The No-Inner-Horizon Theorem and Interior Structure of Anisotropic Black Holes

Based on Yuxuan Peng & Yu-Sen An, On the absence of inner horizons for anisotropic black holes. *Eur. Phys. J. C* **86**, 445 (2026).





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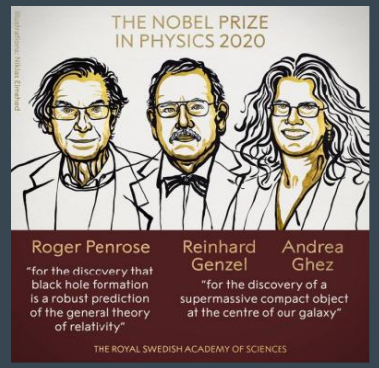
东华理工大学，江西南昌

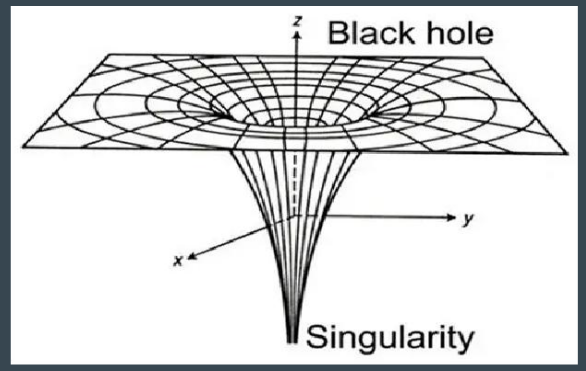
Outline

01	**Motivation**	Why black hole interiors?
02	**Known mechanisms**	Matter hair and anisotropiy excluding inner horizons
03	**A general theorem**	How anisotropy does it
04	**Applications**	Bianchi II horizon and flat-horizon black holes

Why study black-hole interiors?

- Singularity formation and the nature of singularities are fundamental problems in gravity and quantum physics.







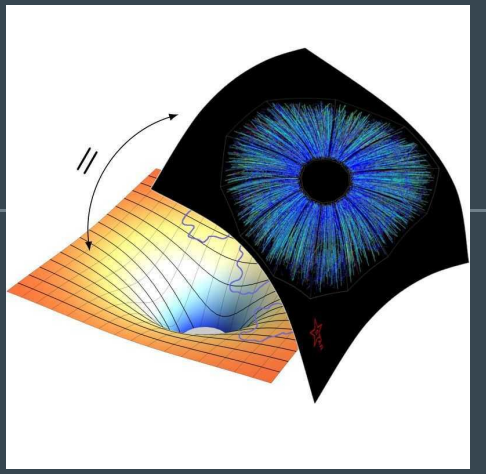


斯蒂芬·霍金(Stephen Hawking)1942-2018

Interior:
inner horizons?
singularities?

Event horizon

Why study black-hole interiors?



Holographic observables
—entanglement entropy and complexity—probe the black-hole interior.

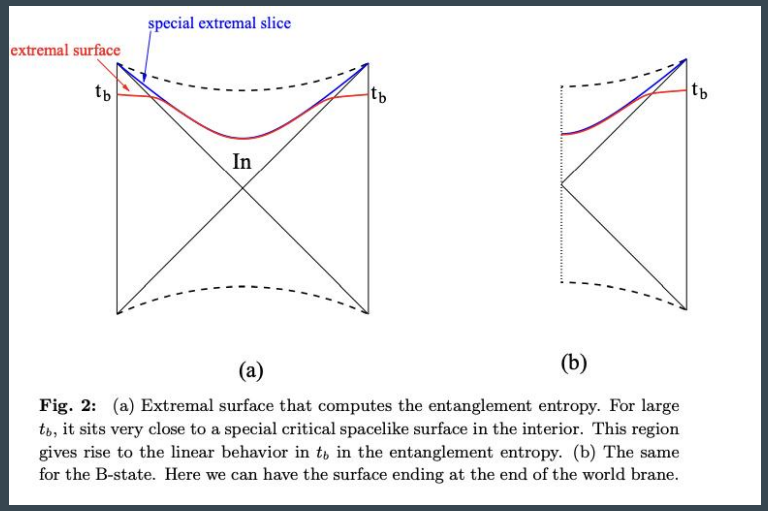


Fig. 2: (a) Extremal surface that computes the entanglement entropy. For large t_b, it sits very close to a special critical spacelike surface in the interior. This region gives rise to the linear behavior in t_b in the entanglement entropy. (b) The same for the B-state. Here we can have the surface ending at the end of the world brane.

Time-dependent entanglement entropy

Hartman & Maldacena, JHEP (2013);

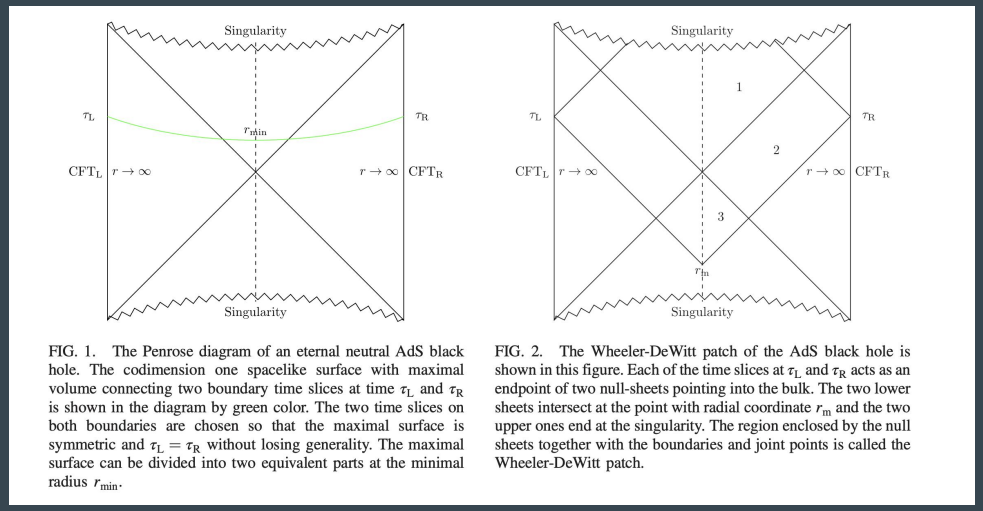


FIG. 1. The Penrose diagram of an eternal neutral AdS black hole. The codimension one spacelike surface with maximal volume connecting two boundary time slices at time τ_L and τ_R is shown in the diagram by green color. The two time slices on both boundaries are chosen so that the maximal surface is symmetric and $\tau_L = \tau_R$ without losing generality. The maximal surface can be divided into two equivalent parts at the minimal radius r_{min}.

FIG. 2. The Wheeler-DeWitt patch of the AdS black hole is shown in this figure. Each of the time slices at τ_L and τ_R acts as an endpoint of two null-sheets pointing into the bulk. The two lower sheets intersect at the point with radial coordinate r_m and the two upper ones end at the singularity. The region enclosed by the null sheets together with the boundaries and joint points is called the Wheeler-DeWitt patch.

An, Cai & Peng, PRD (2018)

Holographic complexity

Stanford & Susskind, PRD (2014)

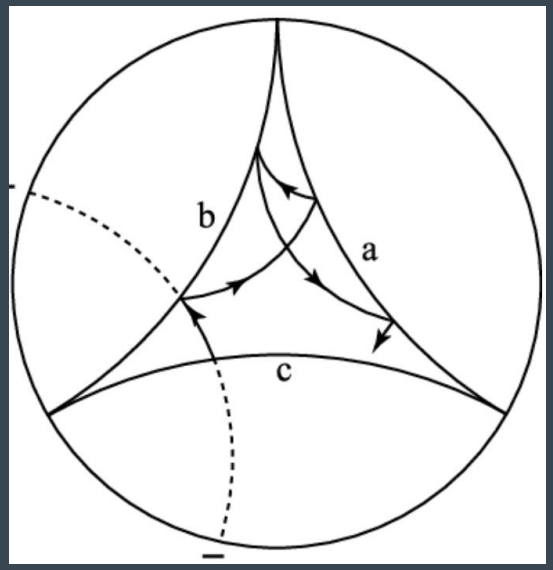
Kasner geometry near a spacelike singularity

$$ds^2 = -d\tau^2 + \sum_i \tau^{2p_i} (dx^i)^2$$

Kasner exponents $\sum_i p_i = 1, \qquad \sum_i p_i^2 = 1$

Anisotropic geometry near the singularity

- Different directions expand or contract at different rates.
- Matter can drive transitions and oscillations between Kasner epochs.



cosmological billiards

Damour, Henneaux & Nicolai, CQG 2003

Kasner (1921); Lifshitz & Khalatnikov (1970); Belinsky, Khalatnikov & Lifshitz (1970).

These Kasner dynamics have not been fully understood, especially when matter fields are present

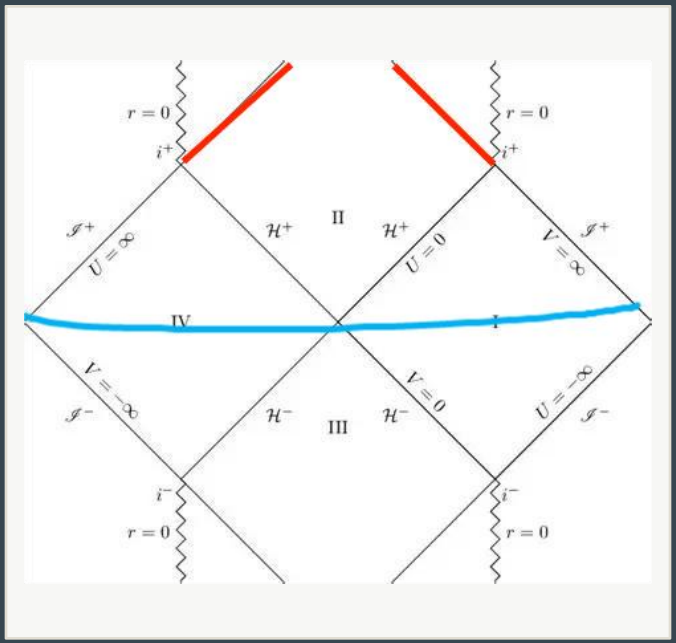
Hartnoll & Neogi SciPost Phys 2023

RG Cai，MN Duan，L Li，FG Yang, JHEP 2024.....

Inner horizons threaten predictability

REISSNER–NORDSTRÖM INTERIOR

- An inner horizon is a Cauchy horizon: initial data do not uniquely determine what lies beyond it.
- The spacetime is not globally hyperbolic, in tension with strong cosmic censorship conjecture.



Perturbations → infinite blueshift → mass inflation

Removing the Cauchy horizon is consistent with a deterministic, spacelike interior.

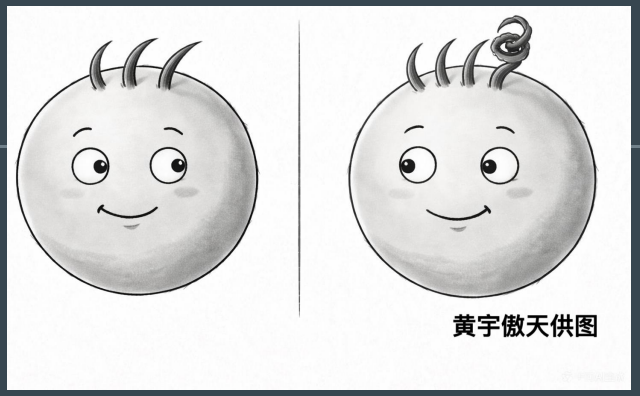
Poisson & Israel, PRL (1989); PRD (1990).

01 What is exactly left after destruction of the inner horizon?

Known mechanisms for matter excluding inner horizons

Matter hair · Einstein–Rosen bridge collapse · Kasner dynamics

Neutral scalar hair removes the inner horizon



METRIC ANSATZ

$$\mathrm{d}s^2 = \frac{1}{z^2}\left(-f(z)e^{-\chi(z)}\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{f(z)} + \mathrm{d}x^2 + \mathrm{d}y^2\right)$$

SCALAR EOM

$$z^2 e^{\chi/2}\left(e^{-\chi/2}z^{-2}f\phi'\right)' = \frac{m^2}{z^2}\phi$$

Gravitational duals to the grand canonical ensemble abhor Cauchy horizons

Sean A. Hartnoll[1], Gary T. Horowitz[2], Jorrit Kruthoff[1] and Jorge E. Santos[3,4]

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INTEGRATE BETWEEN THE ASSUMED OUTER AND INNER HORIZONS

$$0 = \int_{z_{\mathcal{H}}}^{z_{\mathcal{I}}}\left(\frac{f e^{-\chi/2}\phi\phi'}{z^2}\right)'\mathrm{d}z = \int_{z_{\mathcal{H}}}^{z_{\mathcal{I}}}\frac{e^{-\chi/2}}{z^4}\left[m^2\phi^2 + z^2 f(\phi')^2\right]\mathrm{d}z \quad (m^2 < 0)$$

The boundary term vanishes; for , the integrand is negative ⇒ no-inner-horizon theorm.

Hartnoll, Horowitz, Kruthoff & Santos, JHEP (2020).

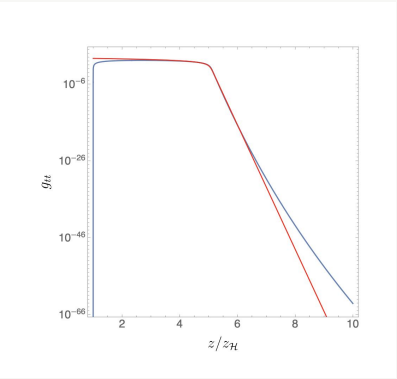
The interior left behind by scalar hair

NEAR-SINGULARITY KASNER FORM

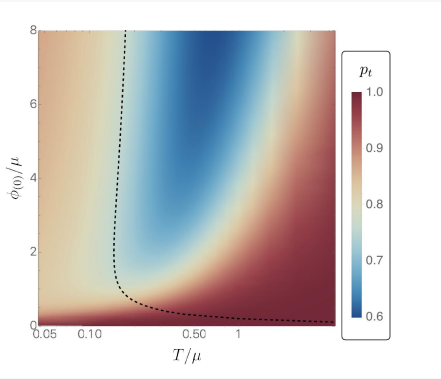
$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + c_t \tau^{2p_t}\,\mathrm{d}t^2 + c_x \tau^{2p_x}\left(\mathrm{d}x^2 + \mathrm{d}y^2\right)$$

$$\phi = -\sqrt{2p_\phi}\log\tau\,,\quad p_t + 2p_x = 1\,,\quad p_\phi^2 + p_t^2 + 2p_x^2 = 1$$

The scalar kinetic term contributes p_ϕ to the second Kasner constraint.



Einstein–Rosen bridge collapse



Constant Kasner exponents

Hartnoll, Horowitz, Kruthoff & Santos, JHEP (2020).

Charged scalar hair: charge and Kasner dynamics

CONSERVED CHARGE FROM SCALING SYMMETRY

$$\mathcal{Q}(z) = z^{2-d} e^{\chi/2} \left[z^{-2} (f e^{-\chi})' - Z A_t A_t' \right] + 2k(d-1) \int^z y^{-d} e^{-\chi(y)/2} \mathrm{d}y$$

EVALUATED AT TWO ASSUMED HORIZONS

$$\frac{f'(z_H)}{z_H^d} e^{-\chi(z_H)/2} - \frac{f'(z_I)}{z_I^d} e^{-\chi(z_I)/2} = 0$$

For $k = 0$: $f'(z_H) < 0$, whereas $f'(z_I) > 0$. The equality cannot hold.

Contradiction ⇒ no-inner-horizon theorem

Cai, Li & Yang, JHEP (2021); Hartnoll et al., SciPost Phys. (2021); Cai et al., JHEP (2024).



No Inner-Horizon Theorem for Black Holes with Charged Scalar Hairs

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Diving into a holographic superconductor

Sean A. Hartnoll[1], Gary T. Horowitz[2], Jorrit Kruthoff[1] and Jorge E. Santos[3,4]

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Towards classifying the interior dynamics of charged black holes with scalar hair

Rong-Gen Cai[1,2,3], Mei-Ning Duan[2,4], Li Li[1,2,4,5], Fu-Guo Yang[1,4]

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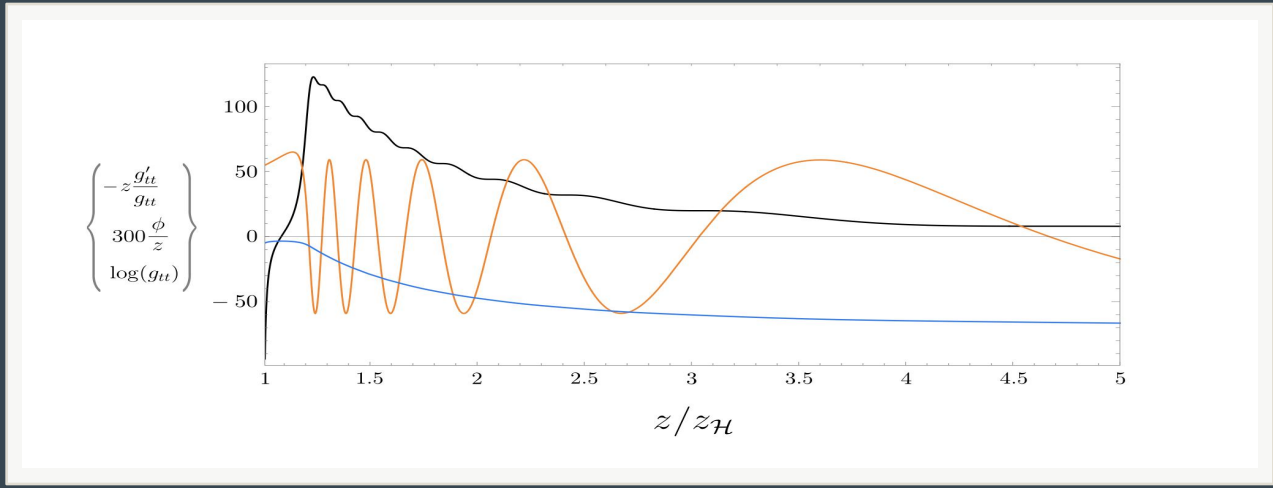
No inner horizon: scalar oscillations and Kasner transitions

For $k = 0$: conserved scaling charge ⇒ no second zero of $f(z)$ ⇒ no inner Cauchy horizon.

$$ds^2 = -d\tau^2 + c_t \tau^{2p_t} dt^2 + c_s \tau^{2p_s} d\Sigma_{d,k}^2$$

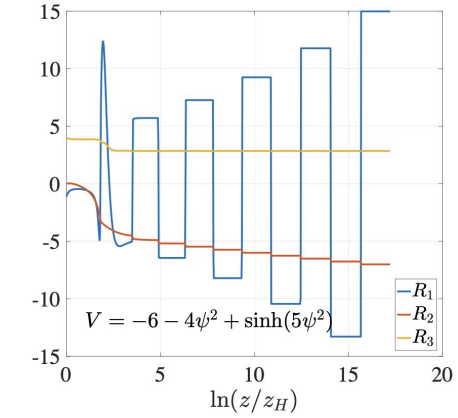
$$\phi = -p_\phi log\tau, p_t + dp_s = 1, p_\phi^2 + p_t^2 + dp_s^2 = 1$$

JOSEPHSON OSCILLATIONS



Josephson oscillations of the scalar hair, driven by electromagnetic–scalar coupling.

SUCCESSIVE KASNER EPOCHS



Infinitely many Kasner transitions/inversions can develop toward the singularity.

Cai, Li & Yang, JHEP (2021); Hartnoll et al., SciPost Phys. (2021).

Charged vector hair and anisotropic dynamics

VECTOR-HAIR ANSATZ

$$ds^2 = \frac{1}{z^2}\left[-f(z)e^{-\chi(z)}\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{f(z)} + u(z)\mathrm{d}x^2 + \mathrm{d}\vec{y}_{d-1}^2\right],$$

$$\rho_\nu dx^\nu = \rho_x(z)dx, \quad A_\nu dx^\nu = A_t(z)dt\,.$$

$u(z)$ describes anisotropic scaling; $\rho_x(z)$ is the vector condensate.

MATTER LAGRANGIAN

$$L = R - 2\Lambda - \frac{1}{4}F^2 - \frac{1}{2}\rho^\dagger_{\mu\nu}\rho^{\mu\nu} - m^2\rho^\dagger_\mu\rho^\mu + iq\gamma\rho_\mu\rho^\dagger_\nu F^{\mu\nu}$$

Conserved charge excludes the RN inner horizon.

Interior: ER collapse, Josephson oscillations, and Kasner transitions.

Cai, Ge, Li & Yang, JHEP (2022); Cai et al., JHEP (2025).

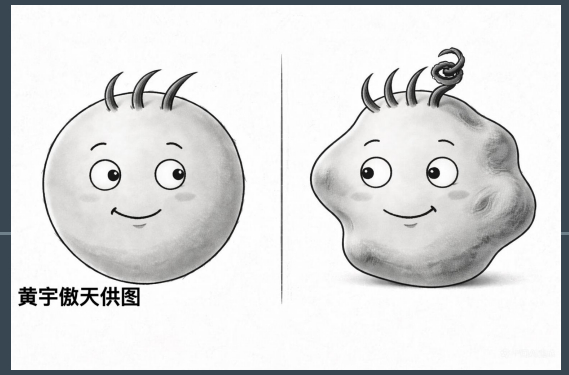
02

Indications for anisotropy excluding inner horizons

Bianchi VII_0 horizons

Flat horizons with anisotropic matter

Bianchi VII$_0$: anisotropy without hair



BIANCHI VII$_0$ METRIC ANSATZ

$$ds^2 = -gf^2dt^2 + \frac{dr^2}{g} + h^2(\boldsymbol{\theta}^3)^2 + r^2(e^{-2\alpha}(\boldsymbol{\theta}^1)^2 + e^{2\alpha}(\boldsymbol{\theta}^2)^2)$$

INTEGRAL BETWEEN THE OUTER AND ASSUMED INNER HORIZON: from the gravitational eoms

$$0 = \int_{r_i}^{r_h} dr \left(\frac{r^2 fgh\alpha'}{\alpha}\right)' = \int_{r_i}^{r_h} dr \left(\frac{k^2 r^2 f}{h}\frac{\sinh 4\alpha}{\alpha} - \frac{r^2 fhg\alpha'^2}{\alpha^2}\right)$$

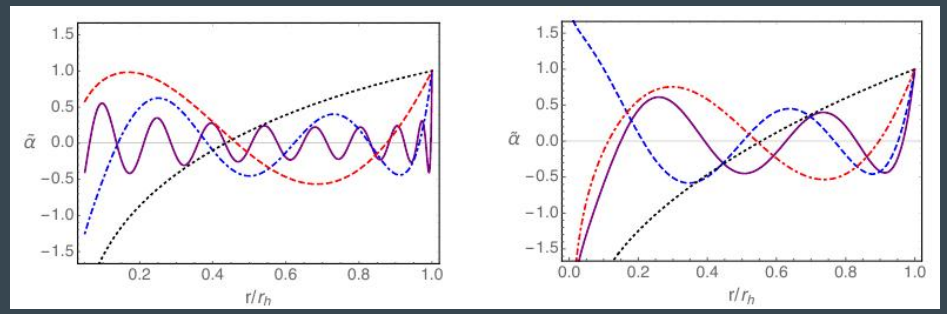
Interior of helical black holes

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The boundary terms vanish; the remaining integral supplies the contradiction.



Oscillating scale factors;
a stable Kasner epoch near the singularity

Liu & Lyu, JHEP (2022).

Shear introduced by axions: pressure anisotropy on a flat horizon

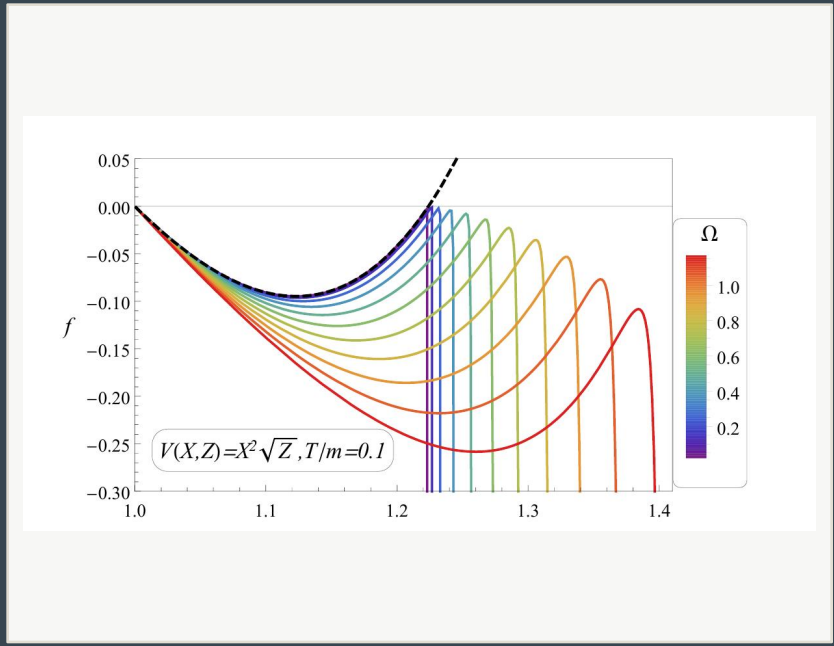
SHEARED-AXION ANSATZ

$$ds^2 = \frac{1}{z^2}\left[-f(z)e^{-\chi(z)}\mathrm{d}t^2 + \frac{1}{f(z)}\mathrm{d}z^2 + \gamma_{ij}(z)\mathrm{d}x^i dx^j\right]$$

$$\phi^I = M^I{}_i x^i\,, \quad i = 1\,,2$$

$$M^I{}_i = \begin{pmatrix} \cosh(\Omega/2) & \sinh(\Omega/2) \\ \sinh(\Omega/2) & \cosh(\Omega/2) \end{pmatrix} \qquad \gamma_{ij}(z) = \begin{pmatrix} \cosh[h(z)] & \sinh[h(z)] \\ \sinh[h(z)] & \cosh[h(z)] \end{pmatrix}$$

Ω controls the shear; $h(z)$ controls metric anisotropy.



Increasing shear removes the second zero of the blackening function.

No shear: an inner horizon can remain.

Shear: an integral criterion removes it.

Xu, Li & Li, JHEP (2026).

03

A general theorem

Static anisotropic black holes · A unified integral criterion

From isolated examples to a general criterion

QUESTION

Can anisotropic scaling and horizon geometry exclude an inner horizon in a unified way?

RESULT

A nonzero cumulative effective-pressure difference excludes an inner horizon.



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https://doi.org/10.1140/epjc/s10052-026-15638-8

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

On the absence of inner horizons for anisotropic black holes

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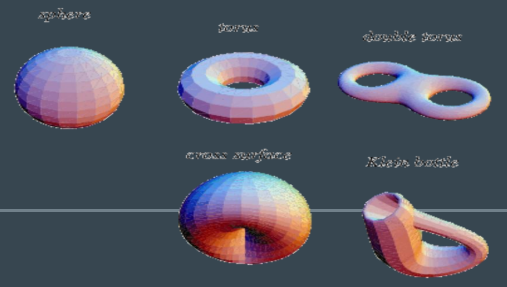
Received: 17 February 2026 / Accepted: 1 April 2026
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Abstract We establish a general no-inner horizon theorem for a wide class of static anisotropic black holes in Einstein gravity. Working in an orthonormal frame, we show that anisotropic scaling inevitably induces a non-vanishing "effective pressure difference" between spatial directions apart from the radial direction. We prove that, provided this effective pressure difference satisfies a simple integral condition which holds in a general setting, inner (Cauchy) horizons are excluded. As an application, we demonstrate that a large variety of black holes which possess homogeneous but anisotropic three-dimensional horizons can not have inner horizons. These anisotropic horizon geometries fall within

3 Existence of inner horizons for various Bianchi type black holes
3.1 General analysis of all Bianchi types
3.2 Example: the charged black hole with a Bianchi type II horizon
4 Conclusion and discussion
Appendix A: Existence of inner horizons for anisotropic black branes
1. The holographic solid model under shear strain
2. The holographic p-wave superconductor
References

Yuxuan Peng & Yu-Sen An, "On the Absence of Inner Horizons for Anisotropic Black Holes" (2026).

Static anisotropic black holes with nontrivial horizons



STATIC ANISOTROPIC METRIC

$$\mathrm{d}s^2 = -F(z)\mathrm{d}t^2 + G(z)\mathrm{d}z^2 + \sum_{i=1}^{n} h_i(z)^2(\boldsymbol{\theta}^i)^2\,, \qquad F(z) \equiv -f(z)^2\,, \qquad G(z) \equiv -g(z)^2$$

HORIZON STRUCTURE

$$\mathrm{d}\boldsymbol{\theta}^i = -\frac{1}{2}c_{jk}{}^i\,\boldsymbol{\theta}^j \wedge \boldsymbol{\theta}^k$$

c^i_{jk} determine the homogeneous horizon geometry.

ORTHONORMAL FRAME

$$\boldsymbol{e}^0 = f(z)\mathrm{d}t\,, \quad \boldsymbol{e}^{\hat{z}} = g(z)\mathrm{d}z\,, \quad \boldsymbol{e}^i = h_i(z)\boldsymbol{\theta}^i$$

Distinct $h_i(z)$ encode anisotropic scaling in the transverse directions.

For n = 3, the homogeneous horizon geometries are classified by the nine Bianchi types.

Luigi Bianchi, Sugli spazi a tre dimensioni che ammettono un gruppo continuo di movimenti, Memorie di Matematica e di Fisica della Societa Italiana delle Scienze, Serie Terza, vol. 11, p.267-352, 1898.

Effective pressure and the key equation

TRANSVERSE RICCI DIFFERENCE FROM THE EINSTEIN EQUATIONS

$$R_{ii} - R_{jj} = \frac{1}{\sqrt{-g}}\left[-\sqrt{-g} g^{zz} (\ln \frac{h_i}{h_j})' \right]' + X_{ii} - X_{jj} = \kappa^2 (T_{ii} - T_{jj})$$

$$T_{\text{eff}\, ij} \equiv T_{ij} - \frac{1}{\kappa^2} X_{ij}$$

X is just the curvature of the constant t,z surface

$$\Delta_{ij} \equiv \kappa^2 (T_{\text{eff}\, ii} - T_{\text{eff}\, jj})$$

$$\frac{1}{\sqrt{-g}}\left[-\sqrt{-g}\, g^{zz} \left(\ln \frac{h_i}{h_j} \right)' \right]' = \Delta_{ij}$$

Anisotropic scaling and anisotropic effective pressures come in pairs.

No-inner-horizon theorem: the integral criterion

At the event horizon z_h and a hypothetical inner horizon z_i, $g^{zz} = 0$. Integration gives

$$0 = \left[-\sqrt{-g}\, g^{zz} \left(\ln \frac{h_i}{h_j} \right)' \right]_{z_h}^{z_i} = \int_{z_h}^{z_i} \mathrm{d}z \sqrt{-g}\, \Delta_{ij}$$

If some i, j have $h_i \propto h_j$ and, for every $z > z_h$ inside the horizon,

$$\int_{z_h}^{\tilde{z}} \mathrm{d}z \sqrt{-g}\, \Delta_{ij} \neq 0$$

the cumulative effect of Δ leads to a contradiction: no inner horizon exists.

What the criterion means

If the effective-pressure difference is never cancelled by radial integration, an inner horizon cannot form.

A simple sufficient condition: one Δ_{ij} is sign-definite throughout the interior.

Two simple cases:

1 ISOTROPIC MATTER PRESSURE

$$T_{ii} = T_{jj} \implies \Delta_{ij} = -(X_{ii} - X_{jj})$$

2 ISOTROPIC FLAT HORIZON

$$X_{ij} = 0 \implies \Delta_{ij} = \kappa^2 (T_{ii} - T_{jj})$$

Application: homogeneous Bianchi inner horizons: existence

Isotropic matter pressure: $\Delta_{ij} = -(X_{ii} - X_{jj})$

BIANCHI TYPES classified by c^{i}_{jk}	CRITERION	INNER HORIZON
I, V, IX	Vanishing Δ in the isotropic limit	Possible
II, III, IV, VI	Sign-definite Δ	Excluded
VII	Sign-definite combination for most types including the helical black hole	Excluded
VIII	Sign-definite Δ for only one different h_i	Excluded

Bianchi II: an analytical and numerical check

$$\Delta_{1j} = -\frac{h_1^2}{h_2^2 h_3^2} < 0 \qquad (j \neq 1)$$

The sign-definite difference excludes an inner horizon.

NUMERICAL EVIDENCE

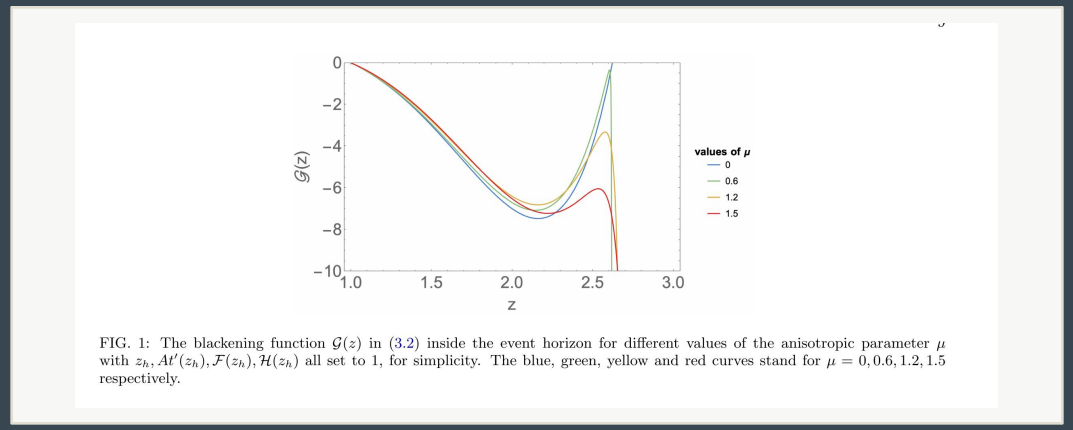


FIG. 1: The blackening function $\mathcal{G}(z)$ in (3.2) inside the event horizon for different values of the anisotropic parameter μ with $z_h, At'(z_h), \mathcal{F}(z_h), \mathcal{H}(z_h)$ all set to 1, for simplicity. The blue, green, yellow and red curves stand for $\mu = 0, 0.6, 1.2, 1.5$ respectively.

No second zero for the lapse funciton inside the event horizon

ANALYTIC KASNER EPOCH

$$p_t = \frac{n_{\mathcal{H}}^2 - 1}{n_{\mathcal{H}}^2 + 2}, \quad p_1 = \frac{1 + 2n_{\mathcal{H}}}{n_{\mathcal{H}}^2 + 2}, \quad p_2 = \frac{1 - n_{\mathcal{H}}}{n_{\mathcal{H}}^2 + 2}$$

$$p_t + p_1 + 2p_2 = 1, \quad p_t^2 + p_1^2 + 2p_2^2 = 1$$

Numerics indicate $n_H > 1$; the Kasner exponents therefore remain constant.

Charged Solv/Nil black-hole solution: Arias & Landea, JHEP (2017); analysis from Peng & An (2026).

Flat horizons: the sheared-axion model

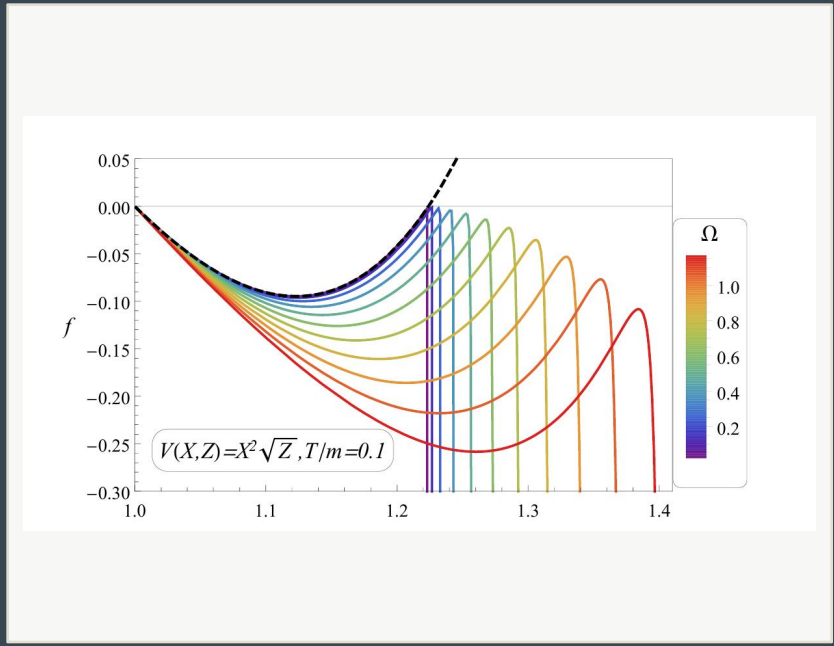
PRESSURE DIFFERENCE: NOT SIGN-DEFINITE BY ITSELF

$$\begin{aligned}\Delta_{12} &= \kappa^2(T_{11}-T_{22}) = m^2 z^2 \frac{\partial V}{\partial X}(e^{\Omega_1-h_1}-e^{\Omega_2-h_2}) \\ &= 2m^2 z^2 \frac{\partial V}{\partial X}\sinh(\Omega-h)\,.\end{aligned}$$

TOTAL-DERIVATIVE EQUATION

$$\frac{\left[-\sqrt{-g}g^{zz}h'\right]'}{\sinh(\Omega-h)} = 2\sqrt{-g}m^2 z^2 \frac{\partial V}{\partial X}\,.$$

After constructing the total derivative, radial integration gives a positive-definite contribution ⇒ no inner horizon.



Shear removes the would-be inner horizon.

Xu, Li & Li, JHEP (2026); criterion recovered from Peng & An (2026).

Flat horizons: vector hair needs a different argument

EFFECTIVE PRESSURE DIFFERENCE FOR CHARGED VECTOR HAIR

$$\Delta_{ij} = \kappa^2(T_{ii} - T_{jj}) = \partial_z \rho^\dagger_{x^i} \partial_z \rho_{x^i} g^{zz} g^{x^i x^i} + (q^2 A_t A^t + m^2) \rho^\dagger_{x^i} \rho_{x^i} g^{x^i x^i}$$

Gradient term: $g^{zz} < 0$ inside $\Rightarrow$ negative.

Mass / electrostatic term: positive inside.

The sign of Δ_{ij} is not fixed, so the integral criterion is not directly conclusive.

No inner horizon follows instead from the conserved-charge criterion.

Cai, Ge, Li & Yang, JHEP (2022); Cai et al., JHEP (2025).

Conclusions

- Theorem: cumulative effective-pressure anisotropy can exclude an inner horizon.
  A new mechanism in addition to the known scalar eom and the conserved charge proof.

- Mechanism: horizon anisotropic geometry alone can suffice; matter hair can be absent.

- Applications: most Bianchi types and shear axions; possibly other types of anisotropic matter models

$$\text{nonzero cumulative anisotropy} \implies \text{no inner horizon}$$

Outlook

- Which anisotropic matter pressures satisfy the criterion directly? Which do not?
- How does the theorem change in modified gravity?
- Can the argument be extended to stationary black holes?
- What controls Kasner dynamics in general Bianchi interiors?



Thank you!