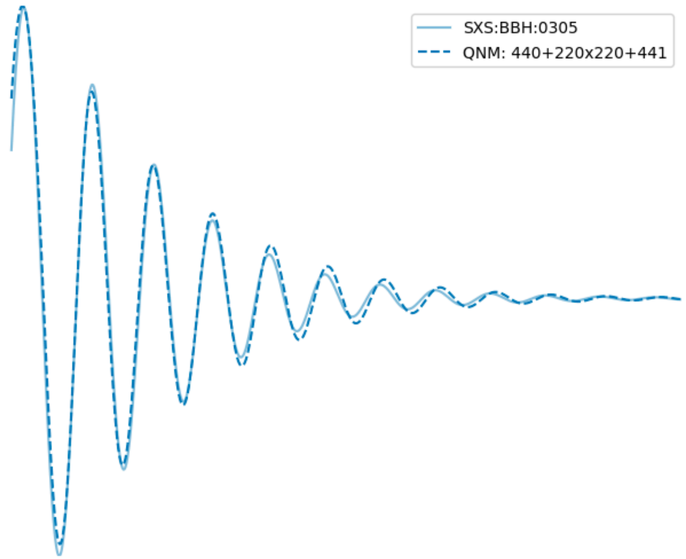
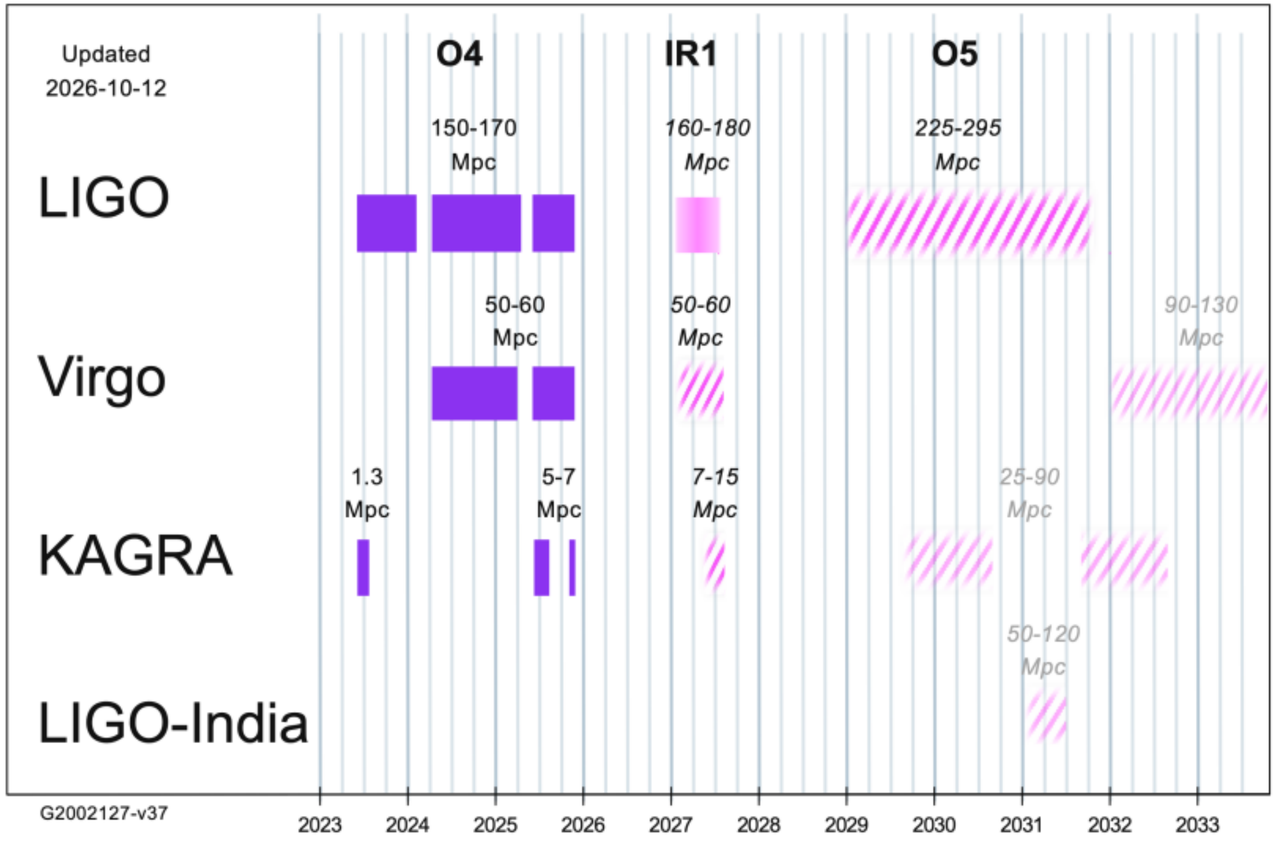
Black Hole Spectroscopy: A Nonlinear Voice From GW250114 Ringdown

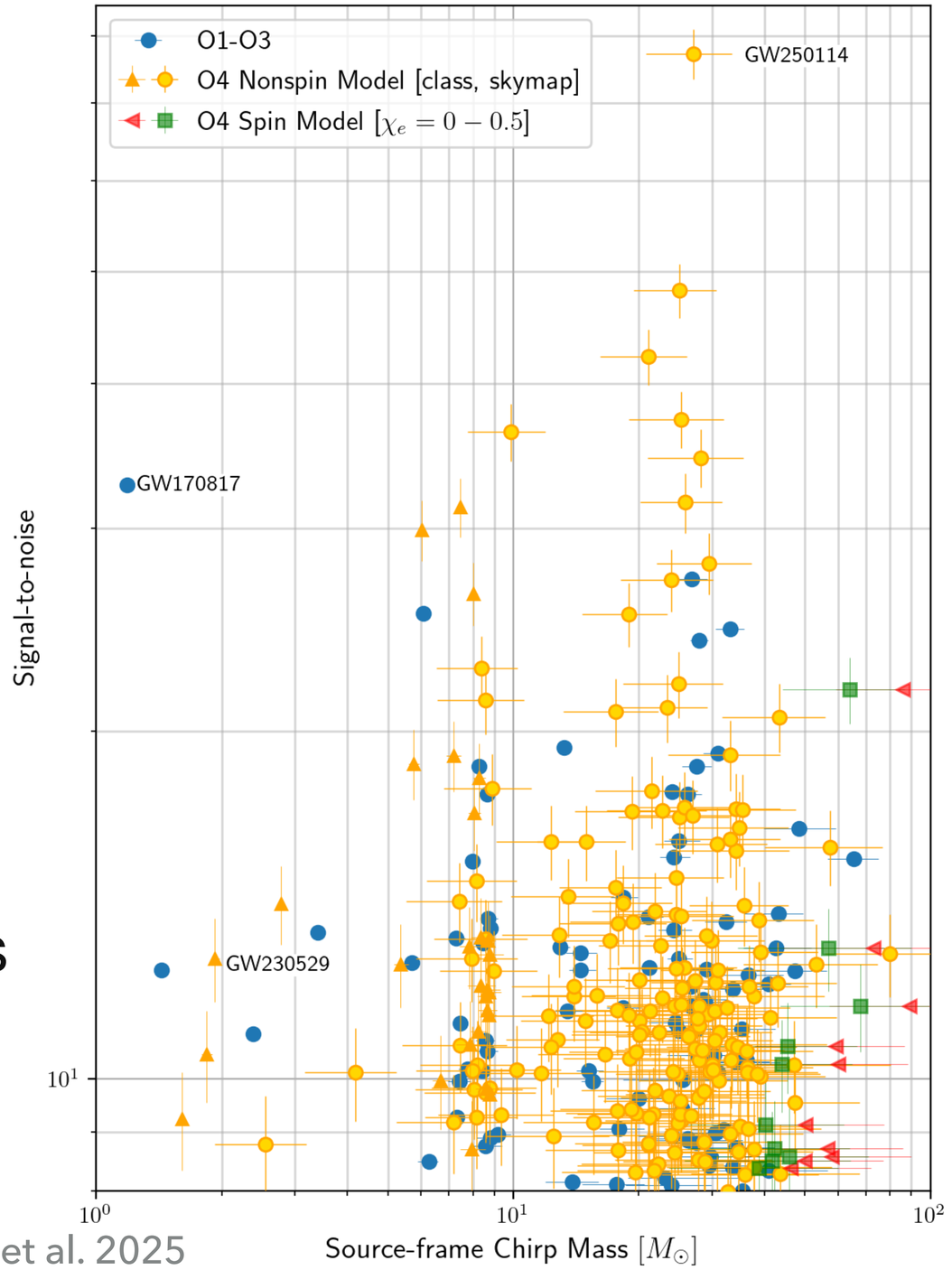


Yifan Wang

Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing

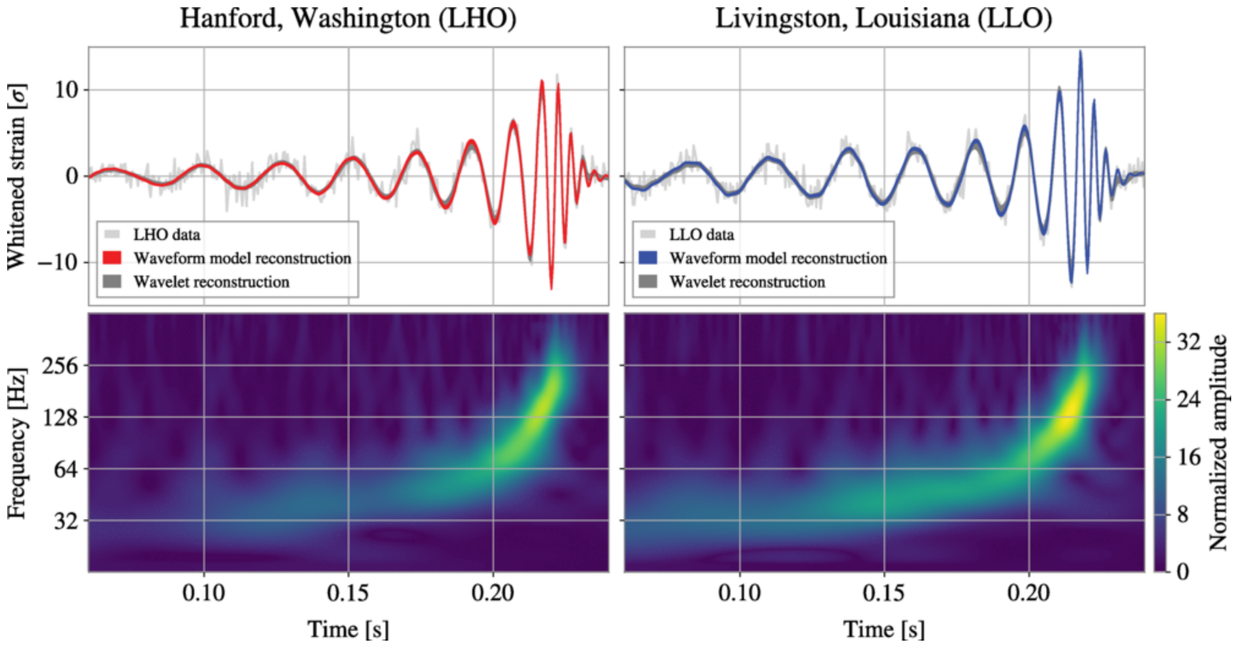
2026.11.07 @ Hangzhou



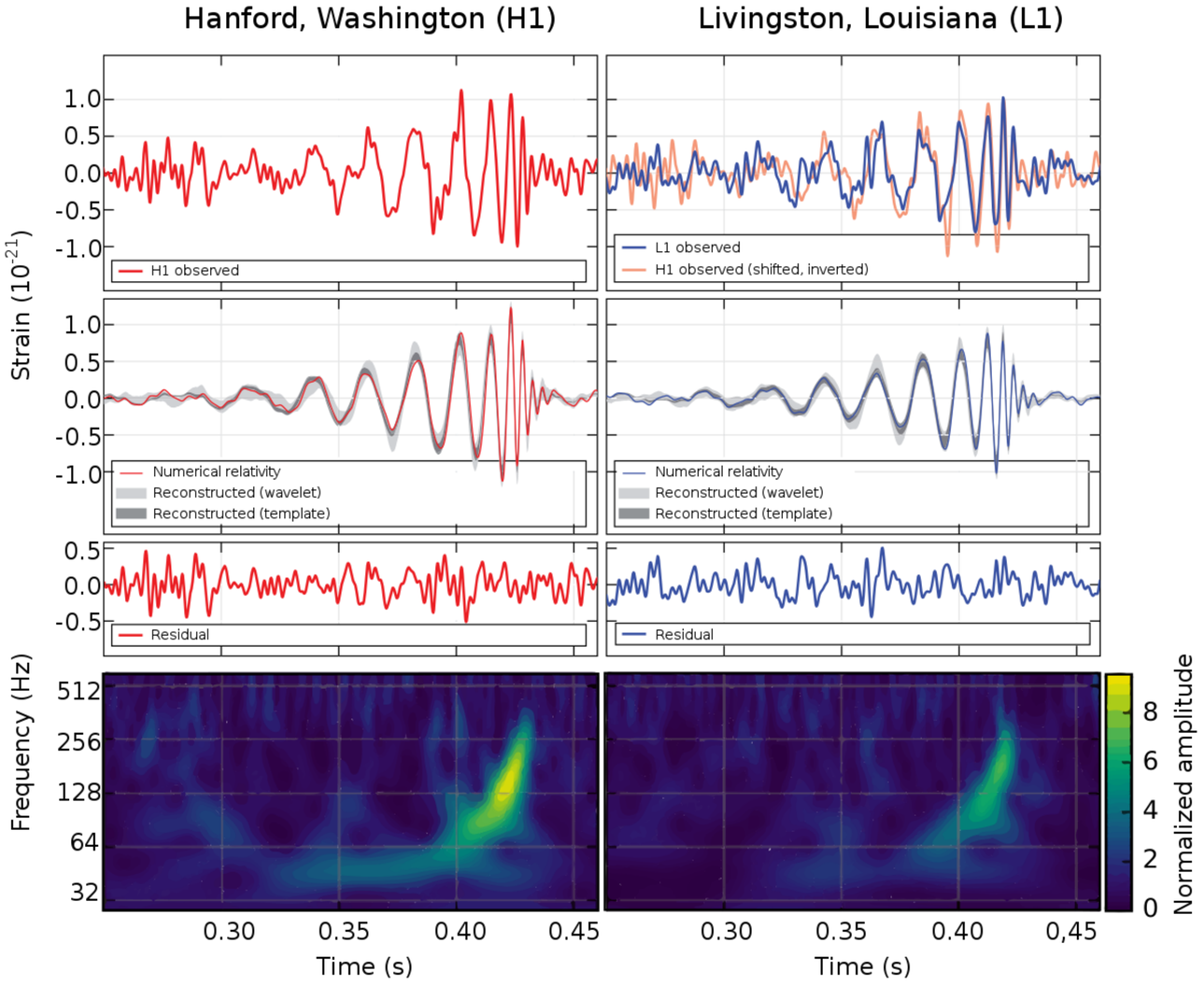


More than 300 Gravitational Wave Signals

Akyuz et al. 2025

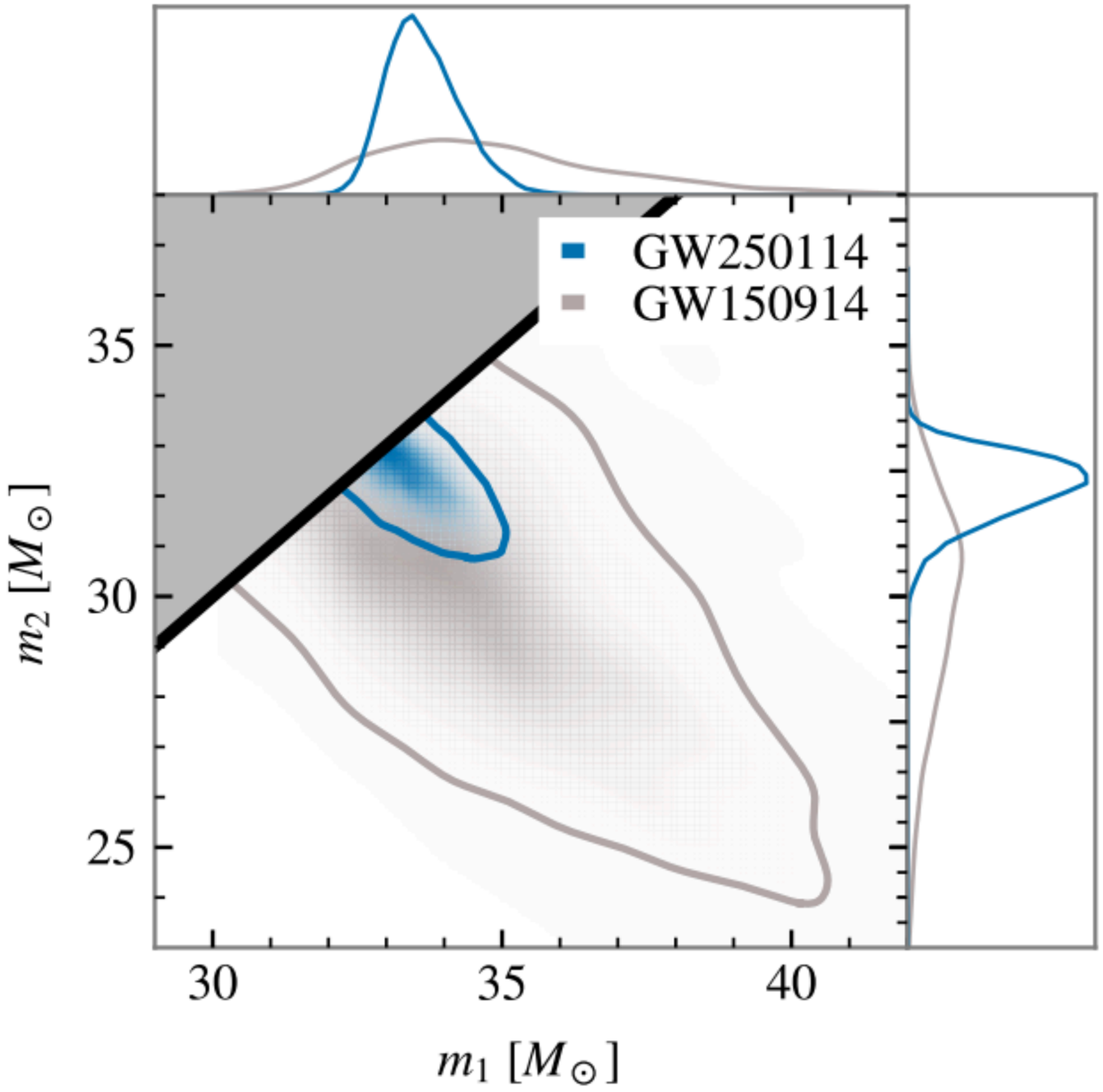


GW250114



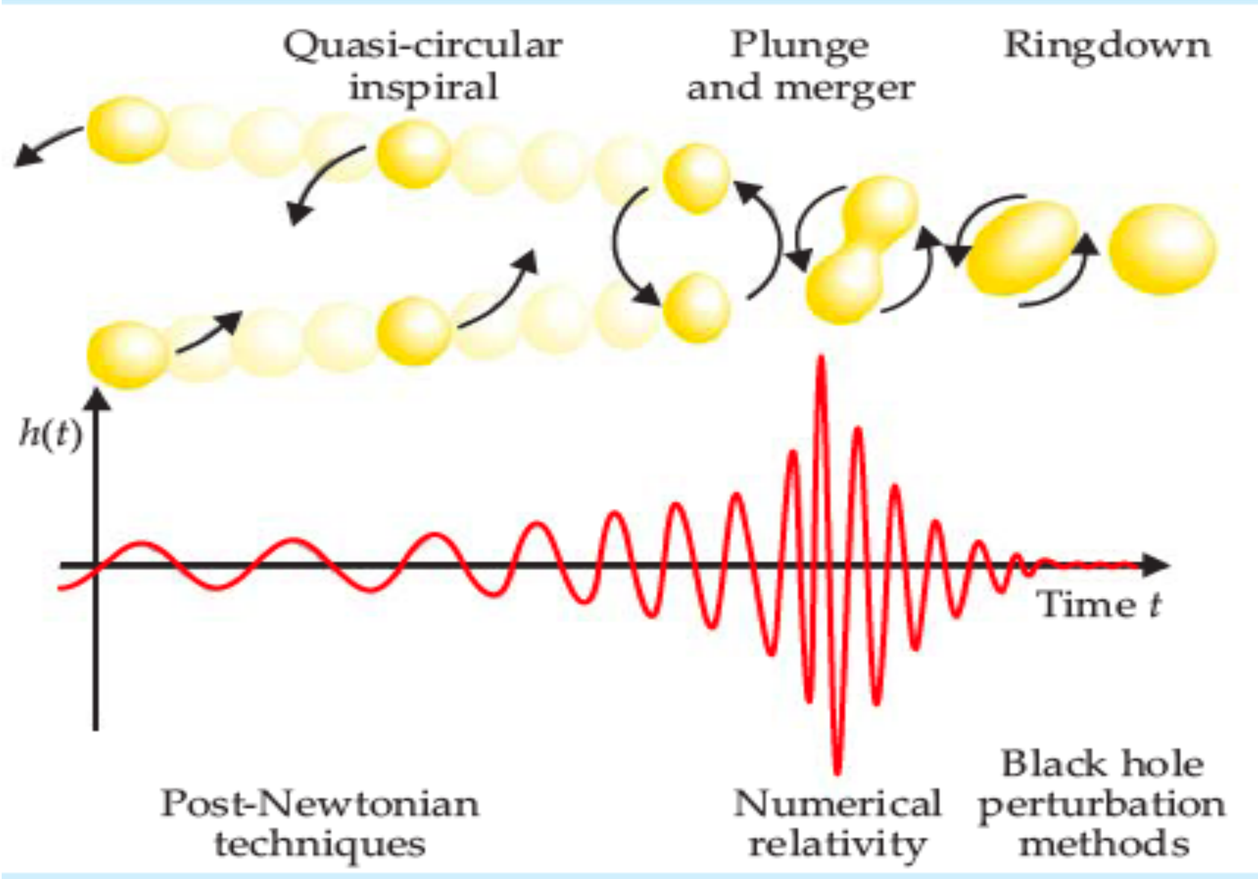
GW150914

More accurate parameter estimation



A golden opportunity for Black Hole Ringdown

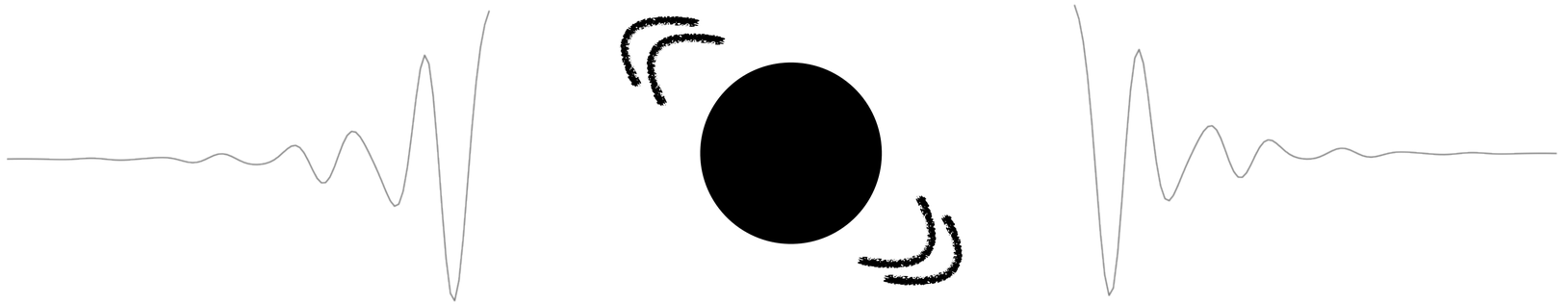
- A lot of SNR in black hole ringdown = the entire GW150914's SNR



LVK, GW250114 TGR paper, arXiv: 2112.06878

Black Hole Perturbation

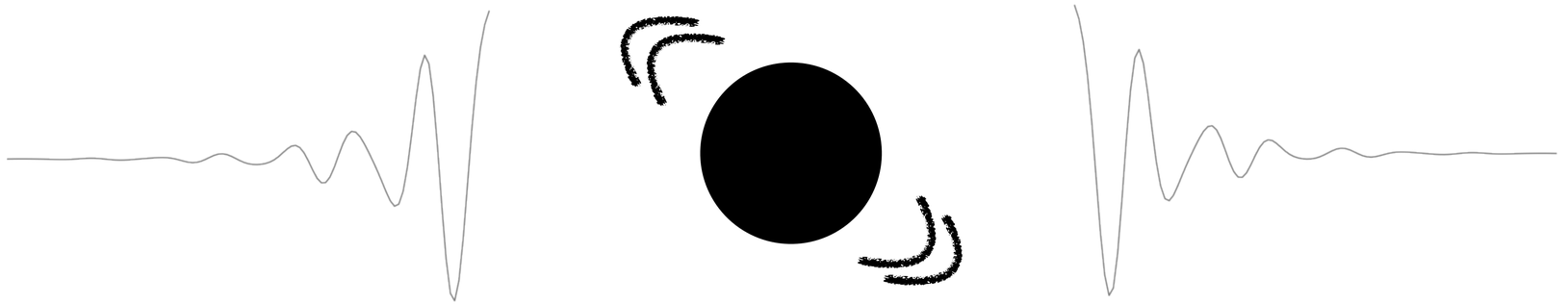
- $G_{\mu\nu} = 0$ --> $g_{\mu\nu} = g^{\text{Kerr}}_{\mu\nu} + h_{\mu\nu}$
- Boundary conditions: *ingoing at the horizon* and *outgoing at the infinity*
- **Teukolsky equation:** $[\partial_t^2 - \partial_{r_*}^2 + V(r)]\psi_{lm} = 0$



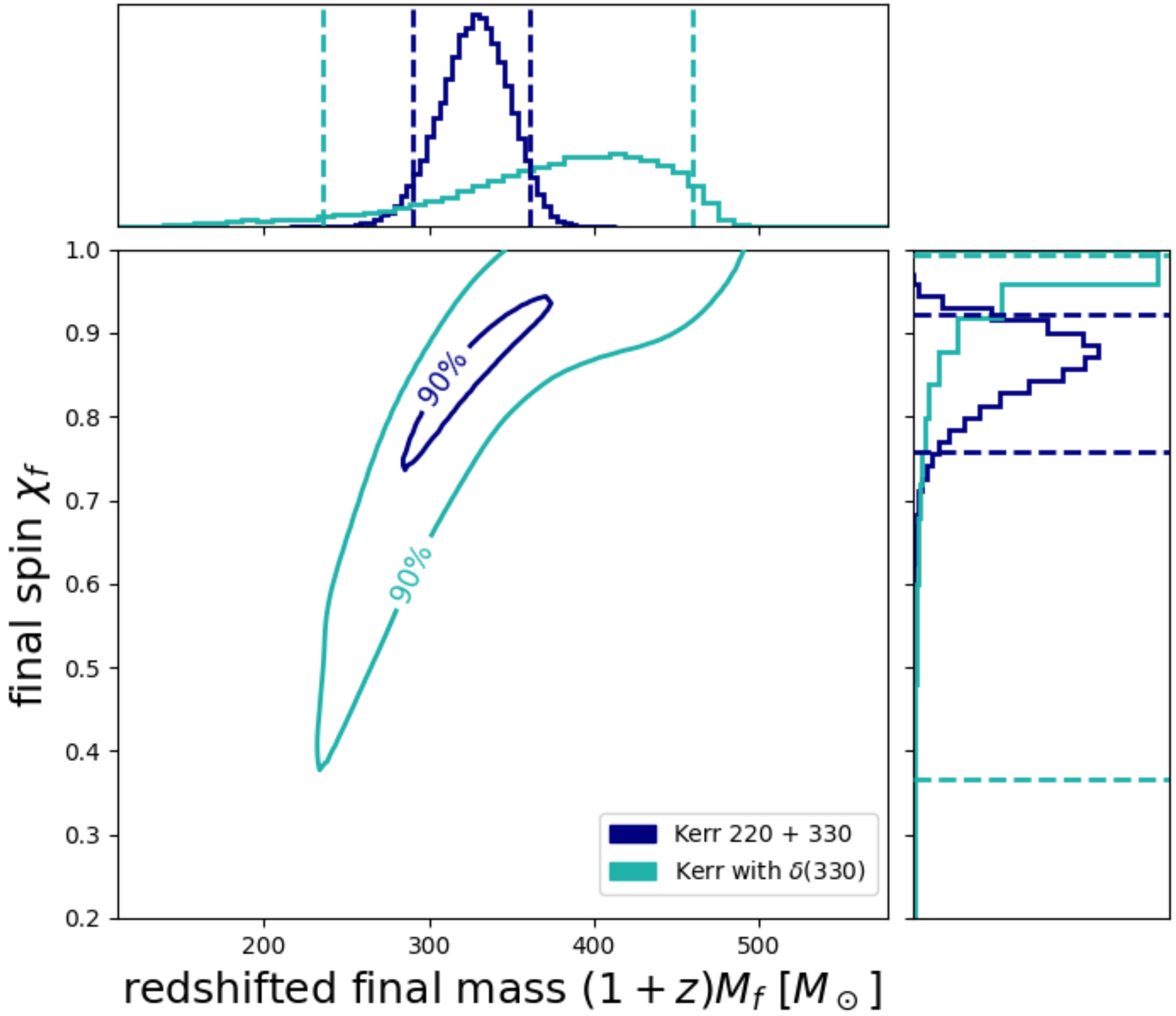
Introduction to Black Hole Ringdown

- Quasi-normal modes: $h_+ + ih_\times = \frac{M_f}{D_L} \sum_{\ell mn} {}_{-2}S_{\ell mn}(\iota, \varphi, \chi_f) A_{\ell mn} e^{i(\Omega_{\ell mn} t + \phi_{\ell mn})}$
  - ℓ, m: angular mode numbers
  - n=0: fundamental mode, n>0: overtones
  - $(\ell, m) \neq (2,2)$: higher angular modes

- Black Hole Spectroscopy: Ω_{lmn} uniquely determined by mass and spin



- Mass and spin from Ω_{lmn}



Capano et al. (including YFW) 2021

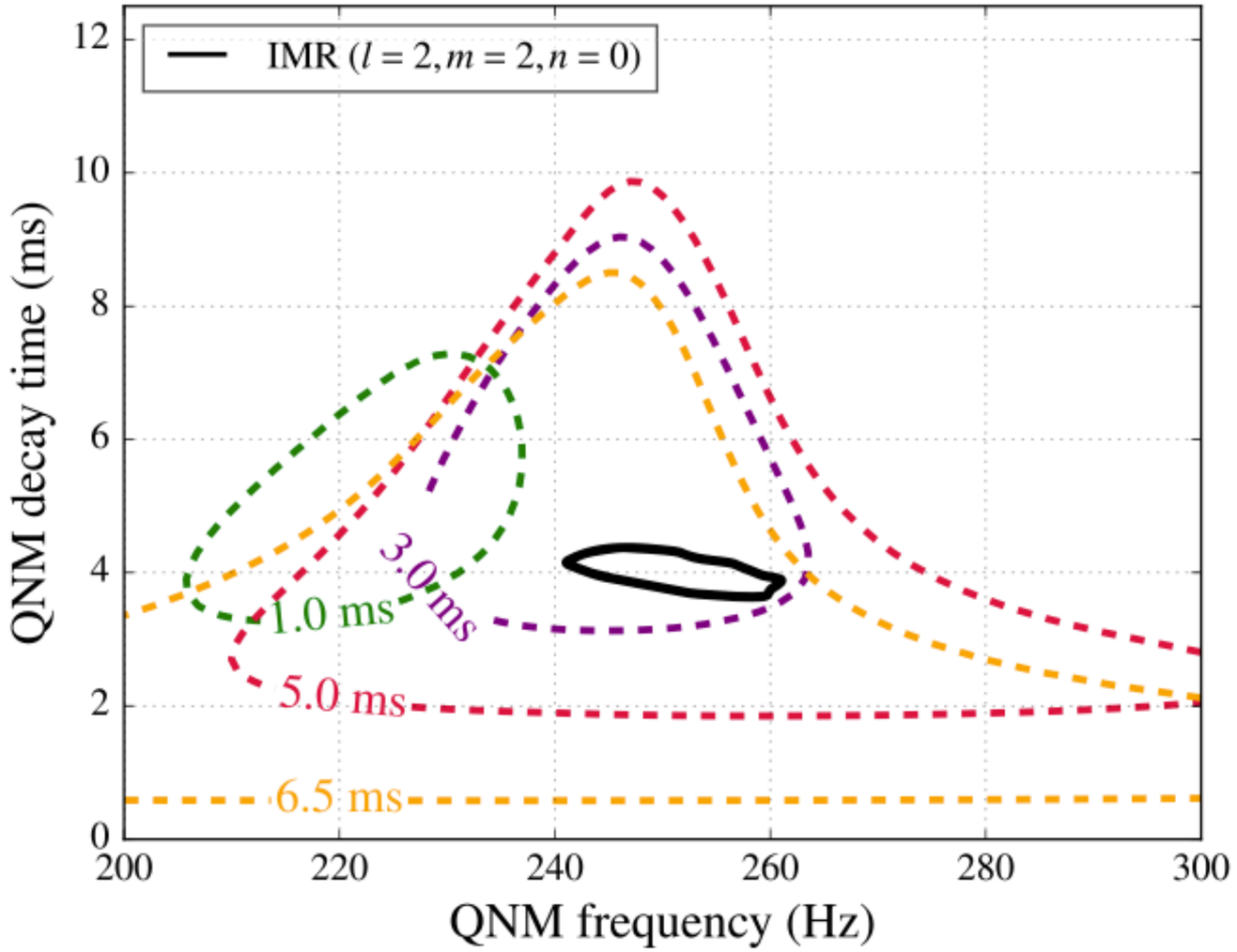
Technical Hurdles in Data Analysis

- Starting time?
  - Too early: contamination, not perturbation
  - Too late: barely any signals

- Ringdown Amplitude?
  - Introducing two free parameters per mode
  - Paradox: the more SNR the modes needed

Brief overview of observational evidences for BH ringdown

- LVK GW150914 ringdown analysis:

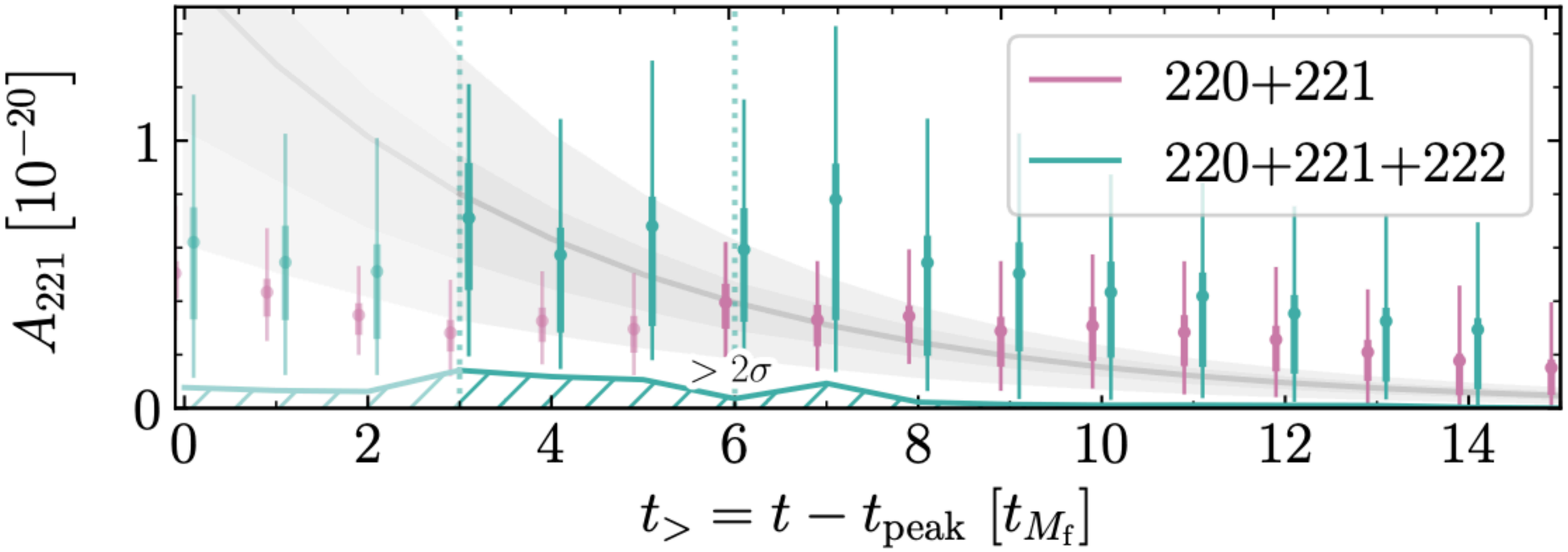


- When is a proper starting time for ringdown?

arXiv:1602.03841

GW250114

- LVK work: 220 and its first overtone 221 are clearly identified, no more modes are identified

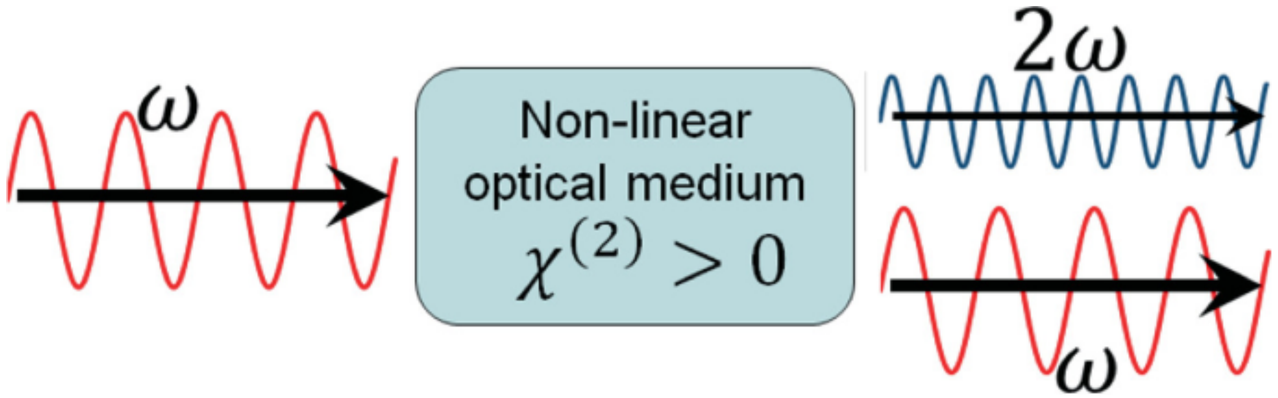


arXiv: 2509.08099, LVK, GW250114 testingGR paper

Second-order black hole perturbation

- Expand the spacetime metric: $g_{\mu\nu} = \bar{g}_{\mu\nu} + \epsilon h^{(1)}_{\mu\nu} + \epsilon^2 h^{(2)}_{\mu\nu}$
- Up to second order, Einstein equation reads: $G^{(1)}_{\mu\nu}[h^{(1)}] = 0$, and $G^{(2)}_{\mu\nu}[h^{(2)}] = -G^{(1)}_{\mu\nu}[h^{(1)}, h^{(1)}]$, where $G^{(1)}$ and $G^{(2)}$ are Einstein tensor's linear and bilinear parts, respectively

Nonlinear Optics

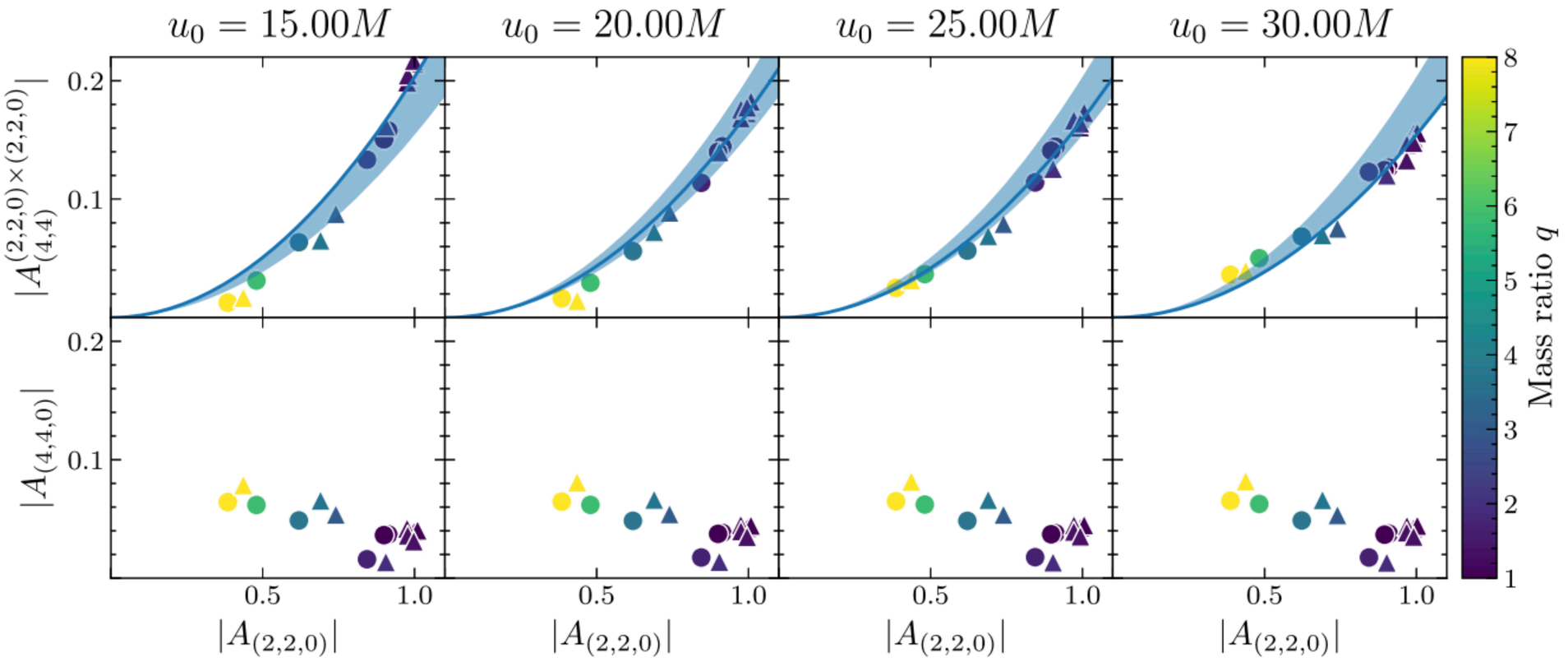


A high-intensity laser beam (e.g., 1064 nm infrared) passes through a crystal with a nonlinear susceptibility, generating new light at exactly twice the original frequency

Su, Mu & Li 2024

Quadratic Quasi-normal modes

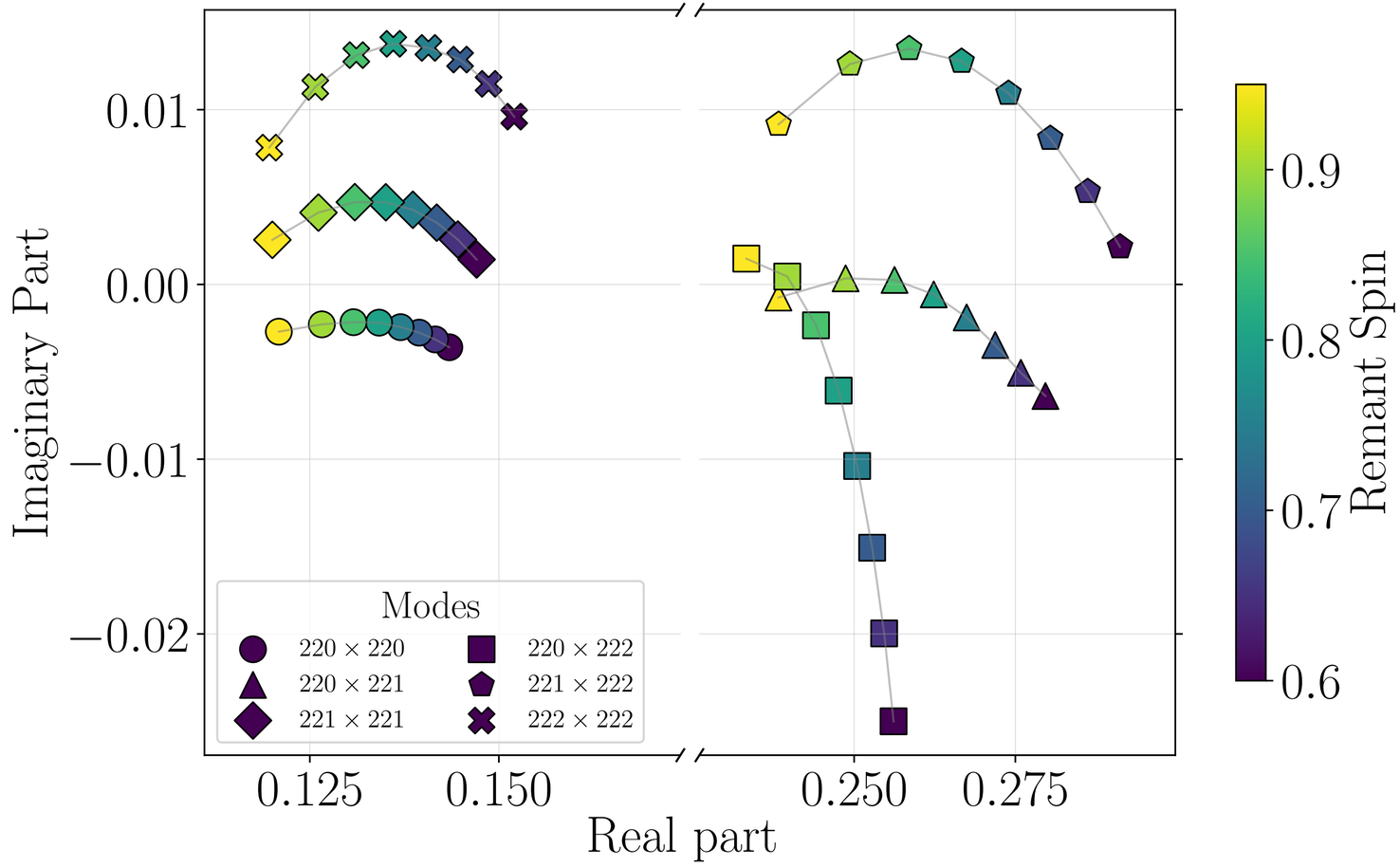
- Evidences of quadratic modes are found in numerical relativity simulations
- (2,2,0) × (2,2,0) mode



Mitman 2023, Cheung 2023, Khera 2023

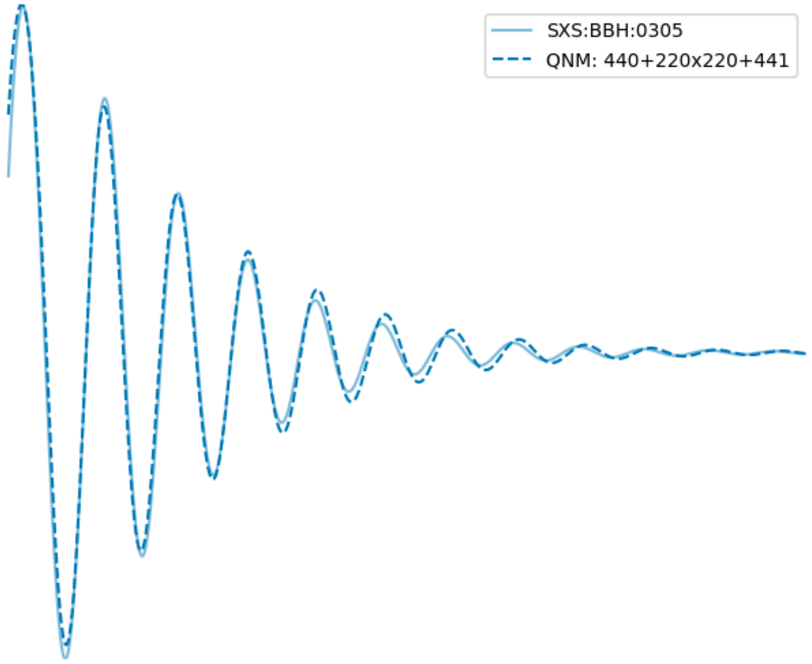
Theoretical Progress on quadratic mode coupling ratio

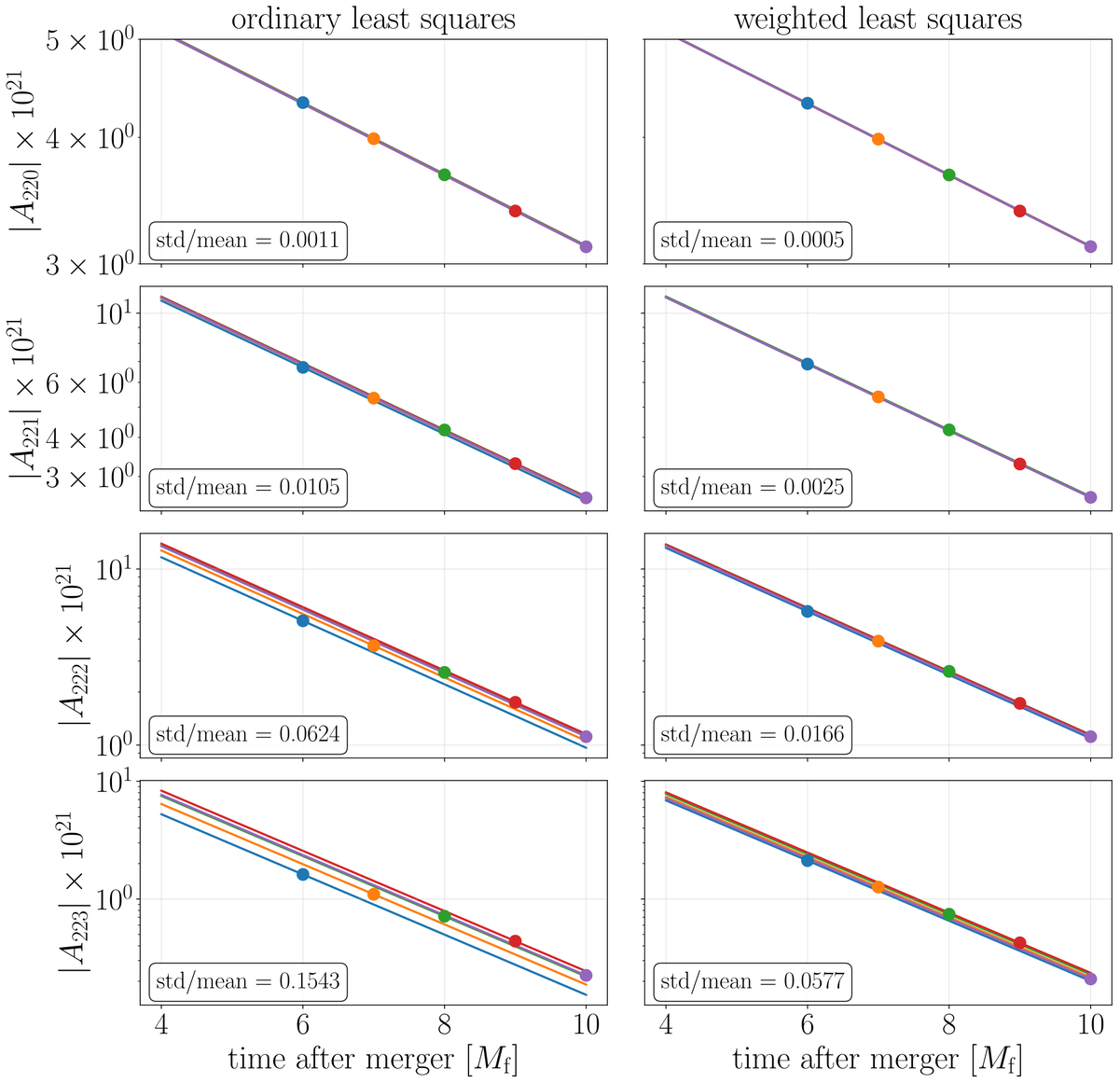
- Recently, a theoretical progress was made by Kehra et al. (PRL, 2410.14529) by solving Teukolsky equation for second order perturbation
- $$\sum_{n,m} rA_{22n\times 22m} e^{-i\omega_{22n\times 22m}t} = \sum_{n,m} \mathcal{R}_{22n\times 22m}(a_f) \times rA_{22n} \times rA_{22m} e^{-i\omega_{22n}t} e^{-i\omega_{22m}t}$$



Infer the Ringdown Amplitudes with Weighted Least Square Fits

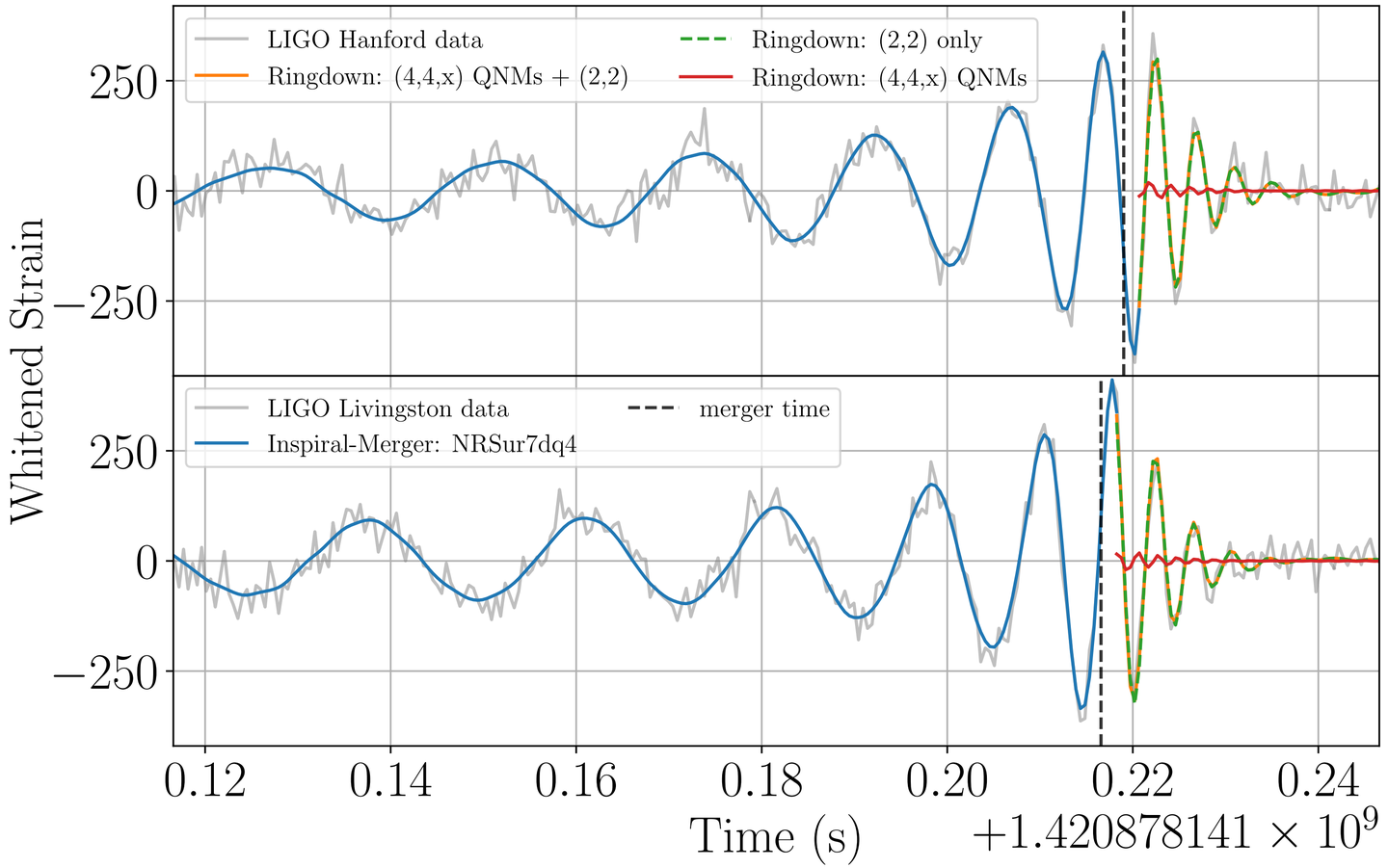
$$\widehat{\boldsymbol{A}} = \arg\min_{\boldsymbol{A}} \sum_j \frac{\left| h(t_j) - \sum_n A_n e^{-i\omega_n (t_j - t_{\rm ref})} \right|^2}{\epsilon^2 + \sigma^2(t_j)} .$$





Reinforce the Ringdown Inference with Pre-merger

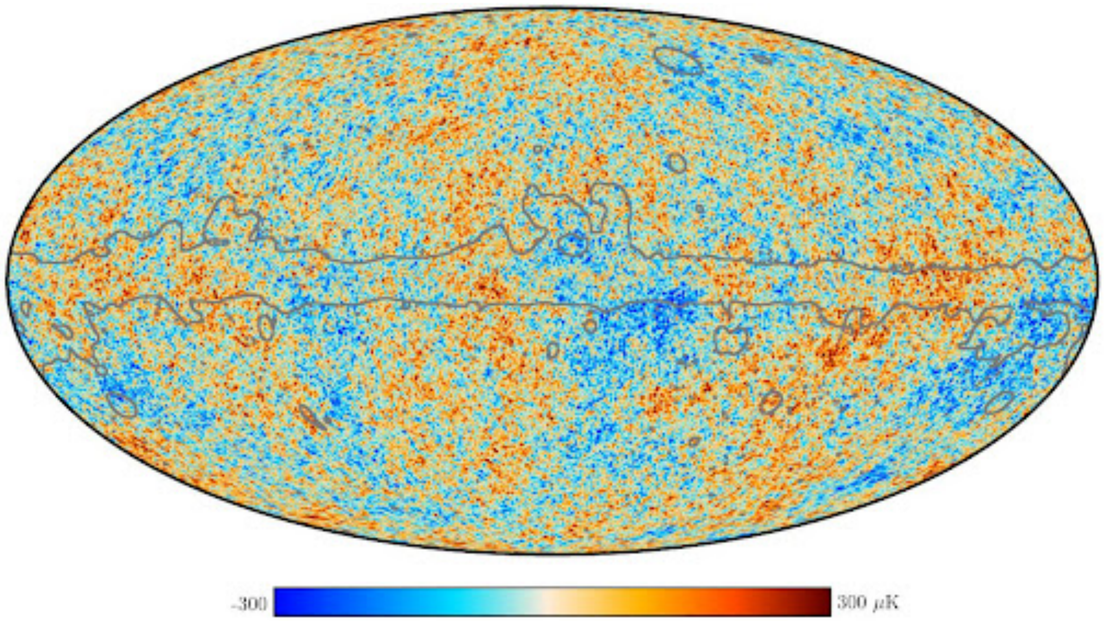
- $\mathscr{L}(\{d_{\rm IM}, d_{\rm R}\}\,|\,\vartheta) = \mathscr{L}_{\rm IM}(d_{\rm IM}\,|\,\vartheta) \times \mathscr{L}_{\rm R}(d_{\rm R}\,|\,\vartheta)$



Gating and inpainting

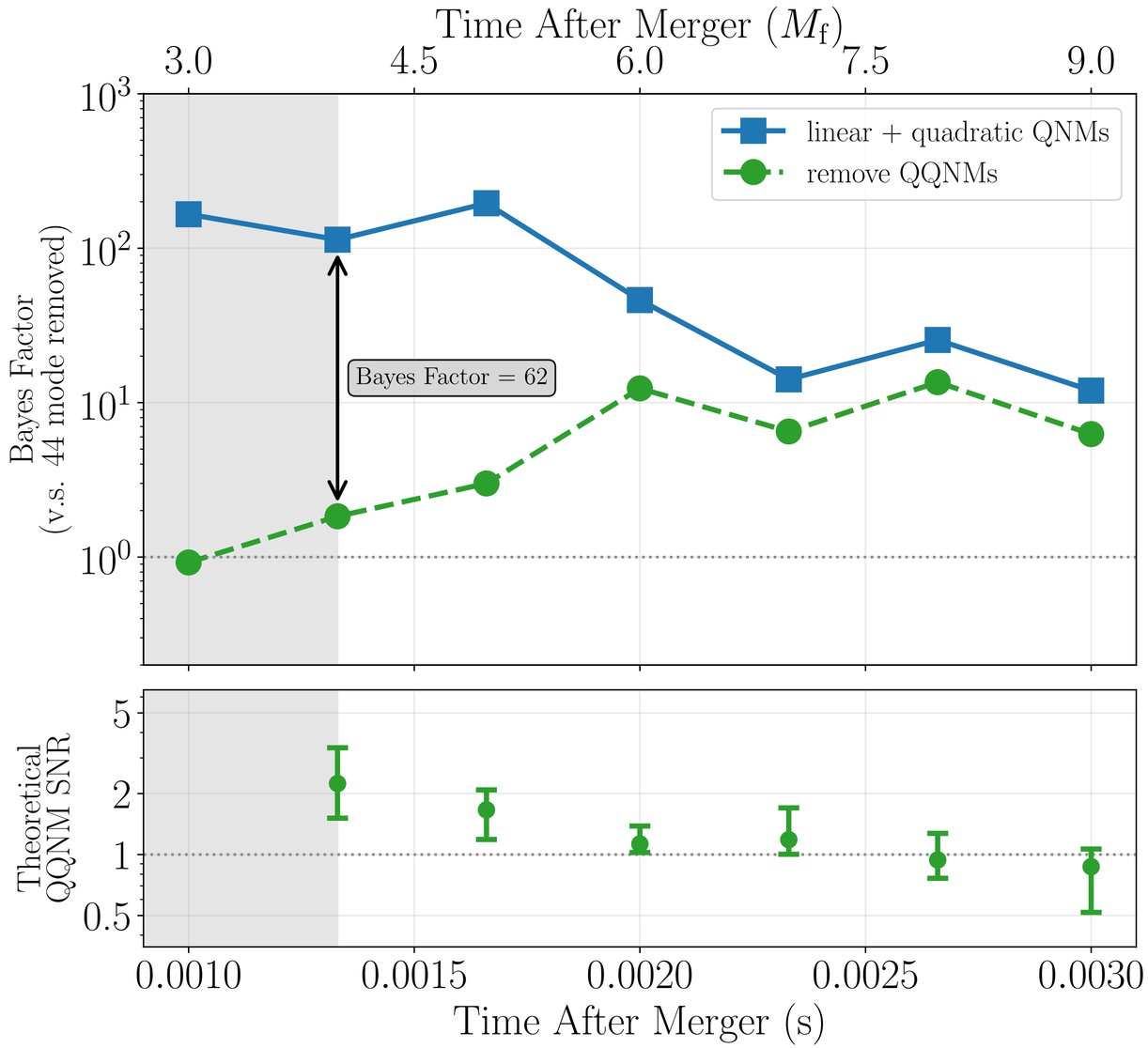


"Digital Image Restoration" (from wikipedia)



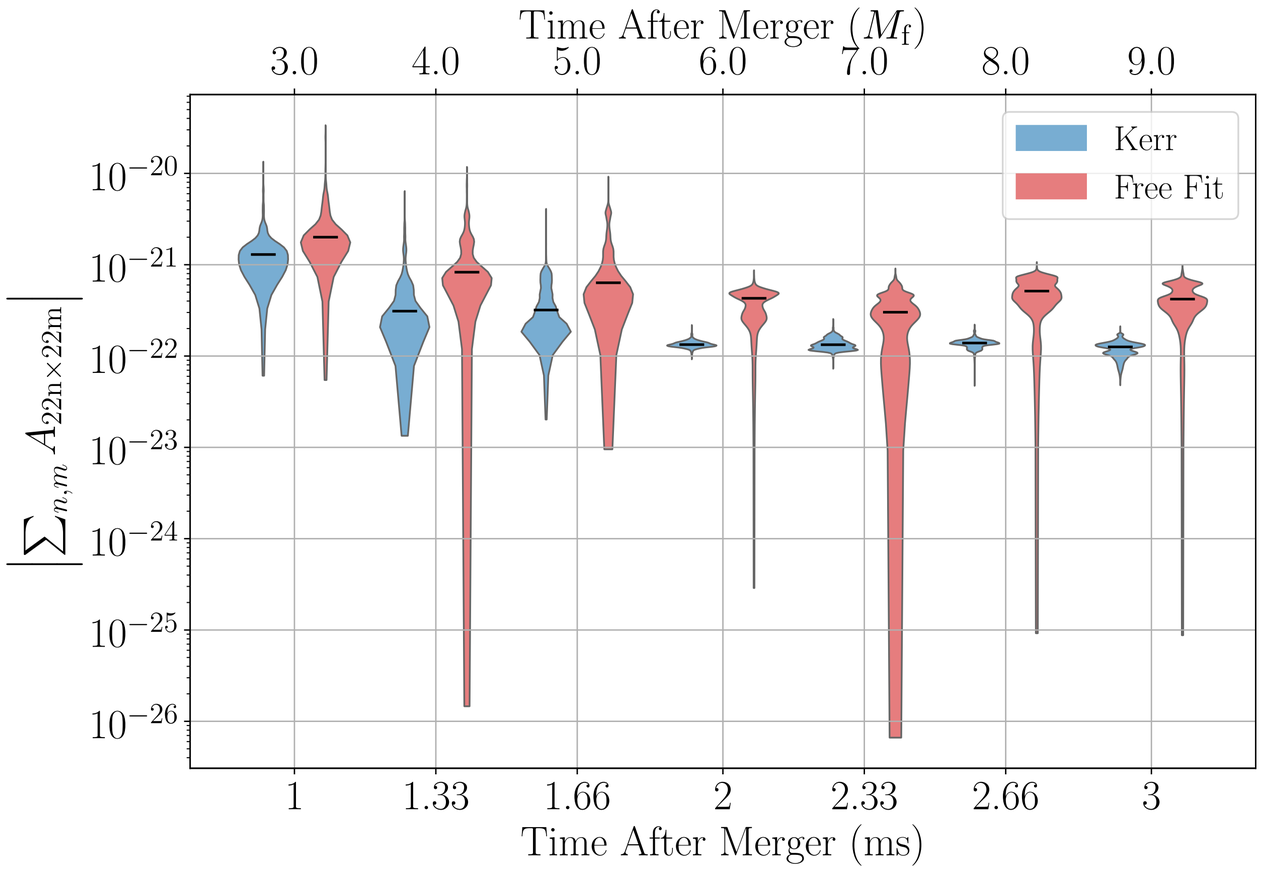
Another example: CMB inpainting

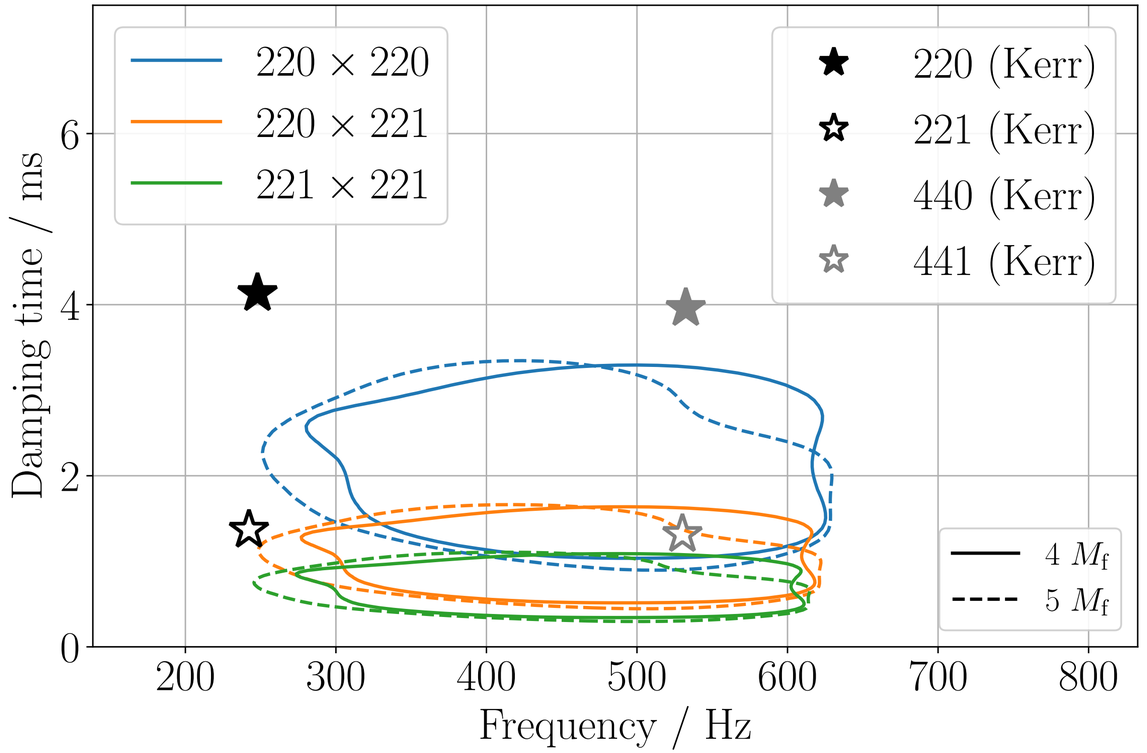
Observational Evidence for QQNMs



- Consider a bundle of 10 QQNMs, sourced by (2,2,0) through (2,2,3)

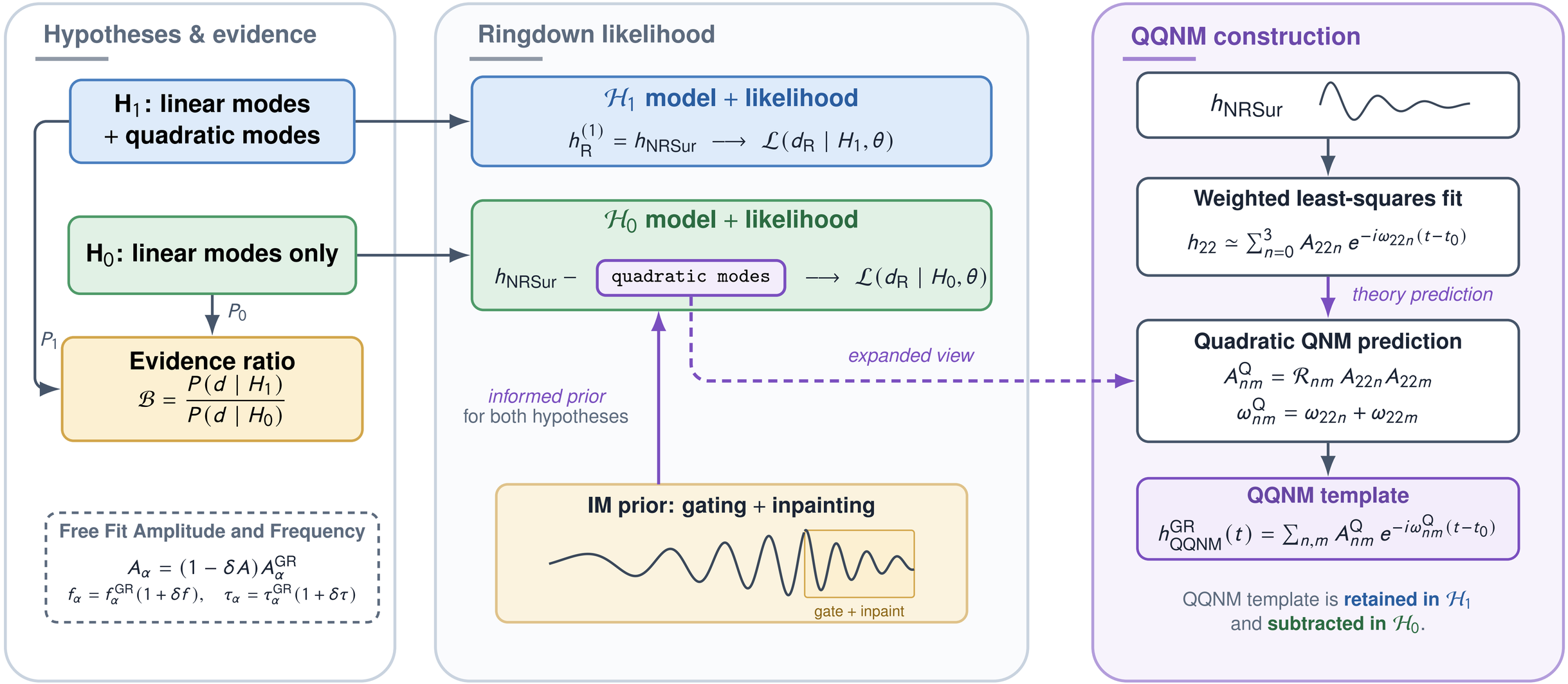
Testing General Relativity





$$A_\alpha \to (1+\delta)A_\alpha$$

$$f_\alpha \to (1+\delta)f_\alpha$$



- https://github.com/yi-fan-wang/TestingGR_with_Gravwaves
- **pip install tgr**

PyCBC

Summary: how to dissect GWs?

- We probed a new phenomenon in black hole ringdown, nonlinearity, using GW250114 due to its high SNR
- As the detection of high SNR events becomes increasingly common, ringdown analysis faces a general challenge of isolating individual contributions within a set of signals, particularly at early times close to the merger, when multiple components coexist without a clear hierarchy of dominance.
  - non-modal signals (direct wave, Lu 2025)
  - linear modes (fundamental + overtone, Giesler 2021)
  - nonlinear modes (quadratic, Mitman 2023, Cheung 2023, Khera 2023) , cubic?

$$\text{Null Hypothesis: } H_0 : d = n + \sum_{i=1}^{N-1} h_i$$

$$\text{Alternative Hypothesis: } H_1 : d = n + \sum_{i=1}^{N} h_i$$

Softwares

- PyCBC: https://github.com/gwastro/pycbc
  - Core package to analyze gravitational-wave data, find signals, and study their parameters

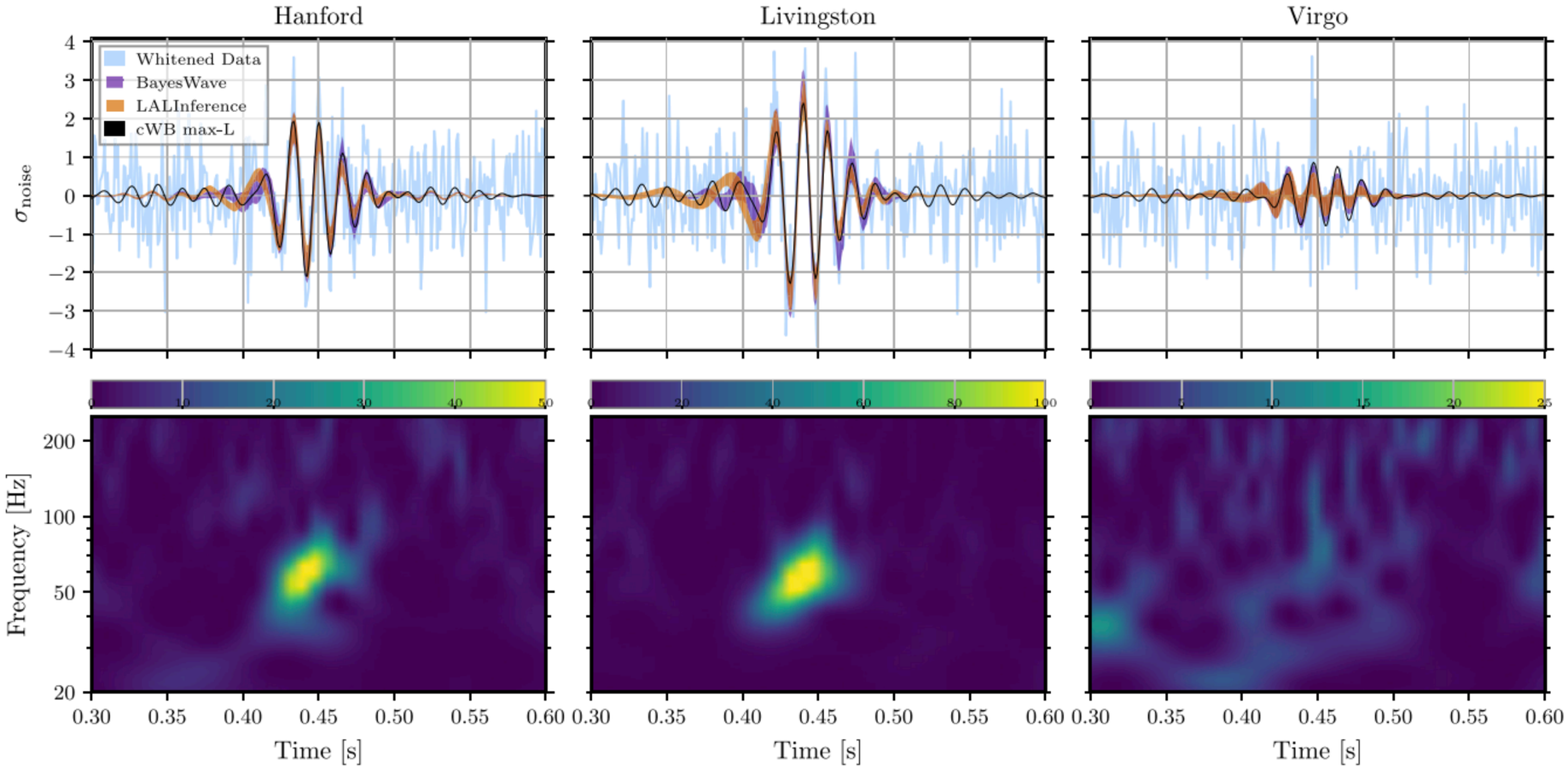
- PyTGR: https://github.com/yi-fan-wang/TestingGR_with_Gravwaves
  - A waveform plugin, removing (4,4) mode in NRSur and replacing with Quadratic modes, QNM fitting

- Data release for GW150914 overtone studies, Wang et al. PRD
  - https://github.com/gwastro/gw150914-overtone

- Data release for GW190521 (3,3,0) mode detection, Capano et al. (including YW) PRL
  - https://github.com/gwastro/BH-Spectroscopy-GW190521

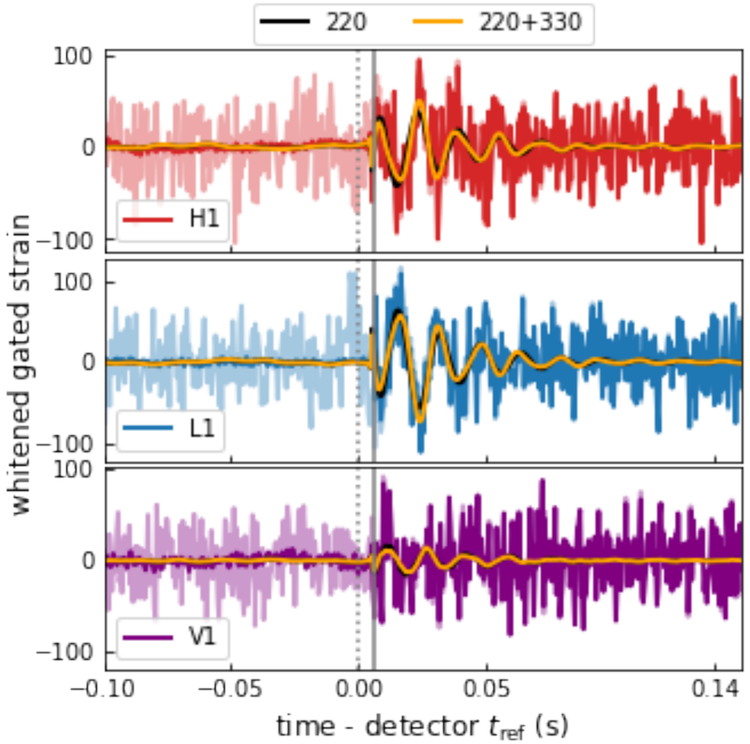
BACKUP SLIDES

GW190521



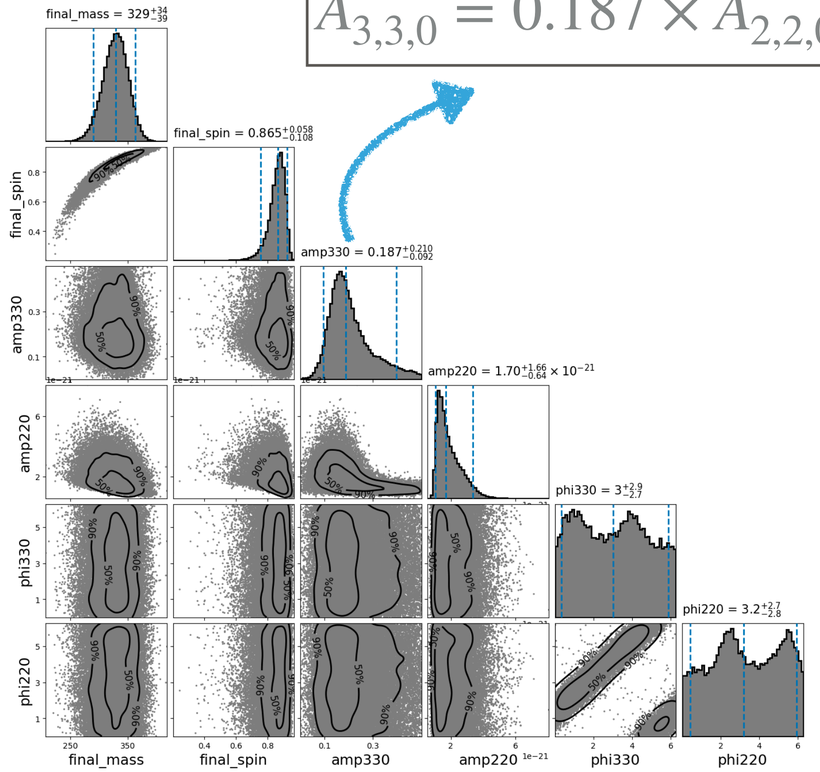
LVK collaboration, PRL, arXiv: 2009.01075

Analysis on GW190521

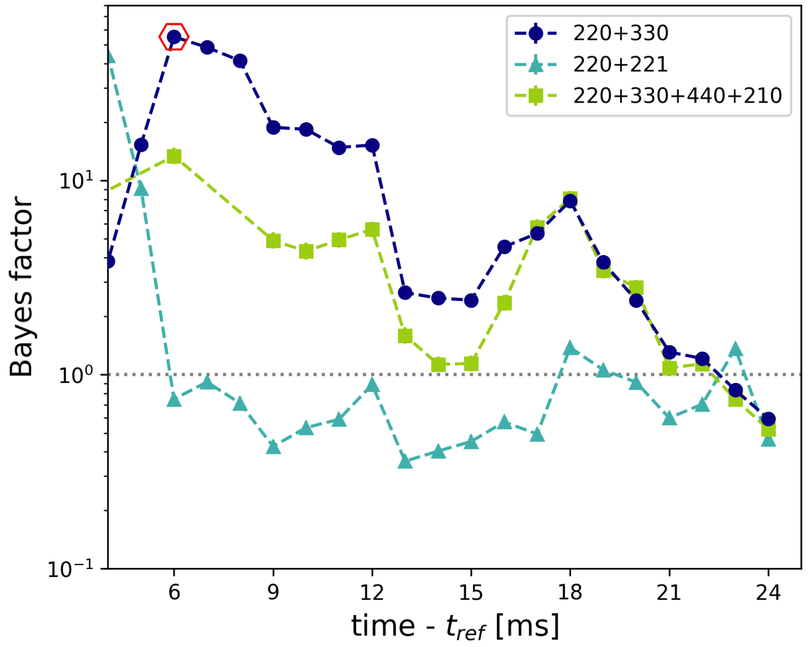


- Posterior:

$$A_{3,3,0} = 0.187 \times A_{2,2,0}$$



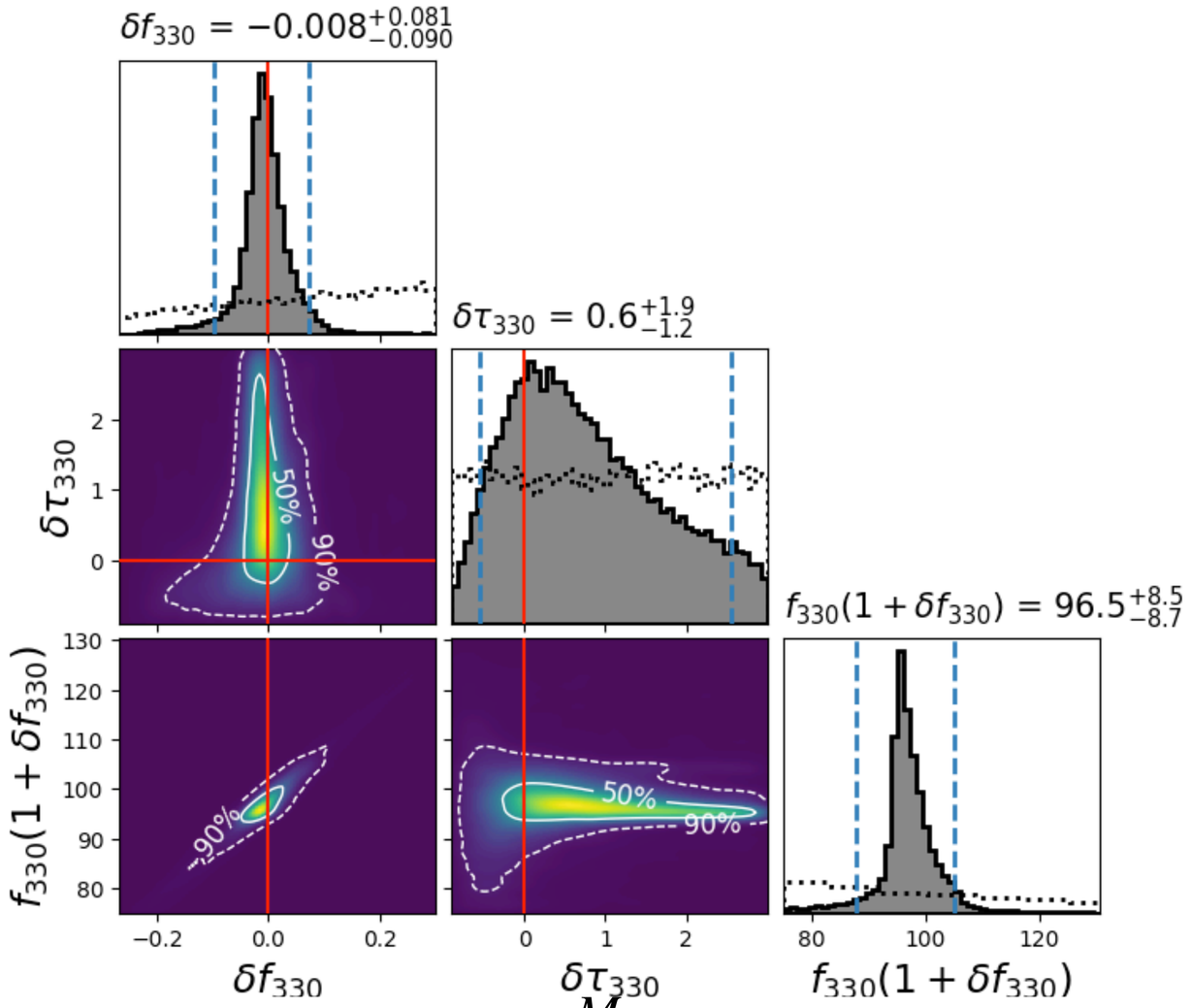
- Find the evidence of (3,3,0) with a Bayes Factor 56!



- Waveform model: $h_+ + ih_\times = \frac{M_f}{D_L} \sum_{\ell mn} {}_{-2}S_{\ell mn}(\iota, \varphi, \chi_f) A_{\ell mn} e^{i(\Omega_{\ell mn} t + \phi_{\ell mn})}$

Capano, ..., YW, et al, PRL, arXiv: 2105.05238

Analysis on GW190521: Testing GR

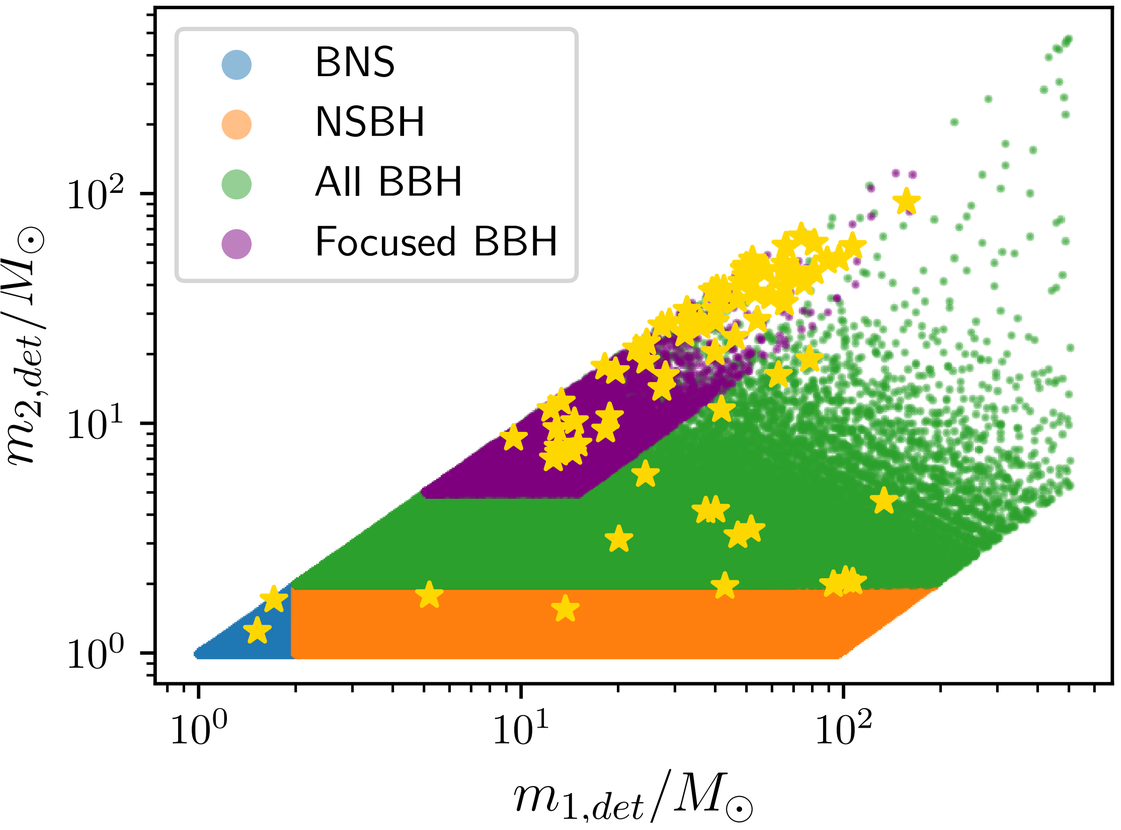


- Waveform model: $h_+ + ih_\times = \frac{M_f}{D_L}\sum_{\ell mn} {}_{-2}S_{\ell mn}(\iota,\varphi,\chi_f)A_{\ell mn}e^{i(\Omega_{\ell mn}t+\phi_{\ell mn})}$

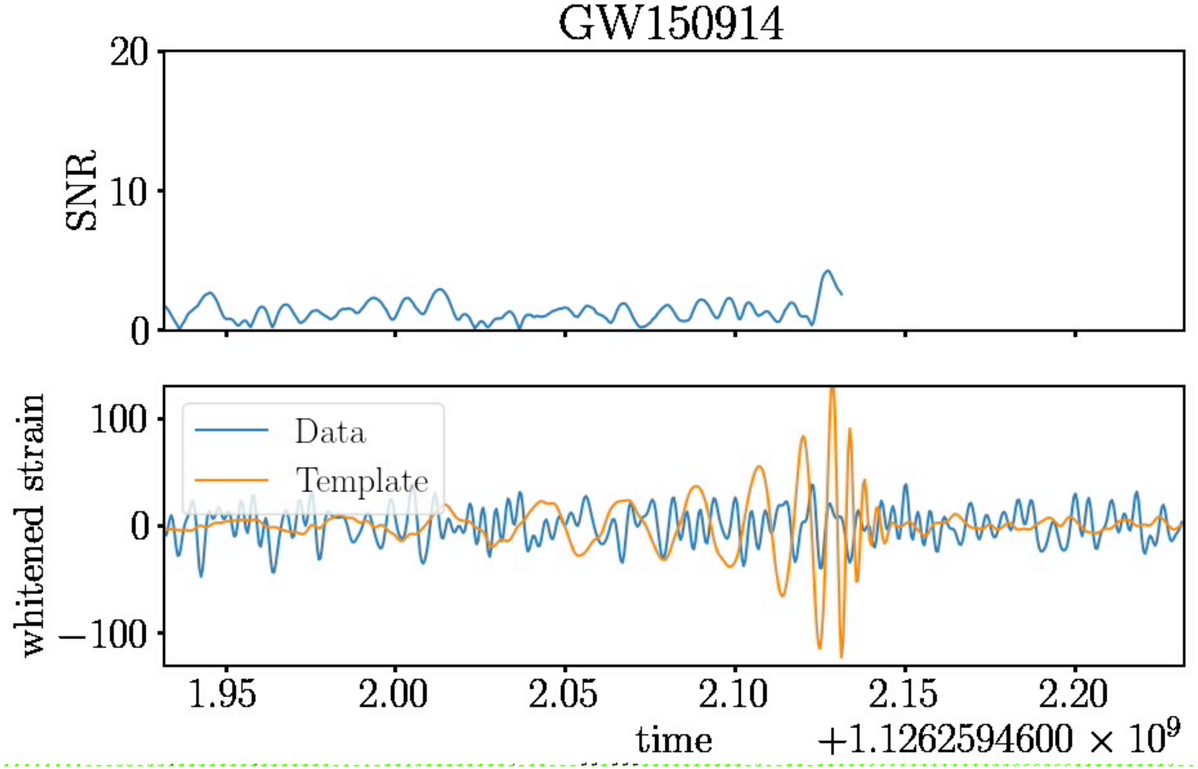
Capano, ..., YW, et al, PRL, arXiv: 2105.05238

Template bank

- The template bank used for OGC (Open Gravitational-wave Catalog), which identified 94 events, among which 14 events are detected for the first time



Nitz et al. (including YW), ApJ, arXiv: 2112.06878



- In a recent work, Wang & Nitz (ApJ, 2508.05018), we performed the first matched-filtering search for eccentric binary black holes using SEOBNRv5E