Modulated FOPT as the origin of large-scale curvature perturbations and gravitational wave anisotropies

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Based on: 2610.18975 (WA, Gabriele Franciolini, Xin Wang & Xiang-Xi Zeng)



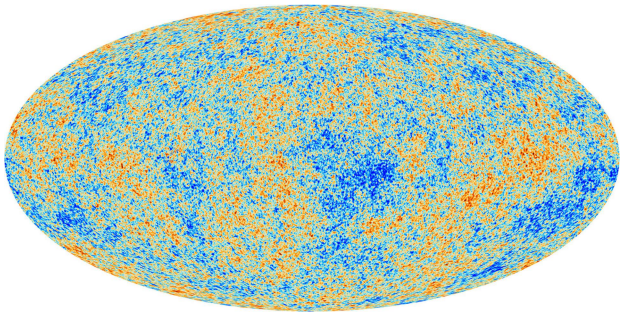
The 6th BRICS-AGAC Symposium 2026, Hangzhou, China

Background: Precision cosmology and GW era

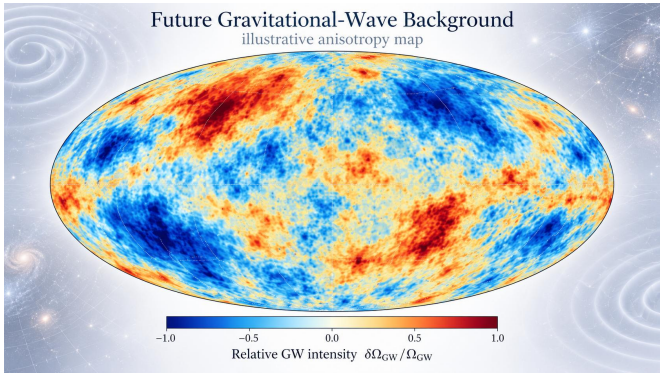
There must be new physics beyond the Standard Model of particle physics:

Dark Matter, Inflation, Hierarchy problem, Matter-anti-matter asymmetry…Where is it?

Energy frontier | Cosmic frontier | Intensity frontier



1965’ precision cosmology



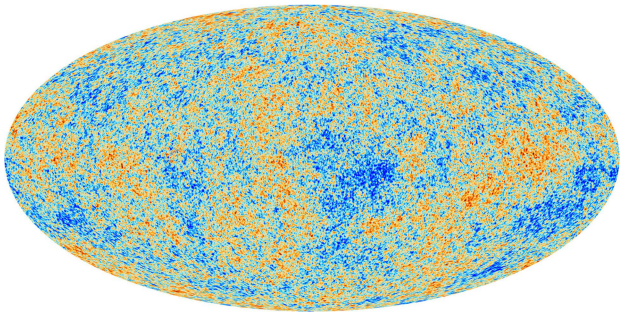
2015’ even more precision cosmology?

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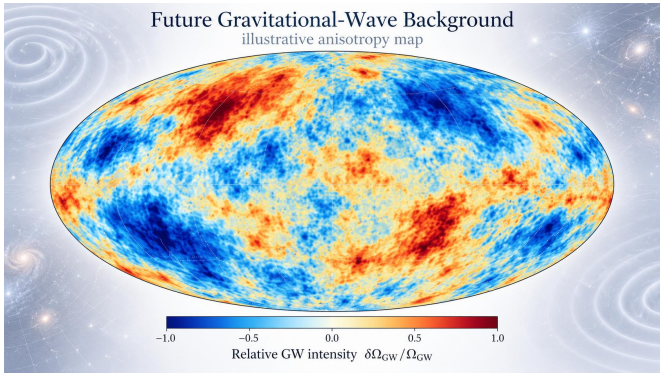
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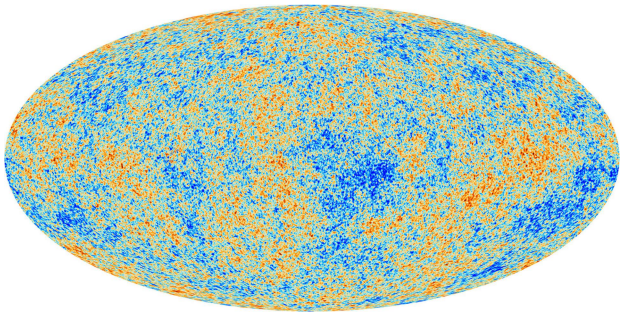
Question: Can we have a unified origin for CMB and a stochastic GW background?

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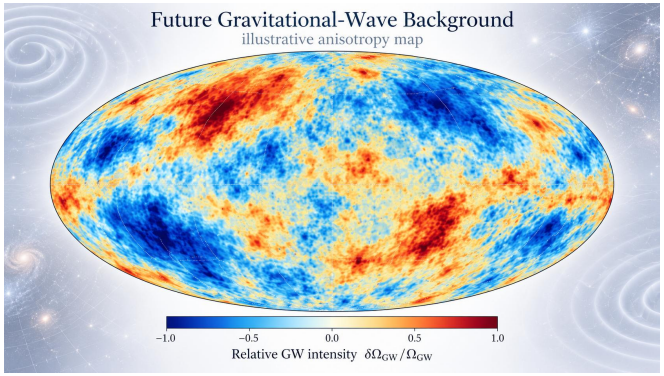
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⇒ Correlated signals!

We show that a first-order phase transition (FOPT) can do this!

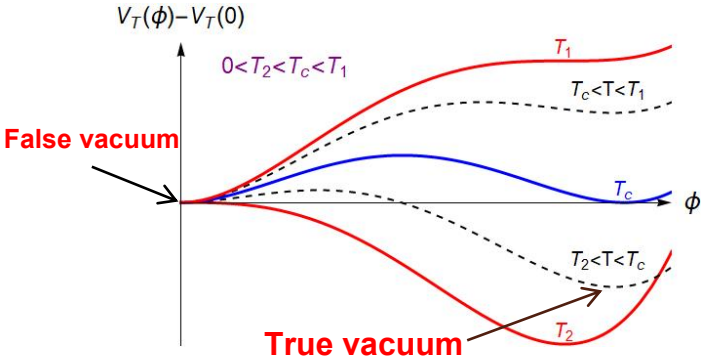
Outline

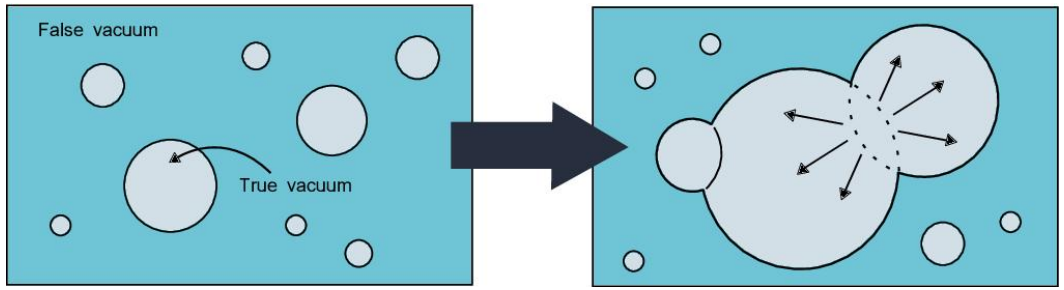
- What is a first-order phase transition?
- Small-scale curvature perturbations generated from a FOPT
- **Modulated FOPT**: large-scale curvature perturbations
- An example (particle-physics) model

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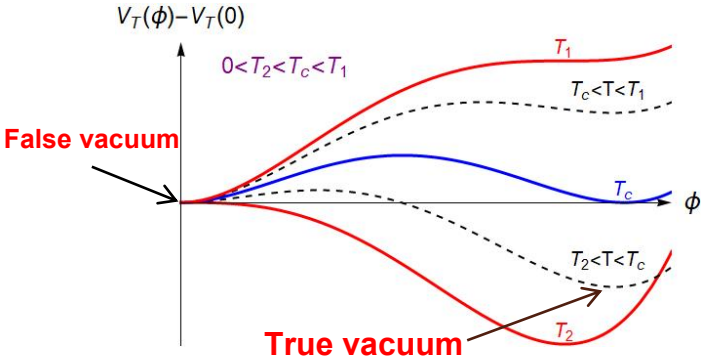
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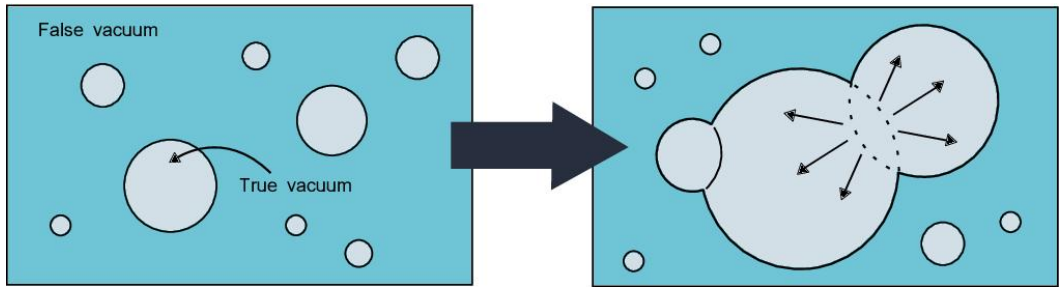




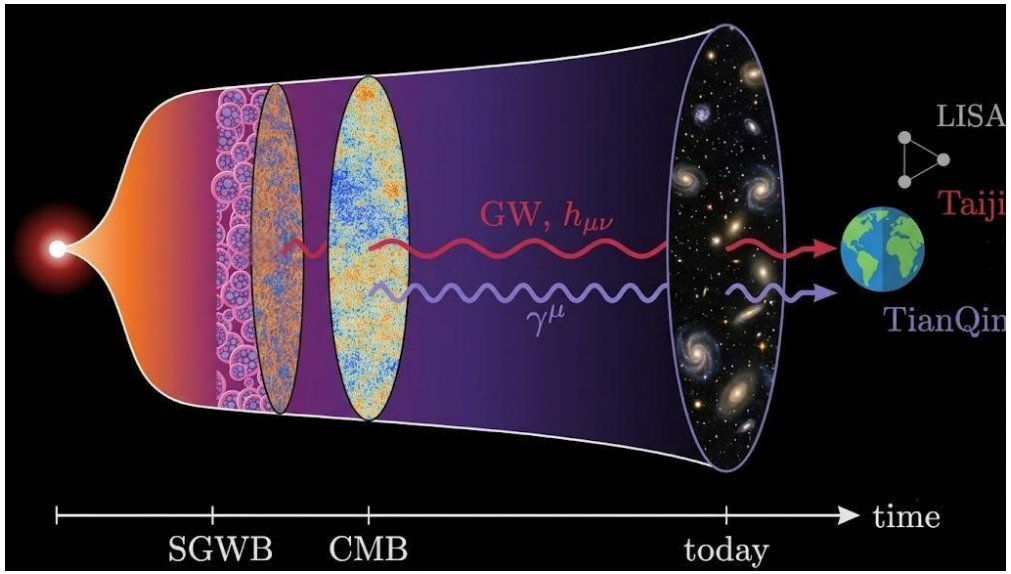
GWs generated

What is a first-order phase transition?





GWs generated



[adapted from Philipp Schicho's slide]

Bubble collision and percolation

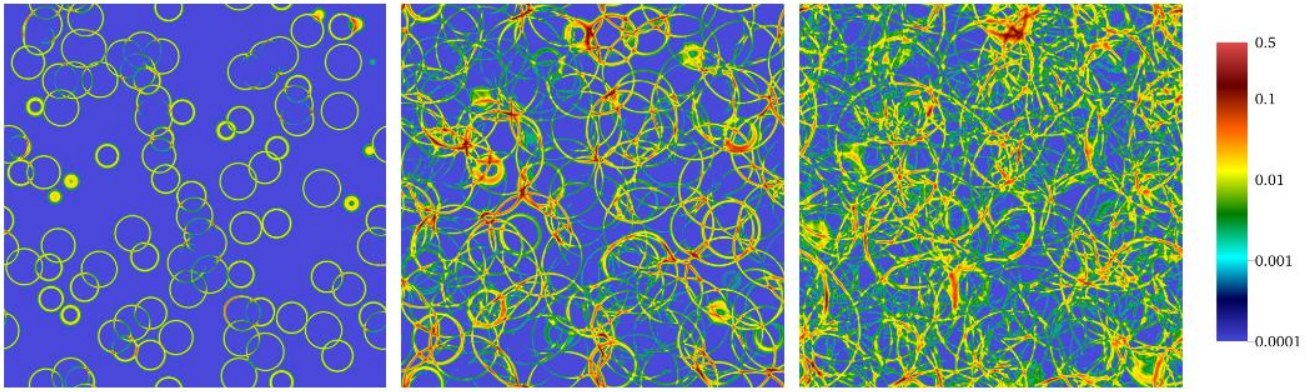


FIG. 4. Slices of fluid kinetic energy density E/T_c^4 at $t = 500\,T_c^{-1}$, $t = 1000\,T_c^{-1}$ and $t = 1500\,T_c^{-1}$ respectively, for the $\eta/T_c = 0.15$, $N_b = 988$ simulation.

[Hindmarsh, Huber, Rummukainen, Weir 2014]

GW sources:

- Bubble wall collisions
- Sound waves
- (Magneto)hydrodynamic turbulence

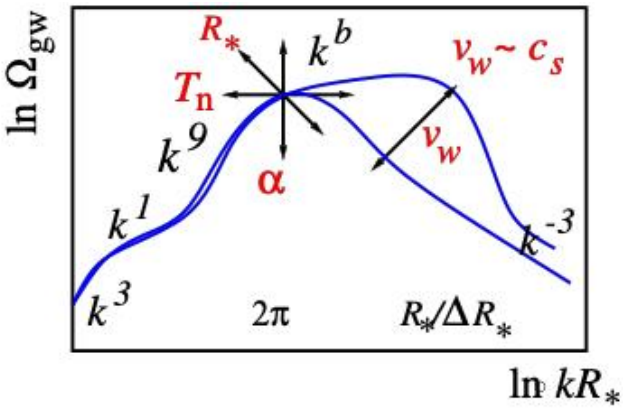
T_n nucleation temperature

α transition strength

β/H avg. bubble separation

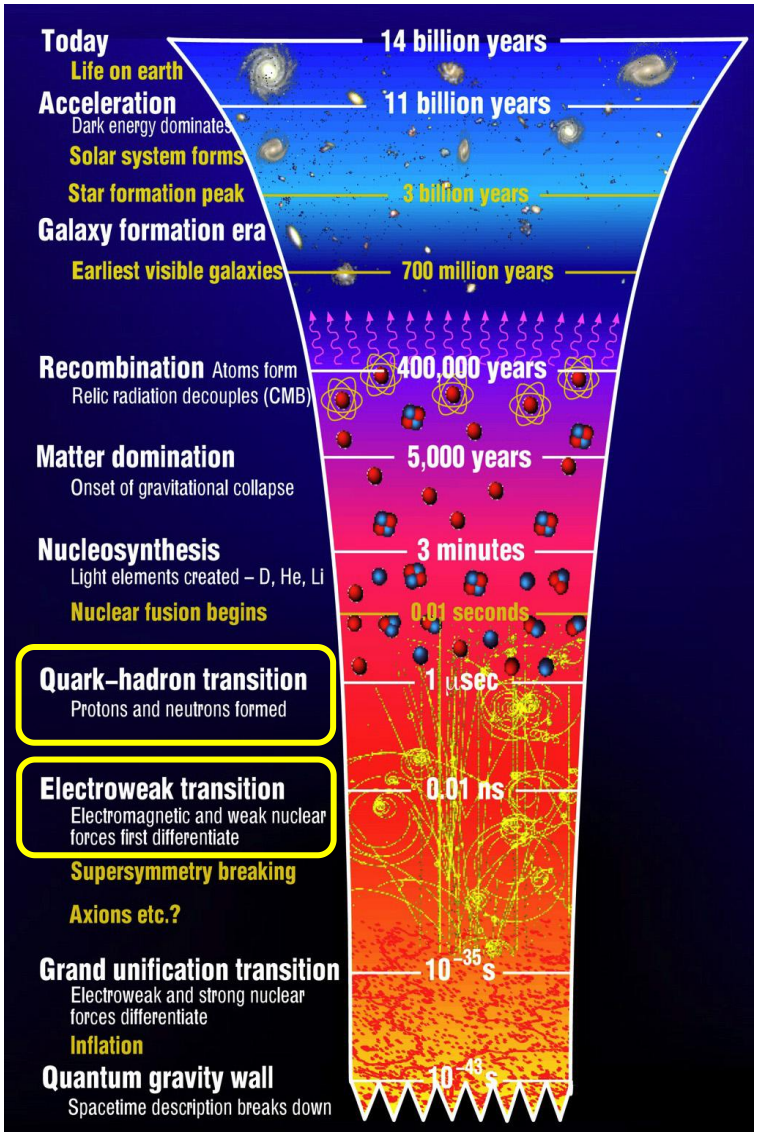
v_w bubble wall speed

....



[Cai, Wang & Yuwen (2023); Sharma, Dahl, Brandenburg, Hindmarsh (2023); Roper-Pol, Procacci, Caprini 2023; Giombi, Dahl, Hindmarsh 2024]

Phase transitions predicted by the SM



Credit: CTC

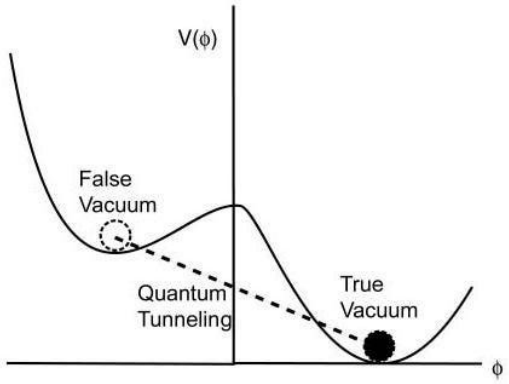
- Both are predicted to be crossover!
- Any GW signals of a FOPT will be a smoking gun of BSM!

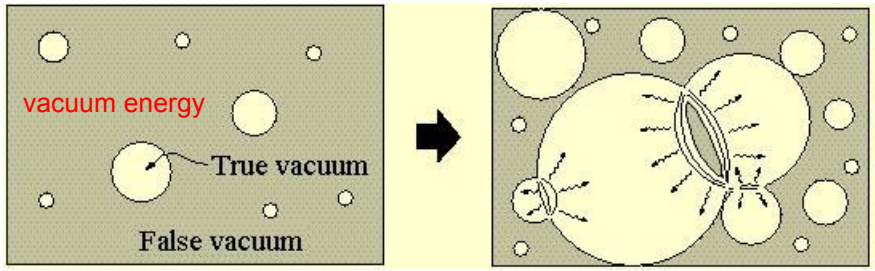
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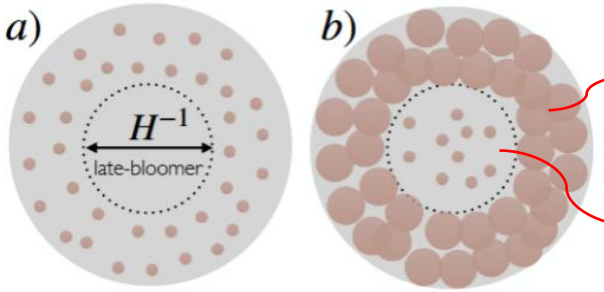
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Curvature perturbations generated from a FOPT

Original: randomness of bubble nucleation + different equations of state in the true and false vacua [Liu, Bian, Cai, Guo & Wang 2021; 2022]







radiation, fast redshift of the energy density

"false-vacuum island", slow redshift of the energy density

overdensity in the region with delayed transition

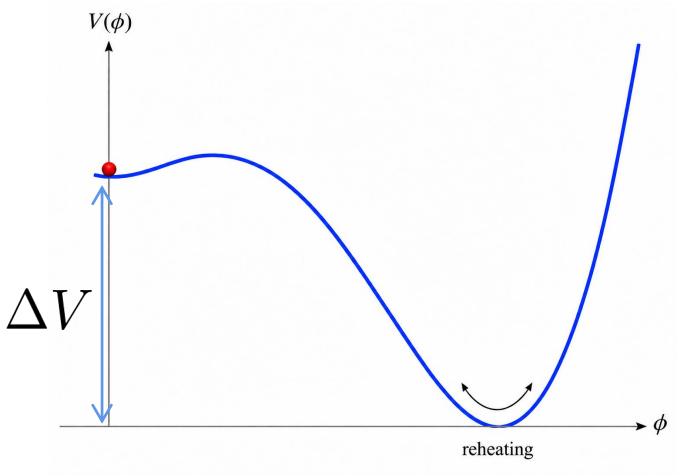
$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

[Gouttenoire & Volansky 2023]

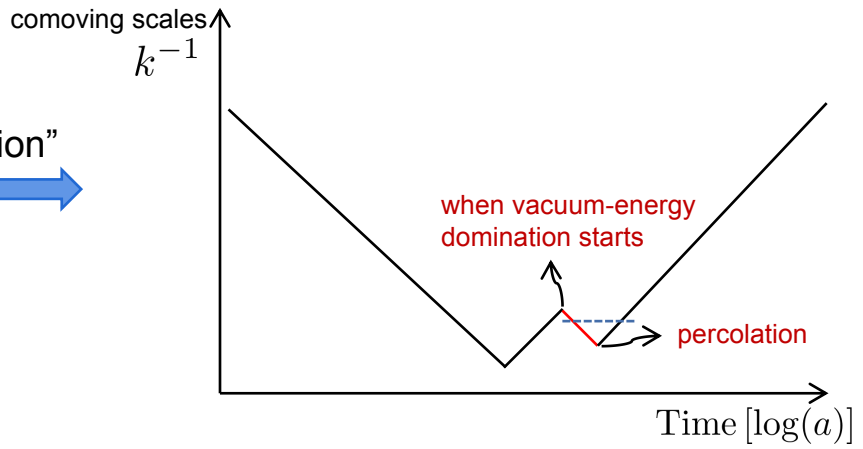
Curvature perturbations generated from a FOPT

New: Curvature perturbations from “old inflation” (low-scale, finite T)

[Lewicki, Toczek & Vaskonen 2023]



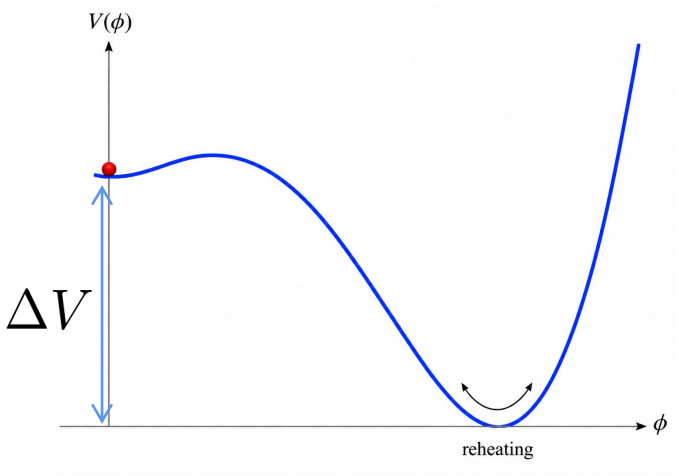
“old inflation”



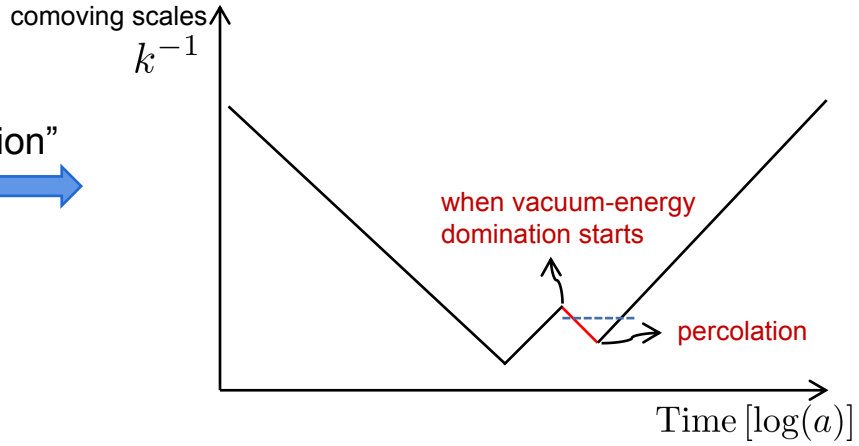
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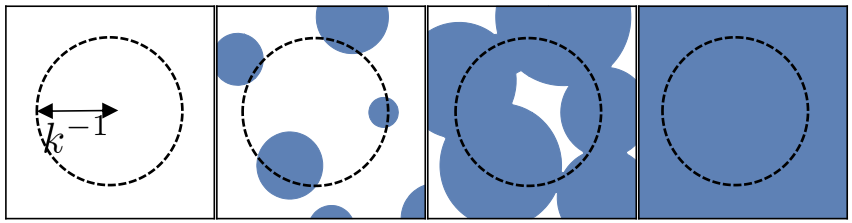


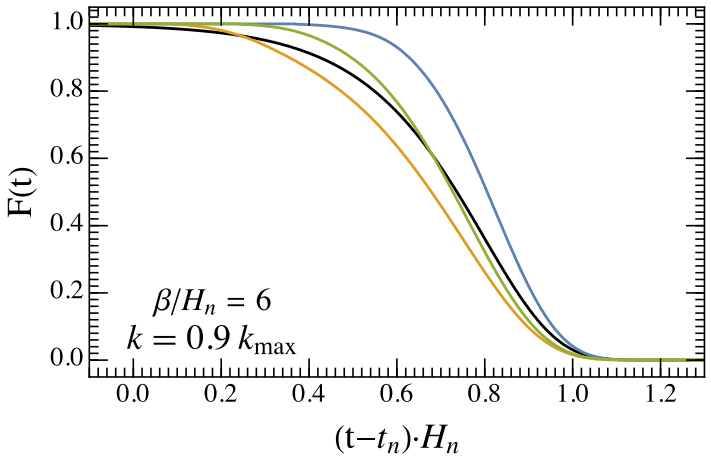
EoM:

$$\dot{\rho}_{\rm rad} + 4H\rho_{\rm rad} = -\dot{\rho}_{\rm vac}$$

$$\rho_{\rm vac}(t) = F(t)\,\Delta V$$

false-vacuum volume fraction: requires simulations!

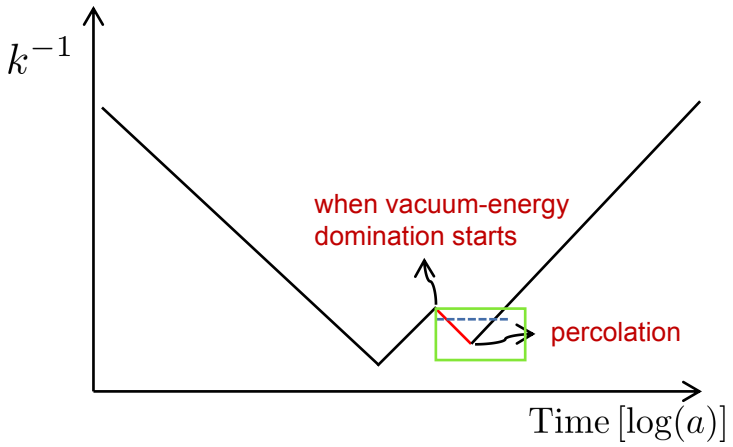


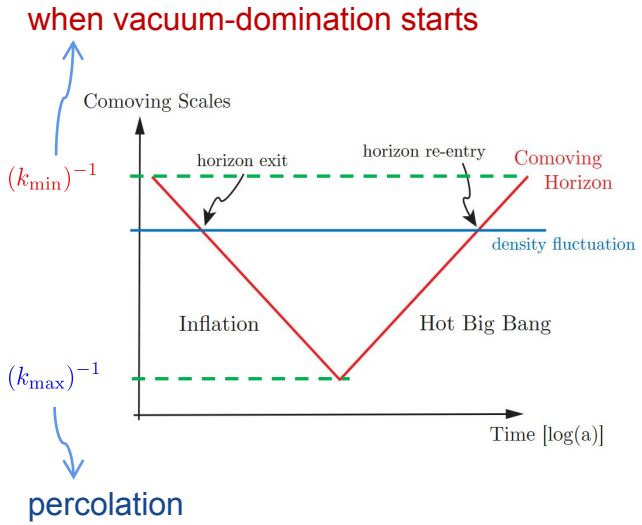


$$\overline{F}(t) = \exp\left[-\frac{4\pi}{3}\int_{-\infty}^{t} dt_n\,\Gamma(t_n)a(t_n)^3\left(\int_{t_n}^{t}\frac{d\tilde{t}}{a(\tilde{t})}\right)^3\right]$$

Curvature perturbations generated from a FOPT

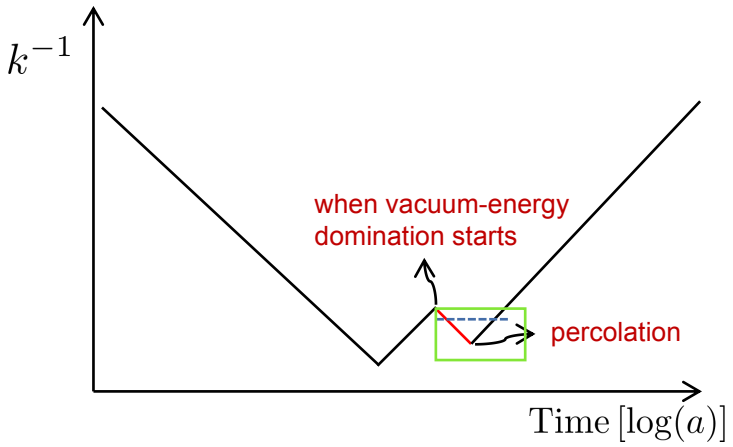
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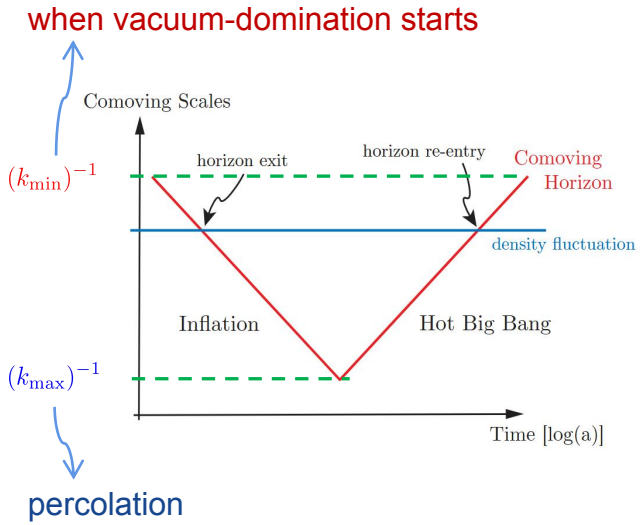




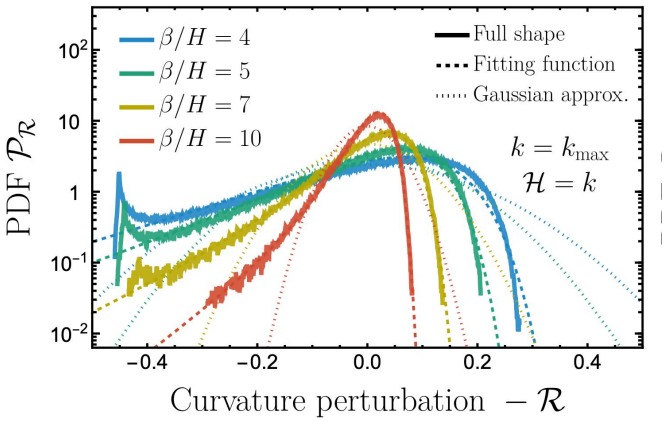
Curvature perturbations generated from a FOPT

New: Curvature perturbations from "old inflation" (low-scale, finite T)





[Franciolini, Gouttenoire & Jinno 2025]



a FOPT can naturally generate curvature perturbations at length scales:

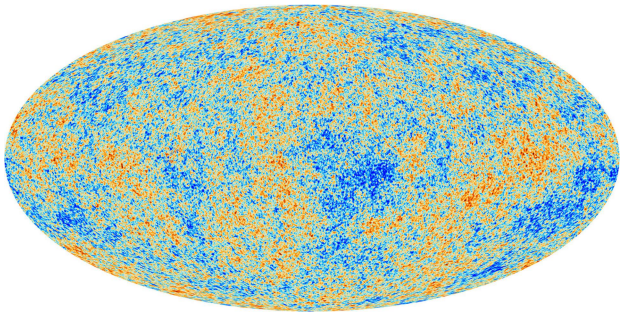
$$((k_{\max})^{-1}, (k_{\min})^{-1}) \sim 1/T_n$$

scales at the phase transition

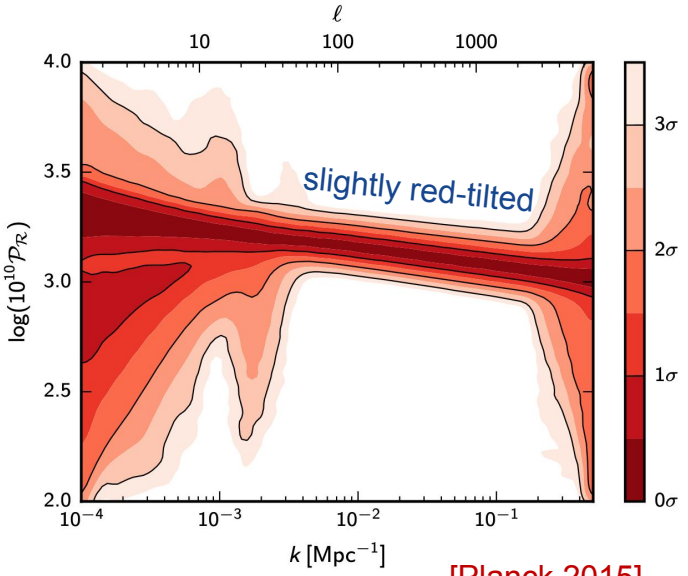
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Large-scale density perturbations

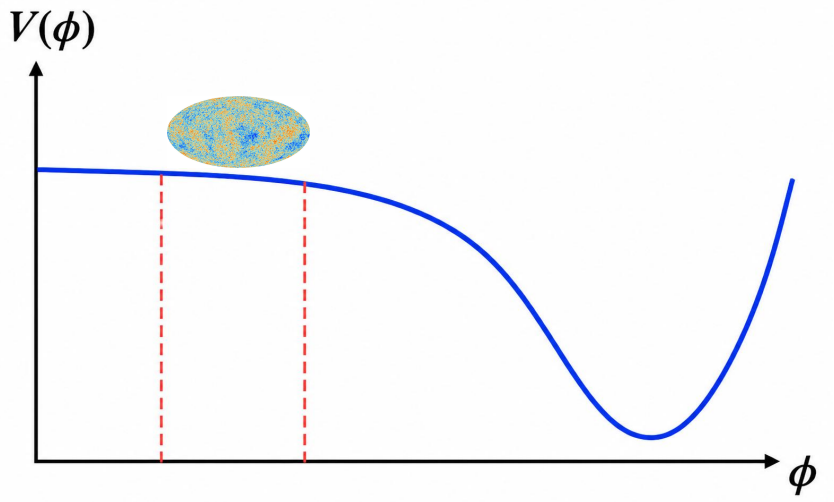


largest scale of our observable Universe

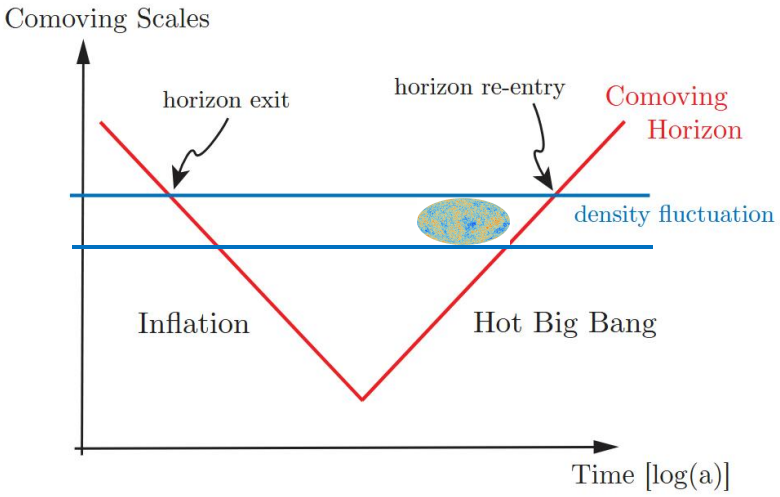


[Planck 2015]

Standard mechanism:



inflaton (not phase transition field!)

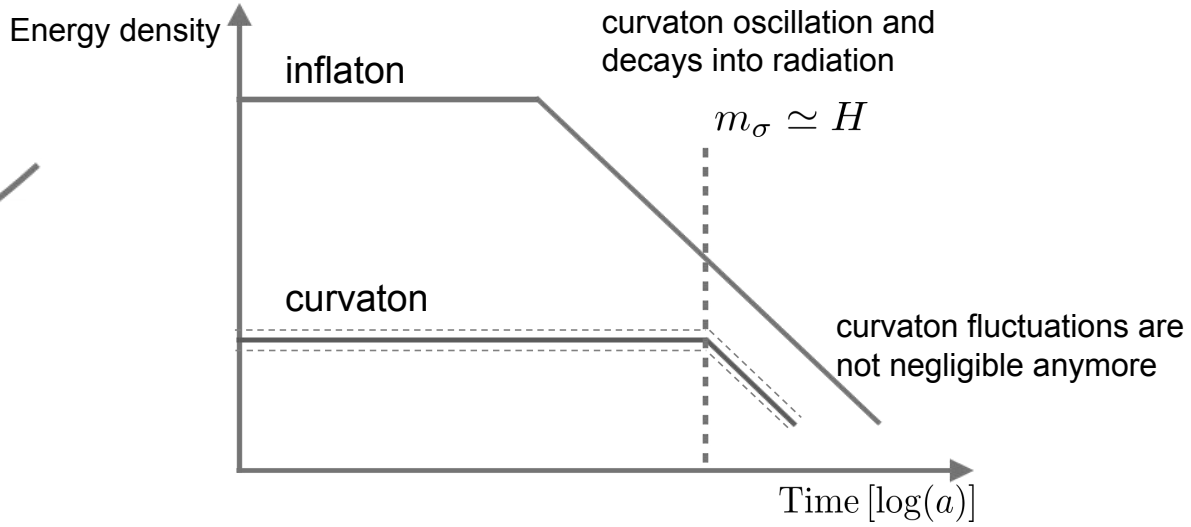


The curvaton mechanism

What is a curvaton? [Lyth & Wands 2002]

- a light (spectator) field during inflation $m_\sigma \ll H_{\rm inf}$
- its averaged value $\bar{\sigma}$ can be random
- obtains almost scale-invariant fluctuations during inflation $\delta\sigma \sim \frac{H_{\rm inf}}{2\pi}$
- decays at a later time: isocurvature to curvature perturbations



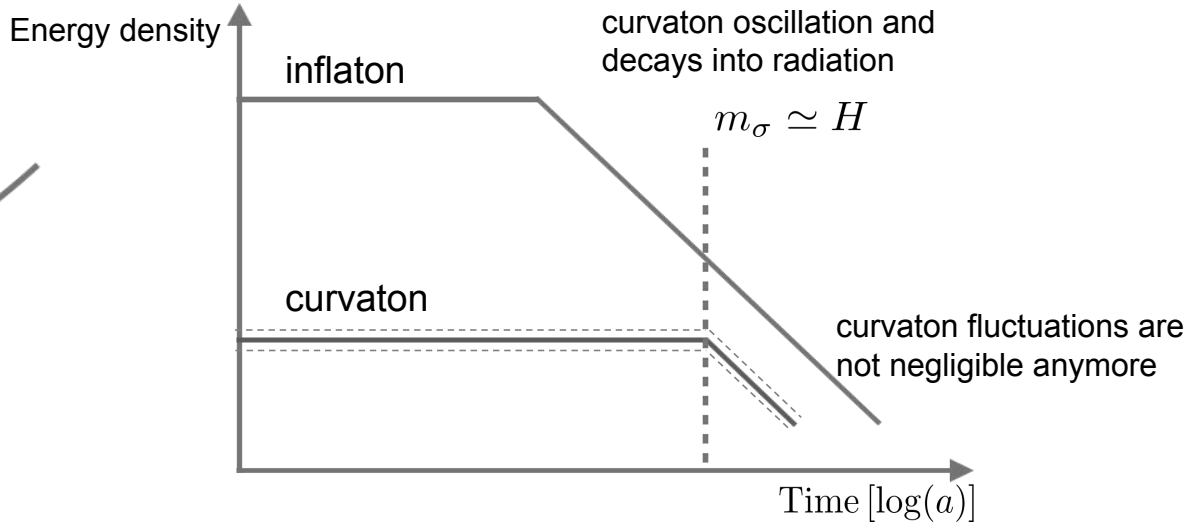


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(flat gauge)

$$\zeta = -H\frac{\delta\rho}{\dot{\rho}}$$

$$\zeta_{\rm r} = -H\frac{\delta\rho_{\rm r}}{\dot{\rho}_{\rm r}} = \frac{1}{4}\frac{\delta\rho_{\rm r}}{\rho_{\rm r}}$$

$$\zeta_\sigma = -H\frac{\delta\rho_\sigma}{\dot{\rho}_\sigma} = \frac{1}{3}\frac{\delta\rho_\sigma}{\rho_\sigma} \equiv \frac{1}{3}\delta$$

$$\Rightarrow \zeta = \frac{4\rho_{\rm r}\zeta_{\rm r} + 3\rho_\sigma\zeta_\sigma}{4\rho_{\rm r} + 3\rho_\sigma} \xrightarrow{\zeta_r \to 0} \zeta = \frac{\rho_\sigma}{4\rho_{\rm r} + 3\rho_\sigma}\delta \Rightarrow \begin{cases} \zeta = \frac{1}{3}\delta & (\rho_\sigma \gg \rho_r) \\ \zeta = \frac{1}{4}\frac{\rho_\sigma}{\rho_r}\delta & (\rho_\sigma \ll \rho_r) \end{cases}$$

The curvaton mechanism

[Lyth & Wands 2002]

Large-scale density perturbations can entirely come from a curvaton

The curvaton mechanism

[Lyth & Wands 2002]

Large-scale density perturbations can entirely come from a curvaton

Our question:

Can CMB (perturbations) and SGWB (stochastic GW background) have one and the same origin?



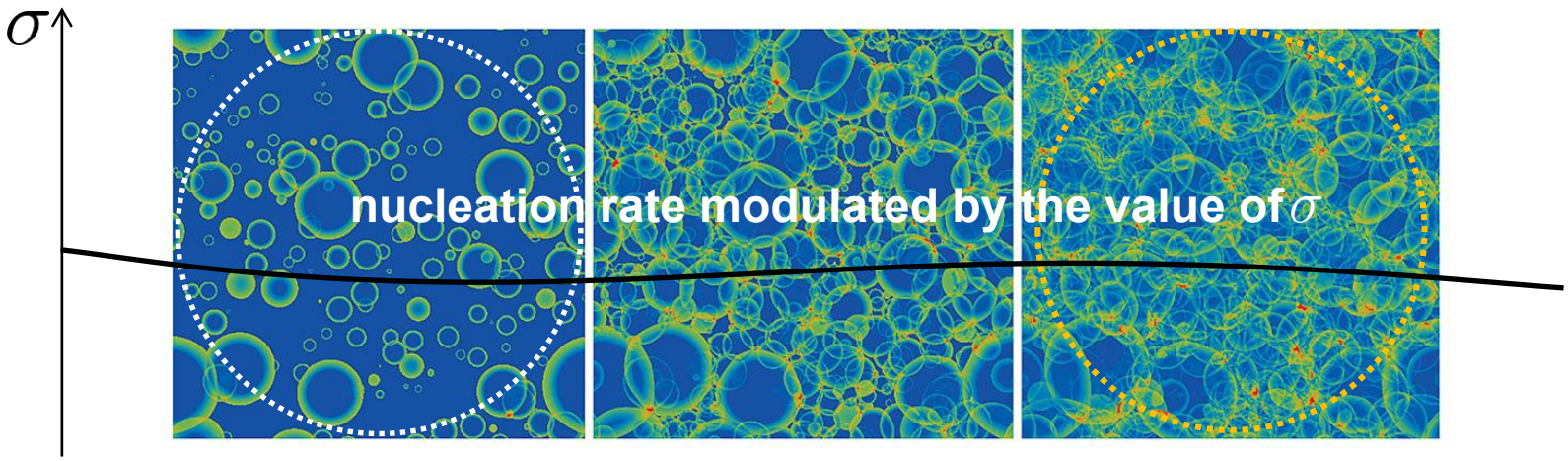
couple curvaton with a FOPT order-parameter!



modulated FOPT & audible curvaton

The picture

σ couples with the order-parameter field ϕ



We are going to compute the curvature perturbations using the δN formalism

δN formalism

$$\zeta = -\psi - \frac{H}{\dot{\bar{\rho}}}\delta\rho$$

where ψ is the scalar curvature perturbation, defined as

$$ds^2 \supset -dt^2 + a^2(t)e^{-2\psi(\vec{x})}d\vec{x}^2$$

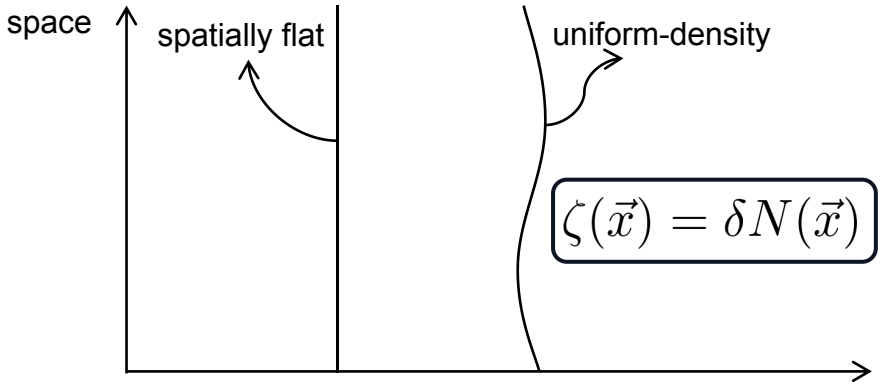
$$\Rightarrow\ a^{\rm loc}(\vec{x},t) = a(t)e^{-\psi(\vec{x})}\ \Rightarrow\ N(\vec{x}) = \ln\frac{a^{\rm loc}(\vec{x},t_f)}{a^{\rm loc}(\vec{x},t_i)} = \bar{N} - \psi_f + \psi_i$$

We are interested in the curvature perturbation at the final time

$$\zeta_f = -\psi_f - \left.\frac{H}{\dot{\bar{\rho}}}\delta\rho\right|_{t_f} = -\psi_f = \delta N - \psi_i = \delta N$$

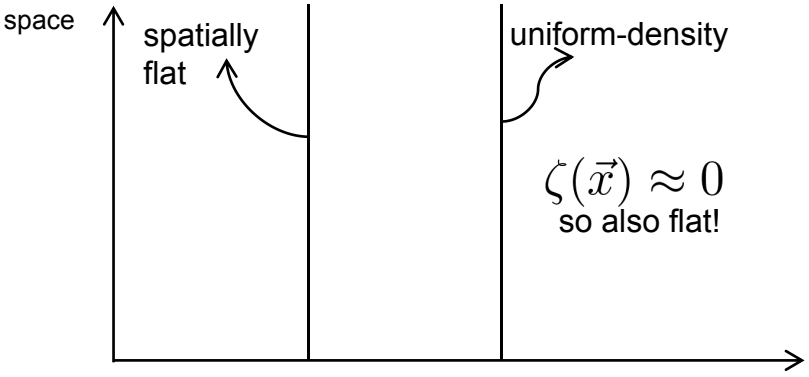
choose $\delta\rho(t_f) = 0$ (final uniform-density slice)

choose $\psi_i = 0$ (initial flat slice)

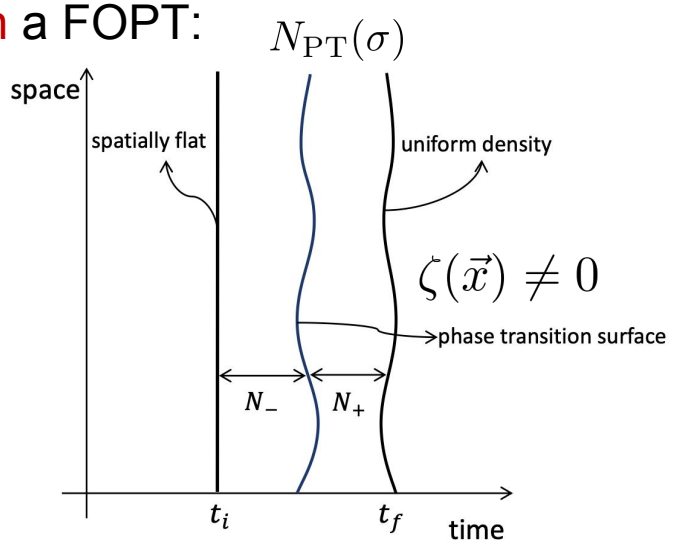


δN formalism for a modulated FOPT

Without a FOPT:



With a FOPT:



$$\zeta(\vec{x}) = \frac{\bar{\alpha}}{1+\bar{\alpha}} \delta N_{\rm PT} + \mathcal{O}(\delta N_{\rm PT}^2)$$

[Ai, Franciolini, Wang & Zeng 2026]

where $\bar{\alpha} = \dfrac{\rho_{\rm vac}}{\rho_r(\bar{T}_{\rm PT})}$ (phase transition strength) $\quad \delta N_{\rm PT}(\vec{x}) = \ln \dfrac{N_{\rm PT}(\vec{x})}{\bar{N}_{\rm PT}} \simeq -\dfrac{\delta T_{\rm PT}(\vec{x})}{\bar{T}_{\rm PT}}$

(We take $T_{\rm PT}$ as the percolation temperature)

Non-Gaussianity

Introduce a dimensionless field variable $\xi = \dfrac{\sigma}{\Lambda}$

Λ: a UV scale that may appear in the coupling between σ and ϕ

$$\zeta(\vec{x}) = \delta N_f = \mathcal{N}_1 \delta\xi(\vec{x}) + \frac{1}{2}\mathcal{N}_2 \delta\xi^2(\vec{x}) + \dots$$

where $\mathcal{N}_n = \left.\dfrac{d^n N_f(\xi)}{d\xi^n}\right|_{\xi=\bar{\xi}}$ can be computed by studying the phase transition evolution

(see our paper for details)

Even though ξ is Gaussian, which we assume, the curvature perturbations are still non-Gaussian!

[Ai, Franciolini, Wang & Zeng 2026]

local bispectrum amplitude: $f_{\rm NL}^{\rm loc} = \dfrac{5}{6}\dfrac{\mathcal{N}_2}{\mathcal{N}_1^2}$ sudden-transition limit

$$f_{\rm NL}^{\rm loc} = \frac{10}{3\bar{\alpha}} + \frac{5(1+\bar{\alpha})}{6\bar{\alpha}} \left.\frac{\partial_\xi^2 N_{\rm PT}}{(\partial_\xi N_{\rm PT})^2}\right|_{\xi=\bar{\xi}}$$

$$f_{\rm NL}^{\rm loc} = -0.9 \pm 5.1 \ @68\% \ \text{C.L.}$$ [Planck 2018]

GW annisotropies

The modulation also affects GW annisotropies

total fractional annisotropy $\delta^{\rm tot}_{\rm GW}(f,\hat{\boldsymbol{n}}) \equiv \dfrac{\Omega^{\rm obs}_{\rm GW}(f,\hat{\boldsymbol{n}}) - \bar{\Omega}_{\rm GW}(f)}{\bar{\Omega}_{\rm GW}(f)}$

sky directions

$$\delta^{\rm tot}_{\rm GW} = \delta^{\rm src}_{\rm GW}(f,\boldsymbol{x}_{\rm f}) + \delta^{\rm prop}_{\rm GW}(f,\hat{\boldsymbol{n}})$$

$$\delta^{\rm src}_{\rm GW}(f,\boldsymbol{x}_{\rm f}) = \left.\frac{\partial \ln \Omega_{\rm GW}}{\partial \xi}\right|_{\xi=\bar{\xi}} \delta\xi(\boldsymbol{x}_{\rm f}) \equiv \mathcal{T}_{\rm src}(f)\zeta(\boldsymbol{x}_{\rm f})$$

$$\mathcal{T}_{\rm prop}(f) = -\frac{4-n_\Omega(f)}{5}$$

(Integrated) Sachs-Wolfe effect
(Hubble refshift + gravitational redshift)

$$\Omega_{\rm GW}(f;\xi) = \Omega_{\rm peak}(\xi) S(f/f_{\rm peak})$$

$$\mathcal{T}_{\rm src}(f) = \frac{G_\Omega - n_\Omega(f) G_f}{\mathcal{N}_1}$$

where $G_\Omega \equiv \partial_\xi \ln \Omega_{\rm peak}|_{\xi=\bar{\xi}}$, $n_\Omega \equiv \partial \ln \bar{\Omega}_{\rm GW}/\partial \ln f$, $G_f \equiv \partial_\xi \ln f_{\rm peak}|_{\xi=\bar{\xi}}$

Outline

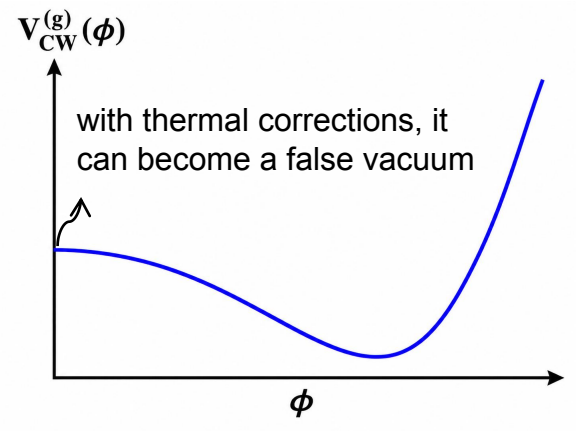
- What is a first-order phase transition?
- Small-scale curvature perturbations generated from a FOPT
- Modulated FOPT: large-scale curvature perturbations
- **An example (particle-physics) model**

Classically scale-invariant model

A complex scalar field Φ , charged under $U(1)_D$

two parameters: g_D , v_ϕ order-parameter: $\phi = \sqrt{2}|\Phi|$

Coleman-Weinberg potential: $V_{\rm CW}^{(g)}(\phi) = \frac{\beta_\lambda}{4}\phi^4\left[\ln\left(\frac{\phi}{v_\phi}\right) - \frac{1}{4}\right] + \frac{\beta_\lambda v_\phi^4}{16}$ $\beta_\lambda \simeq 6(g_D^2/4\pi)^2$

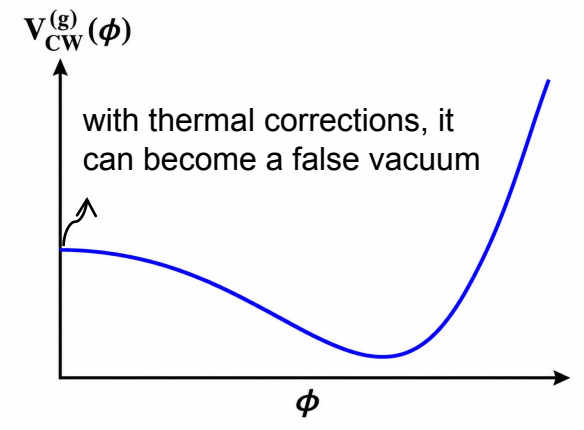


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coupling between curvaton and PT field

$-m_\psi(\phi,\xi)\bar{\psi}\psi$ (recall $\xi = \frac{\sigma}{\Lambda}$)

- fermion one-loop corrections to ϕ introduce σ-dependence in the effective potential
- the fermion also contribute to the potential of σ!

⇩

- so that σ is frozen before the PT, and its potential energy remains negligible after PT

we consider a phenomenological mass term:

$m_\psi = \xi\phi(1 - \phi^2/v_\phi^2)$ (UV completion needed!)

Results

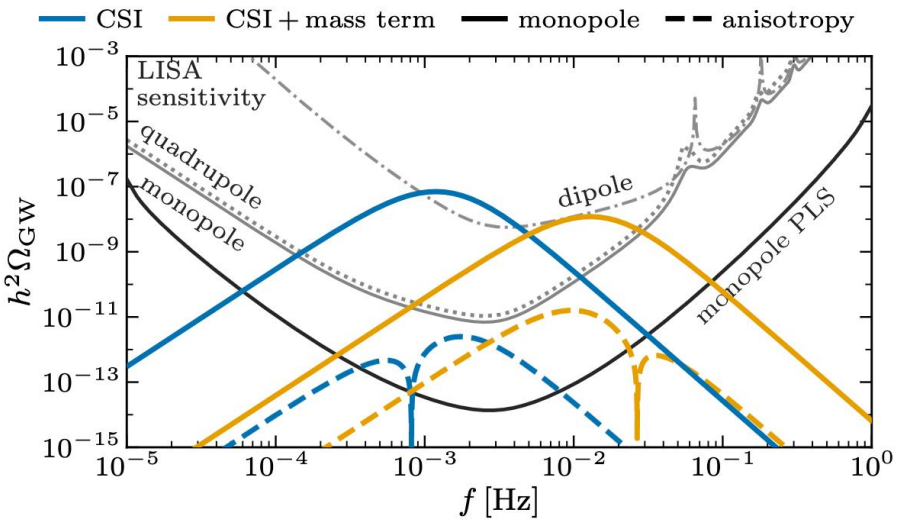


FIG. 1. GW monopole (solid) and absolute quadrupole anisotropy (dashed) for the CSI benchmark (blue) and the benchmark with the mass term (orange). Gray: LISA instrument-noise sensitivity to the monopole, dipole and quadrupole [61]; black: monopole power-law integrated sensitivity (PLS) [62] ($T_{\rm obs} = 3\,{\rm yr}$, SNR = 1).

$$g_D = 0.745, \quad v_\phi = 10\,{\rm TeV}, \quad \bar{\xi} = 0.536$$

$$r_M \equiv \frac{M^2}{\beta_\lambda v_\phi^2} = 0.025$$

Adding a mass term can enhance the detectability of anisotropies

$$V_{\rm CW}^{(g)} \to V_{\rm CW}^{(g)} - \frac{M^2}{4v_\phi^2}(\phi^2 - v_\phi^2)^2$$

$$\Downarrow$$

$$f_{\rm NL}^{\rm loc} = 9.48$$

$${\rm SNR}_{\ell=2} \simeq 1.05$$

for a 5 yr observation with the LISA–Taiji network

Summary

- FOPTs are a very interesting phenomenon:
  - Unique GW power spectrum
  - Smoking gun of new physics
- A modulated FOPT may be the origin for large-scale density perturbations
- It can also generate a loud stochastic GW background, giving correlated signals in CMB and SGWB (audible curvaton!)
- GW annisotropies may be enhanced by the modulation

Ultimate goal: make GWs as good as CMB!

Thank you!