Testing the Dark Sector at Fixed H_0 and S_8

The Cosmological Consistency Triangle

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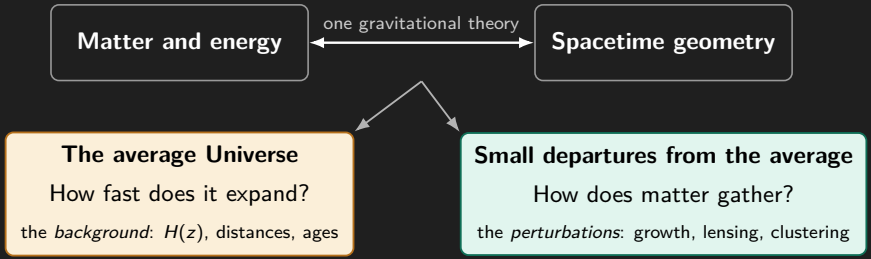
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I. What else must a model get right?

Geometry, gravity and cosmic signatures



The same action, or the same conservation equations, fixes both. A model is therefore tested twice by one set of equations, and the two tests are not interchangeable.

Expansion is the background story; clustering is the perturbation story. The talk asks what a model must get right once it has been tuned to match a number from each.

Two tensions, as they stand in 2026 . . .

H_0, in $\mathrm{km\,s^{-1}\,Mpc^{-1}}$

Local distance network (Cepheids, TRGB, supernovae):

73.50 $\pm$ 0.81

CMB, inferred within flat ΛCDM:

67.24 $\pm$ 0.35

7.1σ *for this specified comparison*.

DESI DR2 BAO + BBN in ΛCDM: 68.51 ± 0.58, still 5.0σ.

$S_8 \equiv \sigma_8 \sqrt{\Omega_m/0.3}$

Planck 2018	0.832 ± 0.013	reference in flat ΛCDM
DES Y6 3 × 2pt	0.789 ± 0.012	2.6σ projected onto S_8; 1.8σ in the full space
KiDS-Legacy	$0.815^{+0.016}_{-0.021}$	0.73σ from *Planck*

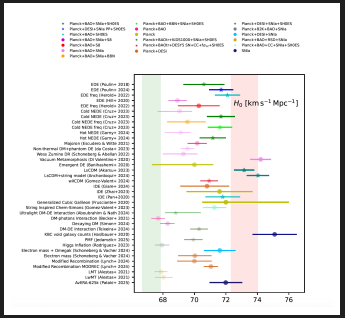
. . . And a geometric one: BAO versus CMB

Within ΛCDM, DESI DR2 BAO and the CMB prefer parameters in mild 2.3σ tension. A $w_0 w_a$ dark energy is preferred at 3.1σ (BAO + CMB) and 2.8 - 4.2σ with supernovae.

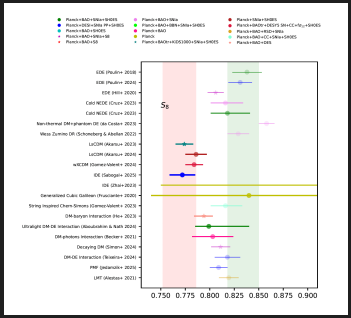
H0DN, Casertano et al., *A&A* **708**, A166 (2026); Planck, *A&A* **641**, A6 (2020); DES, arXiv:2601.14559; Wright et al., *A&A* **703**, A158 (2025); DESI, arXiv:2503.14738.

A significance belongs to a stated pair of measurements; the CMB value is model conditional and S_8 is survey dependent. Both are summaries to be matched, not crises to be solved.

Fixing one summary does not fix the other



H_0 predicted by proposed solutions



S_8 for the same families

early-Universe range late-Universe range

Credit: CosmoVerse White Paper, Di Valentino, Said et al., arXiv:2504.01669 (2025).

A proposed solution inherits responsibility for every other observation. The question is not whether a model can be built; it is what else it must then get right.

Two ways to change the cosmic story

Modified gravity

A different relation between geometry and matter. The same new function then governs both the expansion and the growth of structure.

Example, $f(Q)$ with $Q = 6H^2$: $6f_Q H^2 - \frac{1}{2}f = \kappa^2\rho$, and the same f_Q sets $G_{\rm eff} = G_{\rm N}/f_Q$ in the growth equation.

Interacting dark sectors

General relativity intact, but energy and momentum exchanged between dark matter and dark energy. The exchange law enters the expansion and the clustering of both fluids. Example:

$\dot\rho_c + 3H\rho_c = +Q_{\rm int}, \quad \dot\rho_d + 3H(1+w)\rho_d = -Q_{\rm int};$

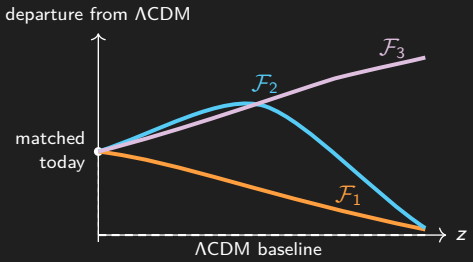
$Q_{\rm int} > 0$ transfers energy from dark energy to dark matter.

The wider field (CosmoVerse White Paper, arXiv:2504.01669): early dark energy, extra relativistic species and modified recombination; evolving or interacting dark energy; $f(R)$, $f(T)$, $f(Q)$ and scalar-tensor gravity; interacting, decaying or warm dark matter; local voids and varying constants. Hundreds of models, most addressing one problem.

A background fit, however good, does not say which route is right; it leaves the perturbation sector untested.

II. The cosmological consistency triangle

The same endpoint can hide a different history



Three schematic *modification histories*, tuned to return the same fitted H_0 and S_8; the shared point today is a drawing convention.

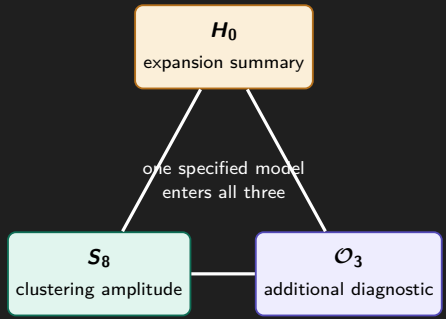
A modification history

How a model departs from ΛCDM over time, written $\mathcal{F}(z)$: the factor f_Q weakening or strengthening gravity in $f(Q)$ (ΛCDM: 1); the exchange rate $Q_{\rm int}/(H\rho_d)$ in interacting dark energy (0); $w(z)$ for dynamical dark energy (-1). Useful, but not a complete set of degrees of freedom: constants, initial conditions, transfer direction and sound speed are not in it.

Matching today's expansion scale and clustering amplitude does not establish that a model gets their *evolution* right. Which additional observation separates the histories once the parameters are fitted?

The cosmological consistency triangle

- Specify the model: action or conservation equations, branch, pressure closure, parameters $\vartheta \in \mathbb{R}^n$, perturbation initial conditions.[1]
- $\mathcal{O}_1 = H_0$ and $\mathcal{O}_2 = S_8$ are useful *summaries*, not complete likelihoods or complete sectors.
- $\mathcal{O}_3$ is a model-specific diagnostic that may carry many redshift or scale bins.
- Additional information is assessed along parameter directions that leave $\mathcal{O}_1$ and $\mathcal{O}_2$ fixed.



An organising device for a joint consistency test, not a theorem that three observables suffice.

[1] **AA**, *The Cosmological Consistency Triangle as a Model-Discrimination Framework*, preprints.org/manuscript/202606.1423/v2 (2026); under review

What makes the third test useful?

Which observable can distinguish histories that share the fitted (H_0, S_8)? Require three things:

(i) Sensitive

$\mathcal{O}_3$ must respond to $\mathcal{F}$ where $\mathcal{F}$ actually departs from ΛCDM, locally or through accumulated growth.

(ii) Additional

Allowed histories with the same (H_0, S_8) must differ in $\mathcal{O}_3$. Independence is *checked*, not assumed because a quantity has a new name.

(iii) Measurable

Measurable now or in near-future probes, at a signal-to-noise that can discriminate the proposed deviation.

Requirement (ii) is the one usually asserted rather than checked, and it is not a ban on background observables: $q_0 = -1 + H_0^{-1}\,\mathrm{d}H/\mathrm{d}z|_0$ is not fixed by H_0 and S_8.

When is a third diagnostic informative?

Jacobians at a specified parameter point, and a basis Z of the directions that preserve the first two summaries:

$$J_{12} = \frac{\partial (H_0, S_8)}{\partial \vartheta}, \qquad J_3 = \frac{\partial \mathcal{O}_3}{\partial \vartheta}, \qquad \mathsf{Z} \text{ spans } \ker J_{12}.$$

Additional-information criterion

$\mathcal{O}_3$ is a locally informative third diagnostic, relative to the model, point and summaries, when

$$J_3 \mathsf{Z} \neq 0 \quad \Longleftrightarrow \quad \operatorname{rank} \begin{pmatrix} J_{12} \\ J_3 \end{pmatrix} > \operatorname{rank} J_{12}$$

A worked example

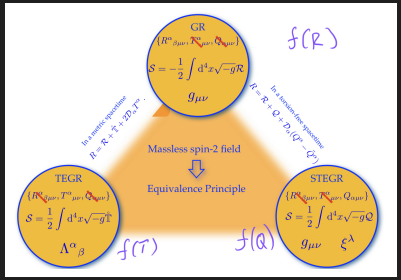
Flat ΛCDM, $\vartheta = (h, \Omega_m, \sigma_8)$: $\mathsf{Z} = (0,\ 1,\ -\sigma_8/2\Omega_m)$ raises Ω_m and lowers σ_8 at fixed S_8. For $\mathcal{O}_3 = q_0 = \frac{3}{2}\Omega_m - 1$, $J_3\mathsf{Z} = \frac{3}{2} \neq 0$: informative. Fix Ω_m and the verdict reverses.

A counter-example

Any $\mathcal{O}_3 = g(H_0, S_8)$ gives $J_3\mathsf{Z} = 0$ by the chain rule: it may sharpen precision, but adds no local rank.

III. Modified gravity: where branch, scale and viability enter

$f(Q)$ gravity: a conditional growth phenomenology



The geometrical trinity: curvature, torsion and non-metricity describe GR equivalently; $f(R)$, $f(T)$ and $f(Q)$ do not. Adapted from Beltrán Jiménez et al., arXiv:1903.06830.

Background, $Q = 6H^2$

Coincident connection, action $\int \sqrt{-g}\,[-f(Q)/2\kappa^2 + \mathcal{L}_m]$:

$$6 f_Q H^2 - \frac{f}{2} = \kappa^2 \rho, \qquad f = Q + 2\Lambda \;\Rightarrow\; \Lambda\text{CDM}.$$

Quasi-static growth

Sub-horizon, scale independent on this branch, $\mu = G_{\rm eff}/G_{\rm N}$:

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm N}\,\mu\,\rho_m \delta_m = 0, \qquad \mu = \frac{1}{f_Q}.$$

A redshift-resolved growth or velocity prediction is a plausible $\mathcal{O}_3$ once H_0 and S_8 are matched; whether it adds rank is tested by the criterion, not by the name of the theory.

The published background fit

Pure power law $f(Q) = \alpha\,(Q/Q_0)^n$ with a phenomenological fluid $p = A\rho - B\sqrt{\rho}$; the background solves exactly (C an integration constant):[1]

$$H(z) = H_0\left[C\,(1+z)^{\frac{3(1+A)}{2}} + \frac{B}{1+A}\right]^{1/n}.$$

What was fitted

- $N = 273$ late-time points (BAO, chronometers, SNIa, quasars, GRBs); the R19 H_0 as a Gaussian prior *inside* the likelihood
- $n = 2.484^{+0.179}_{-0.574}$ on a prior $[1.5, 3]$; derived $\Omega_{m,0} \simeq 0.255$, $z_{\rm tr} \simeq 0.68$
- $\chi^2_{\rm min} = 255.9$ against 260.5 for ΛCDM; five parameters, $\Delta\mathrm{AIC} = 1.4$ favouring ΛCDM

What it does not reach

- The BAO ruler is described two ways: r_d fixed or derived in the text, a prior and an interval in the table
- No perturbation analysis; the strong-coupling and ghost problems of the branch are not addressed (Aguiar Gomes et al. 2024)
- One connection branch; inequivalent branches give different dynamics

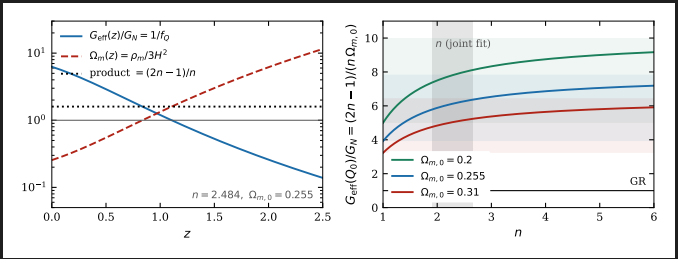
A competitive background fit, honestly reported. The question is what it establishes about the other two vertices.

[1] Özansoy et al., *EPJC* **86**, 134 (2026); **AA**, PoS(CORFU2025)219.

A growth coefficient fixed by the background

Quasi-static growth of pressureless matter has $G_{\rm eff}/G_{\rm N} = 1/f_Q$. The $z = 0$ Friedmann constraint fixes α, which then cancels:

$$\boxed{f_Q(Q_0) = \frac{n\,\Omega_{m,0}}{2n-1}} \quad \Longrightarrow \quad \frac{1}{\Omega_{m,0}} \le \frac{G_{\rm eff}(Q_0)}{G_{\rm N}} < \frac{2}{\Omega_{m,0}} \quad (n \ge 1).$$



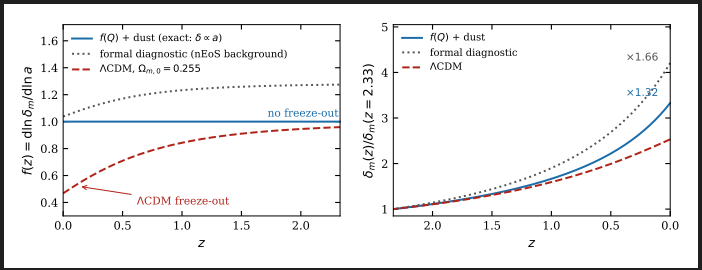
At the central values $f_Q(Q_0) = 0.160$ and $G_{\rm eff}/G_{\rm N} = 6.26$ (5.79 - 6.37 varying n alone): a conditional structural deduction, not an S_8 prediction. $\Omega_{m,0} = 0.255$ is the fitted fluid ratio in the action normalisation, not a CDM fraction.

What the fitted background implies for growth

With $E \equiv H/H_0$: $\rho_m \propto E^{2n}$, $f_Q \propto E^{2(n-1)}$, and the powers cancel in the growth source,

$$\boxed{\tfrac{3}{2}\,\Omega_m(z)\,\frac{G_{\rm eff}(z)}{G_{\rm N}} = \tfrac{3}{2}\left(2-\frac{1}{n}\right)}$$

exact at every z, independent of $\Omega_{m,0}$, α and the equation of state; in $[\frac{3}{2}, 3)$ for $n \geq 1$, where ΛCDM's $\frac{3}{2}\Omega_m(a) \to 0$.



Conserved dust:
$\delta'' + (2 - \frac{3}{2n})\delta' - (3 - \frac{3}{2n})\delta = 0$, growing mode $\delta \propto a$, $f = 1$, no freeze-out; contrast ratio 1.32 today against ΛCDM matched at $z = 2.33$. The fitted fluid's own pressure ($c_a^2 > 1$ for $z > 0.66$) changes its perturbation problem again: strong pressure support.

The scorecard: what the analysis establishes

Vertex	Status	Basis
H_0	conditional	Local H_0 prior inside the likelihood; the BAO-ruler treatment is stated two ways in the paper
S_8	not tested, adverse indications	No S_8 computed. At the central values $G_{\rm eff}/G_{\rm N} = 6.26$ with a source that never decays; separately, strong pressure support
$\mathcal{O}_3$	not tested	No perturbation analysis; curves from the fitted background are not a new test

Tested: computed and confronted with data. **Conditional**: an assumption is doing the work. **Not tested**: no calculation exists. **Inconsistent**: a stated equation, assumption or test is violated.

"Not tested" is not "ruled out". Restoring a linear term, $f = Q + \alpha Q^n$, breaks the cancellation and recovers freeze-out: an exclusion of a branch, not of $f(Q)$.

$f(R)$ gravity: the third vertex can be a scale

Quasi-static, sub-horizon, with $F = f_R$ and $y = k^2 f_{RR}/(a^2 F)$:

$$\mu(k,z) = \frac{G_{\rm eff}}{G_{\rm N}} = \frac{1}{F}\,\frac{1+4y}{1+3y}, \qquad \eta \equiv \frac{\Phi}{\Psi} = \frac{1+2y}{1+4y}, \qquad \Sigma \equiv \tfrac{1}{2}\mu(1+\eta) = \frac{1}{F}.$$

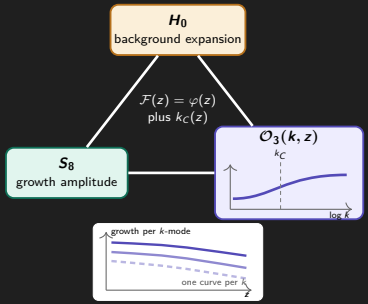
μ rises from $1/F$ to $4/(3F)$ across $k \sim a m_s$, $m_s^2 = \frac{1}{3}(F/f_{RR} - R)$: a transition in *scale*, not in redshift. Photons feel $\Phi + \Psi$, whose response Σ is scale independent: lensing is blind to the scalaron, growth is not.

The natural third diagnostic: E_G

$E_G \equiv \nabla^2(\Phi+\Psi)/(3H_0^2 a^{-1}\theta)$, lensing over velocities (Zhang et al. 2007): $\Omega_{m,0}/f(z)$ in GR, scale free; $\Omega_{m,0}/[F\, f(k,z)]$ in $f(R)$, scale dependent through the velocities alone. Measured since Reyes et al. 2010.

A convention to declare

"$f(k,z)\,\sigma_8(z)$" needs a definition: σ_8 already integrates over k. Forward-model spectra and survey window functions.



One amplitude from S_8; growth reshaped in k.

The same audit applies across theories

Model	Mechanism	Irreducible $\mathcal{F}(z)$	Third vertex $\mathcal{O}_3$
$f(Q)$	non-metricity	$f_Q(Q(z))$	growth shape, velocity growth
Interacting DE	dark-sector energy transfer	$Q_{\rm int}/(H\rho_{\rm DE})$	the velocity amplitude $f_\theta\sigma_8(z)$
Early DE	high-z density injection	$\rho_{\rm EDE}/\rho_{\rm tot}$	sound horizon r_s
Dynamical DE	w_0 - w_a fluid	$w_0 + w_a z/(1+z)$	redshift evolution of $w(z)$
$f(R)$	higher-derivative scalar	$\varphi(z)$	scale-dependent $E_G(k,z)$

At this meeting

Ye (modified gravity and the tensions), Tiwari (distance duality, late-time solutions), Upadhyay (infrared-cutoff models), Chakraborty (the f-family): each sits in a row of this table.

What to report

Declare $\mathcal{F}(z)$ with its perturbation prescription. Report $f_Q(Q_0)$ or its analogue: it costs nothing once a background fit exists, and constrains the amplitude vertex before any new data. State which vertices were tested.

IV. Interacting dark energy: density growth is not velocity growth

What the data actually measure

Geometry

BAO, supernovae and cosmic chronometers give distances and $H(z)$: the expansion history, with H_0 as its endpoint.

Lensing

Coherent image distortions trace the projected matter and the metric potentials; the amplitude is compressed into S_8.

Velocities

Peculiar velocities distort the galaxy pattern in redshift space; redshift-space distortions measure the *velocity* growth rate, usually reported as $f\sigma_8(z)$.

All three are inferences through a model of the tracers, not raw readings. With $D(a)$ the linear growth factor, $f \equiv \mathrm{d}\ln D/\mathrm{d}\ln a$; in conserved pressureless matter velocity growth and density growth coincide, so $f\sigma_8$ can be read either way. The next slides show that an interacting dark sector breaks that equality.

Growth *over time* separates histories that share today's amplitude. In interacting models the mapping from densities to velocities is itself part of the model.

A covariant model, not just a background law

GR, uncoupled baryons, CDM and a DE fluid with constant w and rest-frame $c_{s,d}^2 = 1$. The exchange is specified covariantly, aligned with the CDM four-velocity (no force in the CDM rest frame):

$$\nabla_\mu T_c^{\mu\nu} = Q_{\rm int}\, u_c^\nu, \qquad \nabla_\mu T_d^{\mu\nu} = -Q_{\rm int}\, u_c^\nu, \qquad \dot\rho_c + 3H\rho_c = Q_{\rm int},$$

with $Q_{\rm int} > 0$ for transfer from DE to CDM. Two exchange laws, linear and nonlinear in the dark densities, both proportional to the *CDM expansion scalar* $\Theta_c = \nabla_\mu u_c^\mu$:

$$Q_{\rm int}^{\rm lin} = \xi\, \frac{\Theta_c}{3}\, \rho_d, \qquad Q_{\rm int}^{\rm nl} = \gamma_{\rm int}\, \frac{\Theta_c}{3}\, \frac{\rho_c \rho_d}{\rho_c + \rho_d}.$$

Why the expansion scalar matters

Writing H in the exchange law hides a choice. The perturbation of Θ_c is of the same order as the density source and cannot be discarded while claiming to perturb the same covariant model (Gavela et al. 2010). The nonlinear law and its density linearisation go back to Li & Zhang (2014); the expansion prescription here completes the present model. Quantified: switching $\delta\Theta_c$ off in the transfer term moves the velocity amplitude by 0.5 - 1.2%, at or above the separability residuals that follow.

Perturbing the exchange

Newtonian gauge, $\Psi = \Phi$, $N = \ln a$, primes $= \mathrm{d}/\mathrm{d}N$, $u_i = \theta_i/(aH)$. The local CDM expansion perturbation is

$$\frac{\delta\Theta_c}{3H} = \frac{u_c}{3} - \Phi - \Phi'.$$

For $Q_{\rm int} = (\Theta_c/3)\, g(\rho_c, \rho_d)$ with g homogeneous of degree one, and $\alpha_c = \partial \ln|g|/\partial \ln \rho_c$,

$$\frac{\delta Q_{\rm int}}{\bar{Q}_{\rm int}} = \frac{u_c}{3} - \Phi - \Phi' + \alpha_c\delta_c + (1-\alpha_c)\delta_d, \qquad \alpha_c^{\rm lin} = 0, \quad \alpha_c^{\rm nl} = R_d \equiv \frac{\bar\rho_d}{\bar\rho_c + \bar\rho_d}.$$

Inserted into the perturbed continuity equations $u_i^\mu \partial_\mu \rho_i + (\rho_i + p_i)\Theta_i = \pm Q_{\rm int}$ (+ for CDM, − for DE), with $q_{c,d} = \bar{Q}_{\rm int}/(H\bar\rho_{c,d})$, $L = 3(1+w) + q_d$, $K = k/(aH)$: $b_\Theta = 1 - q_c/3$ multiplies the CDM velocity, a self-source $-q_c(1-\alpha_c)\,\delta_c$ appears at first order in the coupling, and CDM drives DE:

$$\boxed{\begin{aligned} \delta_c' &= -b_\Theta\, u_c - q_c(1-\alpha_c)\,\delta_c \;+\; q_c(1-\alpha_c)\,\delta_d \;+\; 3b_\Theta\,\Phi' \\ \delta_d' &= -3(c_{s,d}^2 - w)\Big(\delta_d + L\frac{u_d}{K^2}\Big) - (1+w)\,u_d + L\,\Phi' - \frac{q_d}{3}\,u_c \;-\; q_d\,\alpha_c(\delta_c - \delta_d) \end{aligned}} \qquad \text{(M)}$$

Uncoupled: $q_c = q_d = 0$, $b_\Theta = 1$, $L = 3(1+w)$. Highlighted: terms added by the covariant exchange; $q_d u_c/3$ is the DE counterpart of b_Θ.

Density growth and velocity growth part company

For smooth dark energy on sub-horizon scales ($\delta_d \to 0$, Φ' dropped), (M) reduces to $\delta_c' = -b_\Theta u - r\,\delta_c$ with $r = q_c(1-\alpha_c)$, and $\delta_b' = -u$: CDM and baryons share one reduced velocity u. With evolving weights $x_c = \bar\rho_c/(\bar\rho_c+\bar\rho_b)$, $\delta_m = x_c\delta_c + (1-x_c)\delta_b$ and $q_m = x_c q_c$:

$$f_m \equiv \frac{\delta_m'}{\delta_m}, \qquad f_\theta \equiv -\frac{u}{\delta_m}, \qquad \boxed{f_\theta = \frac{f_m + q_m\,(1-\alpha_c\,\delta_c/\delta_m)}{1-q_m/3}} \tag{I}$$

What redshift-space distortions see

Linear theory (Kaiser 1987), deterministic bias, no velocity bias:

$$\delta_g^s(\boldsymbol{k}) = \left[\, b_g + f_\theta\,\mu_k^2 \,\right] \delta_m(\boldsymbol{k})$$

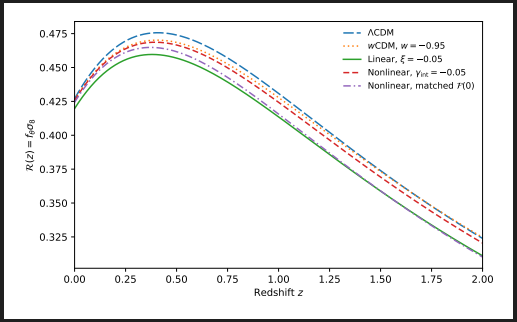
- δ_g^s: galaxy overdensity in redshift space; b_g: linear bias, $\delta_g = b_g\delta_m$
- $\mu_k = \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{n}}$; the μ_k^2 term is the line-of-sight velocity
- $f_\theta\delta_m = -\theta/(aH)$: the velocity divergence

The amplitude of δ_m is degenerate with b_g and f_θ, so the data fix $b_g\sigma_8$ and $\mathcal{R} = f_\theta\sigma_8$, not $\mathcal{G} = f_m\sigma_8$. $\mathcal{R}$ is the same for any tracer (one common velocity); $\mathcal{G}$ is not, since $\delta_c \neq \delta_b$.

Consequence

$f_\theta = f_m$ only when the interaction vanishes; in IDE it is not a conservation identity. The split comes from the energy-transfer terms and the evolving weights, not from a change to Newton's constant. Published $f\sigma_8$ compressions cannot be imported without checking their inference under this mapping.

A benchmark at matched H_0 and S_8



$\mathcal{R} = f_\theta \sigma_8$ in the reduced covariant IDE benchmark. All curves share H_0, $\Omega_{m,0}$ and $\sigma_{8,0}$; IDE curves and the wCDM control have $w = -0.95$. The *Planck*-like values are a convention; any matched pair would do.

Declared conventions

$\Omega_{c,0} = 0.266$, $\Omega_{b,0} = 0.049$, $h = 0.674$, $\sigma_{8,0} = 0.81$, $w = -0.95$, $c^2_{s,d} = 1$; flat, no radiation. Normalised by the same late-time variance, so $H_0 = 67.4$ and $S_8 = 0.830$ by construction.

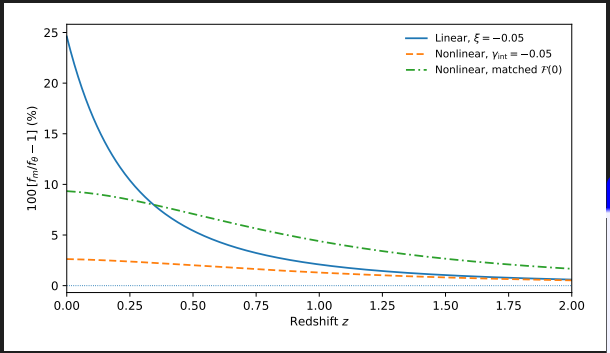
Compared

Linear $\xi = -0.05$; nonlinear with $\gamma_{\rm int} = -0.05$ (equal parameter) and with $\gamma_{\rm int} = -0.179$ (matched $\mathcal{F}(0)$). Equal parameter values are a comparison convention, not equal physical strength.

Theoretical, late-time-normalised curves: not fits, not CMB-calibrated, not forecasts. At $z = 0$ the linear model sits 1.5% below the same-w control; the matched nonlinear model 0.4% below.

Uses (M): integrates $\delta_c' = -b_\Theta u - r\delta_c$, $\delta_b' = -u$ with Euler and Poisson; linear: $q_c = \xi \bar{\rho}_d / \bar{\rho}_c$, $\alpha_c = 0$; nonlinear: $q_c = \gamma_{\rm int} R_d$, $\alpha_c = R_d$.

How large is the density-velocity difference?



$100\,(f_m/f_\theta - 1)$; the common $\sigma_8(z)$ cancels in the ratio.

For the linear benchmark at $z = 0$,

$$\mathcal{G}(0) = 0.523, \qquad \mathcal{R}(0) = 0.420.$$

Reading $\mathcal{G}$ as the RSD combination $\mathcal{R}$ would change even the *qualitative* impression of the interaction's effect.

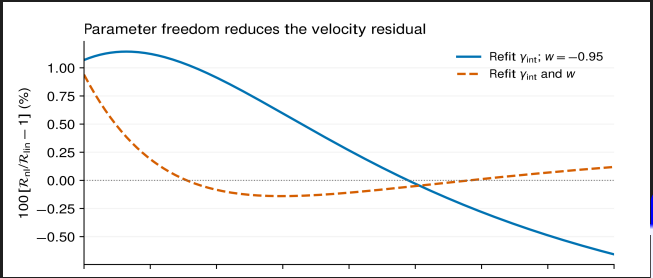
Rank test

With the growing-mode amplitude as a free parameter, the response of $\mathcal{R}$ along the (H_0, S_8)-preserving direction is 0.24 - 0.30 per unit coupling at $z = 0.5$ - 1.5, and it has the *opposite* sign to the response at fixed amplitude: a constructive instance of $J_3\mathsf{Z} \neq 0$.

The distinction vanishes in the uncoupled reduced system; it cannot be neglected for the illustrated interactions.

Uses (M): $\mathcal{G}$ and $\mathcal{R}$ follow from identity (I) evaluated on that solution; the rank-test responses are finite differences of the same integration in the coupling.

Can one exchange law imitate the other?



Solid curve: fractional velocity-amplitude residual of the refitted nonlinear model against the linear model with $\xi = -0.05$, over $0 \le z \le 2$. The dashed curve, with w also refitted, comes next.

Minimise a declared uniform-redshift norm, not a likelihood:

$$\mathcal{E}^2 = \frac{1}{2}\int_0^2 \left[\frac{\mathcal{R}_{\rm nl}}{\mathcal{R}_{\rm lin}} - 1\right]^2 \mathrm{d}z.$$

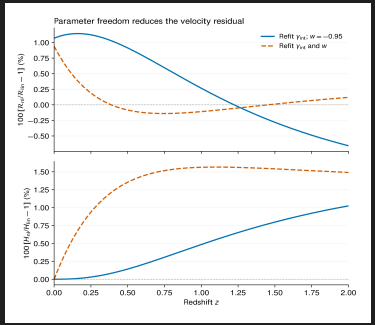
Result

$\gamma_{\rm int}^{\rm refit} = -0.194$; residual RMS 0.68%, maximum 1.14% over $0 \le z \le 2$, with a redshift-dependent shape.

Matching parameter values is not model discrimination; refitting is. A one-parameter refit leaves a resolved shape difference within the specified comparison.

Uses (M): each trial $\gamma_{\rm int}$ reruns the same reduced system; $\mathcal{E}$ compares the resulting $\mathcal{R}_{\rm nl}$ with the linear-law $\mathcal{R}_{\rm lin}$.

Parameter freedom, and what was actually matched



Velocity (top) and expansion (bottom) residuals for the one- and two-parameter refits; only the velocity objective is minimised.

Refit the coupling alone

$\gamma_{\rm int} = -0.194$: velocity residual 0.68% RMS; expansion mismatch up to 1.0%.

Refit the coupling and w

$\gamma_{\rm int} = -0.166$, $w_{\rm nl} = -0.898$: velocity residual 0.21% RMS; expansion mismatch up to 1.6%.

One more free parameter cuts the velocity residual by a factor of three while the expansion-history mismatch *grows*: matching H_0 leaves $H(z > 0)$ unconstrained.

Sensitivity beyond the two endpoint summaries is established; information beyond an already constrained geometric sector is not. That needs conditioning on distance and expansion data with their covariance.

Uses (M): the same system with $\gamma_{\rm int}$ and $w_{\rm nl}$ free; the expansion residuals come from the exact backgrounds that also fix q_c.

V. What follows

A reporting checklist for tension-resolving models

(a) Specify and check the model

Transfer, sign convention, expansion scalar, pressure closure, domain, initial conditions; check conservation, dimensions and the uncoupled limit.

(b) Declare parameter freedom and priors

What is fixed, fitted, marginalised; proper priors for evidences; is the nested point interior, a boundary, or a separate model?

(c) Assess viability separately

Positive densities, regular perturbations, no ghosts or uncontrolled strong coupling; late-time regularity is no all-epoch proof.

(d) Model the observations and release the calculation

Predict the density *and* velocity quantities for the tracers; covariance and nuisance parameters; code, inputs and outputs released.

Tested, conditional, not tested, inconsistent: one line per vertex, with the basis stated.

Instruments across BRICS map the three vertices

Expansion and distances

DESI BAO and redshifts; J-PAS (Brazil-Spain); LSST supernovae; Euclid, LSST and CSST clustering; standard sirens (LIGO-India with the global network).

Clustering amplitude

Euclid, LSST and CSST weak lensing; CMB lensing; cluster counts from SRG (Russia-Germany); 21-cm intensity mapping with BINGO (Brazil), MeerKAT and SKA-Mid (South Africa).

Growth and velocities

Redshift-space distortions from DESI and Euclid; HI galaxy surveys with FAST (China) and MeerKAT; lensing-velocity combinations such as E_G.

Much of the instrumentation for these joint analyses sits in BRICS countries: FAST and the forthcoming CSST in China, MeerKAT and SKA-Mid in South Africa, BINGO and J-PAS with Brazil, SRG in Russia, GMRT and LIGO-India in India.

No survey guarantees the exclusion of a model; what it guarantees is a sharper joint test. Instruments need people who connect measurements to defensible inference.

What has (not) been established

Established within the stated model

- A local rank criterion that makes "additional diagnostic" operational and reparameterisation invariant; non-trivial once the amplitude is a parameter
- For covariant IDE with a perturbed expansion scalar and baryons: $f_\theta \neq f_m$, with an explicit identity; $\mathcal{R}$ is tracer-independent
- At matched H_0 and S_8: refit residuals of 0.68% and 0.21%, $H(z)$ mismatches of 1.0 - 1.6%; the expansion-perturbation prescription alone moves $\mathcal{R}$ by 0.5 - 1.2%

Not established, and not claimed

- Any detection forecast or survey significance
- A resolution of either tension
- All-epoch stability of the IDE completion, or viability of a given $f(Q)$ branch
- Information beyond an already constrained expansion history

Two questions for any model:

What did you fit? **What did you test?**

What is your $\mathcal{F}(z)$, which vertices have you tested, and along which directions?

With thanks



Collaborators and students

Hosts and organisers

Session chair