Quantum Geometry Effects in Quantum Field Theory

How the Hamiltonian Constraint Generates Gravity–Matter Entanglement

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01

Problem

A QFT on a fixed background can only define $|\phi\rangle_{\mathfrak{g}}$. Once the background itself is in a superposition, states on two different backgrounds cannot be added — exactly what black-hole evaporation needs.

02

Approach

Spherically symmetric LQG: geometry is a coherent state $|\mathfrak{g}\rangle$; restrict phase space to a near-horizon subspace U, so Fock representations of different geometries are unitarily equivalent.

03

Result

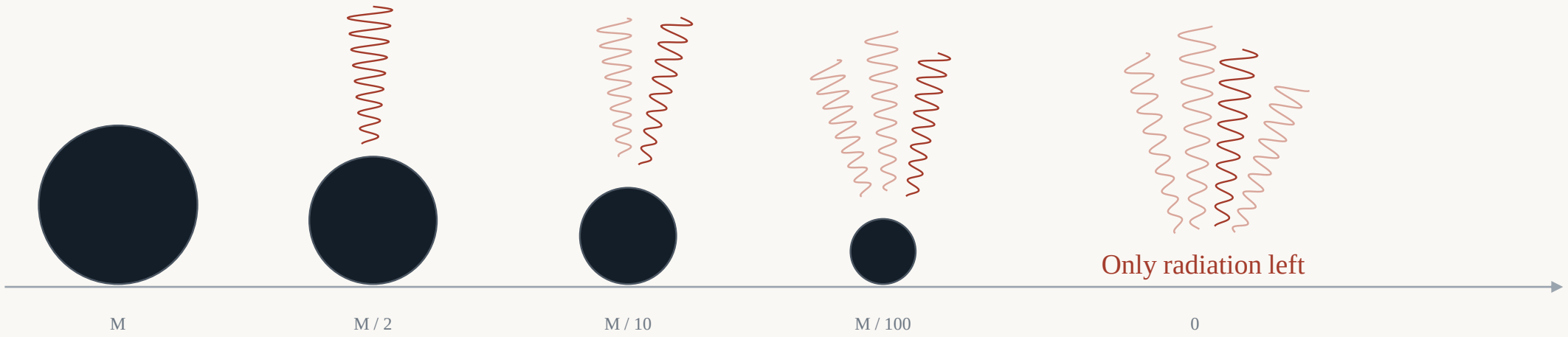
After weakly solving the Hamiltonian constraint, physical states are automatically entangled states of geometry and matter; the Hartle–Hawking vacuum is extended to quantum geometry.

$$|\Psi\rangle = \sum_{\mathfrak{g},\phi} c_{\mathfrak{g},\phi}\ |\mathfrak{g}\rangle \otimes |\phi\rangle$$

Entanglement is not an extra assumption but a necessary solution of the constraints of general relativity.

Evaporation Ends with Radiation Alone

The mass decreases monotonically, the horizon area decreases monotonically, while the radiation Hilbert space grows monotonically.



- At the end of evaporation only radiation is left: a black hole that starts in a pure state must end in a pure radiation state.
- But the semiclassical calculation gives a thermal state — this is the information paradox.

Unitarity

Quantum evolution is generated by a unitary operator: a pure state always evolves into a pure state.

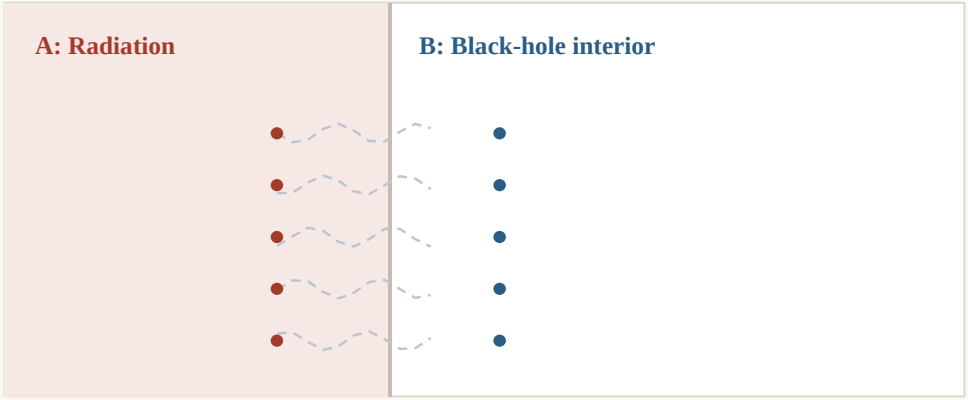
Entanglement Entropy: Who Really Holds the Information?

$$|\Psi\rangle = \sum_i \sqrt{p_i}\, |i\rangle_A \otimes |i\rangle_B,$$

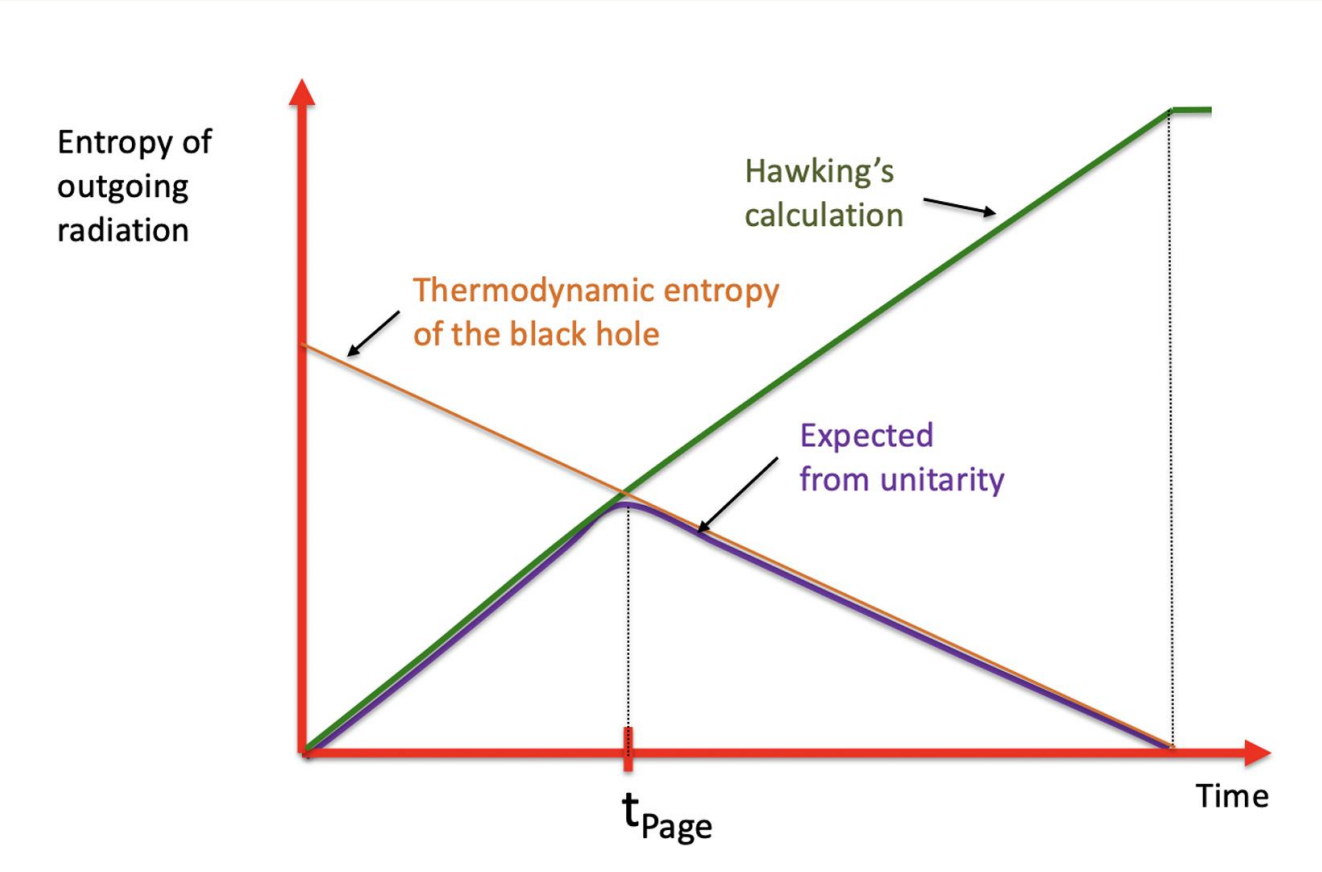
$$S_A = -\mathrm{Tr}(\rho_A \ln \rho_A), \qquad \rho_A = \mathrm{Tr}_B\, |\Psi\rangle\langle\Psi|$$

$$S_B = -\mathrm{Tr}(\rho_B \ln \rho_B), \qquad \rho_B = \mathrm{Tr}_A\, |\Psi\rangle\langle\Psi|$$

- $S_A = S_B$,
- $0 \le S_A \le \ln \dim H_A$,
- $0 \le S_A \le \ln \dim H_B$



The Page Curve: Entropy Must First Rise, Then Fall



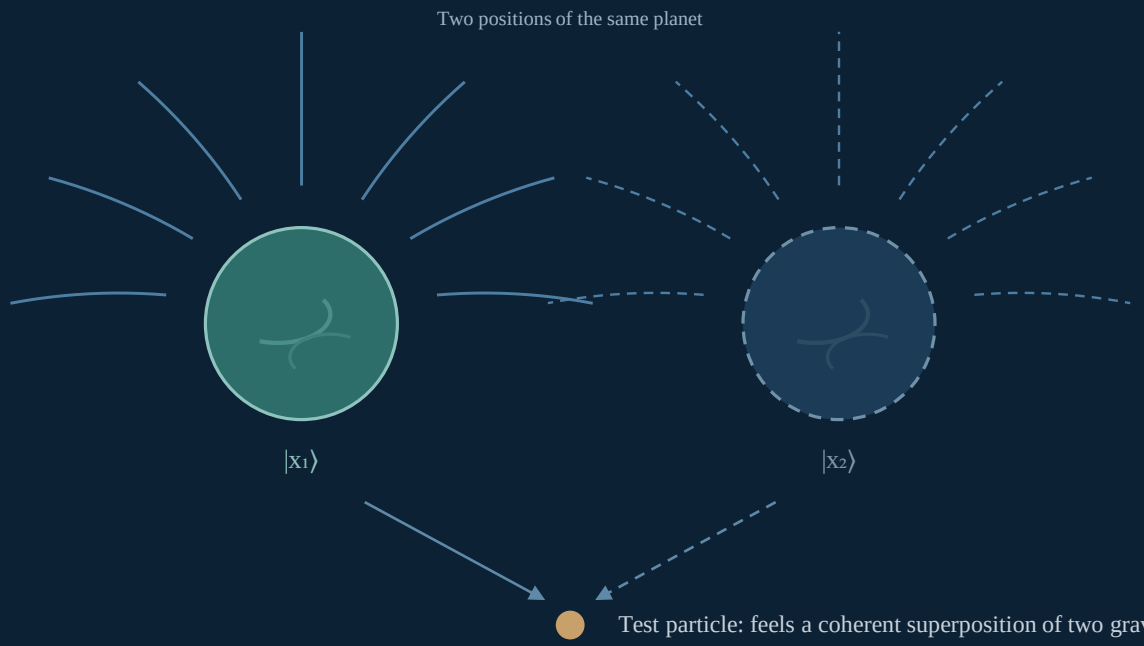
- $t < t_{Page}$: Radiation is the smaller subsystem; its entropy grows with $\ln(\dim H_R)$.
- $t = t_{Page}$: The two sides have comparable dimensions, and the entropy peaks.
- $t > t_{Page}$: Radiation becomes the larger subsystem; the entropy is bounded by the interior dimension.
- **Endpoint**: Entropy returns to zero, and the radiation state becomes pure.

The Page curve tells us that entropy must return to 0, but not which space's degrees of freedom control it, or by what mechanism entropy returns to 0.

Our answer: Quantum Geometry

① **Where are the degrees of freedom of quantum geometry?**	Without a Hilbert space for geometry, geometry cannot be superposed
② **How do $\|g\rangle$ and $\|\varphi\rangle$ span a state space?**	Fock spaces on different geometries may not be unitarily equivalent
③ **How does the constraint select physical states?**	GR is a fully constrained system: states must lie in the constraint kernel

A Superposed Earth Means Superposed Gravity



Source quantum state:

$$|E\rangle = \frac{1}{\sqrt{2}}(|x_1\rangle + |x_2\rangle)$$

Superposition of source positions, hence of two gravitational fields

The gravitational field follows the source

In the Newtonian limit the field is fixed by the source position.

So geometry and source are entangled

$$|\Psi\rangle_{\text{tot}} = \frac{1}{\sqrt{2}}(|g_1\rangle|x_1\rangle + |g_2\rangle|x_1\rangle)$$

The source position and the geometric state correspond one-to-one; the total state is not factorizable.

> As long as gravity is quantum, there is no “fixed classical spacetime.”

Part II · Technical Details

How the Hamiltonian Constraint Generates Entanglement

Weak solutions in spherically symmetric LQG and the generalized Hartle–Hawking vacuum

02

01 Spherically symmetric model and constraint system

02 Failure on a fixed background

03 Quantum geometry and a common Fock space

04 Weak solutions, generalized HH vacuum, and conclusions

Phase Space: Four Geometric Variables Plus One Matter Pair

- Gravity sector consists of $(K_x, E^x, K_\varphi, E^\varphi)$), all defined on the 1-D manifold X.
- E^x and E^φ are frame components of the metric: the spatial metric is fixed entirely by them.
- Matter sector has only one conjugate pair (ϕ, π_ϕ), spherical symmetry implies no angular excitations.
- The total phase space is a direct product: geometry and matter are classically independent.

Non-vanishing Poisson brackets (gravity)

$$\{K_x(x), E^x(x')\} = 2G\delta(x - x'), \quad \{K_\varphi(x), E^\varphi(x')\} = G\delta(x - x')$$

Spatial metric from the variables

$$ds^2 = \frac{(E^\varphi)^2}{|E^x|}dx^2 + |E^x|\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

Matter field

$$\{\phi(x), \pi_\phi(x')\} = \delta(x - x')$$

Fully Constrained System: No Evolution, Only Constraints

Diffeomorphism constraint

$$H_x = H_x^{\mathrm{g}} + H_x^{\phi} =\approx 0$$

Hamiltonian constraint

$$H = H^{\mathrm{g}} + H^{\phi} \approx 0$$

- General relativity is a **fully constrained** system: time evolution is generated by constraints, and physical states must lie in their kernel.
- Both constraints are first class: their Poisson brackets vanish on the constraint surface.

Gravitational part

$$H^{\mathrm{g}} = -2\frac{E^{\varphi}}{\partial_x E^x}\partial_x M + 2\frac{\sqrt{E^x}K_{\varphi}}{\partial_x E^x}H_x^{\mathrm{g}},$$

$$H_x^{\mathrm{g}} = \frac{1}{2G}\left(-K_x\partial_x E^x + 2E^{\varphi}\partial_x K_{\varphi}\right),$$

Matter part

$$H^{\phi} = \frac{1}{2}\left(\frac{\pi_{\phi}^2}{\sqrt{E^x}E^{\varphi}} + \frac{(E^x)^{3/2}}{E^{\varphi}}(\partial_x\phi)^2\right),$$

$$H_x^{\phi} = \pi_{\phi}\partial_x\phi,$$

Misner–Sharp mass

$$M = \frac{\sqrt{E^x}}{2G}\left(1 + K_{\varphi}^2 - \frac{(\partial_x E^x)^2}{4(E^{\varphi})^2}\right)$$

Structure of the Hamiltonian Constraint: Mass as a Total Derivative

Gravitational part

$$H^{\mathrm{g}} = -2\frac{E^{\varphi}}{\partial_x E^x}\partial_x M + 2\frac{\sqrt{E^x}K_{\varphi}}{\partial_x E^x}H_x^{\mathrm{g}}$$

Matter part

$$H^{\phi} = \frac{1}{2}\left(\frac{\pi_{\phi}^2}{\sqrt{E^x}E^{\varphi}} + \frac{(E^x)^{3/2}}{E^{\varphi}}(\partial_x\phi)^2\right)$$

A useful rewrite:

$$H = -2\frac{E^{\varphi}}{\partial_x E^x}\tilde{H}$$

Rewritten Hamiltonian constraint density:

$$\tilde{H} = \partial_x M - \frac{\sqrt{E^x}}{E^{\varphi}}K_{\varphi}H_x^{\mathrm{g}} - \frac{\partial_x E^x}{2E^{\varphi}}H^{\phi}$$

The two forms are completely equivalent.

Smearing the constraint: mass difference equals the field energy in the interval

Hamiltonian Constraint density:

$$\tilde{H} = \partial_x M - \frac{\sqrt{E^x}}{E^\varphi} K_\varphi H_x^{\mathrm{g}} - \frac{\partial_x E^x}{2E^\varphi} H^\phi$$

Integrate along the interval:

$$\tilde{H}^\delta(v, v') := \int_{x(v)}^{x(v')} d\,x\tilde{H} = (M_{v'} - M_v) - \tilde{H}_\phi^\delta(v, v') \approx 0$$

- Constraint**: the mass difference between the two ends equals the field energy inside the interval**.

Near-horizon geometry $S_{v,v'}$: making the problem solvable

Gauge fixing:

$E^x = x^2$: the radial coordinate is the square of the areal radius, and the diffeomorphism constraint is solved.

Condition one:

At $x(v) = x_h$ there is an apparent horizon:$\Theta_{out} = 0$.

Condition two:

In $[x_h, x_h + \delta]$ we have $K_\varphi = 0$, $E^\varphi > 0$, and $\int_{x_h}^{x_h+\delta} dx \frac{(E^\varphi)^2}{x^2} \to \infty$.

The joint consequence of the three conditions:

Geometry is reduced to a single degree of freedom E^φ inside the interval $[x_h, x_h + \delta]$.

Outgoing null expansion:

$$\Theta_{out} = \frac{\partial_x |E^x|}{\sqrt{|E^x|} E^\varphi} - \frac{2K_\varphi}{\sqrt{|E^x|}} = 0$$

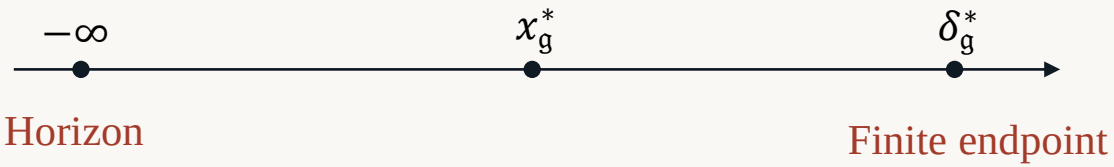
This small patch of phase space is the “laboratory” of the whole paper.

Tortoise Coordinate: Pushing the Horizon to Infinity

Standard tortoise coordinate

$$\frac{\partial x}{\partial x_{\mathrm{g}}^{*}} = \frac{x^{2}}{\left(E^{\varphi}(x)\right)^{2}|_{\mathrm{g}}}$$

- At the horizon $x_{\mathrm{g}}^{*} \to -\infty$;.
- v' corresponds to the finite value δ_{g}^{*}.



So we must “extend” this half-infinite line to the entire real axis.

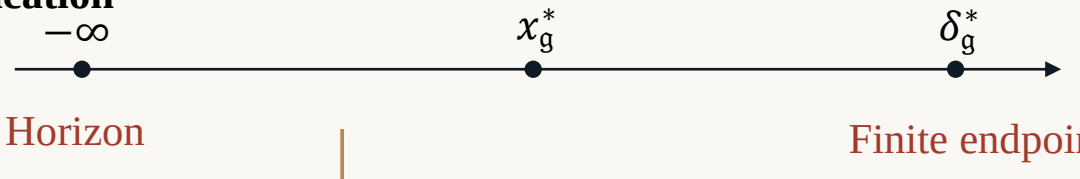
Modified Tortoise Coordinate: Straightening a Finite Interval

Modified tortoise coordinate

$$\breve{x}_{\mathrm{g}}^{*} := x_{\mathrm{g}}^{*} + L - \delta_{\mathrm{g}}^{*}$$

One shift plus one limit turns the finite interval into the whole real axis.

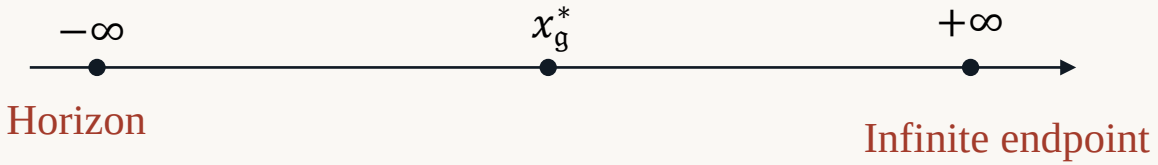
Before modification



half the real axis

Shift by $L - \delta_{\mathrm{g}}^{*}$, then take $L \to \infty$

After modification ($L \to \infty$)



plane waves can be defined

- After the shift, the whole segment $[-\infty,\ \delta_{\mathrm{g}}^{*}]$ is moved to $[-\infty, +\infty]$.
- Define plane waves on this basis.

From Coordinates to Operators: Creation and Annihilation Operators

Annihilation operator

$$a_k^{\mathfrak{g}} := \lim_{L\to\infty} \int_{-L}^{+L} d\,\breve{x}_{\mathfrak{g}}^{*} e^{-ik\breve{x}_{\mathfrak{g}}^{*}} \frac{1}{\sqrt{2\omega_k}} \left(x\omega_k \hat{\phi}\left(\breve{x}_{\mathfrak{g}}^{*}\right) + i \frac{\hat{\pi}_{\phi}\left(\breve{x}_{\mathfrak{g}}^{*}\right)}{x} \right)$$

- **Modes are "exclusive to this geometry": change $\mathfrak{g}$, and you change the set $a_k^{\mathfrak{g}}$.**
- **The same construction works for any $\mathfrak{g} \in S_{v,v'}$—— but different $\mathfrak{g}$ give different Fock representations.**

Canonical commutation relations

$$\left[\hat{\phi}\left(\breve{x}_{\mathfrak{g}}^{*}\right), \hat{\pi}_{\phi}\left(\breve{x}'^{*}_{\mathfrak{g}}\right)\right] = i\hbar\delta\left(\breve{x}_{\mathfrak{g}}^{*} - \breve{x}'^{*}_{\mathfrak{g}}\right)$$

The resulting operator algebra

$$\left[a_k^{\mathfrak{g}}, a_{k'}^{\mathfrak{g},\dagger}\right] \overset{L\to\infty}{=} 2\pi\hbar\delta(k-k')$$

Boulware-Type Vacuum and Fock Space

Definition of the vacuum

$$a_k^{\mathfrak{g}}|0_{\mathrm{B}}\rangle_{\mathfrak{g}} = 0, \quad \forall k$$

Fock space

$$\mathcal{H}_{\mathfrak{g}}^{\phi} := \{|0_{\mathrm{B}}\rangle_{\mathfrak{g}}, a_k^{\mathfrak{g},\dagger}|0_{\mathrm{B}}\rangle_{\mathfrak{g}}, a_k^{\mathfrak{g},\dagger}a_{k'}^{\mathfrak{g},\dagger}|0_{\mathrm{B}}\rangle_{\mathfrak{g}}, a_k^{\mathfrak{g},\dagger}a_{k'}^{\mathfrak{g},\dagger}a_{k''}^{\mathfrak{g},\dagger}|0_{\mathrm{B}}\rangle_{\mathfrak{g}}, \dots\}$$

- Fock space is spanned by the vacuum and all combinations of creation operators.
- It is "**geometry- $\mathfrak{g}$ exclusive**": the subscript $\mathfrak{g}$ reminds us that all this rests on a specific background; change the geometry and both vacuum and modes change.

So far so good: fix $\mathfrak{g}$, and one gets a complete QFT.

Carrying the Constraint Over to a Fixed Geometry

Effective constraint on the fixed geometry $\mathfrak{g}$

$$M_{v'}(\mathfrak{g}) - M_v(\mathfrak{g}) = \frac{\langle \phi' \left| \widehat{\widetilde{H}}^{\delta}_{\phi}(\mathfrak{g}; v, v') \right| \phi \rangle}{\langle \phi' | \phi \rangle}, \quad \forall \langle \phi' |$$

- The left-hand side $M_{v'}(\mathfrak{g}) - M_v(\mathfrak{g})$ is a **purely geometric quantity**: it depends only on $\mathfrak{g}$.
- The right-hand side is the matrix element of the matter Hamiltonian between Fock states.
- **This is not a dynamical equation but a "selection rule": only states satisfying it are physical.**

Diagonalization: The Constraint Becomes an Energy-Spectrum Condition

Matter Hamiltonian (modified coords)

$$\int_{x_h}^{x_h+\delta} dx \left(\frac{\partial_x E^x}{2E^\varphi}\right) \hat{H}^\phi(\mathfrak{g}) = \int \frac{dk}{2\pi} \omega_k a_k^{\mathfrak{g},\dagger} a_k^{\mathfrak{g}}$$

- In modified coordinates the matter Hamiltonian is proportional to $\int \frac{dk}{2\pi} \omega_k a_k^{\mathfrak{g},\dagger} a_k^{\mathfrak{g}}$.
- After normal ordering removes the zero-point energy, it is the total particle energy.
- The matrix element therefore depends only on the particle-number distribution.

Substitute into the constraint

$$M_{v'}(\mathfrak{g}) - M_v(\mathfrak{g}) = \frac{\int \frac{dk}{2\pi} \omega_k \langle \phi' \left| a_k^{\mathfrak{g},\dagger} a_k^{\mathfrak{g}} \right| \phi \rangle}{\langle \phi' | \phi \rangle}$$

Result

The solution must be both an eigenstate and an “energy-matched” state.

Form of the Weak Solution: Mass Difference = Total Field Energy

Particle-number eigenstate

$$|\,\vec{k}_{\vec{n}}(\mathfrak{g})\rangle := \left(\prod_k \frac{\left(a_k^{\mathfrak{g},\dagger}\right)^{n_k}}{(\mathcal{N})^{n_k}\sqrt{n_k!}}\right)|0_{\mathsf{B}}\rangle_{\mathfrak{g}}$$

Constraint condition

$$M_{v'}(\mathfrak{g}) - M_v(\mathfrak{g}) = \sum_k n_k\,\omega_k =: \omega_{\vec{k}_{\vec{n}}(\mathfrak{g})}$$

- The solution is a family of particle-number states.
- **One geometry cannot be paired with arbitrary particle numbers; one particle-number distribution cannot be paired with an arbitrary geometry.**
- Each weak solution is perfectly legal on its own — the problem lies between them.

Geometry ↦ mass difference ↦ allowed particle numbers: the constraint pairs the two sides one-to-one — the origin of both entanglement and the crisis.

Crisis: Two Legal States Whose Superposition Is Undefined

Linear combination of two weak solutions

$$|\vec{k}_{\vec{n}}(\mathfrak{g})\rangle + |\vec{k}'_{\vec{n}}(\mathfrak{g}')\rangle \text{ is ill-defined}$$

Why does the superposition fail?

- Each state is “tied” by the constraint to one classical geometry: a superposition of states equals a superposition of two classical geometries.
- The two states live in two Fock spaces; if these are not unitarily equivalent, there is no common “sum”.

One conclusion

The problem is not computational but **one of setting**: building a quantum system (matter) on a classical background and then demanding that this background take part in a superposition — is logically impossible.

There is only one way out: quantize geometry itself and find a common Hilbert space for both.

Step One: Describing “Semiclassical Geometry” with Coherent States

- LQG kinematical Hilbert space is spanned by cylindrical functions, and coherent states $|\mathfrak{g}\rangle$ can be built there.

- **Three properties of the coherent states**: they peak at the phase-space point$\mathfrak{g}$; their inner product decays exponentially; they satisfy the Ehrenfest theorem.

Cylindrical functions

$$T_{\gamma,\vec{\mu}}(K_\varphi) = \prod_{v\in\gamma} \exp\left(\mathbf{i}\mu_v K_\varphi(v)\right)$$

Inner product

$$\langle \mathfrak{g}|\mathfrak{g}'\rangle \sim \exp\left(-|\mathfrak{g}-\mathfrak{g}'|^2/t\right)$$

Ehrenfest property

$$\langle \mathfrak{g}|\hat{O}|\mathfrak{g}'\rangle \approx O(\mathfrak{g}')\langle \mathfrak{g}|\mathfrak{g}'\rangle$$

“Geometry” is no longer a given classical field but a set of quantum states that can be superposed and entangled.

Step Two: Two Mode Sets Separated by a Bogoliubov Transformation

$$\beta_{kk'}(\mathfrak{g}',\mathfrak{g}) = \frac{1}{2}\sqrt{\frac{\omega_k}{\omega_{k'}}}\left(\lim_{L\to\infty}\int_{-L}^{+L} d\,\check{x}^*_{\mathfrak{g}} e^{-\mathbf{i}k\check{x}^*_{\mathfrak{g}}} e^{-\mathbf{i}k'\check{x}^*_{\mathfrak{g}'}}\right) - \frac{1}{2}\sqrt{\frac{\omega_{k'}}{\omega_k}}\left(\lim_{L\to\infty}\int_{-L}^{+L} d\,\check{x}^*_{\mathfrak{g}'} e^{-\mathbf{i}k\check{x}^*_{\mathfrak{g}}} e^{-\mathbf{i}k'\check{x}^*_{\mathfrak{g}'}}\right)$$

$$a_k^{\mathfrak{g}} = \int \frac{dj}{2\pi}\left(\alpha_{kj}(\mathfrak{g}',\mathfrak{g})a_j^{\mathfrak{g}'} + \beta_{kj}(\mathfrak{g}',\mathfrak{g})a_j^{\mathfrak{g}',\dagger}\right)$$

- The α term: mixes equal-frequency modes and preserves particle-number structure.
- The β term: **pairs creation with annihilation**, corresponding to a vacuum mismatch.

The Shale–Stinespring Criterion: When Can We “Change Background”?

Criterion: β must be a Hilbert–Schmidt operator

$$\int dk\, dj \left|\beta_{kj}(\mathfrak{g}', \mathfrak{g})\right|^2 < \infty$$

Implication

It need not hold for every geometry — holding in a small phase-space region suffices to build a common sub-state-space of geometry and matter fields.

So the problem becomes: find this subspace.

Step Three: Restricted Subspace and Common Fock Space

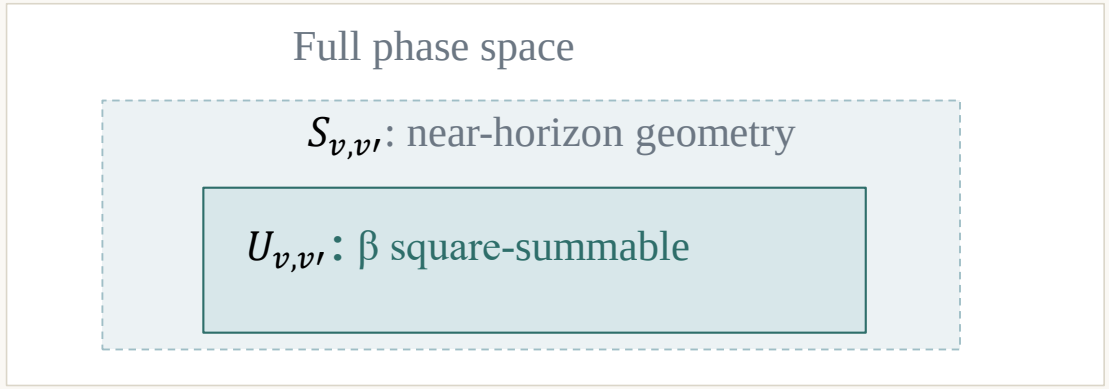
- Within the near-horizon geometry class $S_{v,v'}$, restrict further to the subspace $U_{v,v'}$: between any two geometries β β is square-summable.
- On $U_{v,v'}$, all Fock spaces are unitarily equivalent, so one may write a single $\mathcal{H}^{\phi}$.
- On the geometry side we keep only the coherent states on $U_{v,v'}$, giving the restricted space.

Common Fock space

$$\mathcal{H}^{\phi} \equiv \mathcal{H}^{\phi}_{\mathfrak{g}} \cong \mathcal{H}^{\phi}_{\mathfrak{g}'}, \quad \forall\, \mathfrak{g}, \mathfrak{g}' \in U_{v,v'}$$

Well-defined quantum state space

$$\widetilde{\mathcal{H}}_{\mathfrak{g}} \otimes \mathcal{H}^{\phi}, \qquad \widetilde{\mathcal{H}}_{\mathfrak{g}} = \{|\mathfrak{g}\rangle \in \mathcal{H}_{\mathfrak{g}} \mid \mathfrak{g} \in U_{v,v'}\}$$



From “one Fock space per geometry” to “one Fock space per subspace”:

Figure: $U_{v,v'}$ in $S_{v,v'}$ — a “mutually comparable” patch inside

Step Four: Weakly Solving the Constraint on Quantum Geometry

Quantum Hamiltonian constraint

$$\langle \widehat{M}_{v'} \rangle - \langle \widehat{M}_v \rangle = \langle \widehat{\widetilde{H}}^{\delta}_{\phi}(v, v') \rangle$$

Leading-order constraint

$$M_{v'}(\mathfrak{g}) - M_v(\mathfrak{g}) \approx \frac{\langle \phi' \left| \widehat{\widetilde{H}}^{\delta}_{\phi}(\mathfrak{g}; v, v') \right| \phi \rangle}{\langle \phi' | \phi \rangle}$$

- For a general $|\mathfrak{g}, \phi\rangle$, write the above constraint in matrix-element form.
- Using the Ehrenfest property of coherent states: the matrix element of a geometric operator ≈ classical value × coherent-state overlap.
- **To leading order it returns the fixed-background equation**.

That is: geometry remains "approximately classical," but its quantum nature has already entered the constraint through the coherent state.

What is "weak solution"?

The operator is not required to vanish identically (a strong solution), only its matrix elements between a chosen family of states must vanish.

Strong solutions are out of reach even in reduced models; weak solutions provide a controllable, physically consistent family of approximate states.

Weak Solution: Geometry Labels and Particle Numbers Bound by the Constraint

Form of the weak solution

$$|\mathfrak{g}, \phi\rangle = |\mathfrak{g}\left(\vec{k}_{\vec{n}}\right)\rangle \otimes |\vec{k}_{\vec{n}}\rangle$$

- Left: the coherent state of geometry; right: the particle-number state of the matter field.
- The two are bound through $M_{v'}(\mathfrak{g}) - M_v(\mathfrak{g}) = \sum_k n_k\, \omega_k =: \omega_{\vec{k}_{\vec{n}}(\mathfrak{g})}$: **given the particle number, the geometry is uniquely fixed** (in $U_{v,v'}$).

The "pairing" enforced by the constraint

Geometry: mass difference		Matter: particle numbers
$\Delta M = \omega_1$	→	1 ω_1 mode
$\Delta M = 3\omega_2$	→	3 ω_2 modes
$\Delta M = \omega_1 + \omega_2$	→	one ω_1 and one ω_2

Physical state + linear combination = intrinsic entanglement

- A single $|\mathfrak{g}, \phi\rangle = |\mathfrak{g}\left(\vec{k}_{\vec{n}}\right)\rangle \otimes |\vec{k}_{\vec{n}}\rangle$ is still a product state.
- But a physical state can be linearly combined at will.
- The reduced density matrix of geometry is therefore mixed: geometry and matter "know" each other.
- **Entanglement is not added by hand; it is the geometric structure of the constraint solution space.**

General physical state

$$c \left|\mathfrak{g}\left(\vec{k}_{\vec{n}}\right)\rangle \otimes \left|\vec{k}_{\vec{n}}\right\rangle + c' \left|\mathfrak{g}\left(\vec{k}'_{\vec{n}}\right)\rangle \otimes \left|\vec{k}'_{\vec{n}}\right\rangle + \cdots\right.$$

A Testable Corollary

If the energy distribution of the radiation is measured, the distribution of geometry is fixed accordingly; and vice versa. This is exactly the correlation the Page curve requires.

Mathematically: taking the partial trace over $|\Psi\rangle$ gives a density matrix that is not pure, and its entropy is the geometry–matter entanglement entropy.

"Geometry knows matter, and matter knows geometry" — this statement, in quantum theory, now has a rigorous meaning.

Why This Step Matters

Entanglement is not an assumption we insert, it is the result extracted from the Hamiltonian constraint.

The constraint selects states	Physical states are not arbitrary tensor products but specific combinations in the constraint kernel.
The constraint fixes correlations	Mass difference and particle number correspond one-to-one; this pairing is the “skeleton” of entanglement.
Semiclassical limit	When the coherent state is narrow enough, entanglement reduces to classical geometry + quantum field.

$$|\Psi\rangle = \sum_{\mathfrak{g},\phi} c_{\mathfrak{g},\phi}\, |\mathfrak{g}\rangle \otimes |\phi\rangle$$

Extension: The Hartle–Hawking Vacuum on Quantum Geometry

① Standard form on a fixed background

$$|0_{\mathrm{HH}}\rangle_{\mathfrak{g}} = \sum_{\vec{n}} \left(\prod_k \left(1 - e^{-2\pi \xi_{\mathfrak{g}}^{-1} \omega_k}\right)^{1/2} e^{-\pi n_k \omega_k / \xi_{\mathfrak{g}}} \right) |\vec{k}_{\vec{n}}\rangle_{\mathfrak{g}}$$

The thermal coefficients are set by the surface gravity near the horizon, $\xi_{\mathfrak{g}}$: this is a "thermal state on a given geometry."

Replacement: particle-number states → geometry–matter entangled terms

② Extension to quantum geometry

$$|0_{\mathrm{HHQG}}\rangle := \sum_{\vec{n}} \left(\prod_k \left(1 - \exp(-2\pi \omega_k / \xi_{\mathfrak{g}(\vec{k}_{\vec{n}})})\right)^{1/2} \exp(-\pi n_k \omega_k / \xi_{\mathfrak{g}(\vec{k}_{\vec{n}})}) \right) |\mathfrak{g}(\vec{k}_{\vec{n}}), \vec{k}_{\vec{n}}\rangle$$

Each term now carries its own geometric partner: the spectrum no longer depends on a single fixed $\xi_{\mathfrak{g}}$.

It is not an eigenstate of any annihilation operator — it is defined from the outset on the total "geometry + matter" space.

Semiclassical Consistency: Fixing the Parameter by a Limit

- $\xi_{\mathfrak{g}(\vec{k}_{\vec{n}})}$ is a free parameter; it must make the state reduce, in the semiclassical limit, to the standard HH vacuum.
- The consistency condition is written as a limit of an **annihilation operator**: the closer geometry is to classical, the more the excitations must vanish.
- ε measures the deviation of the coherent state from classical geometry; ε → 0 means "geometry becomes classical."

Semiclassical consistency condition

$$\lim_{\epsilon\left(\mathfrak{g}_c,\mathfrak{g}(\vec{k}_{\vec{n}})\right)\to 0} A_k^{\mathfrak{g}_c}|0_{HHQG}\rangle = 0$$

Deviation measure

$$\epsilon\left(\mathfrak{g}_c,\mathfrak{g}\left(\vec{k}_{\vec{n}}\right)\right) \equiv \sqrt{1-|\langle\mathfrak{g}_c|\mathfrak{g}\left(\vec{k}_{\vec{n}}\right)\rangle|^2}$$

Classical geometry defined by expectation values

$$\mathfrak{g}_c = \left\{\frac{\langle 0_{HHQG}|\hat{E}^{\varphi}|0_{HHQG}\rangle}{\langle 0_{HHQG}|0_{HHQG}\rangle}, \frac{\langle 0_{HHQG}|\hat{K}_{\varphi}|0_{HHQG}\rangle}{\langle 0_{HHQG}|0_{HHQG}\rangle}\right\}$$

Key Difference Between the Two “Generalized Vacua”

Geometry and vacuum simply multiplied

$$|0_{HH}, \mathfrak{g}_c\rangle := |0_{HH}\rangle_{\mathfrak{g}_c} \otimes |\mathfrak{g}_c\rangle$$

It is still a “vacuum” on a classical geometry

$$A_k^{\mathfrak{g}_c}|0_{HH}, \mathfrak{g}_c\rangle = 0$$

But it is a product state: geometry and matter each go their own way, with no entanglement structure.

Property	$\|0_{HH}, \mathfrak{g}_c\rangle$	$\|0_{HHQG}\rangle$
Weakly solves constraint	No	Yes
Entanglement	None	Intrinsic
Semiclassical limit	HH vacuum	HH vacuum

The two give the same physics on the same semiclassical limit, but are entirely different states at the quantum-geometry level.

The constraint not only filters out illegal states but also decides “who entangles with whom” inside the legal ones.

Conclusions: Five Points

01	**The limit of a fixed background**	QFTCS handles only one classical g; it cannot carry a quantum superposition of backgrounds, nor truly solve the Hamiltonian constraint.
02	**The restricted phase space is key**	On the near-horizon subspace $U_{v,v\prime}$, the Fock representations of different geometries are unitarily equivalent.
03	**Entanglement from constraints**	The weak solution binds mass difference to particle number; linear combinations automatically become entangled geometry–matter states.
04	**Upgrading the vacuum**	The HH vacuum is extended to an entangled vacuum on quantum geometry: the semiclassical limit returns the thermal vacuum.
05	**Paradigm shift**	From “QFT on curved spacetime” to “QFT on quantum spacetime.”

In one sentence: the constraint decides not only which states are allowed but also how they entangle.

Open Questions: Where to Continue

① Can the subspace be extended?

$U_{v,v'}$ relies on spherical symmetry and the near-horizon regime. In more general spacetimes, is there a comparable region?

② Time evolution

The present states are “instantaneous.” Describing a full evaporation requires relational time or a sum over histories.

③ Observable corrections

Compute corrections of the new vacuum to the Hawking spectrum; low-mass black holes may be a window onto Planck-scale physics.

④ Information and unitarity

Can the built-in entanglement guarantee information recovery under the full evolution?

The value of this work is not that it “solves” the information paradox, but that it places it back in a well-defined quantum state space.

Closing Remarks

Thank You

Questions and Discussion

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