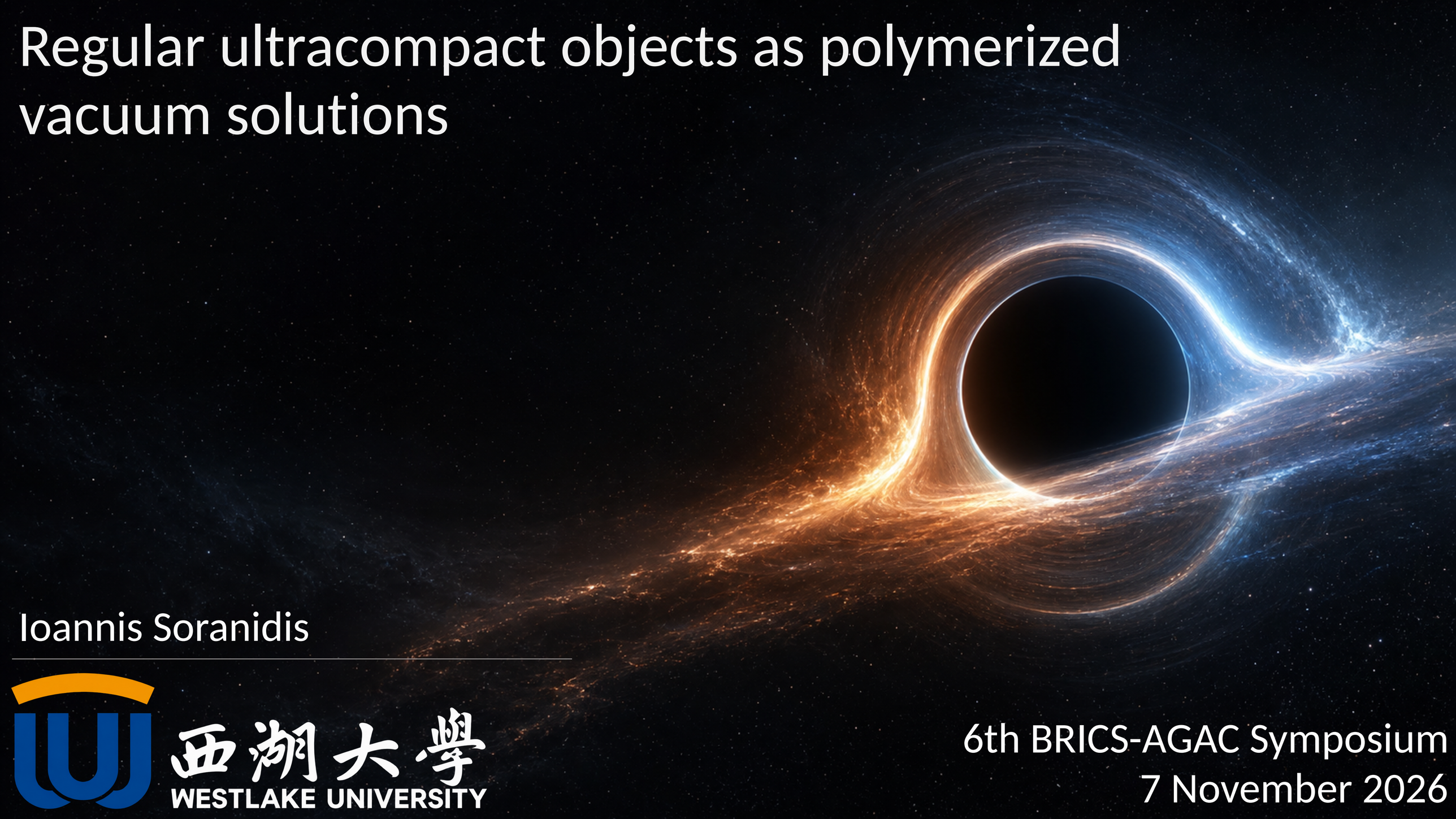
Regular ultracompact objects as polymerized vacuum solutions



Ioannis Soranidis

西湖大學
WESTLAKE UNIVERSITY

6th BRICS-AGAC Symposium
7 November 2026

Why ultracompact objects?

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Cardoso, Pani: 1904.05363

Mann, Murk, Terno: 2112.06515

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Ultracompact objects (UCOs)

are compact enough to trap light and support photon orbits (light rings).



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M87* (EHT Collaboration, 2019)

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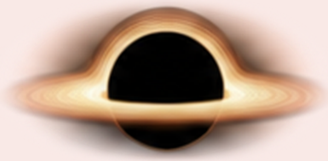
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Singular black holes



- Event horizon(s)
- Curvature singularity

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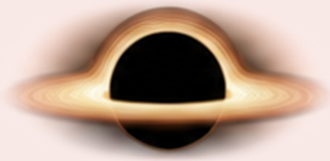
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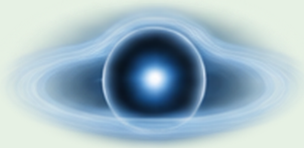
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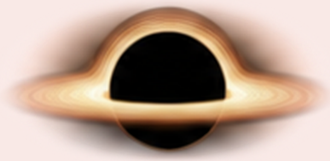
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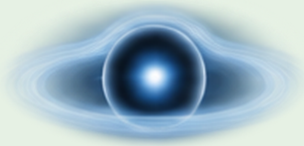
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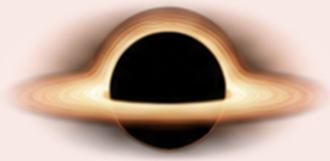
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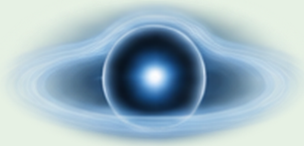
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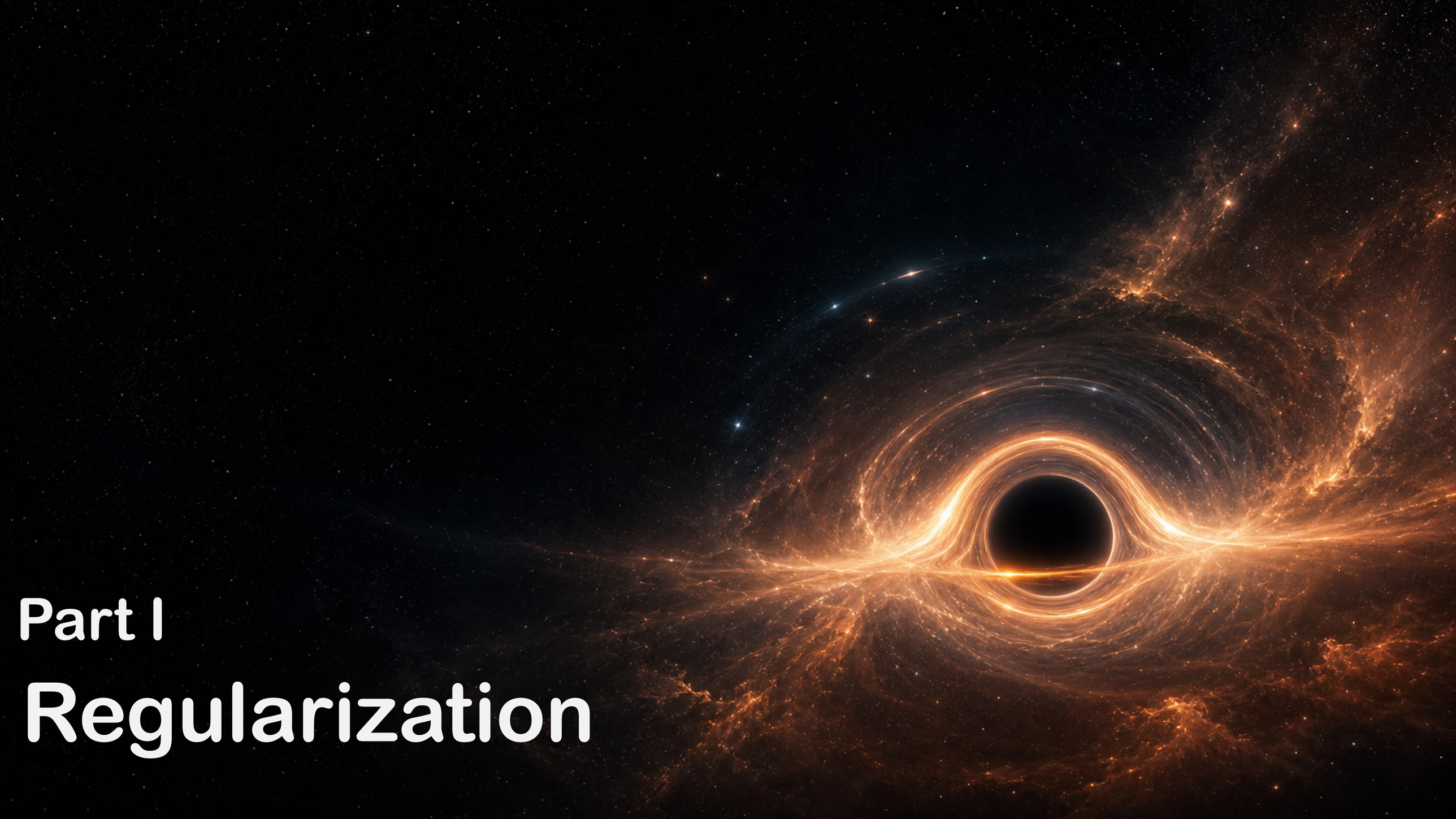
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All three possibilities can be ultracompact and possess light rings.
Observations can provide a way to distinguish between them.



Part I
Regularization

How do we source them?

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Einstein equations: $S = \frac{1}{16\pi G_{\mathrm{N}}} \int d^D x \sqrt{-g}\,(R - 2\Lambda), \quad G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_{\mathrm{N}} T_{\mu\nu}$

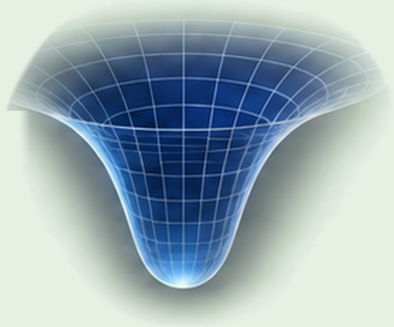
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Gravitational sector

Modify gravity at high curvature scales.

- Quasitopological gravity
- Polymerized Hamiltonians



Bueno, Cano, Hennigar: 2403.04827

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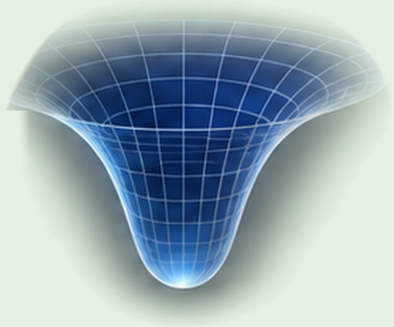
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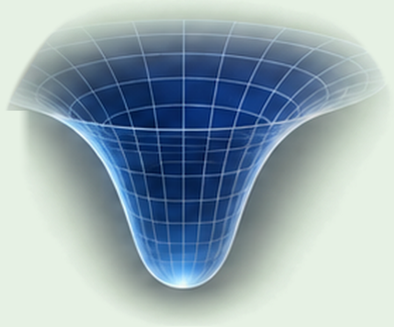
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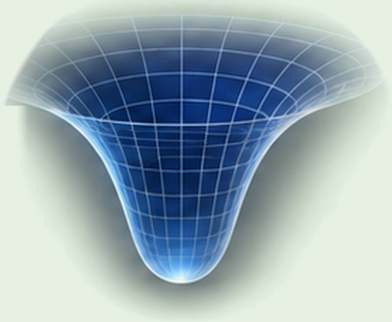
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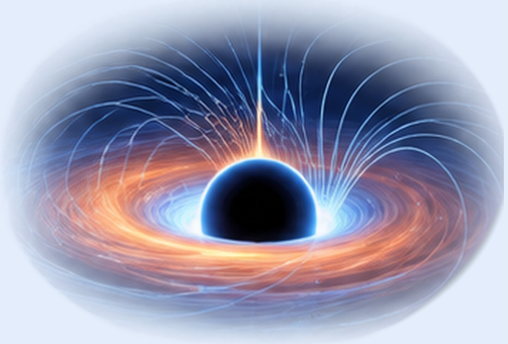
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Matter sector

Introduce non-trivial matter.

- Nonlinear electrodynamics
- Effective anisotropic fluids



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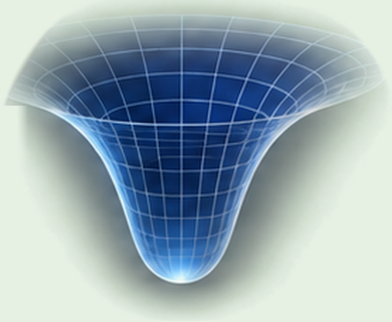
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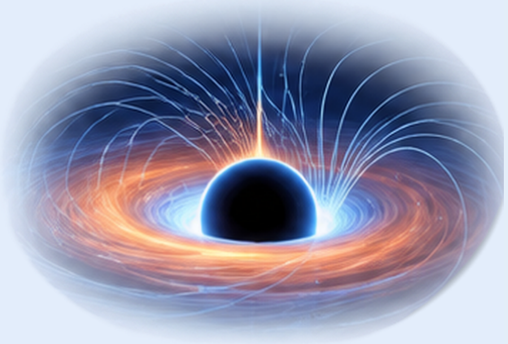
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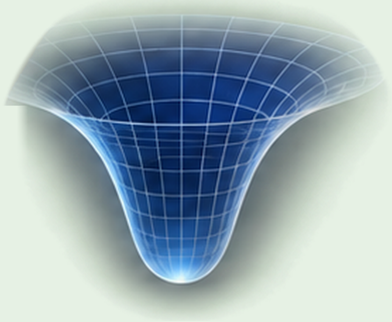
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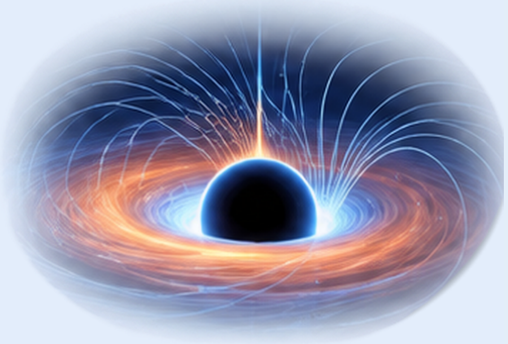
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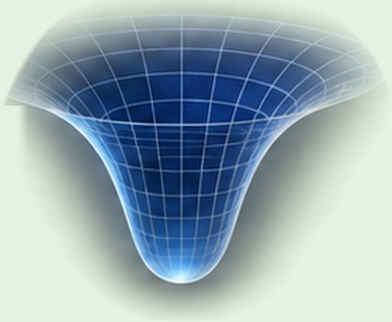
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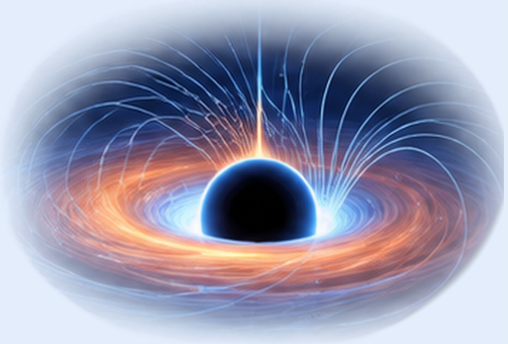
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Geometry supported by matter fields.

Regular black holes can be sourced by modified gravity or by matter fields

Both mechanisms can lead to spacetimes without curvature singularities, with potential observable astrophysical signatures.

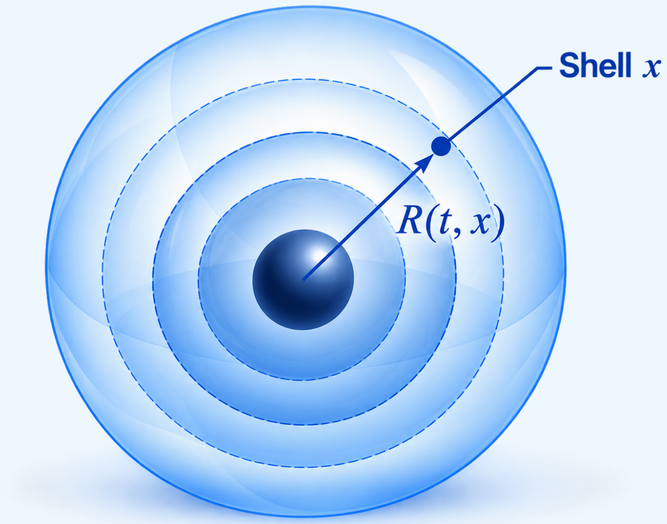
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1 Pressureless dust

Spherically symmetric dust with no pressure.

$$T_{\mu\nu} = \rho u_\mu u_\nu, \quad p = 0$$



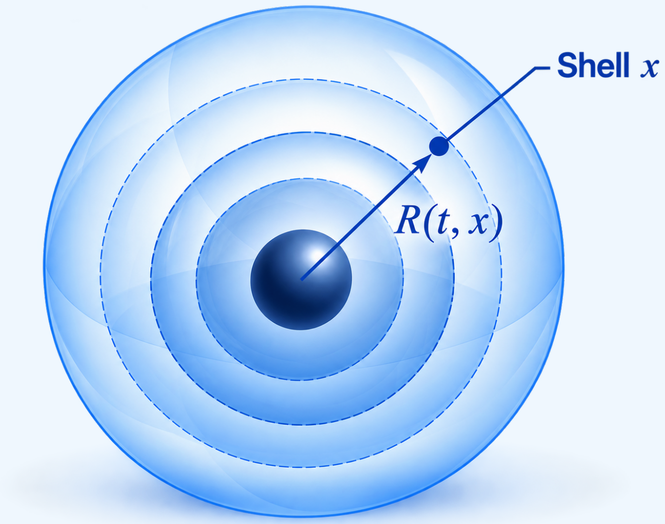
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Einstein's equations reduce to an ordinary differential equation for each shell.

$$\dot{R}^2(t,x) = \frac{2M(x)}{R(t,x)} + \mathscr{E}(x)$$



$$\dot{M}(x) = 0$$

Enclosed mass (conserved)



$$\dot{\mathscr{E}}(x) = 0$$

Energy function (conserved)

- Each shell evolves from its own conserved data.
- Gravity remains encoded in the enclosed mass.

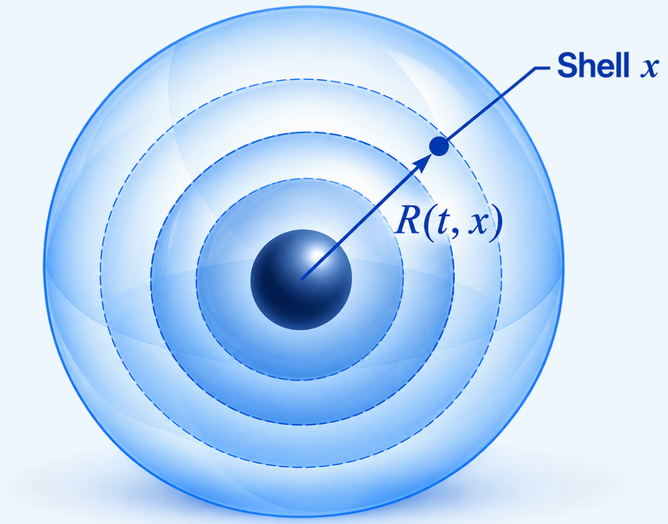
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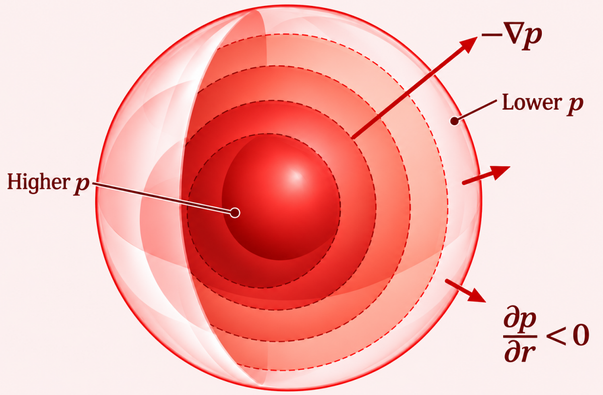
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3 What if there is pressure?

Pressure gradients couple neighboring shells and change the enclosed mass.

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}, \quad p \neq 0$$



- Pressure gradients couple shells.
- Pressure work changes the enclosed mass.

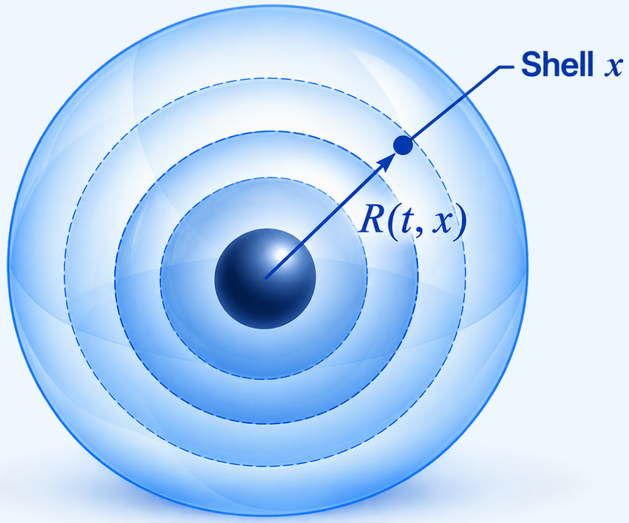
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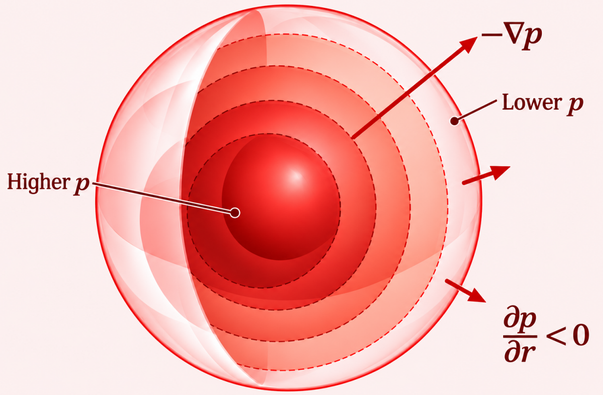
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Decoupling generally fails.

We preserve this independent-shell structure when introducing quantum gravity inspired modifications.

From shells to a polymerized Hamiltonian

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Liu, IS: 2604.27880

LTB geometry

$$ds^2 = -dt^2 + \frac{R'^2}{1+\mathscr{E}(x)}dx^2 + R^2 d\Omega^2$$

$$R = R(t,x)$$

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$$b := \frac{K_\phi}{\sqrt{E^x}} = \frac{K_\phi}{R}$$

Curvature momentum

$$v := \left(E^x\right)^{3/2} = R^3$$

Volume variable

$$\mathscr{R}_1 := \frac{\mathscr{E}(x)}{R^2}$$

Intrinsic curvature

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Preserve decoupling and spatial covariance

$$H = vH_0(b, \mathscr{R}_1)$$

- The same H_0 governs every shell.
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Each shell remains an independent system.

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Set the reference-dust density to zero.

$$\rho_{\text{dust}} = 0 \Rightarrow \partial_x H = 0$$

$$H = vH_0 = 2m$$

- In vacuum, m is constant in time and across shells.
- The dust defines time, with $\rho_{\text{dust}} = 0$

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→ **Vacuum relation:** $H_0 = \dfrac{2m}{R^3}$

Reconstructing the static vacuum

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Static geometry

$$ds^2 = -f(r)dt_s^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 \qquad r = R(t,x)$$

Reconstructing the static vacuum

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1 Compatibility

Require a static vacuum independent of the shell energy $\mathscr{E}(x)$.

$$\frac{1}{4}\left(\partial_b H_0\right)^2 = \mathscr{R}_1 + \frac{1-f(m,r)}{r^2}$$

Define the model function

$$\tilde{f}\left(H_0\right) := \frac{1-f(m,r)}{r^2}$$

The static reconstruction restricts this dependence.

Reconstructing the static vacuum

Liu, IS: 2604.27880

Static geometry

$$ds^2 = -f(r)dt_s^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 \qquad r = R(t,x)$$

1 Compatibility

Require a static vacuum independent of the shell energy $\mathscr{E}(x)$.

$$\frac{1}{4}\left(\partial_b H_0\right)^2 = \mathscr{R}_1 + \frac{1-f(m,r)}{r^2}$$

Define the model function

$$\tilde{f}\left(H_0\right) := \frac{1-f(m,r)}{r^2}$$

The static reconstruction restricts this dependence.

2 Hamiltonian reconstruction

Choose an admissible model function $\tilde{f}(X)$.

$$\left(\partial_b H_0\right)^2 = 4\left[\mathscr{R}_1 + \tilde{f}\left(H_0\right)\right].$$

Solve for $H_0(b, \mathscr{R}_1)$ at fixed $\mathscr{R}_1$.

Shell Hamiltonian: $H = vH_0$

Reconstructing the static vacuum

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→ Fix $\tilde{f}$ to choose the theory. Then m labels its vacuum solutions.

Regularization cores

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Smooth core limit

$$\tilde{f}(X) \longrightarrow F_0, \quad X = \frac{2m}{r^3} \longrightarrow \infty$$

$$f(r) \simeq 1 - F_0 r^2 \quad (r \to 0)$$

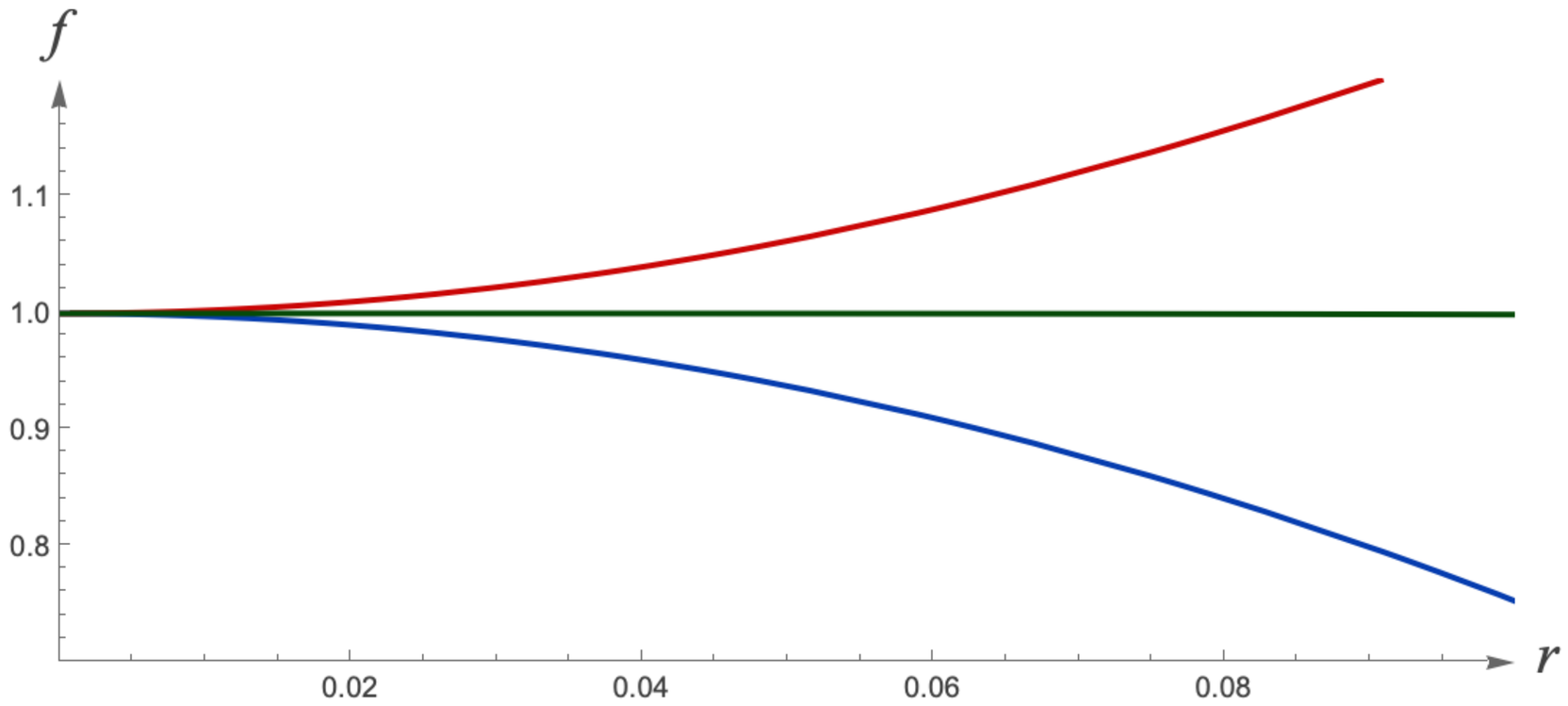
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anti-de Sitter core

$F_0 < 0$

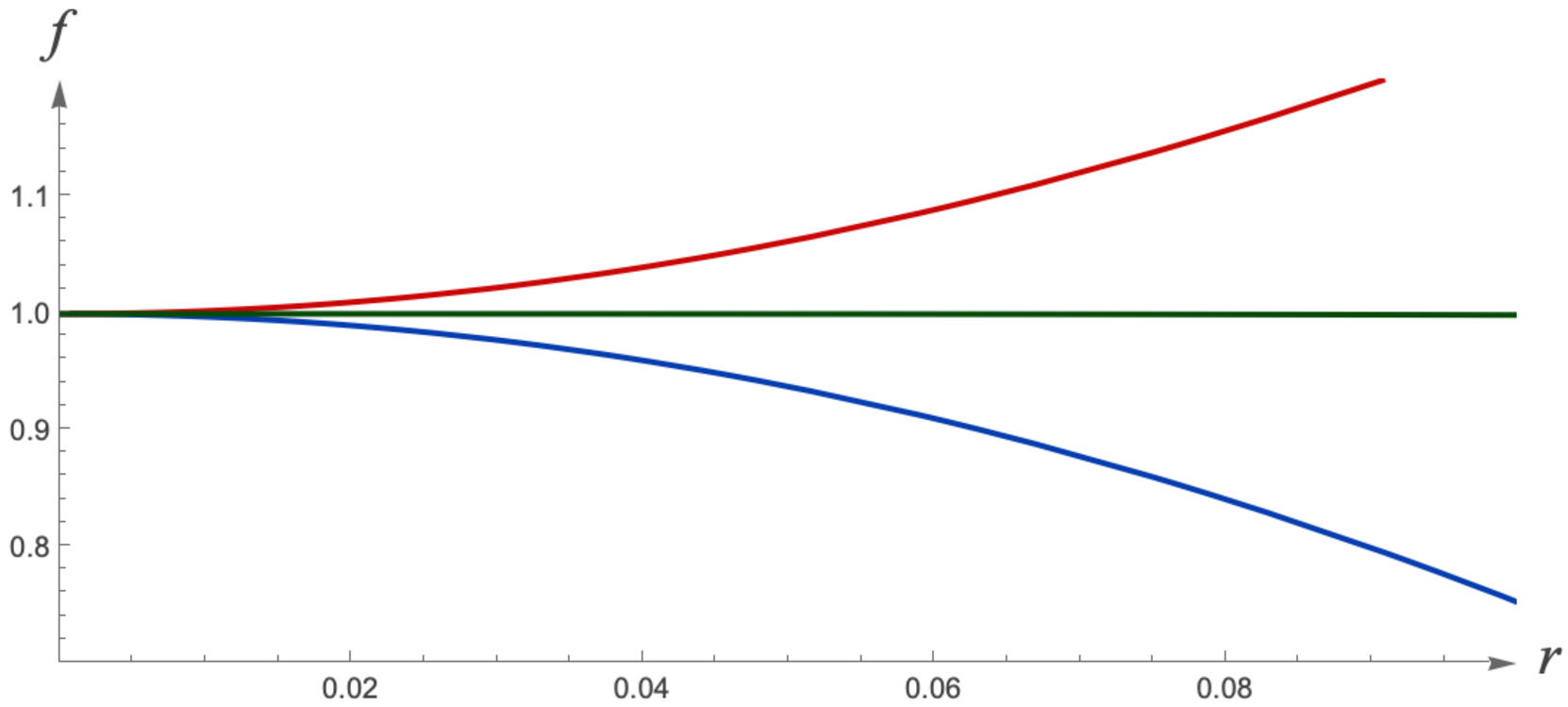
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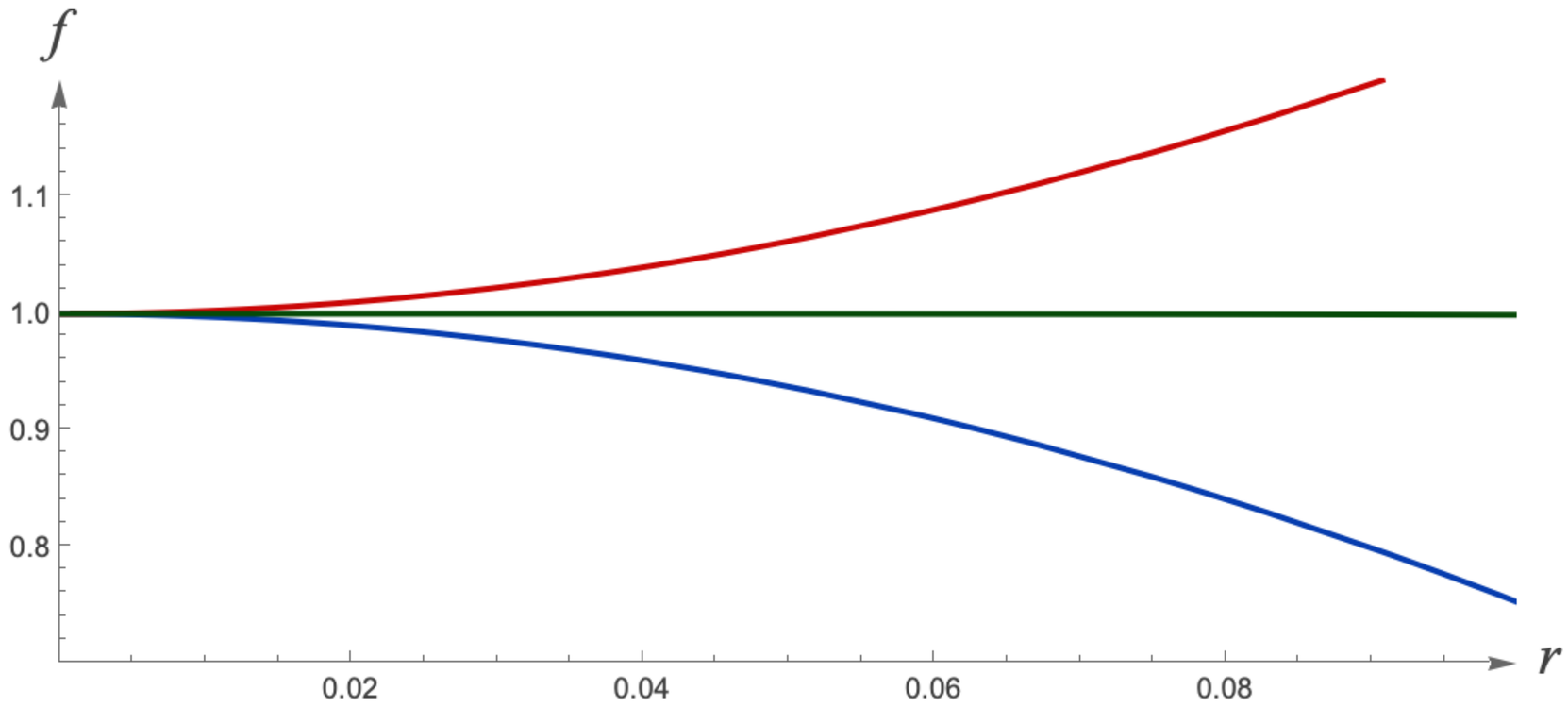
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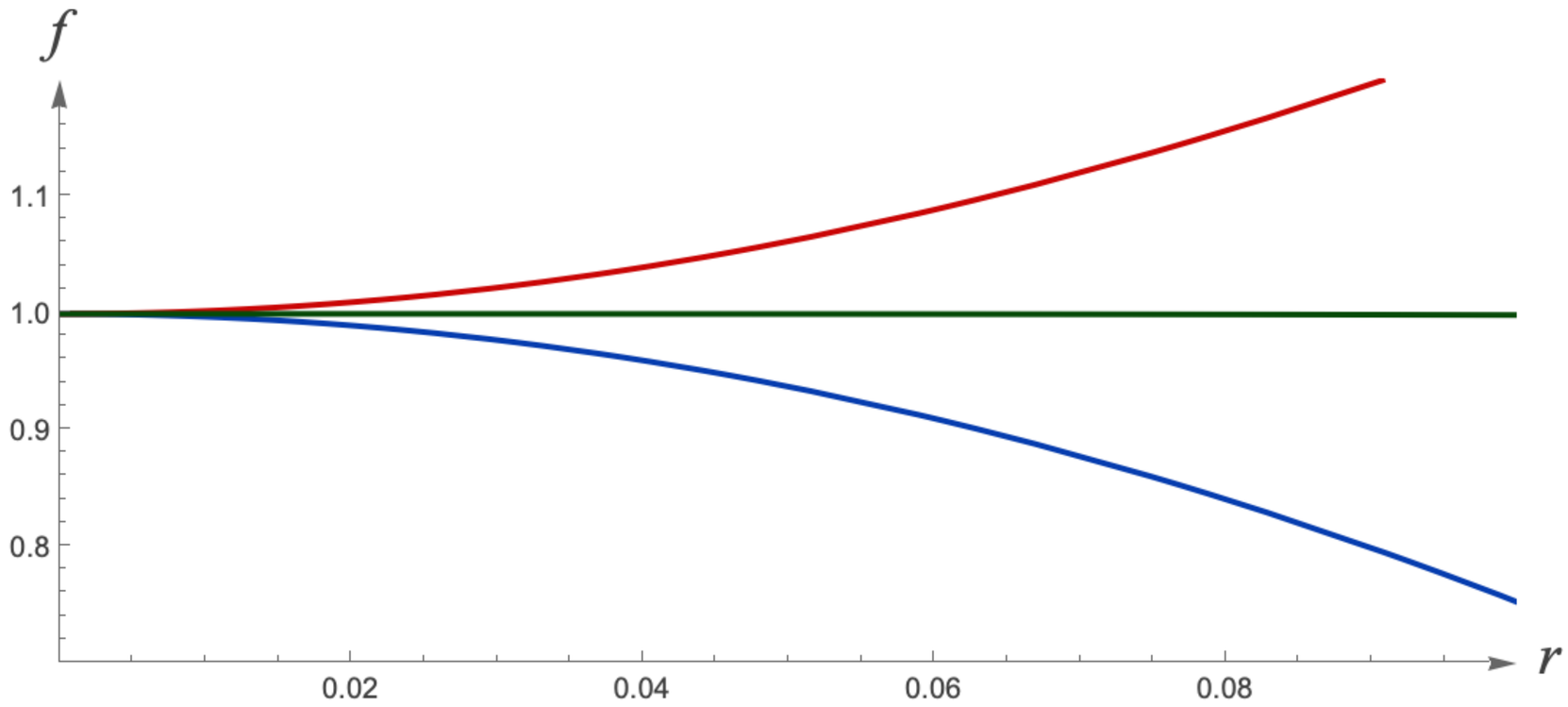
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The sign of F_0 determines the core geometry.

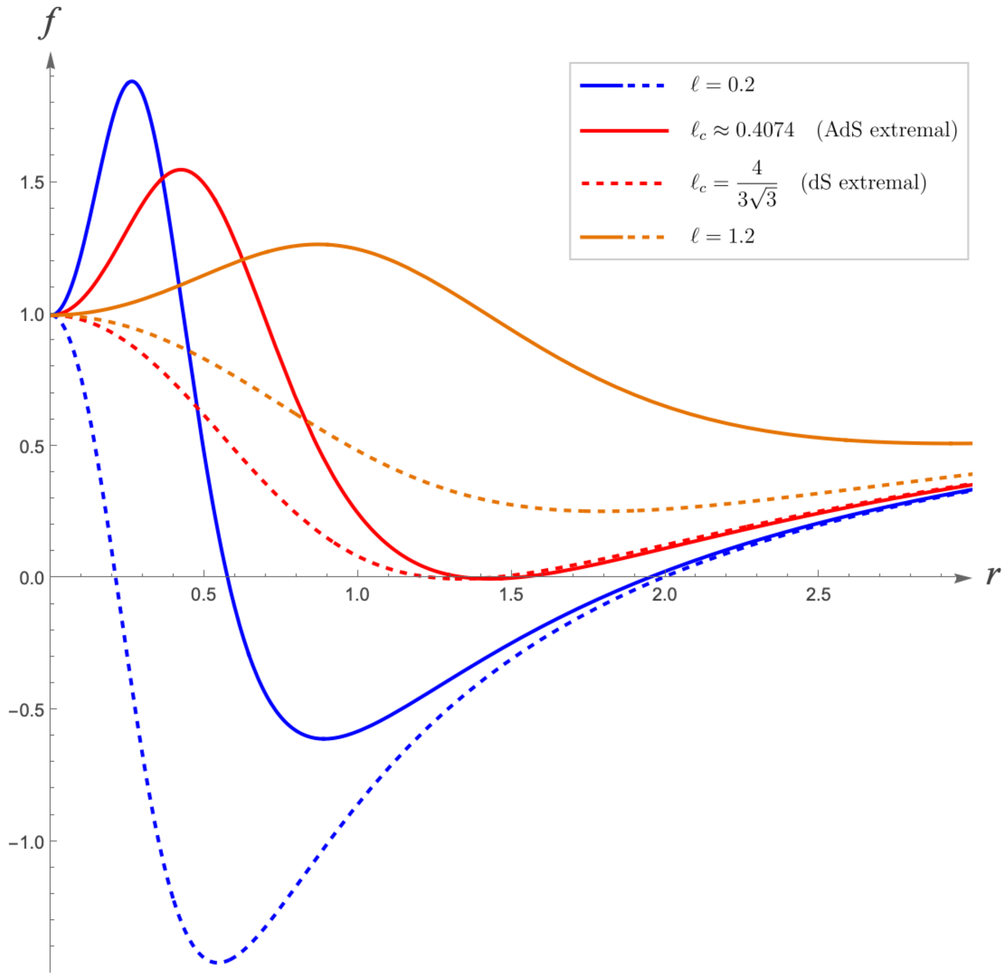
Static solutions: a Hayward example

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Vacuum reconstruction

$$f(r) = 1 - r^2 \tilde{f}_\epsilon \left(\frac{2m}{r^3} \right)$$



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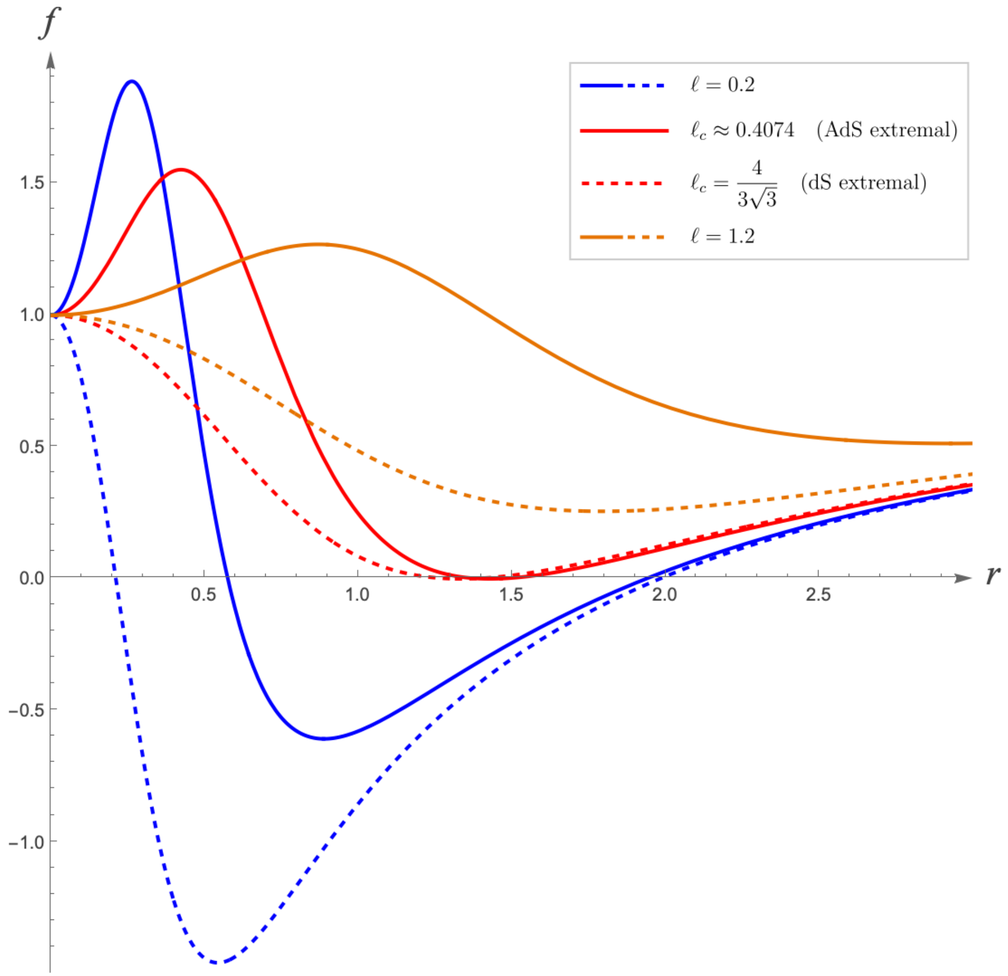
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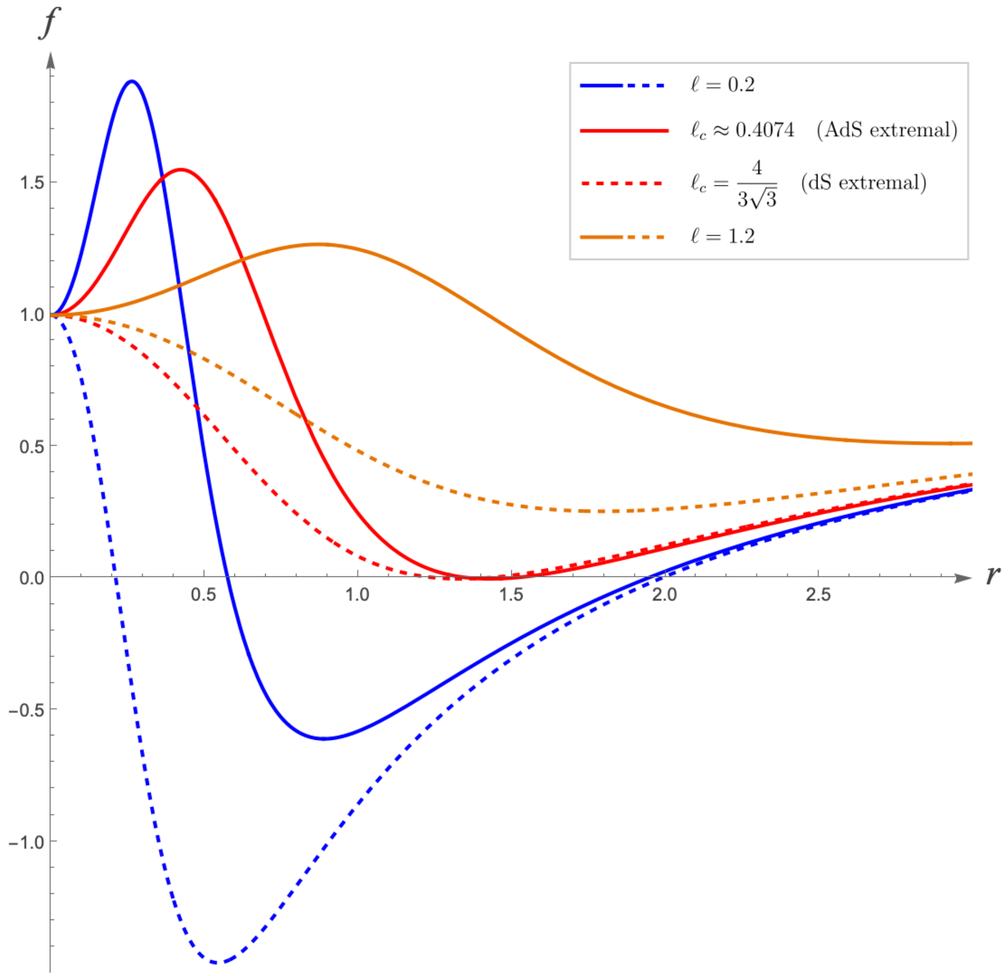
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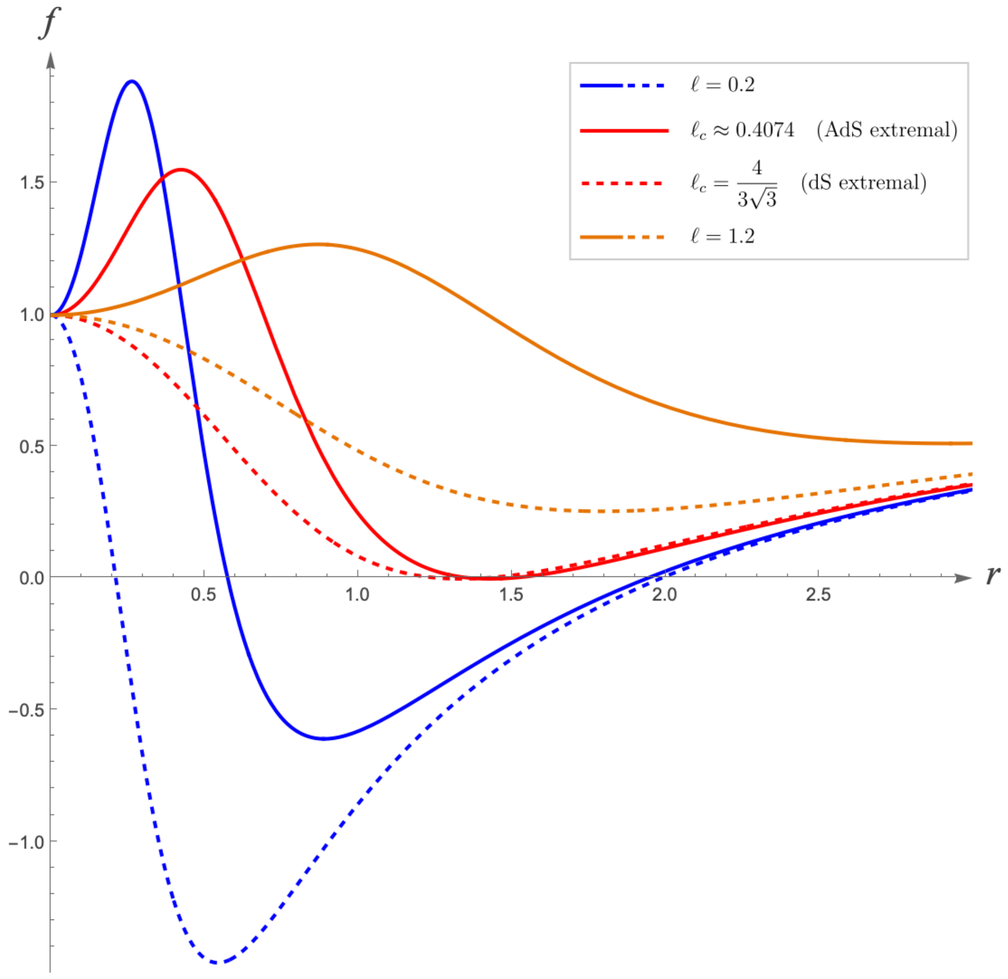
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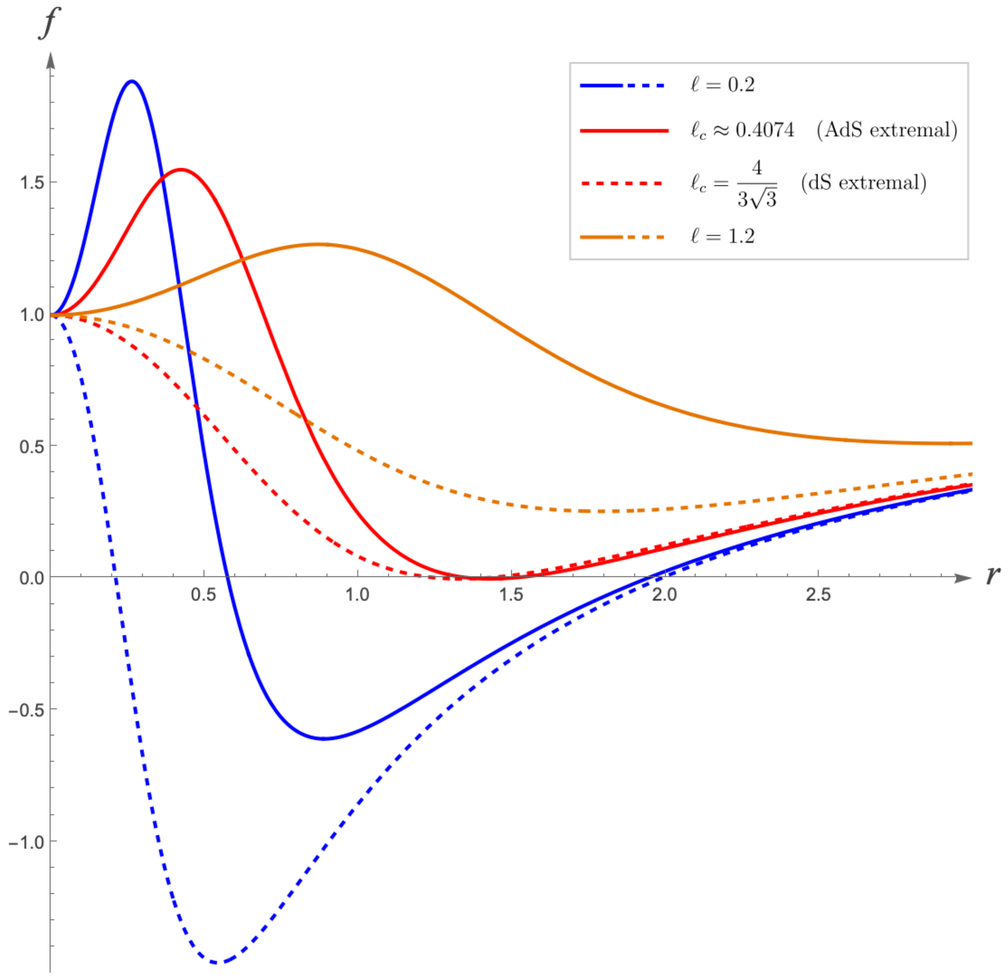
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→ Both core types admit black hole, extremal, and horizonless solutions.

Cosmological constant

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 Bakhoda, IS: 2606.13167

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Vacuum shells: $$\rho_{\rm dust}=0: \quad H_\Lambda = v\left(H_0-\frac{\Lambda}{3}\right)=2m \Longrightarrow H_0=\frac{2m}{r^3}+\frac{\Lambda}{3}\,.$$

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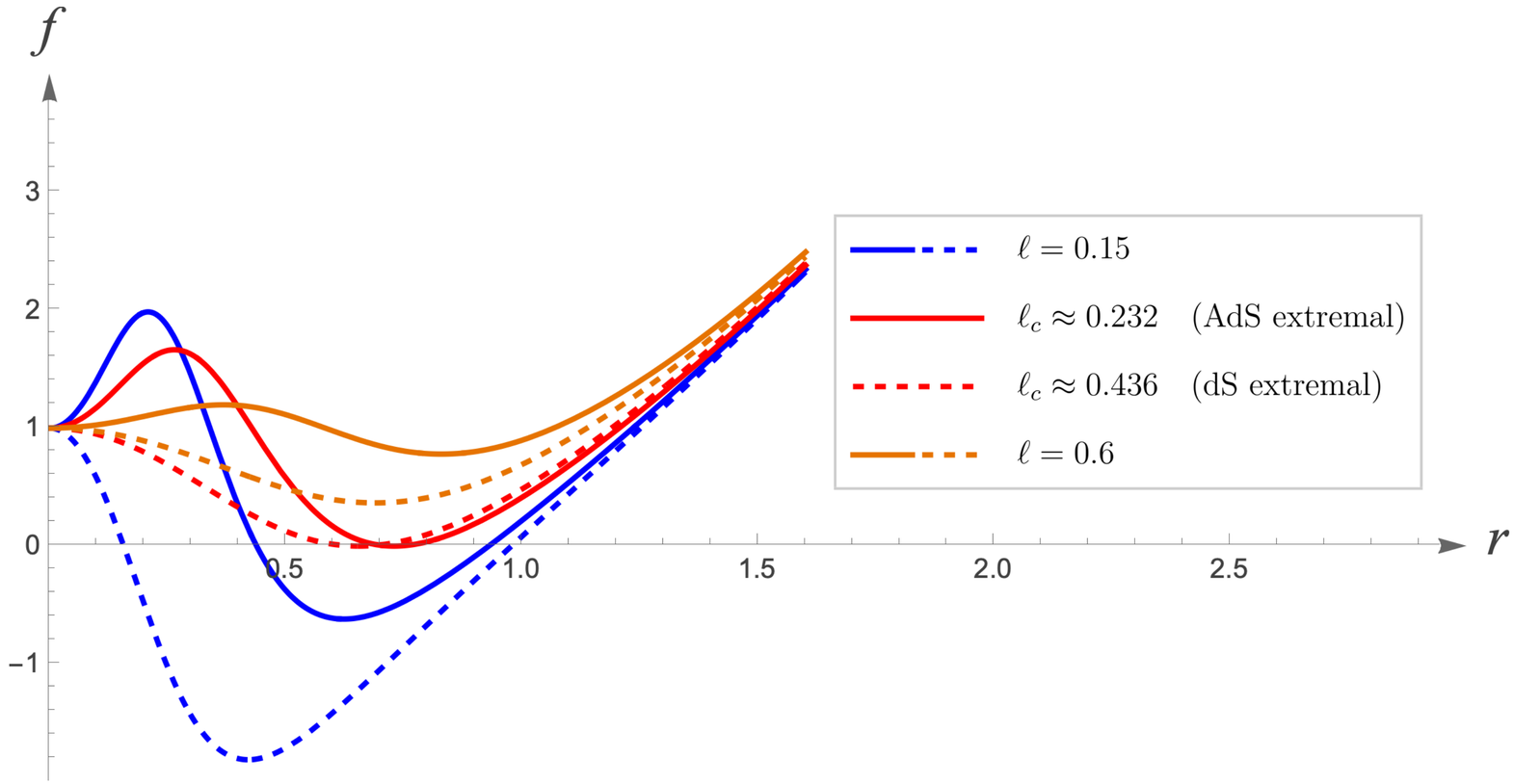
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Covariant completion

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Liu, IS: 2604.27880

Achour, Lamy, Liu, Noui: 1712.03876

Branchwise reconstruction in the spherical sector

1 Reconstruct the spatial Lagrangian

Invert the momentum-curvature relation.

$$\mathscr{K}_1 = K^\theta_\theta = \frac{1}{2}\partial_b H_0$$

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Legendre tranform on the chosen branch.

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Generalized extended mimetic gravity

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Modified metric equations

$$G^{(a)}_{\mu\nu} := G_{\mu\nu} - T^{(\phi)}_{\mu\nu} = -\lambda\partial_\mu\phi\partial_\nu\phi$$

Vacuum branch: $\lambda = 0 \Longrightarrow G^{(a)}_{\mu\nu} = G_{\mu\nu} - T^{(\phi)}_{\mu\nu} = 0.$

Zero dust density; the clock constraint remains.

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→ Regular vacuum geometry from modified gravity.



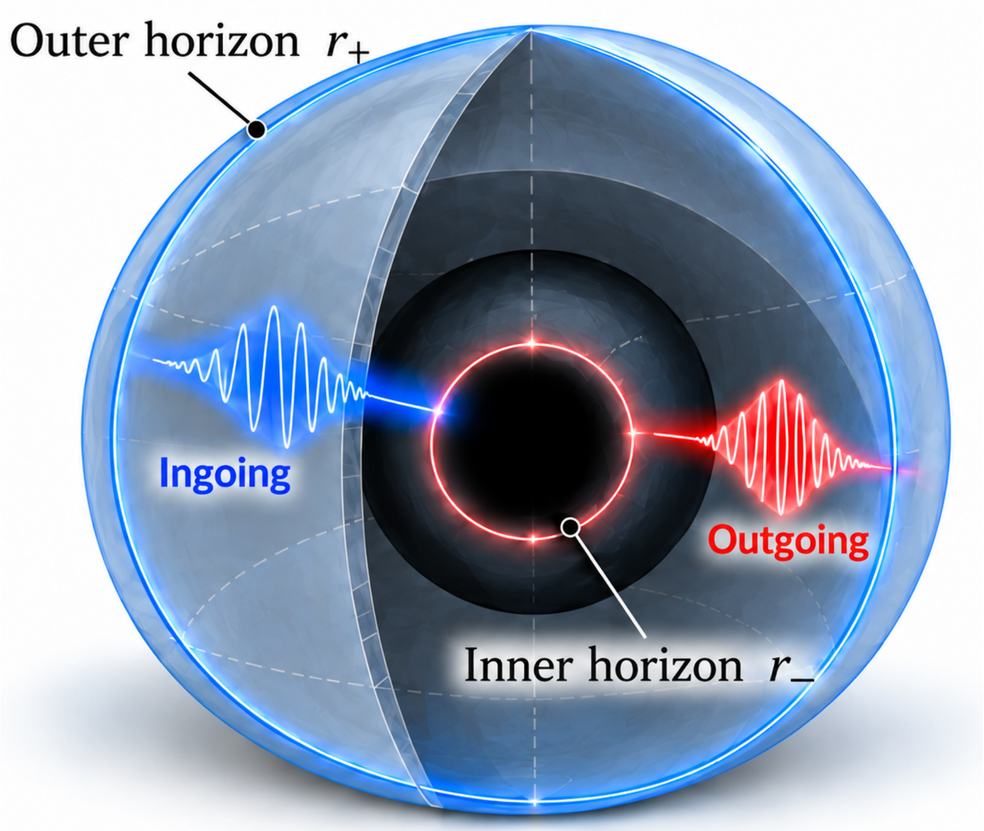
Part II

Mass inflation

Mass inflation

Mass inflation

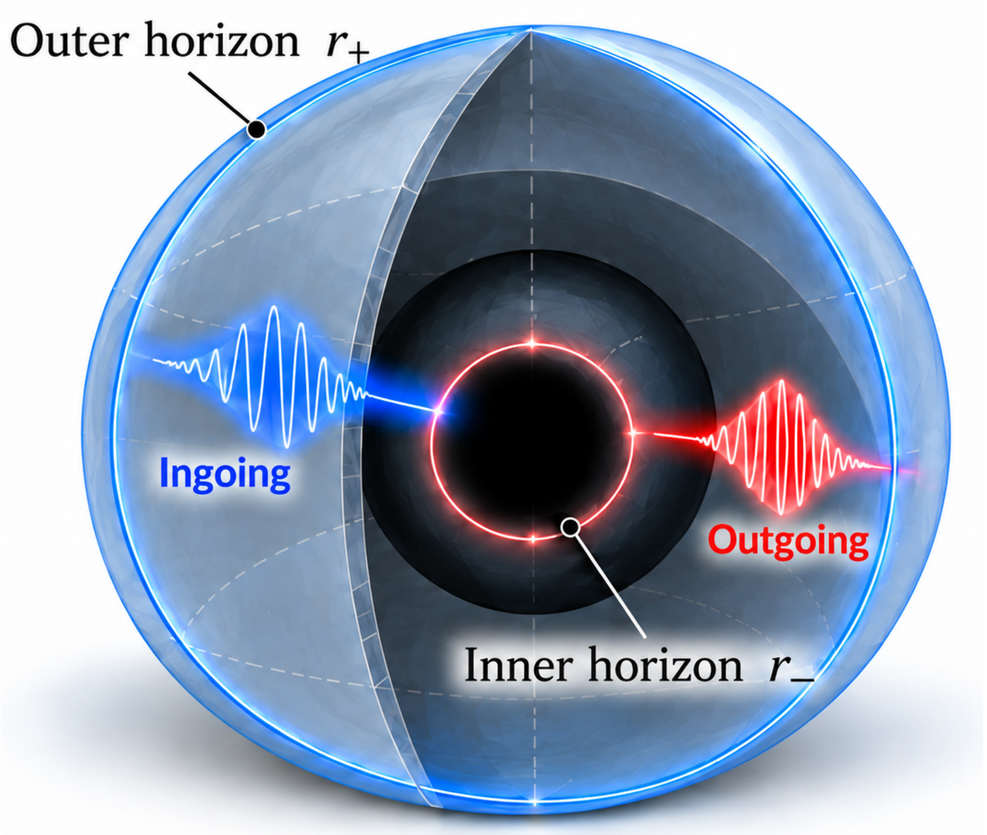
Counter-streaming radiation drives rapid growth of the local mass and curvature near the inner horizon.



Two radiation streams undergo strong relative blueshift near r_-

Mass inflation

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Two radiation streams undergo strong relative blueshift near r_-

Non-degenerate inner horizon: $\kappa_- \neq 0$

Inner horizon surface gravity: $|\kappa_-| = \frac{1}{2}|f'(r_-)|$

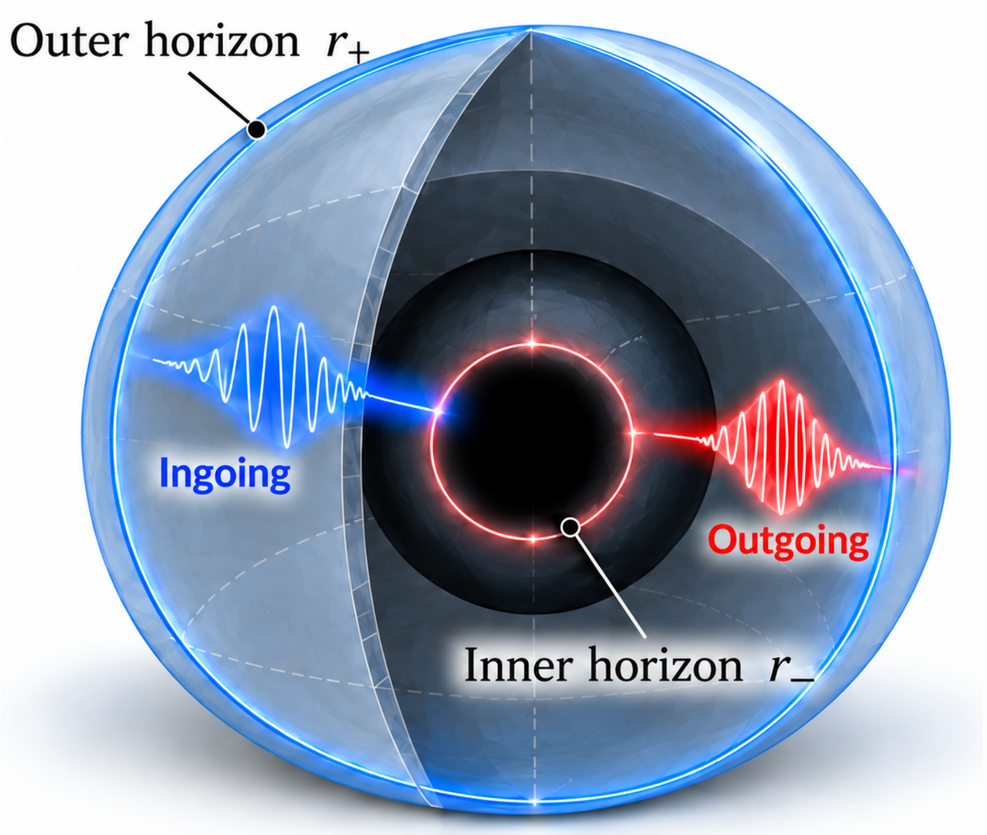
Local Misner-Sharp mass: $M_{\rm MS} = \frac{r}{2}(1-f)$

$$M_{\rm MS}(V) \sim A(V) e^{|\kappa_-|\Delta V}$$

- ΔV : elapsed advanced time
- $A(V)$ contain the decaying radiation tail

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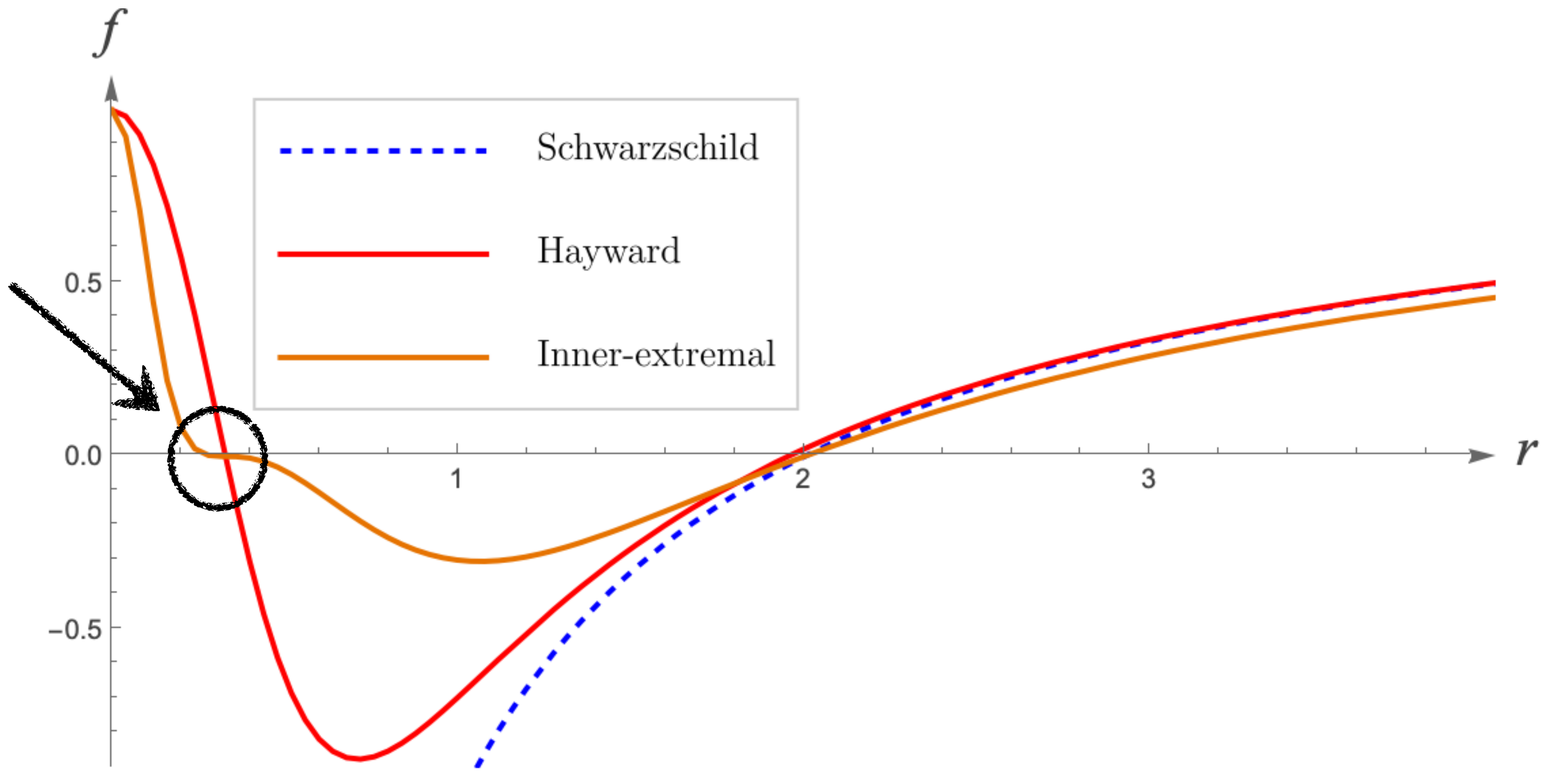
→ Can a degenerate inner horizon ($\kappa_- = 0$) suppress the growth?

Inner-extremal black holes

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Liu, IS: 2604.27897

Triple inner horizon $f(r_-) = f'(r_-) = f''(r_-) = 0, \quad f'''(r_-) \neq 0 \implies \kappa_- = 0$



Inner-extremal black holes

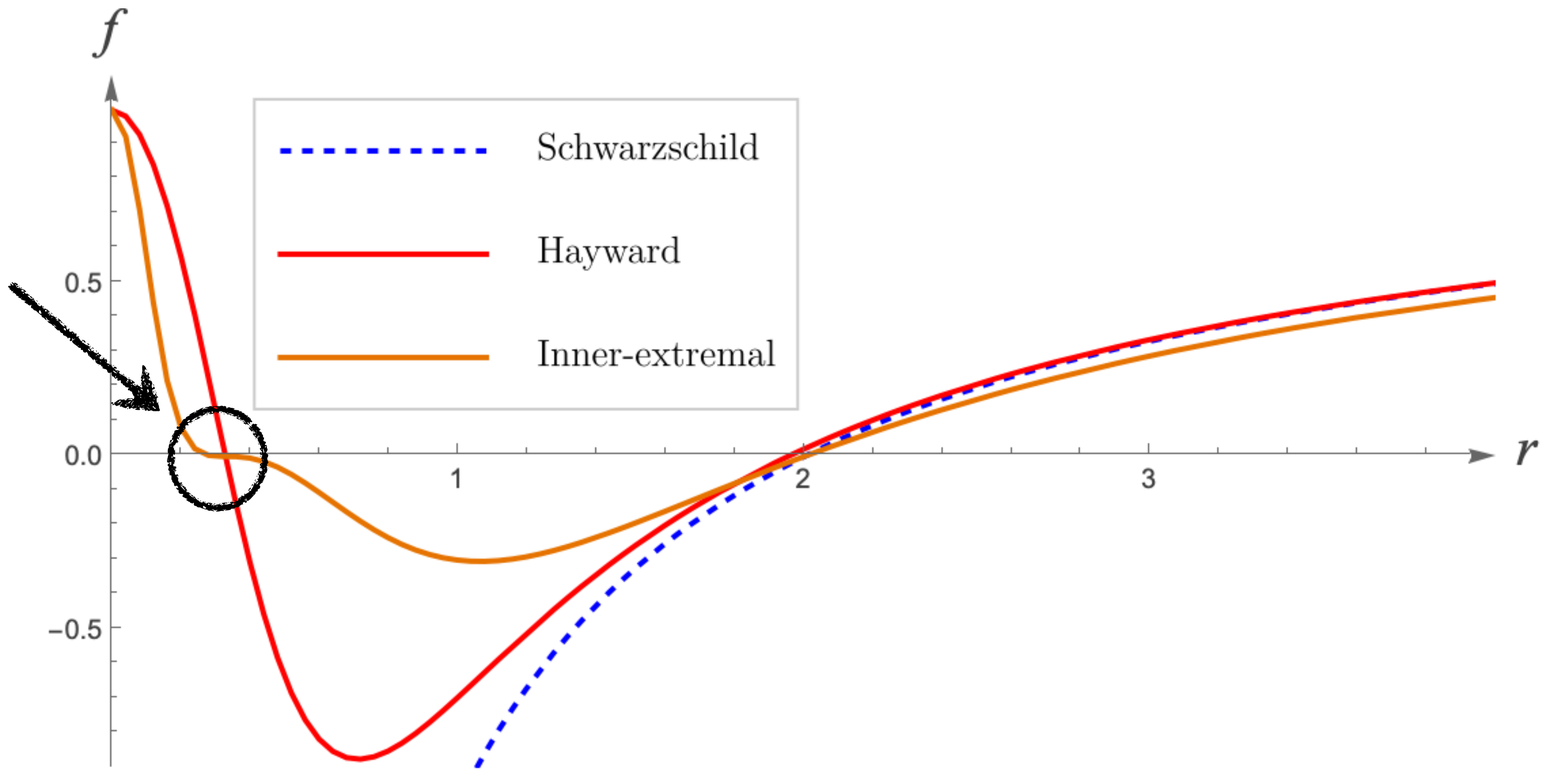
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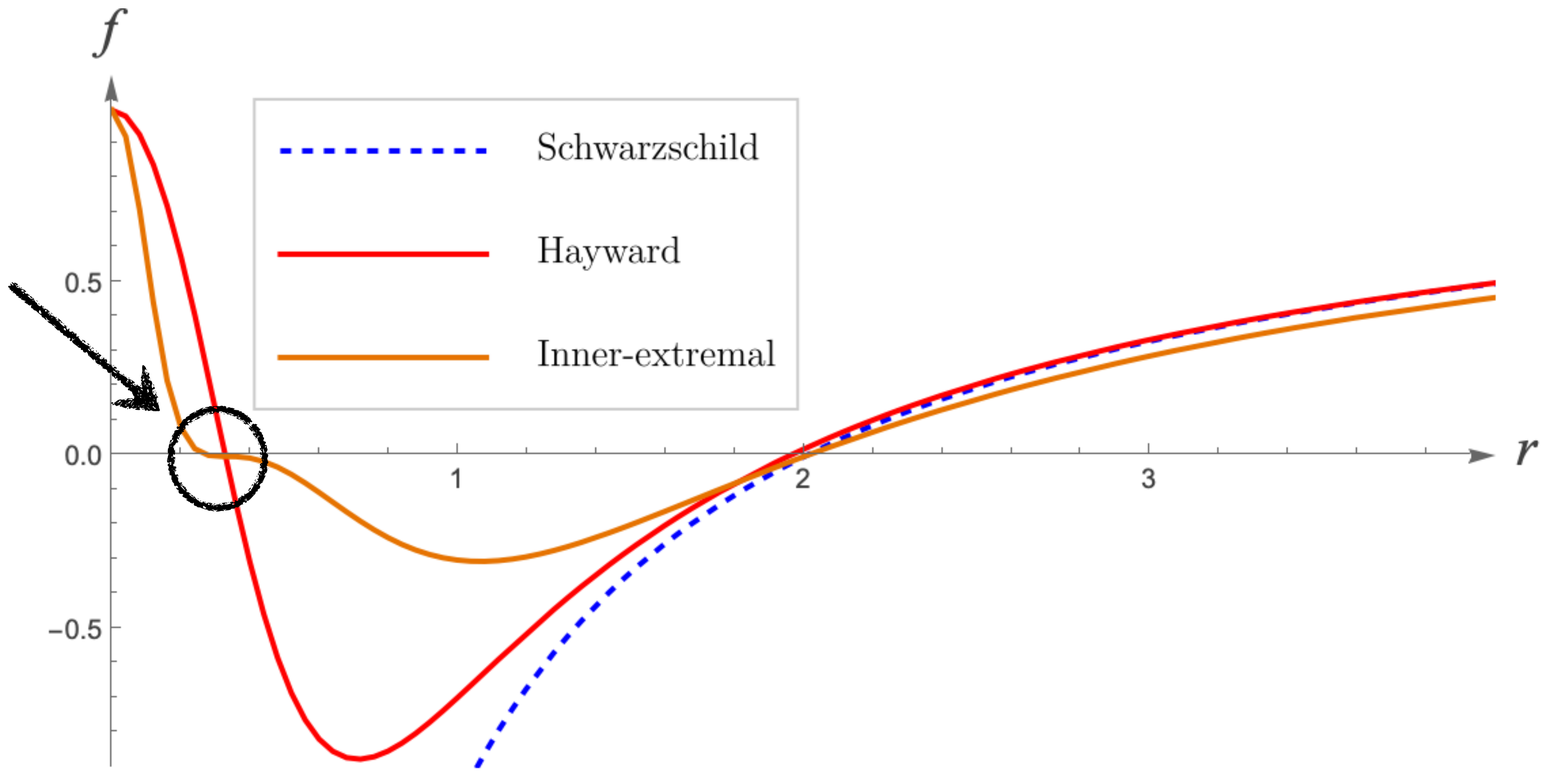
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$$f(r_+) = 0, \quad f'(r_+) \neq 0$$



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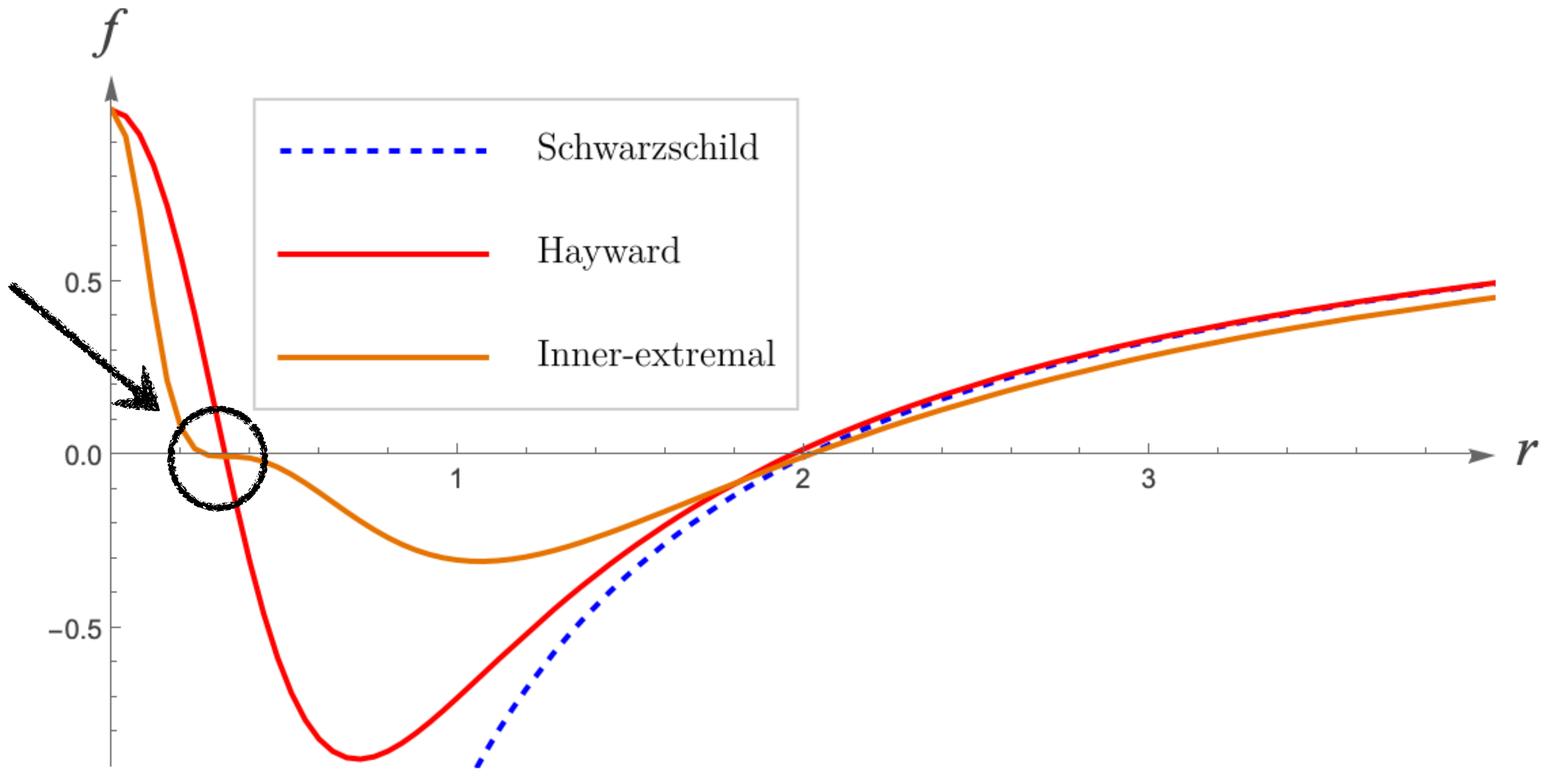
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Tuned mass at fixed theory

$$a_2 = a_2(b_1, b_2), \quad m = m_{\text{ext}}(b_1, b_2)$$

Only a tuned mass gives triple inner horizon. The theory coefficients are fixed during perturbations.



A null shell test for mass inflation

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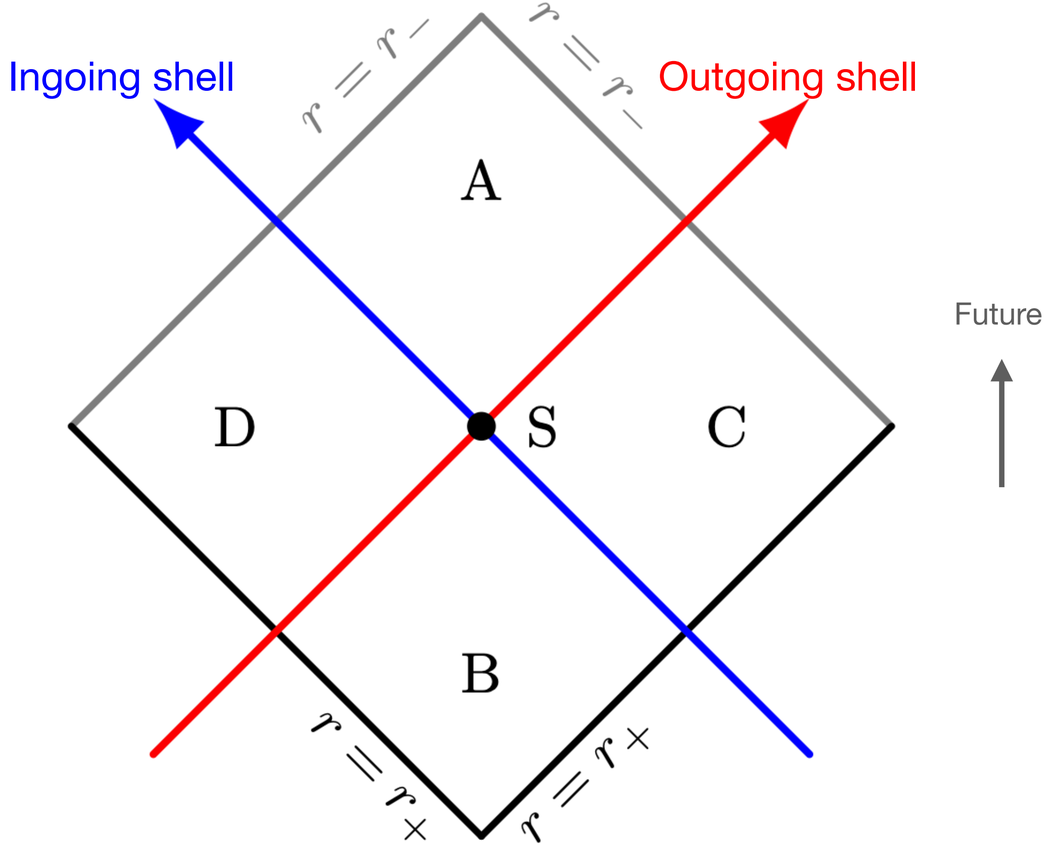
Liu, IS: 2604.27897

Small shells change m; the model function $\tilde{f}$ stays fixed

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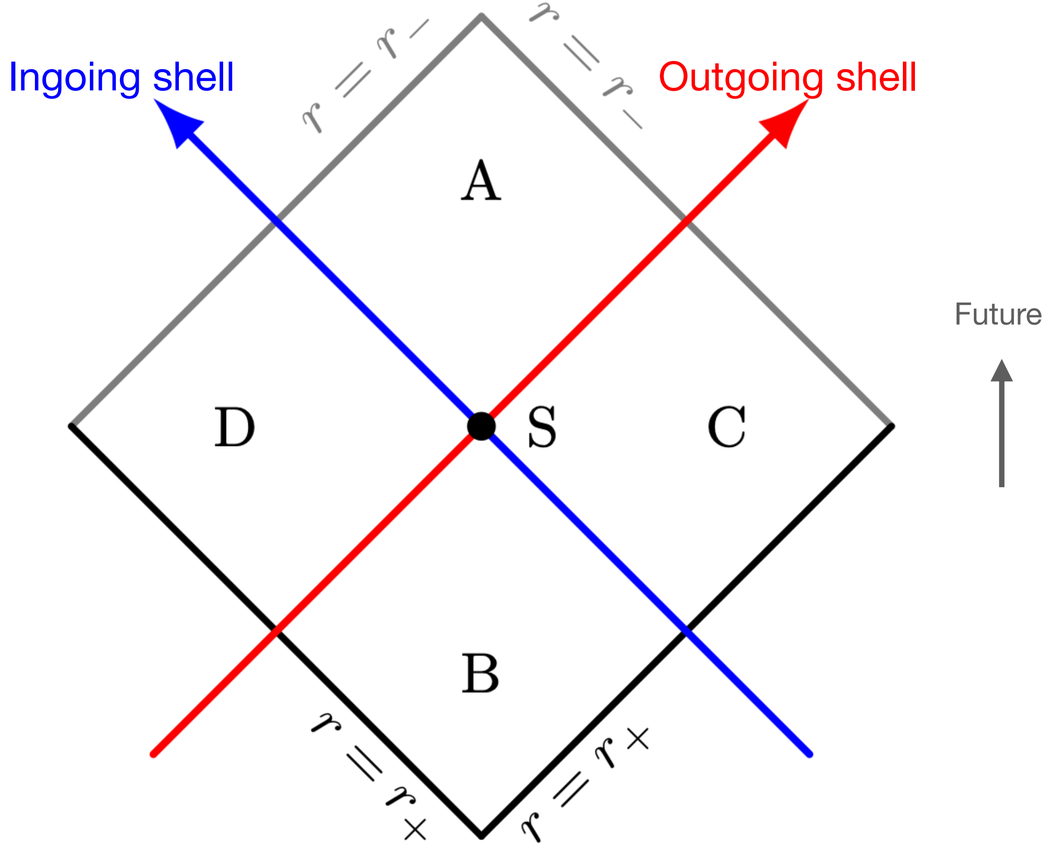
D: after outgoing shell

Effective null shell matching

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Effective null shell matching

1. Matching at the crossing

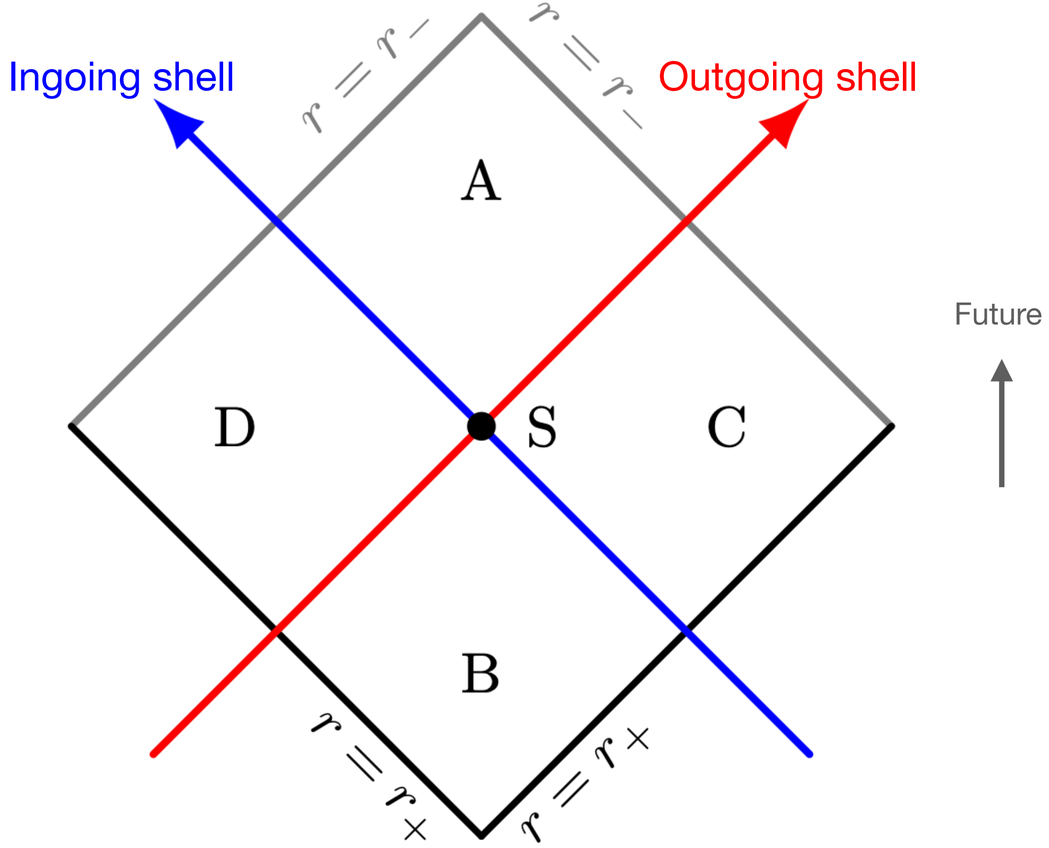
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$$f_B \propto x^k \qquad \delta_{C,D} \propto x^q \qquad x = \frac{r - r_-}{r_0} \to 0^+$$

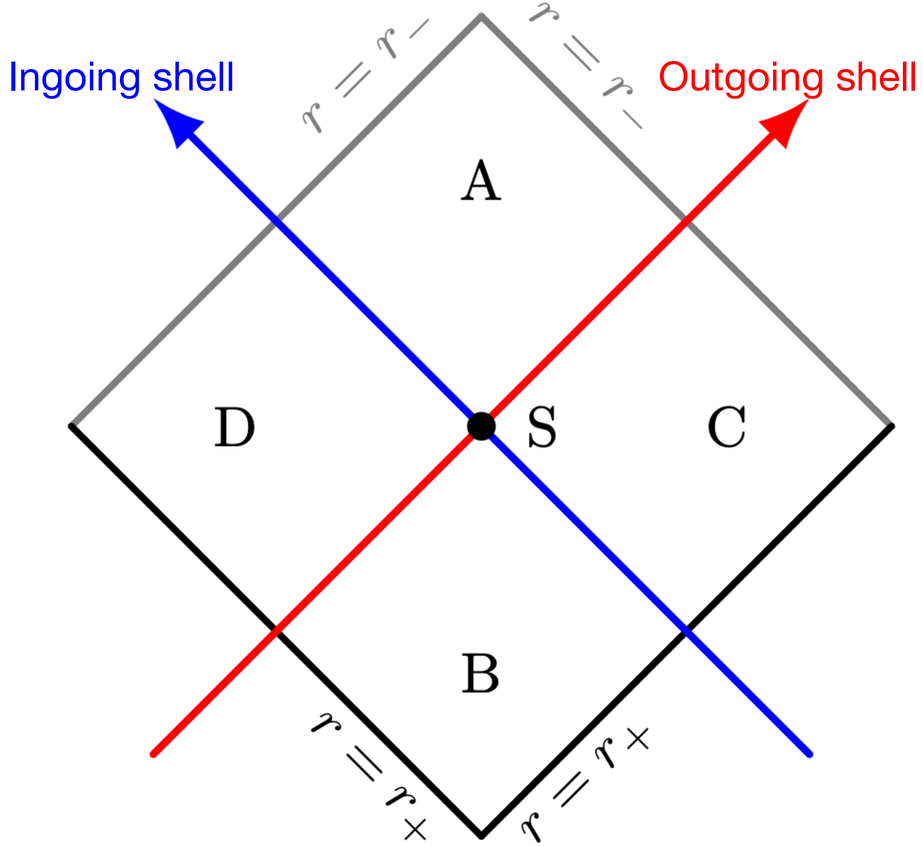
k : horizon-zero order

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3. Crossing contribution

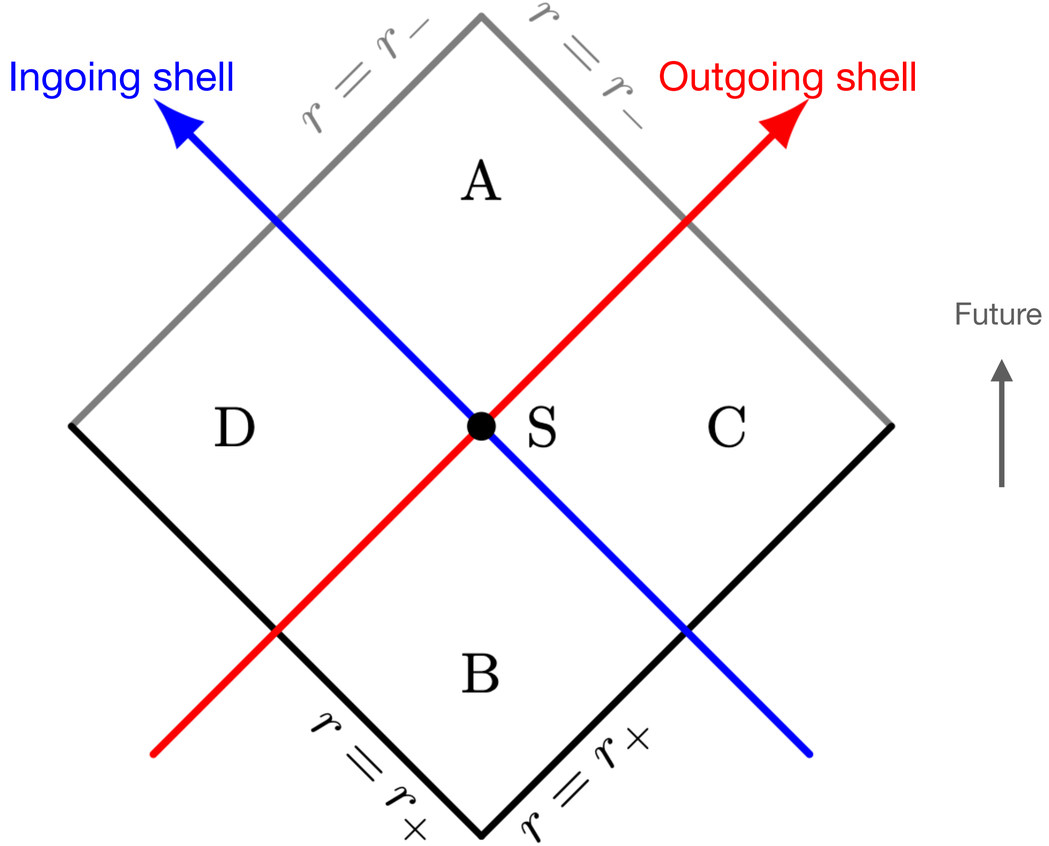
$$\mathcal{J} \equiv \frac{\delta_C \delta_D}{f_B} \propto x^{2q-k}$$

$2q - k < 0$	$2q - k = 0$	$2q - k > 0$
$\lvert\mathcal{J}\rvert$ diverges	Finite contribution	$\lvert\mathcal{J}\rvert$ vanishes

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The horizon order k alone does not determine the amplification

Hayward versus inner-extremal

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Fixed theory: shells change m; all model coefficients stay fixed

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Simple inner horizon

$(k, q) = (1,0)$

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Exponential geometric amplification

$$x \sim e^{-|\kappa_-|\Delta V}$$

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Hayward: where does the amplification become large?

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If $\ell \sim \ell_P$ and $m \gg \ell_P$, for small shells

Areal-radius estimate beyond the effective geometry regime

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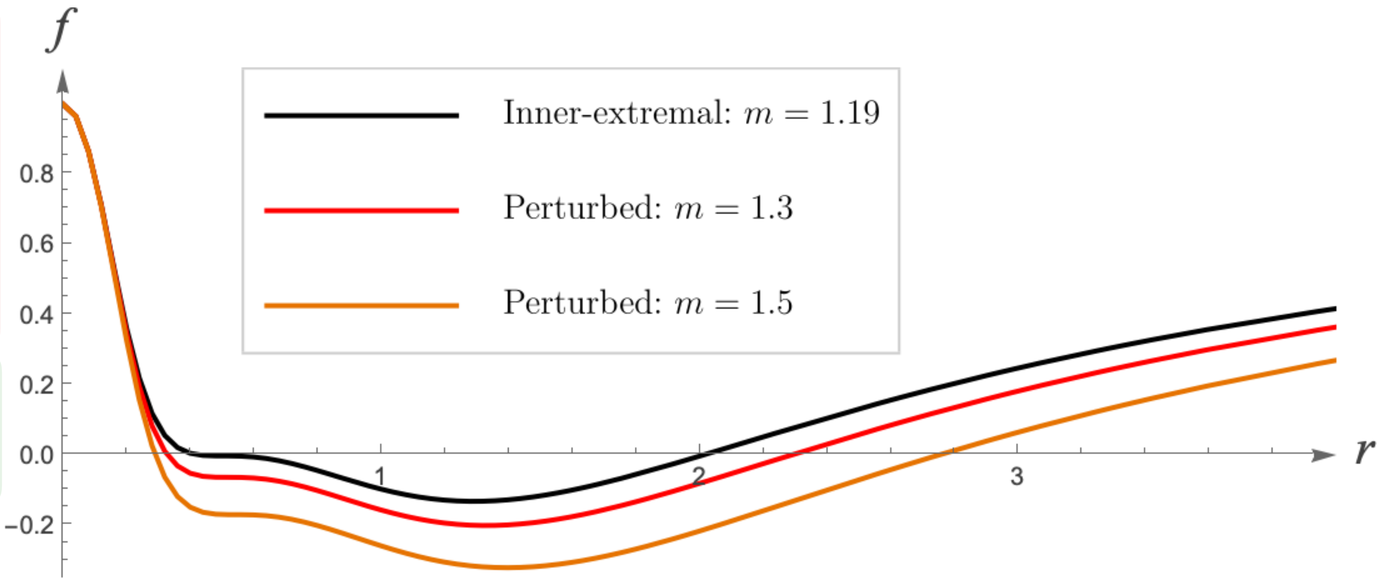
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Here the model coefficients vary with m to preserve the triple horizon



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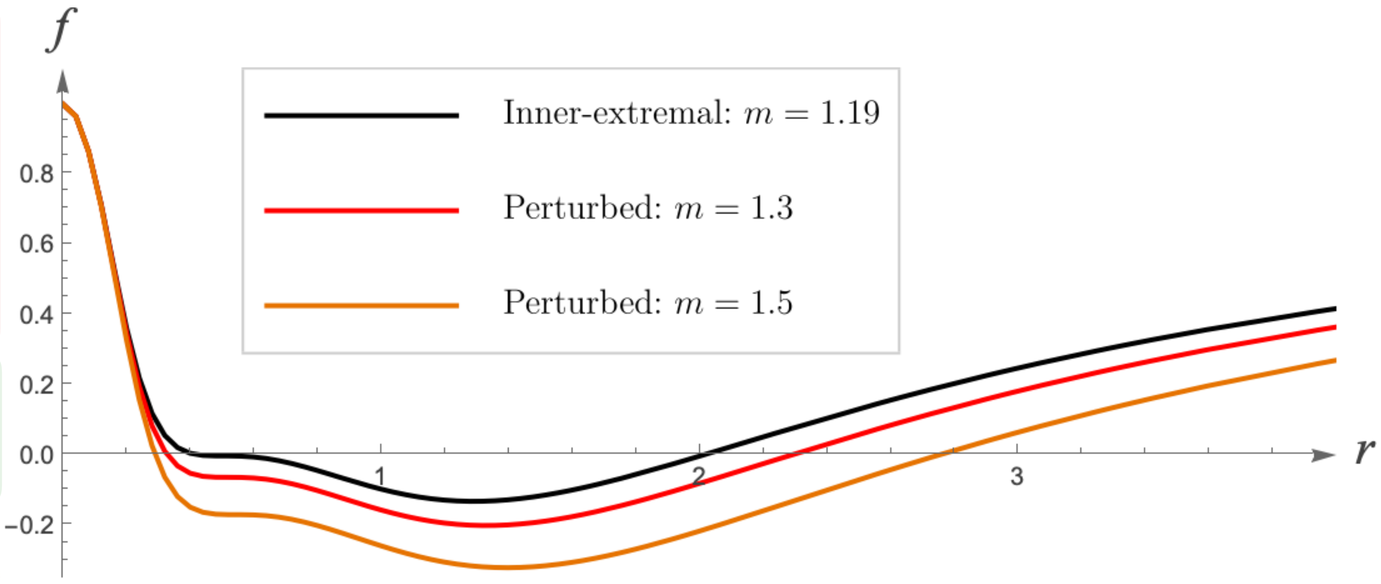
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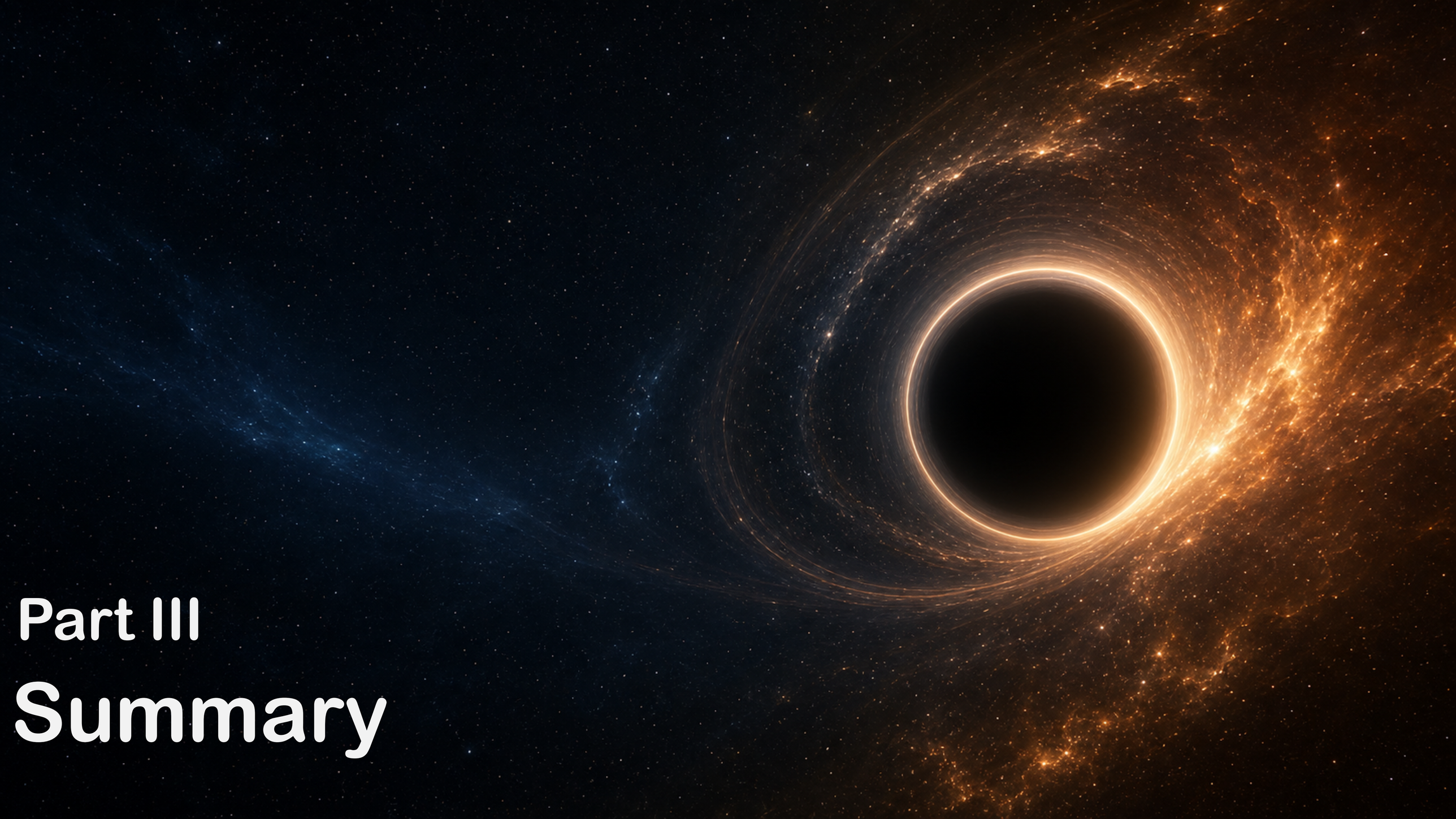
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$\kappa_- = 0$ changes the growth law; it does not by itself establish stability



Part III
Summary

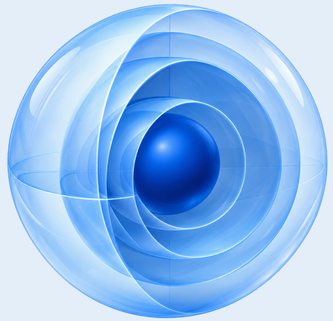
Regular black holes: geometry & stability

Regular black holes: geometry & stability

01

Vacuum reconstruction

Decoupled shells yield a universal vacuum reconstruction.



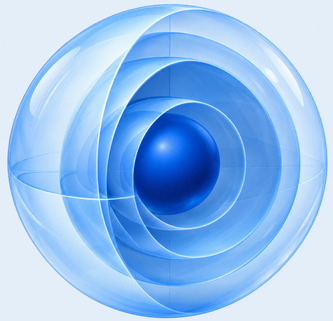
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Fixed theory · mass labeled solutions

Regular black holes: geometry & stability

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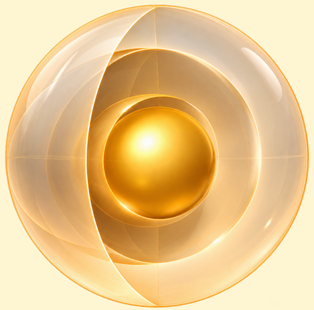


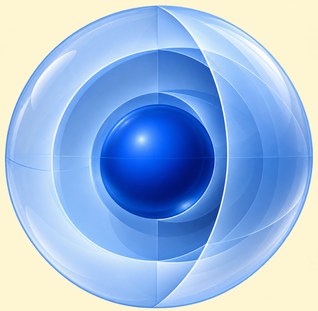
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02 Cores and asymptotic

Regular dS and AdS cores with extensions to nonzero Λ

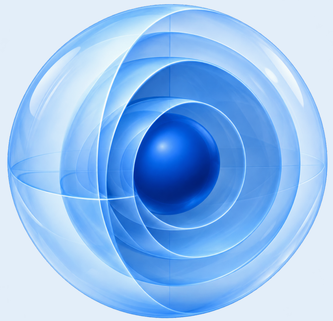




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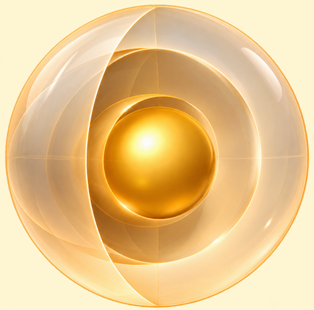


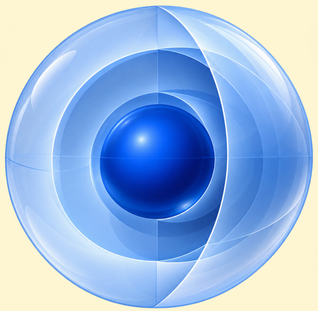
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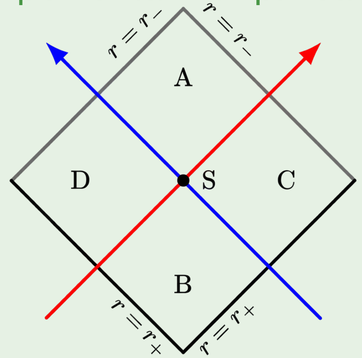
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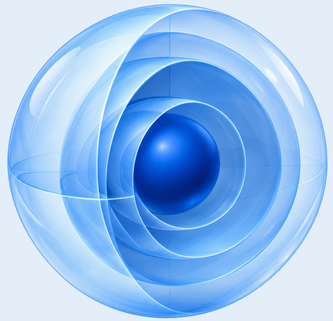
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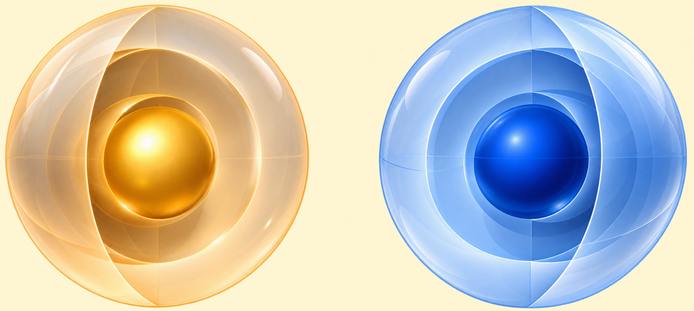


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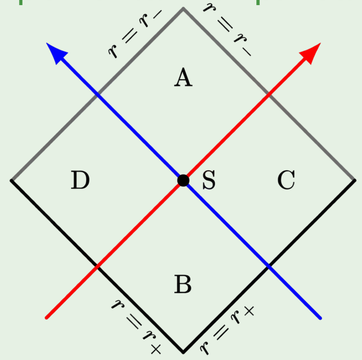
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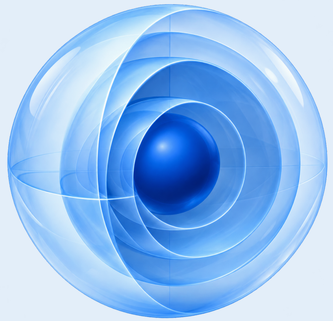
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Both must be assessed within the same effective theory.

Regular black holes: geometry & stability

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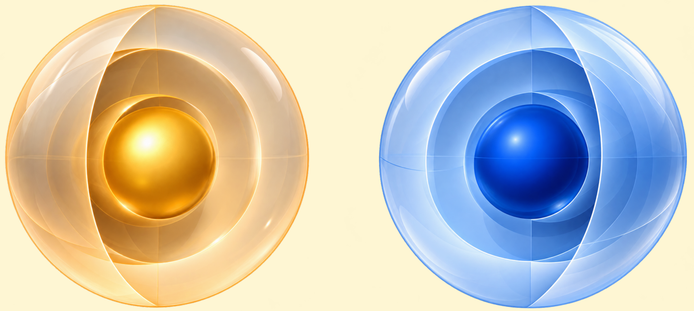


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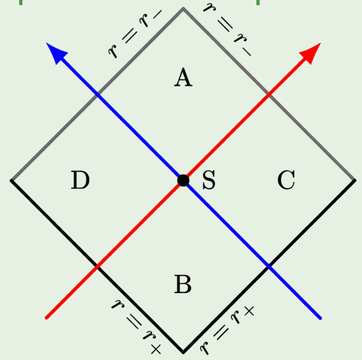
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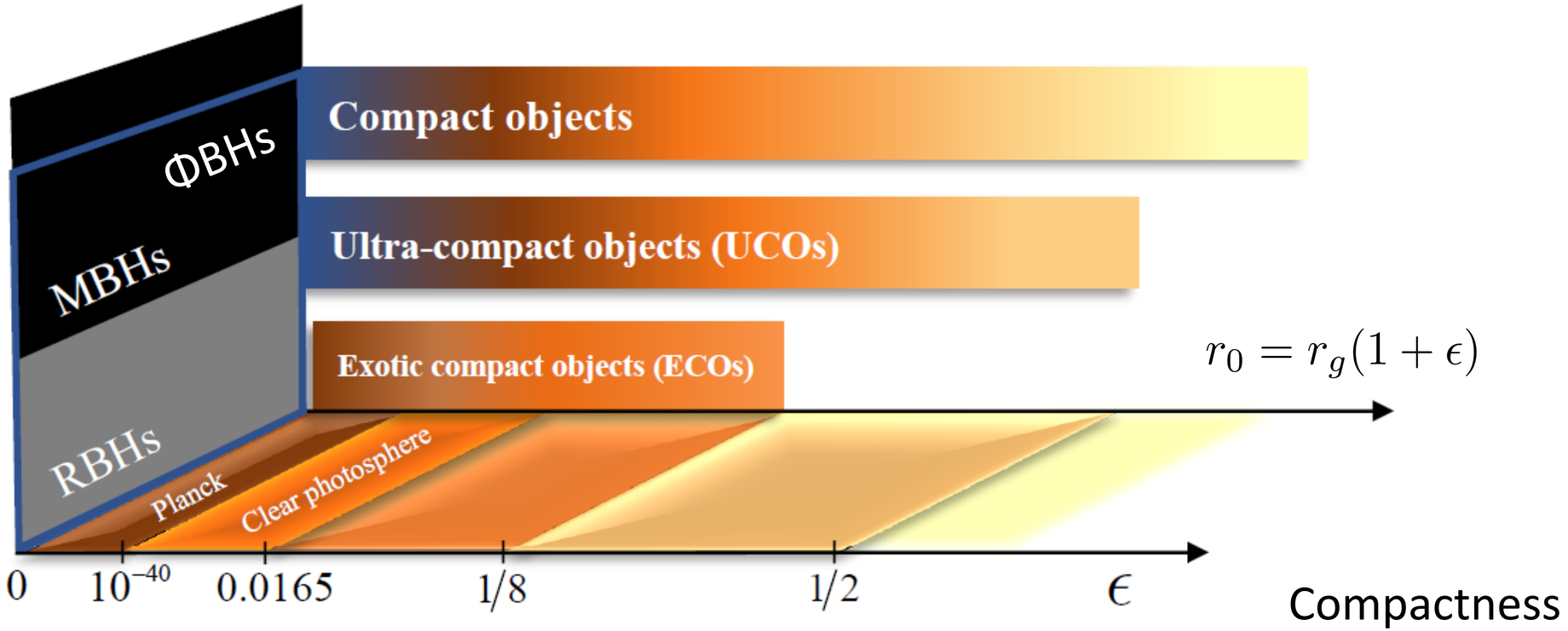
Thank you!

- Vacuum reconstruction: Liu & Soranidis · Phys. Rev. D **114**, 044066 (2026) · arXiv:2604.27880
- Mass inflation: Liu & Soranidis · Phys. Rev. D **114**, 084049 (2026) · arXiv:2604.27897
- AdS thermodynamics: Bakhoda & Soranidis · Phys. Rev. D **114**, 064063 (2026) · arXiv:2606.13167

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Back-up
Slides

Zoo of ultracompact objects

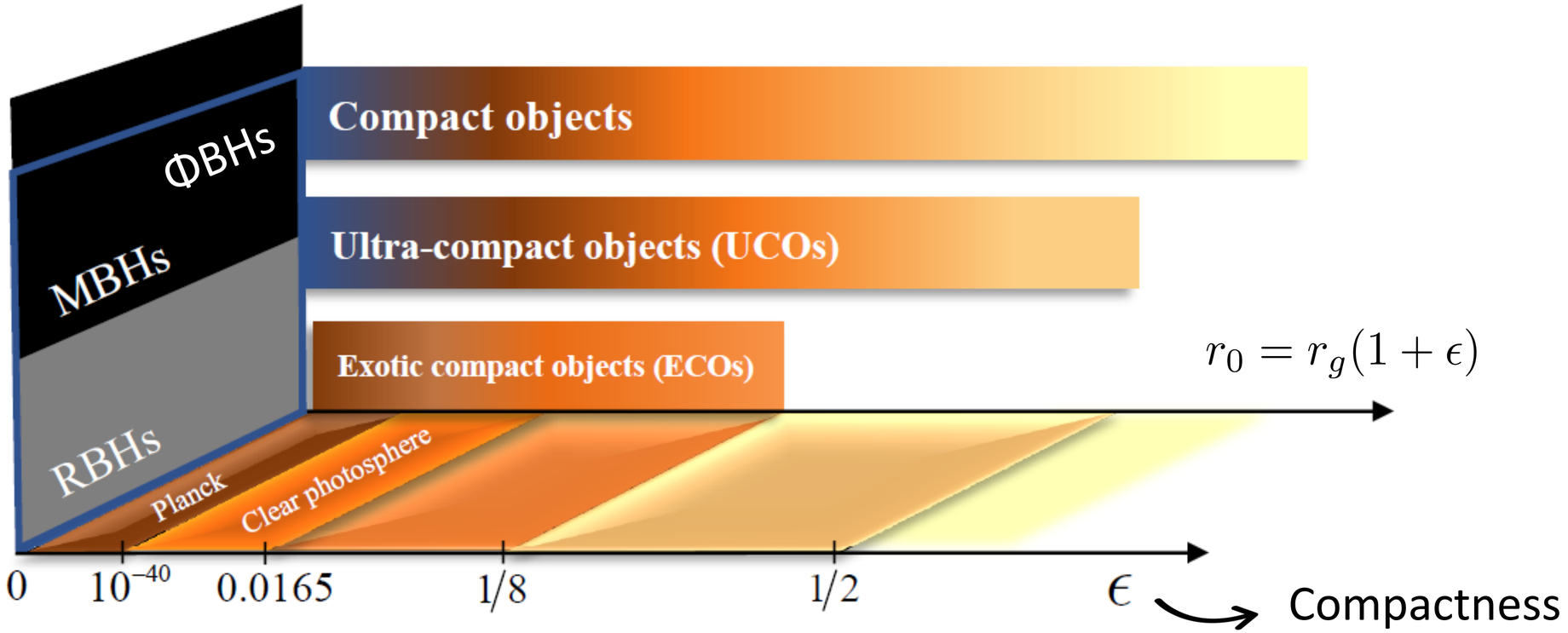


- Perpetual ongoing collapse with an asymptotic horizon $(\epsilon \to 0 \quad \text{as} \quad t \to \infty)$
- Formation of a transient or an asymptotic object, where the compactness reaches a minimum at some finite asymptotic [=distant observer] time $(\epsilon \to \epsilon_{\min} > 0 \quad \text{as} \quad t \to t_{\min} \leq \infty)$
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Cardoso, Pani: 1904.05363

Mann, Murk, Terno: 2112.06515



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Liu, IS: 2604.27897

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Giesel, Liu: 2512.24960

Liu, IS: 2604.27880

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Shell description

Main framework

Main framework

Giesel, Liu: 2512.24960

$$ds^2 = -dt^2 + \frac{1}{1+\mathscr{E}}(dr - \dot{R}dt)^2 + r^2 d\Omega^2 \qquad dr = \dot{R}dt - R'dx$$

$$N^r = -\dot{R}$$

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Giesel, Liu: 2512.24960

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$$ds^2 = -\frac{(1+\mathscr{E}-(N^r)^2)}{(1+\mathscr{E})(\partial_r t_s)^2}dt_s^2 + \frac{1}{1+\mathscr{E}-(N^r)^2}dr^2 + r^2 d\Omega^2 \qquad (N^r)^2 = \mathscr{E} + 1 - f$$

Main framework

Giesel, Liu: 2512.24960

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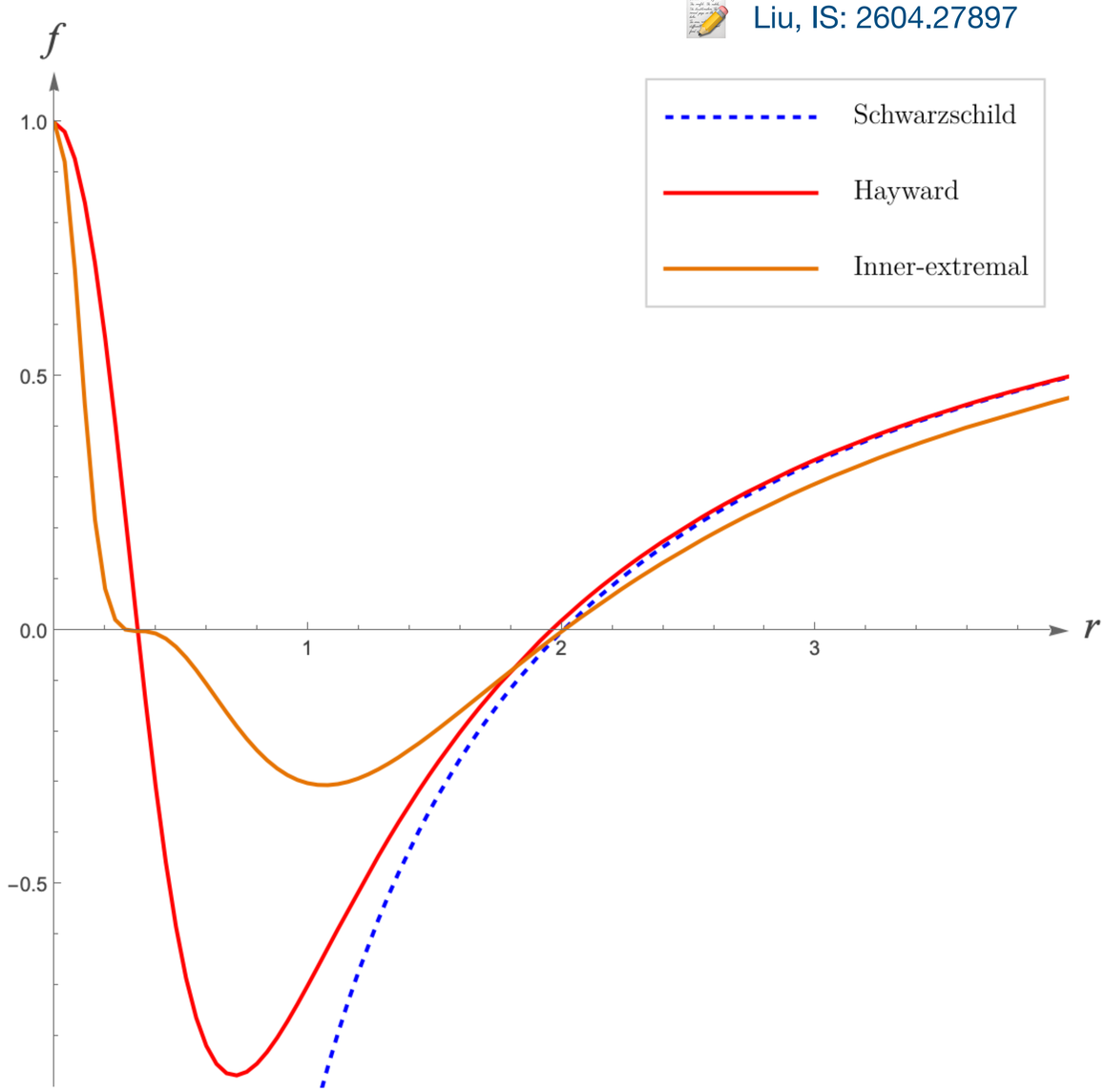
$$N^r = -\frac{1}{2r^2}\partial_b H_s \Rightarrow (\partial_b H)^2 = 4r^2(1+\mathscr{E}-f) \Rightarrow \partial_b H_0 = 2\left(\mathscr{R}_1 + \frac{1-f}{r^2}\right)^{1/2}$$

Inner-extremal black holes

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$$f(r_+) = 0, \quad f(r_-) = f'(r_-) = f''(r_-) = 0$$

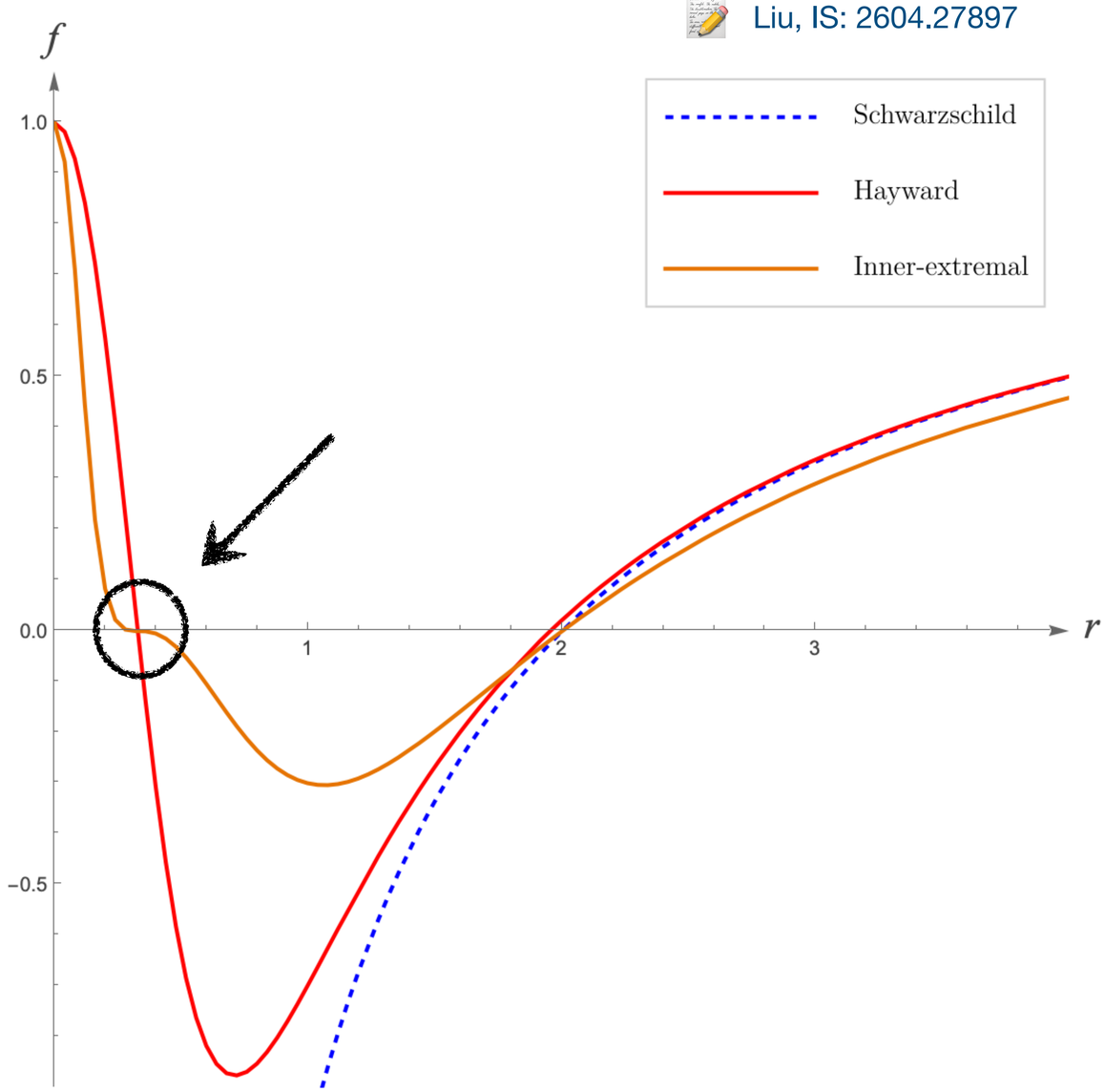
Liu, IS: 2604.27897



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Liu, IS: 2604.27897

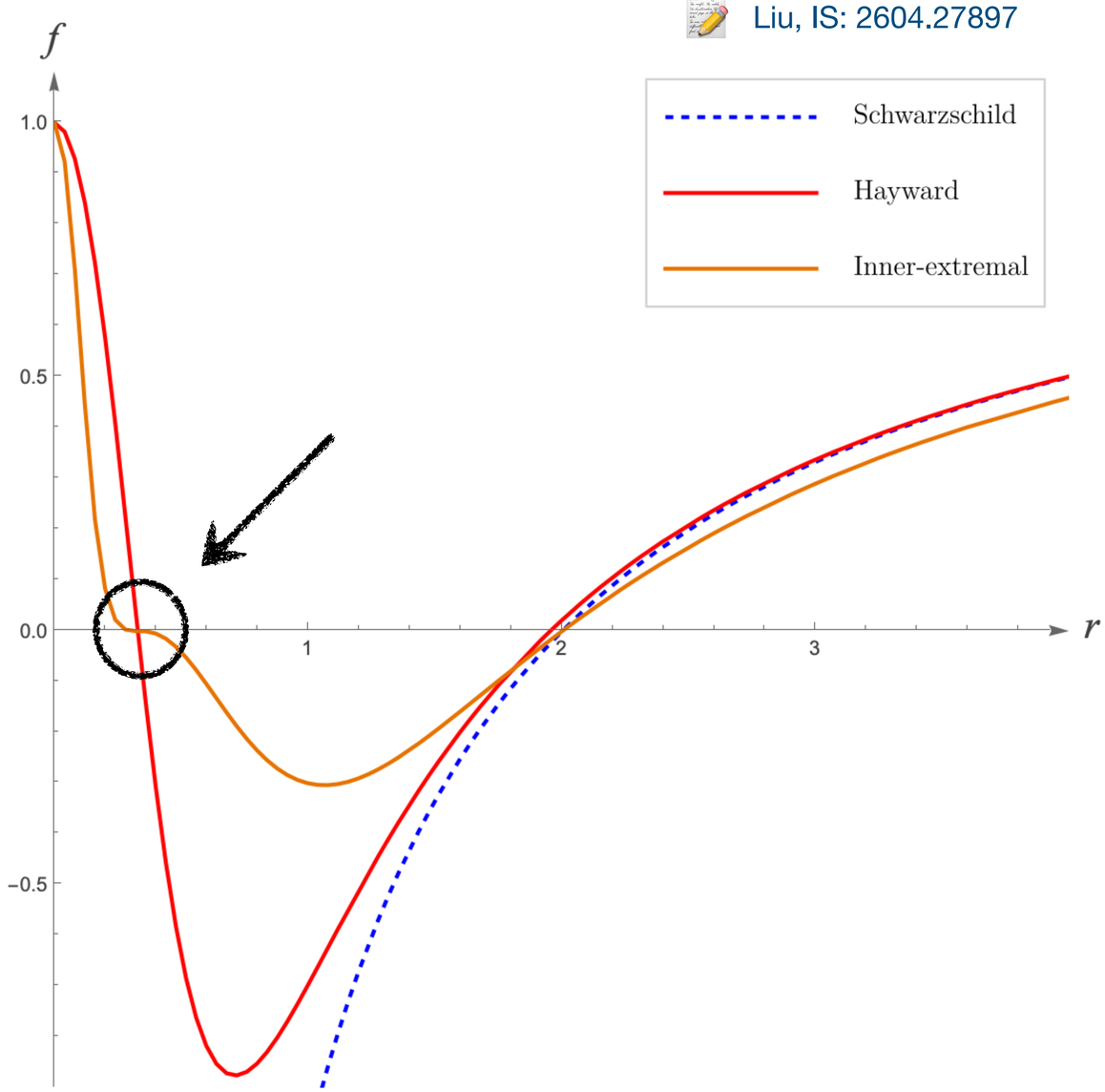


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Liu, IS: 2604.27897



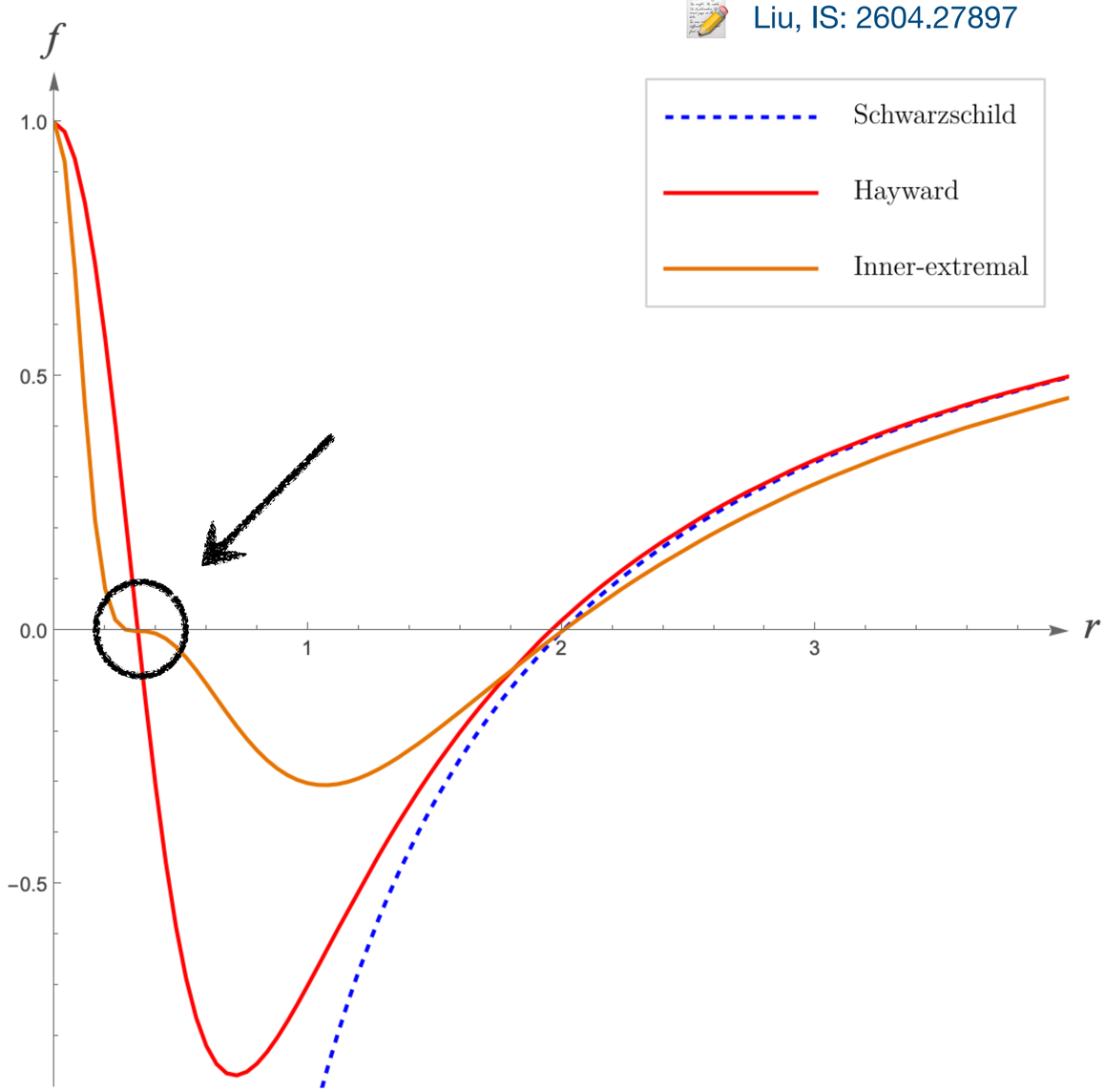
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$$\downarrow \quad \mathcal{X} = \frac{2m}{r^3}$$

Liu, IS: 2604.27897



Inner-extremal black holes

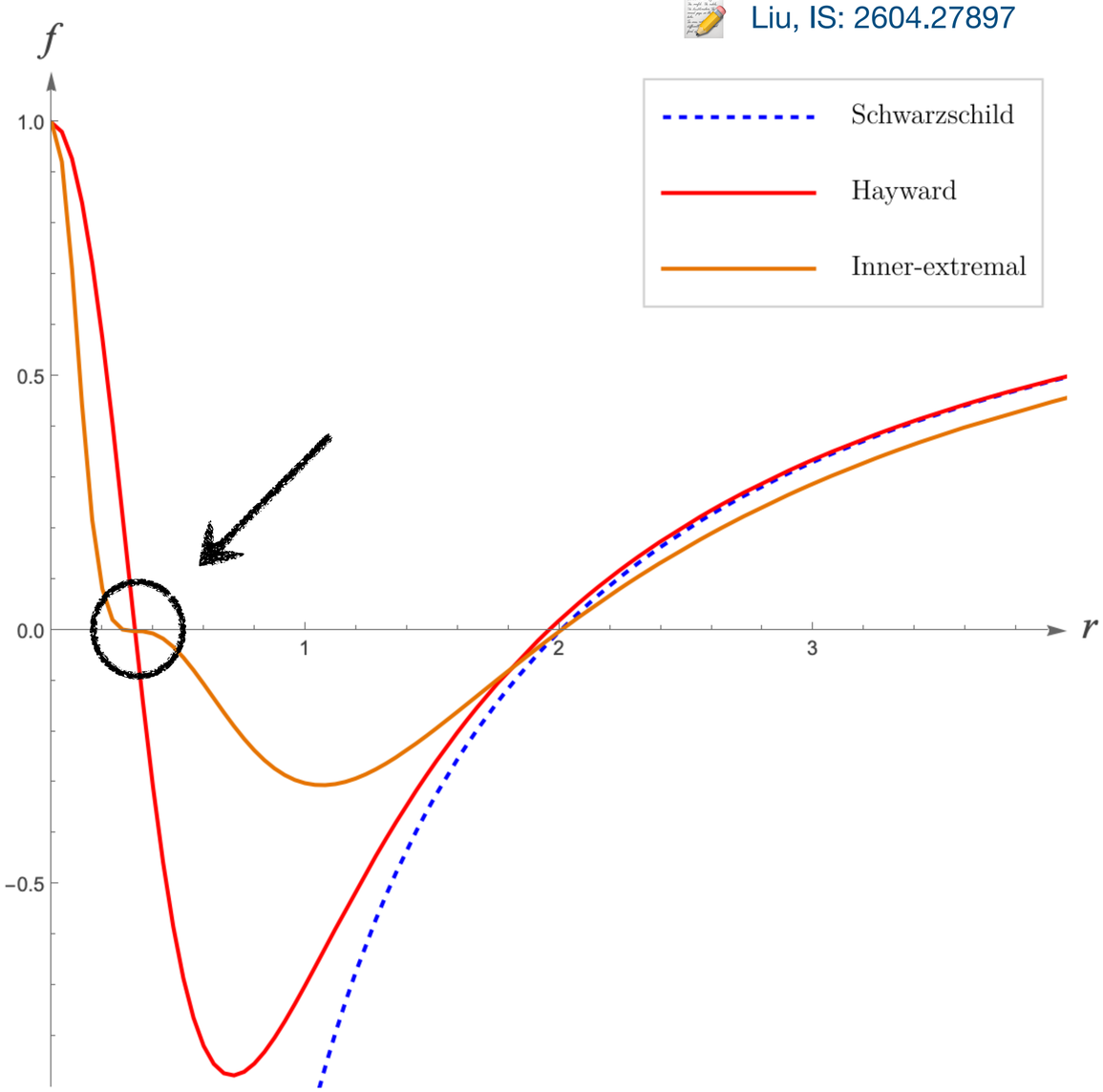
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Liu, IS: 2604.27897



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m : finely-tuned

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