Scalar Induced Gravitational Waves Probing Phase Transitions in the SM and Beyond

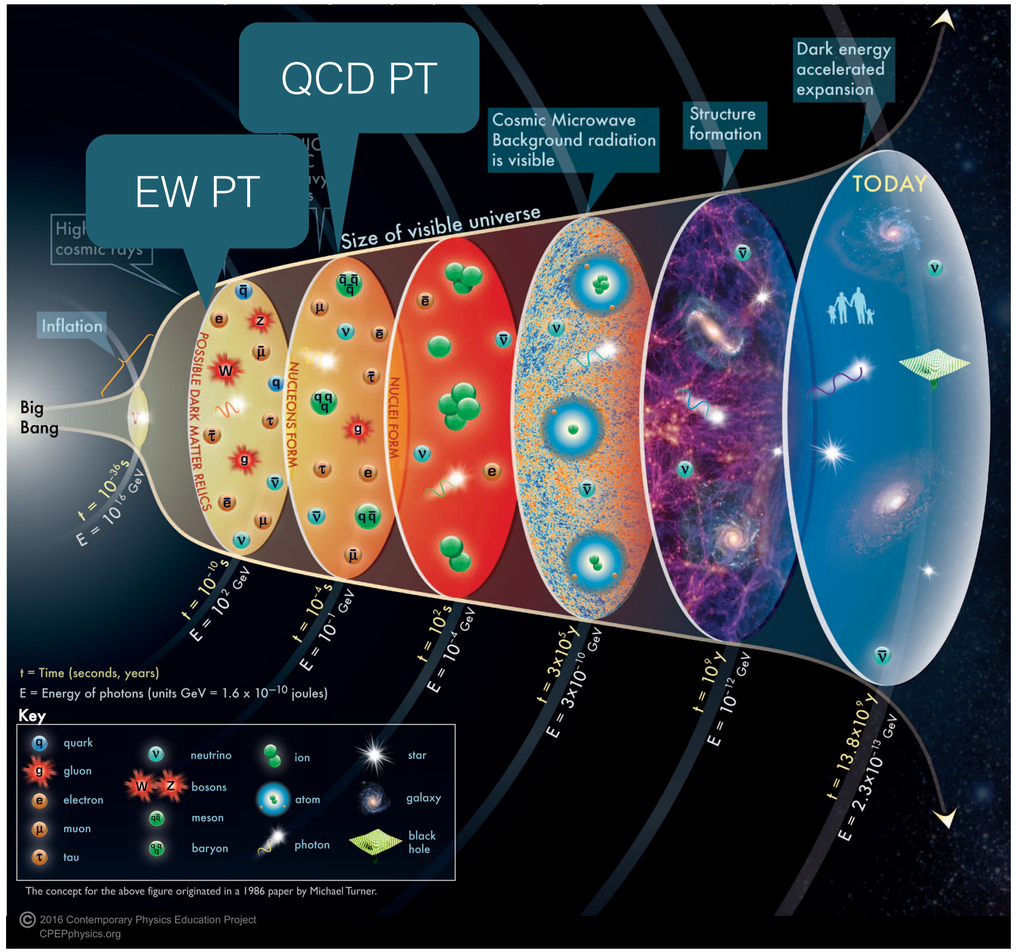
Peizhi Du

University of Science and Technology of China

The 6th BRICS-AGAC Symposium 2026, Hangzhou, China
November 8, 2026

In collabortion with: Xiangxi Zeng and Jiahang Zhong (to appear)

Cosmological Phase Transitons

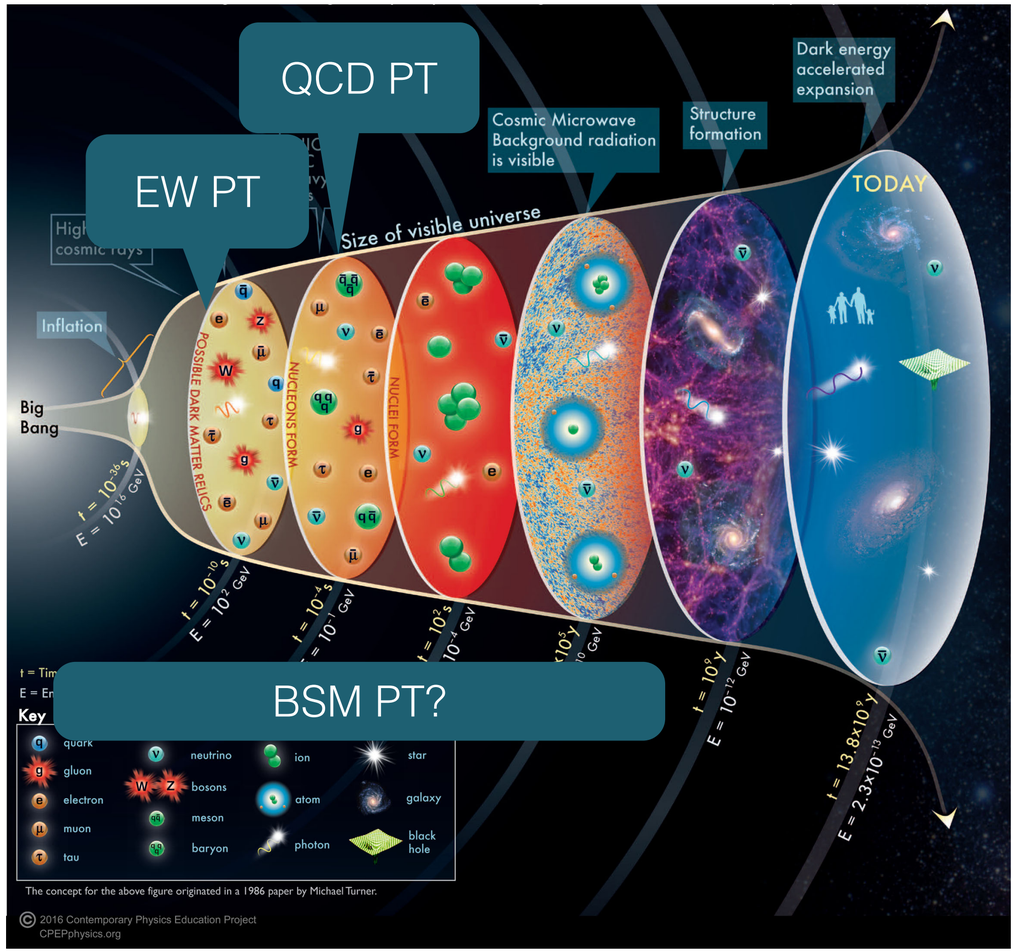


In the SM:

QCD crossover ~100MeV

EW crossover ~100GeV

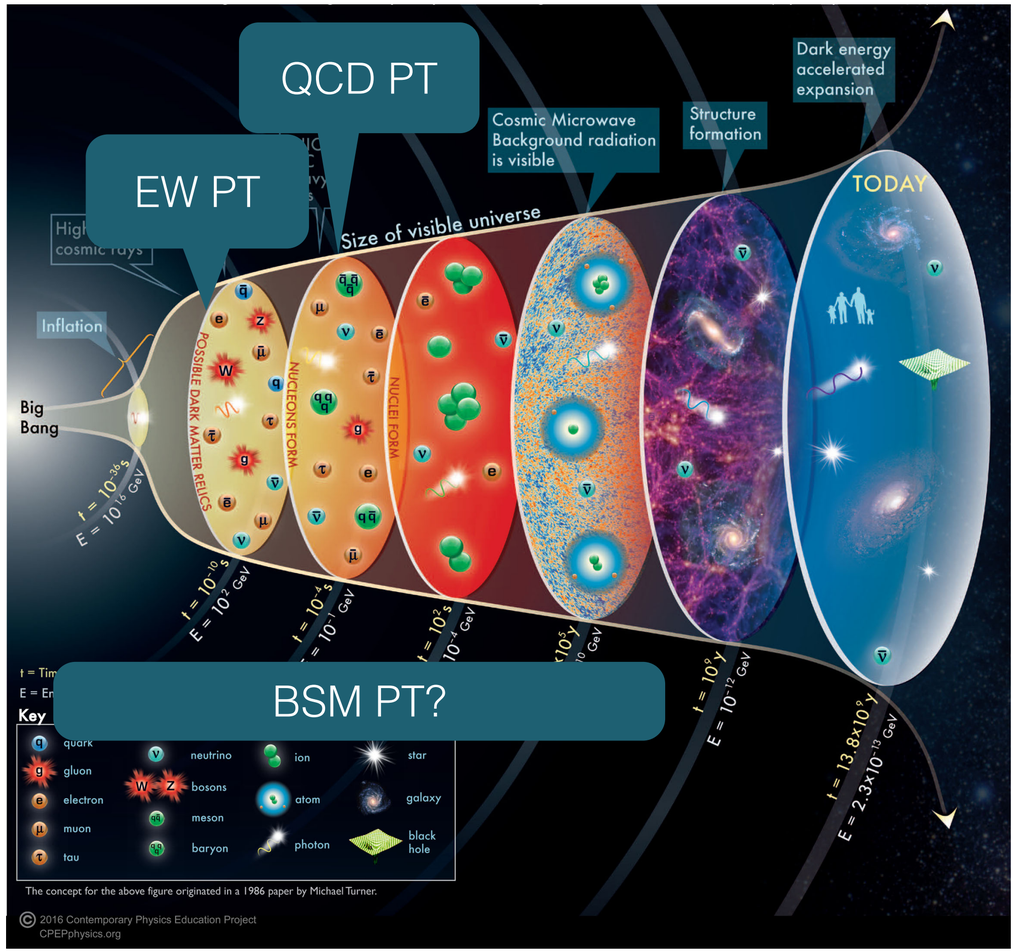
Cosmological Phase Transitons



In the SM: QCD crossover ~100MeV
EW crossover ~100GeV

BSM: EW baryogenesis
Composite Higgs
Dark confining sectors (dark QCD)

Cosmological Phase Transitons

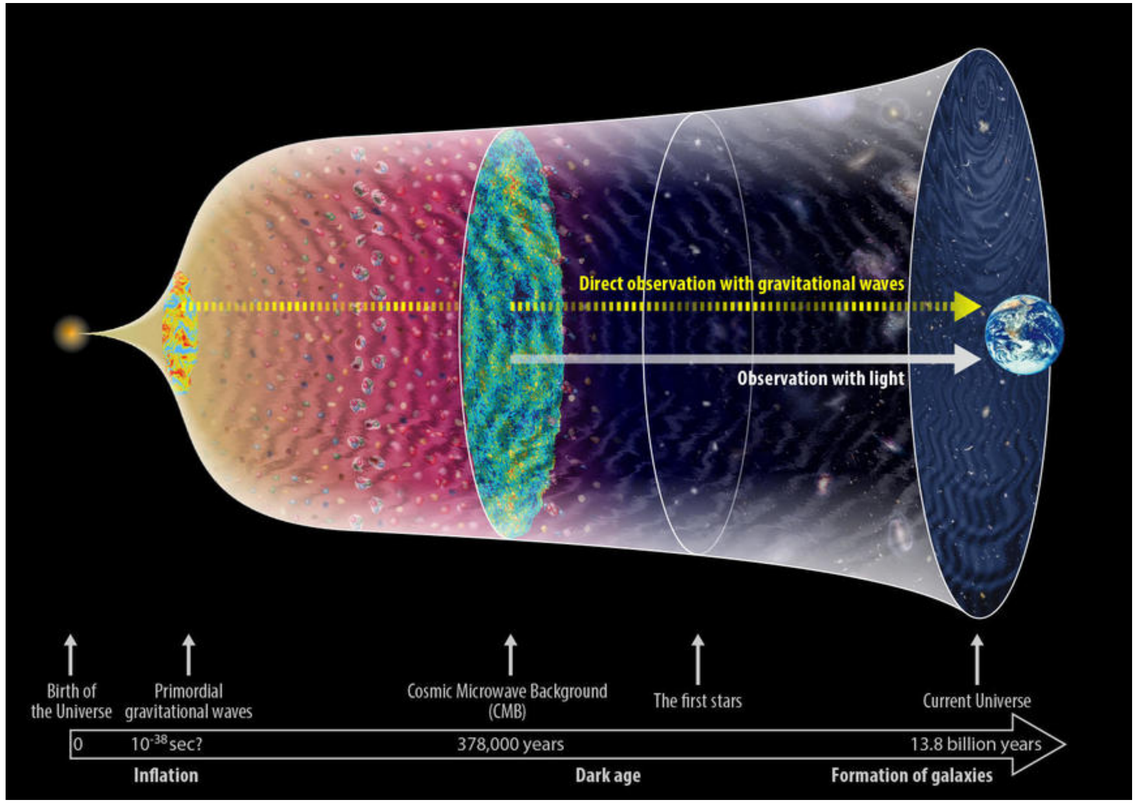


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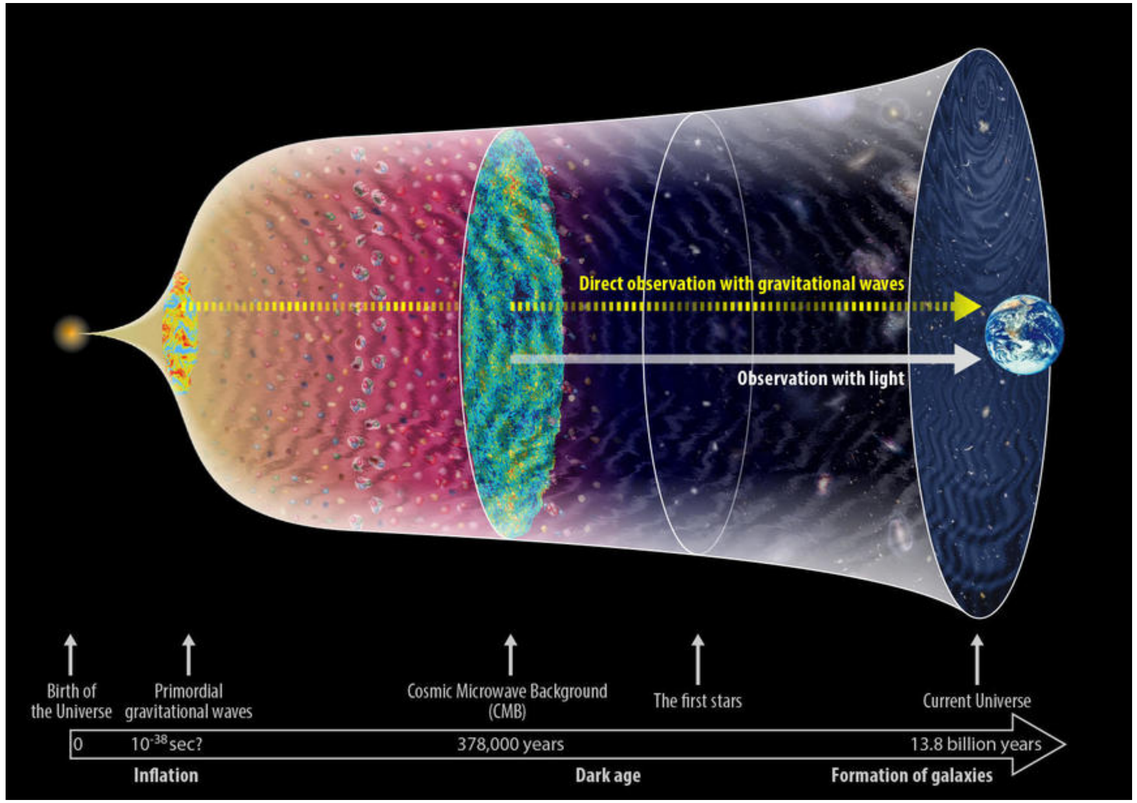
- Non-perturbative dynamics are generally hard to calculate
- Measurements of thermal history of Universe are critical to test the SM and BSM

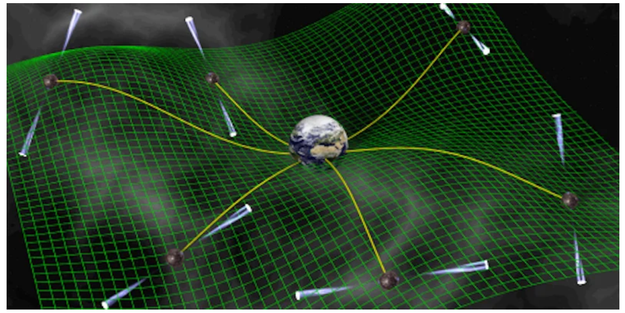
Gravitational Waves



Powerful probes of physics in the early Universe

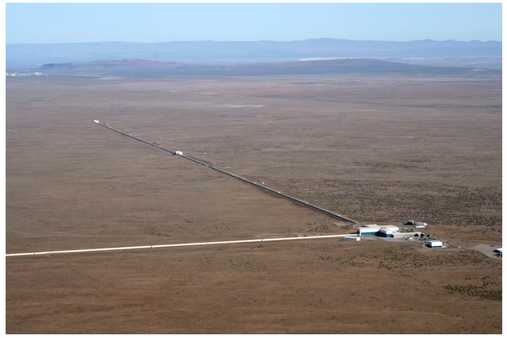
Gravitational Waves

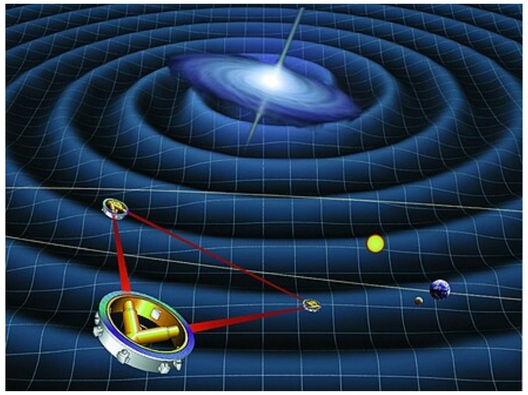




PTA

Ground Interferometers



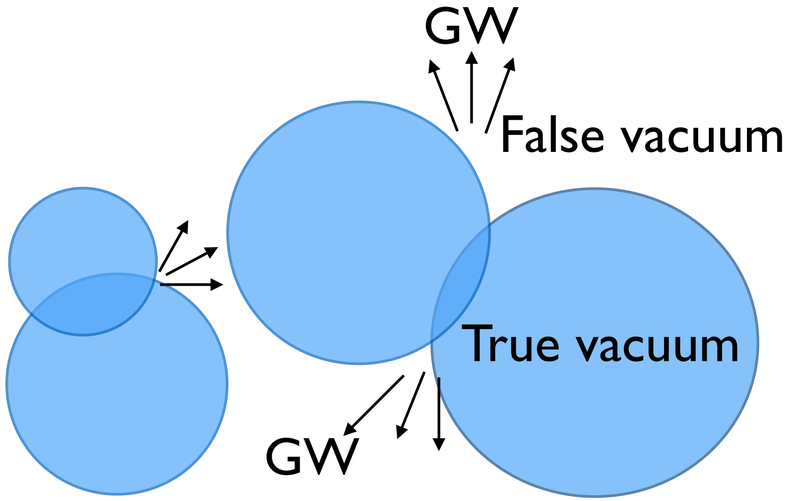


Space Interferometers

Powerful probes of physics in the early Universe

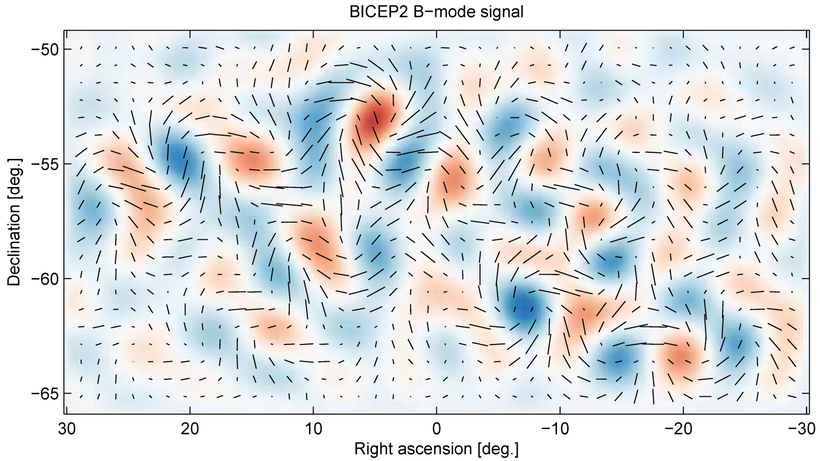
Exciting experimental probes

GW Probing Phase Transitions



- First order phase transitions

- Primordial tensor modes

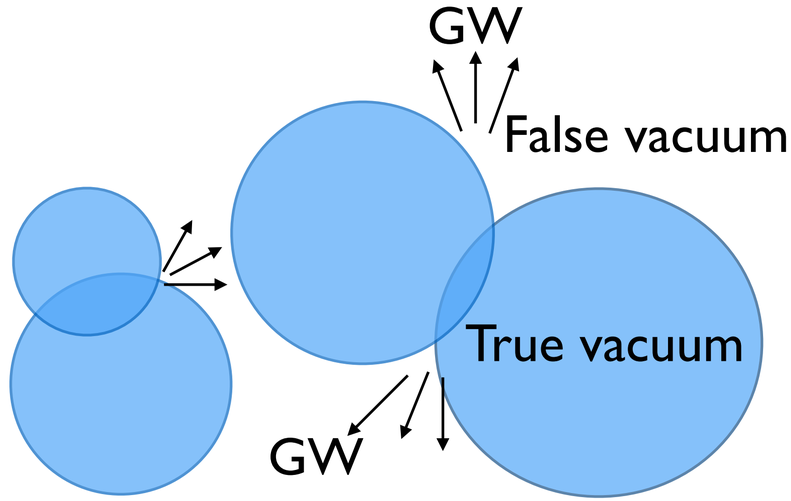


- Scalar induced GWs

Baumann,Steinhardt,Takahashi,Ichiki, *PRD,* 2007
Domenech, *Universe,* 2021

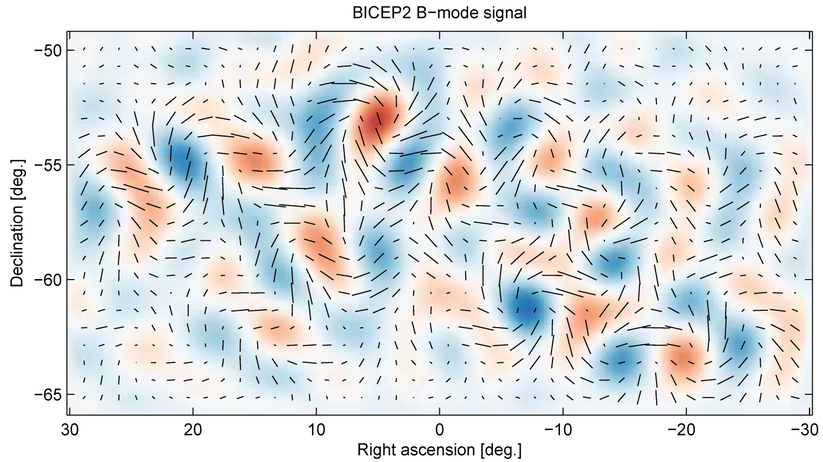
$$\partial_i \Phi^{(1)} \partial_j \Phi^{(1)} \Rightarrow h_{ij}^{(2)}$$

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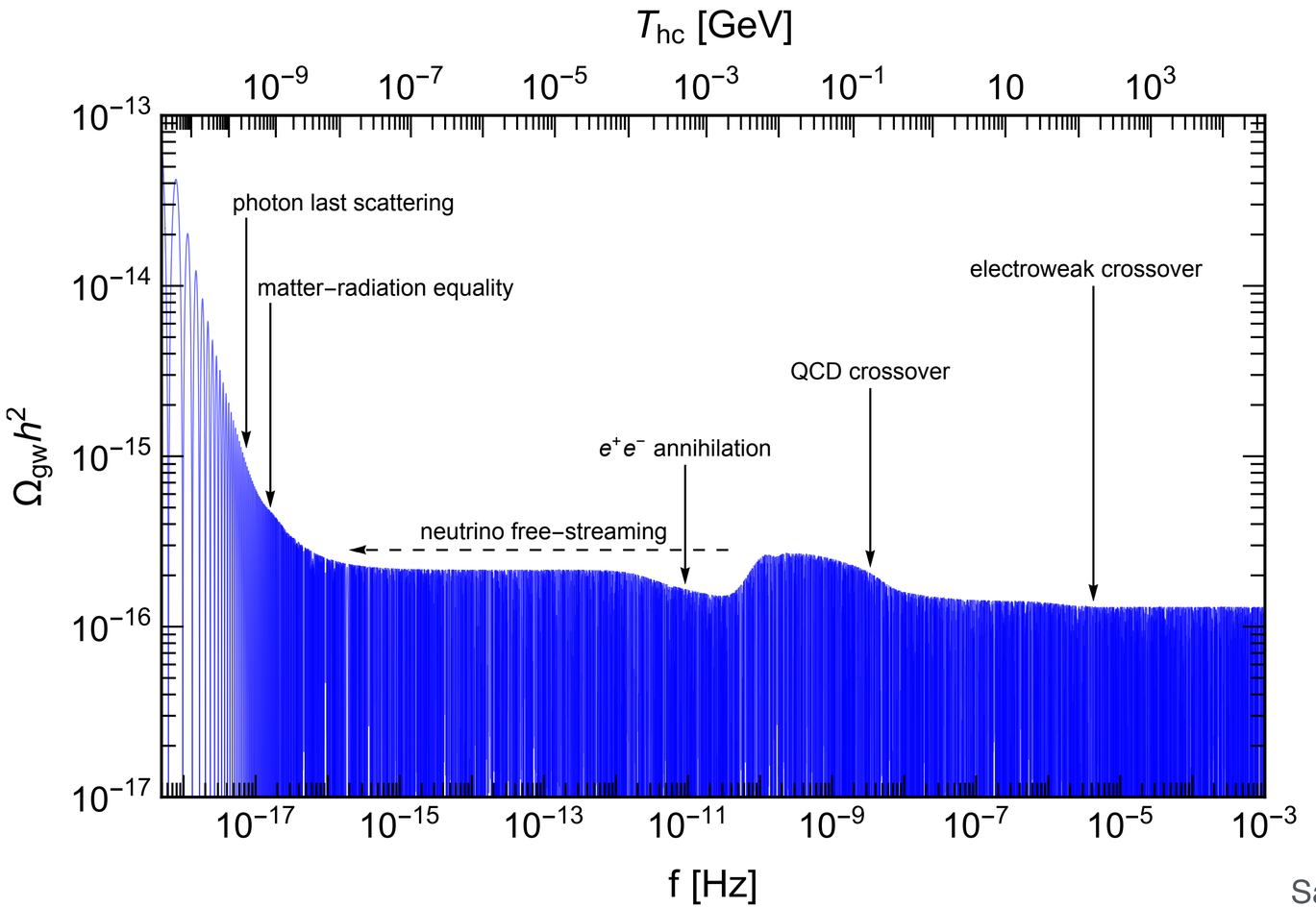
Probing equation of state and sound speed of early Universe

- Scalar induced GWs

Baumann,Steinhardt,Takahashi,Ichiki, *PRD,* 2007
Domenech, *Universe,* 2021

$$\partial_i \Phi^{(1)} \partial_j \Phi^{(1)} \Rightarrow h_{ij}^{(2)}$$

Primordial GW



Saikawa, Shirai, *JCAP,* 2018

- The change of g_* and EoS modifies primoridal GWs
- GW spectrum has imprints of SM PTs and BSM PTs

Scalar Induced GWs

$$ds^2 = a^2(\tau)\left[-(1+2\Phi)d\tau^2 + (\delta_{ij} - 2\Psi\delta_{ij} + h_{ij})dx^i dx^j\right]$$

- First-order perturbations: no source term for GWs
  but a damping term if free-streaming radiation (neutrinos, DR) exists

  Weinberg, *PRD*, 2004

- Second-order perturbations:

$$G_{ij}^{(2)} = 8\pi G T_{ij}^{(2)}$$

$$G_{ij}^{(2)} \supset \partial_i\Phi^{(1)}\partial_j\Phi^{(1)}, h_{ij}^{(2)\prime\prime}$$

$$T_{ij}^{(2)} \supset \bar{\rho}_r \partial_i v^{(1)} \partial_j v^{(1)}$$

Sources for GWs

$$h''_{\vec{k},\lambda} + 2\mathcal{H}h'_{\vec{k},\lambda} + k^2 h_{\vec{k},\lambda} = S_{\vec{k},\lambda}$$

$$S_{\vec{k},\lambda} = 4\int \frac{d^3\vec{q}}{(2\pi)^3} e^{ij}_{\vec{k},\lambda} q_i q_j c(\vec{q}) c(\vec{k}-\vec{q}) f(\tau, q, |\vec{k}-\vec{q}|)$$

- GW energy spectrum

$$\Omega_{\rm GW} \sim \sum_\lambda \langle h_\lambda h_\lambda \rangle \propto A^2_{\rm ad,iso}$$

SIGW in a Time-dependent Background

- Background evolution:

$$\rho' = -3\mathcal{H}(1+w)\rho, \qquad \mathcal{H}^2 = \frac{8\pi G}{3}a^2\rho$$

$$w = p/\rho$$
$$c_s^2 = dp/d\rho$$

- Scalar metric perturbation:

$$\Phi_k'' + 3\mathcal{H}(1+c_s^2)\Phi_k' + \left[c_s^2 k^2 + 3\mathcal{H}^2(c_s^2 - w)\right]\Phi_k = 0$$

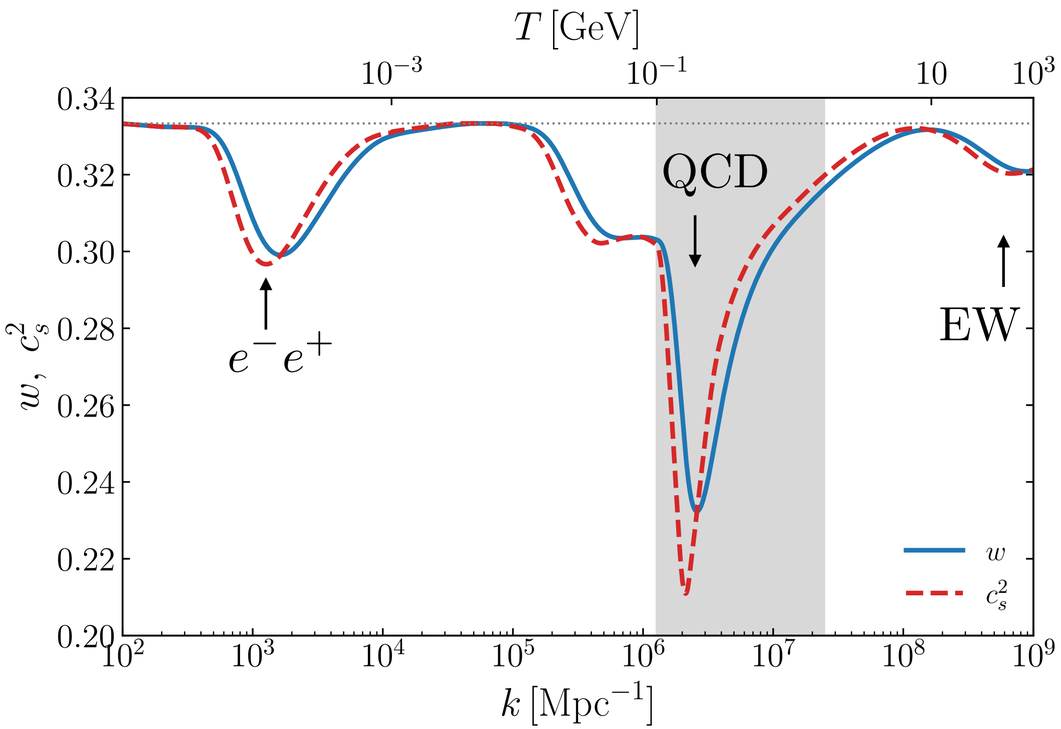
- Tensor perturbations/GWs

$$\left[\frac{\partial^2}{\partial\eta^2} + k^2 - \frac{1-3w(\eta)}{2}\mathcal{H}^2\right][a(\eta)h_{\mathbf{k}}(\eta)] = 4a(\eta)S_{\mathbf{k}}(\eta)$$

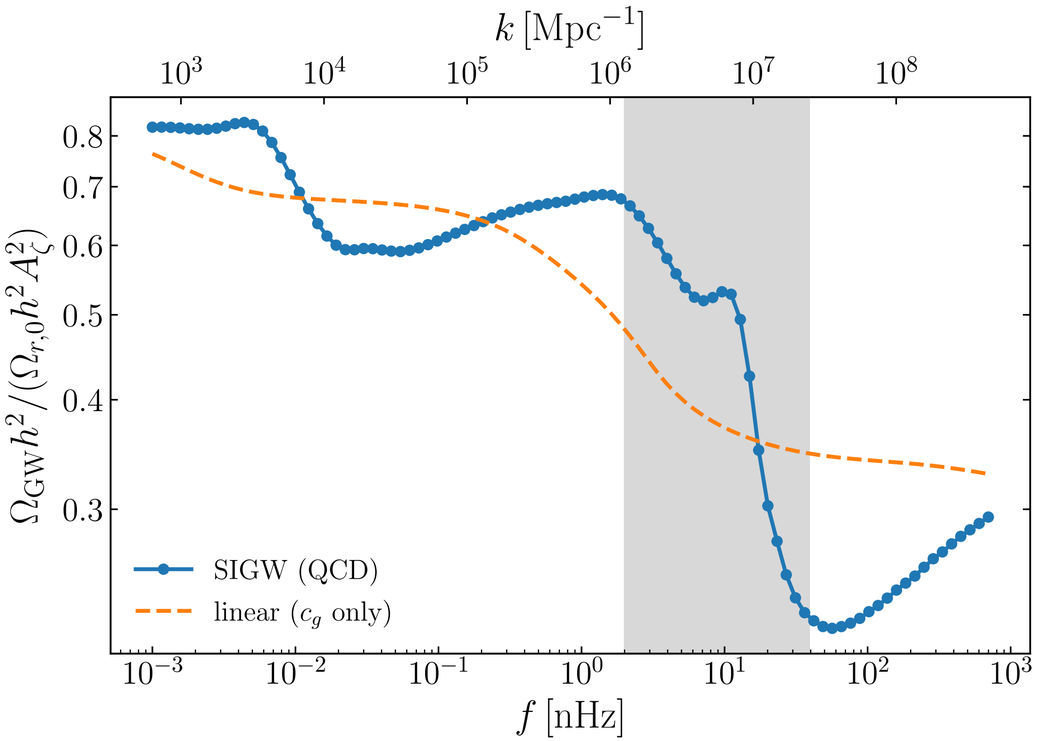
$$S_{\mathbf{k}}(\eta) = \int \frac{d^3\tilde{\vec{k}}}{(2\pi)^3} e^{ij}(\mathbf{k})\tilde{k}_i\tilde{k}_j \Bigg[2\Phi_{\tilde{\mathbf{k}}}\Phi_{\mathbf{k}-\tilde{\mathbf{k}}} + \frac{4}{3[1+w(\eta)]}\left(\Phi_{\tilde{\mathbf{k}}} + \frac{\Phi'_{\tilde{\mathbf{k}}}}{\mathcal{H}}\right)\left(\Phi_{\mathbf{k}-\tilde{\mathbf{k}}} + \frac{\Phi'_{\mathbf{k}-\tilde{\mathbf{k}}}}{\mathcal{H}}\right)\Bigg]$$

SIGW Probing PTs

Thermal History of the SM



SIGW from a flat curvature power spectrum



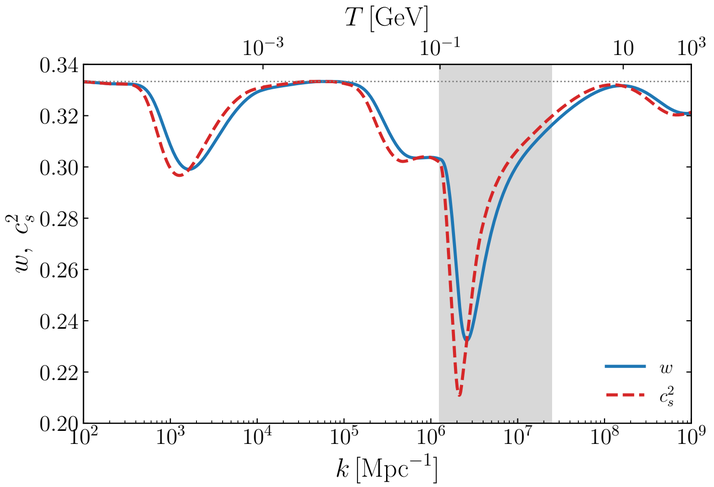
Franciolini, Pritchard, Tada, *arXiv 2609.29078*

Issues of SIGW in a Time-dependent Background

- Entropy non-conservation:

  Current fitting functions of g_*, g_{*s} do not conserve entropy!

  Abe, Tada, Ueda, *JCAP,* 2021



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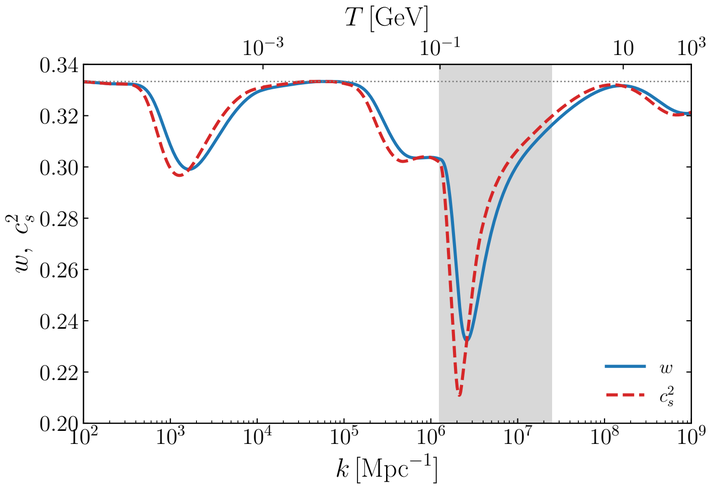
  Current fitting functions of g_*, g_{*s} do not conserve entropy!

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- Initial conditions for metric perturbations:

  Standard approach: $\Phi_k(\eta_{\rm ini}) = 2/3\ ,\ \Phi_k'(\eta_{\rm ini}) = 0$

  Valid in RD, but not in a general time-dependent background



Issues of SIGW in a Time-dependent Background

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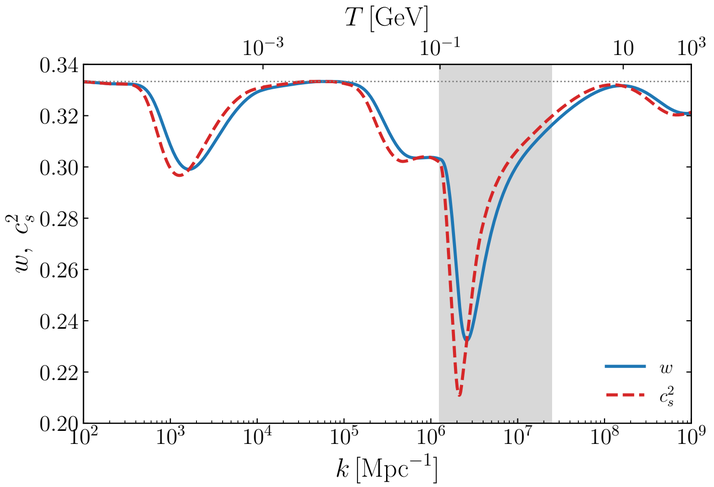
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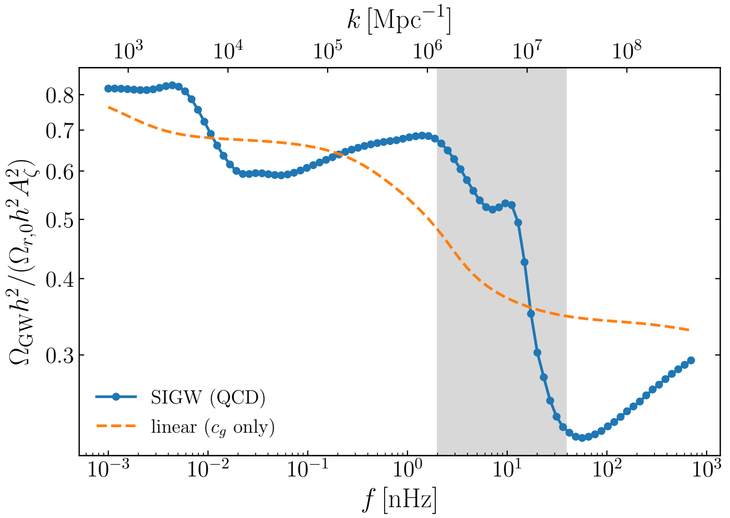
  Standard approach: $\Phi_k(\eta_{\rm ini}) = 2/3\ ,\ \Phi_k'(\eta_{\rm ini}) = 0$

  Valid in RD, but not in a general time-dependent background

- Effects from different transitions in the SM mix in GW spectrum

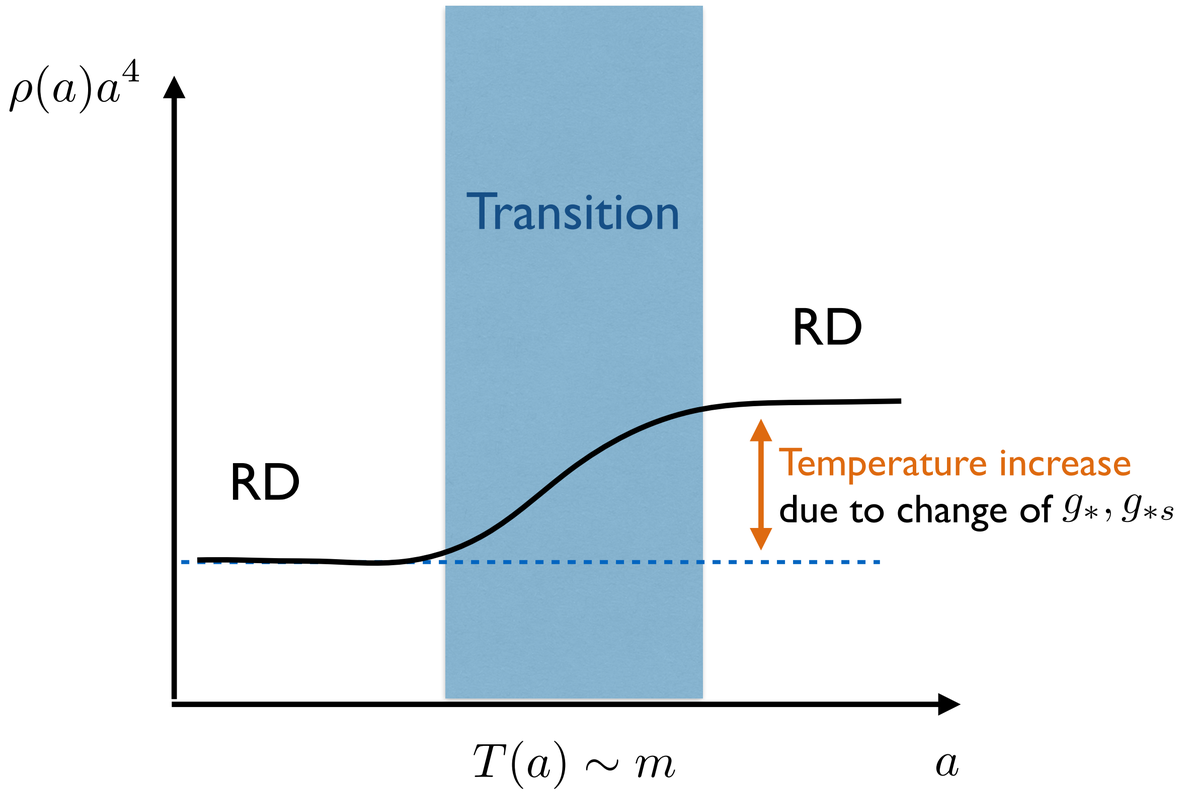
  It is hard to isolate and understand the effect from a single phase transition





Toy Model for Phase Transitions

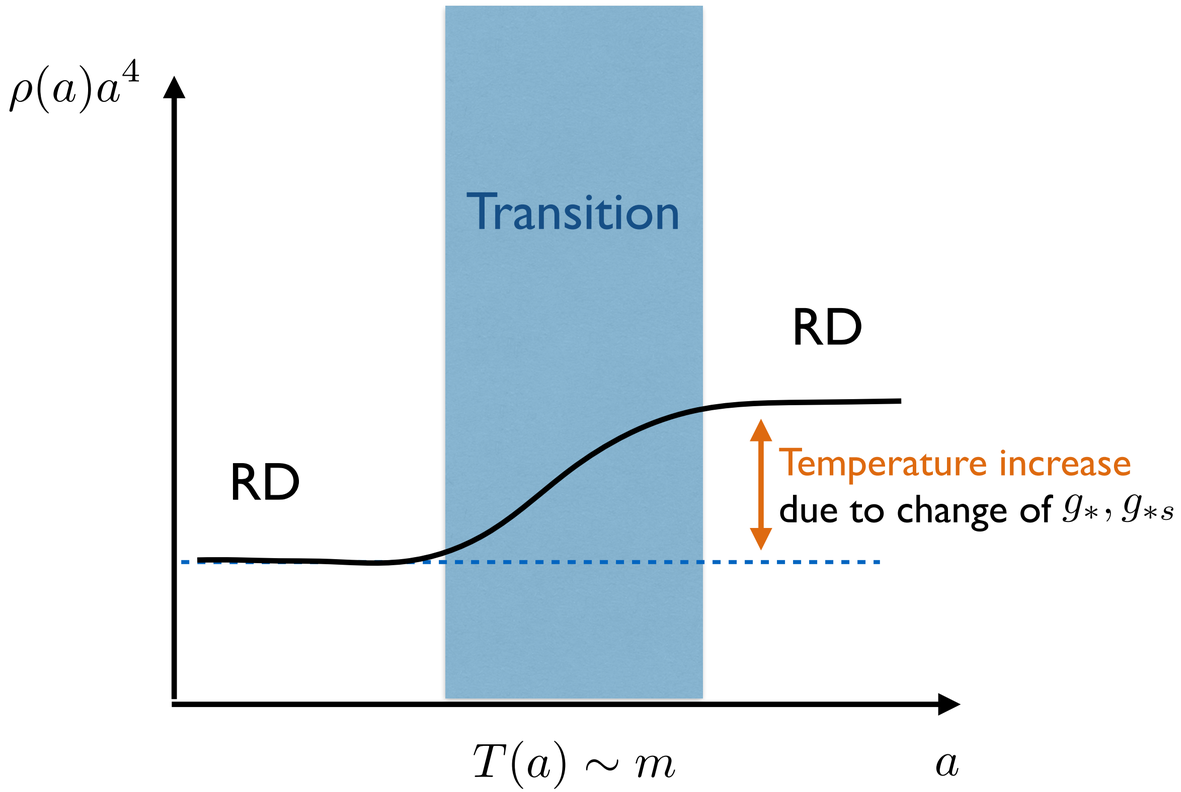
- Coupled fluid of massive particles and massless particles

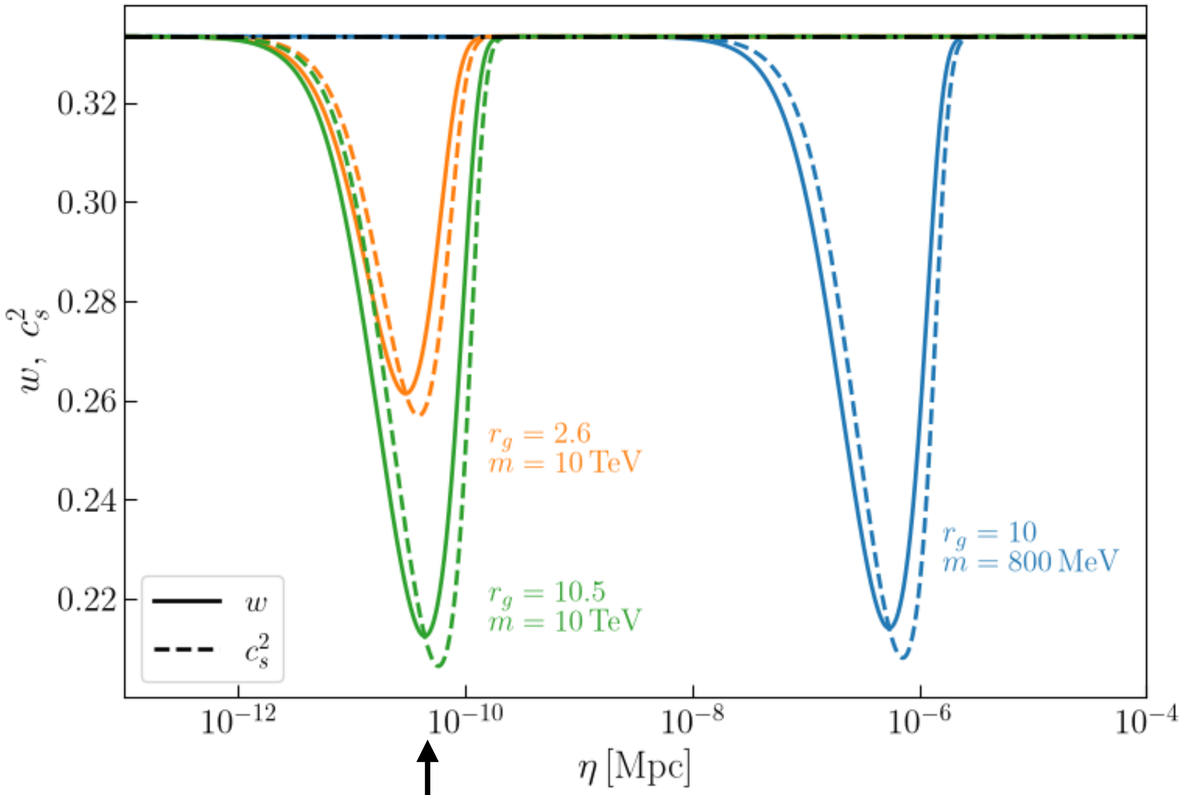


Toy Model for Phase Transitions

- Coupled fluid of massive particles and massless particles

$$r_g = (g_{*,\mathrm{UV}} - g_{*,\mathrm{IR}})/g_{*,\mathrm{IR}}$$

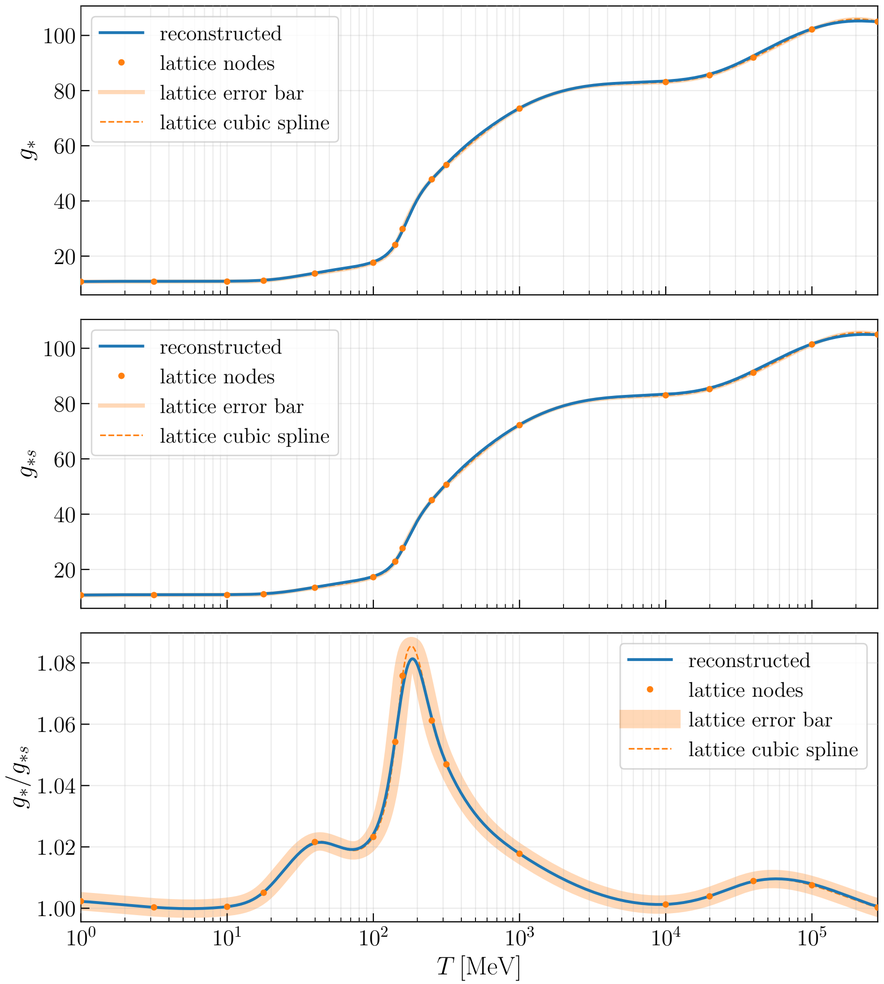


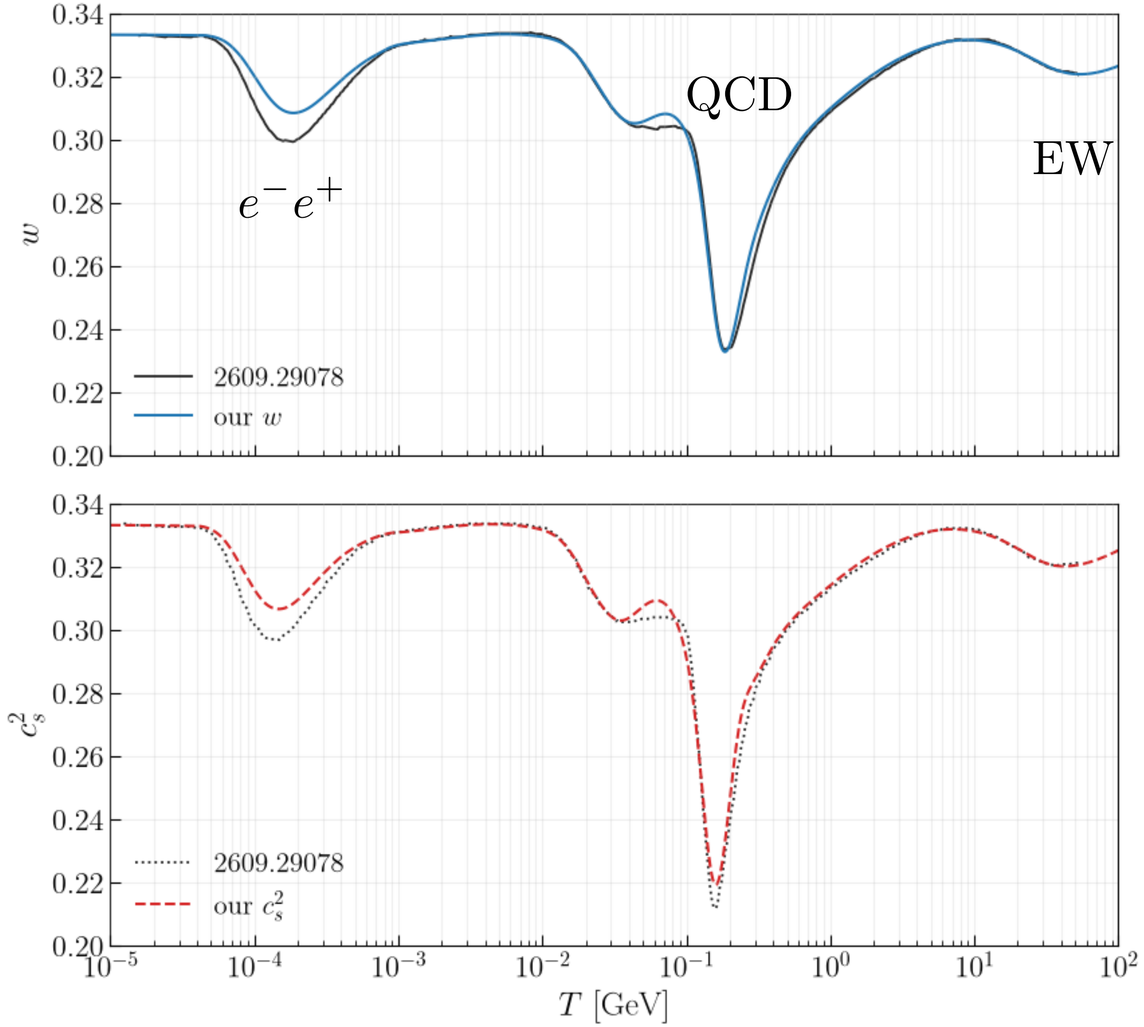


Composite Higgs with $T_c = 10\mathrm{TeV}$

Phase Transitions in the SM

- We construct g_*, g_{*s} based on lattice QCD data with entropy conservation condition

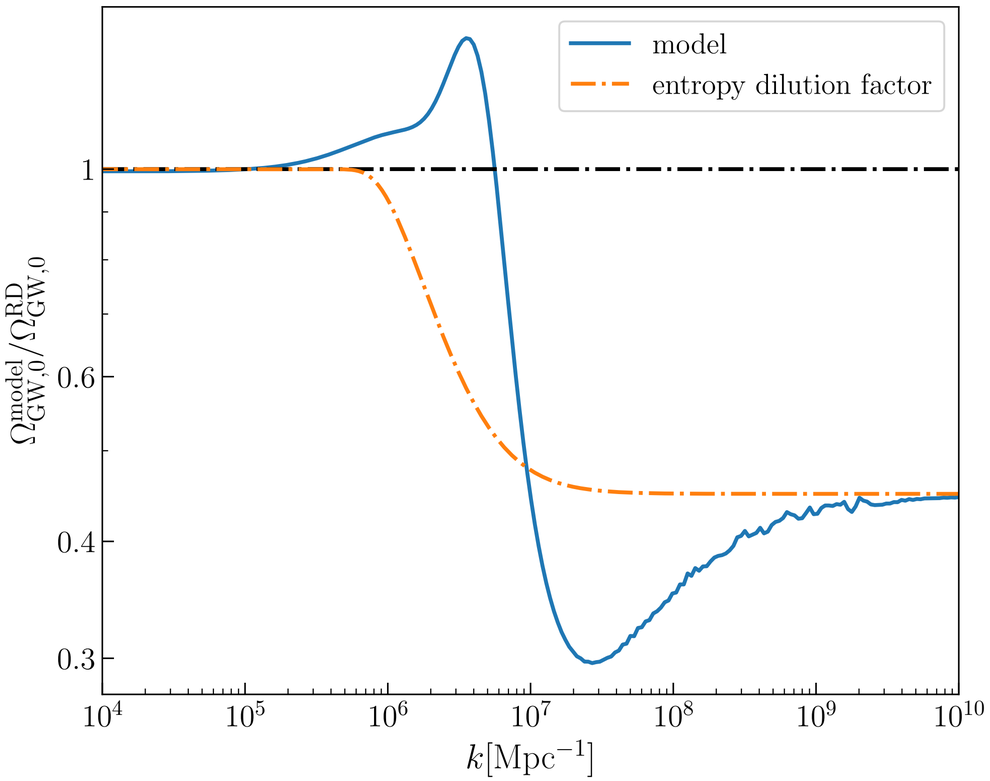




lattice data from Borsanyi et.al, *Nature,* 2016

SIGW Spectrum from PTs

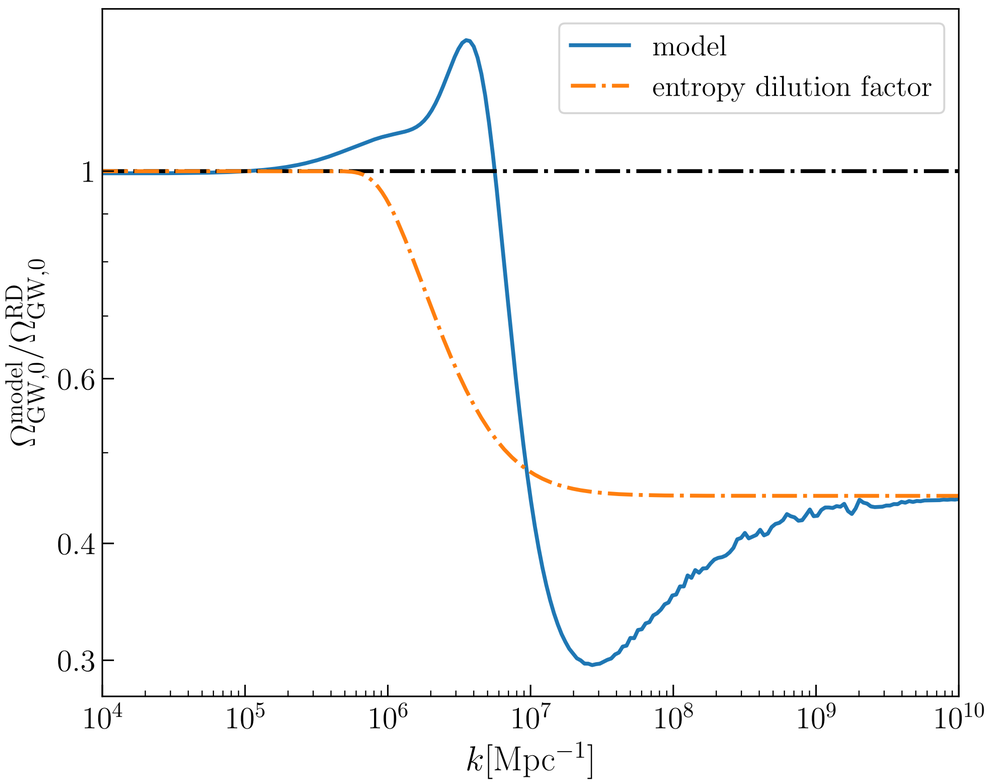
Toy model



General feature: A step-like shape with a peak and a dip

SIGW Spectrum from PTs

Toy model



- Entropy dilution factor

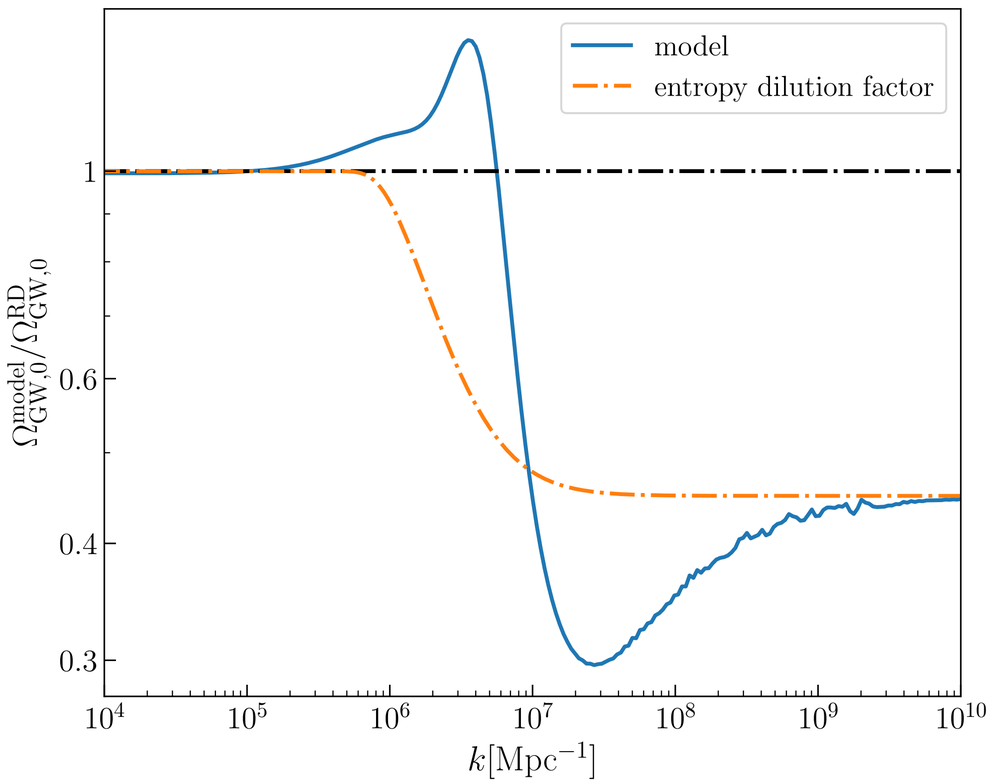
$$D(k) = \frac{g_*(1/k)}{g_*(\eta_{\rm ref})}\left(\frac{g_{*s}(\eta_{\rm ref})}{g_{*s}(1/k)}\right)^{4/3}$$

The heating of photon bath due to change g_*, g_{*s}

General feature: A step-like shape with a peak and a dip

SIGW Spectrum from PTs

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The heating of photon bath due to change g_*, g_{*s}

- Transition frequency

$$f_{\rm tr} \sim \frac{k_{\rm start}}{2\pi}$$

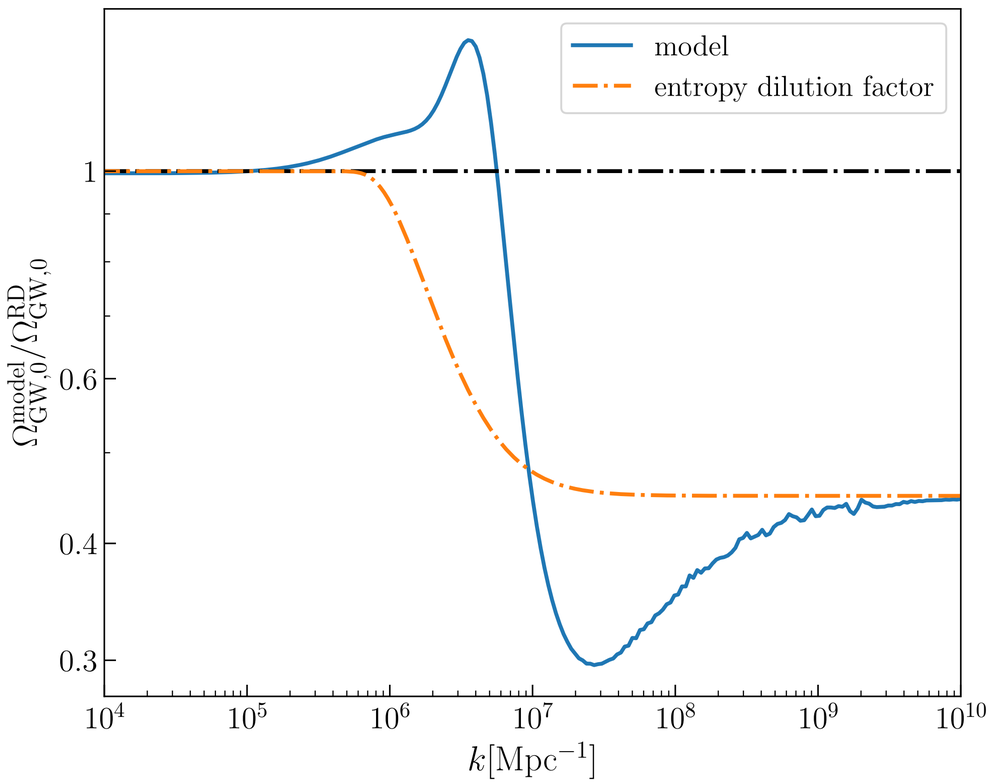
QCD $f_{\rm tr} \sim 10^{-8}\mathrm{Hz}$

10TeV $f_{\rm tr} \sim 10^{-3}\mathrm{Hz}$

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SIGW Spectrum from PTs

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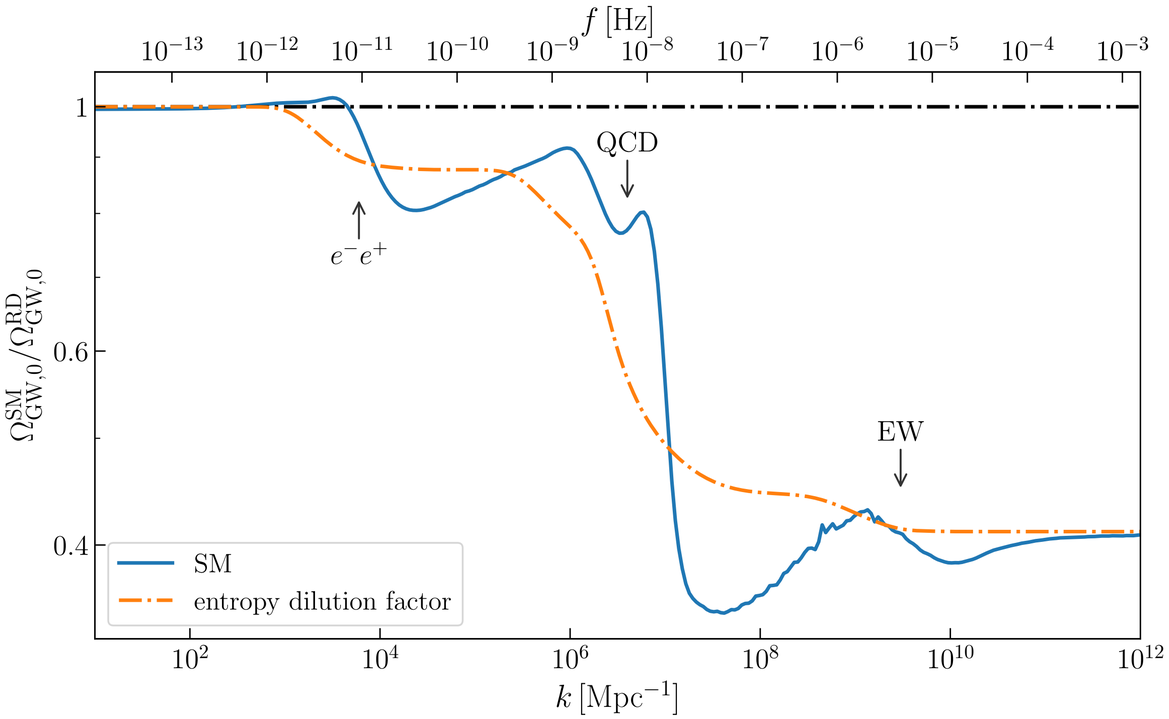
10TeV $f_{\rm tr} \sim 10^{-3}\mathrm{Hz}$

- A peak due to $c_s^2 < 1/3$ and a dip due to $w < 1/3$

General feature: A step-like shape with a peak and a dip

SIGW Spectrum from PTs

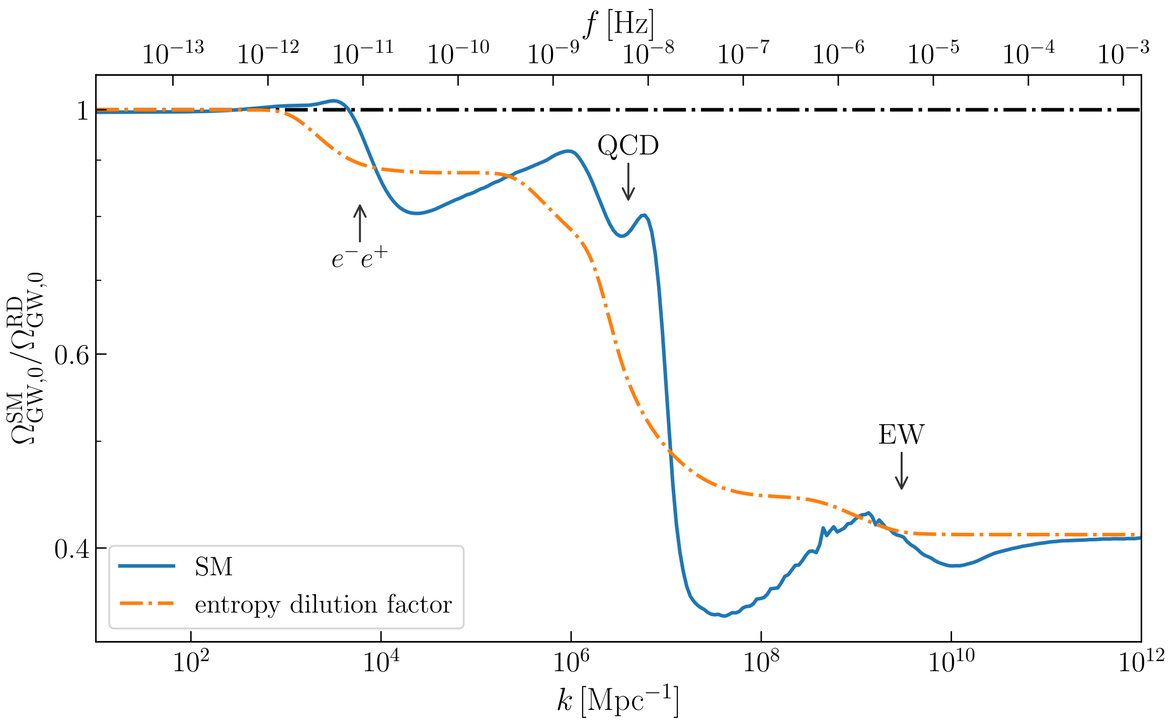
SM



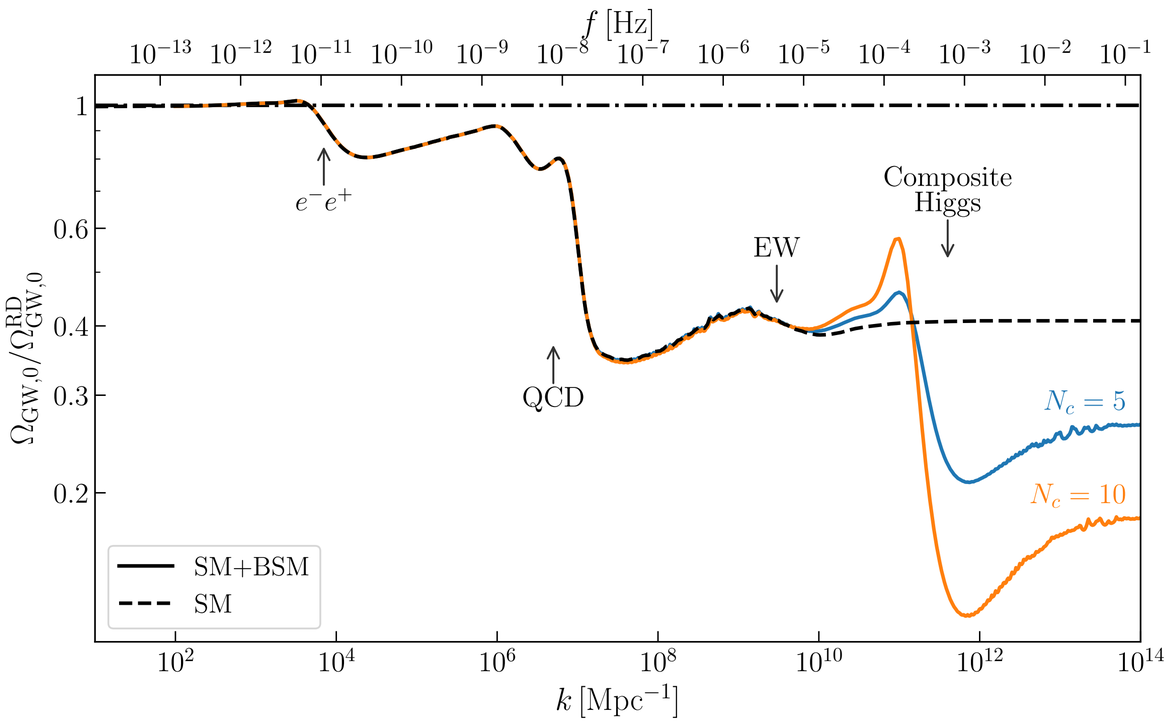
- Combined step-like features from each transition

SIGW Spectrum from PTs

SM



SM+BSM



- Combined step-like features from each transition

Initial Conditions in a Time-dependent Background

- Superhorizon evolution of Φ

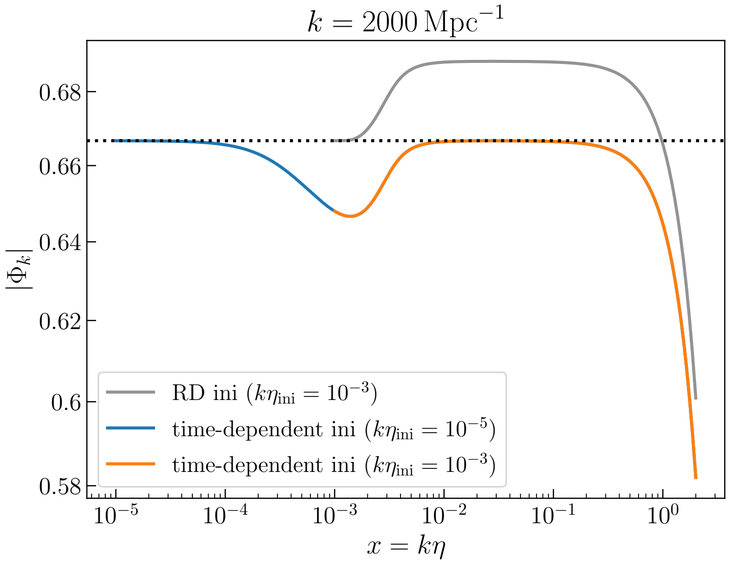
$$\Phi'_{\mathbf{k}} + \frac{5+3w}{2}\mathcal{H}\Phi_{\mathbf{k}} = \frac{3}{2}(1+w)\mathcal{H}\zeta_{\mathbf{k}}$$

- Superhorizon solutions

$$\Phi_k(\eta) = \left(1 - \frac{\mathcal{H}}{a^2}\int_0^\eta a^2(\tilde{\eta})\,\mathrm{d}\tilde{\eta}\right)$$

Standard RD approach:

$$\Phi_k(\eta_{\rm ini}) = 2/3\ ,\ \ \Phi'_k(\eta_{\rm ini}) = 0$$



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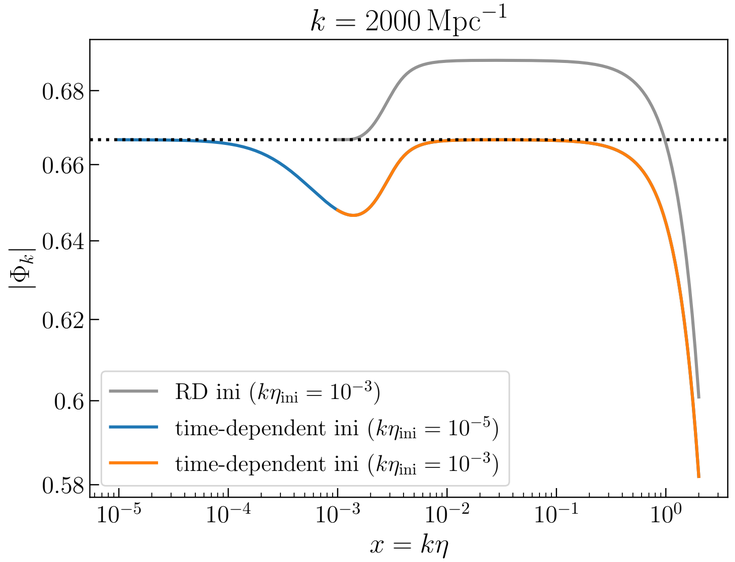
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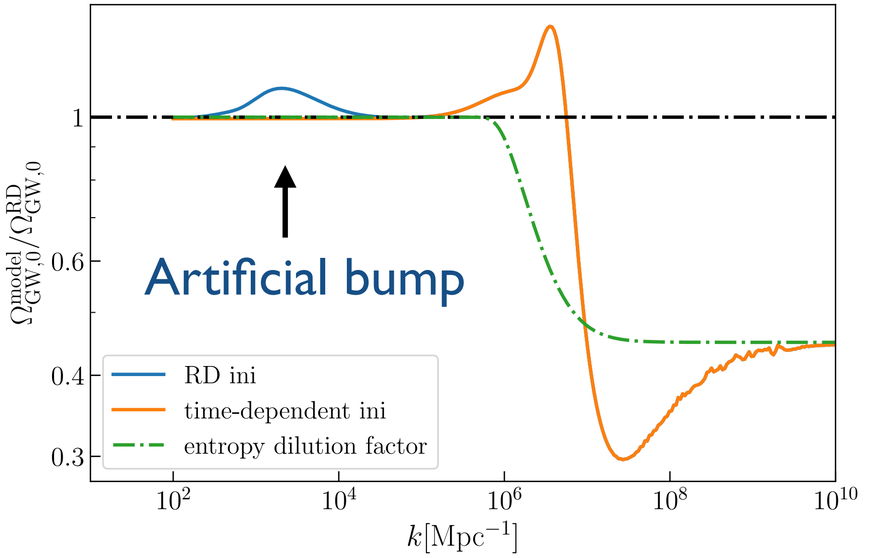
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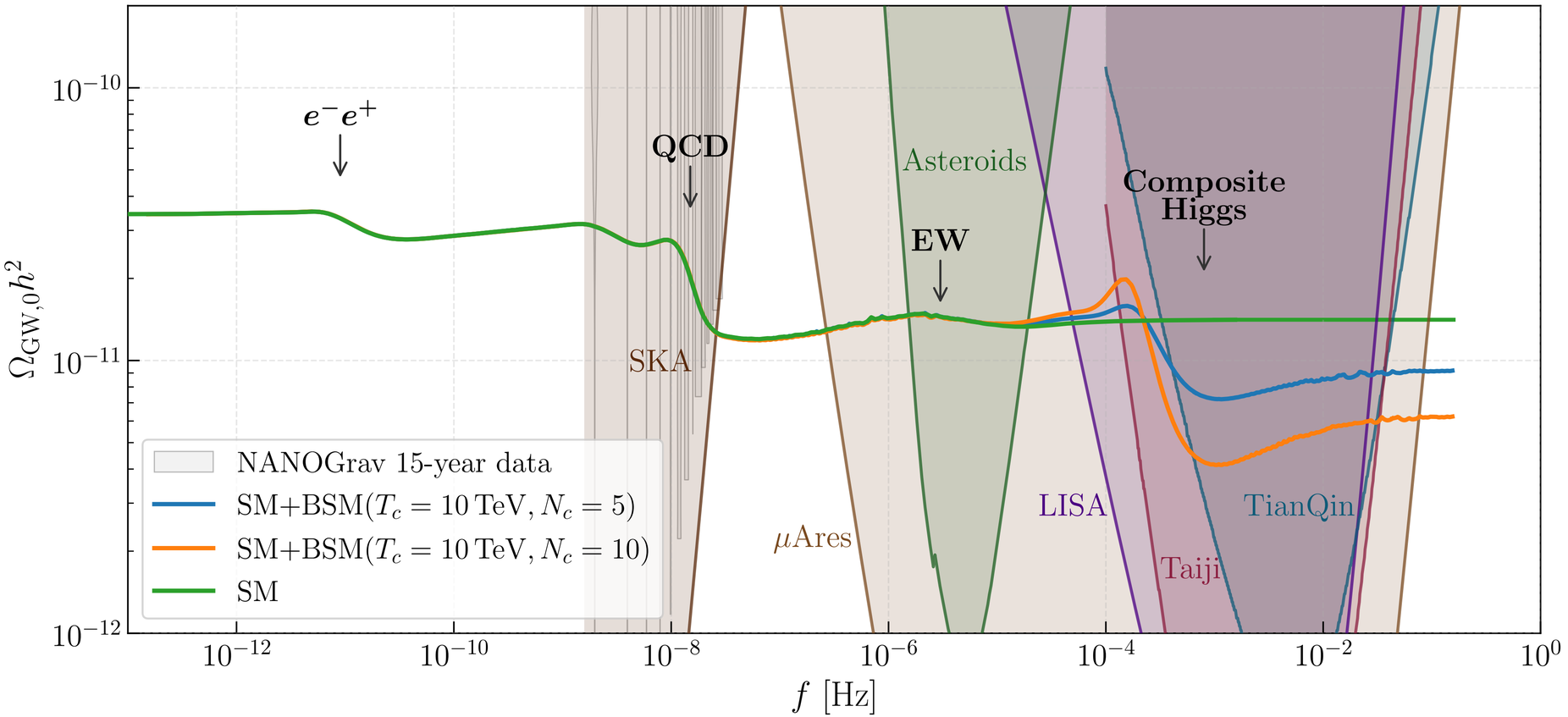
- Artificial enhancement in Φ and a bump in GW spectrum for fixed $k\eta_{\rm ini}$





GW Observations Probing PTs

GW observations across multiple scales can probe SM and BSM PTs

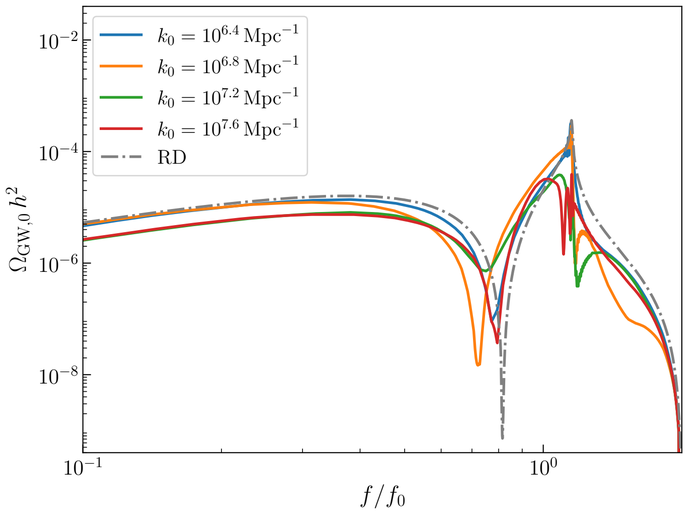


$$\mathcal{P}_\zeta(k) = A$$

QCD: PTA EW: uHz BSM: LISA, Taiji, TianQin

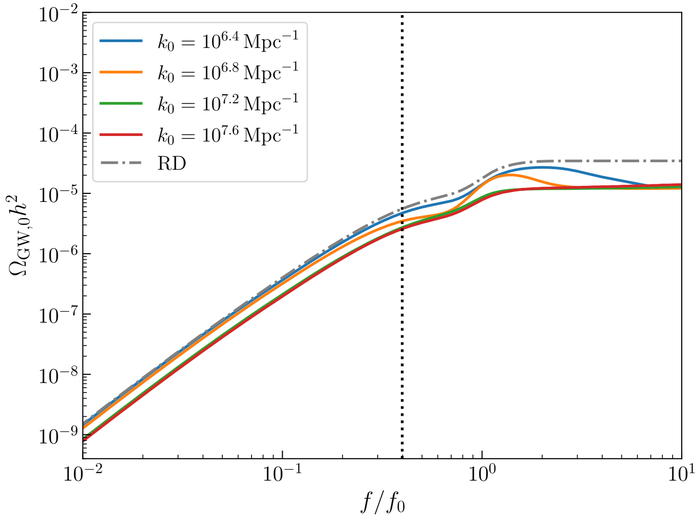
Constraints from PTA

Delta spectrum



$$\mathcal{P}_\zeta(k) = A\,\delta(\ln k - \ln k_0)$$

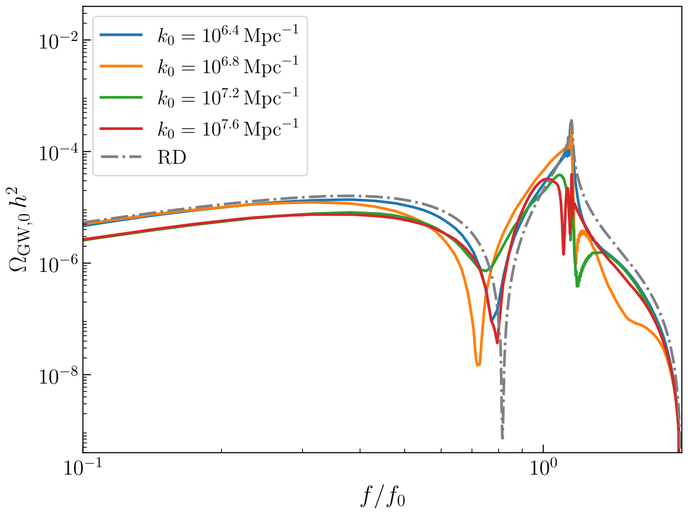
Broken power law



$$\mathcal{P}_\zeta(k) = A \begin{cases} (k/k_0)^3, & k \le k_0, \\ 1, & k > k_0, \end{cases}$$

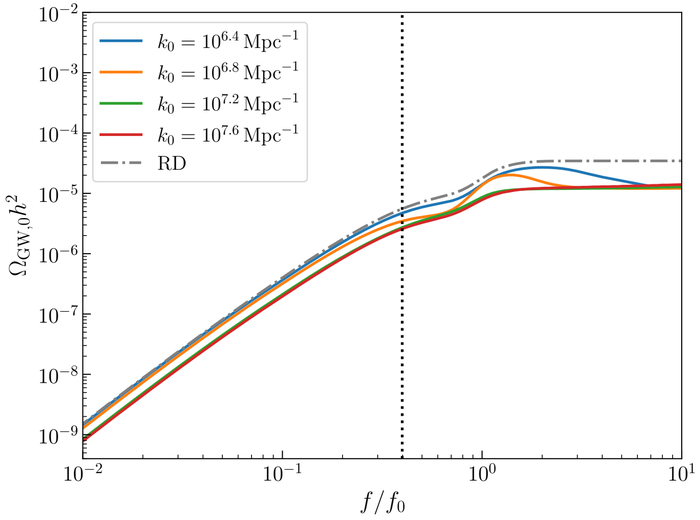
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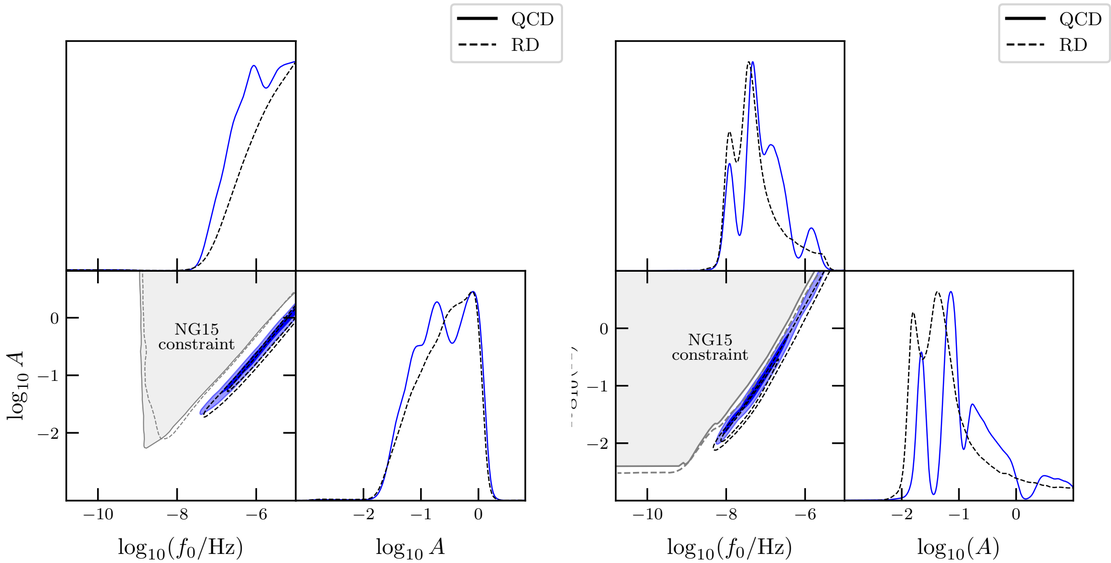
$$\mathcal{P}_\zeta(k) = A\,\delta(\ln k - \ln k_0)$$

Broken power law



$$\mathcal{P}_\zeta(k) = A\begin{cases}(k/k_0)^3, & k \le k_0,\\ 1, & k > k_0,\end{cases}$$

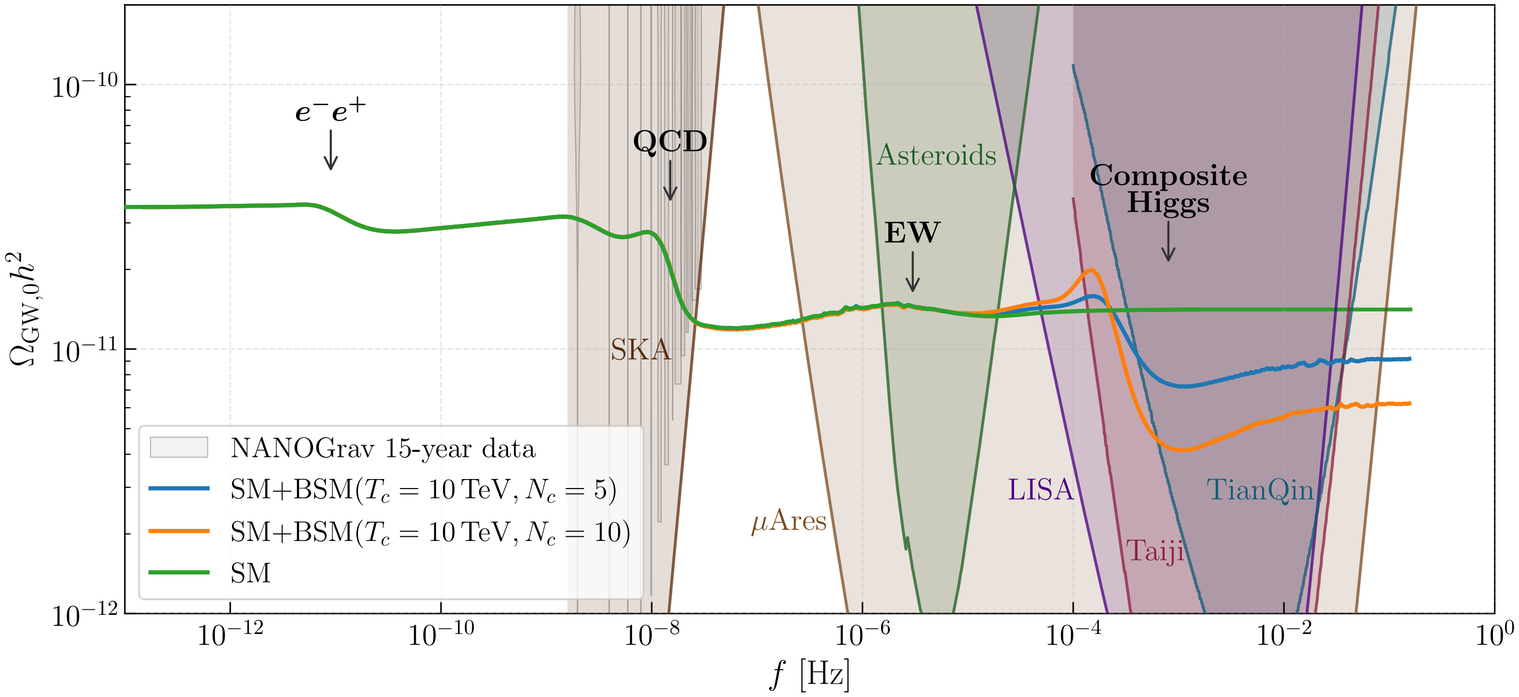
NANOGrav 15 year data



- Current data has only a mild preference for QCD crossover vs standard RD

Discussions and Conclusions

- SIGW spectrm encodes information of PTs in the early Universe
- We addressed issues of entropy non-conservation and initial condtions in SIGW calculation
- Current and future GW observations can probe PTs in the SM and beyond
- Future directions: non-gaussianity and PBH formation in a time-dependent background



Thank you!