Can Distance Duality Violation Save Late-time Solutions to the Hubble Tension?



Yashi Tiwari

Postdoctoral Fellow, ITP-CAS, Beijing

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Revisiting cosmic distance duality with megamasers and DESI DR2 observations: Model-independent constraints on early-late calibration

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Brijesh Kanodia [1,2,*], Ujjwal Upadhyay [1,3,†], and Yashi Tiwari [1,3,‡]

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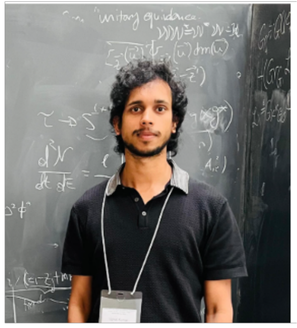
ACCEPTED PAPER

Can distance duality violation save late-time solutions to the Hubble tension?

Yashi Tiwari, Ujjwal Upadhyay, Shao-Jiang Wang, and Vivian Poulin

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Ujjwal Upadhyay



Brijesh Kanodia



Shao-Jiang Wang



Vivian Poulin

Outline

- ✦ The Hubble tension
- ✦ Cosmic Distance Duality Relation (CDDR)
- ✦ Connection between CDDR and H_0 tension
- ✦ Late time solutions: Constraints from CDDR
- ✦ Observational Constraints on Distance Duality Violation
- ✦ Summary

Cosmological Tension ?

Discrepancy between measurement of a cosmological parameter from two different observations

In other terms : **statistical inconsistency** between two different measurements; $X : X_1,\ X_2$

Observation-1: $x_1 \pm \sigma_1$

Observation-2: $x_2 \pm \sigma_2$

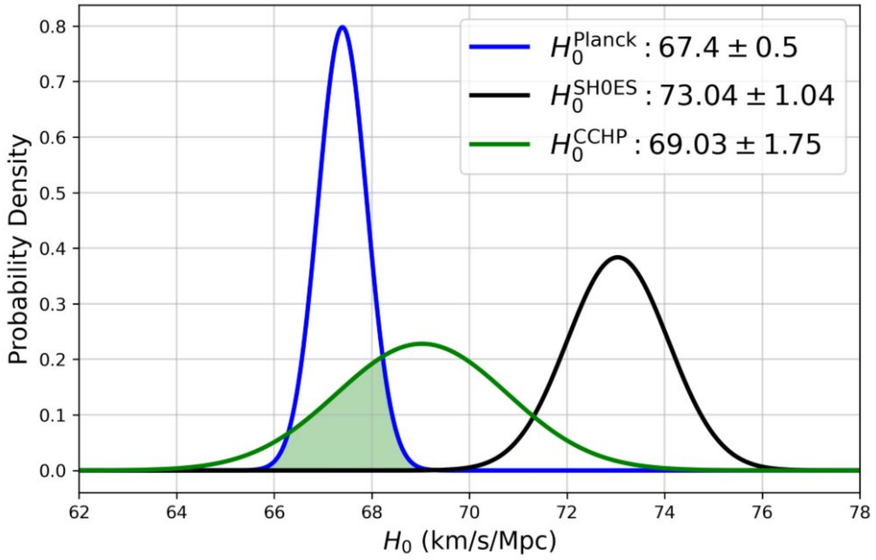
$$|x_1 - x_2| > \sqrt{\sigma_1^2 + \sigma_2^2}$$

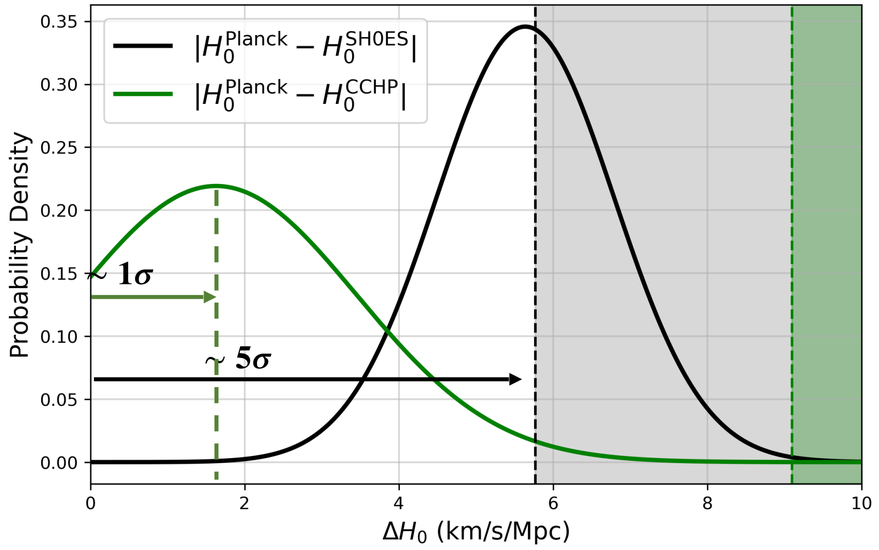
The observations are in tension!

Quantifying Tension

$$S = \frac{|x_1 - x_2|}{\sqrt{\sigma_1^2 + \sigma_2^2}}$$

$< 2\sigma$	Weak
$2\sigma - 3\sigma$	Moderate
$3\sigma - 5\sigma$	Strong
$> \mathbf{5\sigma}$	**Severe**





Hubble Tension:

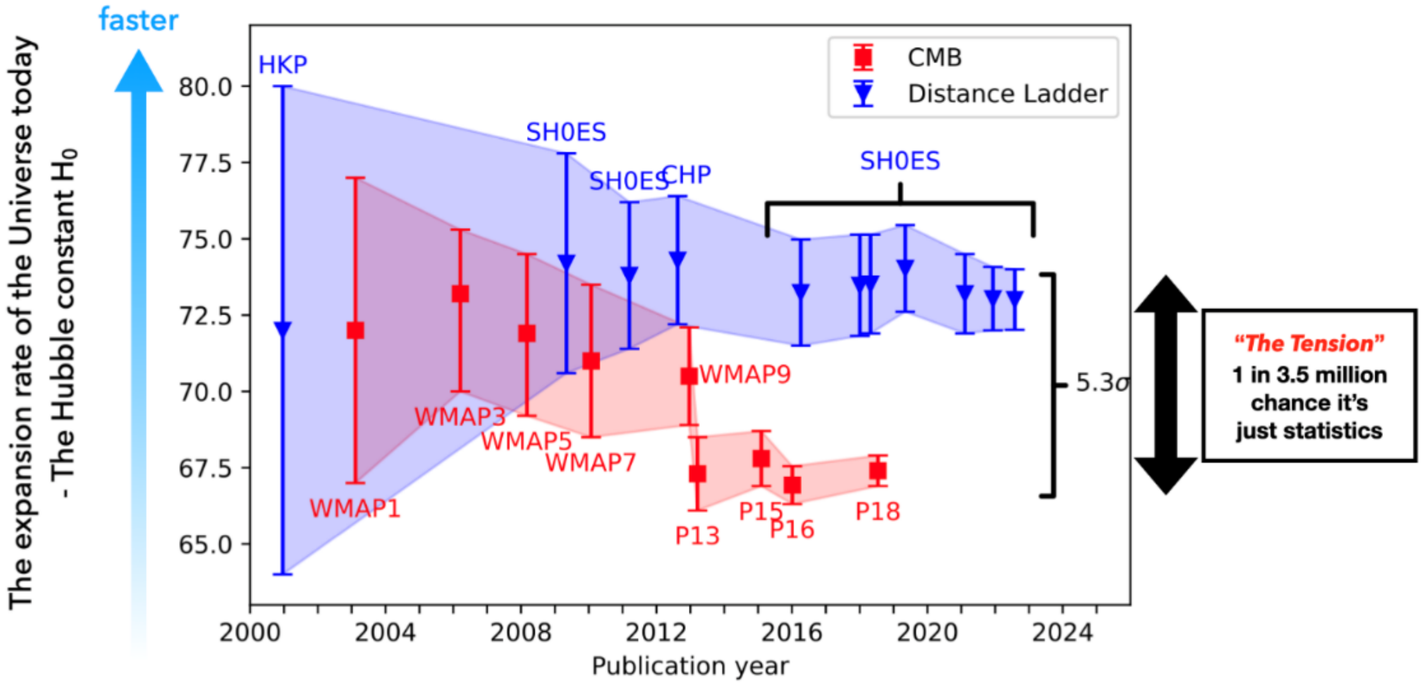
Planck18 + ΛCDM

$H_0 = 67.5 \pm 0.5\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$

$\gtrsim 5\,\sigma$

SH0ES **(Model-independent)**

$H_0 = 73.1 \pm 1.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$



Proposed Solutions: Early, Late, Systematics

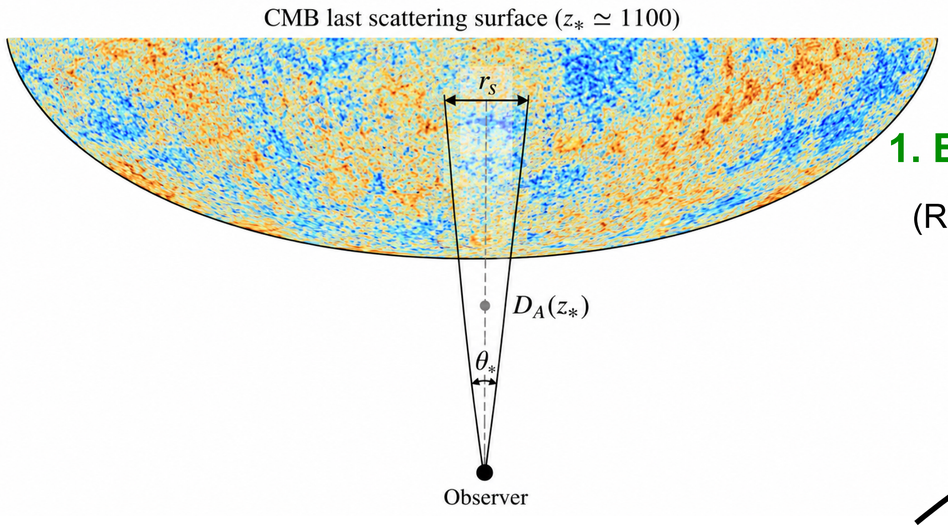
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$$H_0 = 67.5 \pm 0.5\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

$$\gtrsim 5\,\sigma$$

SH0ES **(Model-independent)**

$$H_0 = 73.1 \pm 1.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$



3. Systematics?

(Find source of mismatch)

1. Early-universe Solution

(Reduce sound horizon, r_s)

$$\theta_* = \frac{r_s}{D_A} = H_0 \left(\frac{\displaystyle\int_{z_*}^{\infty} \frac{c_s(z)}{H(z)}\,dz}{\dfrac{1}{1+z_*}\displaystyle\int_0^{z_*} \frac{c\,dz'}{\sqrt{\Omega_{m,0}(1+z')^3 + \Omega_\Lambda}}} \right)$$

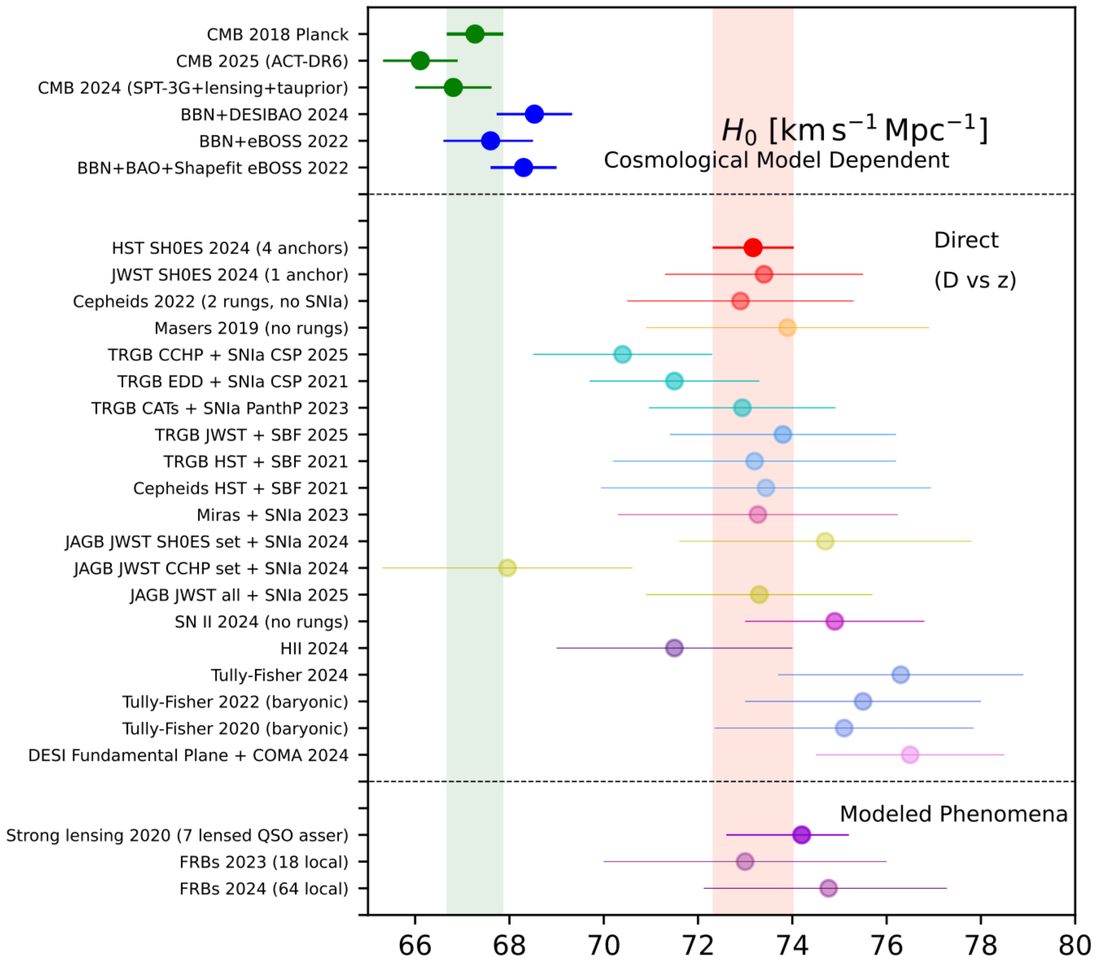
Fixed from CMB Observations

(Angular size of sound horizon)

2. Late-universe Solution

(Modify late-time expansion history)

The Status of Hubble Tension



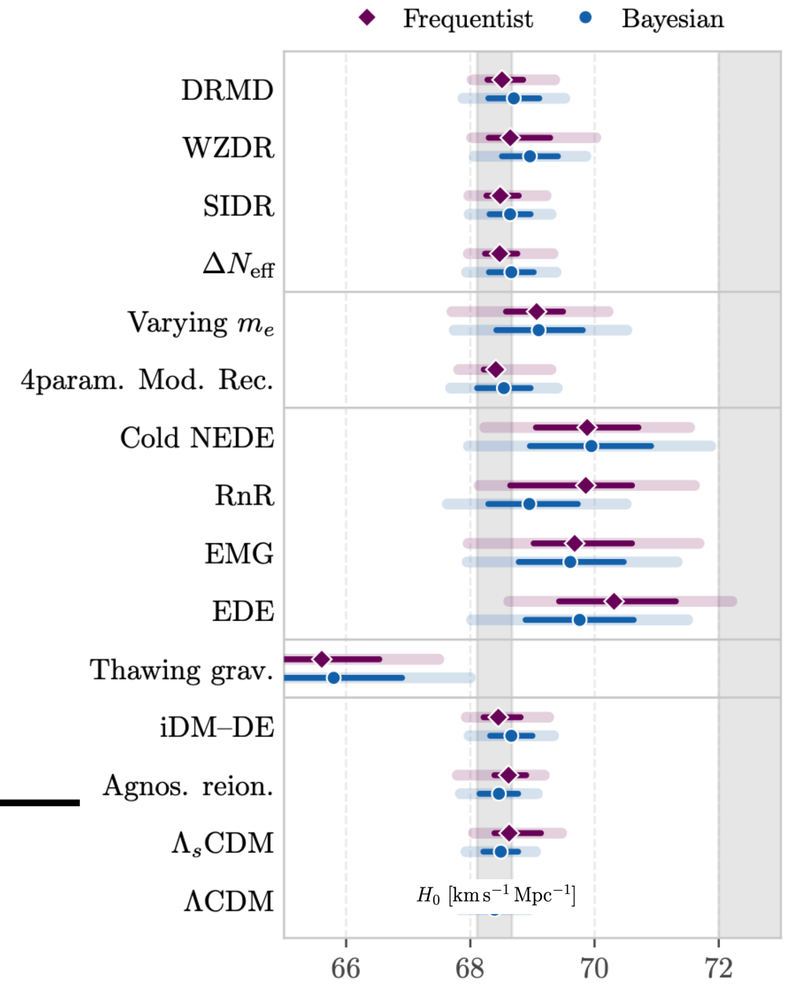
(The CosmoVerse White Paper)

arXiv: 2504.01669

For details of other probes and models

arXiv: 2607.13282

(The H_0 World Cup)



Comparison of models to resolve tension

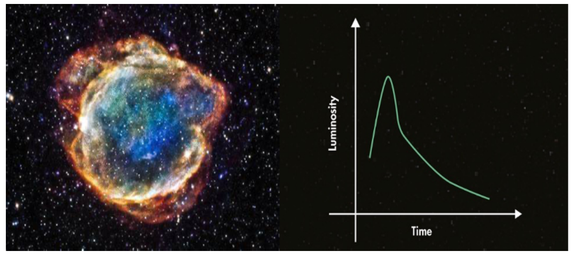
- Late-time modifications: Limited success **(No-go?)**
- Early-time modifications: Partial relief
- Combined early + late physics: An open possibility

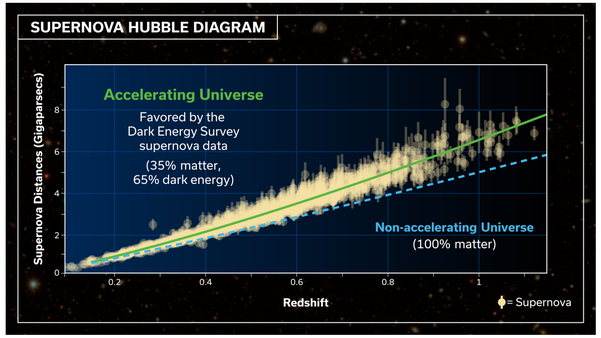
Why do late-time solutions consistently struggle?

Distances in Cosmology & Cosmic Distance Duality Relation

Standard Candle (D_L)

Objects of known intrinsic brightness: Supernovae





Observed brightness (apparent magnitude)

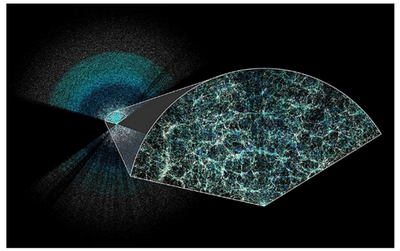
$$m(z) = M_B + 5\log_{10}\left(\frac{D_L(z)}{10\text{pc}}\right)$$

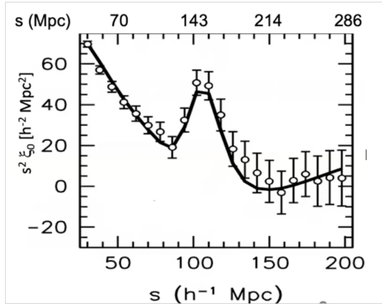
Absolute Magnitude
(Calibration parameter)

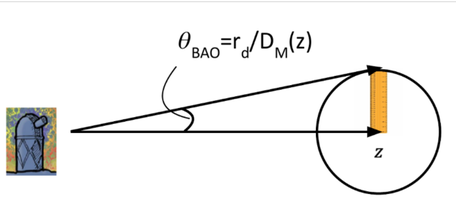
$$D_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')} \quad -(1)$$

Standard Ruler (D_A)

Objects of known physical size: Baryon Acoustic Oscillation







(Observed)

$$\theta_{\text{BAO}}(z) = \frac{r_d}{D_A(z)}$$

Size of sound horizon at baryon drag
(Calibration parameter)

$$D_A(z) = \frac{1}{(1+z)}\int_0^z \frac{dz'}{H(z')} \quad -(2)$$

$$\boxed{d_L = (1+z)^2\, d_A}$$

(Cosmic Distance Duality)

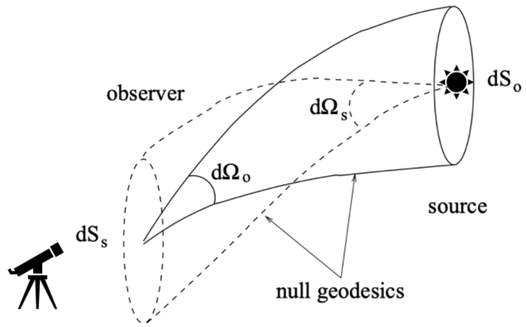
The Cosmic Distance Duality Relation (CDDR)

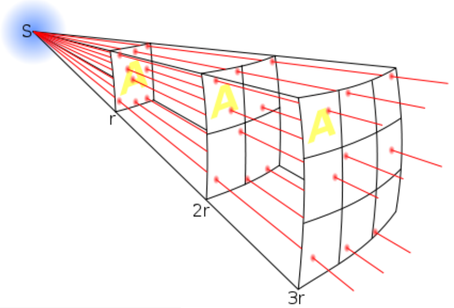
A fundamental relation connecting the luminosity distance and angular diameter distance under the assumptions

(Etherington 1933, Ellis et al 2006, Uzan et al 2004)

Etherington reciprocity

- Metric theory
- Null geodesics





Photon number conservation

- Number is conserved during propagation

$$d_s = (1+z)\, d_A \longrightarrow \boxed{d_L = (1+z)^2\, d_A} \longleftarrow d_L = (1+z)\, d_s$$

What if distance duality is violated: Test of exotic physics

Reciprocity Violation

- Non-metric theories of gravity, deviation from null geodesics
- Possible modification of angular-diameter distances
- **Direct observational tests remain limited.**
  (see for example arXiv: 1301.1312)

Photon number non-conservation

- Absorption, scattering, axion-photon conversion …
- Modifies the observed flux/luminosity distance
- **Tightly constrained:** $T_{CMB}(z)$**,** CMB spectral distortions, etc
  (see for example arXiv:1112.1862)

Test of CDDR ? $\longrightarrow$ $\eta = \dfrac{d_L}{(1+z)^2\, d_A} = 1\,?$

$\downarrow$

constrain exotic physics ?

Astrophysics

[Submitted on 17 Dec 2003 (v1), last revised 15 Apr 2004 (this version, v2)]

Cosmic distance-duality as probe of exotic physics and acceleration

Bruce A. Bassett (Kyoto, ICG), Martin Kunz (Sussex)

Astrophysics

[Submitted on 31 May 2004]

The distance duality relation from X-ray and SZ observations of clusters

Jean-Philippe Uzan (LPT Orsay, IAP), Nabila Aghanim (IAS Orsay), Yannick Mellier (IAP, LERMA)

[Submitted on 7 Jun 2006]

Constraints on Distance Duality Relation from Sunyaev Zel'dovich Effect and Chandra X-ray measurements

Francesco De Bernardis, Elena Giusarma, Alessandro Melchiorri

[Submitted on 30 Oct 2008 (v1), last revised 3 May 2009 (this version, v2)]

Cosmic transparency: A test with the baryon acoustic feature and type Ia supernovae

Surhud More, Jo Bovy, David W. Hogg

$\longrightarrow$ **First test of CDDR using d_A from BAO ?**

[Submitted on 25 May 2010 (v1), last revised 13 Sep 2010 (this version, v2)]

Testing the Distance-Duality Relation with Galaxy Clusters and Type Ia Supernovae

R. F. L. Holanda, J. A. S. Lima, M. B. Ribeiro

[Submitted on 14 Jan 2021 (v1), last revised 15 Jan 2021 (this version, v2)]

Test of the cosmic distance duality relation for arbitrary spatial curvature

Jin Qin, Fulvio Melia, Tong-Jie Zhang

Testing Distance Duality Relation with SN and BAO

$$\eta(z) = \frac{d_L(z)}{(1+z)^2\, d_A(z)}$$ (Distance Duality Parameter)

$d_L(z)$ **from Supernovae**

(M_B: late calibration parameter)

$$\frac{d_L(z)}{\text{Mpc}} = 10^{[m(z)-M_B-25]/5}$$

$d_A(z)$ **from BAO**

(r_d: early calibration parameter)

$$d_A(z) = \frac{r_d}{\theta_{\text{BAO}}(z)(1+z)}$$

$$\eta(z, r_d, M_B) = \underbrace{\frac{10^{-(M_B+25)/5}}{r_d/\text{Mpc}}}_{\text{calibration}} \times \underbrace{\frac{10^{m(z)/5}\,\theta_{\text{BAO}}(z)}{1+z}}_{\text{observed}}$$

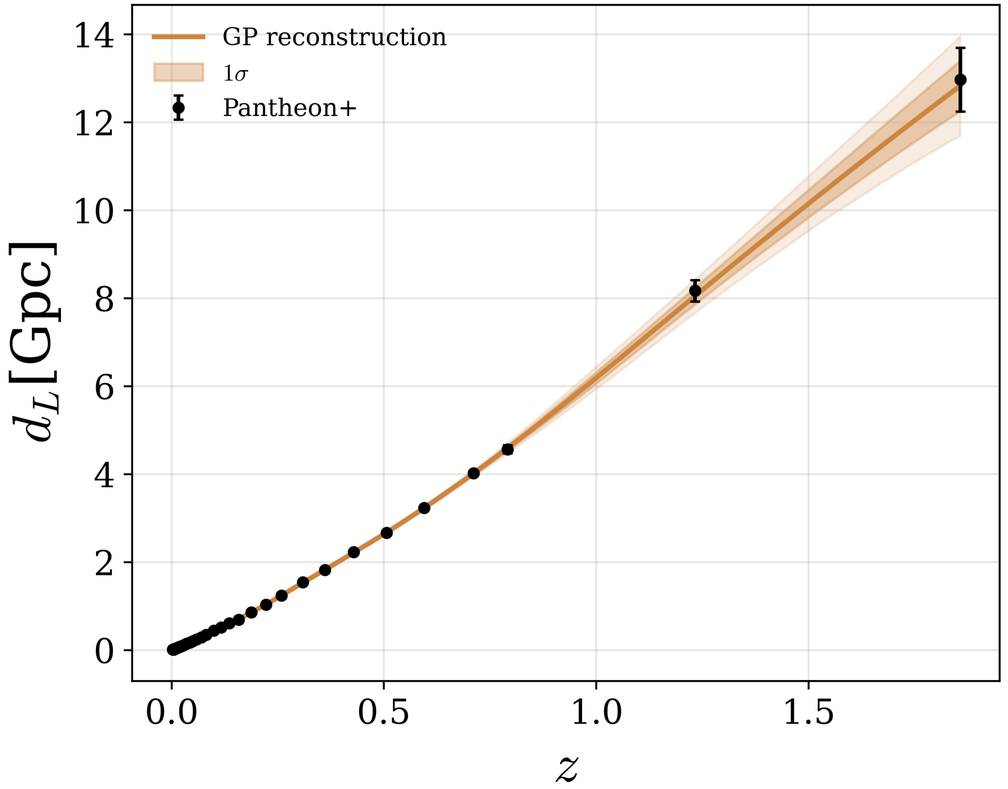


Fig: GP reconstruction of luminosity distances from Pantheon+ SNIa.

(Kanodia, Upadhyay, YT (2025))

CDDR with SN+BAO: the $r_d - M_B$ Degeneracy

$$\eta(z, r_d, M_B) = \underbrace{\frac{10^{-(M_B+25)/5}}{r_d/\mathrm{Mpc}}}_{\text{calibration}} \times \underbrace{\frac{10^{m(z)/5}\,\theta_{\mathrm{BAO}}(z)}{1+z}}_{\text{observed}}$$

- The choice of (r_d, M_b) affects the inference of $\eta(z)$
- **Degeneracy** between calibration choices is seen.

$r_d = 147$ Mpc (Planck+ΛCDM), $M_B = -19.25$ (SHOES)

↓

Strong violation of distance duality

↓

Link to Hubble tension ?

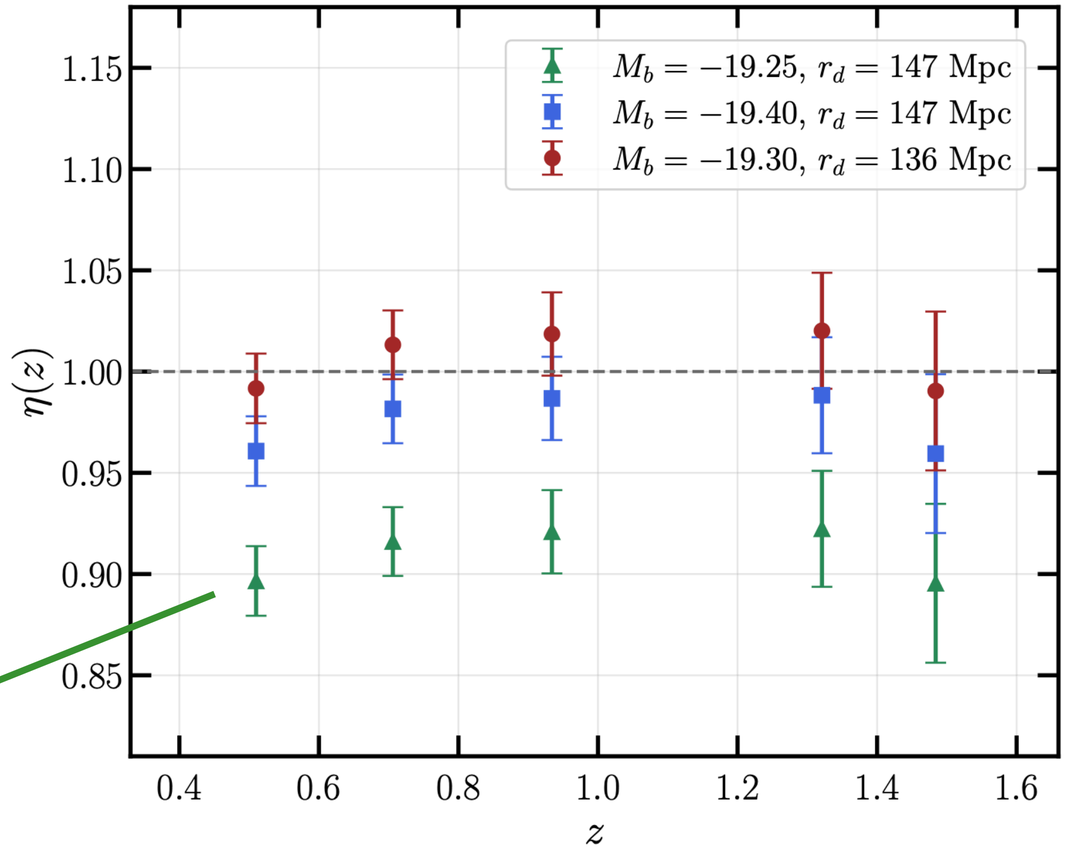


Fig: Calibration dependence of CDDR tests using SNIa and BAO.

(Kanodia, Upadhyay, YT (2025))

CDDR with SN+BAO: the $r_d - M_B$ Degeneracy

- CDDR combined with uncalibrated SNIa and BAO constrains:

$$\mathscr{A} = M_B + 5\log_{10}\left[\frac{r_d}{\mathrm{Mpc}}\eta\right]$$

- Using CDDR as a fundamental anchor ($\eta_0 = 1$):

$$\mathscr{L}_\eta(r_d, M_B) \propto \exp\left[-\frac{1}{2}(\boldsymbol{\eta}-\boldsymbol{\eta}_0)^{\mathrm{T}}\mathbf{C}_\eta^{-1}(\boldsymbol{\eta}-\boldsymbol{\eta}_0)\right]$$

↓

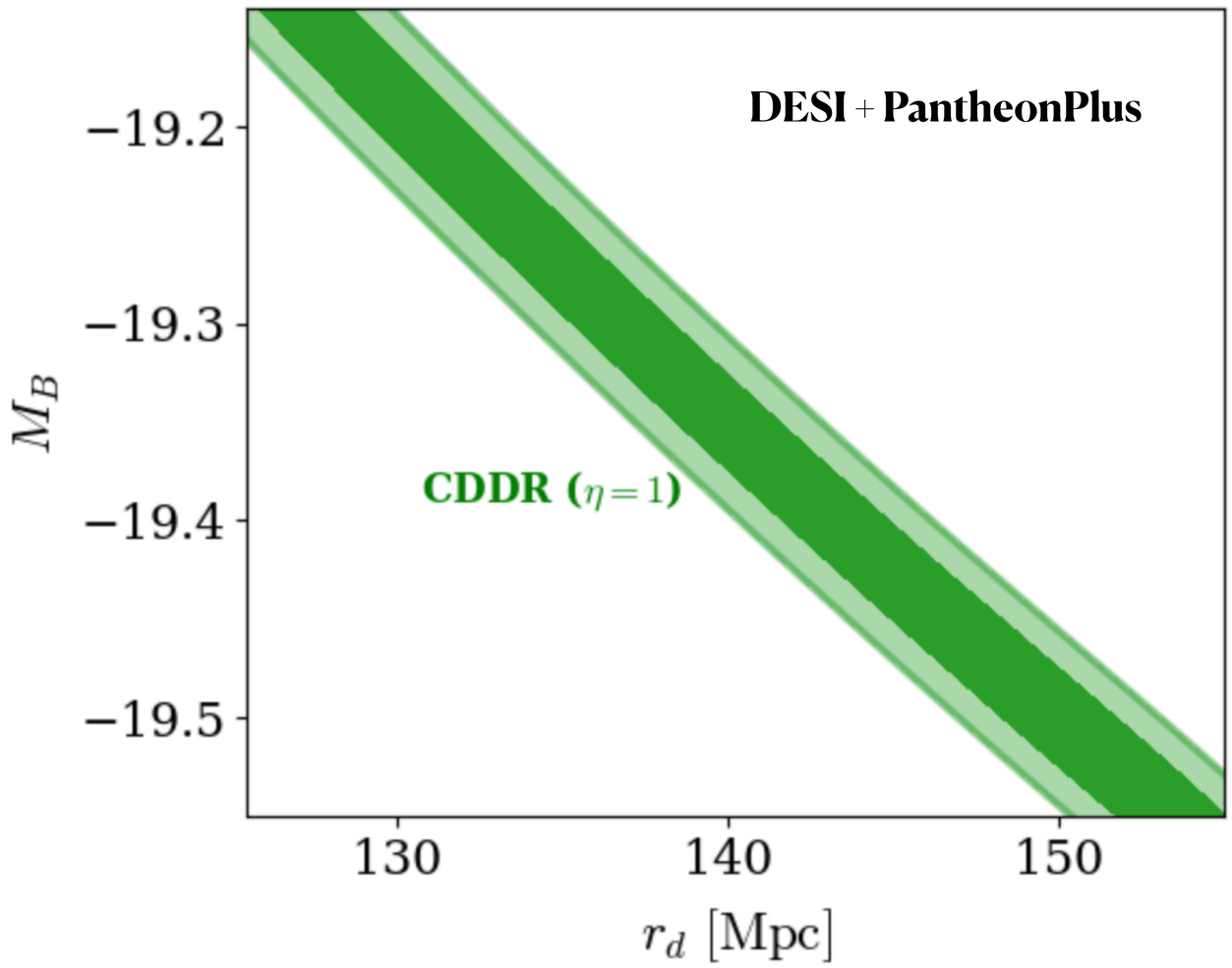
$r_d - M_B$ plane

↓

$$\mathscr{A} = -8.404 \pm 0.021$$

(Kanodia, Upadhyay, YT (2025))

(YT, Upadhyay, Wang, Poulin (2026))



CDDR+BAO+SN fixes H_0 : no room for late-time solutions

(YT, Upadhyay, Wang, Poulin (2026))

- CDDR combined with uncalibrated SNIa and BAO:

$$\mathscr{A} = M_B + 5\log_{10}\left[\frac{r_d}{\mathrm{Mpc}}\,\eta\right]$$

- **SNIa observations constrain the combination:**

$$\mathscr{M} = M_B - 5\log_{10}\left(\frac{H_0}{\mathrm{km\,s^{-1}\,Mpc^{-1}}}\right)$$

- **Late-time expansion history-independent evaluation of H_0 :**

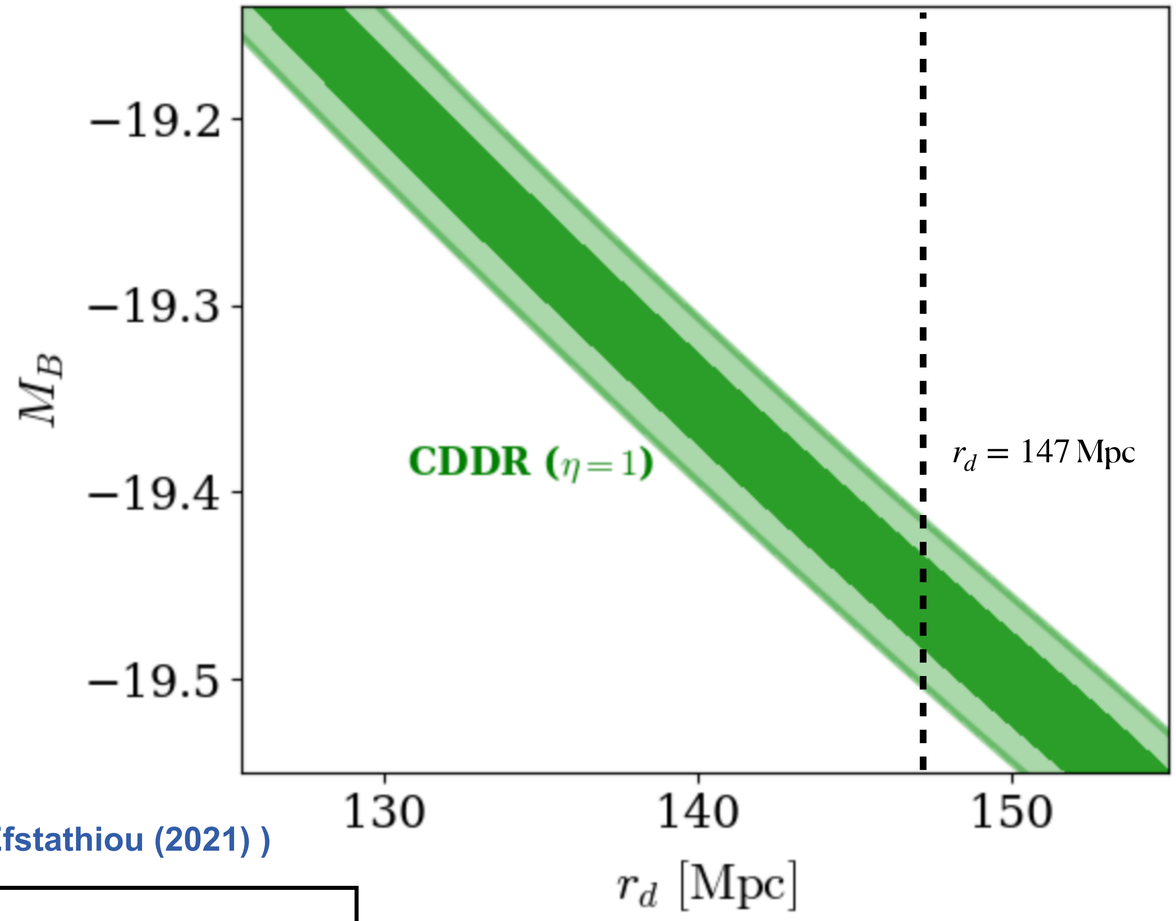
$$\boxed{\frac{H_0}{\mathrm{km\,s^{-1}\,Mpc^{-1}}} = \frac{1}{\eta\,(r_d/\mathrm{Mpc})}\,10^{(\mathscr{A}-\mathscr{M})/5}}$$

- For Late-time models:

$\mathscr{M} = -28.564 \pm 0.009$ (low-z Hubble Diagram)
$\mathscr{A} = -8.404 \pm 0.021$ ($\eta = 1$+SN+BAO)
$r_d = 147 \pm 0.3$ Mpc (Planck+ΛCDM)

(See also Efstathiou (2021))

$$\boxed{H_0 = 66.6 \pm 0.7\ \ \mathrm{km\,s^{-1}\,Mpc^{-1}}}$$



No resolution possible unless $\eta \neq 1$

Recasting Hubble Tension in $r_d - M_B$ Plane

(YT, Upadhyay, Wang, Poulin (2026))

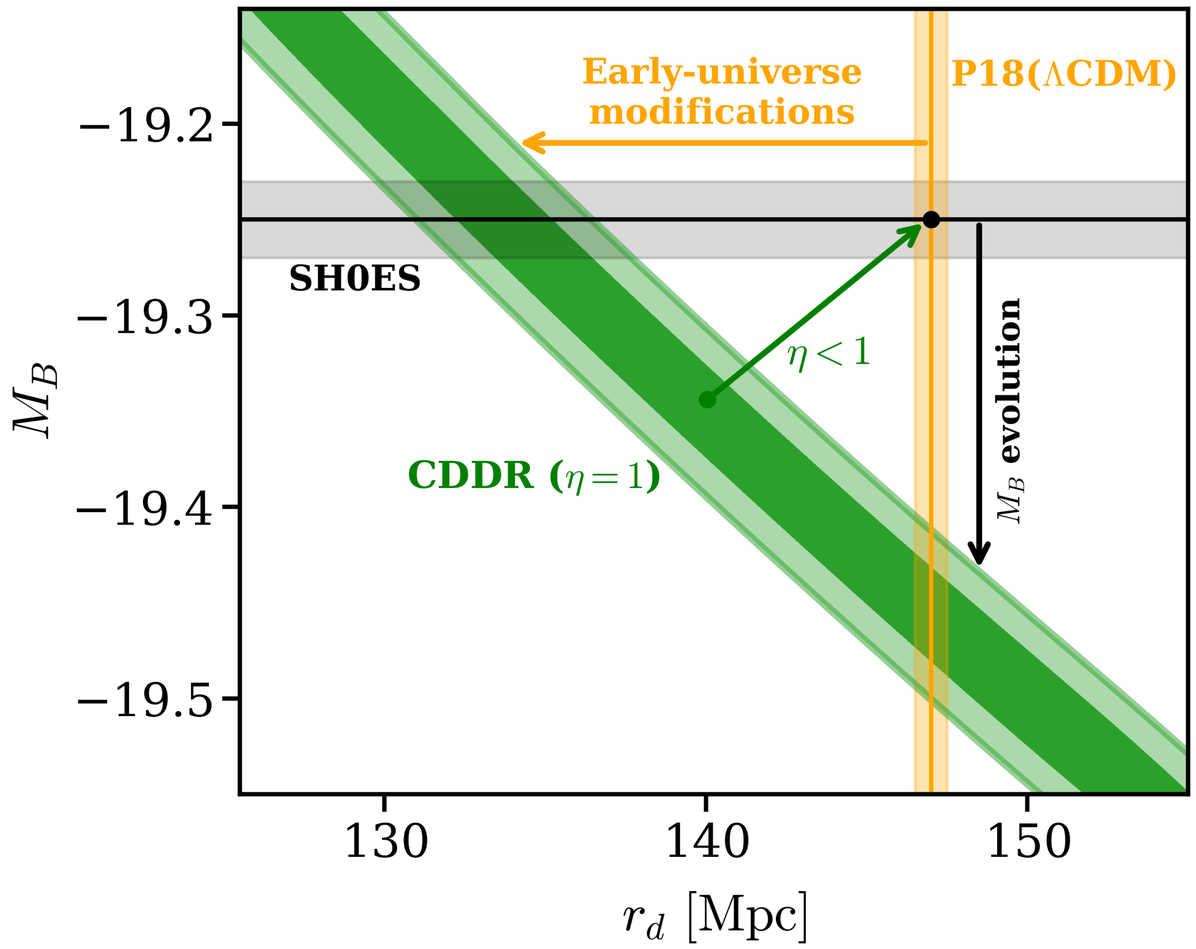
Possible ways forward:

- **Early-universe:** Reducing sound horizon.
- **Systematics:** M_B Evolution.
- **Late-universe:** Violation of distance duality relation.

$$\frac{H_0}{\mathrm{km\,s^{-1}\,Mpc^{-1}}} = \frac{1}{\eta\,(r_d/\mathrm{Mpc})}\,10^{(\mathscr{A}-\mathscr{M})/5}$$

$$\frac{\delta H_0}{H_0} = -\frac{\delta\eta}{\eta}\,.$$

$$H_0 : 67 \to 73 \implies \frac{\delta\eta}{\eta} \sim \mathcal{O}(8-10\%)$$



- **No late-time solutions can resolve the tension!**
- **Unless ($\eta < 1$), i.e. distance duality is violated $\mathcal{O}(8-10\%)$!!!**

Viability of Distance Duality violation?

- **Indirect test** (agnostic to physical cause): $\eta \simeq 1$ within the 2σ statistical significance.
- **Photon Number Non-conservation:** tightly constrained ($\eta \simeq 1$) using CMB temperature measurements from SZ in clusters.

$$T_{\rm CMB}(z) = T_0\,(1+z)^{(1-\epsilon)} \longrightarrow \eta(z) = (1+z)^{-3\epsilon/2}$$

[Submitted on 8 Dec 2011]

Constraints on the CMB temperature redshift dependence from SZ and distance measurements

A. Avgoustidis, G. Luzzi, C. J. A. P. Martins, A. M. R. V. L. Monteiro

$$\epsilon = -\,0.0106 \pm 0.0124 \longrightarrow \eta \simeq 1$$

- **No low-z rest of reciprocity violation.**

(YT, Upadhyay, Wang, Poulin (2026))

Cosmic Distance Duality Relation

OUR FRAMEWORK

Etherington reciprocity

- Metric theory
- Null geodesics

Photon number conservation

- Number is conserved during propagation

$$d_A = \frac{1}{\alpha(z)}\frac{d_S}{(1+z)} \longrightarrow d_L = (1+z)^2\, d_A\, \alpha\, \beta \implies \eta = \alpha \times \beta \longleftarrow d_L = (1+z) d_S\, \beta(z),$$

$\alpha(z) = ?$

A new constraint on Reciprocity Violation from DESI BAO

(YT, Upadhyay, Wang, Poulin (2026))

- **Assuming reciprocity violation through modification to d_A:**

$$d_A = \frac{1}{\alpha(z)} \frac{d_S}{(1+z)}$$

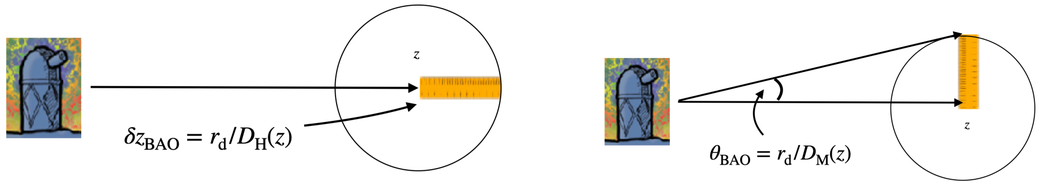
- **Source distance is standard comoving distance in flat universe :**

$$d_A = \frac{1}{\alpha(z)} \frac{1}{(1+z)} \int_0^z \frac{dz'}{H(z')}$$

- **Now, $H(z)$ is independently constrained by line-of-sight BAO :**

$$\alpha(z) = \left[\int_0^z \left(\frac{D_{\rm H}(z')}{r_d} \right)^{\parallel} dz' \right] \left[\left(\frac{D_{\rm M}(z)}{r_d} \right)^{\perp} \right]^{-1}$$

Radial BAO **Transverse BAO**



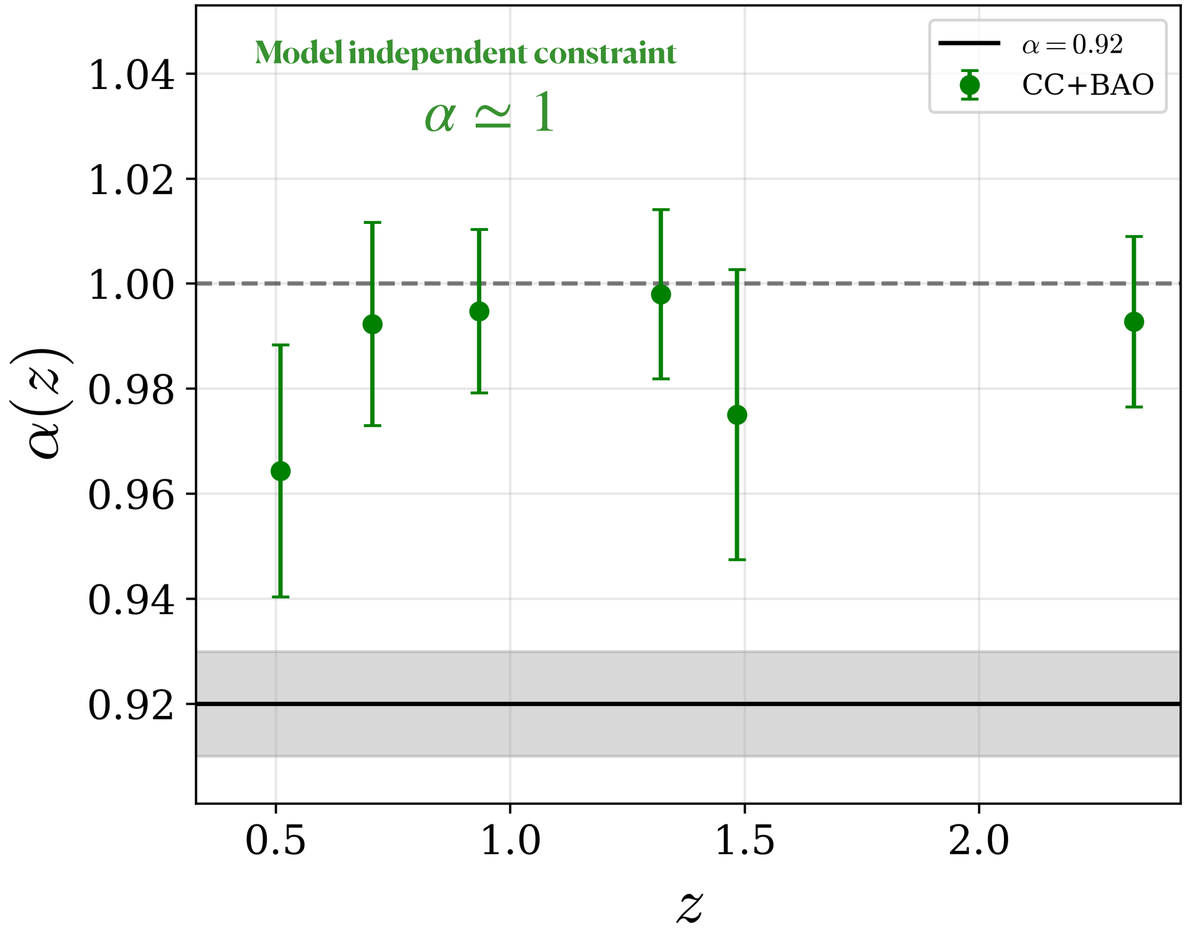
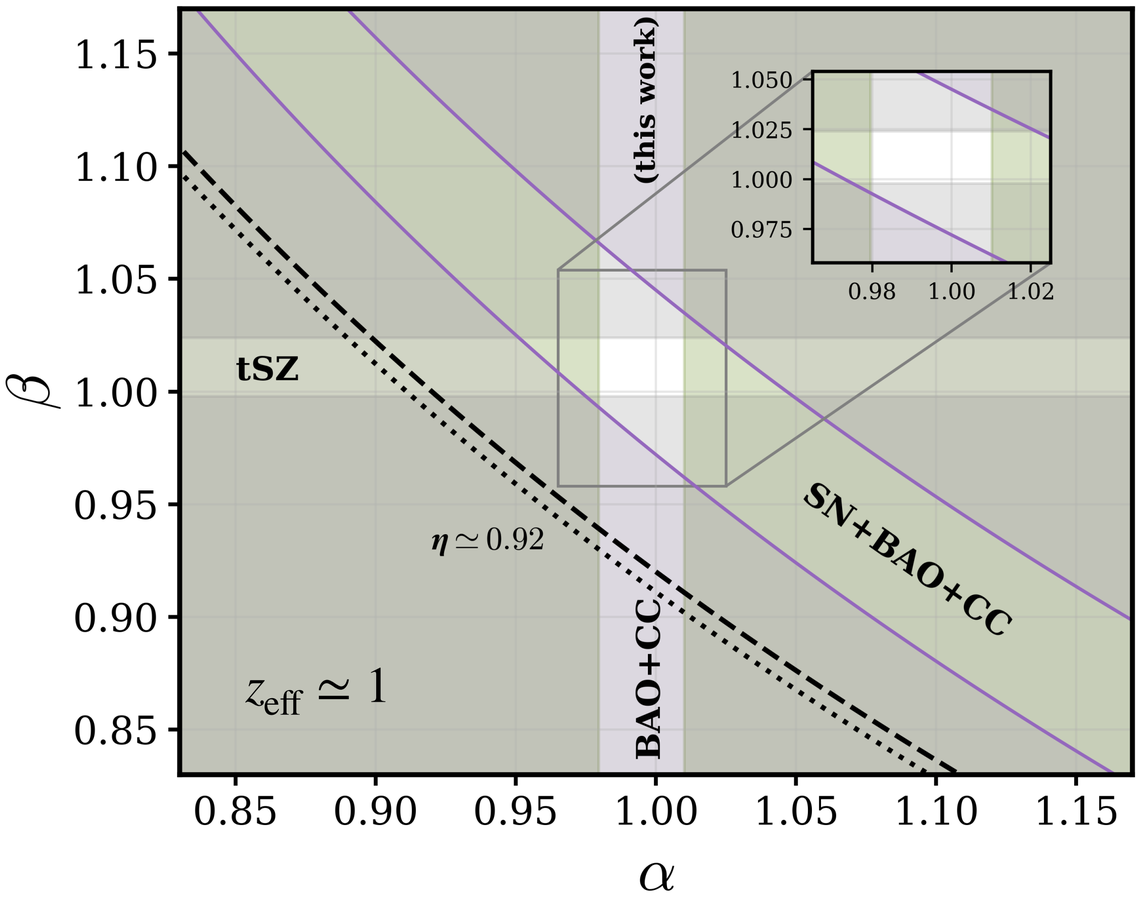


Fig: A new test of reciprocity theorem using CC and BAO

No Observational Evidence of CDDR Violation !

$$\eta(z) = \alpha(z)\,\beta(z)$$

(YT, Upadhyay, Wang, Poulin (2026))



✦ **Previous constraints:**

a) CMB Blackbody Constrains: $|\alpha(z_*) - 1|$
b) Thermal SZ effect Constrains: $\beta(z)$
c) SNIa & BAO measurements Constrains: $\eta(z)$

✦ **This work:**

a) Compiles the existing constraints on CDDR in $\alpha - \beta$ plane.
b) Adds a new constraint on α using radial and transverse BAO, and CC data.

Deviation from CDDR is tightly constrained to allow late-time solution to the Hubble tension!

Summary …

- The Hubble tension: $> 5\sigma$
- CDDR combined with BAO (fixed r_d) and SNIa determines H_0 independent of late-time cosmology.
- Deviations from the distance duality relation strongly constrained.
- Late-time modification to expansion history cannot resolve the tension.
- No late-time Dynamical DE, DM-DE Interaction, Decaying DE, MG solution can help to raise H_0 .

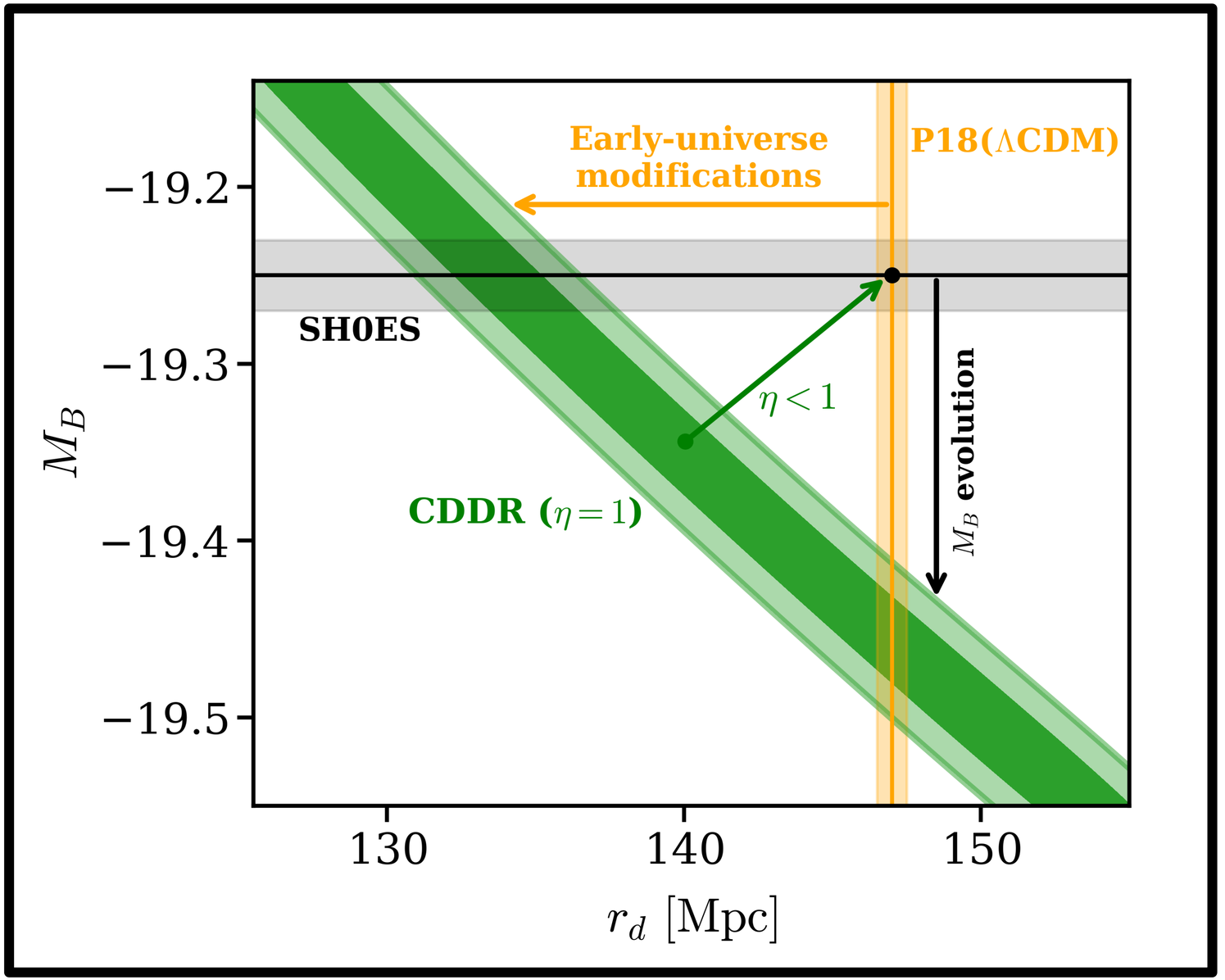
Way ahead on road to Hubble Tension …

- Independent probes and systematics: *Strong lensing, GWs, FRBs, GRBs,…*
- Early-time physics …? Early+Late modifications …?

Interesting prospects from this work …

- Calibration free probes: breaking the $r_d - M_b$ degeneracy!
- Observational tests of Non-metric theories with reciprocity violation.

Thank You!


Early-universe modifications
P18(ΛCDM)
SH0ES
$\eta < 1$
CDDR ($\eta = 1$)
M_B evolution
M_B
−19.2
−19.3
−19.4
−19.5
130
140
150
r_d [Mpc]