QUANTUM FIELD THEORY AND GAME THEORY

Nash Equilibrium from Quantum-Field Entanglement

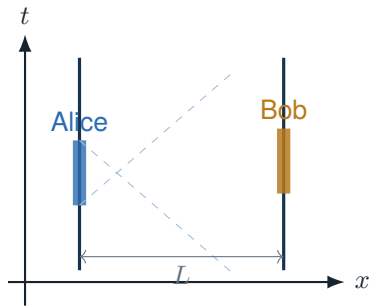
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Entanglement harvesting



Strictly spacelike interaction regions

Preparation. Two uncorrelated two-level detectors begin in $|g_A g_B\rangle$ while the field is in the vacuum $|0\rangle$.

Local sampling. Each detector couples to the field only along its own worldline and only during a finite switching interval.

Harvesting. After the field is traced out, the detector state ρ_{AB} can be entangled even when the two interaction regions are spacelike separated.

$$[\phi(x), \phi(x')] = 0 \quad \text{but} \quad \langle 0|\phi(x)\phi(x')|0\rangle \neq 0$$

Spacelike detectors exchange no signal, yet can acquire correlations already present in the vacuum.

Reznik (2003); Pozas-Kerstjens and Martín-Martínez (2015)

Vacuum correlations and microcausality

The Wightman function has symmetric and antisymmetric parts:

$$W(x,x') = \langle 0|\phi(x)\phi(x')|0\rangle \qquad \text{Wightman function,}$$
$$G^{(1)}(x,x') = \langle 0|\{\phi(x),\phi(x')\}|0\rangle \qquad \text{Hadamard function,}$$
$$[\phi(x),\phi(x')] = \mathrm{i}\Delta(x,x')\mathbb{I} \qquad \text{Pauli–Jordan function } \Delta.$$

$$W(x,x') = \frac{1}{2}G^{(1)}(x,x') + \frac{\mathrm{i}}{2}\Delta(x,x').$$

Correlation

W and $G^{(1)}$ need not vanish at spacelike separation. They describe vacuum fluctuations shared by separated regions.

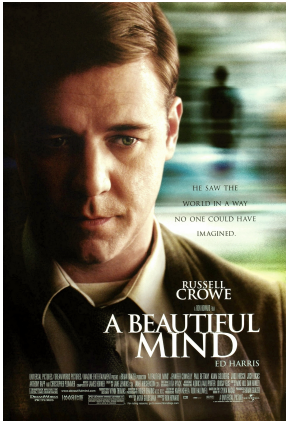
Causal response

Microcausality gives $\Delta(x,x') = 0$ for spacelike separated points. Local operations then cannot transmit a signal between the regions.

Spacelike separation removes communication, not vacuum correlation.

Reeh and Schlieder (1961); Summers and Werner (1985); Witten (2018)

Game theory and Nash equilibrium



Poster: *A Beautiful Mind* (2001)

A Beautiful Mind popularized Nash's idea through a fictionalized bar scene. Although the scene is not a textbook derivation, it poses the right question: which choices remain stable once every player anticipates the others?

A **game** specifies players, available strategies, and payoffs. A **best response** maximizes one player's payoff for the other player's fixed choice.

A pair (a_*, b_*) is a **Nash equilibrium** when both choices are best responses:

$$u_A(a_*, b_*) \geq u_A(a, b_*) \quad \text{for every } a,$$
$$u_B(a_*, b_*) \geq u_B(a_*, b) \quad \text{for every } b.$$

No player can gain by changing strategy alone. The same definition includes probabilistic, or mixed, strategies.

In our game, vacuum entanglement changes the payoffs and can therefore change which detector timings are equilibria.

Nash, Proc. Natl. Acad. Sci. USA **36**, 48 (1950)

From observers to players

Observer
An observer uses a detector to probe the field and record measurement outcomes for a prescribed interaction.

Player
The observer also chooses whether and when to interact. Its payoff depends on both its own choice and the other player's choice.

Several observers can share the same physical system. Treating them as players makes their interdependent choices a game.

Field correlations can then shape stable patterns of participation and timing, revealing physical consequences beyond measurement statistics.

How do field correlations shape the Nash equilibria of the observers' choices?

Detector–field interaction and reduced state

Two identical, static Unruh–DeWitt detectors couple locally to the scalar field:

$$H_I(t) = \lambda \sum_{\nu=A,B} \chi_\nu(t)\, \mu_\nu(t)\, \phi(t, \boldsymbol{x}_\nu),$$

$$\mu_\nu(t) = \sigma_\nu^+ \mathrm{e}^{\mathrm{i}\Omega t} + \sigma_\nu^- \mathrm{e}^{-\mathrm{i}\Omega t}.$$

λ sets the coupling, χ_ν the switching pulse, and Ω the detector energy gap. From $|gg\rangle \otimes |0\rangle$, tracing out the field gives the reduced state at relative delay s:

$$\rho_{AB}(s) = \begin{pmatrix} 1 - 2\lambda^2\mathcal{P} & 0 & 0 & \lambda^2\mathcal{M}^* \\ 0 & \lambda^2\mathcal{P} & \lambda^2\mathcal{L}^* & 0 \\ 0 & \lambda^2\mathcal{L} & \lambda^2\mathcal{P} & 0 \\ \lambda^2\mathcal{M} & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda^4).$$

In the basis $(|gg\rangle, |ge\rangle, |eg\rangle, |ee\rangle)$, the local noise and pair coherence are

$$\mathcal{P} = \int \mathrm{d}t\, \mathrm{d}t'\, \chi(t)\chi(t')\mathrm{e}^{-\mathrm{i}\Omega(t-t')} W(t-t', 0),$$

$$\mathcal{M}(s) = -\int \mathrm{d}t\, \mathrm{d}t'\, \chi(t)\chi(t'-s)\mathrm{e}^{\mathrm{i}\Omega(t+t')} W_F(t-t', L).$$

Here W is the vacuum Wightman function and W_F its time-ordered counterpart. The single-excitation coherence $\mathcal{L}$ is not needed in the following analysis.

Partial transpose and leading negativity

Partial transposition on Bob: $(|ij\rangle\langle kl|)^{T_B} = |il\rangle\langle kj|$.

Partial-transpose spectrum

$$\rho^{T_B} = \begin{pmatrix} 1-2\lambda^2\mathcal{P} & 0 & 0 & \lambda^2\mathcal{L}^* \\ 0 & \lambda^2\mathcal{P} & \lambda^2\mathcal{M}^* & 0 \\ 0 & \lambda^2\mathcal{M} & \lambda^2\mathcal{P} & 0 \\ \lambda^2\mathcal{L} & 0 & 0 & 0 \end{pmatrix}$$

up to $\mathcal{O}(\lambda^4)$, with $\rho = \rho_{AB}$. The single-excitation block has eigenvalues

$$\lambda^2(\mathcal{P} \pm |\mathcal{M}|) + \mathcal{O}(\lambda^4).$$

Its lower eigenvalue is negative at leading order when $|\mathcal{M}| > \mathcal{P}$.

Definition of negativity

$$\mathcal{N}(\rho) = \frac{\|\rho^{T_B}\|_1 - 1}{2},$$

where the trace norm is

$$\|X\|_1 = \mathrm{Tr}\sqrt{X^\dagger X}.$$

For the eigenvalues ξ_k of the Hermitian matrix ρ^{T_B}, $\sum_k \xi_k = 1$. Thus

$$\|\rho^{T_B}\|_1 = \sum_k |\xi_k|, \qquad \mathcal{N}(\rho) = \sum_{\xi_k<0} |\xi_k|.$$

$$\boxed{\mathcal{N}(\rho) = \lambda^2(|\mathcal{M}| - \mathcal{P})_+ + \mathcal{O}(\lambda^4),} \qquad (x)_+ = \max(x, 0).$$

Peres (1996); Horodecki et al. (1996); Vidal and Werner (2002)

Pair coherence: Hadamard and causal contributions

We now separate the pair coherence into Hadamard and causal contributions. Here t, t' are interaction times, s is the relative switching delay, L the detector separation, and Ω the energy gap. The real, even pulse χ has support $[-T, T]$, with half-duration T.

Use the centered mean time $z = (t + t' - s)/2$ and relative pulse time $\zeta = t - t' + s$:

$$\mathcal{C}_\Omega(\zeta) = \int_{\mathbb{R}} \mathrm{d}z\, \chi(z + \zeta/2)\chi(z - \zeta/2)\cos(2\Omega z).$$

This kernel is real, even, and zero for $|\zeta| > 2T$. Split the field correlator into correlation and causal response:

$$W_F(u, L) = \frac{1}{2}G^{(1)}(u, L) + \frac{\mathrm{i}}{2}\,\mathrm{sgn}(u)\Delta(u, L), \qquad u = t - t' = \zeta - s.$$

$$\mathcal{H}(s) = \frac{1}{2}\int_{-2T}^{2T} \mathrm{d}\zeta\, \mathcal{C}_\Omega(\zeta)G^{(1)}(u, L), \qquad \mathcal{Q}(s) = \frac{1}{2}\int_{-2T}^{2T} \mathrm{d}\zeta\, \mathcal{C}_\Omega(\zeta)\,\mathrm{sgn}(u)\Delta(u, L).$$

Removing the known carrier phase separates the coherence into these two real contributions:

$$Z(s) \equiv -\mathrm{e}^{-\mathrm{i}\Omega s}\mathcal{M}(s) = \mathcal{H}(s) + \mathrm{i}\mathcal{Q}(s), \qquad \boxed{|\mathcal{M}| = \sqrt{\mathcal{H}^2 + \mathcal{Q}^2}.}$$

A linear witness and the target payoff

Negativity is nonlinear in ρ. The negative partial-transpose direction instead gives a linear entanglement witness:

$$\theta = \arg \mathcal{M}, \qquad |\eta_\theta\rangle = \frac{|ge\rangle - \mathrm{e}^{\mathrm{i}\theta}|eg\rangle}{\sqrt{2}}, \qquad W_\theta = (|\eta_\theta\rangle\langle\eta_\theta|)^{T_B}.$$

Its expectation detects the same leading entanglement threshold:

$$\mathrm{Tr}(W_\theta \rho) = \lambda^2(\mathcal{P} - |\mathcal{M}|) + \mathcal{O}(\lambda^4).$$

The phase θ depends on the timing history. To use one fixed witness, apply the prescribed local correction:

$$U_\theta = \mathrm{diag}(1, \mathrm{e}^{-\mathrm{i}\theta}), \qquad \widetilde{\rho} = (U_\theta \otimes \mathbb{I})\rho(U_\theta^\dagger \otimes \mathbb{I}).$$

The leading pair coherence is now real and nonnegative, so every timing history uses

$$W_0 = \frac{\mathbb{I}}{2} - \Phi^+, \qquad \Phi^+ = \frac{(|gg\rangle + |ee\rangle)(\langle gg| + \langle ee|)}{2}.$$

Choose a positive reward scale Γ. The target mean payoff is

$$\boxed{G = -\Gamma\,\mathrm{Tr}(W_0\widetilde{\rho}) = -\Gamma\,\mathrm{Tr}(W_\theta\rho) = \Gamma\lambda^2(|\mathcal{M}| - \mathcal{P}) + \mathcal{O}(\lambda^4).}$$

Thus $|\mathcal{M}| > \mathcal{P}$ gives both entanglement and a positive leading reward. A trusted controller calibrates after the field interactions and before drawing question labels, adding no player strategy.

Next, realize this fixed linear expectation through local answers and payments.

Quantum questions and the local protocol

The referee keeps labels $q, r \in \{0, 1, 2, 3\}$ and sends trusted question states:

$$\tau_q = \frac{\mathbb{I} + \boldsymbol{r}_q \cdot \boldsymbol{\sigma}}{2}, \qquad \tau_r = \frac{\mathbb{I} + \boldsymbol{r}_r \cdot \boldsymbol{\sigma}}{2}.$$

Question ensemble and dual operators

$$\boldsymbol{r}_0 = (1,1,1)/\sqrt{3}, \qquad \boldsymbol{r}_1 = (1,-1,-1)/\sqrt{3},$$
$$\boldsymbol{r}_2 = (-1,1,-1)/\sqrt{3}, \quad \boldsymbol{r}_3 = (-1,-1,1)/\sqrt{3}.$$

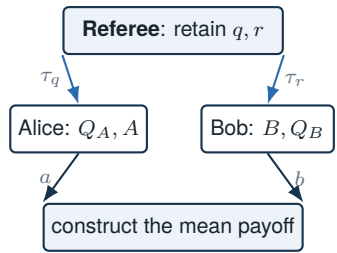
The states are nonorthogonal: $\mathrm{Tr}(\tau_q \tau_r) = 1/3$ when $q \neq r$. Define the dual operators by

$$D_q = \frac{3\tau_q - \mathbb{I}}{2}, \qquad \mathrm{Tr}(D_q \tau_r) = \delta_{qr}.$$

This biorthogonal relation extracts the witness coefficients on the next slide.

Local answers

Question registers Q_A, Q_B are measured with A, B, returning bits a, b without communication.



Buscemi (2012); Branciard et al. (2013)

Witness expansion and Bell probabilities

Witness expansion

$$W_0 = \sum_{q,r=0}^{3} \beta_{qr} \tau_q^T \otimes \tau_r^T ,$$

$$\beta_{qr} = \mathrm{Tr}[(D_q^T \otimes D_r^T) W_0].$$

$$\beta = \frac{1}{8} \begin{pmatrix} -1 & -1 & 5 & -1 \\ -1 & -1 & -1 & 5 \\ 5 & -1 & -1 & -1 \\ -1 & 5 & -1 & -1 \end{pmatrix} .$$

Local Bell readout

Each player uses $\{\Phi^+, \mathbb{I} - \Phi^+\}$ and reports 1 for the Bell projection.

$$\langle \Phi^+ | X \otimes Y | \Phi^+ \rangle = \frac{1}{2} \mathrm{Tr}(X^T Y).$$

Applying this identity for both players gives

$$p(1,1|q,r) = \frac{1}{4} \mathrm{Tr}\left[(\tau_q^T \otimes \tau_r^T) \widetilde{\rho} \right] .$$

$$G = -\Gamma \,\mathrm{Tr}(W_0 \widetilde{\rho}) = -\Gamma \sum_{q,r} \beta_{qr} \,\mathrm{Tr}[(\tau_q^T \otimes \tau_r^T) \widetilde{\rho}]$$

$$= \boxed{-4\Gamma \sum_{q,r} \beta_{qr} p(1,1|q,r)} .$$

The single-round payment

The referee samples question pairs uniformly ($1/16$ each). The expected payment $\mathbb{E}[r]$ is each player's mean payoff, averaged over question pairs and measurement outcomes. With zero payment unless $a = b = 1$,

$$\begin{aligned}\mathbb{E}[r] &= \frac{1}{16} \sum_{q,r,a,b} r(q,r,a,b)\, p(a,b|q,r) \\ &= \frac{1}{16} \sum_{q,r} r(q,r,1,1)\, p(1,1|q,r).\end{aligned}$$

Matching this mean to $G = -4\Gamma \sum_{q,r} \beta_{qr} p(1,1|q,r)$ gives $r(q,r,1,1)/16 = -4\Gamma\beta_{qr}$.

The single-round payment to each player is

$$\boxed{r(q,r,a,b) = -64\Gamma\beta_{qr}\delta_{a1}\delta_{b1}.}$$

$\mathbb{E}[r] = G$ **exactly for the prescribed Bell readouts, with all rounds included in the average.**

Field correlations determine the timing payoffs

Having implemented the witness payoff exactly, use the leading detector state to evaluate it. Define

$$\boxed{g(s) = \Gamma\lambda^2\bar{g}(s), \qquad \bar{g}(s) = \sqrt{\mathcal{H}(s)^2 + \mathcal{Q}(s)^2} - \mathcal{P}.}$$

Thus $G(s) = g(s) + \mathcal{O}(\lambda^4)$. For this detector family, $(G/\Gamma)_+ = \mathcal{N}(\rho) + \mathcal{O}(\lambda^4)$.

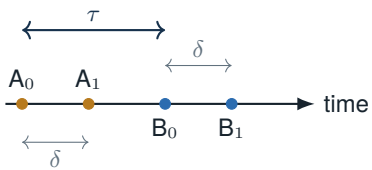
- $\mathcal{H}$ Hadamard correlations contribute to the shared coherence.
- $\mathcal{Q}$ The commutator adds a causal-response quadrature when its support is sampled.
- $\mathcal{P}$ Local excitation noise sets the threshold for a positive reward.

Coherence must exceed local noise to support a positive active payoff. All three ingredients matter.

Timing, participation, and the complete payoff table

Before the interactions, each player commits to E (exit), 0 (early entry), or 1 (late entry). For entry choices $i, j \in \{0, 1\}$,

$$t_A(i) = i\delta, \qquad t_B(j) = \tau + j\delta, \qquad s_{ij} = \tau + (j - i)\delta.$$



Rows: Alice. Columns: Bob. Each entry is (u_A, u_B).

	E	0	1
E	$(0, 0)$	$(0, -1)$	$(0, -1)$
0	$(-1, 0)$	(g_0, g_0)	(g_+, g_+)
1	$(-1, 0)$	(g_-, g_-)	(g_0, g_0)

Exit pays 0. Entering alone pays -1.

$$g_0 = g(\tau), \qquad g_\pm = g(\tau \pm \delta).$$

The common joint-entry rewards form the timing matrix G. Equal lone-entry losses make timing comparisons depend only on G whenever the opponent enters with positive probability.

The baseline τ varies externally. Pulse shape, gap, separation, δ, verification, and Γ stay fixed. Phase calibration equalizes the diagonal entry rewards. Entry and timing are simultaneous choices.

1. Count fixed early–late choices

Assume $g_0 \neq g_\pm$. A fixed timing pair is an equilibrium only if neither player can gain by switching early $\leftrightarrow$ late:

Fixed pair (i, j)	Mutual best responses when	Common reward
$(0,0)$ and $(1,1)$	$g_0 > \max\{g_+, g_-\}$	g_0 for each
$(0,1)$	$g_+ > g_0$	g_+
$(1,0)$	$g_- > g_0$	g_-

For example, at $(0,1)$ either unilateral timing switch changes the reward from g_+ to g_0. Thus $g_+ > g_0$ is sufficient, regardless of g_-.

Retain only positive rewards. Count each pair separately: the diagonal row contributes *two* equilibria when $g_0 > 0$.

Let n_+ count all positive-reward timing equilibria. Start with the fixed pairs; next check for a positive timing mixture.

2. Check for a mixed timing equilibrium

Let x, y be Alice's and Bob's early probabilities, conditional on entry. If both use both timings, early and late must tie:

$$yg_0 + (1-y)g_+ = yg_- + (1-y)g_0 \qquad \text{(Alice)},$$
$$xg_0 + (1-x)g_- = xg_+ + (1-x)g_0 \qquad \text{(Bob)}.$$

For $2g_0 \neq g_- + g_+$, solving gives

$$x_* = \frac{g_0 - g_-}{2g_0 - g_- - g_+}, \qquad y_* = \frac{g_0 - g_+}{2g_0 - g_- - g_+}.$$

Check $0 < x_*, y_* < 1$: this holds exactly when g_0 is above *both* $g_\pm$ or below *both*. If g_0 lies between them, there is no timing mixture. Its common reward, when it exists, is

$$u_* = \frac{g_0^2 - g_- g_+}{2g_0 - g_- - g_+}.$$

Add one to n_+ only if the interior mixture exists and $u_* > 0$.

Under $g_0 \neq g_\pm$, no equilibrium has just one player mixing timings.

3. Allow entry to mix with exit

Fix a timing equilibrium (x, y) with reward $u > 0$. Let α, β be Alice's and Bob's entry probabilities. Keeping the same conditional timings, entry pays

$$\text{Alice: } \beta u - (1 - \beta), \qquad \text{Bob: } \alpha u - (1 - \alpha),$$

whereas exit pays 0. There are exactly two possibilities:

Both enter certainly

$$\alpha = \beta = 1.$$

Entry gives $u > 0$, so exit is strictly worse. The original timing equilibrium is retained.

Both mix entry with exit

$$\beta u - (1 - \beta) = 0, \qquad \alpha = \beta = \frac{1}{1 + u}.$$

Entry and exit tie. Conditional on entry, the original timing equilibrium is again retained.

There is no extra asymmetric branch: if one player enters certainly, $u > 0$ makes the other enter certainly too. If $u < 0$, entry loses even against certain entry, so neither branch exists.

Positive timing equilibria, fixed or mixed, each give two participation equilibria.

4. Assemble the count of perfect equilibria

Mutual exit adds one more equilibrium: entering alone loses 1. The full count is

$$N_{\mathrm{THP}} = \underbrace{1}_{\text{mutual exit}} + \underbrace{n_+}_{\text{certain entry}} + \underbrace{n_+}_{\text{entry mixed with exit}} = \boxed{1 + 2n_+ \in \{1, 3, 5, 7\}.}$$

An abstract payoff example

$$\mathsf{G} = \begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix}.$$

Two fixed equilibria: $(0, 0), (1, 1)$, each with reward $1 > 0$.

One timing mixture: $x_* = y_* = 1/2$, with $u_* = -1$; do not count it.

Thus $n_+ = 2$ and $N_{\mathrm{THP}} = 1 + 2 + 2 = 5$.

Every counted branch is perfect

Supported actions stay optimal against suitable completely mixed strategies approaching equilibrium:

- Mutual exit and strict timing preferences persist.
- Mixed timings retain their early–late tie.
- Adjusting entry probabilities preserves the tie with exit.

Assumptions: $g_0 \neq g_\pm$ and no timing equilibrium has zero reward.

Selten (1975)

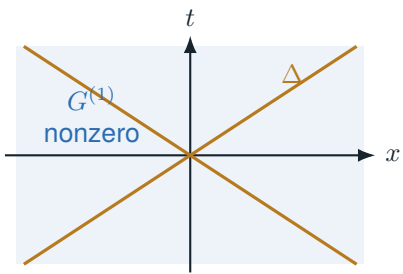
Massless fields: correlation versus response

For a free massless scalar field in $(3+1)$-dimensional Minkowski spacetime,

$$G^{(1)}(t,L) = \frac{1}{2\pi^2}\,\mathrm{PV}\,\frac{1}{L^2-t^2},$$
$$\Delta(t,L) = -\frac{\delta(t-L)-\delta(t+L)}{4\pi L}.$$

$G^{(1)}$ is the vacuum anticommutator expectation. Its correlations need not vanish outside the light cone and contribute to $\mathcal{H}$.

Δ is the Pauli–Jordan commutator. Microcausality makes it vanish at spacelike separation. The massless Huygens property confines its support to the light cone.



Compact switching keeps the calculation analytic

Use the raised-cosine pulse

$$\chi(t) = \cos^2\left(\frac{\pi t}{2T}\right), \qquad |t| \le T,$$

and zero otherwise. Introduce $x = \zeta/T$, the dimensionless relative pulse-time variable. At $\Omega T = 3\pi/2$, the carrier convolution becomes

$$\frac{\mathcal{C}_\Omega(Tx)}{T} = \frac{\sin^5(\pi|x|/2)}{15\pi}, \qquad |x| \le 2.$$

The response quadratures are then

$$\mathcal{H}(s) = \frac{1}{4\pi^2}\,\mathrm{PV}\int_{-2T}^{2T} \frac{\mathcal{C}_\Omega(\zeta)\,\mathrm{d}\zeta}{L^2 - (\zeta - s)^2},$$
$$\mathcal{Q}(s) = -\frac{\mathcal{C}_\Omega(s - L) + \mathcal{C}_\Omega(s + L)}{8\pi L}.$$

The analytic kernel determines the response peaks and cancellation. We use these, together with the support geometry, to select the baseline delays.

Why these separation and timing-menu values?

Fix one detector separation and one early–late menu throughout:

$$L = 2.1T, \qquad \delta = 0.09T, \qquad s_{ij} \in \{\tau - \delta, \tau, \tau + \delta\}.$$

Each pulse lasts $2T$. In a history with center separation s_{ij}, the event time difference Δt lies in $[s_{ij} - 2T, s_{ij} + 2T]$.

At $\tau = 0$: all spacelike

$$\max_{i,j,\,\text{events}} |\Delta t| = 2T + \delta = 2.09T < L.$$

Even the largest time difference is smaller than the spatial separation.

At $\tau = 2L$: all past the shell

$$\min_{i,j,\,\text{events}} \Delta t = 2L - \delta - 2T = 2.11T > L.$$

Even the smallest time difference is larger than the spatial separation.

$$\boxed{\text{Both clearances} = L - 2T - \delta = 0.01T > 0.}$$

Geometry requires only $0 < \delta < L - 2T$. Separately, evaluating the field response at $L/T = 2.1$ gives $\mathcal{H}(0) > \mathcal{P}$, so the spacelike starting point has positive reward.

Why choose these five baseline delays?

For $L > 2T$ and $s \geq 0$, the second kernel term vanishes. Thus

$$|\mathcal{Q}(s)| = \frac{T}{120\pi^2 L} \sin^5\left(\frac{\pi|s-L|}{2T}\right), \qquad |s-L| \leq 2T,$$

and $\mathcal{Q} = 0$ outside this interval. Its two maxima and internal zero select three baselines; the two geometric endpoints complete the comparison:

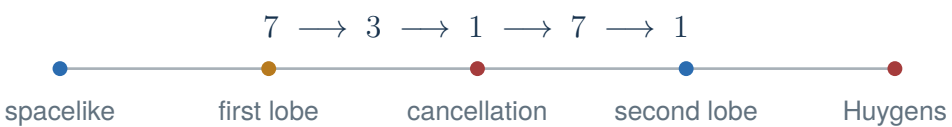
Baseline τ	τ/T	Reason for choosing it
0	0	Every timing history is spacelike
$L-T$	1.1	First maximum of $\lvert\mathcal{Q}\rvert$
L	2.1	Carrier cancellation: $\mathcal{Q}(\tau) = 0$, but $\mathcal{Q}(\tau \pm \delta) \neq 0$
$L+T$	3.1	Second maximum of $\lvert\mathcal{Q}\rvert$
$2L$	4.2	Every timing history is beyond the wavefront

Payoff peaks depend on the full combination $\sqrt{\mathcal{H}^2 + \mathcal{Q}^2} - \mathcal{P}$.

Five baseline settings of one fixed protocol

τ/T	$10^4(\bar{g}_-, \bar{g}_0, \bar{g}_+)$	positive active type	N_{THP}
0	$(0.2818, 0.2646, 0.2818)$	anti-coordination	7
1.1	$(2.3556, 2.1709, 1.8397)$	directional	3
2.1	$(-1.7413, -1.5438, -1.3458)$	no positive equilibrium	1
3.1	$(1.9418, 2.0370, 1.9891)$	coordination	7
4.2	$(-1.0507, -1.1437, -1.2195)$	no positive equilibrium	1

$g = \Gamma\lambda^2\bar{g}$. Only the baseline delay changes between rows.



Robustness and limits of the result

Finite-game result	The equilibrium classification is exact for the leading payoff matrices. It excludes timing degeneracies and zero active rewards.
Field calculation	The detector state and payoff are calculated to order λ^2. No higher-order correction or numerical coupling threshold has been calculated.
Small-coupling limit	For fixed Γ, $u = \mathcal{O}(\lambda^2)$ and the mixed-entry probability $1/(1+u)$ approaches one. The two entry branches approach each other as the coupling decreases.
Parameter scope	Strict payoff signs and timing gaps give open neighborhoods of the five settings. These settings do not exhaust the delay axis.

Continuous timing, strategic readouts, and different entry contracts require new equilibrium analyses.

Conclusion

A fixed quantum-input verification rule converts detector entanglement into positive payoffs excluded for separable resources under the same conditions.

For the nondegenerate timing and participation game,

$$\boxed{N_{\mathrm{THP}} = 1 + 2n_{+} \in \{1, 3, 5, 7\}.}$$

Here n_{+} counts all positive-reward timing equilibria, including both fixed and mixed early–late choices. Each gives two participation equilibria; mutual exit adds one.

Hadamard correlations, causal response, and local noise determine the timing payoffs and thereby the equilibrium count.

Illustration: five selected baselines of one massless-field protocol give $7 \to 3 \to 1 \to 7 \to 1$.

Thank you

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