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Semileptonic $B_{(s)} \to D_{(s)}$ decays in the AdS/QCD softwall Model

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Outline

1. Motivation: Semileptonic $B_{(s)} \to D_{(s)}$ Mesons
2. Heavy Quark Physics in a nutshell
3. Bottom-up Holography
4. Semileptonic Form factors
5. Results
6. Conclusions

Motivation: Semileptonic $B_{(s)} \to D_{(s)}$ Mesons

Semileptonic heavy-meson transitions

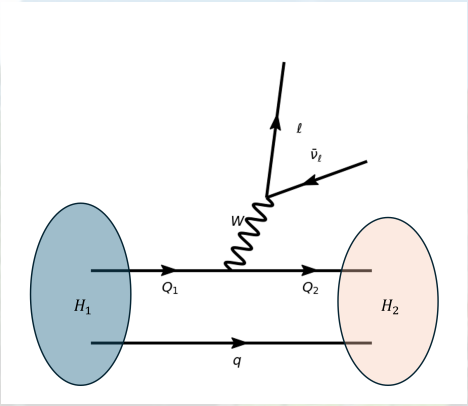


Figure: Semileptonic transition $B \to D\ell\bar{\nu}_\ell$

Motivation

Can a holographic model capture the *nonperturbative structure* underlying heavy-meson transitions?

- The nonperturbative nature of heavy-light hadrons is encoded in Form Factors (FF).

- Task 1: Flavor-changing current written with bulk objects.
- Taks 2: Hadron dual description.
- Task 3: Electroweak FFs extraction.

Heavy Quark Physics in a nutshell

Semileptonic heavy-meson transitions

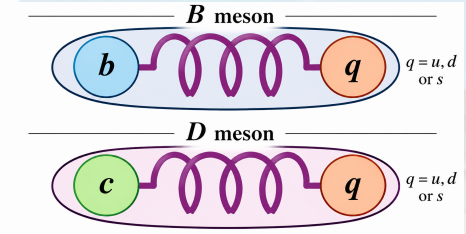


Figure: Heavy-light mesons cartoon

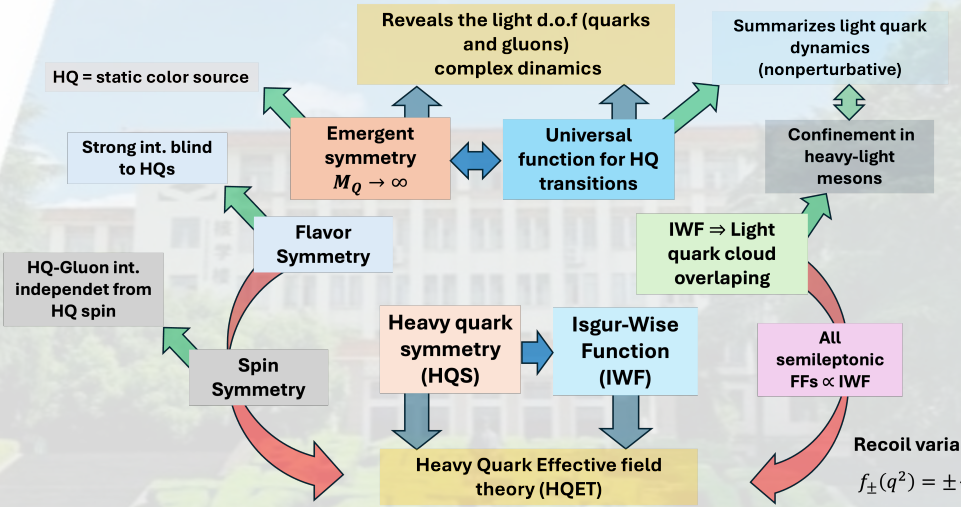
- Initial states: B mesons, composed by a heavy quark (b) and a light *spectator* (u, d or s).
- Final states: D mesons, compose by a heavy quark c and a light spectator.
- b transitions into c *via* a weak current.

EW FFs describe the nonperturbative nature of these transitions.

$$\langle D(p_2)|J^\mu|B(p_1)\rangle = f_+\left(q^2\right)\left(p_1^\mu + p_2^\mu\right) + f_-\left(q^2\right)\, q^\mu,$$

$f_\pm\left(q^2\right)$ are the *semileptonic FF*.

Heavy-quark symmetry and the Isgur–Wise function



IWF $= \zeta(\omega)$

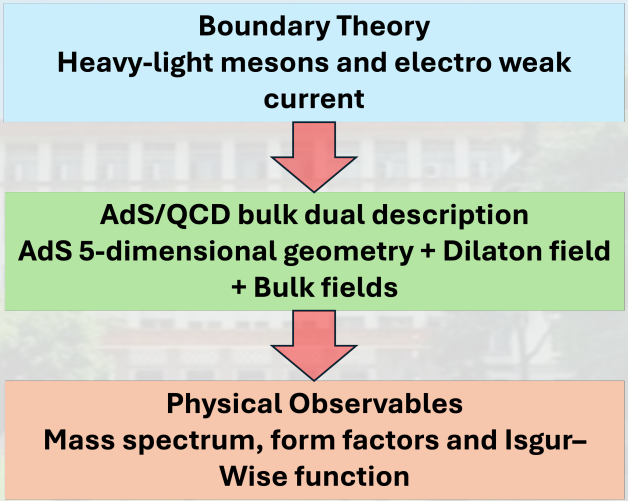
Recoil variable $\omega = \vec{v}_1 \cdot \vec{v}_2$

$$f_\pm(q^2) = \pm \frac{M_B \pm M_D}{2\sqrt{M_B M_D}} \zeta(\omega)$$

HQS: $\zeta(\omega) = 1$

Bottom-up Holography

Bottom-up approach: The Road map



Hadrons *a la* bottom-up

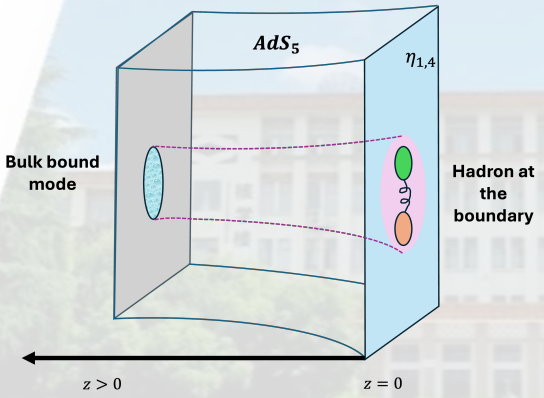


Figure: Hadron/bulk mode duality

Hadrons in AdS/QCD ⇒ **Bags of constituents**

$$\dim \widehat{\boldsymbol{O}}_{\boldsymbol{p}} \Leftrightarrow \psi_p\Big|_{z\to 0} \sim \boldsymbol{z}^{\boldsymbol{\Delta}-\boldsymbol{p}}$$

$$\boldsymbol{M_5} = \boldsymbol{M_5}(\Delta - p)$$

- Hadrons are *bound states* of quarks and gluons, with $\dim \hat{\mathcal{O}} = 3$.
- Hadrons are taxonomically organized as *Regge Trajectories*, i.e., $M^2_{n,J} = a[n + b(J)]^{\nu}$.
- Hadronic states are:
  - a confinement manifestation.
  - nonperturbative phenomena.
- Confinement: break conformal symmetry $\Rightarrow$ introduce an energy scale κ in the AdS_5 bulk *i.e.*, $M^2_{n,J} \propto \kappa^2$.

Bottom-up Softwall Model (SWM)

AdS Geometry

$$dS^2 = \frac{R^2}{z^2}\left[\eta_{\mu\nu}\, dx^\mu\, dx^\nu - dz^2\right]$$

UV Bndry at $z \to 0$

Linear Confinement

$$\Phi_X(z) = \kappa_X^2\, z^2$$

with $X = \left\{B_{(s)},\, D_{(s)}\right\}$

Dominates IR

- We introduce a dimensionful scale into the holographic dynamics.
- Confinement is introduced phenomenologically through a background dilaton field (KKSS 2006).
- Probe approximation $\Rightarrow$ the dilaton does not modify the AdS geometry.
- Dynamical Extensions: See Batel & Gherghetta, 2008 and D. Li et al. 2013.

From Dilaton to Hadrons

5-dimensional scalar bulk field action

$$I_{\text{Scalar Meson}} = -\frac{1}{2\,g_s^2}\int d^4x\,\sqrt{g}\,e^{-\Phi(z)}\left[g^{MN}\partial_M S\,\partial_N S + M_5^2\,S^2\right]$$

Field equation to Schrodinger-like equation

$$-u''(z) + V_{\text{hol}}(z)\,u(z) = M_n^2\,u(z)$$

Holographic Confining Potential

$$V_{\text{hol}}(z) = \frac{1}{4}\,B'(z)^2 - \frac{1}{2}\,B''(z) + \frac{M_5^2\,R^2}{z^2}$$

Eigenvalues

Mass spectrum M_n^2

Sch. mode spectrum $u_n(z)$

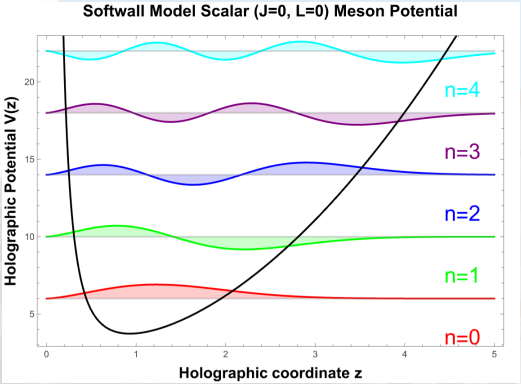
Fourier-transformed bulk fields
$S(z,q) = \widetilde{S}(q)\,\psi(z)$
On-shell mass condition
$M_n^2 = q^2$

Bogoliubov transformation
$u(z) = e^{B(z)/2}\psi(z)$

Bulk Mass
$M_5^2\,R^2 = (3+L)(L-1)$

$B(z) = \Phi(z) - 3\,\log\left(\frac{R}{z}\right)$

Scalar Heavy-Light mesons $J^P = 0^-$ in the SWM



$B_{(s)}$ and $D_{(s)}$ are heavy-light mesons with $J^P = 0^-$, *i.e.*, $J = 0$ and $L = 0$.

- $\Phi_X(z) = \kappa_X^2\, z^2$, with $X = \{B_{(s)},\, D_{(s)}\}$.
- Confining holographic potential (see Colangelo et al. 2008):

$$V(z) = \frac{3}{4\, z^2} + \kappa_X^4\, z^2 + 2\, \kappa_X^2$$

- Eigenspectra:

$$u_{n,X}(z) = \sqrt{\frac{2\, \kappa_X^4\, n!}{(n+1)!}}\, e^{-\frac{1}{2}\, \kappa_X^2\, z^2}\, z^{3/2}\, L_n^1\left(\kappa_X^2\, z^2\right)$$

$$M_{n,X}^2 = 2\, \kappa_X^2\, (2\, n + 3)$$

EW Current Holographic Dual

Boundary:

- EW current: $J^{\mu}_{\rm EW} = \bar{c}\,\gamma^{\mu}\,b$.
- $\dim J_{\rm EW} = 3$.
- Current divergence $\partial_{\mu}\,J^{\mu}_{\rm EW} = i\,(m_b - m_c)\,\bar{c}\,b$.
  - Non-conservation.
  - m_Q breaks the flavor symmetry explicitly.
- Ward-Identity:

$$q_{\mu}\langle D|J^{\mu}_{\rm EW}|B\rangle = (m_b - m_c)\langle D\,|\bar{c}\,b|\,B\rangle.$$

$\Rightarrow$ Should it be $J_{\rm EW}$ dual to a massless field?
In principle: $M_5^2 R^2 = \gamma\,(\gamma+2)$, with $\gamma \equiv$ anomalous. dim.
$\Rightarrow$ Flavor symmetry breaking *is a classical*, i.e., not a quantum fluctuation.

Bulk:

- (Hyp.) Dual bulk field: $U(1)$ vector *massless* field $F_{MN} = 2\,\partial_{[M}\,A_{N]}$.

$$I_{\rm EW} = -\frac{1}{4\,g_J^2}\int d^5x\,\sqrt{g}\,e^{-\Phi_c(z)}\,F_{MN}\,F^{MN},$$

- Gauge Fixing: $A_z = 0$.
- Current Dilaton $\Phi_c(z) = \kappa_c^2\,z^2$, with κ_c related to $f_{\pm}(0)$.
- Bulk to Bndry propagator $A_{\mu}(z,q) = \tilde{A}_{\mu}\,\mathcal{J}(z,q)$, with $\mathcal{J}(z,q) = \Gamma\left(1 - \frac{q^2}{4\,\kappa_c^2}\right)\,U\left(-\frac{q^2}{4\,\kappa_c^2}, 0, \kappa_c^2\,z^2\right)$, with $U(a,b,x)$ the *Tricomi function*

Semileptonic Form factors

Holographic form factors: overlap integral

- Starting point: EW current

$$\langle D(p_2)|J^\mu|B(p_1)\rangle = f_+\left(q^2\right)\left(p_1^\mu + p_2^\mu\right) + f_-\left(q^2\right)\, q^\mu,$$

- Full bulk action

$$\mathcal{S} = \int d^5x\,\sqrt{g}\,\left[e^{-\Phi_B(z)}\mathcal{L}_B + e^{-\Phi_D(z)}\mathcal{L}_D + e^{-\Phi_c(z)}\mathcal{L}_c + e^{-\frac{1}{2}[\Phi_B(z)+\Phi_D(z)]}\mathcal{L}_{\rm int}\right]$$

- Interaction action

$$\mathcal{S}_{\rm int} = g_\pm \int d^5x\,\sqrt{g}\,e^{-\frac{1}{2}[\Phi_1(z)+\Phi_2(z)]}\,g^{MN}\,V_M(x,z)\,S_1^*(x,z)\,\partial_N^\pm\,S_2(x,z).$$

with $\partial_M^\pm = \overrightarrow{\partial}_M \mp \overleftarrow{\partial}_M$.

- Overlapping integral:

$$f_\pm\left(q^2\right) = g_\pm \int_0^\infty dz\,\mathcal{J}(z,q)\,\psi_1(z)\,\psi_2(z) = g_\pm\, I\left(q^2\right).$$

Form Factors Recipe: Overlaping integral

$f_\pm(q^2) = g_\pm\, I(q^2)$

Holographic EW FFs

$\Leftrightarrow$

$f_\pm(q^2) = \pm \dfrac{M_B \pm M_D}{2\sqrt{M_B\, M_D}}\, \zeta(\omega)$

Boundary EW FFs

HQ limit $m_Q \to \infty$

$\zeta(\omega = 1) = 1$

zero-recoil point

$\omega = 1,\ q^2 = q^2_{\max}$

Kinematic Region

$m_\ell^2 \le q^2 \le (M_1 - M_2)^2 \equiv q^2_{\max}$

$\omega = \dfrac{M_1^2 + M_2^2 - q^2}{2 M_1 M_2}$

Recoil variable

$\dfrac{f_-(q^2_{\max})}{f_+(q^2_{\max})} = -\dfrac{M_1 - M_2}{M_1 + M_2} = \dfrac{g_-}{g_+}.$

$\zeta(\omega) = \pm \dfrac{2\sqrt{M_1\, M_2}}{M_1 \pm M_2}\, g_\pm\, I[q^2(\omega)]$

$I(q^2) = \displaystyle\int_0^\infty dz\, \mathcal{J}(q^2, z)\, u_B\, u_D$

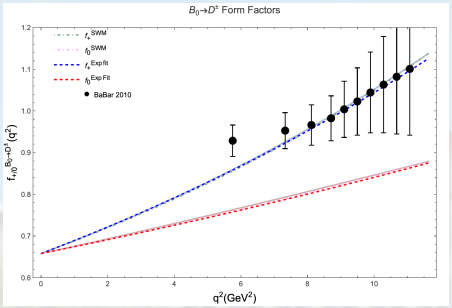
EW form factors

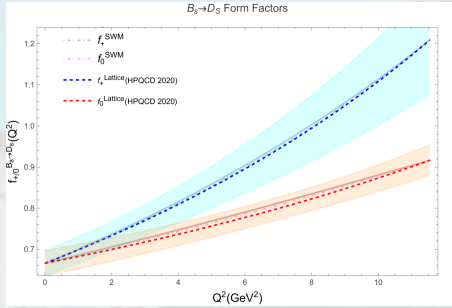
Scalar FF: $f_0(q^2) = \left[g_+ + \dfrac{q^2}{6\,(M_B^2 - M_D^2)}\, g_- \right] I(q^2)$

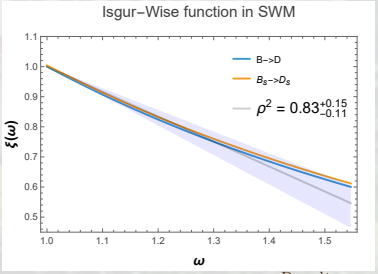
Vector FF: $\boldsymbol{f_+(q^2) = g_+\, I(q^2)}$

Results

SWM Results for $B_{(s)} \to D_{(s)}$







SWM Results for $B_s \to D_s$

Lepton-flavor universality ratio

Observable	Soft-Wall AdS/QCD	Lattice QCD	Experiment
$R(D)$	0.299	0.2938(54)	0.359(24)
$R(D_s)$	0.2965	0.2989(41)	–

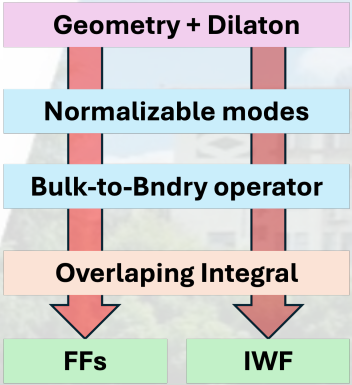
Table: Comparison of the Soft-Wall AdS/QCD predictions with lattice QCD and experimental results. No direct experimental measurement of $R(D_s)$ is included in this comparison.

$$R(D_{(s)}) = \frac{\mathcal{B}\left(B_{(s)} \to D_{(s)}\,\tau\,\nu_\tau\right)}{\mathcal{B}\left(B_{(s)} \to D_{(s)}\,\ell\,\nu_\ell\right)}$$

- SWM predictions agree well with lattice QCD results for both transitions.
- For $B \to D$, the prediction is approximately 1.8% above the lattice result.
- For $B_s \to D_s$, the prediction differs from lattice QCD by approximately 0.8%.
- The $R(D)$ prediction is below the experimental central value.

Conclusions

Conclusions



Conclusions

- Soft-Wall AdS/QCD provides a framework to describe heavy-light mesons.
- Holographic wavefunction overlaps determine the transition form factors.
- Heavy Quark Symmetry constrains the Isgur–Wise function.
- We obtain predictions for branching fractions and the lepton-flavor universality ratios $R(D)$ and $R(D_s)$.
- We compare our results with experimental data and lattice QCD.

Outlook

- Include finite heavy-quark mass corrections.
- Extend the analysis to other heavy-flavor transitions.


Thank you
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Спасибо
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terima kasih
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Благодаря
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Köszönöm
Ďakujem
Vielen Dank
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Parameter Summary SWM

Data Input SWM

Initial Hadron		**Final Hadron**	
H_1	κ_1 **(GeV)**	H_2	κ_1 **(GeV)**
B^0	2.1554(1)	$D^{\pm}$	0.7632(1)
B_s^0	2.1910(1)	$D_s^{\pm}$	0.8036(1)
$B \to D$: $f_+(0) = f_0(0)$			0.658
$B_s \to D_s$: $f_+(0) = f_0(0)$			0.665

Weak current Fitted Data

Transition	κ_J **(GeV)**	g_+	g_-
$B \to D\,\ell\,\bar{\nu}_\ell$	4.097(1)	1.661(1)	$-0.793(1)$
$B_s^0 \to D_s^{\pm}\,\ell\,\bar{\nu}_\ell$	4.1452(1)	1.743(1)	$-0.832(1)$

Table: SWM mass scale in GeV calculated from the expression $M_0^2 = 2\,\kappa^2(2\,n+3)$ for the scalar meson spectra, using the experimental masses (PDG 2026). $f_{+,0}(0)$ is used to extract $g_{\pm}$.

Bottom-up Softwall Model (SWM)

AdS-like Background

$$dS^2 = \frac{R^2}{z^2}\left[\eta_{\mu\nu}\, dx^\mu\, dx^\nu - dz^2\right]$$

with R defined as the AdS radius and $\eta_{mn} = \text{diag}(1, -\vec{1})$.

General Action with minimal coupling for $p-$forms

$$I_{\text{H}} = \int d^5x\, \sqrt{-g}\, e^{-\Phi(z)}\, \left[\nabla_m\, \phi^{m_1 \dots\, m_p}\, \nabla^m\, \phi_{m_1 \dots\, m_p} + M_5^2\, \phi^{m_1 \dots\, m_p}\, \phi_{m_1 \dots\, m_p}\right],$$

with $\Phi(z)$ defined as a static dilaton field (confinement). The bulk field $\phi^{m_1 \dots\, m_p}$ carries the information about the hadron living at the boundary.
In general, for bulk $p-$forms (dual to hadrons at the boundary), the equations of motion have the structure:

$$\partial_z \left[e^{-B(z)}\, \partial_z\, \psi(z)\right] + \left(-q^2\right)\, e^{-B(z)}\, \psi(z,q) - \frac{M_5^2\, R^2}{z^2}\, e^{-B(z)} \psi(z,q) = 0$$

Where in Fourier space, the $p-$form is written as: $\phi_{\mu_1 \dots \mu_p}\,(z,q) = \tilde{\phi}_{\mu_1 \dots \mu_p}\,(q)\; \psi\,(z,q)$.

Bottom-up Softwall Model (SWM)

with the following definitions:

$$
\begin{aligned}
B(z) &= \Phi(z) + \beta \log\left(\frac{R}{z}\right) \\
M_5^2\, R^2 &= (\Delta + L - p)\,(\Delta + L + p - 4) \\
\beta &= -(3 - 2\,p), \text{ Defines hadronic Total angular momentum } J \leftrightarrow p. \\
M_n^2 &= -q^2, \text{ on-shell condition.}
\end{aligned}
$$

Bogoliubov transformation: $\psi(z) = e^{\frac{1}{2}B(z)}\, u(z)$.
From the action, we construct the Schrödinge-like form as (Karch et al., 2005):

$$-u''(z) + V(z)\, u(z) = M_n^2\, u(z),$$

where $V(z)$ is the holographic potential written in terms $\Phi(z)$, and M_n^2 is the hadronic spectrum (Regge trajectory).

Bottom-up Softwall Model (SWM)

Holographic Confining Potential

In the case of $p-$form bulk fields, as follows:

$$V(z) = \frac{M_5^2\, R^2}{z^2} + \frac{1}{4}\left[-\frac{\beta}{z} + \Phi'(z)\right]^2 - \frac{1}{2}\left[\frac{\beta}{z^2} + \Phi''(z)\right], \tag{1}$$

with M_5 the bulk mass associated with the hadronic dual bulk field, and $\beta = -(3-2\,p)$.
The next step is to fix a dilaton $\Phi(z)$ and compute the spectrum M_n^2.

SMW Form Factors Analytic Expression

Semileptonic form factors, $0 < q^2 < q^2_{\rm max} = 6\,(\kappa_1 - \kappa_2)^2$

$$f_\pm\left(q^2\right) = \pm \frac{(\kappa_1 \pm \kappa_2)\,(1+a_{\rm max})_2}{2\,(\kappa_1\,\kappa_2)^{\frac{1}{2}}\;{}_2F_1\left(a_{\rm max}, 2; 3+a_{\rm max}; 1-\frac{2\,\kappa^2}{\kappa_1^2+\kappa_2^2}\right)} \frac{{}_2F_1\left(-\frac{q^2}{4\,\kappa_c^2}, 2; 3-\frac{q^2}{4\,\kappa_c^2}; 1-\frac{2\,\kappa_c^2}{\kappa_1^2+\kappa_2^2}\right)}{\left(1-\frac{q^2}{4\,\kappa_c^2}\right)_2}$$

$$f_0\left(q^2\right) = \frac{(\kappa_1+\kappa_2)\,(1+a_{\rm max})_2\left[1-\frac{q^2}{6\,(\kappa_1+\kappa_2)^2}\right]}{2\,(\kappa_1\,\kappa_2)^{\frac{1}{2}}\;{}_2F_1\left(a_{\rm max}, 2; 3+a_{\rm max}; 1-\frac{2\,\kappa_c^2}{\kappa_1^2+\kappa_2^2}\right)} \frac{{}_2F_1\left(-\frac{q^2}{4\,\kappa_c^2}, 2; 3-\frac{q^2}{4\,\kappa_c^2}; 1-\frac{2\,\kappa_c^2}{\kappa_1^2+\kappa_2^2}\right)}{\left(1-\frac{q^2}{4\,\kappa_c^2}\right)_2} \text{ with } a_{\rm max} = -\frac{3\,(\kappa_1-\kappa_2)^2}{2\,\kappa_c^2}.$$

$$f_\pm^{\text{Exp., Lat.}}(0) = \frac{(\kappa_1 \pm \kappa_2)\,(1+a_{\rm max})_2}{2\,(\kappa_1\,\kappa_2)^{\frac{1}{2}}\;{}_2F_1\left(a_{\rm max}, 2; 3+a_{\rm max}; 1-\frac{2\,\kappa_c^2}{\kappa_1^2+\kappa_2^2}\right)}, \text{ used to fix } \kappa_c$$

Isgur-Wise function

$$\zeta\left(\omega\right) = \frac{(1+a_{\rm max})_2}{{}_2F_1\left(a_{\rm max}, 2; 3+a_{\rm max}; 1-\frac{2\,\kappa^2}{\kappa_1^2+\kappa_2^2}\right)} \frac{{}_2F_1\left(-\frac{q(\omega)^2}{4\,\kappa^2}, 2; 3+\frac{q(\omega)^2}{4\,\kappa^2}; 1-\frac{2\,\kappa^2}{\kappa_1^2+\kappa_2^2}\right)}{\left(1-\frac{q(\omega)^2}{4\,\kappa^2}\right)_2}$$

$$q^2\left(\omega\right) = 6\left(\kappa_1^2+\kappa_2^2\right) - 12\,\kappa_1\,\kappa_2\,\omega.$$