BLACK-HOLE INTERIOR SPECTROSCOPY

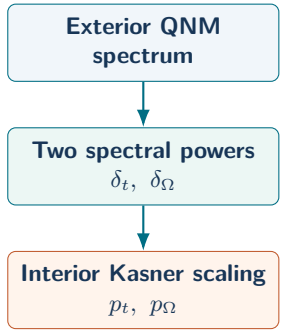
Quasinormal Modes Decode Kasner Scaling inside Black Holes

From exterior spectral powers
to interior geometric exponents

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Based on: Zi-Qing Xiao, Jun Nian and Li Li
Channel-Resolved High-Overtone Quasinormal Modes Decode Kasner Scaling inside Black Holes
arXiv:2609.17426 [hep-th]



What can exterior observables reveal about a hidden interior?

CAUSALLY HIDDEN **Different exterior observables provide different information**

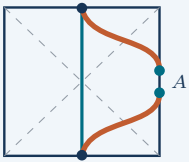
Black-hole shadow



Exterior optical sensitivity

Primarily exterior optical geometry; no direct Kasner measurement.

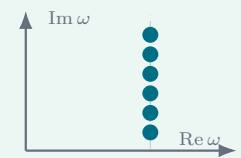
Holographic probes



Holographic sensitivity

Interior signatures require a holographic dictionary.

Our approach: QNMs



Spectral sensitivity

High-n spectral corrections can encode Kasner scaling indirectly.

Our result: two independently fitted spectral powers $\longrightarrow$ two Kasner exponents.

What interior geometry do we want to reconstruct?

Kasner geometry: anisotropic power laws

$$ds^2 \simeq -d\tau^2 + c_t \tau^{2p_t} dt^2 + c_\Omega \tau^{2p_\Omega} d\Omega_2^2$$

$\tau \to 0^+$: proper time remaining to the singularity.

p_t: longitudinal (t) scaling $\quad$ p_Ω: angular (S^2) scaling

A familiar example: Schwarzschild interior

Inside the horizon, $f(r) = 1 - r_h/r < 0$: r becomes timelike.

$$d\tau = \frac{dr}{\sqrt{-f(r)}}, \qquad \tau \propto r^{3/2} \ (r \to 0)$$

Since $-f \propto \tau^{-2/3}$ and $r^2 \propto \tau^{4/3}$, we obtain

$$p_t = -\frac{1}{3}, \qquad p_\Omega = \frac{2}{3}$$

Our QNM target: reconstruct p_t and p_Ω from exterior QNM spectra.

Our strategy: three complementary tests

1 Reconstruct

Schwarzschild scalar

Can two independent spectral powers recover two known interior exponents?

A controlled inverse problem.

2 Extend

Schwarzschild spin-2

Does the leading fractional power also appear in physical gravitational perturbations?

Beyond a scalar probe.

3 Vary the geometry

Gibbons–Maeda scalar

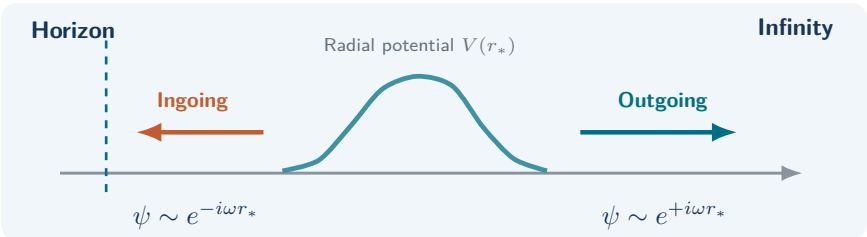
Does the reconstruction track a continuously changing Kasner interior?

Beyond a fixed answer.

All three tests use four-dimensional, asymptotically flat backgrounds.

High-overtone QNMs are a global connection problem

One radial wave must satisfy two exterior conditions.



Only special complex ω satisfy both conditions.

At high overtone, can complex-plane continuation determine them?

Monodromy consistency determines high-overtone QNMs

One QNM solution: ingoing at the horizon, outgoing at infinity

Same analytically continued QNM solution

Horizon-side evaluation

$$e^{\beta_H \omega}$$

Ingoing horizon solution
and its logarithmic branch

Global connection

$$-3$$

Outgoing matching, near-$r = 0$
Bessel connection and outer return

Same relative monodromy $\Longrightarrow$ the two evaluations agree

$$\boxed{e^{\beta_H \omega} = -3}$$

Complex-log branches select a discrete QNM spectrum.

$$\omega_n^{(0)} = T_H \ln 3 - 2\pi i T_H \left(n + \tfrac{1}{2}\right)$$

Why can the singularity affect an exterior QNM spectrum?

The singularity enters the calculation, not the boundary conditions.

Exterior QNM problem



Global connection

Near $r = 0$

$$V_0 \sim -1/(4r_*^2)$$

Bessel-type local solution
enters the connection calculation

Near-zero Bessel behavior + outgoing outer matching

$$\Longrightarrow \quad \mathcal{R}^{(0)}_{\text{global}} = -3$$

What can the subleading near-singularity terms reveal about Kasner scaling?

Kasner geometry fixes two local correction powers

Two scalar-wave responses carry two distinct near-singularity powers.

Kasner metric scaling $a_t \sim \tau^{p_t}, \quad R \sim \tau^{p_\Omega}, \quad r_* \sim \tau^{1-p_t}$

Angular response

$V_\Omega \propto L^2 a_t^2 / R^2 \sim r_*^{-2+\delta_\Omega}$

$$\delta_\Omega = \frac{2(1-p_\Omega)}{1-p_t}$$

Scalar-mass response

$V_t \propto \mu^2 a_t^2 \sim r_*^{-2+\delta_t}$

$$\delta_t = \frac{2}{1-p_t}$$

Kasner exponents predict the local powers before any QNM calculation.

Kasner corrections in the radial wave equation

Near-singularity scalar radial equation

$$\left[\frac{d^2}{dr_*^2} + \omega^2 + \frac{1}{4r_*^2} - \sum_{a=\Omega,t} \lambda_a \, r_*^{-2+\delta_a} + \cdots\right]\psi = 0$$

$a = \Omega$: angular response $\quad$ $a = t$: scalar-mass response

Rescale with $z = \omega r_*$ and divide the equation by ω^2

$$\frac{\lambda_a r_*^{-2+\delta_a}}{\omega^2} = \lambda_a \omega^{-\delta_a} \, z^{-2+\delta_a}$$

A local $\omega^{-\delta_a}$ factor enters QNMs only through global monodromy matching.

How does the local power enter the QNM spectrum?

$$e^{\beta_H \omega} = -3 + \sum_a C_a \omega^{-\delta_a} + \cdots$$

Local geometry predicts δ_a
Which powers are possible?

Global matching determines C_a
Which corrections survive?

Expand around a leading QNM root $\omega_n^{(0)}$

$$-3\beta_H \Delta\omega_n \sim C_a (\omega_n^{(0)})^{-\delta_a} \quad \Rightarrow \quad \boxed{\Delta\omega_n \propto N^{-\delta_a}}$$

The exterior spectrum inherits the Kasner power only if the global connection permits it.

Two QNM responses reconstruct two Kasner exponents

Two independent spectral powers, two interior geometric exponents

Angular-momentum response

$$D_\Omega \equiv \partial_{L^2}\Omega_n \sim N^{-\delta_\Omega}$$

Scalar-mass response

$$D_t \equiv \partial_{\bar{\mu}^2}\Omega_n \sim N^{-\delta_t}$$

$\Omega_n = r_h\omega_n$, $\bar{\mu} = r_h\mu$, $L^2 = \ell(\ell+1)$, $N = n + \frac{1}{2}$

Extract δ_Ω and δ_t from the QNM spectrum

$$p_t = 1 - \frac{2}{\delta_t}, \qquad p_\Omega = 1 - \frac{\delta_\Omega}{\delta_t}$$

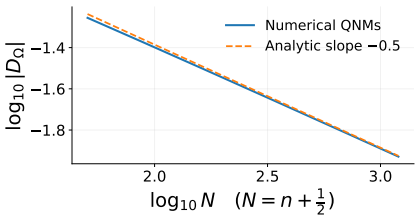
Independent reconstruction: the Kasner constraint is *not* used as an input.

Four-dimensional Schwarzschild: scalar reconstruction

Independent Leaver QNMs $\longrightarrow$ two fitted response powers

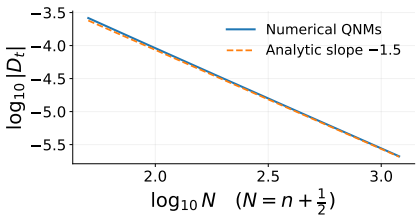
$$(p_t, p_\Omega) = (-\tfrac{1}{3}, \tfrac{2}{3}) \quad \Longrightarrow \quad \delta_\Omega = \tfrac{1}{2}, \quad \delta_t = \tfrac{3}{2}.$$

Angular response



$$\delta_\Omega^{\mathrm{fit}} = 0.5000032$$

Scalar-mass response



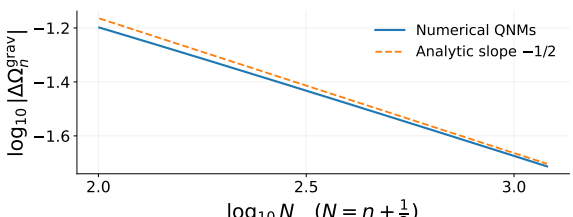
$$\delta_t^{\mathrm{fit}} = 1.4999952$$

$$p_t^{\mathrm{fit}} = -0.3333376, \qquad p_\Omega^{\mathrm{fit}} = 0.6666635 \quad \textbf{without a Kasner constraint.}$$

Four-dimensional Schwarzschild: beyond scalar probes

Axial gravitational perturbations: $s = 2$, $\ell = 2$

$$\Omega_n^{\rm grav} = \underbrace{\frac{\ln 3}{4\pi} - \frac{iN}{2}}_{\Omega_n^{(0)}} + C_g N^{-1/2} + O(N^{-1}), \qquad \Delta\Omega_n^{\rm grav} \equiv \Omega_n^{\rm grav} - \Omega_n^{(0)}.$$



Analytic prediction

$\delta_{\rm grav} = \frac{1}{2}$

Free high-n fit

$\delta_{\rm grav}^{\rm fit} \simeq 0.5000$

The leading exponent is not fixed in the fit.

The same fractional power appears in physical spin-2 QNMs.

Gibbons–Maeda: a tunable Kasner interior

A controlled family: varying the coupling changes the interior Kasner exponents

$$\mathrm{d}s^2 = -\Delta(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\Delta(r)} + R(r)^2\mathrm{d}\Omega_2^2$$

GM metric functions

$$\Delta = f_+ f_-^{\frac{1-A}{1+A}}, \qquad R = r f_-^{\frac{A}{1+A}}$$

$$f_+ = 1 - \frac{1}{r}, \quad f_- = 1 - \frac{b}{r}$$

$r_+ = 1$, singularity at $r = b$ $(0 < b < 1)$.

$A = a^2 > 1$: dilaton coupling.

Exact Kasner target near $r = b$

$$p_t(A) = \frac{1-A}{1+3A}$$

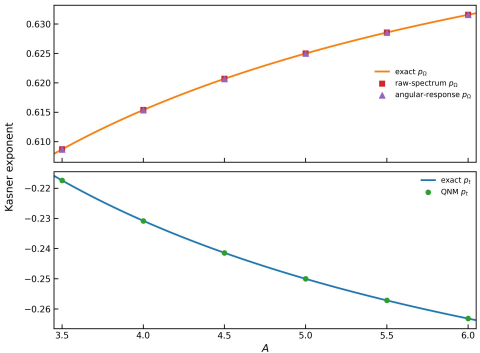
$$p_\Omega(A) = \frac{2A}{1+3A}$$

The two exponents vary continuously with A.

Can exterior QNM data reconstruct a changing Kasner target?

QNM spectroscopy tracks a changing black-hole interior

GIBBONS–MAEDA **Two QNM spectral powers recover two changing Kasner exponents**



Compare two things:

Solid curves
Exact Kasner exponents from the metric.

Colored markers
Reconstruction from QNM spectral powers.

p_Ω rises, while p_t falls as A increases.

QNM data follow both changing exponents, without fitting the Kasner constraint.

An independent test of the Kasner relation

Only after the two-index inversion do we test an unused geometric identity

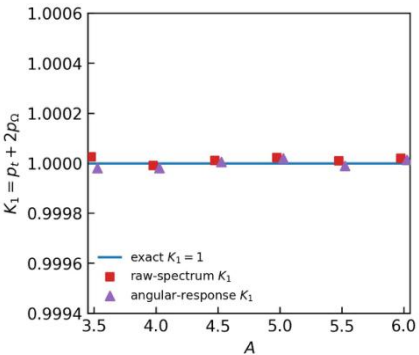
Predicted by the metric:

$$\boxed{K_1 = p_t + 2p_\Omega = 1}$$

The check:

Extract p_t and p_Ω from QNMs first.
Then compute K_1.

Notice the strongly zoomed vertical scale.



The reconstructed values satisfy $K_1 \simeq 1$, although the fit never imposed it.

Summary

Exterior high-overtone QNMs encode reconstructible Kasner information in the backgrounds tested.

1	**Mechanism**	**Local Kasner scaling** fixes candidate powers; **global complex-plane matching** controls their appearance in exterior QNMs.
2	**4D tests**	In **4D asymptotically flat** Schwarzschild, scalar QNMs reconstruct both indices; **spin-2 QNMs** verify the predicted fractional power.
3	**Kasner test**	In the **Gibbons–Maeda family**, QNMs track changing Kasner indices and check the **unused Kasner relation** after inversion.

Bulk-only inference, without a holographic dictionary.

Outlook

Three directions for future research

Beyond spherical symmetry
Can this approach extend to **rotating, dynamical, or BKL-like** interiors?

Global spectral visibility
When does **global complex-plane matching** preserve or suppress interior signatures?

Toward observational accessibility
Can such signatures survive at **accessible overtones** in gravitational-wave data?

Thank you!