not only Love is in the Air

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The yielding of the Earth to disturbing forces. By A. E. H. Love, D.Sc., F.R.S., Sedleian Professor.

[*Abstract of a paper in the Proceedings of the Royal Society, A., vol. lxxxii.*, 1909, *pp.* 73–88.]

The Earth, not being an absolutely rigid body, must be deformed by the tide-generating forces of the Sun and Moon. The object of the first part of the paper is to determine the amount by which the surface actually yields to these forces, or, what is the same thing, the height of the corporeal tide raised by them on the Earth.

Outline

- Static tidal Love numbers of black holes.
- Dynamical tidal Love number of black holes.
- Effective Field Theory of tidal Love numbers.

References

SC and Pani, arXiv: 2604.08679.

- Biswas, Singha and **SC**, **PRD 109**, 064043 (2024); **PRD 106**, 124003 (2022).
- **SC**, de Luca, Gualtieri and Pani, **PRD 112**, 104015 (2025)
- **SC**, Heidmann and Pani, **PRD 113**, L061503 (2026).
- **SC**, Saketh, Hinderer and Steinhoff, arXiv: 2605.00693.

Tides are Everywhere

- Tides are ubiquitous and captures the true nature of gravity, as it depends on the Riemann.
- Deformation produced by the tides depends on the detail of the constituent of the object being deformed.
- What about black holes? Can they be deformed?
- In the context of GWs, the tidal effects are dynamic. Implications?
- Effective field theory of tidal deformations.

[Hinderer, arXiv: 0711.2420]

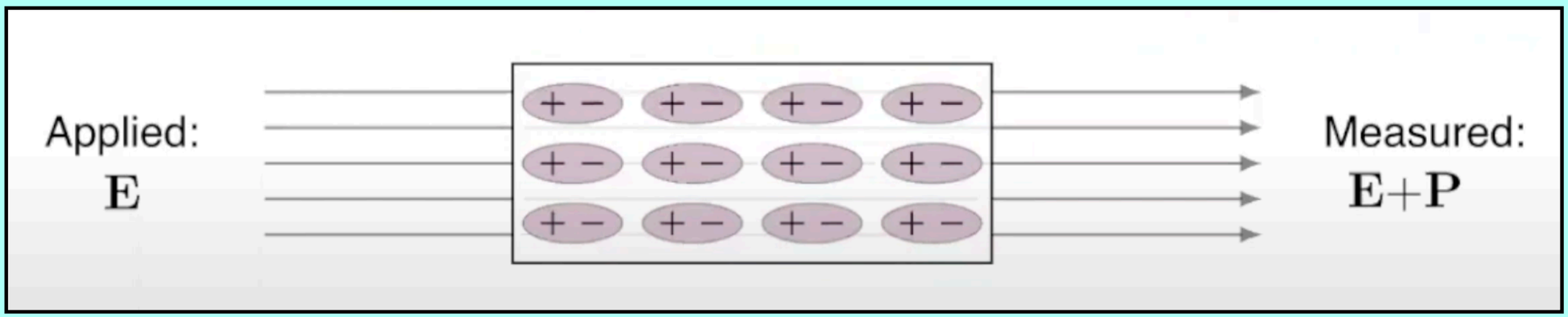
[Binnington and Poisson, arXiv: 0906.1366]

[Damour and Nagar, arXiv: 0906.0096]

Electric Love

- The more familiar situation with non-zero Love corresponds to polarisation due to an external electric field.

$$\mathbf{P} = \chi_E \mathbf{E}$$



[Figure Courtesy: https://www.youtube.com/watch?v=3mXo8xqAKs4]

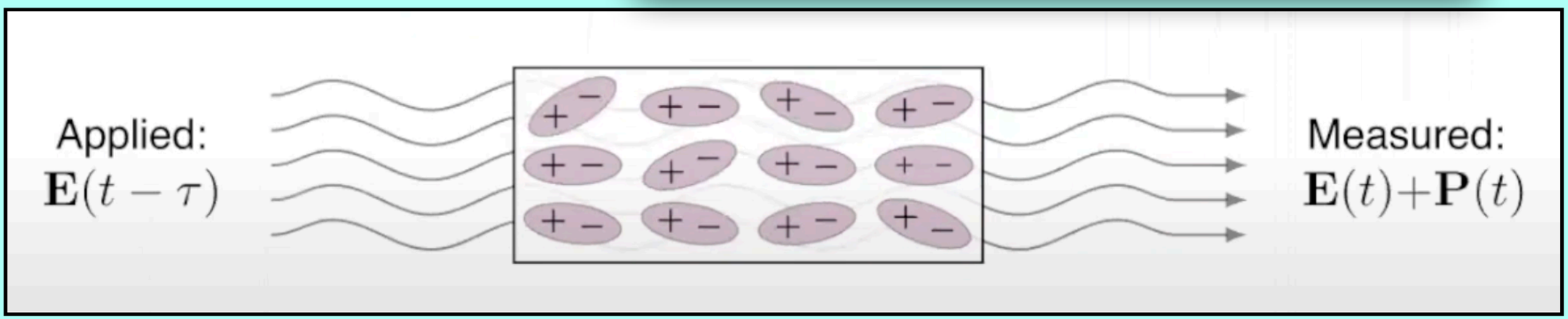
Electric Love

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$$\mathbf{P} = \chi_E \mathbf{E}$$

- In the dynamical situation:

$$\begin{aligned}\mathbf{P}(t) &= \int_0^\infty \mathrm{d}\tau\, \chi_E(\tau)\, \mathbf{E}(t-\tau)\,, \\ &= \chi_E^{(0)}\, \mathbf{E}(t) - \tau_0\, \chi_E^{(1)}\, \dot{\mathbf{E}}(t) + \cdots\,,\end{aligned}$$



[Figure Courtesy: https://www.youtube.com/watch?v=3mXo8xqAKs4]

Newtonian Deformation

- Newtonian gravity is solely governed by the potential.
- The tidal field arises from double derivative of the potential.

$$\mathcal{E}_{ij} = \frac{\partial^2 \Phi_{\rm ext}}{\partial x^i \partial x^j}.$$

- The total potential will consist of potential due to the external tidal field and the potential of the deformed object.

$$\frac{(1-g_{tt})}{2} = -\frac{M}{r} - \frac{3Q_{ij}}{2r^3}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right) + O\left(\frac{1}{r^3}\right) + \frac{1}{2}\mathcal{E}_{ij}x^i x^j + O\left(r^3\right),$$

- Quadrupole moment of the deformed object arises due to tidal field.

$$Q_{ij} = -\lambda \mathcal{E}_{ij}.$$

$$k_2 = \frac{3}{2} G \lambda R^{-5}.$$

[Thorne, Phys. Rev. D 58, 124031 (1998)]

Love number of BHs

- BH perturbation theory becomes directly applicable.

$$g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + h_{\alpha\beta},$$

$$h_{\alpha\beta} = \mathrm{diag}\left[e^{-\nu(r)}H_0(r),\ e^{\lambda(r)}H_2(r),\ r^2K(r),\ r^2\sin^2\theta K(r)\right] Y_{2m}(\theta,\varphi).$$

- Among the three unknown functions, perturbed Einstein's equations demand $H_2 = H_0 \equiv H$, and K can be expressed in terms of H.

$$(x^2-1)\,H'' + 2xH' - \left(6 + \frac{4}{x^2-1}\right) H = 0.$$

[Hinderer, arXiv: 0711.2420]

- Exact solution exist, in terms of associated Legendre polynomials.

$$\begin{aligned} H &= c_1 \left(\frac{r}{M}\right)^2 \left(1-\frac{2M}{r}\right)\left[-\frac{M(M-r)(2M^2+6Mr-3r^2)}{r^2(2M-r)^2} + \frac{3}{2}\log\left(\frac{r}{r-2M}\right)\right] \\ &\quad +3c_2\left(\frac{r}{M}\right)^2\left(1-\frac{2M}{r}\right). \end{aligned}$$

[near horizon]

$$H = \frac{8}{5}\left(\frac{M}{r}\right)^3 c_1 + O\left(\left(\frac{M}{r}\right)^4\right) + 3\left(\frac{r}{M}\right)^2 c_2 + O\left(\left(\frac{r}{M}\right)\right),$$

[intermediate]

Questions to Address

- **Q1**: Can we provide a formalism based on scalars?
- **Q2**: What happens in a dynamical situation?
  - The GW observations of tidal Love number happens in the in-spiral phase, which is inherently dynamical.
- **Q3**: Do BHs have vanishing dynamical tidal Love numbers?
- **Q4**: What about effective field theory matching?

Tidal Love number from scalar

- For the Love number to be gauge invariant, one solves the Teukolsky equation for the Weyl scalars and impose boundary conditions near the horizon.

[Le Tiec +, arXiv:2010.15795]

$$\lim_{c\to\infty} c^2\psi_0 = \sum_{\ell m} \alpha_{\ell m}(t)\, r^{\ell-2} \left[1 + 2k_{\ell m}\left(\frac{R}{r}\right)^{2\ell+1}\right] {}_2Y_{\ell m}(\theta,\varphi)\,,$$

$$F_{\ell m}(\omega) = 2\,k_{\ell m} + i\omega\tau_0\,\nu_{\ell m} + \mathcal{O}(\omega^2)\,.$$

- The second term arises from single time derivative of the tidal field with respect to co-rotating frame of reference. For Kerr BH

$$\omega \rightarrow \omega' = \omega - m\Omega_h$$

[Chia, arXiv:2010.07300]

Static Love Number of a BH

- The radial Teukolsky equation can be solved exactly in the static limit.

$$R_s(z) = c_1 z^{-s}(z+1)^{-s}\, {}_2F_1\left(1+\ell-s, -s-\ell; 1-s+2iP_0; -z\right)$$

- The corresponding Weyl scalars need to expanded for large r/z, providing the following response function:

$$\mathcal{F}_{s\ell m} = \begin{cases} i\,(-1)^{1-s}\,\dfrac{am}{r_+}\,\dfrac{(\ell-s)!\,(\ell+s)!}{(2\ell+1)!\,(2\ell)!}\,\displaystyle\prod_{k=1}^{\ell}\left[k^2\left(1-\dfrac{r_-}{r_+}\right)^2+\left(\dfrac{2am}{r_+}\right)^2\right] & \text{for bosons, } (s,\ell,m) \text{ integers,} \\ \dfrac{(-1)^{\frac{1}{2}-s}}{2}\,\dfrac{(\ell-s)!\,(\ell+s)!}{(2\ell+1)!\,(2\ell)!}\,\displaystyle\prod_{k=1}^{\ell+\frac{1}{2}}\left[(k-\tfrac{1}{2})^2\left(1-\dfrac{r_-}{r_+}\right)^2+\left(\dfrac{2am}{r_+}\right)^2\right] & \text{for fermions, } (s,\ell,m) \text{ half-integers,} \end{cases}$$

[Chia, arXiv:2010.07300]

[Le Tiec, Casals, Franzin, arXiv: 2010.15795]

[Bhatt, SC and Bose, arXiv: 2406.09543]

[SC, Heidmann and Pani, arXiv: 2508.11096]

Dynamical Love number of BHs

- The previous analysis did not perform the correct approximation scheme.

- In small frequency and near horizon limit:

$$R(z) = (z+1)^{2-N_3}\left[c_1 z^2\, {}_2F_1\left(3+l-N_2, 2-l-N_1; 3+2iP_+; -z\right) \right.$$
$$\left. + c_2 z^{-2iP_+}\, {}_2F_1\left(l-2iP_+-N_2+1, -l-2iP_+-N_1; -1-2iP_+; -z\right)\right]$$

- For BH, imposing purely ingoing boundary condition near the horizon, the second solution identically vanishes. Also, N_1 and N_2 are both $\mathcal{O}(M\omega)$.

$$P_\pm = \frac{am - 2r_\pm M\omega}{r_+ - r_-}$$

$$N_3 = \frac{12\omega\left(r_+ - M\right)^2}{am + 2i(r_+ - M)}$$

[Bhatt, SC and Bose, arXiv: 2406.09543]

Love of Arbitrary Rotating BH

- For an arbitrary rotating BH, the response function with all linear-in-frequency term become:

[Bhatt, SC and Bose, arXiv: 2406.09543]

$$F_{lm} = \frac{\Gamma\left(-2l - N_1 + N_2 - 1\right)\Gamma\left(l + 2iP_+ + N_1 + 1\right)\Gamma\left(3 + l - N_2\right)}{\Gamma\left(2 - l - N_1\right)\Gamma\left(-l + 2iP_+ + N_2\right)\Gamma\left(2l + N_1 - N_2 + 1\right)}$$

- The Gamma functions can be simplified using reflection formula, as well as doing a power series expansion, leading to di-gamma functions.

- The above response function turns out to have a real part. The associated tidal Love numbers become

$$k_{lm} = (am)^2\, k_{lm}^{(0)} + am\omega\, k_{lm}^{(1)} + \mathcal{O}(M^2\omega^2)$$

A Perturbative Approach

- An alternative approach is to expand the perturbation equation in powers of frequency.

$$R_\ell = R_\ell^{(0)} + m\omega\, R_\ell^{(1)} + m^2\omega^2\, R_\ell^{(2)} + \cdots$$

- This leads to a set of recursive relations for Schwarzschild BH:

$$\begin{aligned}
\mathcal{D}_0 R_\ell^{(0)} &= 0\,, \\
\mathcal{D}_0 R_\ell^{(1)} &= 4i\mathcal{D}_1 R_\ell^{(0)}\,, \\
\mathcal{D}_0 R_\ell^{(2)} &= 4i\mathcal{D}_1 R_\ell^{(1)}\,,
\end{aligned}$$

$$\begin{aligned}
\mathcal{D}_0 &= z(1+z)\frac{d^2}{dz^2} - (1+2z)\frac{d}{dz} - (\ell-1)(\ell+2)\,, \\
\mathcal{D}_1 &= (1+z)^2\frac{d}{dz} - \left(\frac{2}{z} + 1 - z\right)\,.
\end{aligned}$$

- This eventually leads to the following dynamical response function:

$${}_{-2}\mathcal{F}_2(\omega) = \frac{im\omega}{15} + \frac{2m^2\omega^2}{15}\left[\ln\left(\frac{r}{2m}\right) + \frac{137}{21}\right]$$

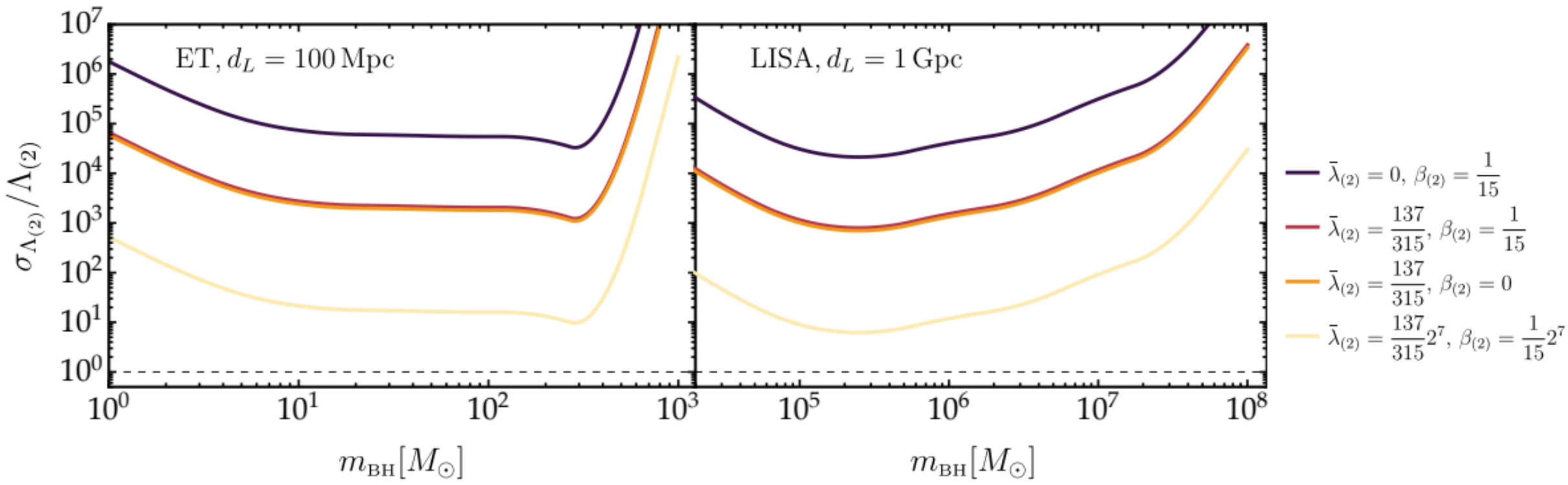
[Katagiri, Yagi, Cardoso, arXiv: 2409.18034]

[SC, de Luca, Gualtieri, Pani, arXiv: 2507.22994]

Implications for Future GW

- The effect of dynamical tides on GW phasing corresponds to:

$$\psi(x) \equiv \frac{3}{128\,\eta\,x^{5/2}}\left[1-\frac{39}{2}\Lambda_{(0)}x^5+\frac{15}{11}\Lambda_{(2)}x^8 +\frac{15}{11}B_{(2)1}\,x^8\ln\left(\frac{M}{\bar{r}_1 x}\right)+\frac{15}{11}B_{(2)2}\,x^8\ln\left(\frac{M}{\bar{r}_2 x}\right)\right]$$



[SC, de Luca, Gualtieri, Pani, arXiv: 2507.22994]

Beyond Black Holes

- Black holes can be constructed from normal matter, using simple collapse scenarios.

- Black holes are stable under all possible perturbations.

- Observation of shadows from Event Horizon Telescope and the ringdown signals from LIGO and VIRGO are definitely consistent with the existence of Black Holes.

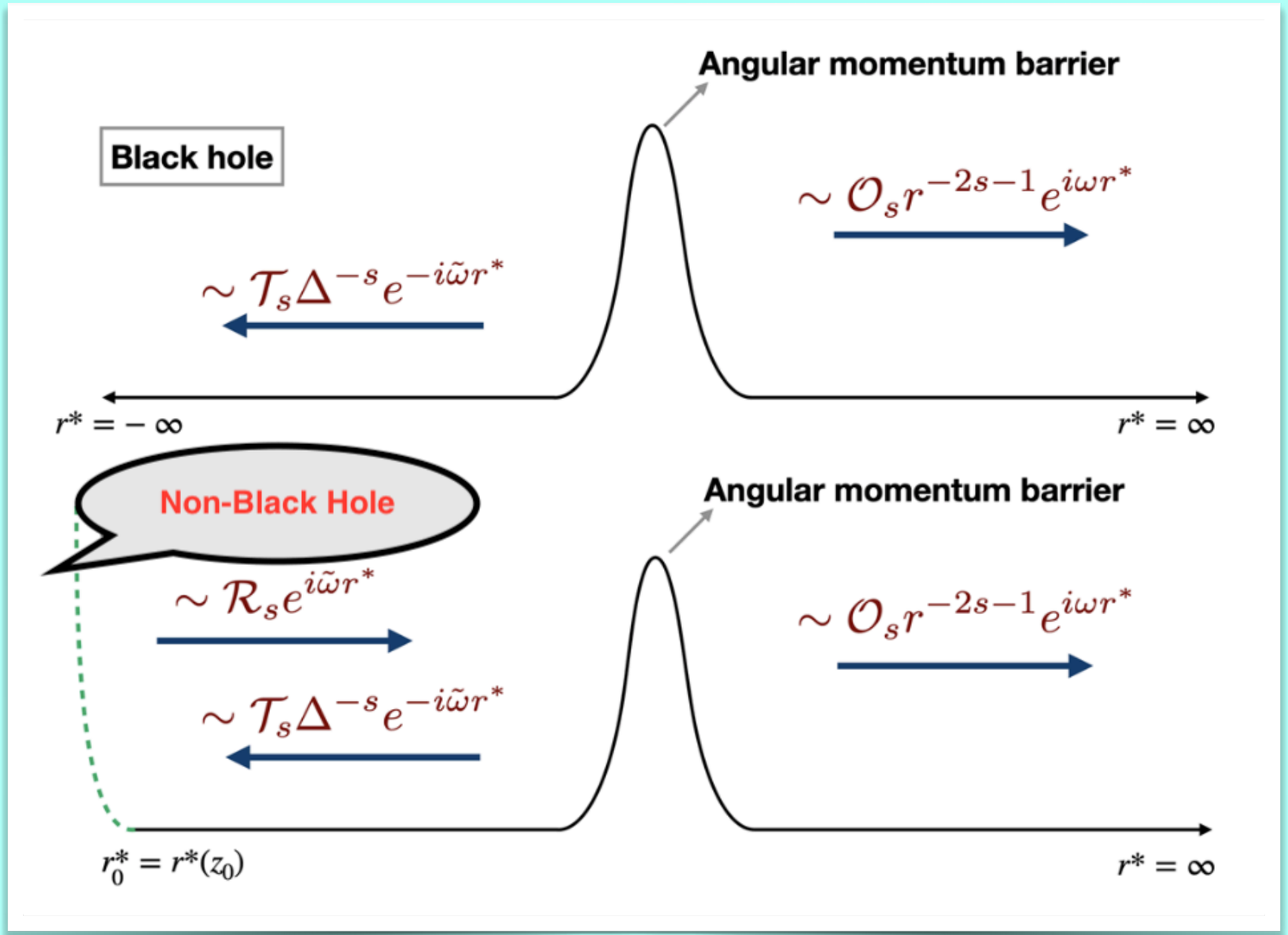
- But, BHs have **Singularities,** existence of Cauchy horizon lead to **Loss of Predictability,** and quantum fields on BH background predict **Information Loss Paradox.**

[Poisson and Israel, Phys. Rev. D 41, 1796 (1990)]
[Hawking, Commun. Math. Phys. 43, 199 (1975)]

- Thus if BHs are really ultra-compact objects whose size is very close to that of the horizon, all the above problems will be solved.

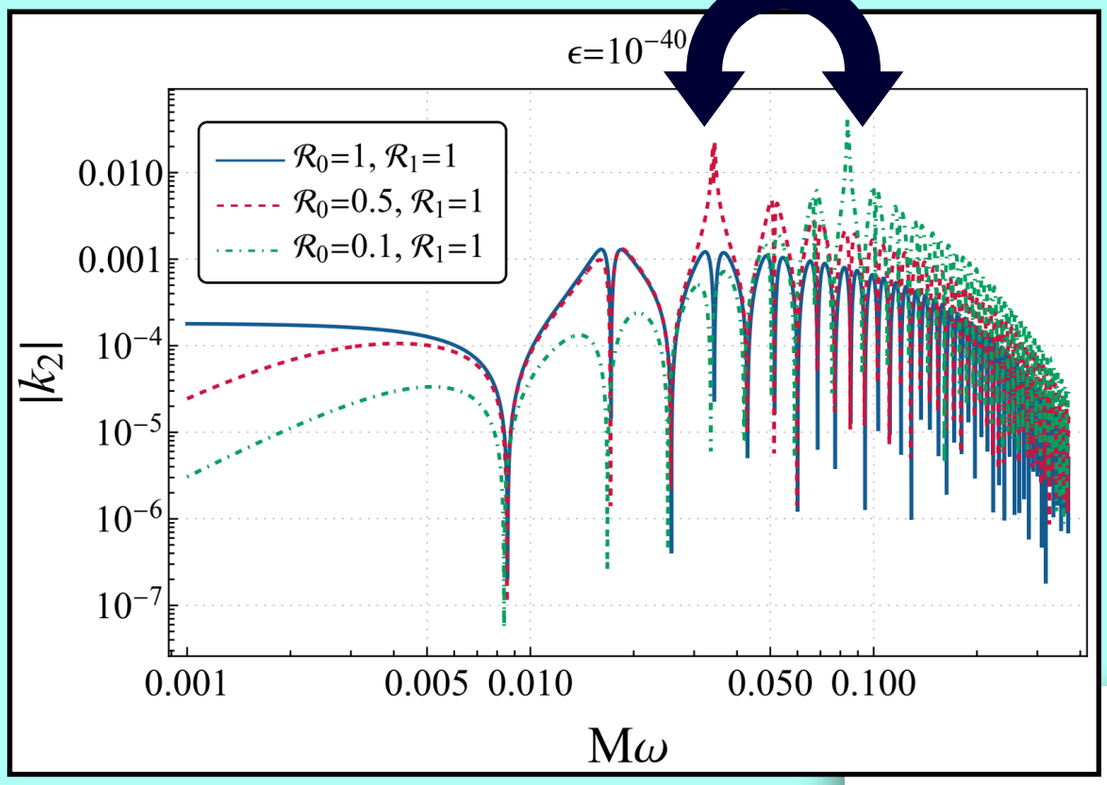
[Maggio +, arXiv: 1807.08840] [Cardoso +, arXiv: 1602.07309]

Reflective Horizon



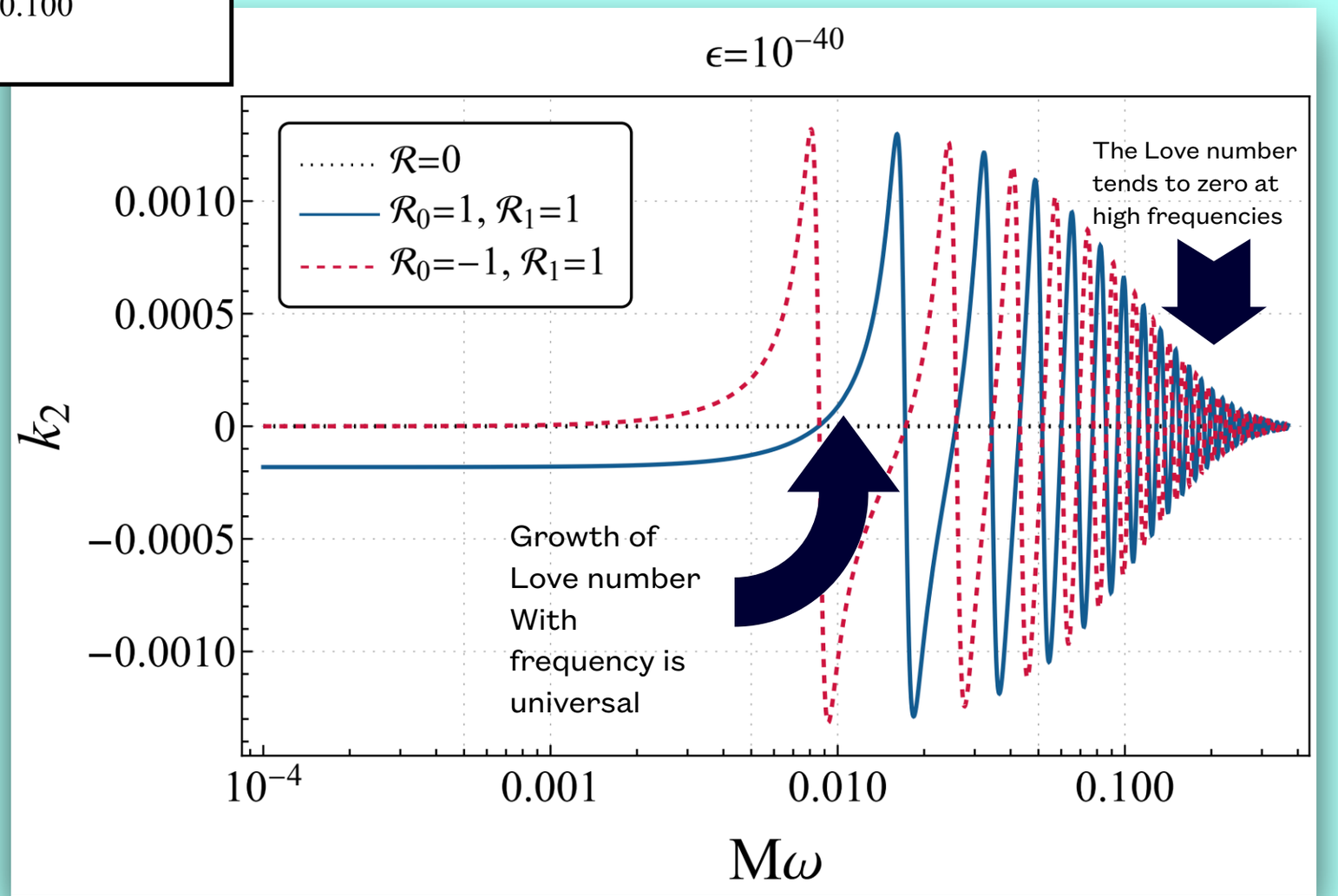
[Figure Courtesy: Dey, Biswas and SC, Phys. Rev. D 103, 084019 (2021)]

Resonances at QNM frequencies



- There are resonances in the Love number at QNM frequencies.
- Increasing rotation and decreasing reflectivity decreases the Love number drastically.

- The tidal Love number grows with frequency.
- At higher frequencies the Love number tends to zero ➡ the compact object effectively behaves as a BH.



[SC + , PRD 110, 084042 (2024)]

Ambiguities and EFT

- BHPT results are ambiguous due to the possibility of re-definition of radial functions.

[Gralla, arXiv: 1710.11096]

- A non-ambiguous result can be obtained by matching the BHPT result with EFT results.

- The EFT consists of a point particle (BH from a large distance) along with finite corrections.

$$\mathcal{A}_{\rm EFT} = \mathcal{A}_{\rm p.p} + \mathcal{A}_{\rm field} + \mathcal{A}_{\rm tidal}.$$

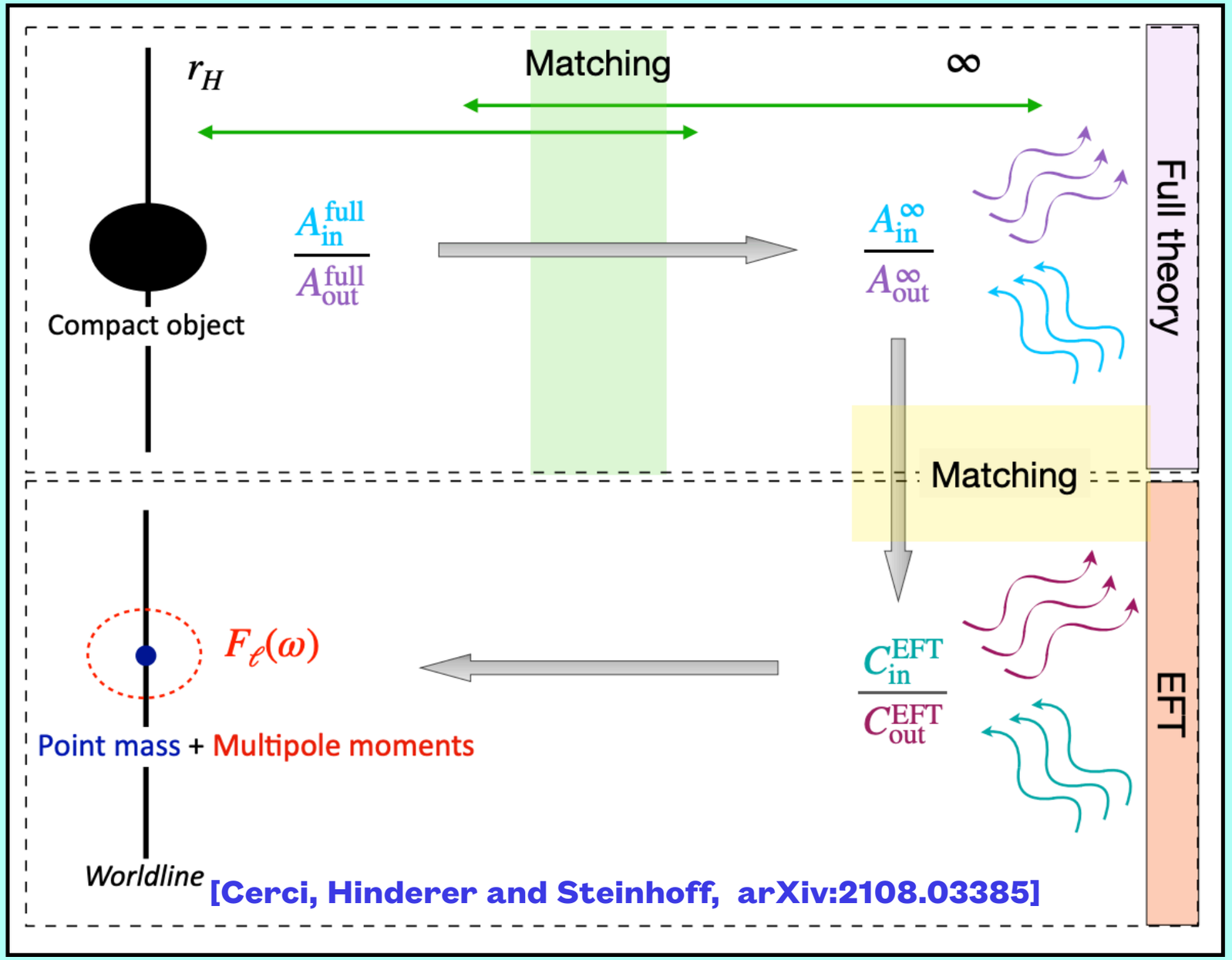
[Goldberger and Rothstein, arXiv: hep-th/0409156]

$$\mathcal{A}_{\rm tidal} = -\int d\tau \sqrt{-u_\mu u^\mu} \sum_{\ell=0}^{\infty} \frac{1}{\ell!} Q_E^{I_\ell} E_{I_\ell} - \int d\tau \sqrt{-u_\mu u^\mu} \sum_{\ell=0}^{\infty} \frac{1}{\ell!} Q_B^{I_\ell} B_{I_\ell}\,.$$

$$E_{\mu_L} = \nabla_{\langle \mu_{L-2}} R_{\mu_{\ell-1}|\alpha|\mu_\ell\rangle\beta} u^\alpha u^\beta\,,$$
$$B_{\mu_L} = \frac{1}{2} \nabla_{\langle \mu_{L-2}} \epsilon_{|\gamma|\mu_{\ell-1}}{}^{\alpha\beta} R_{|\alpha\beta|\mu_\ell\rangle\delta} u^\gamma u^\delta,\ \ell \geq 2\,,$$

[Ivanov and Zhou, arXiv: 2208.08459]

Love number from EFT



EFT for Gravitational Field

- The associated response function is obtained by a series in the spin of the BH,

$$Q_E^{I_\ell}(\omega) = M^{2\ell+1} F^{I_\ell}{}_{I'_\ell} E^{I'_\ell}(\omega) + \chi \Big[M^{2\ell} F^{(-1)I_\ell}{}_{I'_\ell} B^{\langle I_{\ell-1}}(\omega) \hat{s}^{i'_\ell\rangle} + M^{2\ell+2} F^{(1)I_\ell}{}_{I'_\ell} B^{I'_{\ell+1}}(\omega) \hat{s}_{i'_{\ell+1}} \Big] + \chi^2 [\cdots] + \cdots ,$$

$$Q_B^{I_\ell}(\omega) = M^{2\ell+1} F^{I_\ell}{}_{I'_\ell} B^{I'_\ell}(\omega) - \chi \Big[M^{2\ell} F^{(-1)I_\ell}{}_{I'_\ell} E^{\langle I_{\ell-1}}(\omega) \hat{s}^{i'_\ell\rangle} + M^{2\ell+2} F^{(1)I_\ell}{}_{I'_\ell} E^{I'_{\ell+1}}(\omega) \hat{s}_{i'_{\ell+1}} \Big] + \chi^2 [\cdots] + \cdots .$$

[Saketh, Zhou and Ivanov, arXiv: 2307.10391]

- The field equation for gravitational perturbation becomes,

$$\partial_\mu \partial^\mu h_{ij} = 16\pi G |g| \Pi_{ij}^{kl} T_{kl}.$$

[Saketh +, arXiv: 2212.13095]

The EFT Coefficients

- Since the computations have been performed in flat spacetime, it follows that solution to metric perturbations can be expressed in terms of two functions: (a) $\frac{\sin(\omega r)}{\omega r}$ and (b) $\frac{\cos(\omega r)}{\omega r}$. The first one is regular, while the second one is irregular.

$$h_{ij} = \sum_{\ell=2}^{\infty} \frac{1}{\omega^{\ell-2}} \Bigg(C^{K_\ell}_{\mathcal{E},\mathrm{reg}} \Pi_{ij}^{k_{\ell-1}k_\ell} \partial_{K_{\ell-2}} + \omega^{-1} C^{K_\ell}_{\mathcal{B}} \Pi_{ij}^{k_\ell m} \epsilon_{k_{\ell-1}mn} \partial_{K_{\ell-2}n} \Bigg) \psi_{\mathrm{reg}} + (\mathrm{reg} \leftrightarrow \mathrm{irreg}) \,,$$

$$E^{I_\ell} = (-1)^{\ell+1} \mathcal{N}_\ell \omega^\ell C^{I_\ell}_{\mathcal{E},\mathrm{reg}} e^{-i\omega t} \,,$$
$$B^{I_\ell} = i(-1)^{\ell} \mathcal{N}_\ell \omega^\ell C^{I_\ell}_{\mathcal{B},\mathrm{reg}} e^{-i\omega t} \,,$$

$$e^{-i\omega t} C^{L}_{\mathcal{E},\mathrm{irreg}} = 4(\ell!)^{-1}(-1)^{\ell} \omega^{\ell+1} Q^L_E \,,$$
$$e^{-i\omega t} C^{L}_{\mathcal{B},\mathrm{irreg}} = i4(\ell!)^{-1}(-1)^{\ell} \omega^{\ell+1} Q^L_B \,.$$

- These leads to a relation between regular and irregular coefficients.

$$C^{\ell m}_{\mathrm{irreg}} = (i\chi)^{|\ell-\ell_0|} \epsilon^{\ell+\ell_0+1} f^{(\ell-\ell_0)}_{\ell_0 m} C^{\ell_0 m}_{\mathrm{reg}} \,.$$

Connecting to BHPT

- The connection to BHPT comes from the connection between spheroidal and spherical Harmonics.

$$ {}_{-2}Y_{\ell m} = {}_{-2}S_{\ell m} + \sum_{k=1}^{\ell-2} (\chi\epsilon)^k \, {}_{-2}\tilde{r}^{\ell m}_{-k} \; {}_{-2}S_{\ell-k,m} + \sum_{k=1}^{\infty} (\chi\epsilon)^k \, {}_{-2}\tilde{r}^{\ell m}_{k} \; {}_{-2}S_{\ell+k,m} \,, $$

$$ D^{\ell m} = C^{\ell m} + \sum_{k=1}^{\ell-2} (i\chi\epsilon)^k \, {}_{-2}r^{\ell m}_{-k} C^{\ell-k,m} + \sum_{k=1}^{\infty} (i\chi\epsilon)^k \, {}_{-2}r^{\ell m}_{k} C^{\ell+k,m} \,, $$

- The relation between BHPT and EFT response:

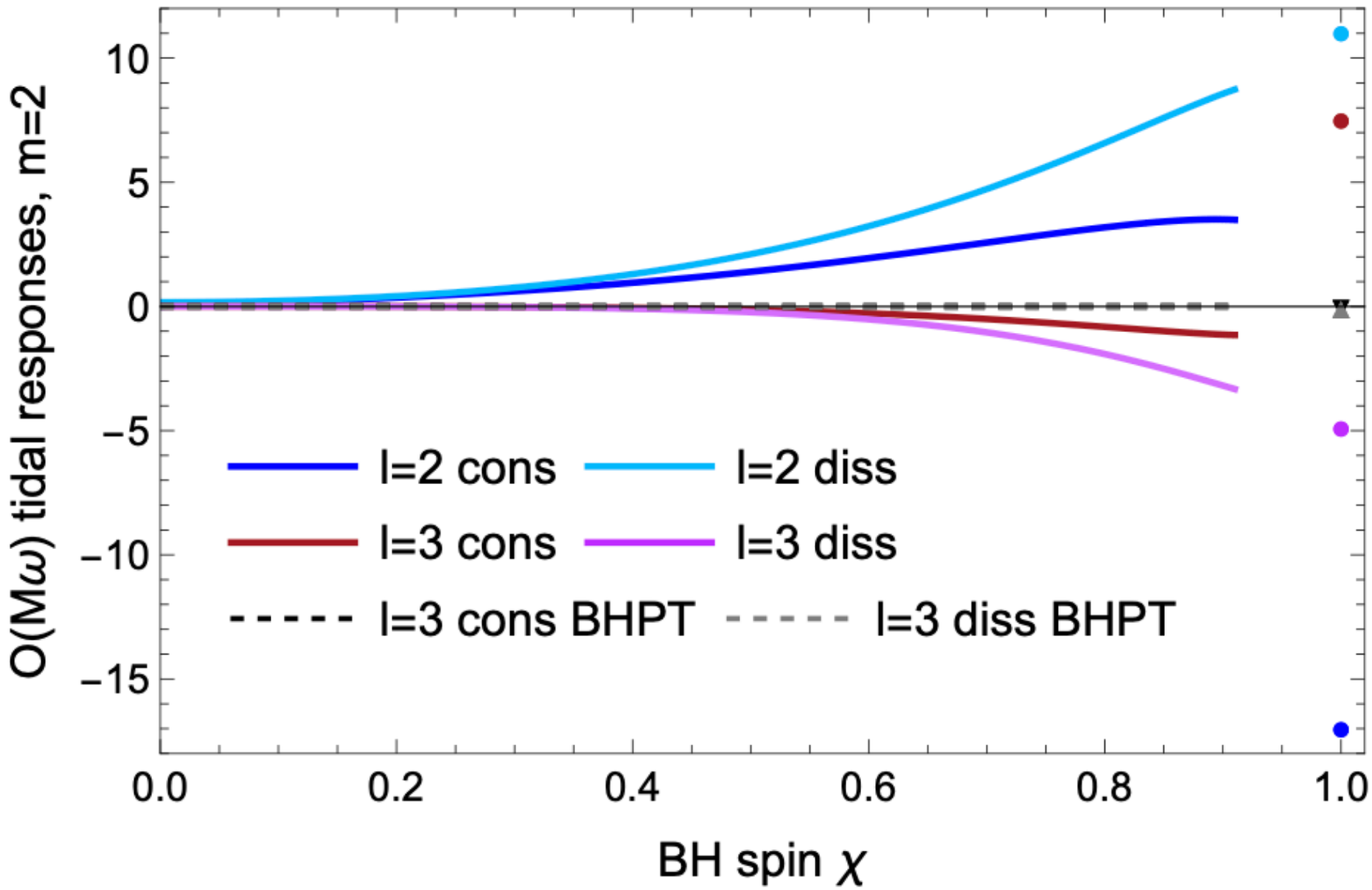
$$ \left[f^{(l-l_0)}_{\ell_0 m} + \sum_{k=1}^{\ell} (i\chi)^{2k} f^{(\ell-k-\ell_0)}_{\ell_0,m} \; {}_{-2}r^{\ell m}_{-k} \right] + \mathcal{O}(\epsilon^2) = {}_{-2}\lambda_{\ell m} \, {}_{-2}r^{\ell m}_{\ell_0-\ell}, \quad \ell_0 \geq \ell \,. $$

$$ f_{\ell m} = {}_{-2}\lambda_{\ell m} \,, \; \ell = 2 \,. $$

$$ f_{3m} = \chi^2 \, {}_{-2}\lambda_{2m} \, {}_{-2}r^{2m}_{1} \, {}_{-2}r^{3m}_{-1} + {}_{-2}\lambda_{3m} \,. $$

[SC et. al., arXiv:2605.00693]

All Inclusive



[SC et. al., arXiv:2605.00693]

Conclusion

- Non-rotating BHs have zero static tidal Love numbers, but non-zero dynamical ones (at quadratic order in frequency).
- Rotating BHs have zero static tidal Love numbers, but non-zero dynamical tidal Love numbers (linear order in frequency).
- The matching with EFT leads to non-trivial mixing between Love numbers for higher multipoles.
- Detection of dynamical Love numbers of BHs is challenging, even for future generations of GW detector. But the same for neutron star is very likely to happen.

Thank You