Relic Gravitational Waves from Cosmic Domain Walls

Dmitry Gorbunov

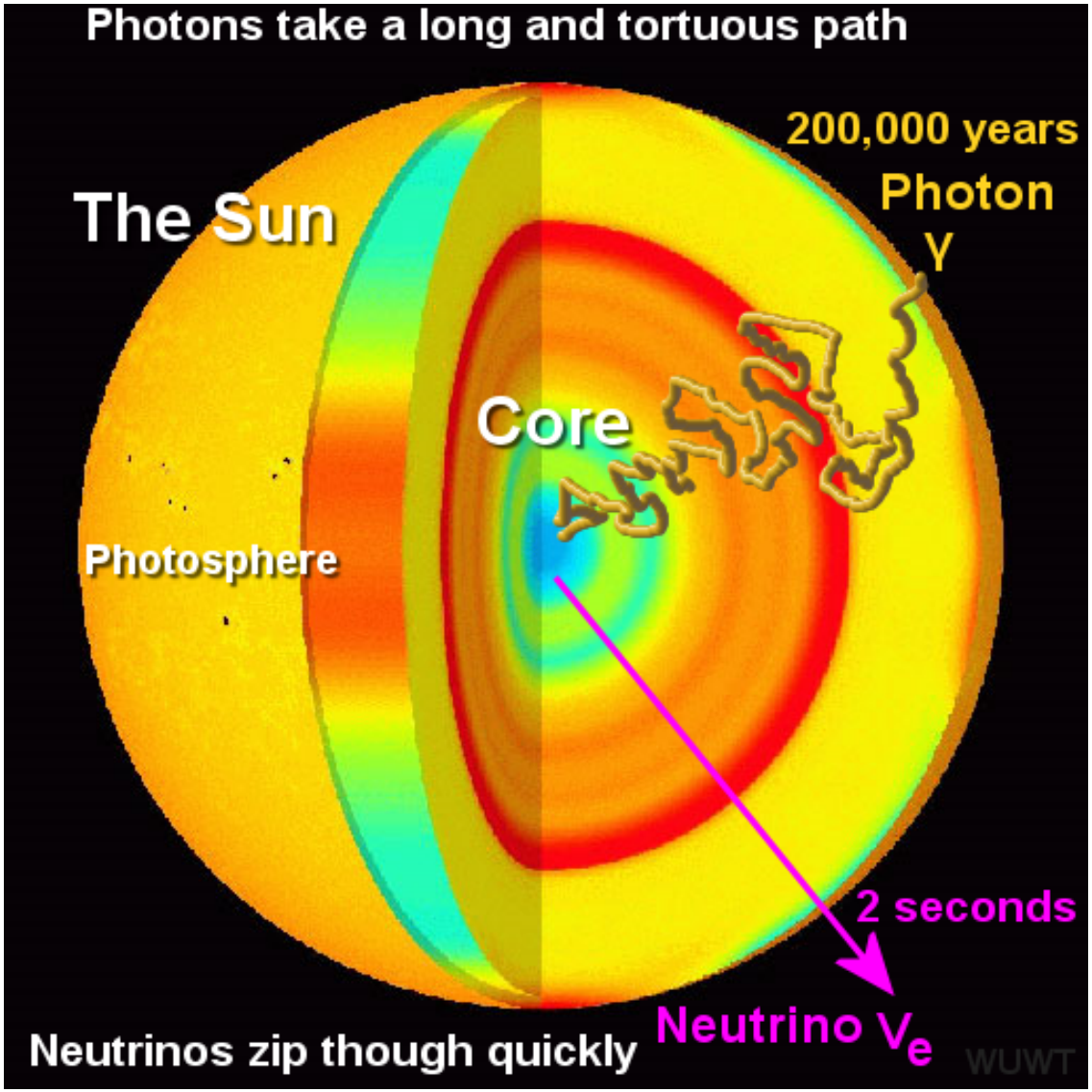
Institute for Nuclear Research of RAS, Moscow

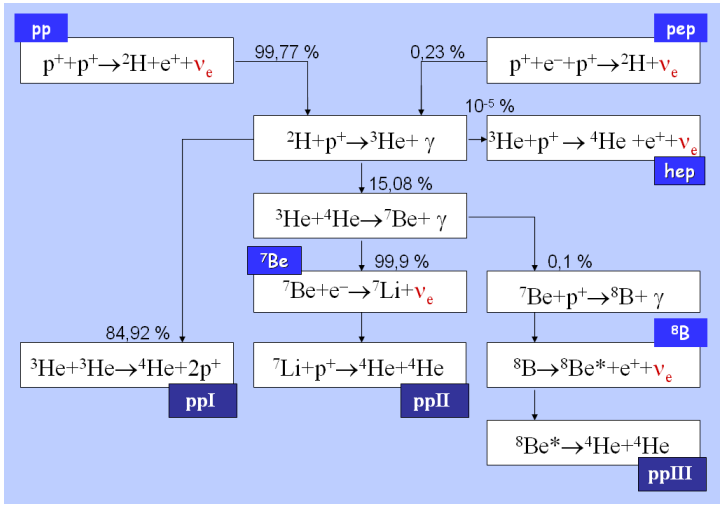
6th BRICS-AGAC Symposium

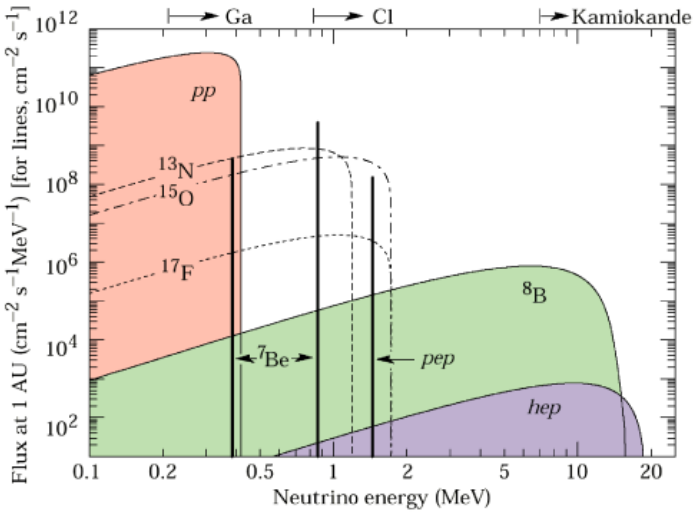
Zhejiang Hotel, Hangzhou, China



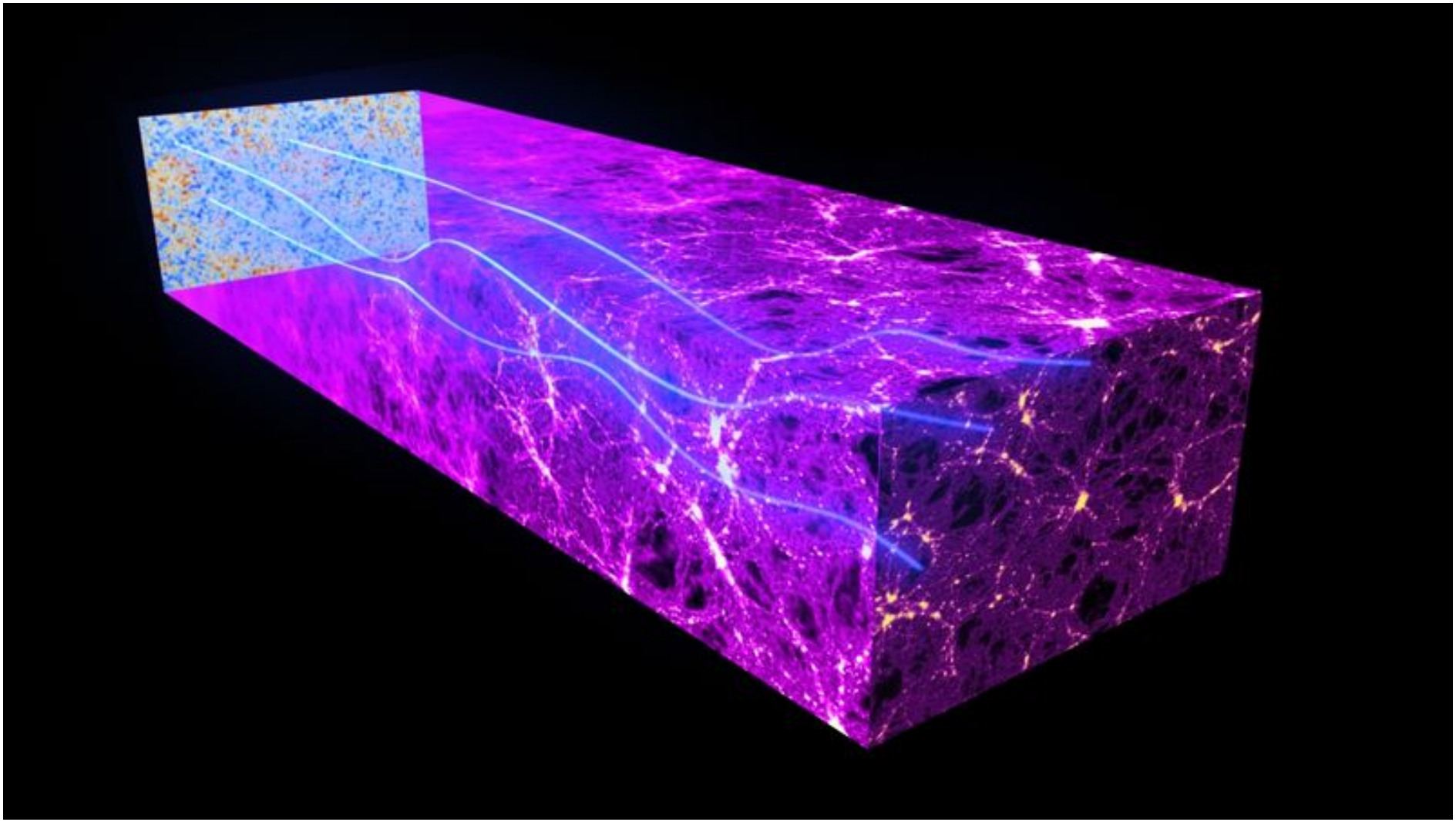
Solar neutrinos: fusion $p + p \rightarrow D + e^+ + \nu_e, \ldots$

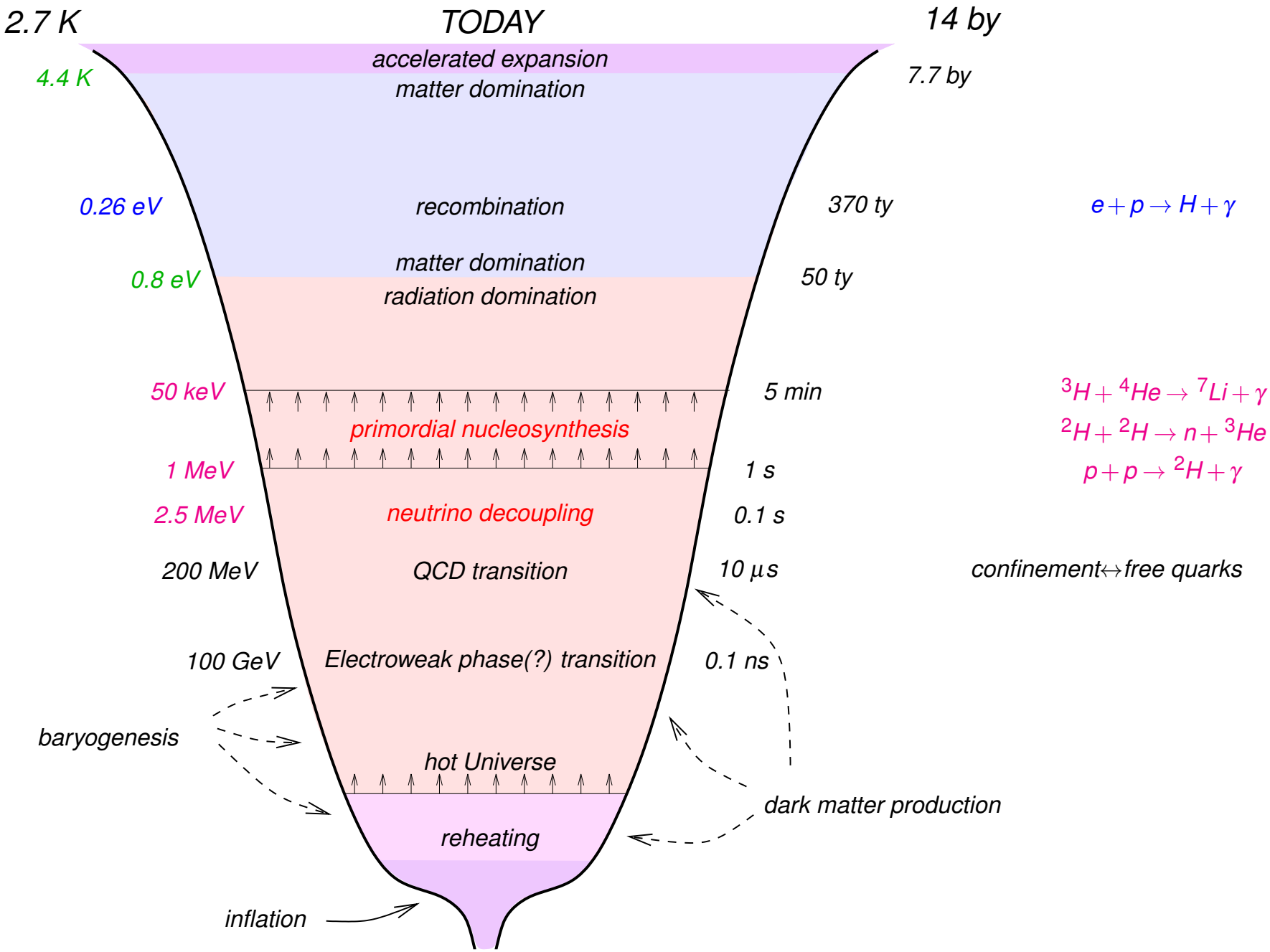






Universe: the surface of last scattering $e + p \to H + \gamma$

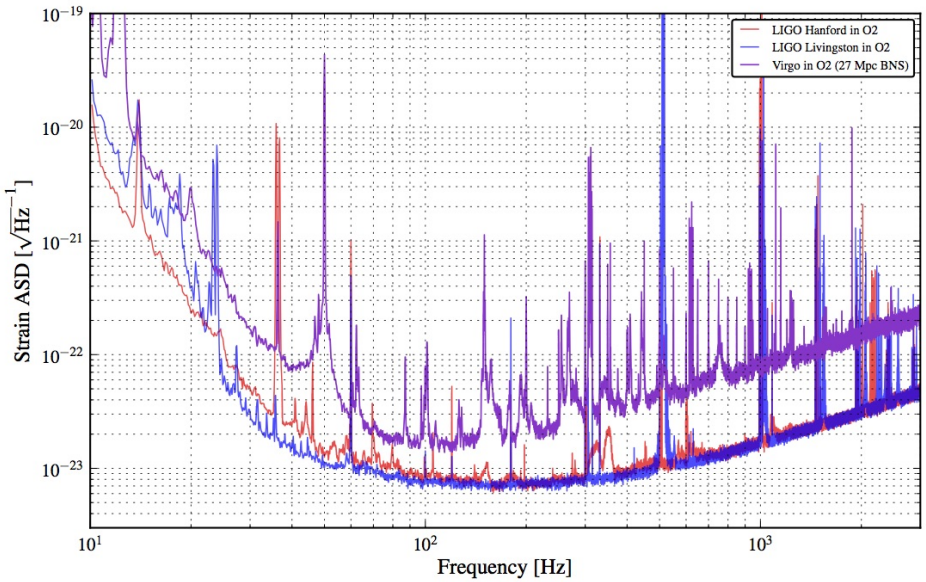


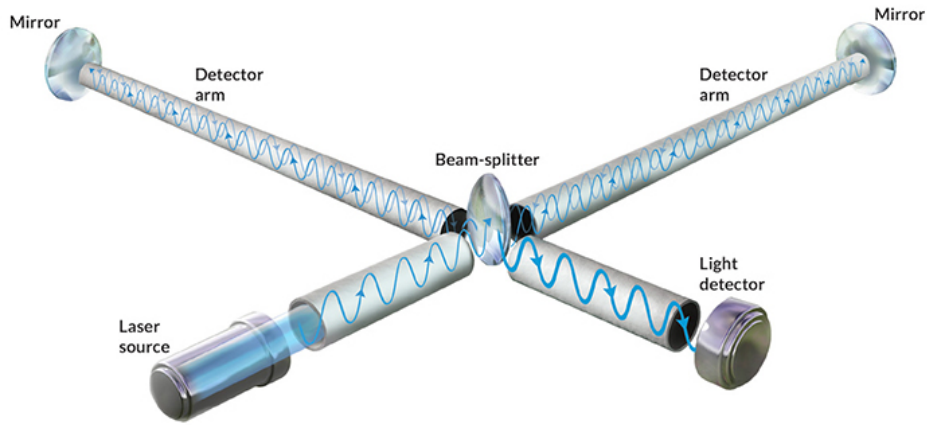

2.7 K
TODAY
14 by
accelerated expansion
matter domination
4.4 K
7.7 by
0.26 eV
recombination
370 ty
$e+p \to H+\gamma$
matter domination
0.8 eV
radiation domination
50 ty
50 keV
5 min
$^3H+^4He \to ^7Li+\gamma$
primordial nucleosynthesis
$^2H+^2H \to n+^3He$
1 MeV
1 s
$p+p \to ^2H+\gamma$
2.5 MeV
neutrino decoupling
0.1 s
200 MeV
QCD transition
10 μs
confinement↔free quarks
100 GeV
Electroweak phase(?) transition
0.1 ns
baryogenesis
hot Universe
dark matter production
reheating
inflation

Nobel prize in 2017: discovery of GW

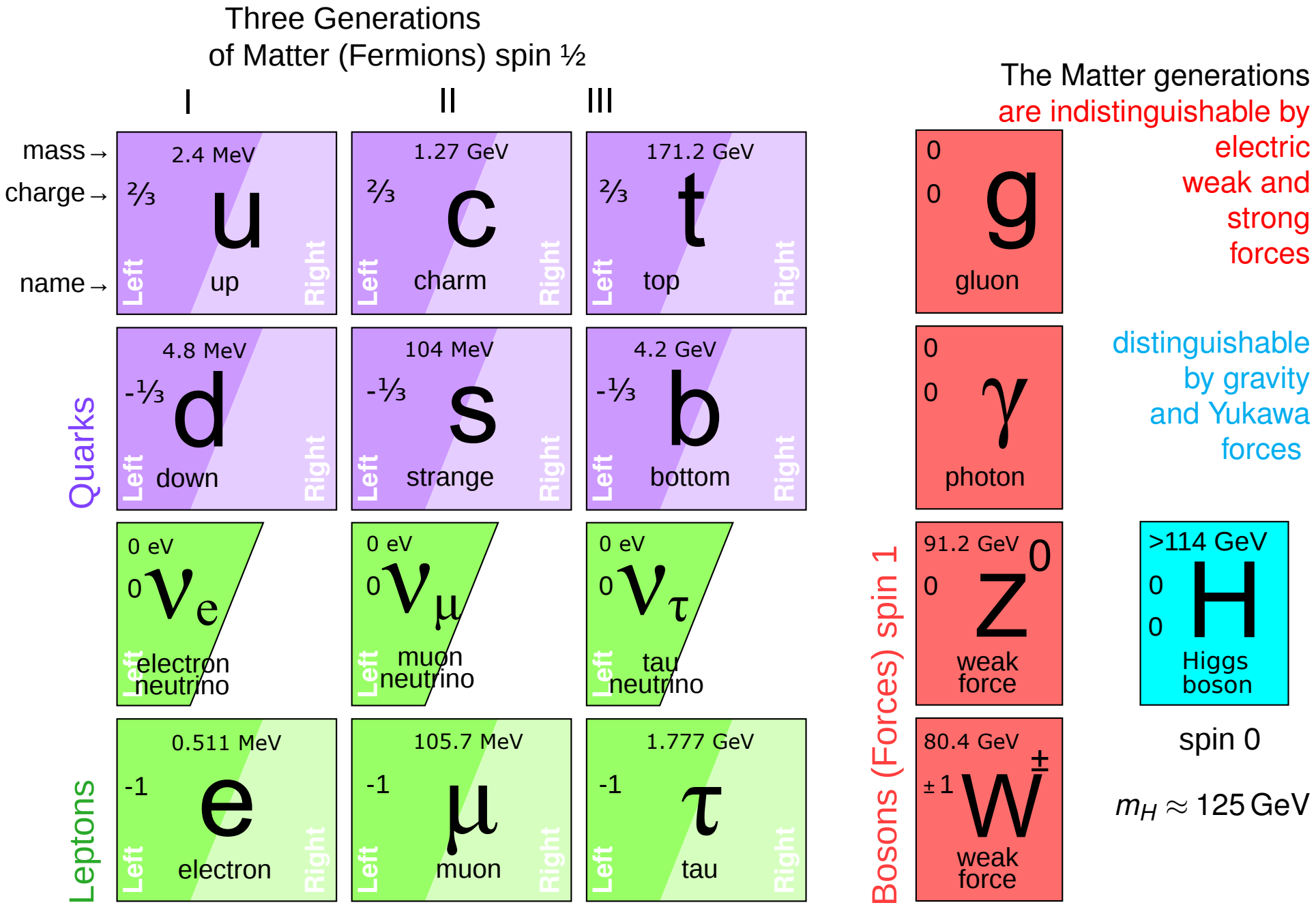











Three Generations
of Matter (Fermions) spin ½
I
II
III
mass →
charge →
name →
2.4 MeV
⅔
u
up
1.27 GeV
⅔
c
charm
171.2 GeV
⅔
t
top
4.8 MeV
-⅓
d
down
104 MeV
-⅓
s
strange
4.2 GeV
-⅓
b
bottom
0 eV
0
ν_e
electron neutrino
0 eV
0
ν_μ
muon neutrino
0 eV
0
ν_τ
tau neutrino
0.511 MeV
-1
e
electron
105.7 MeV
-1
μ
muon
1.777 GeV
-1
τ
tau
Left
Right
Quarks
Leptons
0
0
g
gluon
0
0
γ
photon
91.2 GeV
0
0
Z^0
weak force
80.4 GeV
±1
$W^\pm$
weak force
Bosons (Forces) spin 1
>114 GeV
0
0
H
Higgs boson
spin 0
$m_H \approx 125$ GeV
The Matter generations
are indistinguishable by
electric
weak and
strong
forces
distinguishable
by gravity
and Yukawa
forces

Standard Model + GR : Major Problems

Gauge and Higgs fields (interactions): γ, $W^{\pm}$, Z, g, G, and h

Three generations of matter: $L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$, e_R; $Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$, d_R, u_R

- Describes all experiments dealing with
  - electroweak and strong interactions (anomalies: K-, B-physics, . . .)
- Does not describe (PHENO)
  - Neutrino oscillations
  - Dark matter (Ω_{DM})
  - Baryon asymmetry (Ω_B)
  - Why the Universe is flat and homogeneous?
  - Where did the matter perturbations come from?

(THEORY)

- Dark energy (Ω_Λ)
- Strong CP-problem
- Gauge hierarchy
- Quantum gravity
- Quantization of electric charge
- Why 3 generations?
- Why $Y_e \ll Y_\mu \ll .. \ll Y_t$

In short: we need new physics !!

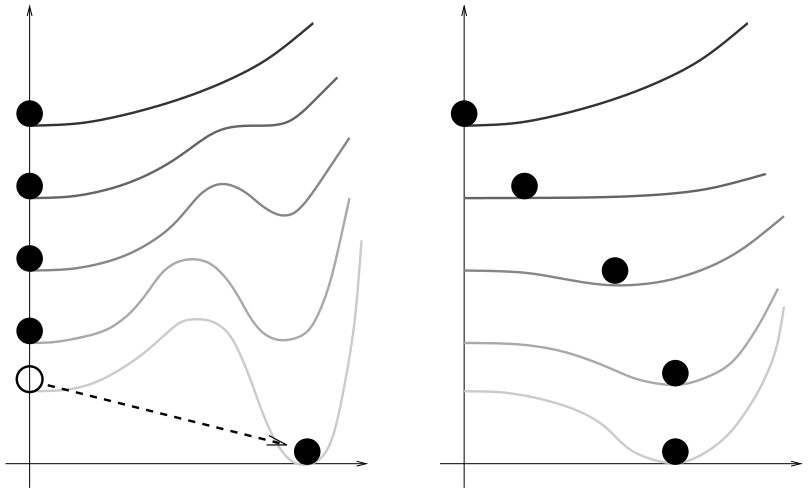
More symmetry !!

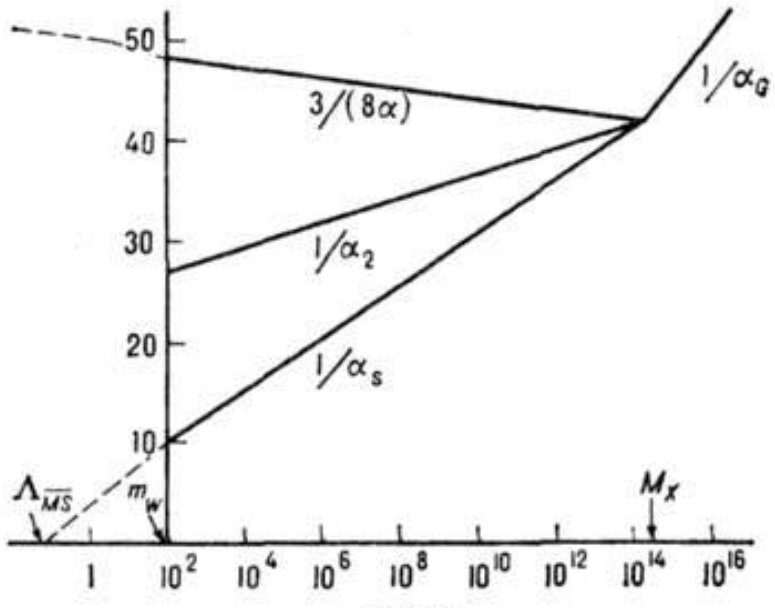
- Electroweak unification
  (transition at $T \sim 100\,\mathrm{GeV}$)

$$U(1)_{em} \to SU(2)_L \times U(1)_Y$$

- Any Ist order phase transition would produce GWs
- Grand Unification ?
  (transition(s) in the early Universe)

$$SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(5)$$
$$SU(3)_c \times SU(2)_L \times U(1)_Y \to$$
$$\to SU(3)_c \times SU(2)_R \times SU(2)_L \times U(1)_Y$$
$$\to SO(10)$$





- Any semisimple group inevitably leads to formation of topological defects:
  – monopoles
  – strings
  – domain walls

Topological defects are dangerous for cosmology !

Naturally overclose the hot Universe

- free monopoles vs radiation

$$\rho_{mon} = M_{mon} \times n_M \propto \frac{1}{a^3(t)} \quad \text{vs} \quad \rho_{rad} \propto \frac{1}{a^4(t)}$$

- free strings (gas of straight strings) vs radiation

$$\rho_{strings} \propto \frac{1}{a^2(t)} \quad \text{vs} \quad \rho_{rad} \propto \frac{1}{a^4(t)}$$

- free domain walls (gas of straight domain walls) vs radiation

$$\rho_{walls} \propto \frac{1}{a(t)} \quad \text{vs} \quad \rho_{rad} \propto \frac{1}{a^4(t)}$$

But the configurations are highly non-linear

They naturally annihilate via various processes

- free monopoles form monopoloniums and annihilate
- free strings intersect, form small closed strings which shrink and disappear leading to the scaling solution: fixed number of long strings inside the horizon

$$\rho_{str-sc} \simeq N\frac{M_s}{l_H^3} \simeq N\frac{\sigma_s l_H}{l_H^3} \propto H^2 \propto \rho_{tot}$$

strings are harmless

- free domain walls get crumpled and shrivel
  domain wall network appoaches scaling regime

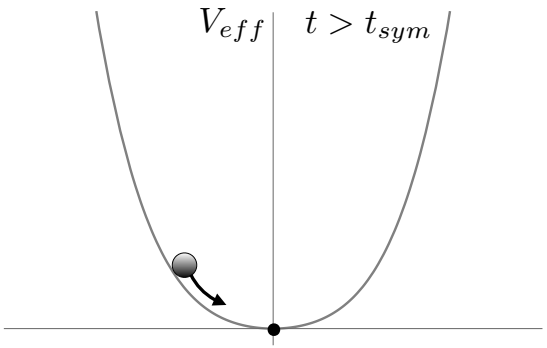
$$\rho_{walls-sc} \simeq N\frac{M_s}{l_H^3} \simeq N\frac{\sigma_s l_H^2}{l_H^3} \propto H \searrow \quad \text{while} \quad \rho_{tot} \propto H^2 \searrow\searrow$$

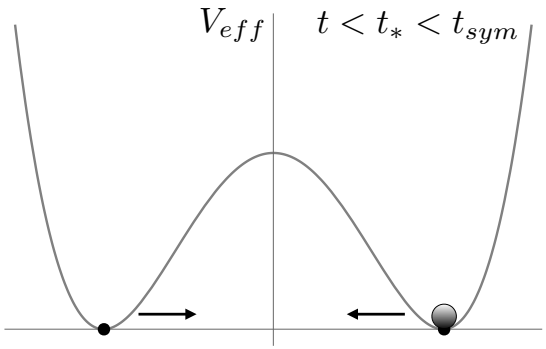
cosmic domain wall network is generically dangerous

hence, must be destroyed

Production of cosmic domain walls is very generic: breaking of any Z_2-symmetry involved in BSM

$t < t_b$ a cosmic scalar with $\langle\phi\rangle \neq 0$ $t > t_b$





$\rightarrow$

$$\mathscr{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{\lambda}{4} \left(\phi^2 - \eta^2\right)^2$$

Straight domain wall in Minkowsky is an analytical solution, kink.

Its width is $\delta_{wall} = \frac{\sqrt{2}}{\sqrt{\lambda}\eta}$, its tension is $\sigma_{wall} = \frac{2\sqrt{2\lambda}\eta^3}{3}$

Simulation of breaking Z_2-symmetry in expanding Universe with CosmoLattice code

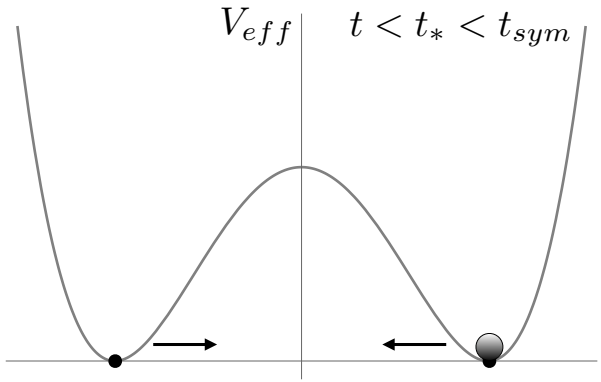
$ds^2 = a^2(\tau)\left(d\tau^2 - \mathbf{dx}^2\right)$ radiation domination start at $\tau = 1$

$$\langle\phi(\mathbf{k})\phi(\mathbf{q})\rangle = A(k)\delta(\mathbf{k}+\mathbf{q}) \quad \langle\dot{\phi}(\mathbf{k})\dot{\phi}(\mathbf{q})\rangle = B(k)\delta(\mathbf{k}+\mathbf{q})$$

with $\phi = 0 + \delta\phi(\mathbf{x}, t)$

1) vacuum fluctuations:

$$A(k) = \frac{\Theta(k - k_{cut})}{16\pi^3 k}, \qquad B(k) = \frac{k}{16\pi^3}\cdot\Theta(k - k_{cut}) .$$

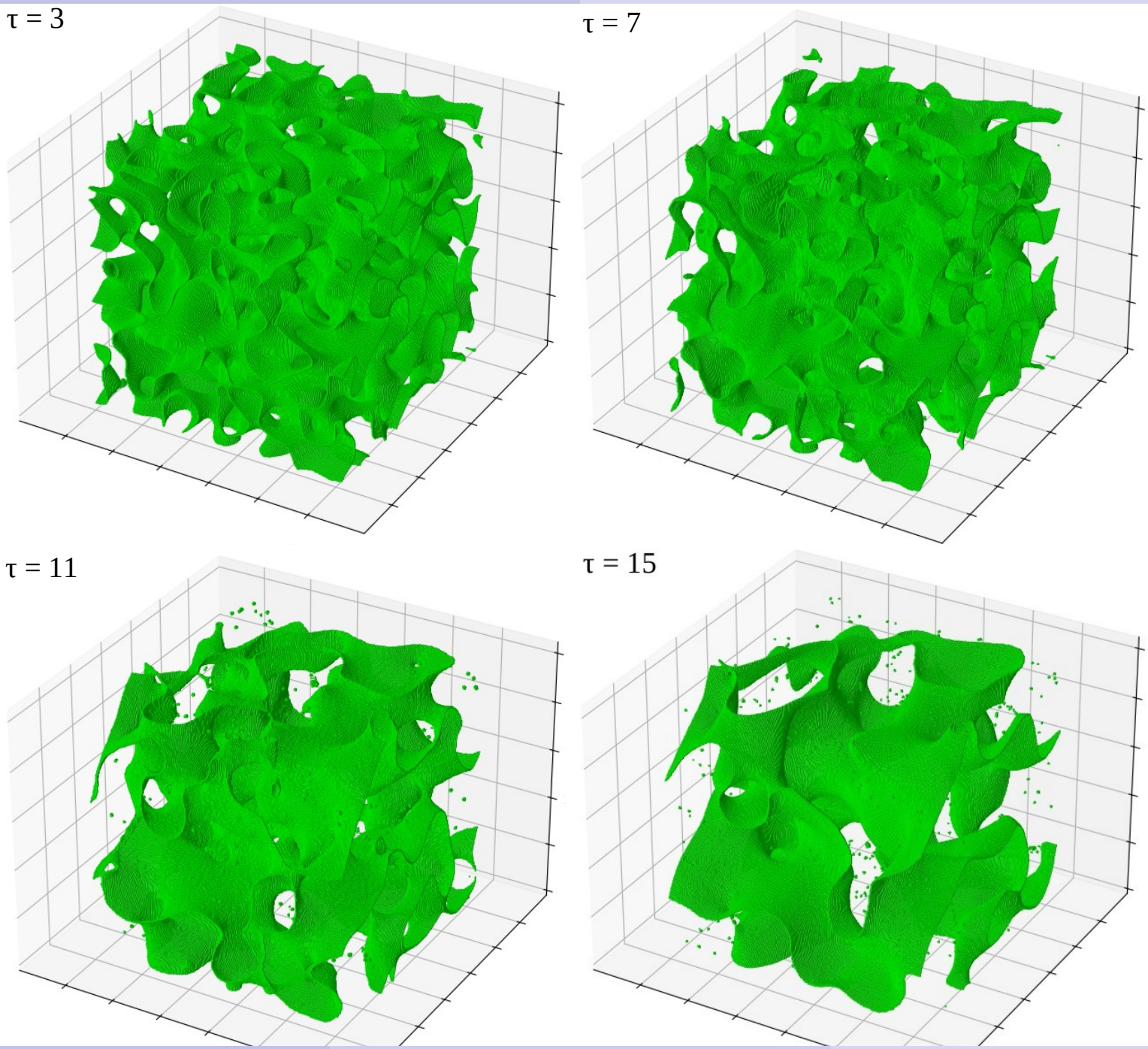


2) thermal fluctuations:

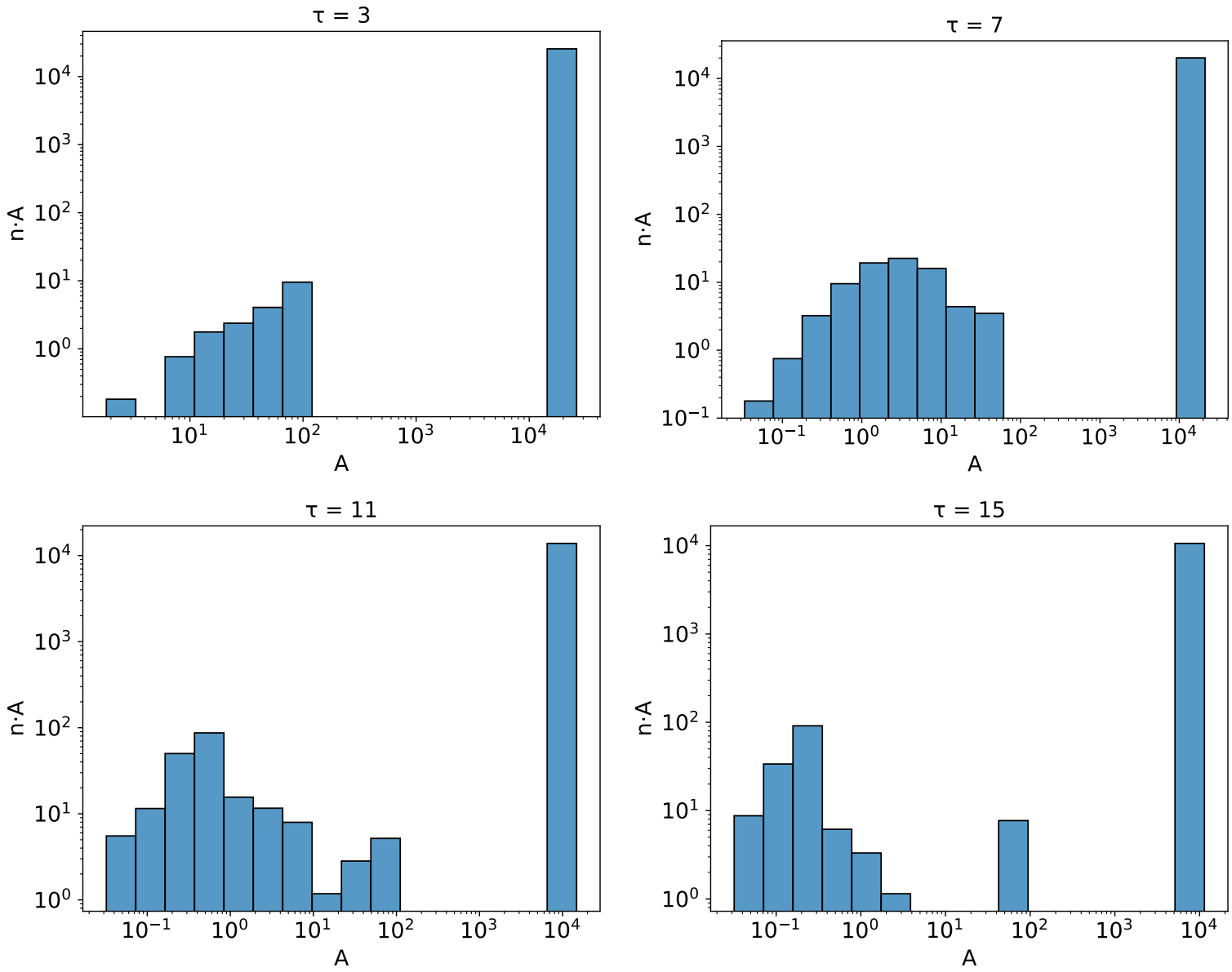
$$A(k) = \frac{1}{8\pi^3 \cdot k\left(e^{\frac{k}{T}} - 1\right)}, \qquad B(k) = \frac{k}{8\pi^3 \cdot \left(e^{\frac{k}{T}} - 1\right)},$$

numerical normalization: in terms of $\sqrt{\lambda}\eta$ 2406.17053

$$\mathscr{L} = \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{4}\left(\phi^2 - \eta^2\right)^2$$



2406.17053



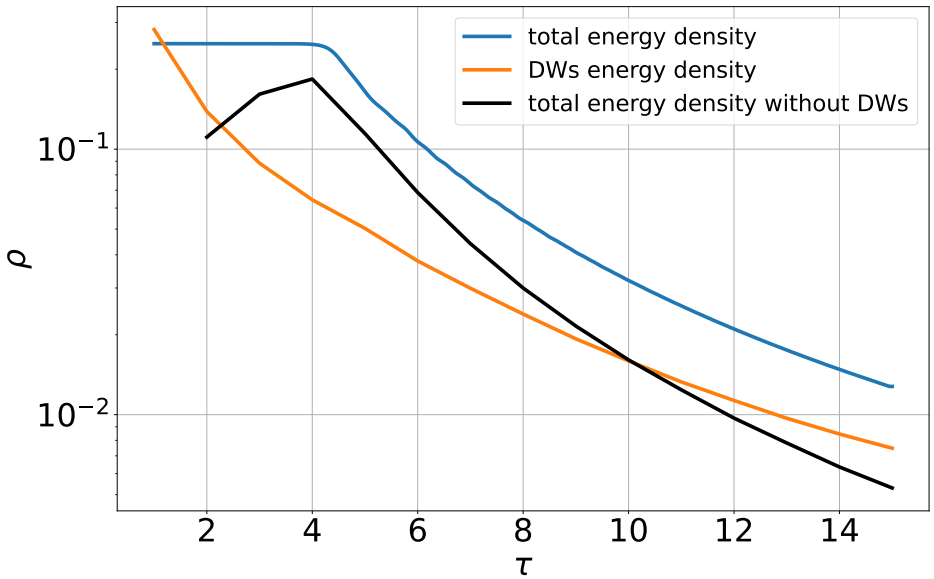
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Budget of scalar energy density

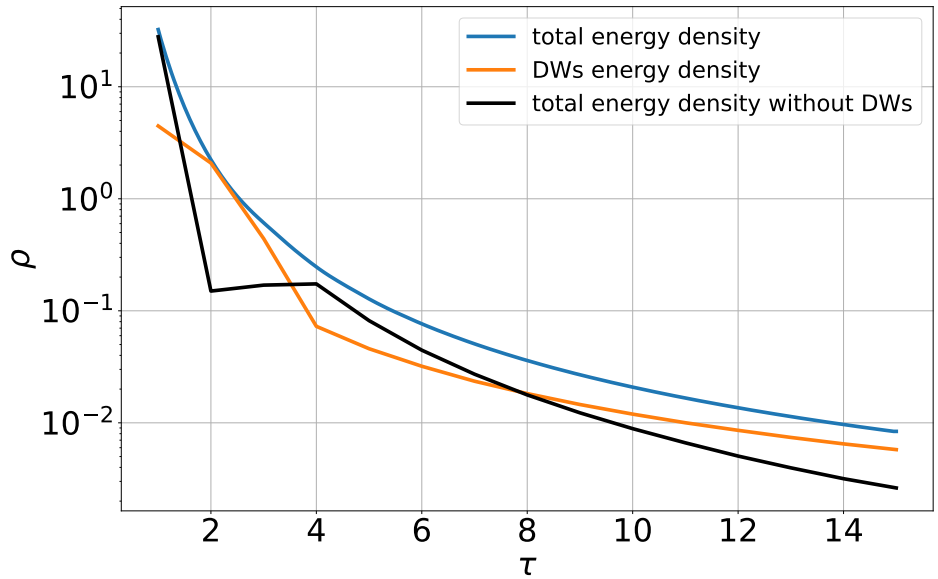
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radiation domination: $a \propto \tau$, $l_H \propto \tau^2$

vacuum perturbations



thermal perturbations



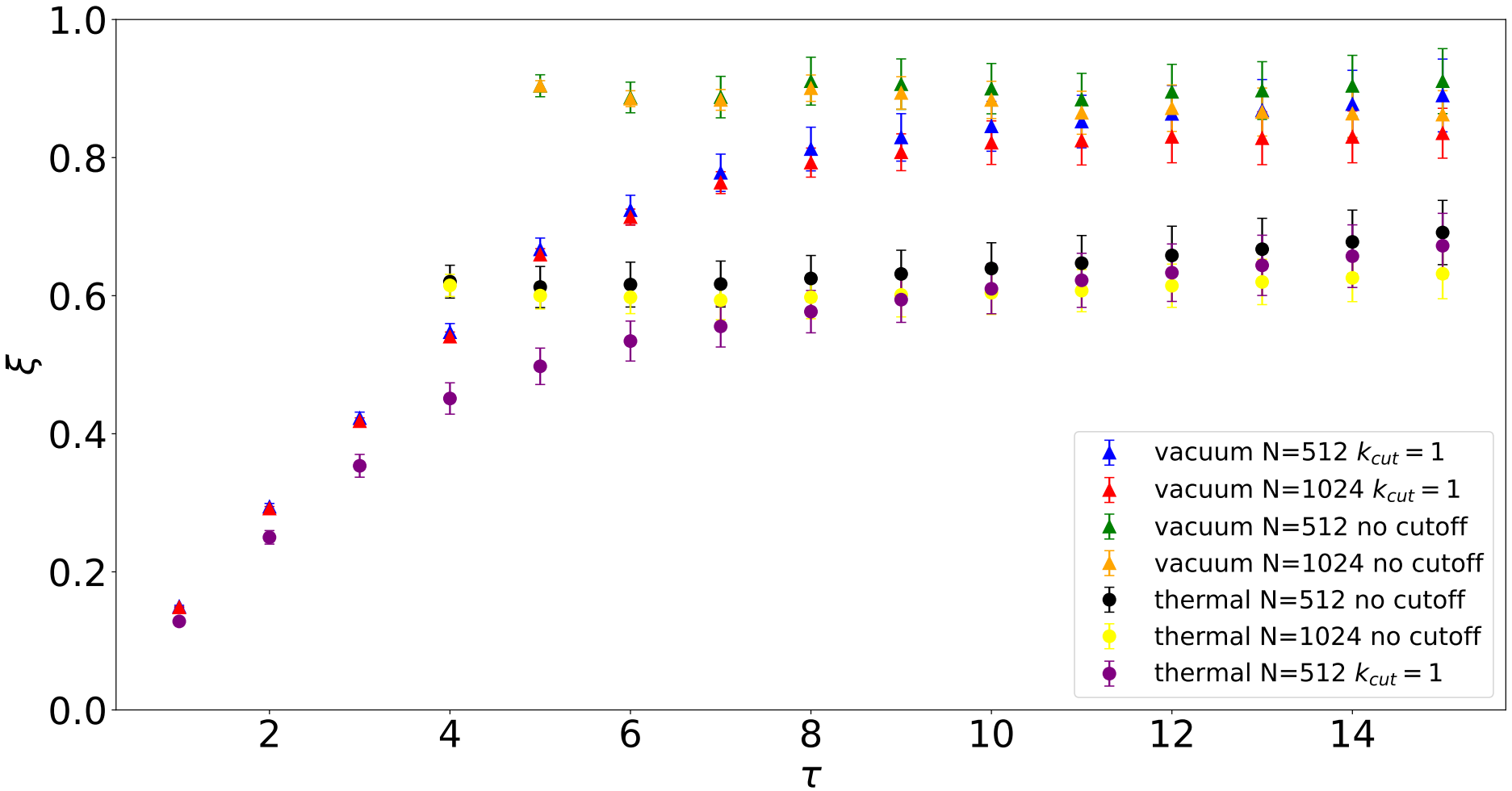
DW energy density: $\rho \propto \frac{1}{l_H} \propto \frac{1}{\tau^2}$

scalar field energy density: $\rho \propto \frac{1}{a^3} \propto \frac{1}{\tau^3}$

Scaling of a DM network: CosmoLattice simulation,

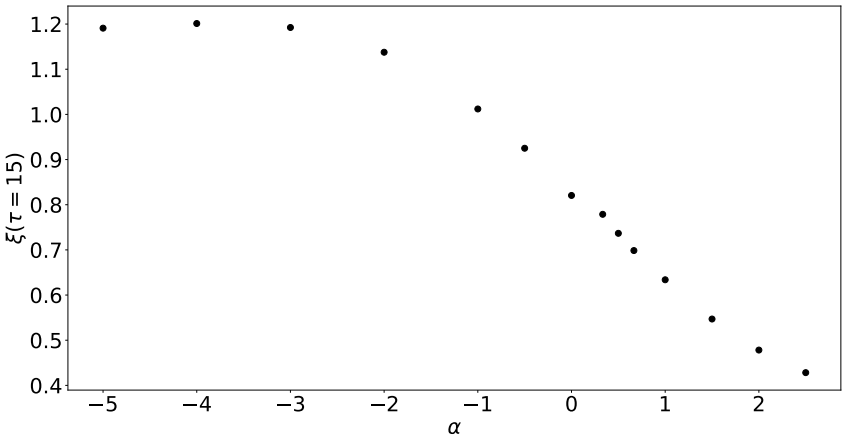
$$\rho_{dw} = \frac{\sigma_{dw} A}{a(t) V} \propto \frac{1}{l_H} \propto \frac{1}{t}, \quad \xi \equiv \frac{At}{a(t)V} \to 1.2$$

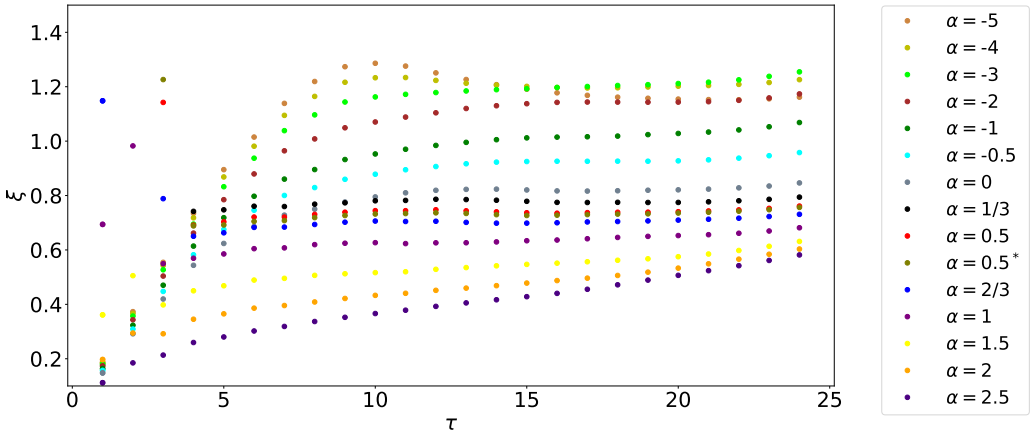
2406.17053, 2509.25367



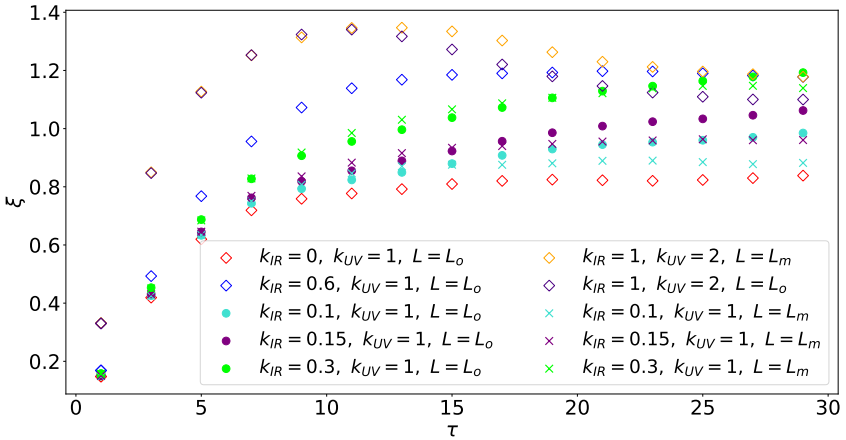
$$\rho_{dw} = \frac{\sigma_{dw} A}{a(t) V} \propto \frac{1}{l_H} \propto \frac{1}{t}, \quad \xi \equiv \frac{At}{a(t)V}$$

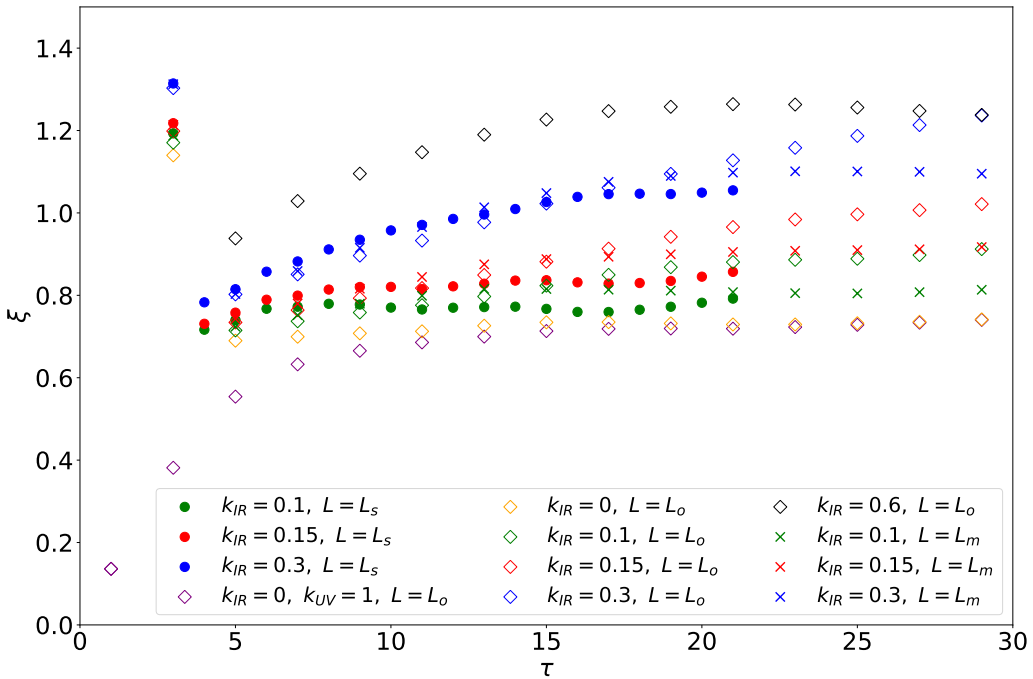
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$$A(k) = \frac{\exp\left(-\frac{k_{IR}^2}{k^2}\right)\cdot\theta(k_{UV}-k)}{k\left(e^{\frac{k}{T}}-1\right)^{\alpha}}, \qquad B(k) = \frac{k\cdot\exp\left(-\frac{k_{IR}^2}{k^2}\right)\cdot\theta(k_{UV}-k)}{\left(e^{\frac{k}{T}}-1\right)^{\alpha}}.$$





Microscopic processes in the expanding Universe

A competition between scattering, decays, etc and expansion

kinetic equations for particle number densities $n_X \propto a^{-3}$

$$\frac{dn_X}{dt} + 3Hn_X = \sum(production - destruction)$$

$$\dot{\rho}_{DW} + H\rho_{DW} = -\Gamma\rho_{DW}^2$$

$\Gamma = 0$ describes a gas of non-interacting DWs:

$$\rho_{DW} = \rho_0 \times \frac{a_0}{a}$$

Assuming $a \propto t^\alpha$, we get — scaling for $\alpha < 1$

$$\rho_{DW} = \left(\frac{1}{\rho_0}\frac{a}{a_0} + \frac{\Gamma t_0}{1-\alpha}\left(\frac{t}{t_0} - \frac{a}{a_0}\right)\right)^{-1} \propto \frac{1}{t} \propto \frac{1}{l_H} \propto H$$

If DWs dominate, we restore the non-interacting regime — then $\alpha > 1$

$$H^2 \propto \rho \propto \frac{1}{a} \propto \frac{1}{t^2}$$

The DWs stretch and “leave the horizon”, rather than ”enter the horizon”

Scaling solution

- Non-linear dynamics + causality leads to a self-similarity see e.g. 2406.17053

$$\frac{\langle \rho(x,\tau)\rho(y,\tau)\rangle}{\langle \rho^2(0,\tau)\rangle} = f\left(\frac{|x-y|}{\tau}\right)$$

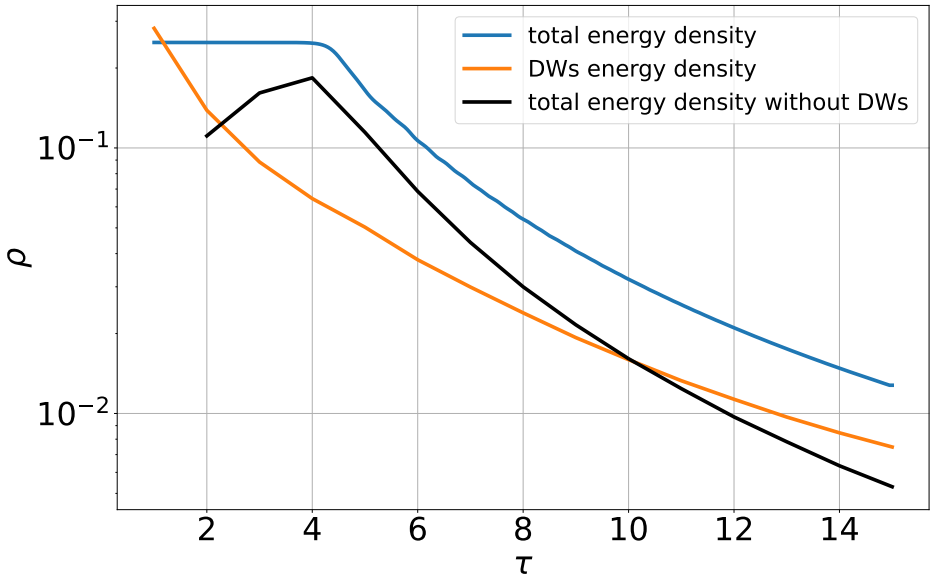
- But the microscopic processes are not determined:
  - DW hitting the heavy particles in the bulk
  - rescattering of waves on the DW and particle emission into the bulk
  - ?
- But who cares, they must be destroyed anyway...

Budget of scalar energy density

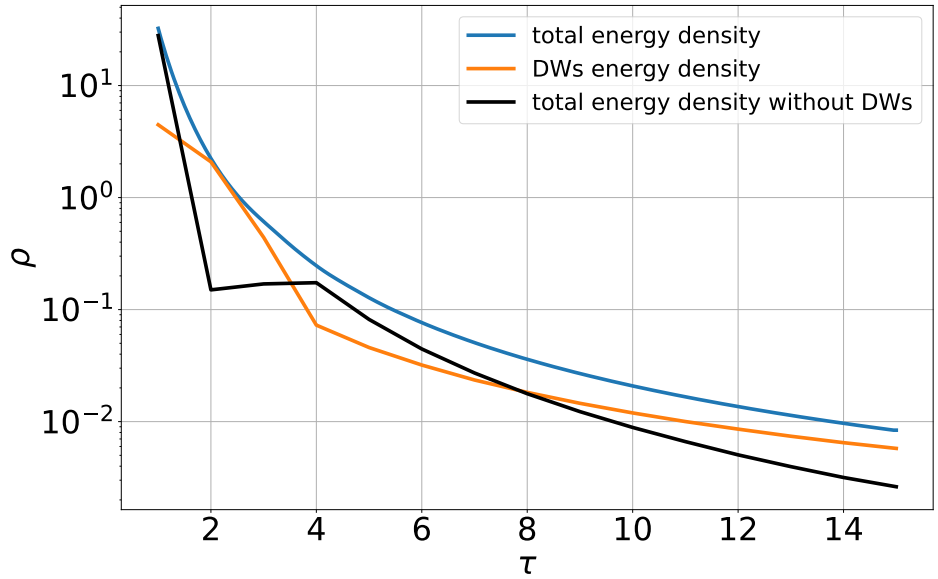
2406.17053

radiation domination: $a \propto \tau$, $l_H \propto \tau^2$

vacuum perturbations



thermal perturbations



DW energy density: $\rho \propto \frac{1}{l_H} \propto \frac{1}{\tau^2}$ $\dot{\rho}_{DW} + H\rho_{DW} = -\Gamma\rho_{DW}^2$ $\rho_{DW} \to \rho_m$

scalar field energy density: $\rho_m \propto \frac{1}{a^3} \propto \frac{1}{\tau^3}$ $\dot{\rho}_m + 3H\rho_m = \Gamma\rho_{DW}^2$ $\rho_m \to \propto \frac{1}{l_H} \propto \frac{1}{\tau^2}$

not enough time to see in simulations...

Domain walls produce Gravitational Waves

In expanding Universe $ds^2 = a^2(\tau)\left(d\tau^2 - dx^2\right)$

$$h''_{ij}(\mathbf{k},\tau) + \frac{2a'}{a} h'_{ij}(\mathbf{k},\tau) + k^2 h_{ij}(\mathbf{k},\tau) = 16\pi G_N a^2 \Pi_{ij}(\mathbf{k},\tau)\,.$$

where all the tensors are transverse and traceless

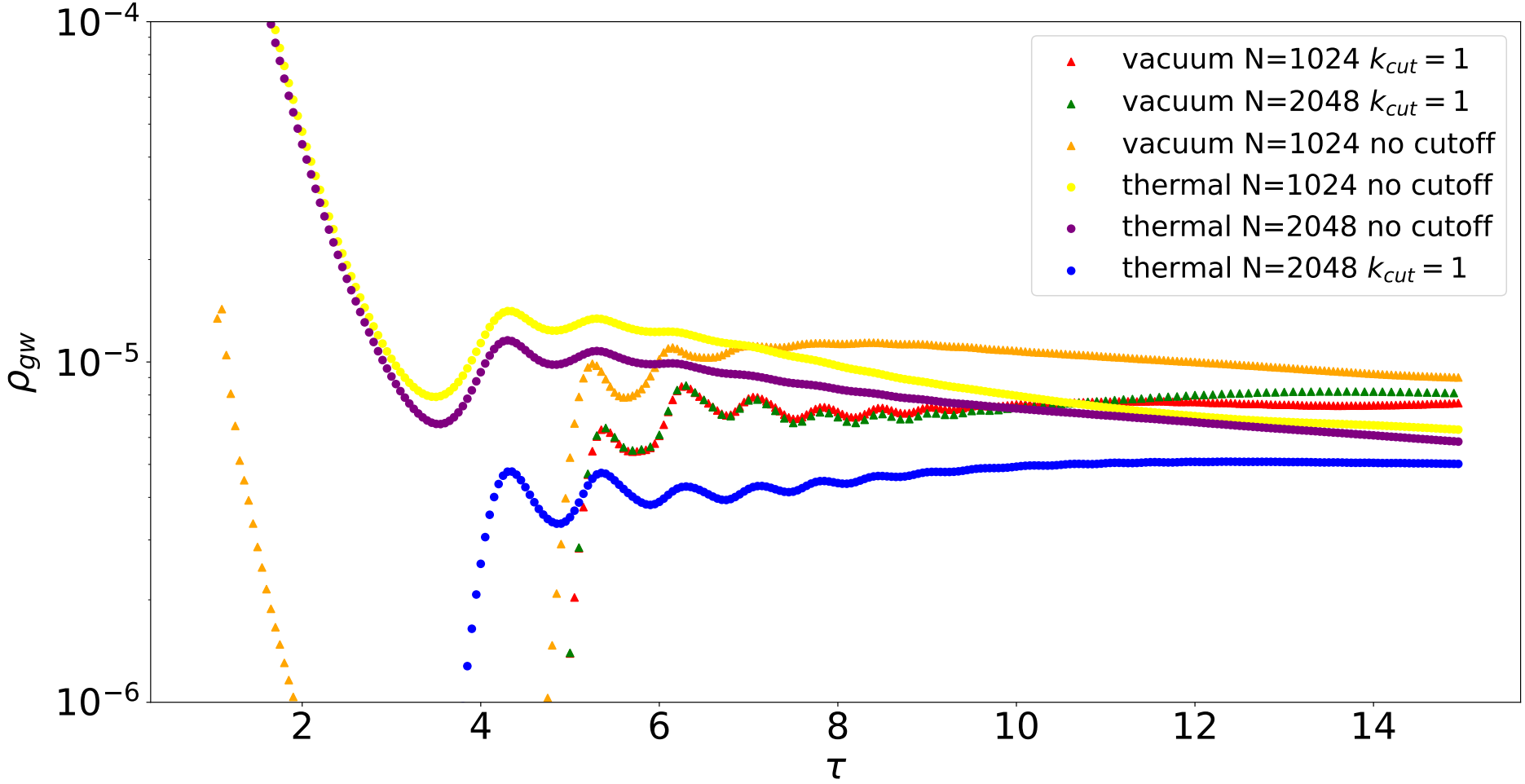
and the source is the scalar field inhomogeneties

$$\Pi_{ij} \to T_{kl} \sim \partial_k \phi \partial_l \phi$$

With DW network in the scaling regime ar RD-stage, $a \propto \tau$:

$$\rho_{GW} \sim \langle \dot{h}_{ij}\, \dot{h}_{ij} \rangle \propto \left(\frac{h'_{ij}}{a} \frac{a'}{a'} \right)^2 \propto \left(a^2 \rho_{DW} \right)^2 = \text{const}$$

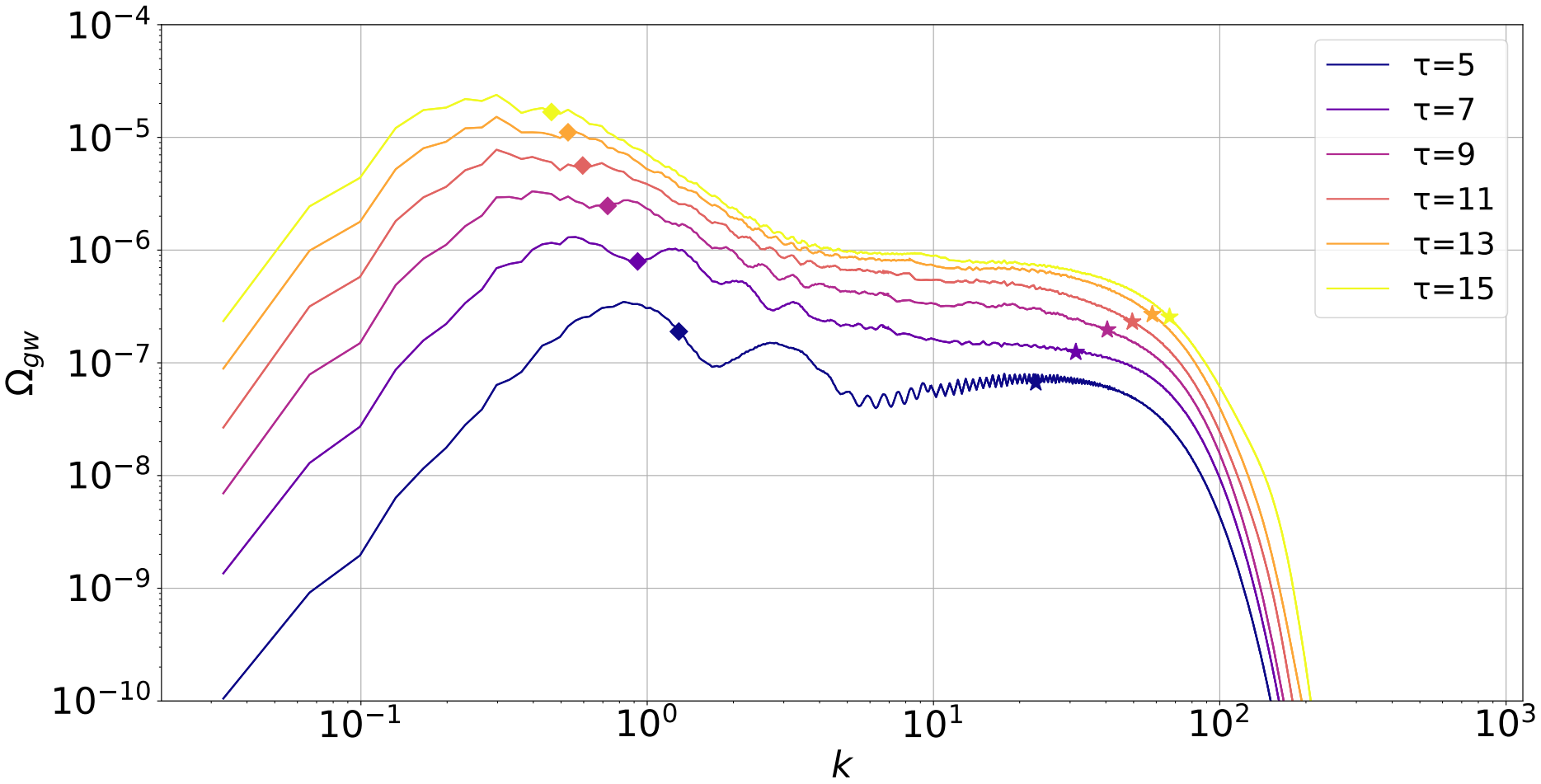
Energy density of GW: simulation with CosmoLattice



2406.17053

GW spectra: simulation with CosmoLattice

2406.17053



box refers to the current Hubble

$k = 2\pi Ha$

maximum is at the horizon (2607.28214)

star refers to the domain wall tension

$k = 2\pi a/\delta_{wall}$

IR tail: $\Omega_{gw} \propto k^{2.5}$

theory predicts $\Omega_{gw} \propto k^3$

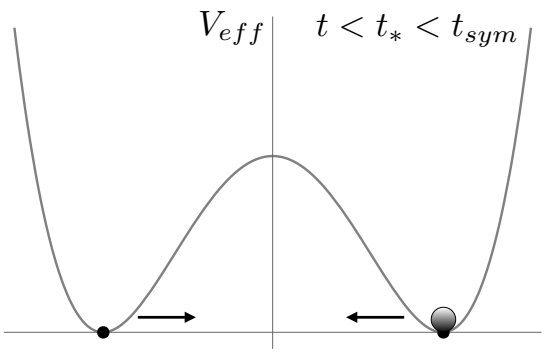
DWs can be destroyed by melting

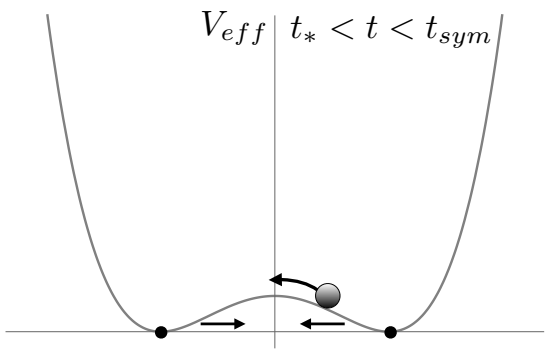
$$\Delta\mathscr{L} = \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \left(g^2 X^\dagger X - \frac{1}{2}M^2\right)\phi^2 - \frac{\lambda}{4}\phi^4$$

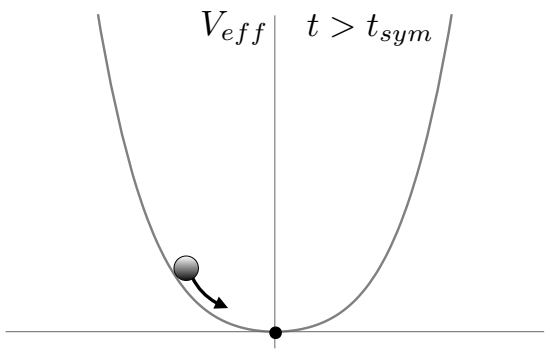
Need non-zero X,
e.g. if it equilibrates in plasma (Higgs ?) changing the potential as:

$$g^2\phi^2 X^\dagger X \;\rightarrow\; g^2\phi^2 T^2/3$$

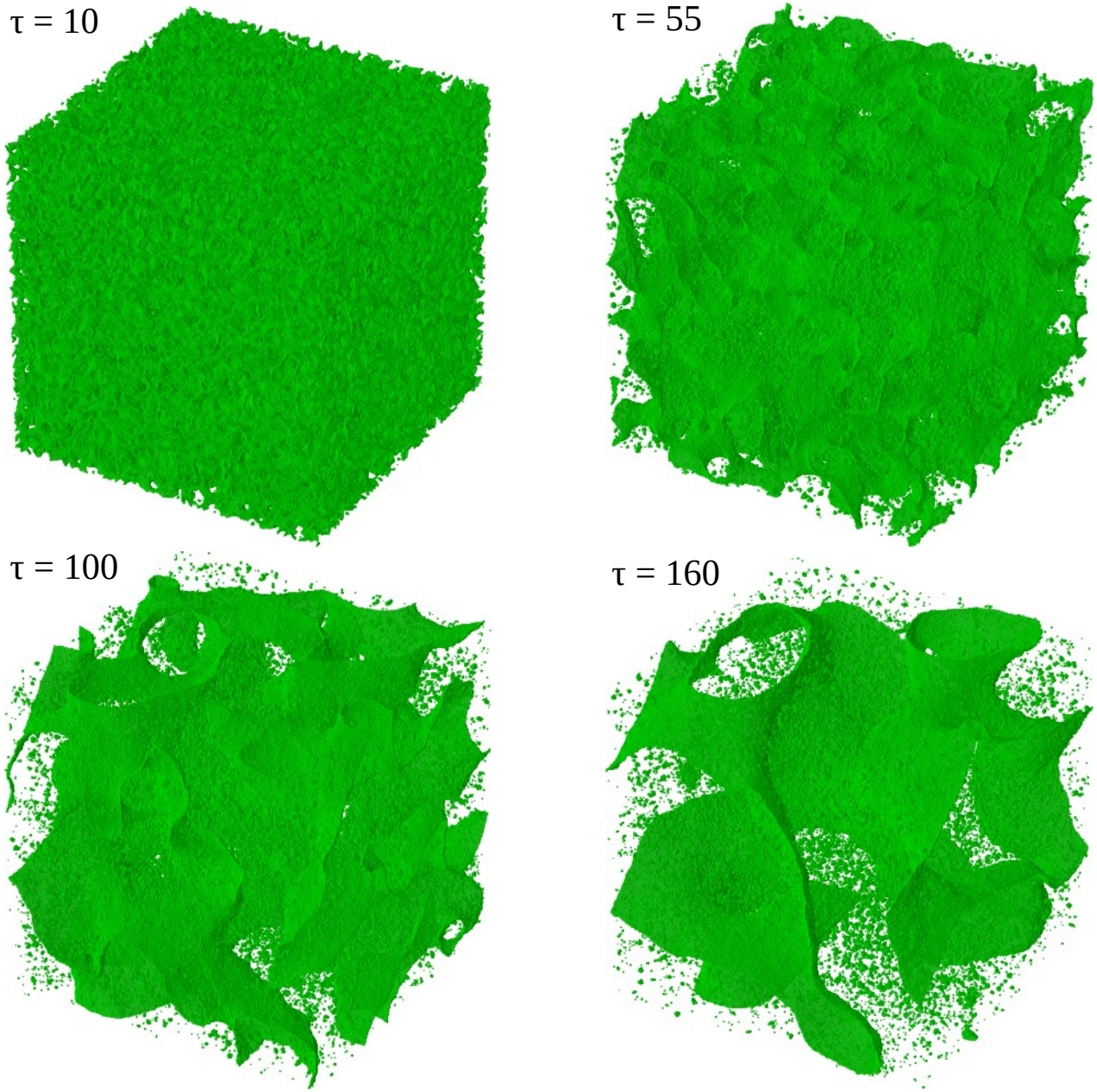
Z_2 symmerty is broken after reheating by the plasma contribution







domain walls with decreasing tension σ_{dw} and width



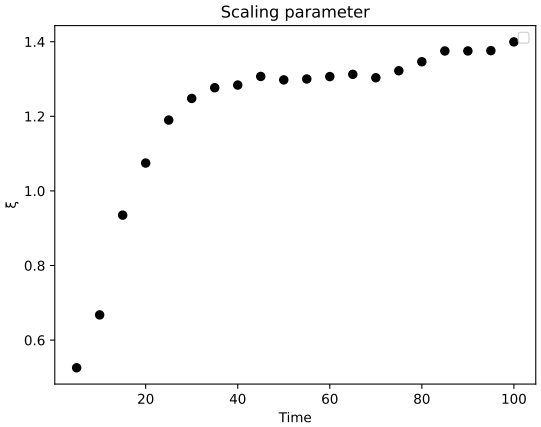
2410.21971

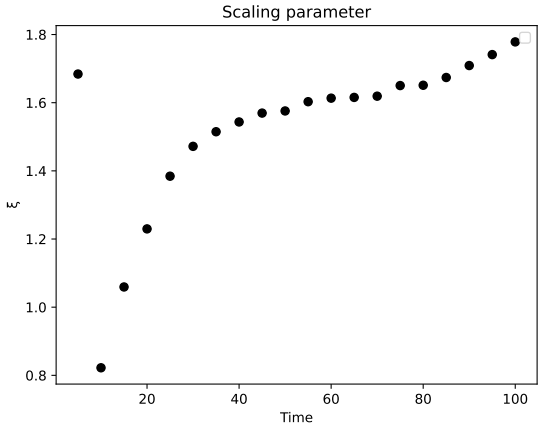
Scaling of melting domain walls

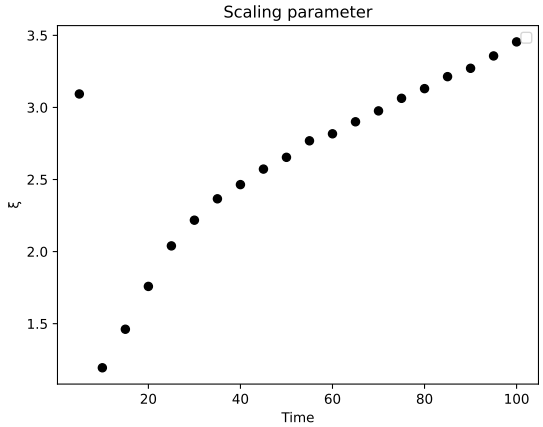
2410.21971

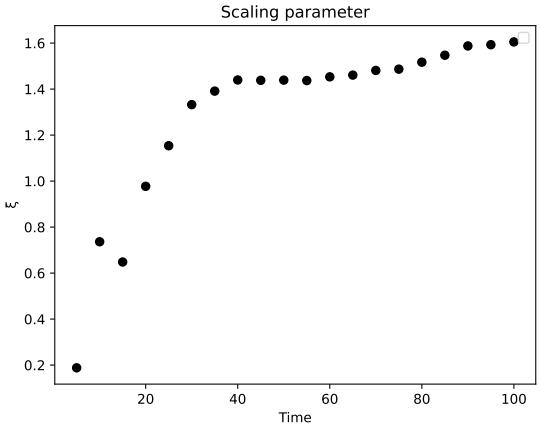
vacuum perturbations

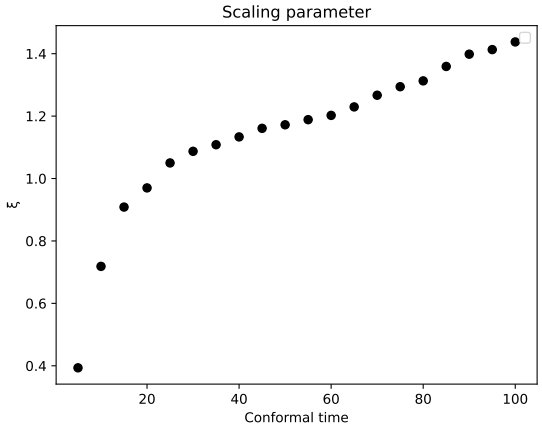
no cuts

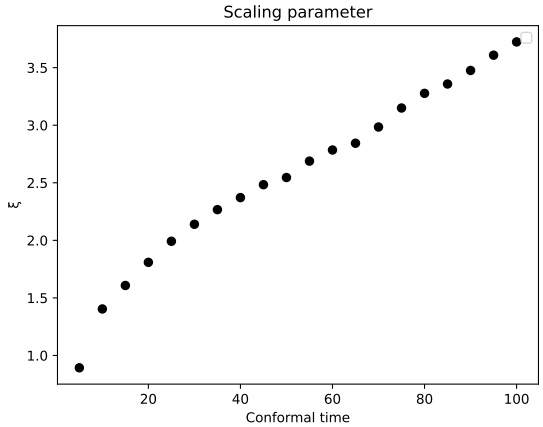












thermal perturbations

no cuts

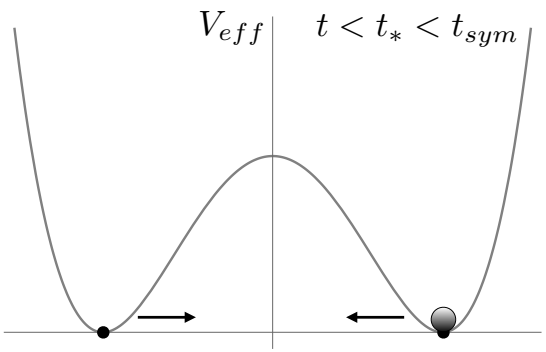
DWs can be destroyed by melting

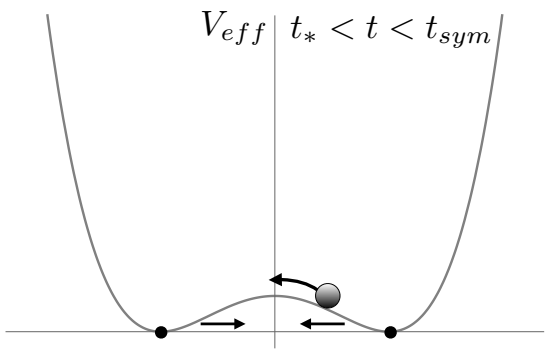
$$\Delta\mathscr{L} = \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \left(g^2 X^\dagger X - \frac{1}{2}M^2\right)\phi^2 - \frac{\lambda}{4}\phi^4$$

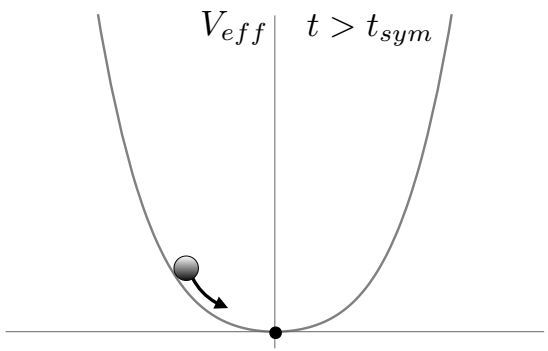
Need non-zero X,
e.g. if it equilibrates in plasma (Higgs ?) changing the potential as:

$$g^2\phi^2 X^\dagger X \;\rightarrow\; g^2\phi^2 T^2/3$$

Z_2 symmerty is broken after reheating by the plasma contribution





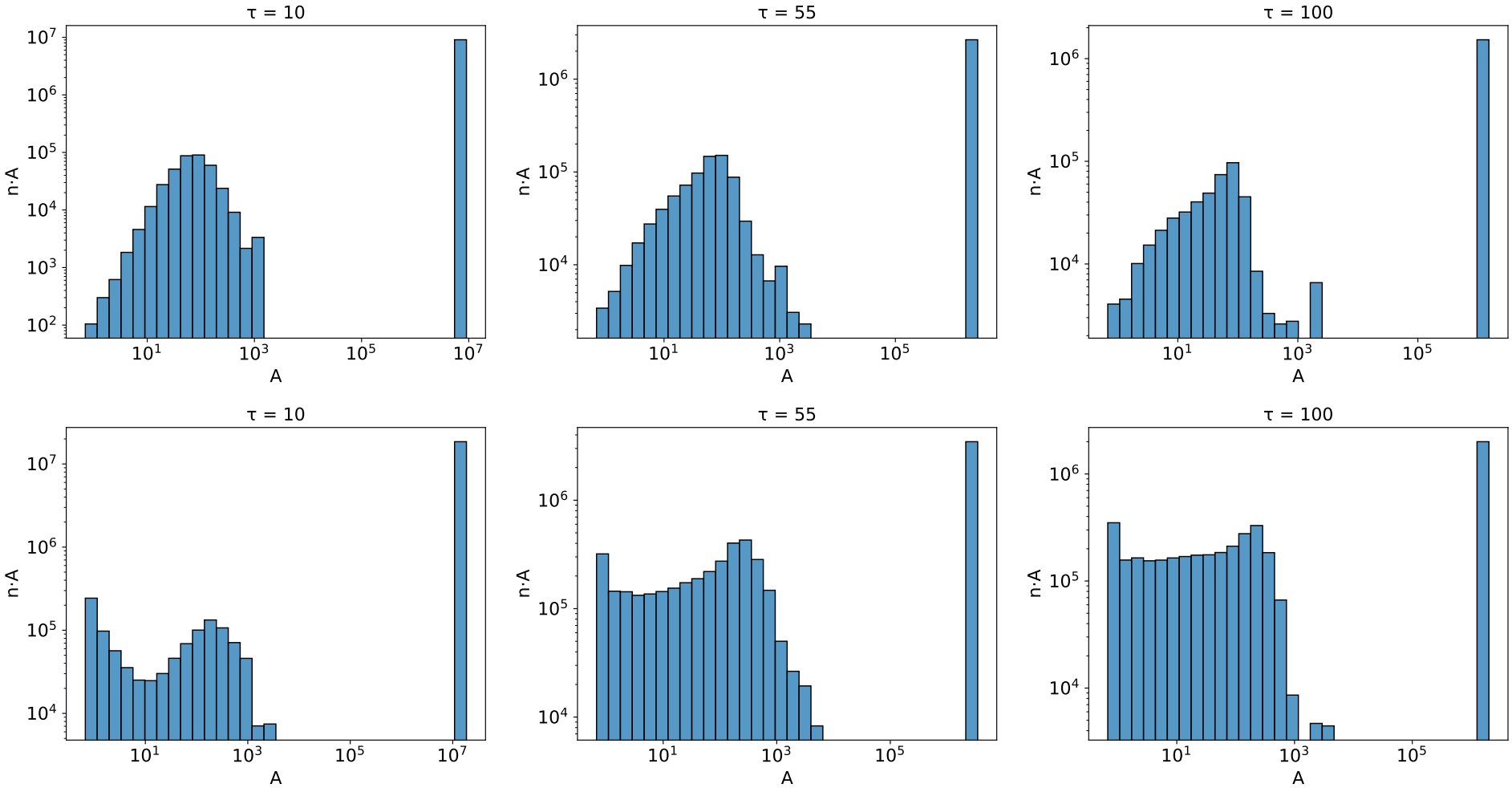


domain walls with decreasing tension σ_{dw} and width

Spectra of domain walls

2410.21971

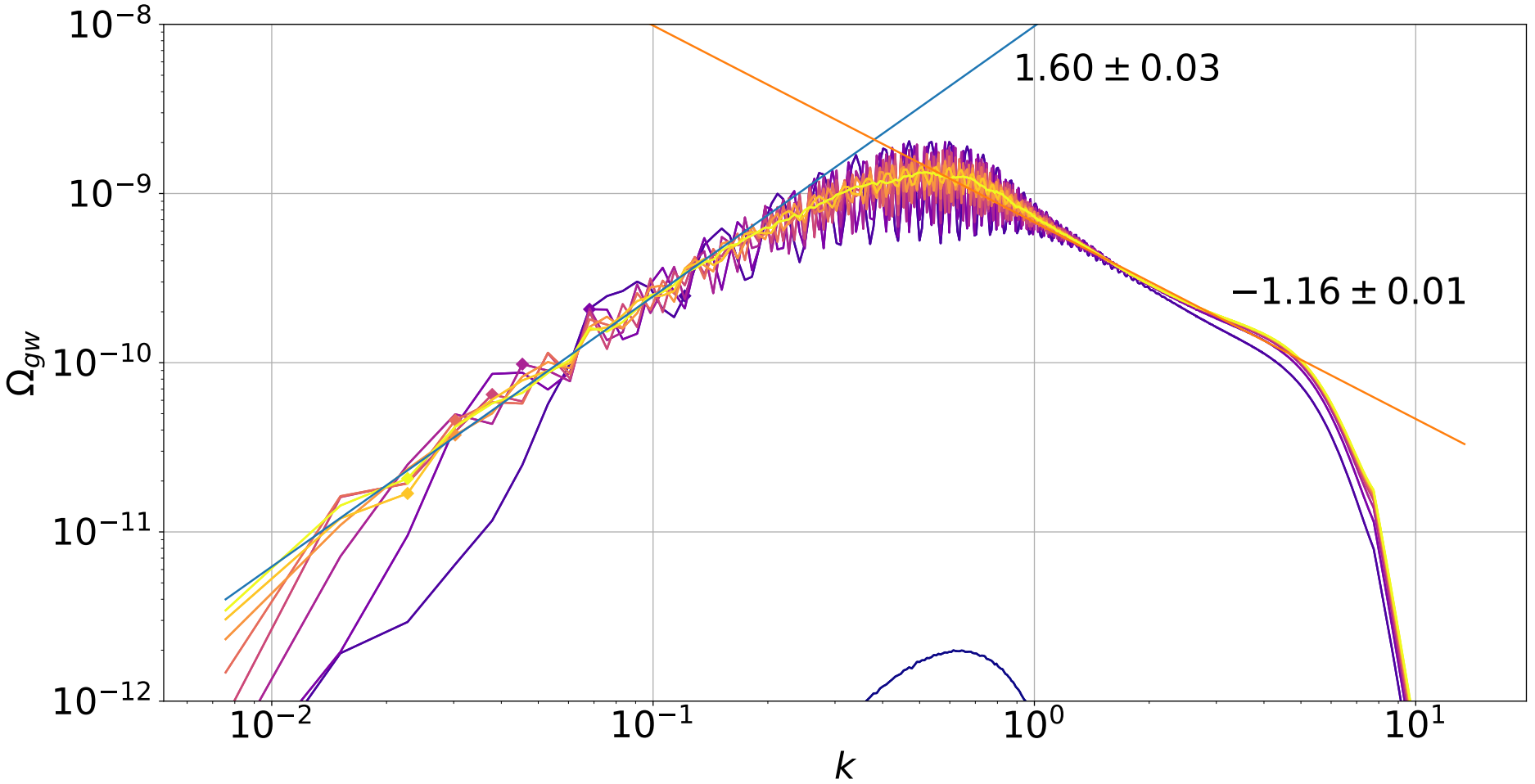
vacuum perturbations



thermal perturbations

GW spectra with vacuum perturbations

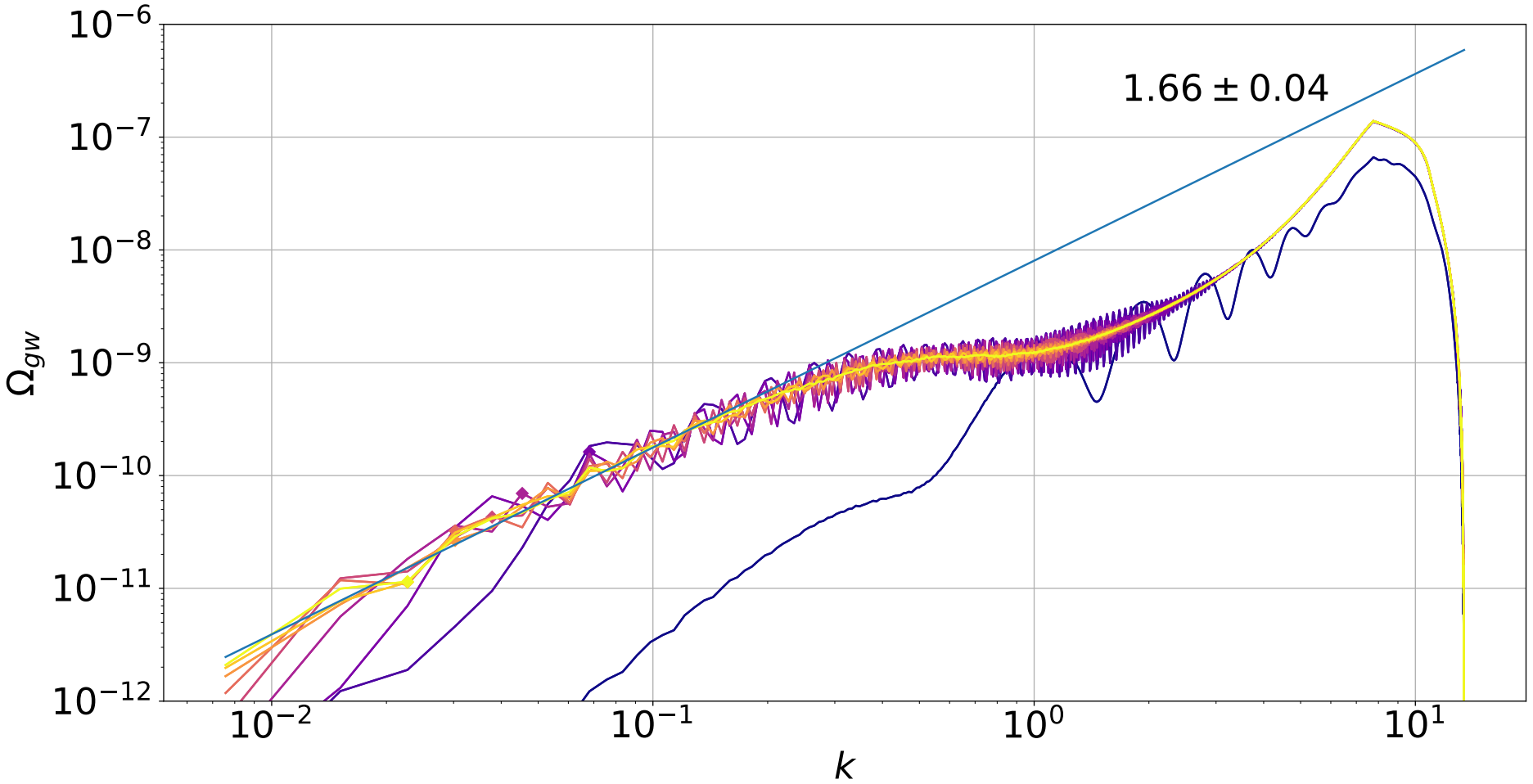
2410.21971



production mostly at the network formation...

GW spectra with thermal perturbations

2410.21971



the same IR-tail: $\Omega \propto k^{1.6}$

theory predicts (2112.12608) $\Omega \propto k^2$

Analog of a well-known effect in CMB: by Rubakov, Sazhin and Veryaskin

From linear approximation to the geodesic equation...

for tensor perturbations

$$\frac{\delta T}{T}(\mathbf{n},\tau_0) = \frac{1}{2}\int_{\tau_r}^{\tau_0} d\tau\, n_i h_{ij}^{TT\,\prime} n_j\,,$$

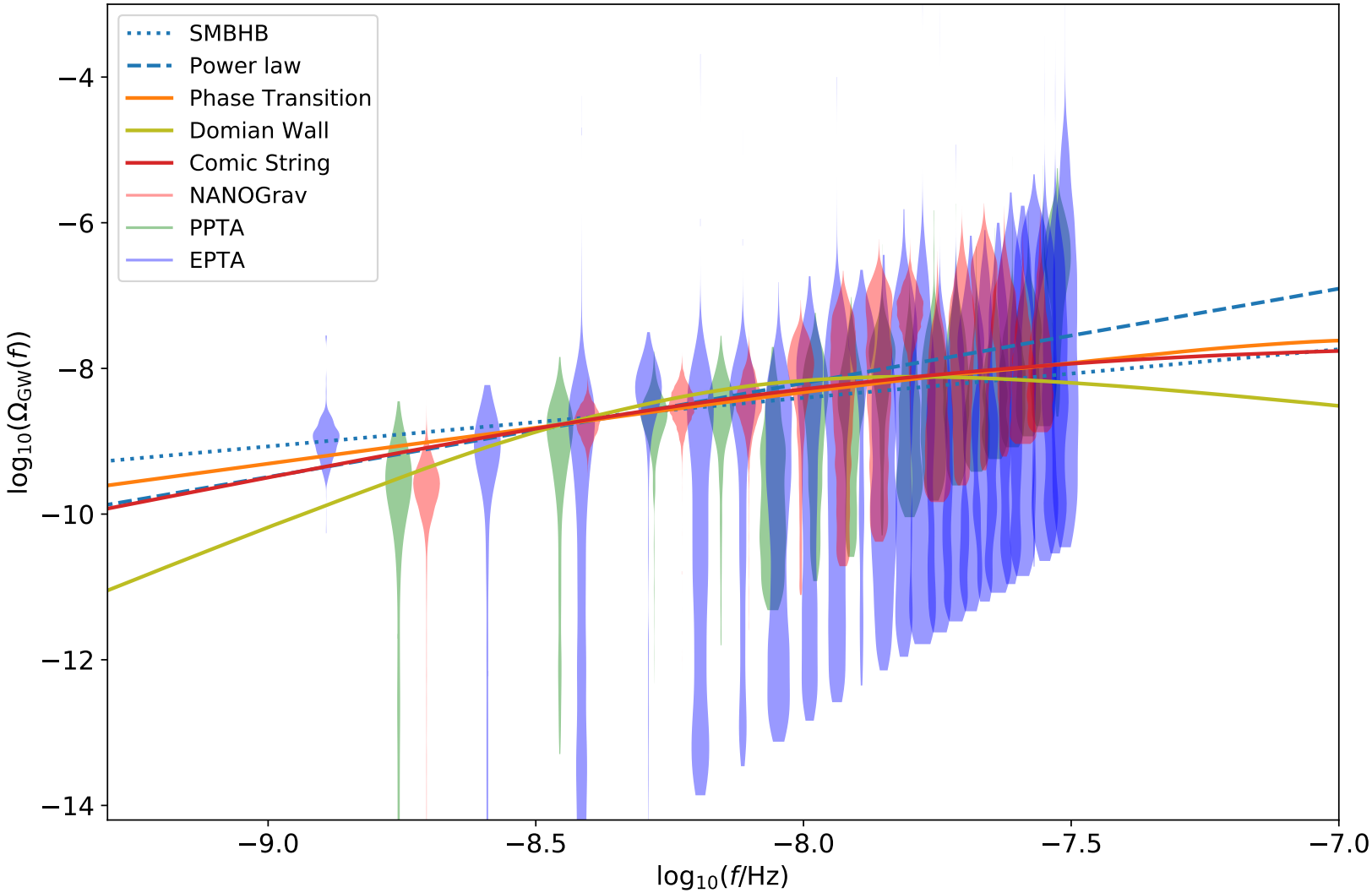
strongest limit on primordial GW and Inflation scale:

$$h_{init} \sim H_{inf}/M_{Pl} \propto \sqrt{\rho_{infl}}$$

Pulsar frequencies are different but stable, and we observe its' time variantion



NANOGrav, T. Klein



$$\Omega_{GW}h^2 = 6.3\cdot 10^{-10}\, A^2 \left(\frac{f}{\mathrm{yr}^{-1}}\right)^d,\quad A = \{6.4^{+4.2}_{-2.7}, 2.9^{+2.6}_{-1.8}, 3.1^{+1.3}_{-0.9}\},\quad d = \{1.8\pm 0.6,\ 0.81^{+0.63}_{-0.73},\ 1.1\pm 0.4\}$$

2307.03141, 2406.02288

DWs with explicitly broken Z_2: No walls, no problem!

$$\mathscr{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{\lambda}{4} \left(\phi^2 - v^2 \right)^2 - \varepsilon \phi^3$$

tilted potential induces some pressure...

$$V_b = \varepsilon \phi^3 , \quad V_{bias} = V_b(\phi = v) - V_b(\phi = -v) \approx 2\varepsilon v^3$$

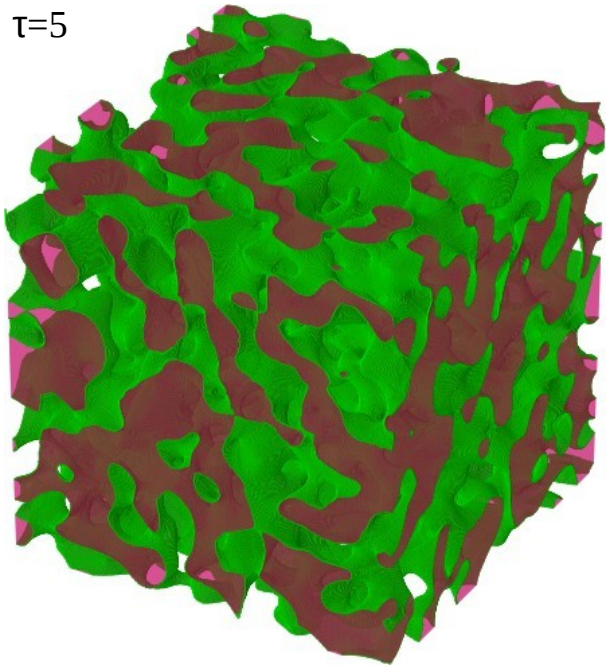
the expected time of DW destruction: e.g. at radiation domination

$$\rho_{DW} \propto \frac{1}{l_H} \propto H \propto \frac{1}{\tau^2}$$

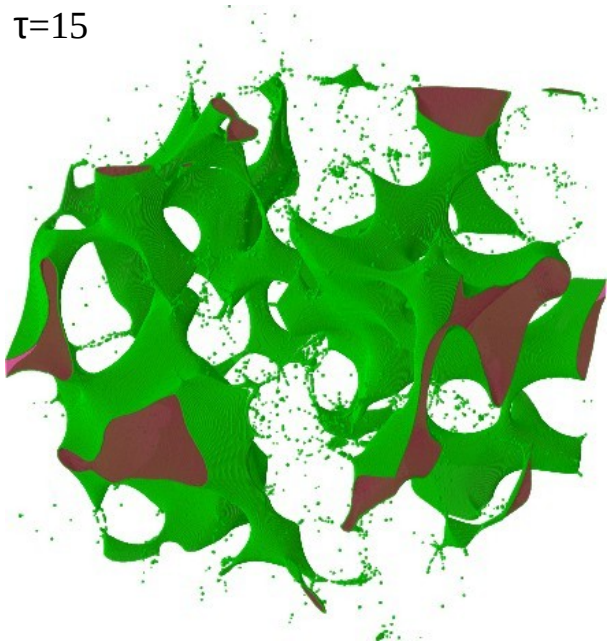
the energy balance

$$\rho_{DW} \sim V_{bias} \quad \rightarrow \tau_{ann} \propto 1/\sqrt{\varepsilon}$$

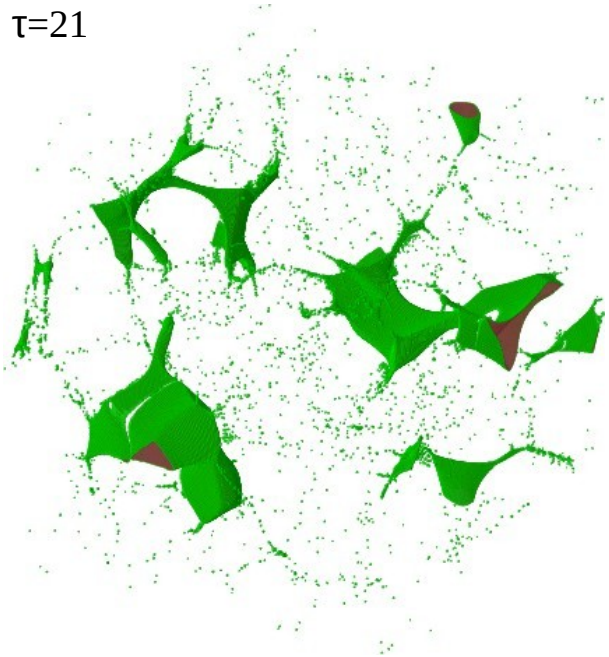
τ=5



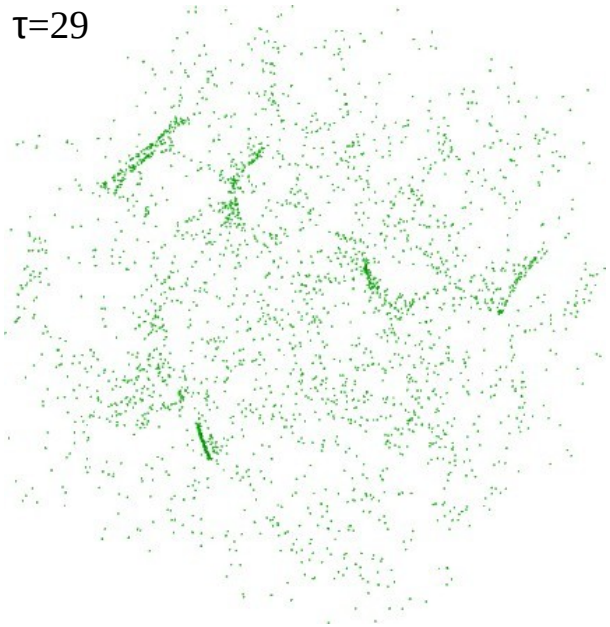
τ=15



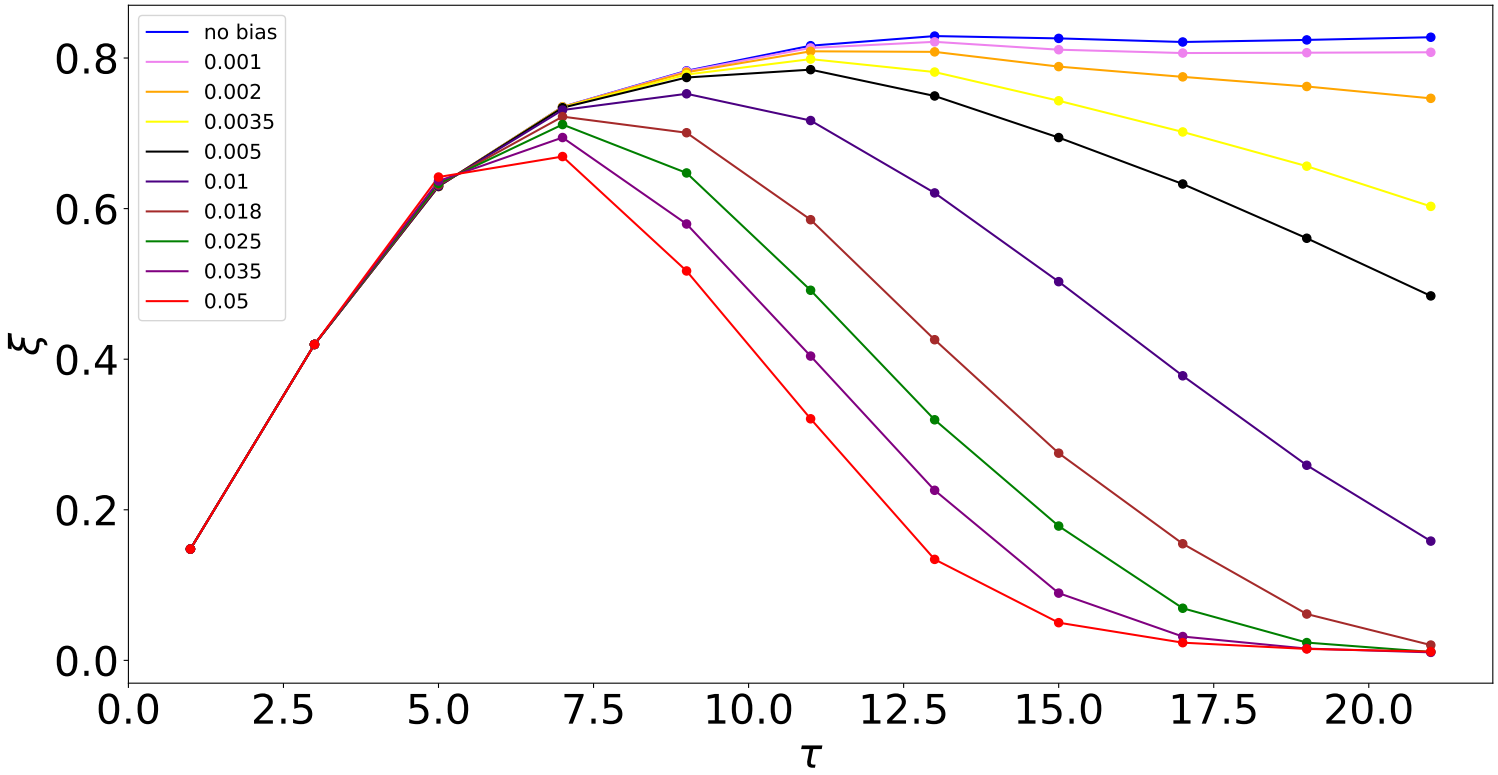
τ=21



τ=29

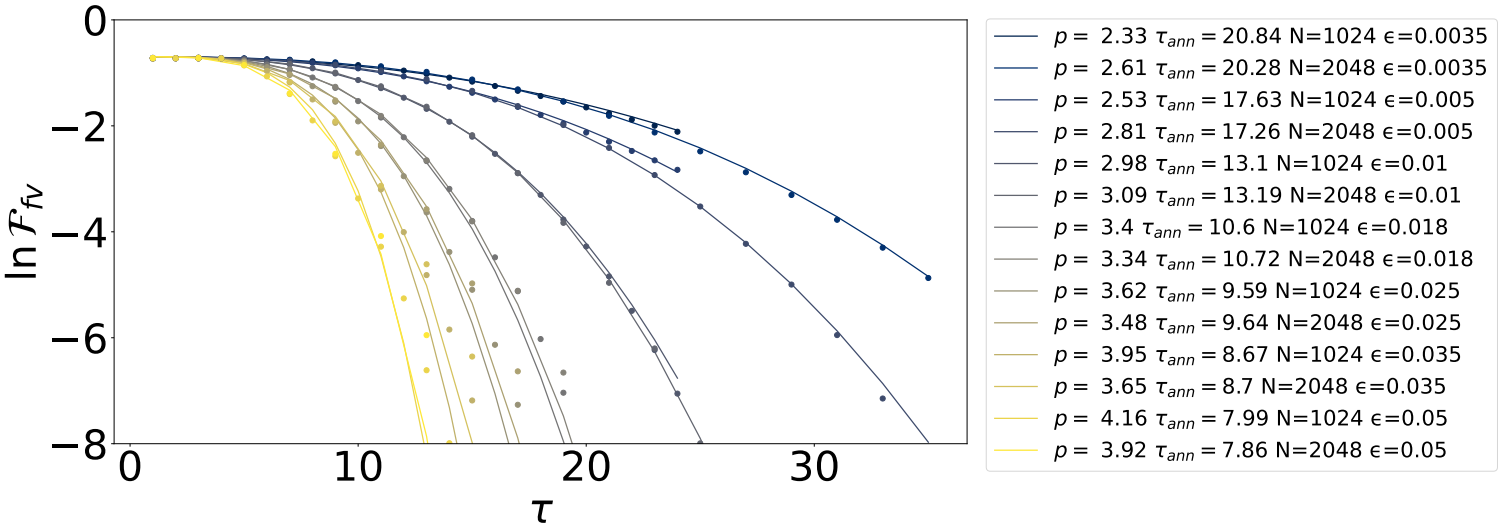


2504.07902

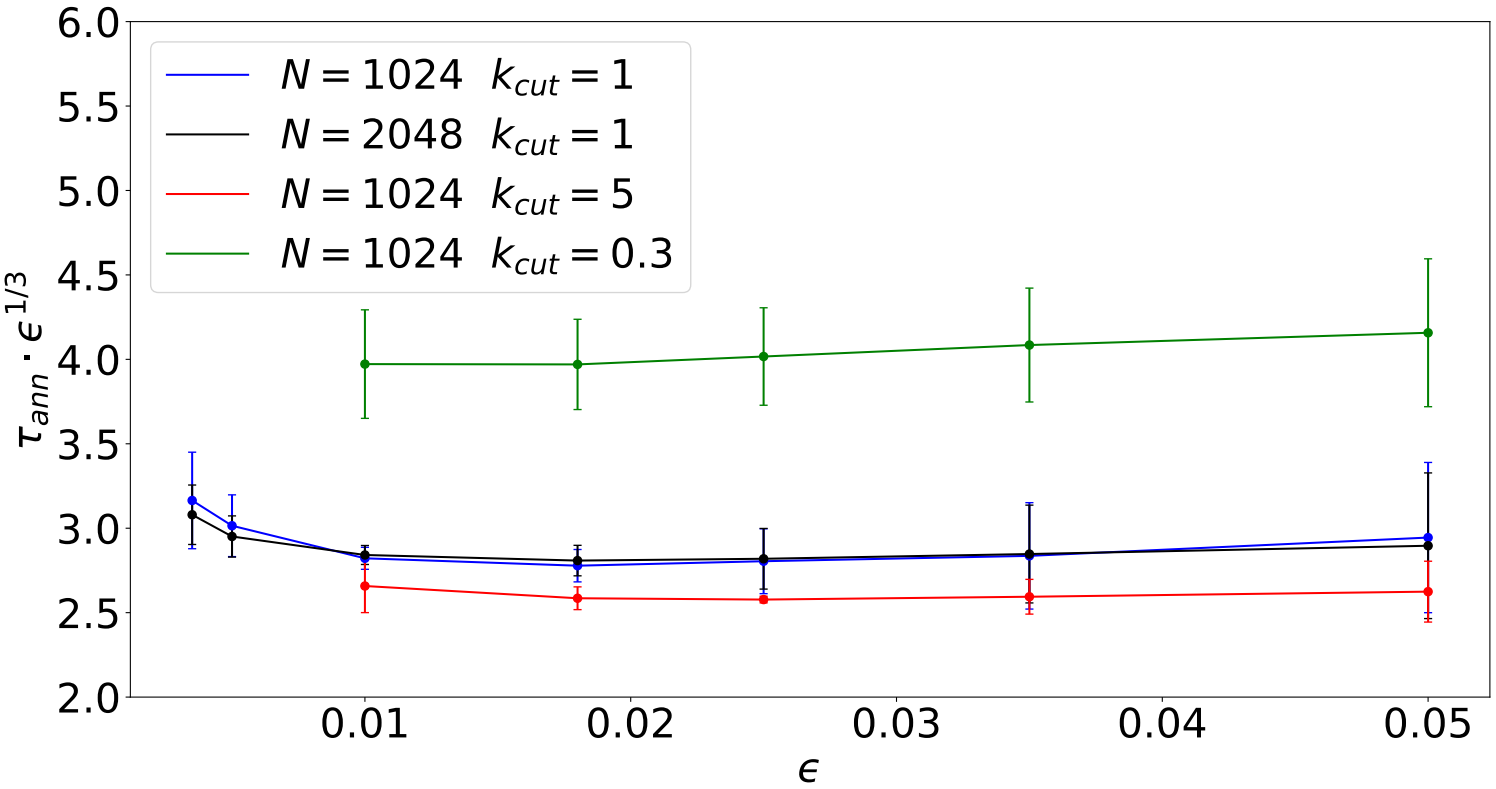


vacuum fraction

$$\mathcal{F}_{fv} = \frac{1}{2} \times \exp\left[-\left(\frac{\tau}{\tau_{ann}}\right)^{p}\right]$$



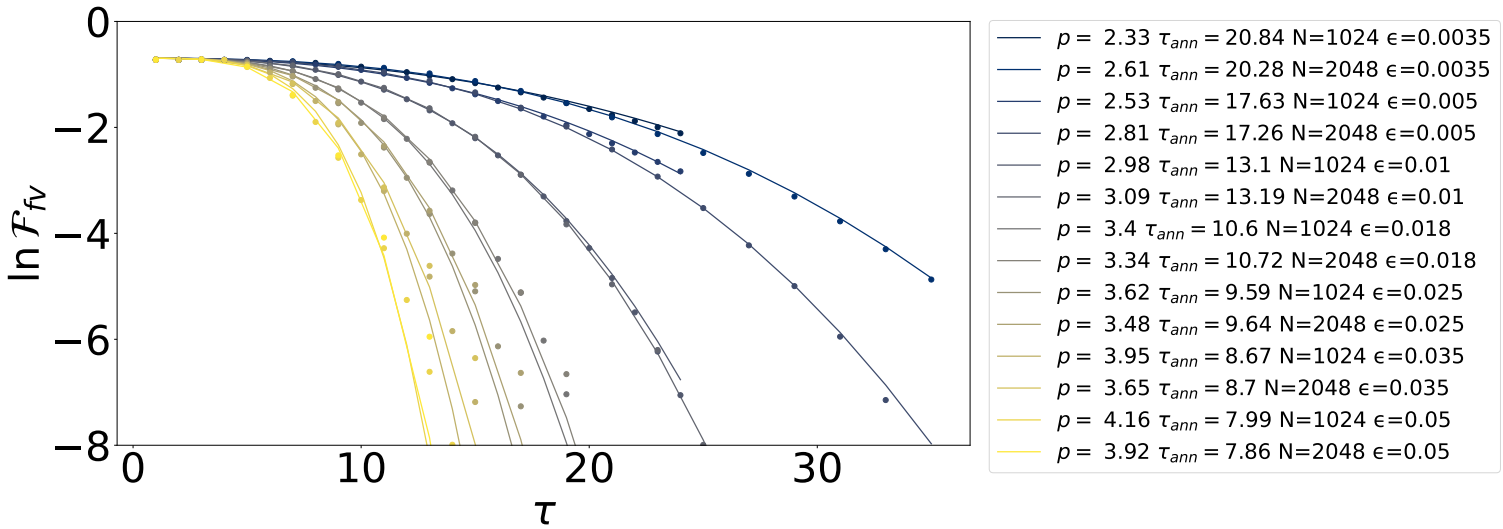
2504.07902



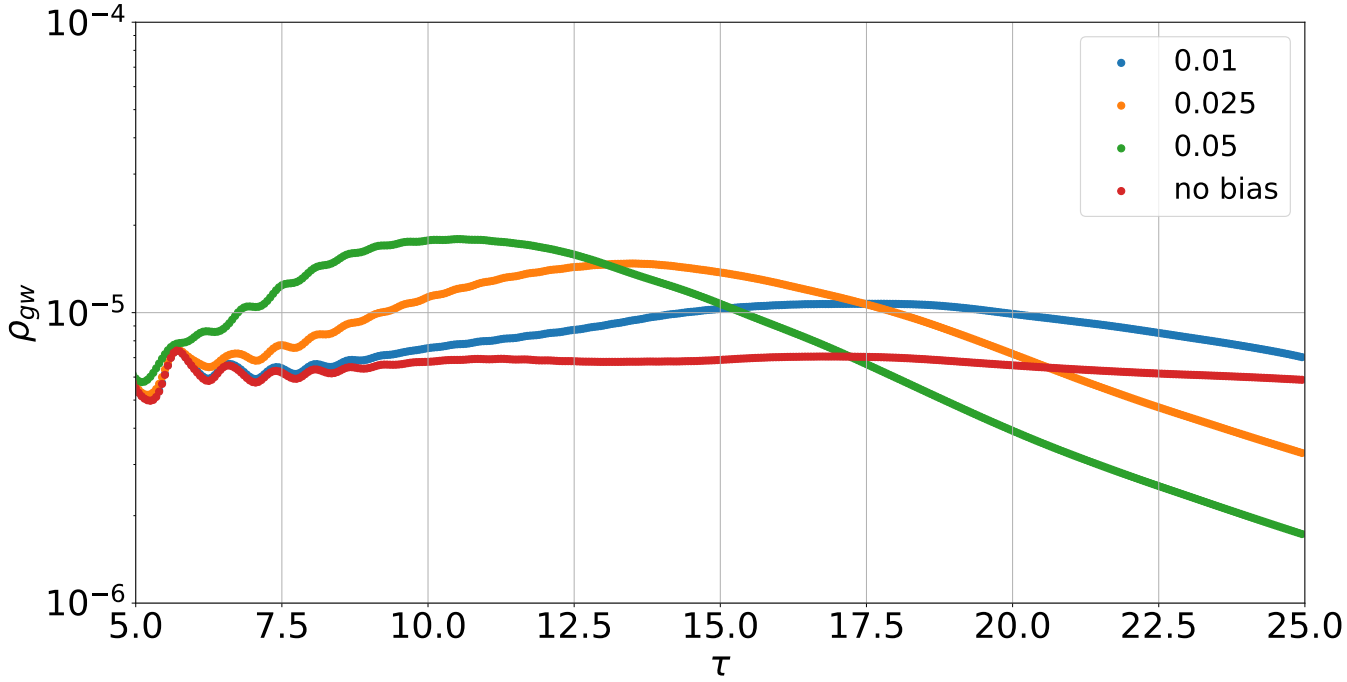
first surprise:

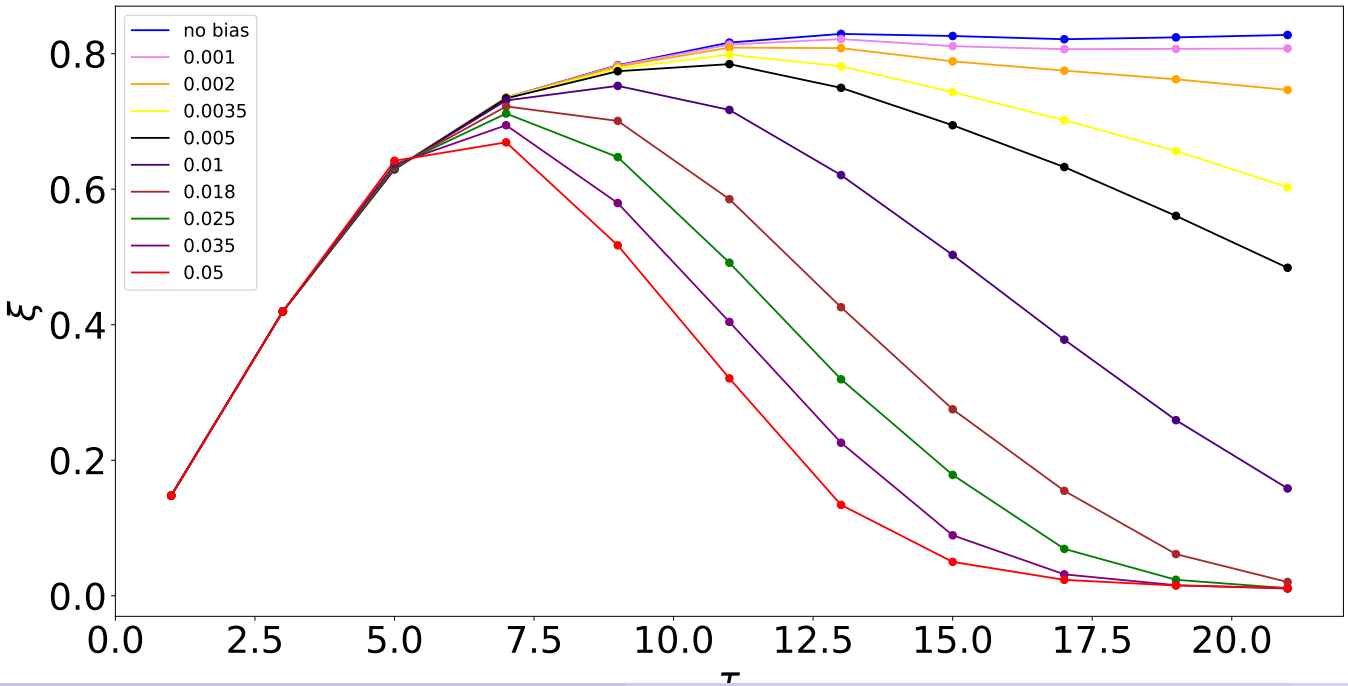
$\tau_{ann} \propto \varepsilon^{-1/3}$

no idea why...



2504.07902





$$\varepsilon = 0.025$$

second surprise:
GW production stops later

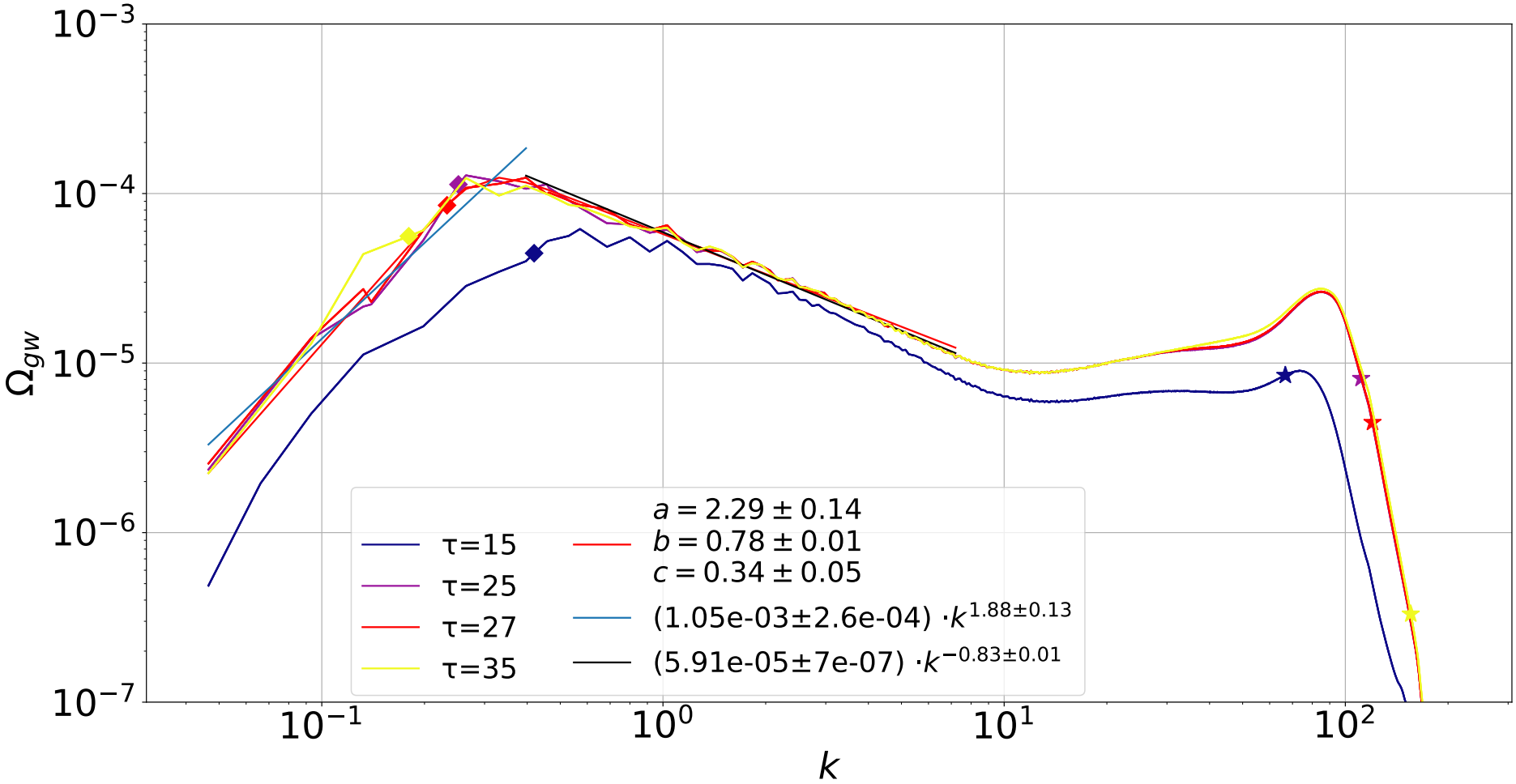
$$\tau_{GW} \simeq 2\,\tau_{ann}$$

while naively at RD

$$\Omega_{GW} = \frac{\rho_{GW}}{\rho_{tot}} \propto a_{GW}^4$$

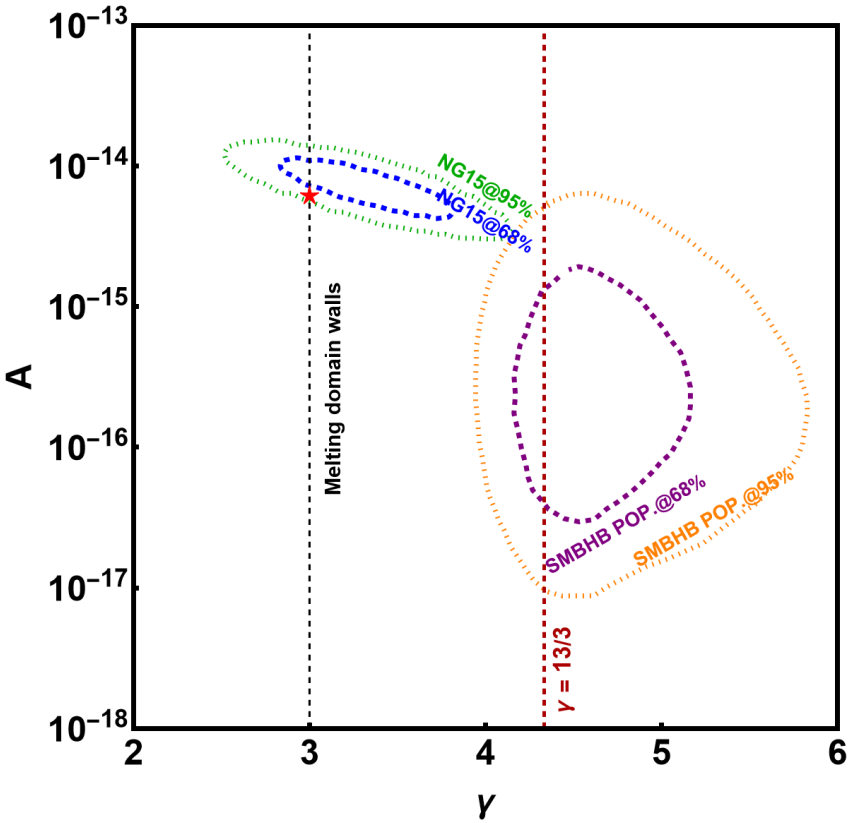
$$\Omega_{GW} \propto \tau_{GW}^4 \propto \frac{1}{\varepsilon^2}$$

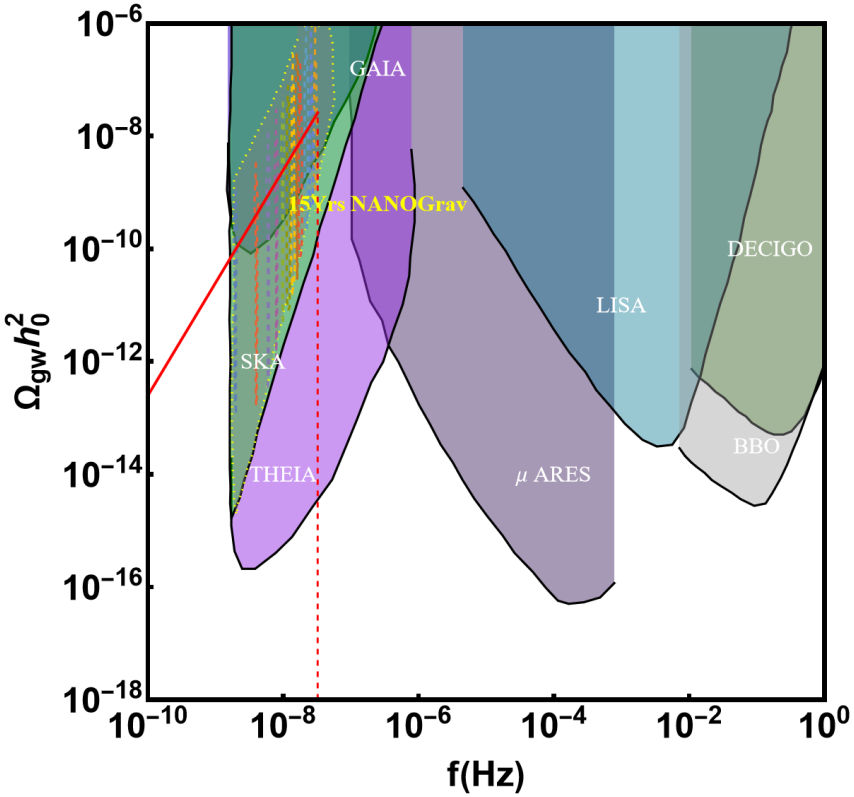
2504.07902



$$f_{peak} \simeq 8\ \text{nHz}\,\lambda^{1/4} \cdot \left(\frac{\varepsilon}{10^{-36} \cdot \lambda v}\right)^{1/3} \cdot \sqrt{\frac{v}{100\ \text{TeV}}} \cdot \left(\frac{100}{g_*(T_{ann})}\right)^{1/12},$$

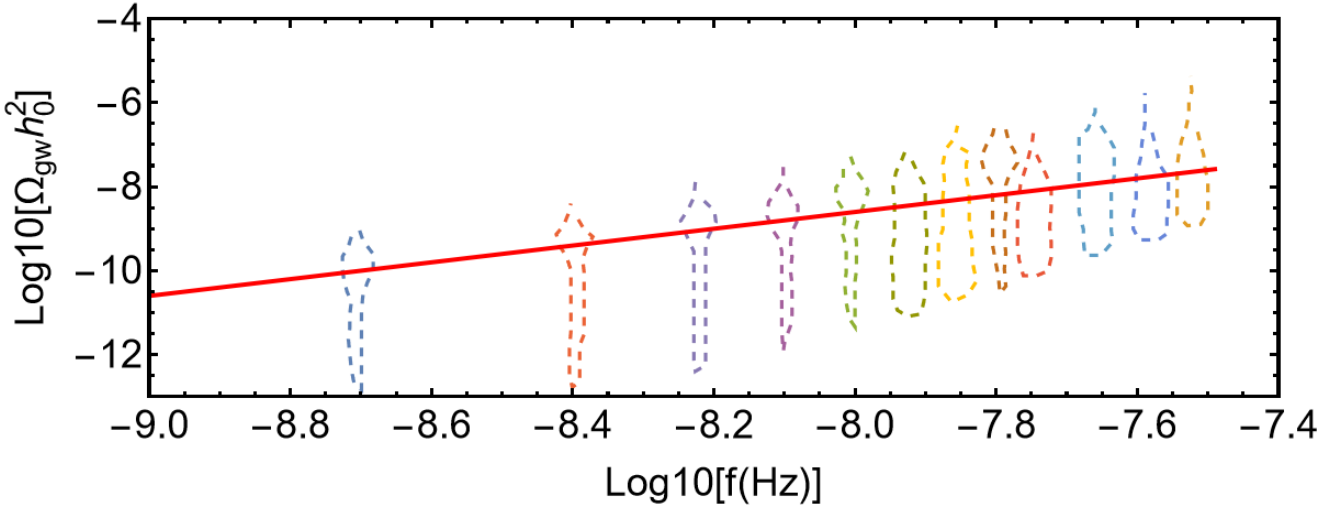
$$\Omega_{gw,peak} h_0^2 \simeq 1 \cdot 10^{-10} \cdot \left(\frac{v}{100\ \text{TeV}}\right)^4 \cdot \left(\frac{10^{-36} \cdot \lambda v}{\varepsilon}\right)^{4/3} \cdot \left(\frac{100}{g_*(T_{ann})}\right)^{1/3}.$$





$$\gamma \equiv 5 - d$$

2307.04582



Conclusion

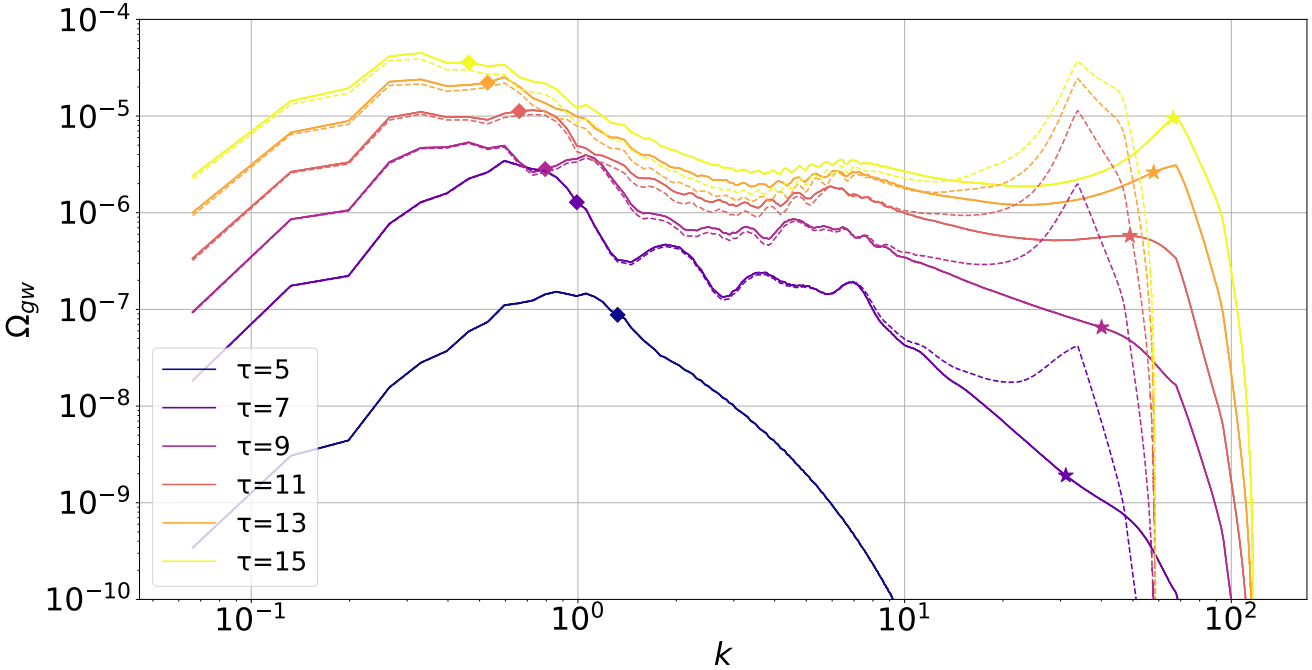
- What NANOGrav and others observe might be explained by the GW from melting domain walls
- Note: $f = 1$ nHz implies GW production at $T \sim 100$ MeV
- field ϕ may serve as dark matter !! see e.g. 2004.03410, 2104.13722, 2112.12608
  In realistic models its mass is of order 10^{-15}-10^{-11} eV
  That predicts super-radiance: instability of rotating black holes with astrophysical masses 10-$10^5\, M_\odot$ see e.g. 0905.4720
  Might explain why the BH in the observed (GW) binary system are not with $J \approx 1$
- Biased DWs... A signal of $G_{WG} \propto 1/\varepsilon^{3/2}$ instead of $G_{WG} \propto 1/\varepsilon^2$ ... Naturally $\varepsilon \lll 1$, e.g. gravity
- Analytical estimates are not that good to describe
  the vast region around the maximum:
  numerical simulations with particular sources are needed
  to predict the shape of the spectrum and fit it to the observed data

Large based on the results from
2004.03410, 2104.13722, 2112.12608, 2307.04582, 2410.21971 , 2406.17053, 2504.07902, 2509.25367, 2607.25073, 2607.28214

with Evgeny Babichev, Ivan Dankovsky, Sabir Ramazanov, Rome Samanta and Alex Vikman

GW with initial vacuum perturbations

2406.17053

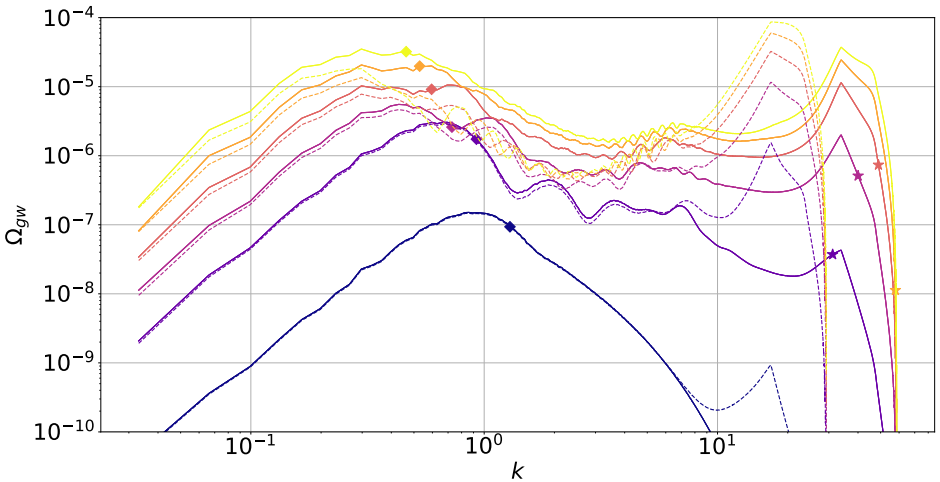


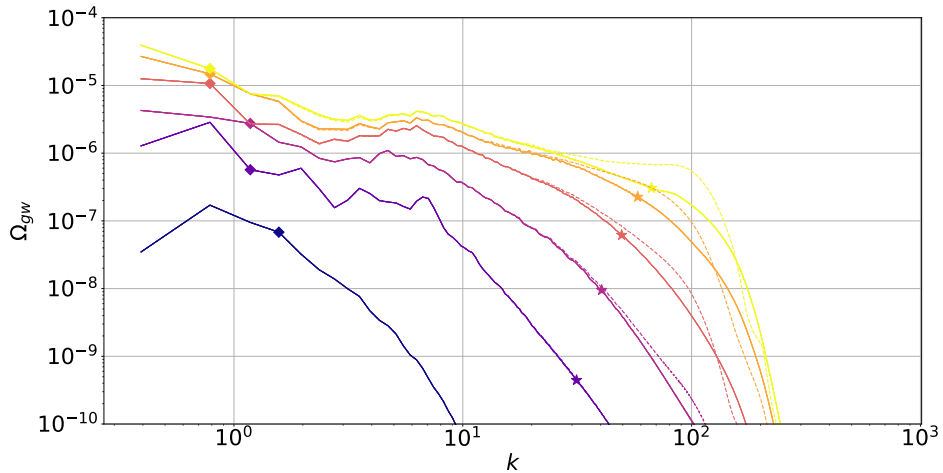
IR peak: horizon size

UV peak: lattice artifact

Middle peak:

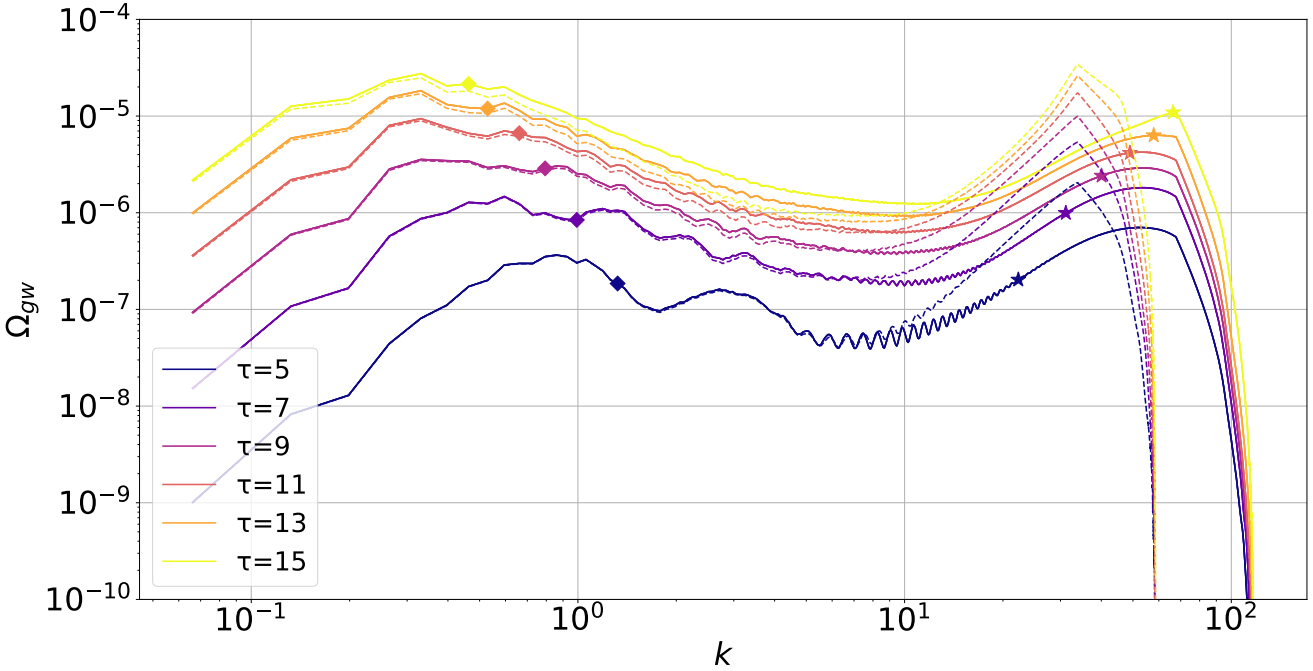
$$k/a \propto 1/a ... ? ... = \sqrt{Hm} \; ?$$





GW with initial vacuum perturbations

2406.17053

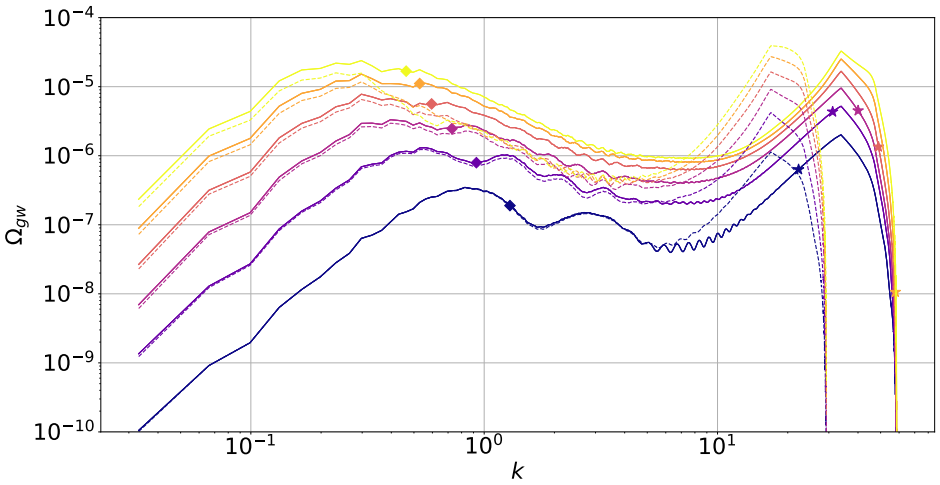


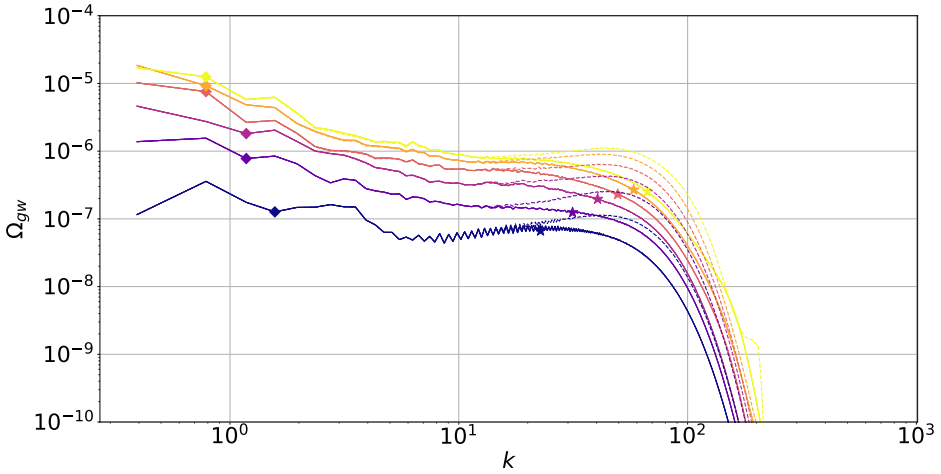
IR peak: horizon size

UV peak: lattice artifact

Middle peak:

$$k/a \propto 1/a ... ? ... = \sqrt{Hm} \; ?$$





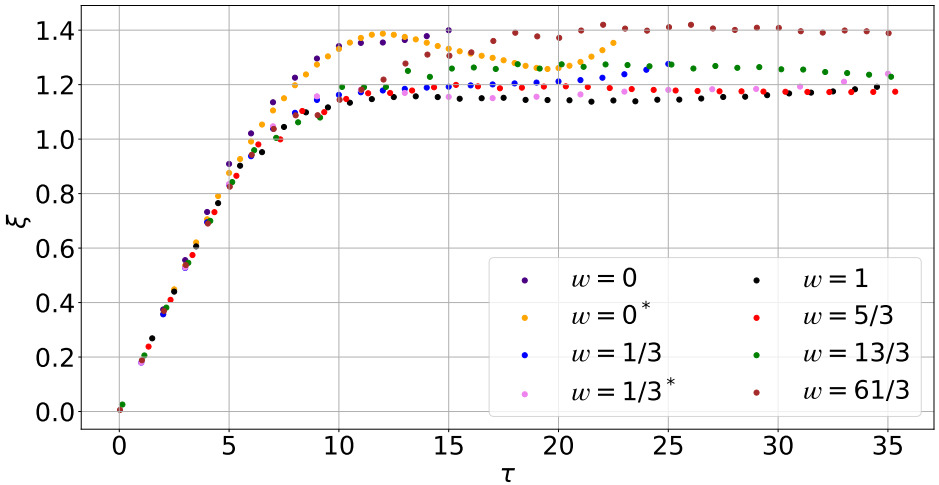
Non-standard cosmology with $P_U = w\rho_w$

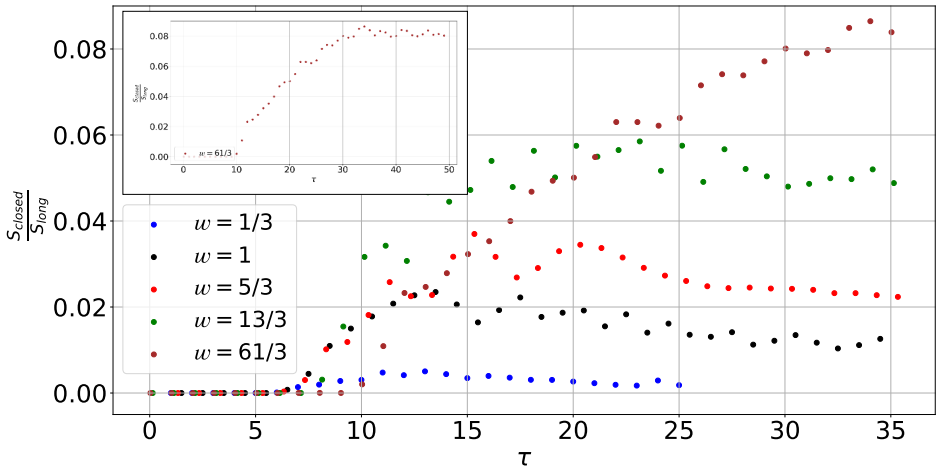
2607.28214

$$l = a(\tau)\tau$$

$$\rho_{wall} \approx \frac{2\xi\sigma_{wall}}{l} \quad \text{with} \quad \xi \equiv \frac{S\tau}{2V}$$

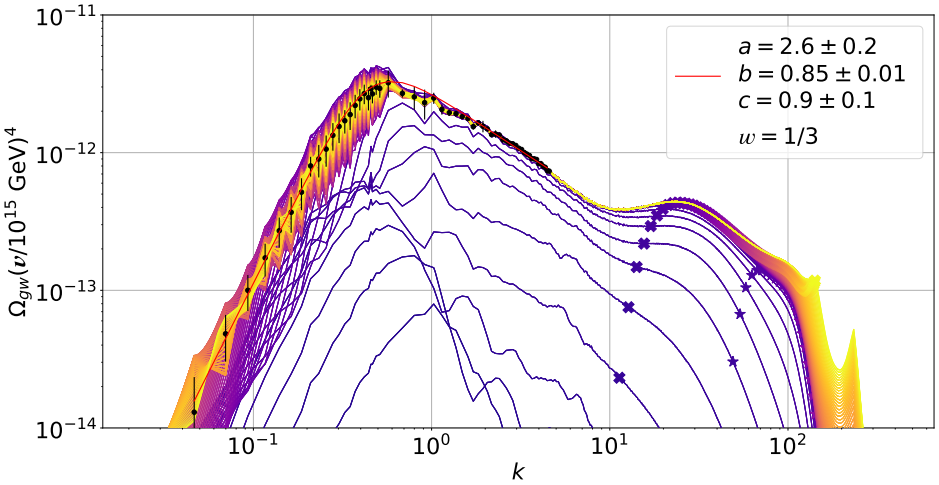
what matters is the horizon scale, not Hubble
the scaling is universal
more closed walls with increasing w:
weaker friction from Hubble for the waves propagating on DWs...?





Non-standard cosmology with $P_U = w\rho_w$

2607.28214

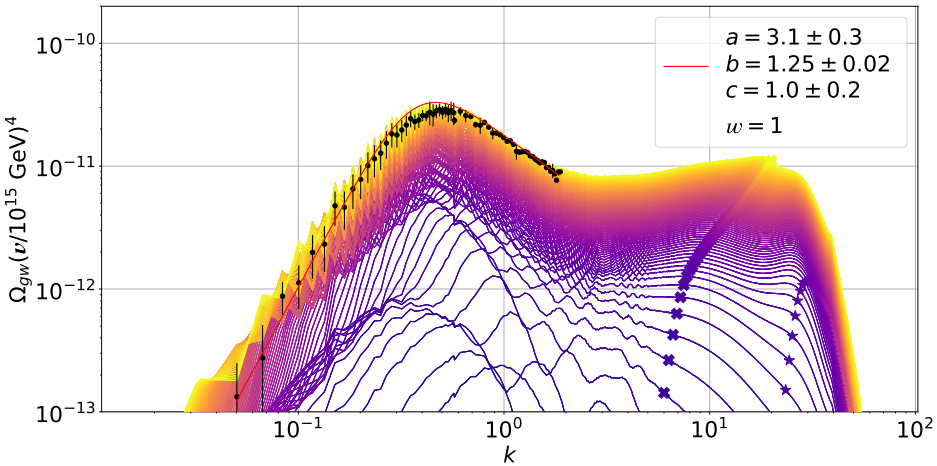


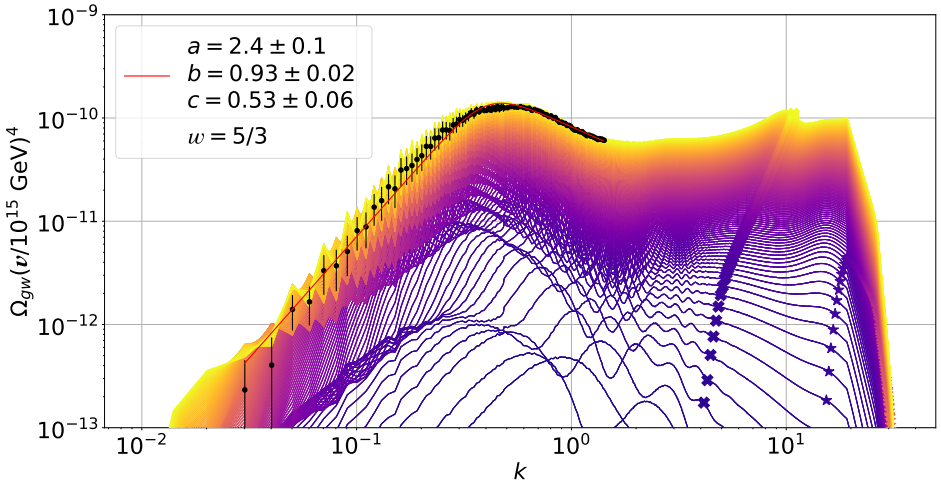
broken power law fit

$$\Omega_{gw} = \frac{\Omega_{gw,peak}\,(a+b)^c}{\left[b\left(\frac{k_{peak}}{k}\right)^{\frac{a}{c}} + a\left(\frac{k}{k_{peak}}\right)^{\frac{b}{c}}\right]^c}$$

UV border: stars indicate DW width
$k = 2\pi a/\delta_{wall} = \pi a m_\chi$

feature in the middle (crosses): $k = a m_\chi$





2-peak shape becomes more pronounced

NANOGrav signal

the second scale $\propto m$ may be due to cooling waves on the walls