Simulation study of ultralight dark matter

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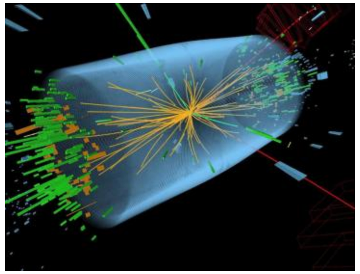
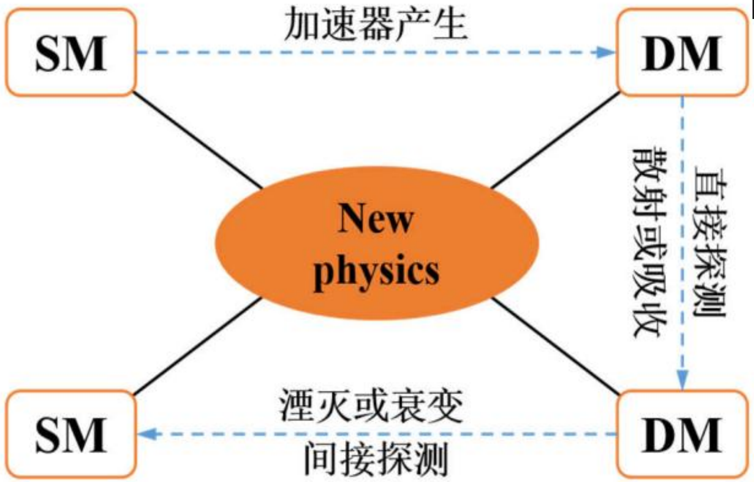
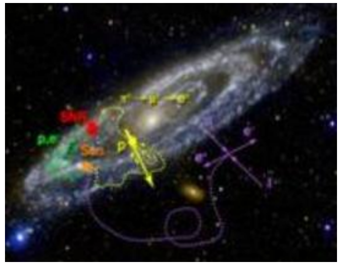
Outline

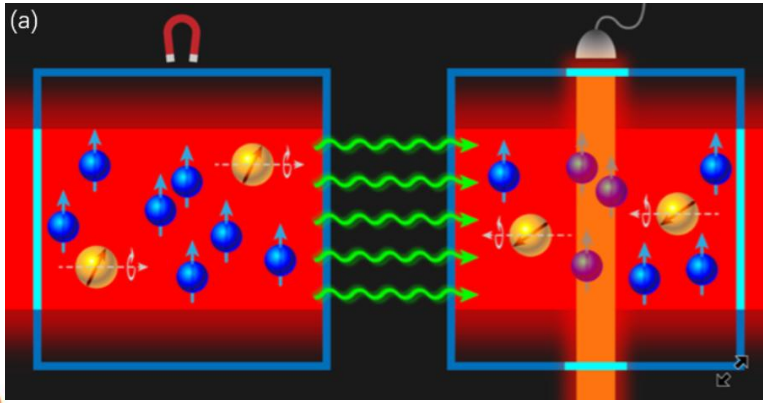
- Brief overview of ultralight dark matter searches
- Our works
  - Explanation of Nube by FDM
  - Suppression of dynamical heating by tidal effects
  - Formation of seague 1
  - Self-interaction and multifield scenarios
- summary

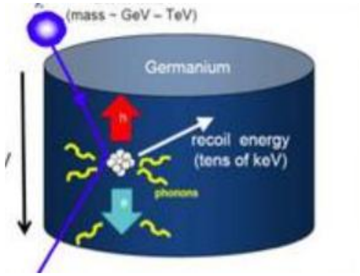
- **Methods to detect dark matter**

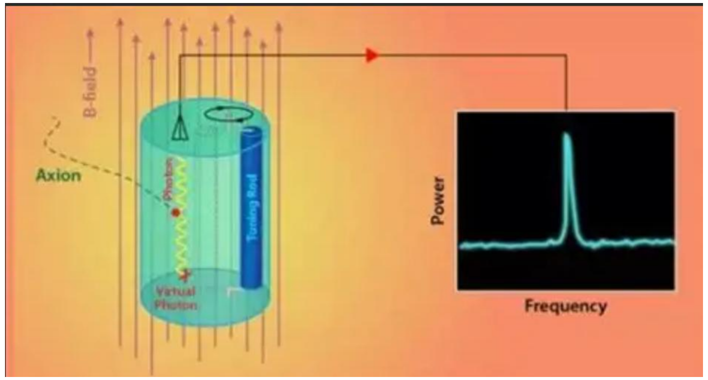


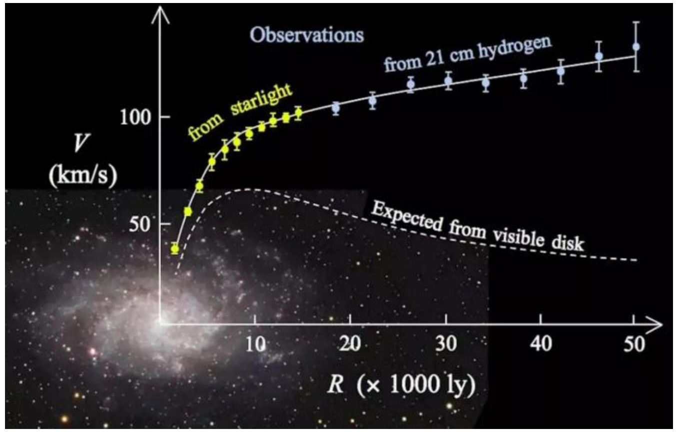








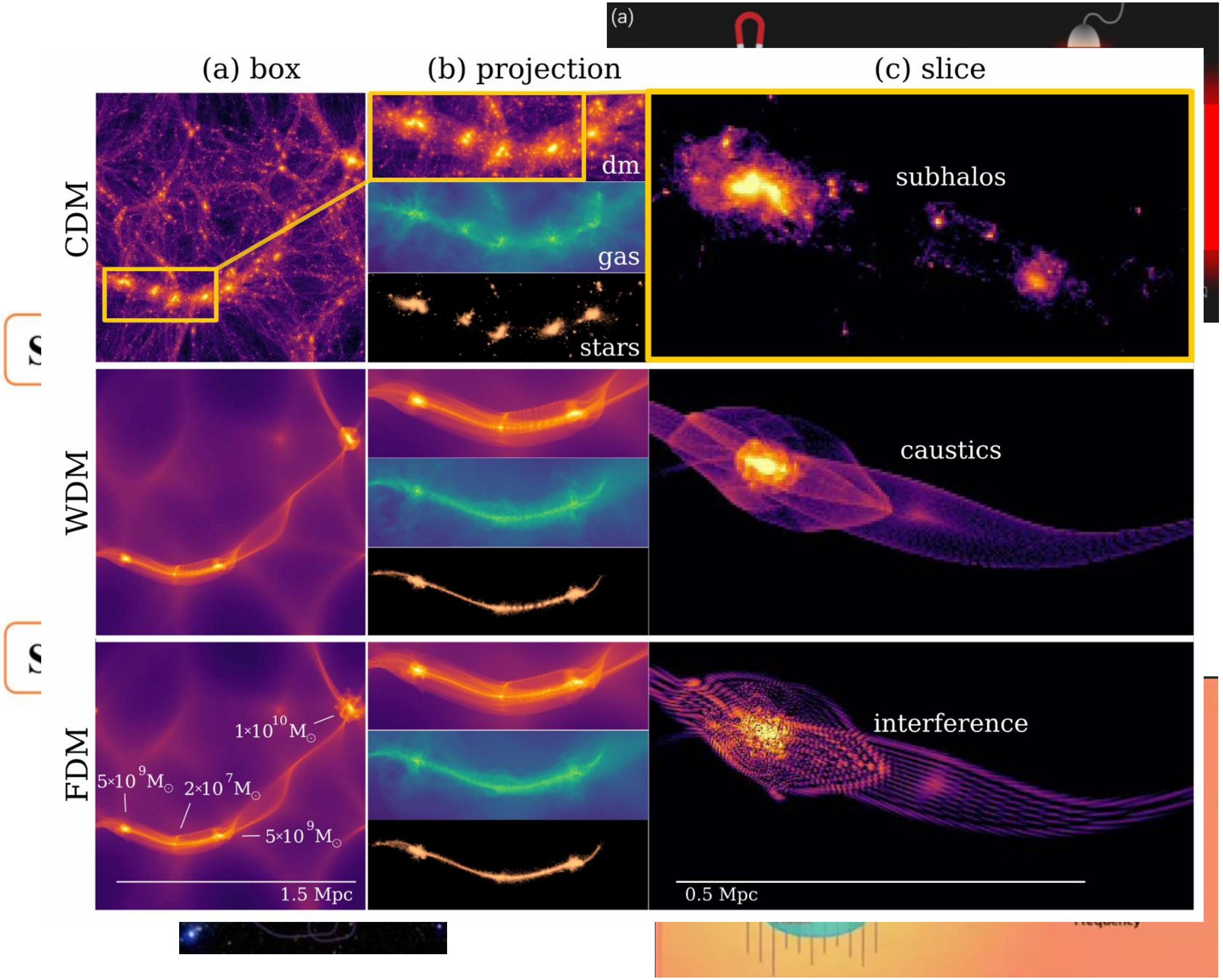


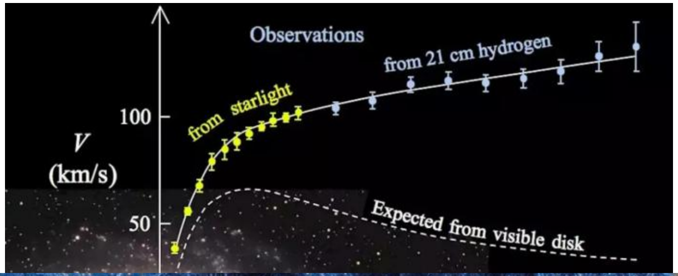


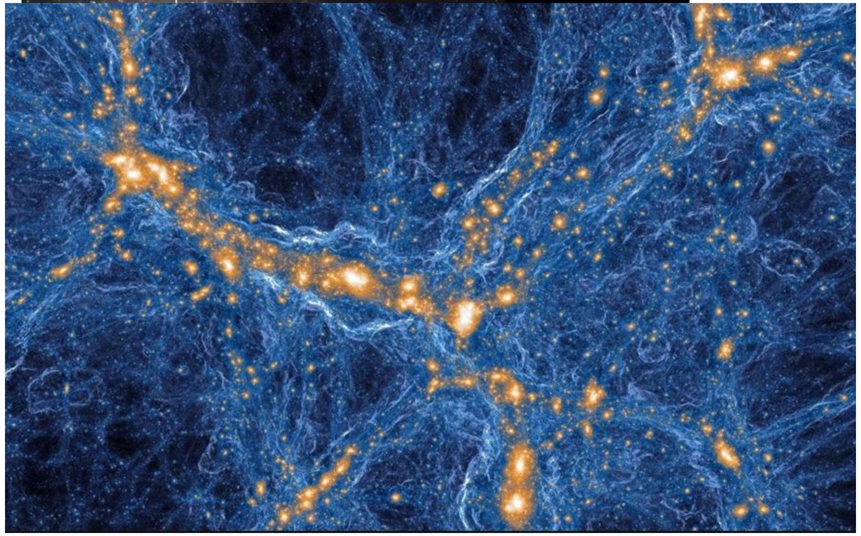
Model dependent searches: WIMP, ALPs

Model independent astronomical observations

- **Methods to detect dark matter**





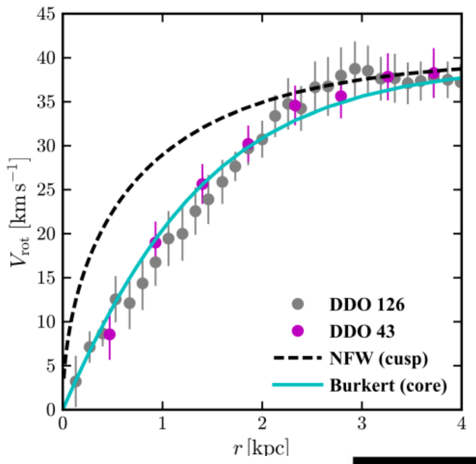


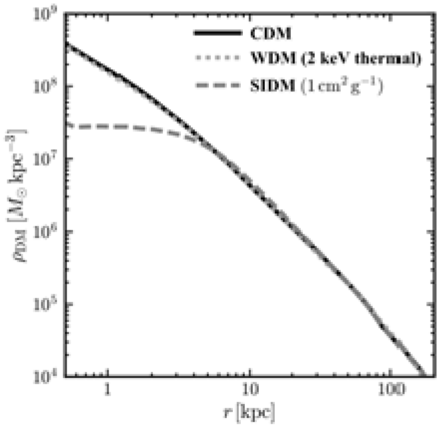
Model dependent searches: WIMP, ALPs

Model independent astronomical observations

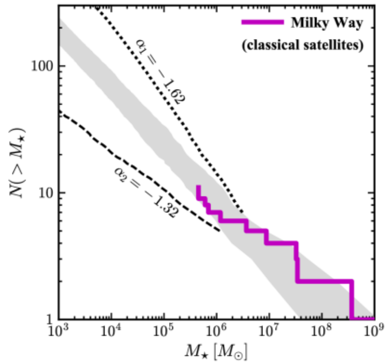
The small scale problems are possible implications on the nature of dark matter particles

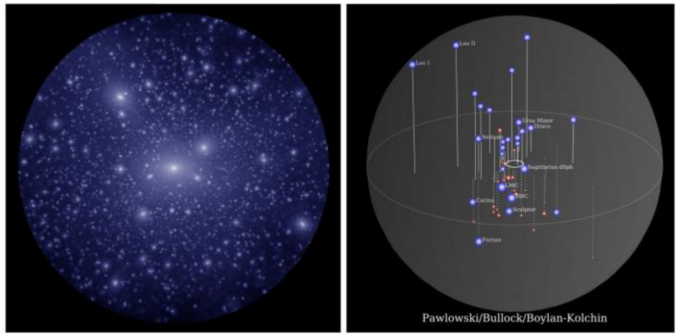
1, core-cusp: The dashed line shows the ΛCDM expectation for a typical rotation curve of a Vmax ≈ 40 km s^{-1} in dwarf galaxies. The data points show the measured rotation curves of two example galaxies of this size requiring a constant DM density core. (LITTLE THINGS survey, Oh et al. 2015)



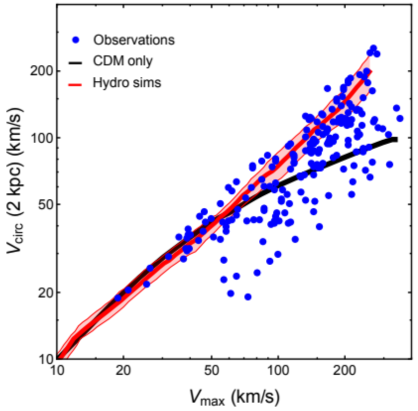


2, missing satellites. Figure show Simulated and observed dwarf galaxies. Deep survey '*seems*' solve the problems. (Garrison-Kimmel et al. 2017)

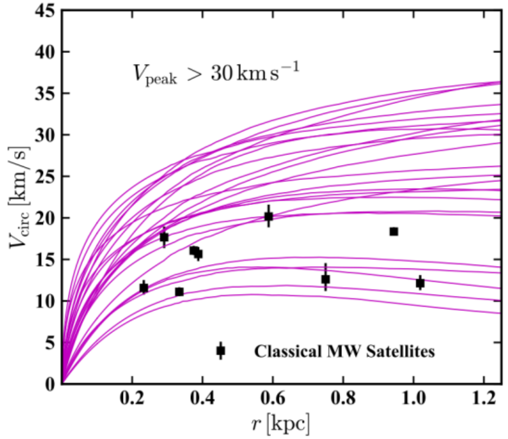




3, diversity problem. The observation of rotation curves shows that the rotation velocity vs maximal velocities have very large dispersion.



4, too-big-to-fail. Simulation gives rotation curves for biggest subhalos, while only smaller dwarf galaxies are observed. No dwarf corresponds to the biggest ones. this is an independent problem to the 1st one.



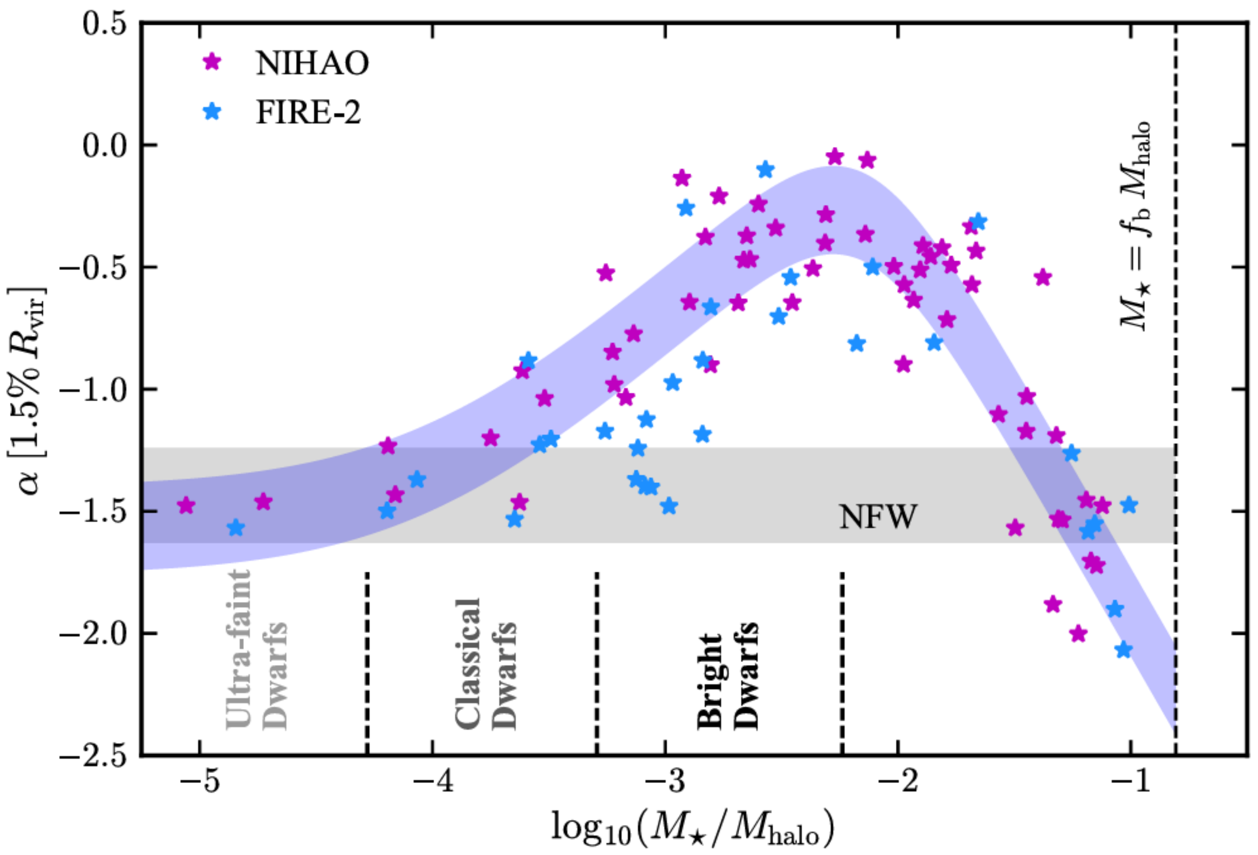
Feedback by baryonic matter

It is suggested that the supernova explosion at the halo center and injection of matter to outside makes gravity potential shallower and generate a flat halo center profile

$\rho \propto r^{\alpha}$, $M_*/M_{\rm halo}$ ~ α relation

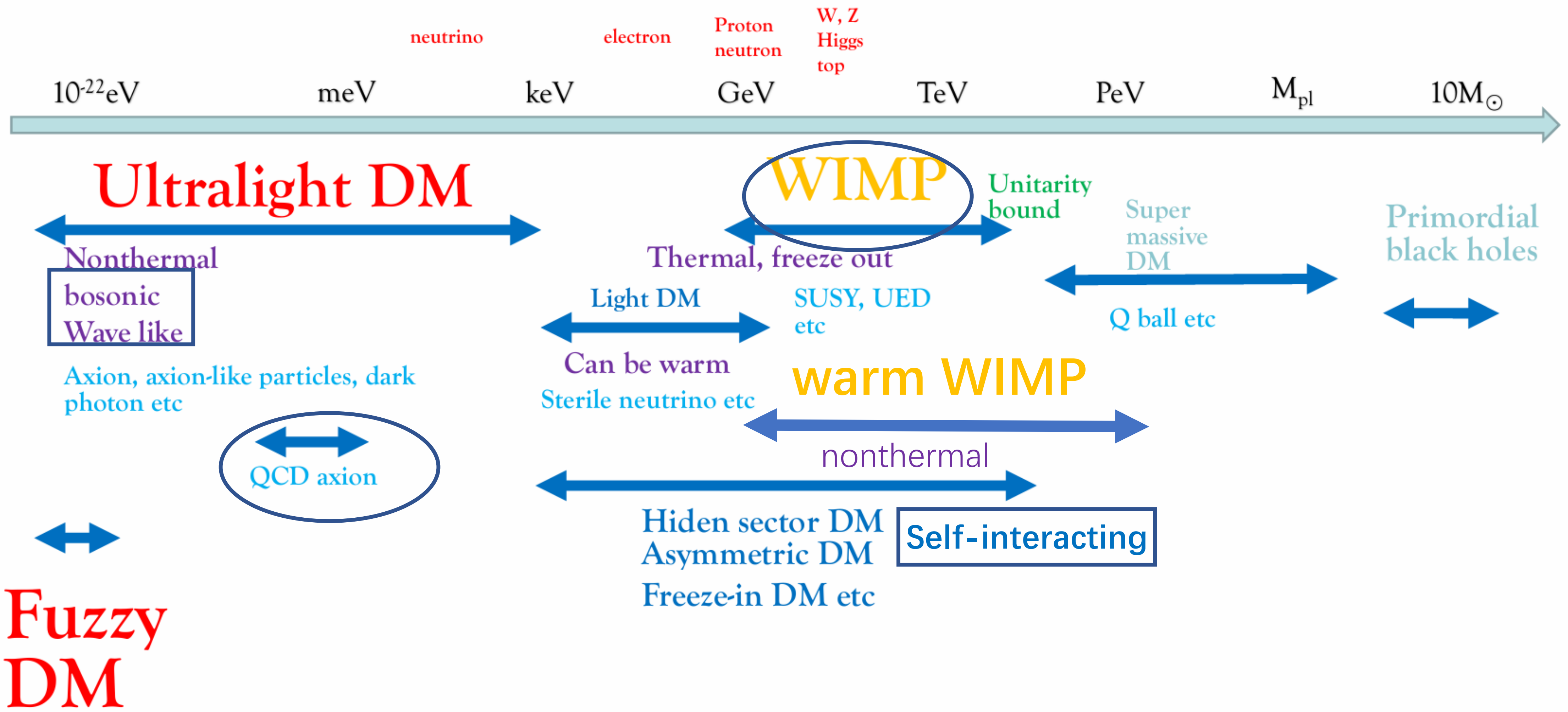
- Low $M_*/M_{\rm halo}$, feedback not enough
- High $M_*/M_{\rm halo}$, deep baryon potential generate halo center
- $M_*/M_{\rm halo} \approx 0.5\%$, the core is effective

Bullock & Boylan-Kolchin. 2017



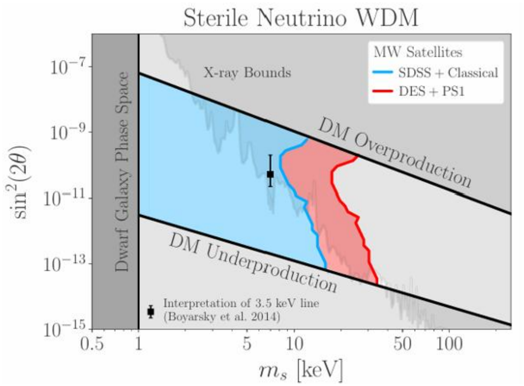
It seems hard to explain the existence of core in a wide range of $M_*/M_{\rm halo}$

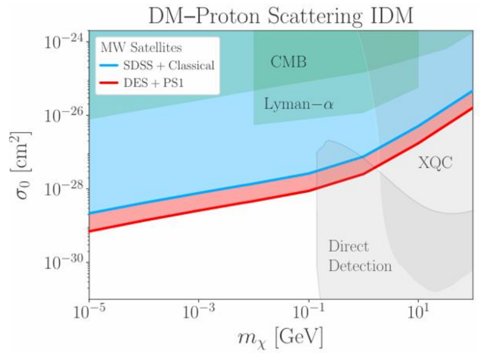
Broad spectrum of dark matter particles

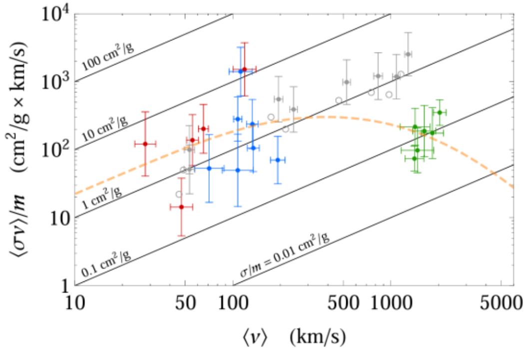


DM models for small scale problems

- Warm dark matter： DM mass at keV moves semi-relativistically and has large free streaming to suppress the small scale structure
- Interacting DM： DM-electron/proton interaction suppress small scale structure at the radiation dominant era
- Self-interaction DM: Strong self-interaction between DM particles and elastic scattering generate a core
- Fuzzy DM： DM mass at 10^{-22}eV pseudoscalar particle, structure at small scales below wave length are suppressed







Wave dark matter

- The number of dark matter particles within a de Broglie volume in the galaxy is

$$N \sim \frac{\rho \lambda^3}{m} = \frac{\rho}{m}\left(\frac{h}{mv}\right)^3 \sim \left(\frac{\rho}{0.4\mathrm{GeV/cm^3}}\right)\left(\frac{37\mathrm{eV}}{m}\right)^4\left(\frac{220\mathrm{km/s}}{v}\right)^3$$

- For m<<30eV, N>>1, so quantum fluctuation is negligible. It should be bosonic particles due to the Tremaine-Gunn bound. Therefore they are well described by classical waves.
- There are many such light bosonic DM candidates from QCD axions to string theory induced axion-like particles. QCD axion has a typical mass at 10^{-6}eV with an ALP down to 10^{-22}-10^{-20}eV
- String compactifications generically yield many axion-like fields (ALP) with mass down to 10^{-22}-10^{-20}eV. Ultralight masses can arise naturally via exponentially suppressed non-perturbative instanton effects induced potential. Shift symmetry protects the potential and small masses are technically natural.

Ultralight dark matter

- Axion and ALPs can be generated by the misalignment mechanism. The relic abundence assuming the initial misalignment angle $\phi_{\rm primordial}/f$ of order 1 $$\Omega_{\rm axion} \sim 0.1 \left(\frac{f}{10^{17}\ {\rm GeV}}\right)^2 \left(\frac{m}{10^{-22}\ {\rm eV}}\right)^{1/2}$$ $$V(\phi) = \Lambda^4 (1 - \cos[\phi/f])$$

- The energy scale of $f \sim 10^{17}$ GeV matches the string eneryg scale; the relic abundence motivates the consideration of the ultralight axions $\sim 10^{-22}$eV arising from string theory.

- Ultralight dark matter ($\sim 10^{-22}$eV Fuzzy DM) is consistent with string axiverse expectation (a way solve diversity problem).

Wave bosonic dark matter dynamics

- Klein-Gordon $\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right)-\sqrt{-g}m^2\phi=0$

  ⇓

  度规： $ds^2=-(1+2\Phi)dt^2+a^2(1-2\Phi)d\mathbf{r}^2$

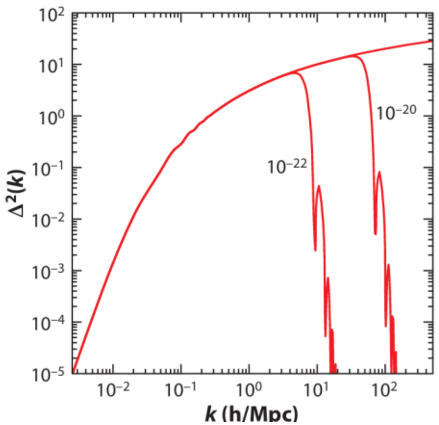
  non-relativistic limit: $\phi=\frac{1}{\sqrt{2m}}\left(\psi e^{-imt}+\psi^\star e^{imt}\right)$

- Schrodinger-Poisson (SP) equation:

$$i\partial_t\psi=-\frac{\nabla^2}{2m}\psi+m\Phi\psi,$$
$$\nabla^2\Phi=4\pi G\rho,\quad \rho=m\left|\psi\right|^2.$$

- de Broglie wave length $\lambda_{\mathrm{dB}}\equiv\frac{2\pi}{mv}=0.48\,\mathrm{kpc}\left(\frac{10^{-22}\ \mathrm{eV}}{m}\right)\left(\frac{250\,\mathrm{km\,s^{-1}}}{v}\right)$

An equivalent description



- The Madelung formulation $\psi = \sqrt{\rho/m}\, e^{i\theta}$
- The fluid velocity is defined as $\vec{v} = \frac{1}{m}\vec{\nabla}\theta = \frac{i}{2m|\psi|^2}(\psi\vec{\nabla}\psi^* - \psi^*\vec{\nabla}\psi)$
- The Schrodinger equation can be equivalently described as mass conservation and Euler equation in fluid description

$$\partial_t \rho + \vec{\nabla}\cdot(\rho\vec{v}) = 0, \qquad \partial_t\vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\vec{\nabla}\Phi + \frac{1}{2m^2}\vec{\nabla}\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$$

- The last term is often referred to as the quantum pressure term
- For large m, the equation reduces to pressureless fluid. While for Fuzzy the pressure is important.

$$k_J = (16\pi G\bar{\rho})^{1/4} m^{\frac{1}{2}}$$

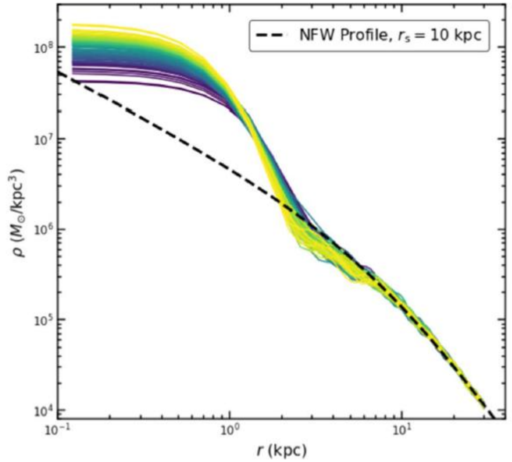
- Considering the linear pertubation, we have that for small k < K_J gravity domnates while for k > k_J qutantum pressure dominates.

Properties of Fuzzy DM: small scale suppression

- Nonlinear bound halos, FDM does not change the structure at large scales and while at scales when quantum pressure dominates the structure is suppressed. $R \sim \frac{1}{GMm^2} \sim 100\,\mathrm{pc}\,\frac{10^9\,\mathrm{M}_\odot}{M}\left(\frac{10^{-22}\,\mathrm{eV}}{m}\right)^2$

- The FDM halo is generally characterized by a profile with a soliton core and an NFW envelope. There is a stationary state of FDM with minimal energy, referred to as soliton;

$$\rho_{\mathrm{FDM}}(r) = \begin{cases} \frac{\rho_c}{[1+0.091(r/r_c)^2]^8} & , r < kr_c \\ \frac{\rho_s}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2} & , r \geq kr_c. \end{cases}$$

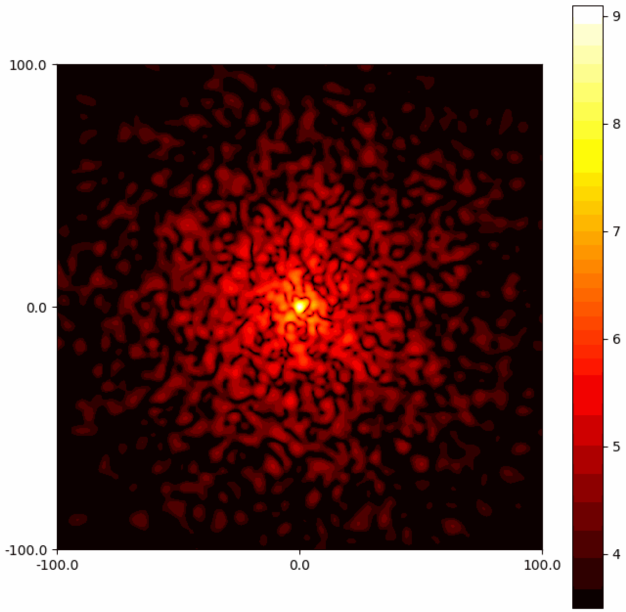


Core-cusp problem

Properties of Fuzzy DM: interference of waves

- Interference between the waves results in many interesting effects.
- These dynamical evolutions lead to **fluctuations of the gravitational field**, which inject energy into the stars residing in it, leading their **velocities to increase** and their spatial distribution to become **more diffuse** over time.
- This effect is referred as the **dynamical heating**.

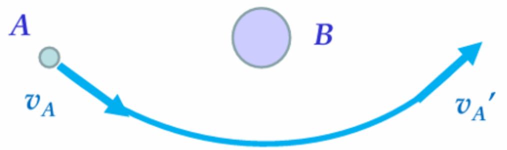
granules evolution



$$\Delta v \sim 2GM/(bv) \qquad M \sim 4\pi\rho(\lambda_{\rm dB}/2)^3/3 \qquad b \sim \lambda_{\rm dB}/2$$

$$\text{rms}\,\Delta v \sim 4\,\text{km/s}\left(\frac{T}{5\,\text{Gyr}}\right)^{1/2}\left(\frac{\rho}{0.01\,\text{M}_\odot\,\text{pc}^{-3}}\right)\left(\frac{250\,\text{km/s}}{v}\right)^2\left(\frac{10^{-22}\,\text{eV}}{m}\right)^{3/2}$$

Hui, 2101.11735



Gravitational Slingshot

$m_A << m_B$

$v_A' > v_A$

Yang, **XJB**, Yin, Phys.Rev.D 111 (2025) 6, 063013

Hints of ULDM in astronomical observations

The Milky Way thick (thin) stellar disc is dominated by older (younger) stellar populations, and exhibits higher (lower) velocity dispersion. This is consistent with heating effect of FDM with $m \sim 0.5 - 0.7 \times 10^{-22}$ eV.

B. T. Chiang, J. P. Ostriker, and H. Y. Schive, MNRAS 518, 4045-4063 (2023)

✓ Within a dwarf galaxy, older stars tend to have a more diffuse distribution compared to younger stars.

C.R.Higgs et al., MNRAS 503, 176-199 (2021); R.Pucha et al 2019 ApJ 880 104

- 3. Dynamical friction. FDM mass of $m \sim 10^{-22}$ eV **helps explain** the survival of globular clusters against orbital decay in the halo of Fornax

Hui L,Ostriker JP, Tremaine S, Witten E. 2017. Phys. Rev. D 95:043541

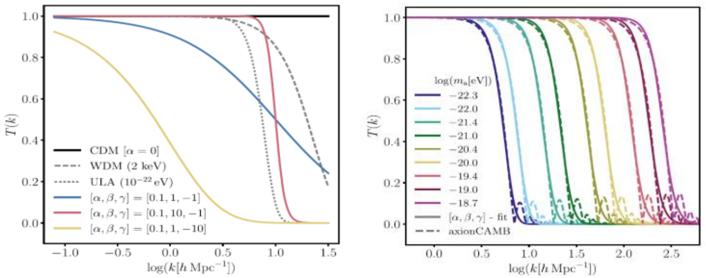
- 4. Extremely extended stellar distribution in the Nube system

Y.M. Yang, Z.C. Zhang, X.J. Bi, and P.F. Yin, 2025 ApJL 981 L26

Constraints on UL DM

- Lyman-alpha forest (FDM压低小尺度功率谱)

$$T(k) \equiv \sqrt{\frac{P_{\rm nCDM}(k)}{P_{\rm CDM}(k)}} = [1 + (\alpha k)^{\beta}]^{\gamma}.$$



- 1. Lyα forest constrain Linear power spectrum at high Z; The constraint has strong model dependence by IGM theremal history, inhomogeneous reionization, UVB fluctuations et al.

目前最强的限制（Phys. Rev. Lett. 126, 071302） $m_{\rm a} > 2 \times 10^{-20}$ eV.

- 2. Dynamical heating of stars.

Nuclear star cluster in Eridanus II (13pc)

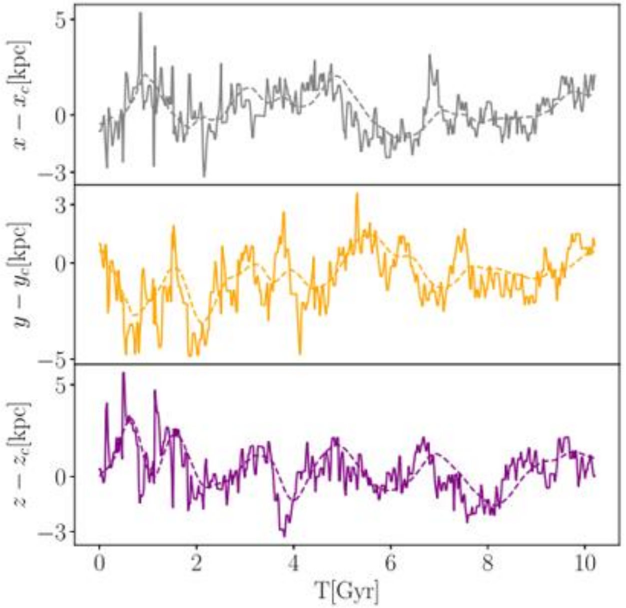
$$m_a \gtrsim 0.6 \to 1 \times 10^{-19} \text{ eV}$$

Segue 1 (24.2pc) & Segue 2 (40.5pc)

$$m_{\rm FDM} > 3 \times 10^{-19} \text{ eV}$$

Fornax (0.668kpc), Carina (0.24kpc), Leo II (0.21kpc), Leo I (0.25kpc), and Draco (0.22kpc) 排除了(低质量的会受到tidal suppression)

$$5 \times 10^{-22} \lesssim m/\text{eV} \lesssim 5 \times 10^{-21}$$

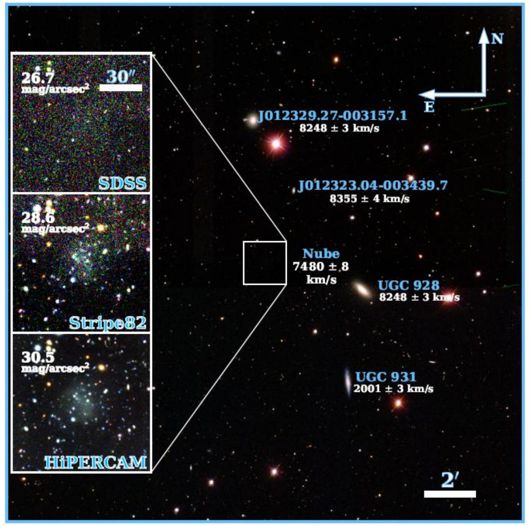


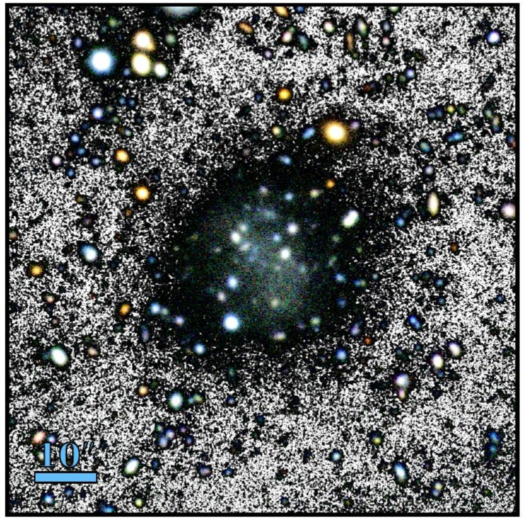
Soliton random walk

Search of ULDM by other effects

- 1. Superradiance. use the measured spin of black holes to put limits on scalars that could drain away their angular momentum
- 2. Boson stars. Gravitational lensing could be used to detect or constrain a population of such objects
- 3. Gravitational potential oscillations. An oscillating scalar produces an oscillating gravitational potential can be searched for in **pulsar timing array**
- 4. Probing interference substructures. One is through the heating and scattering of stars.The other is **through gravitational lensing** by the substructures.
- 5. Precision experiments search for spin-dependent fifth force
- 6. Photon Propagation in Axion Background， this birefringence effect in astronomical data

An isolated and extremely diffuse galaxy: Nube





M. Montes et al. Astronomy & Astrophysics681, A15 (2024).

- Age:

$$10.2^{+2.0}_{-2.5}\ \text{Gyr}$$

- Distance:

$$107 \pm 5\ \text{Mpc}$$

- Total stellar mass:

$$M_* = 3.9 \pm 1.0 \times 10^8\ M_\odot$$

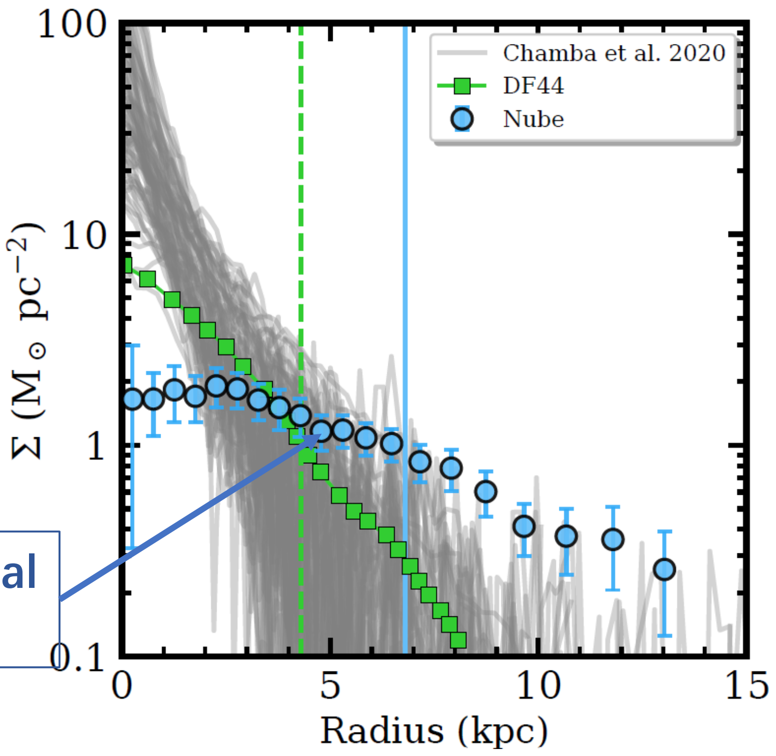
- Half-light radius:

$$R_e = 6.9 \pm 0.8\ \text{kpc}$$

- Dynamical mass:

$$2.6 \pm 1.7 \times 10^{10}\ M_\odot \text{ within } 3\,R_e = 20.7\ \text{kpc}$$

- No host, no interaction.



Dynamical heating?

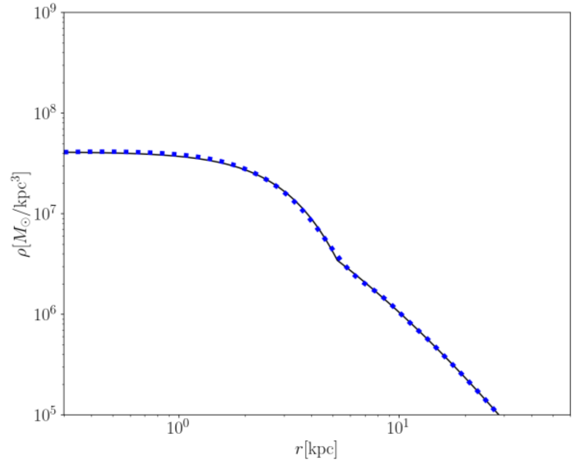
anomalous star distribution: low central brightness, large ***R_e, exceeding UDGs (R_e~1.5-5kpc)***

Numerical simulation of dynamical heating effect of FDM

- Initial wave function

Soliton+NFW profile

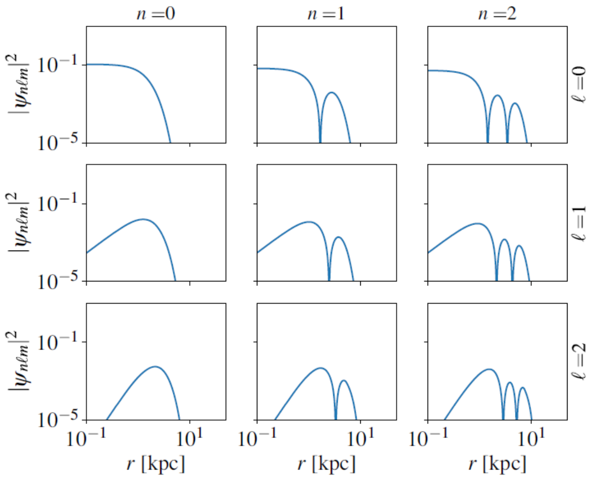
$$\rho_{\rm in}(r) = \begin{cases} \dfrac{\rho_c}{[1+0.091(r/r_c)^2]^8}, & r < kr_c \\ \dfrac{\rho_s}{(r/r_s)\,(1+r/r_s)^2}, & r \geq kr_c, \end{cases}$$



$$\psi(0,\mathbf{x}) = \sum_{nlm} |a_{nl}| e^{i\phi_{nlm}} \Psi_{nlm}(\mathbf{x}),$$

$$-\frac{\hbar^2}{2m_a}\nabla^2\Psi_{nlm}(\mathbf{x}) + m_a\Phi_{\rm in}(r)\Psi_{nlm}(\mathbf{x}) = E_{nl}\Psi_{nlm}(\mathbf{x}).$$

$$\rho_{\rm out}(r) = \frac{m_a}{4\pi}\sum_{nl}(2l+1)|a_{nl}|^2 R_{nl}^2(r).$$



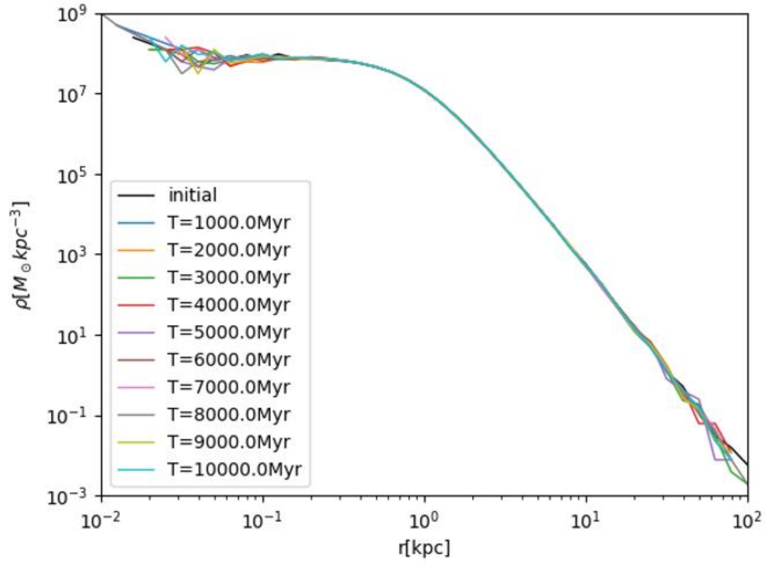
T. D. Yavetz, X Li, and L Hui,
Phys. Rev. D 105 (2022) 2, 023512

- Stellar initial condition
  - Density profile

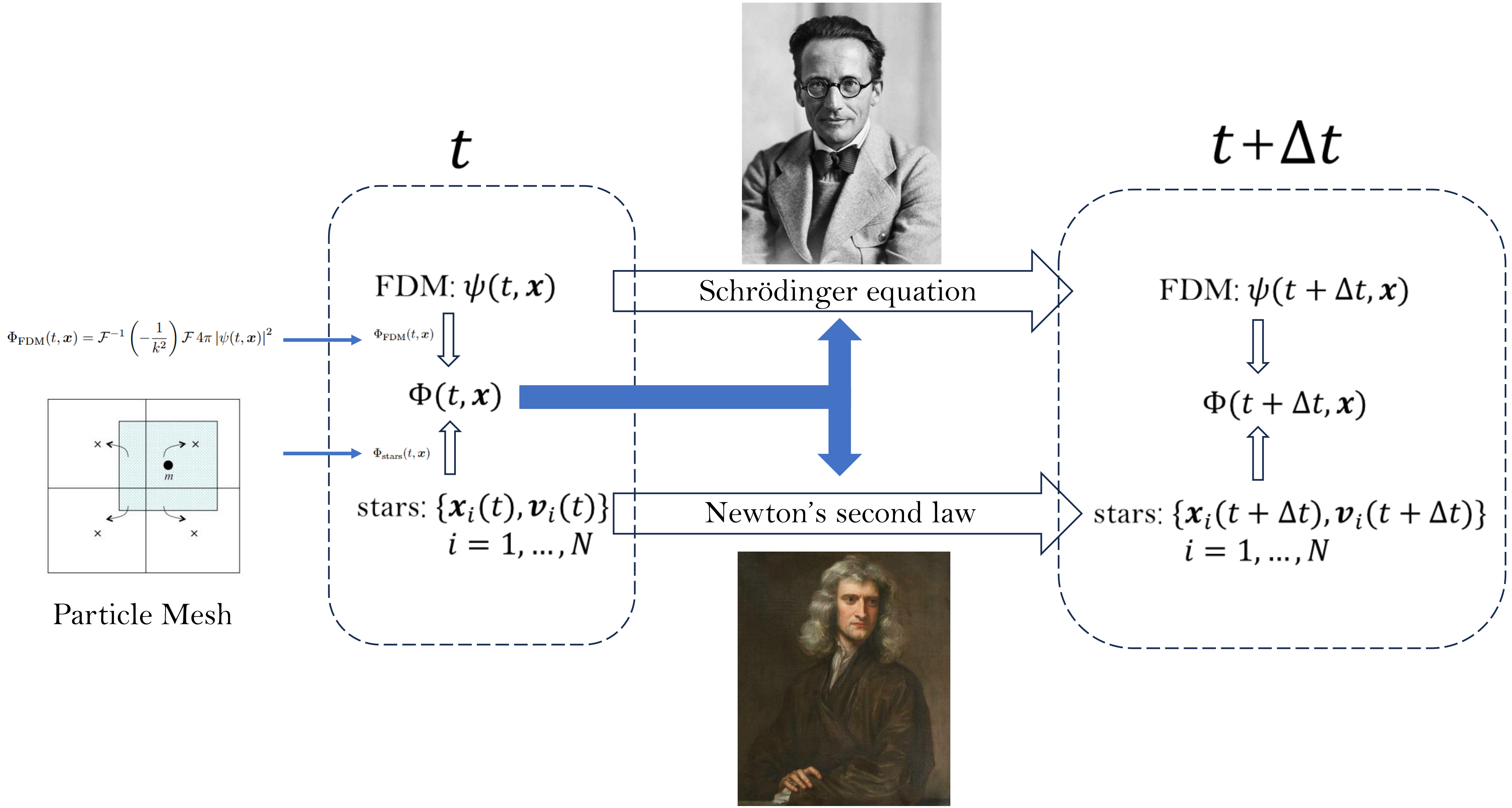
$$\rho_\star(r) = \frac{3M_\star}{4\pi a_i^3}\left(1+\frac{r^2}{a_i^2}\right)^{-5/2}$$

  - Eddington formula

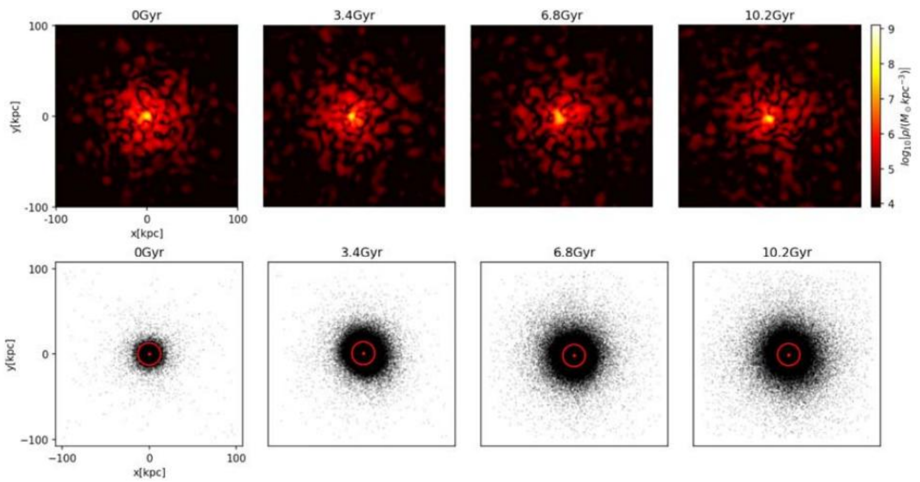
$$f(\mathcal{E}) = \frac{1}{\sqrt{8}\pi^2}\frac{\mathrm{d}}{\mathrm{d}\mathcal{E}}\int_0^{\mathcal{E}}\frac{\mathrm{d}\Phi_0}{\sqrt{\mathcal{E}-\Phi_0}}\frac{\mathrm{d}\rho_\star}{\mathrm{d}\Phi_0},$$

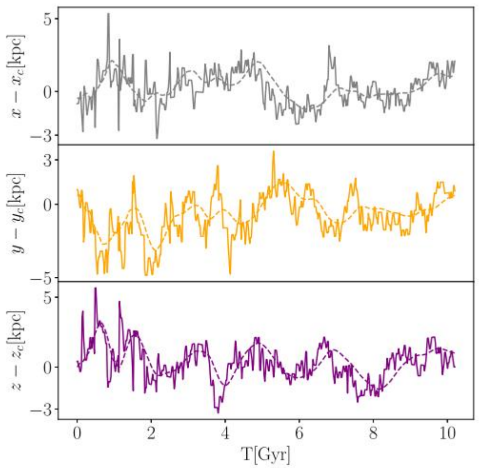


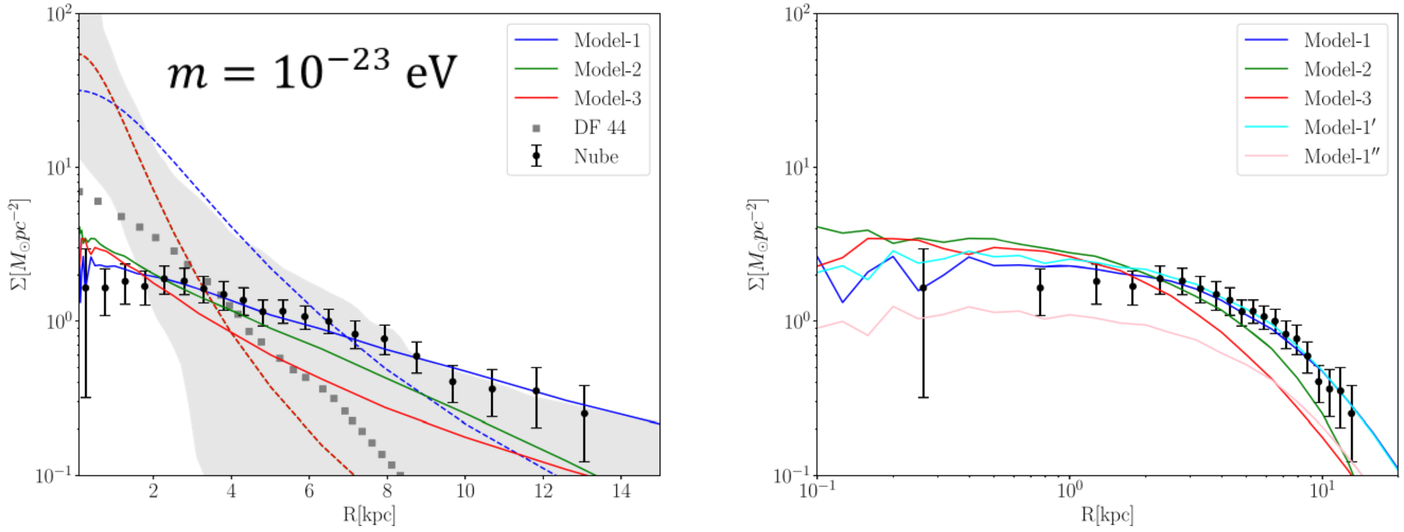
Dynamical heating effect of FDM

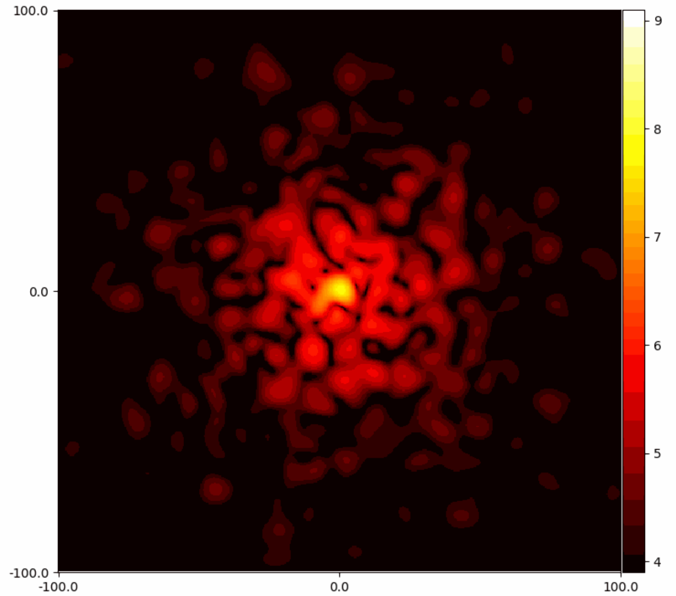


Dynamical heating effect of FDM









Y.M. Yang, Z.C. Zhang, X.J. Bi, and P.F. Yin, 2025 ApJL 981 L26

Tidal suppression of the heating effect

PHYSICAL REVIEW LETTERS **123**, 051103 (2019)

Strong Constraints on Fuzzy Dark Matter from Ultrafaint Dwarf Galaxy Eridanus II

PHYSICAL REVIEW D **106**, 063517 (2022)

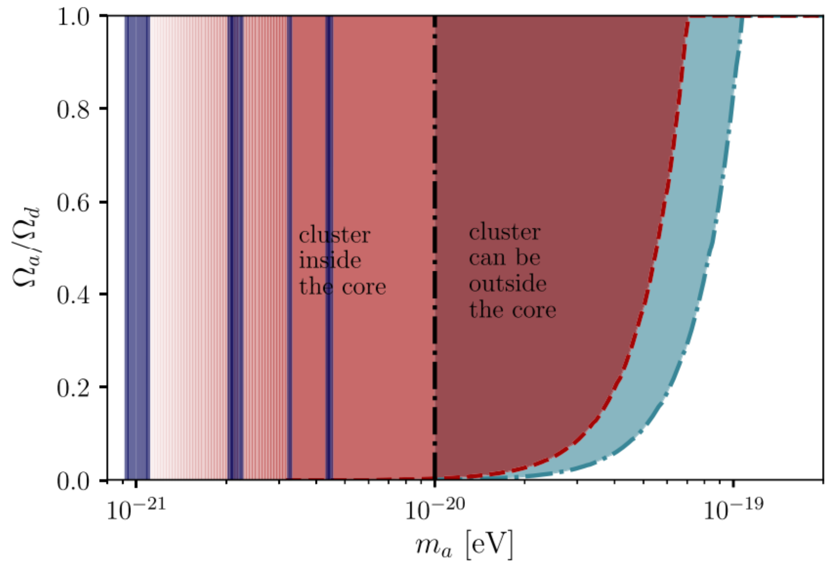
Excluding fuzzy dark matter with sizes and stellar kinematics of ultrafaint dwarf galaxies

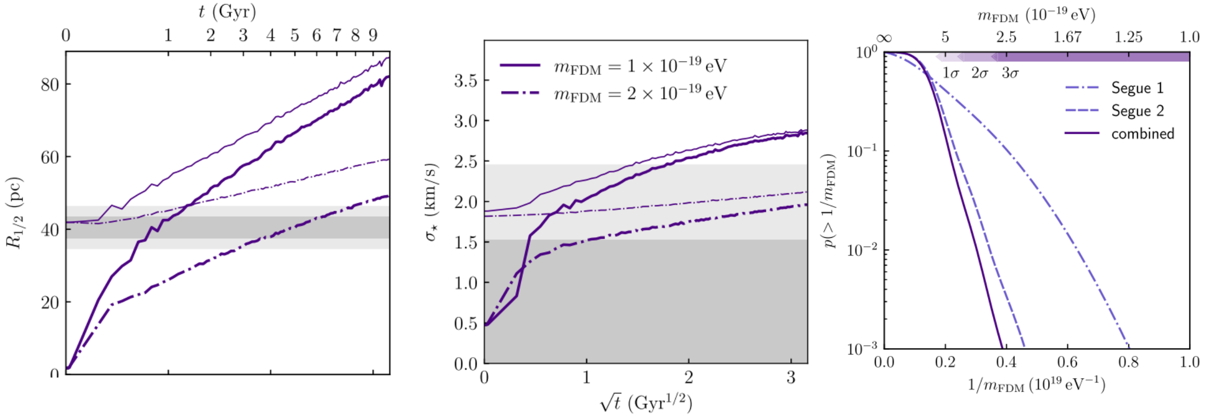
arXiv > astro-ph > arXiv:2501.07631

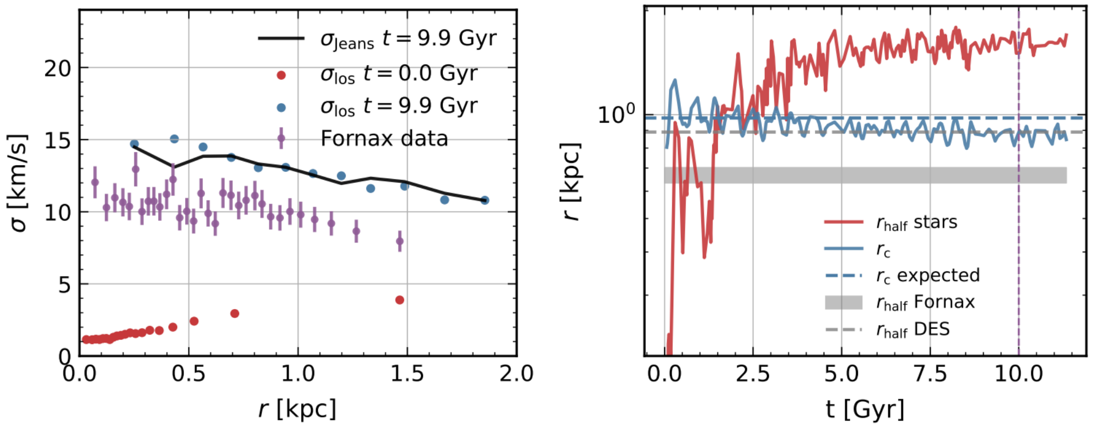
Astrophysics > Astrophysics of Galaxies

[Submitted on 13 Jan 2025]

Ultra-Light Dark Matter Simulations and Stellar Dynamics: Tension in Dwarf Galaxies for $m < 5 \times 10^{-21}$ eV





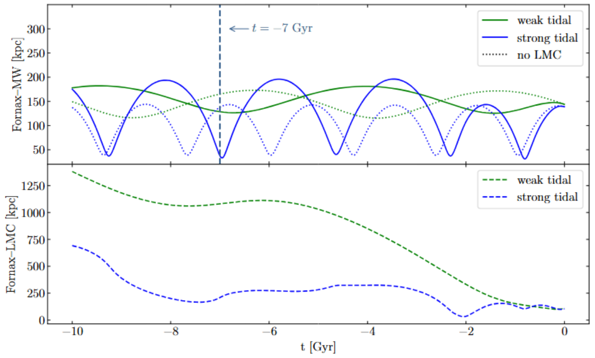


Tidal suppression of the heating effect

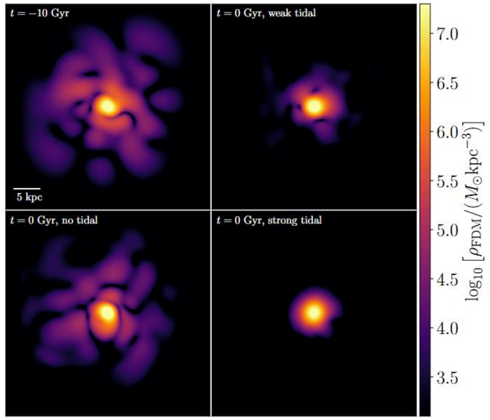
Yang, XJB, Yin, 2507.01686, *APJL.* 990 (2025) 2, L67

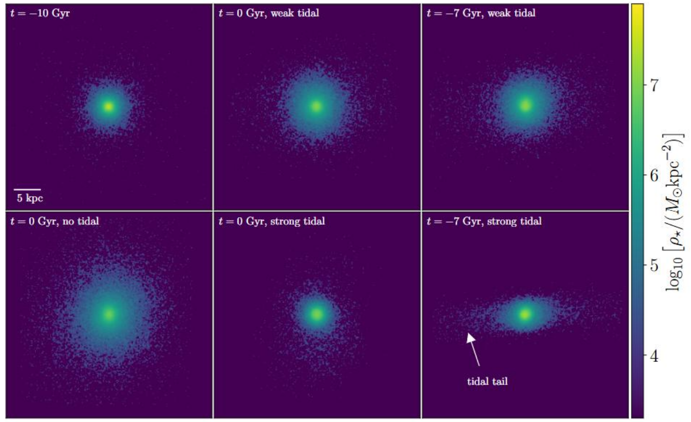
➢ Fornax in the Milky Way

➢ Orbit



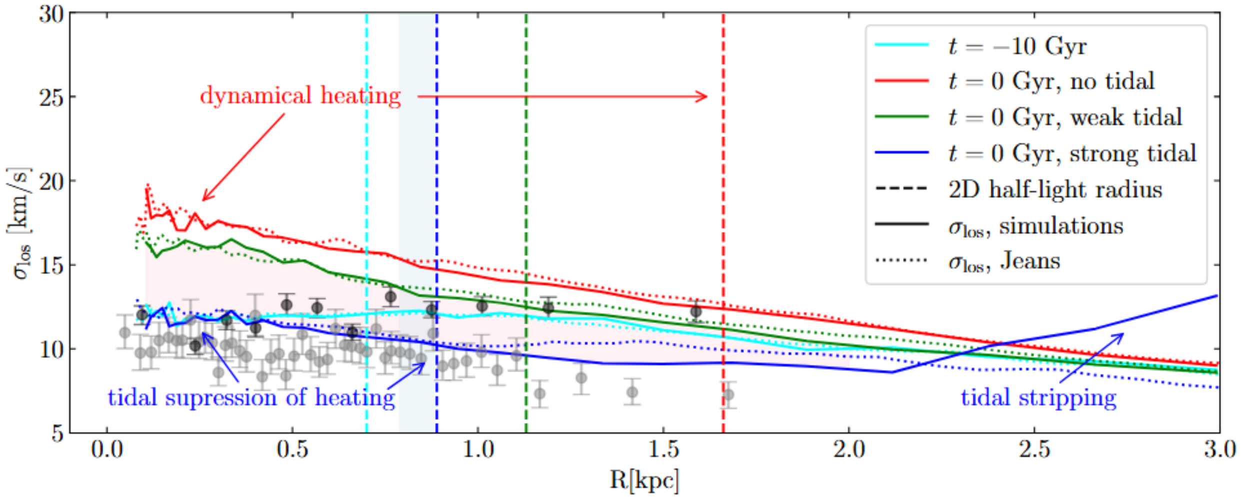
➢ FDM & stellar density





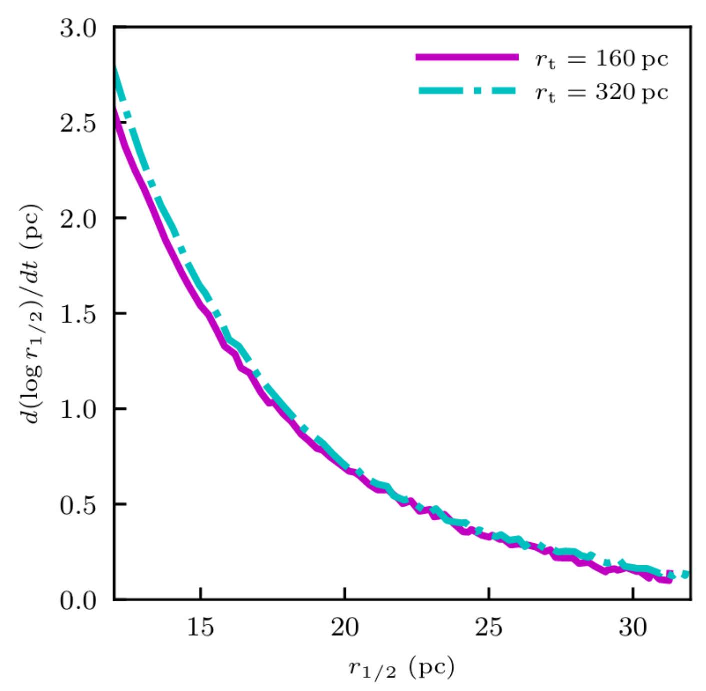
➢ Line-of-sight velocity dispersion

$m=10^{-22}$eV



Argument of UFD as Segue 1

- A paper pointed out that very compact dwarfs like Segue 1 can not be affected by the tidal effect.



Updated bounds on ultra-light dark matter from the tiniest galaxies

Simon May (Perimeter Inst. Theor. Phys.), Neal Dalal (Perimeter Inst. Theor. Phys.), Andrey Kravtsov (Chicago U., Astron. Astrophys. Ctr. and Chicago U., KICP and Chicago U., EFI) (Sep 2, 2025)

e-Print: 2509.02781 [astro-ph.CO]

A recent paper [12] has argued that tidal stripping can weaken, or even invalidate, the bounds on ultra-light dark matter imposed by DK22.[1] The concern raised by [12] is that ultra-faint dwarfs like Segue 1 are found within the Milky Way halo and, as satellites, they are subject to tidal gravitational fields that can strip away their dark matter. In principle, this stripping could suppress FDM heating of these satellites, when the tidal radius becomes as small as the soliton size [30].

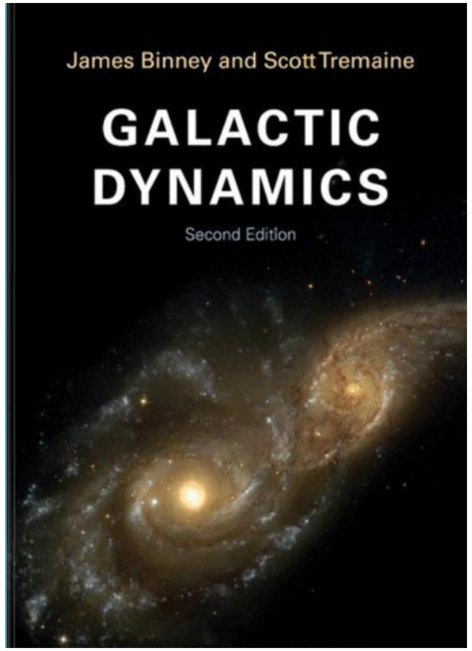
It is quite easy to see that this argument is incorrect for observed UFDs like Segue 1, for the very simple reason that their tidal radii are much larger than the sizes of their stellar distributions [5]. In this regime, the heat-

Gravity between stars has important effect

Collapse versus Disruption: The Fate of Compact Stellar Systems in Ultralight Dark Matter Halos

Yu-Ming Yang (Beijing, Inst. High Energy Phys. and Beijing, GUCAS), Xiao-Jun Bi (Beijing, Inst. High Energy Phys. and Beijing, GUCAS), Long Wang (Zhongshan U., Zhuhai), Peng-Fei Yin (Beijing, Inst. High Energy Phys.) (Jan 14, 2026)

e-Print: 2601.09403 [astro-ph.CO]



- We give a rigorous simulation with a tool simulating stellar clusters, where we also take the FDM effect into account
- Galactic dynamics becomes important for very dense star clusters. For dense systems, gravity induces system relaxation which leads to core collapose.

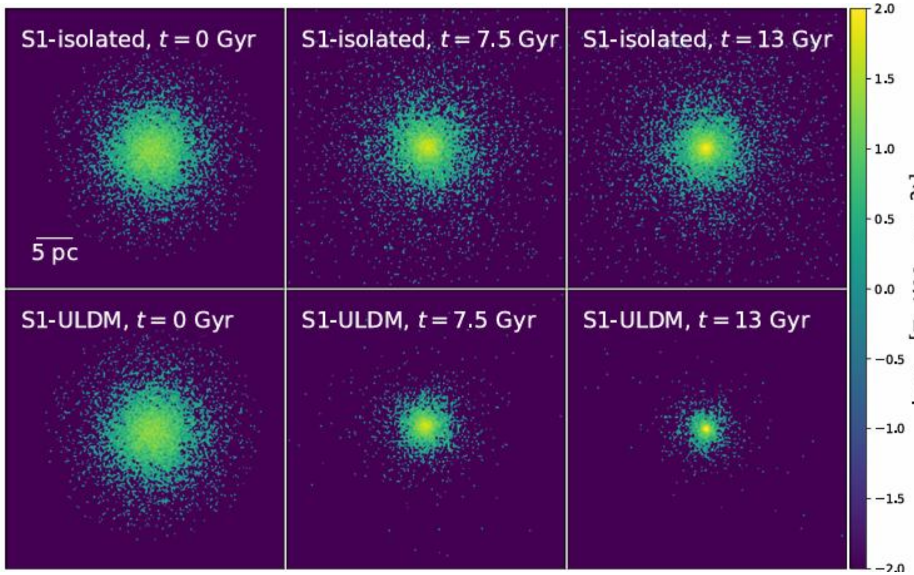
$$t_{\rm rh} = 0.138 \frac{M_\star{}^{1/2} r_{\rm h}^{3/2}}{\langle m \rangle G^{1/2} \ln \Lambda} \simeq \frac{0.212 N R_{\rm h}^{3/2}}{M_\star{}^{1/2} G^{1/2} \ln \Lambda},$$

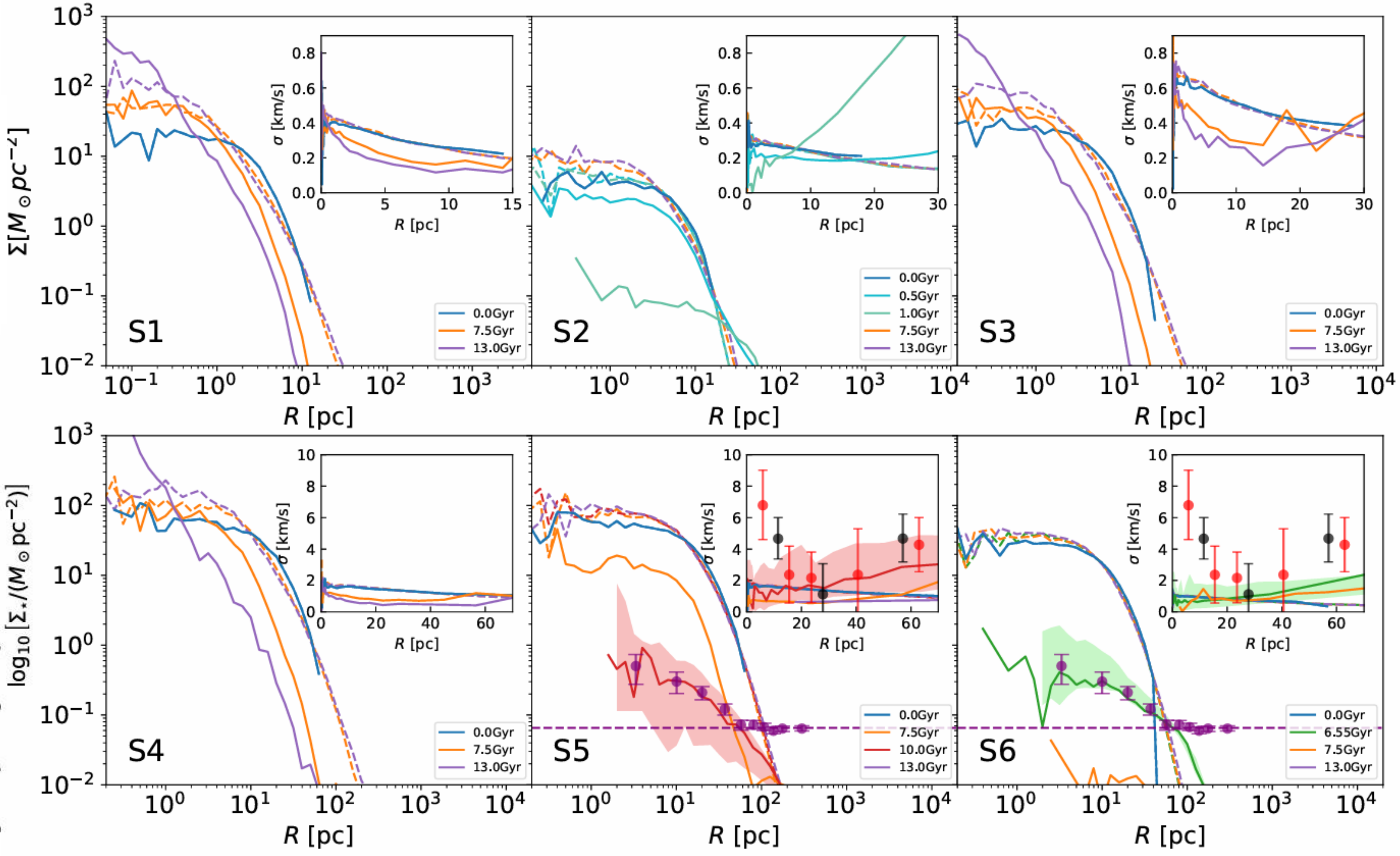
- FDM stripping of outer stars accelerate the core collapose.
- For more extended systems FDM heating may become dominate and disrupt the system.
- We give a phase diagram that shows the fate of the system evolution: collapse v.s. disruption.

Collapse versus disruption

TABLE I. Initial condition parameters of the stellar systems.

	S1	S2	S3	S4	S5	S6
$M_\star[10^3 M_\odot]$	1	1	5	100	100	25
$N[10^4]$	1	1	1	1	3	5
$R_{\rm h}[\rm pc]$	5	10	10	30	30	20



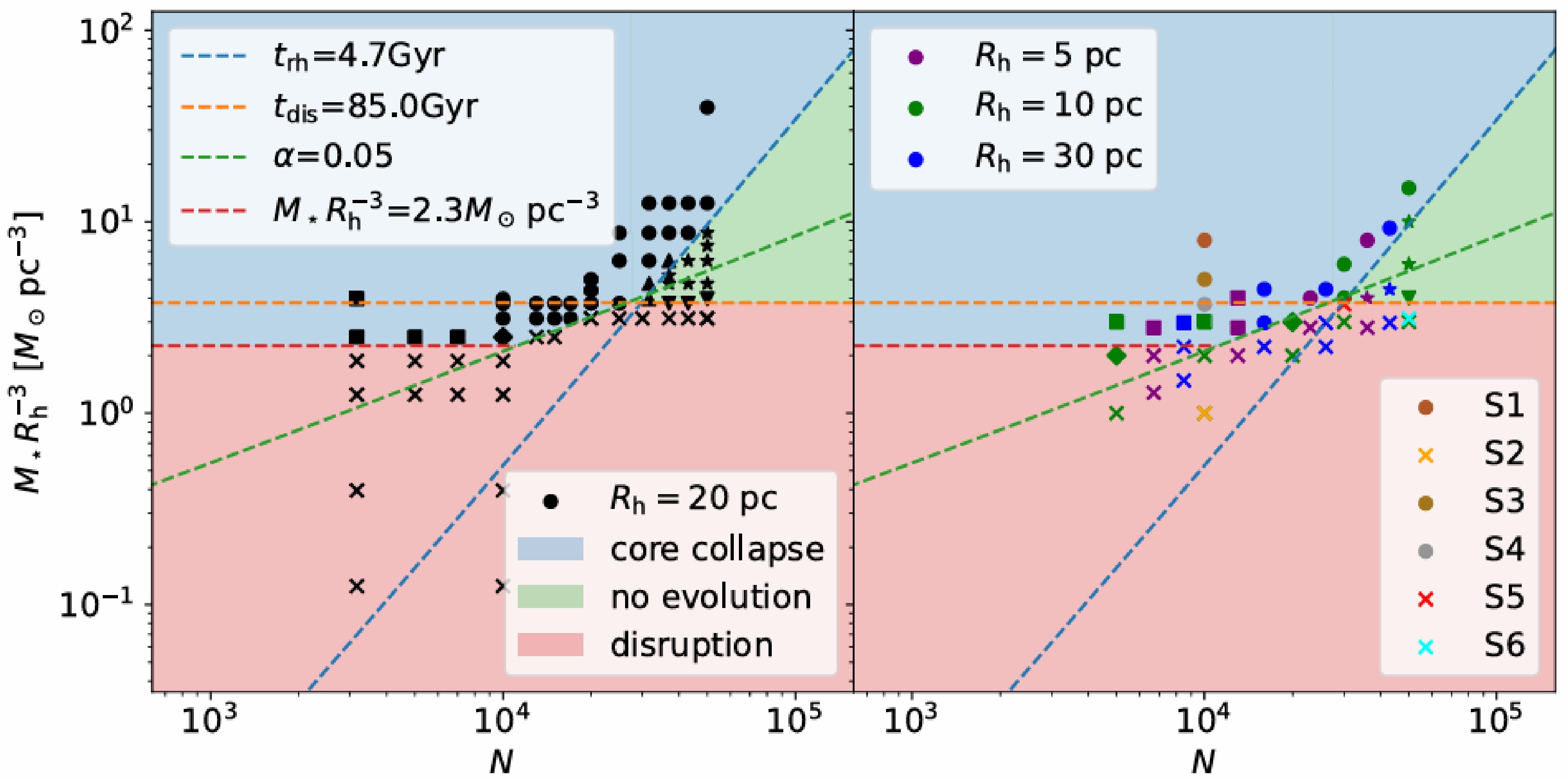


A phase diagram map the parameter space

$$t_{\rm rh} = 0.138\frac{{M_\star}^{1/2} r_{\rm h}^{3/2}}{\langle m\rangle G^{1/2}\ln\Lambda} \simeq \frac{0.212 N R_{\rm h}^{3/2}}{{M_\star}^{1/2} G^{1/2}\ln\Lambda}$$

$$t_{\rm dis} = (3M_\star/4\pi\bar{\rho}_{\rm ULDM}R_{\rm h}^3)(\lambda_{\rm dB}/\sigma_{\rm ULDM})$$

$$\alpha \equiv \frac{t_{\rm rh}}{t_{\rm dis}} = 0.153\left(\frac{N}{10^4}\frac{4}{\lg N}\right)\left(\frac{10^3 M_\odot}{M_\star}\right)^{3/2}\left(\frac{R_{\rm h}}{10\,{\rm pc}}\right)^{9/2} \times\left(\frac{m_a c^2}{10^{-22}\,{\rm eV}}\right)\left(\frac{M(<r_{\rm orbit})}{3.6\times10^8 M_\odot}\right)\left(\frac{3\,{\rm kpc}}{r_{\rm orbit}}\right)^3\left(\frac{\sigma_{\rm ULDM}}{20\,{\rm km\,s^{-1}}}\right)^2$$



Self-Interacting Fuzzy Dark Matter

- Gross-Pitaevskii-Poisson (GPP) equation

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m_a^2 \phi^2 - \frac{\lambda}{4!} \phi^4, \quad \lambda = \pm m_a^2 / f_a^2$$

$$\phi = (1/\sqrt{2m_a})(\psi e^{-im_a t} + \psi^\star e^{im_a t}).$$

$$\Longrightarrow$$

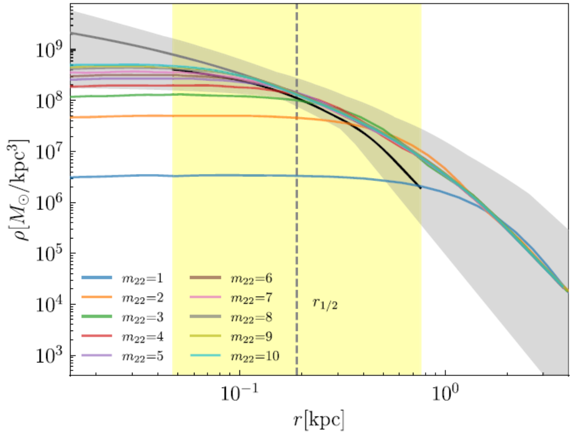
$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m_a} \nabla^2 \psi + m_a \Phi \psi \pm \frac{\hbar^3 c^3}{8 f_a^2} |\psi|^2 \psi,$$

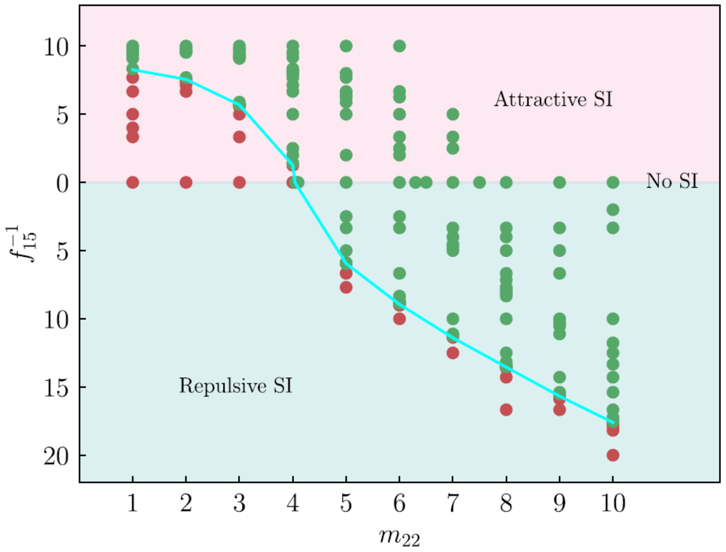
$$\nabla^2 \Phi = 4\pi G \rho.$$

Attractive SI $\Longrightarrow$ Smaller Core

- Revisit the constraint from Leo II

Y. Zhao, Y.M. Yang, X.J. Bi, and P.F. Yin, Phys. Rev. D 114, 023043





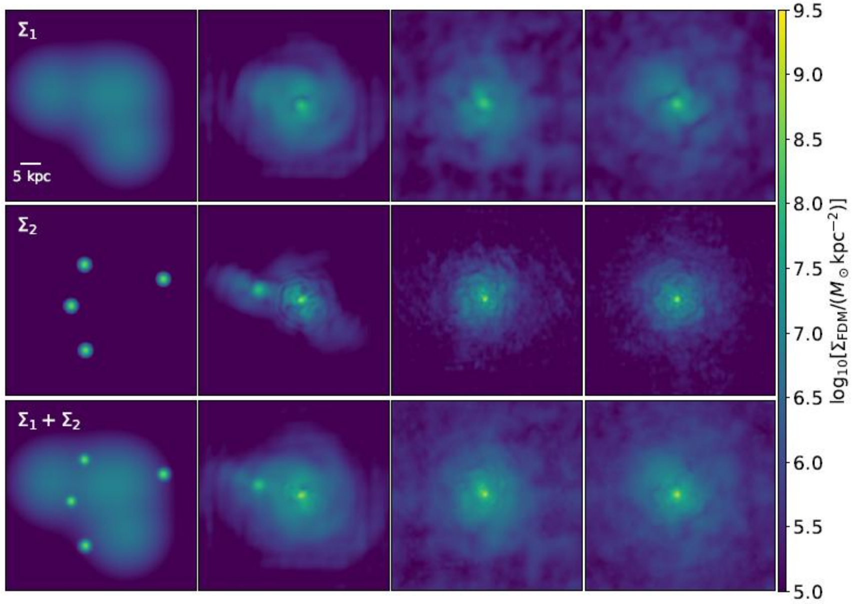
- Attractive SI relax the constraints
- Repulsive SI tighten the constraints

Multifield Fuzzy Dark Matter

- Multifield coupled through gravity

$$i\hbar\frac{\partial\psi_i}{\partial t} = -\frac{\hbar^2}{2m_i}\nabla^2\psi_i + m_i V\psi_i,\ i = 1,\cdots,N \qquad \nabla^2 V = 4\pi G\sum_{i=1}^{N}\rho_i = 4\pi G\sum_{i=1}^{N} m_i|\psi_i|^2.$$

- Solitons merge



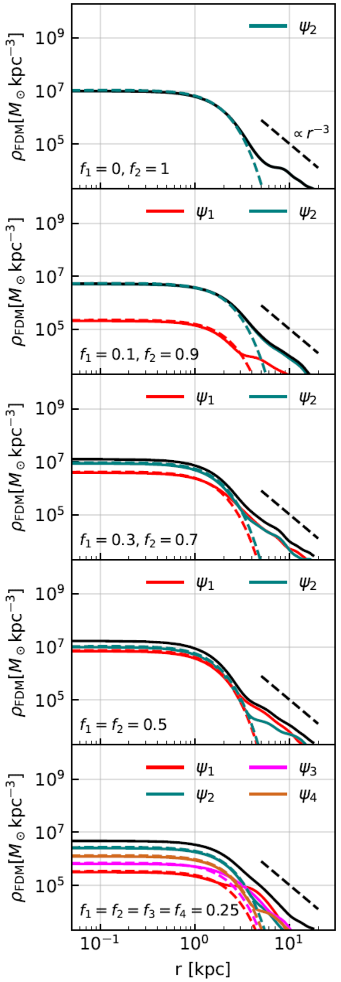
$m_1 = 10^{-22}$ eV

$m_2 = 3\times10^{-22}$ eV

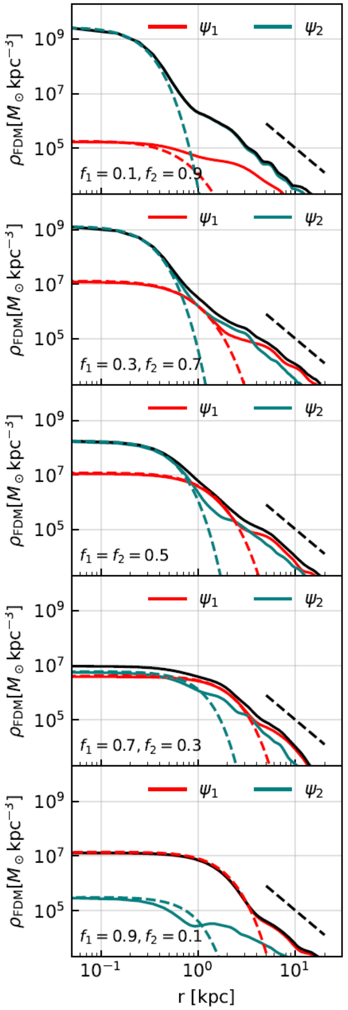
- The initial m_2 solitons are more compact than the m_1 solitons.
- Gravitational attraction drives the initially separated solitons to merge into a virialized halo.
- Each field develops a central core surrounded by granules.
- The m_2 core and granules are noticeably smaller than those of m_1 .
- Total-density fluctuations are visibly weaker than those in either individual field.

Multifield Fuzzy Dark Matter

➢ Density profile



$$m_1 = 10^{-22} \text{ eV}$$
$$m_2 = 10^{-22} \text{ eV}$$



$$m_1 = 10^{-22} \text{ eV}$$
$$m_2 = 3 \times 10^{-22} \text{ eV}$$

- For equal-mass models, each field retains a soliton-like core across all mass fractions.
- For unequal-mass models, the more massive field deviates from the soliton profile at $f_2 = 0.3$, with pronounced deviations at $f_2 = 0.1$.
- The total density follows an NFW profile in the outer halo.
- Increasing the number of field components smooths the transition from the central soliton-like core to the outer NFW envelope.
- Unequal-mass models exhibit diverse density profiles as the field abundances vary.

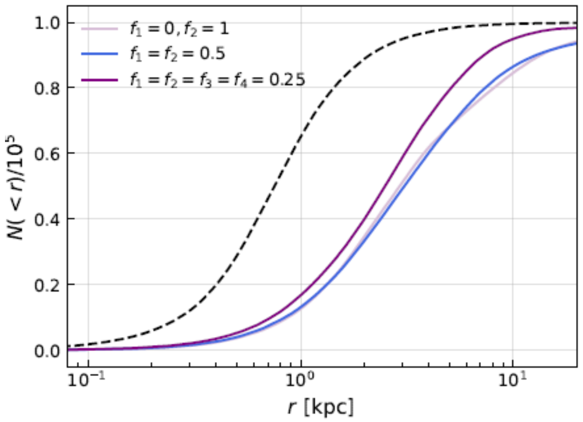
⇩

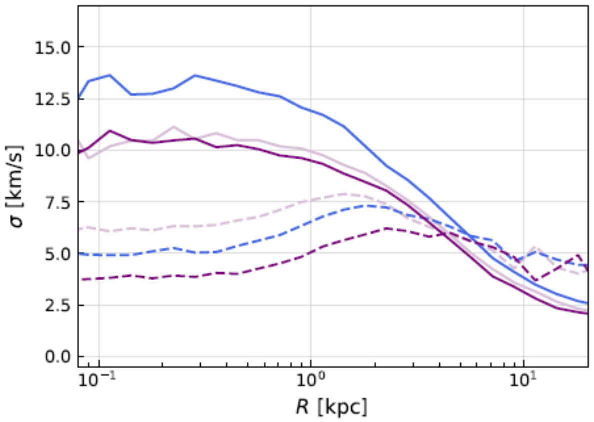
Diversity problem?

Multifield Fuzzy Dark Matter

- Dynamical heating $m_1 = m_2 = 10^{-22}$ eV

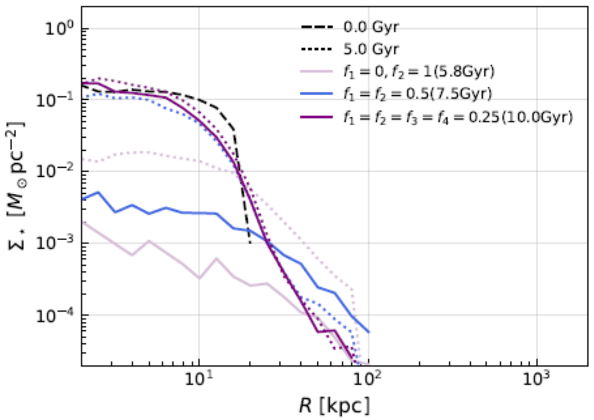
Stellar system initially in equilibrium within a multifield FDM halo





- The single- and two-field cases show very similar stellar distribution expansion, while the four-field case exhibits noticeably weaker expansion.
- The velocity-dispersion profiles show no clear evidence for weaker dynamical heating with increasing number of field components.

Self-gravitating stellar system outside the core



- In the single-field model, the stellar system is nearly disrupted by ~5.8 Gyr.
- In the two-field model, the stellar system is nearly disrupted by ~7.5 Gyr.
- The four-field model shows no clear sign of disruption over 10 Gyr.

Summary

- Wave dark matter (axion/ALPs) is attractive DM candidate.

- Ultralight wave dark matter has a unique property of interference, different from particle DMs, which helps to solve many observations that seem difficult to be explained by CDM or by baryonic processes.

- There are stringent constraints on FDM in the literature. However, we must be very carefule to set limits on the ultralight DM and rigorous simulations are usually necessary.