Inflation from Entropy

Inflationary solutions in the gravity from entropy framework

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Gravity keeps behaving like thermodynamics

- Horizon area *is* an entropy: geometry already counts microstates [Bekenstein; Hawking]
- Jacobson: imposing the Clausius relation $\delta Q = T\delta S$ on every local Rindler horizon *derives* the Einstein equation; it reads as an equation of state.
- Ryu–Takayanagi: entanglement entropy of a boundary region is the area of a bulk extremal surface. Entropy and geometry are the same quantity, computed two ways.
- Operator-algebraic QFT: a subregion algebra is a Type III_1 factor with no well-defined entropy. Including gravity, the crossed product with the modular flow gives a Type II algebra, whose von Neumann entropy is finite and reproduces generalized entropy. [Witten; Chandrasekaran–Penington–Witten]

Gravity is what makes entropy well defined.

Gravity from entropy takes the further step: the metric itself is the operator whose entropy we compute.

That last step is a postulate, not a corollary, and this talk is about what it predicts.

What the derivation tells us, and what it does not

Jacobson: imposing the Clausius relation $\delta Q = T\,\delta S$ on every local Rindler horizon derives the Einstein equation:

$$R_{ab} - \tfrac{1}{2}Rg_{ab} + \Lambda g_{ab} = \frac{2\pi}{\hbar\eta}\,T_{ab}, \qquad G = \frac{1}{4\hbar\eta}$$

- G is fixed by an entropy density.
- Λ enters only as an integration constant. Thermodynamics cannot say what it is.
- The whole argument is kinematics plus thermodynamics. No action is ever written down.
- Every approach so far uses an entropy that is *given*, and Einstein's equations come out as a consistency condition on it.
- None of them is a Lagrangian theory, and prediction is GR.

Gravity from entropy makes the relative entropy the action itself.

Why a model with no scalar field is worth the trouble

Standard inflation works beautifully, but the driver is a field nobody has seen.

- The only fundamental scalar we *do* have is the Higgs: minimal coupling gives no slow roll, and non-minimal coupling introduces unitarity problems and non-renormalizable corrections.
- Higher-curvature modifications work, but the corrections are put in by hand.
- And there is the entropy problem: inflation needs a very low-entropy initial state, with no theory of initial conditions to supply it.

What follows has no potential and no new fields, only one dimensionless coupling β, which sets *where* inflation happens but not *what* it predicts.

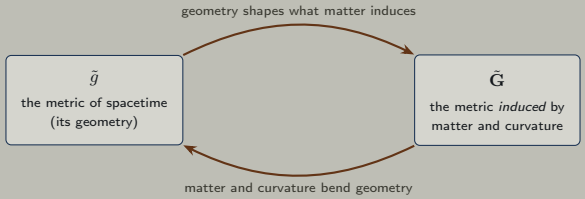
Where we're going

1. The entropic action [Bianconi 2024, 2025]: what it means to derive the entropy of a metric.
2. Put it in vacuum on FLRW, get modified Friedmann equations.
3. Solve them. Three regimes, labelled by a single number $x = GH^2$.
4. One regime inflates and matches the CMB; one is phantom-like; one is excluded.
5. Ask if entropic inflation can reproduce CMB observations.

Ten slides on the framework, then to the cosmology.

The idea: spacetime carries two metrics

G. Bianconi, Phys. Rev. D **111**, 066001 (2025); J. Phys. A **57**, 365002 (2024)



Neither is a plain rank-2 tensor. Each acts on 0-, 1- and 2-forms at once:

$$\tilde{g} = \underbrace{1}_{\text{scalars}} \oplus \underbrace{g_{\mu\nu}}_{\text{vectors}} \oplus \underbrace{g^{(2)}_{\mu\nu\rho\sigma}}_{\text{bi-vectors}}, \qquad g^{(2)}_{\mu\nu\rho\sigma} = \tfrac{1}{2}\big(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\big),$$

and $\tilde{\mathbf{G}} = \mathbf{G}_{(0)} \oplus \mathbf{G}_{(1)} \oplus \mathbf{G}_{(2)}$ has the same three-sector structure.

Both are read as quantum operators, ie. "renormalizable" effective density matrices, one at every point of spacetime. Invertible, but not of unit trace.

What "the entropy of a metric" means

For an operator $\hat{G}$ of this kind, eigenvalues are defined against $\tilde{g}$ rather than against a fixed basis; this is what keeps the notion Lorentz invariant:

$$\hat{G}_{\mu\nu}\big[V^{(\lambda)}\big]^{\nu} = \lambda\, V^{(\lambda)}_{\mu} \qquad \Longleftrightarrow \qquad \lambda \in \operatorname{spec}\big(\hat{G}g^{-1}\big).$$

With $\log$ and Tr defined spectrally, the von Neumann form follows:

$$\big[\log \hat{G}\big]_{\mu\nu} = V^{(\lambda)}_{\mu} V^{(\lambda)}_{\nu} \log\lambda, \qquad \mathrm{Tr}\,\hat{G} = \sum_{\lambda} \lambda, \qquad H[\hat{G}] = \mathrm{Tr}\,\hat{G}\log\hat{G}^{-1} = -\sum_{\lambda} \lambda\log\lambda.$$

Measured this way, every eigenvalue of $\tilde{g}$ is 1, in all three sectors. (backup)

So the spacetime metric carries *no entropy of its own*: $H[\tilde{g}] = 0$.

This is built into the definition rather than discovered, but it is why the action has one term instead of two.

The action: relative entropy

The relative entropy between the two metrics has the usual two terms, and the first one dies:

$$\mathcal{L} \;=\; -\underbrace{\operatorname{Tr}\tilde{g}\log\tilde{g}^{-1}}_{H[\tilde{g}]=0} + \operatorname{Tr}\tilde{g}\log\tilde{\mathbf{G}}^{-1} \;=\; \operatorname{Tr}\tilde{g}\log\tilde{\mathbf{G}}^{-1}.$$

Since $\tilde{g}$ has unit eigenvalues, this cross-entropy is just a sum over the spectrum of $\tilde{\mathbf{G}}$ relative to $\tilde{g}$, and can be written as an ordinary matrix trace:

$$S = \frac{1}{\ell_{\rm pl}^4}\int \sqrt{-g}\,\mathcal{L}\,d^4x\,, \qquad \mathcal{L} = -\operatorname{Tr}_F\log\left(\tilde{\mathbf{G}}\tilde{g}^{-1}\right) = -\sum_{\lambda'}\log\lambda'$$

Tr_F is the ordinary trace of the *flattened* matrix $N = \tilde{\mathbf{G}}\tilde{g}^{-1}$, each sector written as a matrix on a basis of n-forms, the bi-vector sector becoming 6×6 in $d=4$. Its eigenvalues λ' are those of $\tilde{\mathbf{G}}$ measured against $\tilde{g}$.

$\tilde{\mathbf{G}}\tilde{g}^{-1} = \Delta^{1/2}_{\tilde{\mathbf{G}},\tilde{g}}$ is a modular operator: this is an Araki relative entropy, not an analogy to one.

A warm-up: one scalar field

Take $G_{\mu\nu} = g_{\mu\nu} + \alpha\, \nabla_\mu \bar{\phi}\, \nabla_\nu \phi$. The perturbation has rank one, with eigenvalue $|\nabla\phi|^2$, so

$$\mathcal{L} = -\log\bigl(1 + \alpha|\nabla\phi|^2\bigr) \;\simeq\; -\alpha|\nabla\phi|^2.$$

- Low coupling gives the massless Klein–Gordon Lagrangian out of an entropy. In full it is a $P(X)$ kinetic term, and positivity requires $\alpha|\nabla\phi|^2 > -1$.
- Failure 1: no mass term.
- Failure 2: in vacuum the metric equation $-h(|\nabla\phi|^2)\,\mathcal{M}_{\mu\nu} - \frac{1}{2}\mathcal{L}\, g_{\mu\nu} = 0$ is satisfied identically: the geometry is undetermined.

The two failures dictate the two extra terms in the induced metric.

What the induced metric is made of

Matter and curvature both feed it:

$$\tilde{\mathbf{G}} = \tilde{g} + \alpha\tilde{\mathcal{M}} - \beta\tilde{\mathcal{R}}, \qquad \tilde{\mathcal{R}} = R \oplus R_{\mu\nu}dx^\mu \otimes dx^\nu \oplus R_{\mu\nu\rho\sigma}(dx^\mu \wedge dx^\nu) \otimes (dx^\rho \wedge dx^\sigma).$$

- $\tilde{g}$ is a direct sum over scalars, vectors and bi-vectors, so R, $R_{\mu\nu}$ and $R_{\mu\nu\rho\sigma}$ all enter, one per sector.
- Without the bi-vector sector only R and $R_{\mu\nu}$ would enter, and the Weyl curvature (tidal fields and gravitational waves) would be discarded. An entropy of geometry should see it.
  [cf. Penrose, Weyl curvature hypothesis]
- $\tilde{\mathcal{M}} = D|\Phi\rangle\langle\Phi|D + (m^2 + \xi R)|\Phi\rangle\langle\Phi|$ is built from Dirac–Kähler fields.
- Both couplings are fixed by dimensions: $\alpha = \alpha'\ell_{\mathrm{pl}}^4$, $\beta = \beta'\ell_{\mathrm{pl}}^2$.

The matter sector, for completeness

A Dirac–Kähler boson packages forms of three degrees into one state:

$$|\Phi\rangle = \phi \oplus \omega_\mu dx^\mu \oplus \zeta_{\mu\nu} dx^\mu \wedge dx^\nu, \qquad D = d + \delta,$$

$$D|\Phi\rangle = -\nabla_\mu \omega^\mu \;\oplus\; \big(\nabla_\mu \phi - \nabla^\rho \zeta_{\rho\mu}\big) dx^\mu \;\oplus\; \nabla_\mu \omega_\nu \, dx^\mu \wedge dx^\nu .$$

- $d^2 = \delta^2 = 0$, so $D^2 = d\delta + \delta d$ is the Hodge–de Rham Laplacian: D is the natural Dirac operator on forms.
- Abelian gauge fields enter as $\tilde{\mathbf{F}} = 0 \oplus 0 \oplus F_{\mu\nu} F_{\rho\sigma}$ added to $\tilde{\mathcal{M}}$, with minimal coupling inside D.
- Fermions and non-Abelian fields exist in the discrete formulation; in the continuum they are open.

General relativity is the low-coupling limit

For $\alpha', \beta' \ll 1$, expand $-\mathrm{Tr}_F \log(\tilde{\mathbb{I}} - \tilde{\mathbf{U}}) \simeq \mathrm{Tr}_F \tilde{\mathbf{U}}$. The curvature trace gives R in each sector,

$$R_{\mu\nu\rho\sigma} \left[g^{(2)}\right]^{\rho\sigma\mu\nu} = \tfrac{1}{2}\left(R_{\mu\nu}{}^{\mu\nu} - R_{\mu\nu}{}^{\nu\mu}\right) = R \quad \Longrightarrow \quad \mathrm{Tr}_F\, \tilde{\mathcal{R}} \tilde{g}^{-1} = 3R,$$

so the whole thing linearises to

$$\mathcal{L} \simeq 3\beta R - \alpha |\nabla\phi|^2 - \alpha(m^2 + \xi R)|\phi|^2,$$

Einstein–Hilbert with *zero* cosmological constant, plus a scalar in curved spacetime.

- Matching $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ fixes the ratio $\alpha'/3\beta' = 16\pi$ (with $\ell_{\mathrm{pl}}^2 = G$): one free dimensionless coupling remains.

Derivatives stay second order

$\mathcal{L} = -\mathrm{Tr}_F \log \tilde{\mathbf{G}}\tilde{g}^{-1}$ is *non-linear* in $\tilde{\mathcal{R}}$, exactly where Ostrogradsky ghosts usually appear. Impose $\tilde{\mathbf{G}}\tilde{g}^{-1} = \tilde{\Theta}$ with Lagrange multipliers $\mathcal{G}$:

$$\tilde{\mathcal{L}} = -\mathrm{Tr}_F \log \tilde{\Theta} - \mathrm{Tr}_F\, \mathcal{G}\big(\tilde{\mathbf{G}}\tilde{g}^{-1} - \tilde{\Theta}\big) \quad \Longrightarrow \quad \tilde{\Theta}^{-1} = \mathcal{G}, \quad \mathcal{G}^{-1} = \tilde{\mathbb{I}} + \alpha\tilde{\mathcal{M}}\tilde{g}^{-1} - \beta\tilde{\mathcal{R}}\tilde{g}^{-1}.$$

The action then splits into a *dressed* Einstein–Hilbert piece plus matter, with

$$\mathcal{L}_G = \mathcal{R}^G - 2\Lambda_G, \qquad \Lambda_G = \frac{1}{2\beta}\,\mathrm{Tr}_F\big(\mathcal{G} - \tilde{\mathbb{I}} - \log \mathcal{G}\big).$$

- The same manoeuvre that takes $f(R)$ to Brans–Dicke, but with many multipliers.
- $\Lambda_G \geq 0$, vanishing quadratically as $\mathcal{G} \to \tilde{\mathbb{I}}$. It is an emergent, small, positive cosmological constant.

Λ_G is itself a relative entropy

For unnormalised positive operators the Umegaki relative entropy is $S(\rho\|\sigma) = \mathrm{Tr}\big(\rho\log\rho - \rho\log\sigma - \rho + \sigma\big)$. Setting $\rho = \tilde{\mathbb{I}}$, $\sigma = \mathcal{G}$:

$$\Lambda_G = \frac{1}{2\beta}\, S\big(\tilde{\mathbb{I}}\,\|\,\mathcal{G}\big) \;\geq\; 0, \qquad \mathcal{G} = \tilde{\mathbb{I}} + \epsilon \;\Rightarrow\; \Lambda_G \simeq \frac{\mathrm{Tr}_F\,\epsilon^2}{4\beta}.$$

- Positivity is Klein's inequality; the cosmological term measures how far the G-field sits from the identity.
- It is not a parameter, and it runs with curvature; it can diverge.

This phrasing is mine; Bianconi's thermodynamics paper instead reads $2\beta\Lambda_G$ as an internal energy density.

The framework, in one slide

- postulate: the action is the quantum relative entropy between the spacetime metric and the metric induced by matter and curvature.
- Two couplings, fixed by dimensions, of which one combination survives.
- General relativity with $\Lambda = 0$ at low coupling; beyond it, a dressed Einstein–Hilbert action with an emergent, curvature-dependent $\Lambda_G \geq 0$.
- Second-order equations for $g_{\mu\nu}$, Φ and $\mathcal{G}$.
- A domain of validity built into the logarithm.

The rest of the talk asks what this predicts in cosmology.

What goes in, in vacuum

U. Thattarampilly, Y. Zheng, Eur. Phys. J. C **85**, 1433 (2025)

Switch the matter off. The induced metric is then fixed by curvature alone:

$$\tilde{\mathbf{G}} = \tilde{g} - \beta\,\tilde{\mathcal{R}}, \qquad \mathcal{G}^{-1} = \mathbb{I} - \beta\,\tilde{\mathcal{R}}.$$

- I take $\beta = \frac{G}{2}$, to match the coupling used in the Schwarzschild analysis of this theory. [Bianconi, Entropy **27**, 266 (2025)]
- The single dimensionless variable is then $x = 2\beta H^2 = GH^2$. A different β only moves the Hubble scale at which things happen; the structure is unchanged.
- Restricting to the diagonal class $R_{\mu\nu\rho\sigma} = 0$ unless $\mu = \rho,\ \nu = \sigma$ (or the permutation) keeps the flattened matrix diagonal. FLRW and Schwarzschild both qualify.

Modified Einstein equations

$$\mathcal{R}^{G}_{(\mu\nu)} - \tfrac{1}{2} g_{\mu\nu}\big(\mathcal{R}^{G} - 2\Lambda_G\big) + D_{(\mu\nu)} = \kappa\, T_{(\mu\nu)}, \qquad \kappa = \frac{\alpha}{\beta}$$

- $\mathcal{R}^{G}_{\mu\nu}$, $\mathcal{R}^{G}$: Ricci tensor and scalar *dressed* by the G-field, contracted sector by sector with $\tilde{\mathcal{R}}$.
- $D_{\mu\nu}$: second derivatives of the G-field. Its scalar part $(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\mathcal{G}_{(0)}$ is exactly the $f'(R)$ term of metric $f(R)$; the vector and bi-vector sectors are new.
- Λ_G: emergent, non-negative, a function of curvature rather than a constant.
- At $\mathcal{G} = \tilde{\mathbb{I}}$ the left-hand side is $3G_{\mu\nu}$: general relativity is recovered.

The one thing to remember: Λ_G is not a parameter. It runs with curvature, and it diverges.

Making the trace tractable

The Lagrangian needs the trace of $\mathbf{G}^{(2)}_{\mu\nu\rho\sigma} g^{(2)\rho\sigma\eta\theta}$, defined through the flattened 6×6 matrix.

Restrict to metrics whose Riemann components are non-zero only for $R^{\mu\nu}{}_{\mu\nu}$, $\mu \neq \nu$ (and the $\mu \leftrightarrow \nu$ permutation). Then that matrix and $R_{\mu\nu}$ are diagonal, and

$$\mathcal{L} = -\log\Big(1 - \tfrac{G}{2} R\Big) - \sum_{\mu} \log\Big(1 - \tfrac{G}{2} R^{\mu}{}_{\mu}\Big) - \sum_{\mu<\nu} \log\Big(1 - G\, R^{\mu\nu}{}_{\mu\nu}\Big).$$

- FLRW and Schwarzschild both live in this class, so the restriction costs us nothing here.
- Positivity of $\tilde{\mathbf{G}}\tilde{g}^{-1}$ requires every argument to stay positive: the theory has a built-in curvature ceiling of order $1/\beta$.

The dressed curvature terms

$$\mathcal{R}^{G}_{\mu\nu} = \mathcal{G}_{(0)} R_{\mu\nu} + \big[\mathcal{G}_{(1)}\big]_{\mu}{}^{\rho} R_{\rho\nu} - \big[\mathcal{G}_{(2)}\big]_{\rho_1\rho_2\mu\eta} R_{\nu}{}^{\eta\rho_1\rho_2} + 2\big[\mathcal{G}_{(2)}\big]_{\mu}{}^{\eta\rho_1\rho_2} R_{\rho_1\rho_2\nu\eta}$$

$$D_{\mu\nu} = \big(\nabla^{\rho}\nabla_{\rho} g_{\mu\nu} - \nabla_{\mu}\nabla_{\nu}\big)\mathcal{G}_{(0)} - \nabla^{\rho}\nabla_{\nu}\big[\mathcal{G}_{(1)}\big]_{(\rho\mu)} + \tfrac{1}{2}\nabla^{\rho}\nabla_{\rho}\big[\mathcal{G}_{(1)}\big]_{\mu\nu} + \tfrac{1}{2}\nabla^{\rho}\nabla^{\eta}\big[\mathcal{G}_{(1)}\big]_{\rho\eta}\, g_{\mu\nu}$$
$$+ \nabla^{\eta}\nabla^{\rho}\big[\mathcal{G}_{(2)}\big]_{\mu\rho\nu\eta} + \nabla^{\rho}\nabla^{\eta}\big[\mathcal{G}_{(2)}\big]_{\eta\mu\rho\nu} + \tfrac{1}{2}\big[\nabla^{\rho}, \nabla^{\eta}\big]\big[\mathcal{G}_{(2)}\big]_{\rho\eta\mu\nu}$$

G. Bianconi, Phys. Rev. D **111**, 066001 (2025), eqs. (67)–(68)

with

$$\mathcal{G} = \frac{1}{1 - \frac{G}{2}R} \oplus \frac{1}{\delta^{\nu}_{\mu} - \frac{G}{2}R^{\nu}_{\mu}} \oplus \frac{1}{\frac{1}{2}\delta^{\rho\sigma}_{\mu\nu} - \frac{G}{2}R^{\rho\sigma}_{\mu\nu}}, \qquad \mathcal{G}^{-1} = \mathbb{I} - \tfrac{G}{2}\tilde{\mathcal{R}}.$$

Modified Friedmann equations

On FLRW the field equations collapse to two coupled ODEs:

$$\mathcal{R}^G_{00} + \tfrac{1}{2}\big(\mathcal{R}^G - 2\Lambda_G\big) + D_{00} = 0, \qquad \frac{\mathcal{R}^G_{11} + D_{11}}{g_{11}} + \tfrac{1}{2}\big(\mathcal{R}^G - 2\Lambda_G\big) = 0.$$

Highly non-linear, and long; the 22 and 33 components just repeat the second one.

Two useful variables:

$$\epsilon = -\frac{\dot{H}}{H^2} = -\frac{H'(N)}{H(N)}, \qquad \eta = -\frac{d\epsilon}{dN}, \qquad \boxed{x \equiv GH^2}$$

Everything from here is a statement about x: the theory has no other dial. The time component gives H'', which I substitute into the spatial equation to close the system.

FLRW: eleven eigenvalues

For $ds^2 = -dt^2 + a^2(t)\, d\vec{x}^{\,2}$ the non-zero curvature components are

$$R = 6(\dot{H} + 2H^2), \quad R^0{}_0 = 3(\dot{H} + H^2), \quad R^i{}_i = \dot{H} + 3H^2, \quad R^{0i}{}_{0i} = \dot{H} + H^2, \quad R^{ij}{}_{ij} = H^2.$$

Each gives one eigenvalue of $\mathcal{G}^{-1} = \mathbb{I} - \frac{G}{2}\tilde{\mathcal{R}}$. With $\dot{H} = -\epsilon H^2$ and $x = GH^2$:

sector	eigenvalue	multiplicity
scalar	$1 + 3x(\epsilon - 2)$	1
vector, t	$1 + \frac{3x}{2}(\epsilon - 1)$	1
vector, i	$1 + \frac{x}{2}(\epsilon - 3)$	3
bi-vector, ti	$1 + x(\epsilon - 1)$	3
bi-vector, ij	$1 - x$	3

$\mathcal{L} = -\sum \log(\text{eigenvalue})$ is the slow-roll Lagrangian (backup).

The scalar sector vanishes first: $1 + 3x(\epsilon - 2) = 0$ at $x = \frac{1}{3(2-\epsilon)} \simeq \frac{1}{6}$. That is the boundary.

The dressed Ricci tensor on FLRW

$$\mathcal{R}^G_{00} = \frac{1}{G}\left[6 + \frac{1-3GH^2}{-1+6GH^2+3G\dot{H}} + \frac{3}{-1+G(H^2+\dot{H})} + \frac{4}{-2+3G(H^2+\dot{H})}\right]$$

$$\mathcal{R}^G_{11} = \frac{a^2}{3G}\left[-16 + \frac{6}{1-GH^2} + \frac{12}{2-3GH^2-G\dot{H}} + \frac{3}{1-G(H^2+\dot{H})} + \frac{1+3GH^2}{1-3G(2H^2+\dot{H})}\right]$$

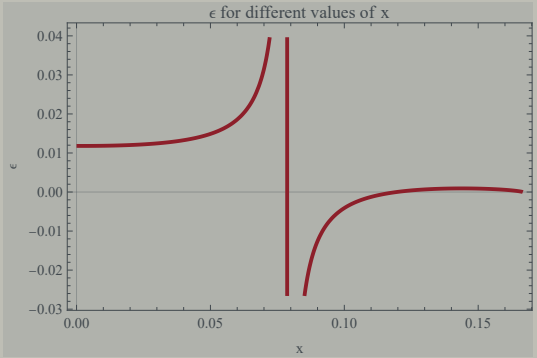
- Every pole is one of the eigenvalues on the previous slide.
- To first order in G: $\mathcal{R}^G_{00} \to -9(\dot{H}+H^2) = 3R_{00}$ and $\mathcal{R}^G_{11} \to 3a^2(\dot{H}+3H^2) = 3R_{11}$, the GR limit $3G_{\mu\nu}$.
- Λ_G and $D_{\mu\nu}$ are built from the same eigenvalues; D_{00} carries $\ddot{H}$.

U. Thattarampilly, Y. Zheng, EPJC **85**, 1433 (2025), supplementary material

Closing the system

- Work in e-folds, $d/dt = H\, d/dN$, so $\dot{H} = HH'$ and $\epsilon = -H'/H$.
- The 00 equation is second order in H through D_{00}: solve it for H''.
- Substitute H'' into the 11 equation.
- Assume slow roll, $H \sim e^{-\epsilon N}$, expand to first order in ϵ and solve for $\epsilon(x, N)$; then $\eta = -d\epsilon/dN$.
- The full expression is long. What we use is its limit $x \to 1/6$.

The whole story lives in $0 < x < 1/6$

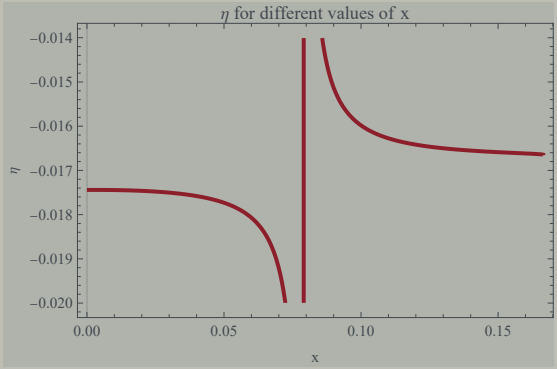


After 50 e-folds. η stays small throughout; see next slides.

- $0 < x \lesssim 0.08$: the slow-roll approximation is not reliable here; I do not use this branch.
- $0.08 \lesssim x \lesssim 0.11$: $\epsilon < 0$, phantom-like.
- $0.12 \lesssim x < 1/6$: slow roll again, and this one survives.
- $x > 1/6$: the action goes complex. The theory stops; it does not merely misbehave.

Near $x \sim 0.08$ the drop is steep and the slow-roll expansion itself breaks down. I do not trust that neighbourhood.

η across the regimes



Away from the pole at $x \simeq 0.08$, $|\eta|$ stays between ~ 1.4 and 2×10^{-2} on both sides of the pole.

At the pole itself ϵ and η both diverge and the slow-roll expansion breaks down.

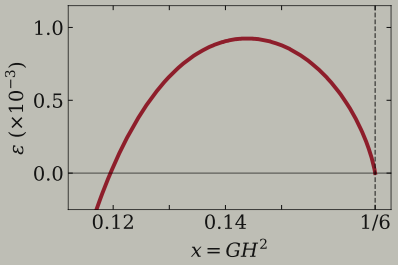
The branch that gives inflation: $x \to 1/6$

Write $u \equiv \log\Big[\dfrac{1/6}{1/6 - x}\Big]$. Near the boundary,

$$\epsilon \;\simeq\; \frac{1.5\,(u + 2.24)(u - 1.98)\big(\frac{1}{6} - x\big)}{0.55 + 0.13N + Nu}, \qquad \eta \;\simeq\; -\,\frac{0.13 + u}{0.55 + 0.13N + Nu}.$$

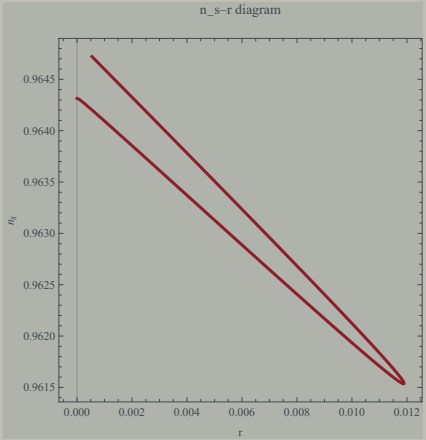
- As $x \to 1/6$, $\mathcal{G} \to 0$ and Λ_G grows without bound.
- The logarithmic corrections supply an effectively enormous potential.
- The slow-roll parameters nevertheless stay small, because they are ratios of logs.

This solution was not something I expected the action to produce.



ϵ after 50 e-folds, zoomed near $x = 1/6$.

And it lands where the data is



$0.12 \lesssim x < 1/6$, 55 e-folds.

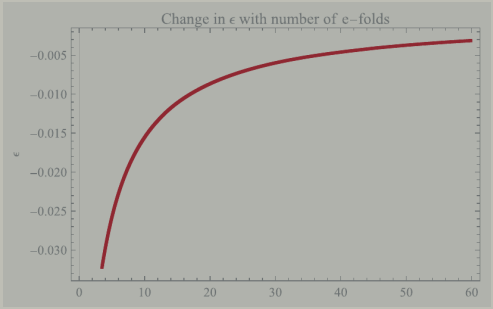
Assuming the usual dictionary $r = 16\epsilon$, $n_s = 1 - 4\epsilon + 2\eta$:

- $0.962 \lesssim n_s \lesssim 0.964$
- $0.000142 \lesssim r \lesssim 0.012$ at 55 e-folds
- at 60 e-folds the window tightens to $0.010 \lesssim r \lesssim 0.012$

Against $r \lesssim 0.036$ and $n_s = 0.96 \pm 0.004$: comfortably inside.

And within reach of Future CMB experiments.

A phantom window sits in between



ϵ against e-folds at $x = 0.1$: small, negative, slowly increasing.

For $0.08 \lesssim x \lesssim 0.11$ the slow-roll parameter turns negative: $w < -1$.

- Null energy condition violated, not shocking given the non-canonical kinetic terms the entropic action generates.
- Same qualitative regime as phantom dark energy and some modified-gravity models.
- Suggestive given DESI/Planck hints of departures from a pure cosmological constant, and the natural place to look for bounce or cyclic solutions.

I am deliberately not over-claiming here; the early dark energy link needs its own paper.

Action counts micro states

The Lagrangian is literally a Boltzmann-like count,

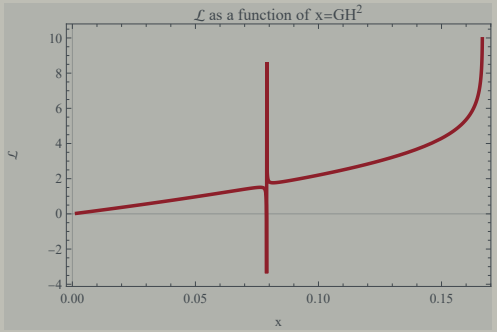
$$\mathcal{L} = -\mathrm{Tr}\log\left(\tilde{\mathbf{G}}\tilde{g}^{-1}\right) = \log W(r),$$

$$S = \frac{1}{\ell_{\mathrm{pl}}^4}\int \sqrt{-g}\,\log W(r)\,d^4x,$$

with $W(r)$ counting degrees of freedom of the geometry. For $a \sim e^{Ht}$,

$$S \simeq \frac{V}{\ell_{\mathrm{pl}}^3}\,\frac{\mathcal{L}}{\ell_{\mathrm{pl}}H}\left(e^{3Ht}-1\right).$$

The observationally viable branch is also the high-entropy branch.



$\mathcal{L}$ against x at 55 e-folds. Ignore the spike near $x \sim 0.08$ where the approximations fail; the maximum sits at $x \sim 1/6$.

What I have not done

- **Perturbations.** Everything above is background dynamics. Reading off r and n_s assumed the standard slow-roll relations carry over; for a theory with a dressed metric and a G-field, that is an assumption, not a result. The scalar and tensor perturbation theory of gravity from entropy still has to be built.
- **Reheating and exit.** How the Universe leaves $x \to 1/6$, and what it reheats into, is open.
- **The coupling.** $\beta = G/2$ was inherited from the Schwarzschild analysis. A different β moves the inflationary scale in H; whether anything picks β out is not known.
- **Stability.** The G-field keeps the equations second order, but a full Hamiltonian analysis of the theory has not been done.

The perturbation calculation is what I am working on now, and it is where this either becomes a real model or does not.

Where that leaves us

- Quantum relative entropy between the spacetime metric and a curvature-induced metric gives modified Friedmann equations that *inflate on their own*, in vacuum.
- One parameter: G. No inflaton, no potential, no exotic matter.
- A viable window at $0.12 \lesssim GH^2 < 1/6$ predicting $n_s \simeq 0.962$–0.964 and $r \lesssim 0.012$: testable, not merely consistent.
- A phantom-like window just below it, with $w < -1$ and NEC violation, pointing towards early dark energy and possibly bouncing solutions.
- The inflationary branch carries the larger geometric entropy, a pleasant thing for a theory built out of entropy to be able to say about initial conditions.

Thank you. Questions welcome.

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Backup

Backup: the Lagrangian in slow roll

For solutions inside the slow-roll approximation, with $x = GH^2$,

$$\begin{aligned}\mathcal{L} = \; & \log 2 + \log 8 - 3\log(1-x) - 3\log\big(2 + x(\epsilon - 3)\big) \\ & - \log\big(1 + 3x(\epsilon - 2)\big) - 3\log\big(1 + x(\epsilon - 1)\big) - \log\big(2 + 3x(\epsilon - 1)\big).\end{aligned}$$

The scalar term $\log\big(1 + 3x(\epsilon - 2)\big)$ is the log of something that vanishes as $x \to 1/6$, which is simultaneously

- why Λ_G diverges,
- why the entropy is maximal there,
- and why the theory has nothing to say for $x > 1/6$.

The three statements are the same statement.