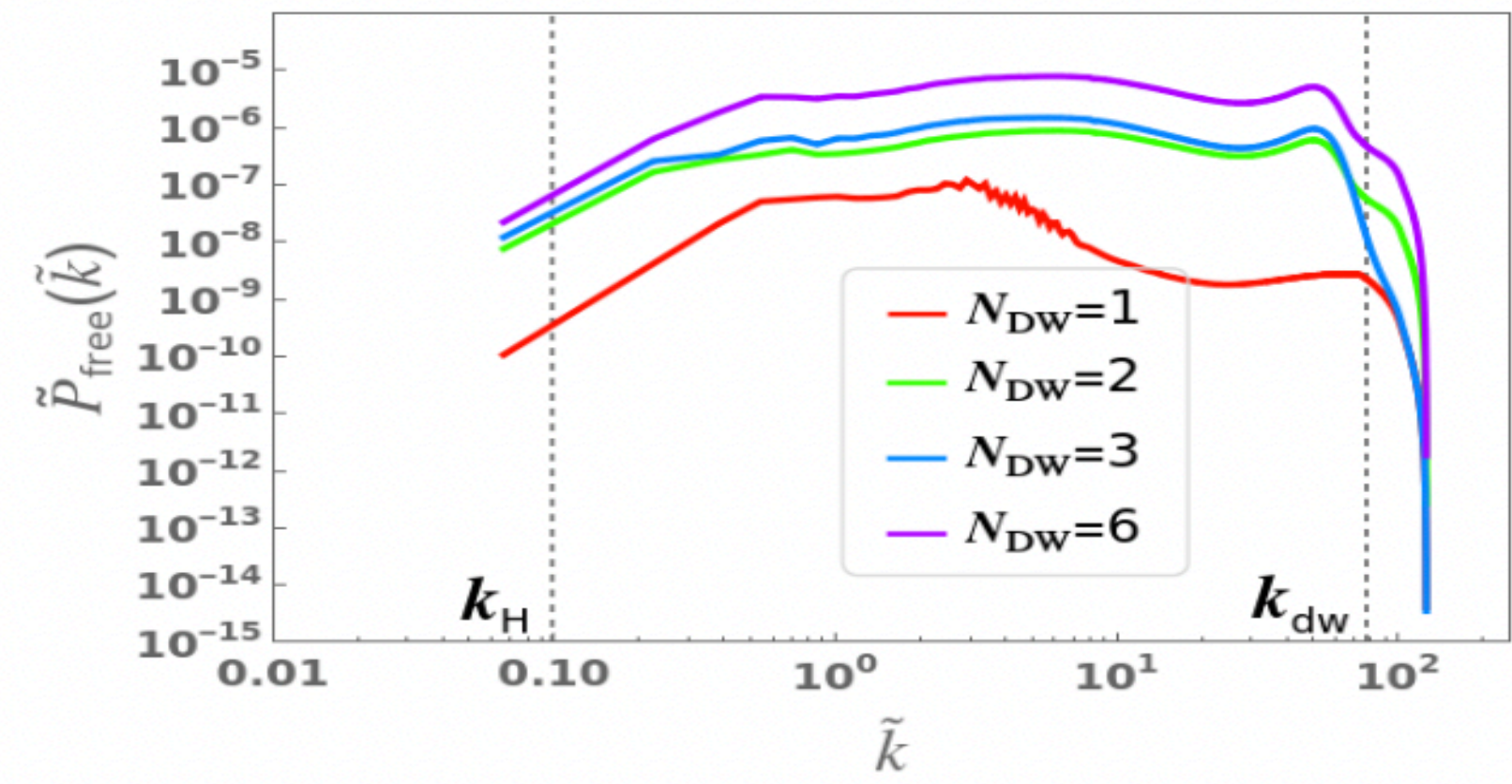
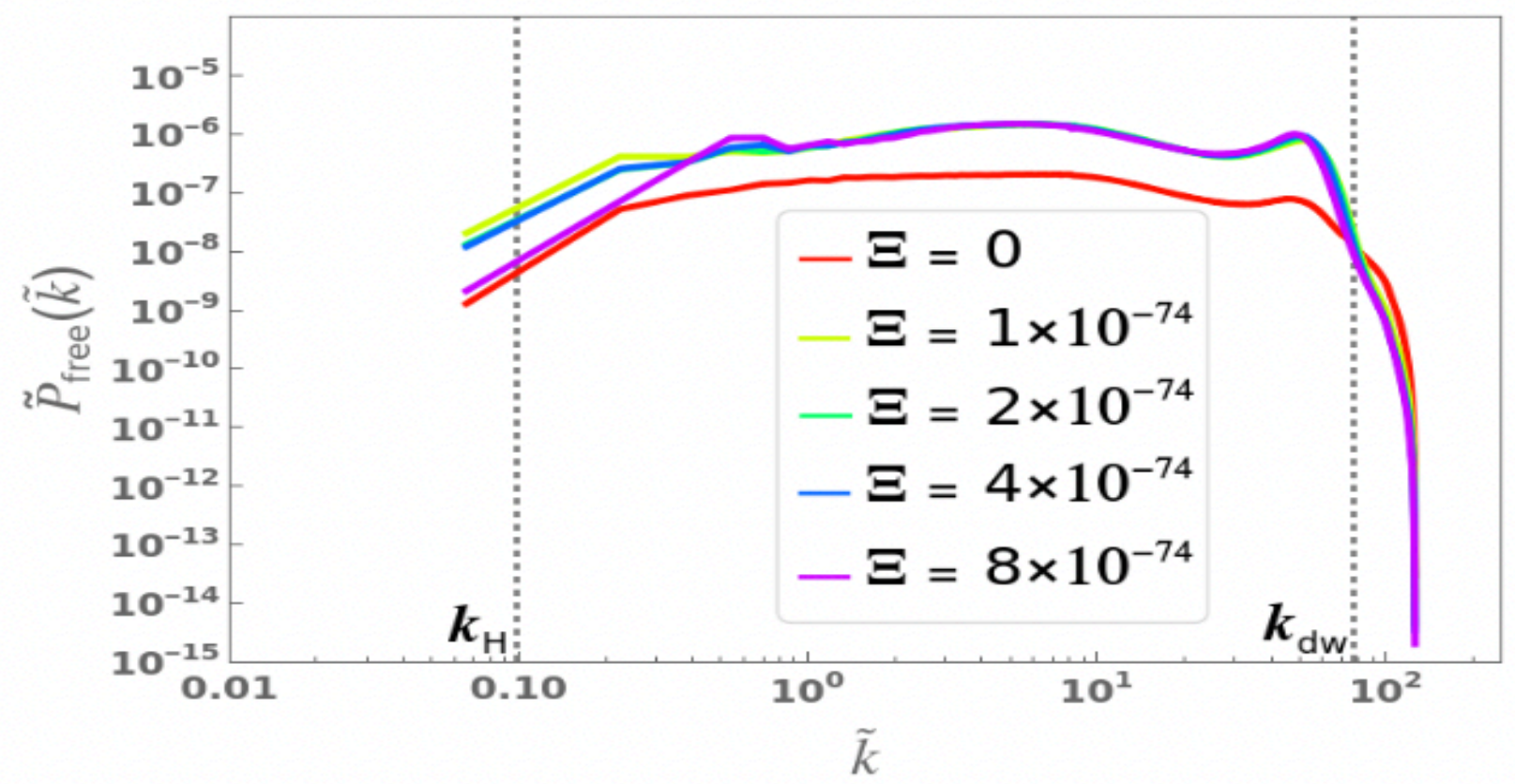
DW decay

$$\Delta V = 2\Xi v^4(1 - \cos(2\pi/N_{DW})) \sim \rho_{\text{wall}} = \mathcal{A}\sigma_{\text{wall}}/t$$

→
$$t_{\text{dec}} = t_{\text{form}} \left(\frac{C_d \mathcal{A}_{\text{form}} \sigma_{\text{wall}}}{t_{\text{form}} \Xi N_{\text{DW}}^4 f_a^4 (1 - \cos(2\pi/N_{\text{DW}}))} \right)^{1/p}$$

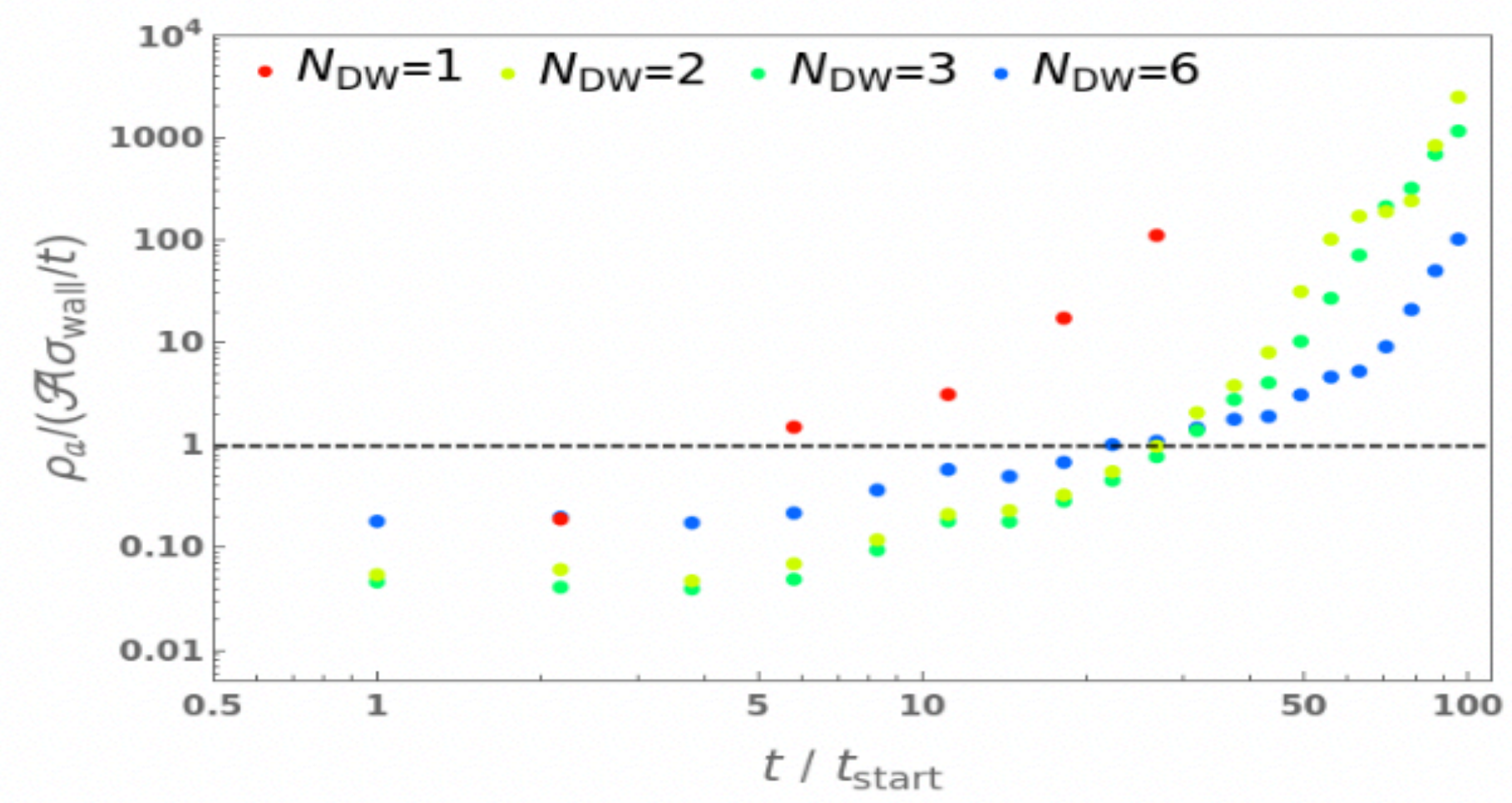
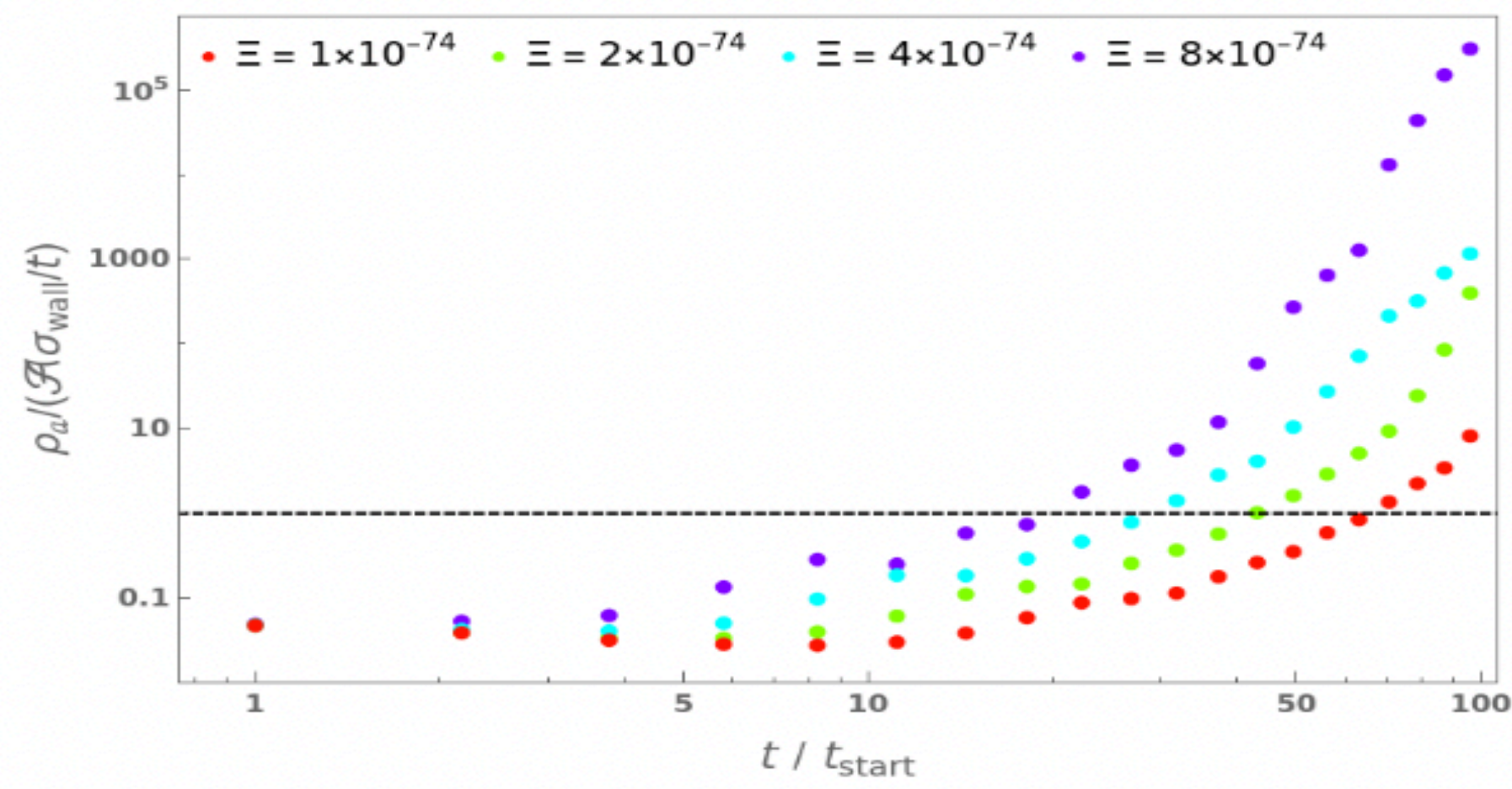
with $p = 1 + \ln C_d / \ln(t_{\text{dec}} / t_{\text{form}})$
 $C_d = \Delta V / (\mathcal{A}\sigma_{\text{wall}}/t) \sim \mathcal{O}(10^2)$

► 畴壁辐射自由轴子



$$\frac{1}{2} \langle \dot{a}_{\text{free}}(\mathbf{k})^* \dot{a}_{\text{free}}(\mathbf{k}') \rangle = (2\pi)^3 \frac{2\pi^2}{k^3} P_{\text{free}}(k) \delta(\mathbf{k} - \mathbf{k}') \quad W(\mathbf{x}) = W_{\text{st}}(\mathbf{x}) \times W_{\text{dw}}(\mathbf{x}), \quad \dot{a}_{\text{free}}(\mathbf{x}) = W(\mathbf{x}) \dot{a}(\mathbf{x}), \quad \dot{a}(\mathbf{x}) = f_a \frac{\phi_1 \dot{\phi}_2 - \dot{\phi}_1 \phi_2}{\phi_1^2 + \phi_2^2}$$

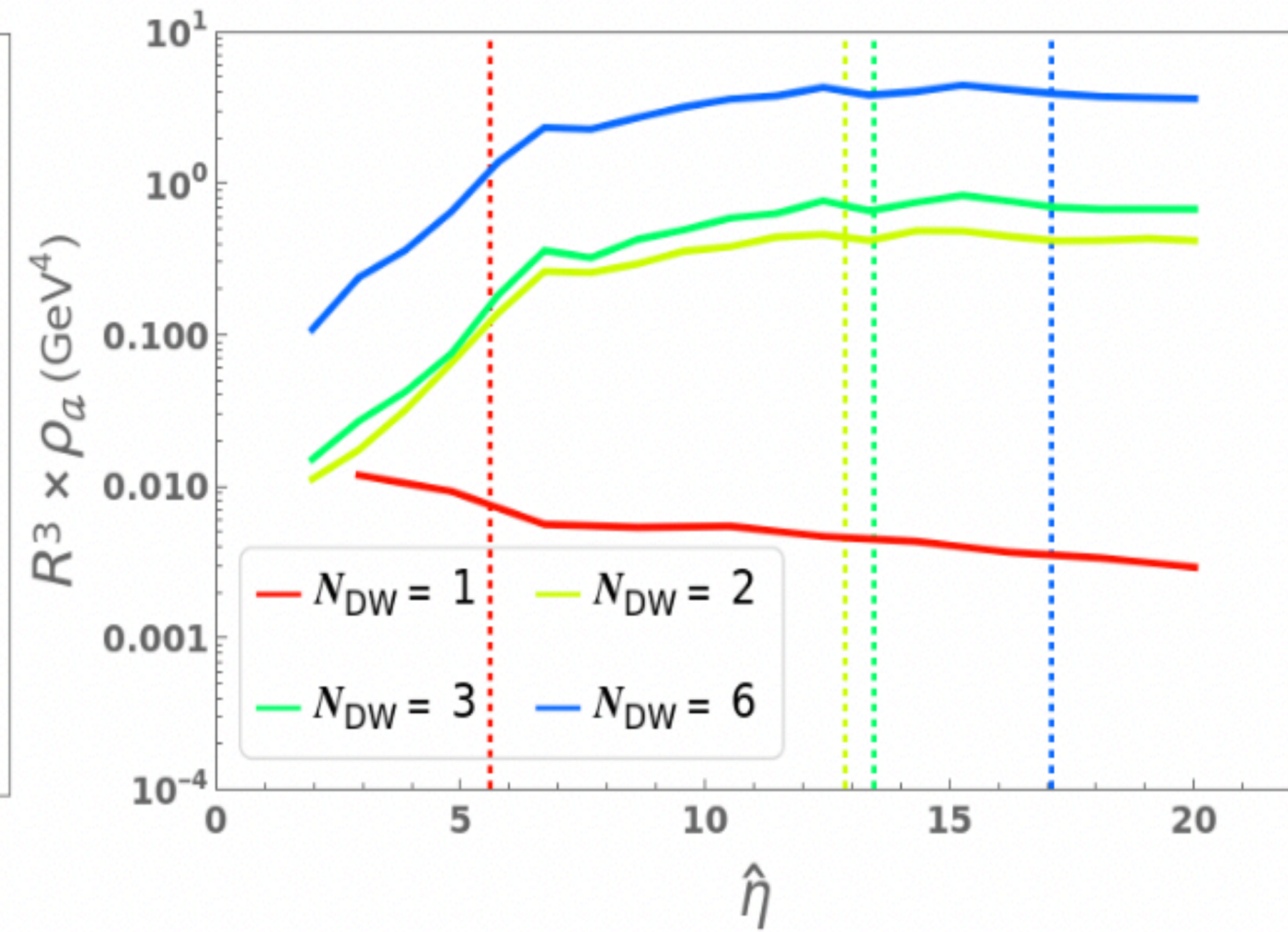
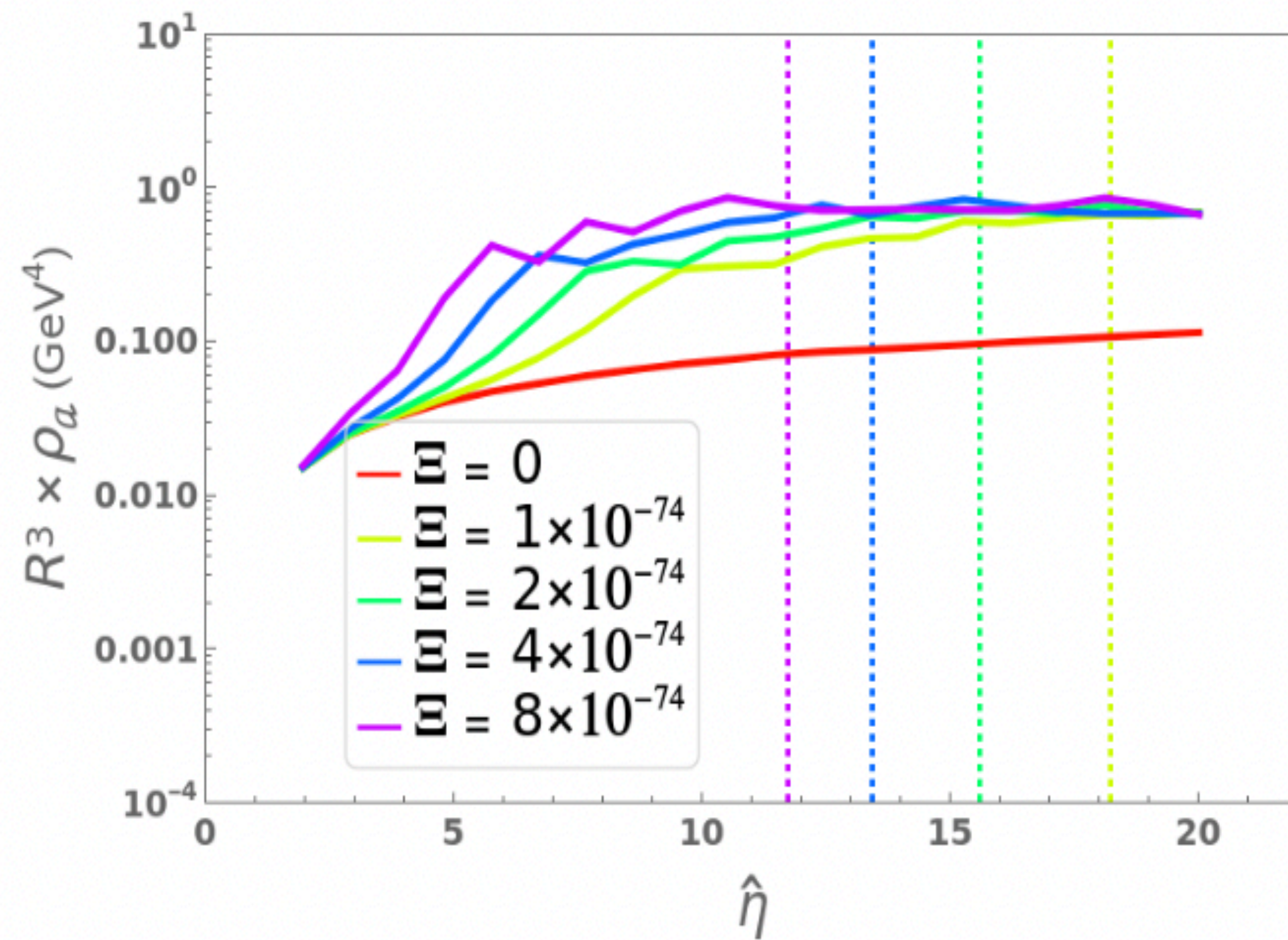
Window function $W_{\text{dw}}(\mathbf{x})$ will take 0 near the DW core and 1 near the true vacuum



$$\rho_a \simeq 2 \times \frac{1}{2} \langle \dot{a}_{\text{free}}^2(\mathbf{x}) \rangle$$

$$\rho_a \sim \mathcal{A} \sigma_{\text{wall}} / t \sim 8 \mathcal{A} m f_a^2 / t_{\text{dec}}$$

► 畴壁辐射自由轴子



$$\rho_a(t_{\text{dec}}) \simeq \frac{2(2p-1)f_a^4 N_{DW}^4}{(3-2p)C_d} \Xi \sin^2(\pi/N_{DW})$$

Axion energy density tends to be a constant at the final moment, and the energy density of the radiated free axion is (almost) proportional to the bias term and N_{DW}

► 畴壁辐射轴子暗物质

$$\Omega_{a,\text{new}}(t_0)h^2 = \Omega_a^{\text{DW}}h^2 + \Omega_{a,0}(t_0)h^2 + \Omega_{a,\text{st}}(t_0)h^2$$

Axion DM from Wall decay:

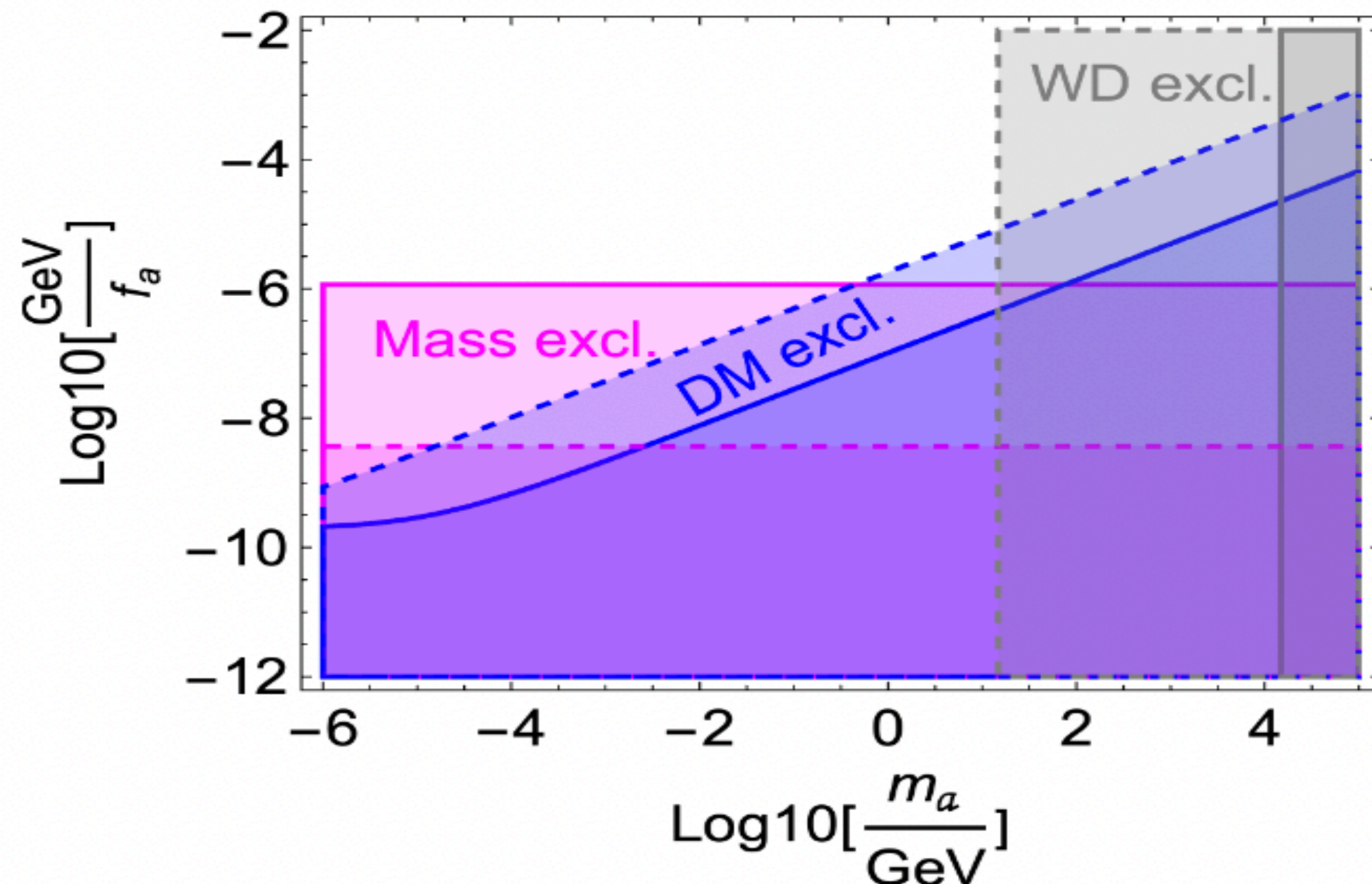
$$\Omega_a h^2 = \frac{\rho_a(t_0)}{\rho_{0,c}/h^2} = 1.024 \times 10^{-19} \left(\frac{8}{3}\right)^{3/(2p)} \left(\frac{2p-1}{3-2p}\right) C_d^{3/(2p)-1} \left(\frac{m}{\text{GeV}}\right)^{3/p-3/2} \left(\frac{f_a}{\text{GeV}}\right)^{4-3/p} \Xi^{1-3/(2p)} \left(\frac{\text{csc}(\pi/N_{\text{DW}})}{N_{\text{DW}}^2}\right)^{3/p-2}$$

Misalignment mechanism:

$$(\Omega_{a,0}(t_0)h^2 \simeq 4.63 \times 10^{-3} (f_{a,\text{new}}/10^{10}\text{GeV})^{1.19})$$

Axion DM from pure axion string:

$$\Omega_{a,\text{st}}(t_0)h^2 = 2.0 \times (f_a / 10^{12}\text{GeV})^{1.19}$$



WD excl.:

$$t_{\text{dec}} < t_{\text{WD}}, \quad t_{\text{WD}} = \frac{3}{16\pi G \sigma_{\text{wall}}}$$

Mass excl.:

bias term dominating over the QCD instanton effect in the contributions to axion mass term

$$\Xi < 2 \times 10^{-45} N_{\text{DW}}^{-2} \left(\frac{10^{10}\text{GeV}}{f_a}\right)^4$$

DM excl.

$$\Omega_a(t_0)h^2 \leq 0.12$$

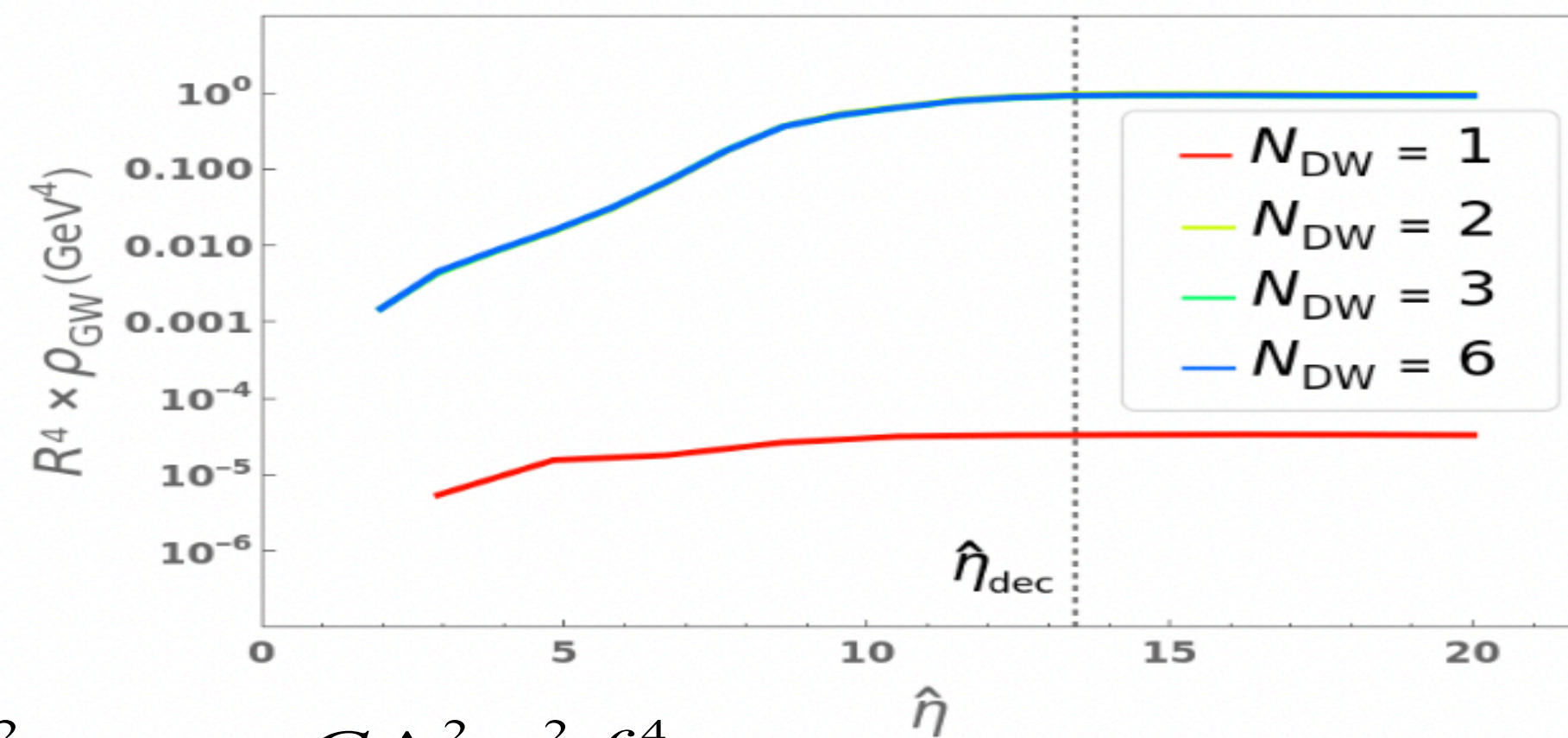
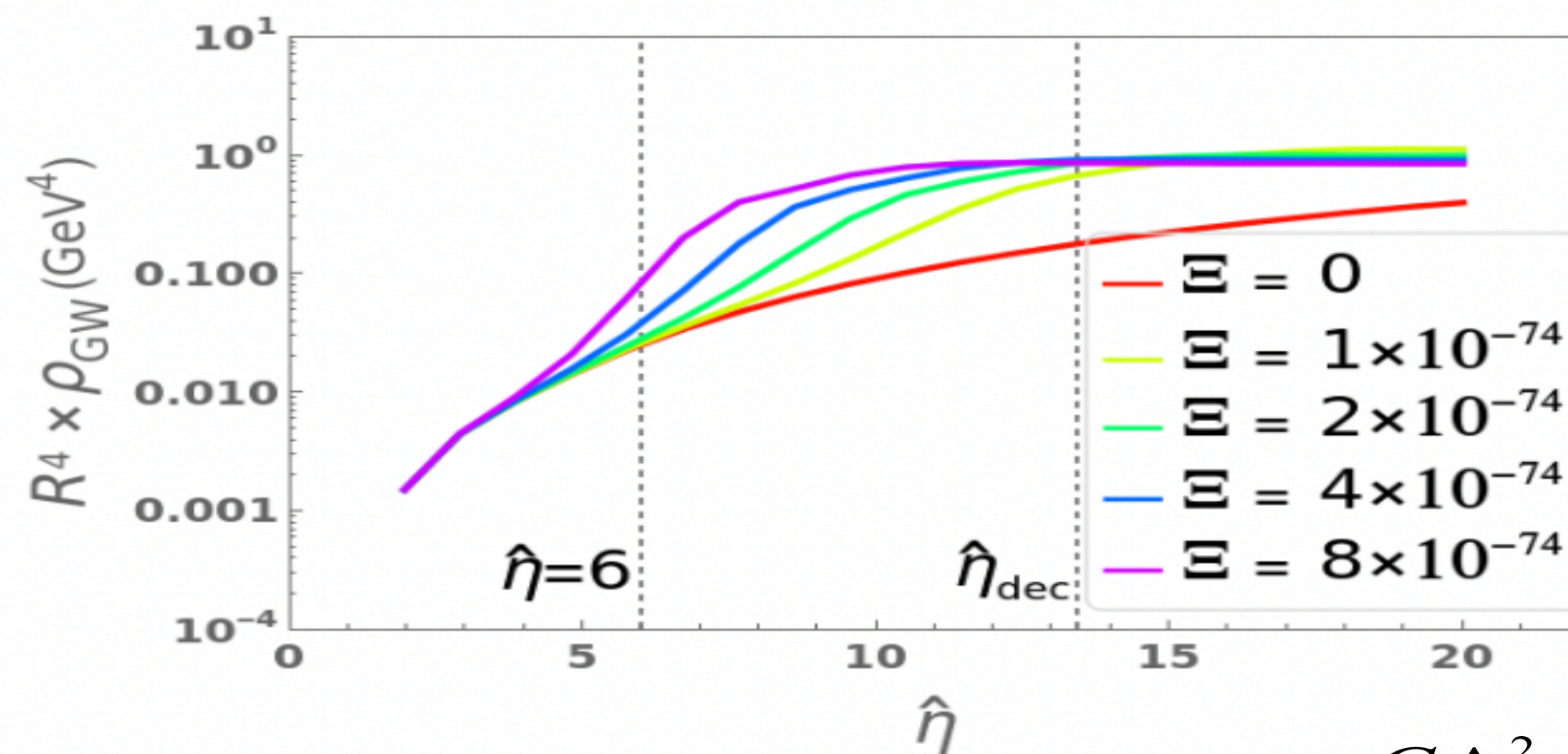
引力辐射

$$\ddot{h}_{ij} + 3\frac{\dot{R}}{R}\dot{h}_{ij} - \frac{\nabla^2}{R^2}h_{ij} = \frac{16\pi G}{R^2}\Pi_{ij}^{\text{TT}}$$

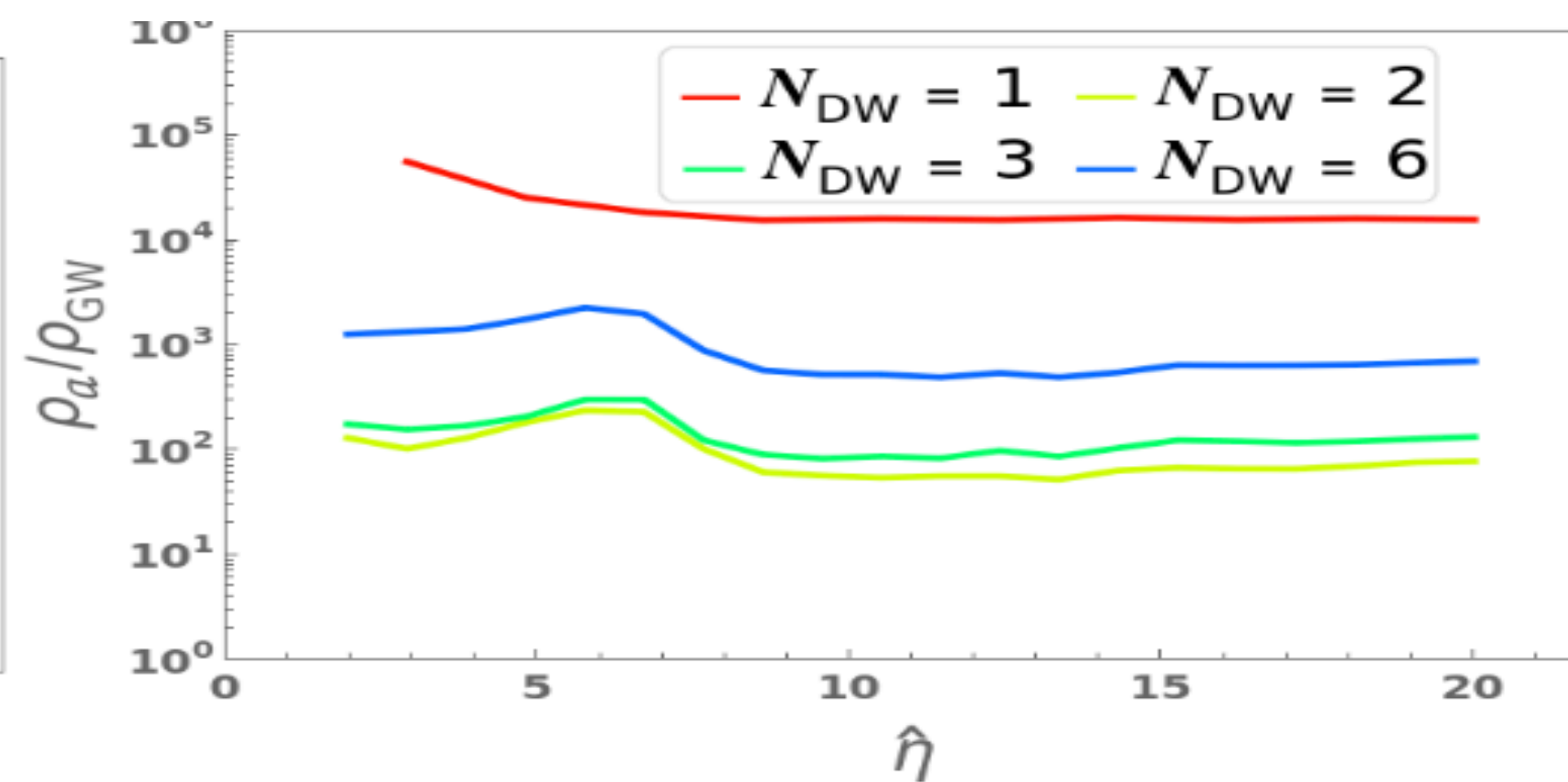
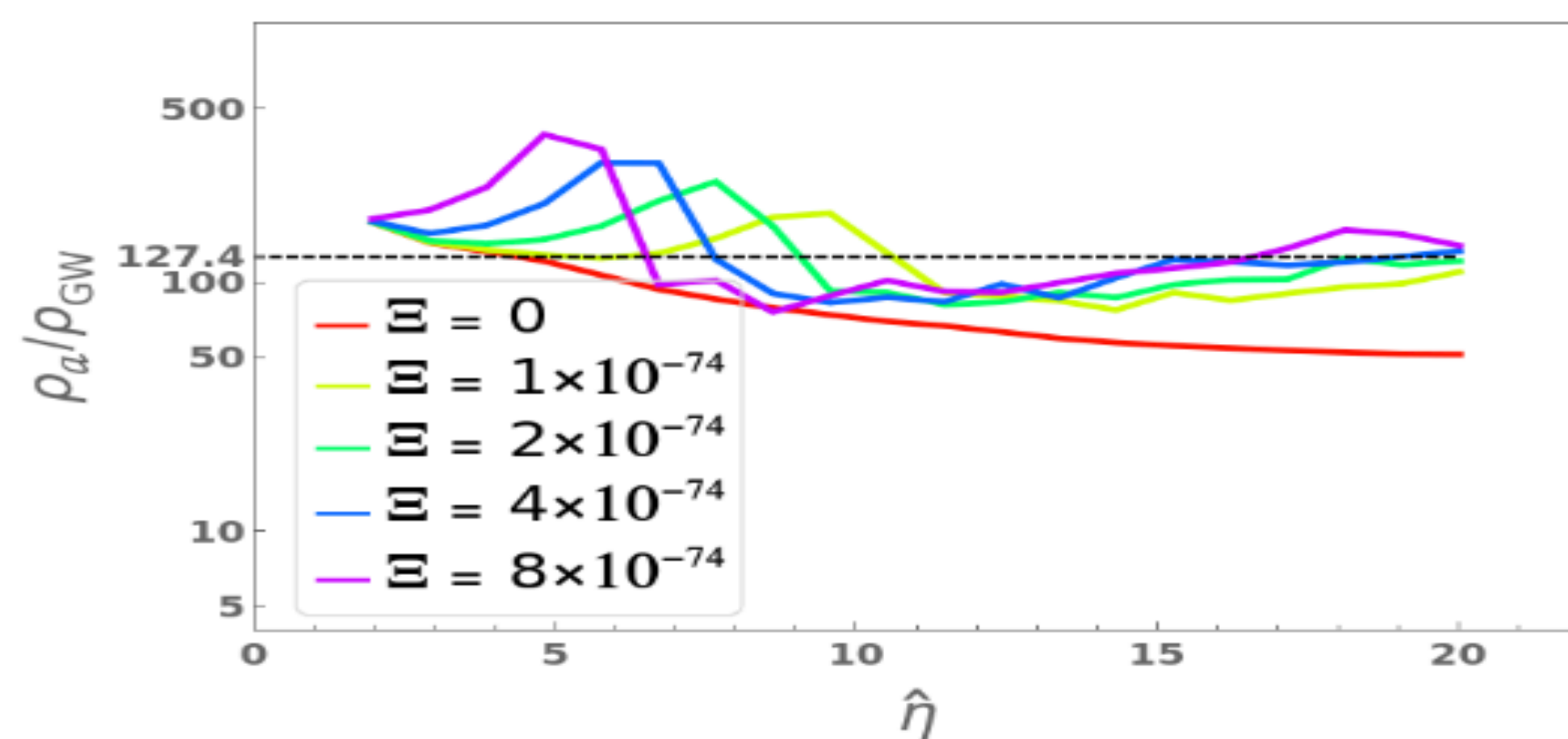
$$\rho_{\text{GW}}(t) = t_{00} = \frac{1}{32\pi G}\langle\partial_\mu h_{ij}^{\text{TT}}(\mathbf{x}, t)\partial_\nu h^{\text{TT},ij}(\mathbf{x}, t)\rangle|_{\mu=\nu=0} = \frac{1}{32\pi G}\langle\dot{h}_{ij}(\mathbf{x}, t)\dot{h}_{ij}(\mathbf{x}, t)\rangle$$

$$\Pi_{ij} \equiv T_{ij} - pg_{ij} = T_{ij} - \frac{\delta_{ij}}{3}\sum_l T_{ll}$$

$$= (\partial_i\phi_1\partial_j\phi_1 + \partial_i\phi_2\partial_j\phi_2) - \frac{\delta_{ij}}{3}\sum_l(\partial_l\phi_1\partial_l\phi_1 + \partial_l\phi_2\partial_l\phi_2)$$

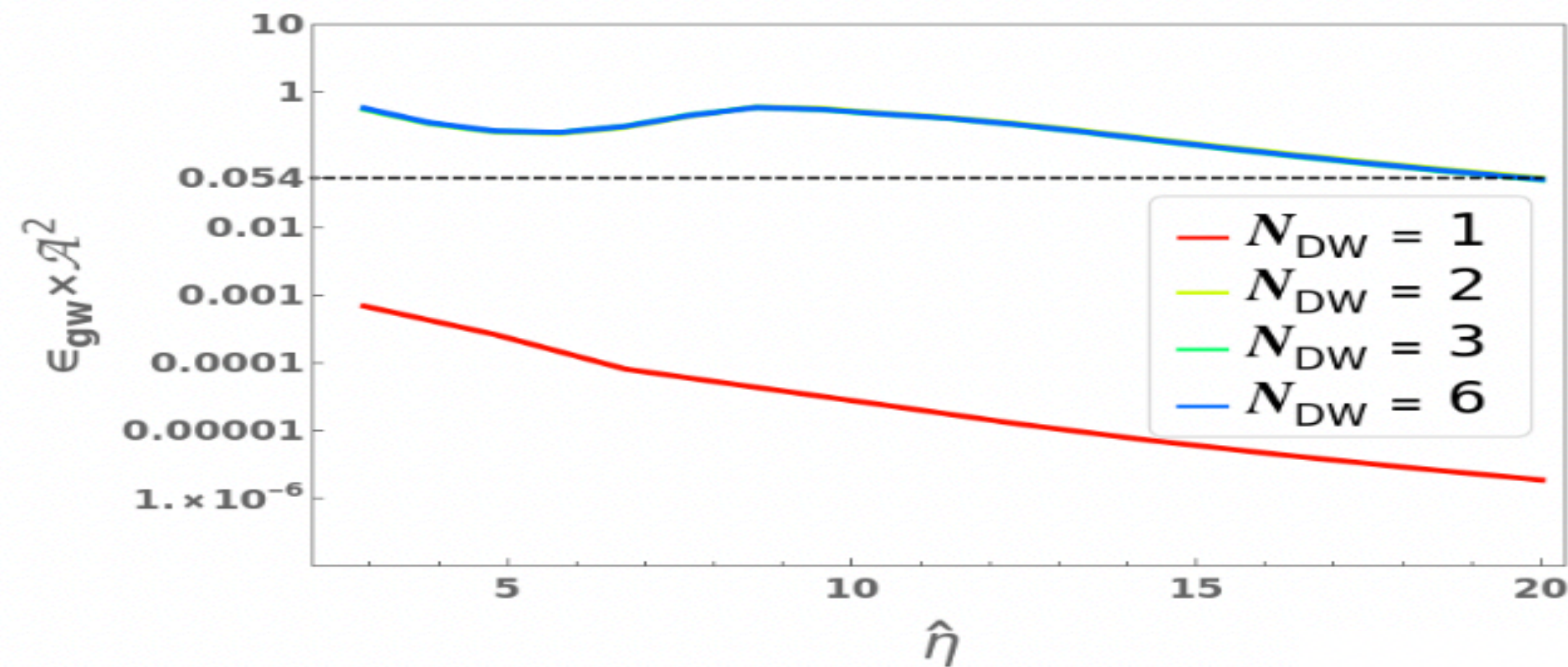


$$\rho_{\text{gw}} = \varepsilon_{\text{gw}} G A^2 \sigma_{\text{wall}}^2 \propto \varepsilon_{\text{gw}} G A^2 m^2 f_a^4$$



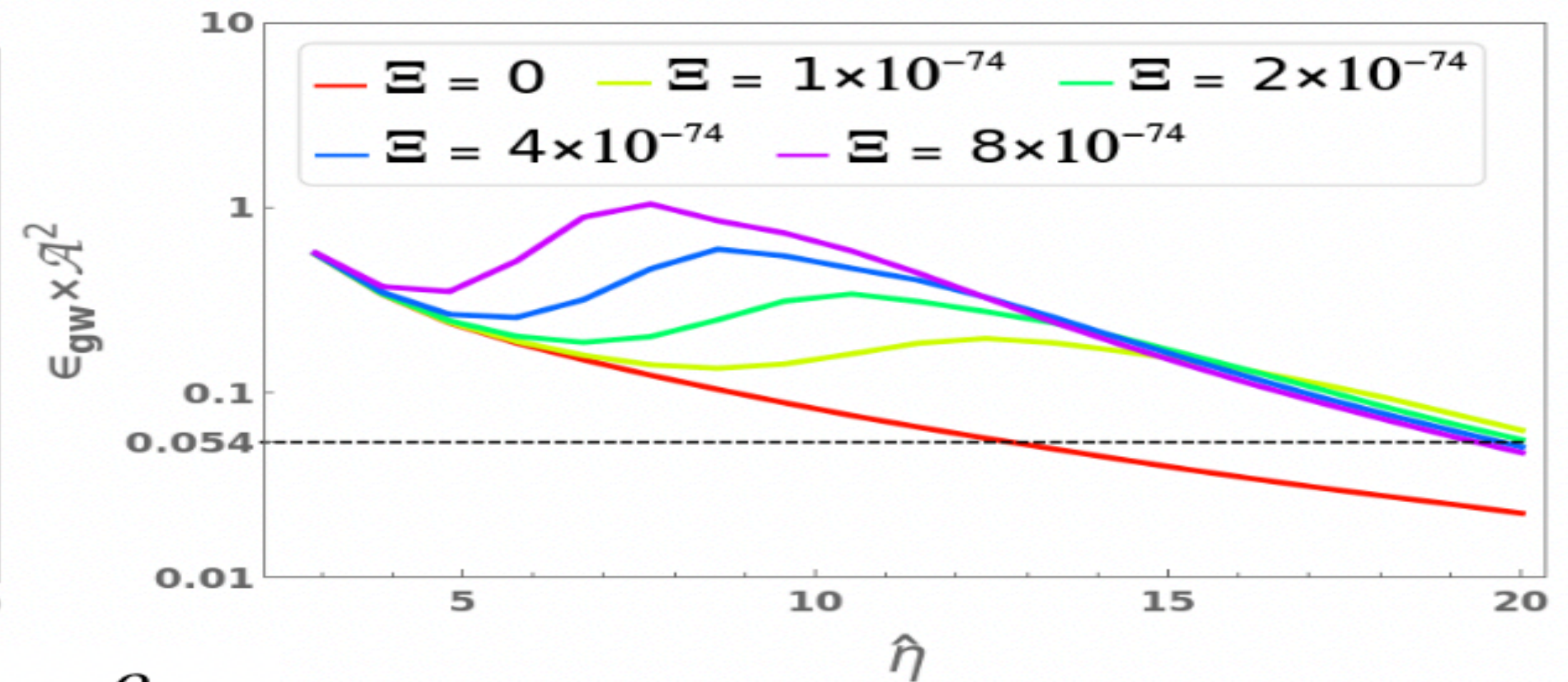
axion string-wall networks mainly decay to axion

► String-wall GW

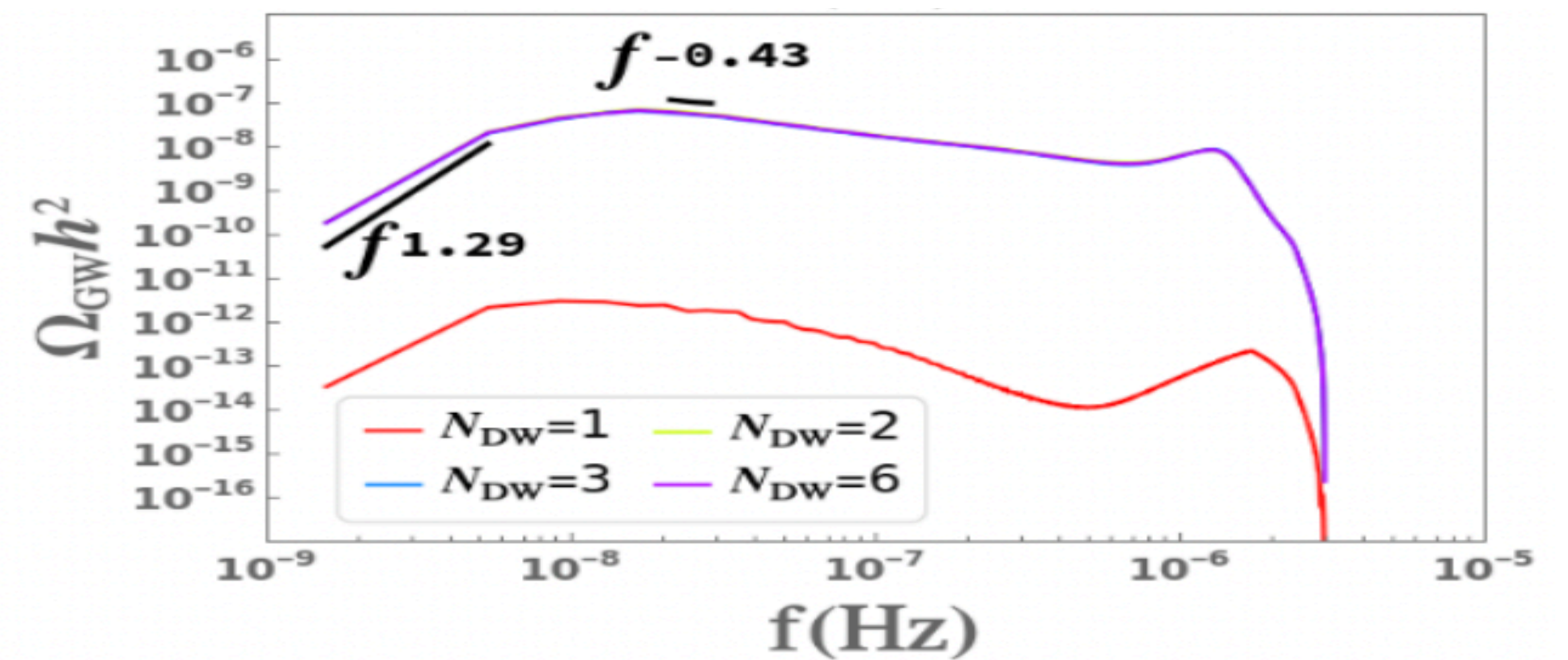
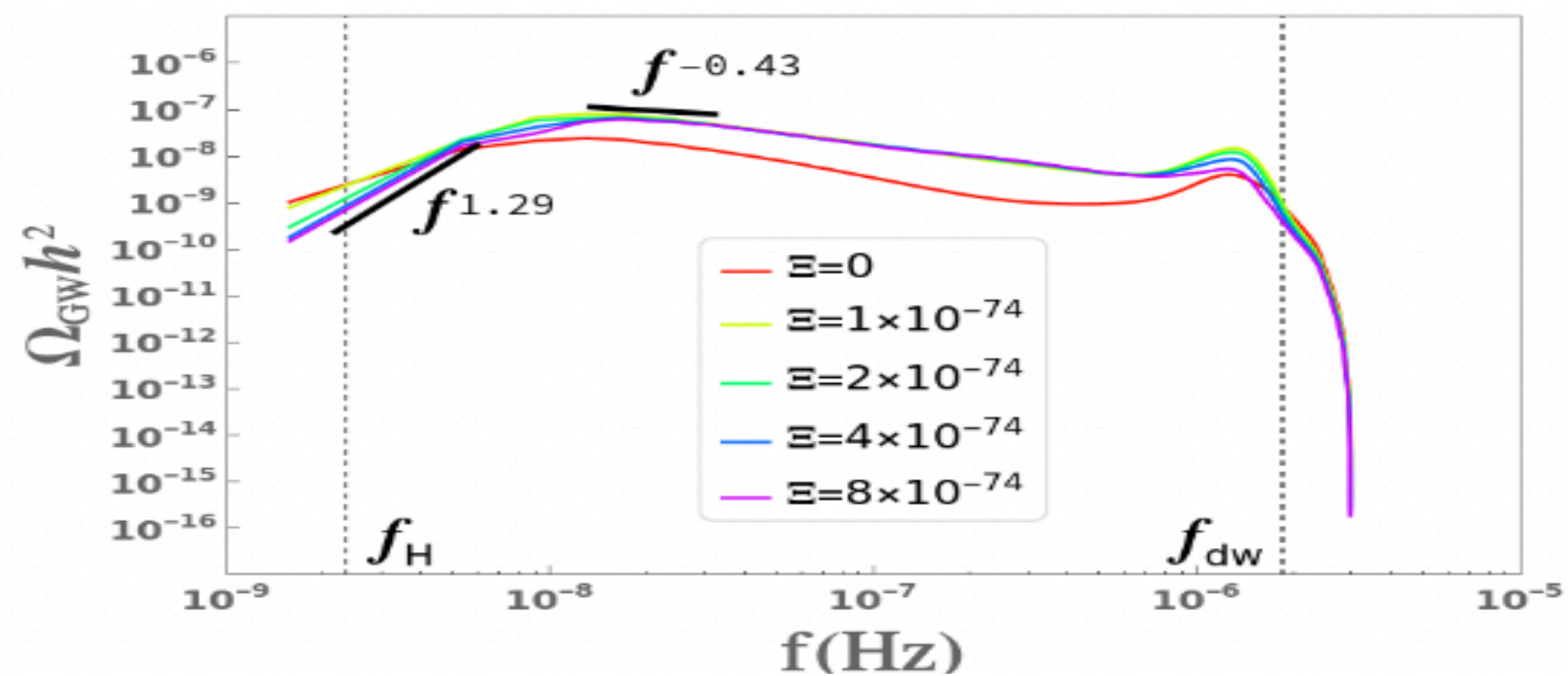


Efficiency parameter

$$\epsilon_{\text{gw}} \mathcal{A}^2 = \frac{\rho_{\text{gw}}}{G\sigma_{\text{wall}}^2} \sim \text{Const.}$$



$$\rho_{\text{gw}} \propto Gm^2 f_a^4$$

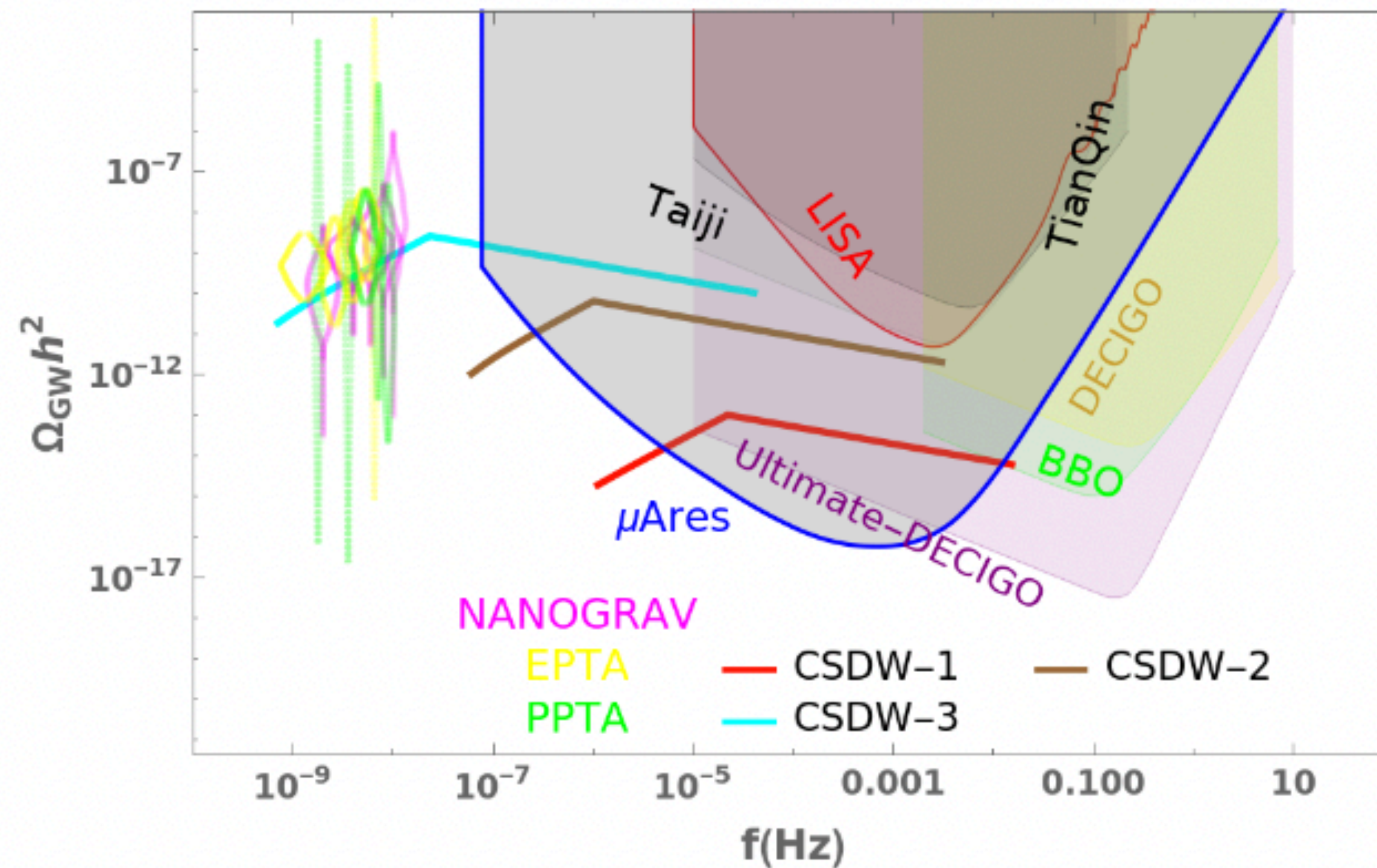
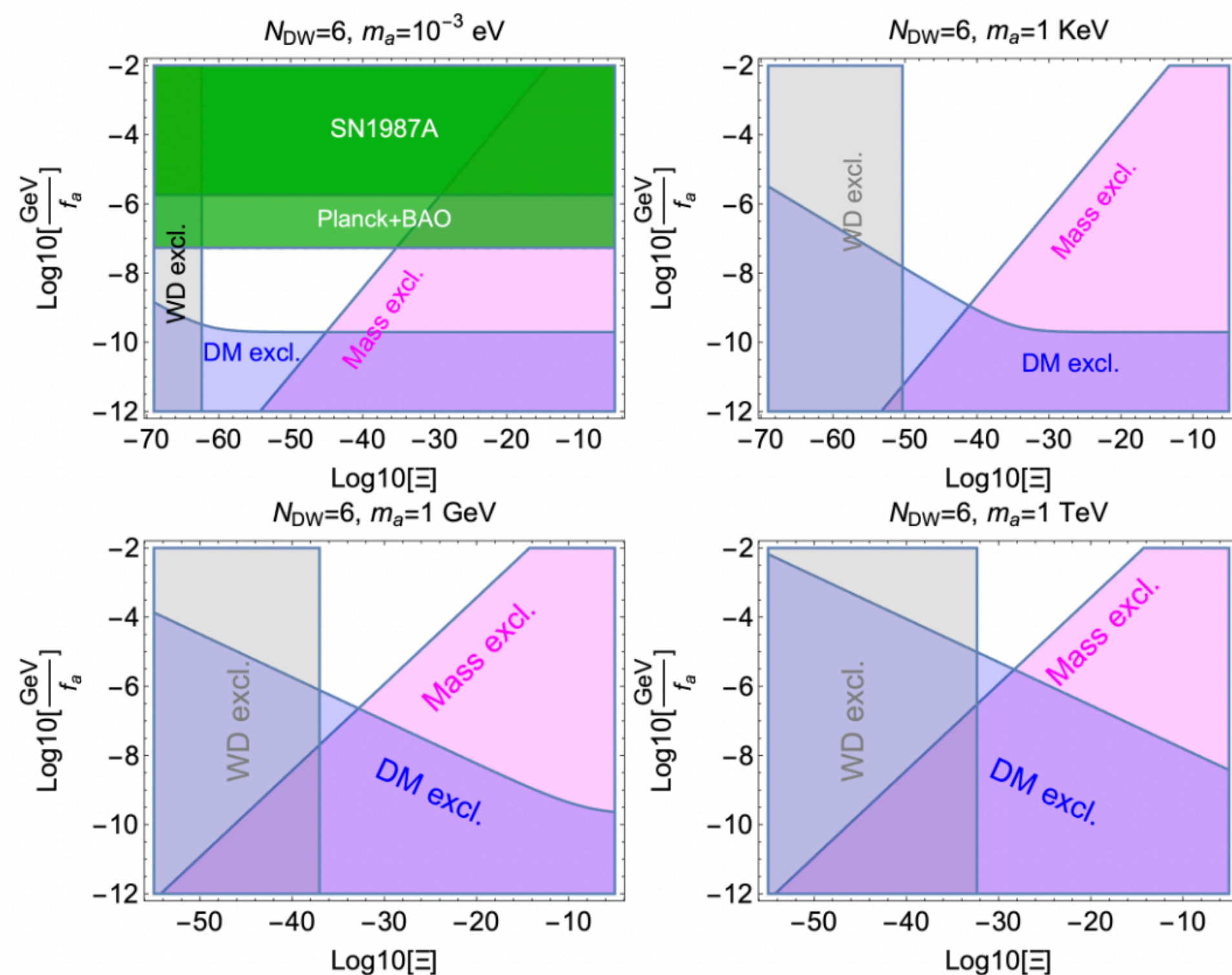


$$\begin{aligned} \Omega_{\text{GW}} &\equiv \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d \ln k} \\ &= \frac{1}{32\pi G} \frac{1}{(2\pi)^3 V} \int d\Omega |\mathbf{k}|^3 |\dot{h}_{ij}(\mathbf{k}, t)|^2. \end{aligned}$$

$$\begin{aligned} \Omega_{\text{GW},0} h^2 &= \frac{h^2}{\rho_{c,0}} \frac{d\rho_{\text{GW},0}}{d \ln k_{p,0}} \\ &= \Omega_{\text{rad},0} h^2 \left(\frac{g_0}{g_e} \right)^{1/3} \left\{ \frac{1}{\rho_{c,e}} \frac{d\rho_{\text{GW},e}}{d \ln k_{co,e}} \right\}. \end{aligned}$$

► Axion DM and GW from String-wall

Our research provides the possibility of searching for axion models or constraining model parameters through GW detection experiments

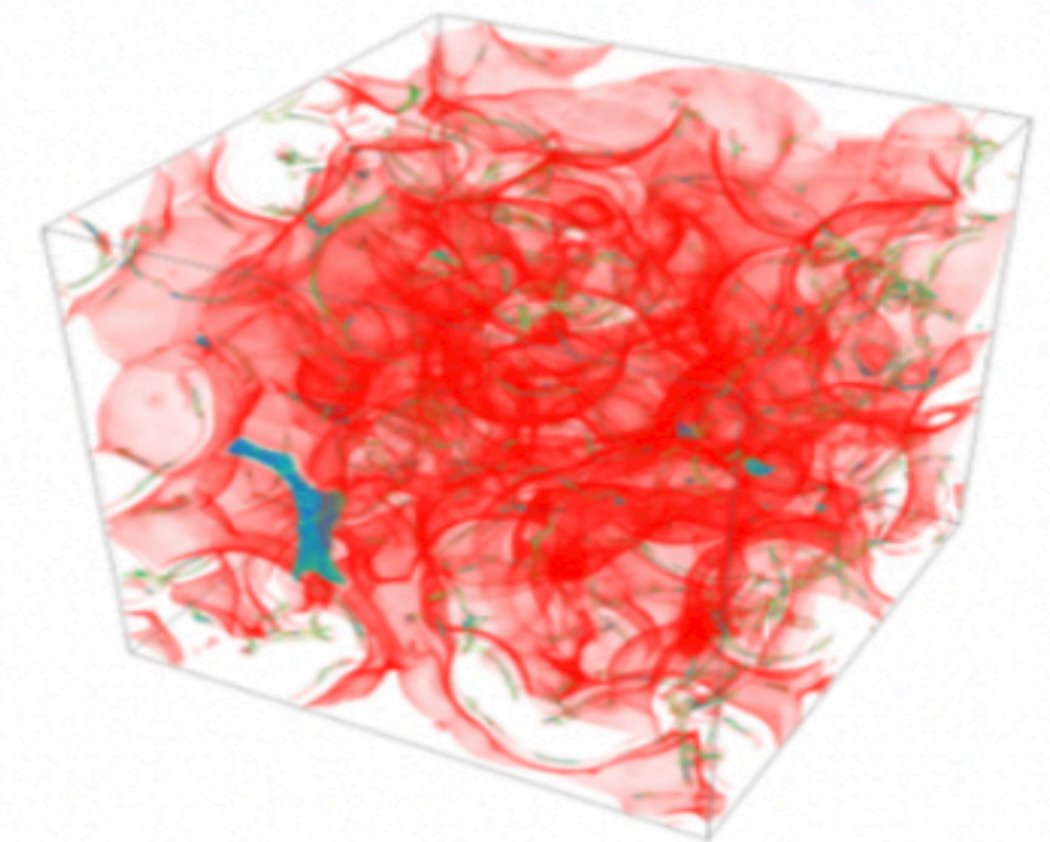
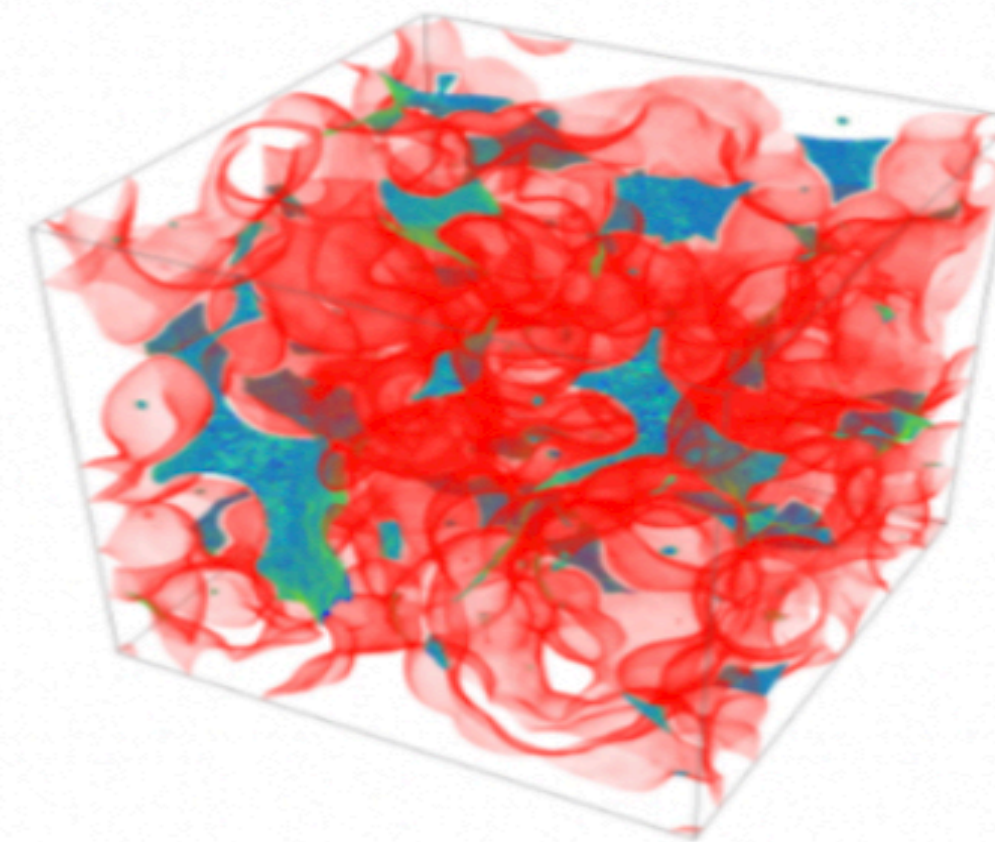
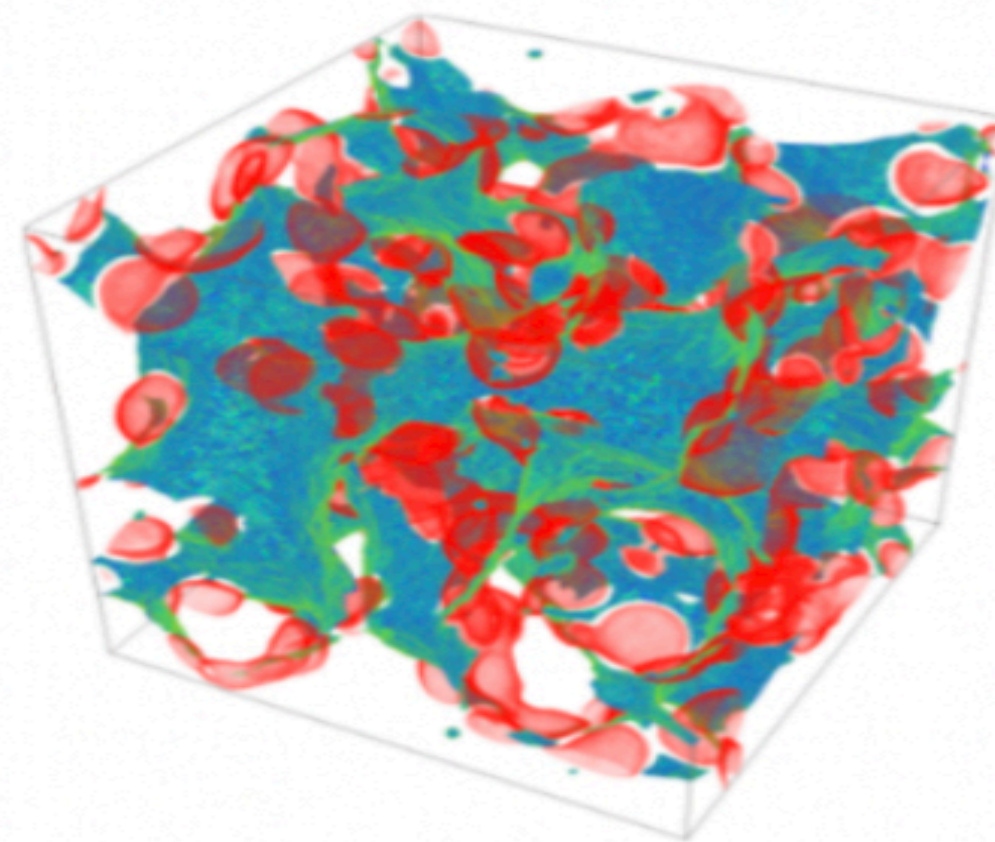
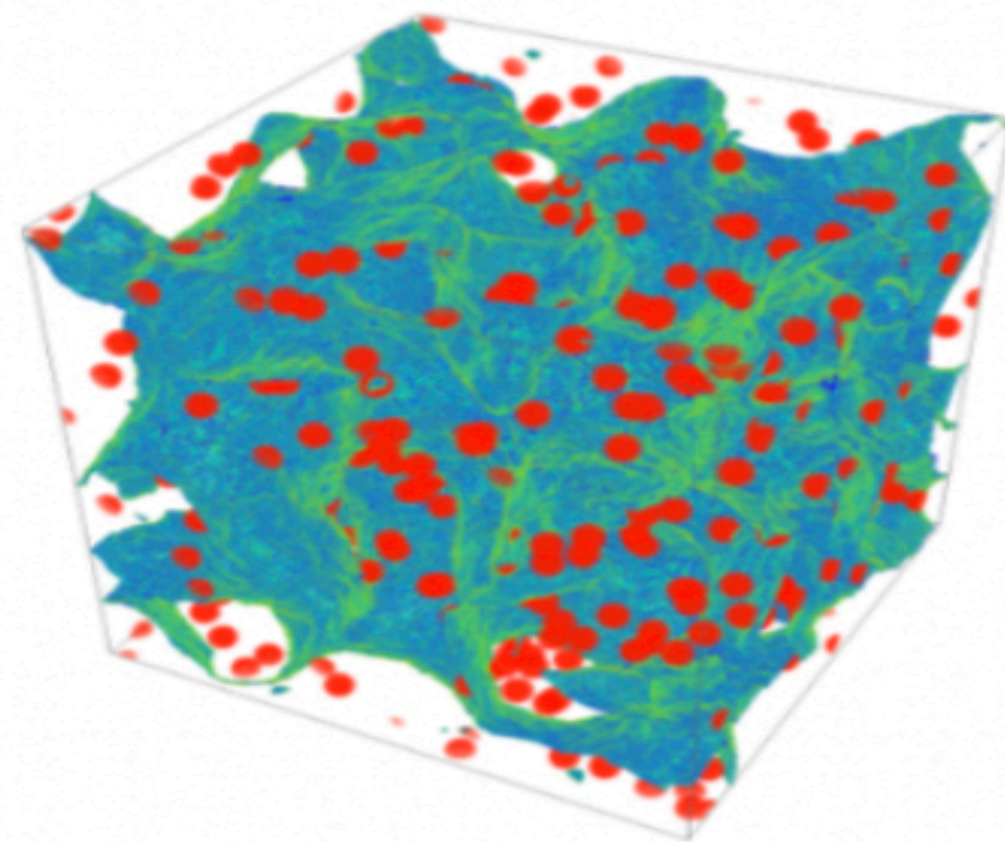
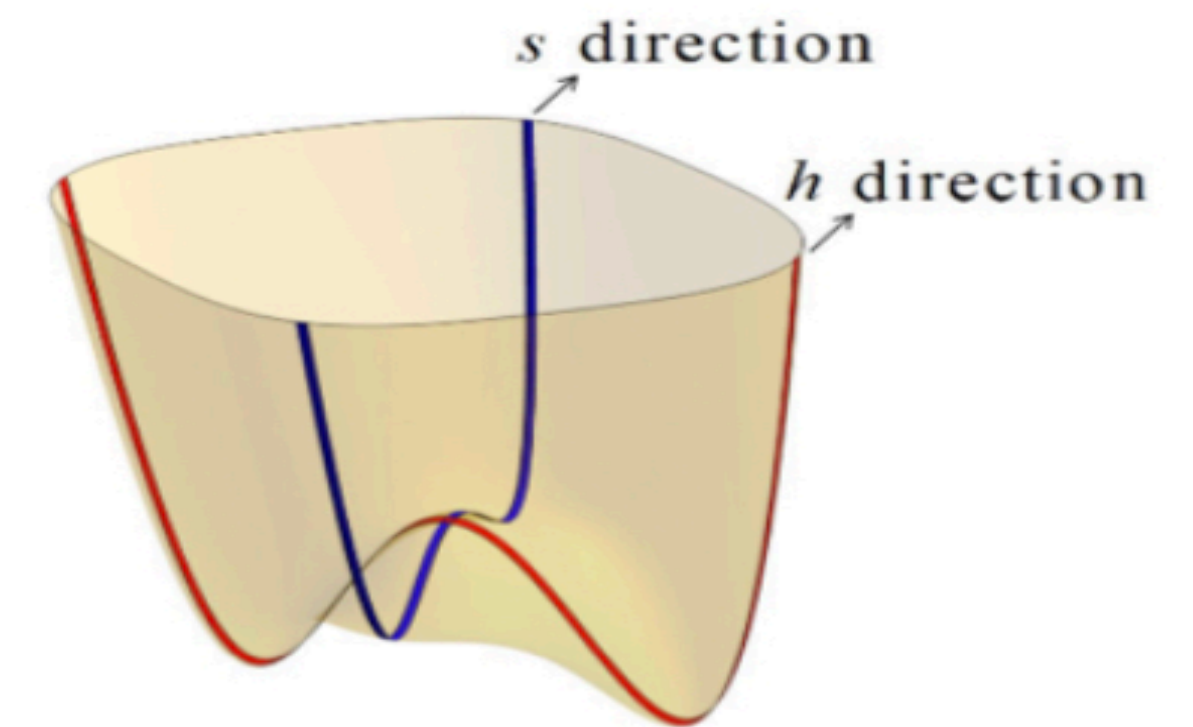
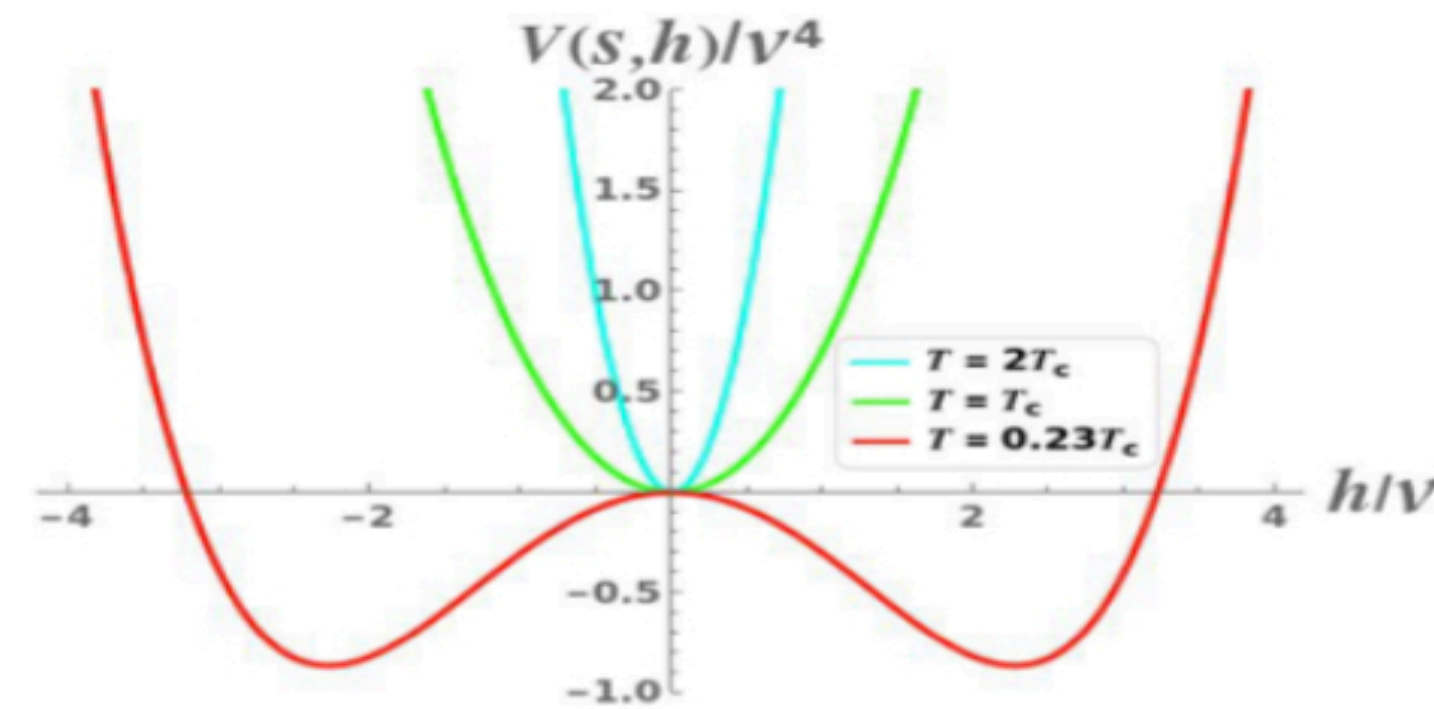
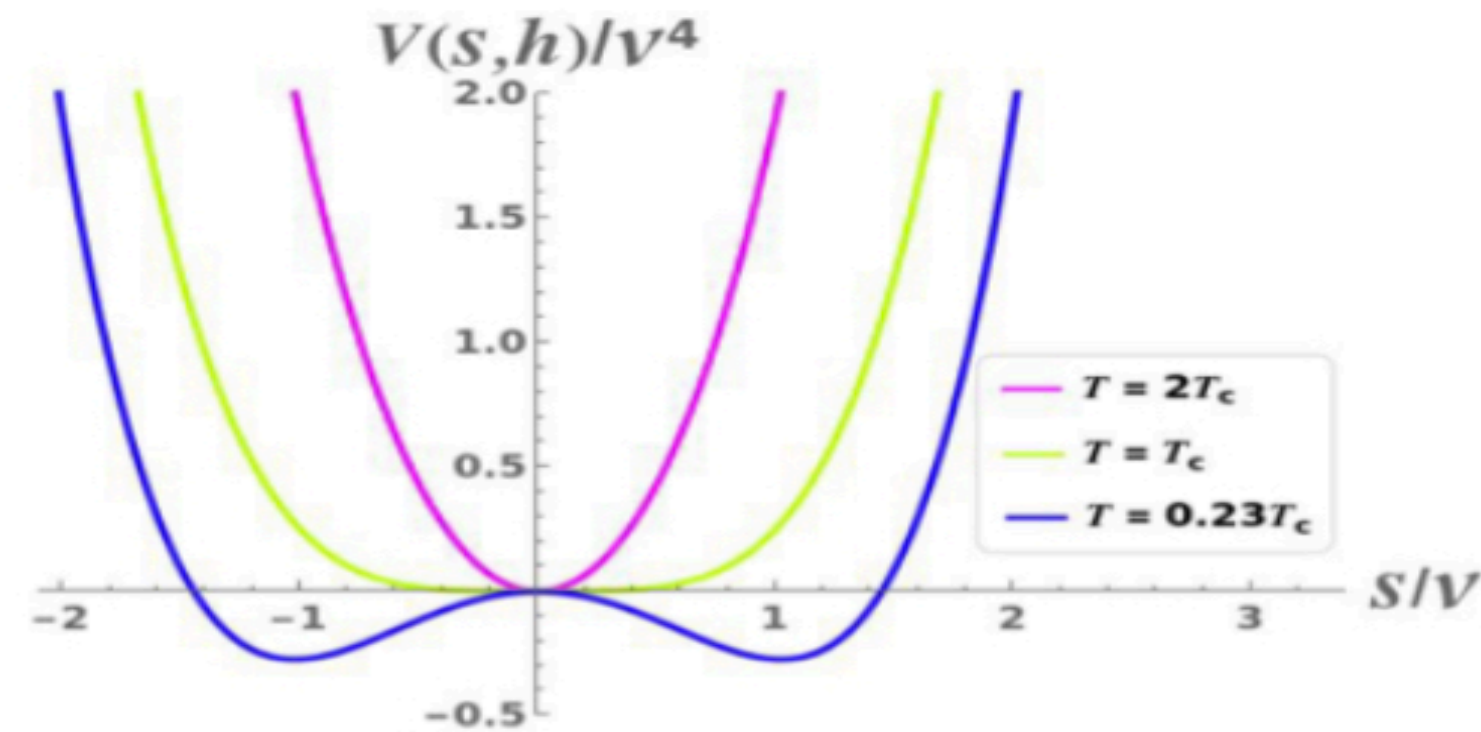


Simultaneously explanation of DM and GWs with axion string-wall networks, **only for $N_{DW} > 1$ of ALP scenarios.**

For $N_{DW} = 1$, the GW energy density appears undetectable for QCD axions and axion-like particles.

► Domain wall solution by FOPT

Second order PT followed by a first-order PT that leads to EWSB

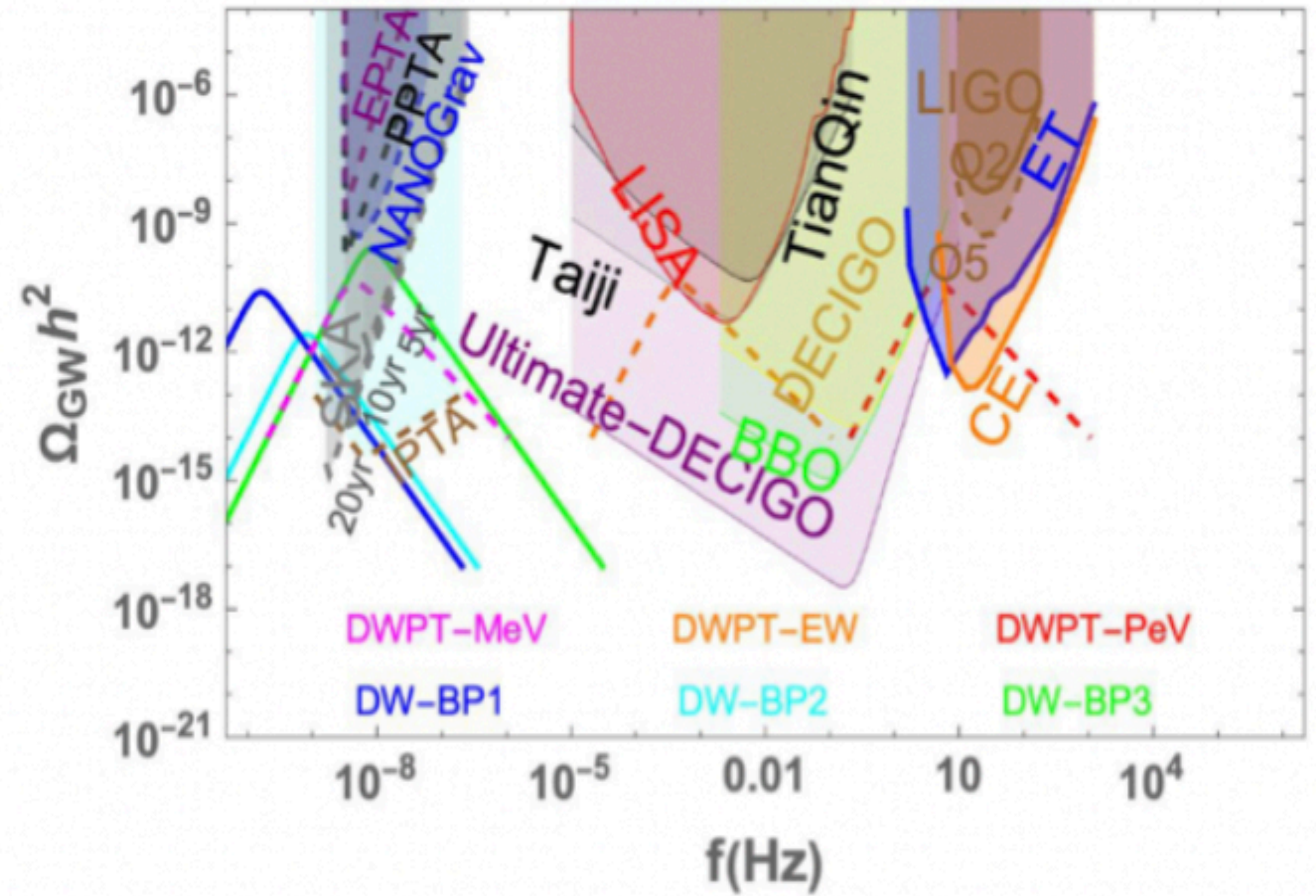
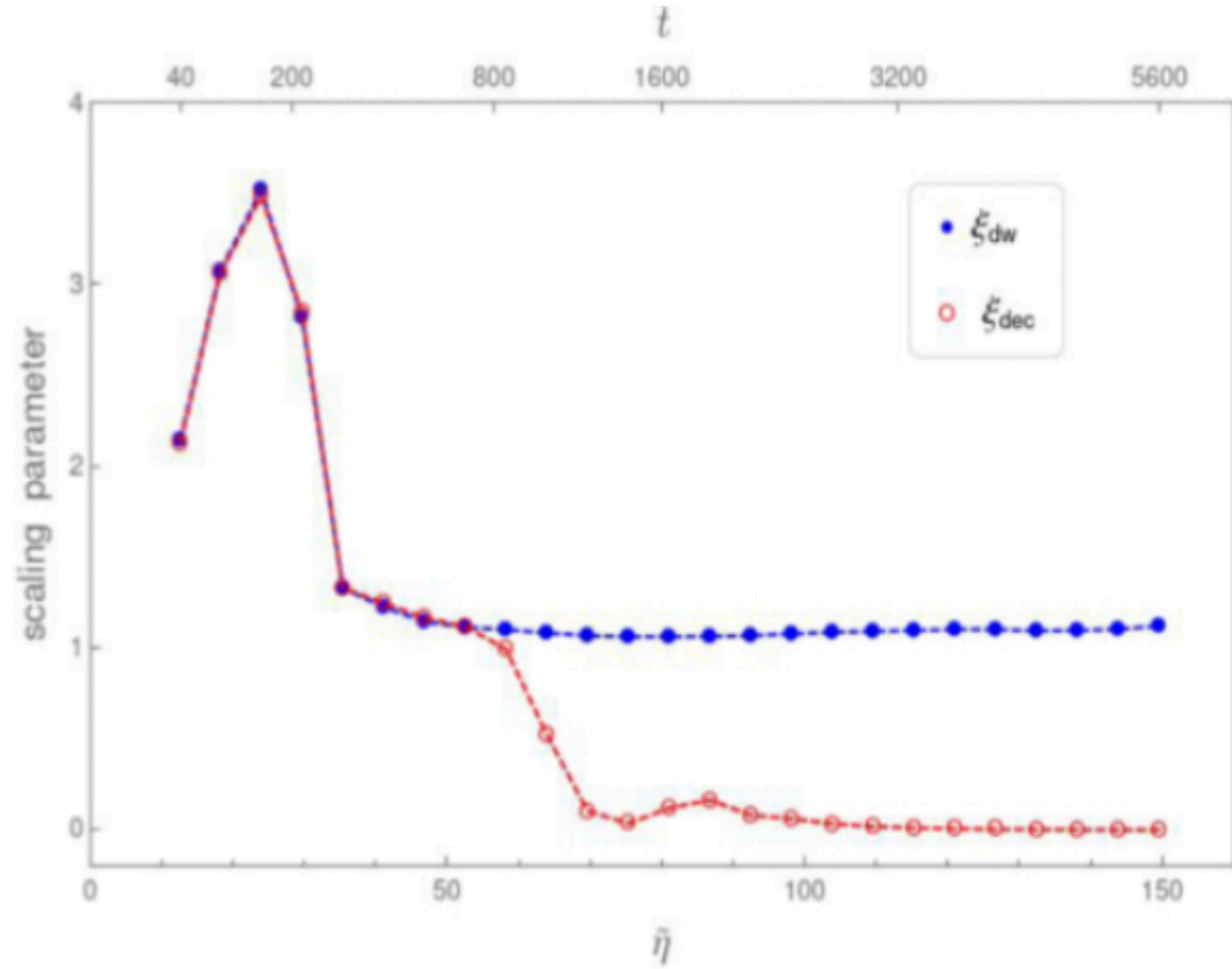


量子隧穿耗散经典的畴壁

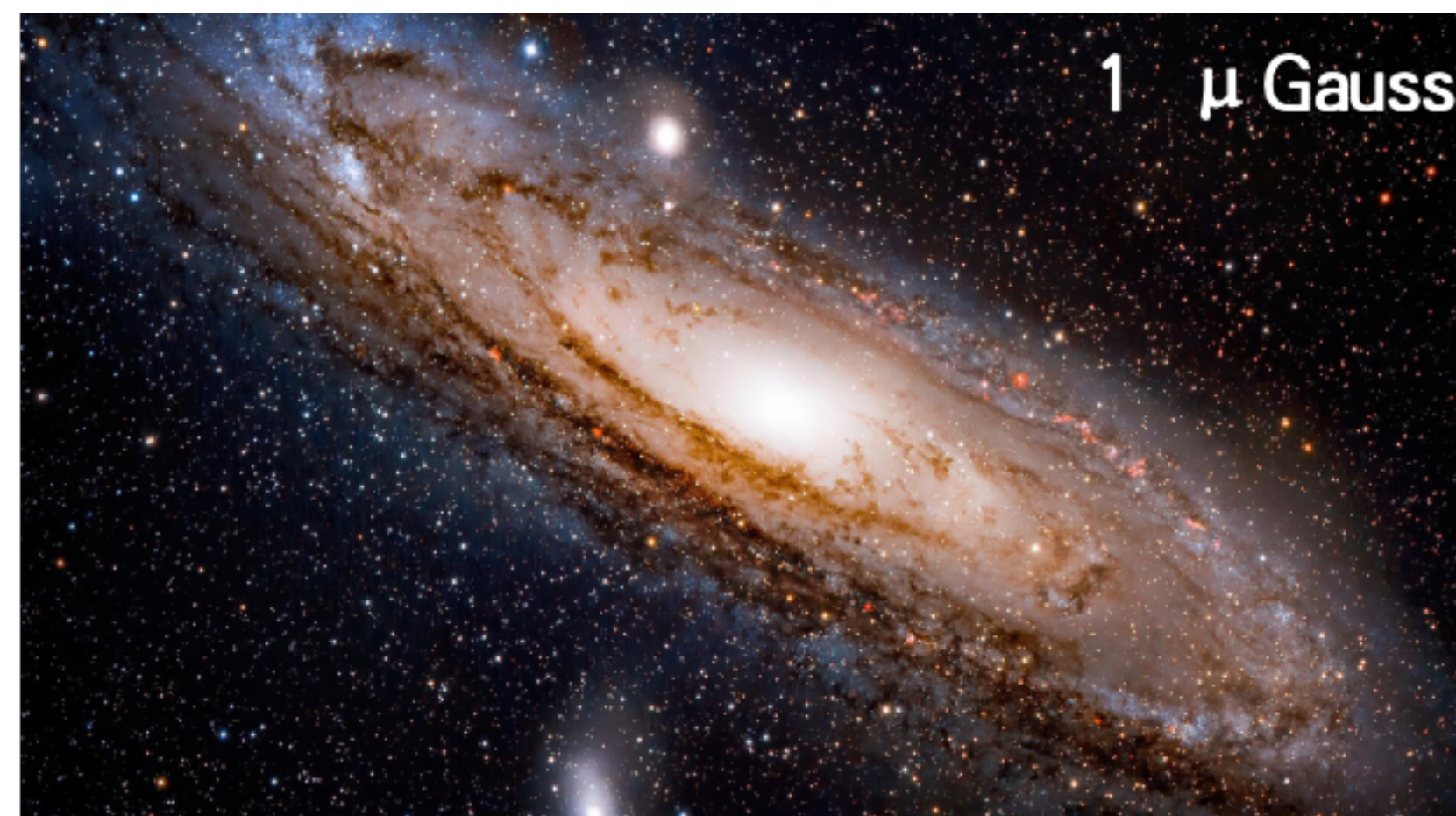
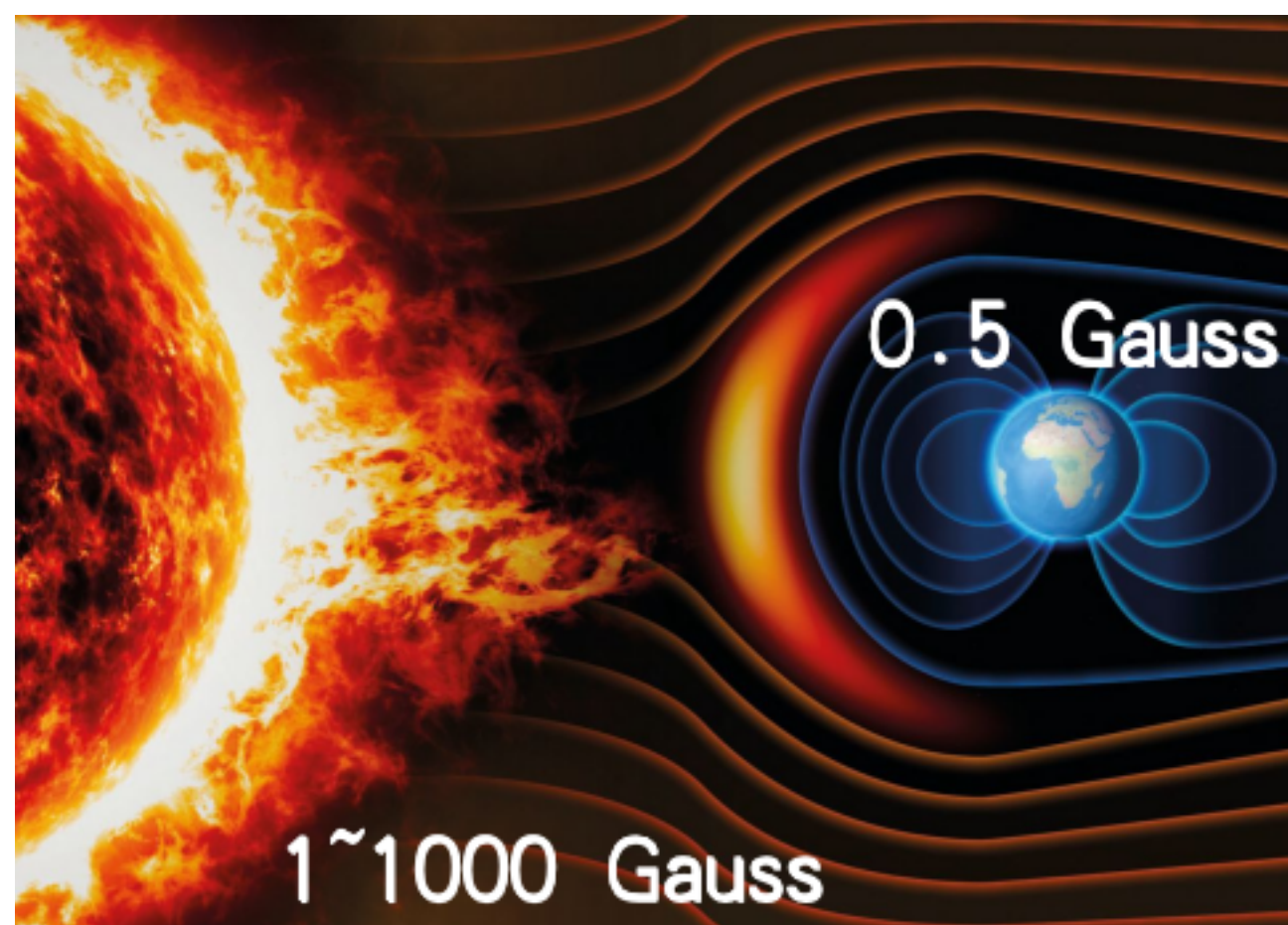
On CPUs

格子大小 1024^3

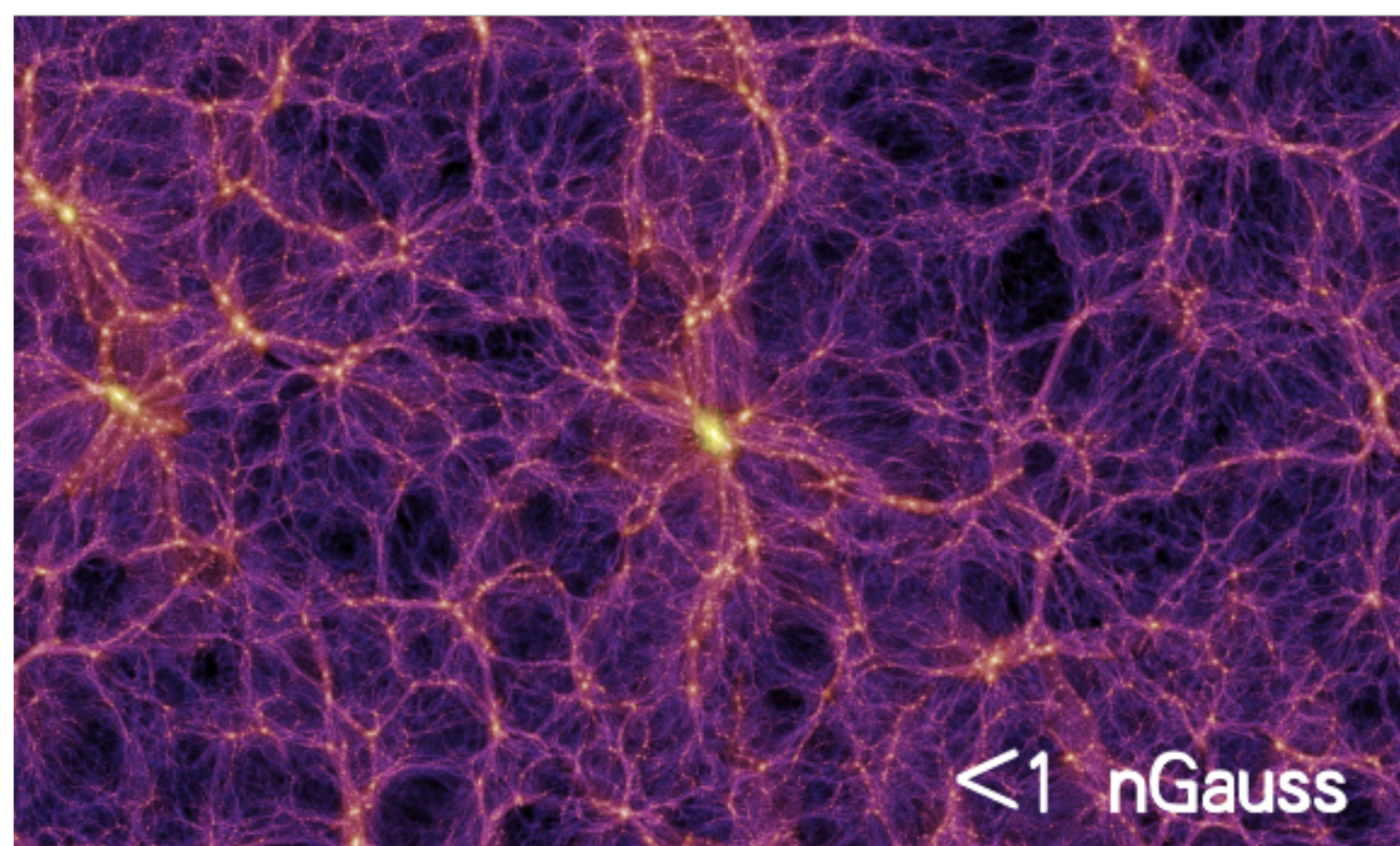
► Domain wall solution by FOPT



► 宇宙中的磁场



- Earth
- Solar System
- Galaxies and galaxy clusters
- **Voids of Large-Scale Structure**



Amplified Magnetic fields



Dynamo and flux conserving compression

Primordial magnetic fields



??? (Inflation, Phase transition, ...)

No magnetic field

► 原初磁场对轴子拓扑缺陷的影响

- The action and potential are

$$S = \int d^4x \sqrt{-g} \left[-\partial_\mu \phi \partial^\mu \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi) - \alpha \frac{\phi}{v} F_{\mu\nu} \tilde{F}^{\mu\nu} \right]$$

$$m_a(T)^2 = \frac{\alpha_a \Lambda^4}{f_a^2 (T/\Lambda)^{6.68}} \quad (T > 100 \text{ MeV}, f_a = \frac{v}{N_{DW}})$$

- The EOMs are

$$\begin{aligned} \phi'' &= \partial_i \partial_i \phi - 2\mathcal{H}\phi' - a^2 \frac{dV}{d\phi} - a^2 \frac{\alpha}{4v} F_{\mu\nu} \tilde{F}^{\mu\nu} \\ E'_i &= -\epsilon_{ijk} \partial_j B_k - \frac{\alpha}{v} \phi' B_i + \frac{\alpha}{v} \epsilon_{ijk} \partial_j \phi E_k \\ \partial_i A'_i &= -\frac{\alpha}{v} \partial_i \phi \epsilon_{ijk} \partial_j A_k \quad E_i = \frac{A'_i}{a^2}, B_i = \frac{1}{a^2} \epsilon_{ijk} \partial_j A_k \end{aligned}$$

Magnetic fields spectrum

$$B_i(\vec{k}) = B_{ini} \Theta(k - k_{UV}) (\delta_{ij} - \hat{k}_i \hat{k}_j - i\sigma_M \epsilon_{ijl} \hat{k}_l) g_j(\vec{k}) k^n$$

$g_j(\vec{k})$ is Gauss random field and $\sigma_M \in [-1, 1]$

determines the helicity of magnetic field. n is the spectrum index

$$\lambda_B = \frac{\int dk E_B(k)/k}{\int dk E_B(k)}$$

$$V(\phi) = \frac{m_a(T)^2 v^2}{N_{DW}^2} \left(1 - \cos \left(N_{DW} \frac{\phi}{v} \right) \right)$$

$$F_{\mu\nu} \tilde{F}^{\mu\nu} = 4 \sum_i E_i B_i \text{ is Chern-Simons (CS)}$$

QCD era - Lattice implementation

$$\begin{aligned} \phi'(\tau + \Delta\tau) &= \phi'(\tau) \frac{a^2(\tau)}{a^2(\tau + \Delta\tau)} + \frac{d\tau}{a^2(\tau + \Delta\tau)} \left[a^2(\tau) \nabla^+ \nabla^- \phi(\tau) - a^4(\tau) \frac{\partial V(\phi)}{\partial \phi}(\tau) \right. \\ &\quad \left. - \frac{\alpha}{v} a^4(\tau) E_i^{(2)}(\tau) (B_i^{(4)}(\tau) + B_i^{(4)}(\tau + \Delta\tau)) \right], \end{aligned}$$

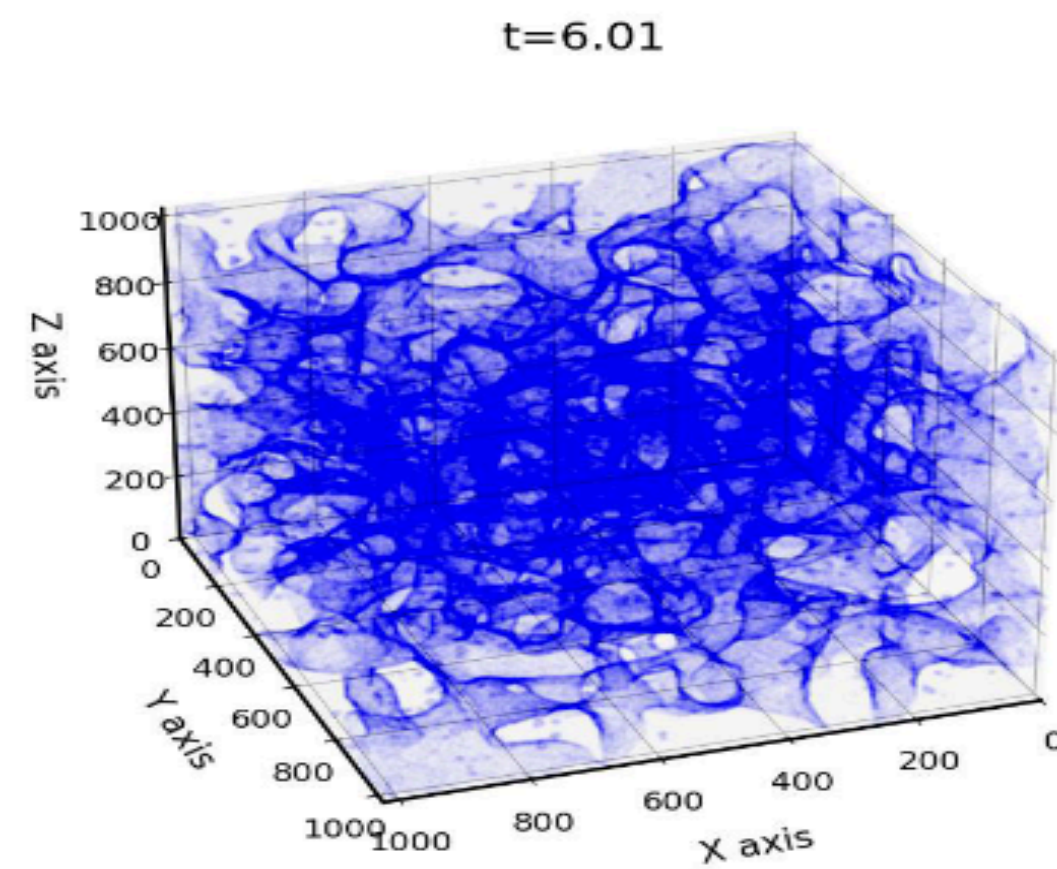
$$A'_i(\tau) = A'_i(\tau - \Delta\tau) + d\tau \cdot a^2(\tau) \left[-\epsilon_{ijk} \partial_j^- B_k(\tau) + \frac{\alpha}{2v} \left(\phi'(\tau) B_i(\tau) + \phi'_{+i}(\tau) B_{i,+i}(\tau) \right) - \frac{\alpha}{4v} \epsilon_{ijk} (K_{jk} + K_{jk,+i}) \right],$$

$$\partial_i^- A'_i(\tau + \Delta\tau) = \frac{\alpha}{2} \left[\partial_i^- \phi(\tau + \Delta\tau) \epsilon_{ijk} \partial_j^+ A_{k,-i}^{(2)}(\tau) + \partial_i^+ \phi(\tau + \Delta\tau) \epsilon_{ijk} \partial_j^+ A_{k,+i}^{(2)}(\tau) \right]$$

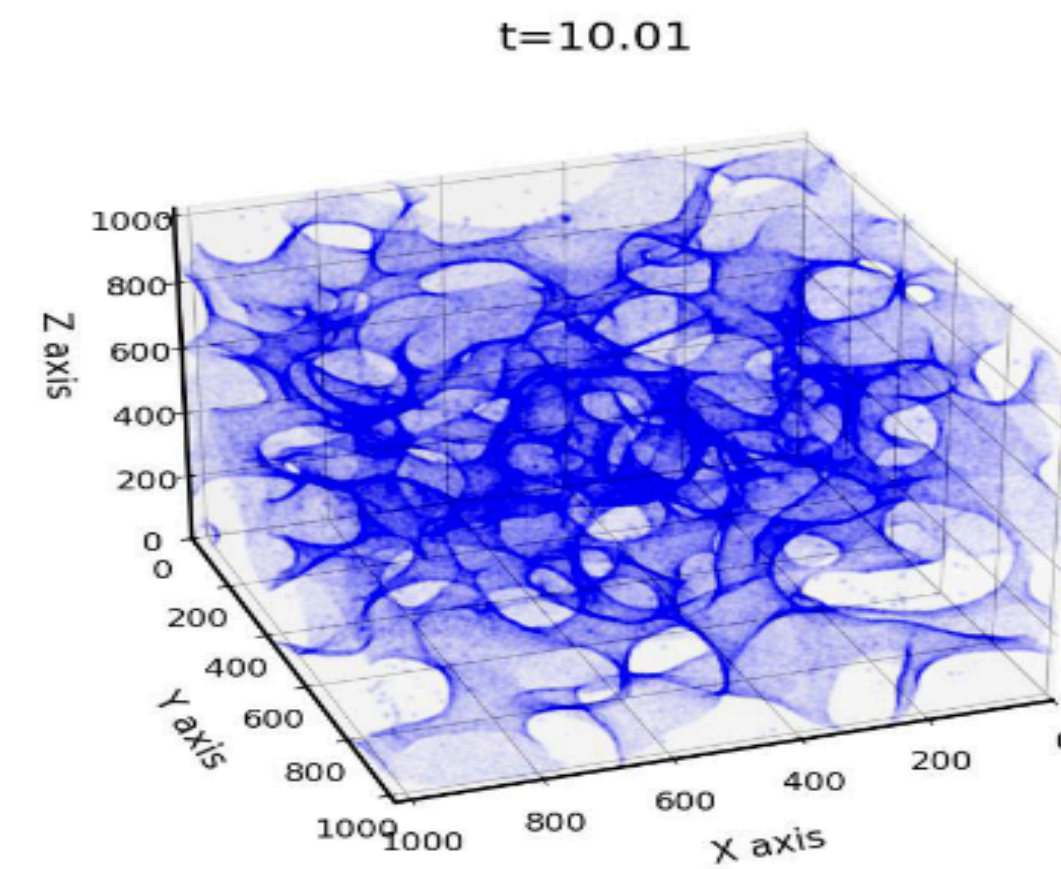
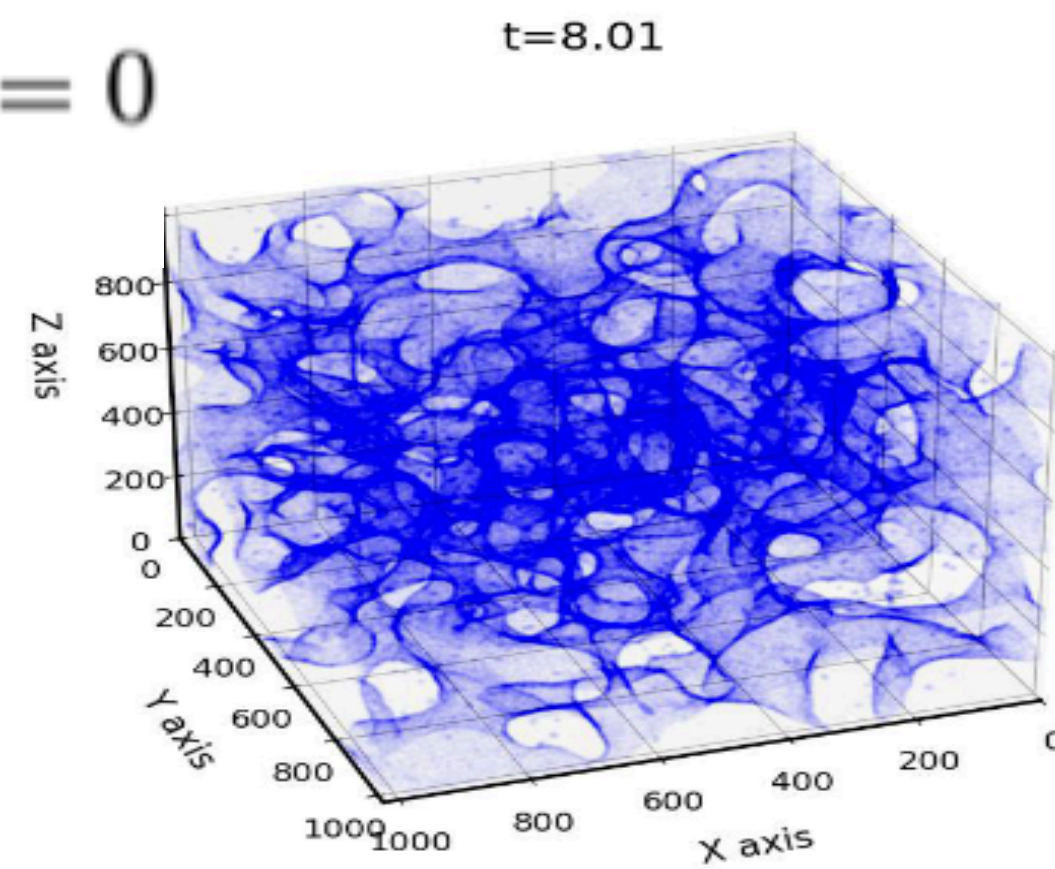
Magnetic fields helicity

$$f_H = \frac{\sum_k h(k)}{\sum_k \frac{2\rho_B(k)}{k}} = \frac{2\sigma_M}{1 + \sigma_M^2}$$

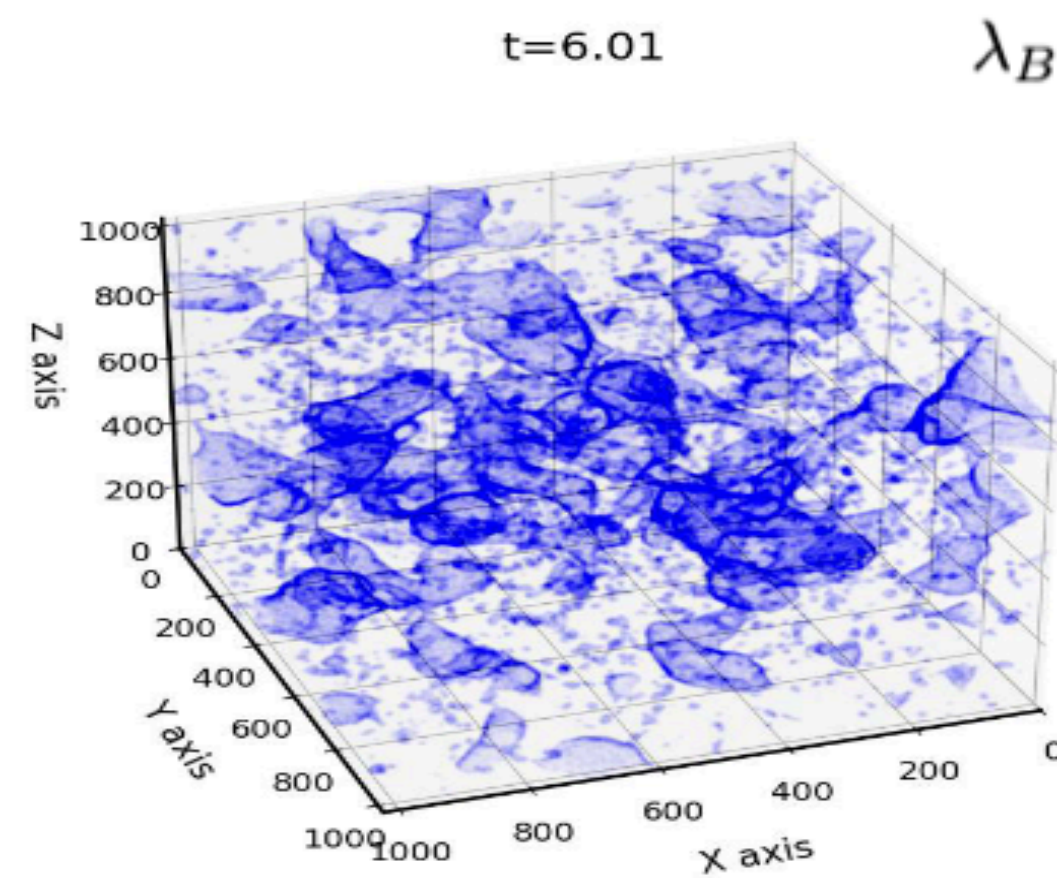
$$\mathcal{H} = \int dx^3 \mathbf{A} \cdot \mathbf{B}_c, \quad \frac{d\mathcal{H}}{d\tau} = 2 \int dx^3 \mathbf{E}_c \cdot \mathbf{B}_c,$$



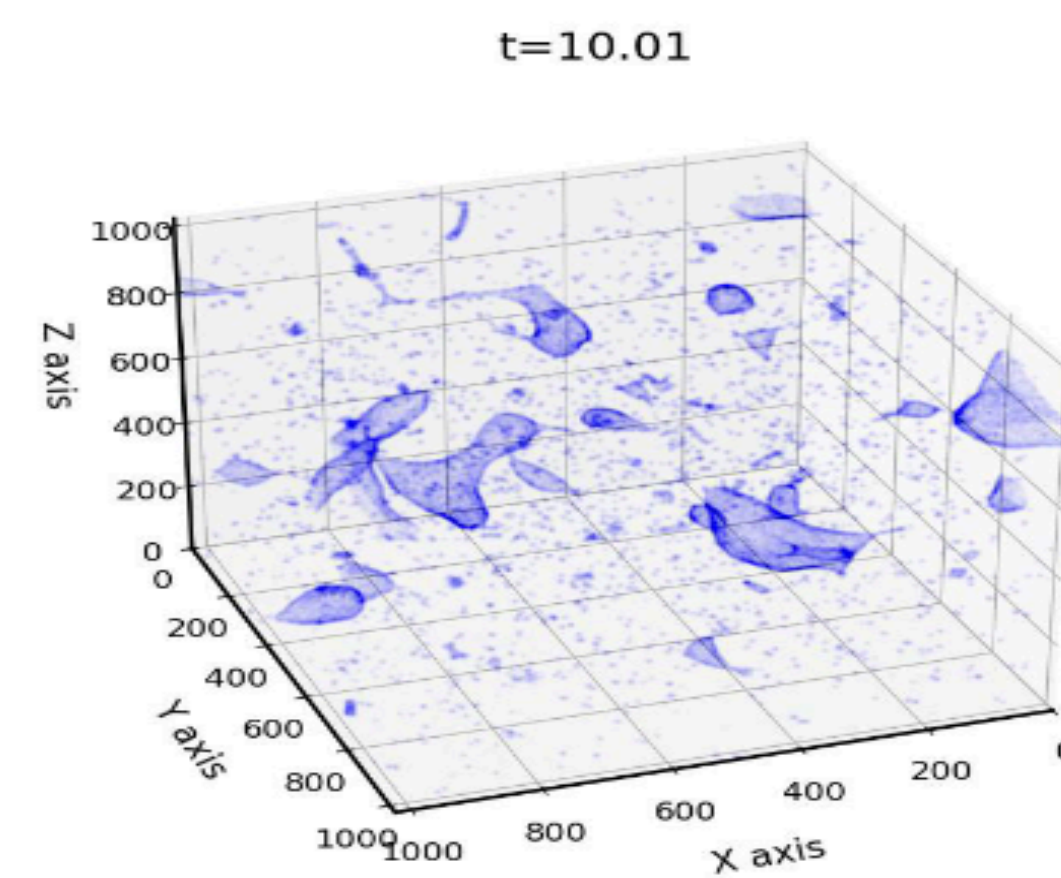
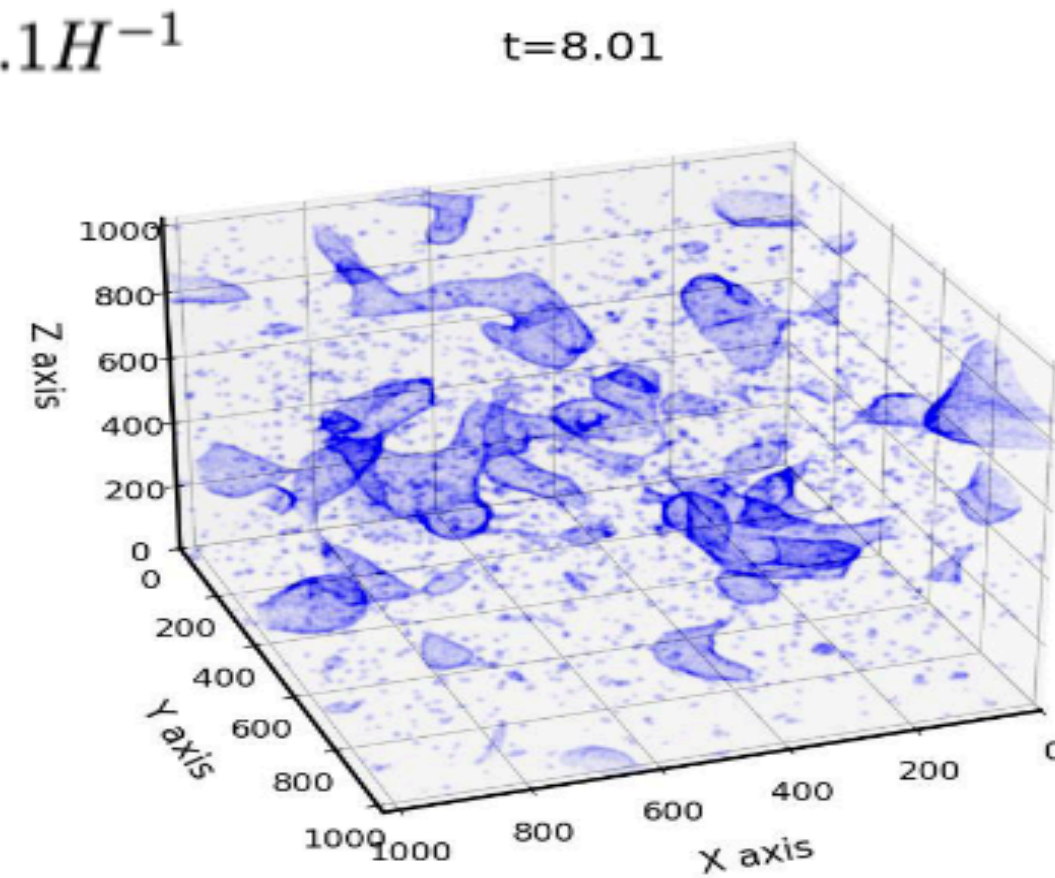
$$\alpha = 0$$



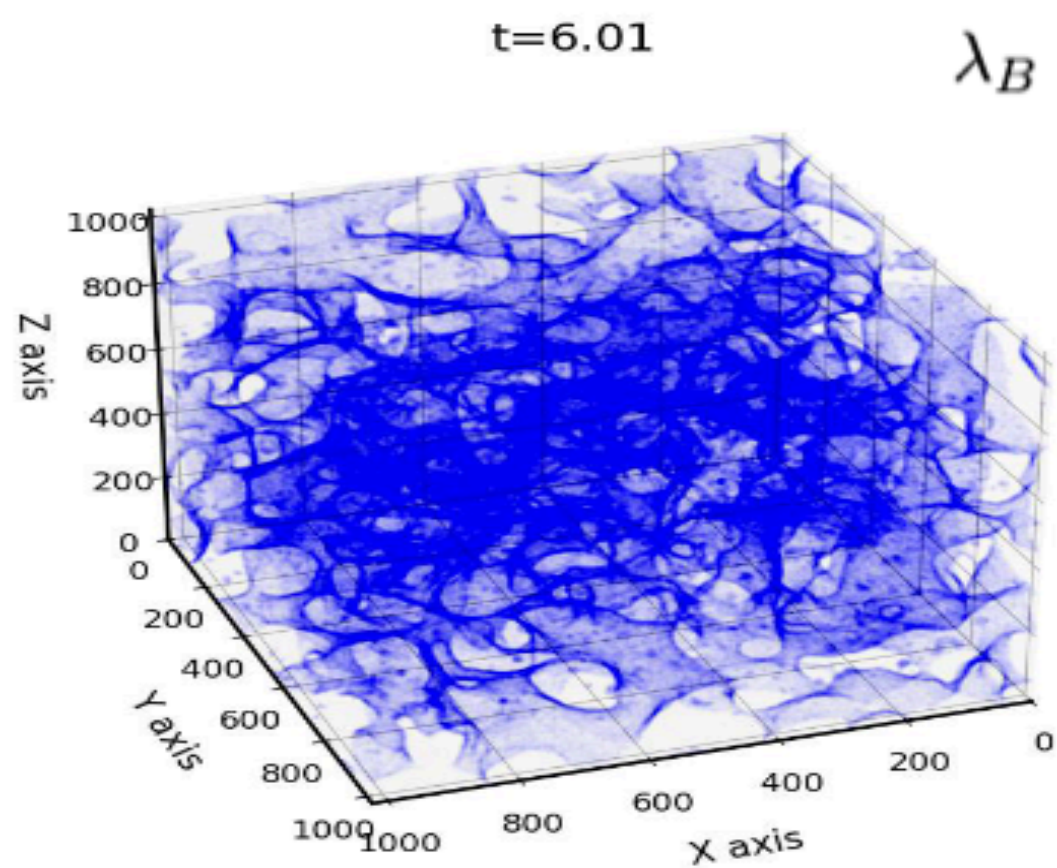
$$N^3 = 1024^3$$



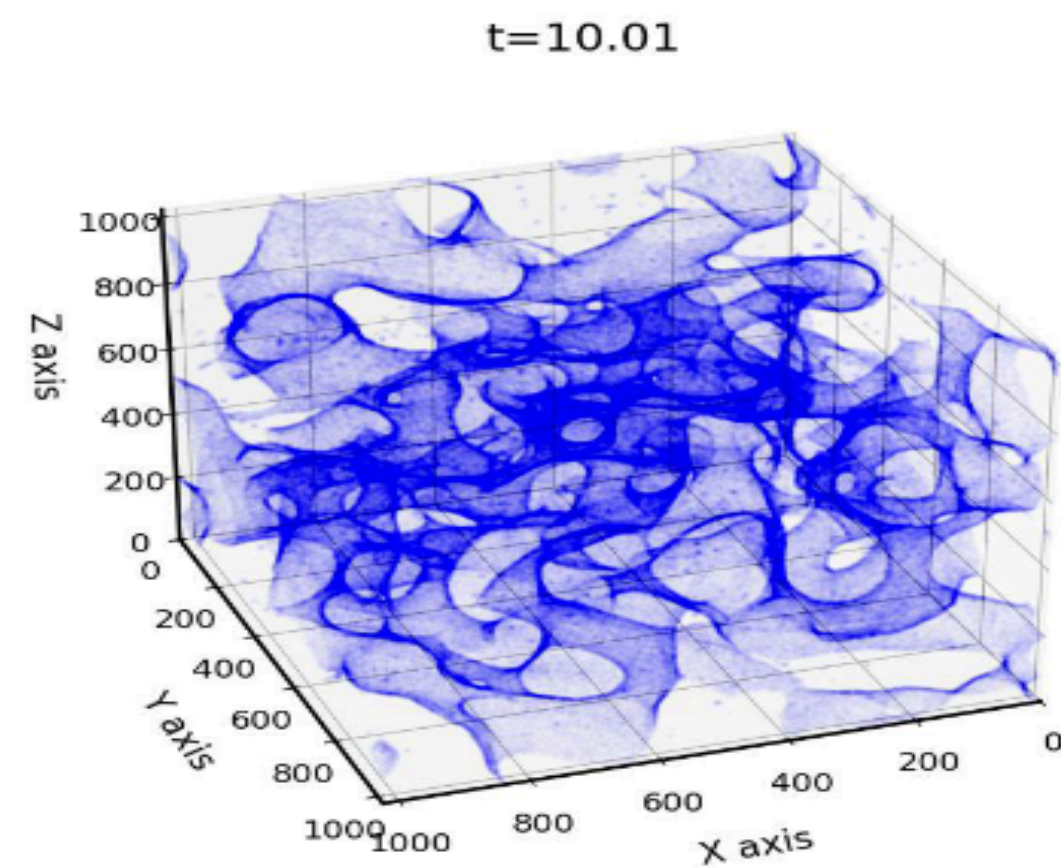
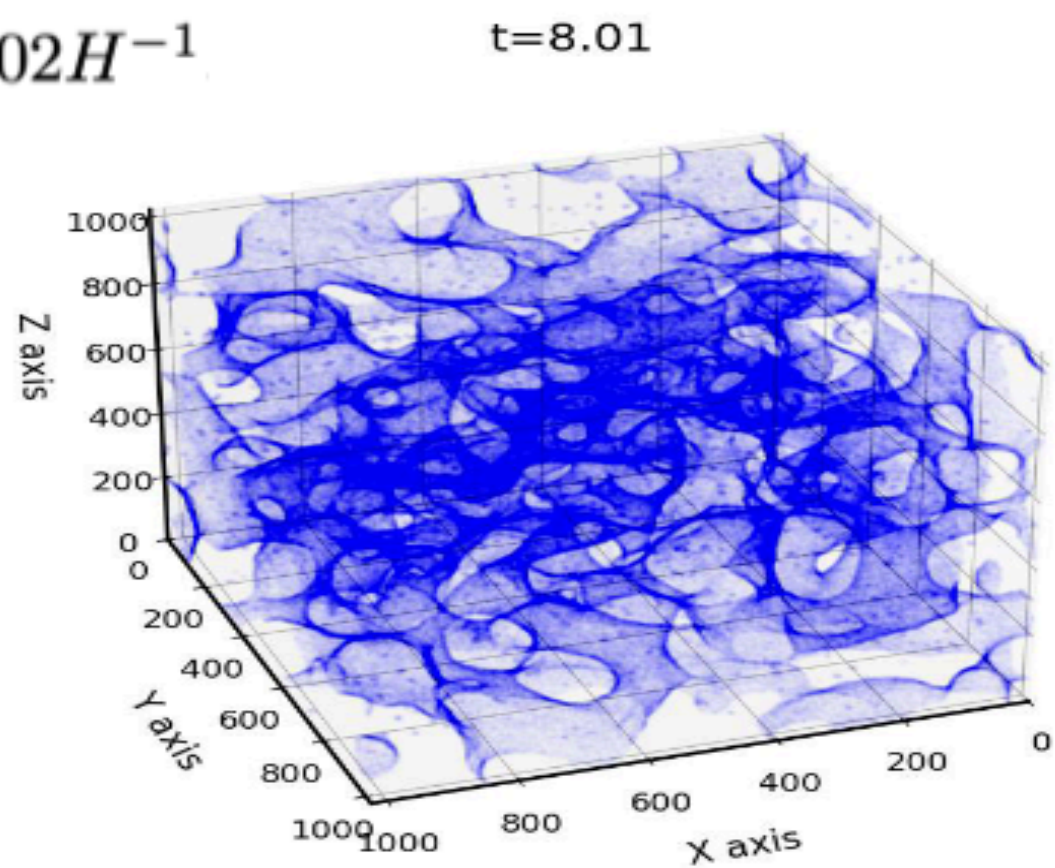
$$\lambda_B = 0.1 H^{-1}$$



$O(10^3)$
Hubble volume

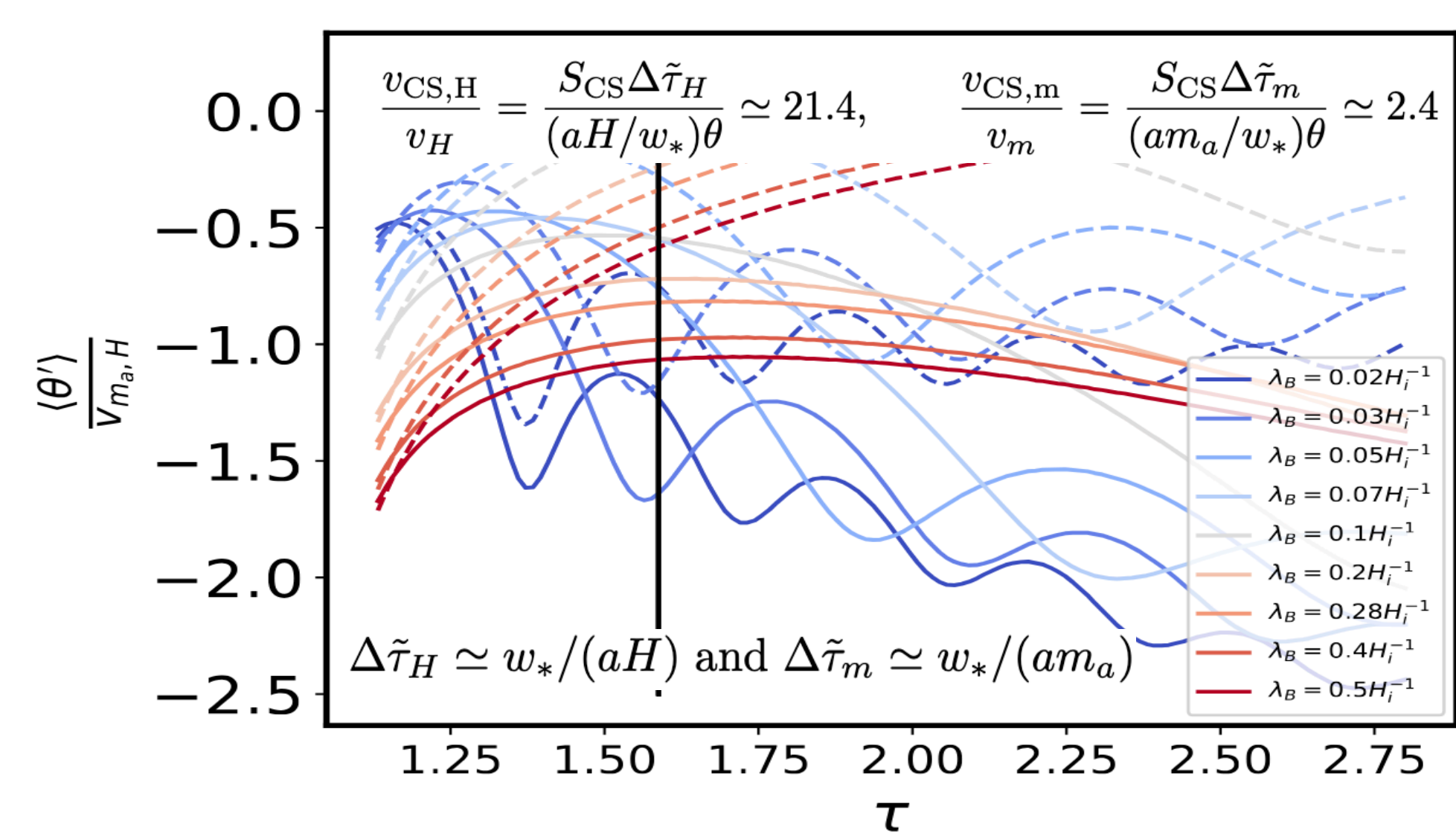
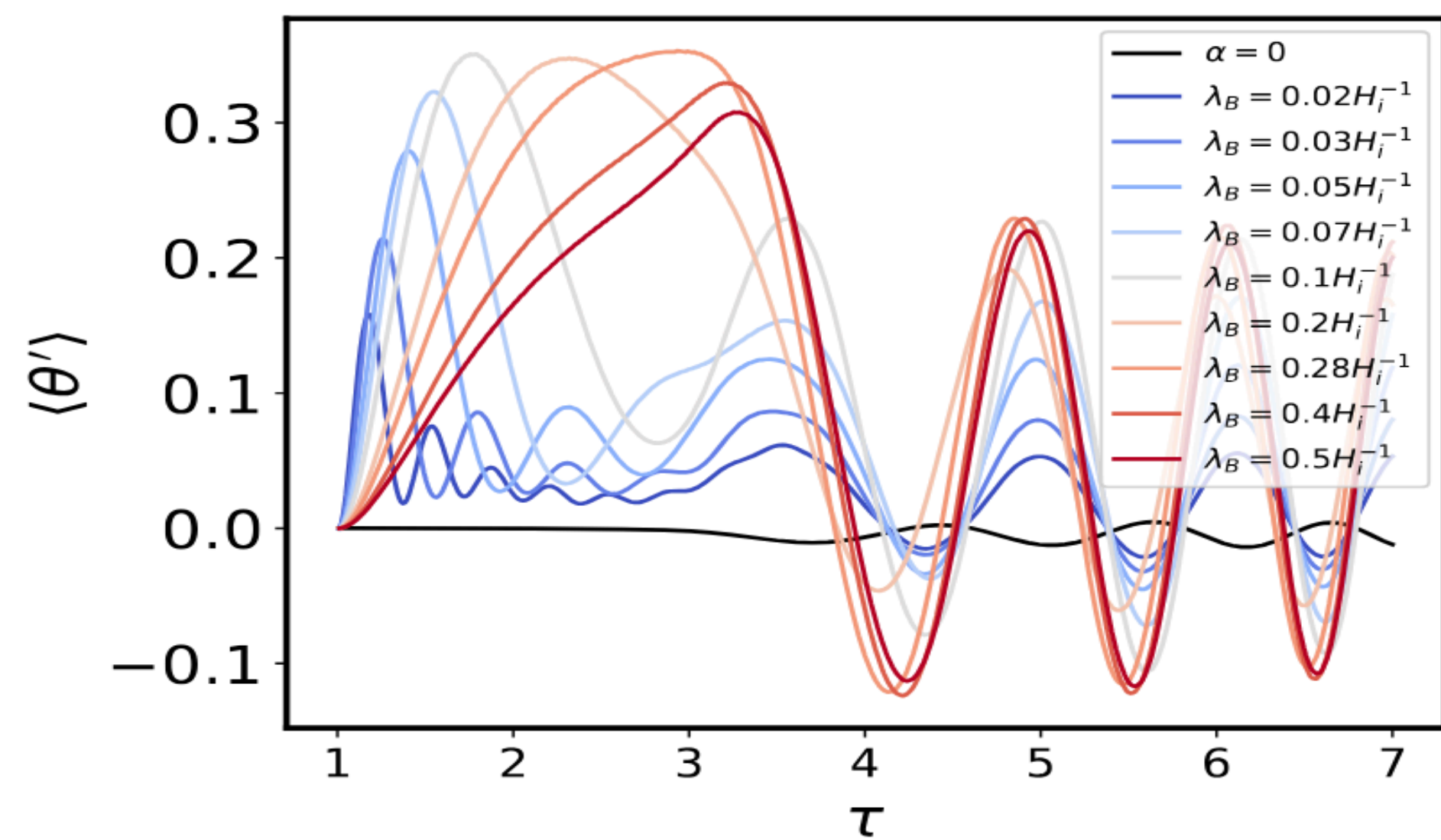
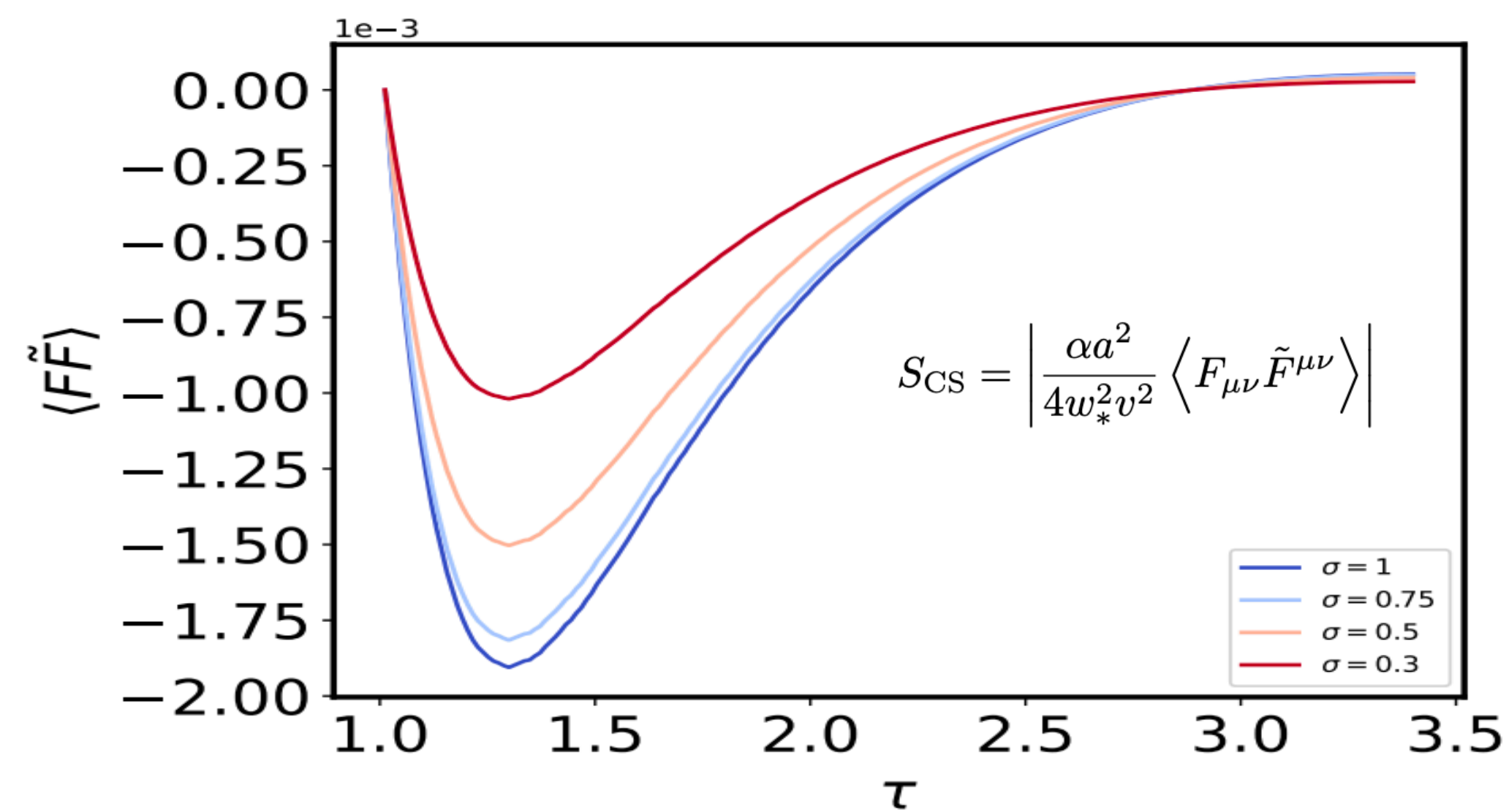
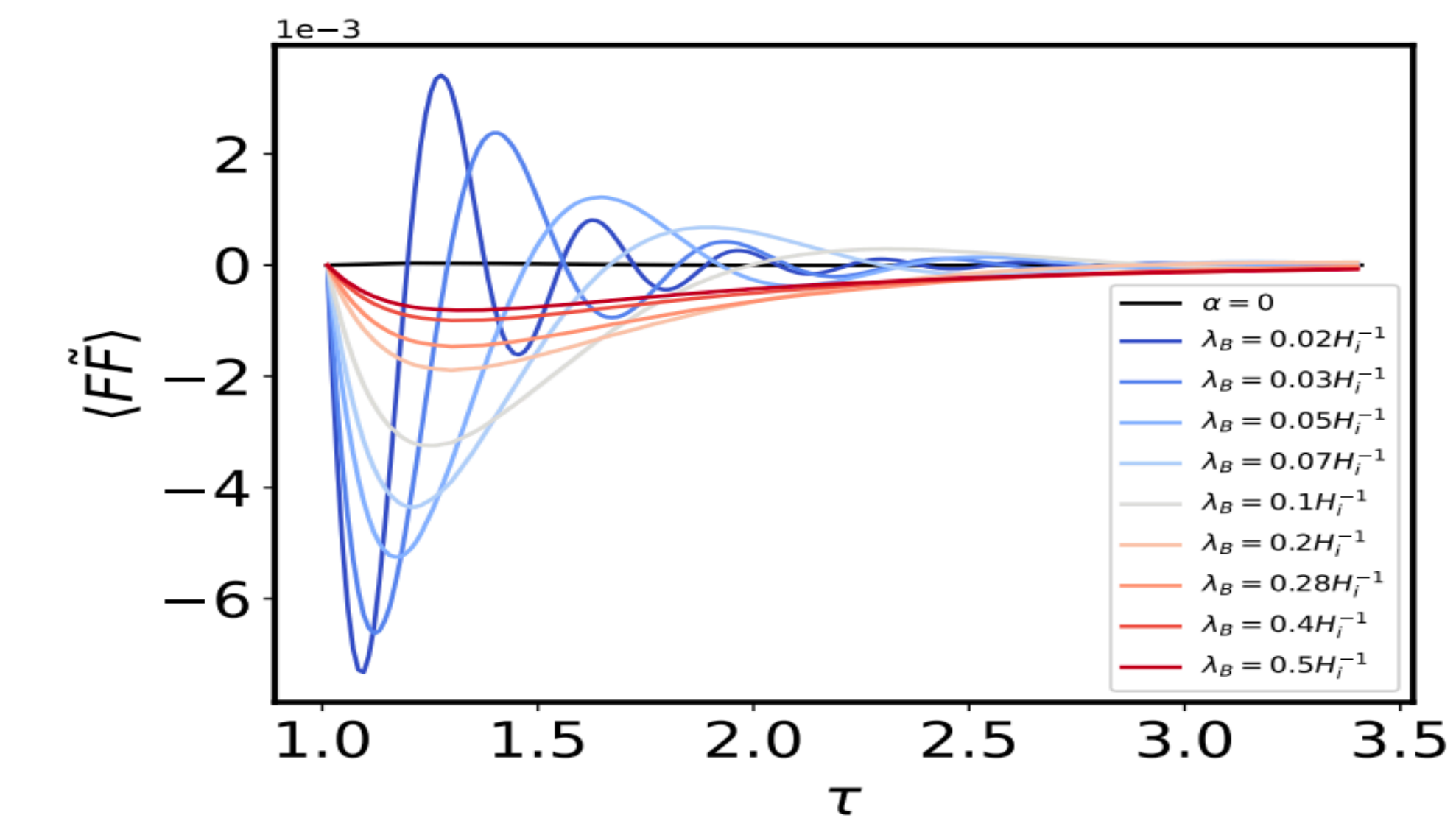


$$\lambda_B = 0.02 H^{-1}$$

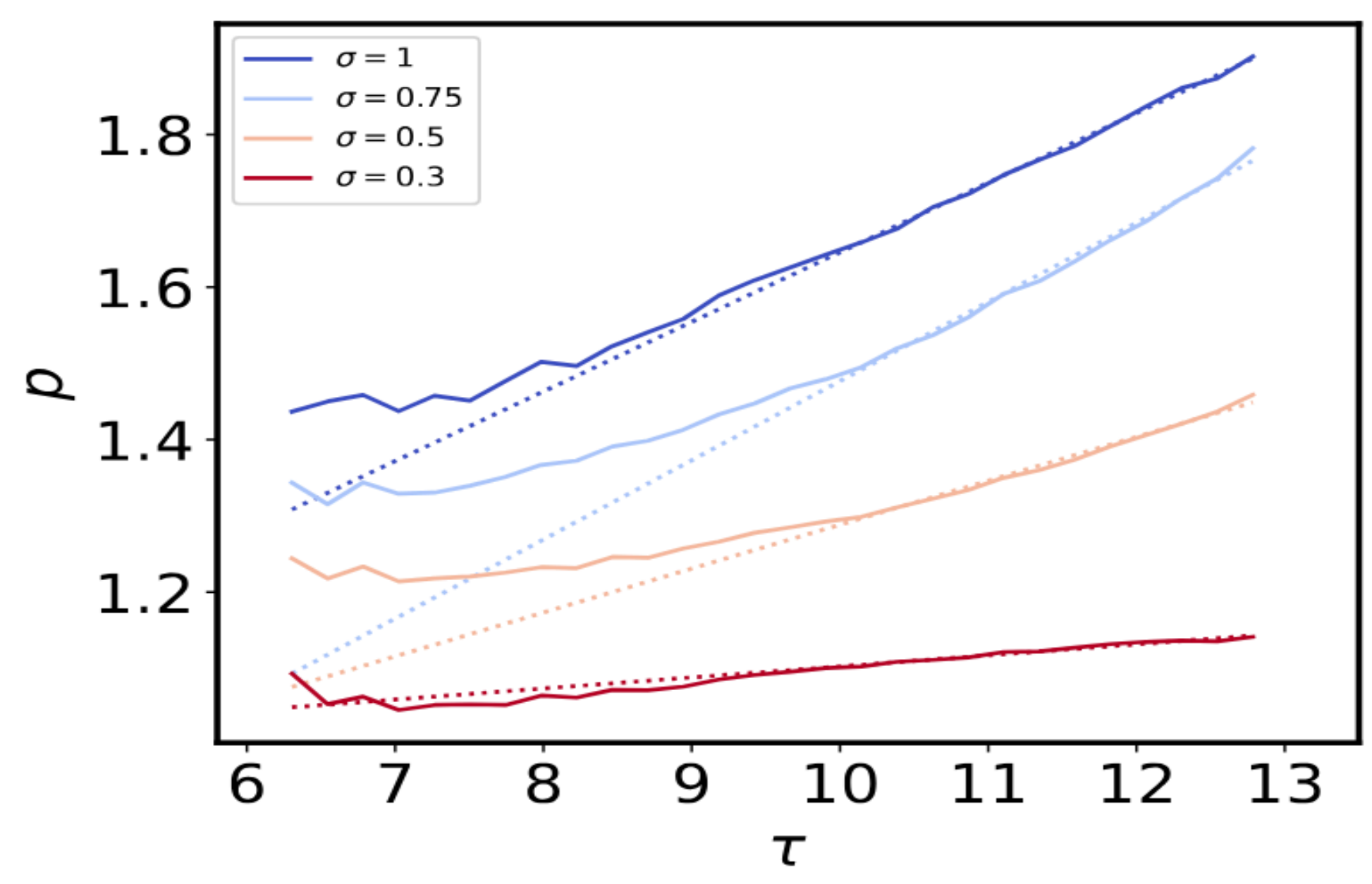
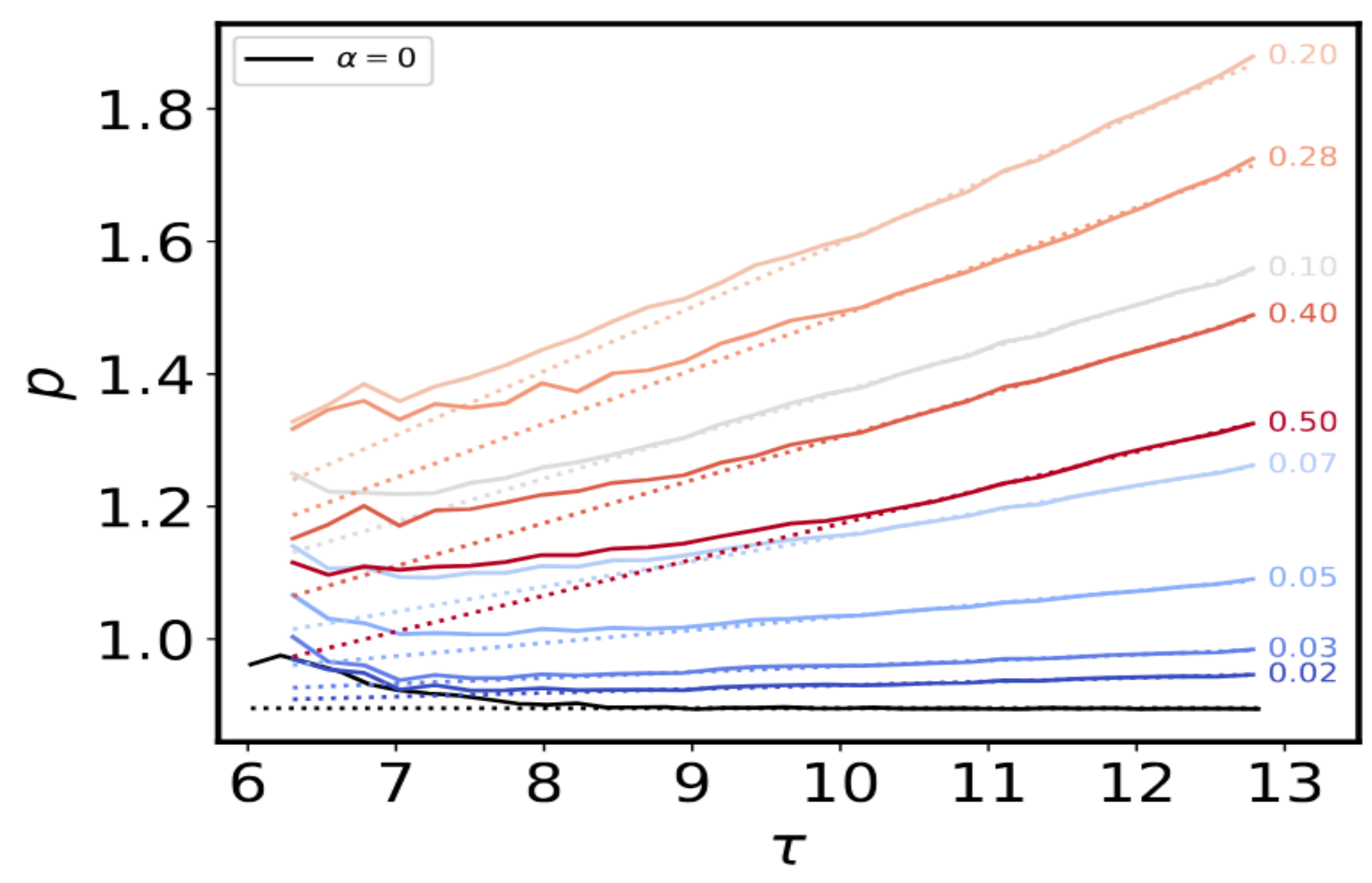
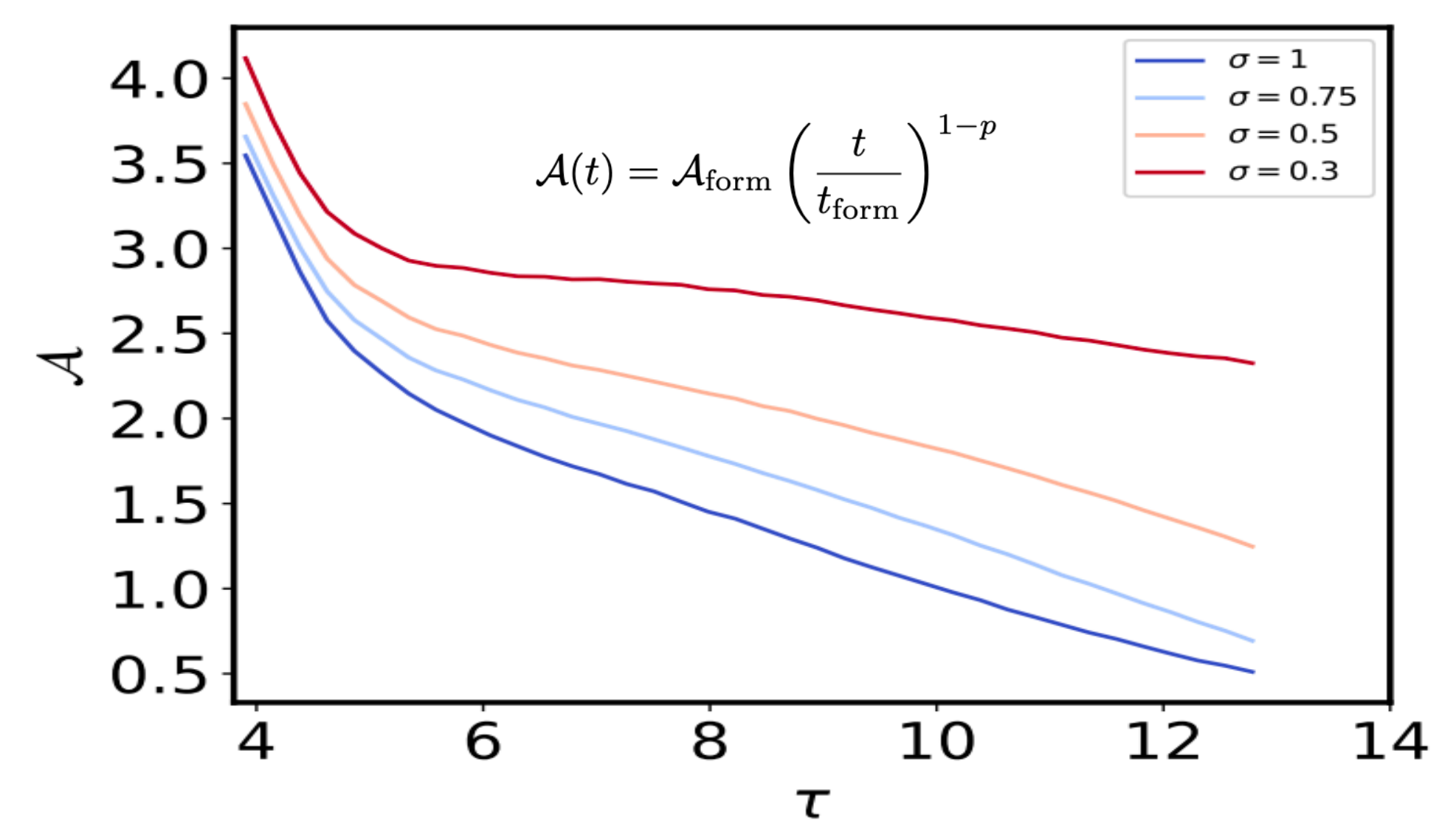
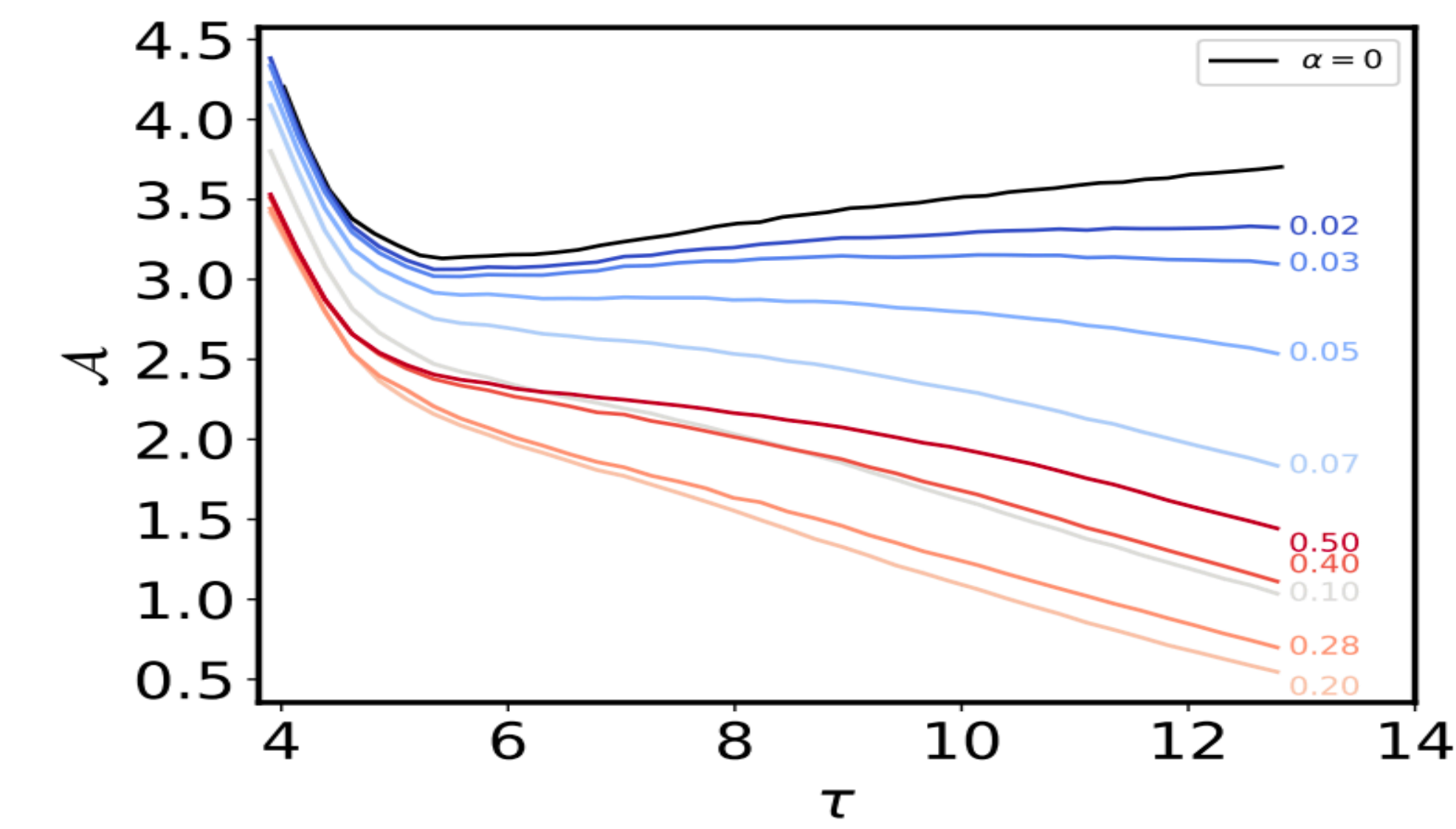


$O(1)$
Hubble volume

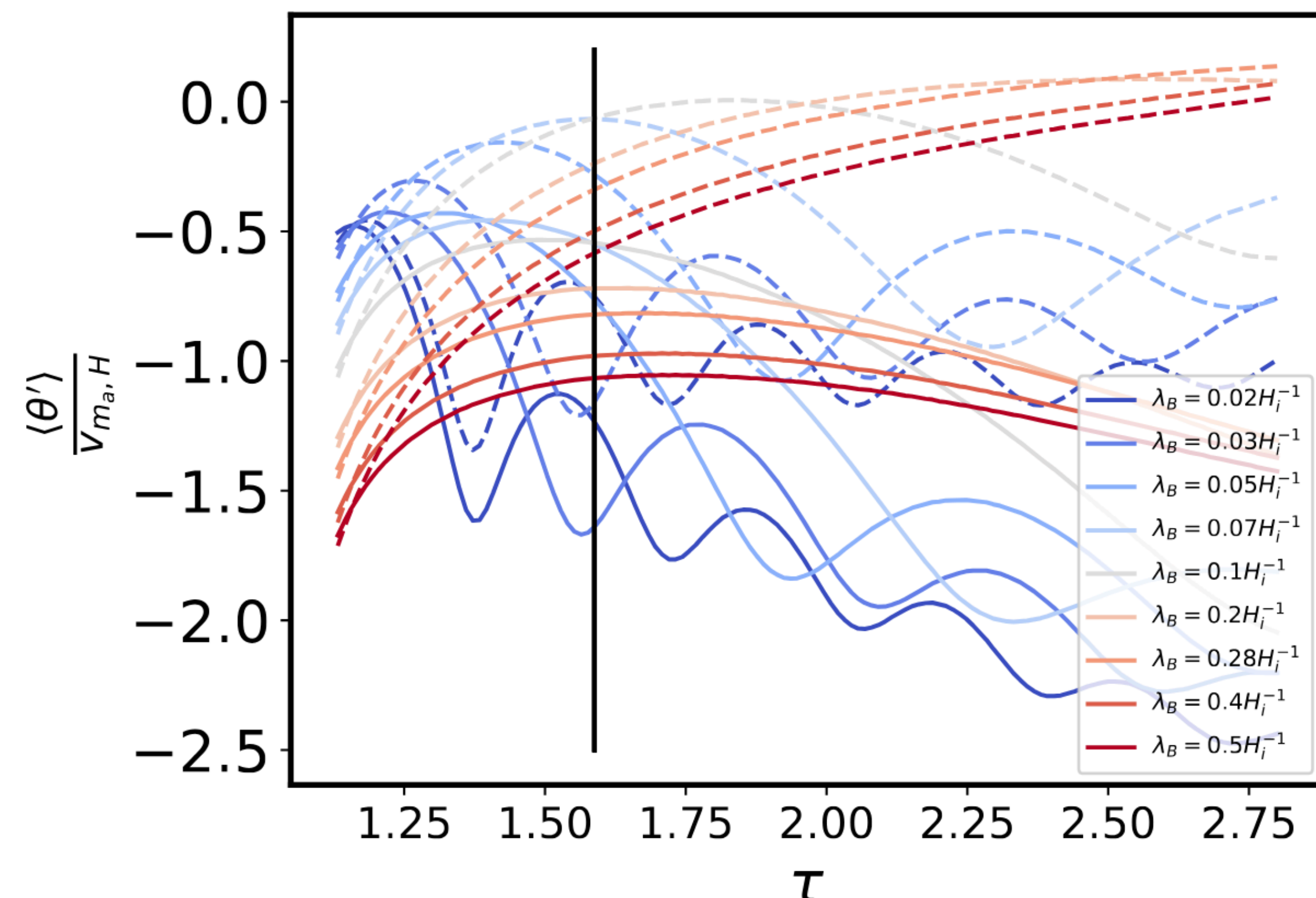
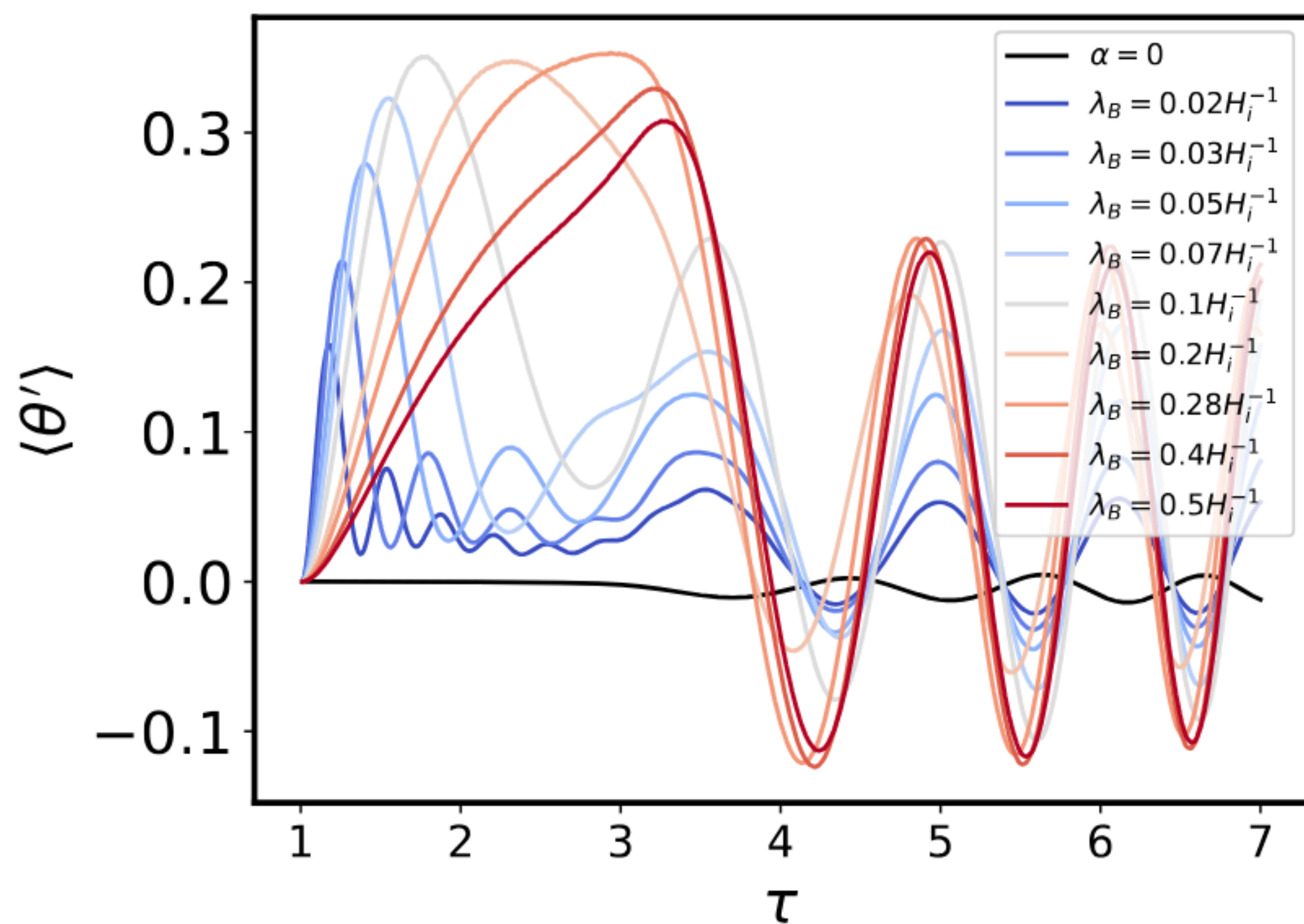
► 陈-西蒙斯项导致畴壁坍塌



► 陈-西蒙斯项导致畴壁坍塌



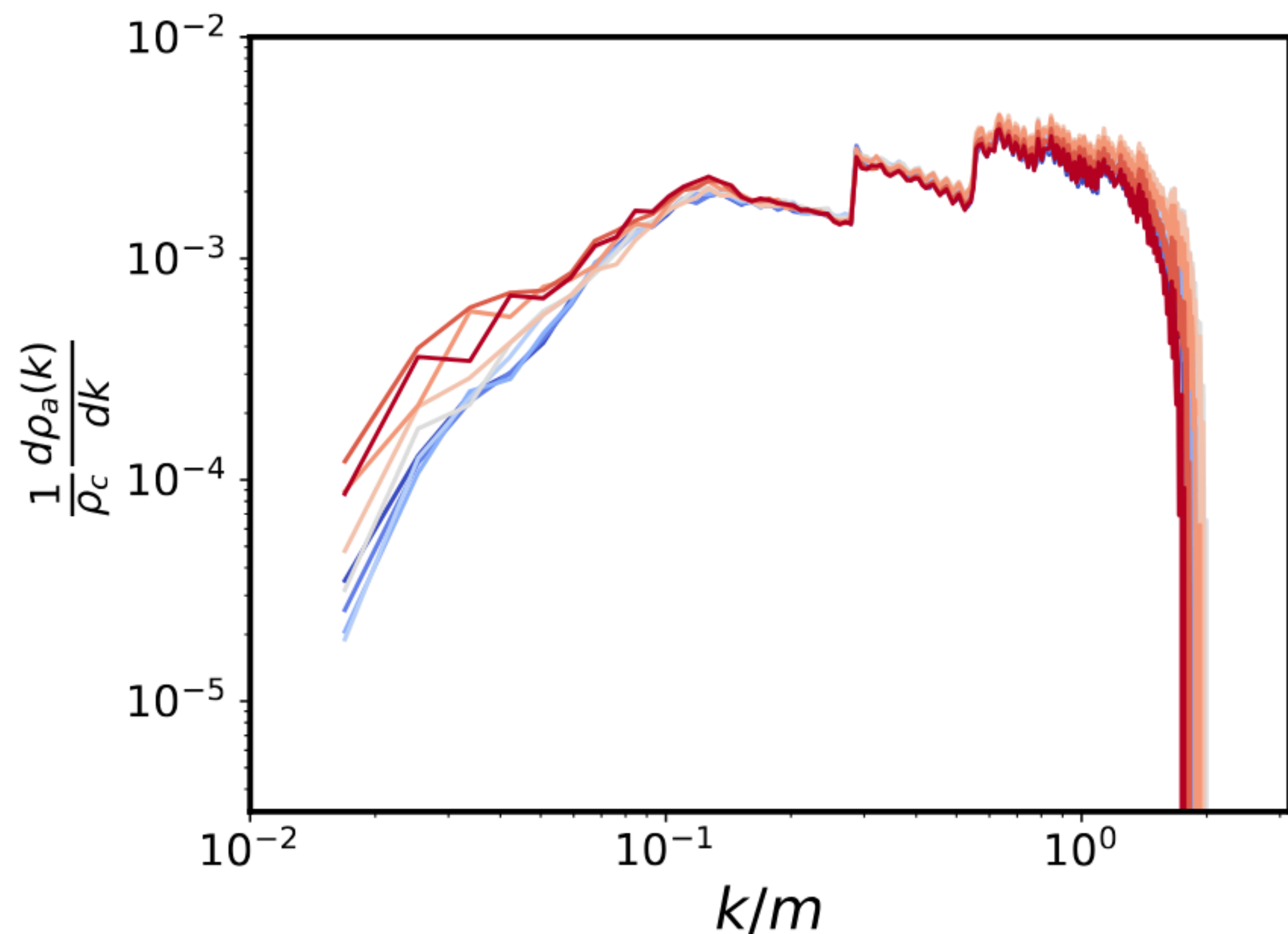
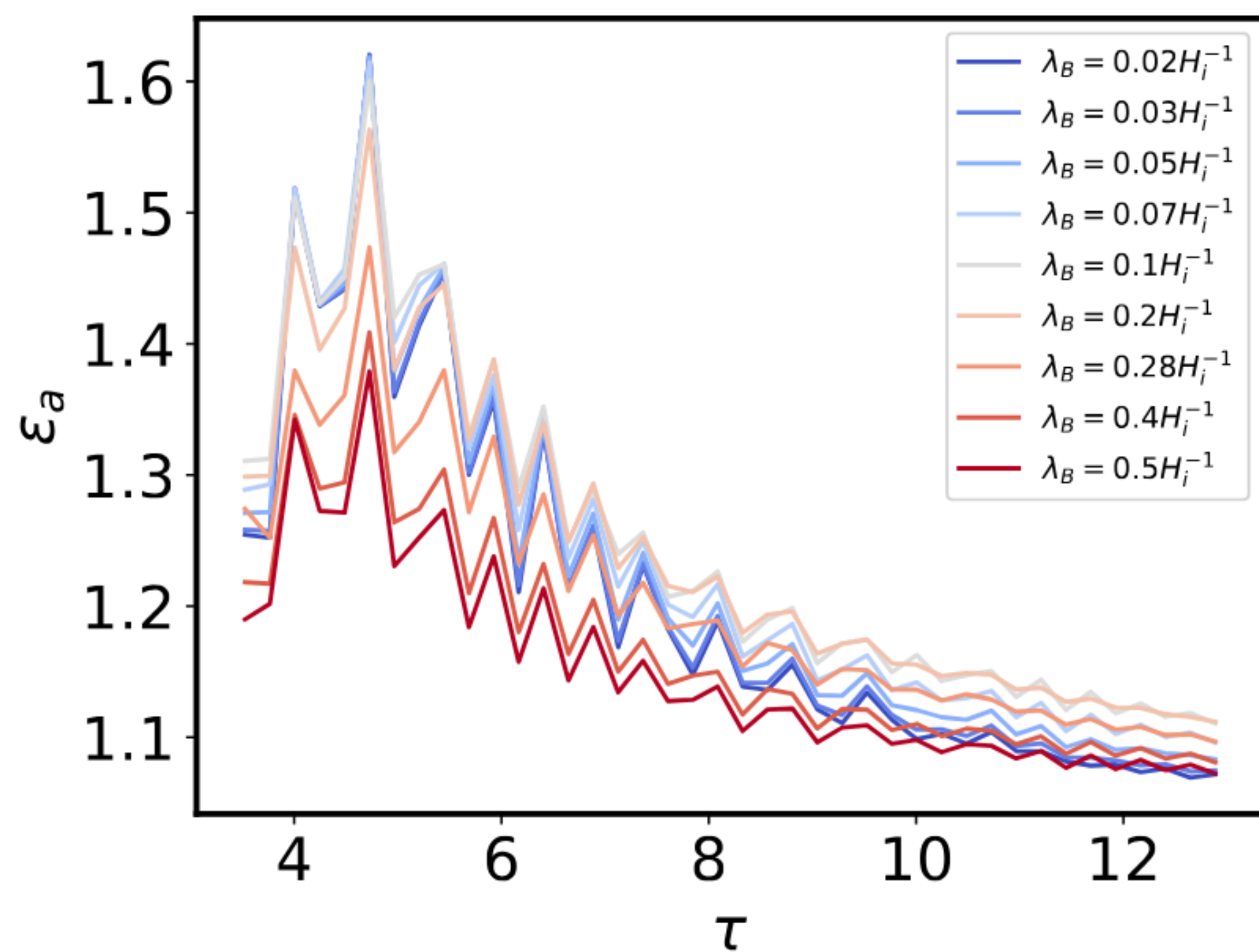
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$$\frac{v_{\text{CS,H}}}{v_H} = \frac{S_{\text{CS}} \Delta \tilde{\tau}_H}{(aH/w_*)\theta} \simeq 21.4, \quad \frac{v_{\text{CS,m}}}{v_m} = \frac{S_{\text{CS}} \Delta \tilde{\tau}_m}{(am_a/w_*)\theta} \simeq 2.4$$

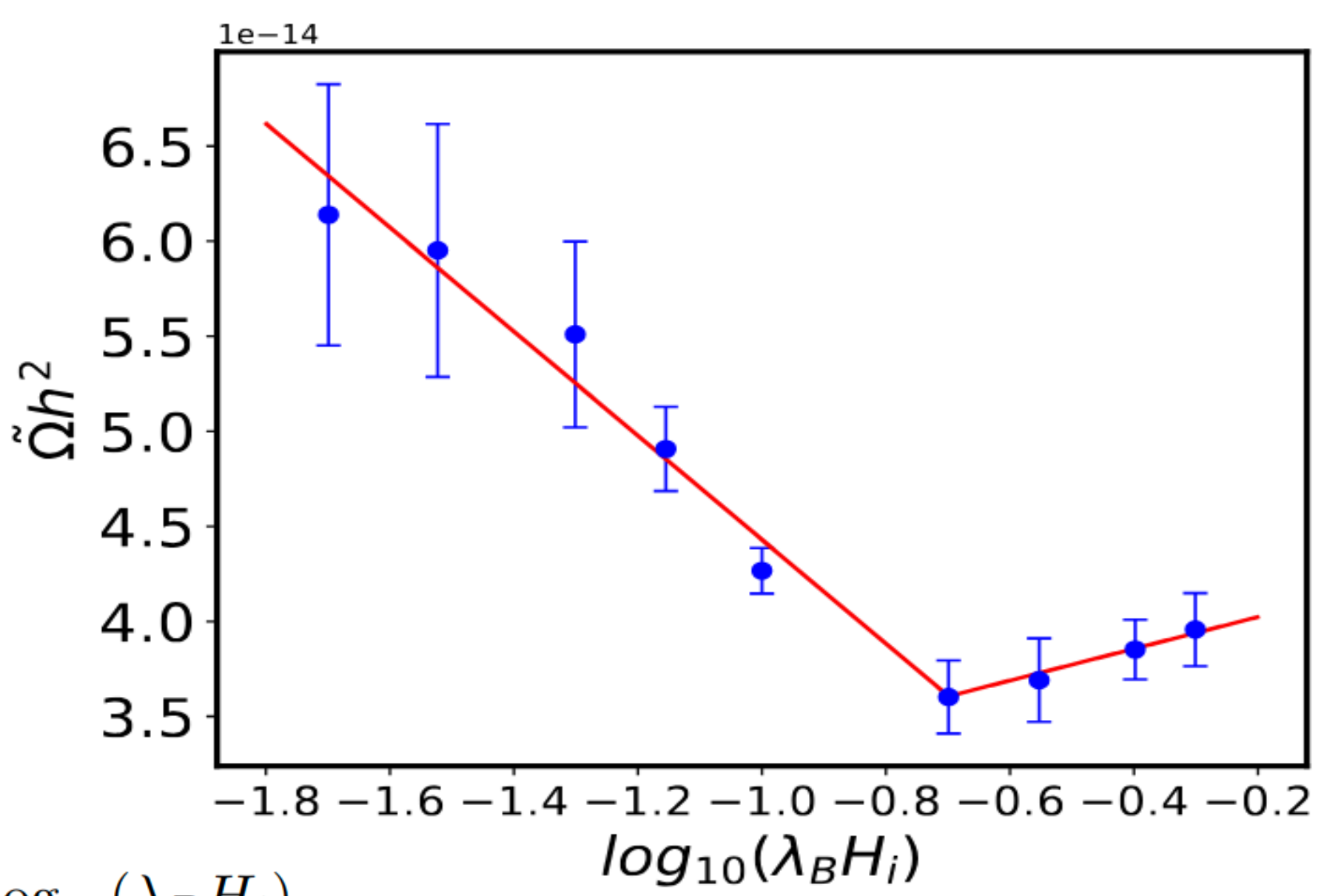
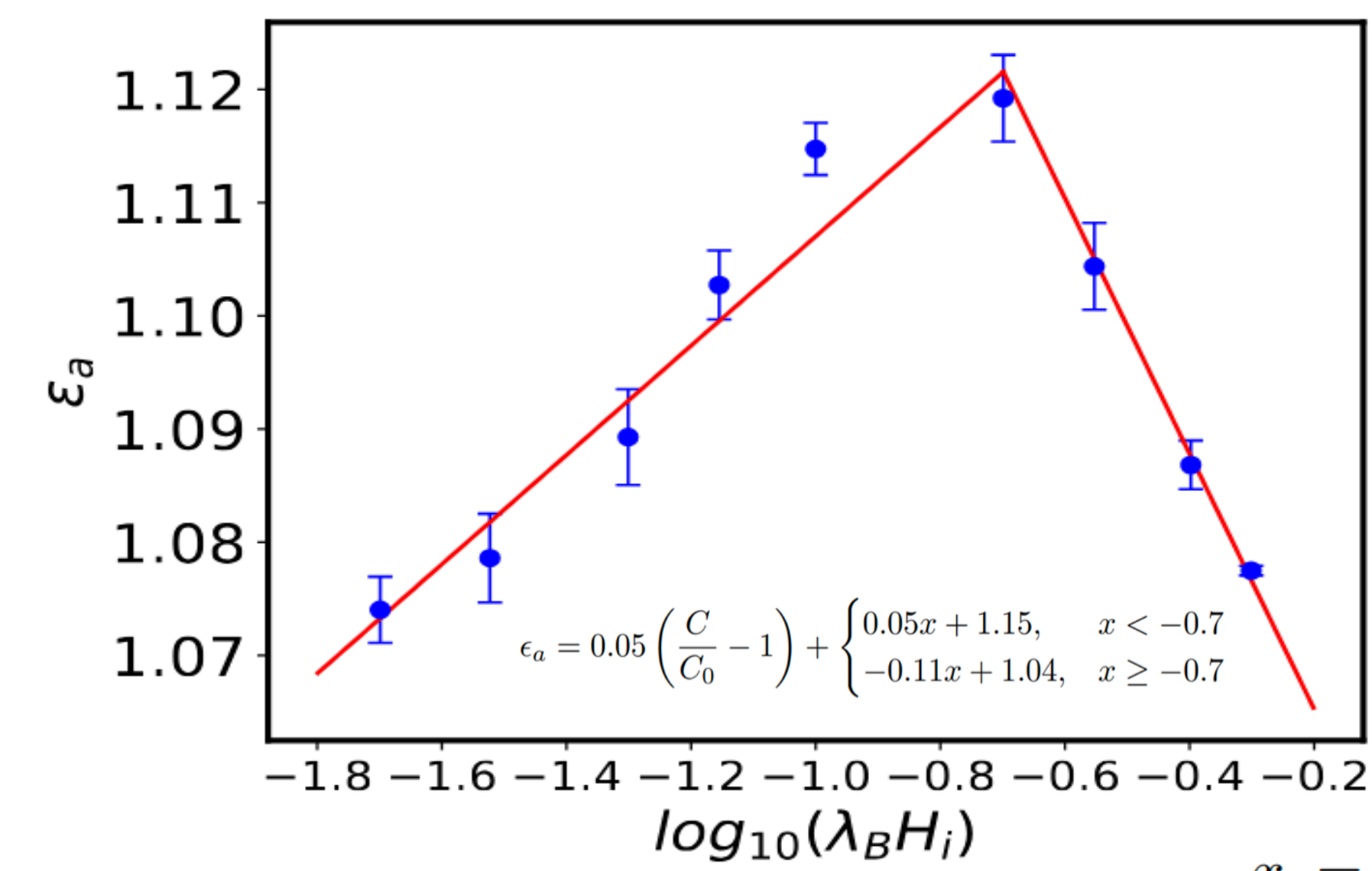
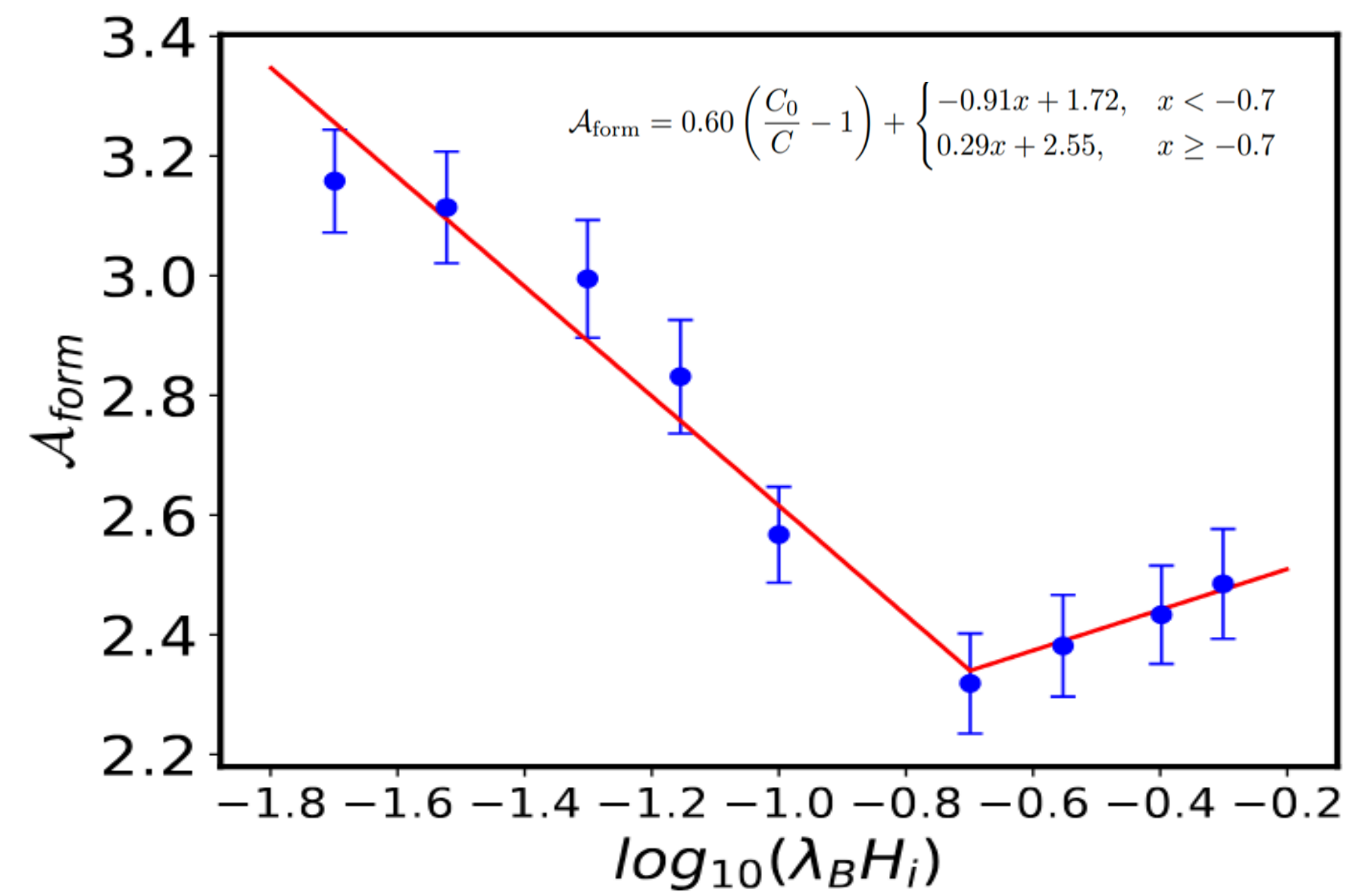
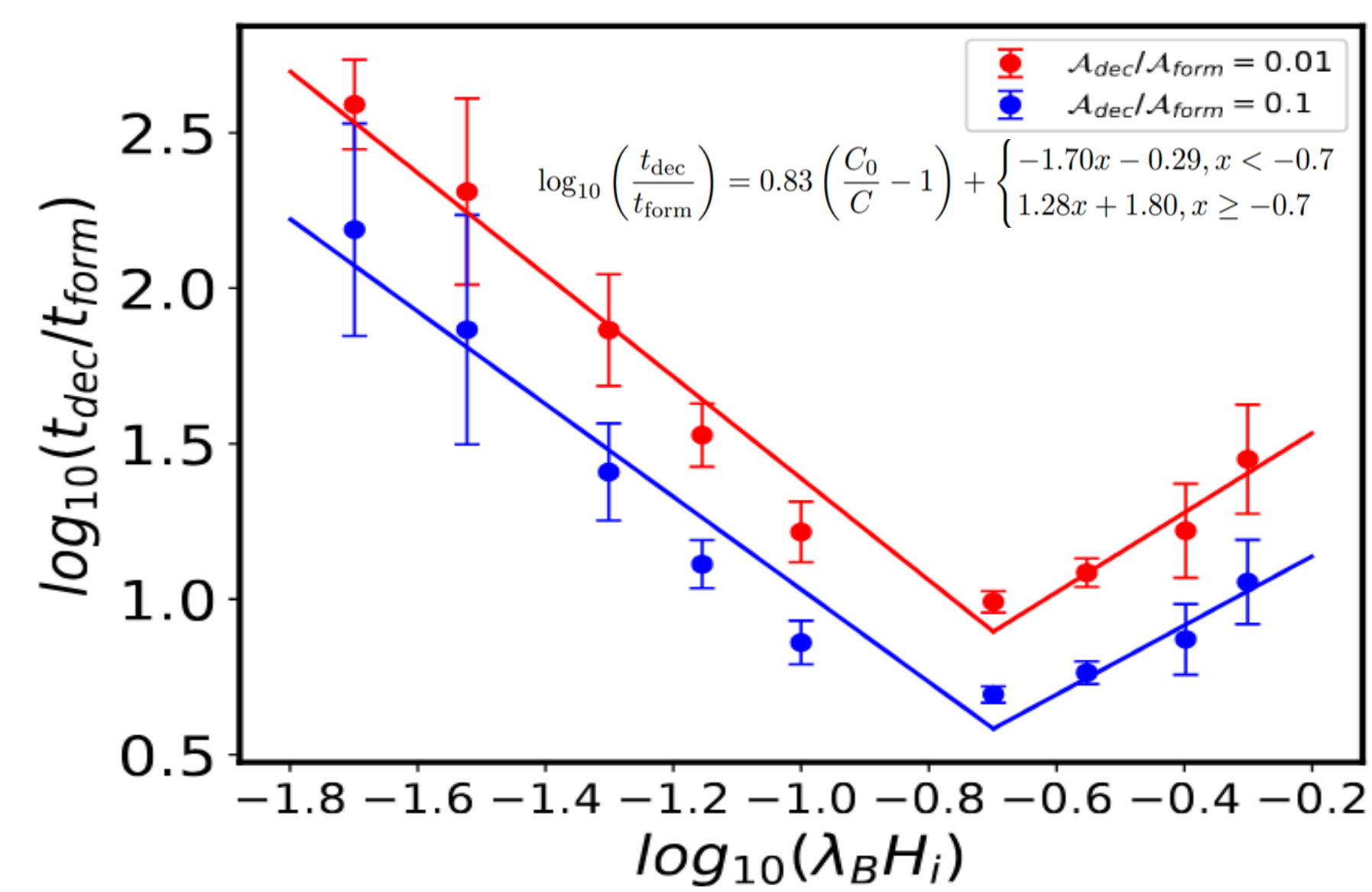
$$S_{\text{CS}} = \left| \frac{\alpha a^2}{4w_*^2 v^2} \langle F_{\mu\nu} \tilde{F}^{\mu\nu} \rangle \right| \quad \Delta \tilde{\tau}_H \simeq w_*/(aH) \text{ and } \Delta \tilde{\tau}_m \simeq w_*/(am_a)$$

► 陈-西蒙斯项导致畴壁坍塌



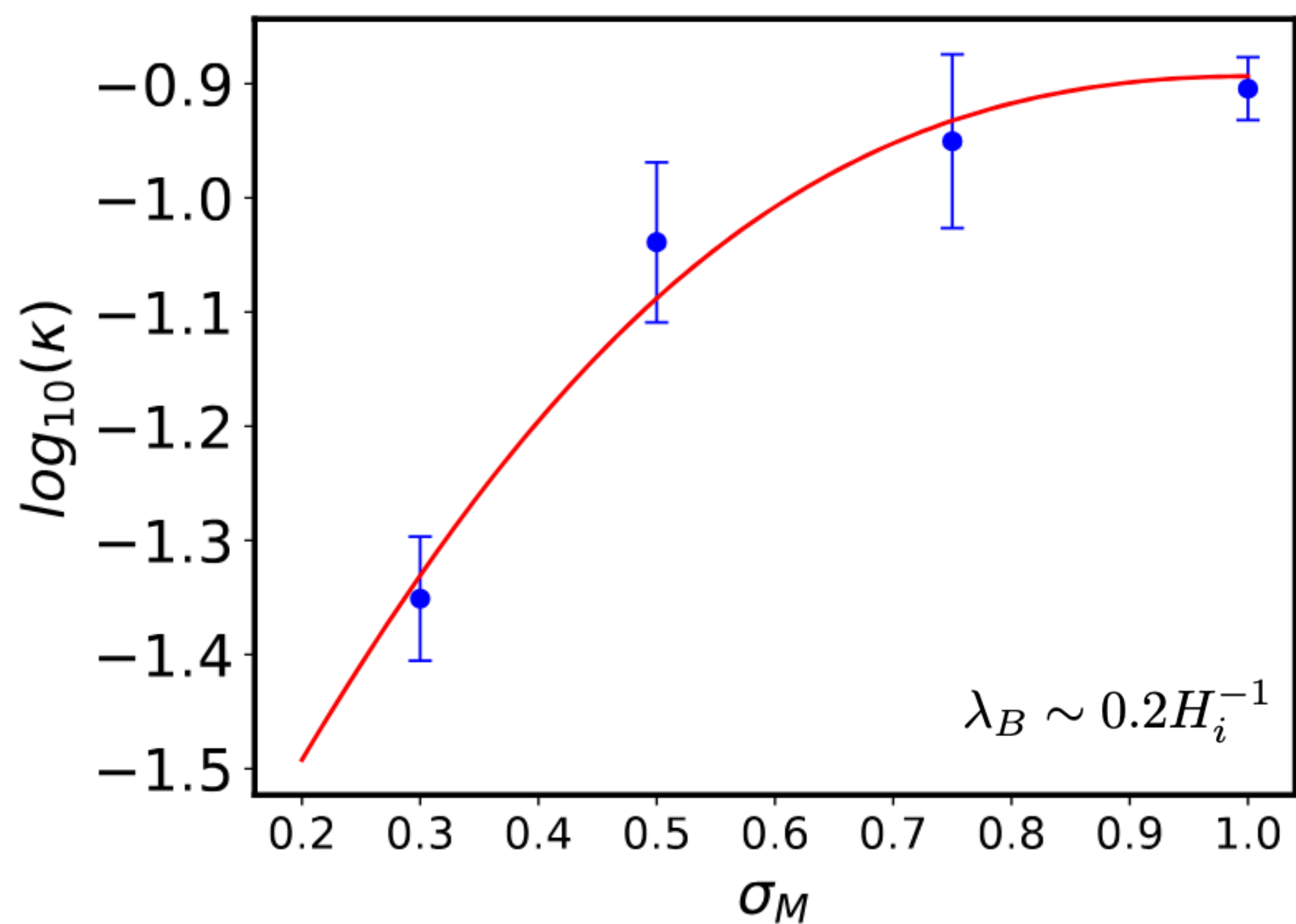
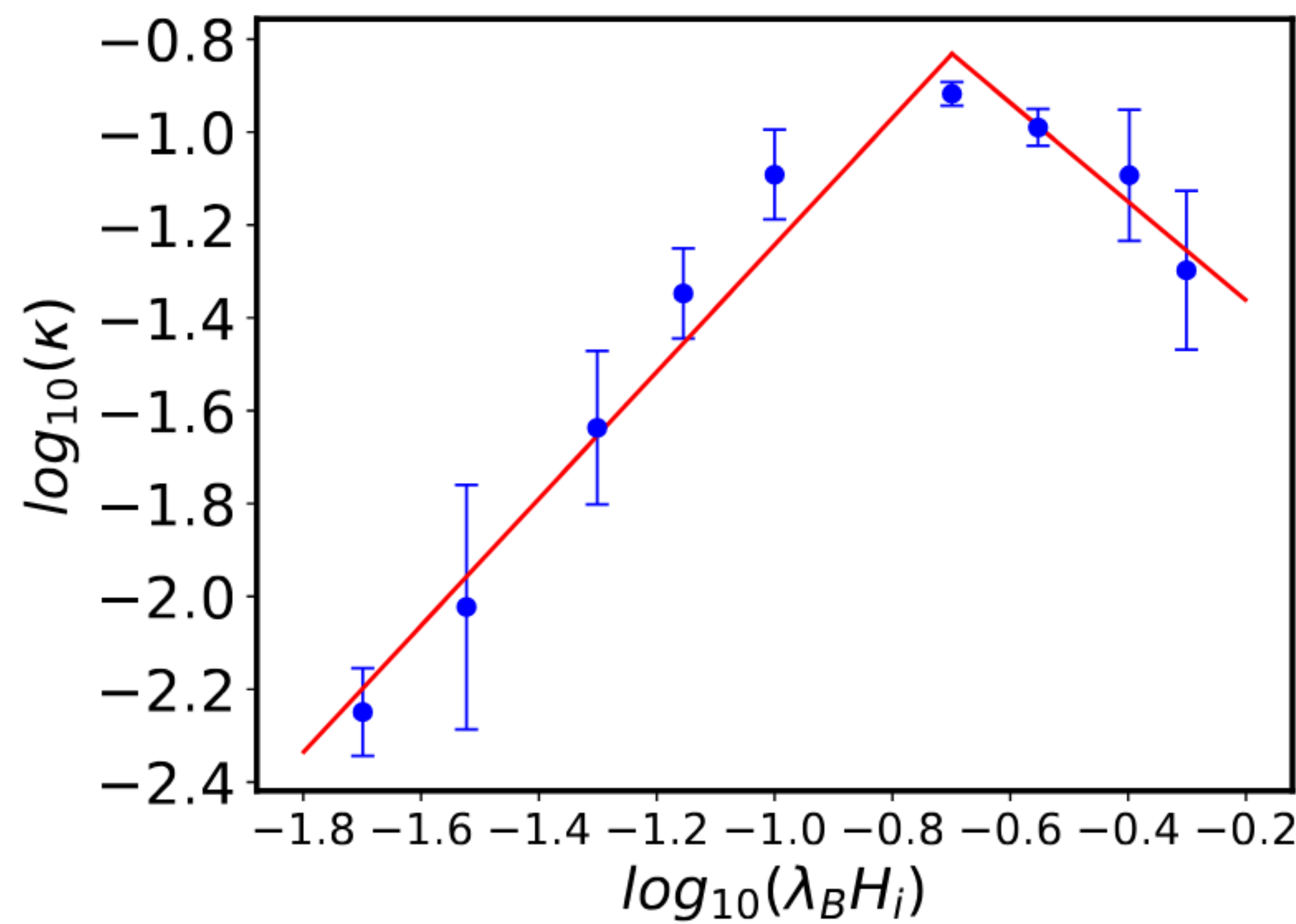
$$\frac{\Omega_a}{\Omega_{DM}} \sim 0.085 \frac{\kappa}{\epsilon_a} \mathcal{A}_{\text{form}} \left(\frac{t_{\text{dec}}}{t_{\text{form}}} \right)^{3/2} \left(\frac{10.75}{g_*(T_{\text{dec}})} \right)^{1/4} \times \left(\frac{f_a}{10^{12} \text{GeV}} \right)^2 \left(\frac{m_a}{10^{-6} \text{eV}} \right)^{1/2}.$$

陈-西蒙斯项导致畴壁坍塌



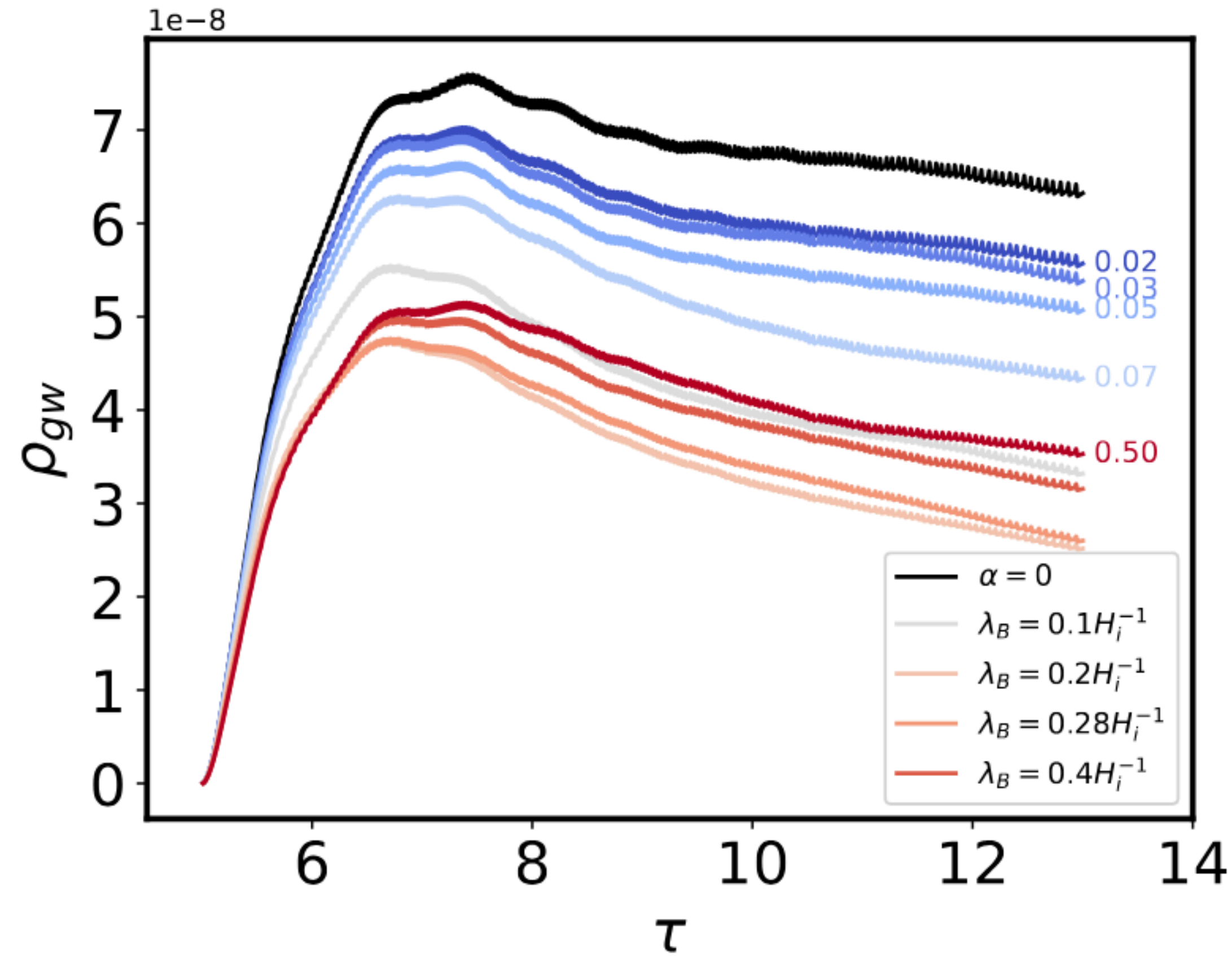
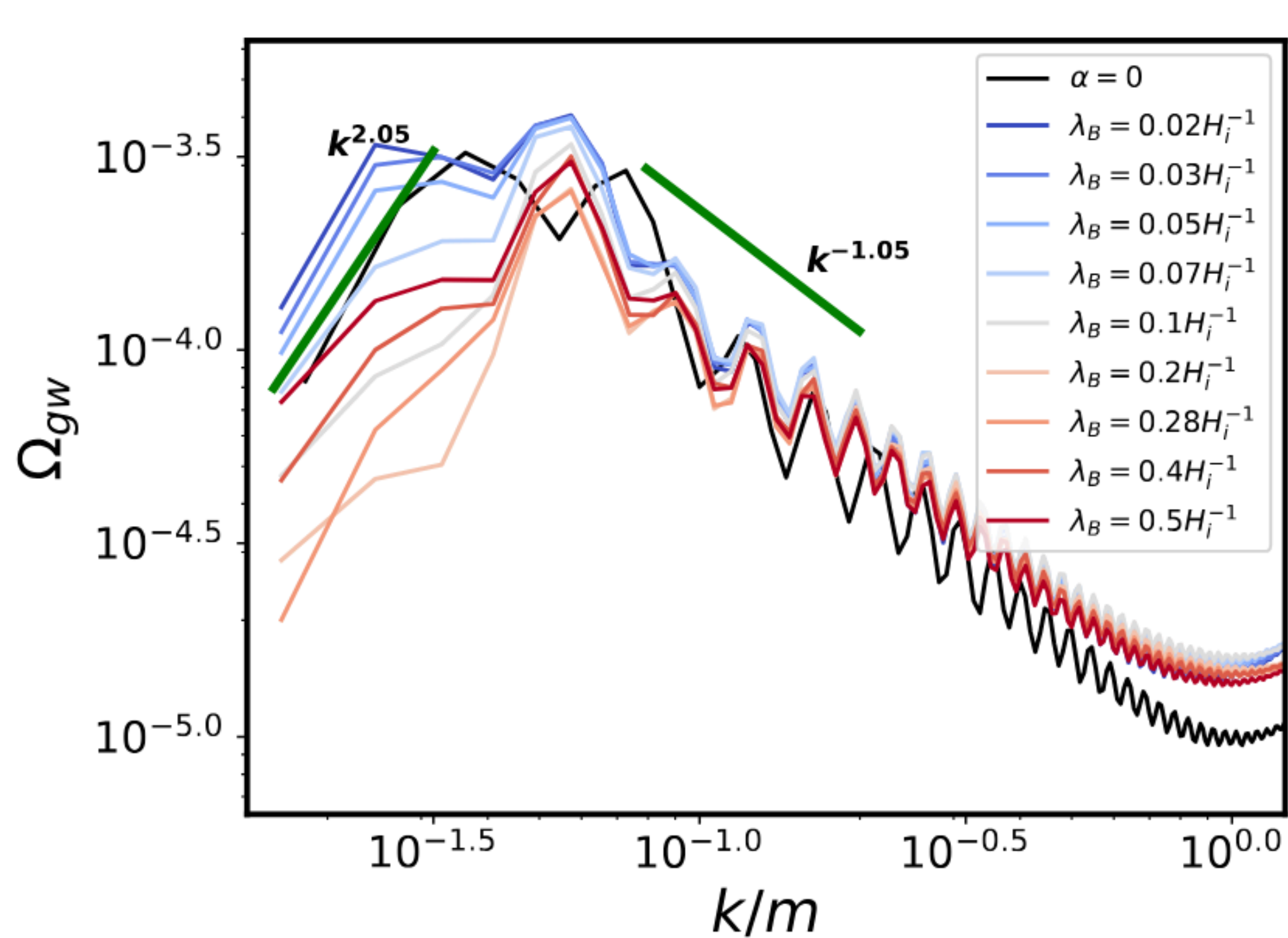
$x = \log_{10}(\lambda_B H_i)$

► 陈-西蒙斯项导致畴壁坍塌



$$\log_{10}(\kappa) = 1.26 \left(\frac{C}{C_0} - 1 \right) + \begin{cases} 1.42x + 0.16, & x < -0.7 \\ -1.06x - 1.58, & x \geq -0.7 \end{cases}$$

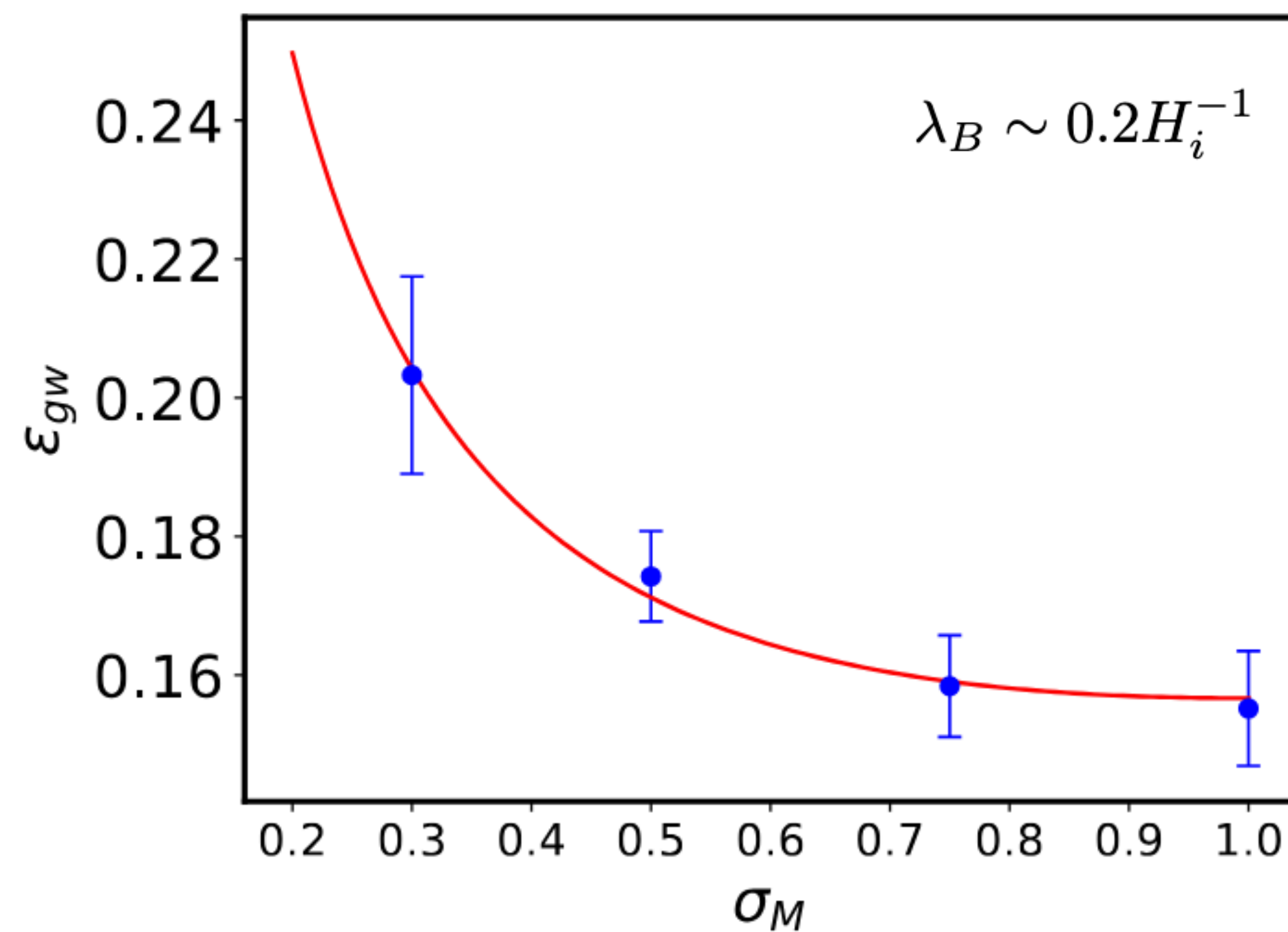
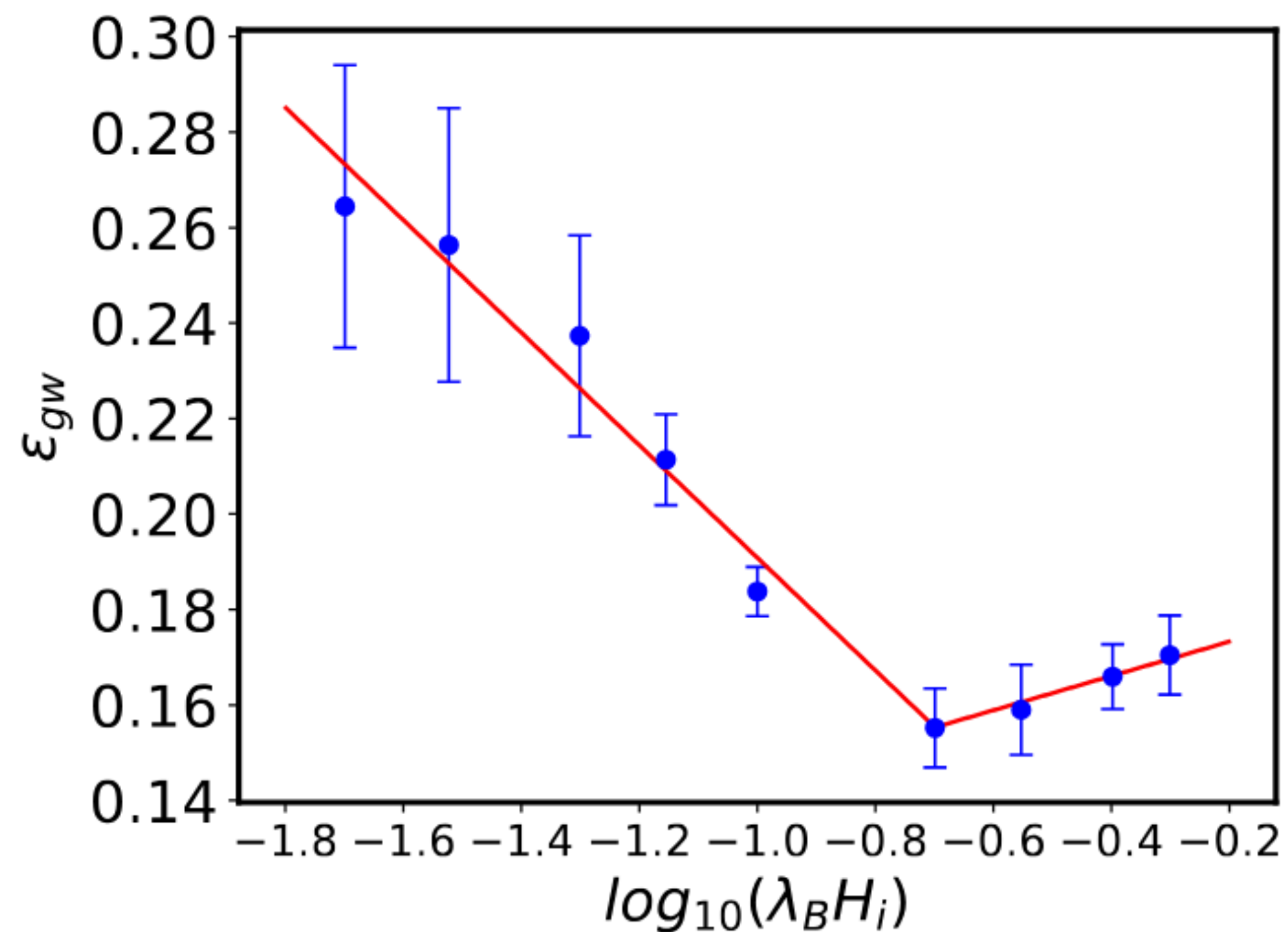
► 陈-西蒙斯项导致畴壁坍塌



$$\Omega_{gw,0}^p = \tilde{\Omega} \left(\frac{t_{\text{dec}}}{t_{\text{form}}} \right)^2 \left(\frac{f_a}{1.8 \times 10^{16} \text{GeV}} \right)^4 \left(\frac{10.75}{g_*(T_{\text{dec}})} \right)^{4/3}$$

$$\tilde{\Omega} = 10^{-14} \left(1.39 \left(\frac{C_0}{C} - 1 \right) + \begin{cases} -2.82x + 1.59, & x < -0.7 \\ 0.85x + 4.16, & x \geq -0.7 \end{cases} \right)$$

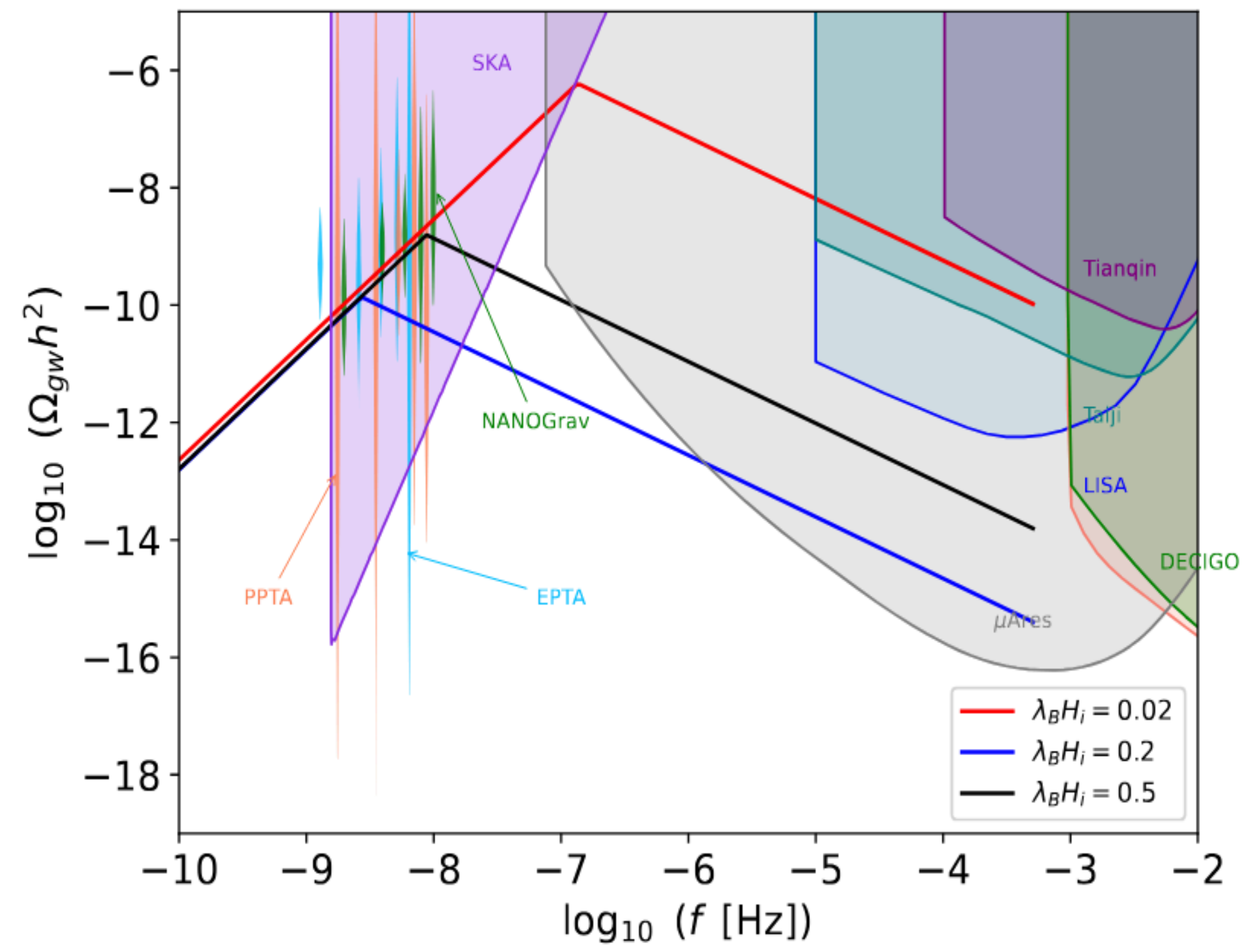
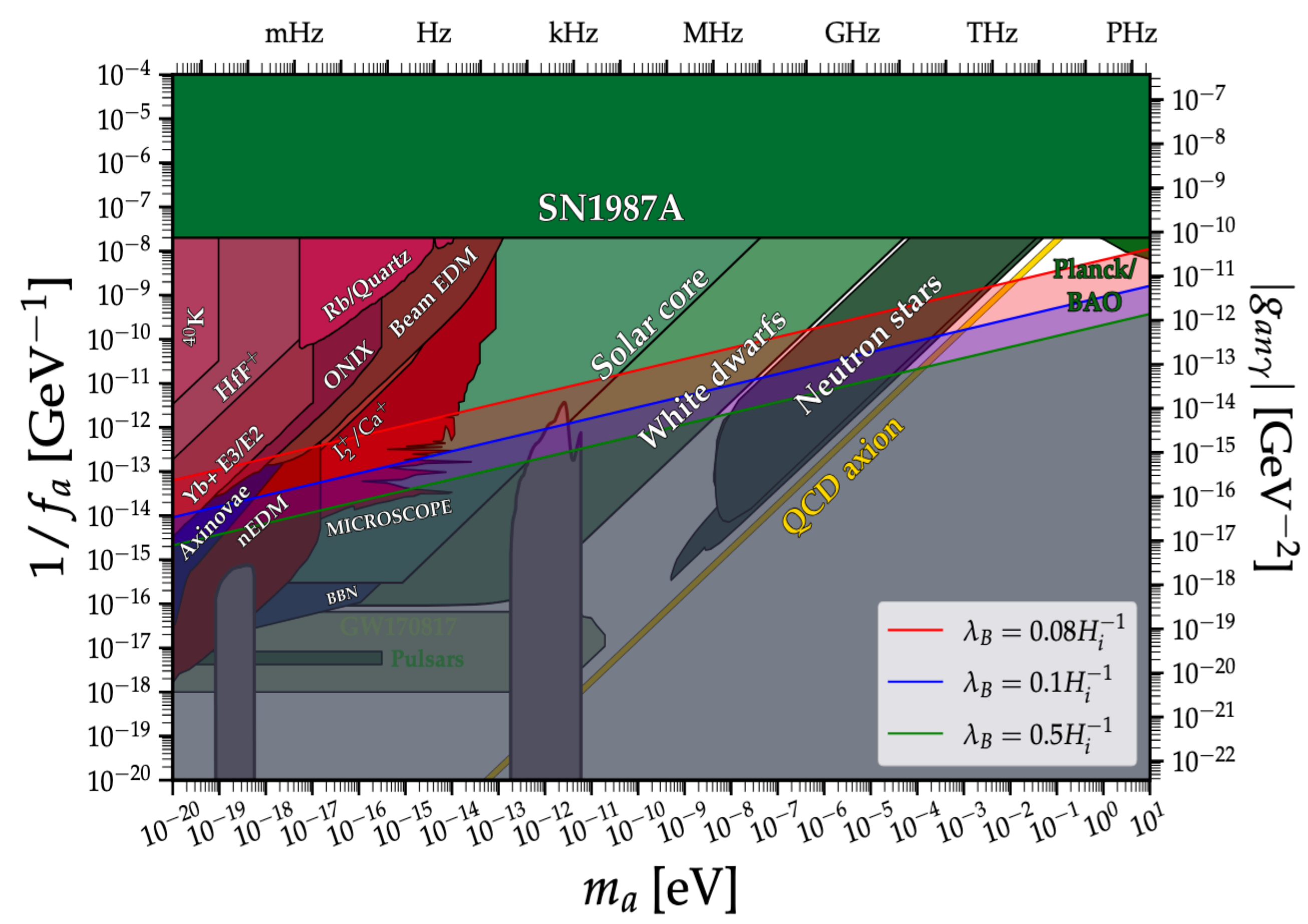
► 陈-西蒙斯项导致畴壁坍塌



$$\Omega_{gw} = \frac{\rho_{gw}}{\rho_c} = \epsilon_{gw} \frac{8\pi G^2 \sigma_{dw}^2}{3H^2} = \epsilon_{gw} \beta^2 \frac{8\pi G^2 m_a^2 f_a^4}{3H^2}$$

$$\epsilon_{gw} \equiv \rho_{gw} / (G \sigma_{dw}^2)$$

► 引力波与暗物质丰度



❖ Summary

- **GW of Axion/ALP dark matter scenario in KSVZ cannot be probed**
- **GW of ALP particles in the DFSZ case can be probed by GW detectors**
- **Primordial magnetic field leads to DW decay**

❖ Future

- **Dark matter and PT**
- **Topological defects: Magnetic monopoles, cosmic strings, domain walls**
- **Magnetic fields, baryogenesis, and Axion dynamics**

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