Neutron Stars More Compact Than Black Holes

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Phys.Rev.D 114, L021504 (2026) with H. Lu, Y. Gao, R. Xu, L. Shao, H. Yu
Phys.Rev.D 114, 024049 (2026) with L. Liang, Z. Luo, H. Yu
2612.***** with L. Liang, F. Yu, D. Doneva, S. Yazadjiev, H. Yu in perparation

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- Physical Background and Motivation
- QTG and the Uniqueness of the Schwarzschild BH
- Stellar Configurations and Properties
- Radial Stability
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- Conclusion and Outlook

Physical Background and Motivation

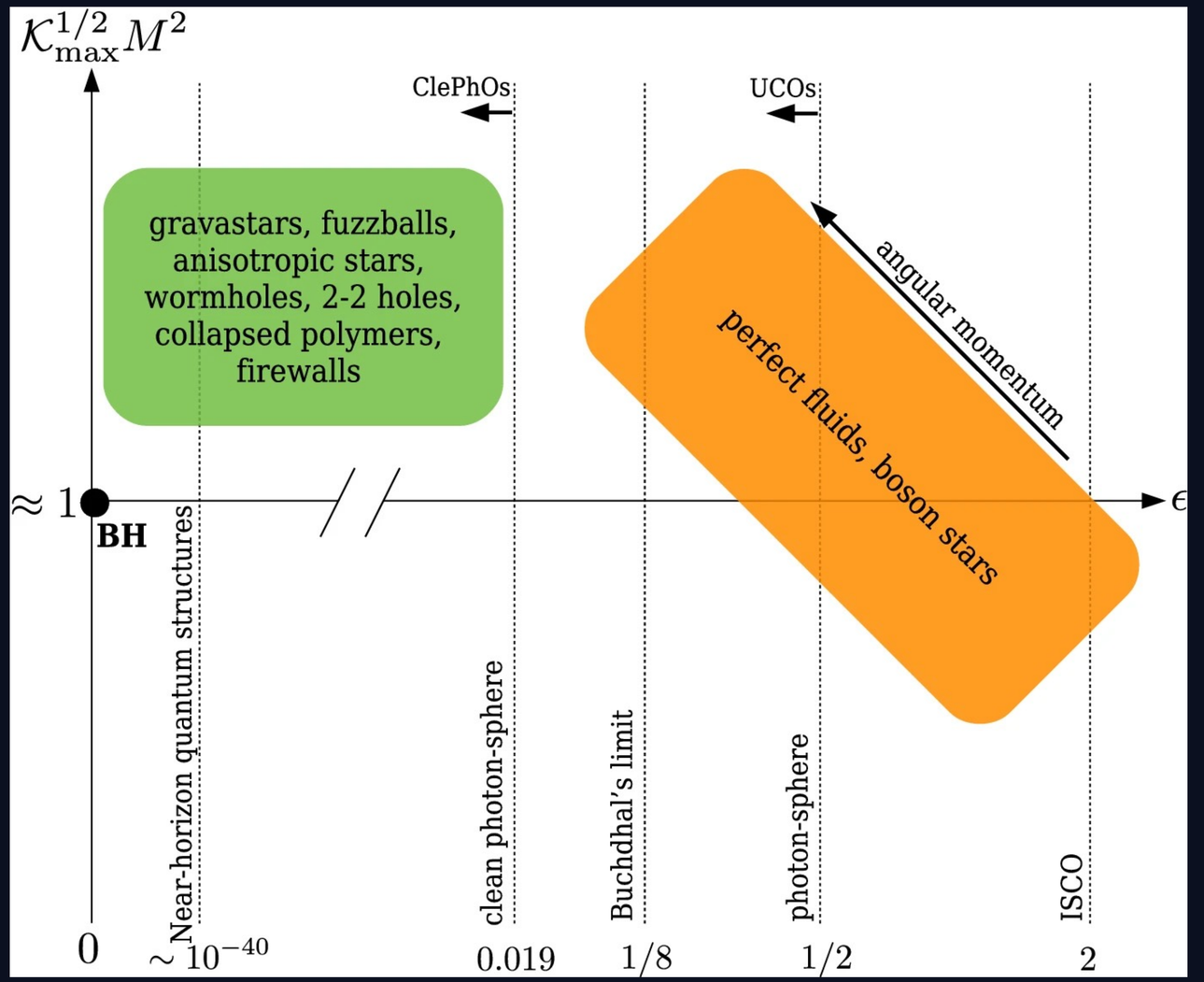
BHs set the compactness limit in GR

$$\text{compactness} = \frac{\text{mass}}{\text{radius}}$$

$$C_{\rm BH} = 0.5\,, \quad \epsilon_{\rm BH} = 0$$

$$C_{\rm NS} \approx 0.2\,, \quad \epsilon_{\rm NS} \approx 1.5$$

$$R = 2M(1+\epsilon)$$



Cardoso & Pani, Living Rev. Rel., 2019]

What happens when $R < 2M$ in GR?

Formation perspective—Penrose singularity theorem

A trapped surface forms.

Under the standard assumptions, continued collapse leads to singularity.

Existence perspective—Birkhoff's theorem

The vacuum exterior must be Schwarzschild.

for a static, spherically symmetric star

If $R < 2M$, the stellar surface lies inside the Schwarzschild horizon.

GR excludes such stars, irrespective of the matter!

How can a star be more compact than BH?

Break Birkhoff uniqueness

Effective higher-derivative gravity

Higher-curvature gravity

Nonminimally coupled gravity

Quasi-topological gravity

$$L = R + \lambda L_{\mathrm{QTG}}$$

$$L_{\mathrm{QTG}} = R^3 - 6RR_{\mu\nu}R^{\mu\nu} + 8R_{\mu}{}^{\nu}R_{\nu}{}^{\gamma}R_{\gamma}{}^{\mu}$$

[Li, Liu & Lu, JHEP, 2018] [Hennigar, Kubiznak & Mann, PRD, 2017]

QTG and the Uniqueness of the Schwarzschild BH

Why quasi-topological gravity?

- SSS vacuum solution is non-unique
  SSS black-hole solution is unique (Schwarzschild)

Straightforward NS–BH compactness comparison

- vacuum in strong-field regions is non-unique
  vacuum in weak-field regions is unique (Schwarzschild)

Compatible with stringent Solar-System tests

QTG has no ghost for SSS spacetime!

SSS: static, spherically symmetric

QTG has no ghost for SSS spacetime!

Theory $L = R + (e_1 R^3 + e_2 R R_{\mu\nu} R^{\mu\nu} + e_3 R_\mu{}^\nu R_\nu{}^\gamma R_\gamma{}^\mu)$

$e_1 = \lambda$
$e_2 = -6\lambda$
$e_3 = 8\lambda$

Weak field limit

$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$

$\bar{R}_{\mu\nu\rho\sigma} \propto \bar{g}_{\mu\rho}\bar{g}_{\nu\sigma} - \bar{g}_{\mu\sigma}\bar{g}_{\nu\rho}\,, \qquad \bar{R}_{\mu\nu} = a\bar{g}_{\mu\nu}$

$$\delta\tilde{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\delta R + a^2(48e_1 + 20e_2 + 9e_3)\delta\tilde{R}_{\mu\nu} - \frac{a^2}{2}(4e_2 + 3e_3)\bar{g}_{\mu\nu}\delta R$$

$$\delta\tilde{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\delta R = 0$$

Linearized EOM

$$-a(4e_2 + 3e_3)\bar{\Delta}_L(\delta\tilde{R}_{\mu\nu} - \frac{1}{2}\delta R)$$

$= 0$

$$-a(24e_1 + 8e_2 + 3e_3)(\bar{\nabla}_\mu\bar{\nabla}_\nu - \bar{g}_{\mu\nu}\bar{\Box})\delta R = 0$$

$h = f = 1 - \frac{2M}{r}$

No extra mode!

On the uniqueness of Sch BH

Metric $ds^2 = -h\,dt^2 + f^{-1}\,dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$

Near horizon

$$h = h_1\,[(r-r_0) + h_2(r-r_0)^2 + h_3(r-r_0)^3 + \ldots]$$

$$f = f_1(r-r_0) + f_2(r-r_0)^2 + f_3(r-r_0)^3 + \ldots$$

$$h_2 = -\frac{1}{r_0}, \quad h_3 = \frac{1}{r_0^2},$$

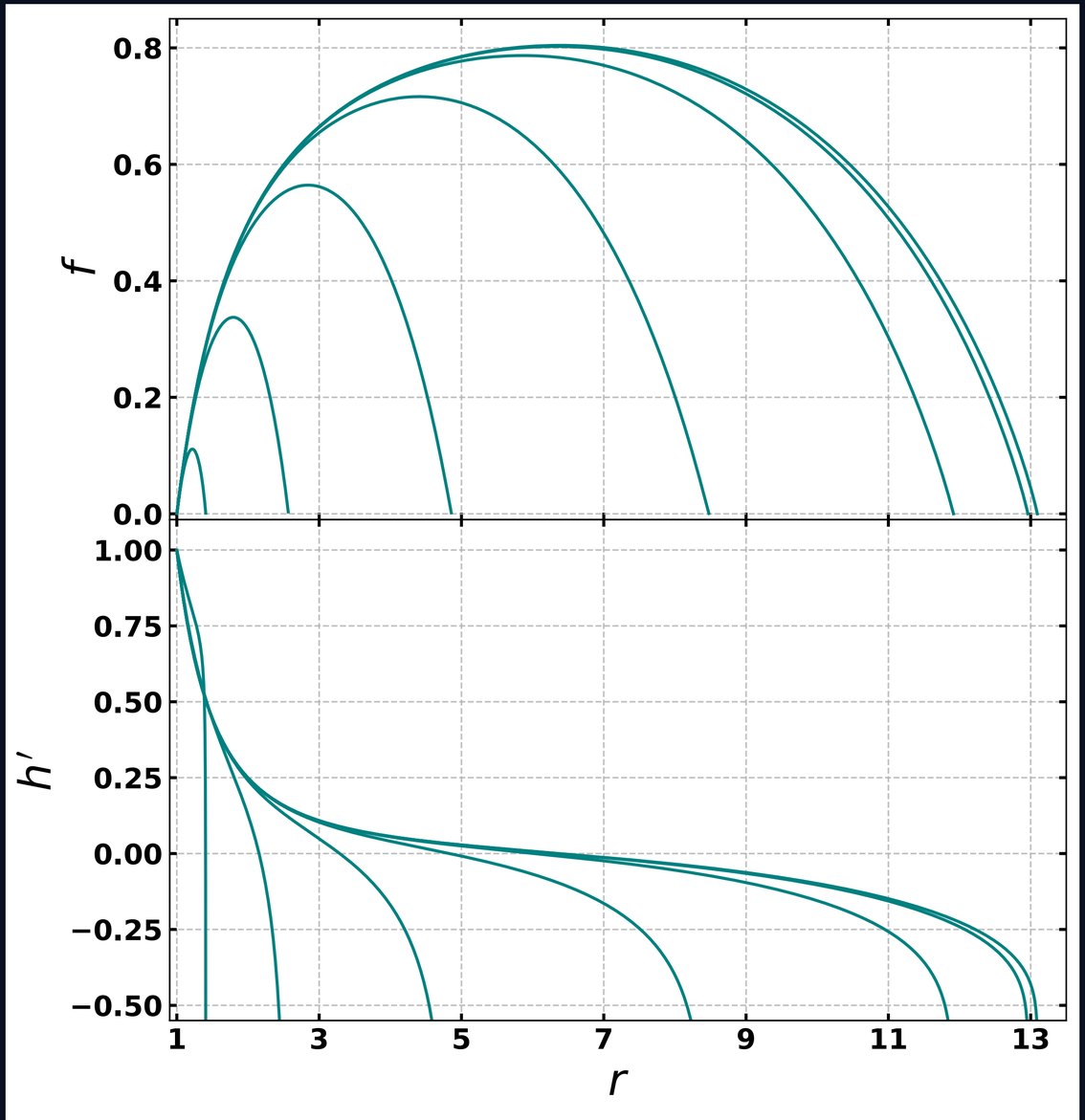
$$f_1 = \frac{1}{r_0}, \quad f_2 = -\frac{1}{r_0^2}, \quad f_3 = \frac{1}{r_0^3},$$

$$h_2 = -\frac{1}{r_0} + \frac{r_0^3}{24\lambda}, \quad h_3 = \frac{1}{r_0^2} - \frac{3r_0^2}{40\lambda} + \frac{r_0^6}{288\lambda^2},$$

$$f_1 = \frac{1}{r_0}, \quad f_2 = -\frac{1}{r_0^2} - \frac{r_0^2}{8\lambda}, \quad f_3 = \frac{1}{r_0^3} - \frac{r_0}{40\lambda} - \frac{r_0^5}{288\lambda^2},$$

weak field $\lim_{r\to\infty} h(r) = \lim_{r\to\infty} f(r) = 1 - 2M/r$

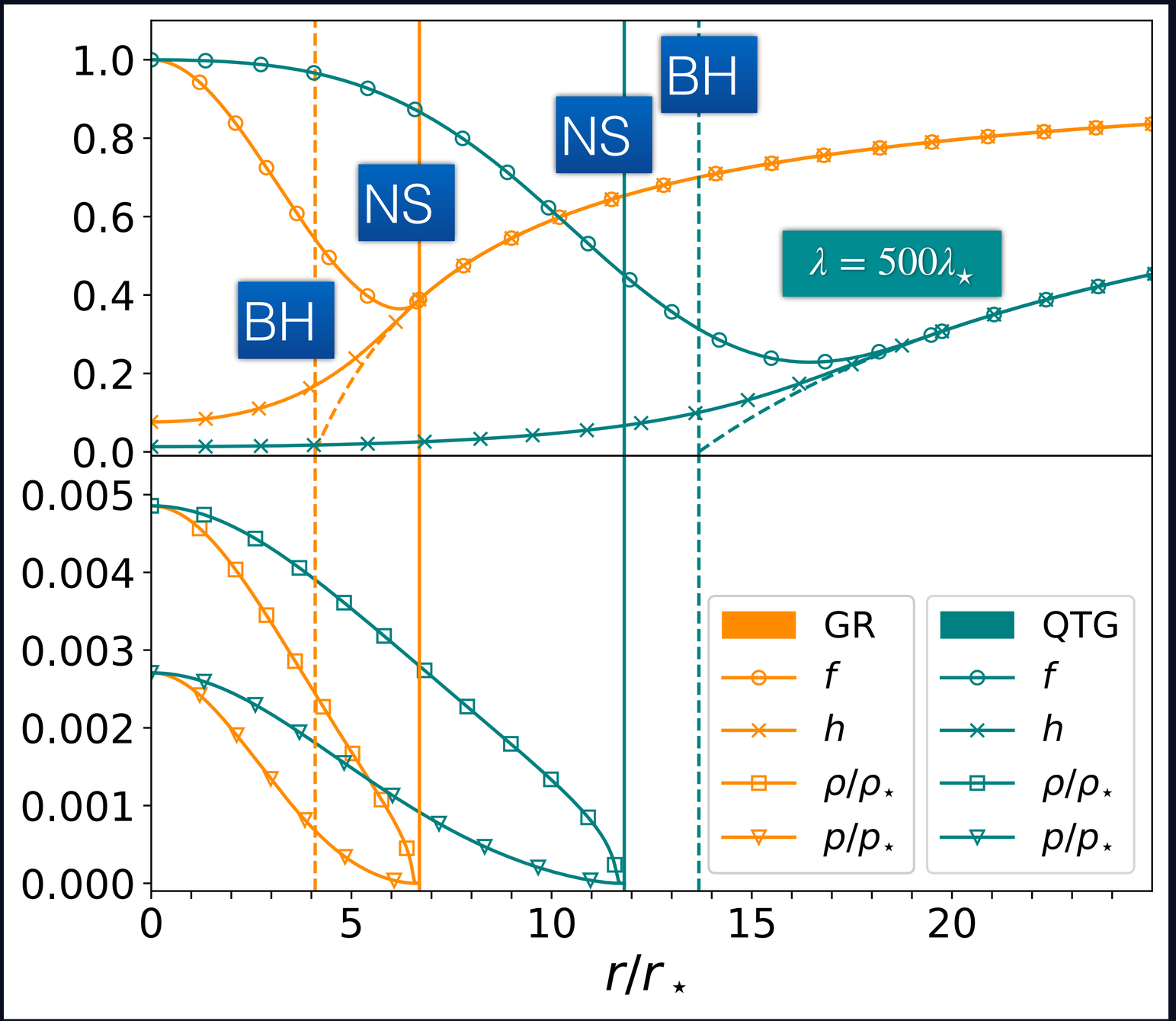
On the uniqueness of Sch BH



$$ds^2 = -h\,dt^2 + f^{-1}\,dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Stellar Configurations and Properties

Stellar solution



$$M = 6.84 M_\odot$$

$$R_{\rm NS} = 11.82 r_\star\,, \quad R_{\rm BH} = 13.68 r_\star$$

$$C_{\rm NS} \approx 0.58 > 0.5$$

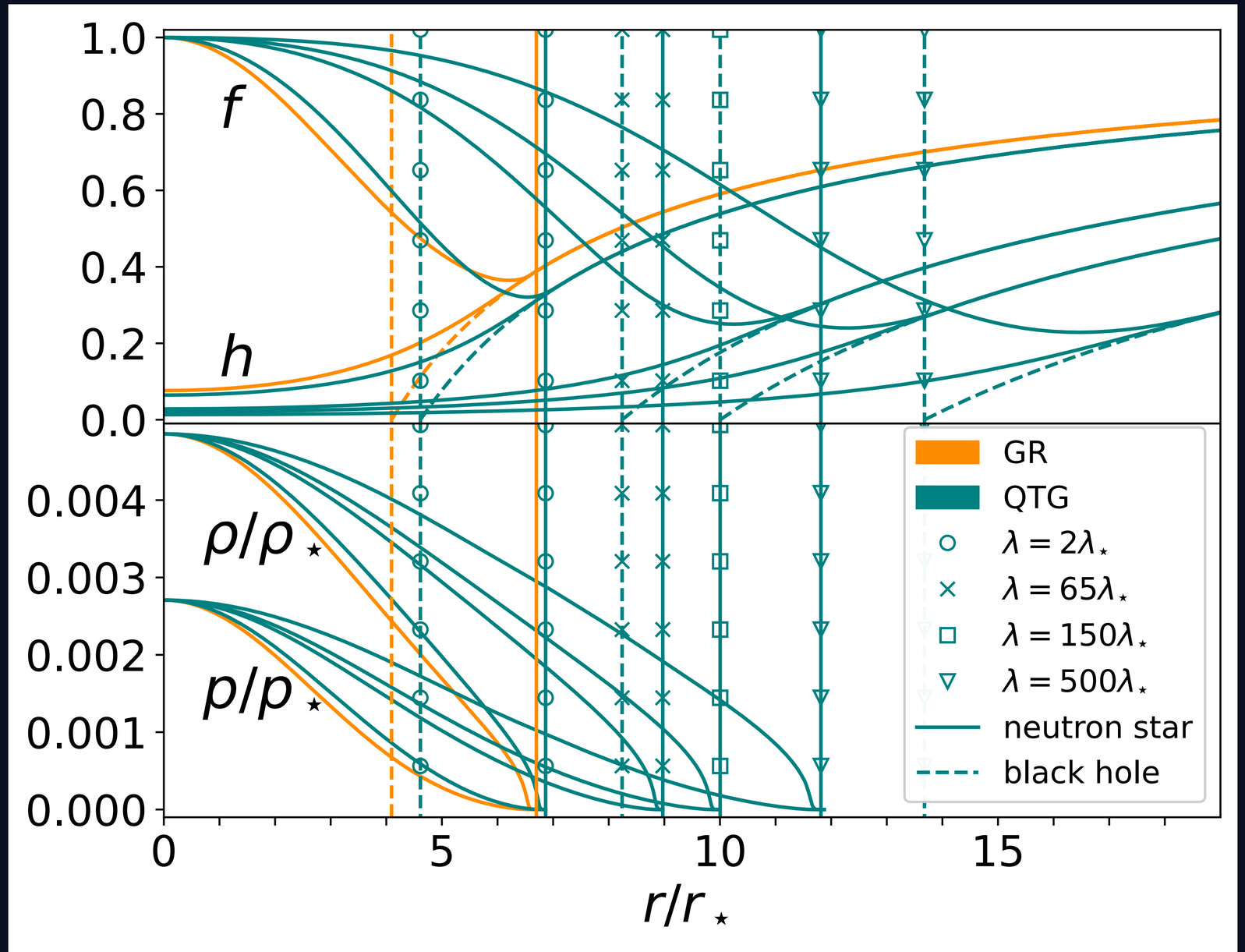
Metric:

$$ds^2 = -h\,dt^2 + f^{-1}\,dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

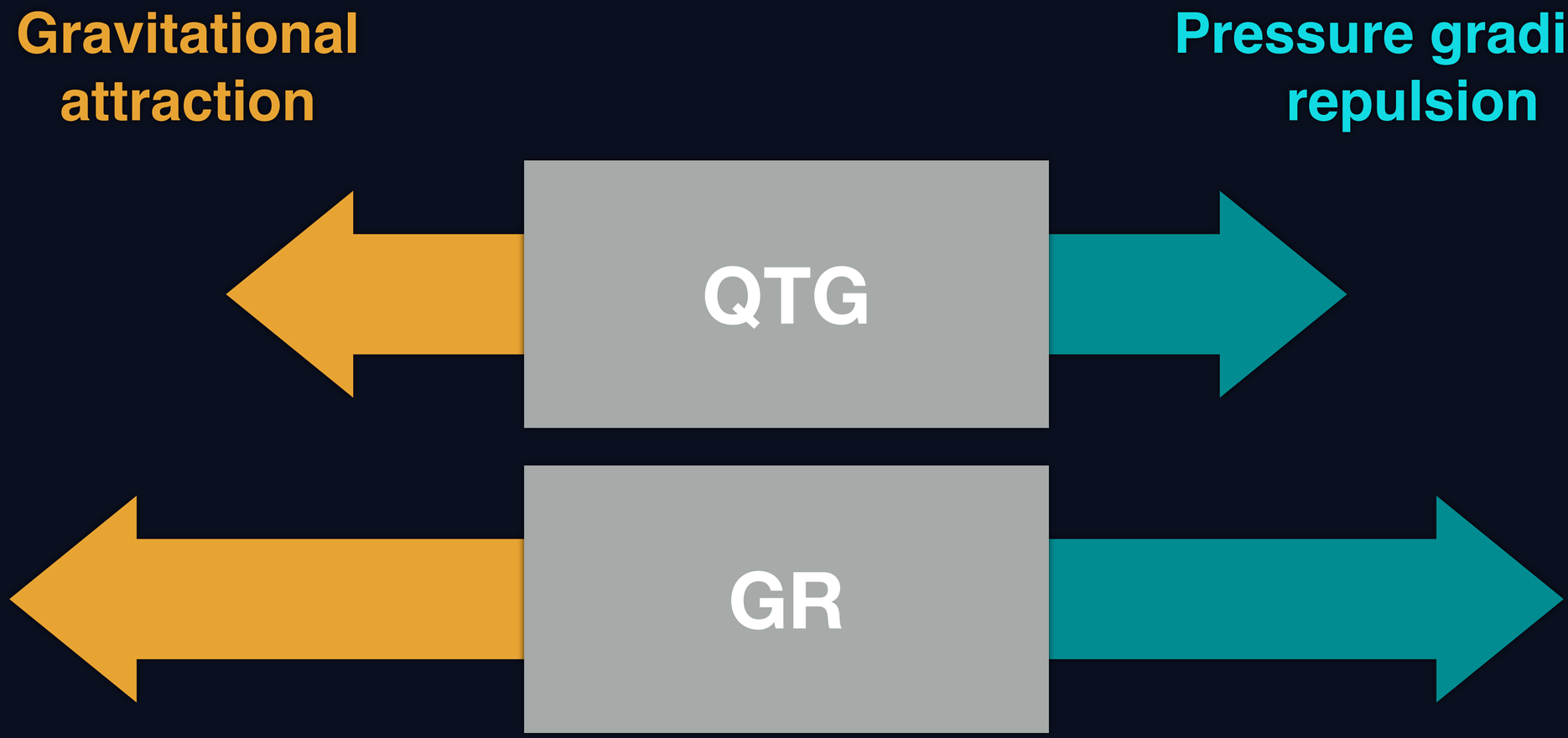
EOS: SLy, $p = P(\rho)$

$$M_\odot \sim r_\star \sim p_\star{}^{-1/2} \sim \rho_\star^{-1/2} \sim \lambda_\star^{1/4}$$

Stellar solution



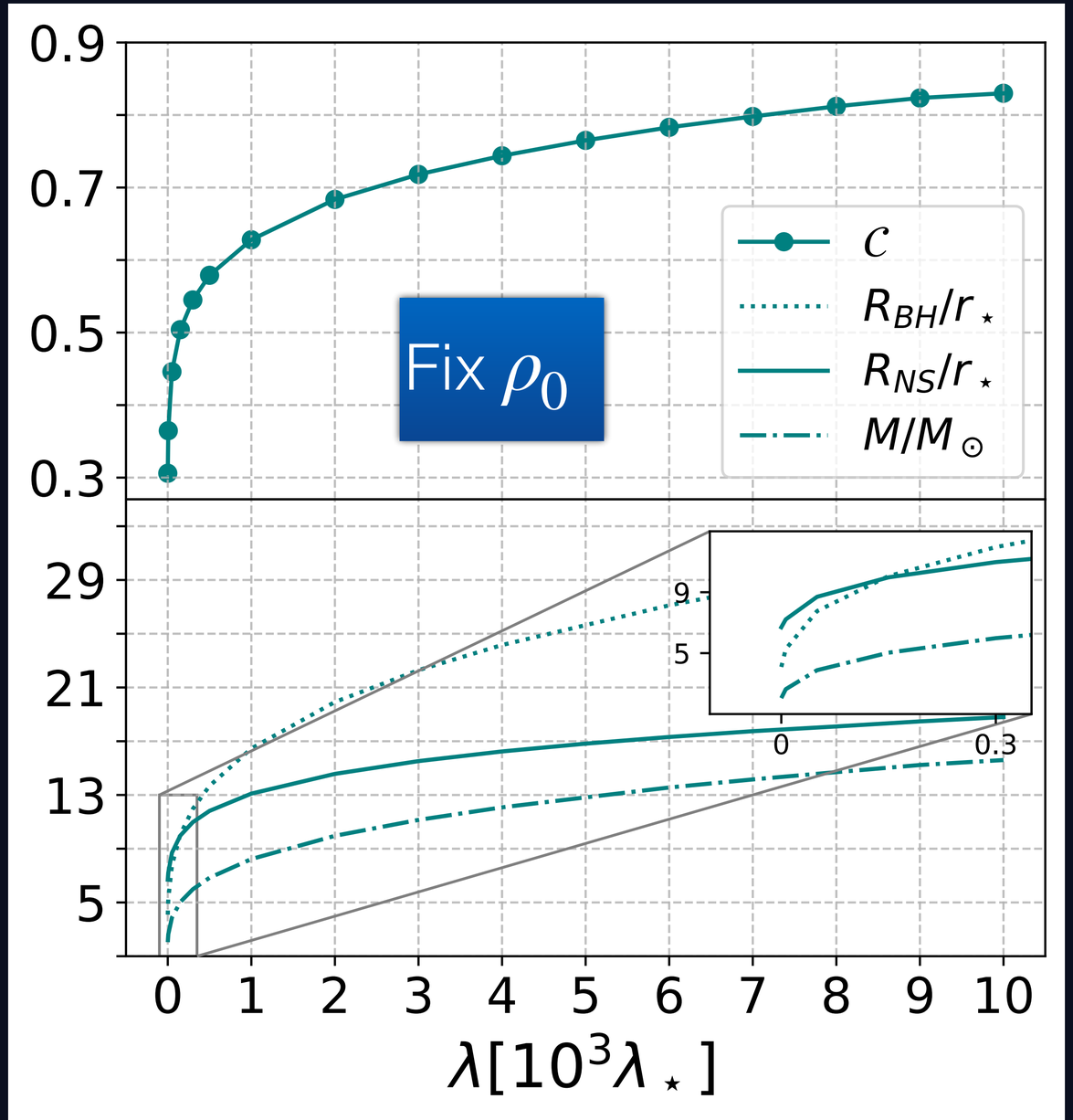
Hydrostatic equilibrium



Fix ρ_0

Keep balance

Influence of QTG on the compactness



Suppose a constant density NS

BH: $M \sim R$

NS: $M \sim R^3$

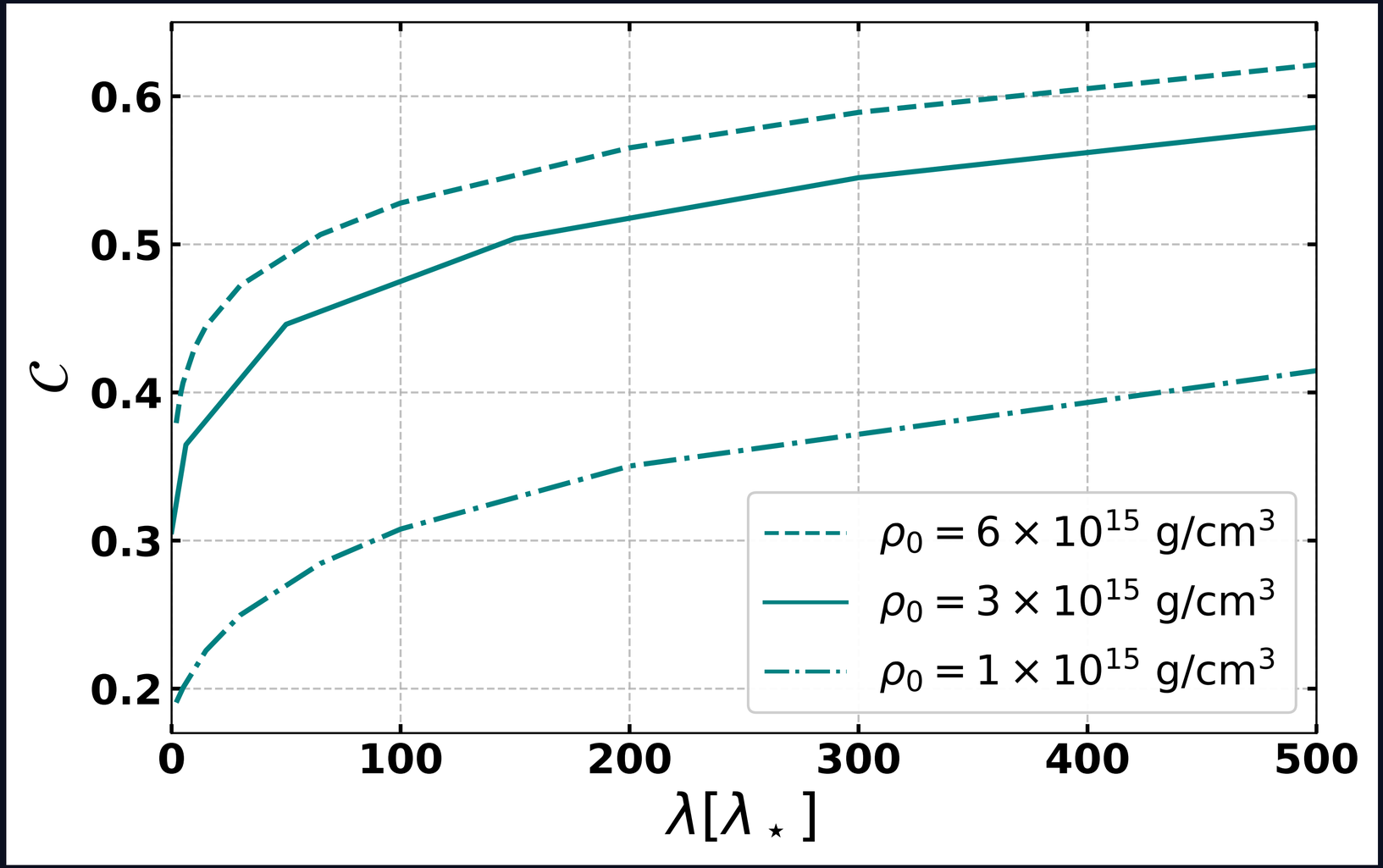
$$M_{\mathrm{BH/NS}} \times 1000$$

$$R_{\mathrm{BH}} \times 1000$$

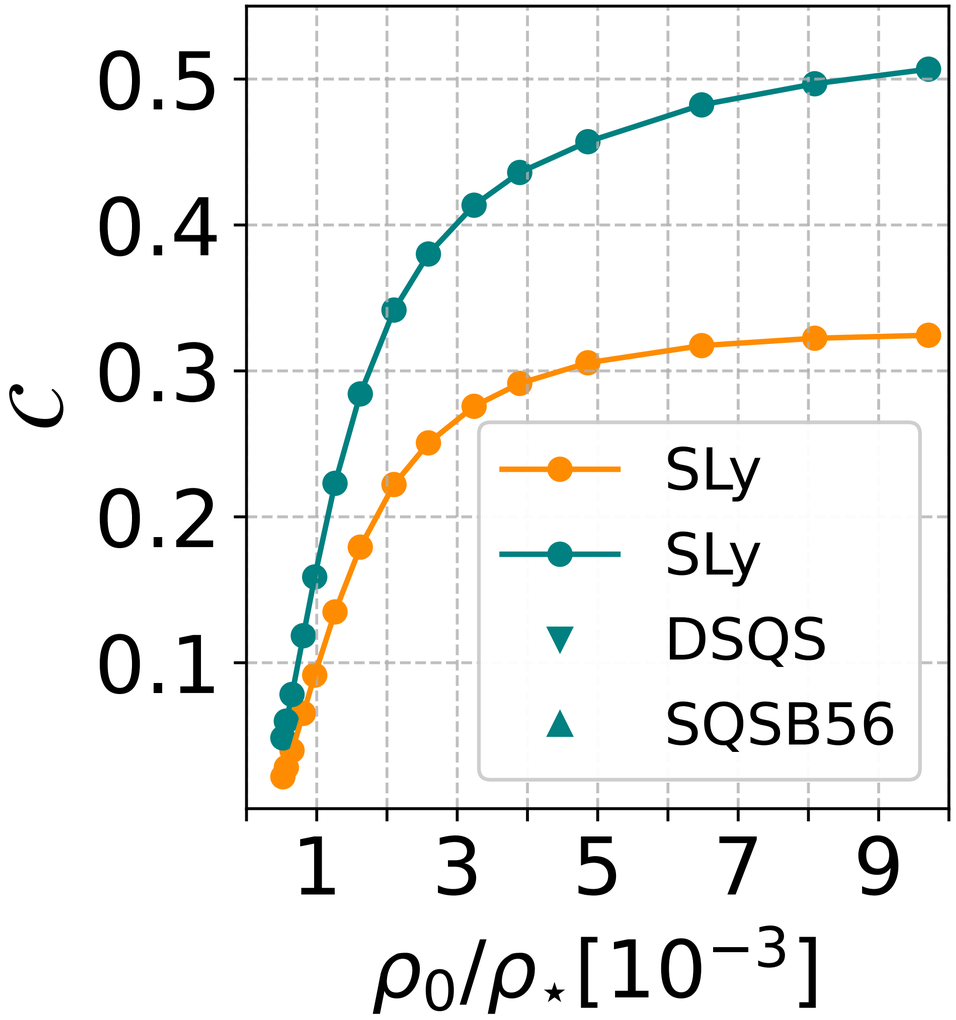
$$R_{\mathrm{NS}} \times 10$$

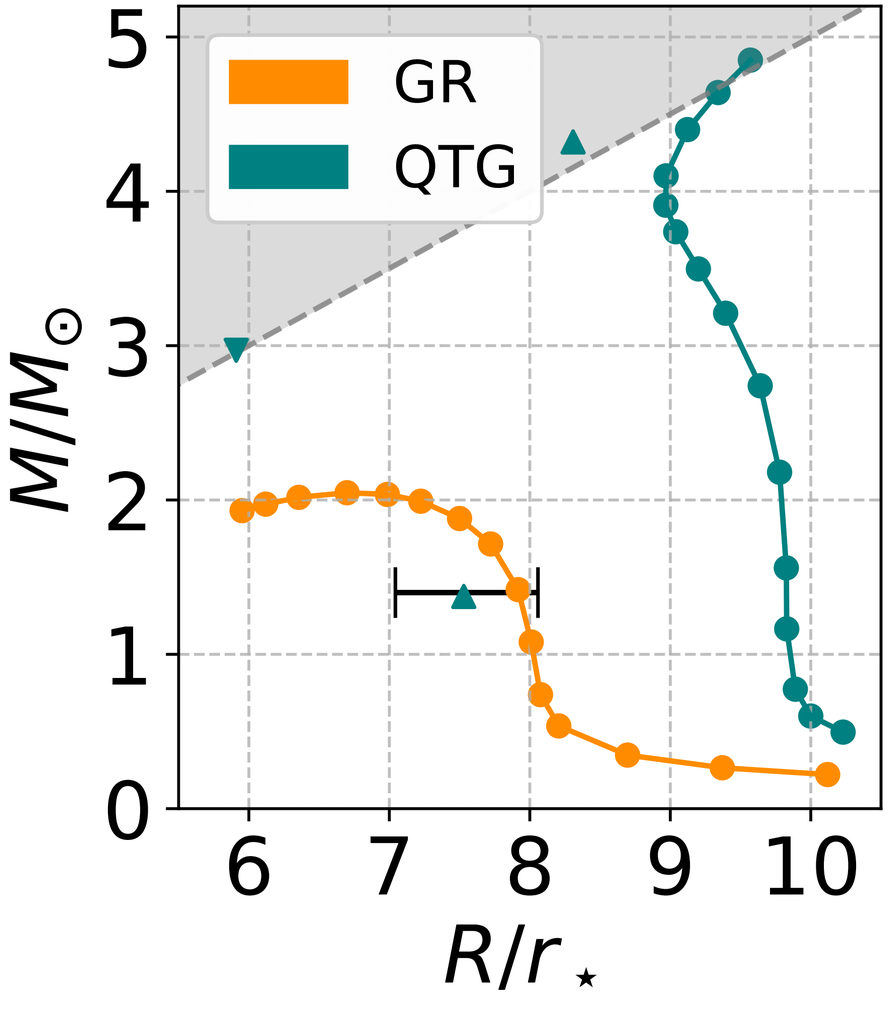
Stronger QTG modifications make NSs more compact than BHs.

Influence of QTG on the compactness



$M - R$ relations





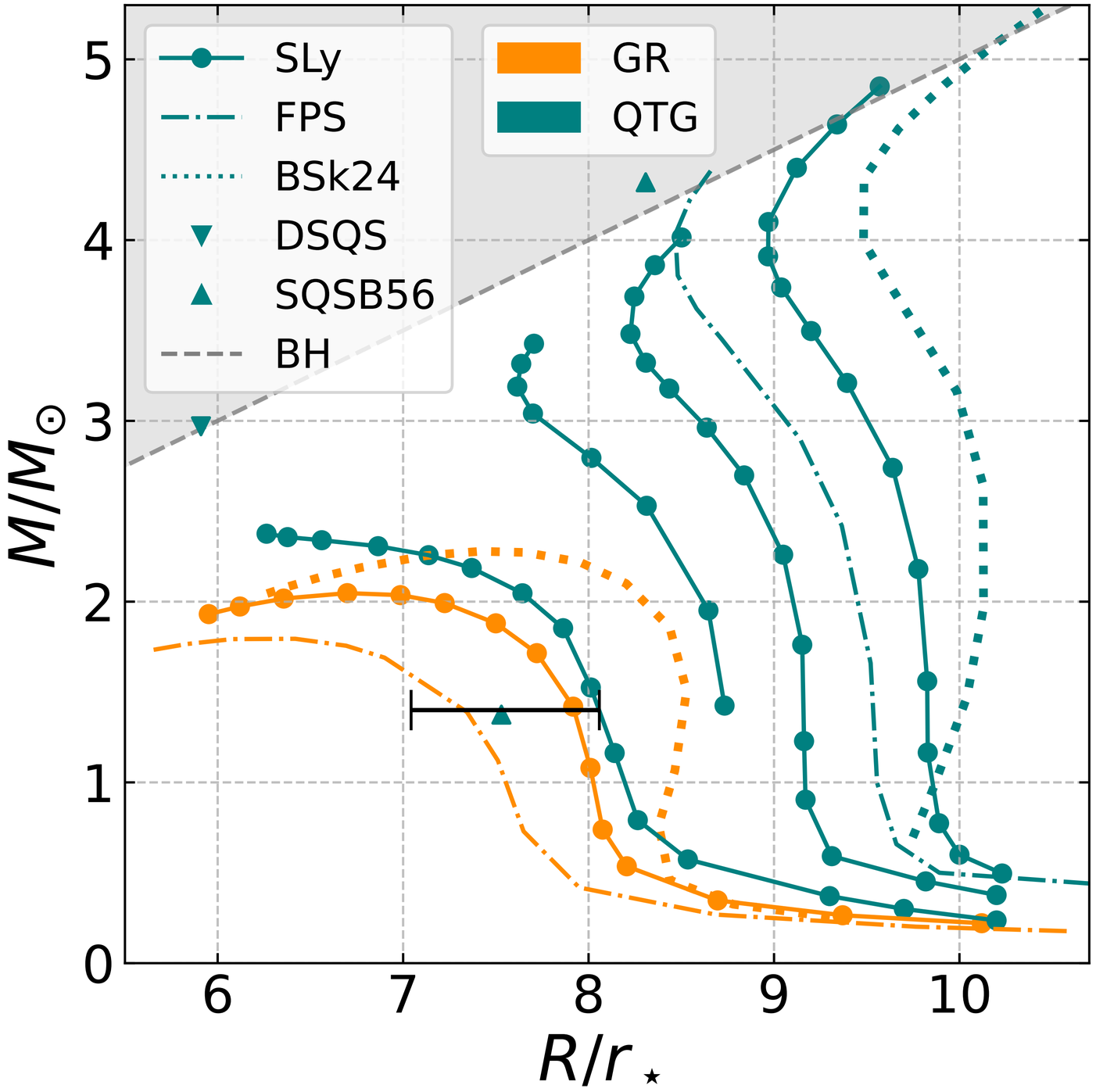
No maximum mass!

High ρ_0: NSs more compact than BHs.

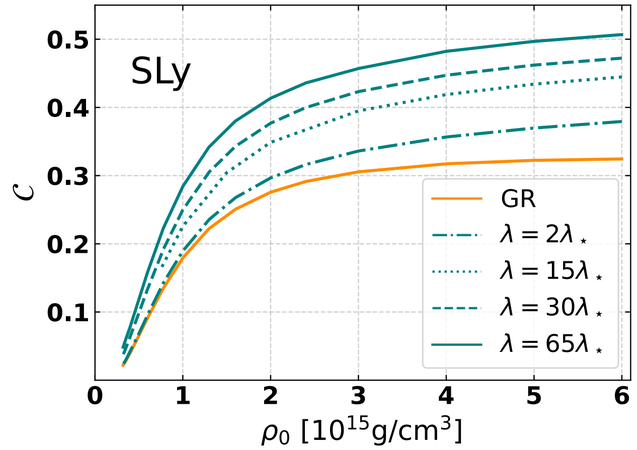
Low ρ_0: NS radii consistent with GW170817.

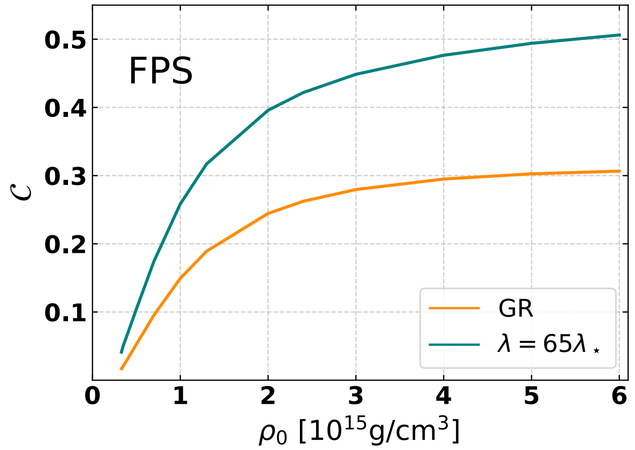
Possible interpretation of mass-gap compact objects and stellar-mass BH candidates.

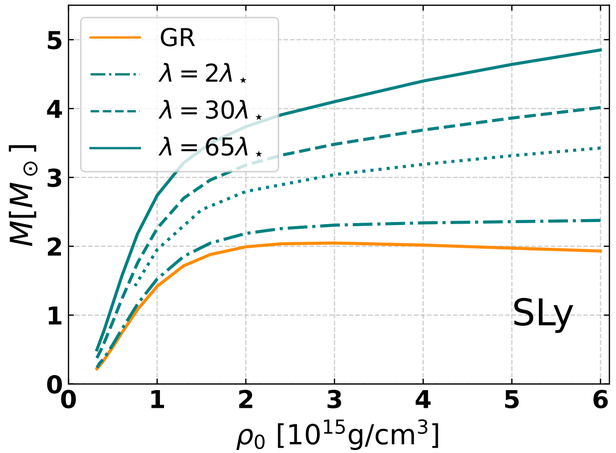
$M - R$ relations

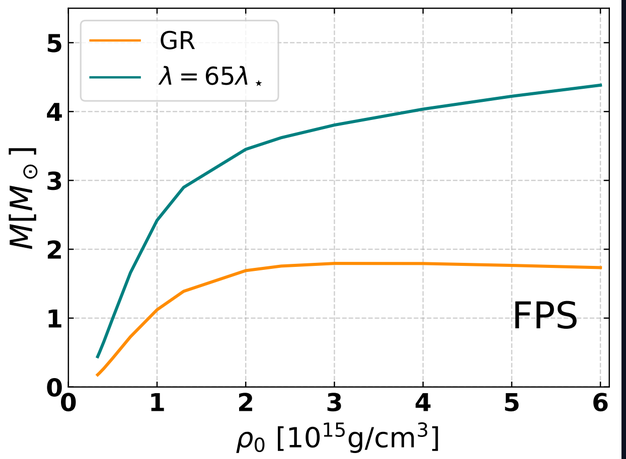


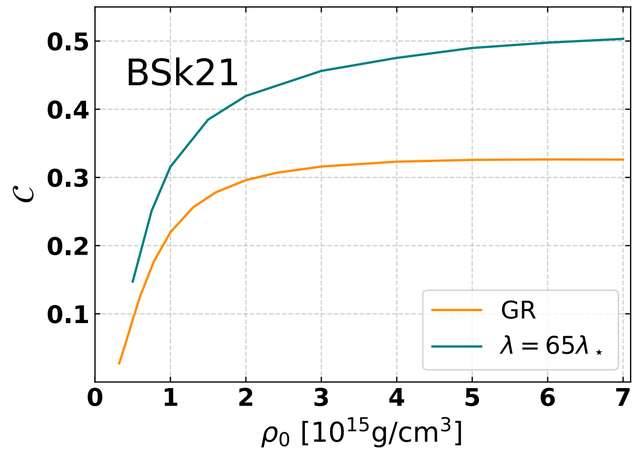
$C - \rho_0$ and $M - \rho_0$ relations

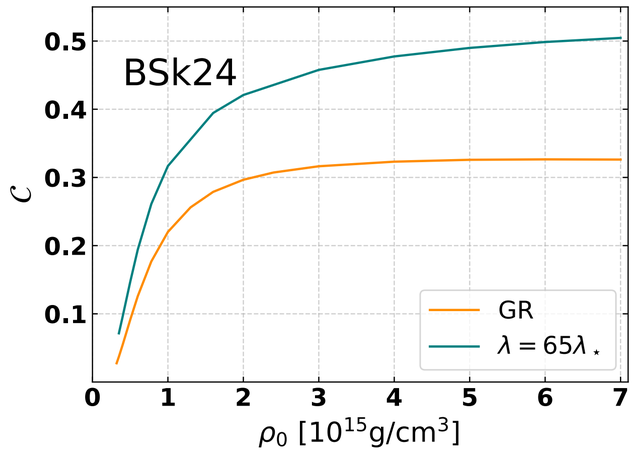


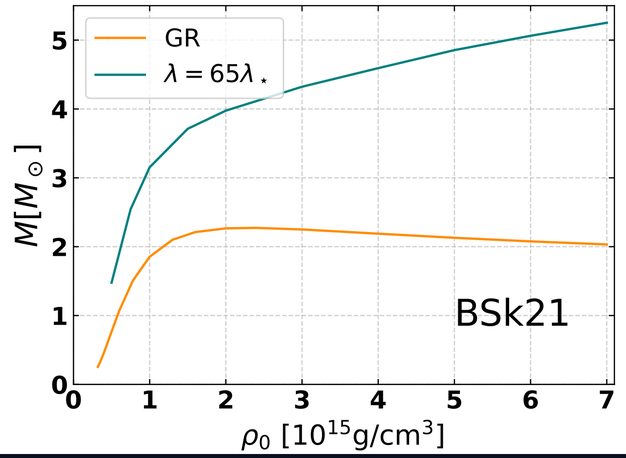


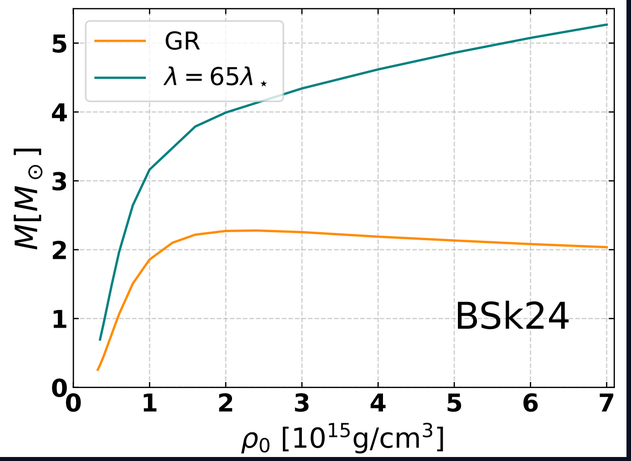




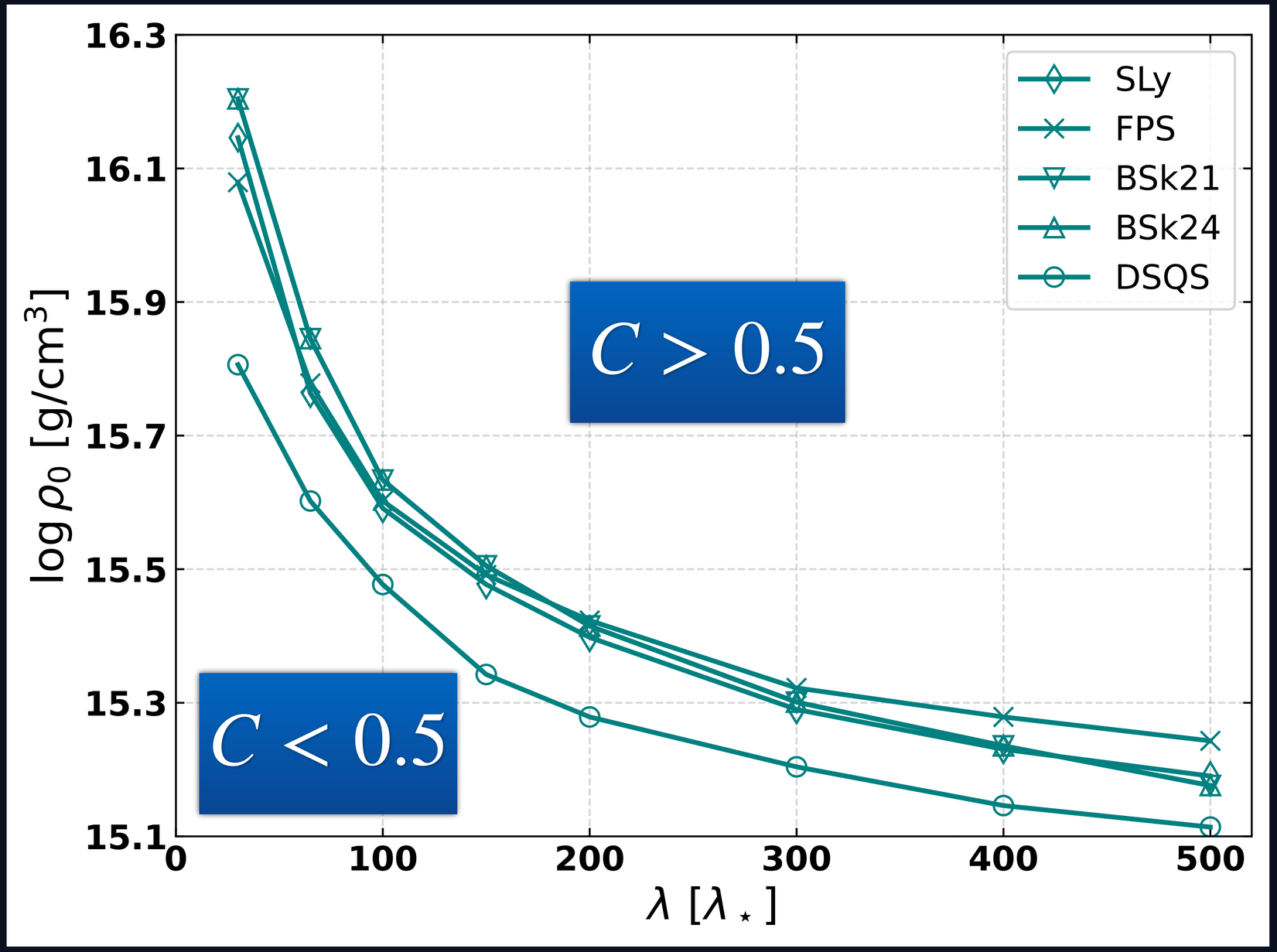






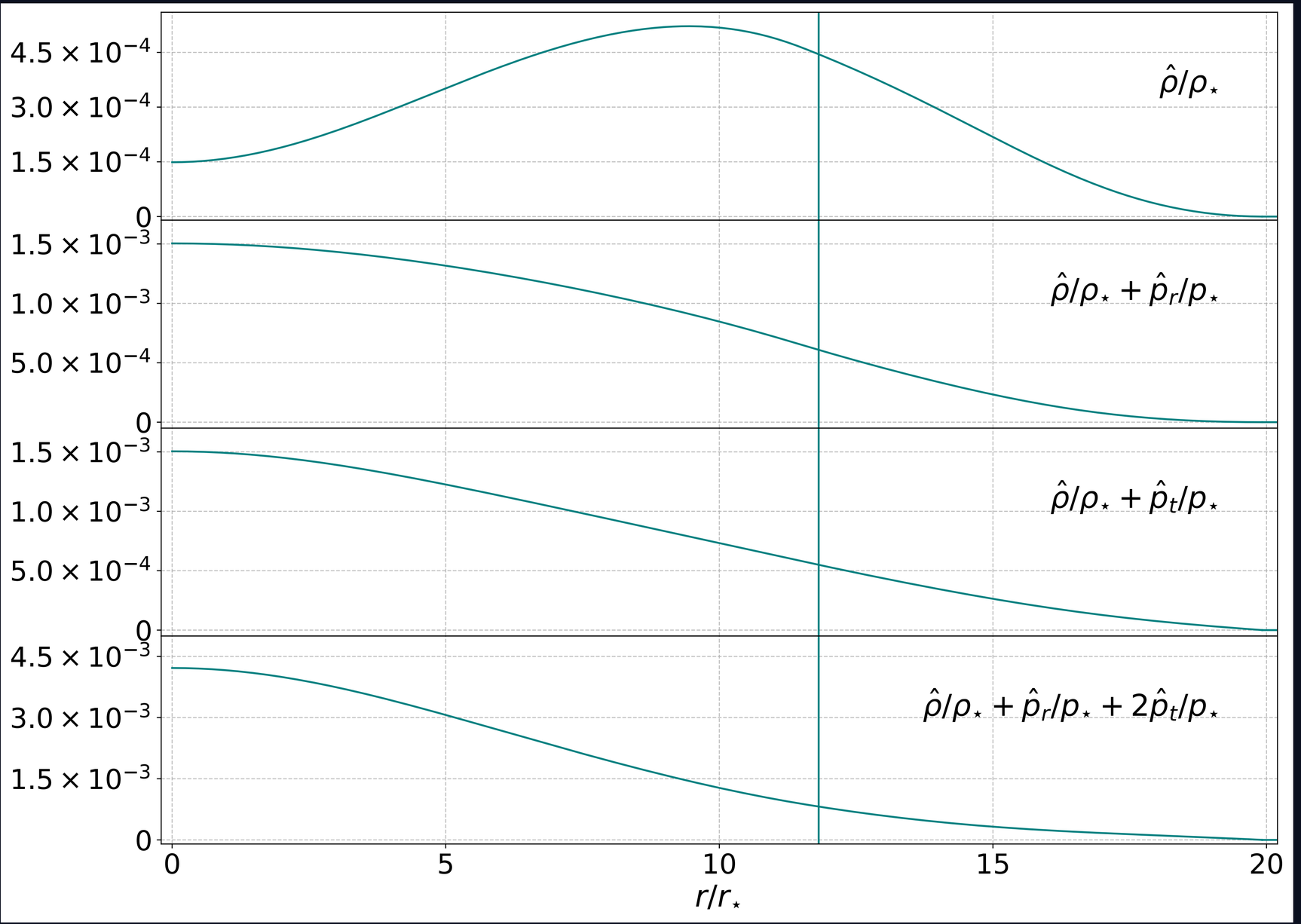


Phase diagram in (ρ_0, λ)



For hadronic EOSs, the result is nearly EOS-independent.

Energy conditions



Treat QTG as an effective matter

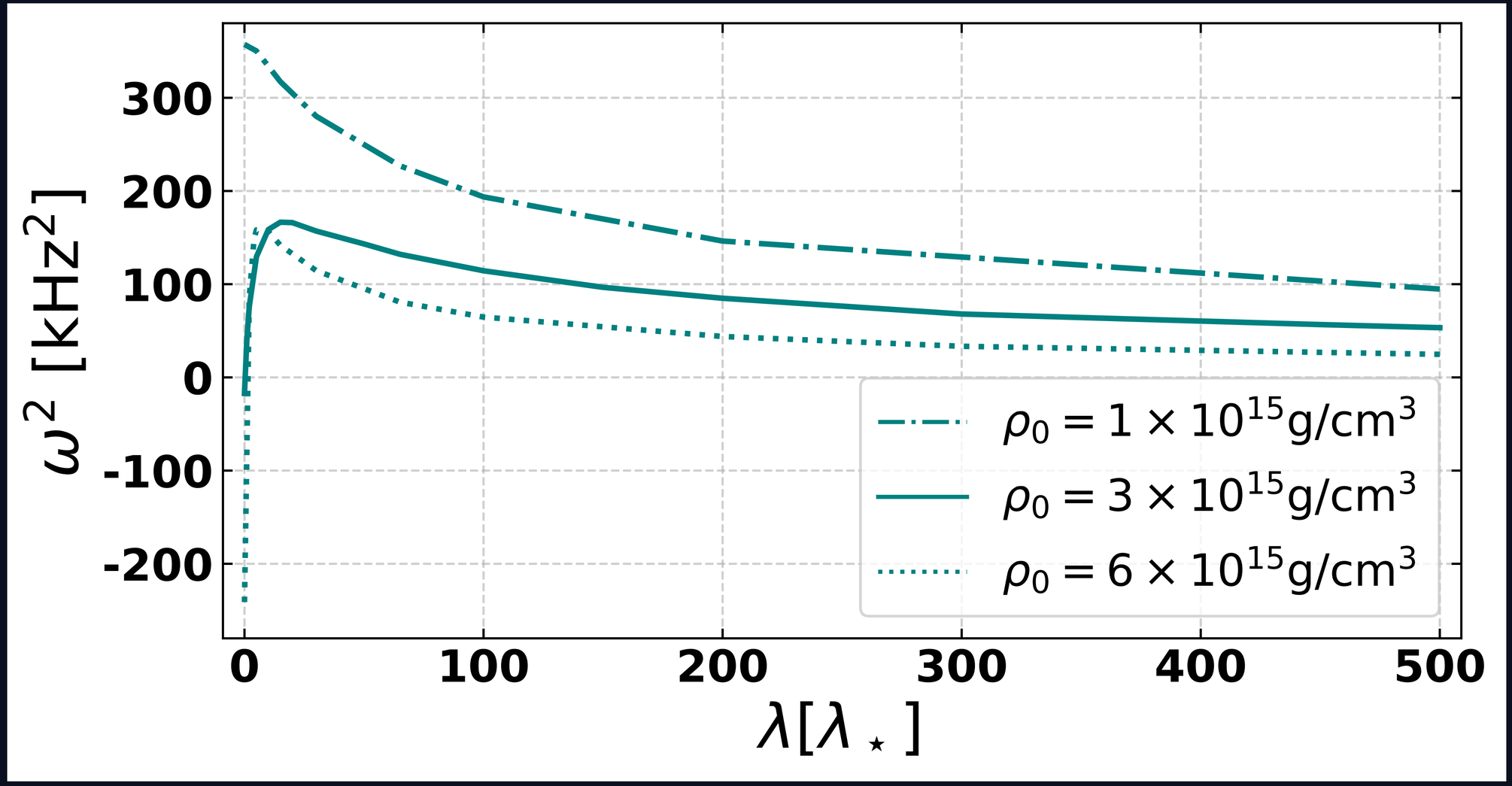
$$\tilde{T}^{\mu}{}_{\nu} = \{\tilde{\rho}, \tilde{P}_r, \tilde{P}_t, \tilde{P}_t\}$$

$$G_{\mu\nu} = 8\pi\tilde{T}_{\mu\nu}$$

SEC, WEC, NEC are satisfied.

Radial Stability

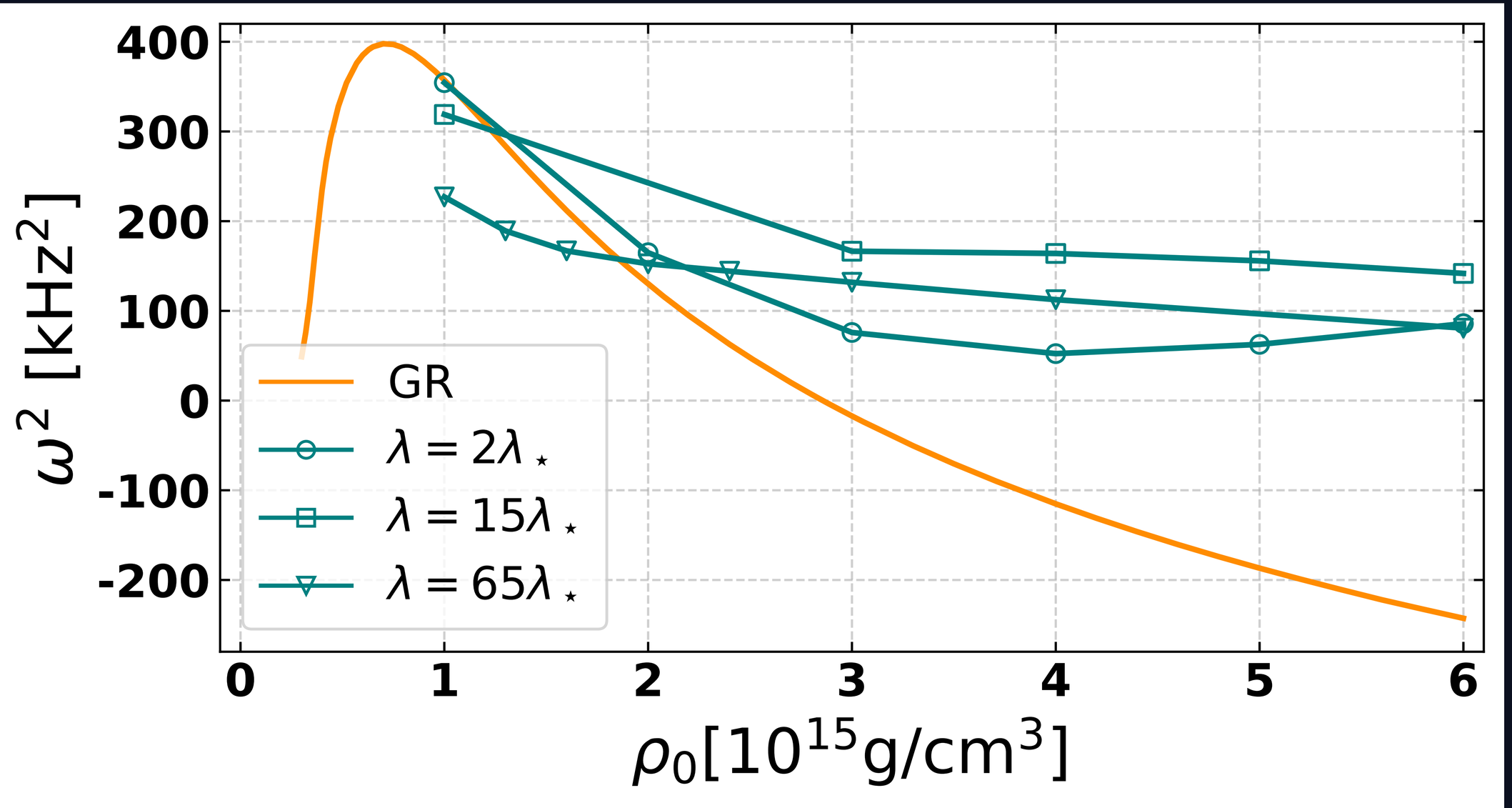
QTG can stabilize NSs



$\omega^2 - \lambda$ **relations**

[Chandrasekhar, PRL, 1964] [MTW, Gravitation] [Kokkotas & Ruoff, A&A, 2001] [Li, Yu, Luo, SL & Yu, PRD, 2025]

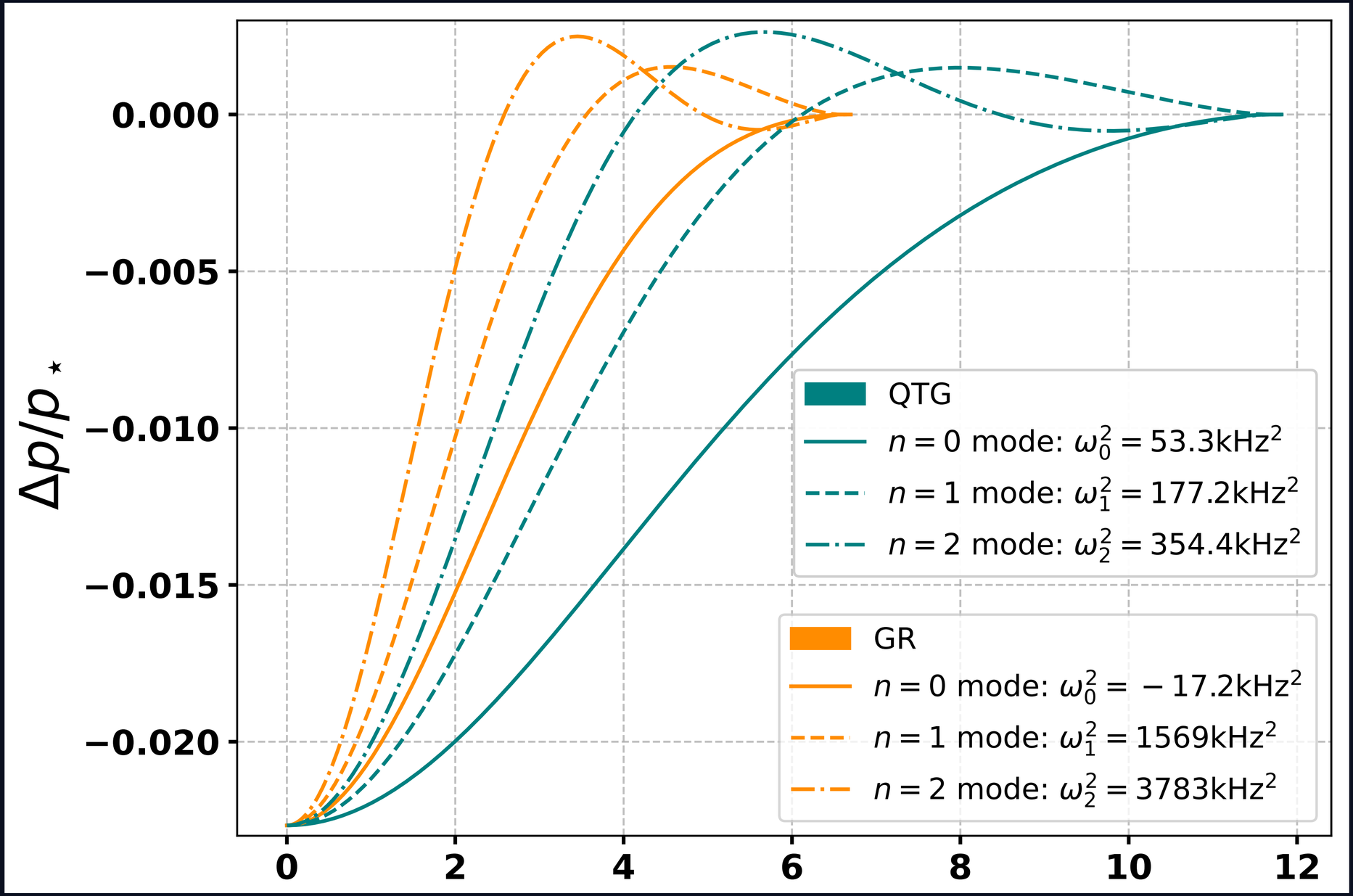
$\omega^2 - \rho_0$ relations



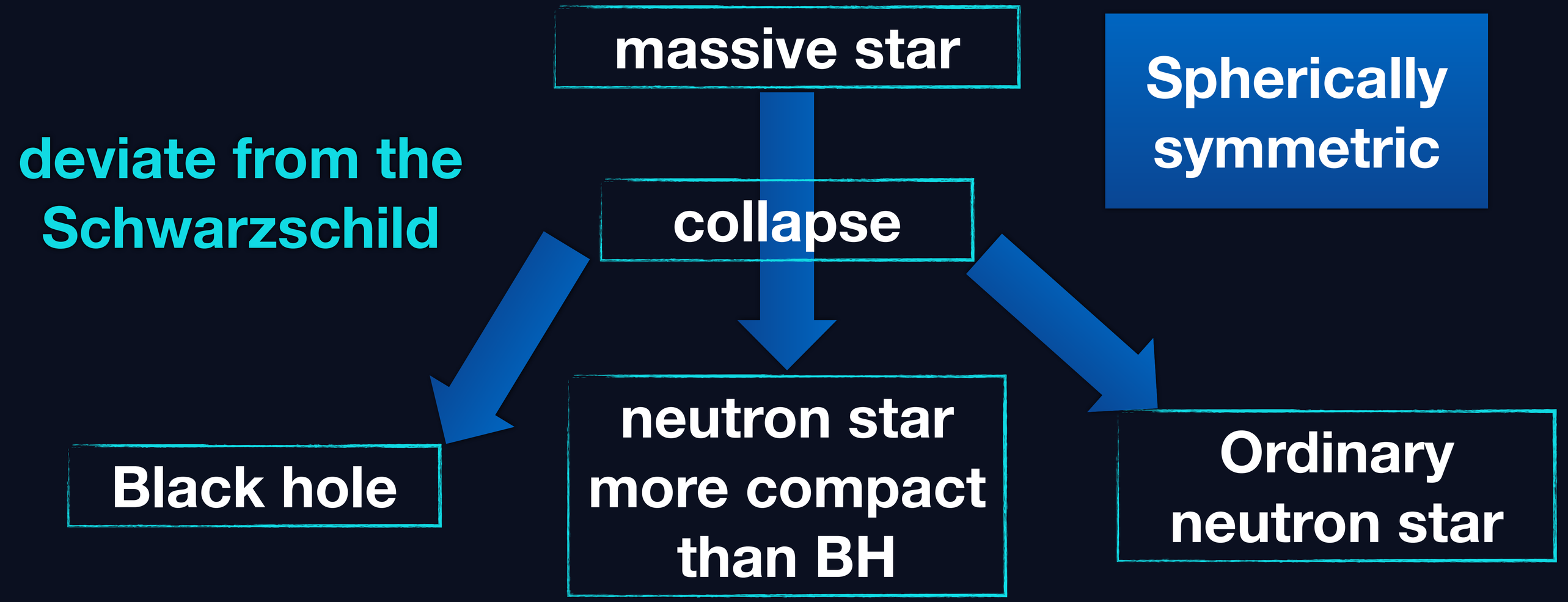
$$\frac{dM}{d\rho_0} > 0$$

remains a reliable stability criterion!

Lagrangian perturbations of pressure

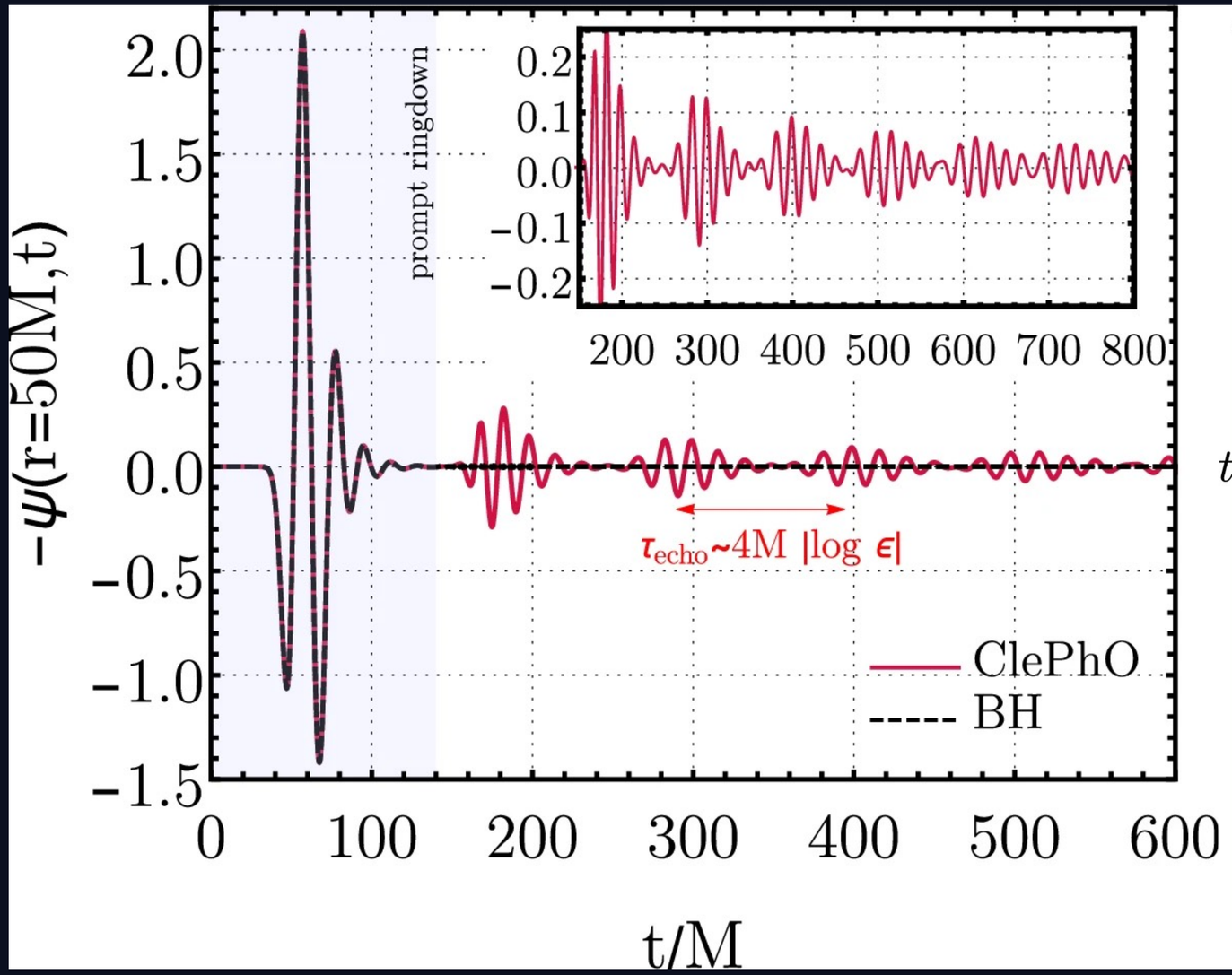


A possible formation scenario

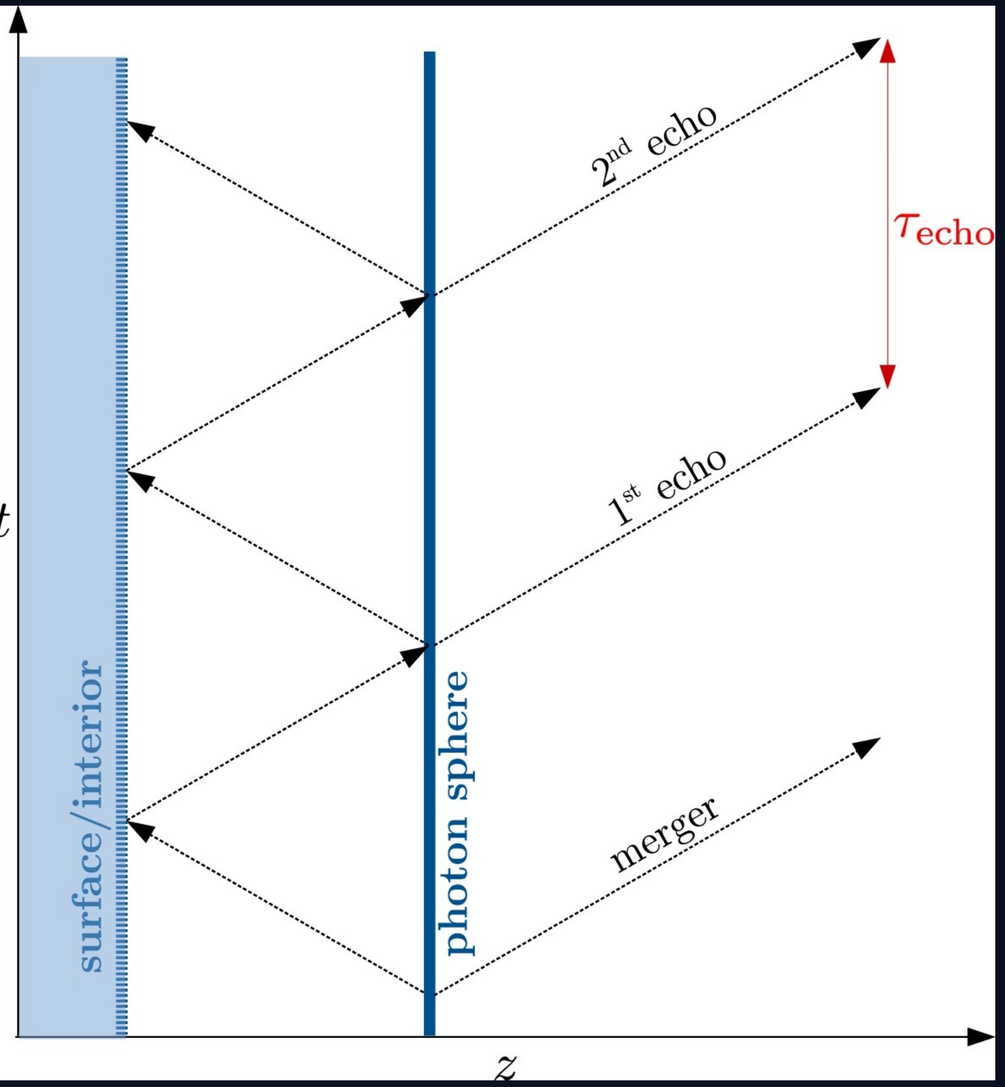


Observational Signatures

GW echoes



Echo waveform in late-time ringdown



Echoes time delay

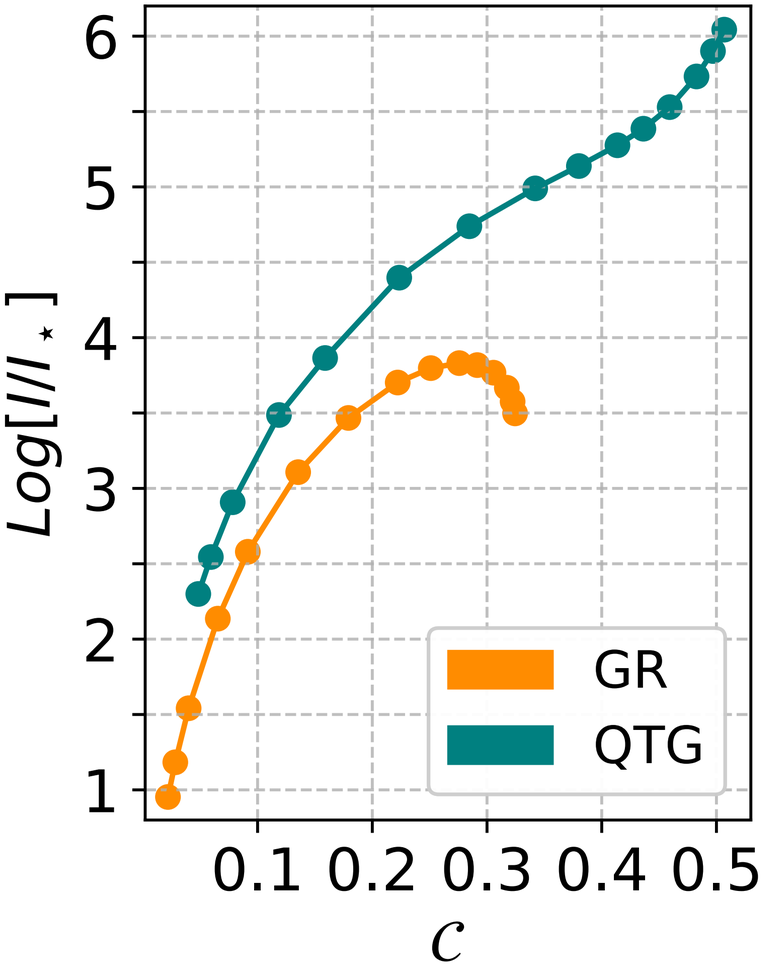
GW echoes

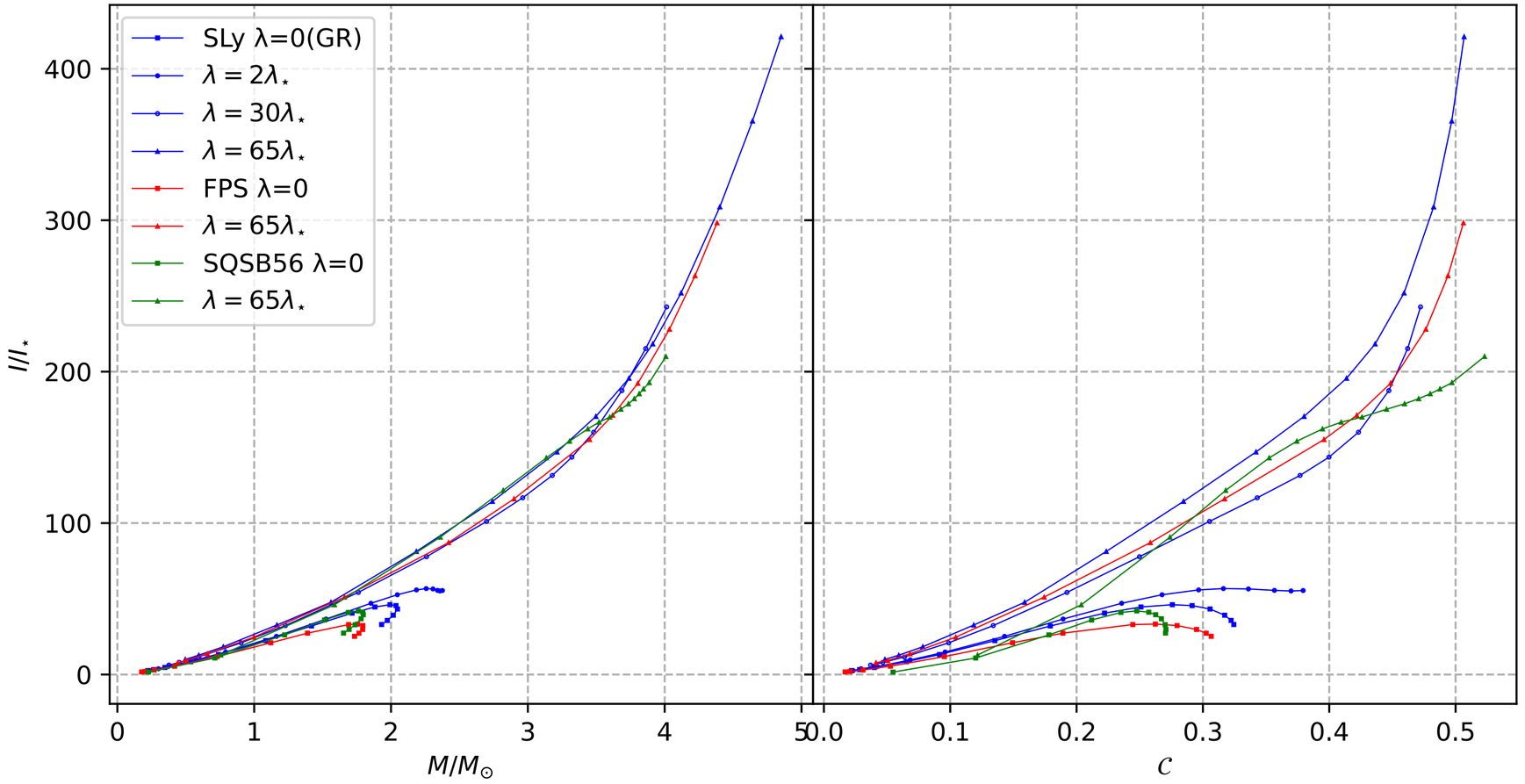
Radius	Compactness	Echo frequency (Hz)	Coupling constant
12.389	0.552	7168	1000
12.103	0.565	6338	800
11.934	0.573	5719	650
11.816	0.579	5032	500
6.84 solar masses, EOS is SLy			

For the same mass, higher compactness leads to lower echo frequencies.

Combined with tidal and multipolar observables, GW echoes can distinguish these objects from BHs.

$I - C$ and $I - M$ relation





2612.*** in perparation with Liang Liang, Fangkang Yu, Daniela Doneva, Stoytcho Yazadjiev, Hongwei Yu**

Conclusion and Outlook

- We propose the concept of stars more compact than bhs and construct the models
- We demonstrate their radial stability
- We identify echoes and other signatures that can distinguish them from bhs
- They provide a new probe of strong-field gravity
- Extend these models to other modified gravities (scalar-tensor gravity?)
- Explore their stability beyond radial perturbations
- Identify additional observational signatures

Thanks!